Rank correlation coefficient (Spearman)

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

ρ Spearmann rank correlation coefficient

d_i difference between two ranks of each observation

n number of observations

Calculate Spearman Rank correlation coefficient for the following data

English	56	75	45	71	62	64	58	80
Mathematics	66	70	40	60	65	56	59	77

① E	② M	Rank 1	Rank 2	d_i	d_i^2
56	66	7	3	4	16
75	70	2	2	0	0
45	40	8	8	0	0
71	60	3	5	-2	4
62	65	5	4	1	1
64	56	4	7	-3	9
58	59	6	6	0	0
80	77	1	1	0	0
					30

$$\rho = 1 - \frac{6 \times 30}{8(8^2-1)} = 1 - \frac{180}{504}$$

$$= 1 - 0.357$$

1.000
0.357
0.643

Repeated Ranks.

$$\rho = 1 - \frac{6\left(\sum d_i^2 + \frac{m(m^2-1)}{12} + \cdots\right)}{n(n^2-1)}$$

m is number of lines Rank is repeated.

Calculate the Rank correlation for the following data

Expenditure on Advertisement	10	15	14	25	14	14	20	22
Profit	6	25	12	18	25	40	10	7

Sol.

X	Y	Rank 1	Rank 2	d_i	d_i^2
10	6	8	6	0	0
15	25	4	2.5	1.5	2.25
14	12	6	5	1	1
25	18	1	4	−3	9
14	25	6	2.5	3.5	12.25
14	40	6	1	5	25
20	10	3	6	−3	9
22	7	2	7	−5	25
					83.5

$$\gamma = 1 - \frac{6\left(64.5 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12}\right)}{8(8^2-1)}$$

$$= 1 - \frac{6\left(83.5 + \frac{24}{12} + 0.5\right)}{8 \times 63}$$

$$= 1 - \frac{6(86)}{8 \times 63} = 1 - \frac{516}{504}$$

$$= 1 - 1.0238 = -0.0357$$

$\frac{1+2}{2}$

$\frac{5+6+7}{3} = 16$

$\frac{357.25}{2.25}$

$\frac{83.5}{}$

$\frac{50.5}{34}$

$\frac{84.5}{}$

$\gamma = 1 - 1.024 = -0.024$