

# Desirable Properties of Decomposition



## Module No. 3

Functional dependency (FD), Closure of FD, Closure of Attributes, Cover, Equivalence of FD, Canonical cover, Key generation, Normalization, **Desirable properties of decomposition.**

# Decomposition of a Relation

## Properties of Decomposition

The following two properties must be followed when decomposing a given relation

### 1. Lossless Decomposition

Lossless decomposition ensures-

1. No information is lost from the original relation during decomposition.
2. When the sub-relations are joined back, the same relation is obtained that was decomposed. Every decomposition must always be lossless.

# Decomposition of a Relation

## Properties of Decomposition

### 2. Dependency Preservation

Dependency preservation ensures:

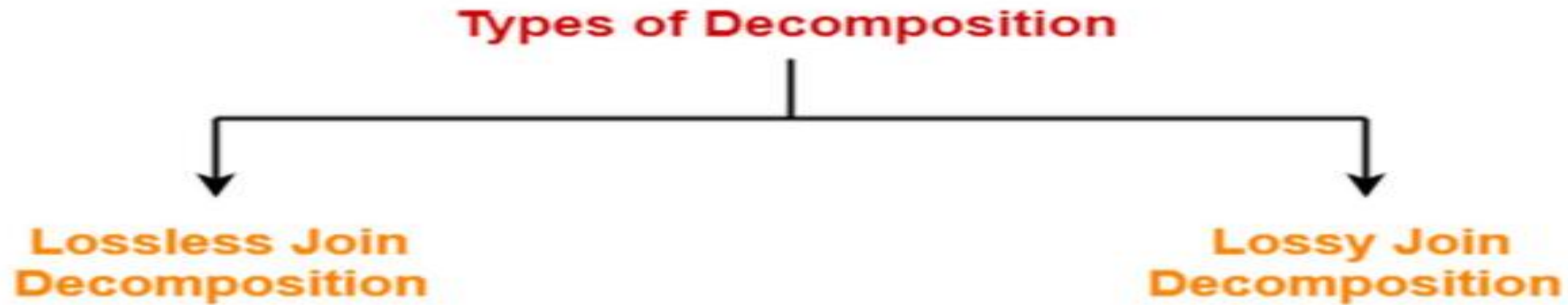
- None of the functional dependencies that hold on the original relation are lost.
- The sub-relations still hold or satisfy the functional dependencies of the original relation.

- Let a relation  $R(A, B, C, D)$  and a set of FDs  $F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$  are given.
- A relation is decomposed into -
- $R_1 = (A, B, C)$  with FDs  $F_1 = \{A \rightarrow B, A \rightarrow C\}$ .
- $R_2 = (C, D)$  with FDs  $F_2 = \{C \rightarrow D\}$
- $F' = F_1 \cup F_2 = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$  So,  $F' = F$ . And so,  $F'^+ = F^+$ .
- Thus, the decomposition is dependency-preserving decomposition.

# Decomposition of a Relation

- The process of breaking up or dividing a single relation into two or more sub-relations is called as decomposition of a relation.

Types of Decomposition



# Decomposition of a Relation

## 1. Lossless Join Decomposition

- Consider there is a relation  $R$  which is decomposed into subrelations  $R_1, R_2, \dots, R_n$ .
- This decomposition is called lossless join decomposition when the join of the sub-relations results in the same relation  $R$  that was decomposed.
- For lossless join decomposition, we always have- .

$$R_1 \bowtie R_2 \bowtie R_3 \dots \bowtie R_n = R$$

where  $\bowtie$  is a natural join operator

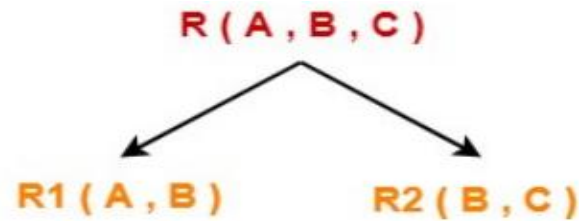
# Decomposition of a Relation

## 1. Lossless Join Decomposition

Example: Consider the following relation  $R(A, B, C)$

A	B	C
1	2	1
2	5	3
3	3	3

Consider this relation is decomposed into two sub relations  $R_1(A, B)$  and  $R_2(B, C)$



The two sub relations are-

A	B
1	2
2	5
3	3

$R_1(A, B)$

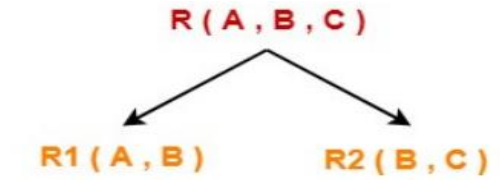
B	C
2	1
5	3
3	3

$R_2(B, C)$

# Decomposition of a Relation

## 1. Lossless Join Decomposition

### Example:



The two sub relations are-

A	B
1	2
2	5
3	3

R<sub>1</sub>(A, B)

B	C
2	1
5	3
3	3

R<sub>2</sub>(B, C)

- Now, let us check whether this **decomposition is lossless or not**. For lossless decomposition, we must have  $R1 \bowtie R2 = R$ .
- Now, if we perform **the natural join ( $\bowtie$ )** of the sub relations **R1 and R2**, we get

A	B	C
1	2	1
2	5	3
3	3	3

# Decomposition of a Relation

## 1. Lossless Join Decomposition

Lossless join decomposition is also known as **non-additive join** decomposition.

- This is because the **resultant relation after joining** the sub-relations is the same **as the decomposed relation**.
- **No extraneous tuples appear** after joining of the sub-relations.



# Decomposition of a Relation

## 2. Lossy Join Decomposition

- Consider there is a relation  $R$  which is decomposed into sub-relations  $R_1, R_2, \dots, R_n$ .
- This decomposition is called **lossy join decomposition** when the join of the sub-relations does not result in the same relation  $R$  that was decomposed.
- The natural join of the sub-relations is always found to have some extraneous tuples.

$$R_1 \bowtie R_2 \bowtie R_3 \dots \bowtie R_n \supset R$$

where  $\bowtie$  is a natural join operator

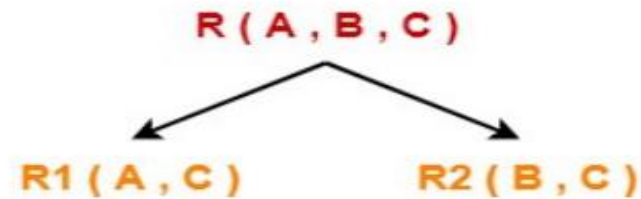
# Decomposition of a Relation

## 2. Lossy Join Decomposition

Example Consider the following relation  $R(A, B, C)$

A	B	C
1	2	1
2	5	3
3	3	3

Consider this relation is decomposed into two sub relations  $R_1(A, C)$  and  $R_2(B, C)$



The two sub relations are-

A	C
1	1
2	3
3	3

$R_1(A, B)$

B	C
2	1
5	3
3	3

$R_2(B, C)$

# Decomposition of a Relation

## 2. Lossy Join Decomposition

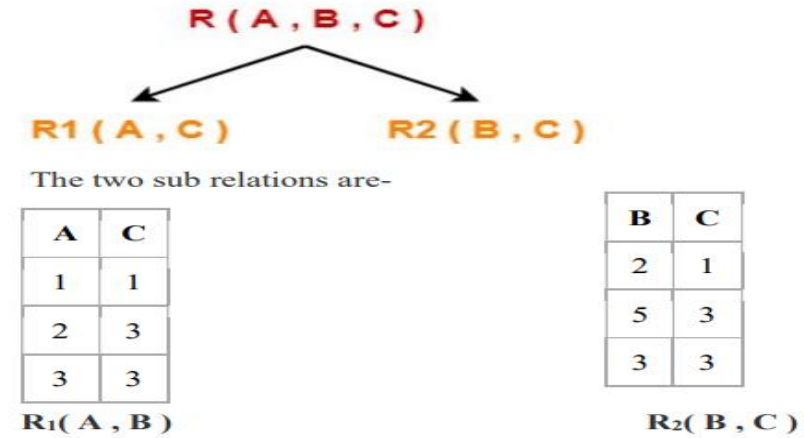
### Example

- Now, let us check whether this decomposition is **lossy or not**.
- For lossy decomposition, we must have  $R1 \bowtie R2 \supset R$

Now, if we perform the natural join ( $\bowtie$ ) of the sub relations R1 and R2

A	B	C
1	2	1
2	5	3
2	3	3
3	5	3
3	3	3

- This relation is **not the same as the original relation R and contains some extraneous tuples**.
- Clearly,  $R1 \bowtie R2 \supset R$ . Thus, we conclude that the above **decomposition is lossy join decomposition**.



# Decomposition of a Relation

## 2. Lossy Join Decomposition

NOTE-

- Lossy join decomposition is also known as careless decomposition.
- This is because extraneous tuples get introduced in the natural join of the sub-relations.
- Extraneous tuples make the identification of the original tuples difficult.

# Decomposition of a Relation

## Determining Whether Decomposition is Lossless or Lossy

Consider a relation R is **decomposed** into two sub relations **R1 and R2**. Then,

- If all the following **conditions are satisfied**, then the decomposition is **lossless**.
- If any of these **conditions fail**, then the **decomposition is lossy**.

### Condition-01

- The **Union of both the sub-relations** must **contain all the attributes** that are present in the original relation R. Thus

$$R1 \cup R2 = R$$

### Condition-02

- The **intersection of both the sub relations** must **not be null**. In other words, there must be **some common attribute that is present** in both the sub relations. Thus

$$R1 \cap R2 \neq \emptyset$$

### Condition-03

- Intersection of both the sub relations **must be a super key of either R1 or R2 or both**. Thus

$$R1 \cap R2 = \text{Super key of R1 or R2}$$

# Decomposition of a Relation

## Examples

- Consider a relation schema  $R ( A , B , C , D )$  with the functional dependencies  $A \rightarrow B$  and  $C \rightarrow D$ . Determine whether the decomposition of  $R$  into  $R1 ( A , B )$  and  $R2 ( C , D )$  is lossless or lossy.

## Solution

**Condition-01:** The union of both the sub relations must contain all the attributes of relation  $R$ .

$$R1 ( A , B ) \cup R2 ( C , D ) = R ( A , B , C , D )$$

- The union of the sub-relations contains all the attributes of relation  $R$ . Thus, **condition-01 satisfies**.

**Condition-01:** the intersection of both the sub relations must not be null.

$$R1 ( A , B ) \cap R2 ( C , D ) = \Phi$$

- The intersection of the sub-relations is null.
- So, condition-02 fails.
- Thus, we conclude that the decomposition is lossy.

# Decomposition of a Relation

## Examples

- Consider a relation schema  $R ( A , B , C , D )$  with the following functional dependencies.

FD :  $A \rightarrow B , B \rightarrow C , C \rightarrow D , D \rightarrow B$

- Determine whether the decomposition of  $R$  into  $R1 ( A , B ) , R2 ( B , C )$  and  $R3 ( B , D )$  is lossless or lossy.

## Strategy to Solve

When a given relation is decomposed into more than two sub relations, then-

- Consider any one possible way in which the relation might have been decomposed into those sub-relations.
- First, divide the given relation into two sub-relations.
- Then, divide the sub-relations according to the sub-relations given in the question.

As a thumb rule,

remember Any relation can be decomposed only into two sub-relations at a time.

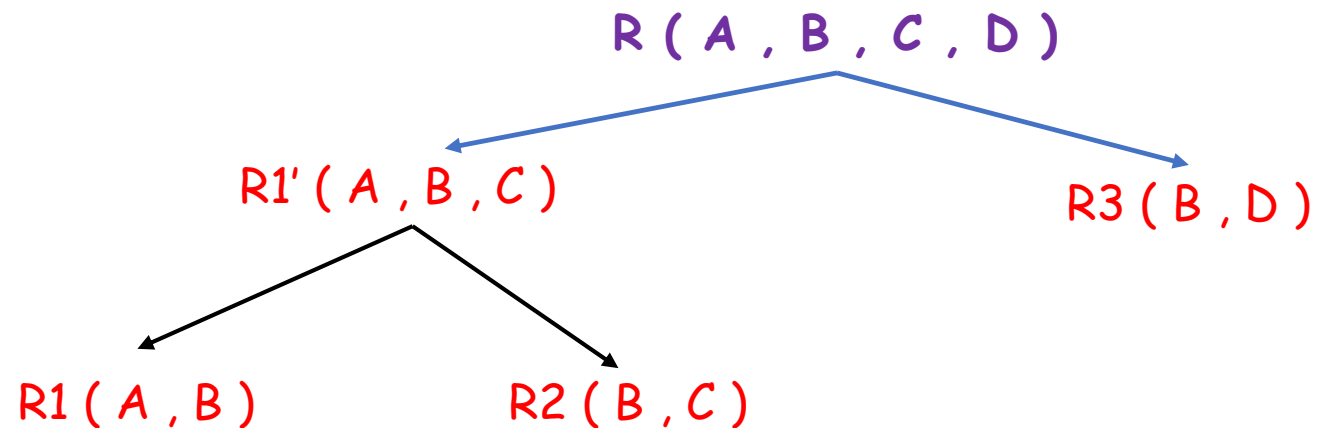
# Decomposition of a Relation

## Examples

- Consider a relation schema  $R(A, B, C, D)$  with the following functional dependencies.

FD :  $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow B$

- Determine whether the decomposition of  $R$  into  $R_1(A, B), R_2(B, C)$  and  $R_3(B, D)$  is lossless or lossy.



## Condition-01

The union of both the sub-relations must contain all the attributes of relation  $R$ .

$$R'(A, B, C) \cup R_3(B, D) = R(A, B, C, D)$$

- Condition-01 satisfies.



# Decomposition of a Relation

## Examples Cont'd

•  $R(A, B, C, D)$

FD :  $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow B$

- Determine whether the decomposition of R into  $R_1(A, B)$ ,  $R_2(B, C)$  and  $R_3(B, D)$  is lossless or lossy.

Condition-02: The intersection of both the sub relations must **not be null**.

$$R'(A, B, C) \cap R_3(B, D) = B$$

Condition-02 satisfies

Condition-03:

- The intersection of both the sub-relations **must be the super key** of one of the two sub-relations or both.

$$R'(A, B, C) \cap R_3(B, D) = B$$

the closure of attribute B is  $B^+ = \{B, C, D\}$

- Attribute 'B' **can not determine** attribute 'A' of sub relation  $R'$ .
- Thus, it is **not a super key** of the sub relation  $R'$ .
- Attribute 'B' **can determine** all the attributes of sub relation  $R_3$ .
- Thus, it is a **super key of the sub relation  $R_3$**

condition-03 satisfies

# Decomposition of a Relation

## Examples Cont'd

•  $R(A, B, C, D)$

FD :  $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow B$

- Determine whether the decomposition of R into  $R_1(A, B)$ ,  $R_2(B, C)$  and  $R_3(B, D)$  is lossless or lossy.

Decomposition of  $R'(A, B, C)$  into  $R_1(A, B)$  and  $R_2(B, C)$ -

Condition-01: According to condition-01, the union of both the sub relations must contain all the attributes of relation  $R'$ . So,

$$R_1(A, B) \cup R_2(B, C) = R'(A, B, C)$$

Clearly, the union of the sub relations contain all the attributes of relation  $R'$ .

Thus, **condition-01 satisfies.**

Condition-02: According to condition-02, intersection of both the sub relations must not be null. So, we have-

$$R_1(A, B) \cap R_2(B, C) = B$$

Clearly,

intersection of the sub-relations is not null. Thus, **condition-02 satisfies.**

# Decomposition of a Relation

## Examples Cont'd

•  $R(A, B, C, D)$

FD :  $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow B$

- Determine whether the decomposition of R into  $R1(A, B)$ ,  $R2(B, C)$  and  $R3(B, D)$  is lossless or lossy.

Decomposition of  $R'(A, B, C)$  into  $R1(A, B)$  and  $R2(B, C)$ -

### Condition-03:

According to condition-03, intersection of both the sub relations must be the super key of one of the two sub-relations or both. So, we have-

$$R1(A, B) \cap R2(B, C) = B$$

Now, the closure of attribute B is  $B^+ = \{B, C, D\}$

- Attribute 'B' can not determine attribute 'A' of sub relation  $R1$ .
- Thus, it is not a super key of the sub relation  $R1$ .
- Attribute 'B' can determine all the attributes of sub relation  $R2$ .
- Thus, it is a super key of the sub relation  $R2$ .

Condition-03 satisfies.  
The decomposition is lossless.

# Decomposition of a Relation

Try this

## EmployeeProjectDetail

Employee_Code	Employee_Name	Employee_Email	Project_Name	Project_ID
101	John	<a href="mailto:john@demo.com">john@demo.com</a>	Project103	P03
101	John	<a href="mailto:john@demo.com">john@demo.com</a>	Project101	P01
102	Ryan	<a href="mailto:ryan@example.com">ryan@example.com</a>	Project102	P02
103	Stephanie	<a href="mailto:stephanie@abc.com">stephanie@abc.com</a>	Project102	P02

## EmployeeProject

Employee_Code	Project_ID	Employee_Name	Employee_Email
101	P03	John	<a href="mailto:john@demo.com">john@demo.com</a>
101	P01	John	<a href="mailto:john@demo.com">john@demo.com</a>
102	P04	Ryan	<a href="mailto:ryan@example.com">ryan@example.com</a>
103	P02	Stephanie	<a href="mailto:stephanie@abc.com">stephanie@abc.com</a>

The **primary key** of the above relation is {**Employee\_Code**, **Project\_ID**}.

## ProjectDetail

Project_ID	Project_Name
P03	Project103
P01	Project101
P04	Project104
P02	Project102

The primary key of the above relation is {**Project\_ID**}.

# Decomposition of a Relation

## Dependency Preservation

### Example:

$R=(A, B, C), F=\{A \rightarrow B, B \rightarrow C\}$

Decomposition of R:  $R_1=(A, C)$   $R_2=(B, C)$

Does this decomposition preserve the given dependencies?

### Solution:

In  $R_1$  the following dependencies hold:  $F_1'=\{A \rightarrow A, C \rightarrow C, A \rightarrow C, AC \rightarrow AC\}$

In  $R_2$  the following dependencies hold:  $F_2'=\{B \rightarrow B, C \rightarrow C, B \rightarrow C, BC \rightarrow BC\}$

The set of nontrivial dependencies hold on  $R_1$  and  $R_2$ :  $F':=\{B \rightarrow C, A \rightarrow C\}$

$A \rightarrow B$  can not be derived from  $F'$ , so this decomposition is NOT dependency preserving.

# Decomposition of a Relation

## Dependency Preservation

### Dependency preservation

#### Example:

$R=(A, B, C), F=\{A \rightarrow B, B \rightarrow C\}$

Decomposition of  $R$ :  $R_1=(A, B)$   $R_2=(B, C)$

Does this decomposition preserve the given dependencies?

#### Solution:

In  $R_1$  the following dependencies hold:  $F_1=\{A \rightarrow B, A \rightarrow A, B \rightarrow B, AB \rightarrow AB\}$

In  $R_2$  the following dependencies hold:  $F_2=\{B \rightarrow B, C \rightarrow C, B \rightarrow C, BC \rightarrow BC\}$

$F' = F_1' \cup F_2' = \{A \rightarrow B, B \rightarrow C, \text{trivial dependencies}\}$

In  $F'$  all the original dependencies occur, so this decomposition preserves dependencies.

# Decomposition of a Relation

## Dependency Preservation

### Example:

$R(A, B, C, D)$ ,  $F = \{A \rightarrow B, B \rightarrow C\}$

Let  $S(A, C)$  be a decomposed relation of  $R$ . What dependencies do hold on  $S$ ?

**Solution:** Need to compute the closure of each subset of  $\{A, C\}$ , wrt  $F^+$

Compute  $\{A\}^+ = \{ABC\}$

- $C$  is in  $S$
- so  $A \rightarrow C$  holds for  $S$

Compute  $\{C\}^+$

- $\{C\}^+ = C$ , no new FD

Compute  $\{AC\}^+$

- $\{AC\}^+ = ABC$ , no new FD

Hence,  $A \rightarrow C$  is the only non-trivial FD for  $S$ ,  $\Pi_S(F^+) = \{A \rightarrow C, + \text{ trivial FDs}\}$

# Decomposition of a Relation

## Dependency Preservation

### Example:

$R(A, B, C, D)$ ,  $F = \{A \rightarrow B, B \rightarrow C\}$

Let  $S(A, C)$  be a decomposed relation of  $R$ . What dependencies do hold on  $S$ ?

**Solution:** Need to compute the closure of each subset of  $\{A, C\}$ , wrt  $F^+$

Compute  $\{A\}^+ = \{ABC\}$

- $C$  is in  $S$
- so  $A \rightarrow C$  holds for  $S$

Compute  $\{C\}^+$

- $\{C\}^+ = C$ , no new FD

Compute  $\{AC\}^+$

- $\{AC\}^+ = ABC$ , no new FD

Hence,  $A \rightarrow C$  is the only non-trivial FD for  $S$ ,  $\Pi_S(F^+) = \{A \rightarrow C, + \text{ trivial FDs}\}$



# Decomposition of a Relation

## Dependency Preservation

### **Example:**

$R(A, B, C, D, E)$ ,  $A \rightarrow D$ ,  $B \rightarrow E$ ,  $DE \rightarrow C$ .

Let  $S(A, B, C)$  be a decomposed relation of  $R$ . What FD-s do hold on  $S$ ?

**Solution:** Need to compute the closure of each subset of  $\{A, B, C\}$

Compute  $\{A\}^+ = AD$ ,  $A \rightarrow D$ , no new FD

Compute  $\{B\}^+ = BE$ , but  $E$  is not in  $S$ , so  $B \rightarrow E$  does not hold

Compute  $\{C\}^+ = C$ , no new FD

Compute  $\{AB\}^+ = ABCDE$ , so  $AB \rightarrow C$  holds for  $S$  ( since  $DE$  are not in  $S$ )

Compute  $\{BC\}^+ = BCE$ , no new FD

Compute  $\{AC\}^+ = ACD$ , no new FD

Compute  $\{ABC\}^+ = ABCDE$ , no new FD

Hence,  $AB \rightarrow C$  is the only nontrivial FD for  $S$ , so  $\Pi_s(F^+) = \{A \rightarrow C, + \text{ trivial FDs}\}$

# Dependency Preservation

## Try

1. R (A, B, C, D) is decomposed into R1(A, B, C), R2(C, D) and  $F = \{B \rightarrow C, AC \rightarrow D\}$ .

What dependencies do hold in R1 and in R2?

Hint: Find the following closures:

$$\{A\}^+ =$$

$$\{B\}^+ =$$

$$\{C\}^+ =$$

$$\{A, B\}^+ =$$

$$\{A, C\}^+ =$$

$$\{A, D\}^+ =$$

$$\{B, C\}^+ =$$

$$\{B, D\}^+ =$$

$$\{C, D\}^+ =$$

$$\{A, B, C\}^+ =$$

$$\{A, B, D\}^+ =$$

$$\{B, C, D\}^+ =$$

$$\{A, C, D\}^+ =$$