Revisiting accepted wisdom in SNARK design

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Talk Outline

1. Survey of SNARK design.
2. What needs to change for scalability.
3. Whirlwind overview of recent developments.
   - Lasso [STW 2023]
   - Jolt [AST 2023]
   - BabySpartan [ST 2023]
   - Binius [DP 2023]

SNARKs: What are they

- Prover P claims to know witness w satisfying some property.
  - For example: A preimage w for a designated output y of a cryptographic hash function h.
    - A w such that $h(w) = y$.

Trivial proof: P sends w to V, who directly checks it satisfies the claimed property.

A SNARK (Succinct Non-interactive ARgument of Knowledge) achieves the same, but with better costs to V.

- SNARK proof π must be shorter than the witness w.
  - This is what "succinct" means.
- Ideally, checking π is faster than checking w directly.
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SNARK design details

General-Purpose SNARKs: Standard Paradigm

- Start with a computer program ψ written in high-level programming language (C, Java, etc.).
  - Hash pre-image example:
    - Program takes w as input, applies h to it, checks that output equals y.

Step 1: Turn ψ into an equivalent model amenable to probabilistic checking.

Typically some variant of circuit-satisfiability.

The "circuit" is called an intermediate representation (IR).

Examples: arithmetic circuits, R1CS, AIR, "Plonkish IR".

Called the Front End of the system.

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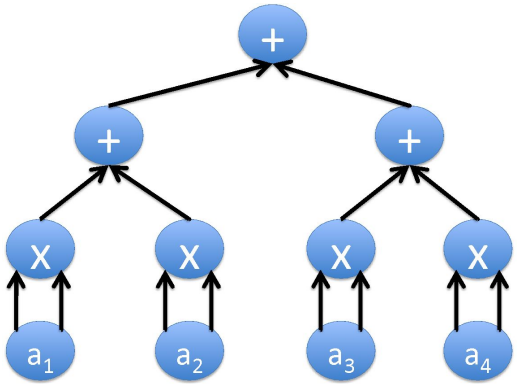
  Called the **backend** of the system.

Front End



P and V run SNARK (back end) for circuit-satisfiability.

Backends

Key Paradigm for Backend Design

1. Give a "**polynomial IOP**" for R1CS or circuit-satisfiability.
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- This is how all SNARKs are designed other than "linear-PCP based" ones (GGPR13, Pinocchio, Groth16, etc.) and "folding-based" ones (Nova, etc.)
  - Even linear-PCP based SNARKs use techniques **very** similar to certain polynomial commitments (KZG).

Polynomial IOPs: Two classes

1. Constant-round, uses univariate polynomials
   - Plonk, Marlin: targeted at *non-uniform* computation (arbitrary circuits)
   - STARKs: targeted at *uniform* computation (e.g., CPU execution)

- Many-round, uses multivariate polynomials
  - Based on the **sum-check protocol**.
  - Spartan, Hyperplonk, Lasso, Jolt, BabySpartan, etc.

Polynomial Commitment Schemes: Three Classes

1. Based on pairings (**not** transparent **nor** post-quantum).
   - e.g., KZG10, PST13, Zeromorph
   - Proof size can be **constant**.

2. Based on discrete logarithm (transparent, **not** post-quantum).
   - Examples: IPA/Bulletproofs, Hyrax, Dory.
     - Proof size can be **logarithmic**.

3. Based on IOPs + hashing (transparent **and** post-quantum)
   - e.g., FRI, Ligero, Brakedown.
     - Proof size can be **polylogarithmic.**
     - $O(\lambda \log^2(n)) \gg O(\log^3(n))$ hash evaluations.

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SNARK performance

Performance bottlenecks

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- But the key scalability bottleneck is P time.
  - 18 months ago, I estimated P overhead as 1 million-100 million-fold.
    - To prove it correctly ran a computer program:
      - If the program runs in 1 second, P runs in 1 million seconds.
      - Though the overhead is highly parallelizable.
      - See https://a16zcrypto.com/posts/article/measuring-snark-performance-frontends-backends-and-the-future/

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- But the key scalability bottleneck is P time.
  - A key contributor to P time is **commitment costs**.
    - i.e., how much time it takes P to cryptographically commit to large vectors.
    - This involves expensive operations like hashing and FFTs, or group operations (depending on the polynomial commitment scheme used).

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    - E.g., Ligero/Brakedown/Binius
- If prover **space** is a key bottleneck, should use a folding scheme like Nova.

What performance can now be achieved?

- Developments since August (Lasso/Jolt/BabySpartan/Binius)
  - P overhead may now be ~50,000-fold.
  - ~20x improvement relative to 18 months ago.

- Compared to today's SNARK deployments, this involves:
  - Different polynomial IOPs.
    - Under the hood: multivariate rather univariate polynomials.
  - Different polynomial commitment schemes.
  - Different hash functions.
  - Different finite fields.
  - Different front-end techniques.

Fast-prover SNARKs: overview

Key tool: the sum-check protocol

Sum-Check Protocol [LFKN90]

- Input: An ℓ-variate polynomial g over field $\boldsymbol{F}$.
- Goal: compute the quantity:

$$\sum_{b_1 \in \{0,1\}} \sum_{b_2 \in \{0,1\}} \dots \sum_{b_\ell \in \{0,1\}} g(b_1, \dots, b_\ell).$$

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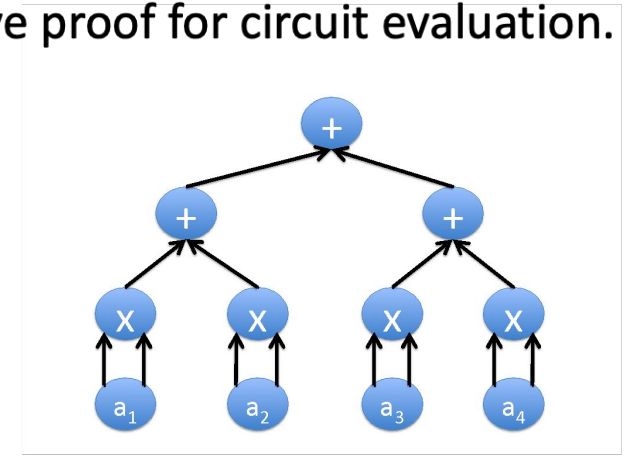
- V only evaluates g at **one** point.
  - Much better than the 2^ℓ evaluations required to compute the sum herself.
- P does just a constant factor extra work compared to "native computation".
  - i.e., computing the sum with no proof of correctness.

The GKR protocol (2008)

- Interactive proof for circuit evaluation.



- Repeatedly applied the sum-check protocol.
- For circuits of size S, P does $O(S)$ field operations [CMT12, T13, XZZPS19].
- In applications to SNARKs, the input to the circuit is committed, and the proof size is $O(\log(S)^2)$.

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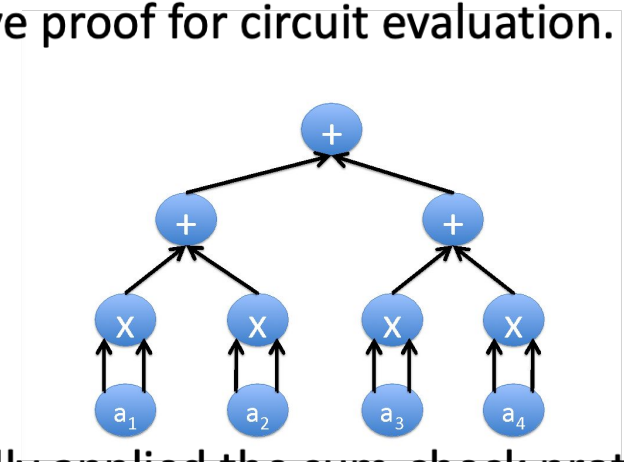

- Repeatedly applied the sum-check protocol.
- For circuits of size S, P does $O(S)$ field operations [CMT12, T13, XZZPS19].
  - No cryptography involved.
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Takeaways for SNARK design

- The sum-check protocol is a tool for minimizing the prover's **commitment costs.**
  - Allows P to commit to **less data** than in SNARKs based on constant-round polynomial IOPs.
    - There are **fewer** committed values.
    - Each committed value is **smaller.**
      - Smaller values can be **much** faster to commit to than big values.
      - E.g., for KZG or Bulletproofs, a small value is over 10x faster to commit to than a big one.

Lasso: A lookup argument with low commitment costs

Indexed lookup arguments

- Let $t \in \boldsymbol{F}^N$ be the vector of N table entries.
- P has committed to a vector $((a_1, b_1), \ldots, (a_m, b_m))$.
- P claims that $a_i = t[b_i]$ for $i = 1 \ldots m$.

- Lasso is a new lookup argument whereby P commits to fewer and smaller field elements than in prior work.
  - Under the hood, it uses the sum-check protocol and the GKR protocol to achieve this.

Lasso Performance

- For m lookups into a table of size N:
  - Lasso P commits to $m + N$ field elements.
    - All of them “small” (in {0, 1, …, m}).
    - No commitment to the table t is needed if the table is “structured”.
    - Can reduce the dependence on N for ”decomposable” tables
  - Comparison points:
    - Plookup [GW20] P commits to $5 * \max\{m, N\}$ field elements.
      - Plus the table commitment.
      - $3 * \max\{m, N\}$ of the committed field elements are random.
    - CQ [EFG22]: P commits to $8m$ field elements (after pre-processing)
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Jolt: a zkVM based on Lasso

Front-ends today for VM execution

- Say P claims to have run a certain computer program for m steps.
  - Say the program is written in the assembly language for a VM.
  - Popular VM's targeted: RISC-V, Ethereum Virtual Machine (EVM)
- Today, front-ends produce a circuit that, for each step of the computation:
  1. Figures out what instruction to execute at that step.
  2. Executes that instruction.

Lasso lets one replace Step 2 with a single lookup.

For each instruction, the table stores the entire evaluation table of the function.

If instruction f operations on two 64-bit inputs, the table stores $f(x, y)$ for every pair of 64-bit inputs (x, y).

This table has size 2^{128}.

All RISC-V instructions are decomposable.

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  - If instruction f operations on two 64-bit inputs, the table stores $f(x, y)$ for every pair of 64-bit inputs (x, y).
    - This table has size 2^{128}.
    - Jolt shows that all RISC-V instructions are structured/decomposable.

Costs of Jolt

- For RISC-V instructions on 32-bit data types (with "multiply extension"):
  - Jolt's P commits to about **60** values per step, requiring **~800 bits** in total to specify.
    - Where a b-bit value is between 0 and $2^b - 1$.
  - **Rough** comparison points:
    - RISC-Zero currently commits to at least 275 values per step, each in a 31-bit field.
      - They use FRI as the polynomial commitment scheme, which is not significantly faster for small values than big ones.
      - So 275 31-bit field elements equations to 8500+ committed bits per step.

Jolt implementation status

- Initial implementation of Jolt (with elliptic-curve-based commitment schemes) is nearing completion.

BabySpartan: Applying Lasso to arbitrary circuits

BabySpartan

- Jolt can be seen as an application of Lasso to VM execution, but Lasso is more generally applicable.
- BabySpartan is a new SNARK for circuit satisfiability (supports a generalization of “Plonkish circuits”).
  - P commits to 4 values per gate of the circuit. All values are small.
  - Comparison point:
    - Plonk P commits to 7 random values per gate (and additional non-random ones).

Binius
[Diamond and Posen 2023]

Context: Ligero/Brakedown commitments

- Lasso/Jolt and other sum-check-based SNARKs can use **any** commitment scheme for multilinear polynomials.
- The best one for P is Ligero/Brakedown.
  - A hashing-based commitment scheme.
    - Same family as FRI.
    - No group operations, works over small fields.
  - But its proofs are somewhat big.
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  - But its proofs are somewhat big.
    - Square root in the size of the polynomial.
  - One can reduce proof size and V work via SNARK recursion.
    - Pushes V's work onto P.
    - But since the Ligero/Brakedown verifier is expensive (lots of hash evaluations), applying recursion will bottleneck P.

Binius (Diamond and Posen, last week)

1. Make Ligero/Brakedown P **even faster** when committing to small values.
    - E.g., committing to a 1-bit value (0 or 1) is now ~32x faster than committing to a 32-bit value.
2. Combine this with a Lasso-based SNARK for circuit-SAT (similar to BabySpartan) to get **much** faster SNARKs for standard hash functions like Keccak.
3. This enables applying recursion to Ligero/Brakedown, addressing proof size issue.
    - Because the Ligero/Brakedown verifier mainly does hashing.
    - So applying recursion to Ligero/Brakedown amounts to P proving it correctly hashed a lot of data.

Side benefits

- Can now use standard hash functions like Keccak in recursive SNARKs, rather than “SNARK-friendly ones” like Poseidon.
    - Faster, better-understood security.
    - Addresses key bottleneck for “Type-1 zkEVMs” (full Ethereum equivalence).
- Takeaway: we’ve been designing hash functions to be friendly to the artificial limitations of today’s popular SNARKs.

Conclusion

- SNARK provers can be sped up well over an order of magnitude, compared to today's popular SNARKs.

- Side benefit of better performance: **better security.**
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