Testing of Hypothesis

Statistical hypothesis is an assertion or conjecture concerning one or more populations.

A random sample from the population of interest and use this data contained in the sample to provide evidence that either supports or does not support the hypothesis.

Rejection means there is a small probability of obtaining information observed when the hypothesis is true

Null hypothesis: is ~~used~~ no statistical significance exists. in a given set of observations.

It is denoted by H_0.

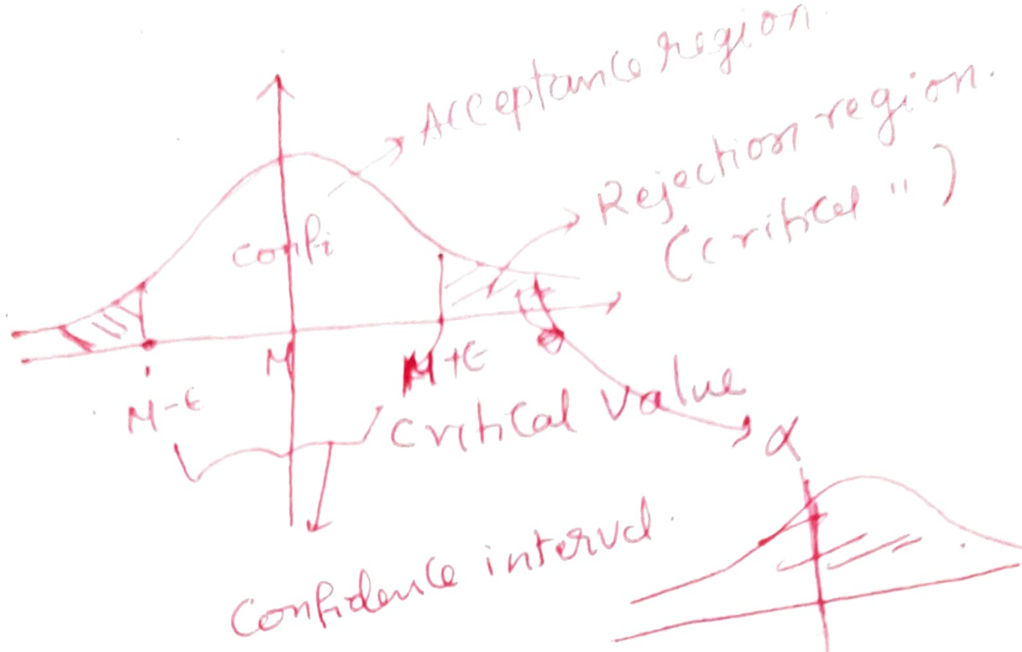
Alternate hypothesis proposes there is a significant difference.

It is denoted by H_1.

Test statistic is a static used in hypothesis testing.

- Critical region (Rejection region) is a set of values ~~of values~~ for the test statistic for which null hypothesis is rejected.

Confidence interval (Acceptance region) is a set of values for the test statistic for which null hypothesis is accepted.



Type 1 error

Rejection of null-hypothesis when Ho is true

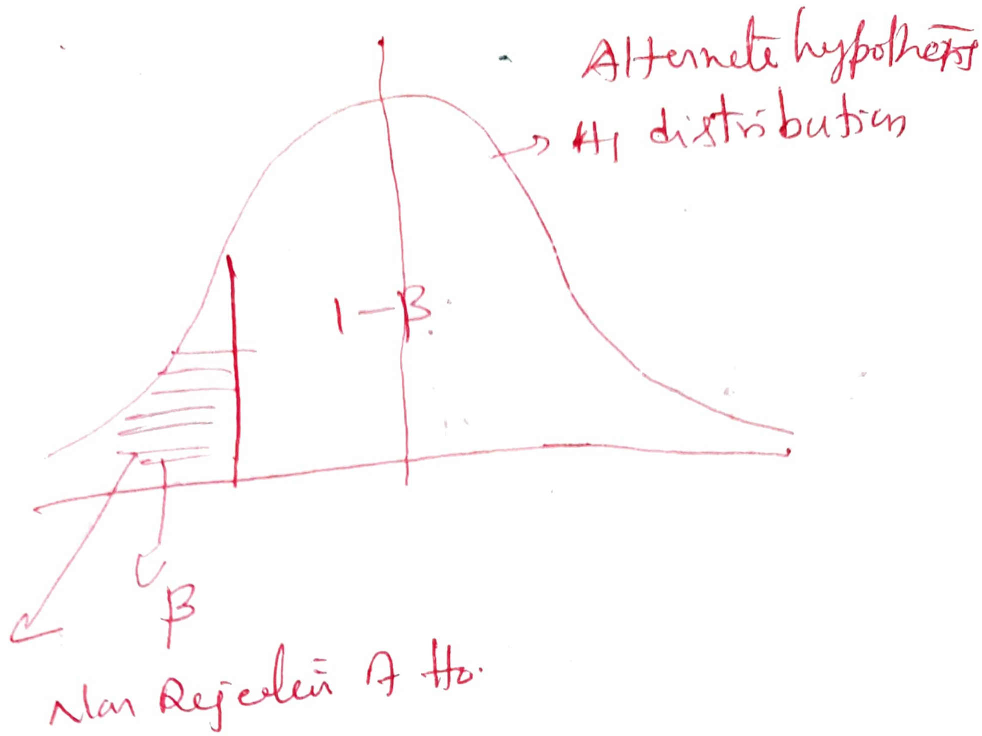
probability of making type 1 error.

$$\alpha = P(\text{Type 1 error})$$

Type 2 error

Failing to reject Ho when Ho is actually false.

$$\beta = P(\text{Type 2 error})$$



A man goes to trial where he is being tried for the murder of his wife. We can put it in a hypothesis testing framework. The hypothesis being tested are

H_0: Not guilty.

H_a: Guilty.

Type 1 error is committed if we reject H_0 when H_0 is true.

Did not kill his wife but was found guilty and is punished for a crime he did not really commit

Type 2 error committed

Did kill his wife but was found not guilty and was not punished.

Test on Single proportion

Test statistic

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

n: sample size.

p: population proportion

$\hat{p}$: sample proportion

when students are asked to pick a number at random from one to twenty. I suspect their selection will show bias in favor of the number seventeen.

In a group of 371 students, 25 choose the number 17. Does this provide convincing statistical evidence of bias in favor of the number 17, in that proportion of students picking 17 is significantly higher than $\frac{1}{20} = 0.05$?

$H_0: p = p_0 = 0.05.$

$H_1: p > p_0.$

$\hat{p} = \frac{25}{371}$

$= 0.067$

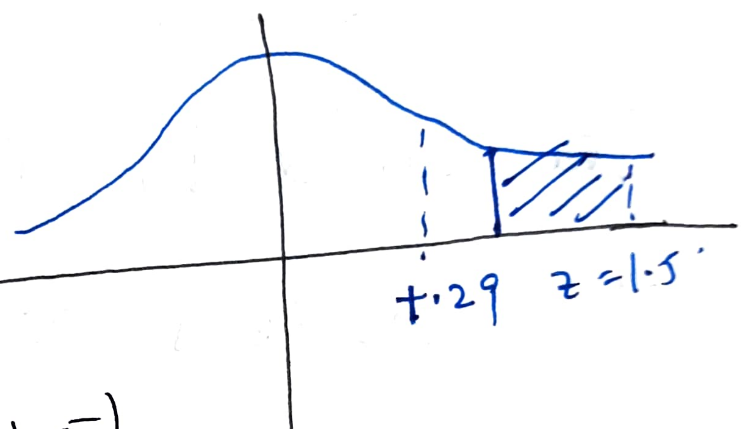
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$Z = \frac{0.067 - 0.05}{\sqrt{\frac{(0.05)(0.05)}{371}}}$$

$$= 1.5$$

P-Value

$= P(Z \geq 1.5)$



$= 1 - P(Z \leq 1.5)$

$= 0.0668$

P-Value $< \alpha$ $\Rightarrow$ Then reject H_0.

~~when $\boxed{\alpha = 0.01}$ Then Reject H_0.~~

Probability of Committing type I error

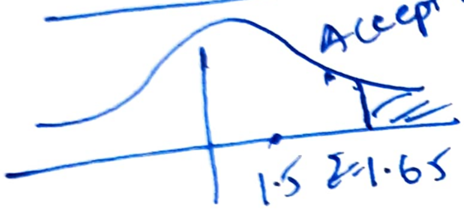
$= \boxed{\alpha > 0.0668}$.

P-value > α

Accept Ho.

$\alpha = 0.05$

Alternative



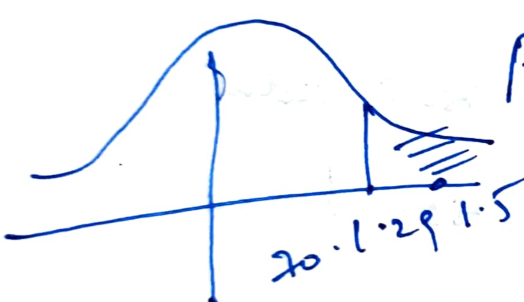
$P(Z < z) = 0.95$

$\boxed{z_0 = 1.65}$

0.9

~~$P(Z < z) = 1.65$~~

$P(Z < z) = 0.9$.



$z_0 = 1.29$.

$> Z \cancel{>} z_0$

1.65 1.5 1.29

Reject Ho.

Test on Difference of proportions

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{y_1 + y_2}{n_1 + n_2}.$$

Prob

Time magazine reported the result of a telephone poll of 800 adult Americans. The question posed of the Americans who were surveyed was "should the federal tax on cigarettes be raised to pay for health care reform?

The results of the Survey were:

Non-Smokers	Smokers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said 'yes'	$y_2 = 41$ said 'yes'
$\hat{p}_1 = \frac{351}{605} = 0.51$	$\hat{p}_2 = \frac{41}{195} = 0.21$.

Is there sufficient evidence at the $\alpha = 0.05$, say, to conclude that the two ~~proportions~~ populations – smokers and non-smokers – differ significantly wrt to their opinions?

Ans:

$$\hat{p} = \frac{41+351}{605+195} = \frac{392}{800}$$

$$= 0.49$$

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

$$Z = \frac{(0.58-0.21)-0}{\sqrt{(0.49)(0.51)\left(\frac{1}{195}+\frac{1}{605}\right)}}$$

$$= 8.99.$$

$$p\text{-value} = P(Z \geq \cancel{8})$$

$$= P(Z \geq 8.99)$$

$$= 1 - P(Z \leq 8.99)$$

$$= 1 - 1 = 0.$$

$$p\text{-value } \cancel{\leq}\ 0 < 0.05 = \alpha$$

Reject H_0.

Yes there is a significant difference in their opinions

Alt: $P(Z < z) = \cancel{0}\,0.95$ $\alpha = 0.05$.

$z = 1.65$

Reject H_0.