18K41A04C8

Assignment -5:

Let consider a sample dataset have one input (X_i^a) and one output (Y_i^a), and number of samples 4. Develop a simple linear regression model using MBGD.

Sample (i)	(X_i^a)	(Y_i^a)
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

Do manual calculations for two iterations with batch size 2.

x	y	
0.2	3.4	→ batch 1
0.4	3.8	
0.6	4.2	→ batch 2
0.8	4.6	

Step 1: $[x, y]$, $m = 1$, $c = -1$, $\eta = 0.1$, epochs $= 2$, $bs = 2$

Step 2: $nb = \frac{ns}{bs} = \frac{4}{2} = 2$

Step 3: iter = 1

Step 4: Batch = 1

Step 5: $\frac{\partial E}{\partial m} = \frac{-1}{bs} \sum_{i=1}^{bs} (y_i - mx_i - c)x_i$

$= -\frac{1}{2}\left[\left[(3.4 - (1)(0.2) + 1)(0.2)\right] + \left[(3.8 - (1)(0.4) + 1)(0.4)\right]\right]$

$= -\frac{1}{2}[0.84 + 1.76]$

$\frac{\partial E}{\partial m} = -1.34$

$$\frac{\partial E}{\partial c} = -\frac{1}{2}\left[(3.4 - 0.2 + 1) + (3.8 - 0.4 + 1)\right]$$

$$\frac{\partial E}{\partial c} = -4.3$$

Step 6: $\Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1)(-1.34)$

$$\Delta m = 0.134$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-4.3)$$

$$\Delta c = 0.43$$

Step 7: $m = m + \Delta m = 1 + 0.134 = 1.134$

$c = c + \Delta c = -1 + 0.43 = -0.57$

Step 8: batch = batch + 1 = 1 + 1 = 2

Step 9: if (batch > nb)

2 > 2 → false

goto step 5

Step 5:
$$\frac{\partial E}{\partial m} = -\frac{1}{2}\left[(4.2 - (1.134)(0.6) + 0.57)0.6 + (4.6 - (1.134)(0.8) + 0.57)0.8\right]$$

$$\frac{\partial E}{\partial m} = -\frac{1}{2}\left[2.4537 + 3.4102\right]$$

$$\frac{\partial E}{\partial m} = -2.9319$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2}\left[(4.2 - (1.134)(0.6) + 0.57) + (4.6 - (1.134)(0.8) + 0.57)\right]$$

$$= -\frac{1}{2}\left[4.0896 + 4.2628\right]$$

$$\frac{\partial E}{\partial c} = -4.1762$$

Step 6: $\Delta m = -(0.1)(-2.932)$

$\Delta m = 0.2932$

$\Delta c = -(0.1)(-4.1762)$

$\Delta c = 0.41762$

Step 7: $m = m + \Delta m = 1.134 + 0.2932 = 1.4272$

$c = c + \Delta c = -0.57 + 0.41762 = -0.1523$.

Step 8: batch = batch + 1 = 2 + 1 = 3

Step 9: if (batch > ns)

3 > 2 → True

goto next step.

Step 10: iter = iter + 1 = 1 + 1 = 2

Step 11: if (iter > epochs)

2 > 2 → false

goto step 4.

Step 4: Batch = 1

Step 5: $\frac{\partial E}{\partial m} = -\frac{1}{2}\left[(3.4 - (1.4272)(0.2) + 0.1523)0.2 + [3.8 - (1.4272)(0.4) + 0.1523)0.4\right]$

$\frac{\partial E}{\partial m} = -\frac{1}{2}[0.653 + 1.352]$

$\frac{\partial E}{\partial m} = -1.0029$

$\frac{\partial E}{\partial c} = -\frac{1}{2}\left[(3.4 - (1.4272)(0.2) + 0.1523) + [3.8 - (1.4272)(0.4) + 0.1523)\right]$

$= -\frac{1}{2}(3.266 + 3.381)$

$\frac{\partial E}{\partial c} = -3.323$

Step 6: $\Delta m = (-0.1)(-1.0029)$

$\Delta m = 0.1002$

$\Delta c = (-0.1)(-3.323)$

$\Delta c = 0.332$

Step 7: $m = m + \Delta m = 1.4272 + 0.1002$

$m = 1.5274$

$c = c + \Delta c = -0.1523 + 0.332$

$c = 0.1797$

Step 8: batch = batch + 1

$= 1 + 1 = 2$

Step 9: if (batch > nb)

$2 > 2$ false

$\Rightarrow$ goto step 5

Step 5: $\frac{\partial E}{\partial m} = \frac{-1}{2}\Big[(4.2 - (1.5274)(0.6) - 0.1797)0.6$

$+ (4.6 - (1.5274)(0.8) - 0.1797)0.8\Big]$

$= \frac{-1}{2}[1.8623 + 2.558]$

$\frac{\partial E}{\partial m} = -2.2101$

$\frac{\partial E}{\partial c} = \frac{-1}{2}\Big[(4.2 - (1.5274)(0.6) - 0.1797) +$

$(4.6 - (1.5274)(0.8) - 0.1797)\Big]$

$= \frac{-1}{2}[3.103 + 3.198]$

$\frac{\partial E}{\partial c} = -3.151$

Step 6: $\Delta m = -(0.1)(-2.2101) = 0.221$

$\Delta c = -(0.1)(-3.151) = 0.315$

Step 7: $m = m + \Delta m = 1.5274 + 0.221 = 1.7484$

$c = c + \Delta c = 0.1797 + 0.315 = 0.4947$

Step 8: batch = batch + 1 = 2 + 1 = 3

Step 9: if (batch > nb)

3 > 2 True

goto step 10

Step 10: iter = iter + 1 = 2 + 1 = 3

Step 11: if (iter > epochs)

3 > 2 True

goto next step

Step 12: print m, c

$m = 1.748$, $c = 0.494$