

Binomial Random Variable

A variable X is said to be Binomial if ① it is discrete,

② Sample size is fixed (no. of trials fixed)

③ on each trial, event is a success or failure.

④ trials are independent

⑤ probability of occurrence is the same on each trial.

pmf

$$P(X=x) = {}^n C_x p^x q^{n-x}$$

$x = 0, 1, 2, \dots, n$

where $q = 1 - p$

p is probability of success

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Example of 20% of the

bolts produced by a machine
are found to be defective.

Determine the probability that

out of 4 bolts chosen at

random (a) one (b) zero

(c) at most two will be

defective.

$n = 4 =$ no. of bolts (fixed)

X : no. of bolts ~~are~~ that
will be defective.

$p = \text{probability of defective bolts}$

$$= \frac{20}{100} = \underline{0.2}$$

$$q = 0.8 = 1 - 0.2$$

(a)

$$P(X=1) = {}^4C_1 (0.2)^1 (0.8)^3$$

$$= 4 (0.2) (0.8)^3$$

$$= \underline{0.256}$$

$$(b) P(X=0) = {}^4C_0 (0.2)^0 (0.8)^4$$

$$= (0.8)^4$$

(c)

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

DECEMBER • Saturday

(361-005) WK 52

$$= 4C_0 (0.2)^0 (0.8)^4 + 4C_1 (0.2)^1 (0.8)^3$$

$$+ 4C_2 (0.2)^2 (0.8)^2$$

X is Binomial R.V.

$$\text{Mean} = E(X)$$

$$=$$

$$= \sum x P(X=x)$$

$$E(X) = \sum_{x=0}^n x \cdot {}^nC_x p^x q^{n-x}$$

$$= np.$$

$$\frac{{}^nC_x}{\sum_{x=0}^n {}^nC_x} = \frac{{}^nC_x}{2^n}$$

$$\text{Variance}(X) = npq$$

$$S.D.(X) = \sqrt{npq}$$

27 Sunday

M	T	W	T	F	S	S
30						
2	3	4	5	6	7	1
9	10	11	12	13	14	8
16	17	18	19	20	21	15
23	24	25	26	27	28	22
						29

Poisson random Variable

Happening of an event a certain number of times in a given interval.

pmf for poisson distribution

$$P(X=x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!} \quad x=0,1,2,\dots$$

factorial $\leftarrow x!$

λ is average value per unit time
 number of times event occurs.

22 23 24 25 26 27 28 29 30 31
DECEMBER
50
WK 01 (365-000)

(*) An average of 0.61 soldiers died by horse kicks per year in each Prussian army corps. you want to calculate the probability that exactly two soldiers died VII army Corps 1898.

$$\lambda = 0.61 \quad x =$$

X : Number of soldiers died in an army Corps.

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{L^x}$$

$$P(X=2) = \frac{e^{-\lambda} \lambda^2}{L^2}$$

2020

No. 222
Date 1/11/20

$$P(X=2) = \frac{e^{-0.61} (0.61)^2}{L_2}$$

$$= 0.101$$

X : y poisson random variable

$$\text{Mean } E(X) = \lambda = np.$$

$$\text{variance } \text{Var}(X) = \lambda = np.$$

(*) Suppose that, on average,

1 person in 1000 makes a

numerical error in preparing

his or her income tax return.

If 10,000 returns are selected

at random and examined.

Find the probability that

6, ~~7~~ or 8 of them contain error.

Sol.

X : no. of persons
making numerical
error.

$$n = 10000$$

$$p = \frac{1}{1000}$$

$$\lambda = np = 10000 \times \frac{1}{1000} = 10$$

$$\lambda = 10$$

$$P(\cancel{6} \leq X \leq 8)$$

$$= P(X=6) + P(X=7) + P(X=8)$$

$$= \frac{e^{-10} 10^6}{6!} + \frac{e^{-10} 10^7}{7!} + \frac{e^{-10} 10^8}{8!}$$



⑦ on average, a text book author makes two word processing errors per page on the first draft of her text book. what is the probability that on the next page she will make

- Ⓐ 4 or more errors
- Ⓑ no errors
- Ⓒ fewer than 3 errors
- Ⓓ at least 2 errors.

⑧ A scientist inoculates mice, one at a time, with a disease germ until he finds 2 that have contracted the disease.

of the probability of contracting
the disease is $\frac{1}{6}$. what is
the probability that 8 mice
are required?