Tutorial for CAT-2

11-09-2019

An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu /= 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a P-value in your answer.

$\star$ Let's notice that the level of significance is not given, so let it be $\alpha = 0.05$

- The manufacturer claim that the lifetime of light bulbs is 800 hours.

Hence, the null hypothesis is:

$$H_0 : \mu = 800$$

The alternative hypothesis is:

$$H_1 : \mu \neq 800$$

The sample size is $n = 30$;
The sample mean is $\overline{x} = 788$;
The population standard deviation is $\sigma = 40$.

We will reject the null hypothesis if P-value is less than 0.05

$\star$ Lets find the z- value:

$$\begin{aligned} z &= \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \\ &= \frac{788 - 800}{40/\sqrt{30}} \\ &\approx -1.64 \end{aligned}$$

The test is two-tailed, so the P-value is:

$$\begin{aligned} P &= 2P(Z \geq |z|) \\ &= 2P(Z \geq |-1.64|) \\ &= 2P(Z \leq -1.64) \end{aligned}$$

Using Normal Table, we get:

$$\begin{aligned} P &= 2 \cdot 0.05 \\ &= 0.1 \end{aligned}$$

- P-value of the test is 0.1, which is greater than the assumed level of significance 0.05.

Hence, we will not reject the null hypothesis.

- Therefore, we can conclude that the average lifetime of each electric bulb is 800 hours.

In the American Heart Association journal Hypertension, researchers report that individuals who practice Transcendental Meditation (TM) lower their blood pressure significantly. If a random sample of 225 male TM practitioners meditate for 8.5 hours per week with a standard deviation of 2.25 hours, does that suggest that, on average, men who use TM meditate more than 8 hours per week? Quote a P-value in your conclusion.

$\star$ Let's notice that the level of significance is not given, so let it be $\alpha = 0.05$.

Let μ denotes the true population mean of meditation time for male.

- Number of males in the sample $n = 225$
- Mean is $\overline{x} = 8.5$
- Standard deviation is $s = 2.25$

The null hypothesis is:

$$H_0 : \mu = 8$$

The alternative hypothesis is:

$$H_1 : \mu > 8$$

- Lets find the corespodending z-value:

$$
\begin{aligned}
z &= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \\
&= \frac{8.5 - 8}{2.25/\sqrt{225}} \\
&= \frac{0.5}{0.15} \\
&\approx 3.33
\end{aligned}
$$

- Using the Normal Table, we can find the P-value as folows:

$$
\begin{aligned}
P &= P(Z > z) \\
&= 1 - P(Z \leq 3.33) \\
&= 1 - 0.9996 \\
&= 0.0004
\end{aligned}
$$

We should reject the null hypothesis if the P-value is less than the significance level.

In our case, $P = 0.0004 < 0.05 =$ level of significance, so we will reject the null hypothesis and conclude that the mean meditation time for male is more than 8 hours/week.

The mean meditation time for male is more than 8 hours/week.

A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\bar{x}_2 = 76$. Test the hypothesis that $\mu_1 = \mu_2$ against the alternative, $\mu_1 \neq \mu_2$. Quote a P-value in your conclusion.

We have:
$n_1 = 25$,
$\sigma_1 = 5.2$,
$\overline{x}_1 = 81$
$n_2 = 36$,
$\sigma_2 = 3.4$,
$\overline{x}_2 = 76$

The null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

The alternative hypothesis is:

$$H_1 : \mu_1 \neq \mu_2$$

Therefore, we have

$$H_0 : \mu_1 - \mu_2 = 0$$

And

$$H_1 : \mu_1 - \mu_2 \neq 0$$

We will use the level of significance $\alpha = 0.05$

- Test statistic is:

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - d_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Hence,

$$\begin{aligned} Z &= \frac{(81 - 76) - 0}{\sqrt{\dfrac{(5.2)^2}{25} + \dfrac{(3.4)^2}{36}}} \\ &= \frac{5}{\sqrt{1.0816 + 0.3211}} \\ &= 4.22 \end{aligned}$$

$\star$ The critical region is $Z < -z_{0.025}$ or $Z > z_{0.025}$

Using z-Table,we see that $z_{0.025} = 1.95996$

Because Z=4.22 (which is larger than $z_{0.025}$), we see that we should reject H_0.

Lets now find the P-value. We have:

$$\begin{aligned}
P &= P(|Z| > 4.22) \\
&= P(Z > 4.22) + P(Z < -4.22) \\
&= 1 - P(Z < 4.22) + P(Z < -4.22) \\
&= 2P(Z < -4.22) \\
&= 2 \cdot 0.000012 \\
&= 0.000024 \\
&\approx 0
\end{aligned}$$

Because P value is smaller than the level of significance, we conclude that we should reject H_0.

We should reject the null hypothesis.

A manufacturer claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 kilograms. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with a standard deviation of 6.28 kilograms, while type B thread had an average tensile strength of 77.8 kilograms with a standard deviation of 5.61 kilograms. Test the manufacturer's claim using a 0.05 level of significance.

$\star$ We have:

$$n_A = 50$$
$$n_B = 50$$
$$\overline{x}_A = 86.7 \text{ kg}$$
$$\overline{x}_B = 77.8 \text{ kg}$$
$$s_A = 6.28$$
$$s_B = 5.61$$

Because the sample size is greater than 30, we can use that $s_A = \sigma_A$ and $s_B = \sigma_B$, i.e. $\sigma_A = 6.28$ and $\sigma_B = 5.61$

- Let μ_1 denotes the population average tensile strength of thread A.
- Let μ_2 denotes the population average tensile strength of thread B.

The null hypothesis is

$$H_0 : \mu_A - \mu_B \leq 12$$

The alternative hypothesis is

$$H_1 : \mu_A - \mu_B > 12$$

We will use the level of significance of $\alpha = 0.05$.
From the normal distribution table, the critical value at 0.05 level for right-tailed is 1.645.

- We will reject the null hypothesis if $z > 1.645$

Lets find z:

$$z = \frac{(\overline{x}_A - \overline{x}_B) - d_0}{\sqrt{\frac{\sigma_A^2}{n_1} + \frac{\sigma_B^2}{n_2}}}$$

Hence,

$$
\begin{aligned}
z &= \frac{(\overline{x}_A - \overline{x}_B) - d_0}{\sqrt{\dfrac{\sigma_A^2}{n_1} + \dfrac{\sigma_B^2}{n_2}}} \\
&= \frac{(86.7 - 77.8) - 12}{\sqrt{\dfrac{(6.28)^2}{50} + \dfrac{(5.61)^2}{50}}} \\
&= \frac{-3.1}{1.190886} \\
&= -2.60
\end{aligned}
$$

Because $z = -2.60$ is less than the critical value 1.645, we fail to reject the null hypothesis.

Therefore, we conclude that the difference of average tensile strength of thread A and thread B is less than 12 kilograms.

The difference of average tensile strength of thread A and thread B is less than 12 kilograms.

For the two-sided hypothesis

$$H_0\colon\ \mu = \mu_0,$$
$$H_1\colon\ \mu \neq \mu_0,$$

we reject H_0 at significance level α when the computed t-statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

exceeds $t_{\alpha/2,n-1}$ or is less than $-t_{\alpha/2,n-1}$.

The reader should recall from Chapters 8 and 9 that the t-distribution is symmetric around the value zero. Thus, this two-tailed critical region applies in a fashion similar to that for the case of known σ. For the two-sided hypothesis at significance level α, the two-tailed critical regions apply. For H_1: $\mu > \mu_0$, rejection results when $t > t_{\alpha,n-1}$. For H_1: $\mu < \mu_0$, the critical region is given by $t < -t_{\alpha,n-1}$.

The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal.

For the two-sided hypothesis

$$H_0\colon\ \mu_1 = \mu_2,$$
$$H_1\colon\ \mu_1 \neq \mu_2,$$

we reject H_0 at significance level α when the computed t-statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{1/n_1 + 1/n_2}},$$

where

$$s_p^2 = \frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

exceeds $t_{\alpha/2,n_1+n_2-2}$ or is less than $-t_{\alpha/2,n_1+n_2-2}$.

Recall from Chapter 9 that the degrees of freedom for the t-distribution are a result of pooling of information from the two samples to estimate σ^2. One-sided alternatives suggest one-sided critical regions, as one might expect. For example, for H_1: $\mu_1 - \mu_2 > d_0$, reject H_1: $\mu_1 - \mu_2 = d_0$ when $t > t_{\alpha,n_1+n_2-2}$.

An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 with a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume the populations to be approximately normal with equal variances.