

Let consider a sample dataset have one input (x_i^2) and one output (y_i^2) and number of samples 4. Develop a simple linear regression model using ADAGRAD optimizer

| Sample (i) | x_i^2 | y_i^2 |
|------------|---------|---------|
| 1 | 0.2 | 3.4 |
| 2 | 0.4 | 3.8 |
| 3 | 0.6 | 4.2 |
| 4 | 0.8 | 4.6 |

Manual Calculations:

Step 1: $[x, y]$, $\eta = 0.1$, epochs = 1, $m = 1$, $c = -1$, $\epsilon = 10^{-8}$,

$$G_m = G_c = 0$$

Step 2: $itr = 1$

Step 3: sample = 1

Step 4:
$$g_m = -(y_i - mx_i - c) x_i$$

$$= -(3.4 - (1 \times 0.2) - (-1)) 0.2$$

$$= -0.84$$

$$g_c = -4.2$$

Step 5:
$$G_m = G_m + (g_m)^2 = 0 + (0.84)^2 = 0.7056$$

$$G_c = G_c + (g_c)^2 = 0 + (4.2)^2 = 17.64$$

$$\Delta m = \frac{-0.1}{\sqrt{0.7056 + 10^8}} \times 0.84$$

$$= 0.09999$$

$$\Delta c = \frac{-0.1}{\sqrt{17.64 + 10^8}} \times -4.2$$

$$= 0.09999$$

$$\text{step 7: } m = m + \Delta m = 1 + 0.9999 = 1.9999$$

$$c = c + \Delta c = -1 + 0.9999 = -0.001$$

$$\text{step 8: } \text{sample} = \text{sample} + 1 \\ = 1 + 1 = 2$$

$$\text{step 9: } 2 > 2 \\ \text{false} \rightarrow \text{goto step 4}$$

$$\text{step 4: } g_m = -(y_i - mx_i - c) x_i \\ = -(3.8 - (1 \times 1.999) + 0.001) \cdot 0.4 \\ = -0.72044$$

$$g_c = -1.8011$$

$$\text{step 5: } G_m = G_m + (g_m)^2 = 0.7056 + 0.5190 = 1.2246$$

$$G_c = G_c + (g_c)^2 = 17.64 + 3.2439 = 20.8839$$

$$\text{step 6: } \Delta m = \frac{-0.1}{\sqrt{1.2246 + 10^8}} \times (-0.72044) = 0.065102$$

$$\Delta c = \frac{-0.1}{\sqrt{20.8839 + 10^8}} \times (-1.8011) = 0.03941$$

step 7: $m = 1.9999 + 0.065102 = 2.0650$

$c = -0.01 + 0.3941 = 0.3937$

step 8: $\text{Sample} = \text{sample} + 1 = 2 + 1 = 3$

step 9: $3 > 2$
true \rightarrow goto next step

step 10: $\text{itr} = \text{itr} + 1 = 1 + 1 = 2$

step 11: $\text{itr} > \text{epochs}$
 $2 > 2$
false \rightarrow goto step 4 3

step 3: $\text{sample} = 1$

step 4: $g_m = -(2.5939)0.2 = -0.5187$

$g_c = -2.5939$

step 5: $G_m = G_m + (g_m)^2$
 $= 1.2246 + 0.2690 = 1.4935$

$G_c = G_c + (g_c)^2$
 $= 20.8839 + 6.7283 = 27.6122$

step 6: $\Delta m = \frac{-0.1}{\sqrt{1.4936 \times 10^8}} \times (-0.5187) = 0.01789$

$\Delta c = \frac{-0.1}{\sqrt{27.6122 \times 10^8}} \times (-2.5939) = 0.04936$

step 7: $m = m + \Delta m = 2.08289$
 $c = c + \Delta c = 0.44246$

step 8: $\text{sample} = \text{sample} + 1 = 2$

step 3: $2 > 2$
false \rightarrow goto step 4

step 4: $g_m = -(3.8 - (2.08289 \times 0.4) - 0.44246) \times 0.4$
 $= -1.00972$

$$g_c = -2.5243$$

step 5: $G_m = 1.4936 + (-1.00972)^2 = 2.5131$

$$G_c = 22.61227 + (-2.5243)^2 = 33.9842$$

step 6: $\Delta m = \frac{-0.1}{\sqrt{2.5131 + 10^8}} \times (-1.00972) = 0.06369$

$$\Delta c = \frac{-0.1}{\sqrt{33.9842 + 10^8}} \times (-2.5243) = 0.0433$$

step 7: $m = m + \Delta m = 2.08289 + 0.06369 = 2.1465$
 $c = c + \Delta c = 0.48576$

step 8: $\text{sample} = \text{sample} + 1 = 2 + 1 = 3$

step 9: $3 > 2$
true \rightarrow goto ^{next} step

step 10: $\%tr = \%tr + 1$
 $= 2 + 1 = 3$

step 11: $3 \geq \text{epochs}$
 $3 > 2$
true \rightarrow go to next step

step 12: print m, c

calculate MSE

$$= \frac{1}{2 \times 2} \sum (y_i - y_p)^2$$

$$= \frac{1}{4} \left[(3.4 - (2.14655 \times 0.2) - 0.48576)^2 + (3.8 - (2.14658 \times 0.4) - 0.48576)^2 \right]$$

$$= 3.05121$$