

FIRST function -

FIRST(α) is a set of terminal symbols that are first symbols appearing at RHS in derivation of α . If $\alpha \Rightarrow \epsilon$ then ϵ is also in FIRST (α).

Following are the rules used to compute the FIRST functions.

1. If the terminal symbol a then $\text{FIRST}(a) = \{a\}$.
2. If there is a rule $X \rightarrow \epsilon$ then $\text{FIRST}(X) = \{\epsilon\}$.
3. For the rule $A \rightarrow X_1 X_2 X_3 \dots X_k$ $\text{FIRST}(A) = (\text{FIRST}(X_1) \cup \text{FIRST}(X_2) \cup \text{FIRST}(X_3) \dots \text{FIRST}(X_k))$.

Where $k \leq n$ such that $1 \leq j \leq k-1$

FOLLOW function -

FOLLOW (A) is defined as the set of terminal symbols that appear immediately to the right of A. In other words

$\text{FOLLOW}(A) = \{ a \mid S \xrightarrow{*} \alpha A a \beta \text{ where } \alpha \text{ and } \beta \text{ are some grammar symbols may be terminal or non-terminal.}$

The rules for computing FOLLOW function are as given below -

1. For the start symbol S place \$ in FOLLOW(S).
2. If there is a production $A \rightarrow \alpha B \beta$ then everything in $\text{FIRST}(\beta)$ without ϵ is to be placed in FOLLOW(B).
3. If there is a production $A \rightarrow \alpha B \beta$ or $A \rightarrow \alpha B$ and $\text{FIRST}(\beta) = \{\epsilon\}$ then $\text{FOLLOW}(A) = \text{FOLLOW}(B)$ or $\text{FOLLOW}(B) = \text{FOLLOW}(A)$. That means everything in FOLLOW(A) is in FOLLOW(B).

$A \xrightarrow{*} \alpha B \beta$
 $\alpha \xrightarrow{*} \beta$

$A \xrightarrow{*} \alpha B \beta \text{ for } \beta$
 $\text{for } \beta = \{ \}$

Let us take some examples to compute FIRST and FOLLOW functions using above rules.

Example 3.7 : Consider grammar

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' | \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' | \epsilon$$

$$F \rightarrow (E) | id$$

Find the FIRST and FOLLOW functions for the above grammar.

Solution : AS $E \rightarrow TE'$ is a rule in which the first symbol at RHS is T. Now $T \rightarrow FT'$ in which the first symbol at RHS is F and there is a rule for F as $F \rightarrow (E) | id$.

$$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F)$$

$$\text{As } F \rightarrow (E)$$

$$F \rightarrow id$$

$$\text{Hence } \underline{\text{FIRST}(E)} = \underline{\text{FIRST}(T)} = \underline{\text{FIRST}(F)} = \{ (, id \} \quad \begin{matrix} \text{left} & \text{A} & \text{Right} \\ (E) & & \end{matrix}$$

$$\text{FIRST}(E') = \{ +, \epsilon \}$$

$$\text{As } E' \rightarrow +TE'$$

$$E' \rightarrow \epsilon \text{ by referring computation rule 2} \quad \begin{matrix} & & F \rightarrow (E) \\ & & A \rightarrow \alpha B \beta \\ & & \delta((\epsilon)) =) \end{matrix}$$

The first symbol appearing at RHS of production rule for E' is added in the FIRST function.

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

$$\text{As } T' \rightarrow *TE' \quad \begin{matrix} T(E) \rightarrow \epsilon \end{matrix}$$

$$T(E) \rightarrow \epsilon$$

The first terminal symbol appearing at RHS of production rule for T' is added in the FIRST function. Now we will compute FOLLOW function.

FOLLOW(E) -

i) As there is a rule $F \rightarrow (E)$ the symbol ')' appears immediately to the right of E.

Hence ')' will be in FOLLOW(E).

ii) The computation rule is $A \rightarrow \alpha B \beta$ we can map this rule with $F \rightarrow (E)$ then $A = F, \alpha = (, B = E, \beta =)$.

$$\text{FOLLOW}(B) = \text{FIRST}(\beta) = \text{FIRST}()) = \{ \) \}$$

$$\text{FOLLOW}(E) = \{ \) \}$$

Since E is a start symbol, add \$ to follow of E.

Hence $\text{FOLLOW}(E) = \{ \}, \$ \}$

$\text{FOLLOW}(E)$ -

i) $E \rightarrow TE'$ the computational rule is $A \rightarrow \alpha B\beta$.

$\therefore A = E', \alpha = T, B = E', \beta = \epsilon$ then by computational rule 3 everything in $\text{FOLLOW}(A)$ is in $\text{FOLLOW}(B)$ i.e. everything in $\text{FOLLOW}(E)$ is in $\text{FOLLOW}(E')$.

$\therefore \text{FOLLOW}(E') = \{ \}, \$ \}$

ii) $E' \rightarrow +TE'$ the computational rule is $A \rightarrow \alpha B\beta$.

$\therefore A = E', \alpha = +T, B = E', \beta = \epsilon$ then by computational rule 3 everything in $\text{FOLLOW}(A)$ is in $\text{FOLLOW}(B)$ i.e. everything in $\text{FOLLOW}(E')$ is in $\text{FOLLOW}(E)$.

$\text{FOLLOW}(E') = \{ \}, \$ \}$

We can observe in the given grammar that) is really following E.

$\text{FOLLOW}(T)$ -

We have to observe two rules

$$E \rightarrow TE'$$

$$E' \rightarrow +TE'$$

i) Consider

$E \rightarrow TE'$ we will map it with $A \rightarrow \alpha B\beta$

$A = E, \alpha = \epsilon, B = T, \beta = E'$ by computational rule 2 $\text{FOLLOW}(B) = \{\text{FIRST}(\beta) - \epsilon\}$.

That is $\text{FOLLOW}(T) = \{\text{FIRST}(E') - \epsilon\}$

$$= \{\{+, \epsilon\} - \epsilon\}$$

$$= \{+\}$$

ii) Consider $E' \rightarrow +TE'$ we will map it with $A \rightarrow \alpha B\beta$

$A = E', \alpha = +, B = T, \beta = E'$ by computational rule 3

$\text{FOLLOW}(A) = \text{FOLLOW}(B)$ i.e. $\text{FOLLOW}(E') = \text{FOLLOW}(T)$

$\text{FOLLOW}(T) = \{ \}, \$ \}$

Finally $\text{FOLLOW}(T) = \{+\} \cup \{ \}, \$ \}$

$$= \{+, \), \$\}$$

We can observe in the given grammar that + and) are really following T.

FOLLOW(T') -

$$T \rightarrow FT'$$

$$\begin{array}{c} T \rightarrow FT' \\ \nwarrow \alpha \quad \nearrow \beta \\ A \quad B \quad \beta \end{array}$$

We will map this rule with $A \rightarrow \alpha B\beta$ then $A = T$, $\alpha = F$, $B = T'$, $\beta = \epsilon$ then
 $\text{FOLLOW}(T') = \text{FOLLOW}(T) = \{+, \), \$\}$

$$T \rightarrow *FT'$$

$$\begin{array}{c} T \rightarrow *FT' \\ \nwarrow \alpha \quad \nearrow \beta \\ A \quad B \quad \beta \end{array}$$

We will map this rule with $A \rightarrow \alpha B\beta$ then $A = T$, $\alpha = *F$, $B = T'$, $\beta = \epsilon^*$ then
 $\text{FOLLOW}(T') = \text{FOLLOW}(T) = \{+, \), \$\}$

Hence $\text{FOLLOW}(T') = \{+, \), \$\}$

FOLLOW(F) -

Consider $T \rightarrow FT'$ or $T' \rightarrow *FT'$ then by computational rule 2,

$$T \rightarrow FT'$$

$$A \rightarrow \alpha B\beta$$

$$A = T, \alpha = \epsilon, B = F, \beta = T'$$

$$\text{FOLLOW}(B) = \{\text{FIRST}(\beta) - \epsilon\}$$

$$\text{FOLLOW}(F) = \{\text{FIRST}(T') - \epsilon\}$$

$$\text{FOLLOW}(F) = \{*\}$$

$$T' \rightarrow *FT'$$

$$A \rightarrow \alpha B\beta$$

$$A = T', \alpha = *, B = F, \beta = T'$$

$$\text{FOLLOW}(B) = \{\text{FIRST}(\beta) - \epsilon\}$$

$$\text{FOLLOW}(F) = \{\text{FIRST}(T') - \epsilon\}$$

$$\text{FOLLOW}(F) = \{*\}$$

Consider $T' \rightarrow *FT'$ by computational rule 3

$$T' \rightarrow *FT'$$

$$A \rightarrow \alpha B\beta$$

$$A = T', \alpha = *, B = F, \beta = T'$$

$$\text{FOLLOW}(A) = \text{FOLLOW}(B)$$

$$\text{FOLLOW}(T') = \text{FOLLOW}(F)$$

$$\text{Hence } \text{FOLLOW}(F) = \{+, \), \$\}$$

Finally $\text{FOLLOW}(F) = \{*\} \cup \{+, \), \$\}$

$$\text{FOLLOW}(F) = \{+, *\}, \$$$

To summarize above computation

Symbols	FIRST	FOLLOW
E	{(, id}	{), \$}
E'	{+, ε}	{), \$}
T	{(, id}	{+,), \$}
T'	{*, ε}	{+,), \$}
F	{(, id}	{+, *,), \$}