Continuous Probability Distributions

- **Continuous Random Variable:**

  Values from Interval of Numbers

  Absence of Gaps

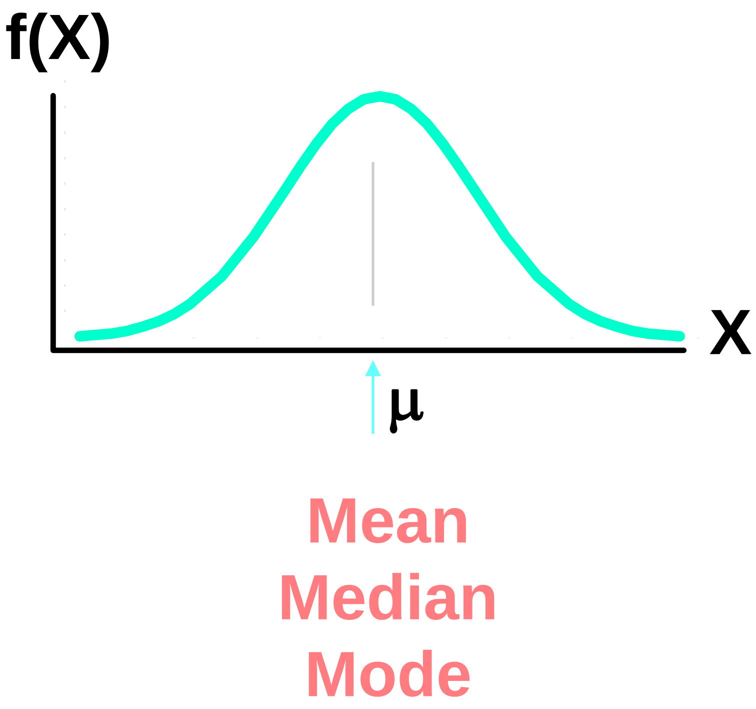
- **Continuous Probability Distribution:**

  Distribution of a Continuous Variable

- **Most Important Continuous Probability Distribution: the Normal Distribution**

The Normal Distribution

- ‘Bell Shaped’
- Symmetrical
- Mean, Median and
- Mode are Equal
- Random Variable has
- Infinite Range

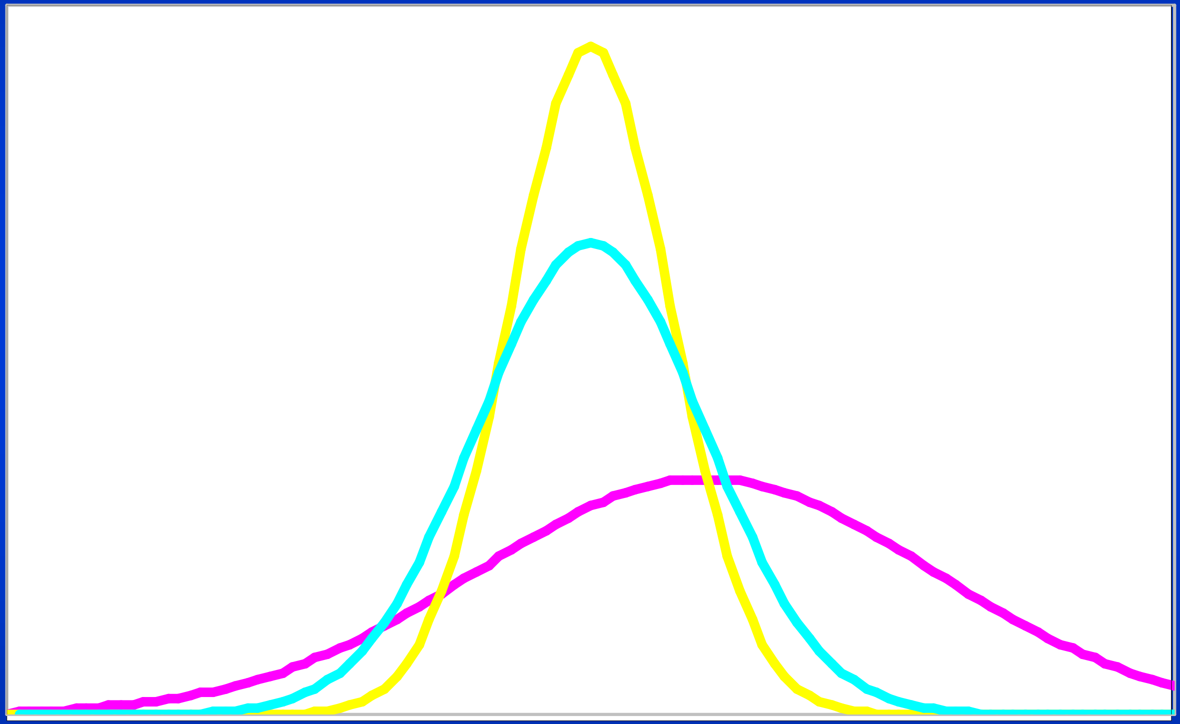


The Mathematical Model

$$f(X) = \frac{1}{\sqrt{2\pi\sigma}}\ e^{(-1/2)((X-\mu)/\sigma)^2}$$

$f(X)$	=	frequency of random variable X
π	=	3.14159; e = 2.71828
σ	=	population standard deviation
X	=	value of random variable ($-\infty < X < \infty$)
μ	=	population mean

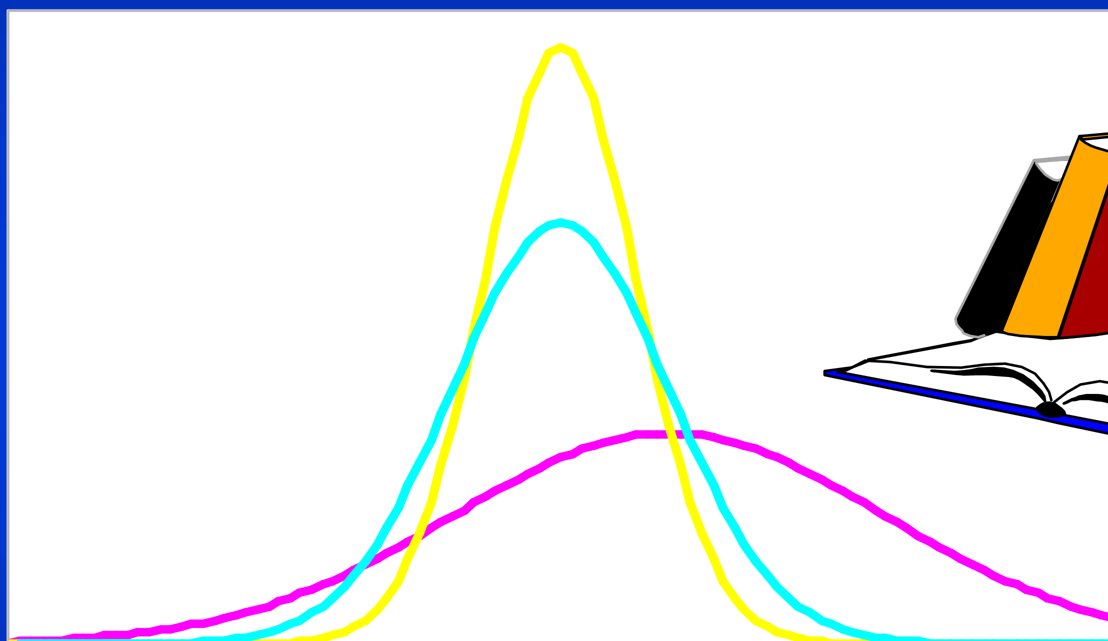
Many Normal Distributions



There are an Infinite Number

Varying the Parameters σ and μ, we obtain Different Normal Distributions.

Which Table?



Each distribution
has its own table?

Infinitely Many Normal Distributions Means
Infinitely Many Tables to Look Up!

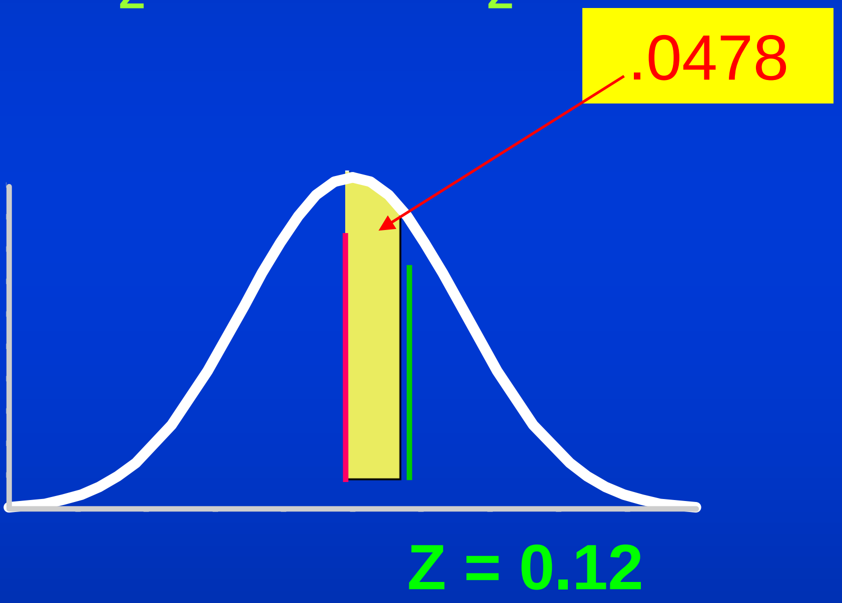
The Standardized Normal Distribution

Standardized Normal Probability Table (Portion)

$\mu_Z = 0$ and $\sigma_Z = 1$

Z	.00	.01	.02
0.0	.0000	.0040	.0080
0.1	.0398	.0438	.0478
0.2	.0793	.0832	.0871
0.3	.0179	.0217	.0255

Probabilities



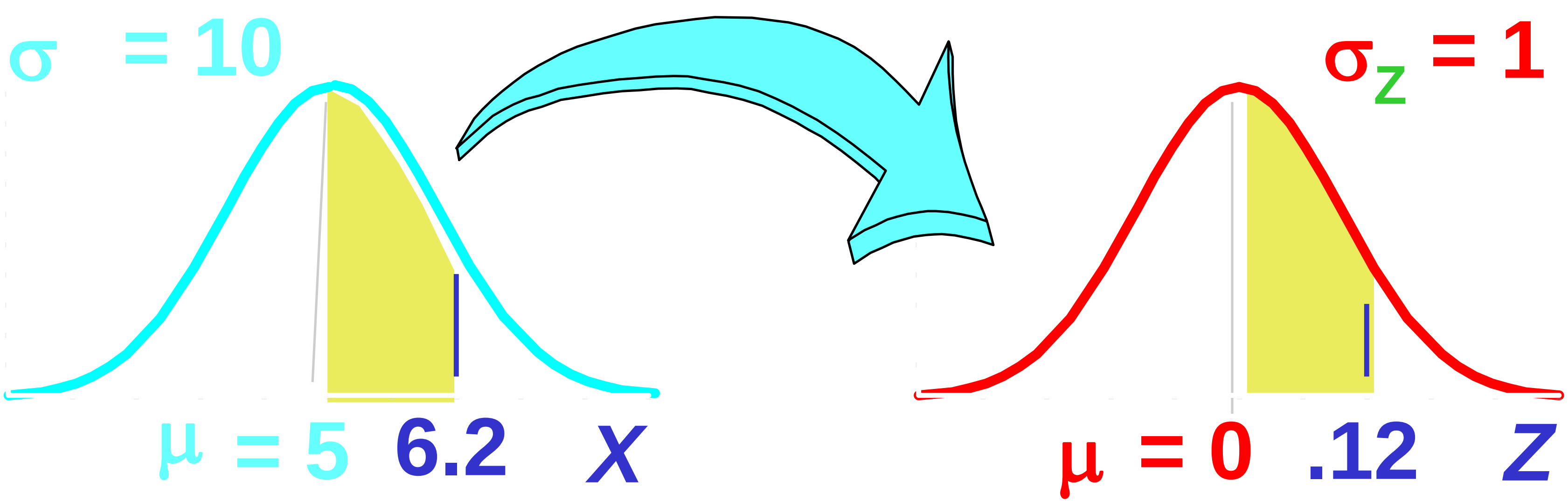
Shaded Area Exaggerated

Standardizing Example

$$Z = \frac{X - \mu}{\sigma} = \frac{6.2 - 5}{10} = 0.12$$

Normal Distribution

Standardized Normal Distribution



Shaded Area Exaggerated

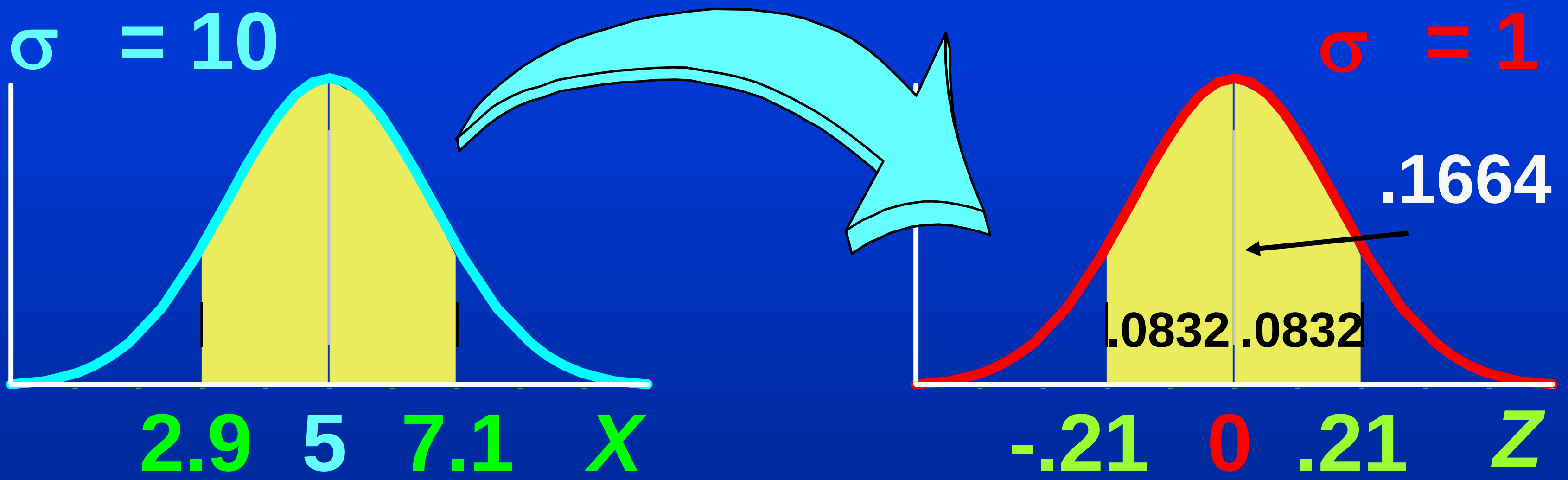
Example:
P(2.9 < *X* < 7.1) = .1664

$$z = \frac{x-\mu}{\sigma} = \frac{2.9-5}{10} = -.21$$

$$z = \frac{x-\mu}{\sigma} = \frac{7.1-5}{10} = .21$$

Normal Distribution

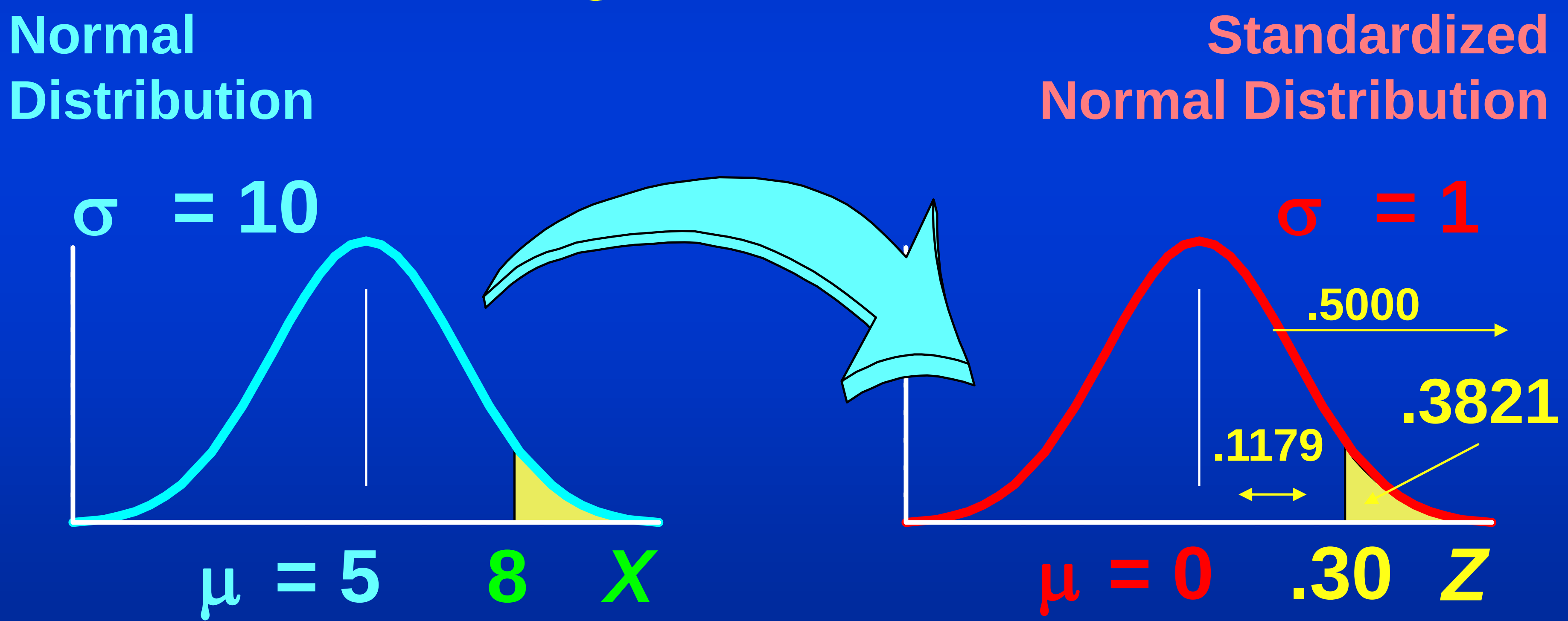
Standardized Normal Distribution



Shaded Area Exaggerated

Example: $P(X \geq 8) = .3821$

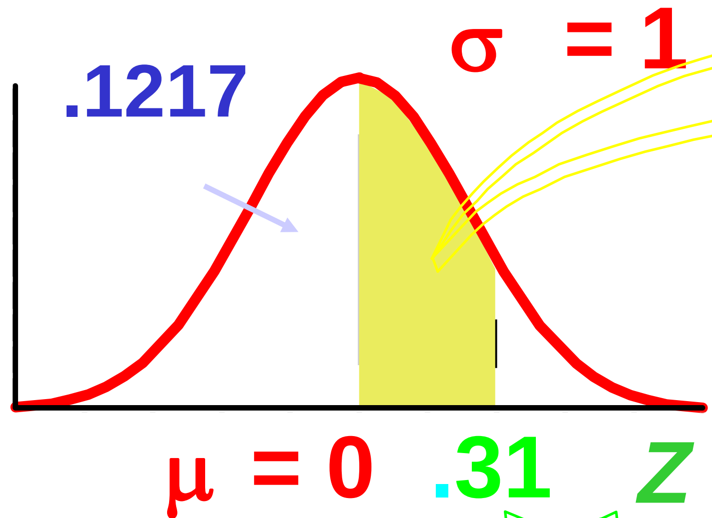
$$z = \frac{x-\mu}{\sigma} = \frac{8-5}{10} = .30$$



Shaded Area Exaggerated

Finding Z Values for Known Probabilities

What Is *Z* Given *P*(*Z*) = 0.1217?

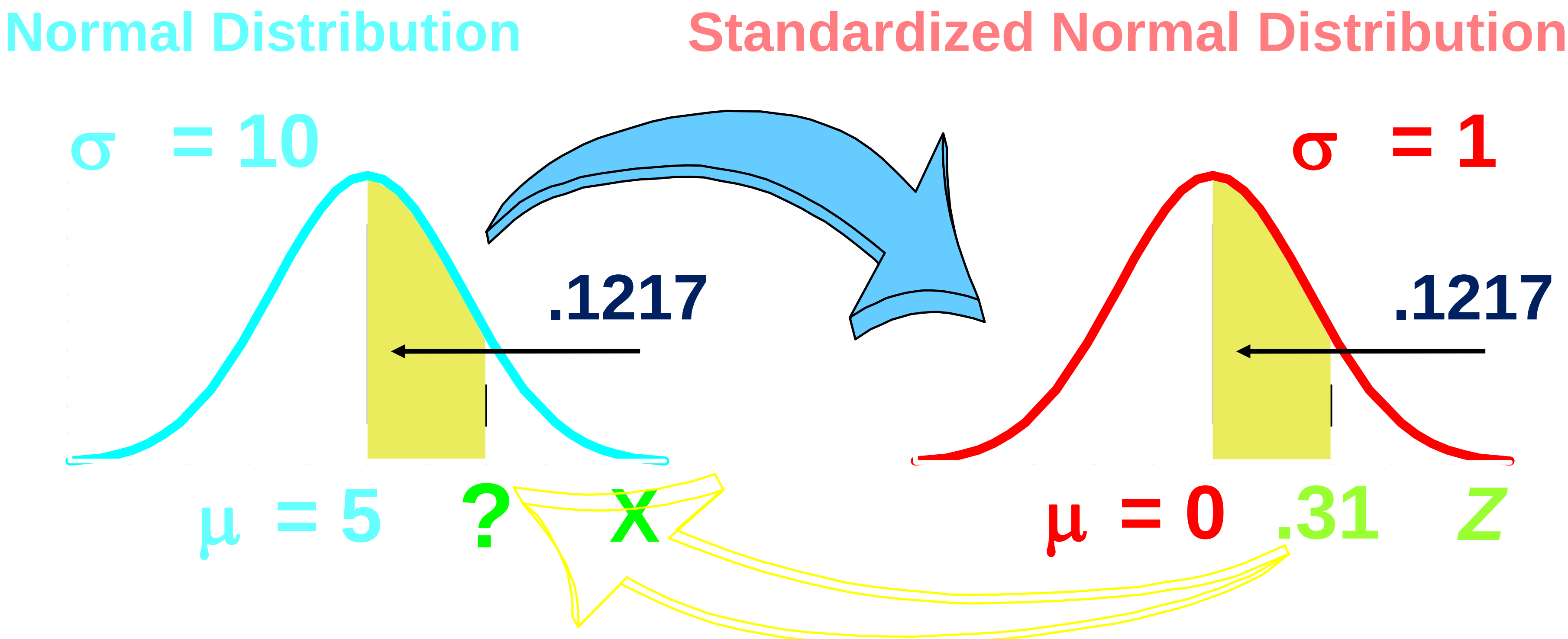


Shaded Area Exaggerated

Standardized Normal Probability Table (Portion)

Z	.00	.01	0.2
0.0	.0000	.0040	.0080
0.1	.0398	.0438	.0478
0.2	.0793	.0832	.0871
0.3	.1179	.1217	.1255

Finding *X* Values for Known Probabilities



$$X = \mu + Z\sigma = 5 + (0.31)(10) = 8.1$$

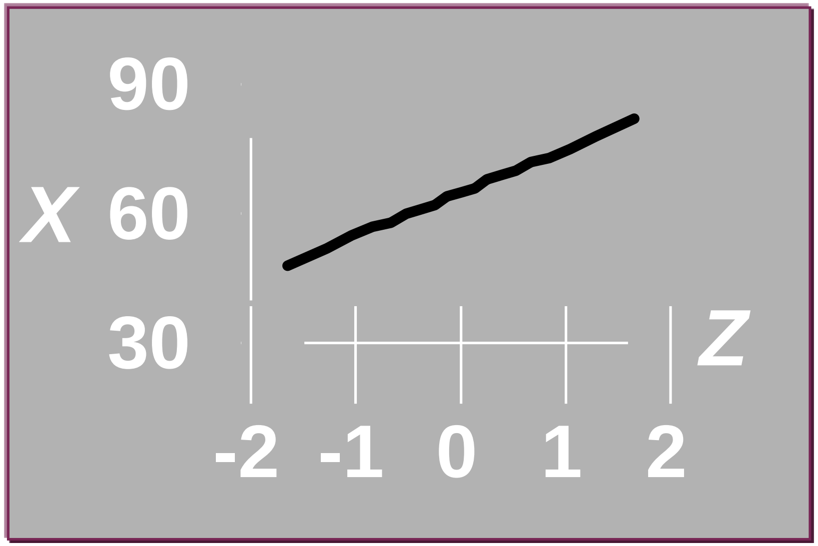
Shaded Area Exaggerated

Assessing Normality

- Compare Data Characteristics
- to Properties of Normal
- Distribution

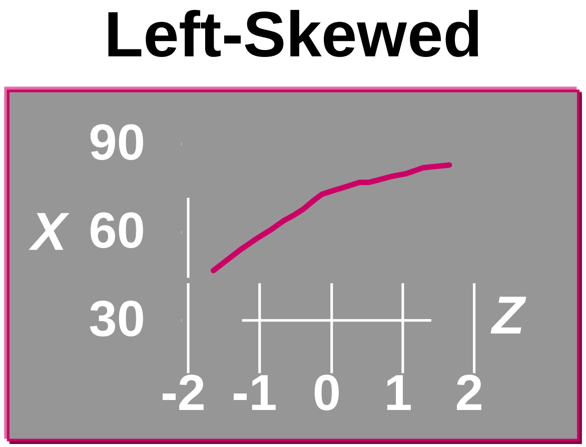
- Put Data into Ordered Array
- Find Corresponding Standard
- Normal Quantile Values
- Plot Pairs of Points
- Assess by Line Shape

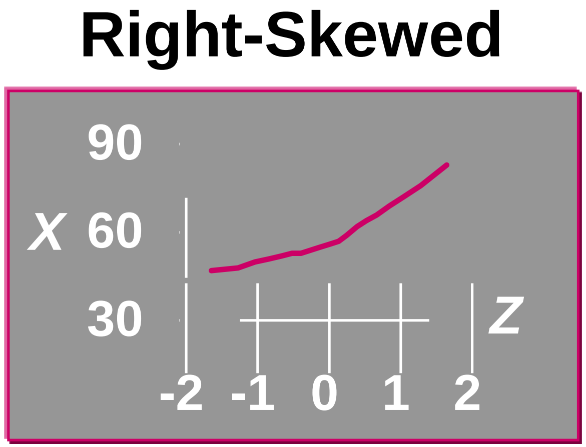
Normal Probability Plot for Normal Distribution

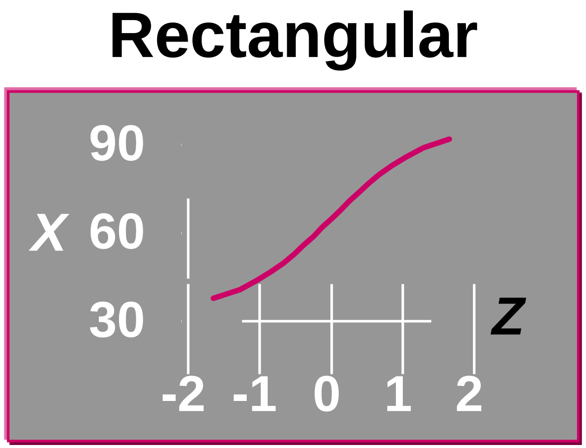


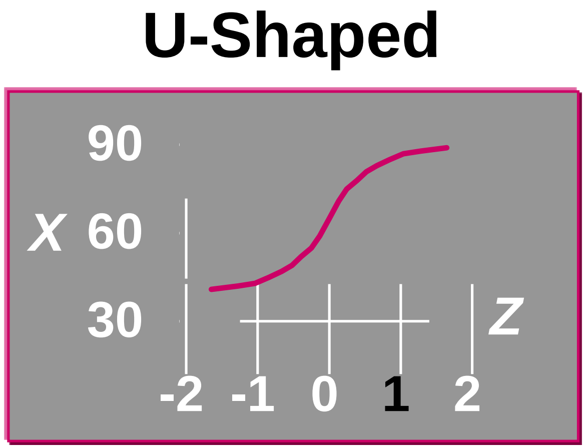
Look for Straight Line!

Normal Probability Plots









Estimation

- **Sample Statistic Estimates Population Parameter**
- **e.g. $\bar{X}$ = 50 estimates Population Mean, μ**
- **Problems: Many samples provide many estimates of the Population Parameter.**
  - **Determining adequate sample size: large sample give better estimates. Large samples more costly.**
- **How good is the estimate?**
- **Approach to Solution: Theoretical Basis is Sampling Distribution.**

Properties of Summary Measures

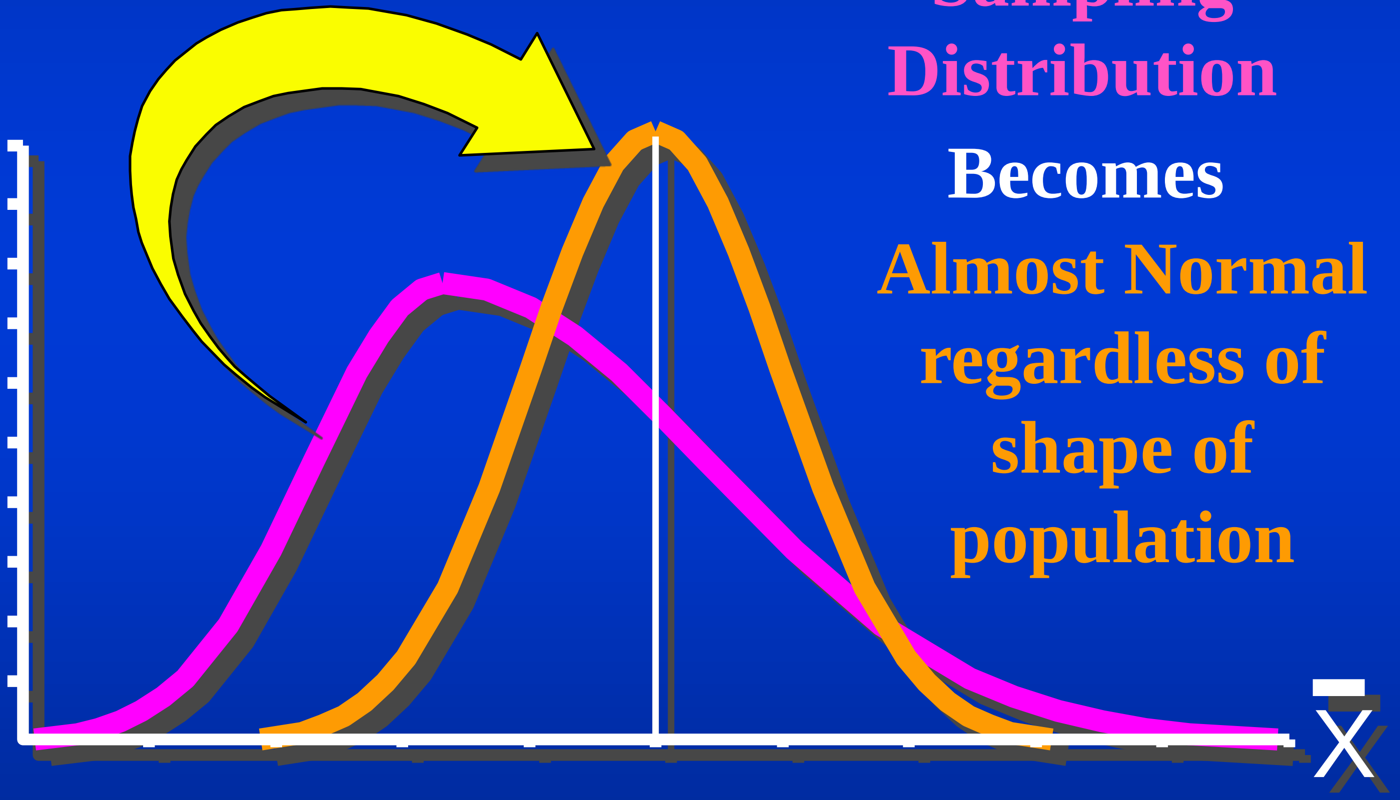
- Population Mean Equal to
- Sampling Mean $\boxed{\mu_{\bar{x}} = \mu}$
- The Standard Error (standard deviation) of the Sampling distribution is Less than Population Standard Deviation
- Formula (sampling with replacement):

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

As n increase, $\sigma_{\bar{x}}$ decrease.

Central Limit Theorem

As Sample Size Gets Large Enough



Sampling Distribution Becomes Almost Normal regardless of shape of population

Population Proportions

- Categorical variable (e.g., gender)
- % population having a characteristic
- If two outcomes, binomial distribution
  - Possess or don’t possess characteristic
- Sample proportion (p_s)

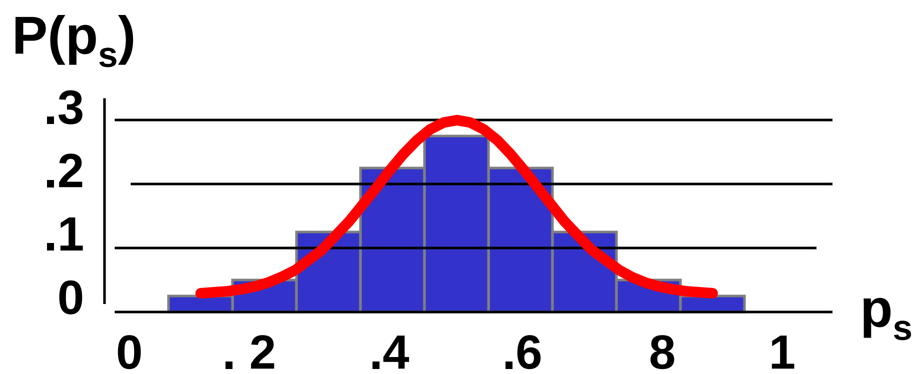
$$P_s = \frac{X}{n} = \frac{\text{number of successes}}{\text{sample size}}$$

Sampling Distribution of Proportion

- Approximated by normal distribution
  - $n \cdot p \geq 5$
  - $n \cdot (1 - p) \geq 5$
- Mean $\mu_P = p$
- Standard error

$$\sigma_P = \sqrt{\frac{p \cdot (1 - p)}{n}}$$

Sampling Distribution



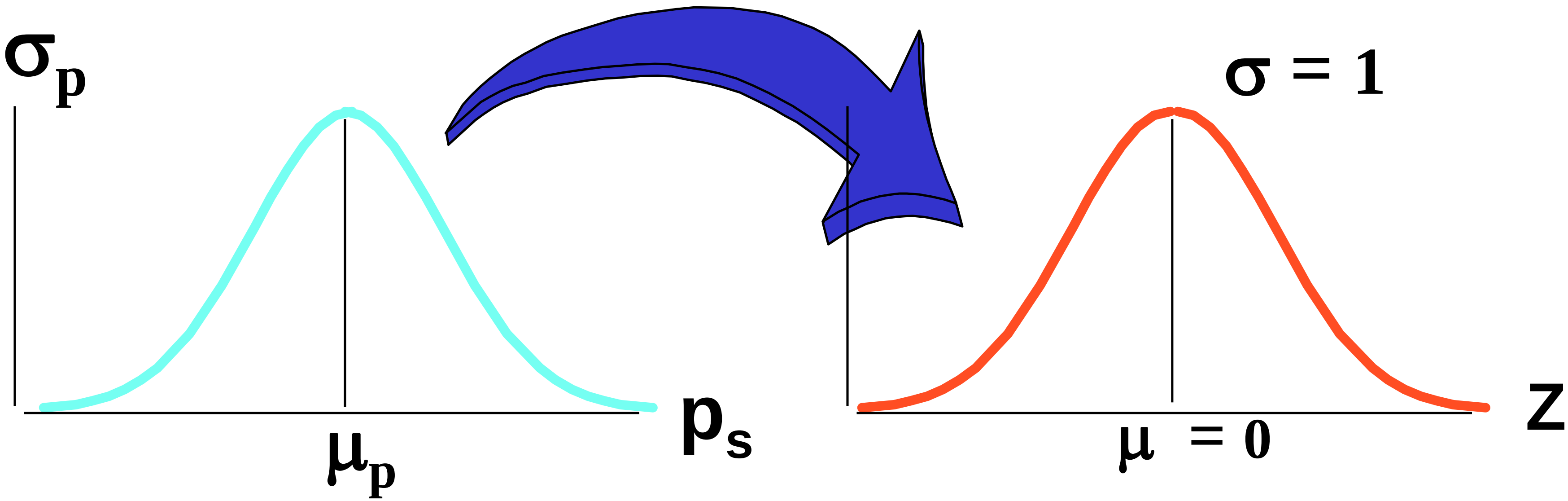
p = *population proportion*

Standardizing Sampling Distribution of Proportion

$$Z \cong \frac{p_s - \mu_p}{\sigma_p} = \frac{p_s - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Sampling Distribution

Standardized Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177

1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974

2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

The mean weight of 500 male students in a certain college is 151 lb. and the standard deviation is 15 lb. Assuming the weights are normally distributed, find how many students weigh (a) between 119.5 and 155.5 lb. and (b) more than 185 lb.

The results of a particular examination are given below. It is known that a candidate gets plucked if he obtains less than 40 marks (out of 100) while he must obtain at least 75 marks in order to pass with distinction. Determine the mean and standard deviation of the distribution of marks assuming this to be normal.

Results	% of candidates
Passed with distinction	**10**
Passed	**60**
Failed	**30**