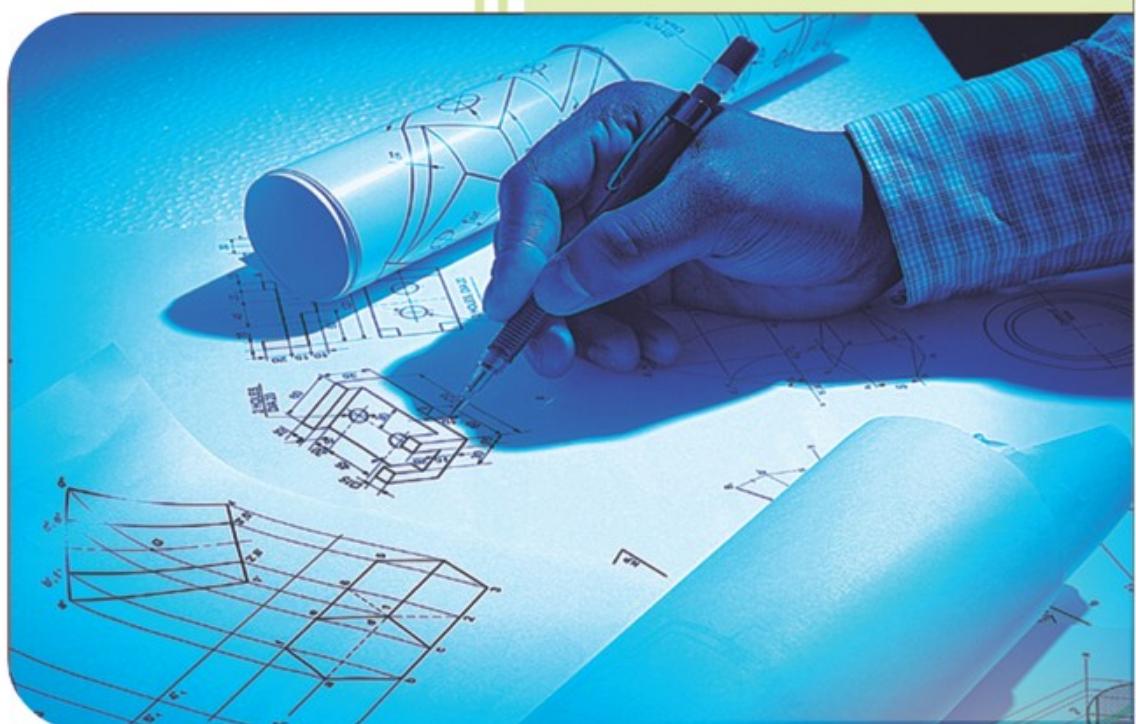


3rd Edition

# Engineering Drawing

K.L. Narayana ~ P. Kannaiah



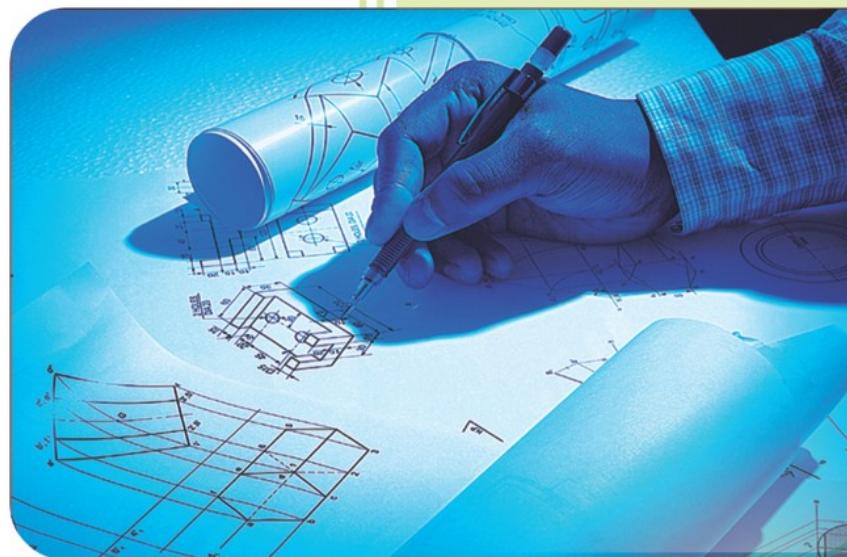
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# **ENGINEERING DRAWING**

**Third Edition**

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# **ENGINEERING DRAWING**

**Third Edition**

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# **Foreword**

One of the major items that we are still importing are engineering text books-text books even in basic engineering subjects. In advanced countries there are as many texts in basic engineering subjects as there are Universities — engineering drawing, thermodynamics and fluid mechanics are some of the subjects in which there are a considerable number of texts available for students to choose from.

In India we have very few good texts for engineering students even in basic subjects. I am very happy that Dr. Narayana and Sri Kannaiah have come forward and have prepared a text book on engineering drawing. The enormous amount of work involved in preparing such books is something that readers are well aware of.

It is my considered view that no engineer can be successful unless he has good grounding in engineering drawing. In this context, I find that this text has been very carefully prepared and will provide good grounding for all engineers. I am also sure that the publication of this text book will enrich all our future engineers.

I congratulate Dr. Narayana and Sri Kannaiah for having made a good contribution towards the development of engineering education.

**G R Damodaran**

*Director*

**PSG Institutions**

# Preface

Engineering drawing is said to be the language of an engineer, and it can be considered as a powerful tool to convey his ideas. Infact, there are evidences that pictorial language compared to other languages, dates back to very old times in its invention and usage. There are certain areas where no language other than engineering drawing can play the vital role of communication.

The subject of engineering drawing, therefore, included in all engineering curricula with the aim of training the students and making them graphically literate. Attempts are made sincerely to serve this purpose by adopting a new style in the form of step-by-step construction procedures to present the subject, so that it can be easily understood.

A number of chapters are designed and written with the specific purpose of training the student and helping him achieve his goal.

For instance, the chapter on principles of graphics incorporating the Bureau of Indian Standards (BIS, SP: 46/1988) is aimed at training the student in standardising the pictorial language.

The first chapter on draughting tools infuses in the student sufficient confidence to use modern drawing equipment. Instruction in free-hand sketching is meant to train the student in various sketching techniques, so as to convey his ideas to others effectively.

The chapters on nomography, miscellaneous topics and computer graphics, as well as the contents of the other chapters are planned to serve the present need and also

the possible future enrichment of the syllabi of various Universities in the country.

A large number of objective questions covering all chapters are also included at the end of each chapter. These questions help the student in self-study, and the teacher in assessing the student's ability.

The present book is a thoroughly revised and enlarged version of the previously published one.

The authors are thankful to the colleagues of the Department of Mechanical Engineering, S.V.U College of Engineering, Tirupati, for their encouragement and valuable suggestions to bring out this book in the present form.

We are also thankful to the Bureau of Indian Standards for permitting us to use certain extracts from their standards.

Finally, we express our heartfelt thanks to Scitech Publications of Chennai, for bringing out the text book in this attractive form.

This book is intended as a class room text book for students of all faculties in degree and diploma, draughtsman's courses and for multipurpose high schools, etc.

Suggestions from teachers and students for the improvement of the book will be highly appreciated.

**August, 1999**

**K. L. Narayana**

**P. Kannaiah**

# **Preface to the Third Edition**

The overwhelming appreciation received from the faculties and the students has cheered us to revise once again the script thoroughly. The purpose of revision is to make the book as simple as possible. We have adopted step-by-step approach for an easy understanding.

University questions have been solved thoroughly inside the book. Suggestions for improvement shall be acknowledged gratefully.

**August, 2012**

**K. L. Narayana**

**P. Kannaiah**

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## 1.1 ENGINEERING DRAWING

A drawing prepared by an engineer, for an engineering purpose is known as an engineering drawing. It is the graphic representation of physical objects and their relationships. It is prepared, based on certain basic principles, symbolic representations, standard conventions, notations, etc. It is the only universal means of communication used by engineers, a language of ever increasing value.

### 1.1.1 Importance of Engineering Drawing

Engineering drawing is a two dimensional representation of a three-dimensional object. It is the graphic language, from which a trained person can visualize the object. As an engineering drawing displays a precise picture of the object to be produced, it conveys the same picture to every trained eye. Drawings prepared in one country may be utilized in

any other country, irrespective of the language spoken there. Hence, engineering drawing is called the universal language of engineers.

Knowledge in engineering drawing is equally essential for the persons holding responsible positions in engineering field. An engineer without adequate knowledge of this language is considered to be professionally illiterate.

## **1.1.2 Role of Drawing in Engineering Education**

The ability to read drawings is the most important requirement of all technical people in engineering profession. The potentialities of drawing as an engineer's language may be made use of as a tool for imparting knowledge and providing information on various aspects of engineering.

The classification of engineering drawings include: Building drawing, machine drawing, electrical drawing , etc.

While teaching majority of subjects; figures or sketches of related objects, machines or systems are made use of, to explain the principles of operation, relation between the parts, etc. Unless the figures are presented, following the norms of draughting practice, the required information cannot be fully conveyed. Hence, the knowledge in engineering drawing is useful in understanding the other subjects as well.

## **1.1.3 Scope of the Subject**

The subject matter presented here relates to basic engineering drawing. It mainly deals with geometrical drawing. It is the art of representation of geometrical objects on a drawing sheet and is the foundation of all engineering drawings.

Plane geometrical drawing deals with the representation of objects having two dimensions. Solid geometrical drawing deals with the representation of objects having three dimensions.

## 1. 2 DRAUGHTING TOOLS

The drawing instruments or draughting tools are used to produce drawings quickly and more accurately. To obtain satisfactory results in the form of accurate drawings, the draughting tools used must be of high quality. The students are advised to procure quality draughting tools, which will facilitate to increase efficiency in their draughting work.

The present chapter deals with description of draughting tools used by professional draughtsmen and their methods of use. The following is the list of a majority of draughting tools used by professional draughtsmen:

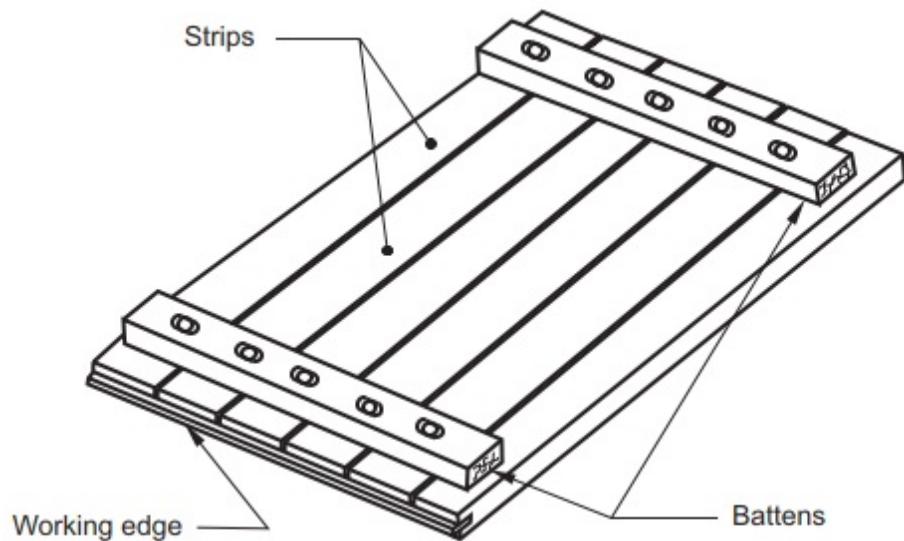
1. Drawing board
2. Mini-draughtsman
3. Instrument box, containing the following:
  - (i) Compass
  - (ii) Bow-compass
  - (iii) Spring bow-compass
  - (iv) Divider

- (v) Bow-divider
- (vi) Bow-pen
- (vii) Inking pen
- 4. 45°-45° and 30° -60° set-squares
- 5. Protractor
- 6. Set of scales
- 7. French curves
- 8. Flexible curve
- 9. Templates
- 10. Drawing sheet
- 11. Paper fasteners
- 12. Pencils
- 13. Eraser
- 14. Erasing shield
- 15. Draughting brush
- 16. Drawing ink
- 17. Tracing paper
- 18. Lettering pens

## 1.3 DRAWING BOARD

Drawing boards are usually made of well-seasoned soft wood. To prevent warping, narrow strips of wood are glued together. Prevention of warping, will also be aided by two battens cleated at the bottom side of the board. In addition, the battens help to give rigidity to the board and raise it

above the surface of the drawing table ([Fig. 1.1](#)). One edge (width) of the drawing board is provided with a working edge, made of hard and durable wood. The working edge is required while using T-square in draughting work.



**Fig.1.1 Drawing board**

The size of the drawing board will depend upon the size of the drawing paper used. Bureau of Indian Standards (BIS) recommends the sizes for the drawing board, as given in [Table 1.1](#).

**Table 1.1 Sizes of drawing board**

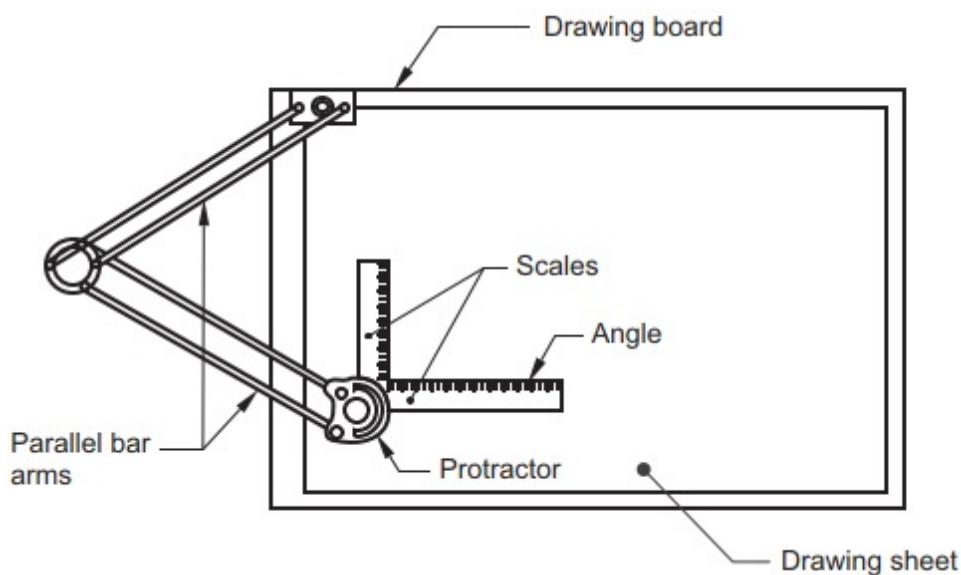
Designation	Size (mm)
B0	1250 × 900
B1	900 × 650
B2	650 × 500
B3	500 × 350

### 1.3.1 Care of the Drawing Board

1. Handle the drawing board carefully so that no dents or holes are made on its surface.
2. Check the working edge at regular intervals and correct it whenever it is found defective. This is required only when T-square is used, instead of mini-draughtsman, for draughting work.
3. Fasten a sheet of paper on the board to keep its surface clean.

## 1.4 MINI-DRAUGHTER

Today in many drawing offices, a unit known as mini-draughtsman, similar to the one shown in Fig. 1.2 is used. It is designed to combine the functions of T-square, setsquares, protractor and scales. This unit, when used, can result in savings of approximately 35% of time in machine drawing and 50% of time in structural drawing work. It consists of an angle, formed by two arms with scales marked and set exactly at right angle to each other. The angle is removable and hence, a variety of scales may be used.



### **Fig.1.2 Mini-draughts**

In the normal position, one of the two arms is horizontal and the other vertical. The arms may also be set and clamped at any desired angle by means of an adjusting head, which has a protractor (with vernier attachment in some cases). The angle of  $90^\circ$  between the arms of course remains unaltered. When the angle formed by the two arms is moved over the drawing sheet, the design of the unit permits the arms to move only into parallel positions.

#### **1.4.1 Use and Care of the Unit**

Before commencing drawing work, the mini-draughts must be so positioned that the edge of the horizontal arm is in-line with the horizontal edge of the drawing sheet, and at the same time, the protractor head should read  $0^\circ$ . Later, whenever work is re-started on the incomplete drawing (sheet), the edge of the horizontal arm should be made to coincide with any horizontal line already drawn, simultaneously ensuring  $0^\circ$  setting of the protractor head.

The elements of the unit are assembled, by using a number of tubular rivets, permitting smooth relative movement. After some time, because of the inevitable wear, the joints become loose, resulting in erroneous movement of the arms. This defect may be rectified by lightly tapping the rivets with a tapered pin. Further, transferring dimensions from the graduated scales of the arms to a drawing should be avoided, as it will damage the scales in due course.

#### **1.5 INSTRUMENT BOX**

## 1.5.1 Compass

The compass is an instrument used for drawing circles and arcs of circles. In any instrument box, there will be two compasses, one big and the other small.

### 1.5.1.1 Big *Compass*

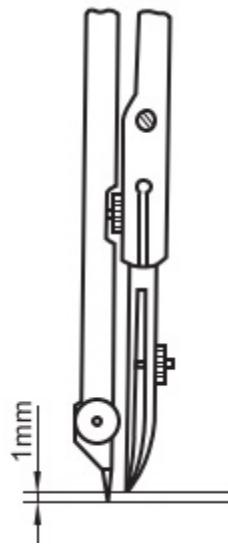
It consists of two legs, hinged at one end. One leg contains a needle at its end. The other leg is so arranged that it can receive either a lead or a pen. In either case, the needle point be at least 1mm longer than the lead or pen point. This is because, when in use, the needle point penetrates into the paper and the pen or lead touches it, as shown in [Fig.1.3](#).

The hardness of the lead used should be the same as that used in the pencil work of the drawing. The lead end may be sharpened, as shown in [Fig. 1.4](#), as it permits to draw smaller circles as well.

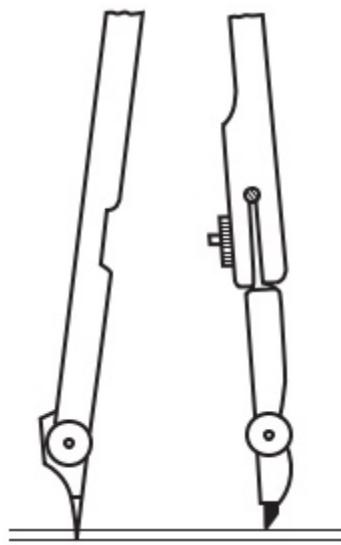
Circles upto, perhaps 50mm radius may be drawn by keeping the compass legs straight. For larger circles, say upto 200 mm radius, it is better if the needle point leg and / or the pen or lead leg are bent at their joints so that they will be perpendicular to the paper ([Fig. 1.5](#)). To draw still larger circles, a lengthening bar is used by placing it between the lead holder and its leg, as shown in [Fig.1.6](#).

To draw a circle, set the compass on the scale and adjust the distance between the lead and needle points to the required radius. Then, guiding with the left hand, place the needle point at the centre and, rolling the handle between the thumb and fore-finger of the right hand, draw the circle clock-wise in one sweep. While drawing a circle, it is better

if the compass is slightly inclined in the direction of the sweep.



**Fig.1.3 Relative positions of compass points**



**Fig.1.4 Sharpening compass lead**



**Fig.1.5 Position of the needle point and the lead leg for drawing larger circles**

### **1.5.1.2 Pen Leg**

Circles are drawn in ink by placing a pen leg in place of the lead leg of the compass. While in use, the nibs are adjusted until almost closed and then ink is added to the tip by means of a quill. The thickness of the line may be controlled, by adjusting the gap between the nibs.

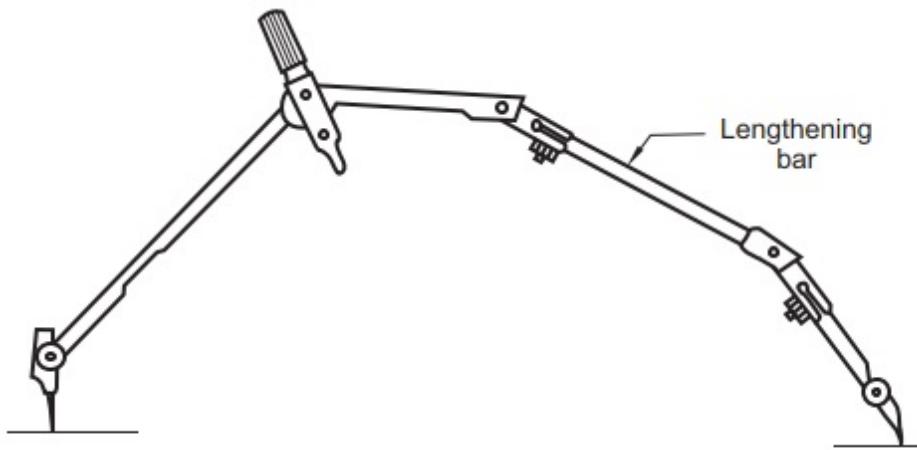
### **1.5.1.3 Testing of a Compass**

The legs of a compass are joined by a pivot joint. The tension in the pivot joint may be checked by holding the legs and slowly opening and closing them. The pivot joint is said to be in good condition, if the tension is constant throughout the motion and just sufficient to enable easy setting of the distance between the points. If required, tension can be adjusted by means of the adjusting screw.

The alignment of the compass is said to be good if the needle point touches at the centre of the closed nibs of the pen leg. If this is not satisfied, the instrument is said to be of inferior quality.

#### 1.5.1.4 Bow - *compass*

The bow-compass is used for drawing small circles upto approximately 25 mm radius. This operates on jack-screw principle, by turning a knurled nut at its centre ([Fig. 1.7](#)). In some designs, the adjusting nut will be at one side of the compass. The advantages of this compass are that the accuracy with which it can be set to the required radius is more and the setting will not be lost, even by keeping it aside temporarily.



**Fig.1.6 Use of lengthening bar**



**Fig.1.7 Bow-compass**

#### **1.5.1.5 *Spring Bow - compass***

This is mostly used for drawing very small circles. In this, the centre rod containing the needle point, remains stationary, while the pencil leg spins around it, as shown in [Fig.1.8](#). The setting of the compass to the required radius is done by turning the adjusting screw on the side. While in use, the centre rod is held between the thumb and first finger of the left hand and the pencil leg is given a spin with the thumb and index finger of the right hand.

#### **1.5.2 Divider**

The divider is used for transferring measurements from one part of the drawing to another part and also for dividing curved or straight lines into any number of equal parts. The divider is different from the compass in that, both its legs do not have knee joints but contain needle points at their lower ends, as shown in [Fig.1.9](#). In some designs, a

hairspring provided in one of the legs, is used for minute adjustment, while setting the measurement.

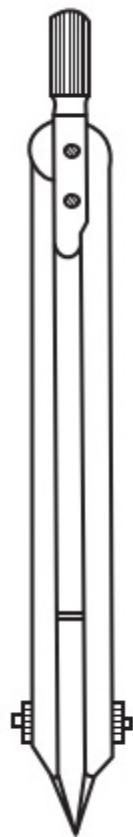
### 1.5.2.1 ***To Divide a Line by Trial***

#### ***Construction (Fig.1.10)***

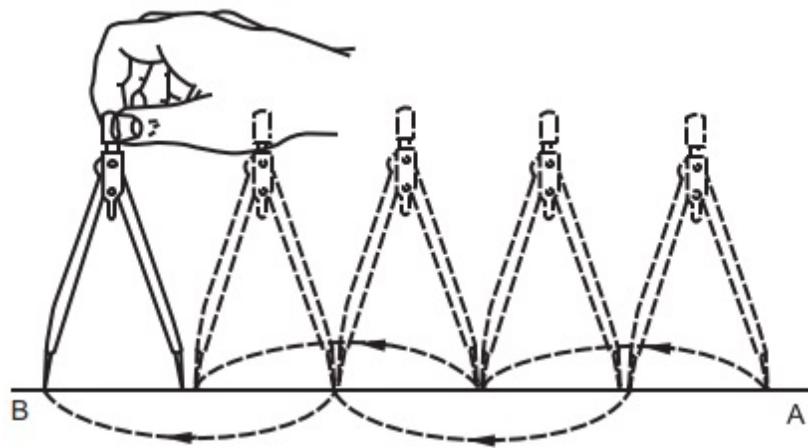
1. Adjust the distance between the legs, approximately equal to the probable length of a division.
2. Step-off the given line lightly.
3. If the last prick mark does not coincide with the end point of the line, increase or decrease the setting by an amount, estimated to be equal to the total error divided by the required number of divisions.
4. Step-off the given line again.
5. Repeat the procedure till the last prick mark coincides with the end point of the line.



**Fig.1.8 Spring bow-compass**



**Fig.1.9 Divider**



**Fig.1.10 Division of a line by trial**

- ☞ A divider should not be used to transfer a dimension from a scale, as the method is slow and damages the graduation marks on the scale.

### **1.5.2.2 Bow - divider**

The bow-divider is similar to the bow-compass except that both the legs contain needle points, and the distance between the legs is adjusted by a knurled nut ([Fig.1.11](#)). It is used for the same purpose and in the same manner as the big-divider. Similar to the bow-compass, bow-divider holds a setting better and is convenient for marking small and accurate dimensions.

### **1.5.3 Bow - pen**

[Figure 1.12](#) shows the bow-pen used for drawing small circles in ink. This is similar to the bow-compass in design except that, in place of lead leg there is a pen leg.

### **1.5.4 Inking Pen**

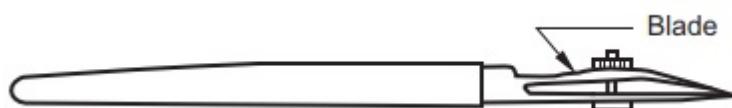
An inking pen or a ruling pen is used for drawing lines and non-circular curves in ink ([Fig.1.13](#)). It consists of a blade, fixed rigidly to a handle. The two nibs of the blade end may be opened or closed by means of a screw. This adjustment permits drawing of lines of different widths or thicknesses. It is, filled by a quill.



**Fig.1.11 Bow-divider**

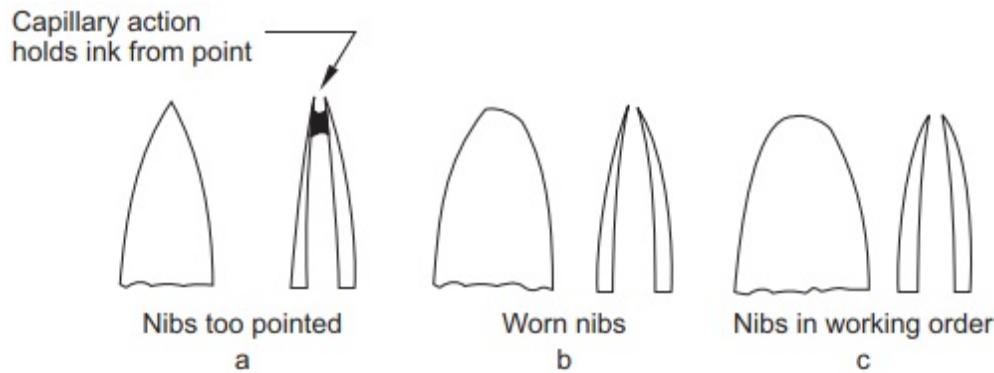


**Fig.1.12 Bow-pen**



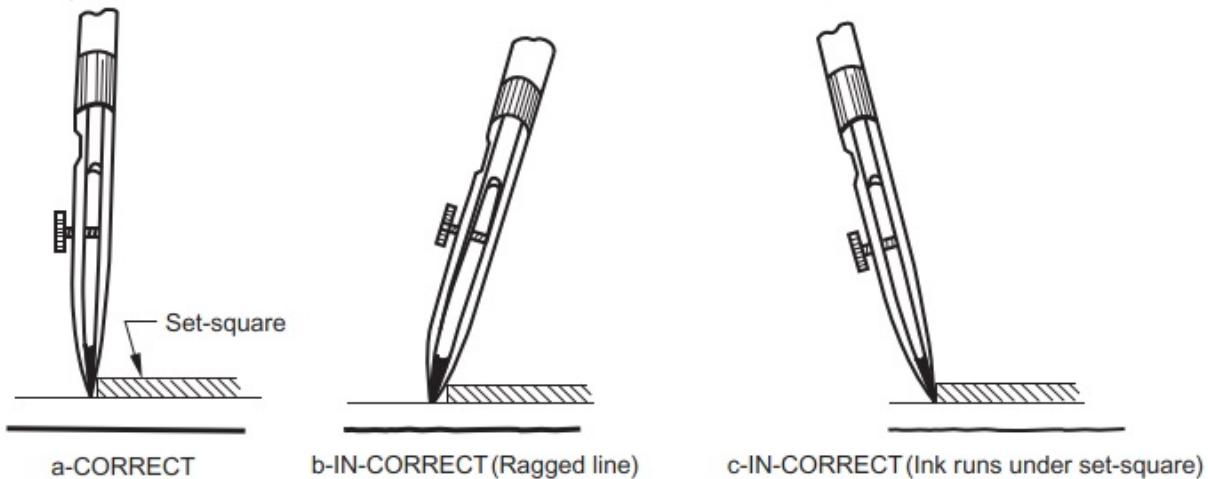
**Fig.1.13 Inking pen**

Figure 1.14 shows the ruling pen nib ends, and Fig.1.15, the use of the ruling pen.



**Fig.1.14 Ruling pen nib ends**

Sometimes, the manufacturers supply pens with the nib ends sharp, as shown in Fig.1.14a. The nib end of this type will not permit the ink to flow freely. Also, after sufficient use, the nib end will get worn-out on one side, as shown in Fig.1.14b. Both of these, may be rectified by grinding the ends properly, as shown in Fig.1.14c.

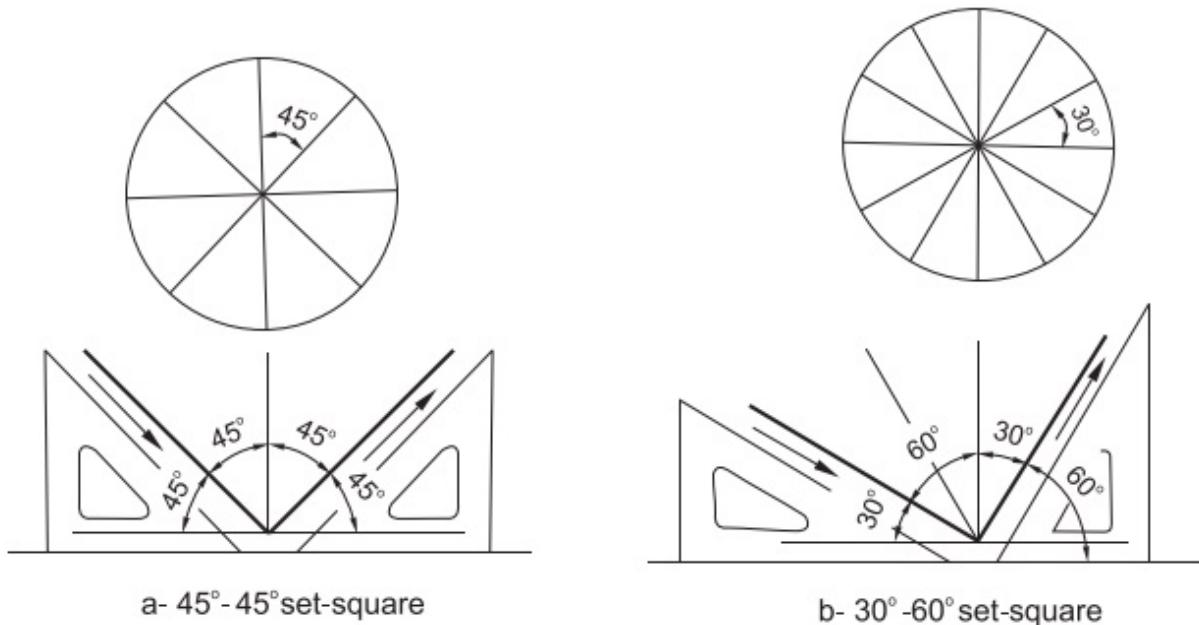


**Fig.1.15 Use of ruling pen**

## 1.6 SET-SQUARES

Set-squares, normally two in number, are the instruments used to draw lines, inclined with the horizontal. Usually,

these are used in conjunction with the T-square and these may also be used with the mini-draughtsman. The two common types are  $45^\circ$ - $45^\circ$  setsquare and  $30^\circ$ - $60^\circ$  set-square (Fig.1.16). These are so called, because of the angles at the corners of each set-square.



**Fig.1.16 Set-squares**

These are made of transparent celluloid or other plastic materials and are beveled on one side. The size of a set-square is designated by the length of the longer side containing the right angle.

The two set-squares may be used in combination to draw a line parallel to any given line, not very far apart and also to draw a line perpendicular to any given line, either from a point on it or outside it.

## 1.7 PROTRACTOR

It is a device used for measuring and laying-off angles, other than those obtained with the set-squares. Usually, it is semi-circular in shape and made of celluloid or other plastic materials. It is beveled along the curved edge. The divisions provided are usually in degrees and half degrees and are readable from both the ends. The line joining  $0^\circ$ - $180^\circ$ , is called the base of the protractor.

## 1.8 SCALES

Scales are used to transfer the true or relative dimensions of an object on to the drawing. For this, place the scale with its edge on the line on which the dimension is to be marked and with a sharp pencil, mark the point opposite the required graduation mark.

Scales are required to make drawings accurately to any desired scale. The recommended scales for use on technical drawings are specified in Chapter 4 on Scales.

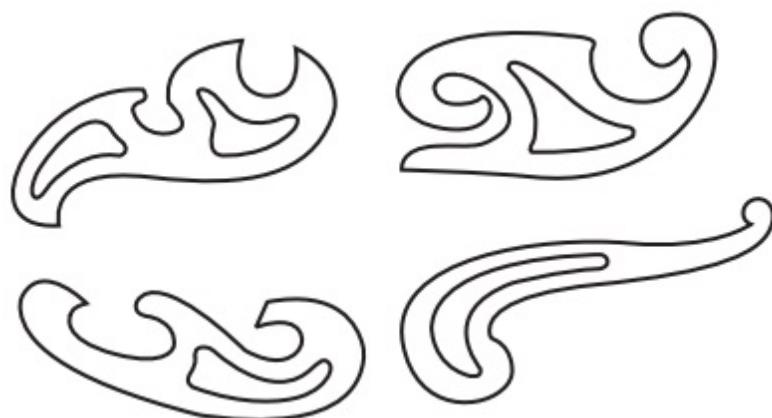
- 
1. A scale should never be used as a straight edge for drawing lines.
  2. If a number of dimensions are to be marked from end-to-end, all should be done at one setting of the scale, by adding each successive dimension to the preceding one. This reduces the total error which, otherwise may accumulate if the distances are marked individually.
  3. If, in the given set of scales, the desired scale is not present, it may be constructed and then used. For construction of a scale, follow the methods presented in Chapter 4 on Scales.

## 1.9 FRENCH CURVES

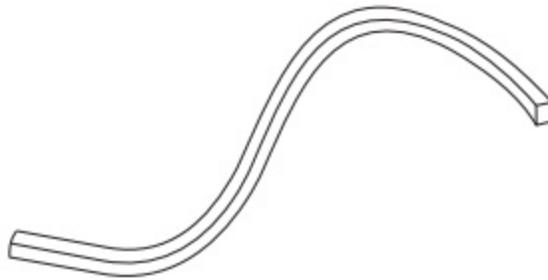
French curves are used for drawing irregular curved lines, the radius of which is not constant. These curves are available in various shapes and sizes, a few of which are shown in [Fig.1.17](#). While using, first a series of points are marked along the desired path and then the most suitable curve is made to fit along it. A smooth curve is then drawn along the edge of the curve. This is repeated till a continuous smooth curve is obtained through all the points marked.

## 1.10 FLEXIBLE CORD

A flexible cord shown in [Fig.1.18](#), is extremely useful in drawing a smooth curve through any given points. It consists of a lead bar enclosed in rubber. The flexibility of the material allows it to bend to any contour.



**Fig.1.17 French curves**



**Fig.1.18 Flexible cord**

## 1.11 TEMPLATES

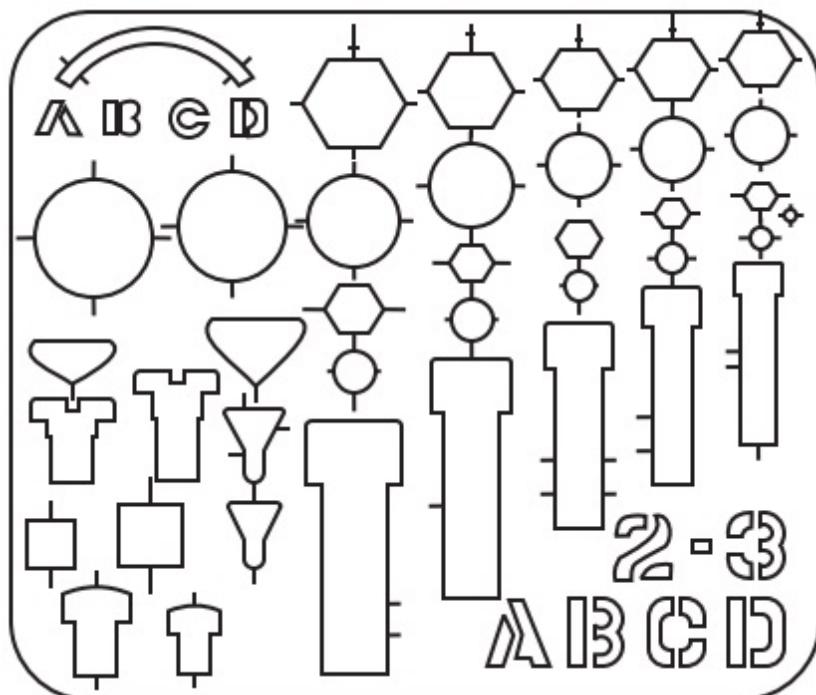
Templates are special celluloid tools used by draughtsmen for drawing small circles, arcs, triangular, square, hexagonal and elliptical shapes and also symbols used in various science and engineering faculties such as chemistry, electrical and architectural engineering and tooling, etc. [Figure 1.19](#) shows a typical tooling template. The use of the templates results in saving of valuable draughting time.

## 1.12 DRAWING SHEET

Drawing sheets are available in six preferred standard sizes, as specified by the Bureau of Indian Standards ([Table 1.2](#)). The sheet should be tough and strong and, when an eraser is used on it, its fibres should not get disintegrated.

**Table 1.2 Preferred drawing sheet sizes**

Designation	Size (mm)
A0	1189 x 841
A1	841 x 594
A2	594 x 420
A3	420 x 297
A4	297 x 210
A5	210 x 148



**Fig.1.19 Typical tooling template**

## 1.13 PAPER FASTENERS

For better drawing work, the drawing sheet must be properly fixed on the drawing board. The various means available for the purpose are: Thumb tacks, clips and adhesive tape.

Thumb tacks or drawing pins are easy to use and remove and these offer a firm grip on the drawing sheet. However, the usage of these, damages the drawing board surface and the heads of these, obstruct the free movement of the mini-draughtsman.

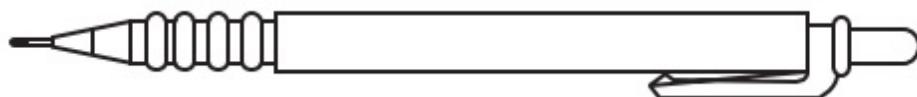
Clips, made of steel, with spring action may be used for fixing the sheet on the board. However, small drawing sheets cannot be fixed with them, on all the four corners.

Now-a-days, adhesive tape is preferred for fixing the sheet on the board. Its smooth surface permits mini-draughtsman, set-squares, etc., to slide easily over the entire drawing sheet.

## 1.14 PENCILS

Special quality pencils are normally used in drawing work. Pencils with different degrees of hardness are available in the market. The selection of a particular hardness or grade, depends upon the line quality desired for the drawing. The grade HB denotes medium hardness of the lead. The hardness increases as the value of the numeral put before the letter H increases. Similarly, the lead becomes softer as the value of the numeral put before the letter B increases.

Pencils of grades B or H may be used for finishing a pencil drawing, as they give a sharp black line. For sketching and artistic work, softer grade pencils are recommended. HB grade is suitable for lettering and dimensioning.



**Fig.1.20 Mechanical pencil**

Both wood and mechanical pencils are available in the market. Wood pencils require frequent sharpening and shaping of the lead end. Mechanical pencils (Lead holders) can hold leads of very small diameter (0.3 mm and above). These pencils are cleaner to use, much more convenient and are likely to produce a line of consistent thickness. Typical mechanical pencil along with lead are shown in [Fig.1.20](#).

## 1.15 ERASER

A variety of erasers are available in the market, which are used to remove ink or pencil lines. A soft pencil eraser should be used for erasing pencil lines and cleaning soiled spots on the drawings. To avoid frequent erasing, careful planning in drawing is needed. Ink erasers are hard in nature and may damage the paper while erasing. Hence, inking work should be taken up only when the pencil drawing is satisfactory in all respects.

## 1.16 ERASING SHIELD

It is a thin piece of metal or celluloid, having openings ([Fig. 1.21](#)). This, when used for erasing a particular spot on the drawing, prevent wrinkling of the paper and protects other nearby lines in the drawing.

## 1.17 DRAUGHTING BRUSH

The draughting brush is used to keep the drawing area clean by removing eraser and graphite particles or any

accumulated dirt before they get spread over the drawing sheet by the T-square or the arms of the mini-draughtsman. Fingers should never be used for the above purpose as they only spoil the drawing, instead of making it clean.

## **1.18 DRAWING INK (INDIAN INK)**

Drawing inks, or Indian inks as usually called, are available in a wide variety of colours; the black ink being the mostly used one in draughting work. The black Indian ink is a finely ground carbon with natural or synthetic gum added to make the mixture waterproof. The carbon is in suspension and not in solution, and thus results in a very thick fluid.

Indian ink dries up very rapidly and forms thin cakes of carbon on the pen nibs. Therefore, the pen nibs must often be cleaned.

## **1.19 TRACING PAPER**

For the purpose of duplicating drawings, the pencil drawings are to be traced in ink on a tracing paper. Tracing papers are available in a wide variety of colours, thicknesses and surface qualities. One must select the tracing paper carefully so that the tracing of the drawing in ink produces a good drawing.

### **1.19.1 Inking Technique**

Inking of drawings or tracing of drawings in ink requires a certain order to achieve better results. Just like pencil

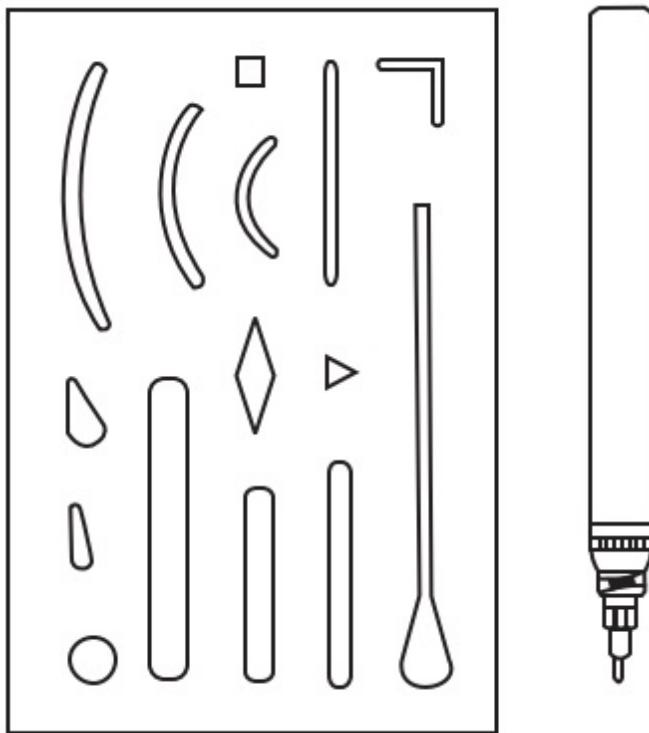
drawings, a good ink drawing is one in which there exists uniformity among the lines of one type and contrast between the lines of different types. The following is the order of the inking technique of any drawing:

1. Centre lines
  - (i) Horizontal lines from top of the sheet
  - (ii) Vertical lines from left side of the sheet
  - (iii) Inclined lines from left to right
2. Visible lines
  - (i) Circles, arcs of circles and other curved lines
  - (ii) Horizontal lines from top of the sheet
  - (iii) Vertical lines from left side of the sheet
  - (iv) Inclined lines from left to right
3. Invisible lines - same order as in 2
4. Cross-hatching lines
5. Extension and dimension lines - same order as in 2
6. Arrow heads
7. Notes and titles
8. Border lines

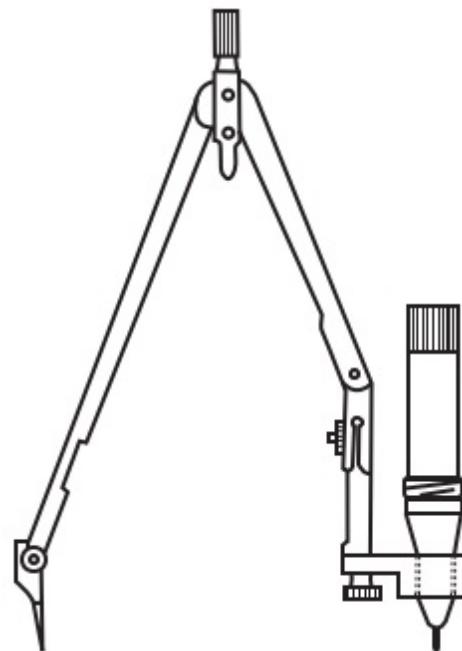
## **1.20 LETTERING PENS**

These are also called the technical fountain pens and are used for lettering. A lettering pen is a needle-in-tube type and with this, lettering of fixed thickness can only be made. Obviously, it is also suitable for drawing straight lines or non-circular curved lines. Different needle-point sizes are

available to suit the width or thickness of the line desired. These are even available as compass attachments to draw circles or arcs of circles ([Fig.1.22](#)).



**Fig.1.21 Erasing shield**

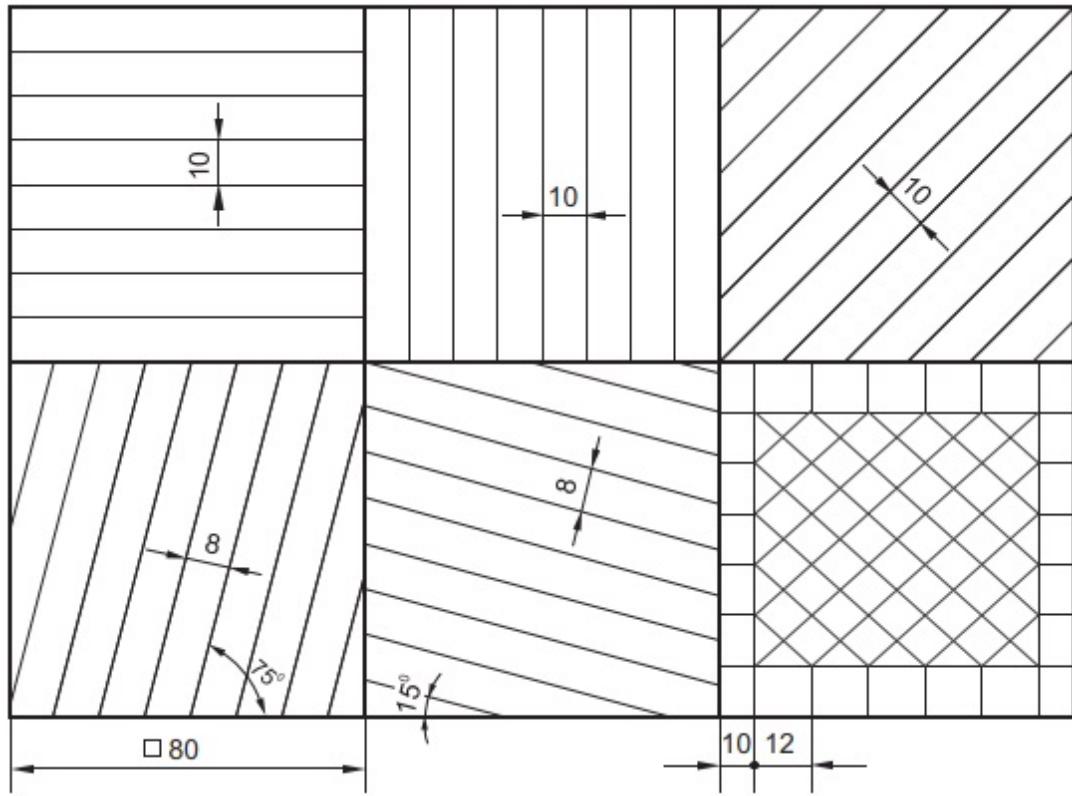


## **Fig.1.22 Lettering pens**

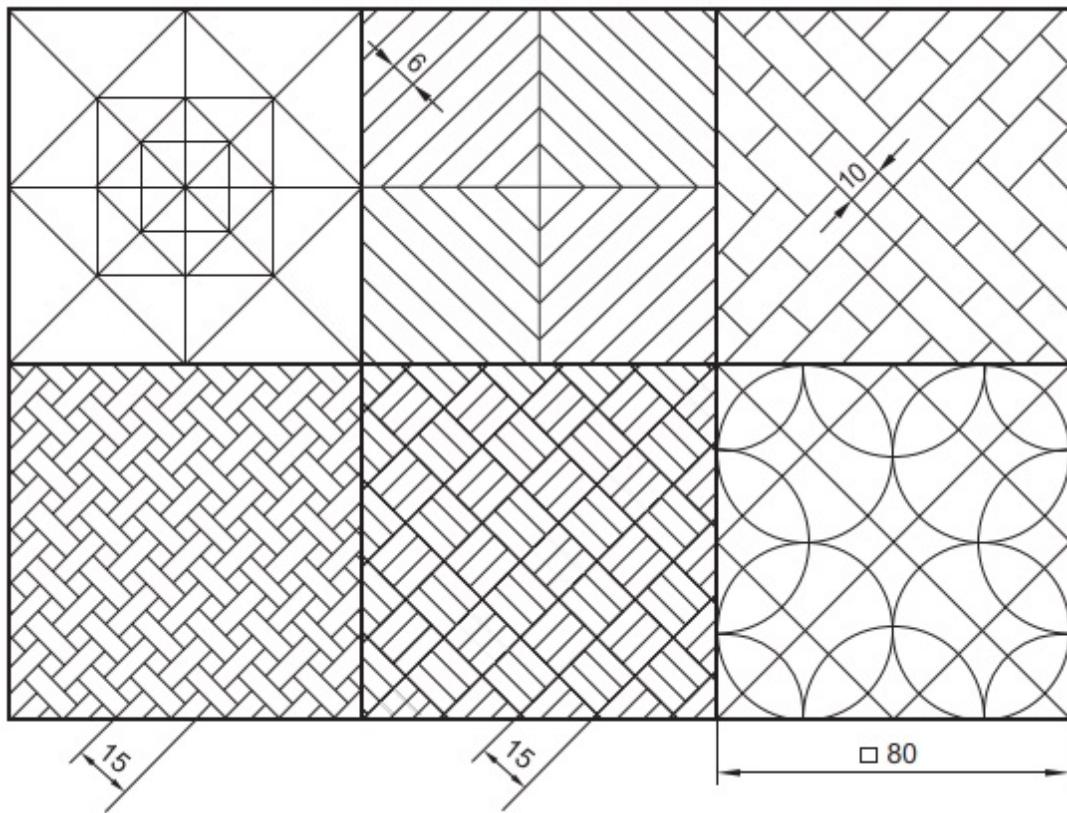
### **EXERCISES**

(All dimensions are in mm)

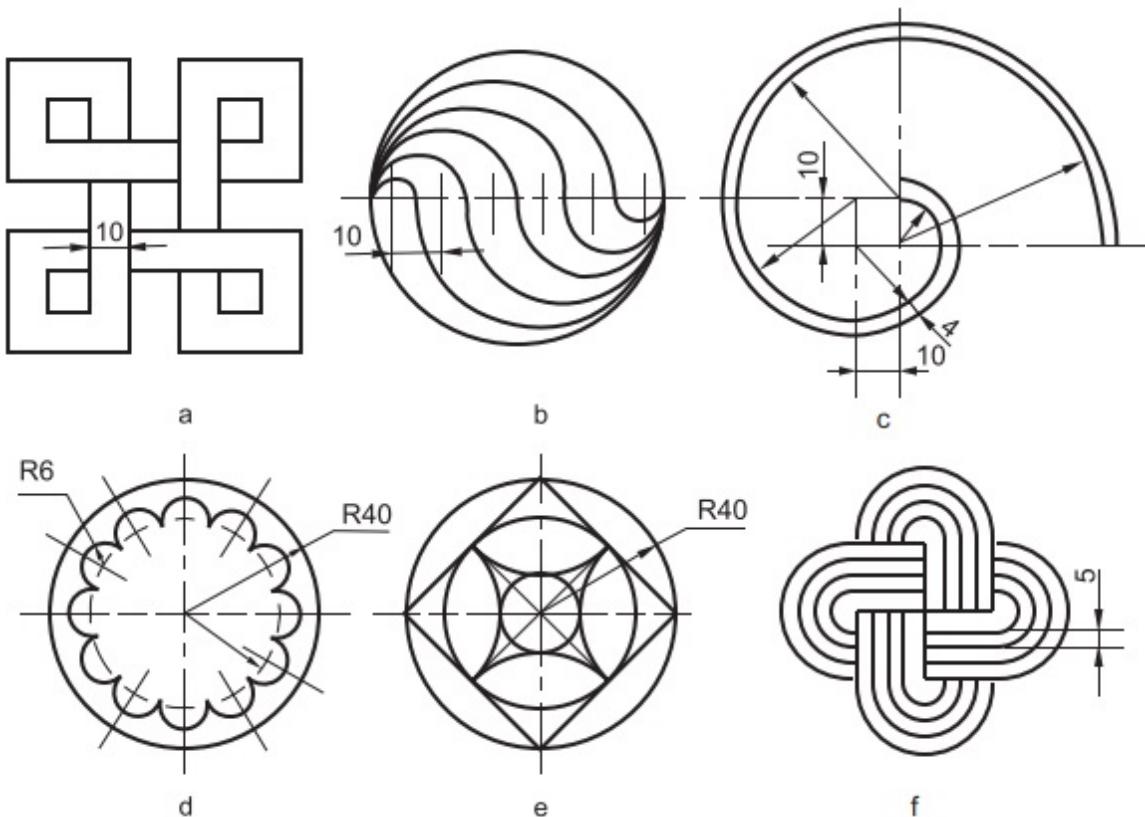
- 1.1 Using a mini-draughtsman, set  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $75^\circ$  angles with a horizontal line.
- 1.2 Using a mini-draughtsman, draw triangles with the following base angles on a 50 long line as the base: (i)  $15^\circ$  and  $45^\circ$ , (ii)  $15^\circ$  and  $60^\circ$ , (iii)  $30^\circ$  and  $60^\circ$ , (iv)  $45^\circ$  and  $75^\circ$ , (v)  $60^\circ$  and  $75^\circ$  and (vi)  $120^\circ$  and  $15^\circ$ .
- 1.3 Draw two lines parallel to each other, separated by 15 and making an angle of  $30^\circ$  with the horizontal.
- 1.4 Draw squares of sides 30, 50 and 70.
- 1.5 Draw rectangles of sizes  $50 \times 30$ ,  $60 \times 40$  and  $70 \times 50$ .
- 1.6 Draw a circle of 100 diameter and without using a protractor, divide it into 4, 8, 12 and 24 equal parts.
- 1.7 Draw a line 150 long and divide it into 9 equal parts by means of a divider.
- 1.8 Reproduce the line formations shown in Figs.1.23 and 1.24.
- 1.9 Reproduce the geometrical features shown in Fig.1.25.



**Fig.1.23**



**Fig.1.24**



**Fig.1.25**

## REVIEW QUESTIONS

- 1.1 What is an engineering drawing?
- 1.2 Engineering drawing is called the universal language of engineers. Explain.
- 1.3 Discuss the role of drawing in engineering education. Name various types of engineering drawings.
- 1.4 Differentiate between plane and solid geometrical drawings.
- 1.5 What is the purpose of lengthening bar ?
- 1.6 Compare the bow-compass with the big compass.
- 1.7 What is the main purpose of the spring bow-compass?

- 1.8 Mention the uses of a divider.
- 1.9 What are the uses of set-squares?
- 1.10 What are the applications of a protractor?
- 1.11 Explain the method of using French curves.
- 1.12 Differentiate between the uses of a French curve and a flexible cord.
- 1.13 What are the uses of templates in draughting practice?
- 1.14 What are the different means of fastening the drawing sheet to the drawing board?
- 1.15 What for drawings are traced on a tracing paper?
- 1.16 Differentiate between an inking pen and a lettering pen.

## OBJECTIVE QUESTIONS

- 1.1 Engineering drawing is a three dimensional representation of an object.  
(True /False)
- 1.2 The working edge of the drawing board should be on the \_\_\_\_\_ side of the draughtsman.
- 1.3 \_\_\_\_\_ are cleated at the bottom, to prevent the warping of the drawing board.
- 1.4 \_\_\_\_\_ is designed to combine the functions of T-square, set-squares, protractor and scale.
- 1.5 The working edge of the drawing board should be in proper condition, while using mini-draughtsman.  
(True /False)

1.6 While drawing inclined lines, the angle between the two arms of the mini-draughtsman is: (a)  $90^\circ$ , (b) less than  $90^\circ$ , (c) more than  $90^\circ$ , (d) depends upon the inclination of the line.

( )

1.7 To draw circles and arcs of circles, \_\_\_\_\_ is used.

1.8 The needle point of the compass must be equal to / longer than / shorter than the lead or pen point.

1.9 The thickness of a line may be controlled by changing the gap between the nibs of the pen leg.

(True/False)

1.10 \_\_\_\_\_ is used to draw circles of small radii.

1.11 \_\_\_\_\_ is used for marking-off short equal distances.

1.12 Divider may be used to divide a straight line into a number of equal parts.

(True/False)

1.13 A scale should not be used as a \_\_\_\_\_ for drawing lines.

1.14 French curves are used to draw regular / irregular curved lines.

1.15 To remove a particular spot on the drawing, \_\_\_\_\_ is used.

1.16 The hardness of the lead increases as the value of numeral before the letter H increases.

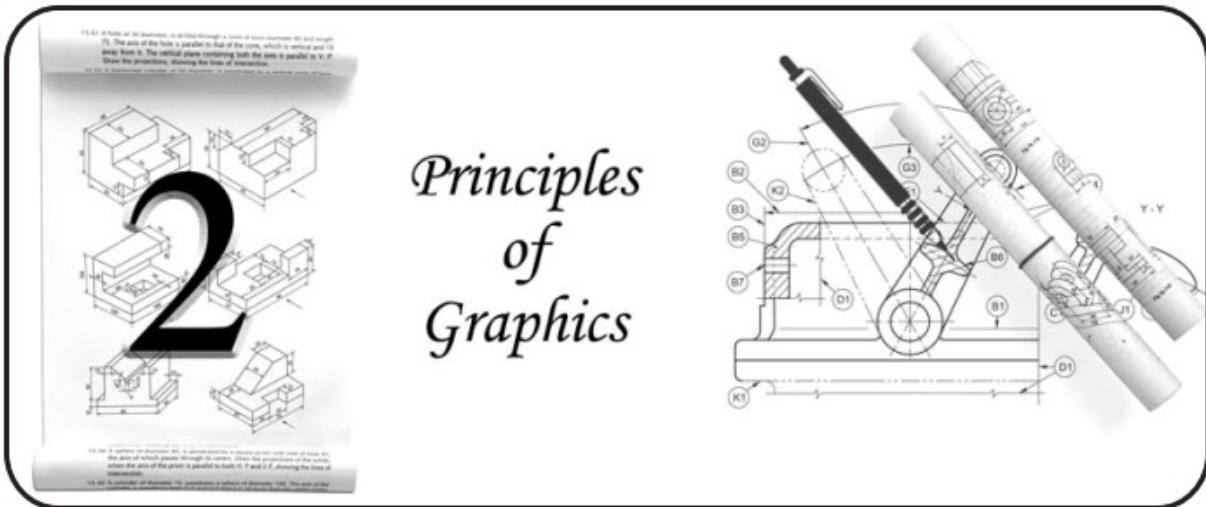
(True/False)

1.17 The lead becomes softer/ harder as the value of the numeral before the letter H increases/decreases.

1.18 Inking pen is used for drawing \_\_\_\_\_ and \_\_\_\_\_.

# ANSWERS

- 1.1 False
- 1.2 left
- 1.3 Battens
- 1.4 Mini-draughtsman
- 1.5 False
- 1.6 a
- 1.7 compass
- 1.8 longer than
- 1.9 True
- 1.10 Spring bow-compass
- 1.11 Bow-compass
- 1.12 True
- 1.13 straight edge
- 1.14 irregular
- 1.15 erasing shield
- 1.16 True
- 1.17 a - softer, decreases, b-harder, increases
- 1.18 straight lines, non-circular curves



## 2.1 INTRODUCTION

Any language to be communicative, should follow certain rules so that it conveys the same meaning to every one. Similarly, draughting practice must follow certain rules, if it is to serve as a means of communication. For this purpose, Bureau of Indian Standards (BIS), adopting the International Standards on code of practice for drawing, has formulated certain rules and published through BIS, SP: 46-1988, titled, "Engineering drawing practices for schools and colleges". These are included in this Chapter and followed throughout the text.

The other foreign standards are: DIN (Germany), ANSI (USA) and BS (UK).

## 2.2 DRAWING SHEET

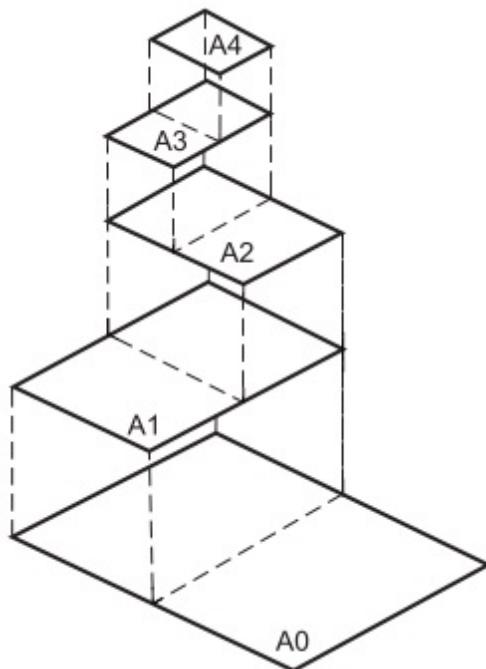
### 2.2.1 Drawing Sheet Sizes

In metric units, drawing sheet sizes are based on A0 size with an area of 1 square metre, having a length to width ratio of  $\sqrt{2}:1$ . The area of the succeeding smaller size is half of the preceding one, the length to width ratio being constant ([Fig.2.1](#)). The preferred standard sizes for drawing sheets as specified by Bureau of Indian Standards are given in section 1.12.

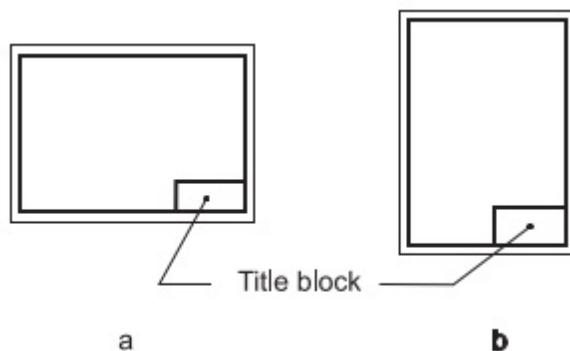
## 2.2.2 Title Block

The title block should lie within the drawing space such that, its location containing the identification of the drawing is at the bottom right hand corner. This is followed both for sheets positioned horizontally or vertically ([Fig.2.2](#)).

The direction of viewing the title block should correspond in general to that of the drawing. It can have a maximum length of 170 mm. [Figure 2.3](#) shows a typical title block, providing the following information: Title of the drawing, drawing number, scale, symbol denoting the method of projection, name of the firm and initials of staff who have drawn, checked and approved.



**Fig.2.1 Drawing sheet formats**



**Fig.2.2 Location of title block**

**Fig.2.3 Typical title block**

## 2.2.3 Drawing Sheet Layout

The layout of a drawing sheet should facilitate easy reading and understanding of the drawings presented there-in. A standard arrangement should make sure that all the required information is included and facilitate easy filing and binding, if required. [Figure 2.4](#) shows a typical layout of a drawing sheet, following the grid reference system. The grid reference system is recommended to permit easy location on the drawing, of details, additions, modifications, etc.

### 2.2.3.1 *Borders and Frames*

It is recommended that the borders have a minimum width of 20 mm for the sizes A0 and A1 and minimum width of 10 mm for the sizes A2, A3, A4 and A5. A filing margin of 20 mm on the left hand side has to be provided (including the border), irrespective of the size of the sheet.



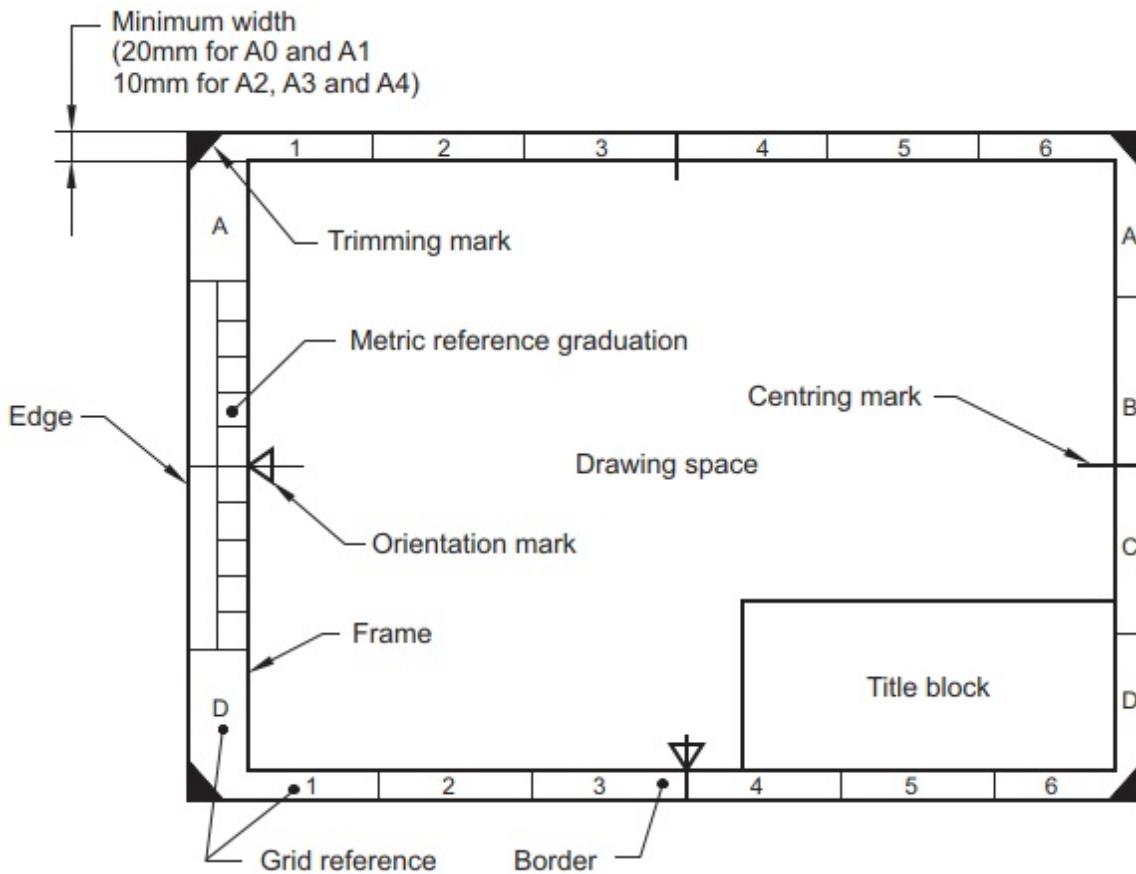
- It is the usual practice to file the drawing sheets in the filing cabinets, without folding.

### ***2.2.3.2 Centring Marks***

Four centring marks may be provided, in order to facilitate positioning of the drawing when reproduced. Two orientation marks may be provided to indicate the orientation of the drawing sheet on the drawing board.

### ***2.2.3.3 Metric Reference Graduation***

It is recommended to provide a figure-less metric reference graduation, with a minimum length of 100 mm and divided into 10 intervals on all the drawing sheets, which are intended to be microfilmed. The metric reference graduation may be disposed symmetrically about a centring mark, near the frame at the border, with a minimum width of 5 mm.



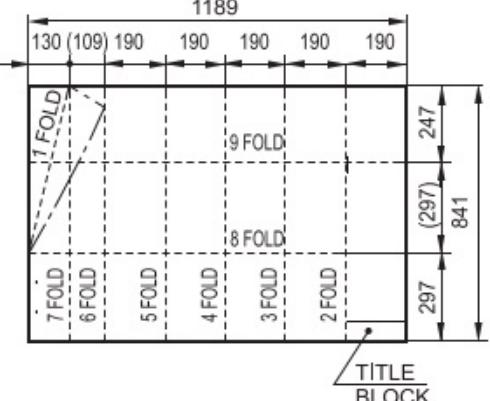
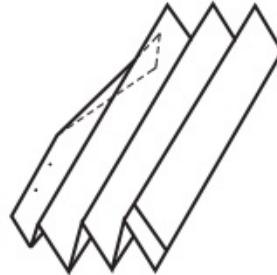
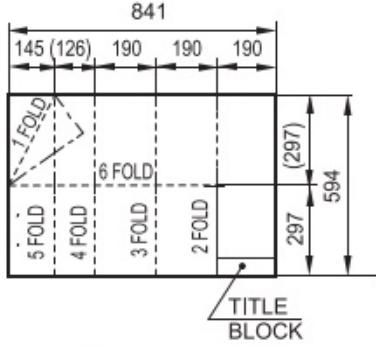
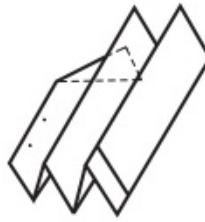
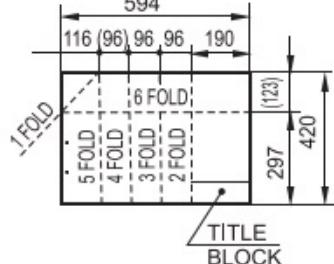
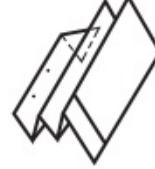
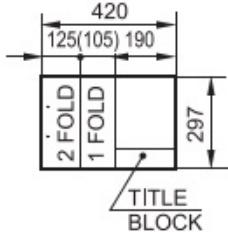
**Fig.2.4 Drawing sheet layout**

#### **2.2.3.4 Grid Reference System (Zoning)**

The grid reference system is recommended for all sizes, in order to permit easy location on the drawing, any details, additions, modifications, etc. The number of divisions should be divisible by two and be chosen in relation to the complexity of the drawing. It is recommended that the length of any side of the grid should not be less than 25 mm and not more than 75 mm. The rectangles of the grid should be referenced by means of capital letters along the edge and numerals along the other edge. The numbering direction may start at the corner, opposite to the title block and be repeated on the opposite sides.

### **2.2.3.5 Trimming Marks**

Trimming marks are provided in the borders at the four-corners of the sheet, in order to facilitate trimming. These marks may be in the form of right-angled isosceles triangles or two short strokes at each corner ([Fig.2.4](#)).

SHEET DESIGNATION	FOLDING DIAGRAM	LENGTH-WISE FOLDING	CROSS-WISE FOLDING
A0 841×1189	 <p>1189</p> <p>130 (109) 190 190 190 190 190</p> <p>1 FOLD 7 FOLD 6 FOLD 5 FOLD 4 FOLD 3 FOLD 2 FOLD</p> <p>9 FOLD</p> <p>8 FOLD</p> <p>TITLE BLOCK</p>		
A1 594×841	 <p>841</p> <p>145 (126) 190 190 190</p> <p>1 FOLD 5 FOLD 4 FOLD 3 FOLD 2 FOLD</p> <p>6 FOLD</p> <p>TITLE BLOCK</p>		
A2 420×594	 <p>594</p> <p>116 (96) 96 96 190</p> <p>1 FOLD 5 FOLD 4 FOLD 3 FOLD 2 FOLD</p> <p>6 FOLD</p> <p>TITLE BLOCK</p>		
A3 297×420	 <p>420</p> <p>125(105) 190 190</p> <p>2 FOLD 1 FOLD</p> <p>TITLE BLOCK</p>		—

**Fig.2.5 Folding of drawing sheets for filing or binding**

SHEET DESIGNATION	FOLDING DIAGRAM	LENGTH-WISE FOLDING	CROSS-WISE FOLDING
A0 841×1189	<p>1189</p> <p>139 (210) 210 210 210 210</p> <p>7 FOLD</p> <p>6 FOLD</p> <p>5 FOLD</p> <p>4 FOLD</p> <p>3 FOLD</p> <p>2 FOLD</p> <p>1 FOLD</p> <p>TITLE BLOCK</p>		
A1 594×841	<p>841</p> <p>210 (211) 210 210</p> <p>4 FOLD</p> <p>3 FOLD</p> <p>2 FOLD</p> <p>1 FOLD</p> <p>297 (297)</p> <p>594</p> <p>TITLE BLOCK</p>		
A2 420×594	<p>594</p> <p>174 (210) 210 (210)</p> <p>3 FOLD</p> <p>2 FOLD</p> <p>1 FOLD</p> <p>297</p> <p>420</p> <p>TITLE BLOCK</p>		
A3 297×420	<p>420</p> <p>(210) 210</p> <p>1 FOLD</p> <p>297</p> <p>TITLE BLOCK</p>		

**Fig.2.6 Folding of drawing sheets for storing in filing cabinet**

## **2.3 FOLDING OF DRAWING SHEETS**

There are two methods of folding of drawing sheets. In the first method ([Fig.2.5](#)), the drawing sheets are so folded that they can be filed or bound. In the second method ([Fig.2.6](#)), the sheets are folded so that they can be kept individually in filing cabinet.

## **2.4 SCALES**

For detailed subject content, refer Chapter 4 on Scales.

## **2.5 LINES**

The various types of lines used in drawing, form the alphabet of the draughting language. Just like the letters of the alphabet, the various lines differ in appearance, thickness and construction. For better understanding of the drawing, the contrast between the various lines must be good. [Table 2.1](#) shows the various types of lines with the recommended applications. [Figure 2.7](#) shows a drawing, indicating the typical applications of different types of lines.

**Table 2.1 Types of lines and their applications**

Line	Description	General applications
A —————	Continuous thick	A1 Visible outlines and edges
B —————	Continuous thin (straight or curved)	B1 Imaginary lines of intersection B2 Dimension lines B3 Projection lines B4 Leader lines B5 Hatching lines B6 Outlines of revolved sections B7 Short centre lines
C ~~~~~	Continuous thin, free-hand	C1 Lines of partial views and sections
D —~~~~~—	Continuous thin with zig-zags	D1 Line (see Fig. 2.7)
F -----	Dashed thin	F1 Hidden outlines and edges
G - - - - -	Chain thin	G1 Centre lines G2 Lines of symmetry G3 Trajectories
H ——— L — —	Chain thin, thick at ends and changes of direction	H1 Cutting planes
K - - - - -	Chain thin, double dashed	K1 Outlines of adjacent parts K2 Alternate and extreme positions of movable parts



1. Centre lines should project only a short distance beyond the outline to which they refer. But, in case of necessity such as for dimensioning, the centre line may further be extended.
2. The dotted lines, representing the invisible features, must be shown only when their presence aids in the interpretation of the drawing. Otherwise they may be omitted, as their presence only causes confusion. However, until perfection is achieved, the student is advised to follow the principles of graphics in toto.

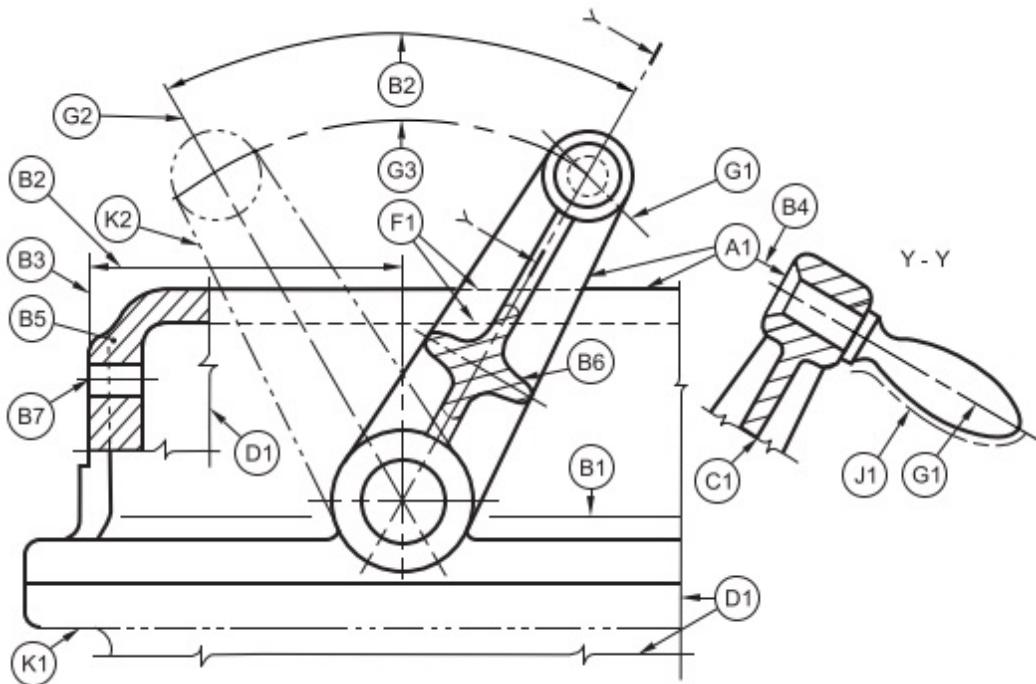
Two line thicknesses are used. The ratio of the thickness of thick and thin lines shall not be less than 2:1. The

thickness of the lines should be chosen according to the size and type of the drawing, from the following range: 0.25, 0.35, 0.5, 0.7, 1.0, 1.4 and 2 mm.

## 2.5.1 Order of Priority of Coinciding Lines

When two or more lines of different types coincide, the following order of priority may be observed while representing them:

- (i) Visible outlines and edges
- (ii) Hidden outlines and edges
- (iii) Centre lines and axis of symmetry



**Fig.2.7 Applications of different types of lines**

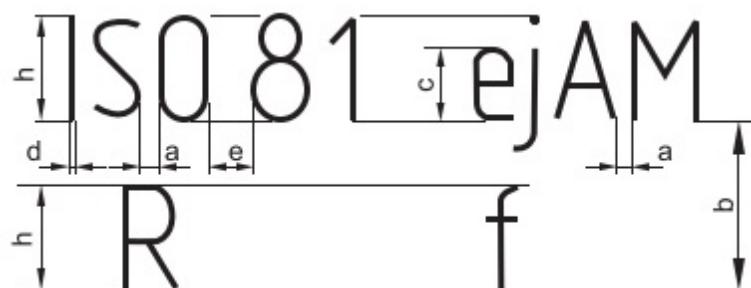
## 2.6 LETTERING

Lettering is an important feature of all engineering drawings, meant for indicating notes, dimensions, etc., on the drawing. The most important requirement for lettering on engineering drawings are legibility, uniformity and ease of execution. These requirements are satisfied by the following rules:

1. The characters are to be clearly distinguishable from each other in order to avoid any confusion between them, even in the case of slight mutilations.
2. Photographic reproductions require the distance between two adjacent lines or the space between letters to be at least equal to twice the line thickness.
3. The line thickness for lower-case and capital letters shall be the same in order to facilitate lettering.

## 2.6.1 Dimensions

The height  $h$  of capital letters is taken as the base of dimensioning (Fig.2.8). The two standard ratios for  $d/h$ ; 1/14 and 1/10 are the most economical ( $d$  represents line thickness) as they result in a minimum number of thicknesses as shown in Table 2.2.



**Fig.2.8 Dimensions of lettering**

The range of standard heights  $h$  for lettering is also shown in the above table, the ratio between the consecutive values being 2, which is derived from the dimensions for drawing sheet sizes.

The lettering may be inclined at  $15^\circ$  to the right or may be vertical. [Figures 2.9](#) and [2.10](#) show the specimens of inclined and vertical lettering of type B; illustrating the above principles.

## 2.7 SECTIONS

Orthographic views when carefully selected may reveal the external features of even the most complicated objects. However, there may be a few objects with complicated interior details; and when represented by hidden lines, may not effectively reveal the true interior details. This may be overcome by representing one or more of the views in section.

A sectional view is obtained by imagining the object, cut by a section plane and the portion between the observer and section plane being removed. The exposed cut surface is represented by section lining or cross-hatching. Cutting planes are designated by capital letters and the direction of viewing the section, is indicated by arrows.

A B C D E F G H I J K L M N O P

Q R S T U V W X Y Z

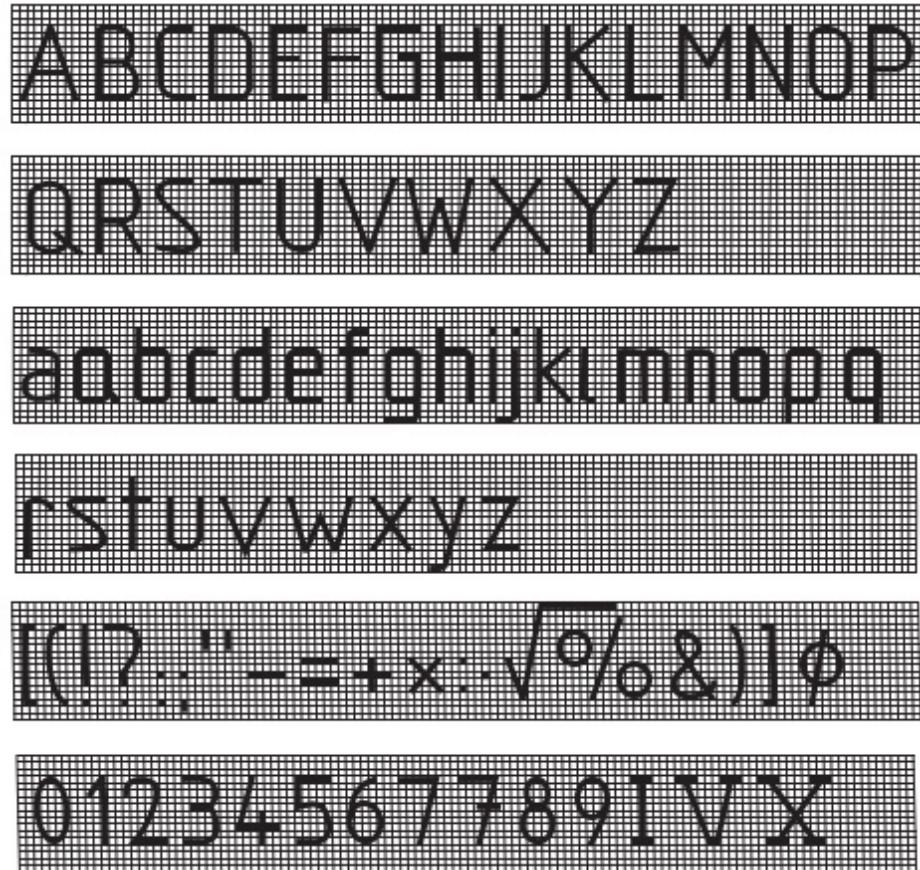
a b c d e f g h i j k l m n o p q

r s t u v w x y z

[ ( ) ? . : ; ' - = + × ÷ √ % & ) ] φ

0 1 2 3 4 5 6 7 7 8 9 I V X

**Fig.2.9 Inclined lettering**



**Fig.2.10** Vertical lettering

**Table 2.2 Standard ratios of lettering**

Characteristics		Ratio	Dimensions, mm						
<i>Lettering A (d=h/14)</i>									
Height of capitals	h	(14/14)h	2.5	3.5	5	7	10	14	20
Height of lower-case letters (without stem or tail)	c	(10/14)h	2.5	3.5	5	7	10	14	
Spacing between characters	a	(2/14)h	0.35	0.5	0.7	1.0	1.4	2	2.8
Min. spacing of base lines	b	(20/14)h	3.5	5	7	10	14	20	28
Min. spacing between words	e	(6/14)h	1.05	1.5	2.1	3	4.2	6	8.4
Thickness of lines	d	(1/14)h	0.18	0.25	0.35	0.5	0.7	1	1.4
<i>Lettering B (d=h/10)</i>									
Height of capitals	h	(10/10)h	2.5	3.5	5	7	10	14	20
Height of lower-case letters	c	(7/10)h	2.5	3.5	5	7	10	14	
Spacing between characters	a	(2/10)h	0.5	0.7	1	1.4	2	2.8	4
Min. spacing of base lines	b	(14/10)h	3.5	5	7	10	14	20	28
Min. spacing between words	e	(6/10)h	1.5	2.1	3	4.2	6	8.4	12
Thickness of lines	d	(1/10)h	0.25	0.35	0.5	0.7	1	1.4	2

## 2.7.1 Full Section

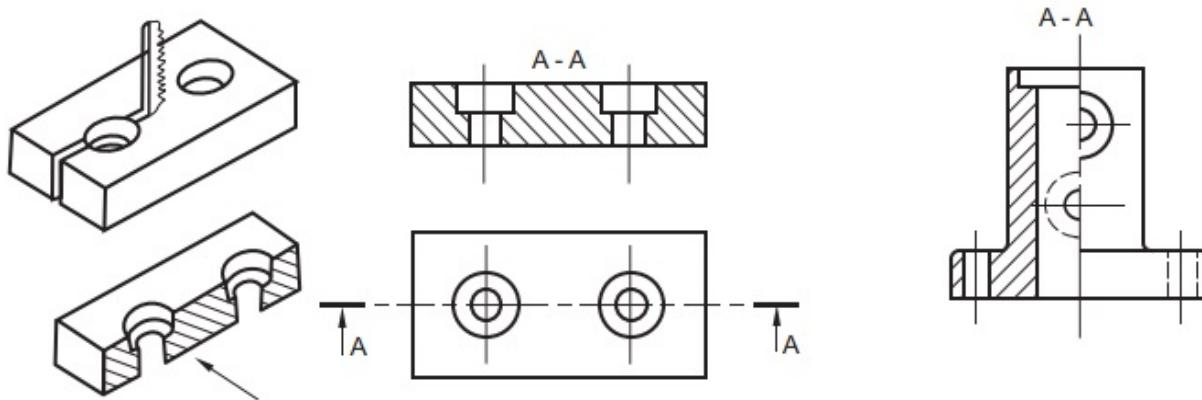
When the cutting plane passes through the entire object; the sectional view obtained is called a full section or simply a sectional view. That is, in order to obtain a sectional view, one half of the object is imagined to be removed. But, it is not actually shown removed anywhere except in the sectional view. As stated above, only in sectional view, the cut surface is shown by cross-hatched lines, leaving the other visible parts behind the cutting plane un-hatched. [Figure 2.11](#) shows the stages in obtaining a sectional view.

## 2.7.2 Half Section

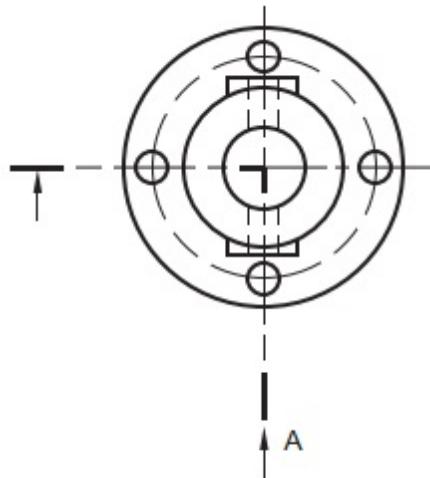
A half section is a view of an object showing half of the view in section. The cutting planes obviously are perpendicular to each other, and remove only one quarter of the object. Thus, a half sectional view shows both the interior and exterior details of the object. [Figure 2.12](#) shows the principles in obtaining a half sectional view.

### 2.7.3 Local Section

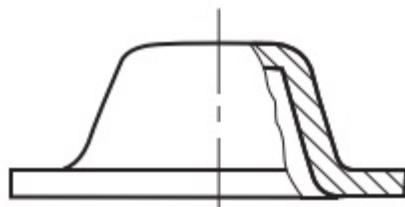
A local section is different from both the full and half sections ([Fig.2.13](#)). For obtaining a local section of a component, only a part of it is imagined to be cut by an irregular section plane. A local section may be drawn when the full or half section is either not convenient or unnecessary.



**Fig.2.11 Stages in obtaining a sectional view**



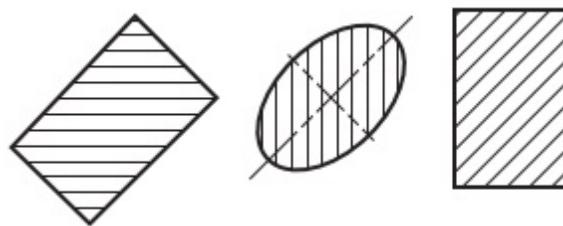
**Fig.2.12 Method of obtaining a half sectional view**



**Fig.2.13 Local section**

#### 2.7.4 Hatching Lines

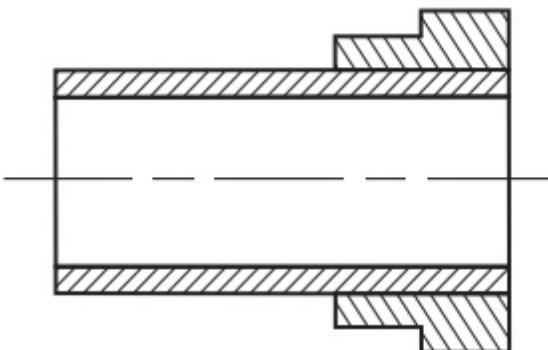
The sectioned portions in sectional views are represented by thin hatching lines, drawn usually at an angle of  $45^\circ$  to the major outline of the drawing. The spacing of the hatching lines should, as far as possible be uniform to give a better appearance to the drawing. The exact distance between the lines will depend upon the size of the drawing. Figure 2.14 shows the preferred hatching angles to be used, depending upon the orientation of the outline of the drawing.



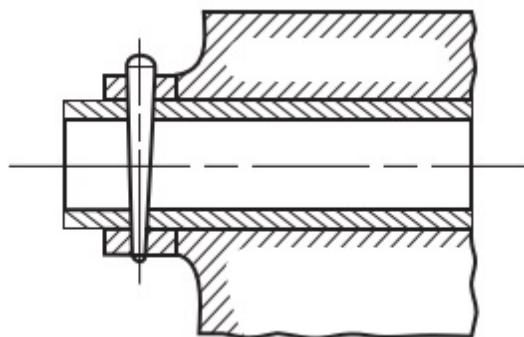
**Fig.2.14 Preferred hatching angles**

Separated areas of a section of a single component must be hatched in the same direction. But two components, adjacent to each other in a section must be represented, by hatching lines in opposite directions, as shown in [Fig.2.15](#).

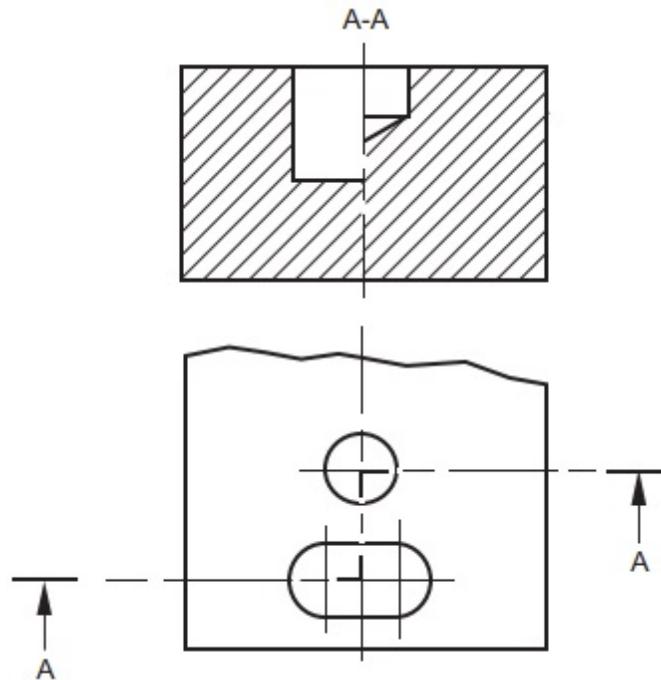
In case of large areas under section, the hatching lines may be limited to a small zone along the contour, as shown in [Fig.2.16](#).



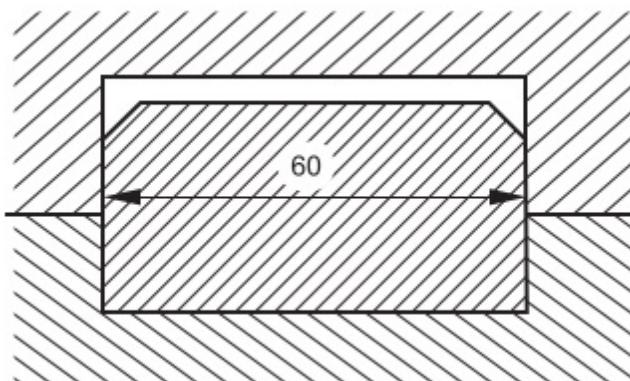
**Fig.2.15 Hatching of adjacent components**



**Fig.2.16 Hatching of large areas**



**Fig.2.17 Sectioning along two parallel planes**



**Fig.2.18 Hatching interrupted for dimensioning**

When a component undergoes sectioning along two parallel planes, the hatching lines in the sectional view must be spaced equally, but must be off-set along the dividing line between the sections, as shown in Fig. 2.17.

When a dimension is required to be placed in the sectioned zone, the hatching must be locally removed to place the dimension, as shown in Fig. 2.18.

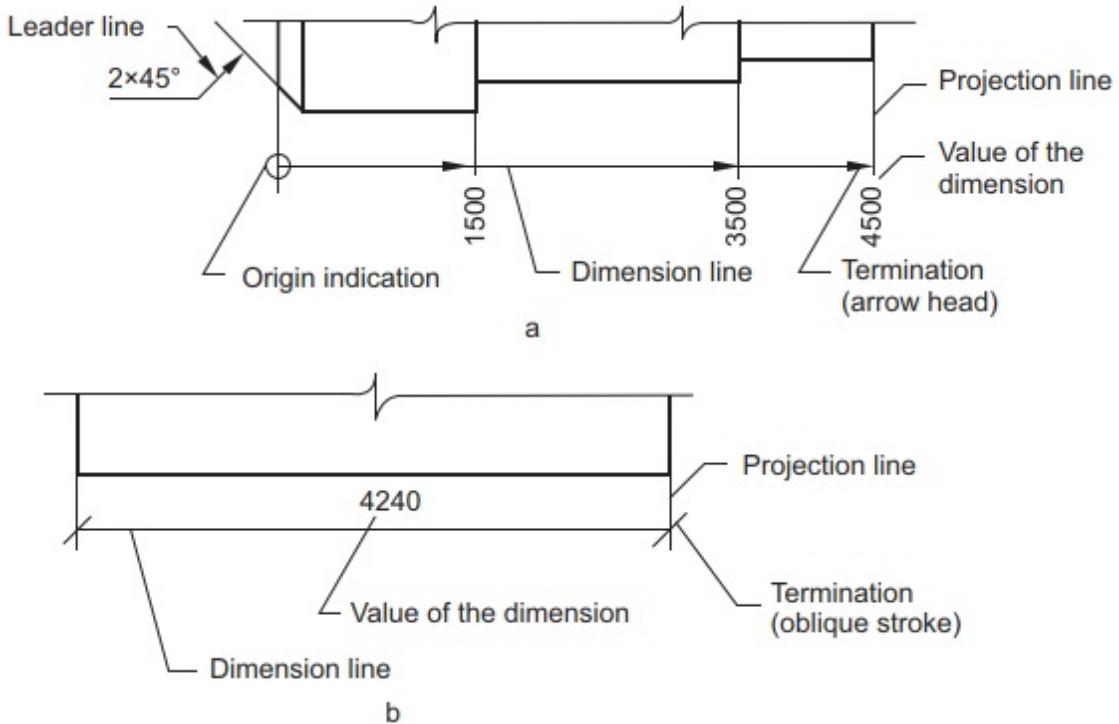
## **2.8 DIMENSIONS**

An object can be represented by a number of views. The entire drawing, consisting of number of views is called a working drawing, provided it gives complete information necessary for the trades person to produce the object. The working drawing must then consist of the views necessary to describe the shape of the object, the dimensions required for the trades person and other specifications such as material, quality, types of machining operations, etc. Any later information is normally provided in the form of a note either on the drawing or in the title block.

Dimensions are indicated on the drawing to define geometric characteristics such as lengths, diameters, radii, angles and locations.

### **2.8.1 Elements of Dimensioning**

The elements of dimensioning include the projection line, dimension line, leader line, dimension line termination, the origin indication and the dimension itself. The various elements of dimensioning are shown in Fig. 2.19.

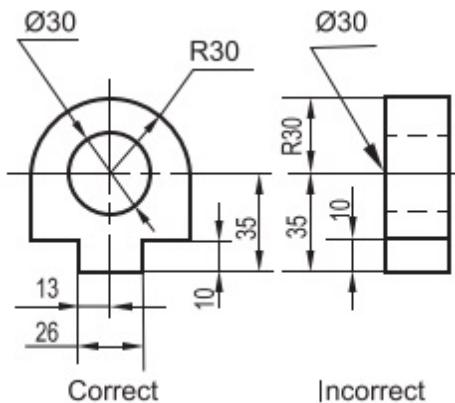


**Fig.2.19 Elements of dimensioning**

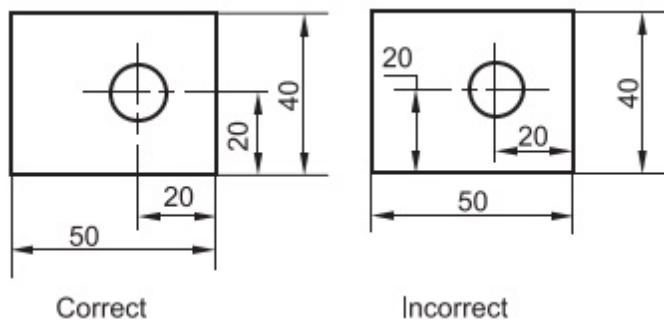
## 2.8.2 Principles of Dimensioning

The following are some of the principles to be applied while dimensioning:

1. Any dimension given, must be clear and permit only one interpretation.
2. Dimensions indicated in one view need not be repeated in another view, except for the purpose of identification, clarity or both.
3. Dimensions shall be placed on the view, where the shape is best shown ([Fig.2.20](#)).

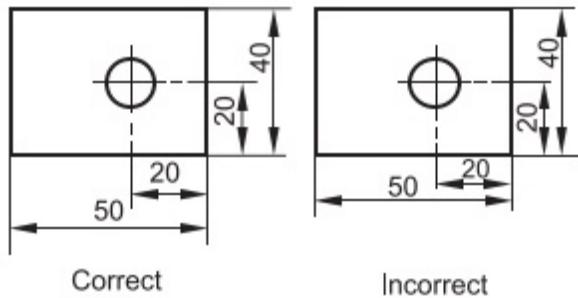


**Fig.2.20 Dimensioning where the shape is best shown**

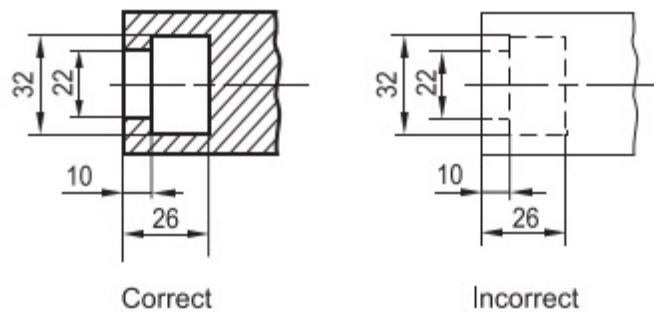


**Fig.2.21 Placing dimensions outside the view**

4. As far as possible, dimensions should be placed outside the view, as shown in [Fig.2.21](#).
5. Dimensions should not be placed very near to the parts being dimensioned, as shown in [Fig. 2.22](#).
6. Dimensions should be marked from visible outlines rather than from hidden lines, as shown in [Fig.2.23](#).

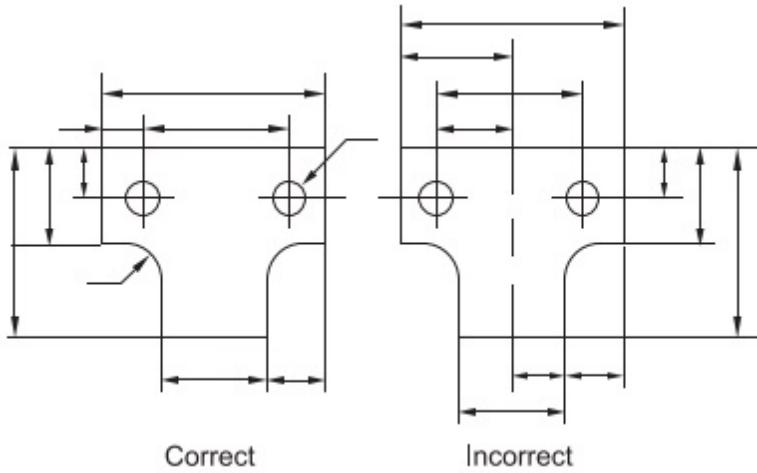


**Fig.2.22 Spacing of dimension lines**

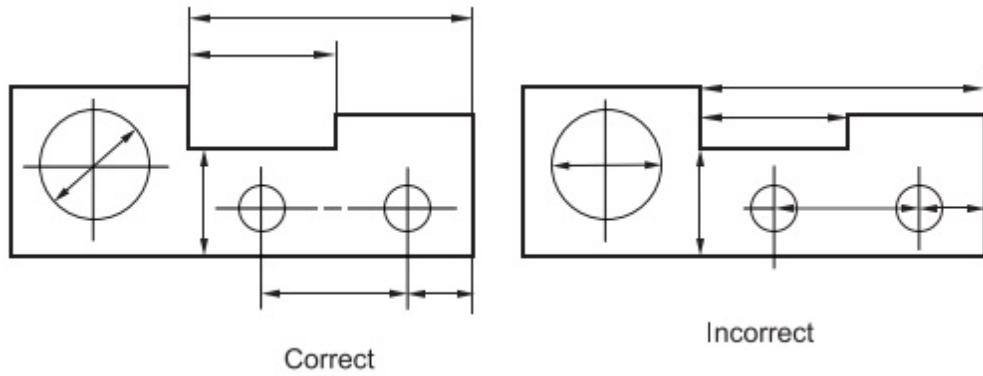


**Fig.2.23 Placing dimensions from the visible outlines**

7. Dimensions should be marked from a base line or centre line of a hole or cylindrical part or finished surfaces, etc. Dimensioning to a centre line should be avoided except when the centre line passes through the centre of a hole or a cylindrical part, as shown in Fig.2.24.



**Fig.2.24 Avoiding centre line for dimensioning**

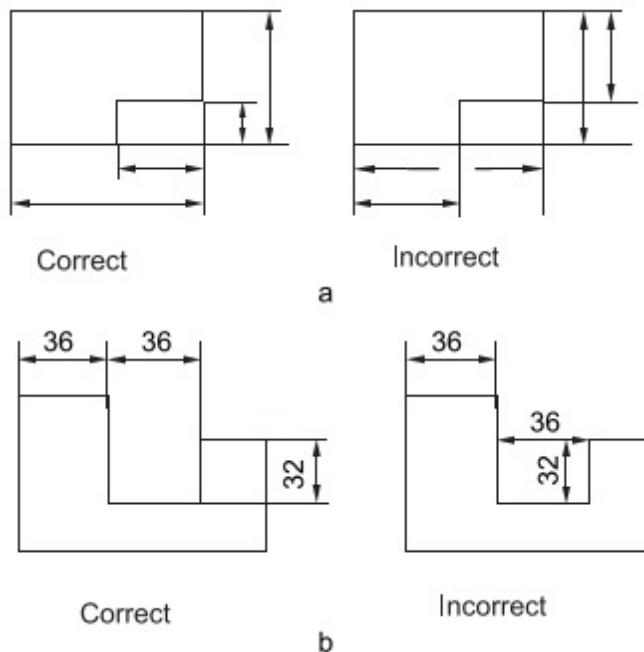


**Fig.2.25 Avoiding contour lines for dimensioning**

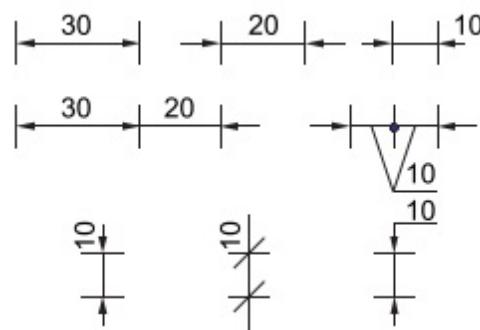
8. An axis or a contour line should never be used as a dimension line but may be used as a projection line, as shown in [Fig.2.25](#).
9. The crossing of dimension lines should be avoided as far as possible. Intersecting projection and dimension lines should also be avoided ([Fig.2.26a](#)). However, when it is unavoidable, the lines should not be shown with a break. Dimension line should not be used as an extension line ([Fig.2.26b](#)).
10. Dimensions should be expressed in one unit only, preferably in millimetres. The unit “mm” can then be

dropped at the end of each dimension, by adding a note separately that all dimensions are in millimetres.

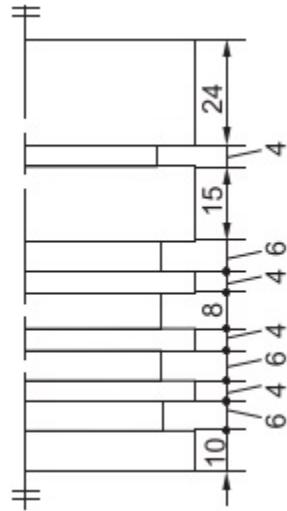
11. When the space for dimensioning is insufficient, the arrow-heads may be reversed, as shown in Fig. 2.27 or adjacent arrow heads may be replaced by a dot, as shown in Fig. 2.28.



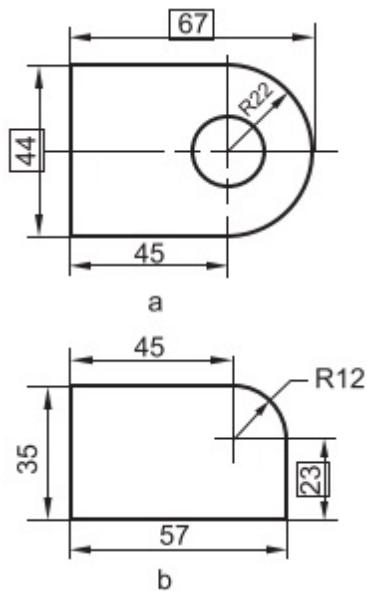
**Fig.2.26 Avoiding crossing of dimension lines**



**Fig.2.27 Different methods of dimensioning a length**



**Fig.2.28 Use of dots for dimensioning**



**Fig.2.29 Indication of only necessary dimensions**

12. No more dimensions than are necessary to describe a part shall be shown on a drawing. No feature of a part shall be defined by more than one dimension in any one direction ([Fig.2.29](#)). For example, in [Fig.2.29](#), the dimensions shown in boxes are redundant.

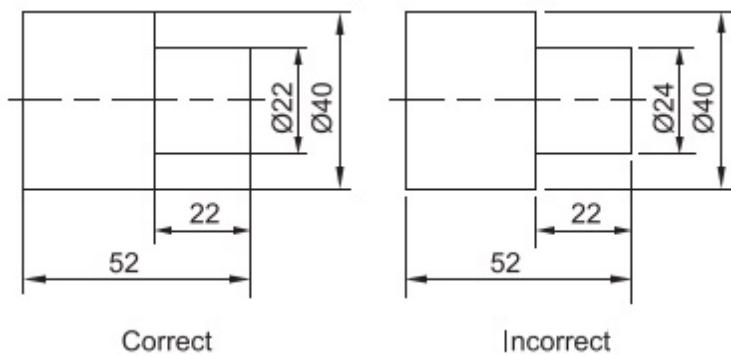
The unit “mm” is dropped both in dimensioning and

 in the script throughout the book.

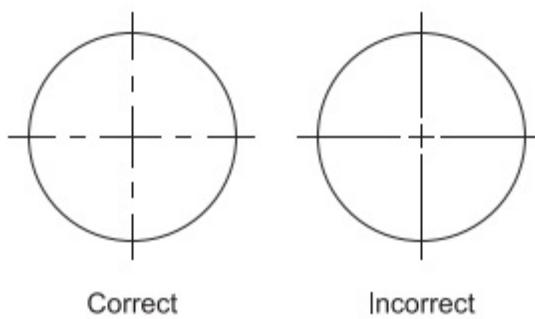
### 2.8.3 Execution of Dimensions

The following are the points to be observed while executing dimensions:

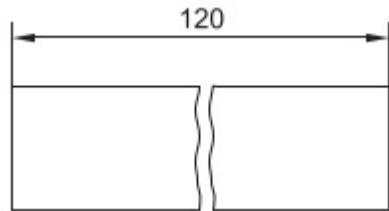
1. Projection lines should be drawn from the visible features of the object, without a gap and extend slightly beyond the dimension line ([Fig.2.30](#)).
2. Crossing of centre lines should be done by a long dash and not a short dash ([Fig.2.31](#)).



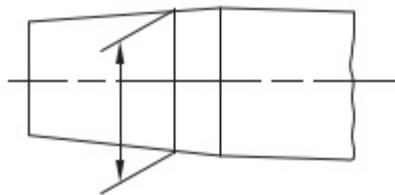
**Fig.2.30 Avoiding gap between feature and projection line**



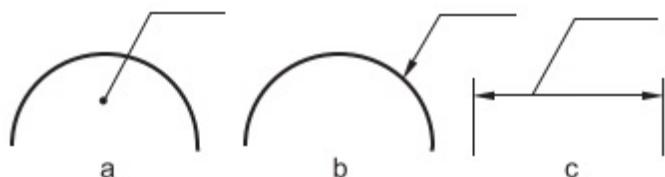
**Fig.2.31 Crossing of centre lines**



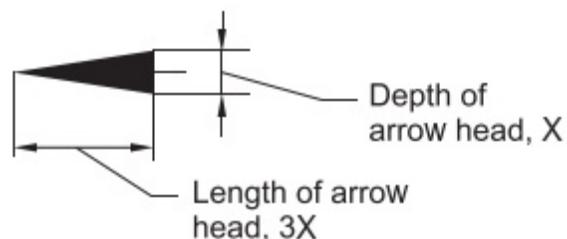
**Fig.2.32 Unbroken dimensions line**



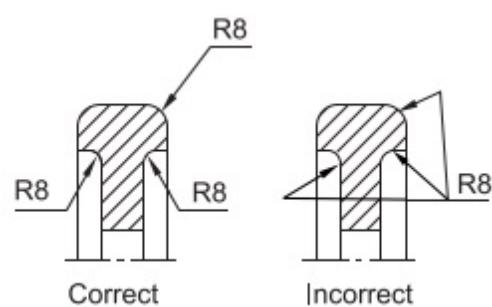
**Fig.2.33 Projection lines on a tapered feature**



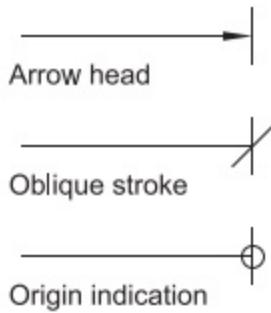
**Fig.2.34 Termination of leader lines**



**Fig.2.35 Proportions of an arrow head**



**Fig.2.36 Dimensioning several arcs**



**Fig.2.37 Terminations of dimension lines**

3. A dimension line should be shown unbroken, even where the feature to which it refers is shown broken ([Fig.2.32](#)).
4. Projection lines must be drawn perpendicular to the outline of the feature to be dimensioned. In some cases such as on tapered features, these may be drawn obliquely but parallel to each other, as shown in [Fig.2.33](#).
5. A leader line is a line referring to a feature (dimension, object, outline, etc.). It is drawn at an angle greater than  $30^\circ$ . Leader lines ([Fig.2.34](#)) should terminate :
  - (i) with a dot, if they end within the outline of an object.
  - (ii) with an arrow head, if they end on the outline of an object.
  - (iii) without dot or arrow head, if they end on a dimension line.
6. For arrow heads used at the ends of dimension and leader lines, the length may be taken as three times the depth, and the space is filled. The size of the arrow

head should be proportionate to the size of the drawing ([Fig.2.35](#)).

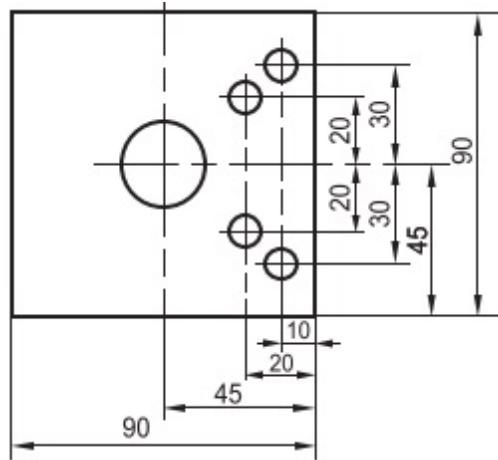
7. When several arcs are dimensioned, it is preferable that separate leaders be used rather than extending the leaders ([Fig.2.36](#)).
8. Dimension lines should show distinct termination in the form of an arrow head or oblique stroke or where applicable, an origin indication ([Fig.2.37](#)).

## 2.8.4 Placing of Dimensions

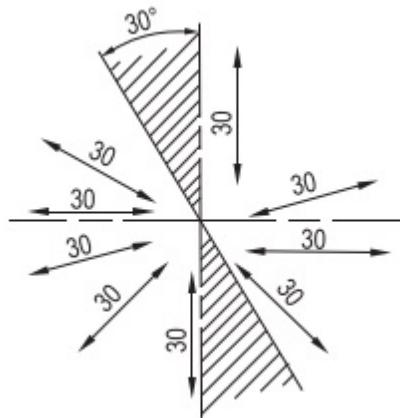
Dimensions may be placed according to either of the following recommended systems:

### 2.8.4.1 *Aligned System*

In an aligned system, all the dimensions are placed above the dimension lines such that, they may be read either from the bottom or from the right hand side of the drawing, as shown in [Fig.2.38](#). Dimensions on oblique dimension lines shall be oriented as shown in [Fig.2.39](#) and except where unavoidable, they should not be placed in the  $30^\circ$  zone. Angular dimensions may be oriented as shown in [Fig.2.40](#).



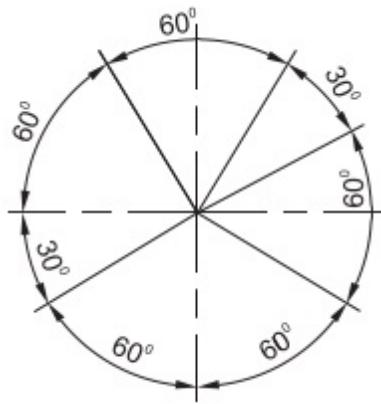
**Fig.2.38 Dimensioning-Aligned system**



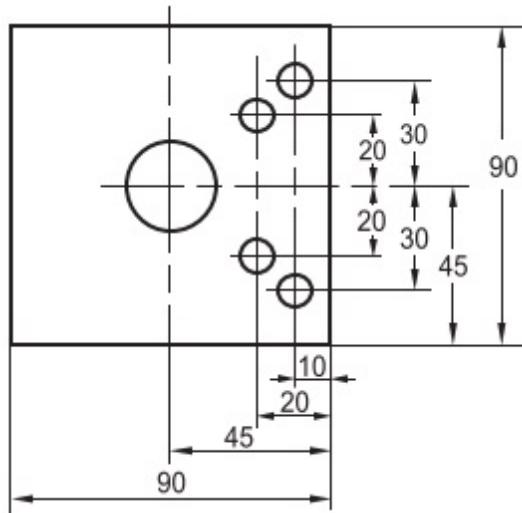
**Fig.2.39 Dimensioning on oblique dimension lines**

#### 2.8.4.2 *Uni-directional System*

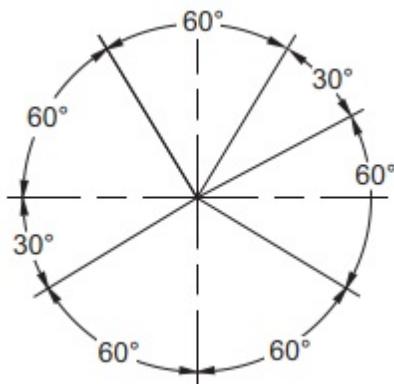
In uni-directional system, all the dimensions are placed in one direction such that, they may be read from the bottom of the drawing only. Also, in this system, the non-horizontal dimension lines are interrupted, preferably in the middle, for insertion of the dimension, as shown in [Fig.2.41](#). This system is useful for very big drawings, where it is inconvenient to read the dimensions from two sides. Angular dimensions may be oriented as shown in [Fig.2.42](#).



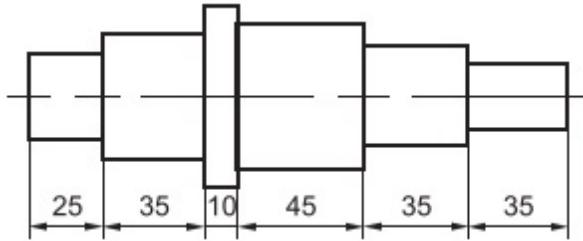
**Fig.2.40 Angular dimensions-Aligned system**



**Fig.2.41 Dimensioning-Uni-directional system**



**Fig.2.42 Angular dimensions-Uni-directional system**



**Fig.2.43 Chain dimensioning**

## 2.8.5 Arrangement of Dimensions

### 2.8.5.1 *Chain Dimensions*

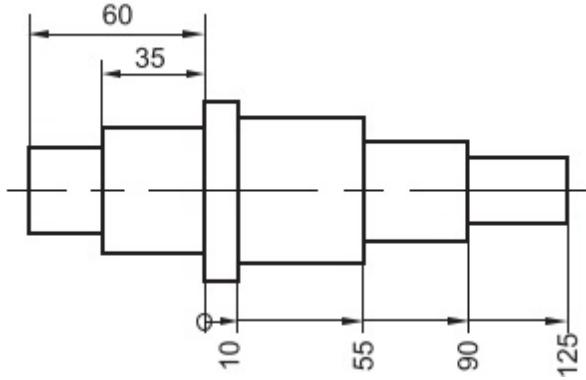
Chain dimensioning should be used only where the possible accumulation of tolerances does not endanger the functional requirement of the object, as shown in Fig.2.43.

### 2.8.5.2 *Parallel Dimensions*

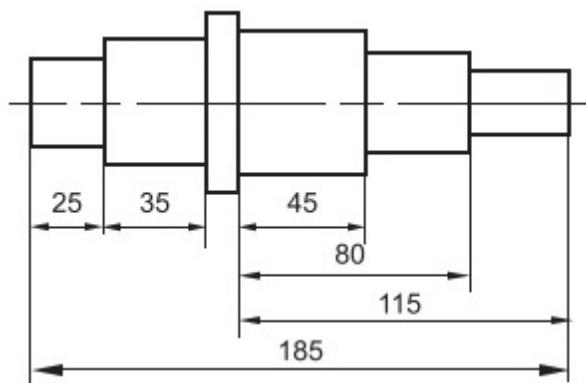
Parallel dimensioning is followed when a number of dimensions have a common datum feature, as shown in Fig. 2.44.

### 2.8.5.3 *Combined Dimensions*

In combined dimensioning, both the chain and parallel dimensions are followed, as shown in Fig.2.45.



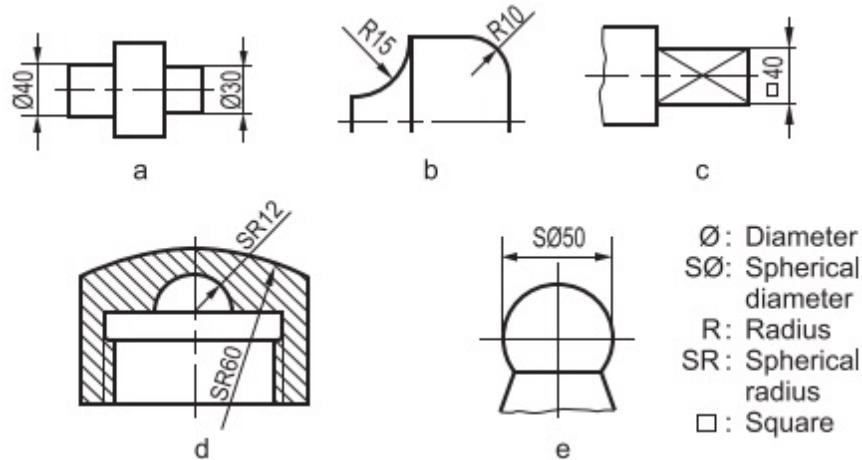
**Fig.2.44 Parallel dimensioning**



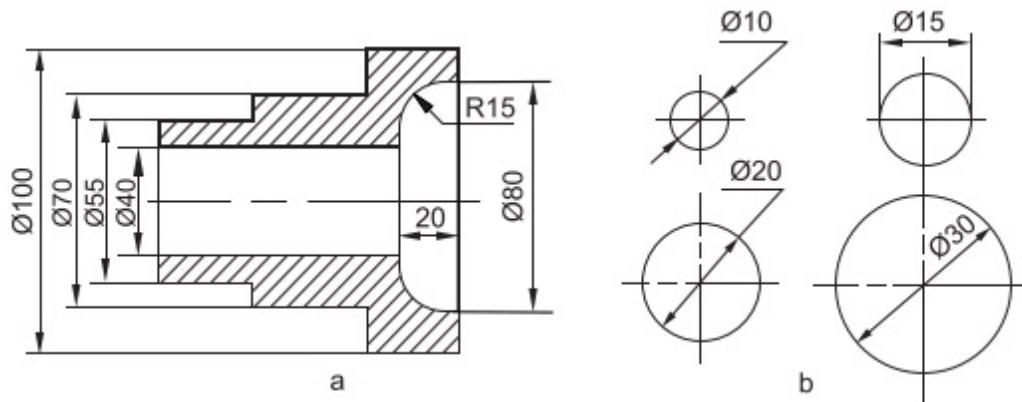
**Fig.2.45 Combined dimensioning**

## 2.8.6 Dimensioning of Common Features

The following indications are used with dimensions for shape identification and to improve the drawing interpretation ([Fig.2.46](#)). Dimensioning of holes and circles may be made as shown in [Fig.2.47](#). [Figure 2.48](#) shows the method of dimensioning an arc, when the centre falls beyond the limits of the space permitted.

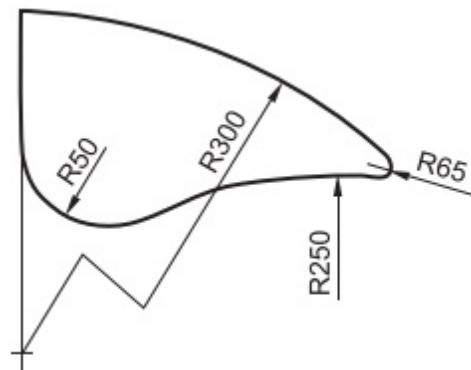


## **Fig.2.46 Shape identification symbols**

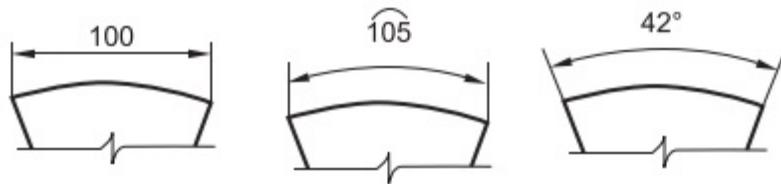


## **Fig.2.47 Dimensioning of diameters**

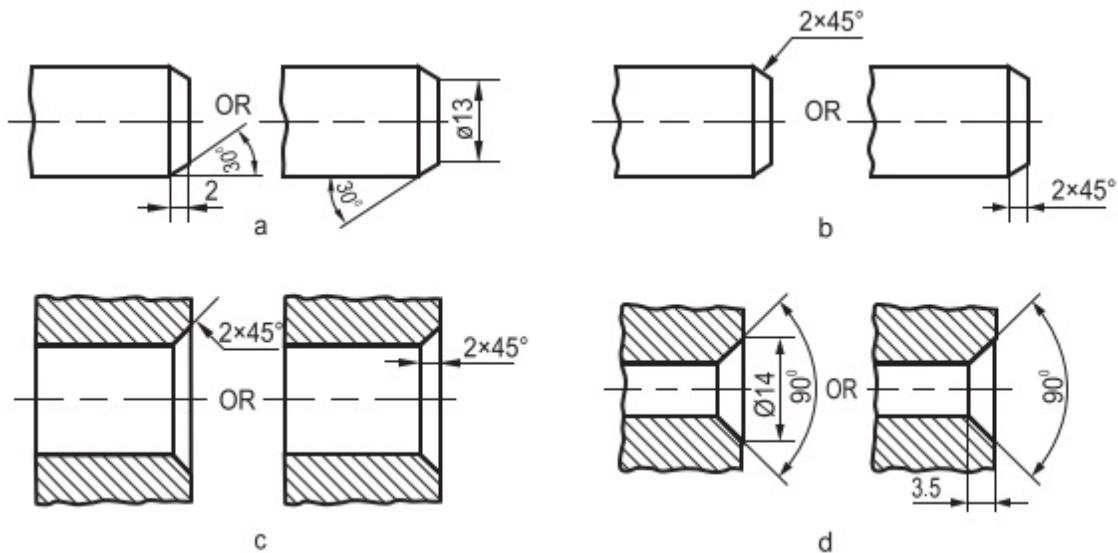
Figure 2.49 represents the method of dimensioning chords, arcs and angles. Figure 2.50 shows some methods of dimensioning chamfers and countersinks.



**Fig.2.48 Dimensioning of large radii**



**Fig.2.49 Dimensioning of chords, arcs and angles**



**Fig.2.50 Dimensioning of chamfers and countersinks**

## 2.9 CONVENTIONAL REPRESENTATION

Certain draughting conventions are used to represent materials in section. As a variety of materials are used in engineering applications, it is preferable to have different conventions of section lining to differentiate between various materials. [Figure 2.51](#) shows the recommended conventions in use.

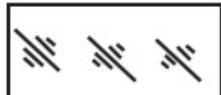
## **EXERCISES**

- 2.1 Draw the centre lines for the views shown in Figs. 2.52 A, B, C and D.
- 2.2 Dimension the views shown in Figs. 2.52 E and F.
- 2.3 For the views shown in Fig. 2.52 G, indicate the dimensions, by both the aligned and uni-directional system of dimensioning.
- 2.4 For the view shown in Fig. 2.52 H, indicate the dimensions, by both the chain and parallel dimensioning.

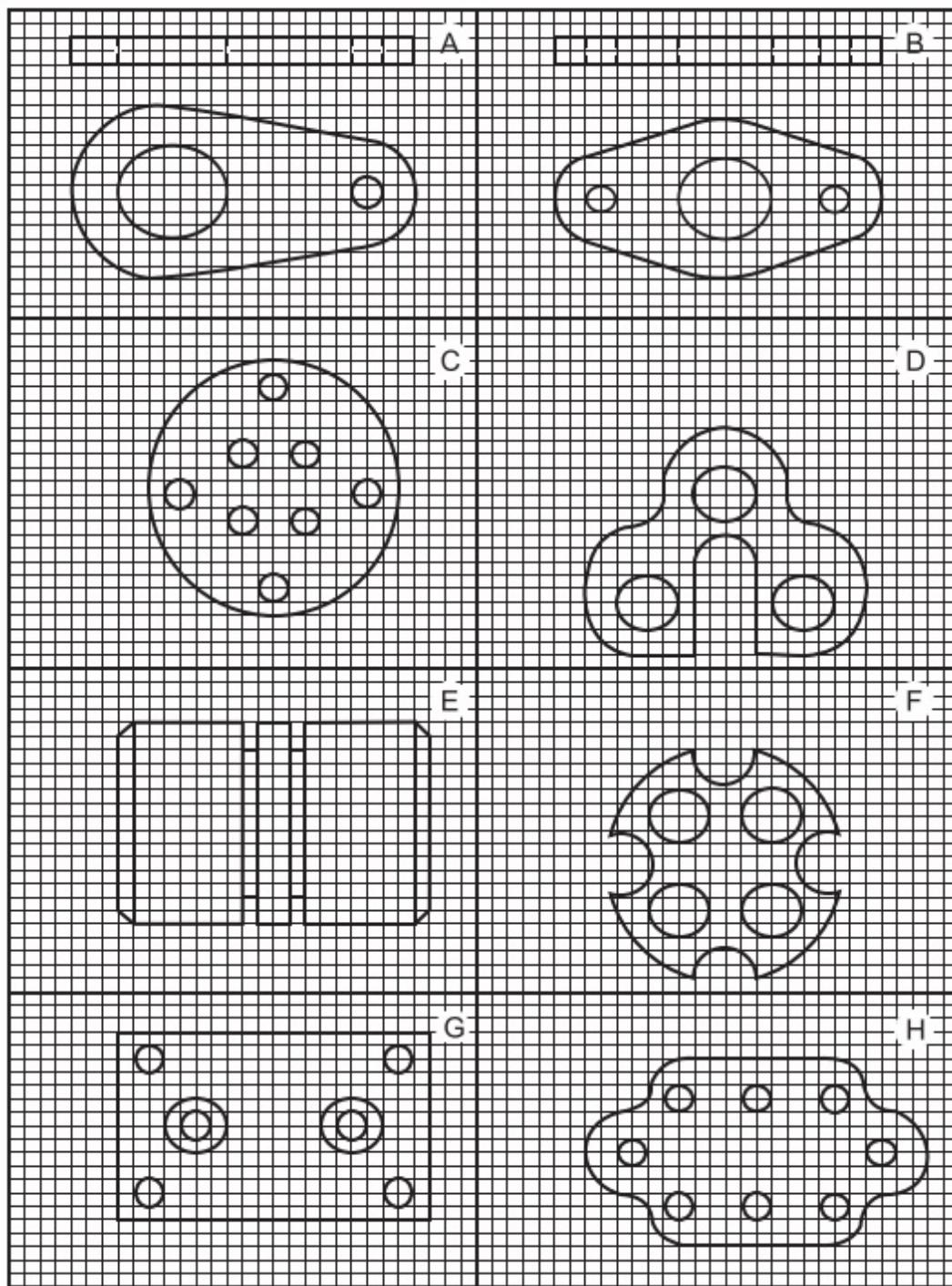
## **REVIEW QUESTIONS**

- 2.1 Explain the following terms with respect to the drawing sheet layout: (i) Borders, (ii) centring marks, (iii) metric reference graduation, (iv) grid reference system and (v) trimming marks.
- 2.2 What is the purpose of sectioning an object?
- 2.3 Differentiate between a local section and a full or half section.
- 2.4 List out the rules to be followed while dimensioning.
- 2.5 What is a leader line?
- 2.6 How are leader lines terminated?
- 2.7 How are origins and terminations of dimensions indicated?
- 2.8 Explain the two systems of dimensioning.

- 2.9 List the order of priority in representation, when two or more different lines coincide.
- 2.10 When is chain dimensioning preferred?
- 2.11 When is parallel dimensioning preferred?
- 2.12 Show the different methods of indicating diameters.
- 2.13 Indicate the methods of dimensioning chamfers and countersinks.

Type	Convention	Material
Metals		Steel, Cast Iron, Copper and its alloys, Aluminium and its alloys, etc.
		Lead, Zinc, Tin, White-metal, etc.
Glass		Glass
Packing and Insulating material		Porcelain, Stoneware, Marble, Slate, etc.
Liquids		Asbestos, Fibre, Felt, Synthetic resin products, Paper, Cork, Linoleum, Rubber, Leather, Wax, Insulating and Filling materials, etc.
Wood		Water, Oil, Petrol, Kerosene, etc.
Concrete		Wood, Plywood, etc.
		A mixture of Cement, Sand and Gravel

**Fig.2.51 Conventional representation of materials**



**Fig.2.52 (Take the side of each square as 10)**

## OBJECTIVE QUESTIONS

- 2.1 What is the area of the drawing sheet of A0 size?
- 2.2 The length to width ratio of all the drawing sheet sizes is \_\_\_\_\_.
- 2.3 Give the location of the title block in a drawing sheet.
- 2.4 Visible outlines and edges are drawn as \_\_\_\_\_.
- 2.5 Dimension lines, leader lines, projection and hatching lines are drawn as \_\_\_\_\_.
- 2.6 Centre lines are drawn as \_\_\_\_\_.
- 2.7 The size of the letters must be constant, irrespective of the size of the drawing.

(True / False)

- 2.8 For type *A* lettering, the ratio  $h/d$  is \_\_\_\_\_, where  $h$  is the height of capital letter(the base dimension) and  $d$  is the line thickness.
- 2.9 The inclination of inclined lettering is \_\_\_\_\_.
- 2.10 For obtaining a sectional view, the part of the object between \_\_\_\_\_ and \_\_\_\_\_ is assumed to be removed.
- 2.11 For a full sectional view, half of the object is assumed to be removed.

(True / False)

- 2.12 For a half sectional view, half of the object is assumed to be removed.

(True / False)

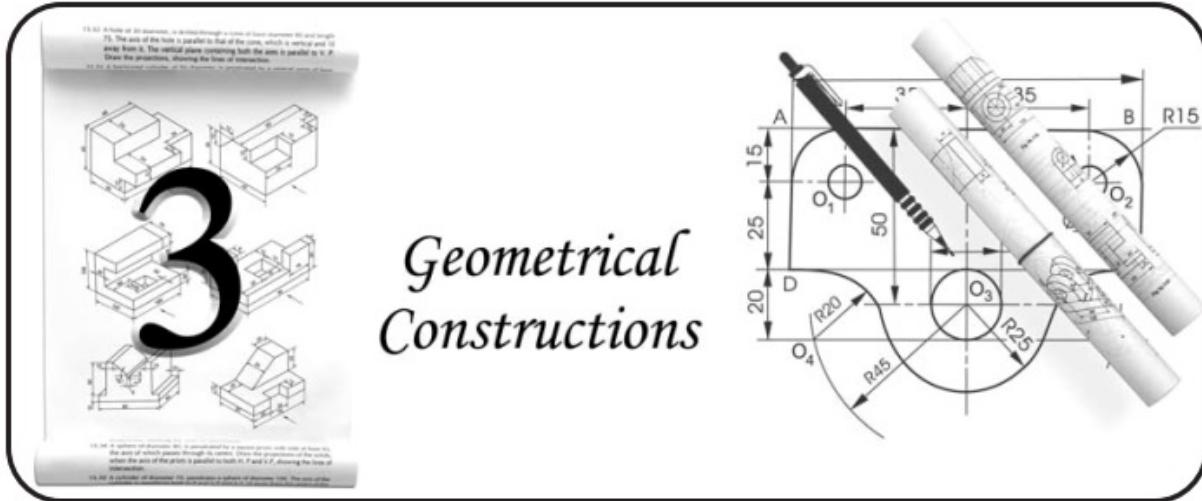
- 2.13 How is the sectioned portion represented?
- 2.14 Hatching lines are usually drawn at an angle of \_\_\_\_\_, to the major outline of the drawing.
- 2.15 Dimension lines should not \_\_\_\_\_ each other.

- 2.16 The centre line should end at the outline of the drawing. (True / False)
- 2.17 How are hidden lines represented?
- 2.18 Dimensions may be marked from hidden lines. (True /False)
- 2.19 Dimensions indicated in one view, need not be repeated in another view.  
(True /False)
- 2.20 An axis or a contour line may be used as a dimension line. (True /False)
- 2.21 What is the proportion of an arrow head?
- 2.22 The projection lines should end at the dimension line.  
(True /False)
- 2.23 In aligned system, the dimensions may be read either from \_\_\_\_\_ or from \_\_\_\_\_ side of the drawing.
- 2.24 In uni-directional system, the dimensions may be read from the top of the drawing.  
(True /False)
- 2.25 How the diameters and radii are designated?

## ANSWERS

- 2.1 One square metre
- 2.2  $\sqrt{2}:1$
- 2.3 Right hand bottom corner
- 2.4 Continuous thick lines
- 2.5 Continuous thin lines
- 2.6 Chain thin lines

- 2.7 False
- 2.8 14
- 2.9  $15^\circ$  to the right
- 2.10 Section plane, observer
- 2.11 True
- 2.12 False
- 2.13 Cross-hatched lines
- 2.14  $45^\circ$
- 2.15 Intersect
- 2.16 False
- 2.17 Dotted lines
- 2.18 False
- 2.19 True
- 2.20 False
- 2.21 Length: Depth = 3:1
- 2.22 False
- 2.23 Bottom, right hand
- 2.24 False
- 2.25 Diameter by  $\phi$  and radius by R



## 3.1 INTRODUCTION

There are a number of geometrical constructions with which a draughtsman or an engineer should be familiar, as they frequently occur in engineering drawing. The methods presented in this chapter are actually applications of the principles of plane geometry. Since the subject of plane geometry is a pre-requisite for a course in "Engineering Graphics," the mathematical proofs are omitted here.

## 3.2 SIMPLE CONSTRUCTIONS

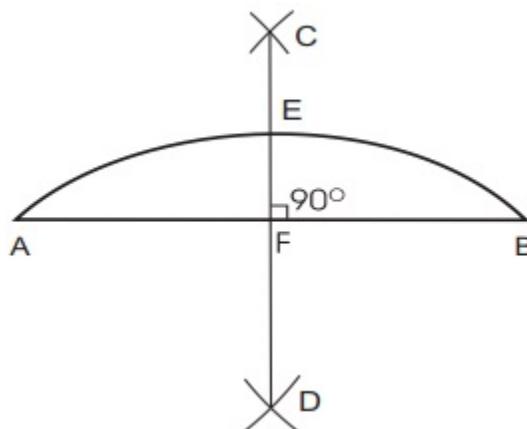
### 3.2.1 To Bisect a Given Arc

#### ***Construction (Fig.3.1)***

1. Draw the given arc AB.
2. With centre A and radius equal to more than half the chord length AB, draw arcs on either side of AB.

3. With centre B and the same radius, draw arcs intersecting the above arcs at C and D.
4. Draw a line through C and D to intersect the given arc at E.

The point E bisects the given arc.



**Fig.3.1 Bisection of an arc**



1. The above procedure may be followed to bisect the given line AB.
2. The point F bisects the line AB and the line CD is called the perpendicular bisector of the line AB.

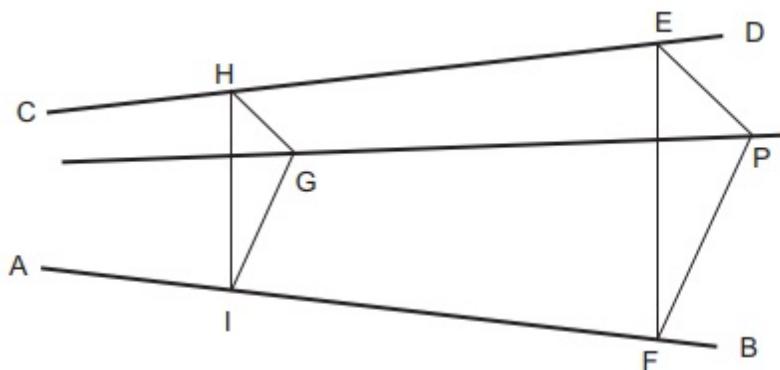
### 3.2.2 To Draw a Line Through a Given Point and the In-accessible Intersection of Two Given Lines

#### ***Construction (Fig.3.2)***

1. Draw the given lines AB and CD.
2. Locate the given point P in its correct position.

3. Construct any triangle PEF such that, the vertices E and F fall on the given lines.
4. At some other convenient location, construct another triangle GHI, similar to the triangle PEF.
5. Join G, P.

The line GP extended is the required line.



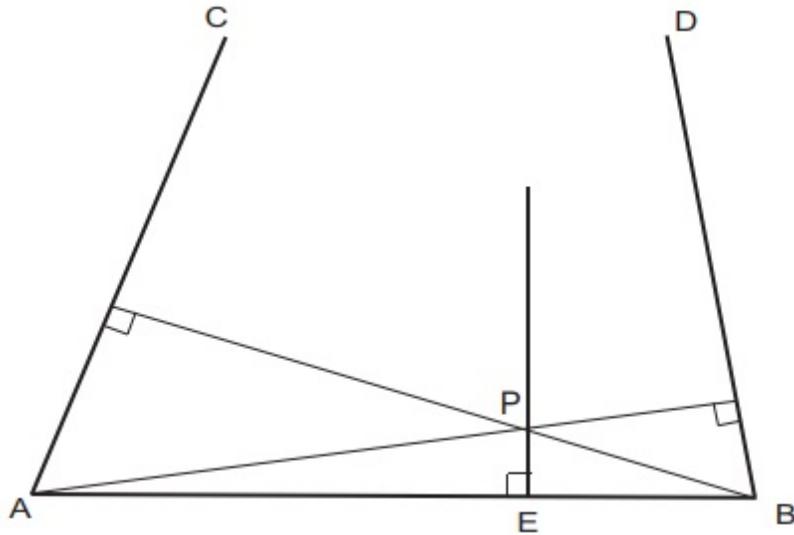
**Fig.3.2 Line through in-accessible intersection of two lines**

### 3.2.3 To Draw a Line Perpendicular to a Given Line and Through the Inaccessible Intersection of Two Other Given Lines

#### ***Construction (Fig.3.3)***

1. Draw the three given lines AB, AC and BD in their correct positions.
2. Through the points A and B, draw lines perpendicular to BD and CA respectively, intersecting at P.
3. Through P, draw a line perpendicular to AB.

The line EP extended is the required line.

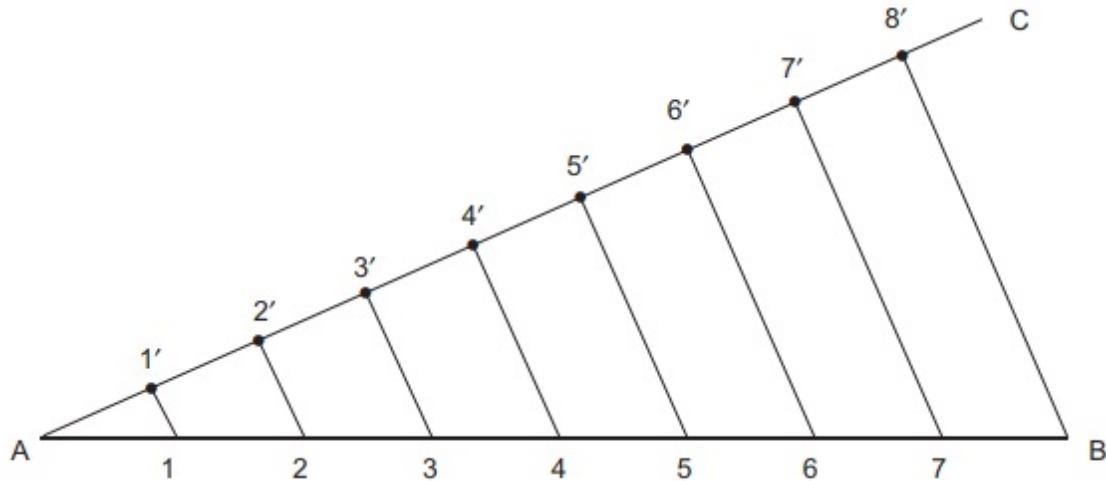


**Fig.3.3 Line through in-accessible intersection of two lines and perpendicular to the other**

### 3.2.4 To Divide a Given line into a Specified Number of Parts, Say Eight

#### ***Construction (Fig.3.4)***

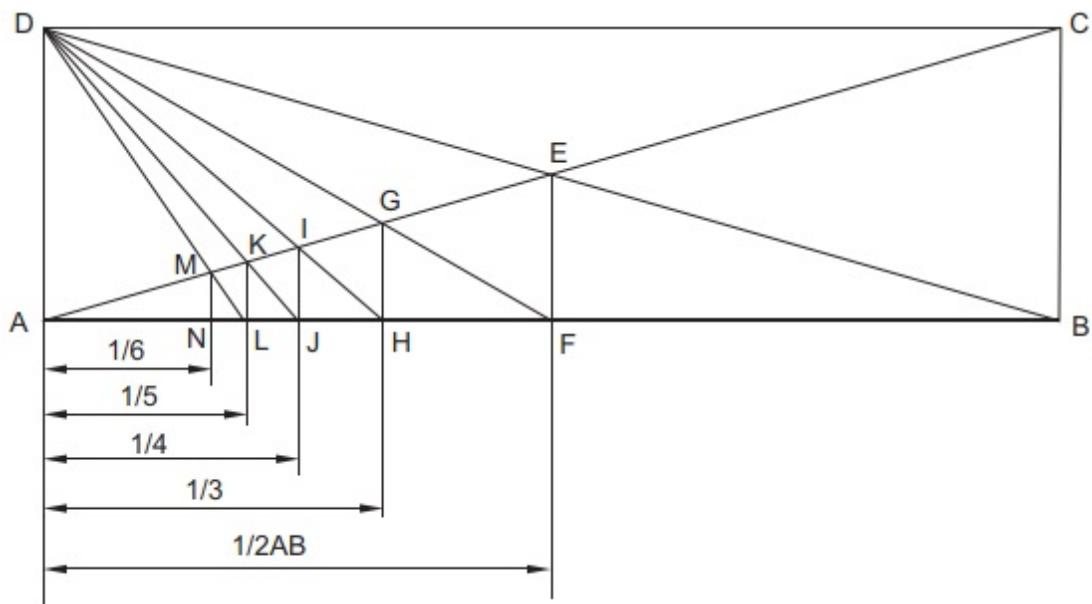
1. Draw the given line AB.
  2. Through A, draw a line AC, making any convenient angle with AB.
  3. Set the compass to any convenient length and lay-off eight equal divisions  $1'$ ,  $2'$ , ..... $8'$ , starting from A.
  4. Join  $8'$ , B.
  5. Draw lines through  $7'$ ,  $6'$ ,  $5'$ , etc., and parallel to  $8'B$ , to meet the line AB at 7, 6, 5, etc.
- The points 1, 2, 3, etc., divide the line AB into eight equal parts.



**Fig.3.4 Division of a line into a number of equal parts**

### 3.2.5 To Divide a Given Line into Unequal Parts by Proportioning

**Construction (Fig.3.5)**



**Fig.3.5 Proportionate division of a line**

1. Draw the given line AB.
2. Construct a square/rectangle ABCD, choosing the side AD as desired.
3. Draw diagonals AC and DB and locate the mid - point E at the intersection of the diagonals.
4. Through E, draw a perpendicular to AB, meeting at the mid-point F of the line AB.
5. Join D,F and locate the intersection point G on the line AC.
6. The line through G and perpendicular to AB intersects it at H ( $AH=1/3 AB$ ).
7. Similarly, locate the sub-divisions  $1/4$ ,  $1/5$ ,  $1/6$ , etc.

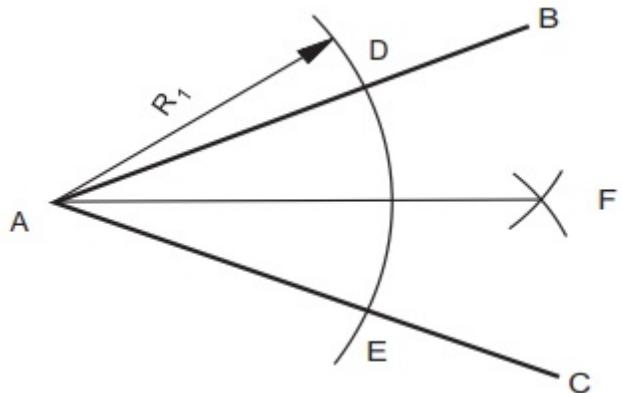
### 3.2.6 To Bisect a Given Angle

*Case I When the vertex of the angle is accessible*

**Construction (Fig.3.6)**

1. Draw the lines AB and AC, making the given angle.
2. With centre A and any convenient radius  $R_1$ , draw an arc intersecting the sides of the angle at D and E.
3. With centres D and E and radius larger than half the chord length DE, draw arcs intersecting at F.
4. Join A, F.

Now,  $\angle BAF = \angle FAC$ .



**Fig.3.6 Bisection of an angle**



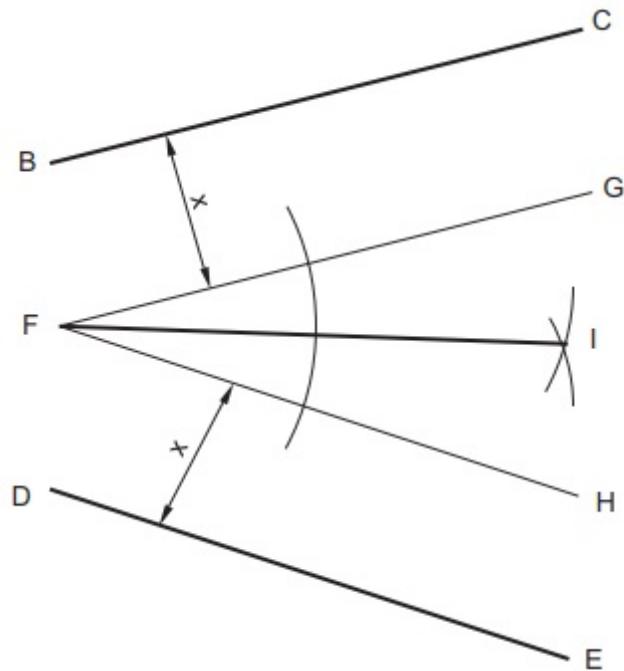
This procedure may be repeated to divide the given angle into 4, 8, 16, etc., equal parts.

*Case II When the vertex of the angle is in-accessible*

**Construction (Fig.3.7)**

1. Draw the lines BC and DE, inclined at the given angle.
2. Draw a line FG parallel to BC, at any suitable distance x.
3. Draw a line FH parallel to DE, at the distance x.
4. Bisect the  $\angle GFH$ , following the method proposed in the preceding case.

The line FI bisects the angle between the lines BC and DE.



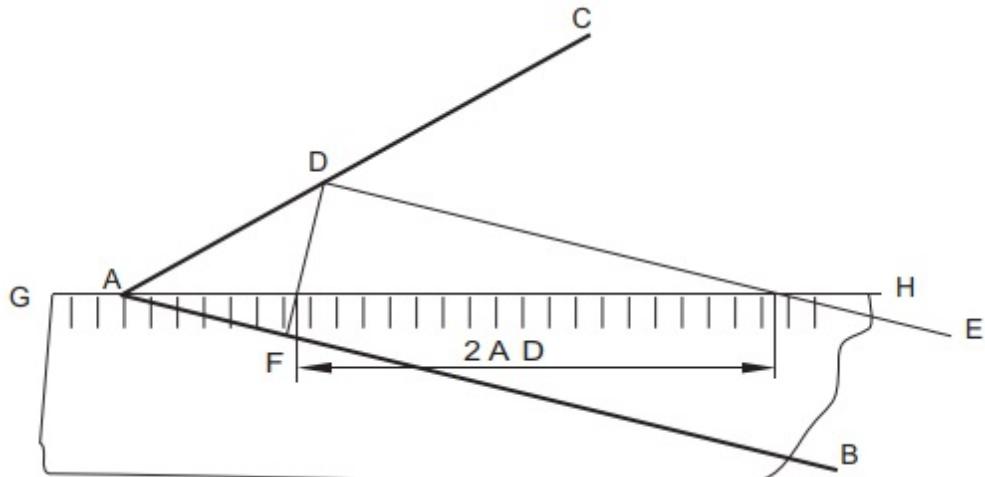
**Fig.3.7 Bisection of an angle with in-accessible vertex**

### 3.2.7 To Trisect a Given Angle

#### ***Construction (Fig.3.8)***

1. Construct the given angle CAB.
2. Locate a point D on AC, at any convenient distance.
3. Draw a line DF perpendicular to AB.
4. Draw a line DE parallel to AB.
5. Place a scale GH such that, it passes through A and a distance equal to  $2AD$  is intercepted between the lines DF and DE.

Now,  $\angle HAB = \frac{1}{3} \angle CAB$ .

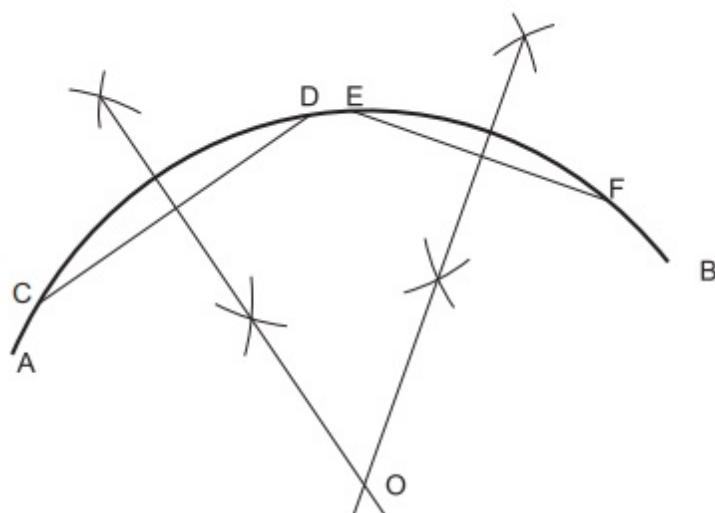


**Fig.3.8 Trisection of an angle**

### 3.2.8 To Locate the Centre of a Given Arc

#### ***Construction (Fig.3.9)***

1. Draw any two chords CD and EF to the given arc AB.
  2. Draw perpendicular bisectors to CD and EF, intersecting each other at O.
- Point O is the required centre.



### **Fig.3.9 Location of the centre of an arc**



The above procedure may be followed to locate the centre of a circle.

### **3.2.9 To Draw a Curve Parallel to Another Curve at a Given Distance**

#### ***Construction (Fig.3.10)***

1. With radius equal to the given distance, draw overlapping arcs, by selecting number of points as centres on the given curve AB.
2. Draw a smooth curve CD, touching the above arcs tangentially.

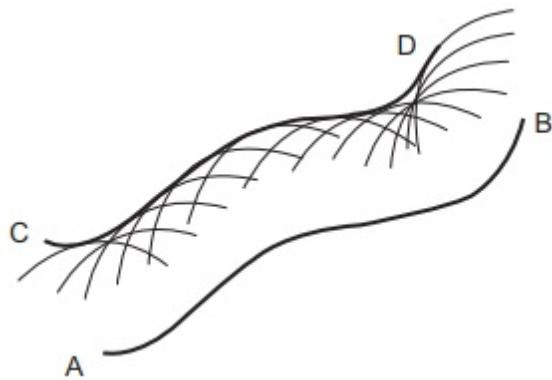
The curve CD is the required parallel curve.

### **3.2.10 To Find the Length of a Given Arc**

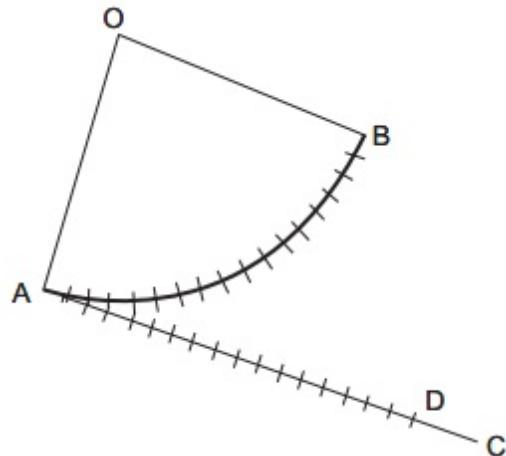
#### ***Construction (Fig.3.11)***

1. With centre O, draw the given arc AB.
2. Divide the given arc into a number of equal parts (the more the number, the better the accuracy).
3. Set the divider equal to the chord length of one of the parts.
4. On any line AC, step-off the divisions equal to the divisions on the curve, reaching the point D on it.

The length of the line AD is approximately equal to the arc length of the curve AB.



**Fig.3.10 Drawing a curve parallel to a given curve**



**Fig.3.11 Determination of the length of an arc**



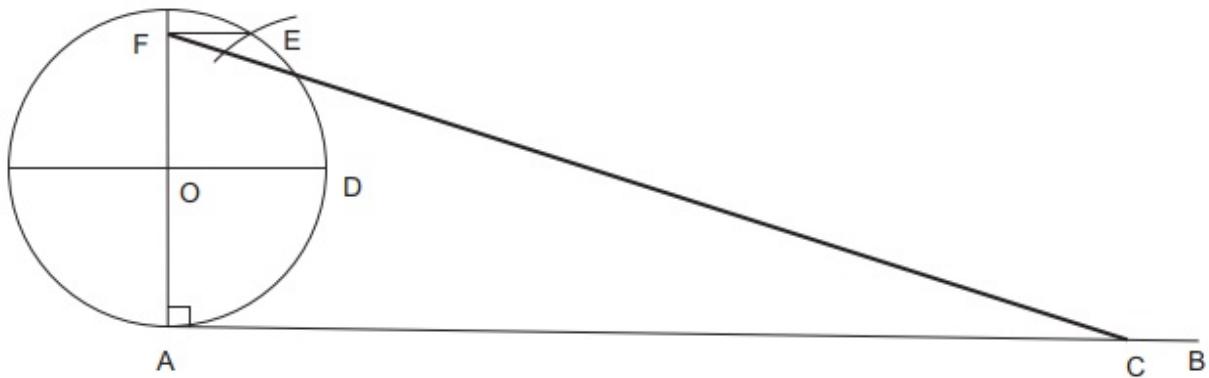
The above procedure may be followed to find the approximate length of the circumference of a circle.

### 3.2.11 To Find the Approximate Length of the Circumference of a Given Circle

### **Construction (Fig.3.12)**

1. With centre O, draw the given circle.
2. Draw the vertical diameter through O, meeting the circle at A.
3. Through A, draw a line AB at right angle to AO.
4. Along the line AB, set-off distance AC equal to three times the diameter of the circle.
5. With centre D on the horizontal diameter and radius equal to the radius of the circle, draw an arc intersecting the circumference of the circle at E.
6. Draw the line EF, parallel to AB.
7. Join FC.

The length FC is approximately equal to the circumference of the circle.



**Fig.3.12 Determination of the circumference of a circle**

**3.2.12 To Draw an Arc, Passing Through Three Points, Not in a Straight Line**

### ***Construction (Fig.3.13)***

1. Locate the given points A, B and C.
2. Draw lines AB and BC.
3. Draw perpendicular bisectors of the lines AB and BC, intersecting at O.
4. With centre O and radius OA ( $=OB =OC$ ), draw an arc. The arc passes through the points A, B and C.

## **3.3 POLYGONS**

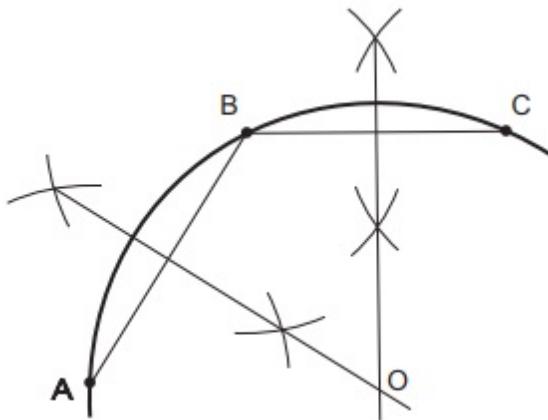
A polygon is a plane figure, bounded by straight edges. When all the edges are of equal length, the polygon is said to be a regular polygon.

### **3.3.1 To Construct an Equilateral Triangle, Given the Altitude**

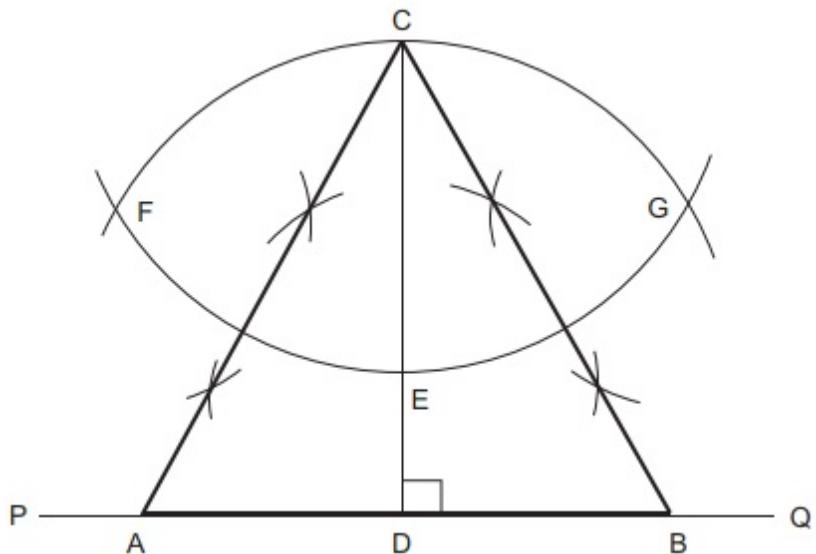
#### ***Construction (Fig.3.14)***

1. Draw any line PQ and select a point D on it.
2. Through the point D, draw a perpendicular DC to the line PQ such that, DC is the altitude.
3. With centre C and any convenient radius, draw an arc intersecting CD at E.
4. With centre E and the same radius, draw arcs intersecting the above arc at F and G.
5. Draw the bisectors of the arcs EF and EG, passing through C, and meeting the line PQ at A and B respectively.

ABC is the required triangle.



**Fig.3.13 Drawing an arc through three points**



**Fig.3.14 Drawing an equilateral triangle of known altitude**

### 3.3.2 To Construct a Square, Given the Length of Diagonal

*Construction (Fig.3.15)*

- With centre O, and the diagonal length as diameter,
1. draw a circle.
  2. Through the centre O, draw two diameters AC and BD and at right angle to each other.
  3. Join A, B; B, C; C, D and D, A.
- ABCD is the required square.

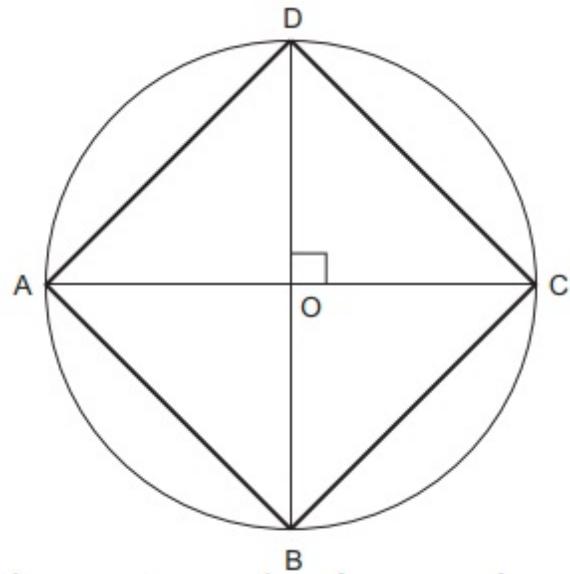
### 3.3.3 To Construct a Regular Polygon of Any Number of Sides N, Given the Length of its Side

#### ***Method I***

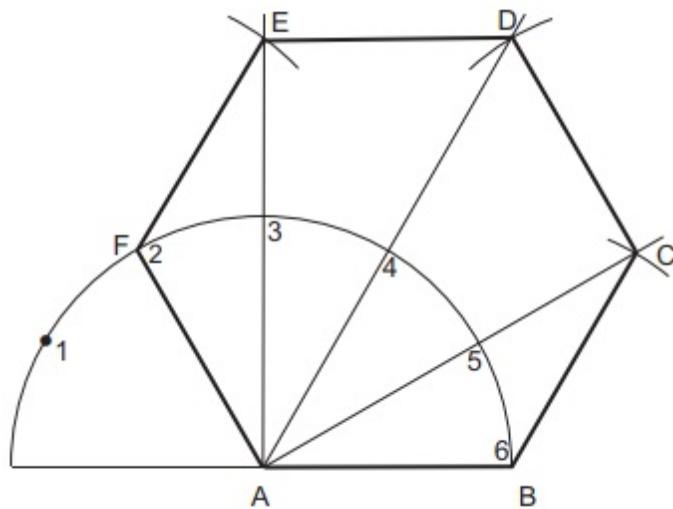
##### ***Construction (Fig.3.16)***

1. Draw a line AB equal to the length of the side.
2. With centre A and radius AB, draw a semi-circle.
3. Divide the semi-circle into the same number of equal parts, as the number of sides N, say 6.
4. Draw radial lines through 2, 3, 4, etc. (second division point 2 will always be a vertex of the polygon).
5. With centre B and radius equal to the side, draw an arc intersecting the radial line through 5 at C.
6. Repeat the procedure till the point on the radial line through 3 is obtained.

The figure obtained by joining the points A, B, C, etc., is the required polygon.



**Fig.3.15 Construction of a square of known diagonal**



**Fig.3.16 Construction of a regular polygon-Method I**

### ***Method II***

#### ***Construction (Fig.3.17)***

1. Follow the steps 1 to 4 as above.

2. Draw perpendicular bisectors of lines  $2A$  and  $AB$ , intersecting at  $O$ .
3. With centre  $O$  and radius  $OA$ , draw a circle passing through the points  $2$  and  $B$ .
4. Locate the corners  $C$ ,  $D$ , etc., of the polygon, where the circle meets the radial lines.

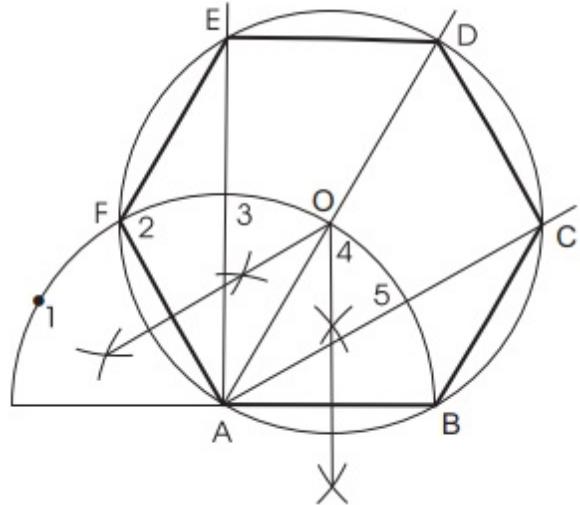
The figure obtained by joining the points  $A$ ,  $B$ ,  $C$ , etc., is the required polygon.

### **Method III**

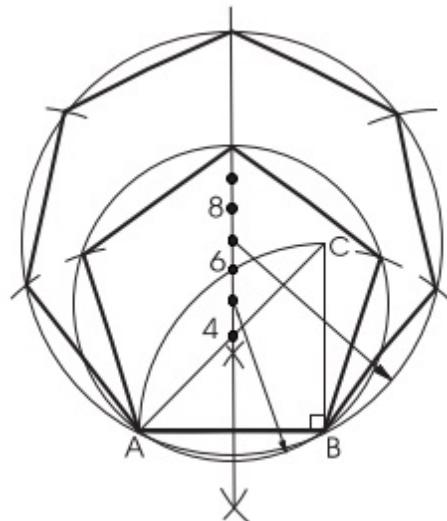
#### **Construction (*Fig.3.18*)**

1. Draw a line  $AB$  equal to the given length of side.
2. At  $B$ , erect a perpendicular  $BC$  of length  $AB$ .
3. Join  $A$ ,  $C$ .
4. With centre  $B$  and radius  $BA$ , draw the arc  $AC$ .
5. Draw a perpendicular bisector to  $AB$ , intersecting the line  $AC$  at  $4$  and the arc  $AC$  at  $6$ .
6. Locate the mid-point of the line  $4-6$  and number it as  $5$ .
7. Along the bisector, locate the points  $7$ ,  $8$ , .... $N$ , such that, the distances  $4-5 = 5-6 = 6-7$ , etc.
8. A pentagon of side equal to  $AB$  can be inscribed in a circle drawn with centre  $5$  and radius  $5A$ .
9. A heptagon of side equal to  $AB$  can be inscribed in a circle drawn with centre  $7$  and radius  $7A$ .

A polygon of any number of sides,  $N$  can be inscribed in a circle drawn with centre  $N$  and radius  $NA$ .



**Fig.3.17 Construction of a regular polygon-Method II**



**Fig.3.18 Construction of a regular polygon-Method III**

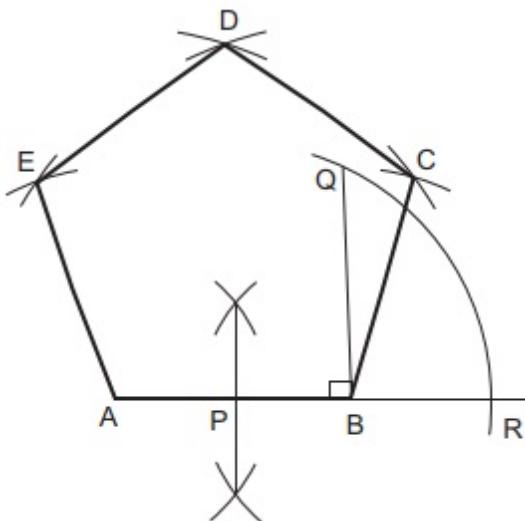
## 3.4 SPECIAL METHODS

### 3.4.1 To Construct a Pentagon, Given the Length of Side

### **Construction (Fig.3.19)**

1. Draw a line AB, equal to the given length of side.
2. Bisect AB at P.
3. At B, erect a perpendicular BQ, equal in length to AB.
4. With centre P and radius PQ, draw an arc intersecting AB produced at R (AR is equal to the diagonal length of the pentagon).
5. With centres A and B and radii equal to AR and AB respectively, draw arcs intersecting at C.
6. With centres A and B and radius AR, draw arcs intersecting at D.
7. With centres A and B and radii equal to AB and AR respectively, draw arcs intersecting at E.

The figure obtained by joining the points B, C, D, E and A is the required pentagon.



**Fig.3.19 Construction of a pentagon**

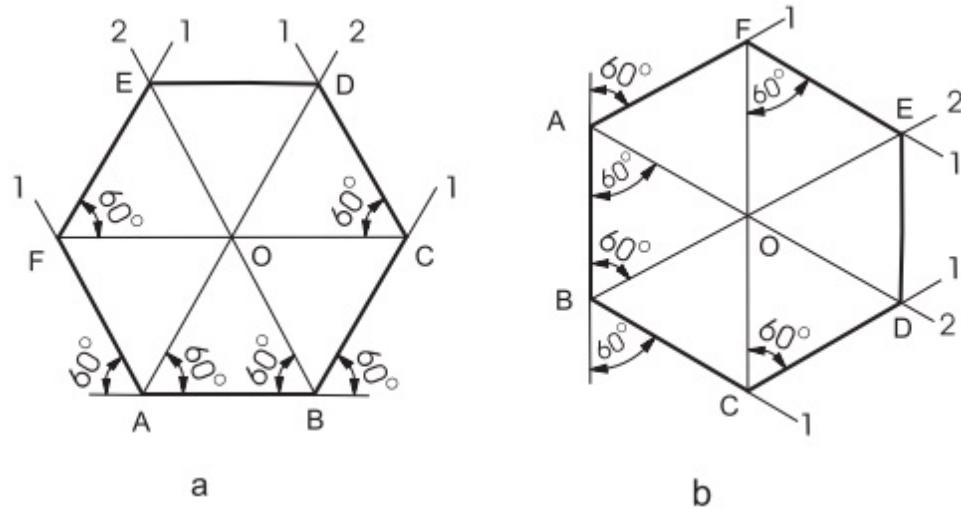


The lines joining the alternate corners of a pentagon are called diagonals.

### 3.4.2 To Construct a Hexagon

**Case I** Length of side is given

**Construction (Figs.3.20a and b)**



**Fig.3.20 Construction of a hexagon-Given a side**

1. Draw a line and mark AB on it, equal to the side of the hexagon.
2. Using  $30^\circ$ -  $60^\circ$  set-square, draw lines A1, A2, B1 and B2 as shown.
3. Through the point of intersection O, of the lines A2 and B2, draw a line parallel to AB intersecting A1 and B1 at F and C respectively.
4. Draw lines F1 and C1 at  $60^\circ$  as shown, intersecting B2 at E and A2 at D.
5. Join E, D.

ABCDEF is the required hexagon.

**Case II** Distance across corners is given

**Construction (Fig.3.21)**

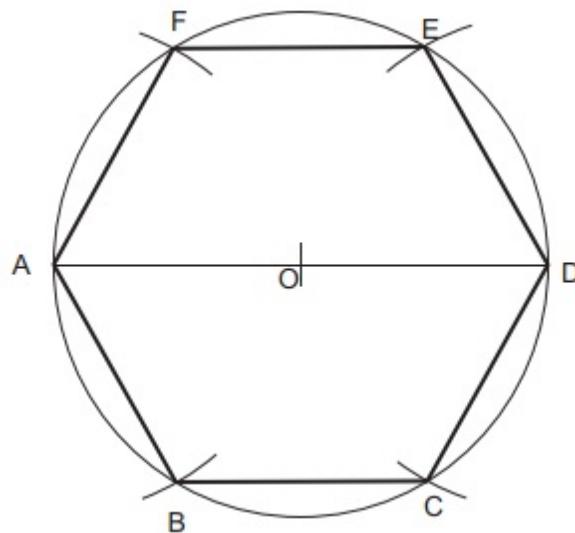
1. Draw a circle, with the distance across corners AD as diameter.
2. With centres A and D and radius equal to the radius of the circle, draw arcs intersecting the circumference at B, F and C, E respectively.

The figure obtained by joining the points A, B, C, etc., is the required hexagon.

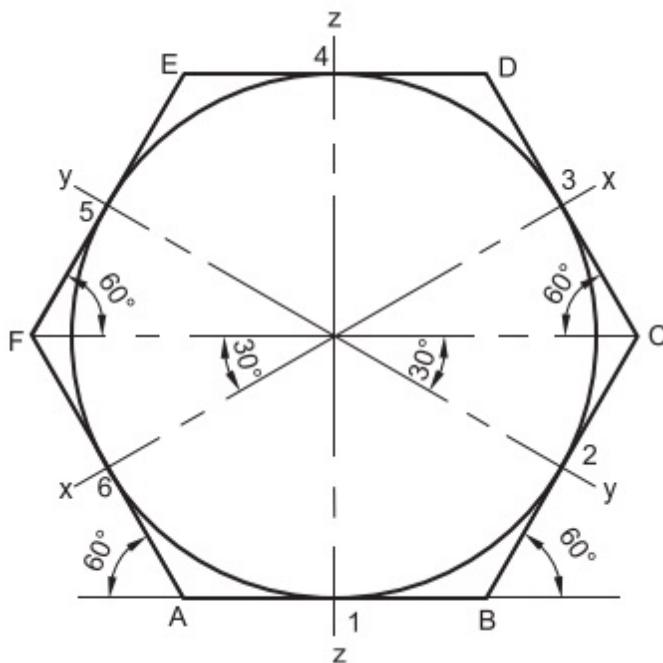
**Case III Distance across flats is given**

**Construction (Fig.3.22)**

1. With centre O, draw a circle, with distance across flats as diameter.
2. Through O, draw a vertical line z-z and using  $30^\circ - 60^\circ$  set-square, draw the lines xx and yy at  $30^\circ$  from the horizontal.
3. Locate the points of intersection 1, 2, 3, etc., between the circle and the above lines.
4. Draw tangents to the circle at the above points, viz., AB, BC, etc., forming the required hexagon.



**Fig.3.21 Construction of a hexagon-Given distance across corners**



**Fig.3.22 Construction of a hexagon-Given distance across flats**

### 3.4.3 To Construct an Octagon, Given the Distance Across Flats

#### *Construction (Fig.3.23)*

1. Draw a square, with its side equal to the distance across flats.
2. Draw the diagonals to the square.
3. With corners as centres and one half of the diagonal as radius, draw arcs intersecting the sides of the square.  
The figure obtained by joining these points in the order is the required octagon.

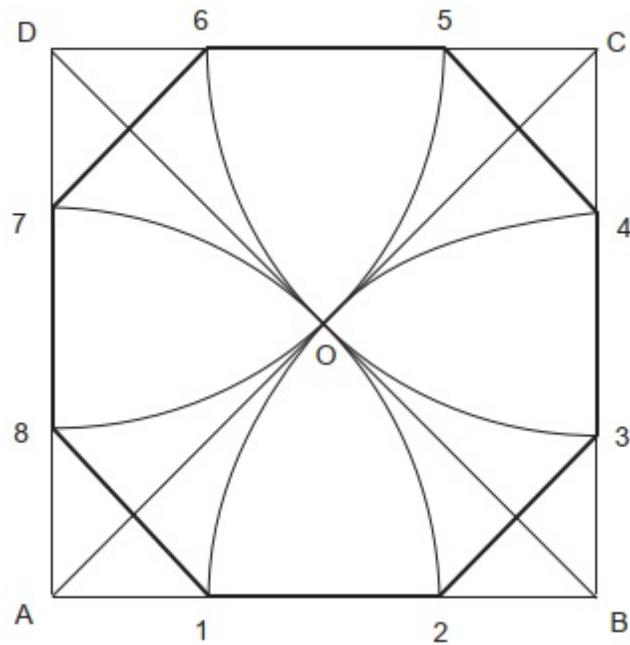
## **3.5 INSCRIBED AND DESCRIBED FIGURES**

### **3.5.1 To Inscribe a Regular Polygon of Any Number of Sides, N in a Given Circle**

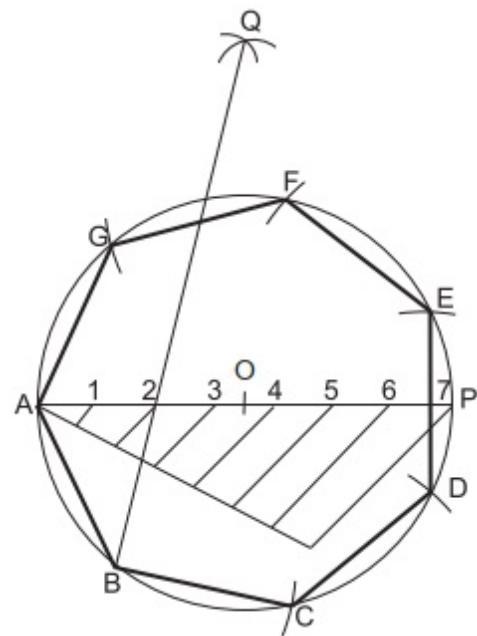
#### ***Construction (Fig.3.24)***

1. Draw the given circle of diameter AP.
2. Divide the diameter AP into N equal parts, say 7.
3. With radius AP and centres A and P, draw arcs intersecting each other at Q.
4. Join Q, 2 and extend to meet the circle at B.
5. Join A, B; which is the side of the required polygon.
6. Starting from B, mark-off length AB on the circumference of the circle and obtain the points C, D, E, etc.

The figure obtained by joining the points in the order is the required polygon.



**Fig.3.23 Construction of an octagon-Given distance across flats**



**Fig.3.24 Inscribing a regular polygon**

### **3.5.2 To Describe any Regular Polygon about a Given Circle**

#### ***Method I***

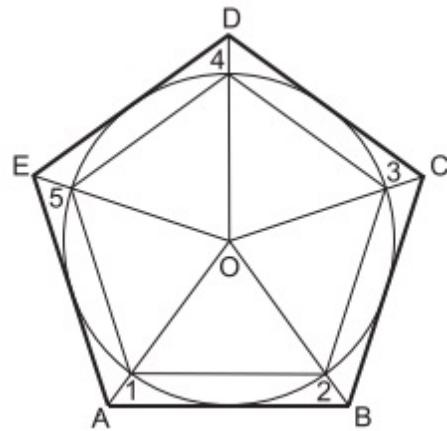
##### ***Construction (Fig.3.25)***

1. With centre O, draw the given circle.
2. Divide the circumference of the circle into a number of equal parts, equal to the sides of the required polygon, say 5.
3. Join O, 1; O, 2; etc., and extend.
4. Join 1, 2 which is equal to the side of the inscribed pentagon.
5. Draw a line, touching the circle and parallel to 1-2, intersecting the lines O1 and O2 extended, at A and B respectively.
6. Mark OC, OD and OE equal to OB ( $=OA$ ).
7. Join B, C; C, D; D, E and E, A.

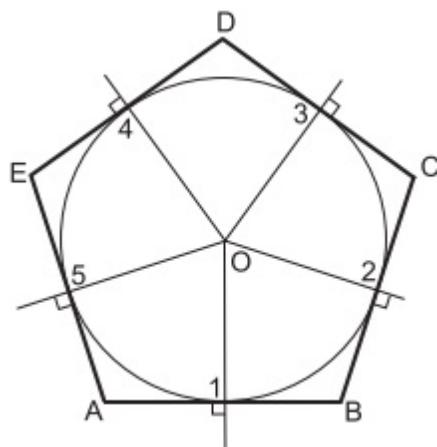
The figure ABCDEA is the required pentagon.

#### ***Method II***

##### ***Construction (Fig.3.26)***



**Fig.3.25 Describing a regular polygon-Method I**



**Fig.3.26 Describing a regular polygon-Method II**

1. Follow the steps 1 to 3 as above.
2. Through the points 1, 2, 3, 4 and 5, draw perpendiculars to the lines O1, O2, O3, O4 and O5, intersecting at points B, C, D, E and A respectively.

The enclosed figure thus formed is the required pentagon.

### 3.5.3 To Draw the Circum-circle of a Triangle

The circle passing through the vertices of a triangle is called the circum-circle of the triangle.

### ***Construction (Fig.3.27)***

1. Draw the given triangle ABC.
2. Draw perpendicular bisector of any two sides, say AB and BC, intersecting at O.
3. With centre O and radius OA ( $=OB=OC$ ), draw a circle.  
Circle passing through the points A, B and C is the required one.

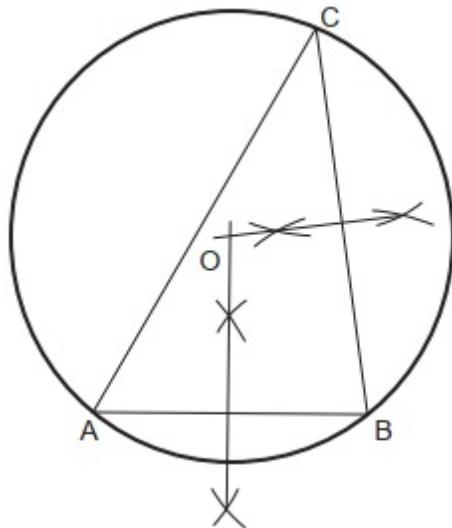
### **3.5.4 To Draw the In-circle of a Triangle**

The circle drawn inside a triangle such that, all the sides of the triangle are tangential to it, is called the in-circle of the triangle.

### ***Construction (Fig.3.28)***

1. Draw the given triangle ABC.
2. Draw bisectors of any two angles, say  $\angle CAB$  and  $\angle ABC$ , intersecting at O.
3. Draw OD, perpendicular to any side, say BC.
4. Draw a circle with centre O and radius OD.

The circle, touching all the three sides is the required one.



**Fig.3.27 Drawing the circum-circle of a triangle**



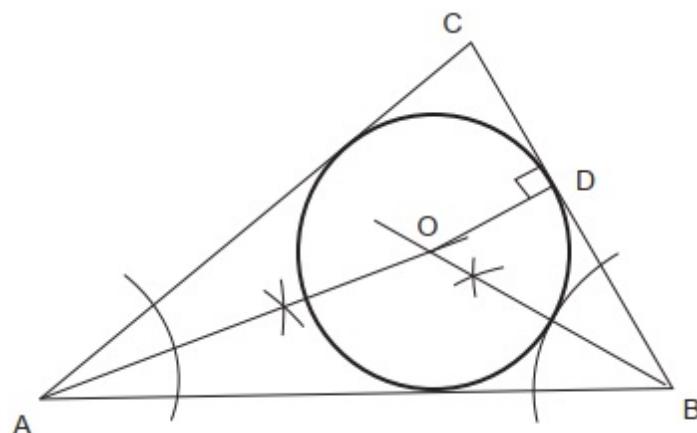
The above procedure may be followed to inscribe a circle in a regular polygon of any number of sides.

### 3.5.5 To Draw an Equilateral Triangle in a Given Circle

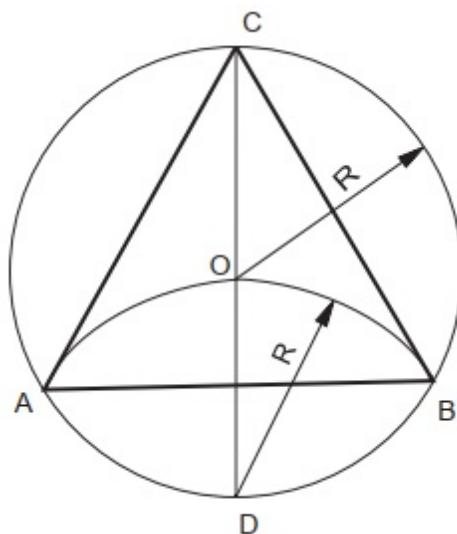
#### ***Construction (Fig.3.29)***

1. Draw the given circle of radius  $R$  and mark any diameter  $CD$  of it.
2. With centre  $D$  and radius  $R$ , draw an arc intersecting the circle at  $A$  and  $B$ .
3. Join  $A, B$ ;  $B, C$  and  $C, A$ .

$ABC$  is the required equilateral triangle.



**Fig.3.28 Drawing the in-circle of a triangle**



**Fig.3.29 Inscribing an equivalent triangle**

## 3.6 TRANSFER OF FIGURES

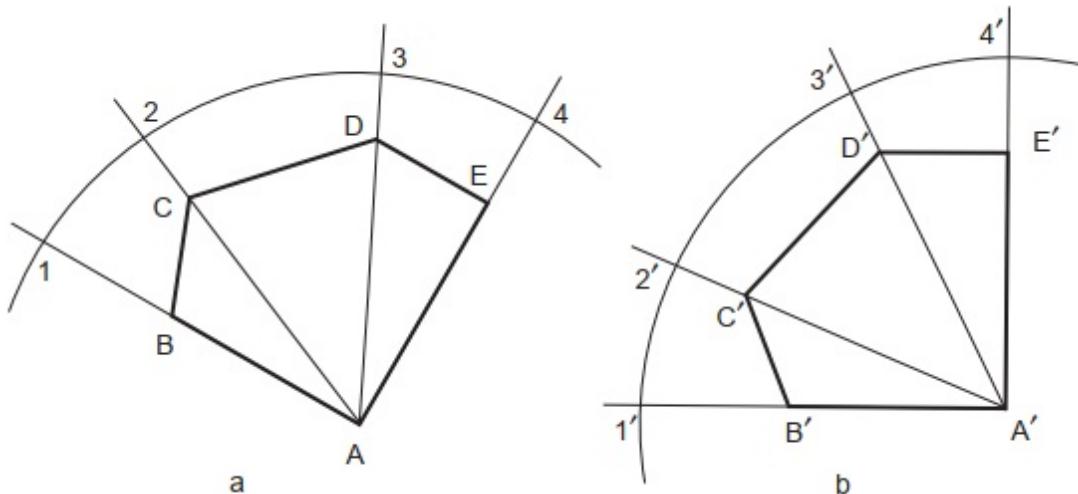
### 3.6.1 To Transfer any Plane Figure, Consisting of Straight Lines

*Construction (Fig.3.30)*

1. Draw the given plane figure ABCDEA.

2. Join A, B; A,C; A, D and A,E and extend.
3. With centre A and any convenient radius (preferably greater than the longest length of AB, AC, AD and AE), draw an arc intersecting the above lines at 1, 2, 3 and 4.
4. Draw a line  $1'A'$  ( $= 1A$ ) in the desired position, with respect to which it is required to transfer the figure.
5. With centre  $A'$  and radius  $A1'$ , draw an arc.
6. Mark points  $2'$ ,  $3'$  and  $4'$  on the above arc such that,  $1'-2' = 1-2$ , etc.
7. Join  $A', 2'; A', 3'$  and  $A', 4'$ .
8. On the lines  $A'1'$ ,  $A'2'$ ,  $A'3'$  and  $A'4'$ , locate the points  $B'$ ,  $C'$ ,  $D'$  and  $E'$  such that,  $A'B' = AB$ ;  $A'C' = AC$ ;  $A'D' = AD$  and  $A'E' = AE$ .

The figure obtained by joining the points in the order is the required one.

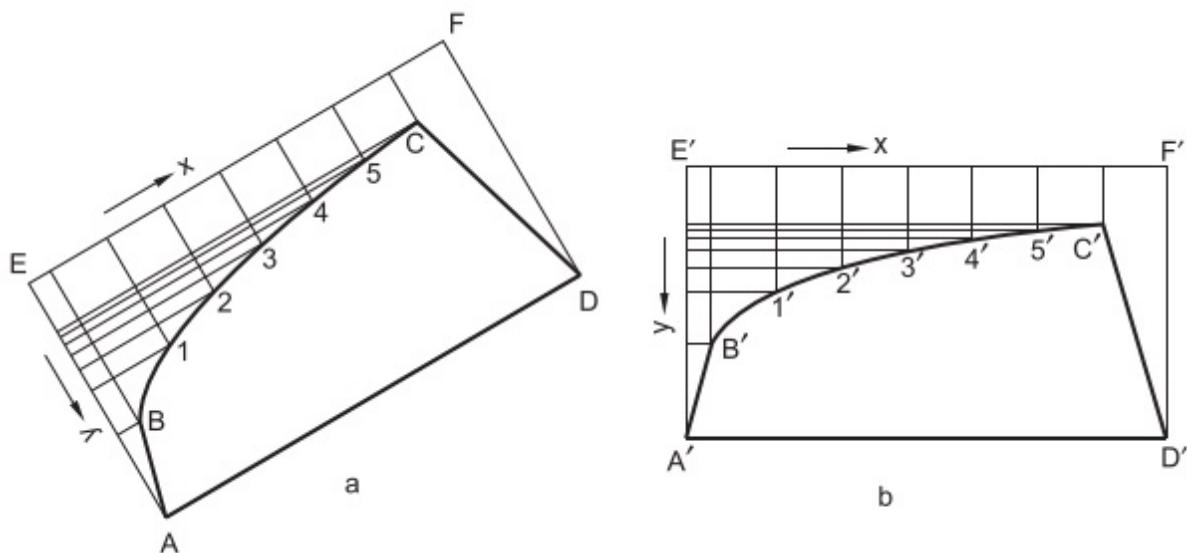


**Fig.3.30 Transferring a plane figure-Bounded by straight lines**

### 3.6.2 To Transfer any Plane Figure having Irregular Curves

#### ***Construction (Fig.3.31)***

1. Draw the given plane figure ABCD.
2. Construct a rectangle AEFD, of suitable size, enclosing the given figure.
3. Select a number of points 1, 2, 3, etc., on the curve such that, the curve can be located accurately.
4. Treating, say E as the origin, measure the co-ordinates of the points B, 1, 2, 3, etc.
5. Reconstruct a rectangle A'E'F'D' (=AEFD) in the required position.
6. Using the co-ordinates, locate the points B', 1', 2', 3', etc., in the rectangle A'E'F'D'. The figure obtained by joining the points in the order is the required one.



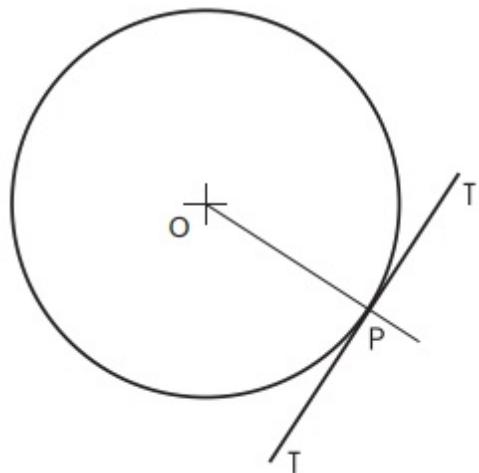
**Fig.3.31 Transferring a plane figure-Bounded by curved lines**

## 3.7 TANGENTS

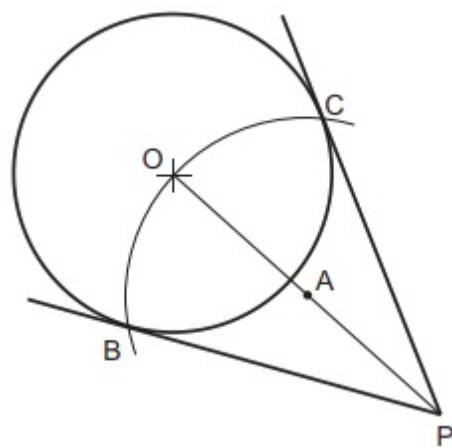
A line, meeting a curve at a point on it is called the tangent to the curve.

### 3.7.1 To Draw a Tangent to a Given Circle, at any Point on it

**Construction (Fig.3.32)**



**Fig.3.32 Tangent to a circle at a point on it**



### **Fig.3.33 Tangent to a circle from an outside point**

1. With centre O, draw the given circle and locate the given point P on it.
2. Join O, P and extend.
3. Draw a perpendicular TT to the above line at P.  
The line TT is the required tangent.

### **3.7.2 To Draw a Tangent to a Given Circle, from any Point Outside it**

#### ***Construction (Fig.3.33)***

1. With centre O, draw the given circle.
2. Locate the given point P in its correct position.
3. Join P, O and locate its mid-point A.
4. With centre A and radius AO, draw an arc intersecting the given circle at B and C.
5. Join P, B; P, C and extend.

The lines PB and PC extended are the two possible tangents.

### **3.7.3 To Draw a Tangent to a Given Arc of In-accessible Centre, at any Point on it**

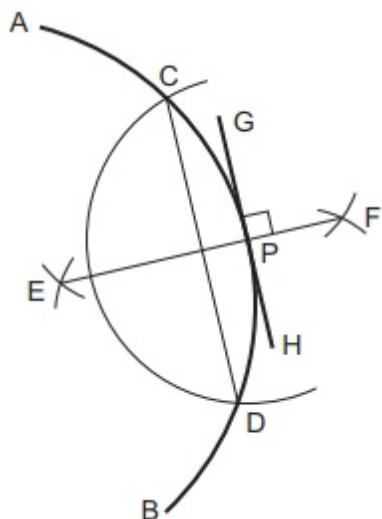
#### ***Construction (Fig.3.34)***

1. On the given arc AB, locate the given point P.

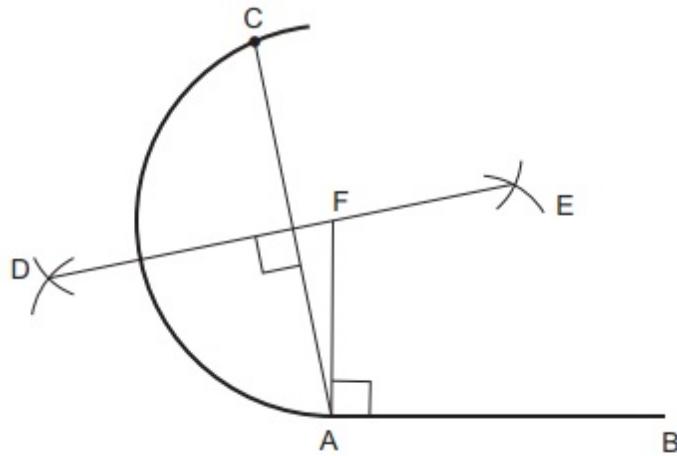
2. With centre P and any convenient radius, draw an arc intersecting the given arc at C and D.
  3. Draw perpendicular bisector EF of the chord CD, passing through P.
  4. Through P, draw a line GH, perpendicular to EF.
- GH is the required tangent.

### 3.7.4 To Connect a Given Point with a Given Line, by an Arc Smoothly (Tangentially)

**Construction (Fig.3.35)**



**Fig.3.34 Tangent to an arc having in-accessible centre**



**Fig.3.35 Connecting a point to a line by an arc**

1. Draw the given line AB.
2. Locate the given point C in its correct position.
3. Join A, C and draw the perpendicular bisector DE to it.
4. Through A, draw a perpendicular to AB, intersecting the line DE at F.
5. With centre F and radius FA ( $=FC$ ), draw an arc.  
The arc passing through C is the required one.

### 3.7.5 To Draw an Arc of given Radius $r$ through a Given Point, Tangential to a Given Circle of Radius R

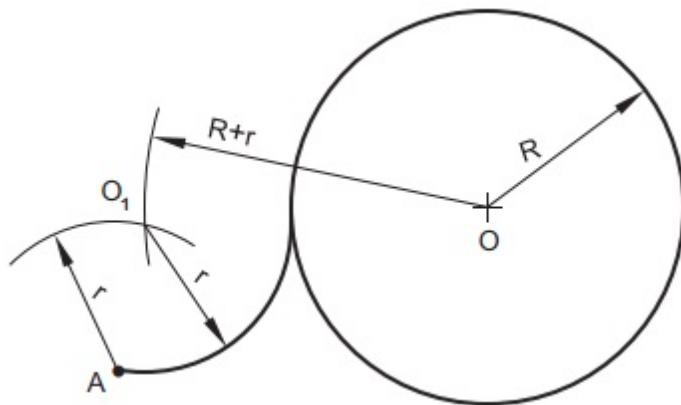
#### ***Construction (Fig.3.36)***

1. With centre O and radius R, draw the given circle.
2. Locate the given point A in its correct position.
3. With centres A and O and radii  $r$  and  $R + r$  respectively, draw arcs intersecting each other at  $O_1$ .

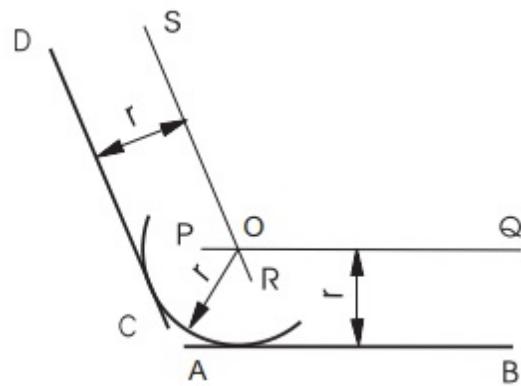
4. With centre  $O_1$  and radius  $r$ , draw an arc.  
 The arc passing through point A is the required one.

### 3.7.6 To Connect Two given Straight Lines, With an Arc of Given Radius r

**Construction (Fig.3.37)**



**Fig.3.36 Connecting a circle and a point by an arc of known radius**



**Fig.3.37 Connecting two straight lines by an arc of known radius**

1. Draw the given lines AB and CD.
2. Draw two lines PQ and RS parallel to AB and CD respectively and at a distance  $r$ , intersecting at O.
3. With centre O and radius  $r$ , draw an arc.

The arc, meeting the given lines tangentially, is the required one.

### 3.7.7 To Draw an Arc of Given Radius $R$ , Touching a Given Arc and a Given Straight Line

**Case I** The centres of the two arcs lie on the same side

**Construction (Fig.3.38)**

1. Draw the given straight line AB.
2. With centre  $O_1$  and radius  $R_1$ , draw the given arc CD.
3. Draw a line EF, parallel to the line AB and at a distance  $R$  from it .
4. With centre  $O_1$  and radius  $(R_1 - R)$ , draw an arc intersecting the line EF at O.
5. With centre O and radius R, draw an arc.

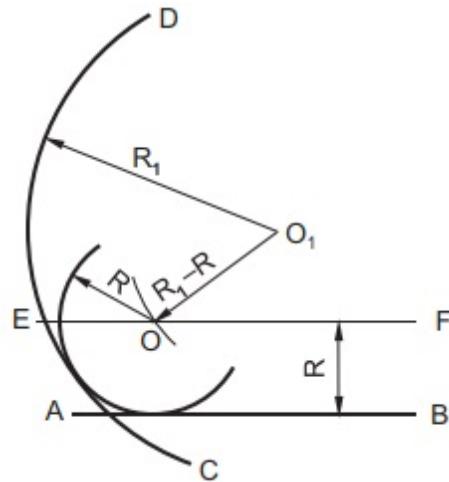
The arc meets the given arc and the line tangentially.

**Case II** The centres of the two arcs lie on opposite sides

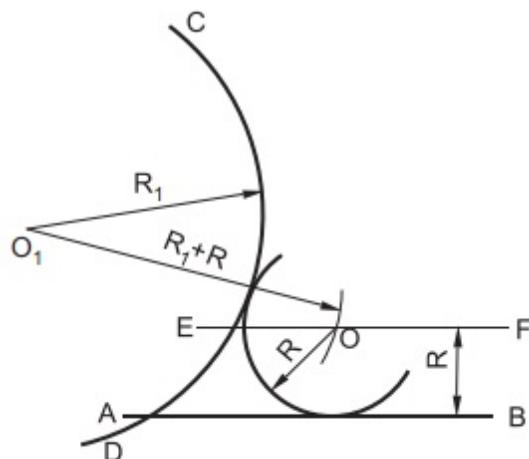
**Construction (Fig.3.39)**

1. Follow the steps 1 to 3 as above.
2. With centre  $O_1$  and radius  $(R_1 + R)$ , draw an arc intersecting the line EF at O.

3. With centre O and radius R, draw an arc.
- The arc meets the given arc and the line tangentially.



**Fig.3.38 Connecting a line and an arc by an internal arc**



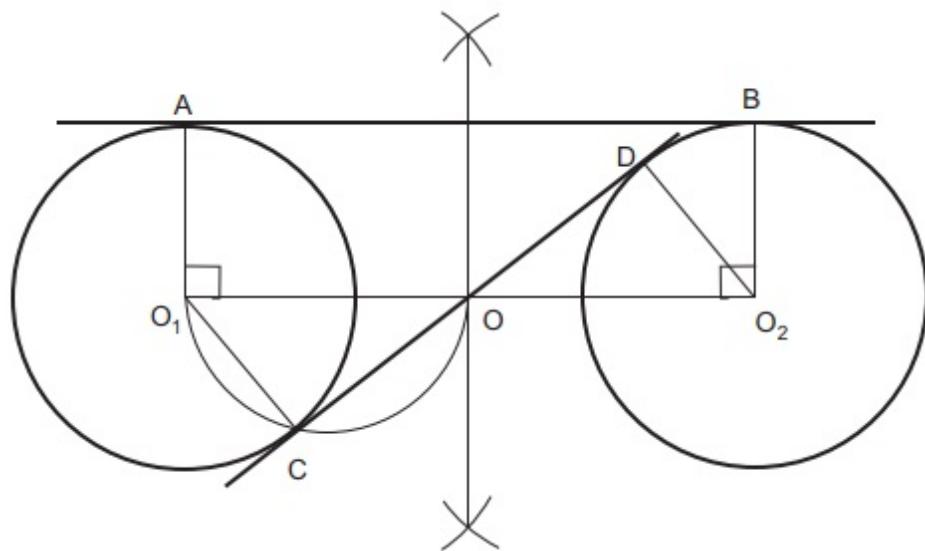
**Fig.3.39 Connecting a line and an arc by an external arc**

### 3.7.8 To Draw Exterior and Interior Tangents to Two Circles of Equal Radius

### **Construction (Fig.3.40)**

1. With  $O_1$  and  $O_2$  as centres, draw the two given circles.
2. Join  $O_1, O_2$ .
3. Draw perpendiculars  $O_1A$  and  $O_2B$ , to the line  $O_1 O_2$ .
4. Join A, B.
5. Bisect the line  $O_1 O_2$  at O.
6. Draw a semi-circle with  $O_1 O$  as diameter, intersecting the circle at C.
7. Join  $O_1, C$ .
8. Through  $O_2$ , draw a line  $O_2 D$  and parallel to  $O_1 C$ .
9. Join C, D.

The lines AB and CD extended are respectively the exterior and interior tangents.



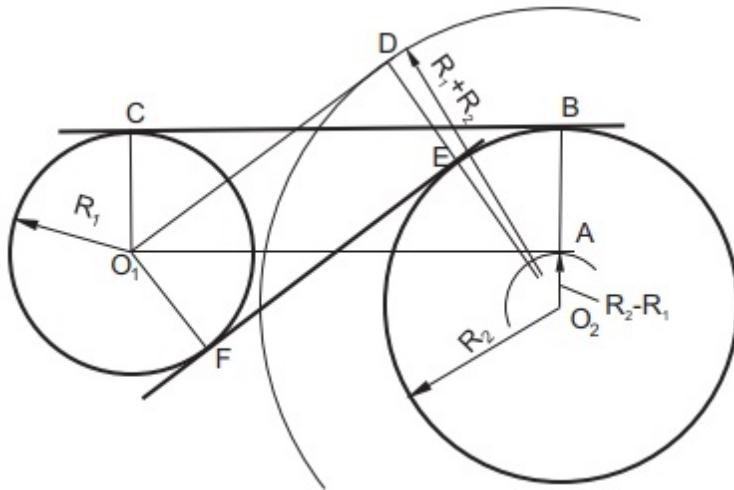
**Fig.3.40 Exterior and interior tangents to two circles of equal radius**

### 3.7.9 To Draw Exterior and Interior Tangents to Two Given Circles of Unequal Radii

#### ***Construction (Fig.3.41)***

1. With centres  $O_1$  and  $O_2$  and radii  $R_1$  and  $R_2$  respectively, draw the given circles (let  $R_2 > R_1$ ).
2. With centre  $O_2$  and radius  $(R_2 - R_1)$ , draw an arc.
3. Through  $O_1$ , draw tangent  $O_1 A$  to the above arc (Construction: Fig. 3.33).
4. Join  $O_2, A$  and extend, meeting the circle at B.
5. Through B, draw a line parallel to  $O_1 A$ , touching the smaller circle at C.
6. With centre  $O_2$  and radius  $(R_1 + R_2)$ , draw an arc.
7. Through  $O_1$ , draw a line  $O_1 D$ , tangential to the above arc.
8. Join  $O_2, D$ , intersecting the circle at E.
9. Through  $O_1$ , draw a line  $O_1 F$ , parallel to  $O_2 D$  such that, the lines  $O_2 D$  and  $O_1 F$  lie on the same side of  $O_1 D$ .
10. Join E, F.

The lines BC and EF extended are the exterior and interior tangents respectively.



**Fig.3.41 Exterior and interior tangents to two circles of unequal radii**

### 3.7.10 To Connect a Given Straight Line to a Circle, With an Arc Tangentially

**Case I Using a small tangent arc**

**Construction (Fig.3.42)**

1. With centre O and radius R, draw the given circle.
2. Draw the given straight line AB in its position.
3. Through B, draw a perpendicular CD to AB.
4. Locate a point E on CD such that,  $BE = R$ .
5. Join E, O.
6. Draw perpendicular bisector FG of EO, intersecting the line CD at H.
7. With centre H and radius HB, draw an arc.

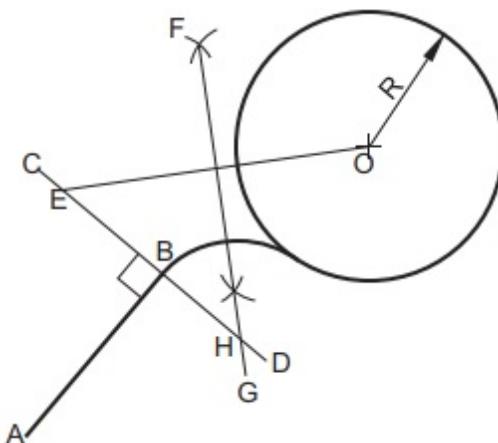
The arc connects the given line and the circle tangentially.

**Case II** Using a big tangent arc

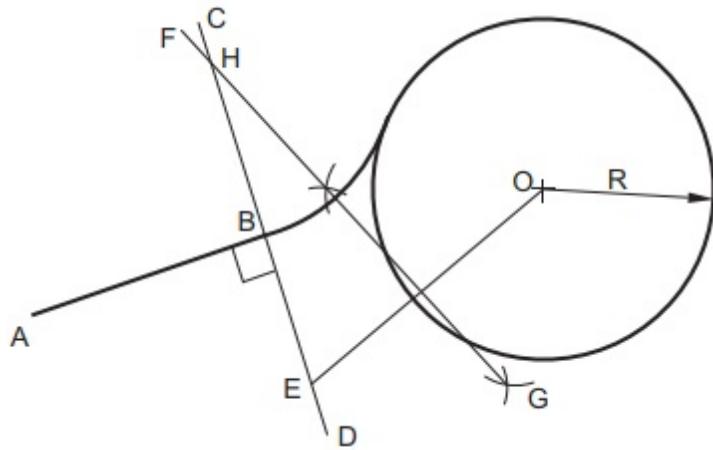
**Construction (Fig.3.43)**

1. Follow steps 1 to 3 as above.
2. Locate a point E on CD, to the right of AB such that,  $BE = R$ .
3. Join E, O.
4. Draw perpendicular bisector FG of EO, intersecting the line CD at H.
5. With centre H and radius HB, draw an arc.

The arc connects the given line and the circle tangentially.



**Fig.3.42 Connecting a straight line to a circle by a small arc**

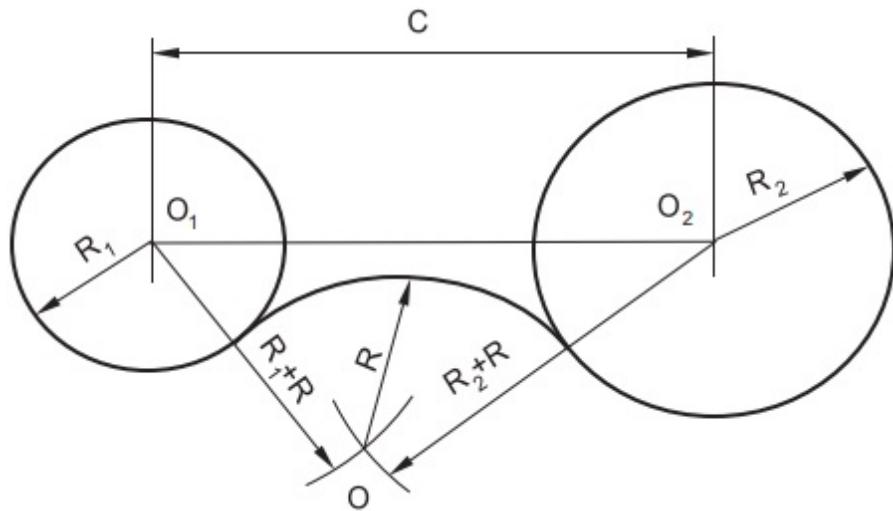


**Fig.3.43 Connecting a straight line to a circle by a big arc**

### 3.7.11 To Draw an Arc of Given Radius R, Touching Two Given Circles

**Case I Internal arc**

**Construction (Fig.3.44)**



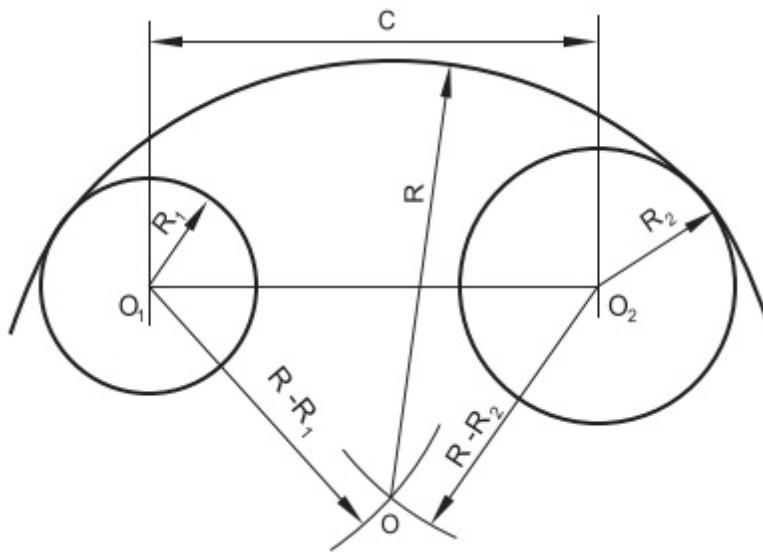
**Fig.3.44 Connecting two circles by an internal arc of known radius**

- With centres  $O_1$  and  $O_2$  and radii  $R_1$  and  $R_2$  respectively, draw the given circles.
- With centres  $O_1$  and  $O_2$  and radii  $(R_1+R)$  and  $(R_2+R)$  respectively, draw arcs intersecting at  $O$ .
- With centre  $O$  and radius  $R$ , draw an arc.  
The arc meets the two circles tangentially.

**Case II External arc**

**Construction (Fig.3.45)**

- With centres  $O_1$  and  $O_2$  and radii  $R_1$  and  $R_2$  respectively, draw the given circles.
- With centres  $O_1$  and  $O_2$  and radii  $(R - R_1)$  and  $(R - R_2)$  respectively, draw arcs intersecting at  $O$ .
- With centre  $O$  and radius  $R$ , draw an arc.  
The arc meets the two circles tangentially.

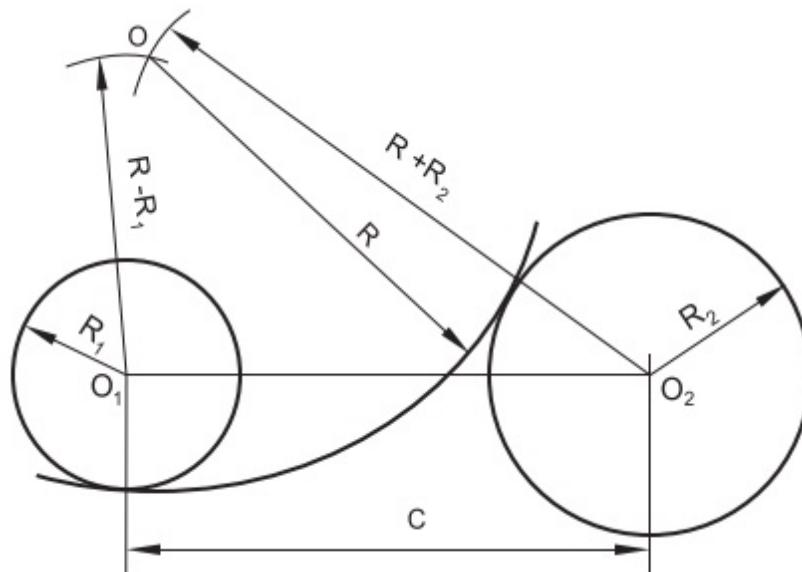


**Fig.3.45 Connecting two circles by an external arc of known radius**

The conditions to be satisfied for drawing the internal and external arcs are:

 (i)  $R < \frac{C - R_1 + R_2}{2}$  and (ii)  $R > \frac{R_1 + R_2}{2}$  respectively,  
where C is the distance between centres  $O_1$  and  $O_2$

**Case III Tangent internal to one and external to the other**  
**Construction (Fig.3.46)**



**Fig.3.46 Connecting two circles by an arc-Internal to one and external to another**

1. With centres  $O_1$  and  $O_2$  and radii equal to  $R_1$  and  $R_2$  respectively, draw the given circles.
2. With centres  $O_1$  and  $O_2$  and radii  $(R - R_1)$  and  $(R + R_2)$  respectively, draw two arcs intersecting at  $O$ .
3. With centre  $O$  and radius  $R$ , draw an arc.

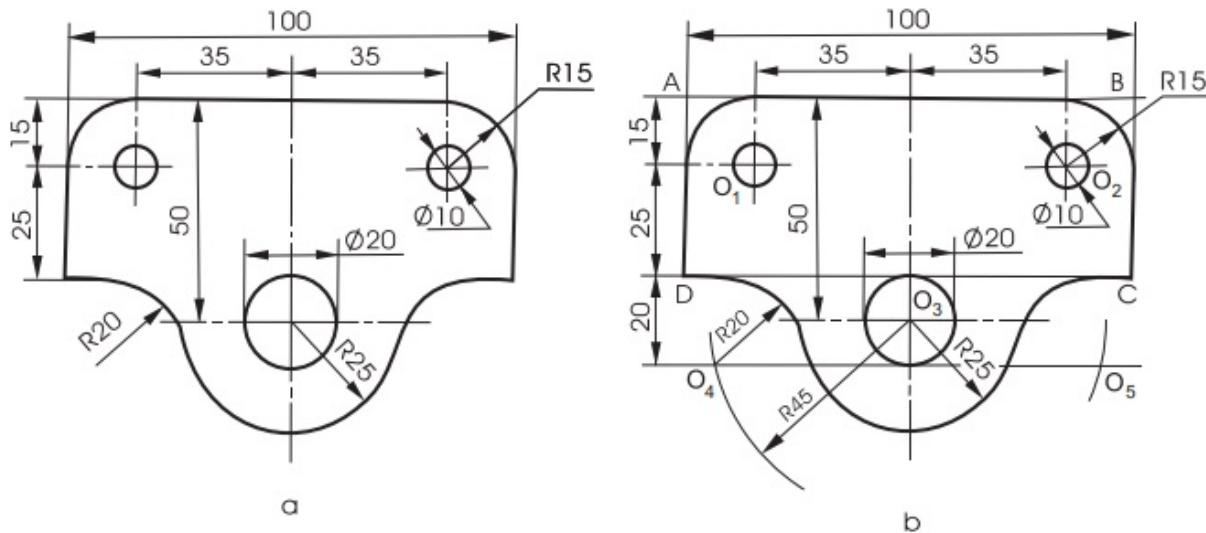
The arc forms an internal tangent to one and external tangent to the other circle.

## 3.8 APPLICATIONS OF GEOMETRICAL CONSTRUCTIONS

The various principles studied so far are used while developing the views in advanced engineering draughting work. Two examples are chosen to illustrate the applications of some of the principles discussed above.

### 3.8.1 To Construct the View Shown in Fig.3.47a

#### ***Construction (Fig.3.47b)***



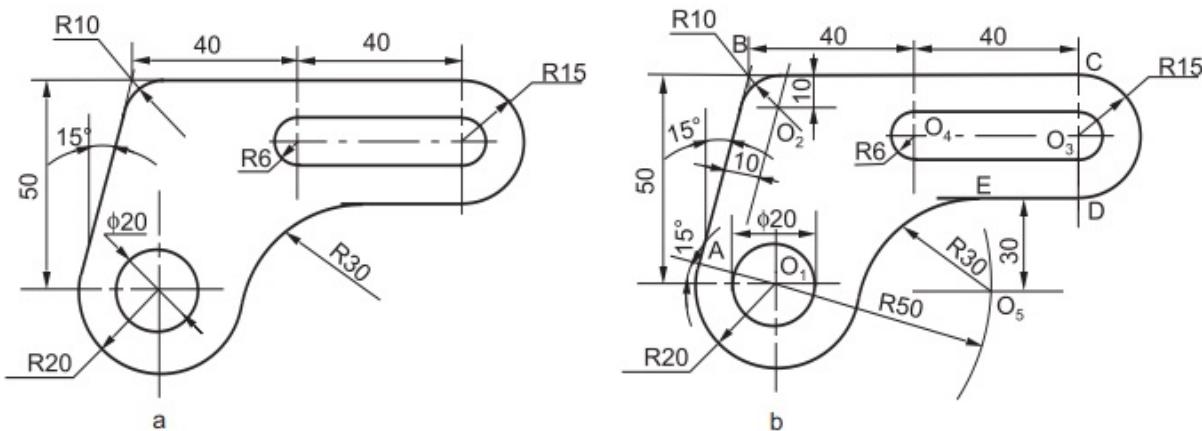
**Fig.3.47**

1. Draw a rectangle of size  $100 \times 40$  and locate the corners A, B, C and D.
2. Locate the centres  $O_1$  and  $O_2$ , which lie at 15 from AB and AD; and BA and BC respectively.

3. With centres  $O_1$  and  $O_2$  and radius 15, draw arcs meeting the lines AB and AD; and BA and BC tangentially.
4. Locate the centre  $O_3$  and draw the circle of diameter 20 and an arc of radius 25.
5. Draw a line parallel to CD and at 20 below it.
6. With centre  $O_3$  and radius  $45(= 25 + 20)$ , draw arcs intersecting the above line at  $O_4$  and  $O_5$ .
7. With centres  $O_4$  and  $O_5$  and radius 20, draw arcs meeting the line CD and the arc of radius 25, tangentially.

### 3.8.2 To Construct the View Shown in Fig.3.48a

**Construction (Fig.3.48b)**



**Fig.3.48**

1. With centre  $O_1$ , draw a circle of diameter 20 and an arc of radius 20.

- Locate the point A on the above arc, which lie at  $15^\circ$  from the horizontal.
- 2.
  3. From A, draw a line at  $15^\circ$  to the vertical and locate the point B on it, at a height 50 from  $O_1$ .
  4. Draw the line BC and locate the centres  $O_3$  and  $O_4$ , 15 below it and at 80 and 40 from B respectively.
  5. With centre  $O_3$  and radius 15, draw the semi-circle CD and draw the line DE, parallel to CB.
  6. With centres  $O_3$  and  $O_4$  and radius 6, draw the semi-circles and complete the slot.
  7. Draw a line parallel to DE and at 30 below it.
  8. With centre  $O_1$  and radius 50 ( $= 20 + 30$ ), draw an arc intersecting the above line at  $O_5$ .
  9. With centre  $O_5$  and radius 30, draw an arc meeting the arc of radius 20 and the line DE tangentially.
  10. Locate the centre  $O_2$ , which is at 10 from the lines AB and BC.
  11. With centre  $O_2$  and radius 10, draw an arc meeting the lines AB and BC tangentially.

## EXERCISES

- 3.1 By the method of bisection, divide a line AB, 125 long into four equal parts.
- 3.2 Draw a line AB, 100 long and trisect it.
- 3.3 Draw a line AB, 150 long and divide it into 11 equal parts.

- 3.4 With centre O and radius 50, draw two arcs of any length on either side. Show that the bisectors of the two arcs meet at O.
- 3.5 Show the construction of dividing a circle into 6 equal parts, by means of a compass.
- 3.6 Draw two lines AB and AC, making an angle of  $40^\circ$  at A and trisect it.
- 3.7 Determine the angle subtended at the centre, by an arc of 100 radius and length 80. Check by calculation.
- 3.8 Determine the circumference of a circle of 80 diameter. Show the construction.
- 3.9 Construct the following triangles:
- (i) Altitude 50, base angles  $60^\circ$  and  $30^\circ$
  - (ii) Base 60, altitude 50 and one side 60
  - (iii) Altitude 50, two sides, 60 and 50
  - (iv) Equilateral triangle having altitude 60
- 3.10 Construct a parallelogram, having diagonals 50 and 80, the angle between the diagonals being  $60^\circ$ .
- 3.11 Construct regular polygons of 5, 6, 7 and 8 sides, with the length of the side as 25, by different methods.
- 3.12 Inscribe regular polygons of 3, 4, 5, 6, 7 and 8 sides, in a circle of 75 diameter.
- 3.13 Construct a regular hexagon, with the distance across
  - (i) corners as 100 and (ii) flats as 60.
- 3.14 Construct a regular octagon, with the distance across flats as 75.
- 3.15 Describe regular polygons of 3, 4, 5, 6, 7 and 8 sides, on a circle of 50 diameter.

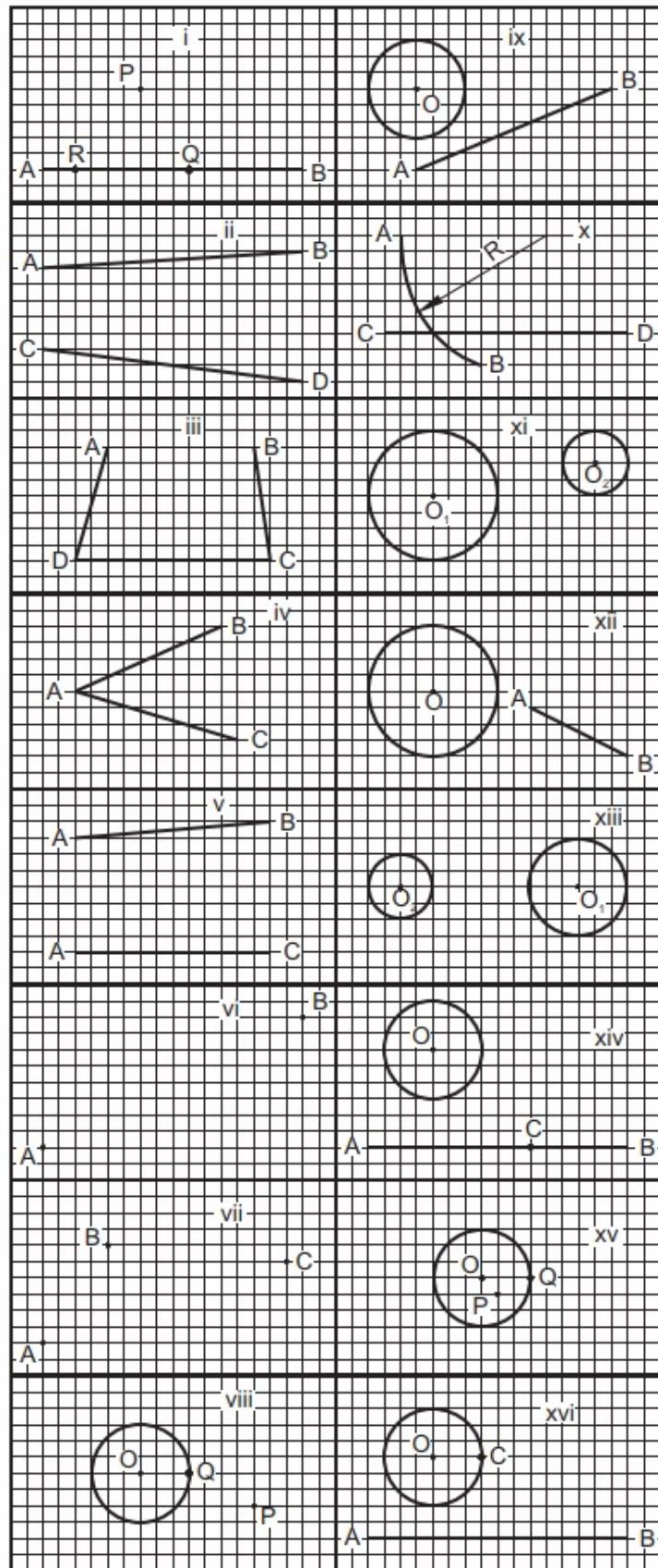
- 3.16 Two lines are 65 apart. Join the lines by any two arcs of equal radius.
- 3.17 Draw perpendiculars to the line AB, through the points P, Q and R ([Fig.3.49i](#)).
- 3.18 Draw a line parallel to AB and passing through P ([Fig.3.49i](#)).
- 3.19 Draw a line through the point P and the in-accessible intersection of the lines AB and CD ([Fig.3.49 ii](#)).
- 3.20 Draw a line perpendicular to BC and through the in-accessible intersection of the lines AB and CD ([Fig.3.49 iii](#)).
- 3.21 Bisect the angle between the lines AB and AC ([Figs.3.49 iv and v](#)).
- 3.22 Trisect the  $\angle BAC$  ([Fig.3.49 iv](#)).
- 3.23 Draw an arc of radius 100, passing through the points A and B ([Fig.3.49 vi](#)).
- 3.24 Draw an arc passing through the points A, B and C([Fig.3.49 vii](#)).
- 3.25 Draw tangents to the circle, through the points P and Q ([Fig.3.49 viii](#)).
- 3.26 Connect the point P to the line AB by an arc ([Fig.3.49 i](#)).
- 3.27 Draw a tangent to the circle and parallel to the line AB ([Fig.3.49 ix](#)).
- 3.28 Draw an arc of radius 40 through the point P and tangential to the given circle ([Fig.3.49 viii](#)).
- 3.29 Join the lines AB and AC by two arcs of equal radius ([Fig.3.49 v](#)).
- 3.30 Draw a circular arc of radius 25, tangential to the two lines AB and AC ([Fig.3.49 v](#)).

- 3.31 Draw a circle touching the three lines AB, BC and CD (Fig.3.49 iii).
- 3.32 Draw an arc of radius 25, touching the given arc AB and the line CD (Fig.3.49 x).
- 3.33 Draw exterior and interior tangents to the two given circles (Fig.3.49 xi).
- 3.34 Connect the circle and the line AB by an arc (Fig.3.49 xii).
- 3.35 Join the two circles tangentially, by arcs of radii 25, 60 and 80 (Fig.3.49 xiii).
- 3.36 Draw a circle passing through the points P and Q and tangential to the line AB (Fig.3.49 i).
- 3.37 Draw a circle, to touch the given circle and the point C on the line AB (Fig.3.49 xiv).
- 3.38 Draw a circle, to pass through the point P and touching the circle at point Q (Figs.3.49 viii and xv).
- 3.39 Draw a circle, touching the line AB and the circle at C (Fig.3.49 xvi).
- 3.40 Construct the views shown in Figs.3.50 to 3. 52.

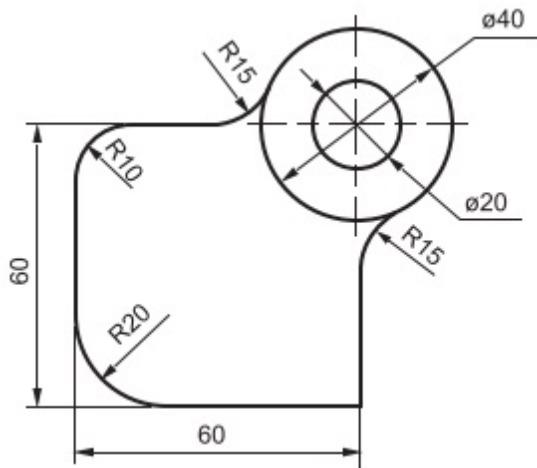
## REVIEW QUESTIONS

- 3.1 Discuss the method of dividing a line into any number of equal parts.
- 3.2 With the help of a sketch, explain how a given angle can be bisected?
- 3.3 What is a regular polygon?
- 3.4 What is meant by the circum-circle of a triangle?
- 3.5 What is meant by in-circle of a triangle?

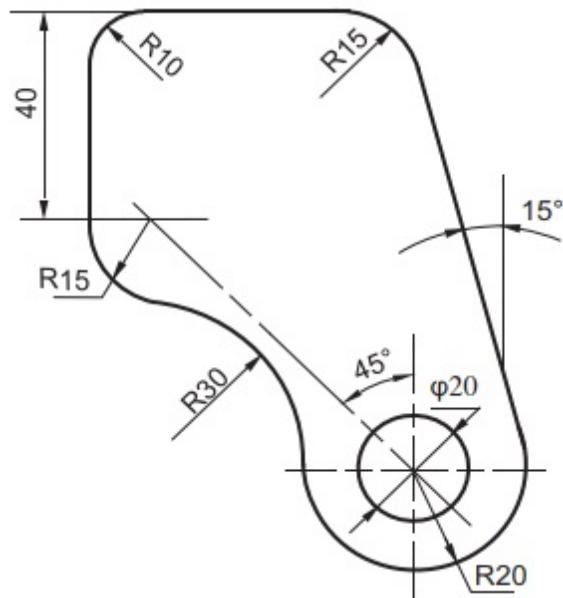
3.6 Give the definition of tangency.



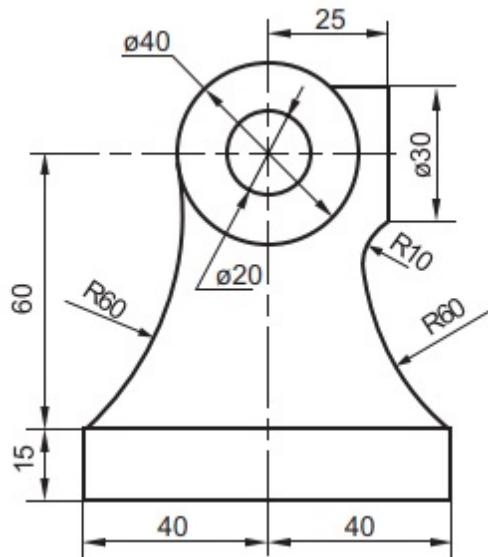
**Fig.3.49 (Take the side of each small square as 10)**



**Fig.3.50**



**Fig.3.51**



**Fig.3.52**

## OBJECTIVE QUESTIONS

- 3.1 The bisector of a line meets it at  $90^\circ$  / less than  $90^\circ$  / greater than  $90^\circ$ .
- 3.2 The bisector of an arc passes through its centre.  
(True / False)
- 3.3 Two sides of a right angled triangle are 3 and 4 units in length. What is the length of the third side?
- 3.4 The distance between any two parallel lines is constant.  
(True / False)
- 3.5 The bisector of an angle of in-accessible vertex may not pass through its vertex when extended.  
(True / False)
- 3.6 The distance between any two parallel curves is constant/ varying.

- 3.7 Three points are situated such that two lie on a line and the third away from it. Is it possible to draw a circle passing through the three points?
- 3.8 The lines joining the alternate corners of a pentagon are called \_\_\_\_\_.
- 3.9 The angle at any corner of a pentagon is \_\_\_\_\_.
- 3.10 The distance across the corners of a hexagon is equal to the diameter of its circum-circle.  
(True / False)
- 3.11 The distance across flats of a hexagon is equal to the diameter of its in-circle.  
(True / False)
- 3.12 A tangent to a circle must intersect it at least at two points.  
(True / False)
- 3.13 Is it possible to draw a tangent to a circle from a point inside it?
- 3.14 How many tangents can be drawn to a circle from a point outside it?
- 3.15 A circle may be connected to a given outside point, only by an arc of fixed radius.  
(True / False)
- 3.16 It is not possible to connect two straight lines, at an angle less than  $90^\circ$ , by an arc of any radius.  
(True / False)
- 3.17 A given arc and a given straight line may be connected by either an internal arc or an external arc.  
(True / False)

- 3.18 The exterior tangent to two circles of equal radii is parallel to the line joining the centres of the circles.  
(True / False)
- 3.19 A given straight line and a circle, may be connected by an arc of fixed radius.  
(True / False)
- 3.20 Two circles of different diameters are separated by a certain distance. They may be connected by a)  
internal arc, b) external arc, c) an arc internal to one and external to the other, d) all are correct.  
( )

## ANSWERS

- 13.1 at  $90^\circ$   
3.2 True  
3.3 5 units  
3.4 True  
3.5 False  
3.6 constant  
3.7 Yes  
3.8 diagonals  
3.9  $108^\circ$   
3.10 True  
3.11 True  
3.12 False  
3.13 No

3.14 Two

3.15 False

3.16 False

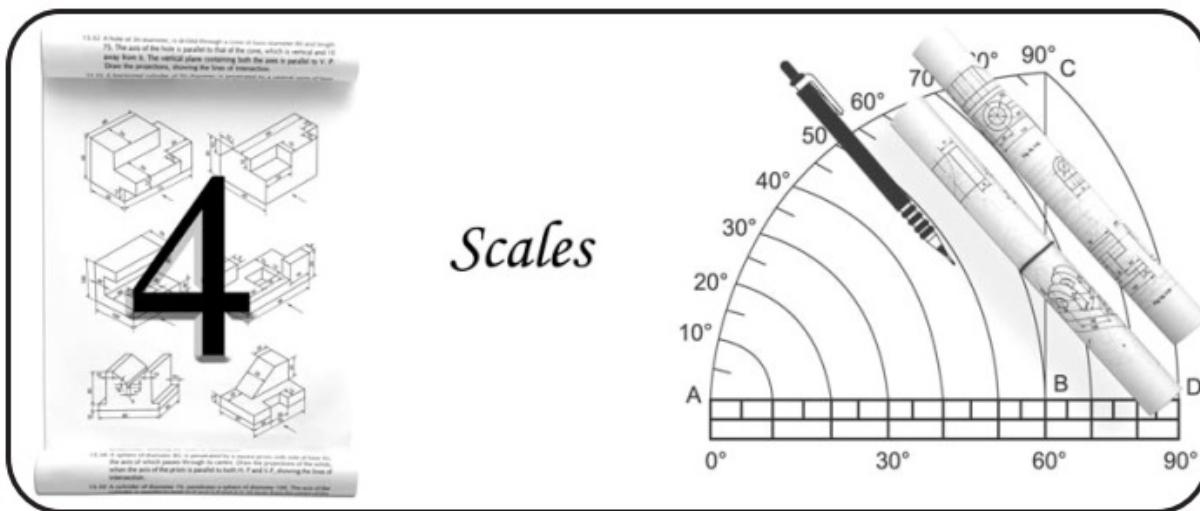
3.17 True

3.18 True

3.19 False

3.20 d

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## 4.1 INTRODUCTION

All engineering drawings are prepared to some scale. For clarity of a drawing, the scale adopted should be the largest possible one.

The term, “ Scale” is defined as the ratio of the length of a line in the drawing to the actual length of the edge on the object. This ratio is also called scale factor (S.F). Thus,

$$\text{Scale or S.F} = \frac{\text{length of a line in the drawing}}{\text{actual length of the edge on the object}}$$

Or, it can also be represented as, SCALE or S.F = Length of a line in the drawing: Actual length of the edge on the object.

### 4.1.1 Full Size Scale

It is customary to represent objects to their actual or full size in drawings, if their sizes permit. This gives an idea

about the size of the object, without leaving anything for imagination. A drawing is said to be prepared to full size scale, if the actual sizes of the edges of the object are used in its preparation.

### **4.1.2 Reducing Scale**

It is not always possible or convenient to prepare drawings of an object to its full size. For instance, drawings of very big objects such as buildings, large machine parts, town plans, etc., cannot be prepared to their actual size. In such cases, it is necessary to prepare the drawings to a reduced size, in some proportion. These drawings are said to be prepared to reducing scale.

### **4.1.3 Enlarging Scale**

The drawings of very small objects, like parts of precision instruments such as watches, electronic devices, etc., cannot be prepared even to full size, as they would be too small to draw and read as well. In such cases, they are represented by enlarging scales.

### **4.1.4 Scale (Scale Factor)**

The complete designation of a scale consists of the word 'SCALE', followed by the indication of its value, as follows:

SCALE 1: 1 for full size scale

SCALE 1: X for reducing scales ( $X > 1$ )

SCALE X: 1 for enlarging scales ( $X > 1$ )

**Table 4.1** gives scales with standard scale factors that are available for use. If in the given set of scales, the desired scale is not available, it may be constructed and then used.

**Table 4.1 Scale factors (S.F) of standard scales**

Category	Recommended scale factors		
Full size scale	1:1		
Reducing scale	1:2	1:5	1:10
	1:20	1:50	1:100
	1:200	1:500	1:1000
	1:2000	1:5000	1:10000
Enlarging scale	50:1	20:1	10:1
	5:1	2:1	



1. The scale of a drawing shall always be indicated on the drawing sheet, either below the drawing or in the title block.
2. Irrespective of the scale used, the actual dimensions of the object only should be indicated on the drawing.

## 4.2 TYPES OF SCALES

The following are the types of scales used in engineering draughting practice: (i) Plain scales, (ii) diagonal scales, (iii) comparative scales, (iv) isometric scale, (v) vernier scales and (vi) scale of chords.

The method of constructing the isometric scale is not presented here, as it is dealt in Chapter 16, on pictorial projections.

### 4.2.1 Scale on a Drawing

Any scale to be used with the drawing, may be constructed on the drawing sheet. This requires the following information:

1. The scale factor,
2. The units to be represented, and
3. The maximum length to be measured.

For example, if an actual length of 1m of an object is to be represented by a line of 20 mm length on the drawing,

$$\text{SCALE} = \frac{20 \text{ mm}}{1 \text{ m}} = \frac{20 \text{ mm}}{1000 \text{ mm}} = \frac{1}{50} \text{ or } 1:50$$

If a maximum length of 10 m is required to be measured by the scale, the length of the scale is obtained by multiplying the scale factor with the maximum length required. Thus,

$$\text{The length of the scale} = \frac{1}{50} \times 10,000 = 200 \text{ mm}$$

### 4.2.2 Metric Measurements

The relation between the various units in SI system are given below:

$$10 \text{ mm} = 1 \text{ cm} \text{ (centimetre)}$$

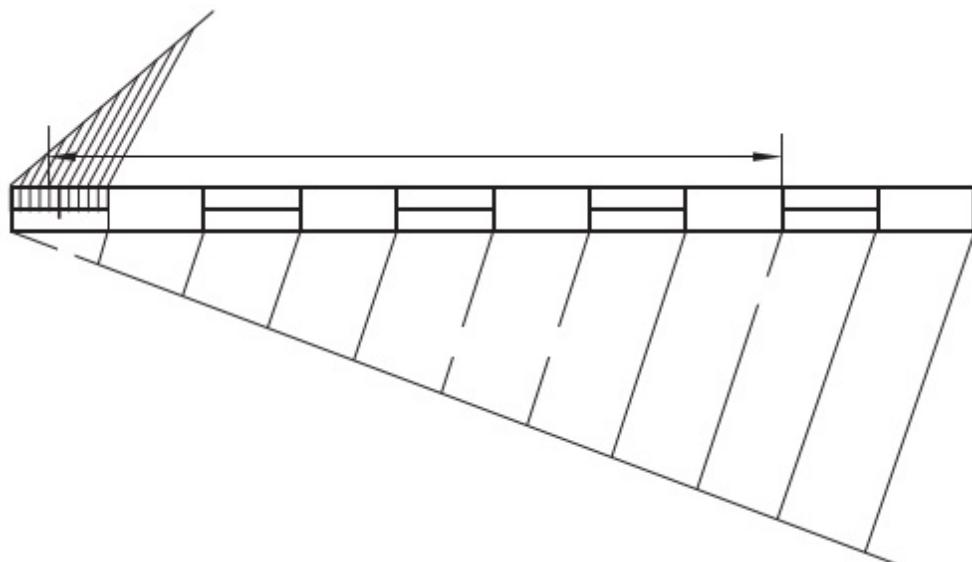
- 10 cm = 1 dm (decimetre)
- 10 dm = 1 m (metre)
- 10 m = 1 dam (decametre)
- 10 dam = 1 hm (hectometre)
- 10 hm = 1 km (kilometre)
- 1 hectare = 10,000 square metres

## 4.3 PLAIN SCALES

A line, suitably divided into equal parts (primary divisions) is called a plain scale. Generally, the first part is sub-divided into smaller parts (secondary divisions). Thus, a plain scale is used to represent either two units or a unit and its fraction such as km and hm; m and dm, etc.

**Problem 1** *Construct a scale of 1: 8 to show decimetres and centimetres and to read upto 1m. Show a length of 7.6 dm on it.*

**Construction (Fig.4.1)**



### **Fig.4.1**

1. Obtain the length of the scale:

$$\text{Scale factor} \times \text{length} = \frac{1}{8} \times 1000 \text{ mm} = 125 \text{ mm}$$

2. Draw a 125 mm long line. Divide it into 10 equal parts (primary divisions) geometrically; each part representing 1 dm.
3. Mark 0 after the first division and continue 1, 2, 3, etc., to the right of the scale.
4. Divide the first division into 10 equal parts (secondary divisions); each part representing 1 cm.
5. Mark the secondary division points from right to left.
6. Write the units at the bottom of the scale in their respective positions and also the scale factor.
7. Mark the distance 7.6 dm (selecting 7 primary and 6 secondary divisions).



For clear indication of the divisions, the scale is shown as a rectangle of about 5 mm width. All alternate divisions are marked with dark lines centrally to help taking the measurements.

**Problem 2** Construct a scale of feet and inches to measure upto 7 feet, to a scale  $3'' = 3' 9''$ . Mark on it a length of  $4' 7''$ .

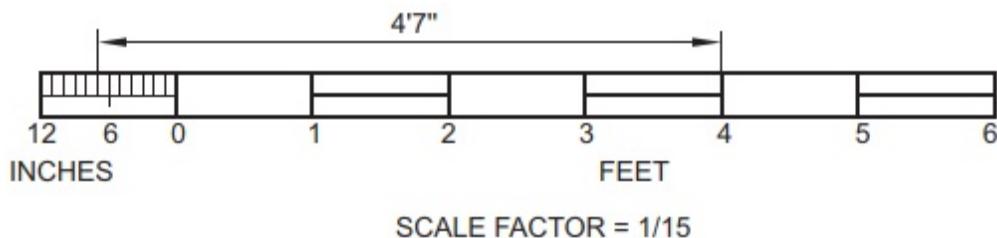
### ***Construction (Fig.4.2)***

1. Obtain the scale factor:  $3''/3'9'' = 1/15$
2. Calculate the length of the scale:  $(1/15) \times 7' = 5.6''$
3. Draw a line 5.6" long and divide it into 7 equal parts; each part representing one foot.

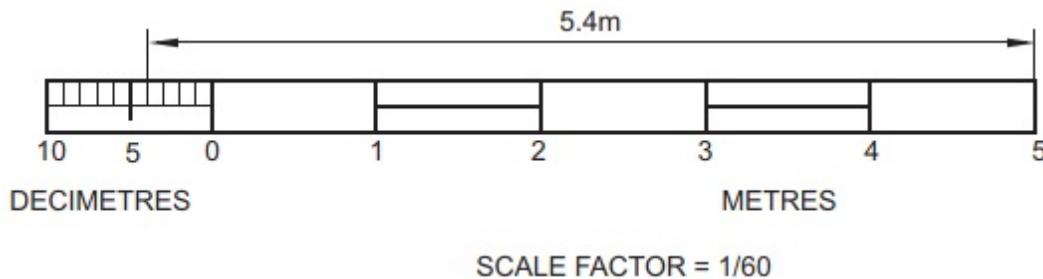
- Repeat steps 3 to 6 of Construction: [Fig.4.1](#), keeping in view that  $1' = 12$  inches. 5. Mark the distance  $4' 7''$  (choosing 4 primary divisions and 7 secondary divisions).

**Problem 3** Construct a scale of  $1/60$  to read metres and decimetres and long enough to measure upto 6 m. Mark on it a distance of 5.4 m.

### **Construction ([Fig.4.3](#))**



**Fig.4.2**



**Fig.4.3**

- Obtain the length of the scale:  $(1/60) \times 6 \text{ m} = 100 \text{ mm}$
- Draw a 100 mm long line and divide it into 6 equal parts, to represent metres.
- Repeat steps 3 to 6 of Construction: [Fig.4.1](#) suitably and obtain the desired scale.
- Mark the distance 5.4 m (choosing 5 primary divisions and 4 secondary divisions).

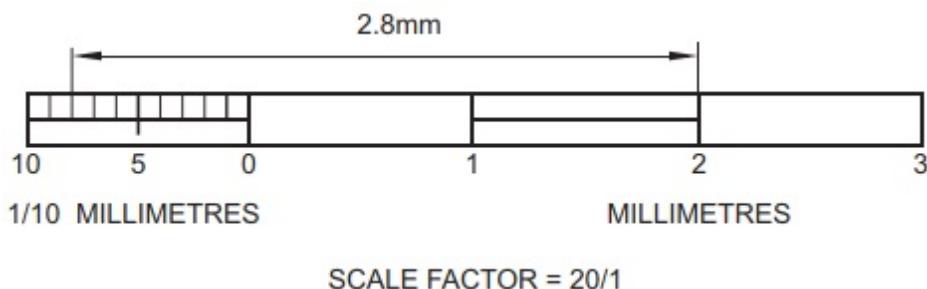
**Problem 4** Construct a scale, to measure one-tenth of mm and mm in which 0.5 mm on a part is represented by a line of 10 mm. Mark on it a length of 2.8 mm.

**Construction (Fig.4.4)**

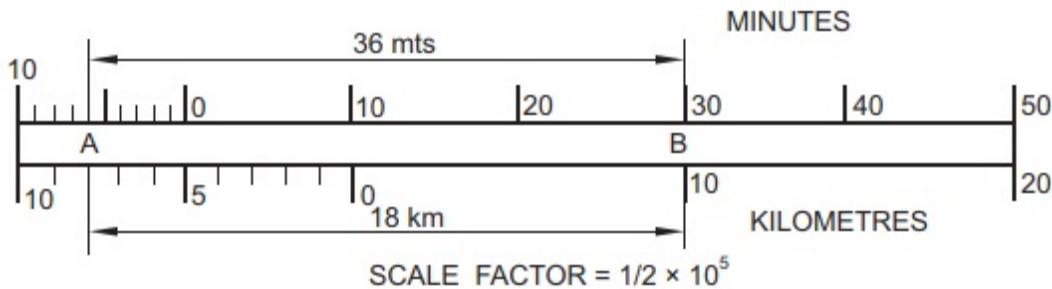
1. Obtain the scale factor:  $10/0.5 = 20/1$
2. Determine the length of the scale representing, say 4mm:  $20 \times 4 = 80$  mm
3. Draw a line 80 mm long and divide it into 4 equal parts; each part representing 1 mm.
4. Repeat steps 3 to 6 of Construction: Fig. 4.1 suitably and obtain the desired scale.
5. Mark the distance of 2.8 mm (selecting 2 primary and 8 secondary divisions).

**Problem 5** The distance between two towns is 120 km. A passenger train covers the distance in 4 hours. Construct a scale to measure-off the distance covered by the train in a single minute and upto 1 hour. The scale factor is  $1/2,00,000$ . Show on it, the distance covered by the train in 36 minutes.

**Construction (Fig.4.5)**



**Fig.4.4**



**Fig.4.5**

1. Obtain the distance covered by the train in 1 hour:  
 $120/4 = 30$  km
2. Calculate the length of the scale to represent 30 km, covered by the train in 1 hour:  $(1/2,00,000) \times 30$  km = 150 mm
3. Draw two parallel lines of 150 mm length, separated by, say 5 mm.
4. Divide one of the lines into 3 equal parts; each part representing 10 kilometres.
5. Divide the other line into 6 equal parts; each representing 10 minutes.
6. Repeat steps 3 to 6 of Construction: [Fig. 4.1](#) suitably and obtain the desired scale.

The distance between the points A and B, i.e., 18 km, represents the distance covered in 36 minutes.

**Problem 6** A rectangular plot of 100 square kilometres is represented on a certain map by a similar rectangular area of 4 square centimetres. Draw a scale to show kilometres and mark a distance of 43 kilometres on it.

### ***Construction (Fig. 4. 6)***

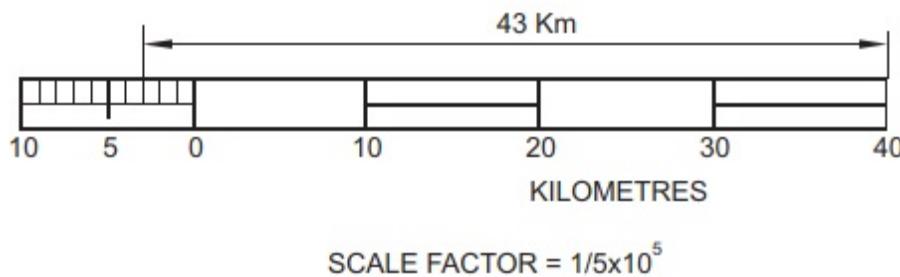
1. Obtain the scale factor as:

$$\frac{\sqrt{4 \text{ cm}^2}}{\sqrt{100 \text{ km}^2}} = \frac{1}{5 \times 10^5}$$

2. Calculate the length of the scale to represent, say 50 km:

$$\frac{50 \text{ km}}{5 \times 10^5} = 100 \text{ mm}$$

3. Draw a line of 100mm long and divide it into 5 equal parts, each part representing 10 kilometres.
4. Repeat steps 3 to 6 of Construction: [Fig. 4.1](#) suitably and obtain the desired scale.
5. Mark the distance 43 kilometres.



**Fig.4.6**

## 4.4 DIAGONAL SCALES

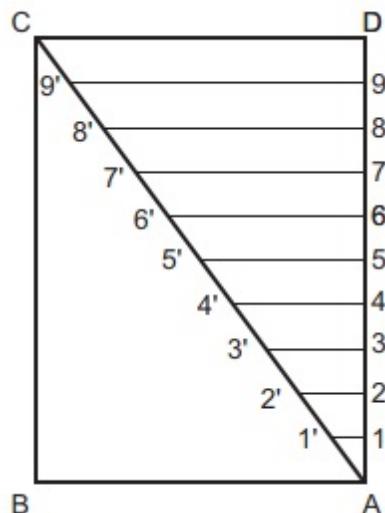
If a fractional portion of a secondary division is needed which is too small to be subdivided, diagonal scales may be used with ease. Hence, diagonal scales are used to measure three consecutive units such as kilometres, hectometres and decametres; metres, decimetres and centimetres; decimetres, centimetres and millimetres, etc., or to read accurately upto two decimals.

## 4.4.1 Principle of Diagonal Scale

It consists of sub-division of the secondary divisions into the required number of equal parts.

### ***Construction (Fig.4.7)***

1. Draw a line AB. Let it represent one of the units of the secondary divisions.
2. Through A, draw a line perpendicular to AB.
3. Starting from A, mark points 1, 2, 3, - - - - D, which are equi-spaced (of suitable distance). The number of divisions is equal to the number of sub-divisions required of AB, say 10.
4. Complete the rectangle ABCD and draw the diagonal AC.
5. Through 1, 2, 3, etc., draw lines parallel to AB; intersecting the diagonal at 1', 2', 3', etc. Considering the similar triangles A-1-1' and ADC;  $1-1'/DC = A1/AD$



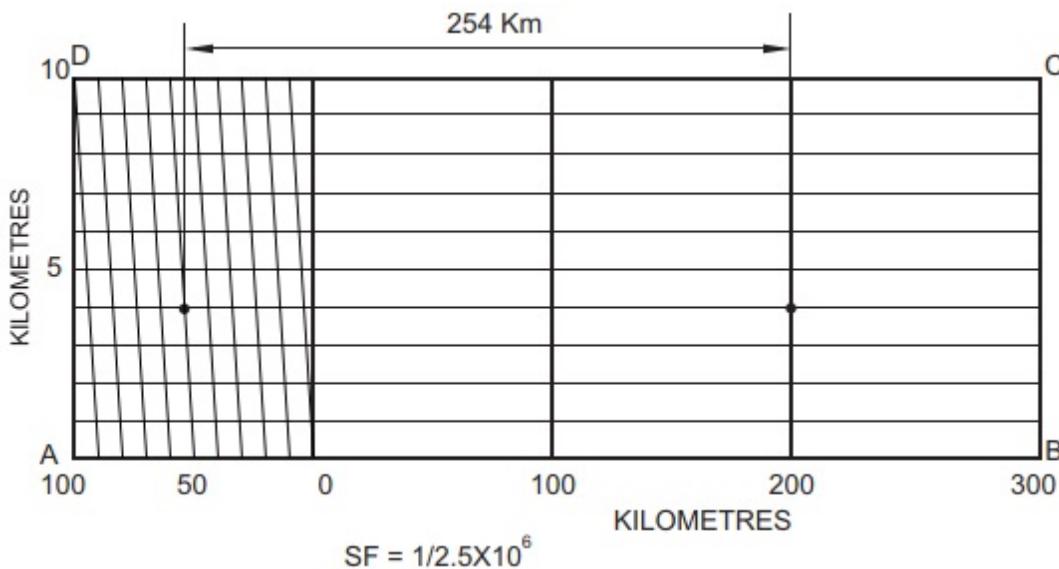
**Fig.4.7 Principle of diagonal division**

Thus,  $1-1' = (A1/AD) DC = (1/10) AB$ , as  $DC = AB$

Similarly, it can be proved that  $2-2' = (2/10) AB$ ;  $3-3' = (3/10) AB$ , etc.

**Problem 7** Construct a diagonal scale of  $S.F=1/(2.5 \times 10^6)$  to read upto a single kilometre and long enough to measure 400 km. Mark a length of 254 km on it.

### Construction (**Fig.4.8**)



**Fig.4. 8**

1. Obtain the total length of the scale:  $(1/2.5 \times 10^6) \times 400 \times 10^6 = 160$  mm
2. Draw a line AB of 160 mm long and divide it into 4 equal parts, to represent 100 km each.
3. Divide the first part into 10 equal parts, to represent 10 km each.
4. Draw 10 equi-distant parallel lines above AB and complete the rectangle ABCD.
5. Join D with the division point 9 of OA, forming the first diagonal line.

- Through the remaining points 8, 7, 6, etc., draw lines parallel to D9.
- Mark the distance 254 km as shown.

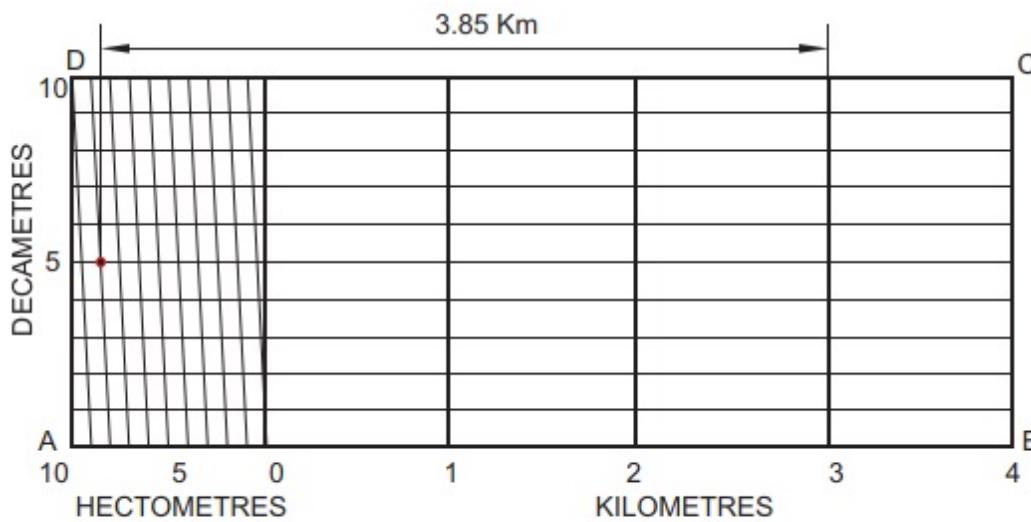


Name the primary, secondary and tertiary divisions and indicate the scale factor as shown.

**Problem 8** Construct a scale to be used with a map, the scale of which is  $1 \text{ cm} = 500 \text{ m}$ . The maximum length to be read is 5 km. Mark on the scale, a distance of 3.85 km.

### **Construction (Fig.4.9)**

- Obtain the length of the scale:  $(1\text{cm}/500 \text{ m}) \times 5 \text{ km} = 100 \text{ mm}$
- Draw a line AB of 100 mm long and divide it into 5 equal parts; representing one km each.
- Repeat steps 3 to 7 of Construction: Fig.4.8 suitably and obtain the desired scale.
- Mark the distance 3.85 km as shown.



$$\text{S.F} = 1/5 \times 10^4$$

**Fig.4.9**

**Problem 9** On a map, the actual distance of 10 m is represented by a line of 50 mm long. Calculate the scale factor. Construct a diagonal scale, long enough to measure 30 m and mark on it, a distance of 26.3 m.

**Construction (Fig. 4. 10)**

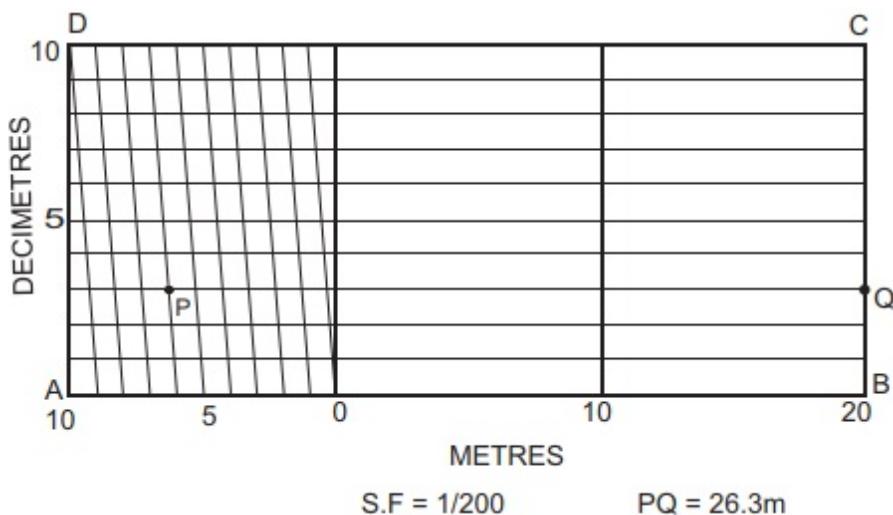
1. Obtain the scale factor:

$$\frac{50}{10} \times \frac{1}{1000} = \frac{1}{200}$$

2. Calculate the length of the scale:

$$\frac{1}{200} \times 30 \times 1000 = 150 \text{ mm}$$

3. Draw a line AB, 150 long and divide it into 3 equal parts; each representing 10 m (1decametre).
4. Repeat steps 3 to 7 of Construction: Fig.4.8 suitably and obtain the desired scale.
5. Mark the distance PQ; representing 26.3 m.

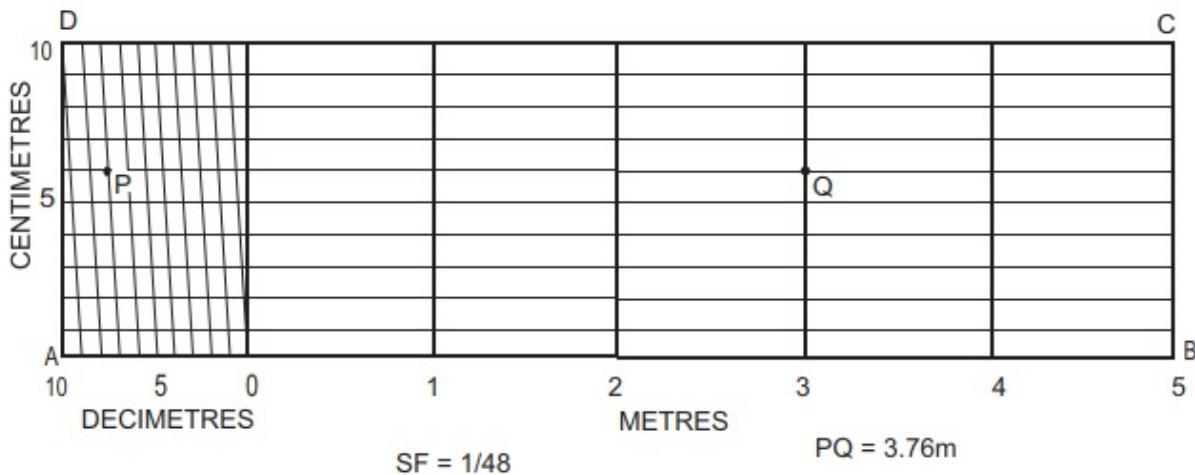


**Fig.4.10**

**Problem 10** Construct a diagonal scale of  $1/48$ , showing metres, decimetres and centimetres and to measure upto 6m. Mark a length of 3.76 m on it.

**Construction (Fig.4.11)**

1. Calculate the length of the scale:  $(1/48) \times 6 \text{ m} = 125 \text{ mm}$
2. Draw a line AB, 125 mm long and divide it into 6 equal parts; each representing 1m.
3. Repeat steps 3 to 7 of Construction: Fig. 4.8 suitably and obtain the desired scale.
4. Mark the distance PQ; representing 3.76 m.



**Fig.4.11**

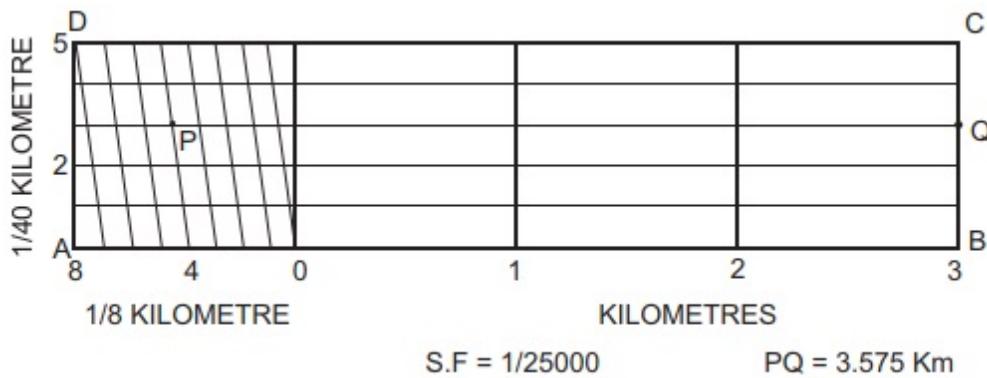
**Problem 11** Construct a scale, to measure km,  $1/8$  of a km and  $1/40$  of a km, in which 1 km is represented by 4 cm. Mark on this scale, a distance of 3.575 km.

**Construction (Fig.4.12)**

1. Obtain the scale factor:  $(4\text{cm}/1 \text{ km}) = 1/25000$
2. Calculate the length of the scale representing, say 4 km:

$$\frac{4 \text{ km}}{25,000} = 160 \text{ mm}$$

3. Draw a line AB of 160 mm long and divide it into 4 equal parts; each representing 1 km.
4. Divide the first part into 8 equal parts; each representing  $1/8^{\text{th}}$  of a km.
5. Draw 5 equi-distant parallel lines above AB and complete the rectangle ABCD.
6. Join D, 7 and draw the other diagonals through 6, 5, 4, etc., and parallel to D7.
7. Mark the distance PQ; representing 3.575 km ( $3 \text{ km} + 4 \times 1/8 \text{ km} + 3 \times 1/40 \text{ km}$ ).



**Fig.4.12**



To obtain  $1/40$  of a km, each secondary division should be divided into 5 equal parts.

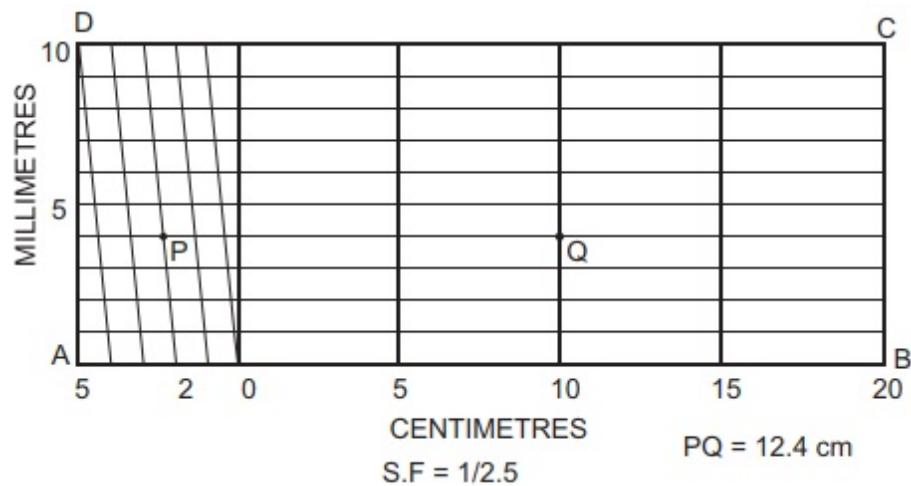
**Problem 12** Draw a diagonal scale of 1:2.5, showing centimetres and millimetres and long enough to measure upto 25 centimetres. Mark a distance of 12.4 cm on the scale.

**Construction (Fig.4.13)**

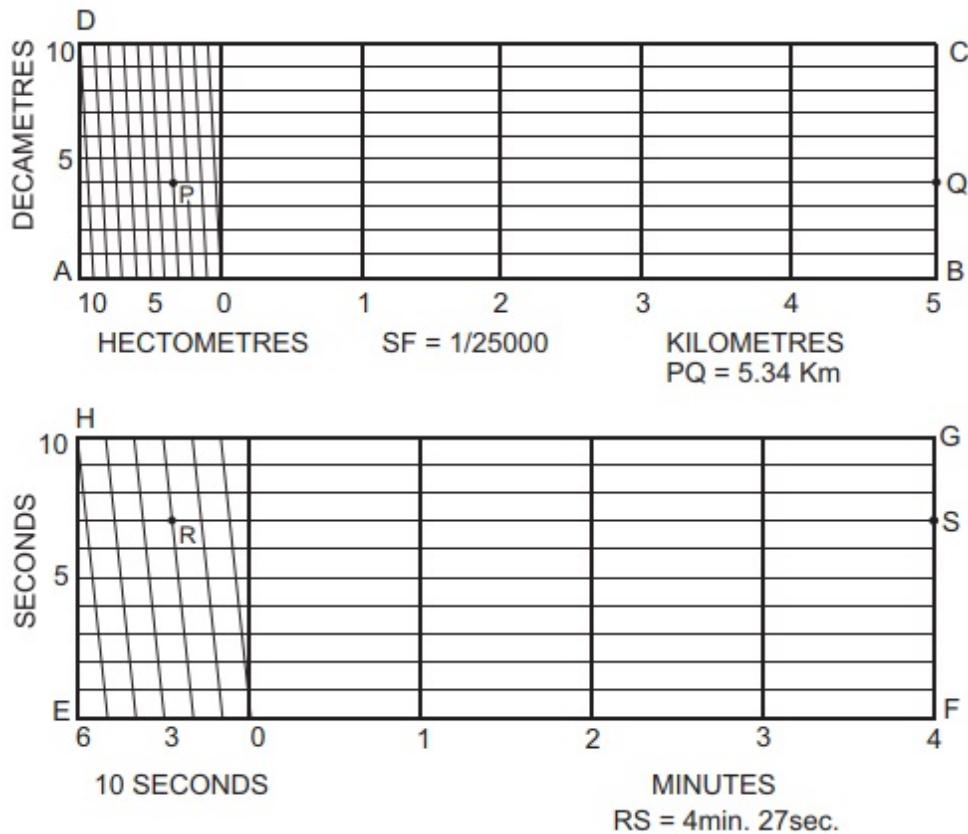
- Obtain the length of the scale:  $(1/2.5) \times 25 \text{ cm} = 100 \text{ mm}$
- Draw a line AB of 100mm long and divide it into 5 equal parts; each representing 5 cm.
- Repeat steps 3 to 7 of Construction: Fig. 4.8 suitably and obtain the desired scale.
- Mark the distance PQ; representing 12.4 cm.

**Problem 13** A train is moving at the rate of 1.2 km per minute. Construct a scale with scale factor 1/25,000, showing minutes and seconds. Indicate on it, the distance moved by the train in 4 minutes and 27 seconds.

**HINT** The scale may be constructed to read the distances covered upto 1 second.



**Fig.4.13**



**Fig.4.14**

**Construction (Fig.4.14)**

1. Calculate the length of the scale, to represent 6 km, covered by the train in, say 5 minutes:  $(1/25,000) \times 6 \text{ km} = 240 \text{ mm}$
2. Draw two parallel lines of 240 mm long, separated by, say 15 mm.
3. Construct two diagonal scales; ABCD, representing the distance and EFGH, the time.
4. Mark the distance PQ covered by the train in 4 mts. and 27 sec, showing 5.34 km.

**Problem 14** A rectangular plot of land of area 0.45 hectare, is represented on a map by a similar rectangle of 5 sq. cm. Calculate the scale factor of the map. Also,

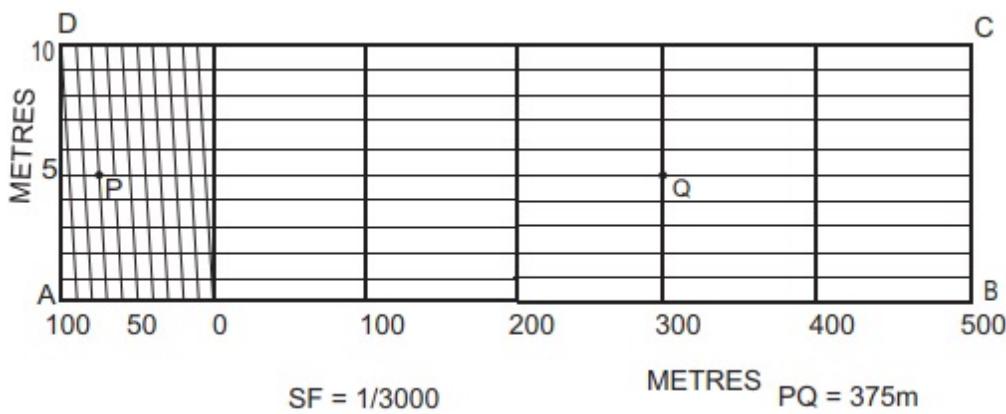
*construct a scale to read upto a single metre and long enough to measure 600 metres. Mark on it, a distance of 375 m.*

### **Construction (Fig.4.15)**

1. Obtain the scale factor of the map:

$$\sqrt{5 \text{ cm}^2 / 0.45 \text{ hectare}} = 1/3000$$

2. Calculate the length of the scale:  $(1/3000) \times 600 \text{ m} = 200 \text{ mm}$
3. Draw a line AB of 200 mm length and divide it into 6 equal parts; each representing 100 m.
4. Repeat steps 3 to 7 of Construction: Fig.4.8 suitably and obtain the desired scale.
5. Mark the distance PQ; representing 375 m.



**Fig.4.15**

**Problem 15** An area covered by a triangle of base 12 cm and altitude 24 cm, represents an area of  $36 \text{ km}^2$ . Find the scale factor and construct a diagonal scale to read kilometres, hectometres and decametres. Mark the distances of 1.05 km and 4.82 km on it.

### **Construction (Fig.4.16)**

- Obtain the scale factor :

$$\frac{\sqrt{\text{area of the triangle}}}{\sqrt{36 \text{ km}^2}}$$

where, Area of the triangle

$$\begin{aligned}&= \frac{1}{2} \times \text{base} \times \text{altitude} \\&= \frac{1}{2} \times 12 \times 24 = 144 \text{ cm}^2\end{aligned}$$

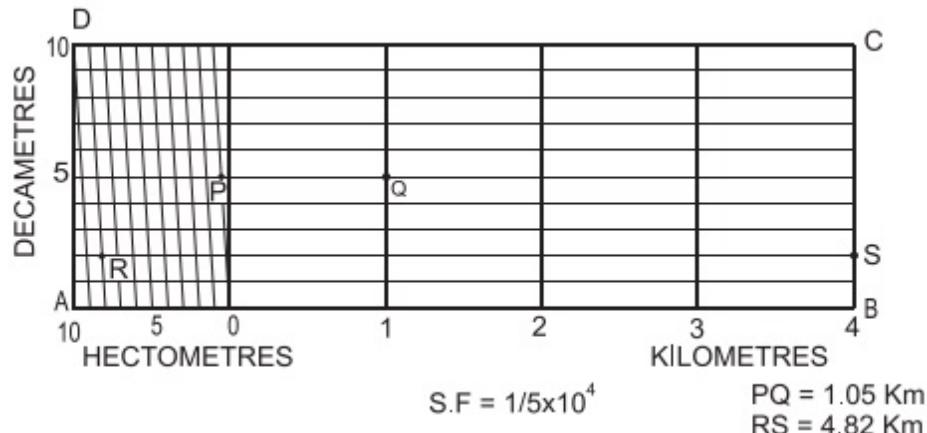
$\therefore$  Scale factor,

$$= \frac{\sqrt{144}}{\sqrt{36 \times (1000 \times 100)^2}} = \frac{1}{5 \times 10^4}$$

- Calculate the length of the scale representing, say 5 km;

$$\frac{5 \times 10^5}{5 \times 10^4} = 10 \text{ cm} = 100 \text{ mm}$$

- Draw a line AB of 100 mm long and divide it into 5 equal parts; each representing 1 km.
- Divide the first part into 10 equal parts; each representing 100 m (1 hectometre).
- Draw 10 equi-distant parallel lines above AB and complete the rectangle ABCD.
- Join D, 9 and draw the other diagonals through 8, 7, 6, etc., and parallel to D9.
- Mark the distances, PQ = 1.05 km and RS = 4.82 km.



**Fig.4.16**

**Problem 16** A block of ice-berg  $1000\text{m}^3$  volume, is represented by a block of  $27 \text{ cm}^3$  volume. Find the scale factor and construct a scale to measure upto 60m. Mark a distance of 42.5 m on the scale.

**Construction (Fig.4.17)**

1. Obtain the scale factor:

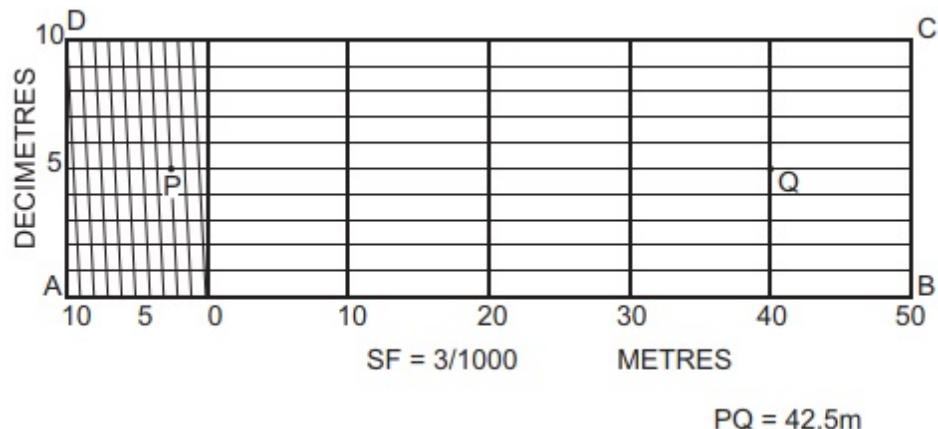
$$\frac{\sqrt[3]{27 \text{ cm}^3}}{\sqrt[3]{1000 \text{ m}^3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1000 \times (100)^3}} = \frac{3}{1000}$$

2. Calculate the length of the scale representing 60 m:

$$\frac{3}{1000} \times 60 \times 1000 = 180 \text{ mm}$$

3. Draw a line AB of 180mm long and divide it into 6 equal parts; each representing 10 m.
4. Divide the first part into 10 equal parts; each representing 1 m.
5. Draw 10 equi-distant parallel lines above AB and complete the rectangle ABCD.

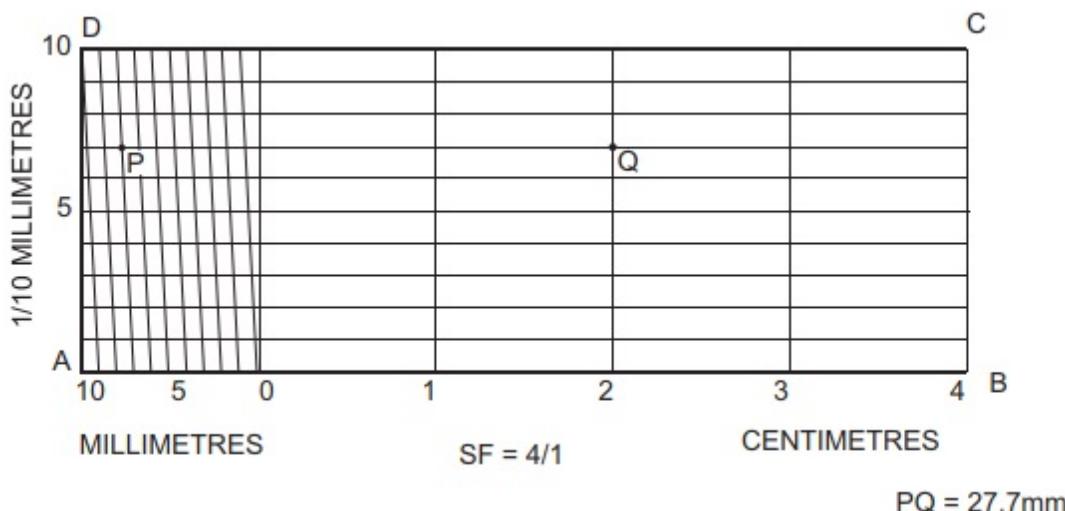
- Join D, 9 and draw the other diagonals through 8, 7, 6, etc., and parallel to D9.
- Mark the distance, PQ = 42.5 m.



**Fig.4.17**

**Problem 17** A distance of 0.5 mm on a part of an instrument is to be represented by a line of 2mm on the drawing. Determine the scale factor of the drawing. Construct a scale, showing cm, mm and one-tenth of mm and long enough to measure 5 cm. Mark on it, a distance of 27.7mm .

**Construction (Fig.4.18)**

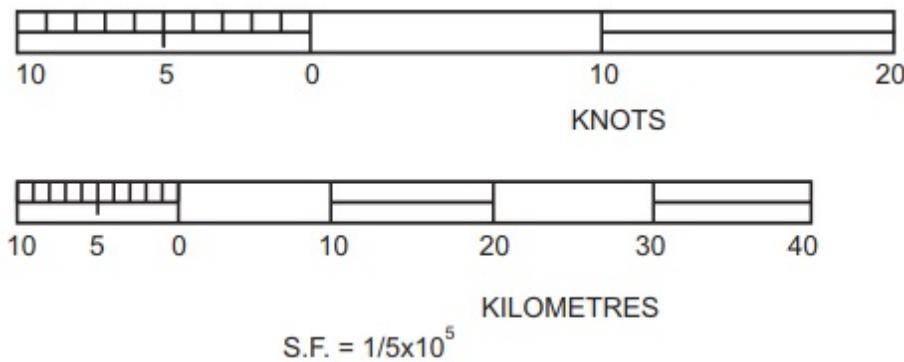


**Fig.4.18**

1. Obtain the scale factor:  $2\text{mm}/0.5\text{mm} = 4/1$
2. Calculate the length of the scale, representing 5 cm:  $4 \times 5 \text{ cm} = 200 \text{ mm}$
3. Draw a line AB of 200 mm length and divide it into 5 equal parts; each representing 1 cm.
4. Divide the first part into 10 equal parts; each representing 1 mm.
5. Draw 10 equi-distant parallel lines, above AB and complete the rectangle ABCD.
6. Join D, 9 and draw the other diagonals through 8, 7, 6, etc., parallel to D9.
7. Mark the distance PQ, measuring 27.7 mm as shown.

## 4.5 COMPARATIVE SCALES

Comparative scales or corresponding scales have the same scale factor, but graduated in different units. On a map or a drawing, made with a scale, reading miles and furlongs; distances can be measured-off directly in kilometres and hectometres with a comparative scale. These scales can be constructed as plain or diagonal, depending upon the requirement. They may be constructed separately or one above the other, as required.



**Fig.4.19**

**Problem 18** A map is drawn to a scale of  $1\text{cm} = 5\text{ km}$ . Draw a comparative scale to read (i) 30 knots and (ii) 50 km, to be used with the above map ( $1\text{ knot} = 1.85\text{ km}$ ).

**Construction (Fig.4.19)**

1. Obtain the scale factor of the map:  $1\text{cm}/5\text{km} = 1/(5 \times 10^5)$
2. Calculate the length of the scale; representing 30 knots:  

$$\{(1/(5 \times 10^5))\} \times 30 \times 1.85 \times 10^6 = 111\text{ mm}$$
3. Calculate the length of the scale to represent 50 km:  

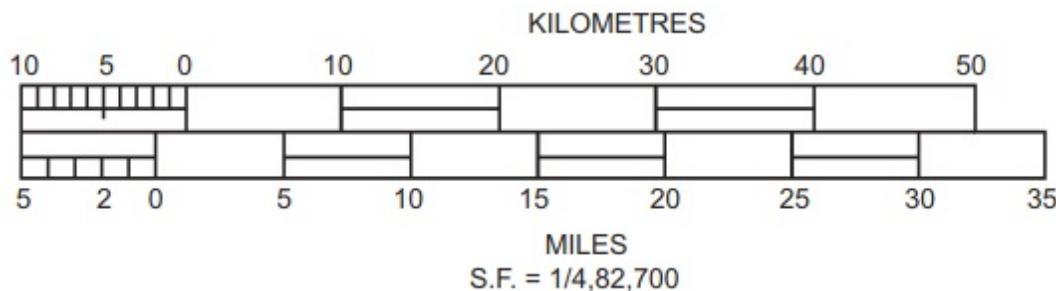
$$\{(1/(5 \times 10^5))\} \times 50 \times 10^6 = 100\text{ mm}$$
4. Repeat steps 2 to 6 of Construction: Fig. 4.1 suitably and draw the plain comparative scales.

**Problem 19** Construct a comparative scale to measure upto (i) 40 miles and (ii) 60 km, to be used with a road map. On this map, a distance of 15 miles is shown by a 5cm long line ( $1\text{ mile} = 1.609\text{ km}$ ).

**Construction (Fig.4.20)**

1. Obtain the scale factor of the map:  $5\text{ cms}/15\text{ miles} = 1/4,82,700$

2. Calculate the length of the scale to represent 40 miles:  
 $(1/4,82,700) \times 40 \text{ miles} = 13.33 \text{ cm}$
3. Calculate the length of the scale to represent 60 km:  
 $(1/4,82,700) \times 60 \text{ km} = 12.5 \text{ cm}$
4. Repeat steps 2 to 6 of Construction: [Fig.4.1](#) suitably and draw plain scales, one above the other.

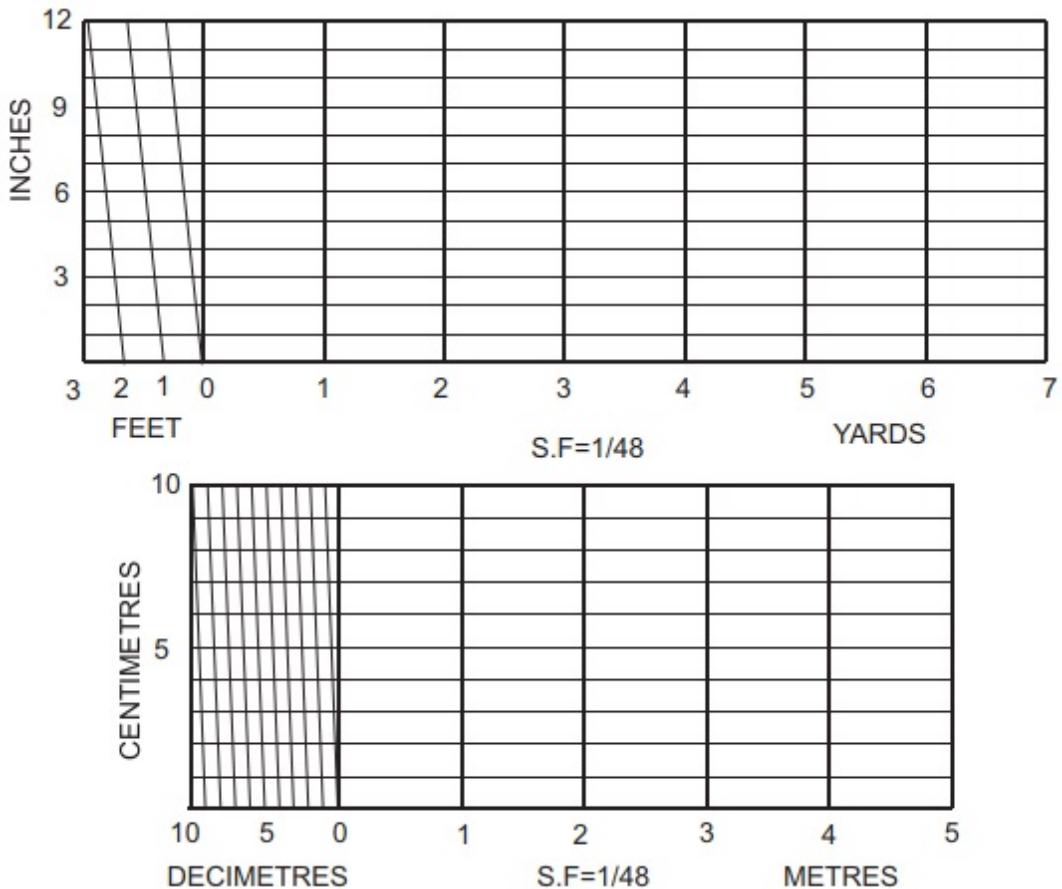


**Fig.4.20**

**Problem 20** Draw comparative diagonal scales of  $1/48$ , to measure upto (i) 8 yards and (ii) 6m. These scales should read (a) yards, feet and inches and (b) metres, decimetres and centimetres.

**Construction ([Fig.4.21](#))**

1. Obtain the length of the scale to represent 8 yards:  
 $(1/48) \times 8 \text{ yds} = 6 \text{ inches}$
2. Calculate the length of the scale to show 6 m:  $(1/48) \times 6 \text{ m} = 12.5 \text{ cm}$
3. Repeat steps 2 to 7 of Construction: [Fig. 4.8](#) suitably and draw the diagonal comparative scales.



**Fig.4.21**

## 4.6 VERNIER SCALES

Vernier scales sometimes replace the diagonal scales and hence they are used when a smaller division of the secondary division is to be read/measured.

A vernier scale consists of two parts, viz., a main scale and a vernier. The main scale is a plain scale, fully divided into primary and secondary divisions. The vernier is used to sub-divide the secondary divisions.

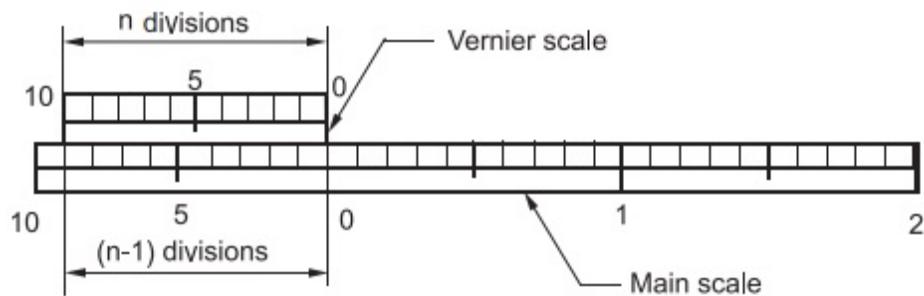
**Least count** It is the smallest distance that can be measured by a vernier scale. It is given by the difference

between one main scale division (msd) and one vernier scale division (vsd).

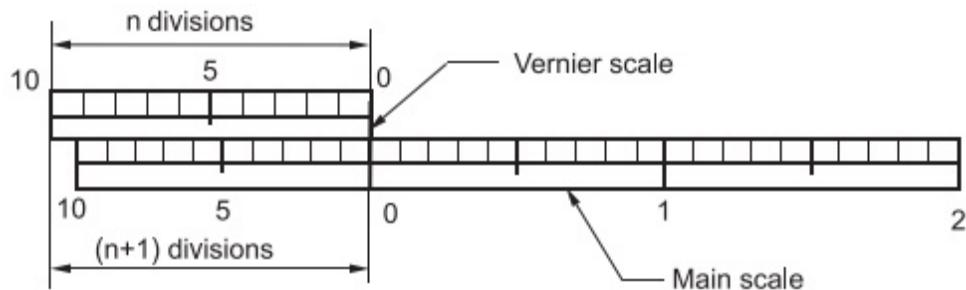
**Types of verniers** The verniers are classified into:  
Direct vernier and retrograde vernier.

## 4.6.1 Direct Vernier

In this,  $n-1$  main scale divisions are divided into  $n$  equal parts on the vernier. Thus, if  $1/10$ th of the secondary division is required; 9 main scale divisions are divided into 10 equal parts on the vernier and, if  $1/20$ th of the secondary division is required; 19 main scale divisions are divided into 20 equal parts on the vernier and so on. Thus, in the case of direct vernier, one vernier scale division is smaller than one main scale division. [Figure 4.22](#) shows the construction of a direct vernier.



**Fig.4.22 Construction of a direct vernier**



**Fig.4.23 Construction of a retrograde vernier**

## 4.6.2 Retrograde Vernier

In this,  $n+1$  main scale divisions are divided into  $n$  equal parts on the vernier. Thus, if  $1/10$ th of the secondary division is required; 11 main scale divisions are divided into 10 equal parts on the vernier and, if  $1/20$ th of the secondary division is required; 21 main scale divisions are divided into 20 equal parts on the vernier and so on. Thus, in the case of retrograde vernier, one vernier scale division is greater than one main scale division. [Figure 4.23](#) shows the construction of a retrograde vernier.

**Problem 21** Construct a vernier scale of 1: 40,000, showing kilometres, hectometres and decametres and long enough to measure 5 km. Mark distances of 2.34 km and 3.92 km on the scale.

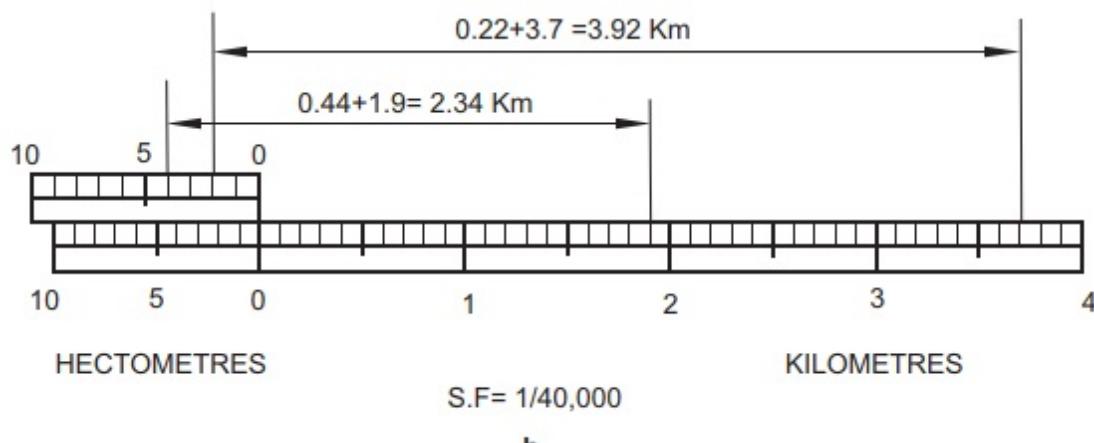
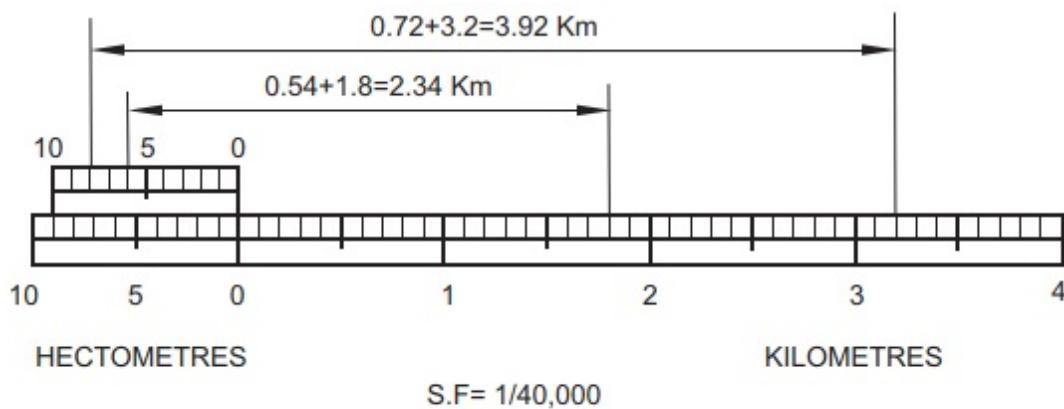
**Construction ([Fig.4.24a](#)) Direct vernier**

1. Calculate the length of the scale:  $(1/40,000) \times 5 \times 10^6 = 125$  mm
2. Draw a line of 125 mm length and divide it into 5 equal parts; each representing 1 km.
3. Divide each part into 10 equal divisions; each representing 1 hectometre.
4. Construct the vernier, by choosing its length equal to 9 main scale divisions and divide it into 10 equal parts.
5. Mark the distances, 2.34 km ( $= 18$  msd + 6 vsd), i.e.,  $(18 \times 0.1$  km +  $6 \times 0.09$  km) and 3.92 km ( $= 32$  msd + 8 vsd) on the scale as shown.

**Construction ([Fig.4.24b](#)) Retrograde vernier**

1. Repeat steps 1 to 3 as above.

2. Construct the vernier, by choosing its length equal to 11 main scale divisions and divide it into 10 equal parts.
3. Mark the distances,  $2.34 \text{ km} (= 19 \text{ msd} + 4 \text{ vsd})$ , i.e.,  $(19 \times 0.1 \text{ km} + 4 \times 0.11 \text{ km})$  and  $3.92 \text{ km} (= 37 \text{ msd} + 2 \text{ vsd})$  on the scale as shown.



**Fig.4.24**

**Problem 22** Construct a vernier scale of 1:50, showing metres, decimetres and centimetres and long enough to measure 5 metres. Mark distances of 2.435m and 3.275 m on the scale.

From the problem, it is evident that the scale



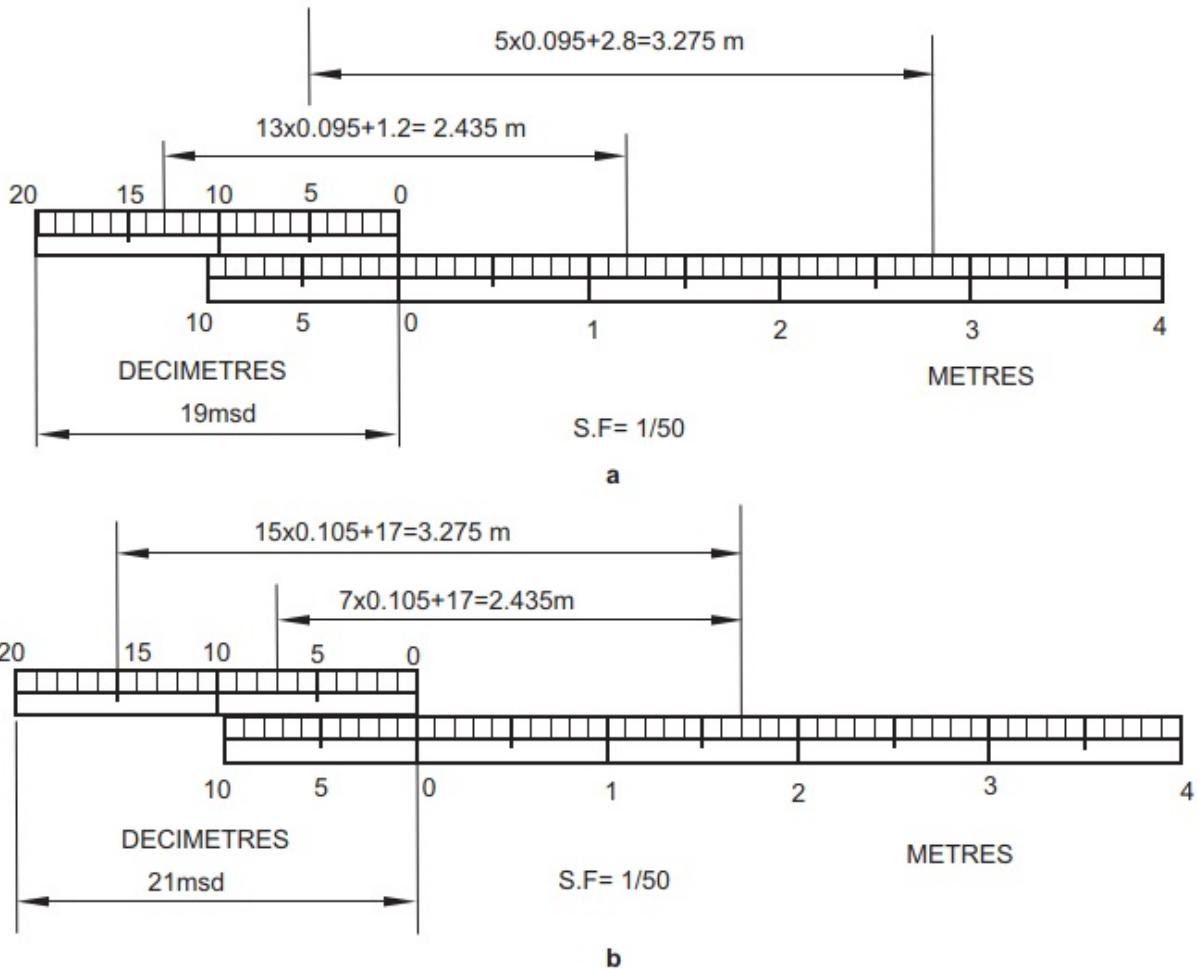
should read correct upto 0.005 m. This is obtained by dividing 19 msd's into 20 vsd's for direct vernier and 21 msd's into 20 vsd's for retrograde vernier.

### ***Construction (Fig.4.25a) Direct vernier***

1. Calculate the length of the scale:  $(1/50) \times 5 \times 1000 = 100 \text{ mm}$
2. Draw a line of 100mm length and divide it into 5 equal parts; each representing 1 m.
3. Divide each part into 10 equal divisions; each representing 1 decimetre.
4. Construct the vernier by choosing its length equal to 19 main scale divisions and divide it into 20 equal parts.
5. Mark the distances, 2.435 m ( $= 12 \text{ msd} + 13 \text{ vsd}$ ) and 3.275 m ( $= 28 \text{ msd} + 5 \text{ vsd}$ ) on the scale as shown.

### ***Construction (Fig.4.25b) Retrograde vernier***

1. Repeat steps 1 to 3 as above.
2. Construct the vernier by choosing its length equal to 21 main scale divisions and divide it into 20 equal parts.
3. Mark the distances, 2.435 m ( $= 17 \text{ msd} + 7 \text{ vsd}$ ) and 3.275 m ( $= 17 \text{ msd} + 15 \text{ vsd}$ ) as shown.



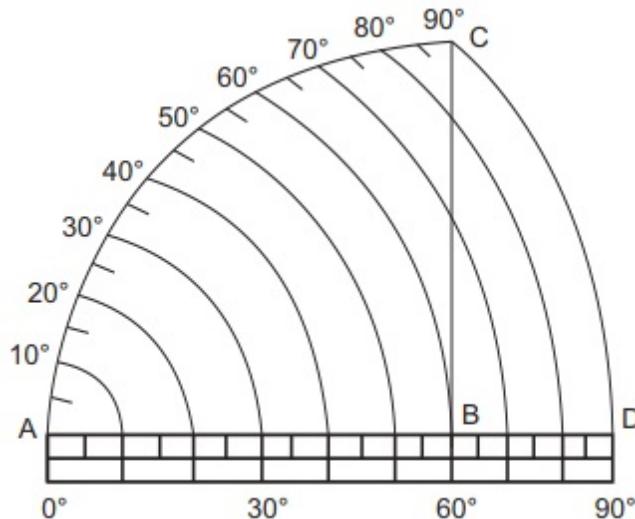
**Fig.4.25**

## 4.7 SCALE OF CHORDS

In the absence of a protractor, a scale of chords may be used to measure an angle or to set the required angle. The scale of chords is a linear scale and the construction is based on the lengths of chords of angles, measured on the same arc.

**Problem 23** Construct the scale of chords, showing  $5^\circ$  divisions.

**Construction (Fig.4.26)**



**Fig.4.26 Scale of chords**

1. Describe an arc of any convenient radius, subtending a right angle at B on a line AB.
  2. Sub-divide the arc into 9 equal parts; representing 10 degree divisions.
  3. Transfer the chord lengths of different angles on to the line AB, by taking A as centre and radii equal to A-10, A-20, etc., to mark 10 degree divisions. The distance A-30° on the scale represents the length of chord A-30°, which subtends an angle of 30° at the point B. Incidentally A-60° along the scale represents the radius of the arc AC.
  4. Similarly, mark 5 degree divisions on the line AB.
- AD is the required scale of chords.

**Problem 24** Measure the given angle ( $\angle ABC$ ) by means of the scale of chords.

**Construction (Fig.4.27)**

1. With centre B and radius equal to  $0^\circ - 60^\circ$  of scale of chords, draw an arc intersecting AB and BC of the

given  $\angle ABC$  at P and Q respectively.

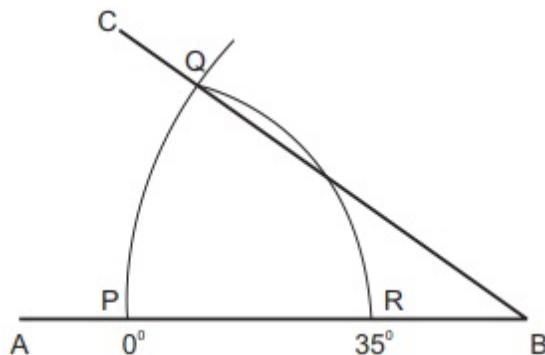
2. Transfer the chord length PQ on to the scale of chords (PR) and read the angle. In this case, it is  $35^\circ$ .

**Problem 25** Set-off an angle, say  $120^\circ$ , using the scale of chords.

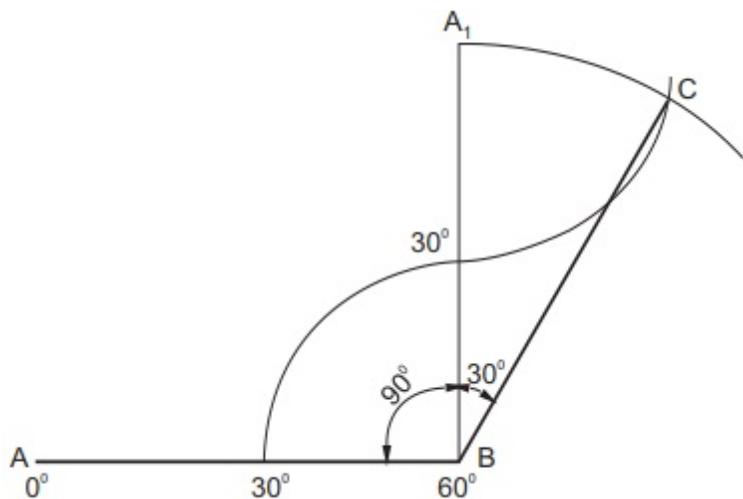
**Construction (Fig.4.28)**

1. Draw a line AB equal to  $0^\circ - 60^\circ$  of the scale of chords.
2. Erect the perpendicular  $BA_1 (=BA)$  to the line BA at B.
3. With centre B and radius  $BA_1$ , draw an arc.
4. With centre  $A_1$  and radius equal to the chord length of the angle  $30^\circ$ , draw an arc intersecting the above arc at C.
5. Join B, C.

$\angle ABC$  is the required  $120^\circ$  ( $= 90^\circ + 30^\circ$ ).



**Fig.4.27**



**Fig.4.28**

## EXERCISES

### Plain scales

- 4.1 Construct a plain scale to show kilometres and hectometres when 2.5 cm represents 1 kilometre and long enough to measure upto 6 kilometres. Find scale factor and indicate a distance of 5 kilometres and 4 hectometres on it.
- 4.2 Construct a scale of scale factor,  $1/6250$  to read kilometres and show on it a distance of 6.5 km.
- 4.3 Construct a scale of  $2 \text{ cm} = 1 \text{ decimetre}$ , to read upto 1 m and show on it a length of 0.66 m.
- 4.4 The distance between two points on a map is 2.1 cm. The points are actually 1 km apart. Construct a scale to measure upto 7 km, and to read in kilometres and hectometres. Show a length of 4.4 km on it.
- 4.5 Construct a scale to be used with a map, the scale of which is  $1 \text{ cm} = 4 \text{ m}$ . The scale should read in metres, upto 60 m. Show on it a distance of 46 m.

- 4.6 Construct the following scales and show below each, its scale factor and the units it represents:
- (i) Scale of  $7/8$ " = 1 yard, to measure upto 10 yards and showing yards and feet. Mark on it, a length of 8 yards and 2 feet.
  - (ii) Scale of  $1\frac{1}{2}$ " = 1 mile, to measure upto 5 miles and showing miles and furlongs. Mark on it, a length of 3 miles and 5 furlongs.
- 4.7 A line of  $1\frac{1}{4}$ " length represents a length of 16' 8". Extend the line to measure upto 60 feet and show on it, units of 10 feet and 1 foot. Mark lengths of 47 feet and 13 feet on it.
- 4.8 In a railway map 100 km measure 2 cm. Construct a scale to measure upto 800km. Show on it, distances of 5.7 km and 7.3 km. What is the scale factor?
- 4.9 A train is moving at the rate of 1.5 km/minute. Construct a scale with scale factor  $1/30,000$ , showing minutes and seconds. Indicate on it, the distance moved by the train in 4 minutes and 35 seconds.
- 4.10 Construct a scale to be used with a map, the scale of which is  $1$  cm = 40 m. The scale should read in metres, upto 500 m. Mark a distance of 456 m on it.
- 4.11 A room of 1728 cu. m of volume is shown by a cube of 216 cu. m volume. Find S.F and construct a plain scale to measure upto 42 m. Mark a distance of 27 m on the scale.

### **Diagonal scales**

- 4.12 Draw a diagonal scale of factor of  $1/48$  to read metres, decimetres and centimetres. Mark a distance of 4.57 m on it.
- 4.13 Construct a scale, to measure km,  $1/8$  of a km and

$1/40$  of a km, in which 1 km is represented by 4 cm.  
Mark a distance of 2.775 km on this scale.

- 4.14 Draw a diagonal scale of 1:4 showing cm and mm and long enough to measure upto 40 cm. Mark on it a length of 32.7 cm.
- 4.15 On a map, the distance between two points is 15 cm. The real distance between them is 20 km. Draw a diagonal scale of this map and show on it a distance of 16.7 km. Also, construct a plain scale to show a distance of 15.6 km on it.
- 4.16 The scale factor adopted for the drawing of a machine part is  $1/10$ . The maximum dimension to be measured is 2 m, and minimum dimension to be measured is 1 cm. Draw a diagonal scale and show a measurement of 1.87 m on it.
- 4.17 Draw a diagonal scale of S.F =  $1/20,000$ , to show kilometres and decimals of a kilometre. Mark on the scale, the distances 1.64 km and 2.37 km.
- 4.18 Construct a scale to read metres and centimetres, upto 3 metres and mark on it, the lengths: (i) 2 metres and 24 centimetres and (ii) 78 centimetres for a given S.F of  $1/20$ .
- 4.19 Construct a diagonal scale to read kilometres and decametres, given that 1 km is represented by 5 cm on the drawing. Mark a distance of 1.54 km on the scale. What is the scale factor?
- 4.20 Construct a diagonal scale of S.F =  $1/40$  and to read upto a centimetre. Show the following distances on the scale: (a) 3.58 metre, (b) 5.79 metre and (c) 4.27 metre.
- 4.21 Construct a diagonal scale of five times the full size, to read accurately upto 0.2 mm and mark on it, the

following lengths: 4.96 cm, 28.8mm and 2.02cm.

- 4.22 On a map drawn in metric units, a line measuring 1.5 cm represents an actual size of 1.2 m length. Draw a diagonal scale, showing the smallest division of 5cm and suitable to measure a maximum length of 20 m. Mark the lengths, 5.5 m and 11.65 m on the scale.
- 4.23 An area covered by a triangle of base 12 cm and altitude 24cm, represents an area of  $36 \text{ km}^2$ . Find the scale factor and construct a diagonal scale to read kilometres, hectometres and decametres. Mark the distances of 1.05 km and 8.82 km on it.
- 4.24 An area of 144 sq. cm on a map represents an area of 36 sq. km on a field. Find the scale factor for this map and draw a diagonal scale and mark a distance of 7.56 km on it.
- 4.25 A block of ice-berg  $1000 \text{ m}^3$  volume, is represented by a block of  $8 \text{ cm}^3$  volume. Find scale factor and construct a scale to measure upto 50 m. Mark a distance of 32.5 m on the scale.
- 4.26 A distance of 1 mm on a part of an instrument is to be represented by a 3 mm long line on the drawing.
- Determine the scale factor of the drawing.
  - Construct a scale using the above scale factor, showing cm, mm and one tenth of mm and long enough to measure 5 cm.
  - Mark the distances 35.7mm and 20.4 mm on the scale.
- 4.27 A map is drawn with the scale of 1: 20,000. Construct a diagonal scale to read in kilometres and upto  $1/100$  of a kilometre. Mark a distance of 4.37 km on the scale.

- 4.28 On a certain map, a 22 cm long line represents a distance of 440 m. Draw a diagonal scale to read upto a metre. Mark a distance of 189 m on the scale.
- 4.29 Construct a diagonal scale to read metres, tenths of a metre and centimetres to a scale of 1/50. Mark a distance of 4.47 m on the scale.
- 4.30 Construct a scale, showing yards, feet and inches by diagonal division, where 12 inches represent 31.5 yards. The scale is to be long enough to measure 20 yards. Mark on this scale, a distance of 6 yards, 2 feet and 6 inches.
- 4.31 A rectangular plot of land, with an area of 0.5 hectares, is represented on a map by a similar rectangle of 5 sq. cm. Calculate the scale of the map. Also, construct a scale to read upto a single metre and long enough to measure 400 metres. Mark on it, a distance of 345 m.

### **Comparative scales**

- 4.32 The distance between two towns is 800 km. A car covers this distance in 10 hours. Construct a scale to measure the distance covered in 5 seconds interval. The scale factor is 1/400,000. Indicate the distance covered by the car in 35 minutes and 40 seconds.
- 4.33 A drawing is made to a scale of 1 inch = 1 ft and another drawing is required on which the dimensions shall be 1/4 th of those on the first. Make a scale for the second drawing to show feet and inches.
- 4.34 What is the scale factor of a scale, which measures 2.5 inches to a mile? Draw a plain comparative scale of kilometres, to read upto 10 km and 16 miles (1 mile = 1.609 km).
- 4.35 A map is drawn to a scale of 1 inch = 4 miles.

Construct a comparative scale of Russian versts to measure 20 versts and 15 miles (1 verst = 1,166.6 English yards).

- 4.36 On a map showing a scale of kilometres; 100 km are found to be equal to 7.5 inches. What is the scale factor? Construct a comparative scale of miles (1 km = 1093.6 yards).
- 4.37 Construct a comparative diagonal scale of metres and yards, having a scale factor of 1/6250, to show upto 800 m and 900 yards (1 m = 1.0936 yards).

### **Vernier scales**

- 4.38 Construct a scale of 1/2.5, to show decimetres and centimetres and, by a vernier to read millimetres and to measure upto 4 dm. Construct, both retrograde and direct vernier scales. Show 3.65 dm and 2.39 dm on the scales.
- 4.39 Construct a vernier scale of 1/80, to show feet and inches. Mark 15 feet and 7.2 inches on it.
- 4.40 Construct a scale of S.F 1:25 to show decimetres and centimetres and by a vernier to read millimetres, to measure upto 4 decimetres. Show on it, lengths representing 3.39 decimetres and 0.91 decimetres.
- 4.41 Draw a full size vernier scale to read 1/8 and 1/64 inches and mark on it, lengths of  $3\frac{7}{32}$  inches,  $2\frac{49}{64}$  inches and  $2\frac{27}{64}$  inches.
- 4.42 Draw a vernier scale of S.F = 1/25, to read centimetres and upto 4 m; and on it, show lengths of 2.39 m and 0.58 m.
- 4.43 Construct a direct vernier with a least count of 2mm. The S.F of the scale is 1/2.5. The scale should be long

enough to measure 20 cm. Show a length of 13.4 cm on the scale.

- 4.44 Construct a direct vernier scale to read distances, correct to a decametre on a map, in which the actual distances are reduced in the ratio of 1:40,000. The scale should be long enough to measure 6 km. Mark the lengths, 3.34 km and 0.59 km on the scale.
- 4.45 The actual length of 600 m is represented by a line of 30 cm on a drawing. Construct a retrograde vernier scale to read upto 400 m. Draw the scale with a least count of 50 cm. Mark the lengths, 205.5 m and 166.5 m on the scale.

### **Scale of chords**

- 4.46 Construct a scale of chords, showing  $5^\circ$  divisions and with its help; set-off angles of  $25^\circ$ ,  $40^\circ$ ,  $55^\circ$  and  $130^\circ$  (Take  $0^\circ - 60^\circ = 8$  cm).
- 4.47 Construct scale of chords of 75 radius and to read upto  $90^\circ$  by  $10^\circ$  divisions. Also, at the end of a line, 100 long, construct angles of  $40^\circ$  and  $50^\circ$ .
- 4.48 Construct a scale of chords to read in tens of degrees and with its help, set angles of (i)  $50^\circ$  and (ii)  $160^\circ$ . Also, determine the remaining angles in a right angle triangle, having the sides of the right angle as 100 and 60.

## **REVIEW QUESTIONS**

- 4.1 What is meant by scale factor?
- 4.2 Name the different types of scales used in engineering practice.

- 4.3 Describe both in M.K.S and F.P.S units, a scale whose scale factor is  $1/2,400$ .
- 4.4 What is a plain scale?
- 4.5 What is the application of a diagonal scale?
- 4.6 Differentiate between a plain and diagonal scale.
- 4.7 What is a comparative scale?
- 4.8 What are the differences between a vernier and a diagonal scale?
- 4.9 Explain the two different types of verniers.
- 4.10 What is least count?
- 4.11 What is the application of scale of chords?

## OBJECTIVE QUESTIONS

- 4.1 When a drawing is made to the same size of the object, the name of the scale is \_\_\_\_\_ .
- 4.2 The relative values of the scale factor of enlarging, full size and reducing scales are \_\_\_, \_\_ and \_\_\_ respectively.
- 4.3 For drawing of small instruments, watch parts, \_\_\_\_\_ scale is used.
- 4.4 When measurements are desired in three units, \_\_\_\_\_ scale is used.
- 4.5 Calculate the scale factor of a scale on which (i) 5 cm represents 2.5 m and (ii) 6 " represents 1 furlong.
- 4.6 Enlarging scales are usually found in civil engineering practice.

(True/False)

- 4.7 \_\_\_\_\_ scale is used for converting miles into kilometres.
- 4.8 The main scale of a vernier scale is a \_\_\_\_\_ scale.
- 4.9 The main scale of a vernier scale is fully divided into primary and secondary divisions.  
(True/False)
- 4.10 In a direct vernier, 19 main scale divisions are divided into 20 equal parts on the vernier.  
(True/False)
- 4.11 In a retrograde vernier, 9 main scale divisions are divided into 10 equal parts on the vernier.  
(True/ False)
- 4.12 For reading upto  $1/20^{\text{th}}$  of a main scale division, the number of divisions on the vernier should be \_\_\_\_\_.  
4.13 The number of divisions on the vernier should be different on direct and retrograde verniers, for a given least count.  
(True/ False)

## ANSWERS

- 4.1 full scale
- 4.2  $>1$ , 1 and  $<1$
- 4.3 enlarging
- 4.4 diagonal
- 4.5  $1/50$  and  $1/1320$
- 4.6 False
- 4.7 Comparative

4.8 plain

4.9 True

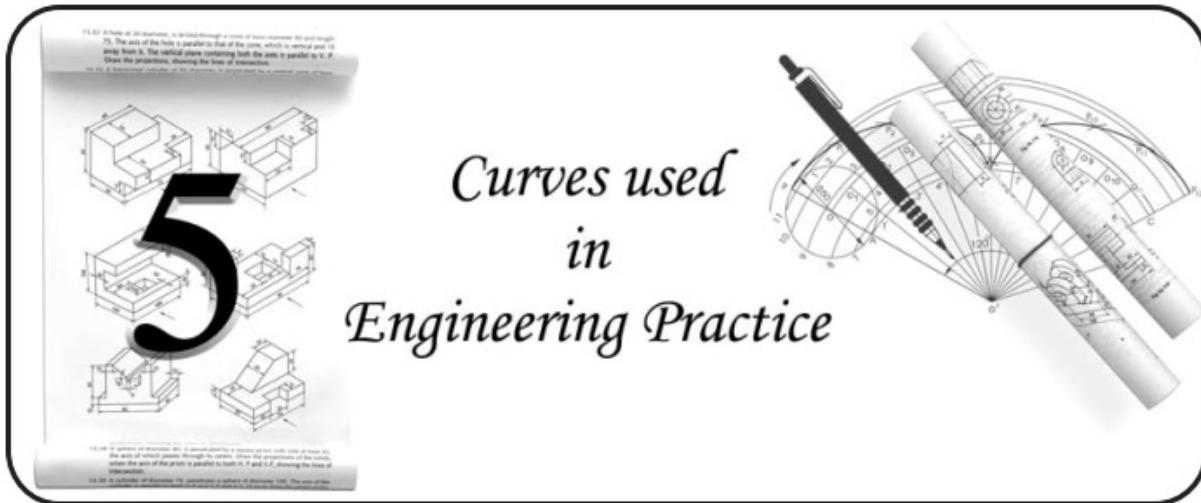
4.10 True

4.11 False

4.12 twenty

4.13 False

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## 5.1 INTRODUCTION

In engineering practice, the profiles of some of the objects contain regular curved features. Some are obtained as intersections, when a plane passes through a cone and some are obtained by tracing the locus of a point moving according to the mathematical relationship, applicable to that particular curve. The following are the types of curves that are considered in this chapter:

Conic sections,

Cycloidal curves,

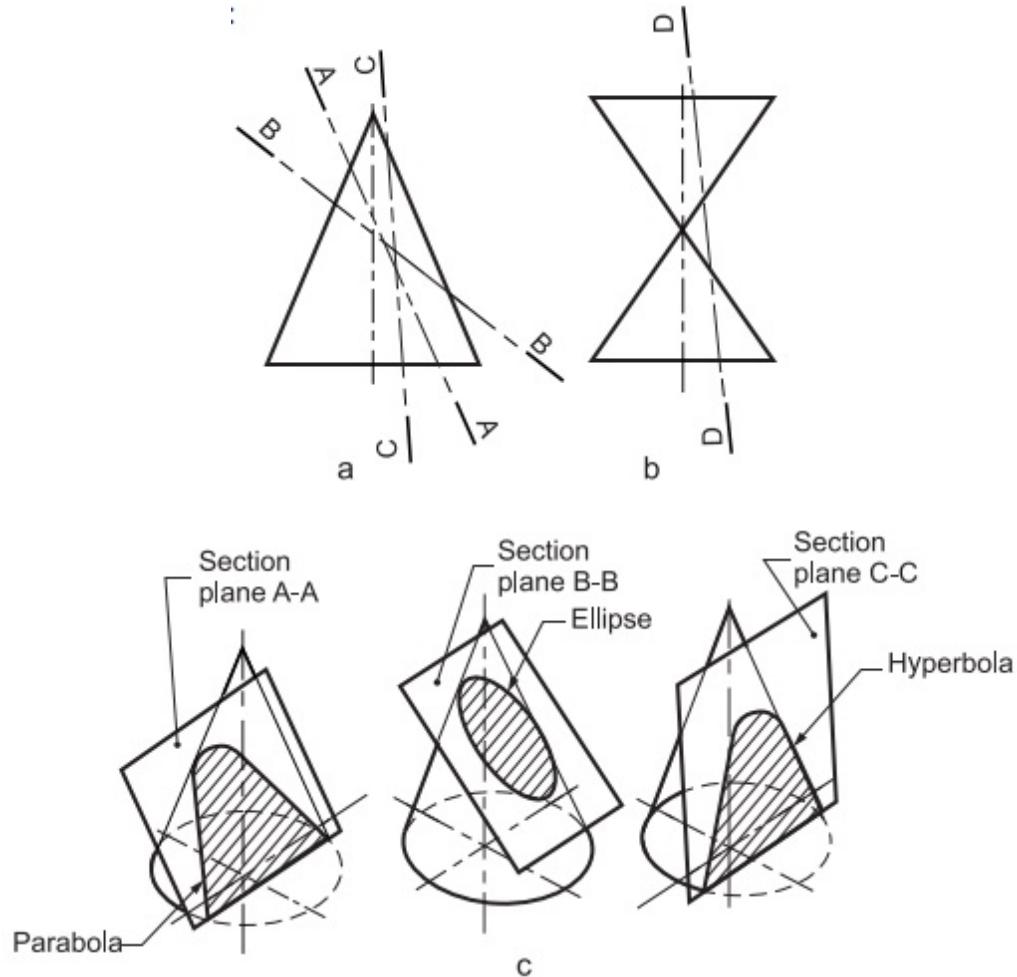
Involutes, and

Spirals.

## 5.2 CONIC SECTIONS

The conic sections are the intersections of a right regular cone, by a cutting plane in different positions, relative to

the axis of the cone. Considering the apex angle of the cone as  $2\theta$ , and the inclination of the cutting plane as  $\alpha$ , the following are the possible conic sections (Fig.5.1a):



**Fig.5.1 Sections of a cone**

## 5.2.1 Parabola

If the cutting plane angle  $\alpha$  is equal to  $\theta$ , i.e., when the section plane A-A is parallel to a generator of the cone, the curve of intersection is a parabola, which is not a closed curve. The size of the parabola depends upon the distance of the section plane from the generator of the cone.

## 5.2.2 Ellipse

When a cone is cut by a section plane B-B at an angle  $\alpha$ , which is more than half of the apex angle  $\theta$  and less than  $90^\circ$ , the curve of intersection is an ellipse. The size of the ellipse depends upon the angle  $\alpha$  and the distance of the section plane from the apex of the cone. Further, it may be noted that only when the section plane cuts all the generators of the cone, the elliptical section formed is a closed curve.

## 5.2.3 Hyperbola

If the angle  $\alpha$  is less than  $\theta$  (section plane C-C), the curve of intersection is a hyperbola. The curve of intersection is hyperbola, even if  $\alpha=0$ , i.e., section plane parallel to the axis, provided the section plane is not passing through the apex of the cone. If the section plane passes through the apex, the section produced is an isosceles triangle.

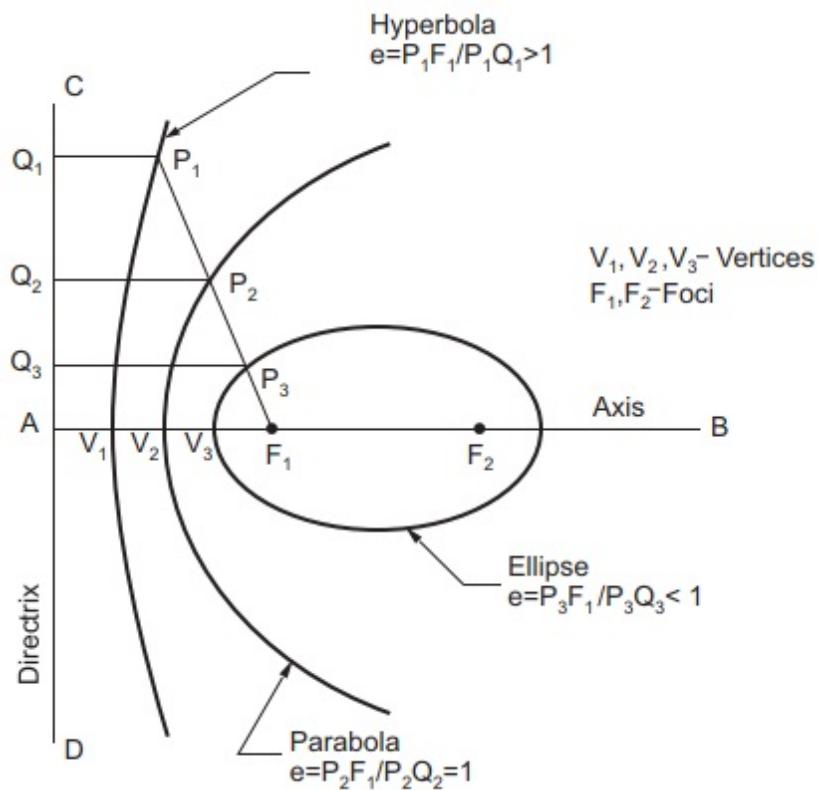
If a double cone is cut by a section plane D-D on one side of the common axis ([Fig.5.1b](#)) the curves of intersection results in two branches of hyperbola. However, if  $\alpha=0$ , the two branches of the curve will be symmetric in form.

[Figure 5.1c](#) shows the different conic sections, produced by the section planes A-A, B-B and C-C of [Fig.5.1a](#).

## 5.3 CONSTRUCTION OF CONICS- ECCENTRICITY METHOD

A conic section may be defined as the locus of a point moving in a plane such that, the ratio of its distance from a fixed point to a fixed straight line is always a constant. The fixed point is called the focus and the fixed straight line, the directrix.

The ratio,  $\frac{\text{distance of the point from the focus}}{\text{distance of the point from the directrix}}$  is called eccentricity,  $e$ . The value of  $e$  is less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola (Fig.5.2).



**Fig.5.2 Eccentricity values for different conics**

The line passing through the focus and perpendicular to the directrix is called the axis. The point at which the conic section intersects the axis is called its vertex.

**Problem 1** Construct a parabola, with the distance of the focus from the directrix as 50. Also, draw normal and tangent to the curve, at a point 40 from the directrix.

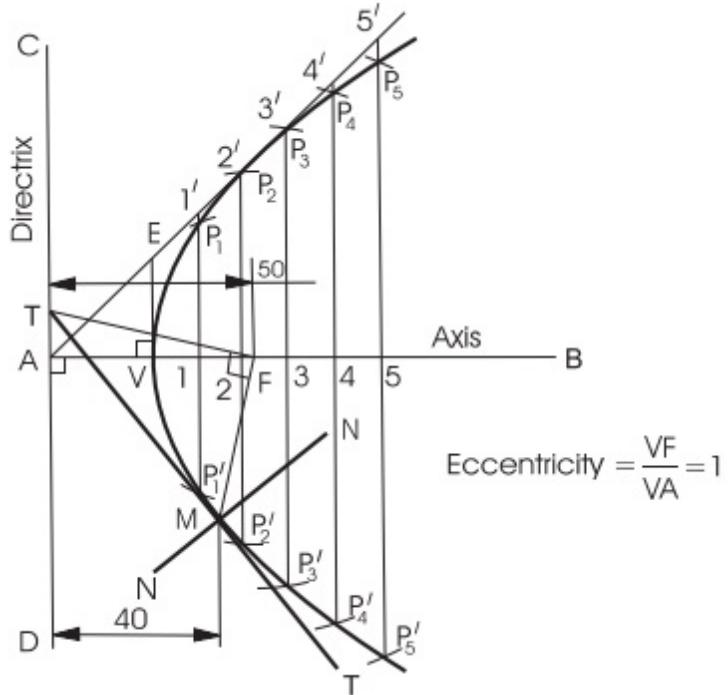
- ⇒ A parabola is a curve traced by a point, moving such that, at any position, its distance from a fixed point (focus) is always equal to its distance from a fixed straight line (directrix).

### **Construction (*Fig.5.3*)**

1. Draw the axis AB and the directrix CD, at right angle to each other.
2. Mark the focus F on the axis such that, AF = 50.
3. Locate the vertex V on AB such that, AV = VF = 25.
4. Draw a line VE, perpendicular to AB such that, VE = VF.
5. Join A, E and extend. By construction,  $\frac{VE}{VA} = \frac{VF}{VA} = 1$ , the eccentricity.
6. Locate a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equi-distant.
7. Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at 1', 2', 3', etc.
8. With centre F and radius 1-1', draw arcs intersecting the line through 1 at P<sub>1</sub> and P<sub>1</sub>'. P<sub>1</sub> and P<sub>1</sub>' are the points on the parabola, because, the distance of P<sub>1</sub> (P<sub>1</sub>') from F is 1-1' and from CD, it is A-1 and,

$$\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = 1$$

9. Similarly, locate the points P<sub>2</sub>, P<sub>2</sub>'; P<sub>3</sub>, P<sub>3</sub>'; etc., on either side of the axis.
10. Join the points by a smooth curve, forming the required parabola.



**Fig.5.3 Construction of parabola-Eccentricity method**

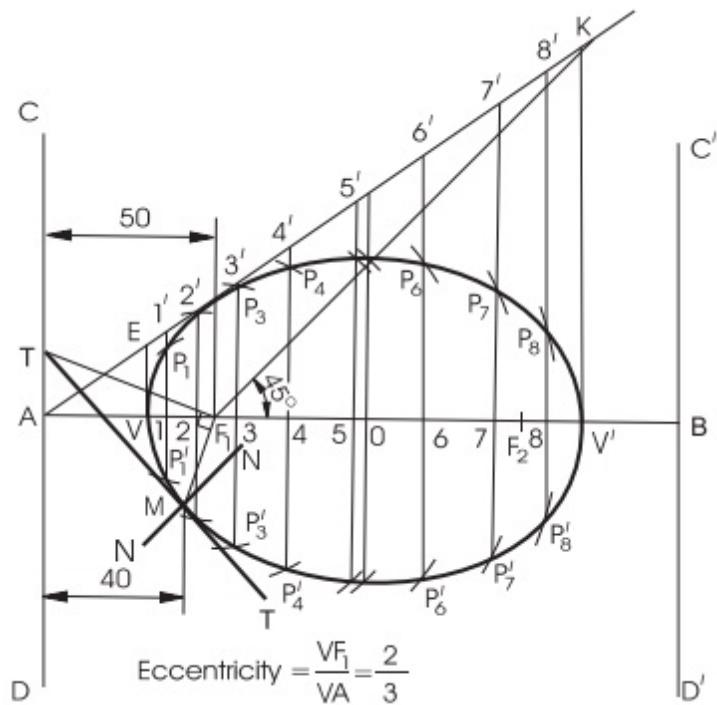
To draw tangent and normal to the parabola, locate the point M, which is at 40 from the directrix. Then, join M to F and draw a line through F, perpendicular to MF, meeting the directrix at T. The line joining T and M and extended (T-T') is the tangent and a line N-N', through M and perpendicular to T-T' is the normal to the curve.

**Problem 2** Construct an ellipse, with distance of the focus from the directrix as 50 and eccentricity as  $2/3$ . Also, draw normal and tangent to the curve at a point 40 from the directrix.

### **Construction (Fig.5.4)**

1. Draw the axis AB and the directrix CD, at right angle to each other.
2. Mark focus  $F_1$  on the axis such that,  $AF_1 = 50$ .

3. Divide  $AF_1$  into 5 equal parts.
4. Locate the vertex V on the third division point from A.
5. Draw a line VE, perpendicular to AB such that,  $VE = VF_1$ .
6. Join A, E and extend. By construction,  $\frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$ , the eccentricity.
7. Mark a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equi-distant.
8. Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at  $1', 2', 3'$ , etc.
9. With centre  $F_1$  and radius  $1-1'$ , draw arcs intersecting the line through 1 at  $P_1$  and  $P_1'$ .  $P_1$  and  $P_1'$  are the points on the ellipse, because the distance of  $P_1$  from  $F_1$  is  $1-1'$  and from CD, it is  $A-1$  and,  $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$ , the eccentricity.
10. Similarly, locate the points  $P_2, P_2'; P_3, P_3'$ ; etc., on either side of the axis.
11. Join the points by a smooth curve, forming the required ellipse.



## **Fig.5.4 Construction of an ellipse-Eccentricity method**

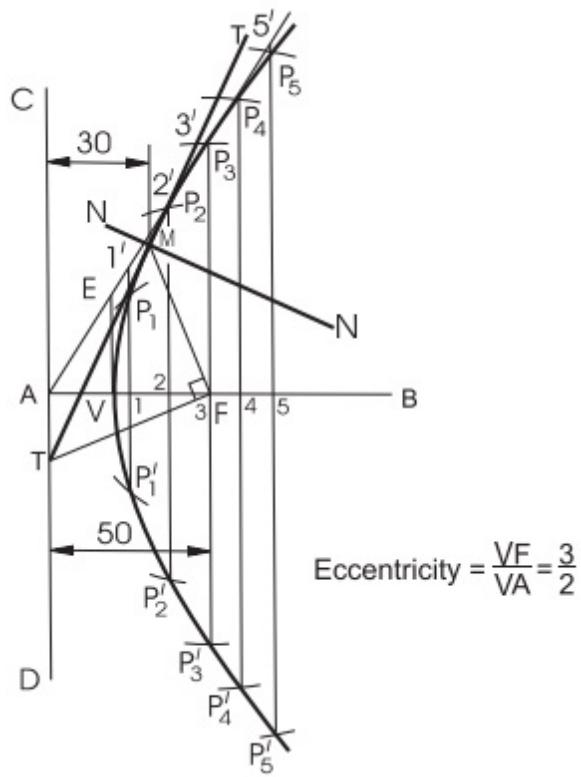
1. The ellipse is a closed curve and has two foci, two directrices and two vertices. To locate the other vertex  $V'$ , draw a line at  $45^\circ$  to the axis passing through  $F_1$  and intersecting  $AE$  produced at  $K$ . A vertical line drawn from  $K$  meets the axis at  $V'$ .
  2. To draw tangent and normal to the ellipse, the construction, similar to the one given for parabola may be followed.
  3. The second focus  $F_2$  may be located such that  $VF_2 = VF_1$ .

**Problem 3** Construct a hyperbola, with the distance between the focus and the directrix as 50 and eccentricity

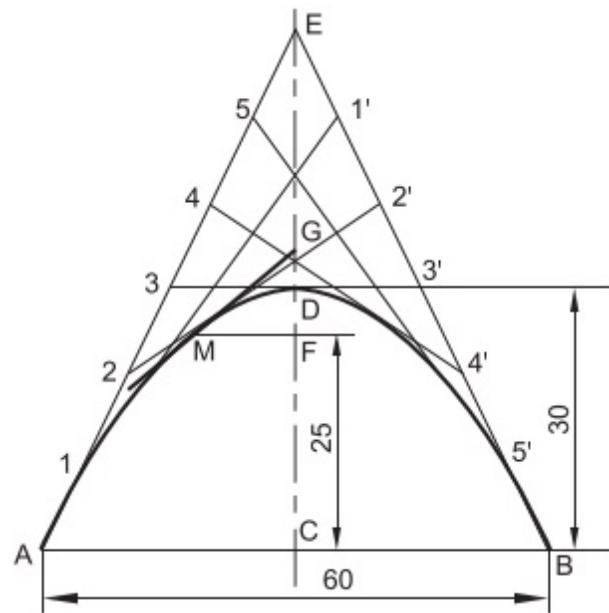
as  $3/2$ . Also, draw normal and tangent to the curve at a point 30 from the directrix.

### **Construction ([Fig.5.5](#))**

1. Draw the axis AB and the directrix CD, at right angle to each other.
2. Mark focus F on the axis such that,  $AF = 50$ .
3. Divide AF into 5 equal parts.
4. Locate the vertex V on the second division point from A.
5. Draw a line VE, perpendicular to AB such that,  $VE = VF$ .
6. Join A, E and extend. By construction,  $\frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$ , the eccentricity.
7. Repeat steps 6 to 8 of Construction: [Fig.5.3](#).  $P_1$  and  $P_1'$  are the points on the hyperbola, because the distance of  $P_1$  from F is  $1-1'$  and from CD, it is  $A-1$ , and  $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$ , the eccentricity.



**Fig.5.5 Construction of hyperbola-Eccentricity method**



**Fig.5.6 Construction of parabola-Tangent method**

8. Similarly, locate the points  $P_2'$ ,  $P_2$ ;  $P_3'$ ,  $P_3$ ; etc., on either side of the axis.
  9. Join the points by a smooth curve, forming the required hyperbola.
-  1. A hyperbola is an open curve and has one focus and one directrix.
2. To draw tangent and normal to the hyperbola, the construction similar to the one given for parabola may be followed.

## 5.4 CONSTRUCTION OF CONICS - OTHER METHODS

### 5.4.1 Parabola

**Problem 4** Construct a parabola, with the length of base as 60 and axis 30 long. Also, draw a tangent to the curve at a point 25 from the base.

*Tangent method*

#### **Construction ([Fig.5.6](#))**

1. Draw the base AB (= 60) and axis CD (=30) such that, CD is a perpendicular bisector to AB.
2. Produce CD to E such that, DE = CD.
3. Join E, A and E, B. These are the tangents to the parabola at A and B.

4. Divide AE and BE into the same number of equal parts and number the points as shown.
5. Join  $1, 1'; 2, 2'; 3, 3'$ ; etc., forming the tangents to the required parabola.

A smooth curve passing through A, D and B and tangential to the above lines is the required parabola.

To draw tangent to the parabola, locate the point M, which is at 25 from the base. Then, draw a horizontal through M, meeting the axis at F. Mark G on the extension of the axis such that,  $DG=FD$ . Join G, M and extend, forming the tangent to the curve at M.

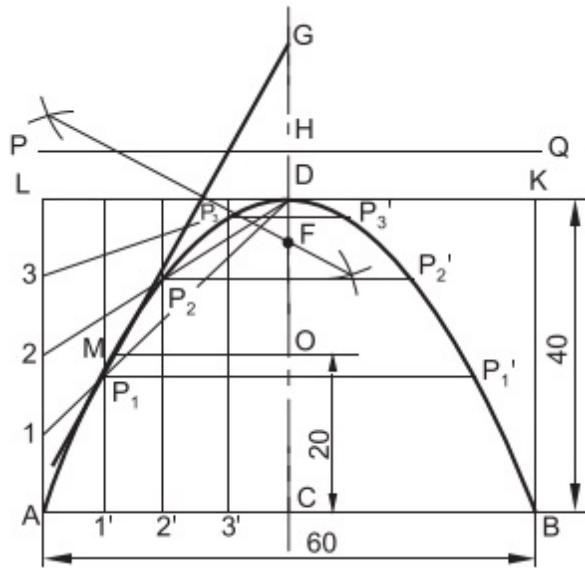


A stone thrown at an angle, traverses the path of a parabola; the distance covered by the stone being equal to the base of the parabola.

**Problem 5** *Construct a parabola with base 60 and length of the axis 40. Draw a tangent to the curve at a point 20 from the base. Also, locate the focus and directrix to the parabola.*

*Rectangle method*

**Construction (Fig.5.7)**



**Fig.5.7 Construction of parabola-Rectangle method**

1. Draw the base AB ( $=60$ ) and axis CD ( $=40$ ) such that, CD is a perpendicular bisector to AB.
2. Construct the rectangle ABKL, passing through D.
3. Divide AC and AL into the same number of equal parts and number the points as shown.
4. Join 1, 2 and 3 to D.
5. Through 1', 2' and 3', draw lines parallel to the axis; intersecting the lines 1-D, 2-D and 3-D at  $P_1'$ ,  $P_2'$  and  $P_3'$  respectively.
6. Obtain the points  $P_1'$ ,  $P_2'$  and  $P_3'$  which are symmetrically placed to  $P_1$ ,  $P_2$  and  $P_3$  with respect to the axis CD.

Join the points by a smooth curve, forming the required parabola.

To draw a tangent to the curve:

- (i) Locate the given point M on the curve, which is at 20 from the base.
- (ii) Draw a horizontal line through M, meeting the axis at O.
- (iii) Locate the point G on the axis such that,  $GD = OD$ .
- (iv) Join G, M and extend, forming the required tangent.

To locate the focus and directrix:

- (i) Draw a perpendicular bisector to the tangent GM, intersecting the axis at F.
- (ii) Mark the point H on the axis such that,  $HD = FD$ .
- (iii) Draw a line PQ, perpendicular to the axis and passing through H.

F is the focus and PQ, the directrix of the given parabola.

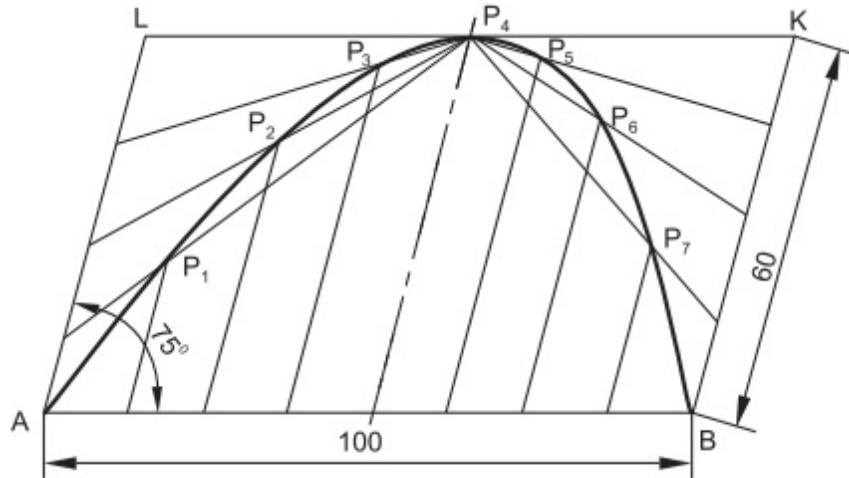
**Problem 6** Construct a parabola in a parallelogram of sides  $100 \times 60$  and with an included angle of  $75^\circ$ .

*Parallelogram method*

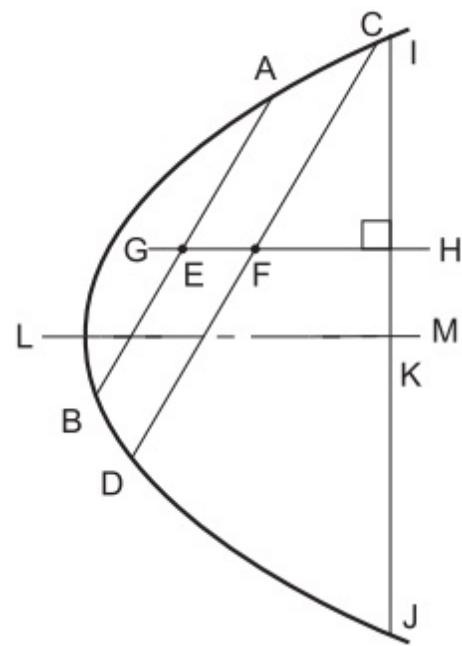
Figure 5.8 shows the construction of a parabola, in the parallelogram ABKL, following the method similar to Construction: [Fig.5.7](#).

**Problem 7** Determine the axis of the parabola, shown in [Fig.5.9](#).

**Construction ([Fig.5.9](#))**



**Fig.5.8 Construction of parabola-Parallelogram method**



**Fig.5.9 Determination of the axis of a parabola**

1. Draw any two parallel chords AB and CD, to the given parabola, separated by any suitable distance.
2. Locate the mid-points E and F of the chords AB and CD respectively.

3. Draw a line GH, passing through E and F. The line GH is parallel to the axis.
4. Draw a chord IJ, perpendicular to GH.
5. Locate the mid-point K of the chord IJ and through it, draw a line LM, parallel to GH.

The line LM is the required axis to the curve.

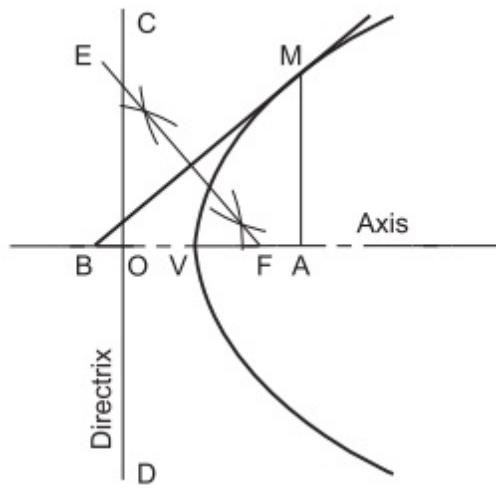
**Problem 8** Determine the focus and directrix of the parabola, shown in [Fig.5.10](#).

**Construction ([Fig.5.10](#))**

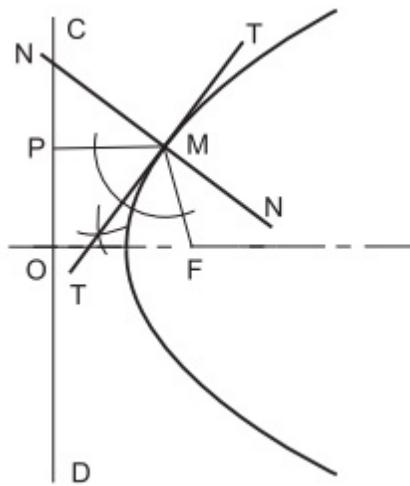
1. Locate any point M on the given parabola.
  2. Draw the line MA, perpendicular to the axis.
  3. Mark the point B on the axis such that,  $BV = VA$ .
  4. Join B, M and extend.
  5. Draw the perpendicular bisector EF to BM, intersecting the axis at F, the focus.
  6. Mark the point O on the axis such that,  $OV = VF$ .
  7. Through O, draw the line CD perpendicular to the axis.
- CD is the directrix of the given parabola.

**Problem 9** Draw normal and tangent to the given parabola ([Fig.5.11](#)) at any point on it, given the focus and directrix.

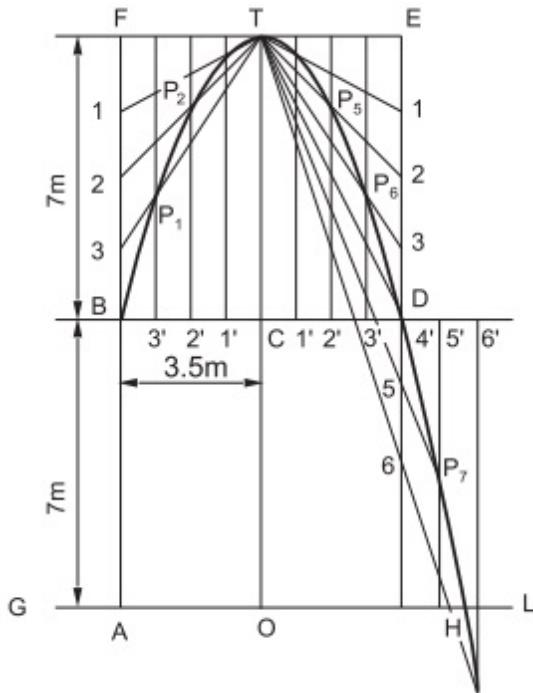
**Construction ([Fig.5.11](#))**



**Fig.5.10 Determination of the focus and directrix of a parabola a parabola**



**Fig.5.11 Drawing normal and tangent to**



**Fig 5.12**

1. Locate a point M on the curve.
2. Join MF and draw MP, perpendicular to the directrix CD.
3. Bisect  $\angle PMF$ .  
The bisector TM produced is the required tangent.
4. Draw the line MN, perpendicular to MT, forming the required normal.

**Problem 10** A stone is thrown from a building of 7m height and at its highest flight, the stone just crosses a palm tree of 14 m height. Trace the path of the stone, if the distance between the building and the tree is 3.5 m.

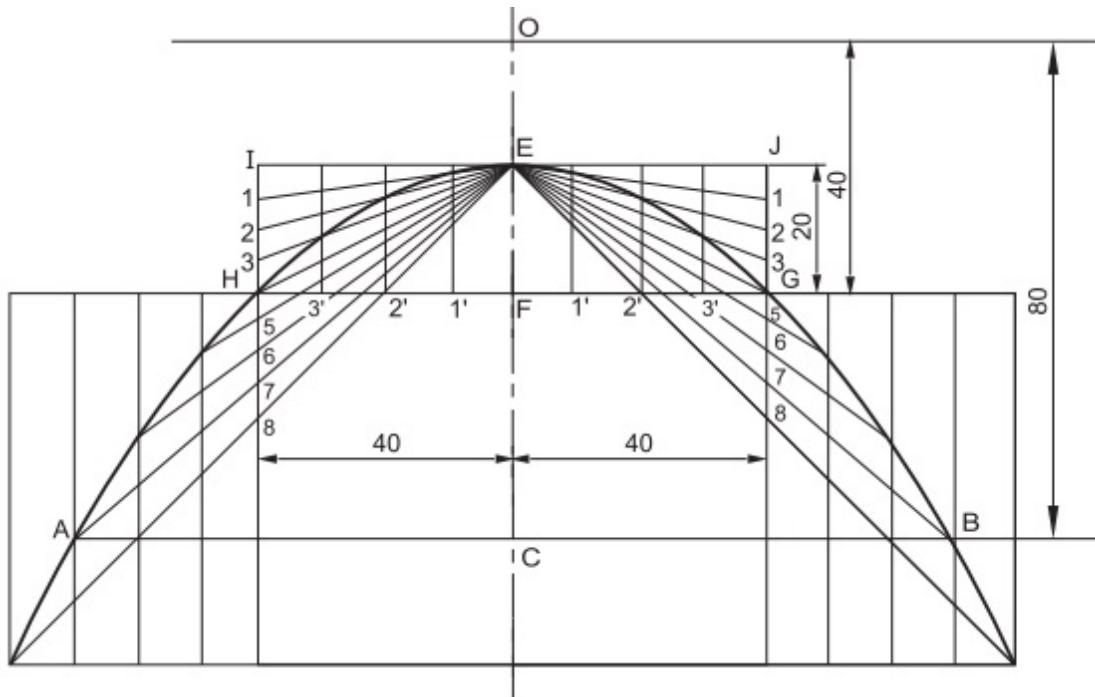
**Construction (Fig.5.12)**

1. Draw lines AB and OT, representing the building and palm tree respectively;

3. 5 m apart and above the ground line GL.
2. Draw a horizontal line through B; intersecting OT at C.
3. Locate D on BC extended such that,  $CD = BC$  and complete the rectangle BDEF.
4. Inscribe the parabola in the rectangle BDEF, by rectangle method.
5. Draw the path of the stone till it reaches the ground (H), extending the principle of rectangle method.

**Problem 11** Construct a right angled triangle EFG such that,  $FG=40$  and  $FE=20$  and  $\angle EFG=90^\circ$ . The point G is on the parabola, whose focus is the point F on the triangle. If FE is a part of parabola's axis, draw the parabola and determine the double ordinate at a distance of 80 from its directrix.

**Construction (Fig.5.13)**



**Fig.5.13**

1. Construct the given right-angled triangle EFG.
2. Draw the axis, by extending EF, on either side.
3. Locate the point O on the axis such that,  $FO = FG$ .
4. Through O, draw a line, perpendicular to the axis; forming the directrix of the hyperbola.



$$\frac{\text{Distance of the point } G \text{ from the focus } F}{\text{Distance of the point } G \text{ from the directrix}} = 1$$

$$\therefore FG = FO$$

5. Complete the rectangle GHIJ, passing through the points F and E, keeping  $FH = FG$ .
6. Inscribe the parabola in the rectangle, by rectangle method.
7. Extend the parabola, beyond the points H and G, by rectangle method, as shown.
8. Locate the point C on the axis such that,  $OC = 80$ .
9. Through C, draw a line perpendicular to the axis, meeting the parabola at A and B.

The length AB is equal to the length of the double ordinate of the parabola.

**Problem 12** *A jet of water is issuing through an orifice of 50 diameter, fitted to a vertical side of a tank. The centre of the orifice is 1.5 m above the ground and centre of the jet touches the ground at a distance of 2.5 m from the vertical orifice. Draw the locus of the centre of the jet, which is just issuing from the orifice, till it reaches the ground. Name the curve.*

**Construction (Fig.5.14)**

1. Draw the ground line GL and locate the point A, the centre of the orifice, on the vertical line through a point D on GL such that,  $DA = 1.5$  m.
2. Locate the point C, the point where the jet touches the ground on GL such that,  $DC = 2.5$  m.
3. Complete the rectangle ABCD.
4. Inscribe the parabolic curve through A and C in the rectangle; following the rectangle method (Refer Construction: [Fig.5.7](#)).

## 5.4.2 Ellipse

The ellipse is a curve traced by a point, moving such that, the sum of its distances from two points, known as foci, is constant and equal to the major axis.

The ellipse, shown in [Fig.5.15](#), has two foci  $F_1$  and  $F_2$ . The line passing through the two foci and terminated by the curve AB, is called the major axis. The line bisecting the major axis at right angle and terminated by the curve CD, is called the minor axis. The foci are at equi-distant from centre O.

The points P, C, Q, etc., are on the curve. By definition,

$$PF_1 + PF_2 = CF_1 + CF_2 = QF_1 + QF_2 = AB$$

$$\therefore CF_1 = CF_2 = \frac{1}{2} AB$$

Hence, the distance from an end of minor axis to the focus is equal to half of the major axis.

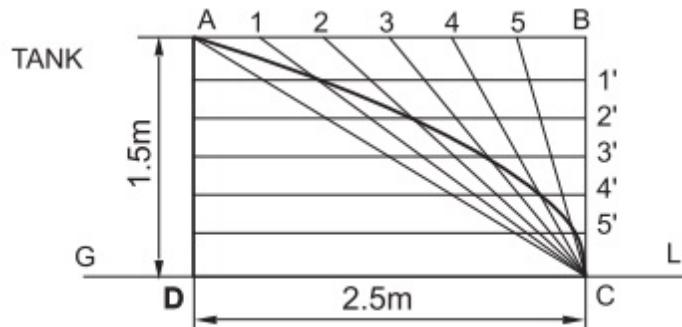


Out of the three details, major axis, minor axis and foci; if any two are given, the third one may be

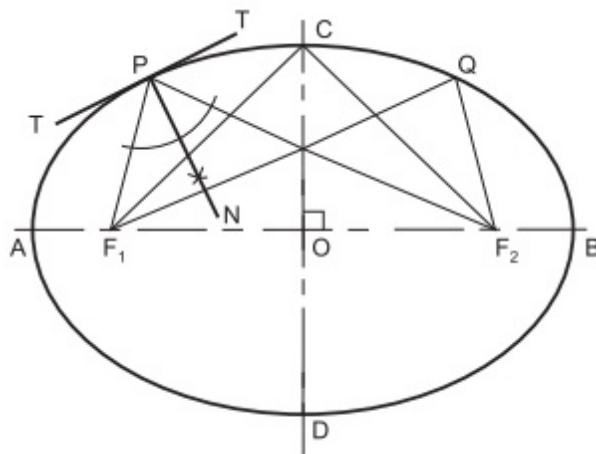
determined by using the above property of the ellipse.

**Problem 13** Draw tangent and normal to the given ellipse, through a given point P on it ([Fig.5.15](#)).

**Construction** ([Fig.5.15](#))



**Fig.5.14**



**Fig.5.15 Drawing tangent and normal to an ellipse**

1. Join the foci  $F_1$ ,  $F_2$  with the given point P.
2. Bisect  $\angle F_1PF_2$ .

The bisector PN is the required normal and the line TT, passing through P and perpendicular to PN, is the tangent to the ellipse.

**Problem 14** The major and minor axes of an ellipse are 120 and 80. Draw an ellipse.

I Foci or arcs of circles method.

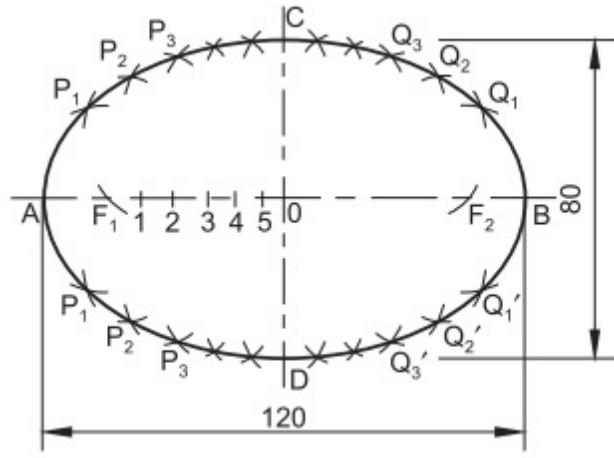
**Construction (Fig.5.16)**

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. With centre C (or D) and radius OA (=OB), draw arcs intersecting the major axis at  $F_1$  and  $F_2$ , the foci.
3. Mark a number of points 1, 2, 3, etc., between  $F_1$  and O, which need not be equidistant.
4. With centres  $F_1$  and  $F_2$  and radii A-1 and B-1 respectively, draw arcs intersecting at points  $P_1$  and  $P_1'$ .
5. With centres  $F_1$  and  $F_2$  and radii B-1 and A-1 respectively, draw arcs intersecting at points  $Q_1$  and  $Q_1'$ .
6. Repeat the steps 4 and 5 with the remaining points 2, 3, 4, etc., and obtain additional points on the curve.

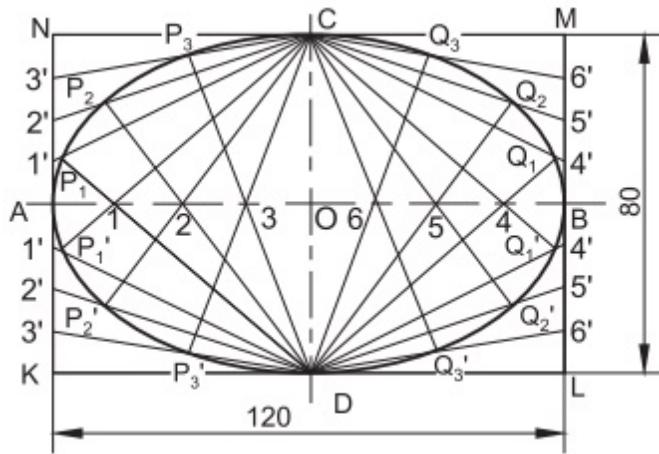
A smooth curve through all the points is the required ellipse.

II Oblong method

**Construction (Fig.5.17)**



**Fig.5.16 Construction of an ellipse-Foci/arcs of circles method**



**Fig.5.17 Construction of an ellipse-Oblong method**

1. Draw the major ( $AB = 120$ ) and minor ( $CD = 80$ ) axes and locate the centre O.
2. Draw the rectangle KLMN, passing through D, B, C and A.
3. Divide AO and AN into the same number of equal parts and number the points as shown.
4. Join C with the points 1', 2' and 3'.

5. Join D with 1, 2, 3 and extend till they meet the above lines C-1', C-2' and C-3' respectively at P<sub>1</sub>, P<sub>2</sub> and P<sub>3</sub>.
6. Repeat steps 3 to 5 and obtain the points in the remaining quadrants.

A smooth curve through all the points is the required ellipse.



If it is required to inscribe an ellipse in a rectangle  $120 \times 80$ , then the major and minor axes of the ellipse are equal to 120 and 80 respectively.

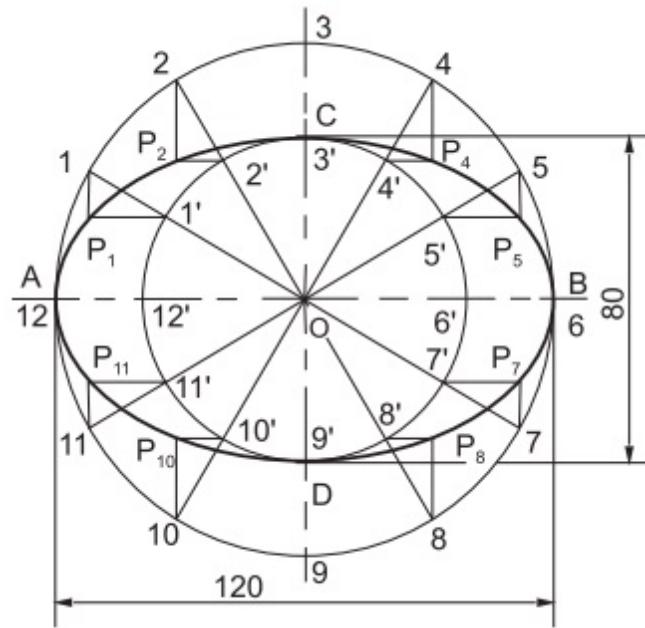
### *III Concentric circles method*

#### **Construction ([Fig.5.18](#))**

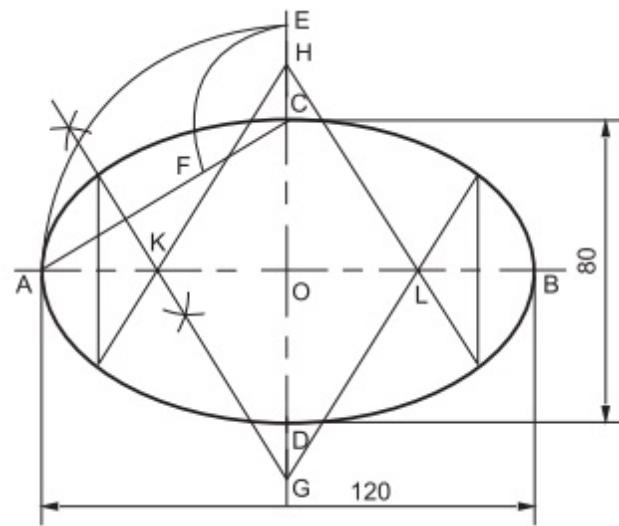
1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. With centre O and major and minor axes as diameters, draw two concentric circles.
3. Divide both the circles into the same number of equal parts, say 12 by radial lines.
4. Considering radial line O-1'-1, draw a horizontal line from 1' and a vertical line from 1, intersecting at P<sub>1</sub>.
5. Repeat the construction through all the points and obtain P<sub>2</sub>, P<sub>3</sub> etc.

A smooth curve through the points A, P<sub>1</sub>, P<sub>2</sub> etc., is the required ellipse.

### **IV Four centre method**



**Fig.5.18 Construction of an ellipse - Concentric circles method**



**Fig.5.19 Construction of an ellipse-Four centre method**

### ***Construction (Fig.5.19)***

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.

2. With centre O and radius OA, draw the arc AE.
  3. With centre C and radius CE, draw an arc meeting the line AC at F.
  4. Draw perpendicular bisector of AF, meeting AB at K and CD extended at G.
  5. Locate point L on AB such that,  $OL = OK$  and H on DC extended such that,  $HC = GD$ . The points K, L, G and H are the four centres that may be used to draw the ellipse.
  6. With G and H as centres and radius CG, draw two arcs.
  7. With K and L as centres and radius KA, draw two arcs.
- The four arcs meet tangentially, forming the required ellipse.

#### *V Parallelogram method*

*A parallelogram has sides 100 and 80, at an included angle of  $70^\circ$ . Inscribe an ellipse in the parallelogram. Find the major and minor axes of the curve.*

#### **Construction ([Fig.5.20](#))**

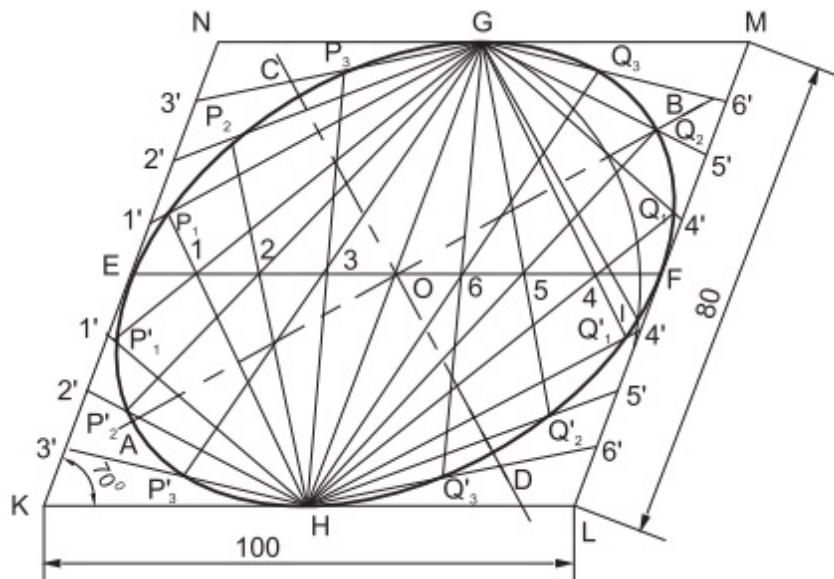
1. Draw the parallelogram KLMN of given sides and included angle. The two axes EF and GH are called the conjugate axes (diameters).
2. Divide EO and EN into the same number of equal parts and number the division points as shown.
3. Join G with 1', 2' and 3'.
4. Join H with 1, 2, 3 and extend till these meet the lines  $G1'$ ,  $G2'$  and  $G3'$  at  $P_1$ ,  $P_2$  and  $P_3$  respectively.
5. Repeat steps 2 to 4 and obtain the points in the remaining quadrants.

A smooth curve through all the points is the required ellipse.

To find the major and minor axes, with centre O and radius OG, draw an arc meeting the ellipse at I. Join G, I and then draw a line CD through O and parallel to GI. The line CD is the minor axis and a line AB drawn through O and perpendicular to CD is the major axis.

*VI Ellipse through three points, not in a straight line*

### **Construction ([Fig.5.20](#))**



**Fig.5.20 Construction of an ellipse-Parallelogram method**

1. Locate the given points E, G and F.
2. Join E and F and locate its mid-point O.
3. Draw GO and extend it to H such that,  $OH = GO$ .
4. Draw the parallelogram KLMN through the points E, G, F and H.
5. Follow the steps given for parallelogram method and obtain the points on the curve.

A smooth curve through the points is the required ellipse.



1. A number of ellipses may be drawn, passing through 3 given points, not in a straight line.
2. EF is taken as a diameter, to draw the unique ellipse passing through the 3 given points.
3. Any line passing through the centre of an ellipse and bound by the curve is a diameter of the ellipse. If the tangents drawn to the ellipse at the ends of a diameter are parallel to another diameter, these diameters are called conjugate diameters. The given ellipse may have unlimited pairs of conjugate diameters. The major and minor axes form one such pair with an included angle of  $90^\circ$ .

### VII Circle method

*Two conjugate diameters EF and GH of an ellipse are 75 and 50 long, with an included angle of  $60^\circ$  between the two. Draw an ellipse, passing through the points E, G, F and H.*

#### **Construction (Fig.5.21)**

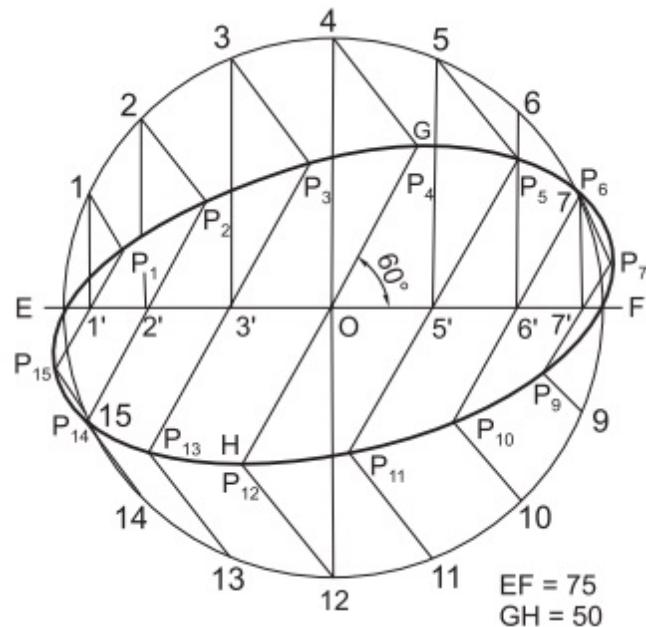
1. Draw the conjugate diameters EF and GH, at the given included angle and bisecting at O.
2. With centre O and diameter EF, draw a circle and divide it into any number of parts, say 16.
3. Draw the lines perpendicular to EF and passing through the above division points; meeting EF at 1', 2', 3', etc.
4. Join 4, G.
5. Through 1, 2, 3, etc., draw lines parallel to 4-G.

6. Through  $1'$ ,  $2'$ ,  $3'$ , etc., draw lines parallel to  $GH$ ; intersecting the above lines at  $P_1$ ,  $P_2$ ,  $P_3$ , etc.

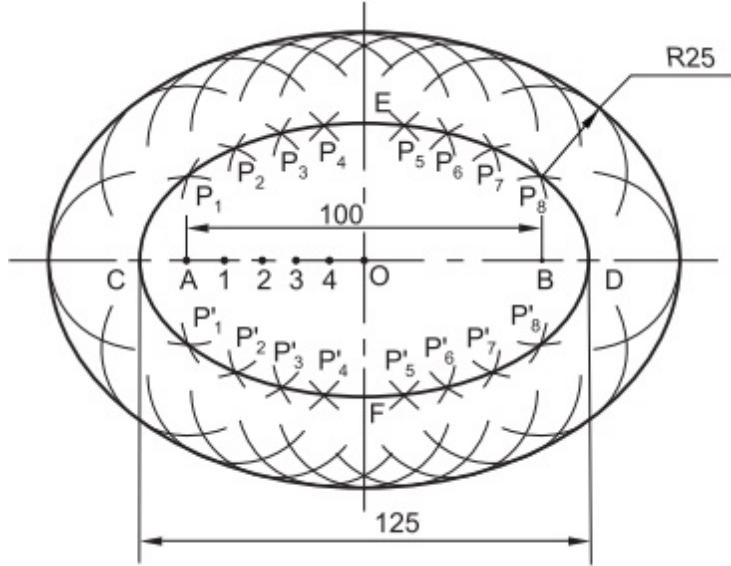
A smooth curve through the points  $E$ ,  $P_1$ ,  $P_2$ ,  $P_3$ , etc., is the required ellipse.

**Problem 15** Two fixed points  $A$  and  $B$  are 100 apart. Trace the complete path of a point  $P$  moving (in the same plane as that of  $A$  and  $B$ ) in such a way that the sum of the distances from  $A$  and  $B$  is always equal to 125. Name the curve. Draw another curve parallel to and 25 away from this curve.

**Construction (Fig.5.22)**



**Fig.5.21 Construction of an ellipse-Circle method**



**Fig.5.22**



The path of the point P is an ellipse and the major axis is 125 (properties of ellipse).

1. Draw a line CD of length 125 (major axis), locate centre O and the fixed points (foci) A and B symmetrically such that, AB is 100.
2. Mark a number of points 1, 2, 3, etc., between AO which need not be equidistant.
3. With centres A and B and radii C-1 and D-1 respectively, draw arcs intersecting at points  $P_1$  and  $P'_1$ .
4. With centres A and B and radii D-1 and C-1 respectively, draw arcs intersecting at points  $P_8$  and  $P'_8$ .
5. Repeat steps 3 and 4 with the remaining points 2, 3, etc., and obtain additional points on the curve.

A smooth curve through all these points is the required curve, the ellipse.

*To draw a parallel curve*

6. Choose a number of points on the ellipse as centres and draw arcs of circles with radius 25.

A smooth curve, drawn tangential to these arcs is the required parallel curve.

### Problem 16

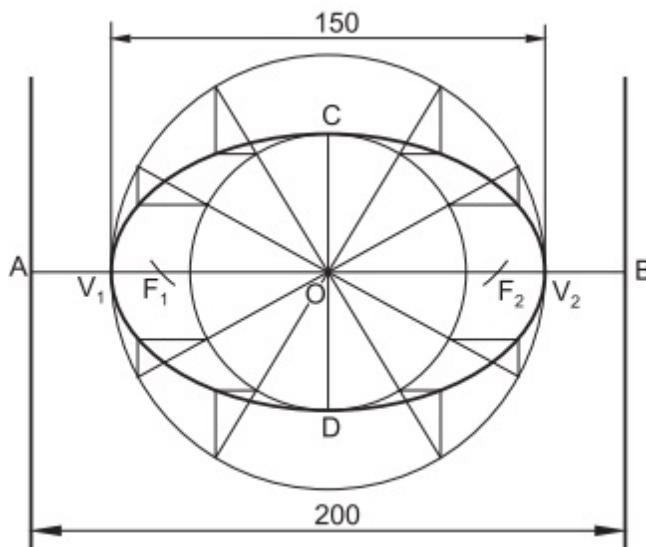
*Construct an ellipse with the following data:*

*Distance between the directrices = 200*

*Distance between the vertices = 150*

*Determine the eccentricity and the minor axis.*

**Construction (Fig.5.23)**



**Fig.5.23**

1. Draw a line AB of 200 long and locate  $V_1 V_2 = 150$  centrally to AB. Let  $F_1$  and  $F_2$  are the two foci of the ellipse. The eccentricity may be obtained from,

$$\frac{V_1F_1}{V_1A} = \frac{V_1F_2}{V_1B}$$

$$\therefore \frac{V_1F_1 + V_1F_2}{V_1A + V_1B} = \frac{V_1V_2}{AB} = \frac{150}{200} = \frac{3}{4}$$

2. Locate  $F_1$  and  $F_2$  such that,

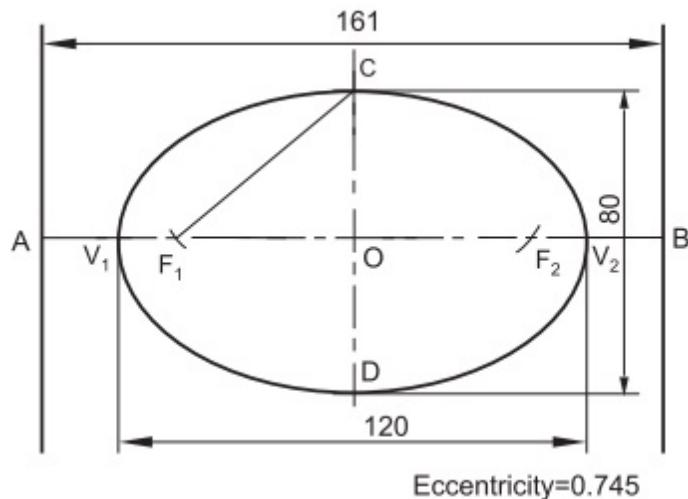
$$\frac{V_1F_1}{V_1A} = \frac{3}{4}$$

$$\therefore V_1F_1 = (3/4) \times 25 = 18.75 = V_2F_2$$

3. Draw the minor axis CD such that,  $CF_1 = V_1O$ .

The ellipse may now be constructed by following any one of the methods of construction discussed above. The ellipse shown in Fig.5.23 is obtained by following the concentric circles method.

**Problem 17** *Figure 5.24 shows an ellipse with major axis 120 and minor axis 80. Determine the eccentricity and the distance between the directrices.*



**Fig.5.24**

Eccentricity,

$$e = \frac{V_1 F_1}{V_1 A} = \frac{V_1 F_2}{V_1 B}$$

$$\therefore \frac{V_1 F_2 - V_1 F_1}{V_1 B - V_1 A} = \frac{F_1 F_2}{V_1 V_2}$$

From the triangle  $F_1 CO$ ,

$OC = 40$  (half the minor axis)

$F_1 C = 60$  (half the major axis)

Thus,  $F_1 O = \sqrt{60^2 - 40^2} = 44.7$

Hence,  $F_1 F_2 = 2F_1 O = 89.4$

On substitution,  $e = 89.4/120 = 0.745$

From the preceding problem; eccentricity,  $e = \frac{V_1 V_2}{AB}$

Hence,  $AB$ , the distance between the directrices =  $V_1 V_2/e = 161$

### 5.4.3 Hyperbola

A hyperbola is a curve generated by a point moving such that, the difference of its distances from two fixed points, called the foci is always constant and equal to the distance between the vertices of the two branches of hyperbola. This distance is also known as the major axis of the hyperbola. Referring to Fig.5.25,

$$P_1F_2 \sim P_1F_1 = V_1V_2, \text{ and}$$

$$P_2F_1 \sim P_2F_2 = V_1V_2$$

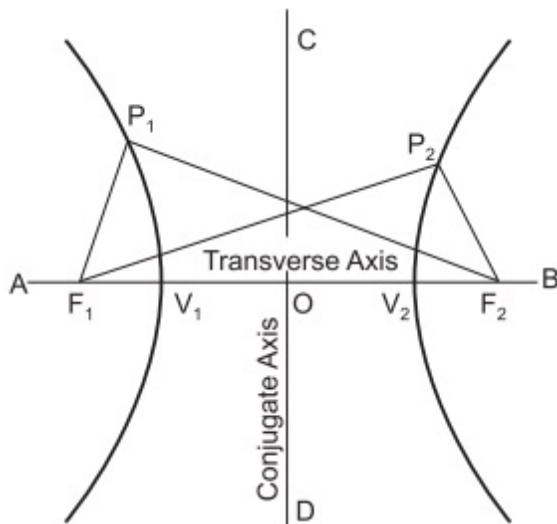


The axes AB and CD are respectively known as transverse and conjugate ' axes of the hyperbola. The curve has two branches, which are symmetric about the conjugate axis.

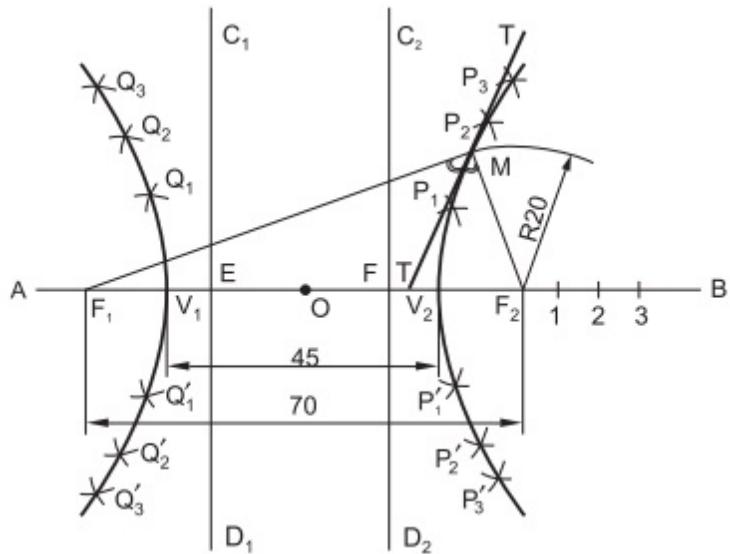
**Problem 18** Construct a hyperbola with its foci, 70 apart and the major axis 45. Draw a tangent to the curve at a point 20 from a focus. Also, determine the eccentricity of the curve.

**Construction (Fig.5.26)**

1. Draw the axis AB and locate a point O on it.



**Fig.5.25 Hyperbola**



**Fig.5.26 Construction of hyperbolaArcs of circles method**

2. Locate the foci  $F_1$ ,  $F_2$  ( $F_1 F_2 = 70$ ) and the vertices  $V_1$ ,  $V_2$  ( $V_1 V_2 = 45$ ) on  $AB$  which are symmetric about  $O$ .
3. Mark a number of points 1, 2, 3, etc., on  $AB$ , to the right of  $F_2$ , which need not be equi-distant.
4. With centre  $F_1$  and radius  $V_1-1$ , draw arcs on either side of the transverse axis.
5. With centre  $F_2$  and radius  $V_2-1$ , draw arcs intersecting the above arcs at  $P_1$  and  $P_1'$ .
6. With centre  $F_2$  and radius  $V_1-1$ , draw arcs on either side of the transverse axis.
7. With centre  $F_1$  and radius  $V_2-1$ , draw arcs intersecting the above arcs at  $Q_1$  and  $Q_1'$ .
8. Repeat steps 4 to 7 and obtain the points  $P_2$ ,  $P_2'$ ; etc., and  $Q_2$ ,  $Q_2'$ ; etc.

Join the points in the order and obtain the two branches of the hyperbola.

To draw a tangent to the hyperbola, locate the point M, which is at 20 from the focus, say  $F_2$ . Then, join M to the foci  $F_1$  and  $F_2$ . Draw a line T-T, bisecting  $\angle F_1MF_2$ ; forming the required tangent.

*To locate the directrices*

1. Determine the eccentricity,

$$e = \frac{OF_1}{OV_1} = \frac{OF_2}{OV_2} = \frac{35}{22.5} = 1.56$$

2. Locate the points E and F on the transverse axis such that,

$$\frac{OV_1}{OE} = \frac{OV_2}{OF} = e$$

$$\therefore OE = OV_1/e = 22.5/1.56 = 14.4 = OF$$

Draw lines through the points E and F, perpendicular to the transverse axis; representing the directrices to the two branches of the hyperbola.

**Problem 19** *Draw (i) asymptotes and (ii) directrices to the hyperbola, shown in Fig.5.27, with the major and conjugate axes indicated.*



Asymptotes are the lines, which pass through the centre of the major axis and tangential to the curve at infinity.

**Construction (Fig.5.27)**

1. Draw perpendiculars to the transverse axis, through the vertices  $V_1$  and  $V_2$ .

2. With centre O and radius  $OF_1$ , draw a circle meeting the above perpendicular lines at P, P' and Q, Q'.

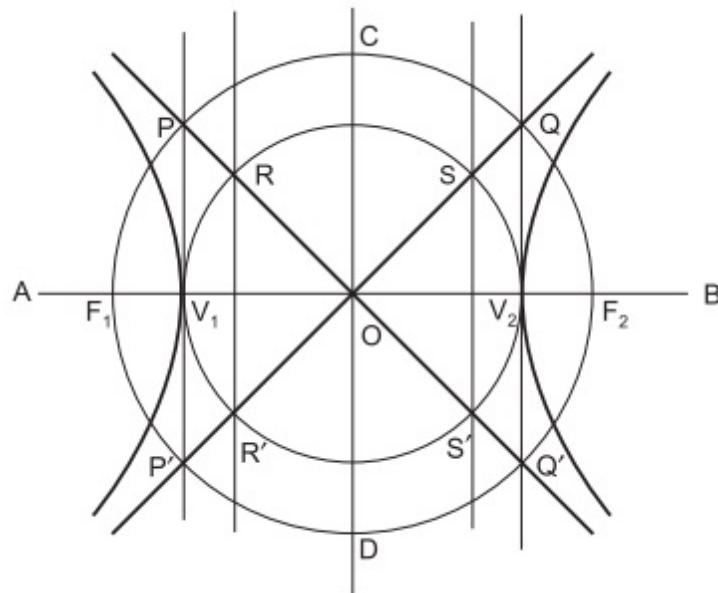
The lines joining P, O, Q' and P', O, Q and extended form the asymptotes to the hyperbola.

3. With centre O and radius  $OV_1$ , draw a circle meeting the asymptotes at R, R' and S, S'.

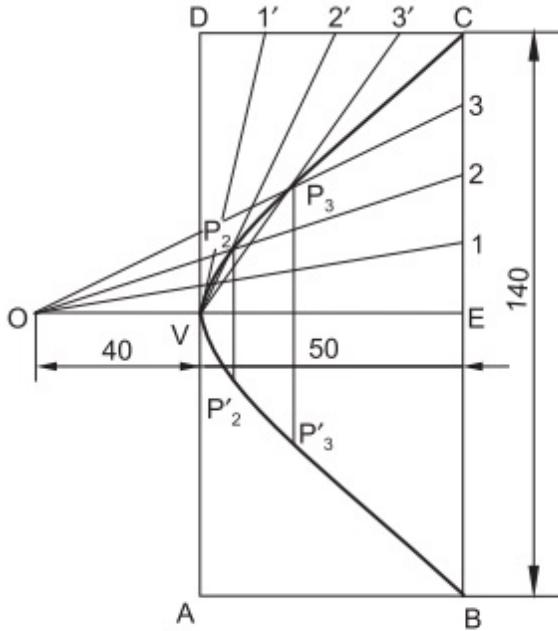
The lines joining R, R' and S, S' and extended form the directrices to the hyperbola.

**Problem 20** Draw a hyperbola with half the major axis as 40, the abscissa 50 and double ordinate 140.

**Construction (Fig.5.28)**



**Fig.5.27 Drawing asymptotes and directrices to a hyperbola**



OV-half the major axis, VE-abscissa,  
BC-double ordinate

**Fig.5.28 Construction of hyperbola-Given major axis and abscissa**

1. Locate the points O, V and E on a horizontal line such that,  $OV = 40$  (half the major axis) and  $VE = 50$  (abscissa).
2. Through E, draw a perpendicular and mark on it, the points B and C such that,  $BE = EC = 70$ .
3. Draw the rectangle ABCD, passing through V, as shown.
4. Divide EC and DC into the same number of equal parts, say 4 and name the parts as shown.
5. Join O, 1'; O, 2' and O, 3'.
6. Join V, 1'; V, 2' and V, 3'; intersecting the above lines at  $P_1$ ,  $P_2$  and  $P_3$ .

7. Follow the steps 4 to 6 and locate the points in the other half of the rectangle.

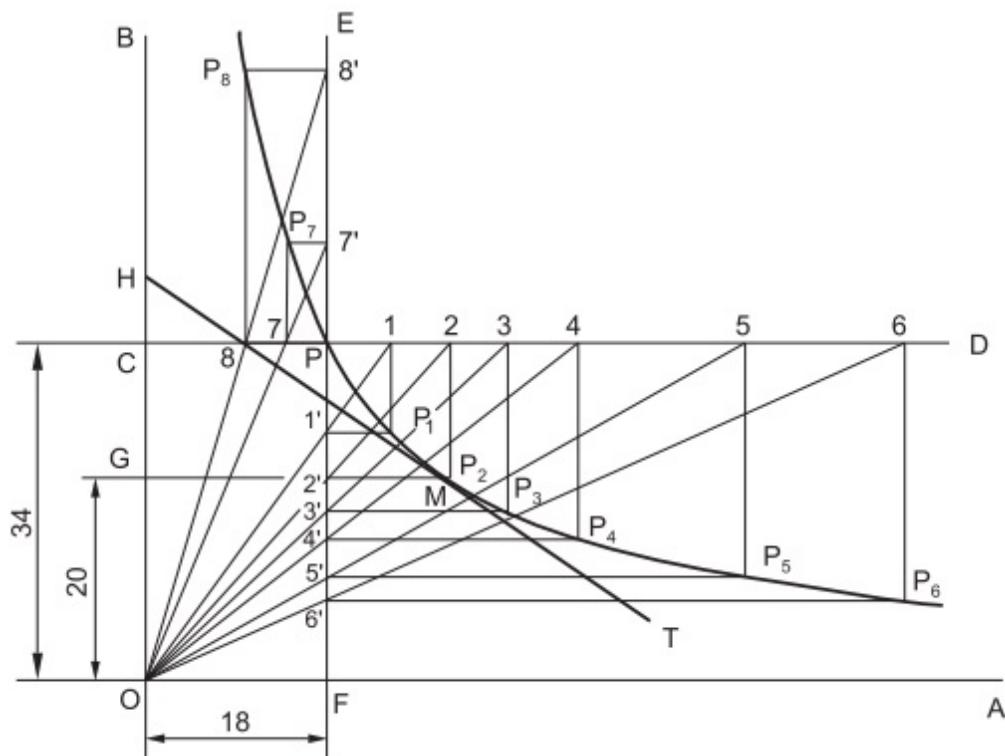
Join the points in the order and obtain the hyperbola.

### *Rectangular hyperbola*

*It is a curve generated by a point which moves in such a way that the product of its distances from two fixed straight lines, the asymptotes at right angle to each other, is a constant.*

**Problem 21** Construct a rectangular hyperbola, when a point  $P$  on it is at a distance of 18 and 34 from two asymptotes. Also, draw a tangent to the curve at a point 20 from an asymptote.

### **Construction (Fig.5.29)**



**Fig.5.29 Construction of rectangular hyperbola**

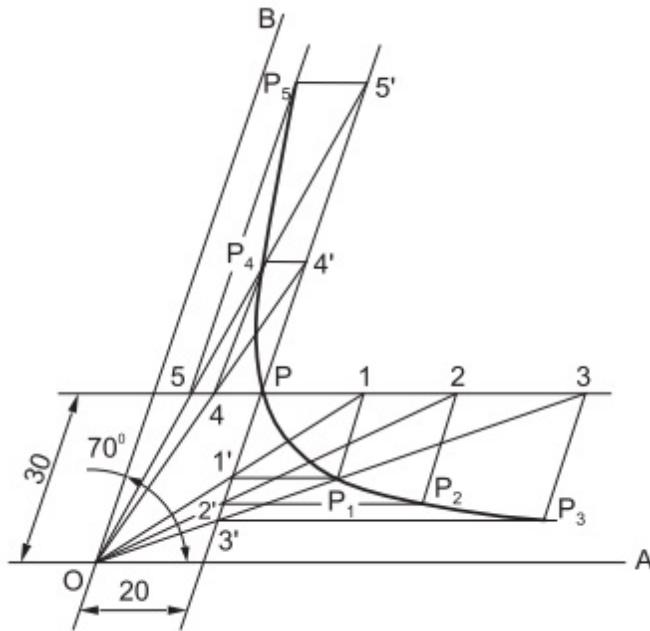
1. Draw the asymptotes OA and OB at right angle to each other and locate the given point P.
2. Draw the lines CD and EF; passing through P and parallel to OA and OB respectively.
3. Locate a number of points 1, 2, 3, etc., along the line CD, which need not be equi-distant.
4. Join 1, 2, 3, etc., to O and extend if necessary, till these lines meet the line EF at points 1', 2', 3', etc.
5. Draw lines through 1, 2, 3, etc., parallel to EF and through 1', 2', 3', etc., parallel to CD, to intersect at  $P_1$ ,  $P_2$ ,  $P_3$ , etc.

A smooth curve passing through these points is the required rectangular hyperbola.

To draw a tangent to the curve, locate the point M on the curve by drawing a line GM, parallel to OA and at a distance 20 from it. Then, locate the point H on OB such that,  $GH = OG$ . The line HT passing through M is the required tangent to the curve.



If the eccentricity for a hyperbola is  $\sqrt{2}$ , the asymptotes will be at right angle to each other and the hyperbola is known as rectangular hyperbola.



**Fig.5.30 Construction of hyperbola-Given asymptotes and a point on it**

**Problem 22** *The asymptotes of a hyperbola are inclined at  $70^\circ$  to each other. Construct the curve when a point P on it is at a distance of 20 and 30 from the two asymptotes.*

A hyperbola passing through any given point, located between the two asymptotes, making any angle other than  $90^\circ$ , may also be constructed (Fig.5.30); following the method similar to Construction: Fig.5.29.

**Problem 23** *Draw one branch each of a hyperbola and conjugate hyperbola, whose major and conjugate axes are 60 and 80 respectively. Also, locate the foci.*

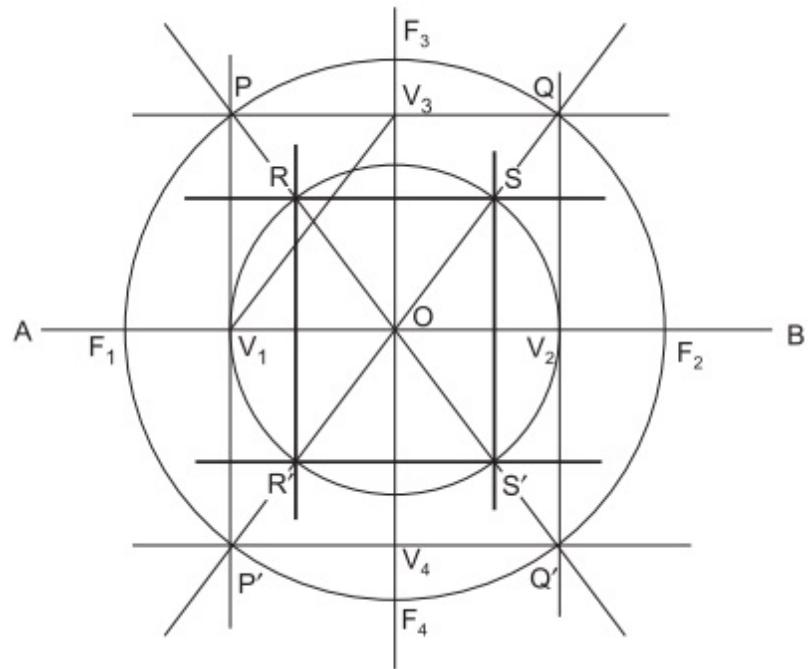
#### **Construction (Fig.5.31a)**

1. Draw transverse axis AB and locate a point O on it.
2. Mark  $V_1, V_2$  such that,  $OV_1 = OV_2 = 30$ , half the major axis.

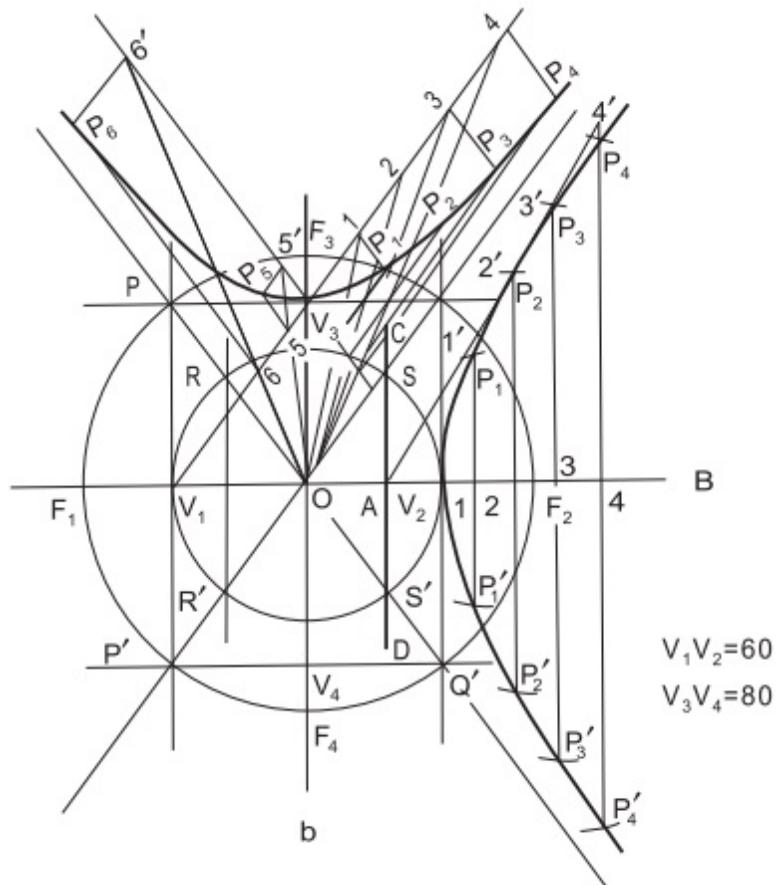
3. Draw a perpendicular bisector to  $V_1V_2$ . Mark on it,  $V_3$ ,  $V_4$  such that,  $OV_3 = OV_4 = 40$ , half the conjugate axis.
4. Locate the foci  $F_1$ ,  $F_2$  such that,  $OF_1 = OF_2 = V_1V_3$ .
5. Complete the rectangle  $PP'Q'Q$ , passing through  $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ .
6. Join  $P$ ,  $Q'$  and  $P'$ ,  $Q$  and extend, forming the asymptotes.
7. With centre  $O$  and diameter  $V_1V_2$ , draw a circle.
8. Draw lines  $RR'$  and  $SS'$ , passing through the intersecting points between the above circle and the asymptotes and extend, forming the directrices of the hyperbola.
9. Draw lines  $RS$  and  $R'S'$  and extend, forming the directrices of the conjugate hyperbola.
10. With centre  $O$  and radius  $OF_1$ , draw a circle.
11. Locate  $F_3$ ,  $F_4$  at the intersection between the above circle and the conjugate axis, forming the foci of the conjugate hyperbola.

*To locate the foci*

**Construction (*Fig.5.31b*)**



**Fig.5.31a**



### **Fig.5.31b Hyperbola and conjugate hyperbola**

1. Draw one branch of hyperbola, with  $S S'$  extended as directrix and  $F_2$  as focus, using the eccentricity method.
2. Draw one branch of conjugate hyperbola, using the method similar to Construction: [Fig.5.30](#), with  $V_3$  as a point on the curve.



If the lengths of major and conjugate axes are equal, the asymptotes will be at right angle and the curves obtained are rectangular hyperbolas.

## **5.5 CYCLOIDAL CURVES**

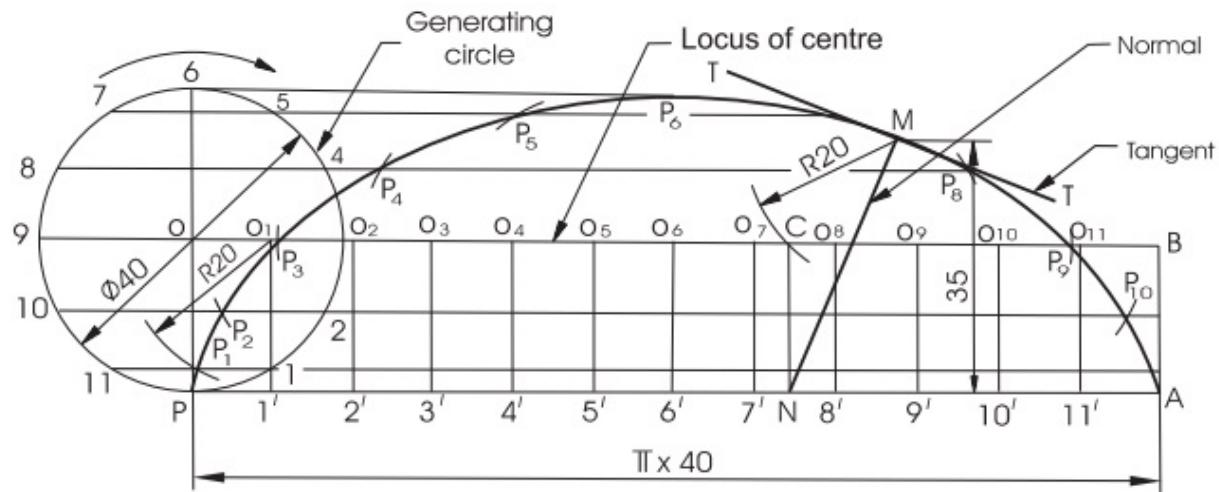
Cycloidal curves are generated by a point on the circumference of a circle, when it rolls without slipping along a straight or curved path. The rolling circle is called the generating circle and the fixed straight line / circle is called the directing line/ circle respectively.

### **5.5.1 Cycloid**

A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls along a straight line without slipping ([Fig.5.32](#)). Obviously, the size of the curve depends upon the diameter of the generating circle.

**Problem 24** A circle of 40 diameter rolls along a line for one revolution clock-wise. Draw the locus of a point on the circle, which is in contact with the line. Also, draw a tangent and a normal to the curve, at a point 35 from the directing line.

### **Construction (Fig.5.32)**



**Fig.5.32 Cycloid**

1. With centre O and radius 20, draw the generating circle.
2. Locate the initial position of the generating point P on the circumference of the circle.
3. Draw a line PA, tangential and equal to the circumference of the circle.
4. Divide the circle and the line PA into the same number of equal parts and number as shown.
5. Draw the line OB, parallel and equal to PA, which is the locus of the centre of the generating circle.
6. Erect perpendiculars at 1', 2', etc., to meet the line OB at O<sub>1</sub>, O<sub>2</sub>, etc.
7. Through the points 1, 2, 3, etc., draw lines parallel to PA.
8. With O<sub>1</sub> as centre and radius 20, draw an arc intersecting the line through 1 at P<sub>1</sub>. P<sub>1</sub> is the position

of the point P, when the centre of the generating circle moves to O<sub>1</sub>.

9. With O<sub>2</sub> as centre and radius 20, draw an arc intersecting the line through 2 at P<sub>2</sub>.
10. Similarly, locate the points P<sub>3</sub>, P<sub>4</sub>, etc.

A smooth curve passing through these points is the required cycloid.

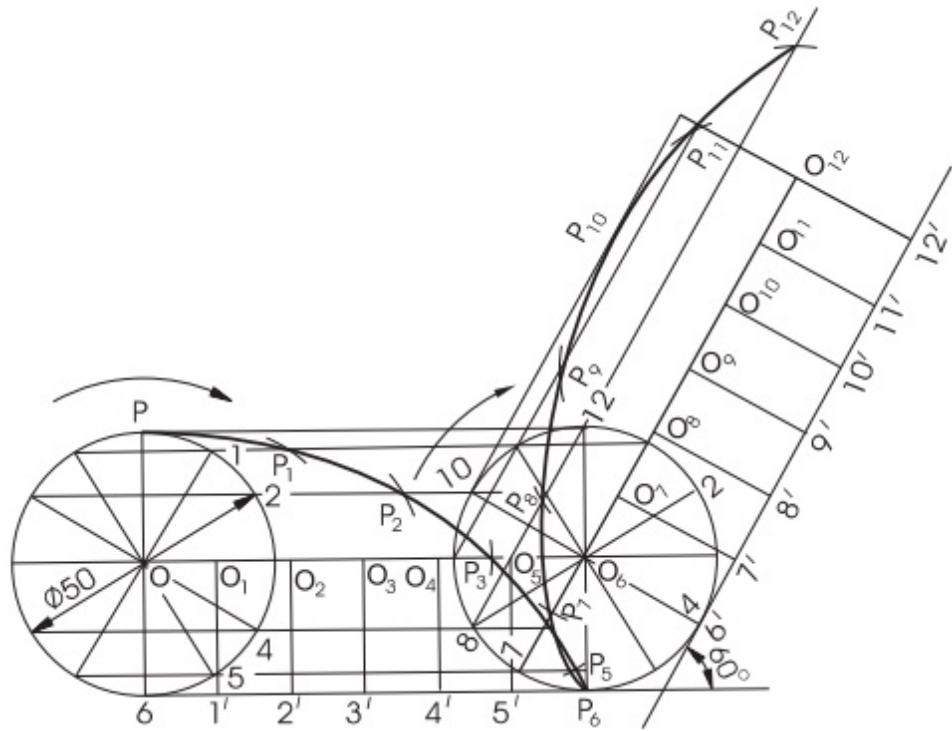
*To draw the tangent and normal*

1. Locate the point M on the curve, which is at 35 from the directing line.
2. With M as centre and radius 20, draw an arc intersecting the locus of the centre (OB) at C.
3. Through C, draw a line perpendicular to the directing line PA, meeting it at N (the point of contact of the generating circle, when its centre moves to C).

The line joining the points M and N is the required normal and a line T-T perpendicular to it and passing through M is the tangent to the cycloid.

**Problem 25** A circle of 50 diameter rolls on a horizontal line for half a revolution clockwise and then on a line inclined at 60° to the horizontal for another half, clock-wise. Draw the curve traced by a point P on the circumference of the circle, taking the topmost point on the rolling circle as the initial position of the generating point.

**Construction (Fig 5.33)**



**Fig.5.33 Cycloid**

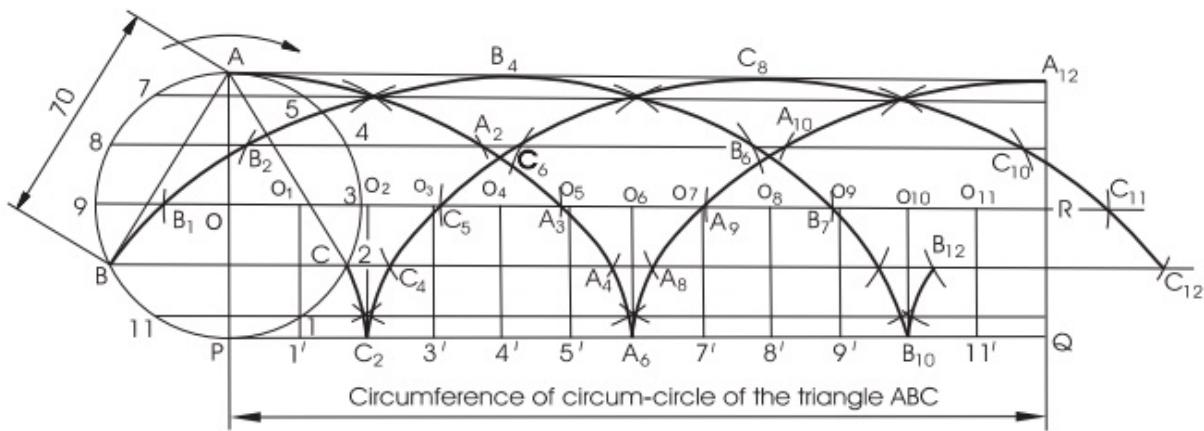
1. With centre O and radius 25, draw the given circle.
2. Divide the circle into a number of equal parts, say 12.
3. Mark the division points on the circle and draw a tangent (directing line) passing through 6.
4. Mark half of the circumference along the tangent and divide it into 6 equal parts.
5. Locate the initial position of the generating point P.
6. Draw a smooth curve, representing the path of P, as the circle rolls along the directing line for half a revolution, following the Construction: [Fig.5.32](#).
7. With  $O_6$  as centre, draw the circle, representing the position of the rolling circle after half revolution.
8. Draw a line tangential to the circle, making  $60^\circ$  with the directing line. This is the directing line for the next

half revolution of the generating circle.

9. Trace the path of P for the next half revolution of the generating circle, as shown.

**Problem 26** ABC is an equilateral triangle of side 70. Trace the loci of vertices A, B and C, when the circle circumscribing ABC, rolls without slipping, along a fixed straight line, for one complete revolution.

**Construction (Fig.5.34)**



**Fig.5.34**

1. Draw the equilateral triangle ABC and draw its circum-circle.
2. Divide the circle into a number of equal parts such that, the corners (vertices) of the triangle A, B and C coincide with the division points.
3. Draw the line PQ, tangential and equal to the circumference of the circle.
4. Divide the line PQ into the same number of equal parts, as that of the circle.
5. Draw the line OR, parallel and equal to PQ, which is the locus of the centre of the (generating) circle.

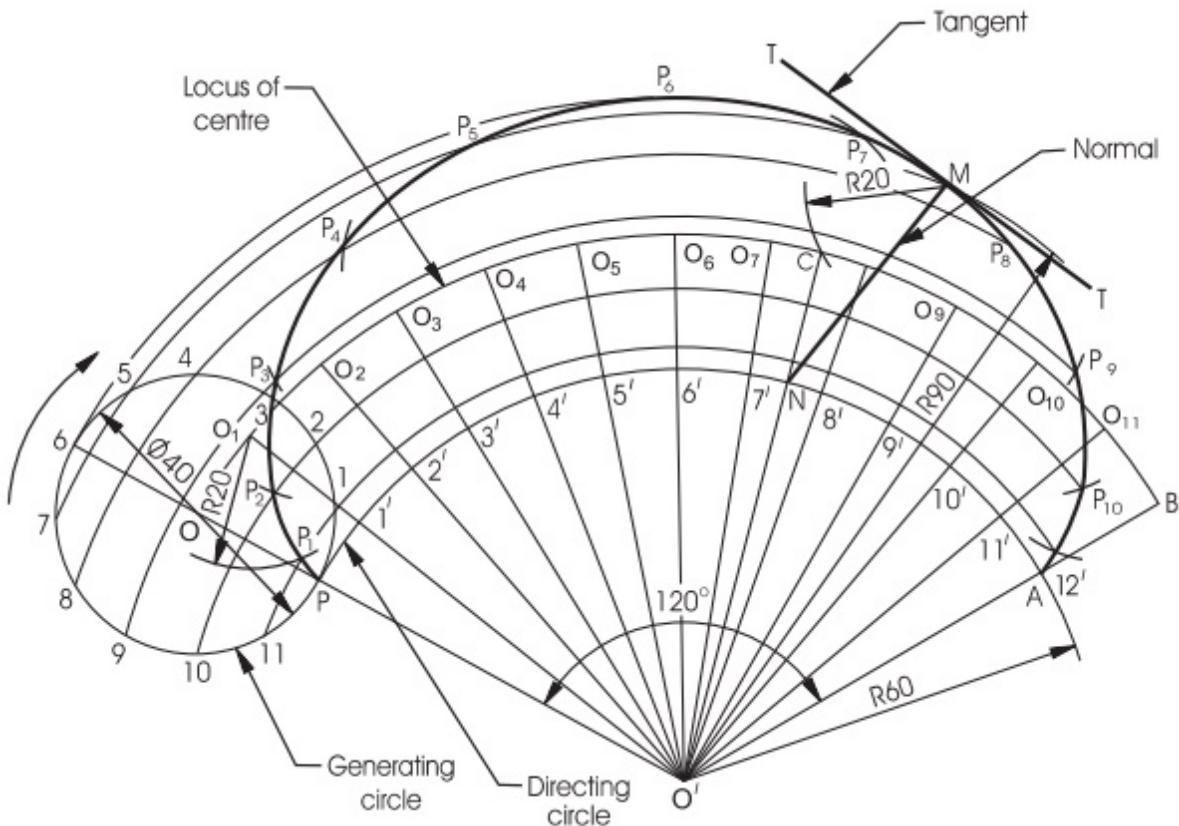
6. Erect perpendiculars at  $1'$ ,  $2'$ , etc., to meet the line OR at  $O_1$ ,  $O_2$ , etc.
7. Follow the steps 7 to 10 of Construction: [Fig.5.32](#) suitably and obtain the loci of vertices A, B and C.

## 5.5.2 Epi-cycloid

An epi-cycloid is a curve traced by a point on the circumference of a circle, which rolls without slipping on another circle (directing circle) outside it.

**Problem 27** *Draw an epi-cycloid of a circle of 40 diameter, which rolls outside on another circle of 120 diameter for one revolution clock-wise. Draw a tangent and a normal to it at a point 90 from the centre of the directing circle.*

**Construction** ([Fig.5.35](#))



**Fig.5.35 Epi-cycloid**

1. Draw a part of the directing circle with  $O'$  as centre and radius 60.
2. Draw any radial line  $O'P$  and extend it.
3. Locate the point  $O$  on the above line such that,  $OP = 20$ , the radius of the generating circle.
4. With  $O$  as centre and radius 20, draw the generating circle.
5. Locate the point  $A$  on the directing circle such that, the arc length  $PA$  is equal to the circumference of the generating circle.

The point  $A$  is obtained by setting  $\angle PO'A = 360^\circ \times 20/60 = 120^\circ$ .

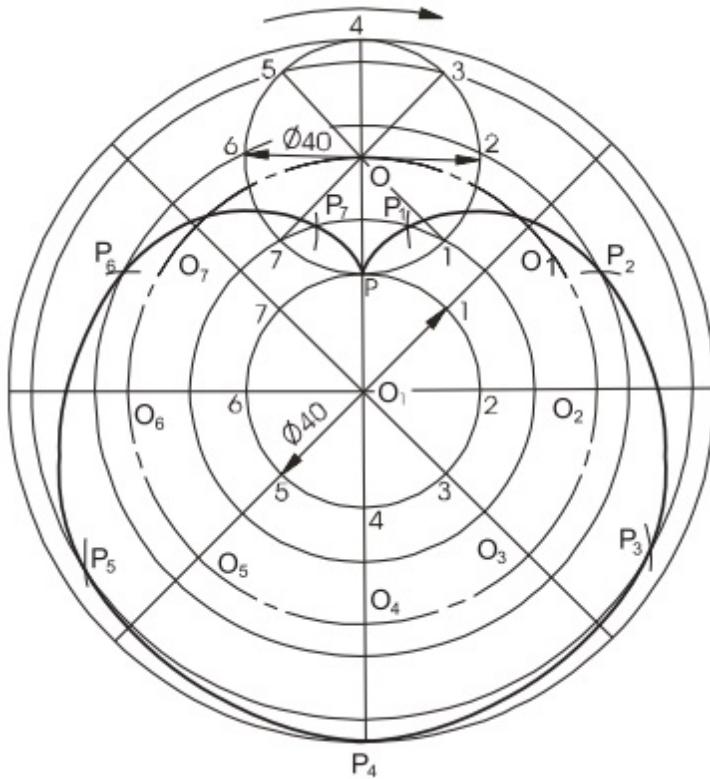
6. With centre  $O'$  and radius  $O'O$ , draw an arc intersecting the line  $O'A$  produced at  $B$ .  
The arc  $OB$  is the locus of the centre of the generating circle.
7. Divide the generating circle and the arc  $PA$  into the same number of equal parts and number as shown.
8. Join  $O', 1'; O', 2';$  etc., and extend, meeting the arc  $OB$  at  $O_1, O_2$ , etc.
9. Through the points 1, 2, 3, etc., on the generating circle, draw arcs with  $O'$  as centre.
10. With centre  $O_1$  and radius 20, draw an arc intersecting the arc through 1 at  $P_1$ .
11. In a similar manner, obtain points  $P_2, P_3$ , etc.

A smooth curve through these points is the required epi-cycloid.

*To draw the tangent and normal*

*Locate the point M on the curve, which is at 90° from the centre of the directing circle.*

With  $M$  as centre and radius 20, draw an arc intersecting the locus of the centre of the generating circle at  $C$ .



**Fig.5.36 Epi-cycloid-Cardioid**

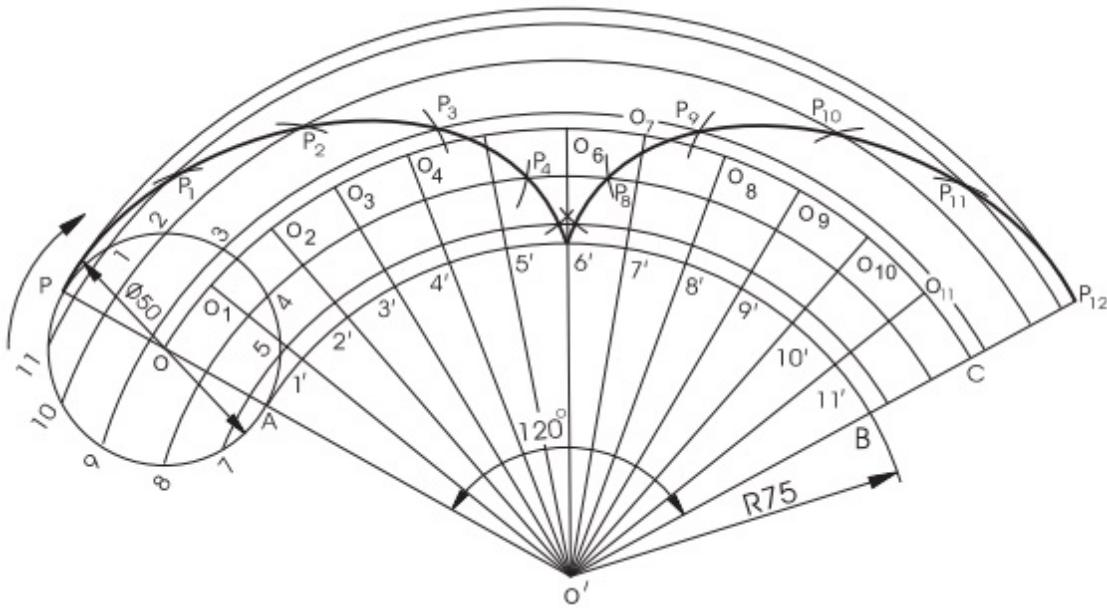
Join C to O', intersecting the circle at N. The line joining N to M is the required normal and a line T-T, perpendicular to it and passing through M is the required tangent.



When the diameters of the generating circle and directing circle are equal, the epi-cycloid traced is called a cardioid, as shown in [Fig.5.36](#).

**Problem 28** A circle of 50 diameter rolls without slipping on the outside of another circle of diameter 150. Show the path of a point on the periphery of the (generating) rolling circle, diametrically opposite to the initial point of contact between the circles.

**Construction ([Fig.5.37](#))**



**Fig.5.37**

1. Draw a part of the directing circle with  $O'$  as centre and radius 75.
2. Draw any radial line  $O' A$  and extend it.
3. Locate the point  $O$  on the above line such that,  $OA = 25$ .
4. With  $O$  as centre and radius 25 ( $=OA$ ), draw the generating circle.
5. Locate the point  $B$  on the directing circle such that, the arc length  $AB$  is equal to the circumference of the generating circle.

The point  $B$  is obtained, by setting  
 $\angle AOB = 360^\circ \times \frac{25}{75} = 120^\circ$

6. With  $O'$  as centre and  $O' O$  as radius, draw an arc intersecting the line  $O' B$  extended at  $C$ . The arc  $OC$  is the locus of the centre of the generating circle.

7. Divide the generating circle and the arc AB into the same number of equal parts and number as shown.
  8. Join O', 1'; O', 2'; etc; and extend meeting the arc OC at O<sub>1</sub>, O<sub>2</sub>, etc.
  9. Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O' as centre.
- Locate the point P on the generating circle, which is lying diametrically opposite to the initial point of contact between the two circles.
10. With O<sub>1</sub> as centre and radius 25, draw an arc intersecting the arc through 1 at P<sub>1</sub>.
  11. In a similar manner, obtain points P<sub>2</sub>, P<sub>3</sub> etc.

A smooth curve through P, P<sub>1</sub>, P<sub>2</sub>, etc., is the required path of the point P, the epicycloid.

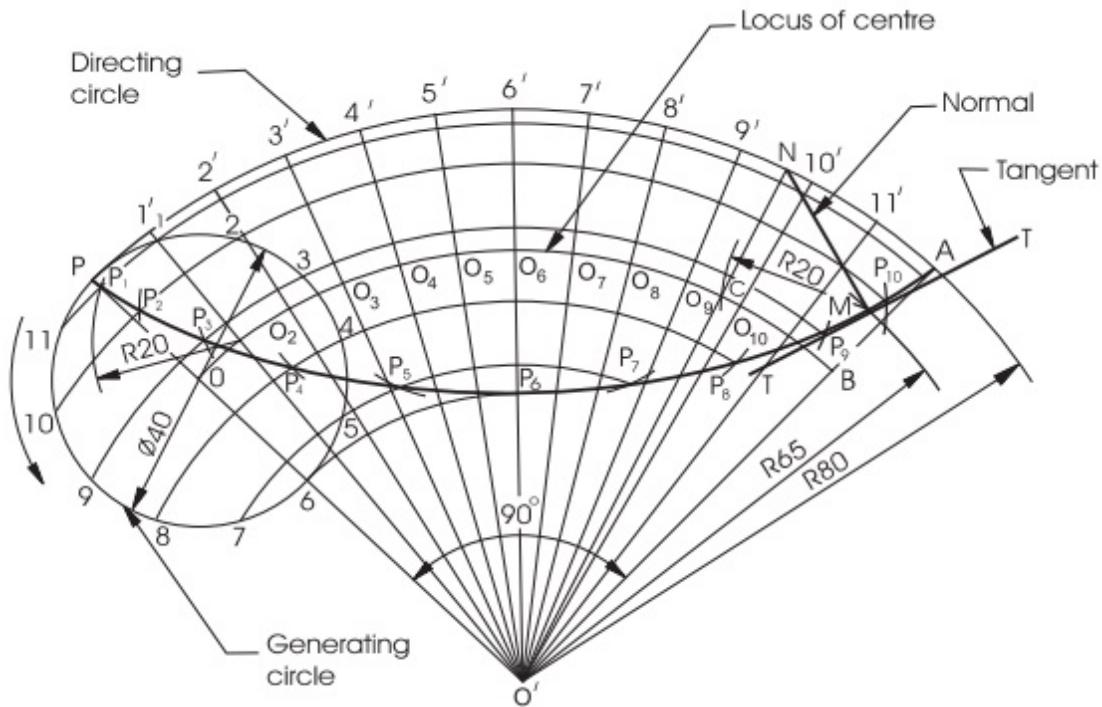
### 5.5.3 Hypo-cycloid

A hypo-cycloid is a curve traced by a point on the circumference of a generating circle, which rolls without slipping on another circle (directing circle), inside it.

**Problem 29** *Draw, a hypo-cycloid of a circle of 40 diameter, which rolls inside another circle of 160 diameter, for one revolution counter clock-wise. Draw a tangent and a normal to it at a point 65 from the centre of the directing circle.*

A procedure similar to the above ([Fig.5.35](#)), may be followed for constructing the hypo-cycloid ([Fig.5.38](#)), keeping in view that the generating circle rolls inside the directing circle.

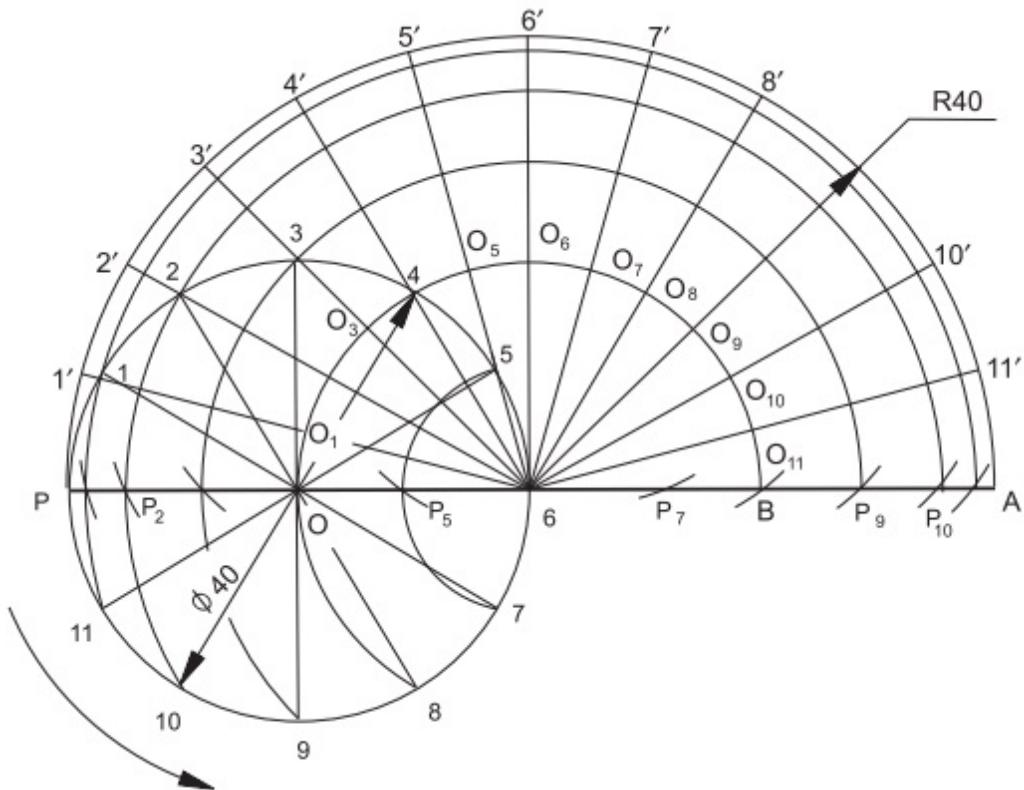
The method of drawing the tangent and normal to the hypo-cycloid is also similar to the one that is followed for the epi-cycloid.



**Fig.5.38 Hypo-cycloid**

**Problem 30** A circle of 40 diameter rolls on the concave side of another circle of 40 radius. Draw the path traced by a point on the generating circle for one complete revolution.

**Construction (Fig.5.39)**



**Fig.5.39 Hypo-cycloid-Straight line**

Construct the hypo-cycloid, following the procedure under Construction: [Fig.5.38](#).

It may be noted that when the diameter of the generating circle is equal to the radius of the directing circle, the hypo-cycloid traced is a straight line.

## 5.5.4 Trochoid

A trochoid is a curve traced by a point, which is situated either inside or outside the generating circle, when it rolls along a straight line, without slipping.

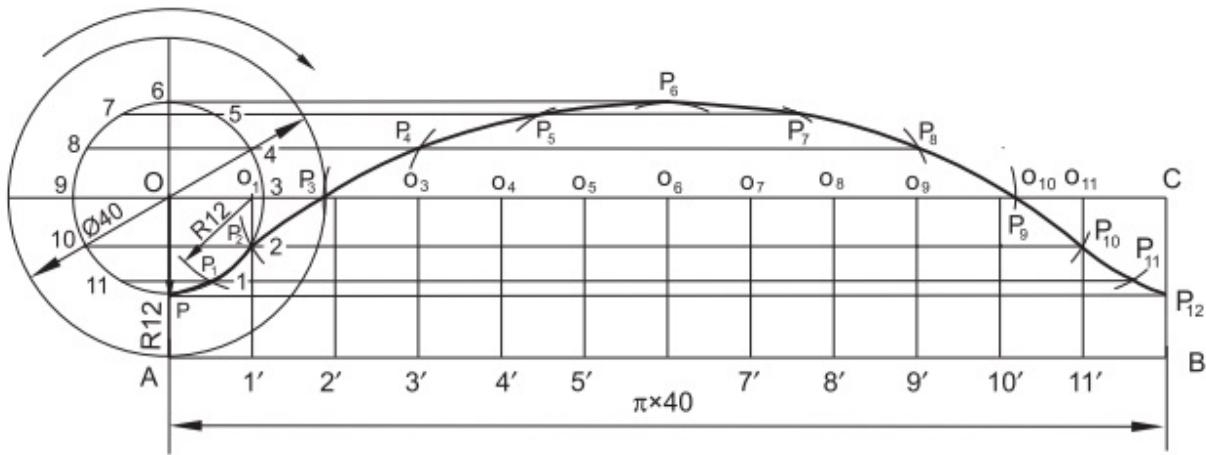
If the tracing point lies within the generating circle, the curve traced is called an inferior trochoid and when it is outside the circle, it is called superior trochoid.

**Problem 31** A circular disc of 40 diameter rolls along a straight line for one revolution clock-wise. Draw the locus of a point, which lies at a distance of 12 from the centre of the disc.

**Construction (Fig.5.40)**

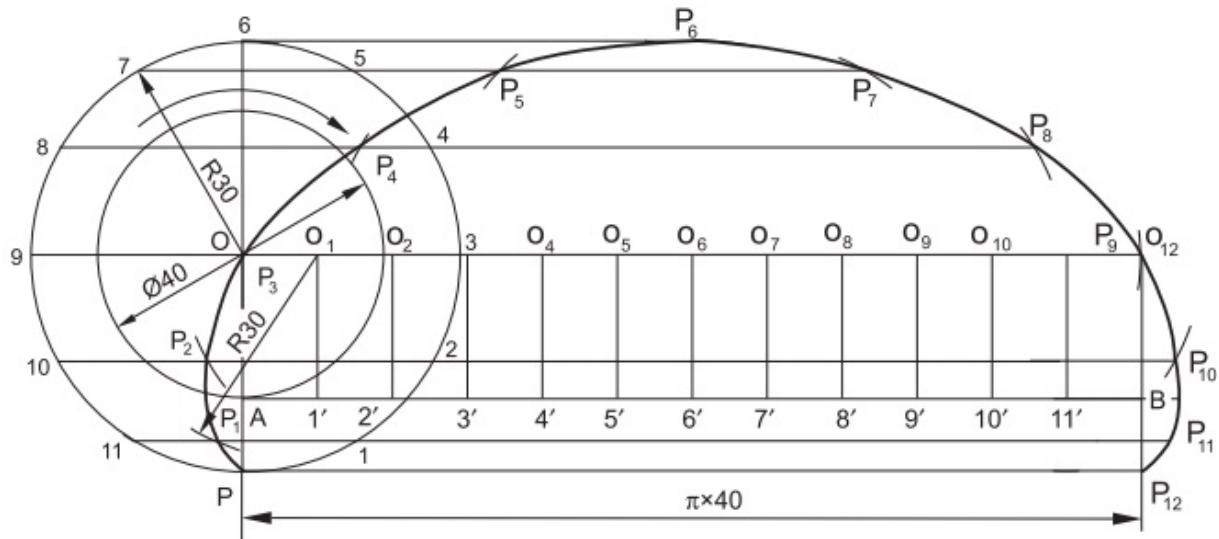
1. With centre O and radius 20, draw a circle representing the disc and locate the generating point P on the vertical diameter such that,  $OP = 12$ .
2. With centre O and radius  $OP (=12)$ , draw a circle.
3. Draw a line AB tangential and equal to the circumference of the generating circle.
4. Divide the above circle ( $R12$ ) and the line AB, into the same number of equal parts and number as shown.
5. Draw the line OC parallel and equal to AB, which is the locus of the centre of the generating circle.
6. Erect perpendiculars at 1', 2', etc., to meet the line OC at  $O_1, O_2$ , etc.
7. Through the points 1, 2, 3, etc., draw lines parallel to AB.
8. With  $O_1$  as centre and radius 12, draw an arc intersecting the line through 1 at  $P_1$ .
9. Similarly, locate the points  $P_2, P_3$ , etc.

A smooth curve passing through these points is the required inferior trochoid.



**Fig.5.40 Inferior trochoid**

**Problem 32** A circular disc of 40 diameter, rolls along a straight line for one revolution clock-wise. Draw the locus of a point, which is at a distance of 30 from the centre of the disc.



**Fig.5.41 Superior trochoid**

A procedure similar to the above may be adopted for constructing the superior trochoid, keeping in view that the generating point P lies outside the disc (Fig.5.41).

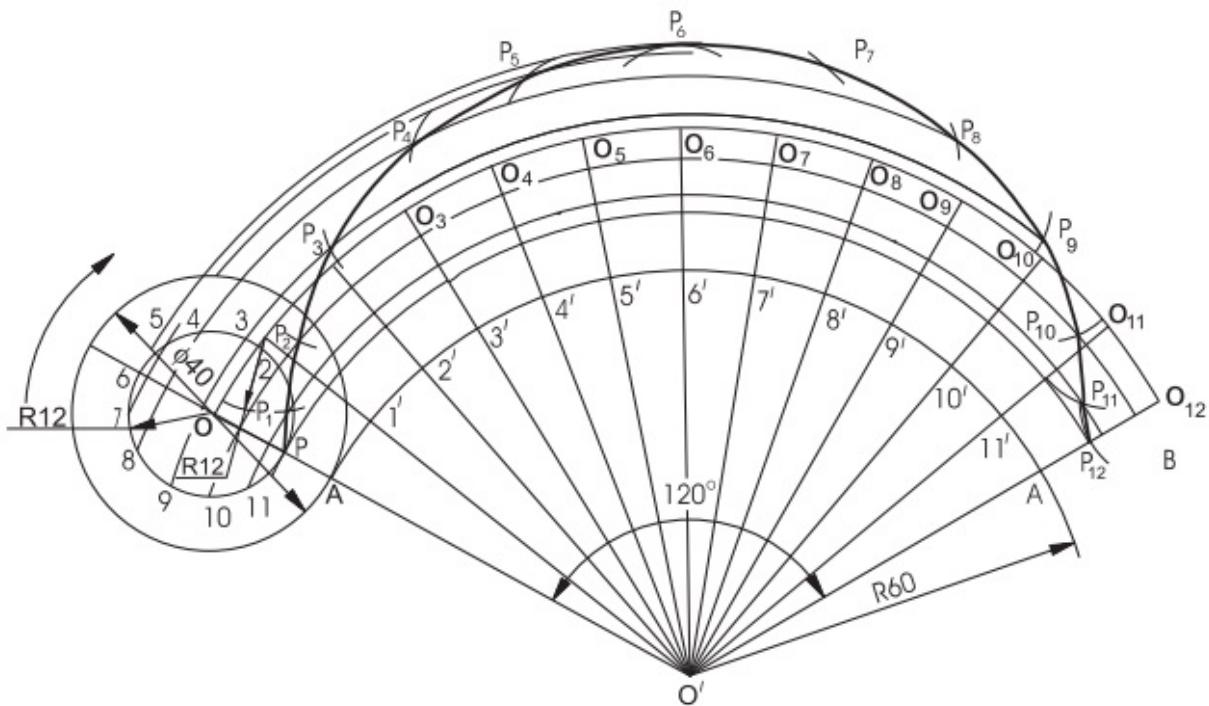
## 5.5.5 Epi-trochoid

An epi-trochoid is a curve traced by a point situated either within or outside the generating circle, when it rolls without slipping on the circumference of another (directing) circle, outside it.

If the tracing point lies within the generating circle, the curve traced is called an inferior epi-trochoid and when it is outside the circle, it is called a superior epi-trochoid.

**Problem 33** A circle of 40 diameter, rolls outside another circle of 120 diameter for one revolution clock-wise. Draw the locus of a point, which lies at a distance 12 from the centre of the generating circle.

**Construction (Fig.5.42)**



**Fig.5.42 Inferior epi-trochoid**

1. Draw a part of the directing circle with  $O'$  as centre and radius 60.
2. Draw any radial line  $O'A$  and extend it.
3. Locate the point  $O$  on the above line such that,  $OA = 20$ , the radius of the generating circle.
4. With  $O$  as centre and radius 20, draw the generating circle.
5. Locate the tracing point  $P$  on  $OA$  such that,  $OP = 12$ .
6. With centre  $O$  and radius 12, draw a circle.
7. Locate the point  $B$  on the directing circle such that, the arc length  $AB$  is equal to the circumference of the generating circle.

Point  $B$  is obtained by setting  $\angle AOB = 360^\circ \times 20/60 = 120^\circ$ .

8. With centre  $O'$  and radius  $O'O$ , draw an arc intersecting the line  $O'B$  produced at  $C$ .

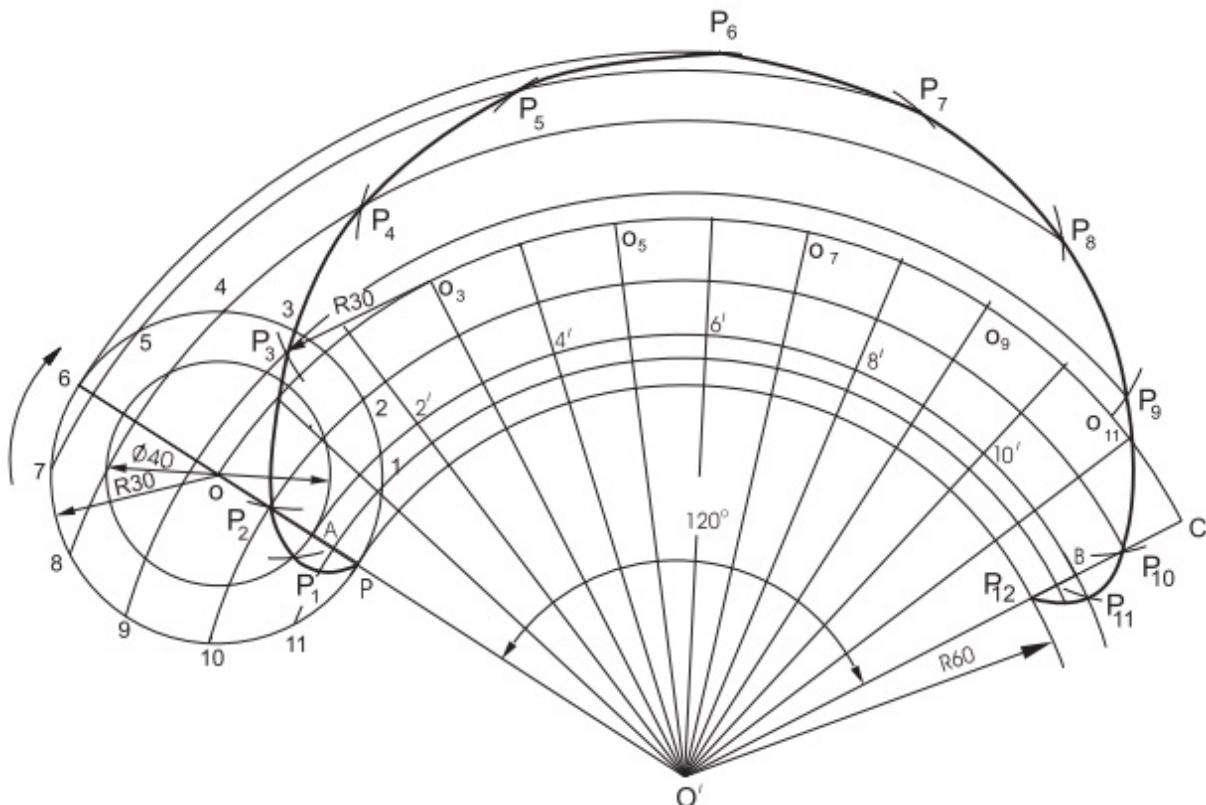
The arc  $OC$  is the locus of the centre of the generating circle.

9. Divide the circle ( $R12$ ) and the arc  $AB$  into the same number of equal parts and number as shown.
10. Join  $O', 1'; O' 2';$  etc., and extend, meeting the arc  $OC$  at  $O_1, O_2$ , etc.
11. Through the points 1, 2, 3, etc., on the circle ( $R12$ ), draw arcs with  $O'$  as centre.
12. With centre  $O_1$  and radius 12, draw an arc intersecting the arc through 1 at  $P_1$ .
13. In a similar manner, obtain points  $P_2, P_3$ , etc.

A smooth curve through these points is the required inferior epi-trochoid.

**Problem 34** A circle of 40 diameter rolls outside another circle of 120 diameter for one revolution clock-wise. Draw the locus of a point, which lies at a distance 30 from the centre of the generating circle.

A procedure similar to the Construction: [Fig.5.42](#) of inferior epi-trochoid, may be followed for constructing the superior epi-trochoid, keeping in view that the tracing point P lies outside the generating circle ([Fig.5.43](#)).



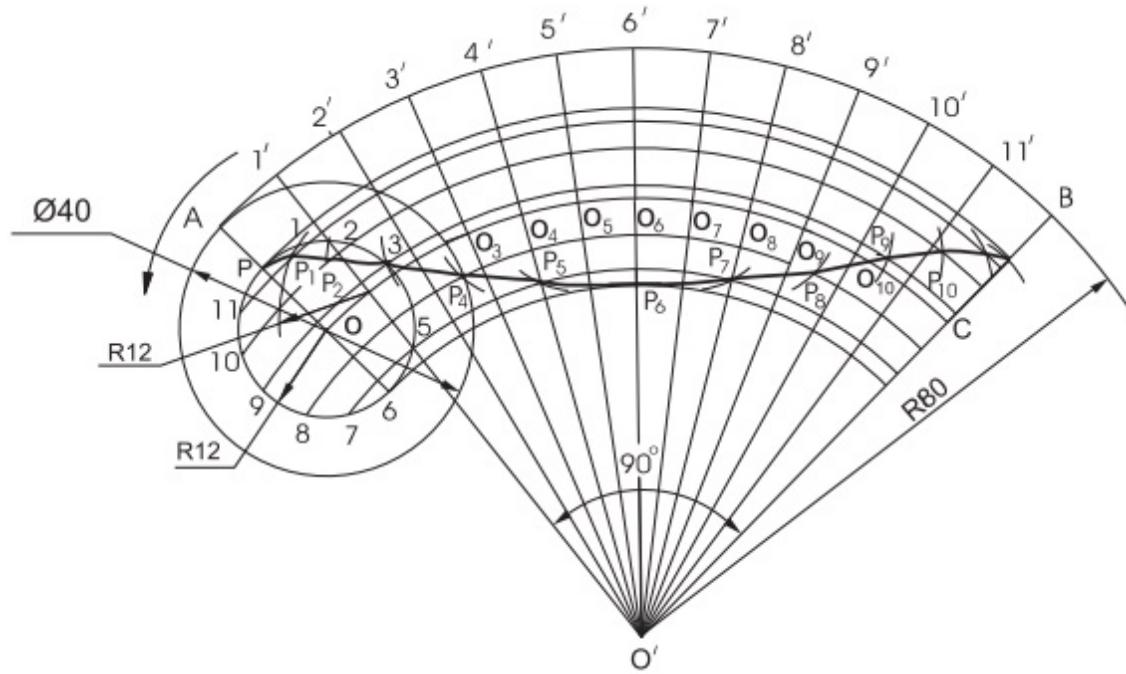
**Fig.5.43 Superior epi-trochoid**

## 5.5.6 Hypo-trochoid

A hypo-trochoid is a curve traced by a point situated either within or outside the generating circle, when it rolls without slipping on the circumference of another (directing) circle, inside it. If the tracing point lies within the generating circle, the curve traced is called an inferior hypo-trochoid and when it is outside the circle, it is called a superior hypotrochoid.

**Problem 35** A circle of 40 diameter, rolls inside another circle of 160 diameter for one revolution counter clockwise. Draw the locus of a point, which is at a distance of 12 from the centre of the generating circle.

A procedure similar to the construction of hypo-cycloid (Fig.5.38) may be followed for constructing the inferior hypo-trochoid, keeping in mind that the tracing point P lies inside the generating circle (Fig.5.44).



**Fig.5.44 Inferior hypo-trochoid**

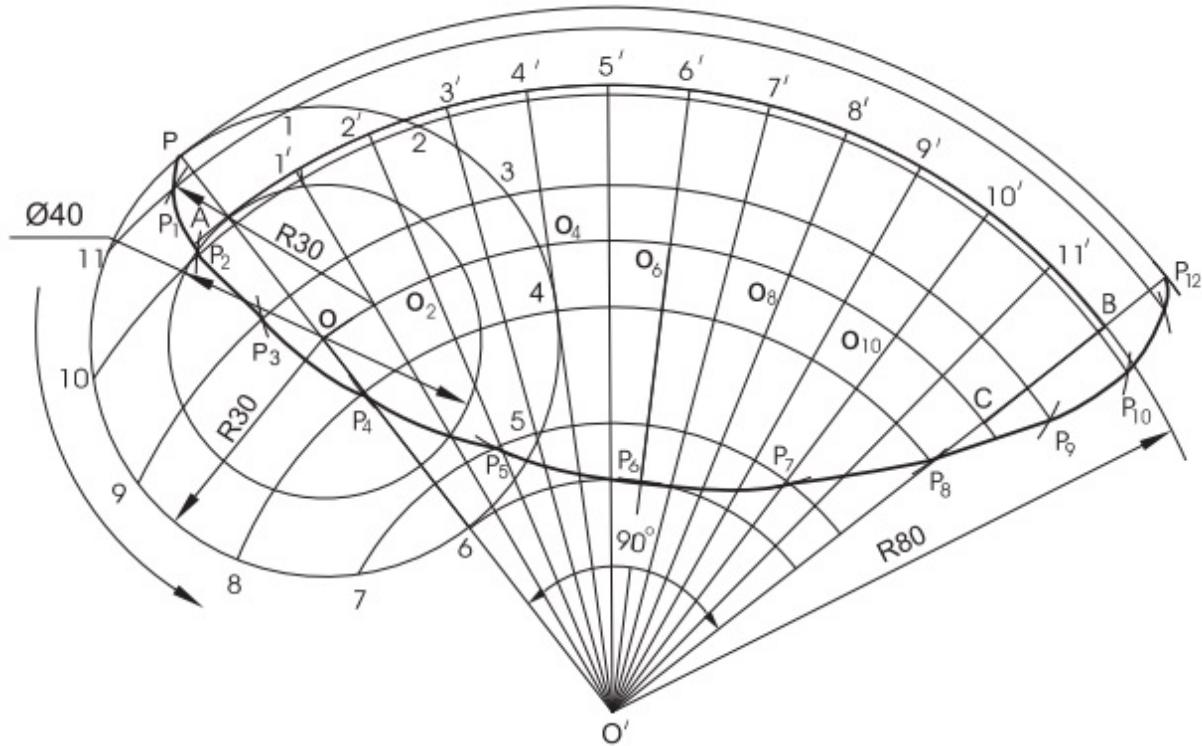
**Problem 36** A circle of 40 diameter, rolls inside another circle of 80 radius for one revolution counter clock-wise.

*Draw the locus of a point, which is at a distance of 30 from the centre of the generating circle.*

A procedure similar to the construction of hypo-cycloid (Fig.5.38) may be followed for constructing the superior hypo-trochoid, keeping in mind that the tracing point P lies outside the generating circle (Fig.5.45).

## 5.6 INVOLUTE

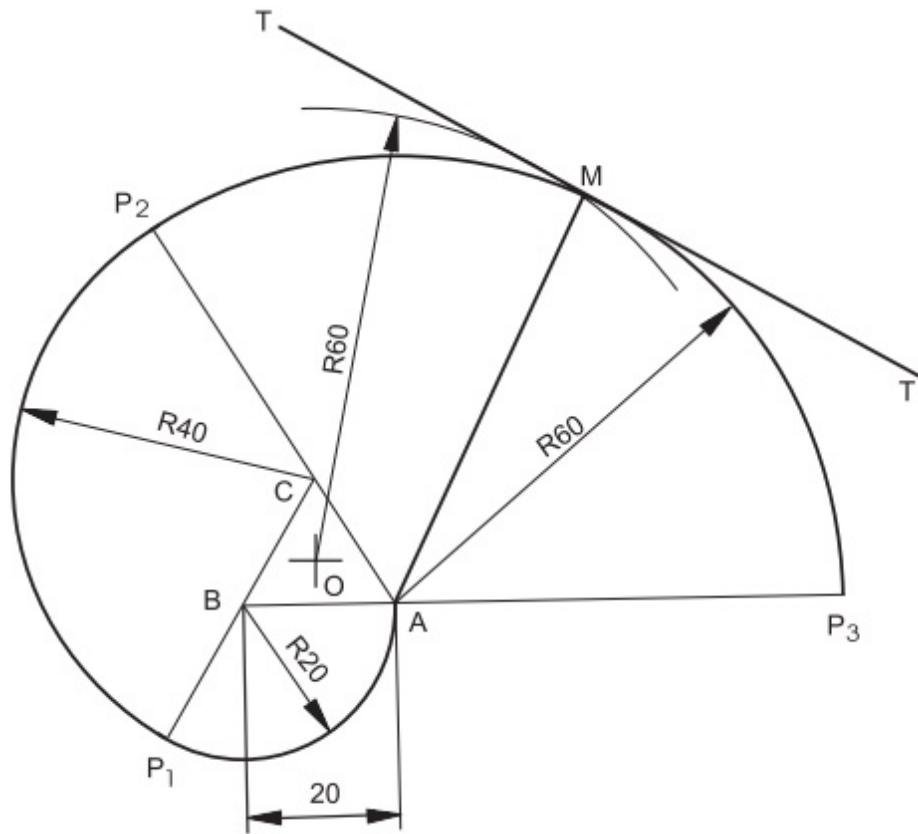
An involute is a curve traced by a point, on a perfectly flexible thread, while unwinding from a circle or a polygon; the thread being kept tight.



**Fig.5.45 Superior hypo-trochoid**

**Problem 37** *Draw the involute of an equilateral triangle of side 20.*

### **Construction (Fig.5.46)**



**Fig.5.46 Involute of a triangle**

1. Draw the given triangle ABC of side 20 and locate its centre O.
2. Assuming A as the starting point, with B as centre and radius BA ( $=20$ ), draw an arc intersecting the line CB extended at P<sub>1</sub>.
3. With centre C and radius CP<sub>1</sub> ( $= 2 \times 20$ ), draw an arc intersecting the line AC extended at P<sub>2</sub>.
4. With centre A and radius AP<sub>2</sub> ( $=3 \times 20$ ), draw an arc intersecting the line BA produced at P<sub>3</sub>.

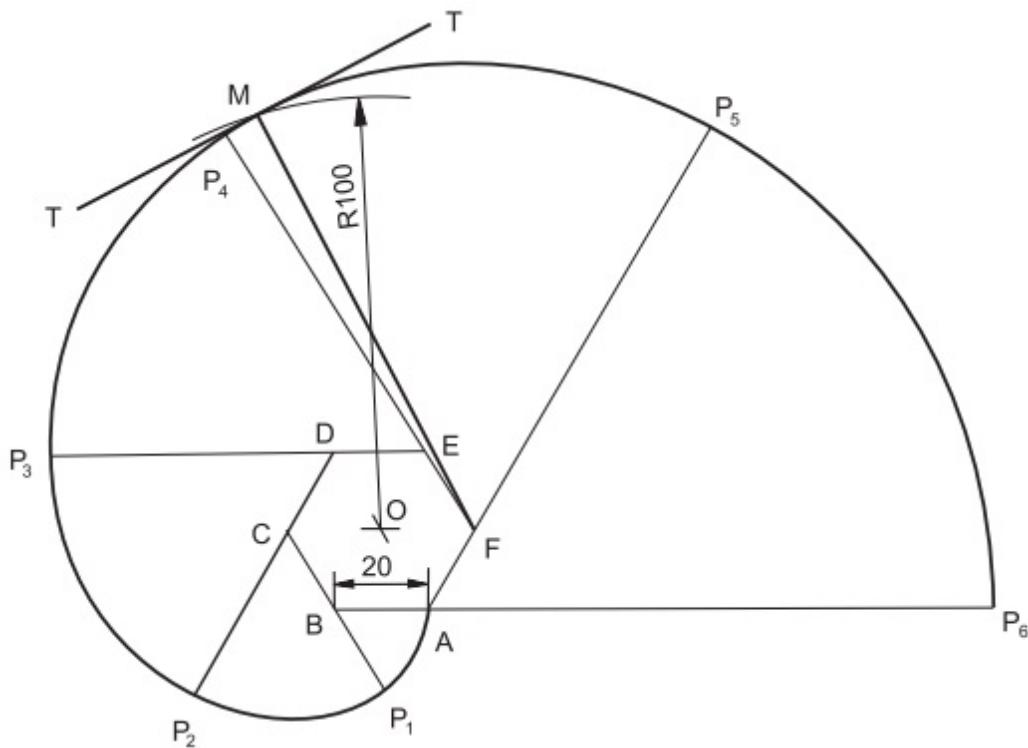
The curve through A, P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub> is the required involute.

*To draw a tangent and a normal to the curve*

- (i) With O as centre (centre of the triangle) and radius 60, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which A is centre.
- (ii) Join A, M; forming the normal to the curve.
- (iii) A line T-T, perpendicular to AM at M is the required tangent.

**Problem 38** *Draw the involute of a regular hexagon of side 20. Draw a tangent and a normal to the curve at a distance 100 from the centre of the hexagon.*

**Construction (Fig.5.47)**



**Fig.5.47 Involute of a hexagon**

1. Draw the hexagon ABCDEF of side 20 and locate its centre O.

Assuming that the thread is unwound from A in the clock-wise direction, the starting point for the involute is A.

2. With centre B and radius BA ( $=20$ ), draw an arc intersecting the line CB extended at  $P_1$ .
3. With centre C and radius  $CP_1$  ( $=2 \times 20$ ), draw an arc intersecting the line DC extended at  $P_2$ .
4. In a similar way, obtain the other points  $P_3, P_4$ , etc.

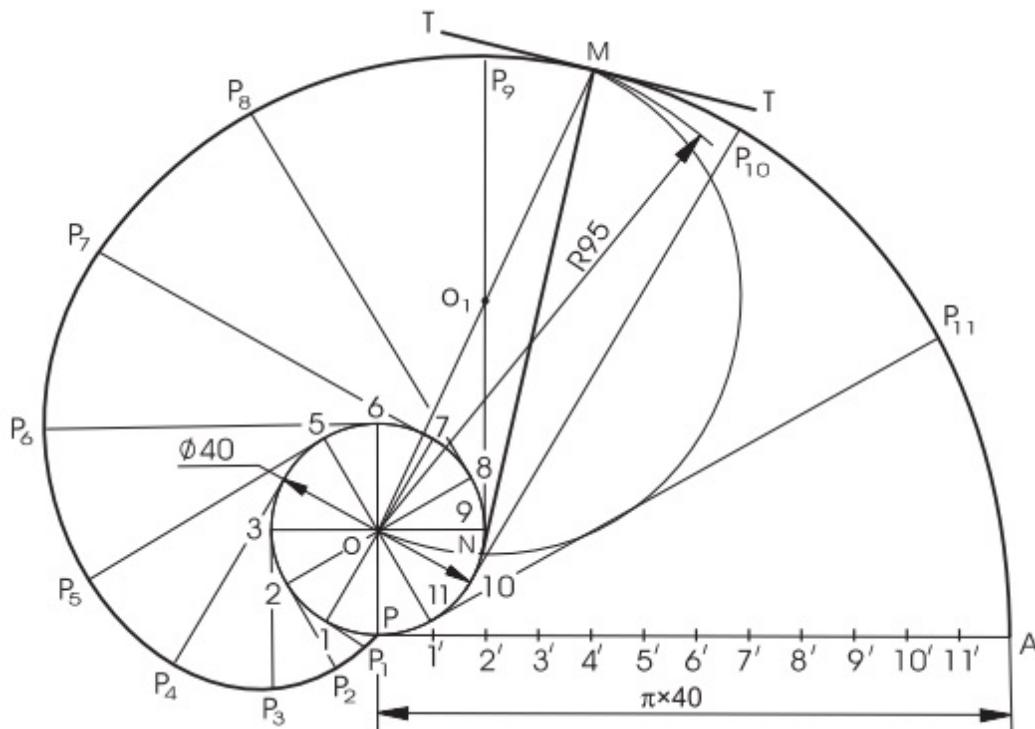
A smooth curve through the above points is the required involute.

*To draw a tangent and a normal to the curve*

- I. With O as centre and radius 100, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which F is the centre.
- II. Join F, M forming normal to the curve.
- III. A line T-T, perpendicular to FM at M is the required tangent.

**Problem 39** *Draw the involute of a circle of 40 diameter. Also, draw a tangent and a normal to the curve at a point 95 from the centre of the circle.*

**Construction (Fig.5.48)**



**Fig.5.48 Involute of a circle**

1. With centre O and diameter 40, draw the given circle.
2. Assuming P as the starting point, draw a line PA, tangent to the circle and equal to the circumference of the circle.
3. Divide the circle and the line PA into the same number of equal parts and number as shown.
4. Draw a tangent to the circle at the point 1 and locate on it  $P_1$  such that,  $1P_1 = P_1'$ .
5. Draw a tangent to the circle at point 2 and locate on it  $P_2$  such that,  $2P_2 = P_2'$ .
6. Locate other points  $P_3, P_4, \dots$ , etc., in a similar way.

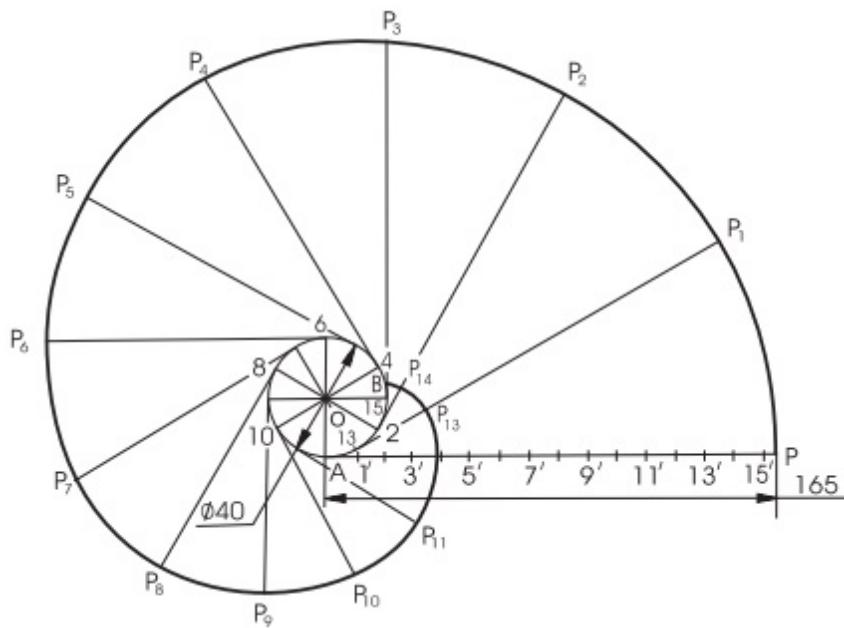
A smooth curve through these points is the required involute.

From the construction, it is obvious that a tangent to the circle is normal to the involute. So, to draw the tangent and normal,

- I. Locate the point M on the curve, which is at 95 from the centre of the circle.
- II. Join M, O and locate its mid-point O<sub>1</sub>.
- III. With centre O<sub>1</sub> and radius O<sub>1</sub> M, draw a semi-circle intersecting the given circle at N.
- IV. Join N, M forming the normal to the curve and a line T-T, perpendicular to NM at M is the tangent to the curve.

**Problem 40** A thread of length 165 is wound round a circle of 40 diameter. Trace the path of end point of the thread.

**Construction (Fig.5.49)**



**Fig.5.49 Path of end point of thread, wound round a circle**

1. With centre O and radius 20, draw the given circle.

2. From point A on the circle, draw a line AP, tangential to the circle and equal to 165, the length of the thread.
3. Divide the circle into 12 equal parts and mark the chord lengths along the line AP.
4. Draw tangents to the circle at points 1, 2, etc.
5. Along the tangent through 1, mark  $P_1$  such that,  $1P_1 = P_1'$ .
6. Along the tangent through 2, mark  $P_2$  such that,  $2P_2 = P_2'$ .
7. In a similar way, locate the points  $P_3, P_4$ , etc.

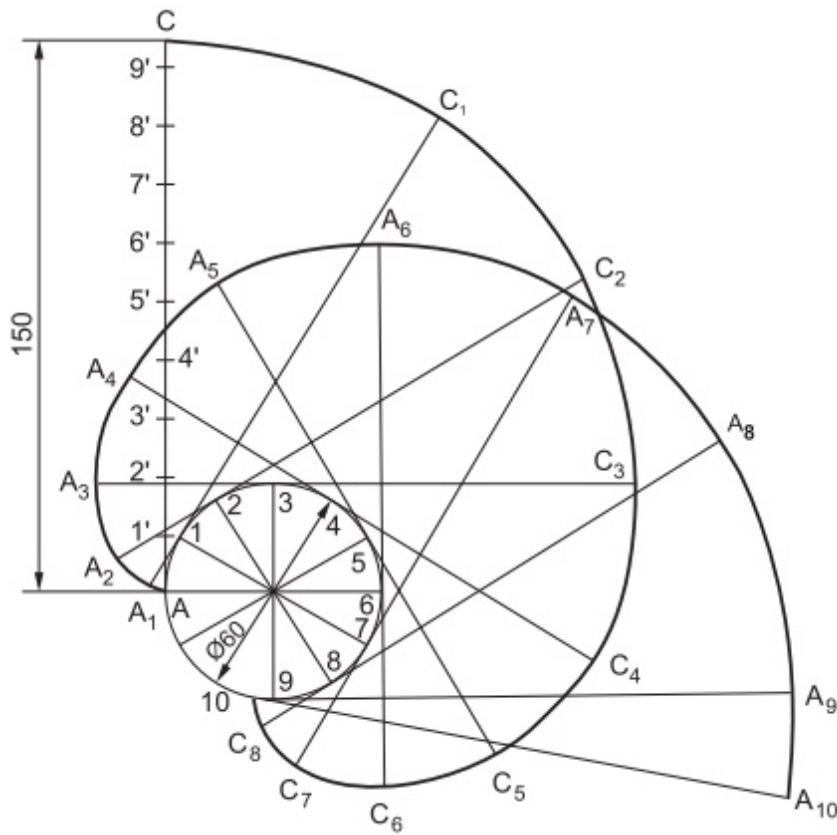
A smooth curve through these points is the required path.



1. The points  $P_{13}, P_{14}$  and  $P_{15}$  are located along the tangents through 1, 2 and 3.
2. The point B on the circle is located such that, the chord length  $15-B = 15'-P$ .

**Problem 41** A line AC of 150 long, is tangential to a circle of diameter 60. Trace the paths of A and C, when the line AC rolls on the circle without slipping.

**Construction (Fig.5.50)**



**Fig.5.50 Paths of end points of a line rolling on a circle**

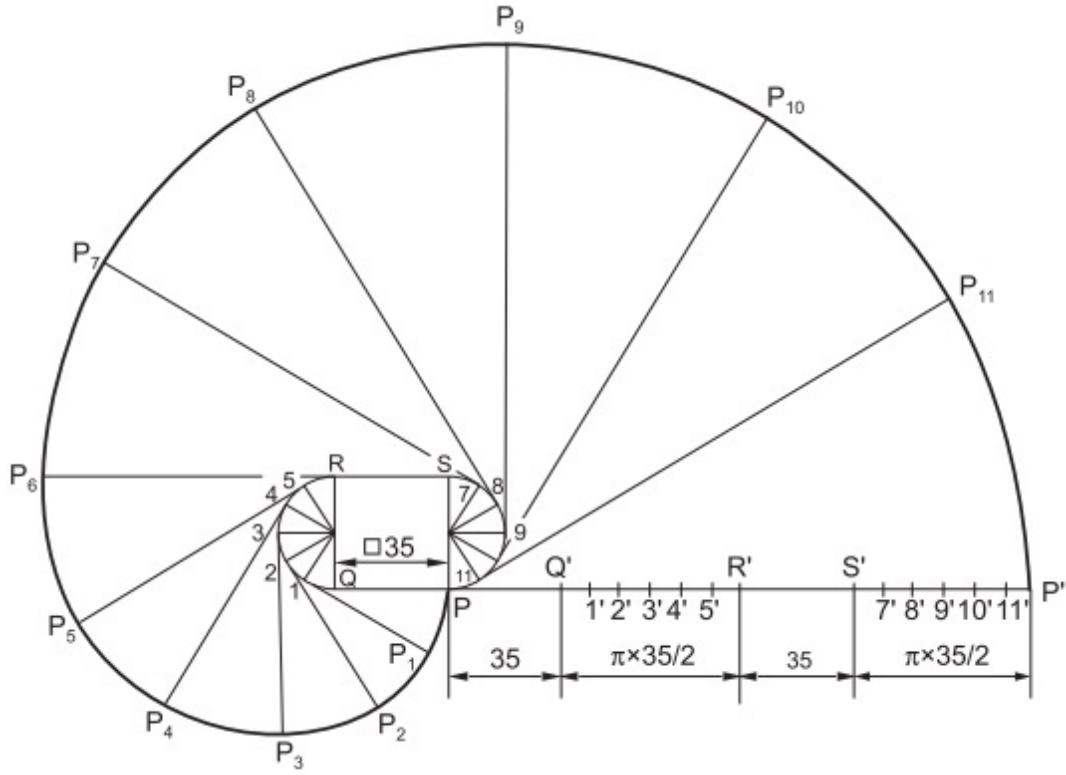
1. Draw a circle of diameter AB (=60).
2. Draw the tangent AC to the circle at A, of length 150.
3. Divide the circle into a number of equal parts, say 12 and number as shown.
4. Mark 1', 2', 3', etc., on AC such that,  $A-1' = 1'-2' = 2'-3'$ , etc., =  $1/12^{\text{th}}$  circumference of the circle.
5. When the line AC rolls on the circle and 1' coincides with 1, locate the positions of A and C; such that,  $1-A_1 = A-1'$  and  $1-C_1 = 1'C$ .
6. Similarly, locate the end points for different positions of the line, as it rolls on the circle.

Join A, A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, etc., and C, C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub>, etc., representing the paths of A and C respectively.

**Problem 42** *A disc in the form of a square of 35 side is surmounted by semi-circles on the two opposite sides. Draw the path of the end of the string, unwound from the circumference of the disc.*

**Construction ([Fig.5.51](#))**

1. Draw the square PQRS of side 35, surmounted by two semi-circles on the two opposite sides.
2. Divide the semi-circles into six equal parts and number as shown.
3. Extend QP to P' such that, PP' = 2 × 35 + π × 35 and mark the division points as shown.
4. Draw tangents to the semi-circles at 1, 2, - - - R, 7, 8, - - - 11.
5. Assuming that the string is unwound from P, locate P<sub>1</sub> along the tangent at 1 such that, 1P<sub>1</sub> = P1'.
6. Locate P<sub>2</sub> along the tangent at 2 such that, 2P<sub>2</sub> = P2' and so on.
7. Join the points P, P<sub>1</sub>, P<sub>2</sub>, etc., by a smooth curve, forming the path of the end of the string.



**Fig.5.51 Path of end point of a string unwound from a disc**

## 5.7 SPIRALS

A spiral is a curve traced by a point, moving continuously along a rotating line. The point about which the line rotates is called the pole. One revolution of the line is called one convolution. The tracing point may follow any number of convolutions before reaching the pole, or to reach any point along the rotating line.

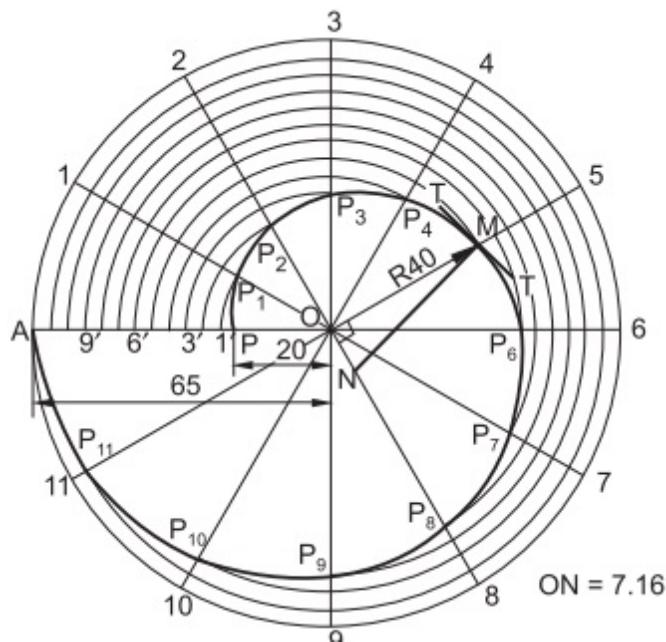
### 5.7.1 Archimedian Spiral

It is a curve traced by a point, moving with uniform velocity along a line, which is also rotating with uniform angular

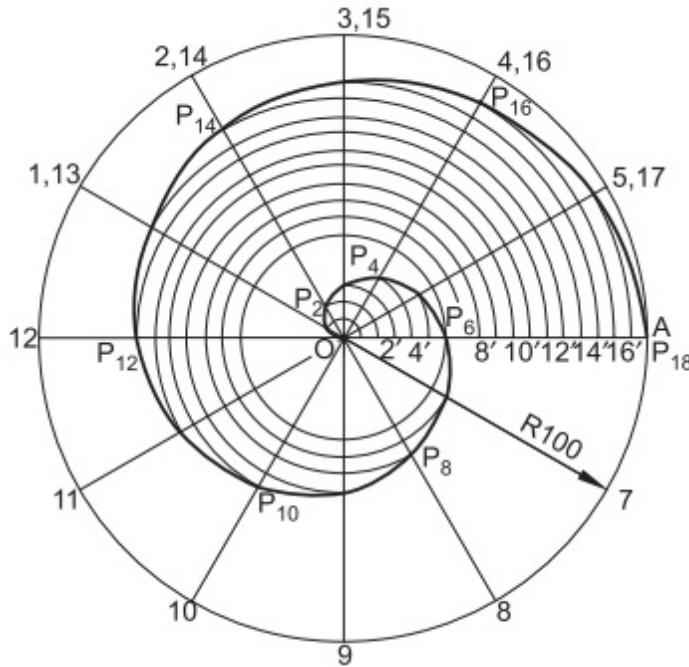
velocity.

**Problem 43** Construct an Archimedian spiral for one convolution. The initial and final radius vectors of the tracing point are 20 and 65 respectively. Draw a tangent and a normal to the curve at a point 40 from the pole.

**Construction (Fig.5.52)**



**Fig.5.52 Archimedian spiral-One convolution**



**Fig.5.53 Archimedian spiral-One and half convolution**

1. With centre O and radius 65 (OA), the final radius vector, draw a circle.
2. Along the radial line OA, mark a point P such that,  $OP = 20$ , the initial radius vector.
3. Divide PA and the circle into the same number of equal parts and number as shown. Draw radial lines from the centre of the circle.
4. With centre O and radii equal to  $O1'$ ,  $O2'$ , etc., draw arcs intersecting the radial lines  $O1$ ,  $O2$ , etc., at  $P_1$ ,  $P_2$ , etc.

A smooth curve through  $P$ ,  $P_1$ ,  $P_2$ , etc., is the required Archimedian spiral.

*To draw a tangent and a normal to the curve*

- I. Locate the given point M, lying at 40 from the pole, and join M, O.

- II. Through the point O, draw a perpendicular ON, to the line OM of length equal to the constant of the curve.
- III. Join N, M, which is the required normal and a line T-T, perpendicular to it, is the tangent to the curve.



The constant of the curve is given by the ratio of the difference between two radius vectors and the angle between them in radians. In the present case, considering the radius vectors OP and OP<sub>3</sub>, separated by 90° ( $= \pi / 2$ ); the constant of the curve is given by,

$$\frac{(OP_3 - OP)}{\pi/2} = \frac{11.2}{\pi/2} = 7.16$$

**Problem 44** Construct an Archimedian spiral for 1-1/2 convolutions. During the period, the tracing point starting from the pole, covers a radial distance of 100.

#### **Construction (Fig.5.53)**

1. With centre O and radius 100(=OA), draw a circle.
2. Divide the total angular movement ( $360^\circ \times 1\frac{1}{2} = 540^\circ$ ) of the tracing point and the distance OA into same number of equal parts, say 18 and number as shown.
3. With centre O and radii equal to O<sub>1</sub>', O<sub>2</sub>', etc., draw arcs intersecting the radial lines O<sub>1</sub>, O<sub>2</sub>, etc., at P<sub>1</sub>, P<sub>2</sub>, etc.

A smooth curve through O, P<sub>1</sub>, P<sub>2</sub>, etc., is the required curve.

**Problem 45** Draw a triangle ABC with AB and AC, 30 and 40 long respectively and  $\angle BAC$  equal to  $45^\circ$ . B and C are the points on the Archimedian spiral of one convolution of

*which A is the pole. Find the initial line and draw the spiral, starting from the pole.*

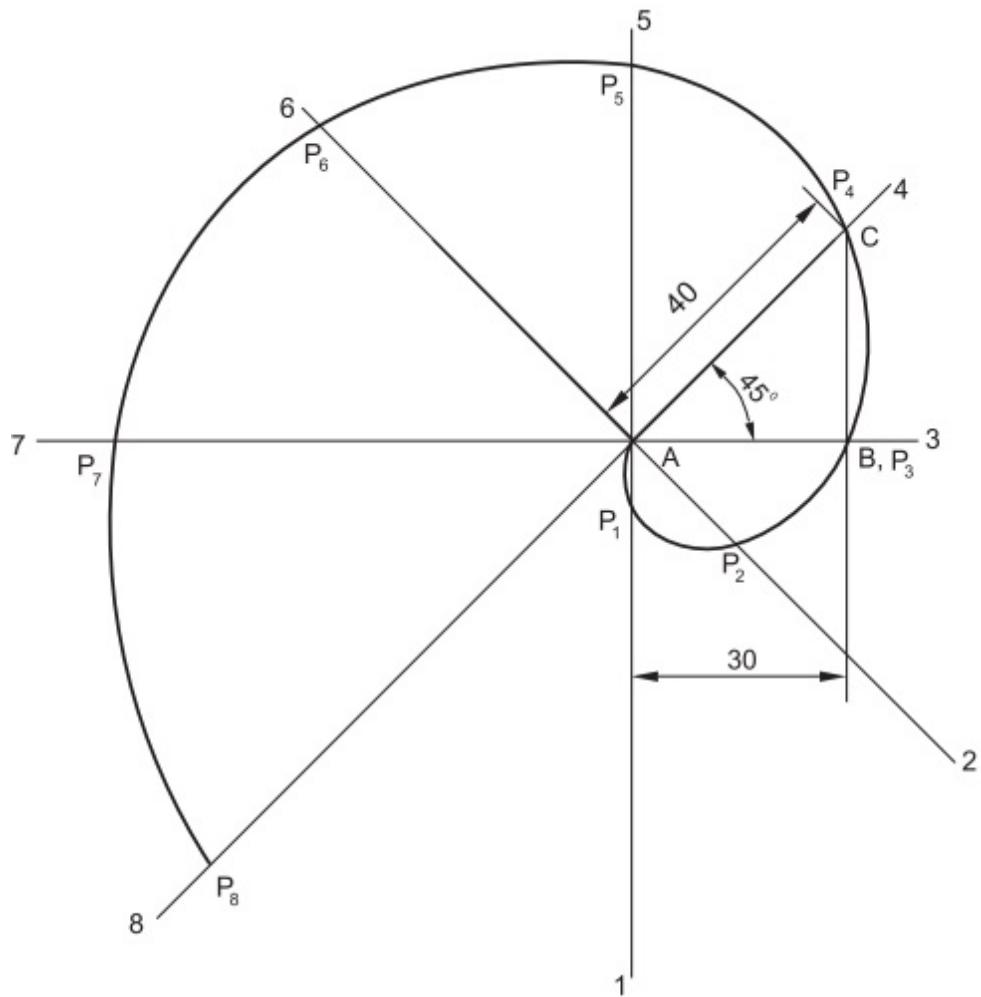
### **Construction ([Fig.5.54](#))**

1. Draw the triangle ABC, satisfying the given information.



- i) B and C are the points on the Archimedean spiral and A is the pole.
- ii) For an angular movement of  $45^\circ$  of the radius vector (B to C), the distance covered is 10.
- iii) As the point moves from the pole, to cover a distance of 40 (upto C), the angular movement shall be  $180^\circ$ .
- iv) Hence, the line CA extended is the initial line.

2. Divide the total angular movement for one convolution, into 8 equal parts and number as shown.
3. Mark  $P_1, P_2, P_3$  along A-1, A-2 and A-3 such that,  $AP_1 = 10$ ,  $AP_2 = 20$ , and  $AP_3 = AB = 30$ .
4. Locate the position of the moving point along the other radial lines,  $P_4, P_5, \dots, P_8$ , suitably.
5. Join the points A,  $P_1, P_2, \dots, P_8$  by a smooth curve, forming the required spiral.



**Fig 5.54**

**Problem 46** A link of 150 long swings about one end, from its vertical position of rest, to the left through an angle of  $40^\circ$ . It returns to the initial position after swinging to the right, through an angle of  $80^\circ$ . A bead traverses from top to bottom of the link at uniform speed during the above motion. Trace the path of the bead, assuming the angular velocity of the link to be uniform.

**Construction (Fig.5.55)**

1. With centre A and radius 150, draw an arc CBD subtending an angle of  $80^\circ$  at A. AB represents the

position of the link at rest and AC and AD are the extreme positions of the link while swinging.

2. Divide the total angular movement ( $160^\circ$ ) and AB into the same number of equal parts, say 16 and number as shown.
3. Draw arcs with radii  $A1'$ ,  $A2'$ , etc., intersecting the radial lines  $A1$ ,  $A2$ , etc., at  $P_1$ ,  $P_2$ , etc.

A smooth curve passing through A,  $P_1$ ,  $P_2$ , etc., is the required path.

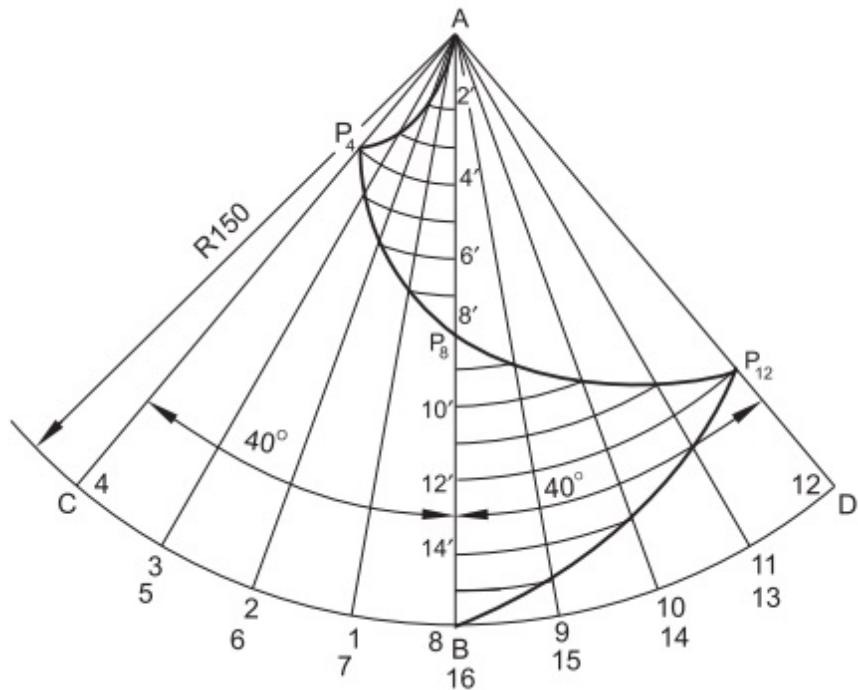
## 5.7.2 Approximate Spiral

A clock spring is made-up of semi-circles and hence the space between the windings is uniform. It is to be noted that the curvature of windings resemble an approximate spiral.

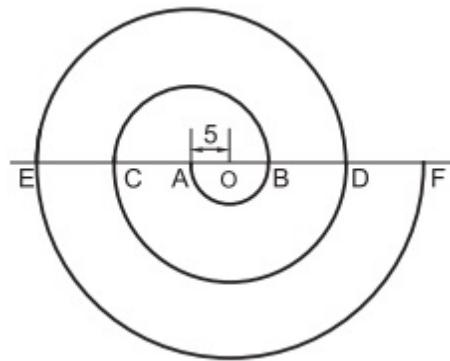
**Problem 47** *Construct an approximate spiral, with the distance between the windings as 10.*

### **Construction (Fig.5.56)**

1. Draw a horizontal line and mark a point O on it.
2. With centre O and radius equal to 5, draw a semi-circle meeting the horizontal line at A and B.
3. With centre A and radius AB, draw a semi-circle meeting the line at C.
4. With centre O and radius OC, draw a semi-circle meeting the line at D.
5. Repeat the above procedure till the required number of windings are obtained.



**Fig.5.55 Path of the bead on the line**



**Fig.5.56 Approximate spiral**

### 5.7.3 Logarithmic Spiral (Equi-angular Spiral)

Logarithmic spiral is the curve traced by a point which is moving along a rotating line such that, for equal angular displacements of the line, the ratio of the lengths of

consecutive radius vectors is constant. Thus, in logarithmic spiral, vectorial angles increase in arithmetic progression and the corresponding radius vectors are in geometric progression.

Logarithmic spiral is also known as equi-angular spiral because of its property that at any point on the curve, the angle between the tangent and the radius vector is constant.

The polar equation for the curve is

$$r = a^\theta$$

where  $r$  = radius vector

$\theta$  = angle turned, and

$a$  = constant

$$\therefore \log r = \theta \log a$$

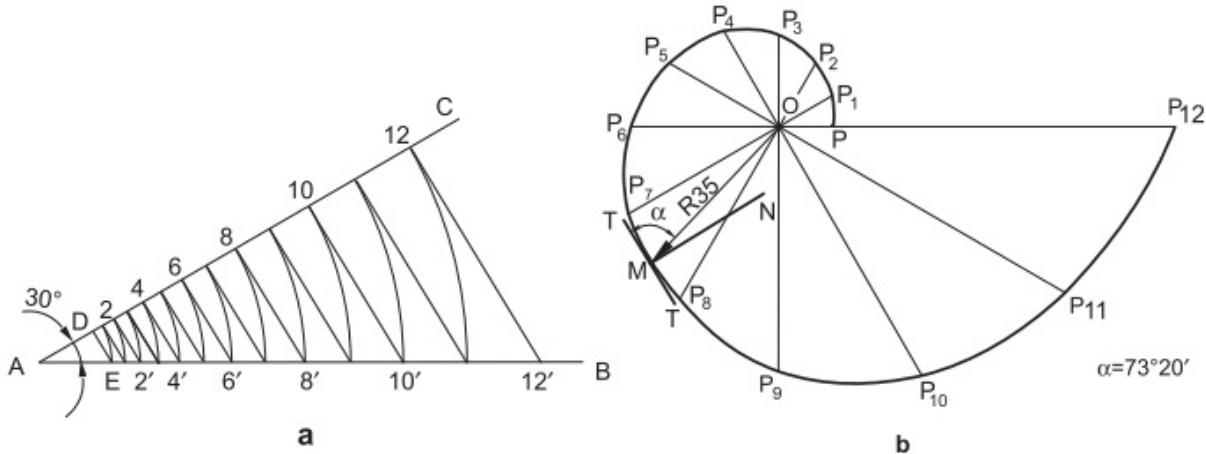
$$\text{when } \theta = 0, \log r = 0$$

$$\therefore r = 1$$

Thus, when  $\theta$  is zero, initial radius vector is unity and hence, the name logarithmic spiral.

**Problem 48** Construct a logarithmic spiral for one convolution such that, the angle between the consecutive radii is  $30^\circ$ , the ratio of lengths of consecutive radii is  $7/6$  and the initial radius vector is 10. Also, draw a normal and a tangent to the curve at a point 35 from the pole.

**Construction (Fig.5.57a)** - to obtain the radial lengths



**Fig.5.57 Logarithmic spiral**

1. Draw two lines AB and AC, making an angle of  $30^\circ$ .
2. Mark a point D on the line AC, such that,  $AD = r = 10$ .
3. Mark a point E on the line AB such that,  $AE = 7/6^{\text{th}} AD = 11.67$ .
4. Join D, E.
5. With centre A and radius AE, draw an arc meeting the line AC at 1.
6. Through the point 1, draw a line parallel to DE and meeting the line AB at 1'.
7. With centre A and radius A1', draw an arc meeting the line AC at 2.
8. Through the point 2, draw a line parallel to DE and meeting the line AB at 2'.
9. Repeat the above construction and obtain the points 3, 4, etc., on the line AC.

The lengths A-1, A-2, etc., correspond to the radial lengths at  $30^\circ$  angular intervals.

**Construction (Fig.5.57b) - to construct the curve**

10. Draw a horizontal line and mark the point P on it such that,  $OP = r = 10$ .
11. Draw radial lines through O at  $30^\circ$  intervals, covering  $360^\circ$ .
12. Mark points  $P_1, P_2$ , etc., along the above lines such that,  $OP_1 = A-1; OP_2 = A-2$ ; etc.

A smooth curve through the points P,  $P_1, P_2$ , etc., is the required curve.

*To draw a tangent and a normal*

- I. Locate the point M on the curve such that,  $OM = 35$ .
- II. Join M, O.
- III. Calculate the angle  $a$ , the angle between radial line and the tangent:

$$\log r = \theta \log a, \text{ and}$$

$$\text{If } \theta = 0, \log r = 0 \text{ and } r = 1$$

$$\text{when } \theta = 30^\circ = \pi/6 \text{ radians, } r = 7/6 \times 1 = 7/6$$

$$\therefore \log 7/6 = \pi/6 \log a$$

$$\text{So } a = 1.342$$

The angle  $\alpha$ , between the tangent and radial line through the point M, is given by,

$$\tan \alpha = \log e / \log a$$

$$\text{where } e = 2.718$$

$$\therefore \alpha = 73^\circ 20'$$

- IV. Draw a line MT at an angle  $\alpha$  with OM.
- V. The line TM extended (T-T) is the required tangent and the line MN, perpendicular to it is the normal.

## 5.7.4 Hyperbolic Spiral

It is a curve traced by a point moving with hyperbolic relation along a line, which is rotating with uniform angular velocity.

The magnitudes of the radius vectors of the moving point may be obtained as follows:

Let the initial position of the point from the pole be  $\mathbf{r}$  and one convolution be divided into  $n$  equal parts.

The length of the radius vector after  $1/n^{\text{th}}$  convolution is  $(n/n-1)\mathbf{r}$

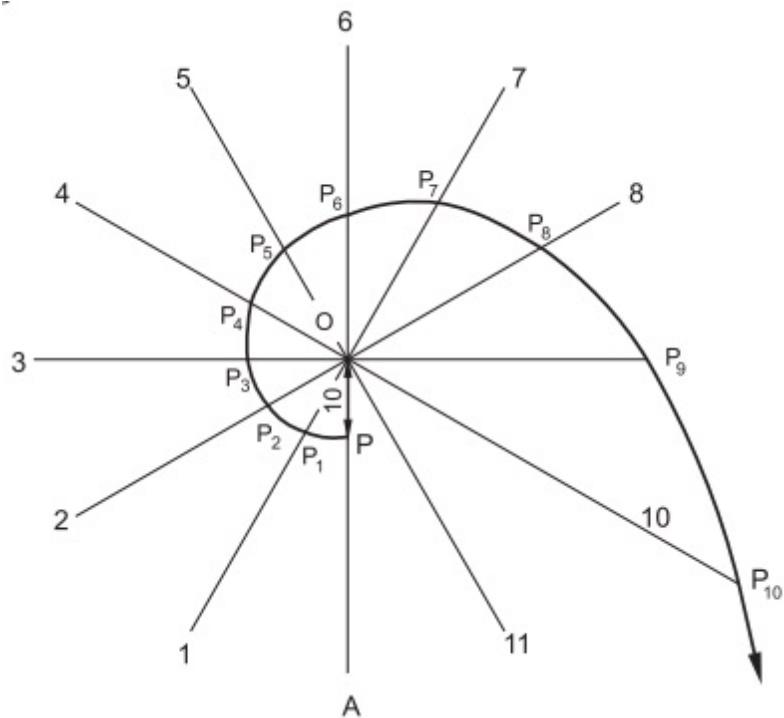
The length of the radius vector after  $2/n^{\text{th}}$  convolution is  $(n/n-2)\mathbf{r}$

The length of the radius vector after  $n-1/n^{\text{th}}$  convolution is  $n\mathbf{r}$

The length of the radius vector after 1 convolution is  $(n/0)\mathbf{r} = \infty$

**Problem 49** Construct a hyperbolic spiral, keeping the initial radius vector as 10.

**Construction (Fig.5.58)**



**Fig.5.58 Hyperbolic spiral**

1. Calculate the lengths of the radius vectors, with respect to their positions ([Table 5.1](#)).
2. Draw a vertical line OA and mark a point P on it such that,  $OP = 10$ .
3. Draw radial lines through O at  $30^\circ$  intervals, covering  $360^\circ$ .
4. Along the radial lines  $O_1, O_2$ , etc., mark the lengths  $(12/11)10, (12/10)10$ , etc., as given in the Table and locate the points  $P_1, P_2$ , etc.

A smooth curve through these points is the required curve.



It may be noted that only a part of a convolution can be covered in constructing a hyperbolic spiral as the distance of the point from the centre increases at an enormous rate.

**Table 5.1 Lengths of radius vectors in relation to their positions**

Position of the radius vector	Length of radius vector
30°	(12/11)r
60°	(12/10)r
90°	(12/9 )r
120°	(12/8) r
150°	(12/7) r
180°	(12/6) r
210°	(12/5) r
240°	3 r
270°	4 r
300°	6 r
330°	12 r
345°	24 r
352. 5°	48 r
356.25°	96 r
358.125°	192 r
360°	∞

## EXERCISES

### Ellipse

- 5.1 Two fixed points A and B are 100 apart. Trace the complete path of the point P moving in such a way that the sum of its distances from A and B is always

the same and equal to 125. Name the curve. Draw a tangent and a normal to the curve at any convenient location.

- 5.2 A point P moves such that, its distances from two fixed points A and B, which are 90 apart remains constant. When P is at equal distances from A and B, its distance from each one is 75. Draw the path traced by the point P.
- 5.3 The major axis of an ellipse is 120 and the distance between the foci is 80. Determine the length of the minor axis. Draw the curve by any four methods. Also, draw a tangent and a normal to the curve through a point P, when (i) it is situated at a distance of 80 from a focus and lying on the curve and (ii) it is situated at a distance of 100 from one focus and 60 from the other.
- 5.4 The foci of an ellipse are 85 apart and the minor axis is 60 long. Determine the length of the major axis and draw the ellipse by oblong method.
- 5.5 The foci of an ellipse are 80 apart and the minor axis is 55 long. Determine the length of the major axis and draw the ellipse by concentric circle method. Draw a curve parallel to the ellipse and 20 away from it.
- 5.6 The major and minor axes of an ellipse are 125 and 100 long respectively. Draw the curve by concentric circles method and locate its foci. Also, determine the eccentricity. Draw a tangent at a point on the curve, 30 above major axis.
- 5.7 Inscribe an ellipse in a rectangle of sides 120 and 80.

- 5.8 The major axis of an ellipse is 120 long and the foci are at a distance of 20 from its ends. Complete the ellipse and draw a tangent at a distance of 35 from focus.
- 5.9 An ellipse has the major axis and minor axis in the ratio of 3:2. Draw the ellipse, when the major axis is 120. Draw a tangent and a normal at any point P on it.
- 5.10 In a triangle ABC; AB, BC and CA are 100,50 and 75 long respectively. Draw an ellipse passing through the points A, B and C. Also, draw a tangent and a normal at any point on it.
- 5.11 In a triangle ABC; AB, AC and BC are 75, 60 and 50 long respectively. Draw an ellipse such that, A and B are the foci and C is a point on the curve.
- 5.12 To the curve in problem 5.4, draw another curve outside it and parallel to it, at a distance of 15.
- 5.13 Inscribe an ellipse in a parallelogram having sides 150 and 100 long and an included of  $120^\circ$ .
- 5.14 The sides of a parallelogram are 100 and 60 and included angle is  $55^\circ$ . Inscribe an ellipse and determine its major and minor axes and locate the foci.
- 5.15 Inscribe an ellipse in a rhombus of side 60. The shorter diagonal of the rhombus is 60.
- 5.16 Two points A and B are 100 apart. A point C is 75 from A and 60 from B. Draw an ellipse passing through A,B and C.
- 5.17 Construct an ellipse having a major axis 100 and minor axis 70. Locate its foci and directrices and find

the eccentricity. Draw a tangent at a distance of 25 from focus.

5.18 The foci of an ellipse are 90 apart and the minor axis is 65 long. Determine the major axis and draw half the ellipse by concentric circles method and the other half by oblong method. Draw a tangent to the ellipse at a point 25 above the major axis.

5.19 A fixed point is at 50 from a fixed straight line. Draw the curves when eccentricity is (i)  $2/3$ , (ii) 1, (iii)  $3/2$ . Name the curves. Draw tangents and normals to the curves through a point P, which is at 60 from the straight line.

5.20 Construct an ellipse when the distance between the focus and the directrix is 30 and the eccentricity is  $3/4$ . Draw a tangent and a normal at any point P on the curve.

5.21 The directrices of an ellipse are 240 apart and the major axis is 190 long. Draw the ellipse and calculate the eccentricity of the ellipse.

## **Parabola**

5.22 Construct a parabola when the distance between focus and directrix is 40. Draw a tangent and a normal at any point P on the curve.

5.23 Draw a straight line AB of any length. Mark a point F, 65 from AB. Trace the path of a point P moving in such a way that the ratio of its distance from the point F, to its distance from AB is 1. Plot at least 10 points. Name the curve. Draw a normal and a tangent to the curve at a point on it, 45 from F.

5.24 A fixed point is 75 from a fixed straight line. Draw the locus of a point P moving in such a way that its

distance from the fixed straight line is equal to its distance from the fixed point. Name the curve. Draw a normal and a tangent at any point on the curve.

5.25 In a triangle ABC; AC and BC are 75 each. The angle at C is  $120^\circ$ . Draw a parabola passing through A, C and B.

5.26 A cricket ball reaching a height of 20 m, covers a distance of 50 m before coming to the ground. Trace the path of the ball. Determine its direction, at a height of 6 m from the ground.

5.27 A shot is discharged from the ground level at an inclination of  $45^\circ$  to the ground, which is horizontal. The shot returns to the ground at a point 250 m from the point of discharge. Trace the path of the shot. Find the direction of the shot, after it has traveled a horizontal distance of 200 m.

5.28 Construct a right angle triangle EFG such that,  $FG = 40$  and  $FE = 20$  and  $\angle EFG = 90^\circ$ . The point G is on the parabola whose focus is the point F of the triangle. If FE is a part of parabola's axis, draw the parabola and determine the double ordinate, at a distance of 80 from its directrix.

5.29 A jet of water is issuing through an orifice of 50 diameter, fitted to a vertical side of a tank. The centre of the orifice is 1.5 m above the ground and centre of the jet touches the ground at a distance of 2.5 m from the vertical orifice. Draw the locus of the centre of the jet, which is just issuing from the orifice till it reaches the ground. Name the curve.

5.30 A fountain jet discharges water from the ground level at an inclination of  $45^\circ$  to the ground. The jet travels a horizontal distance of 7.5 m from the point

of discharge and falls on the ground. Trace the path of the jet. Name the curve.

- 5.31 Inscribe two parabolas in a rectangle of sides 100 and 50, with their axes perpendicular to each other. Determine the focus and directrix of each.
- 5.32 Inscribe a parabola in the parallelogram of problem 5.13, with the longer side of it as the base.

## **Hyperbola**

- 5.33 Two points A and B are 50 apart. Draw the curve traced by a point P moving in such a way that the difference between its distances from A and B is always constant and equal to 20. Draw a tangent to the curve at a point 40 above the transverse axis.
- 5.34 Draw a straight line AB of any length. Mark a point F, 65 from AB. Trace the path of a point P moving in such a way that the ratio of its distance from the point F, to its distance from AB is 3:2. Plot at least 10 points. Name the curve. Draw a normal and a tangent to the curve at a point which is 45 from F.
- 5.35 The vertex of a hyperbola is 60 from its focus. Draw the curve, if the eccentricity is  $\frac{3}{2}$ . Draw a normal and a tangent at a point on the curve, 75 from the directrix.
- 5.36 Draw a hyperbola having the double ordinate of 100, the abscissa of 60 and the transverse axis of 100.
- 5.37 Draw two branches of a hyperbola, with the distance between their foci as 60 and vertices as 35. Also, draw the asymptotes and measure the angle between them.
- 5.38 Draw the two branches of hyperbola, when the distance between the foci is 100 and the vertices are

15 from the foci.

5.39 A point P is 30 and 50 respectively from two straight lines which are at right angle to each other. Draw a rectangular hyperbola from P and within 10 distance from each line.

5.40 The two asymptotes of a hyperbola are at an angle of  $100^\circ$ . A point is 30 and 65 from the asymptotes as measured parallel to them and is on the curve. Draw the two branches of the curve and find the following:

- (a) Length of the transverse axis,
- (b) The eccentricity, and
- (c) The directrices.

Draw a normal and a tangent to the curve at a point on the curve, which is at 50 from the directrix.

5.41 Draw a rectangular hyperbola, through a point P, with the co-ordinates  $x = 25$  and  $y = 30$ .

5.42 The major axis of a hyperbola is 60 and a point P on the curve is at a distance of 75 from the centre of the major axis and 35 from the vertex. Draw one branch of hyperbola.

5.43 Two straight lines OA and OB are at an angle of  $75^\circ$  at O. Draw a hyperbola through a point P, which is 25 from OA and 40 from OB.

5.44 The asymptotes of a hyperbola are inclined at  $70^\circ$  to each other. Construct the curve when a point P on it is at a distance of 20 and 30 from the two asymptotes.

## Cycloidal Curves

- 5.45 A coin of 40 diameter rolls over horizontal table without slipping. A point on the circumference of the coin is in contact with the table surface in the beginning and after one complete revolution. Draw and name the curve. Draw a tangent and a normal at any point on the curve.
- 5.46 A circle of 45 diameter rolls along a straight line without slipping. Draw the curve traced out by a point P on the circumference for 1.5 revolutions of the circle. Name the curve. Draw a tangent and a normal at a point on it 35 from the line.
- 5.47 A circle of 50 diameter rolls on a straight line without slipping. In the initial position, the diameter AB of the circle is parallel to the line, on which it rolls. Draw the loci of the points A and B for one revolution of the circle.
- 5.48 ABC is an equilateral triangle of side 70. Trace the loci of vertices A, B and C, when the circle circumscribing ABC, rolls without slipping along a fixed straight line, for one complete revolution.
- 5.49 Construct a cycloid having a rolling (generating) circle diameter as 50. Draw a normal and a tangent to the curve at a point 35 above the base line.
- 5.50 A rolling circle of diameter 50, rolls without slipping on a horizontal ground. Trace only that part of the locus, traced by a point on the circumference of the rolling circle, as it descends from the highest level, until it touches the ground.
- 5.51 A carriage wheel with a flange of outside diameter 60 rolls on a rail. Draw the path of a point on the flange surface for 1.5 revolutions of the wheel. Name

the curve. Draw a tangent to the curve at any convenient location.

- 5.52 A circle of 50 diameter, rolls on a horizontal line for half a revolution and then on a vertical line for another half. Draw the curve traced out by a point P on the circumference of the circle, taking the top-most point on the rolling circle as the initial position of the generating point.
- 5.53 A circle of 50 diameter, rolls along a circle of diameter 200. Trace the paths of a point P lying on the circumference of the rolling circle, when the rolling circle moves (i) outside and (ii) inside the other circle. Name the curves. Draw normals and tangents to the curves at 125 and 75 respectively from the centre of the directing circle.
- 5.54 Construct a hypo-cycloid, with rolling circle 50 diameter and directing circle 175 diameter.
- 5.55 Draw epi-cycloid of a circle of 60 diameter which rolls outside on another circle of 180 diameter for one revolution. Draw a tangent and a normal at any point P on the curve.
- 5.56 The diameter of the directing circle is twice that of the generating circle. Show that the hypo-cycloid is a straight line, when the diameter of the generating circle is 50.
- 5.57 Construct an epi-cycloid (cardioid); taking the diameter of the rolling circle and directing circle as 50.
- 5.58 A circus man rides on a motor cycle, inside a globe of 4m diameter. The motor cycle wheel is 1m diameter. Draw the locus of a point on the

circumference of the wheel of motor cycle for its one complete turn on the maximum diameter path and name the curve.

## Trochoids

5.59 A wheel of 600 diameter rolls on a straight line without slip. Trace the path of a point on one of the spokes, 100 from the rim and for one convolution. The point is vertically above the centre of the wheel in the starting position.

5.60 The diameter of a generating circle is 50 and a directing circle is 200. Two points P and Q are situated at 20 and 30 from the centre of the generating circle. Trace the paths of the points, when the generating circle rolls (i) outside and (ii) inside the directing circle. Name the curves.

5.61 A circle of 50 diameter, rolls along a straight line, which makes an angle of  $15^\circ$  to the horizontal. Two points X and Y lie on one of the diameters (extended if necessary) such that,  $XZ=20$  and  $YZ=30$ , where Z is the centre of the circle.

Draw the curves traced by the points X and Y, for one complete revolution of the circle.

5.62 Draw a superior trochoid, with the following particulars:

Diameter of generating circle = 240

Diameter of rolling circle = 60

Distance of the tracing point from the centre of the rolling circle = 40

## Involute

5.63 Draw the involute of a hexagon of side 25.

- 5.64 Draw the involute of a circle of 50 diameter. Also, draw a normal and a tangent to the curve at a point 100 from the centre of the circle.
- 5.65 A straight line AB of length 100, initially tangential at A, to a circle of 40 diameter, rolls without slipping on the circle, till the end B touches the circle. Show the paths of the ends A and B of the line and name the curves.
- 5.66 An inelastic string of length L has its one end attached to the circumference of a circle of 50 diameter. Draw the curve traced by the other end of the string, when it is tightly wound round the circle; when L is (i) 100 and (ii) 200.
- 5.67 A disc is in the form of a square of 30 side, surmounted by semi-circles on opposite sides. Draw the path of the end of a string unwound from the circumference of the disc.
- 5.68 AB is the diameter of a semi-circle of radius 80. A string is tied tightly from A to B around the semi-circle starting from A. The end B is unwound, keeping the string taut until it lies along the tangent at A. Trace the path of the end B of the string.
- 5.69 A disc is in the form of a square of 30 side, surmounted by a semi-circle on one side and a half hexagon on the opposite side. Draw the path of the end of a string, unwound from the circumference of the disc, assuming that the length of the string as 140.

## Spirals

- 5.70 Draw one convolution of the Archimedean spiral represented by the polar equation,  $r = 32 + 140$ ,

where  $r$  is in mm and  $\theta$  is in radian measure. Draw a tangent and a normal to the curve at a point 75 from the pole.

- 5.71 Draw an Archimedean spiral of 1-1/2 convolutions, the greatest and least radii being 110 and 20 respectively. Draw a tangent and a normal to the spiral at a point 60 from the pole.
- 5.72 A wheel, 2 m in diameter, has 6 spokes connecting the rim and hub. The hub outer diameter is 300. The wheel is rotating in clock-wise direction at 40 r.p.m. A dust particle covers the length of a spoke in 4 seconds with uniform velocity. Trace the path of the particle.
- 5.73 A link OA of 100 long, rotates about O in clock-wise direction. A point P on the link, which is at 25 from O, moves with uniform velocity and reaches the end A, while the link rotates through 1 - 1/2 revolutions. Trace the path of the point P and name the curve. Draw a normal and a tangent to the curve at a point 75 from O.
- 5.74 Draw half convolution of an Archimedean spiral, with a minimum radius of 25 and radial increment of 6 for each  $30^\circ$ .
- 5.75 Draw two convolutions of an Archimedean spiral, given the maximum radius as 100 and the minimum radius as 28. Draw a tangent and a normal to the curve at a point 45 from the pole.
- 5.76 A circular disc of diameter AB equal to 80, rotates about its centre with uniform angular velocity. During one complete revolution, a point P moves along the diameter AB from A to B. Draw the locus of the point P.

- 5.77 Construct a logarithmic spiral of one convolution, given the shortest distance as 25 and the ratio of the lengths of adjacent radii enclosing  $30^\circ$  as 9: 10. Draw a normal and a tangent to the curve at a point 40 from the pole.
- 5.78 Draw a logarithmic spiral of one convolution, the successive radii being of the ratio 9:8, and final radius vector is 90. The angle between the successive radii is  $30^\circ$ . Draw a tangent at any point on the curve.
- 5.79 Draw one convolution of a logarithmic spiral, given the shortest radius as 30 and the ratio of lengths of radius vectors enclosing an angle of  $30^\circ$  being 9:8.
- 5.80 In a logarithmic spiral, the shortest radius vector is 25. The ratio of lengths of successive radius vectors enclosing  $30^\circ$  is 9:8. Construct one convolution of the spiral. Find the value of the angle, which the tangent at any point makes with the radius vector.

## REVIEW QUESTIONS

- 5.1 Define the terms (i) ellipse, (ii) parabola, (iii) hyperbola, (iv) cycloid, (v) involute and (vi) spiral.
- 5.2 What are the asymptotes to a hyperbola?
- 5.3 What is a rectangular hyperbola?
- 5.4 Define the following terms, with respect to the cycloidal curves: (i) Generating circle, (ii) directing circle and (iii) directing line.
- 5.5 Differentiate between (i) epi-and hypo-cycloids and (ii) epi- and hypo-trochoids.

- 5.6 What is the difference between a spiral curve and an Archimedean spiral curve?
- 5.7 Define the term, "logarithmic spiral".
- 5.8 Differentiate between the logarithmic spiral and hyperbolic spiral.

## OBJECTIVE QUESTIONS

- 5.1 Name the solids of revolution.
- 5.2 When a cone is cut by planes at different angles, the intersection curves obtained are known as \_\_\_\_\_.
- 5.3 Intersection curve between a sphere and any section plane is always a circle.  
(True/False)
- 5.4 Section plane, cutting all the generators of a cylinder and making an angle with the axis other than  $90^\circ$  produces \_\_\_\_\_.
- 5.5 A cone with an apex angle  $2\theta$  is cut by a cutting plane at an angle  $\alpha$ .
  - (i) When  $\alpha$  is greater than  $\theta$ , the intersection curve is \_\_\_\_\_.
  - (ii) When  $\alpha$  is equal to  $\theta$ , the intersection curve is \_\_\_\_\_.
  - (iii) When  $\alpha$  is less than  $\theta$ , the intersection curve is \_\_\_\_\_.
  - (iv) When  $\alpha = 90^\circ$ , the intersection curve is \_\_\_\_\_.
- 5.6 When a cone is cut by a section plane, making the same angle with the axis, as do the generators, the

intersection curve is \_\_\_\_\_.

- 5.7 When the conic section of a cone is elliptical, the section plane need not cut all the generators.

(True/False)

- 5.8 When a cone is cut by a section plane that is parallel to the axis and passing through it, the intersection curve is \_\_\_\_\_.

- 5.9 When a cone is cut by a section plane that is parallel to the axis and away from it, the intersection curve is \_\_\_\_\_.

- 5.10 In a conic, the line passing through the focus and perpendicular to the directrix is called the \_\_\_\_\_.

- 5.11 On a conic, the vertex is the point at which the \_\_\_\_\_ cuts the \_\_\_\_\_.

- 5.12 The distance of the ends of the major axis of an ellipse from the centre is equal to half the minor axis.

(True/False)

- 5.13 The difference of the focal distances from any point on an ellipse is constant.

(True /False)

- 5.14 In a \_\_\_\_\_, the product of distances of any point on the curve, from two fixed lines at right angle, is always constant. The fixed lines are called \_\_\_\_\_.

- 5.15 \_\_\_\_\_ are the curves generated by a fixed point on the circumference of a rolling circle.

- 5.16 The size of the cycloidal curve is the same, irrespective of the size of the generating circle.

(True /False)

- 5.17 The curve generated by a point on the circumference of a rolling circle, rolling along another circle outside it, is called a \_\_\_\_\_.
- 5.18 \_\_\_\_\_ is a curve generated by a point which is situated either inside or outside the generating circle, when it rolls along a straight line or a circle.
- 5.19 The curve generated by a point, fixed to a circle, but inside its circumference, as it rolls along a straight line, is called \_\_\_\_\_ \_\_\_\_\_.
- 5.20 The curve generated by a point, fixed to a circle, but outside its circumference, as it rolls along a circle inside it, is called \_\_\_\_\_ \_\_\_\_\_.
- 5.21 The curve generated by a point, fixed to a circle, but inside its circumference, as it rolls along a circle outside it, is called \_\_\_\_\_ \_\_\_\_\_.
- 5.22 The curve traced by a point on a straight line, when it rolls without slipping, along a circle or a polygon is called \_\_\_\_\_.
- 5.23 The curve traced by a point moving in one direction along a rotating line, is called a \_\_\_\_\_.
- 5.24 Archimedian spiral is a curve traced by a point moving with \_\_\_\_\_ velocity along a line, which is also rotating with \_\_\_\_\_ velocity.
- 5.25 In Archimedian spiral, the difference in lengths between any two radii divided by the angle between the radius vectors, is known as \_\_\_\_\_.
- 5.26 In logarithmic spiral, the \_\_\_\_\_ of lengths of consecutive radii, enclosing equal angles is a constant.

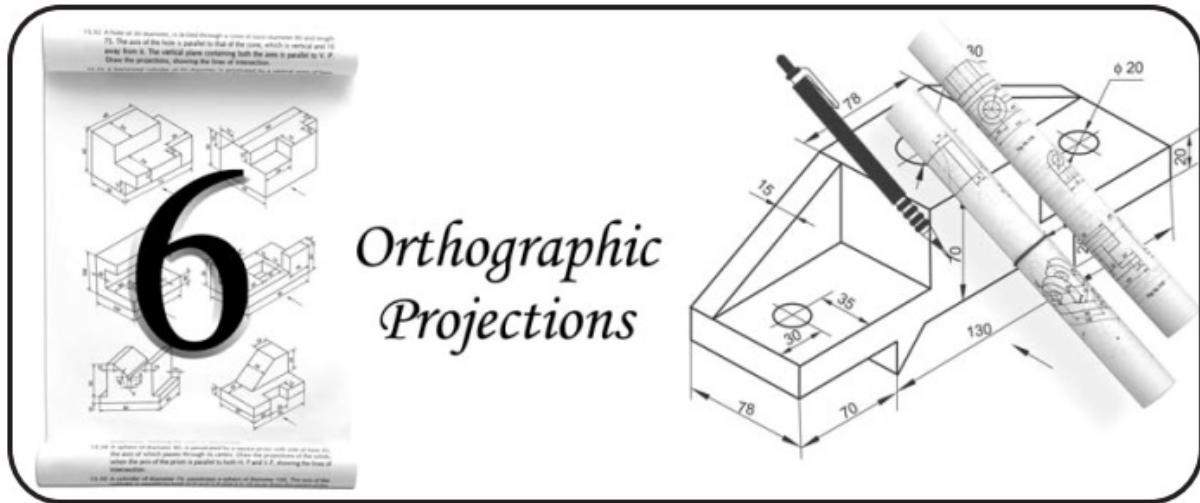
# ANSWERS

- 5.1 Sphere, cylinder and cone
- 5.2 conics
- 5.3 True
- 5.4 ellipse
- 5.5 (i) ellipse, (ii) parabola, (iii) hyperbola, (iv) circle
- 5.6 parabola
- 5.7 True
- 5.8 isosceles triangle
- 5.9 rectangular hyperbola
- 5.10 axis
- 5.11 curve, axis
- 5.12 False
- 5.13 False
- 5.14 rectangular hyperbola, asymptotes
- 5.15 Cycloids
- 5.16 False
- 5.17 epi-cycloid
- 5.18 Trochoid
- 5.19 inferior trochoid
- 5.20 superior hypo-trochoid
- 5.21 inferior epi-trochoid
- 5.22 involute
- 5.23 spiral
- 5.24 uniform, uniform

5.25 constant of the curve

5.26 ratio

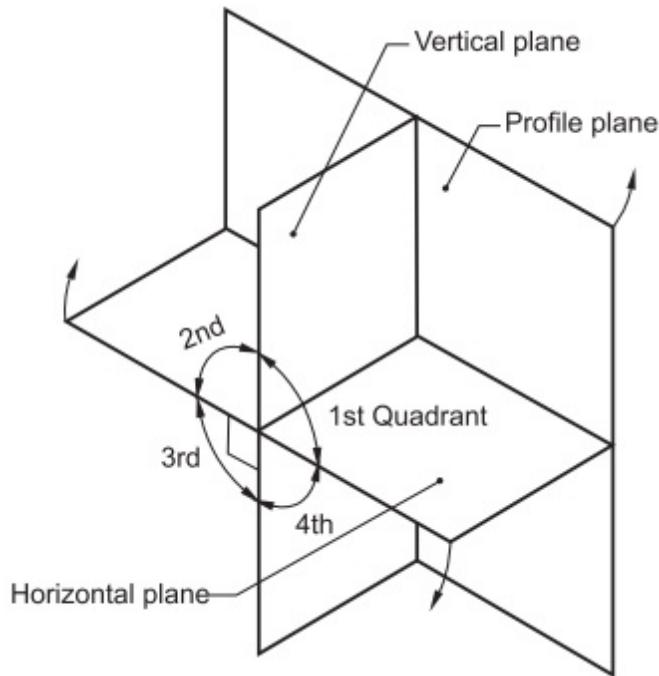
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## 6.1 INTRODUCTION

Any object has three dimensions, viz., length, width and thickness. A projection is defined as a representation of an object on a two dimensional plane. The following are the elements to be considered while obtaining a projection:

1. The object,
2. The plane of projection,
3. The point of sight, and
4. The rays of sight.



**Fig.6.1 Principal planes of projection**

A projection may be obtained by viewing the object from the point of sight and tracing in correct sequence, the points of intersection between the rays of sight and the plane to which the object is projected. A projection is called an orthographic projection, when the point of sight is imagined to be located at infinity so that the rays of sight are parallel to each other and intersect the plane of projection at right angle to it.

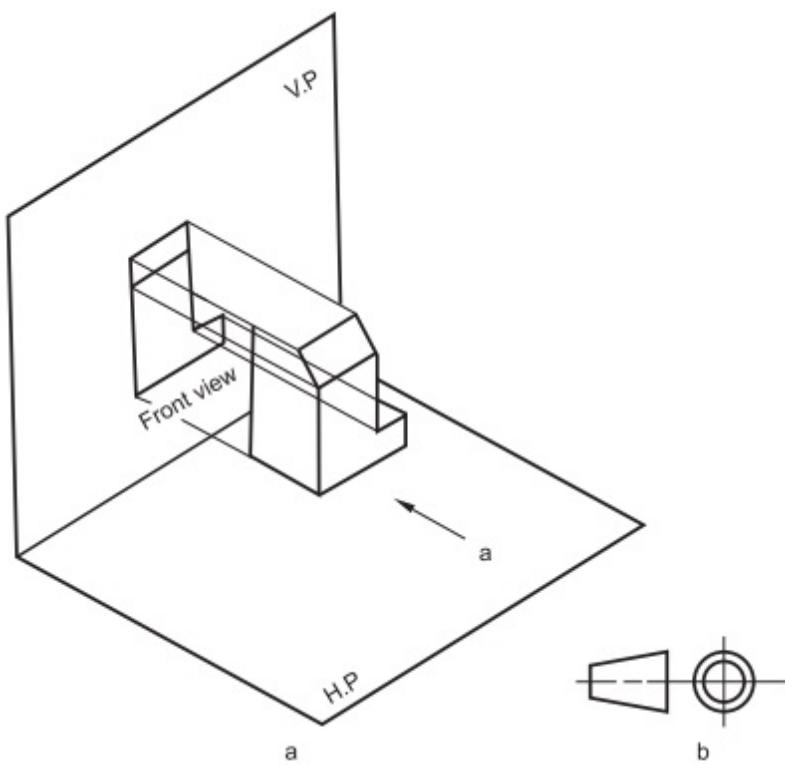
The principles of orthographic projection may be followed in four different angles or systems: First, second, third and fourth angle projections. [Figure 6.1](#) shows the three planes used in orthographic projections in which all the four quadrants are marked. A projection is said to be the first, second, third or fourth angle when the object is imagined to be either in the first, second, third or fourth quadrant respectively. However, only two systems, viz., the first and third angle projections are being followed.

The Bureau of Indian Standards though recommends first angle projection, this chapter deals with the principles involved in both the systems of projection so that the student is in a better position to adopt to any system in his career that is demanded of him.

## **6.2 PRINCIPLES OF THE TWO SYSTEMS OF PROJECTION**

### **6.2.1 First Angle Projection**

In first angle projection, the object is imagined to be positioned in the first quadrant. The front view of the object is obtained by looking at the object from the right side of the quadrant and tracing, in correct sequence, the points of intersection between the projection plane and the lines of sight extended. In this case, the object will be inbetween the observer and the plane of projection (V.P). Here, the object is imagined to be transparent and the projection lines are extended from various points of the



**Fig.6.2 Method of obtaining front view-First angle projection**

object to intersect the plane of projection. Thus, in first angle projection, any view is so placed that it represents the side of the object away from it.

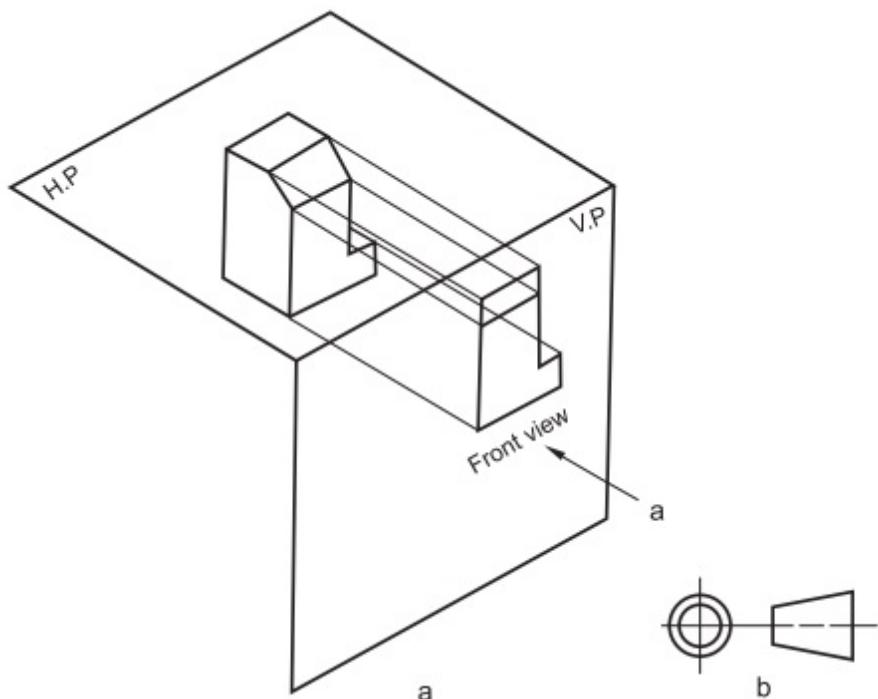
Figure 6.2a shows the method of obtaining the front view of an object in first-angle projection and Fig.6.2b, the symbol used for first-angle projection.

## 6.2.2 Third Angle Projection

In third angle projection, the object is imagined to be positioned in the third quadrant. The front view of the object is obtained by looking at the object from the right side of the quadrant and tracing in correct sequence, the points of intersection between the projection plane and the

lines of sight reaching the object. In this case, the plane of projection (V.P) is in-between the object and observer and so, the projection plane is imagined to be transparent and the rays of sight pass through it and reach the object. Thus, in third angle projection, any view is so placed that it represents the side of the object nearer to it.

[Figure 6.3a](#) shows the method of obtaining the front view of an object in third angle projection and [Fig.6.3b](#), the symbol used for third angle projection.



**Fig.6.3 Method of obtaining front view - Third angle projection**

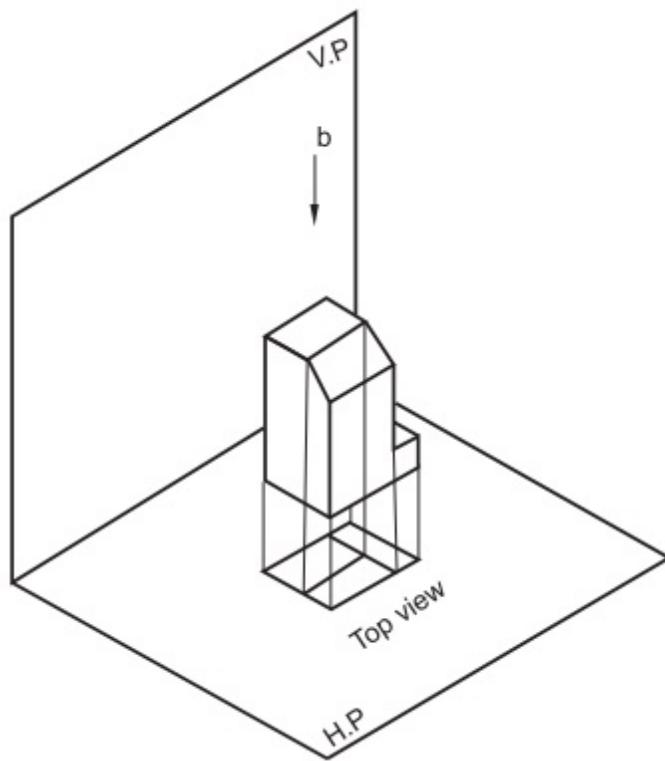
## 6.3 METHODS OF OBTAINING ORTHOGRAPHIC VIEWS

### 6.3.1 Front View

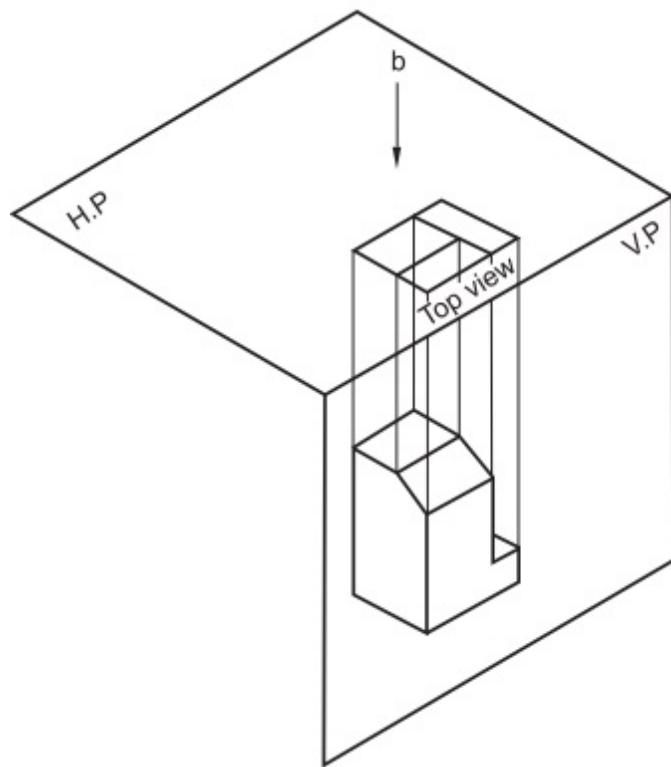
The front view of an object is defined as the view that is obtained as projection on the vertical plane, by looking the object normal to its front surface. It is the usual practice to position the object such that, its front view reveals most of the important features. [Figures 6.2](#) and [6.3](#) reveal the methods of obtaining the front view in first and third angle projections respectively.

### 6.3.2 Top View

The top view of an object is defined as the view that is obtained as projection on the horizontal plane, by looking the object normal to its top surface. [Figure 6.4](#) shows the method of obtaining the top view of the object considered above, in first angle projection. [Figure 6.5](#) shows the method of obtaining the top view of the same object in third angle projection.

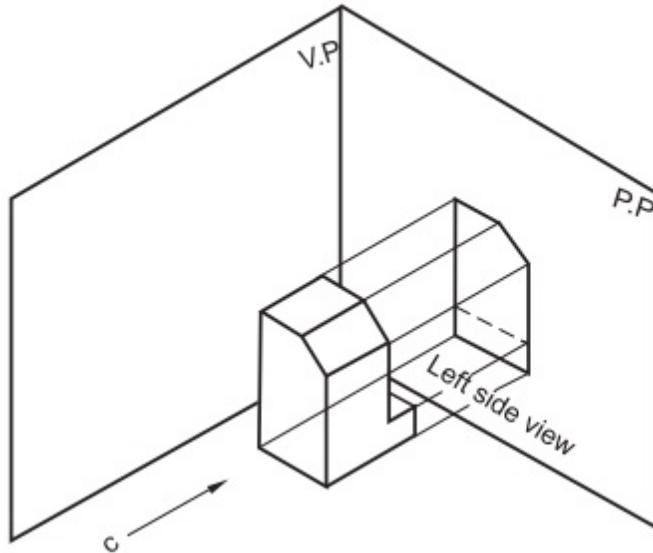


**Fig.6.4 Method of obtaining top view -First angle projection**



**Fig.6.5 Method of obtaining top view -Third angle projection**

### 6.3.3 Side View

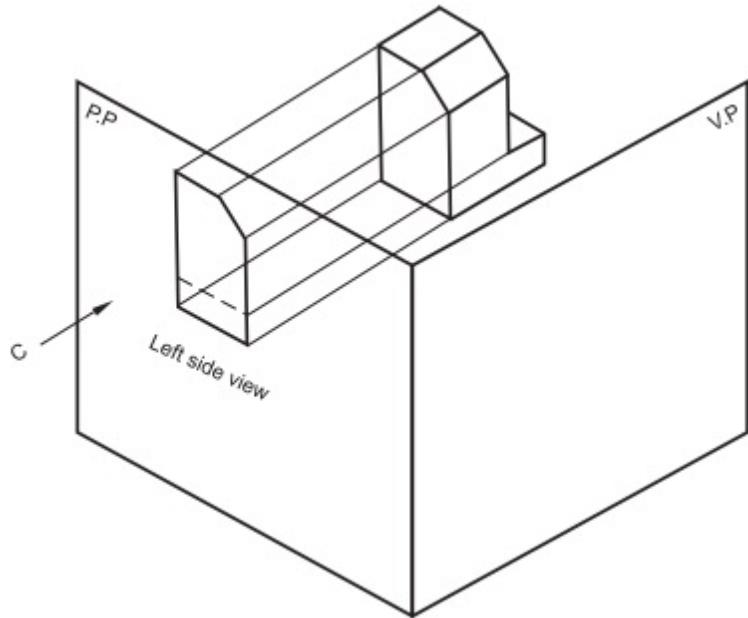


**Fig.6.6 Method of obtaining left side view - First angle projection**

The side view of an object is defined as the view that is obtained as projection on the profile plane, by looking the object, normal to its side surface. As there are two sides for an object, viz., left side and right side; two possible side views, viz., the left side and right side views may be obtained for any object.

In the first angle projection, a left side view is obtained on a profile plane by placing it to the right side of the object. [Figure 6.6](#) shows the method of obtaining a left side view of an object in first angle projection.

[Figure 6.7](#) shows the method of obtaining a left side view in third angle projection. It may be noted that in third angle projection, a left side view is obtained by placing the profile plane to the left side of the object.



**Fig.6.7 Method of obtaining left side view - Third angle projection**

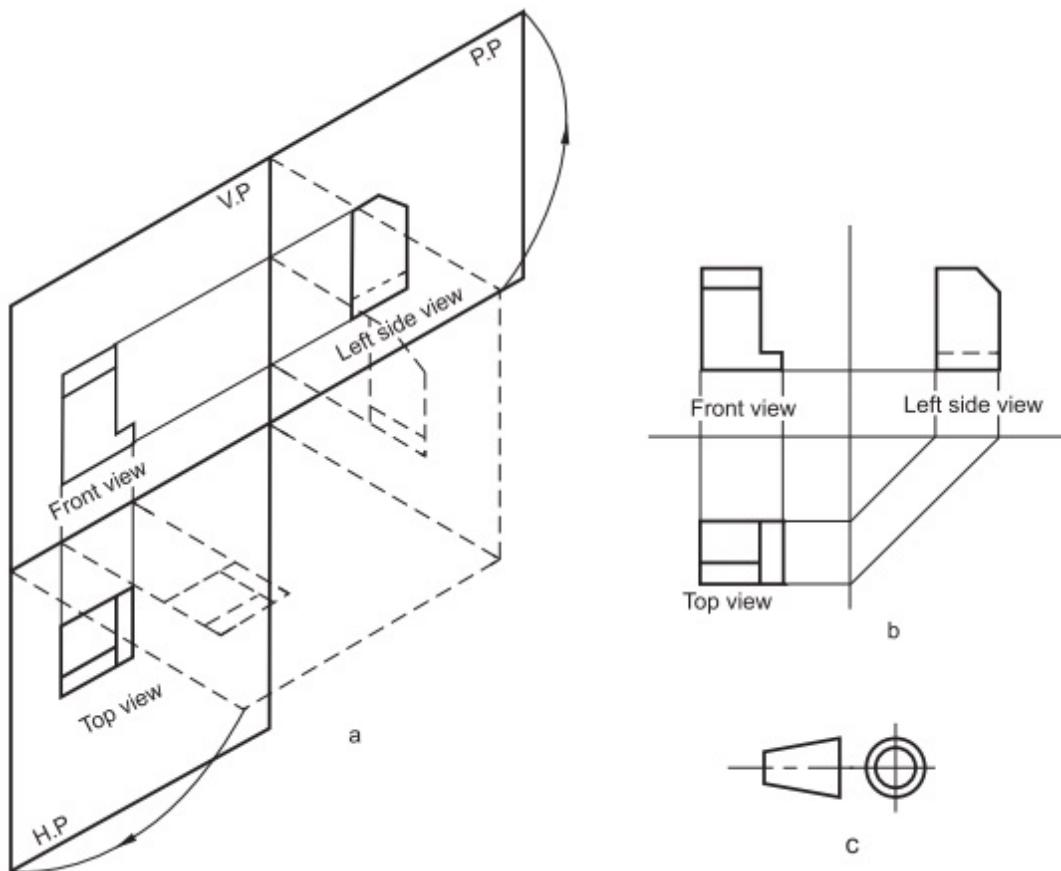
## 6.4 PRESENTATION OF VIEWS

The different views of an object are presented on a drawing sheet which is a two dimensional one, to reveal all the three dimensions of the object. For this, the horizontal and profile planes are rotated till they lie in-line with the vertical plane. In the first and third angle projections, the first and third quadrants are respectively opened for presentation of the views.

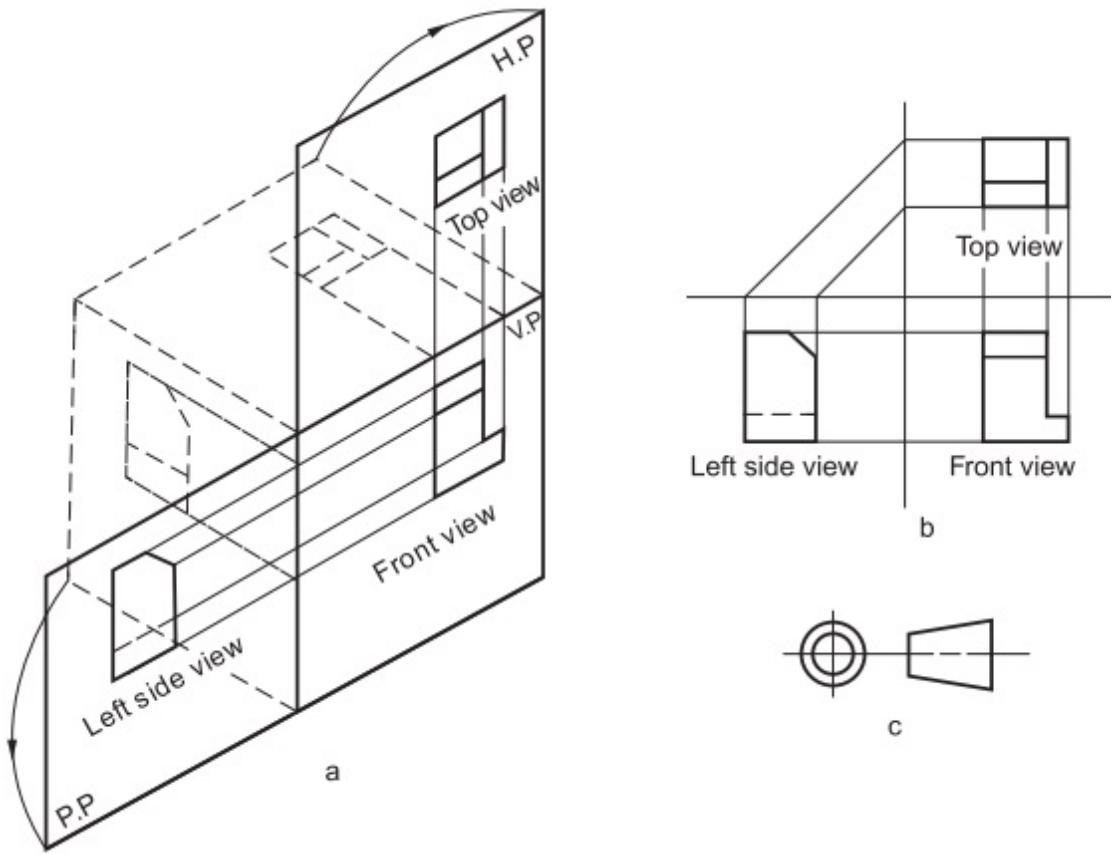
Figures 6.8 and 6.9 show the relative positions of the views, viz., the front, top and left side views in the first and third angle projections respectively.

## 6.5 DESIGNATION AND RELATIVE POSITIONS OF VIEWS

An object positioned in space may be imagined as surrounded by six mutually perpendicular planes. So, for any object, six different views may be obtained by viewing at it along the six directions, normal to these planes. [Figure 6.10](#) shows an object with six possible directions to obtain the different views, which are designated as follows:

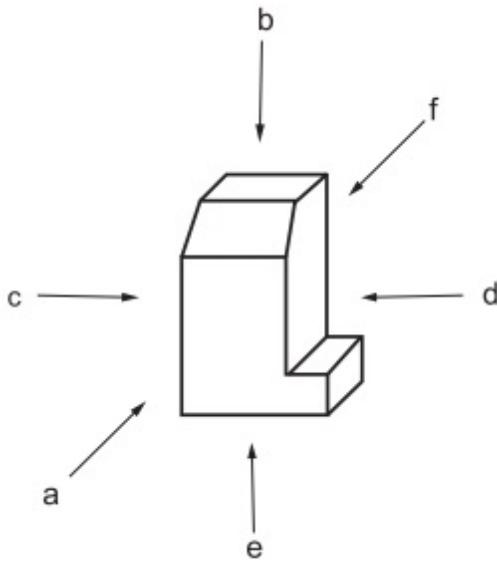


**Fig.6.8 Presentation of views - First angle projection**



**Fig.6.9 Presentation of views - Third angle projection**

1. View in the direction **a** = front view
2. View in the direction **b** = top view
3. View in the direction **c** = left side view
4. View in the direction **d** = right side view
5. View in the direction **e** = bottom view
6. View in the direction **f** = rear view

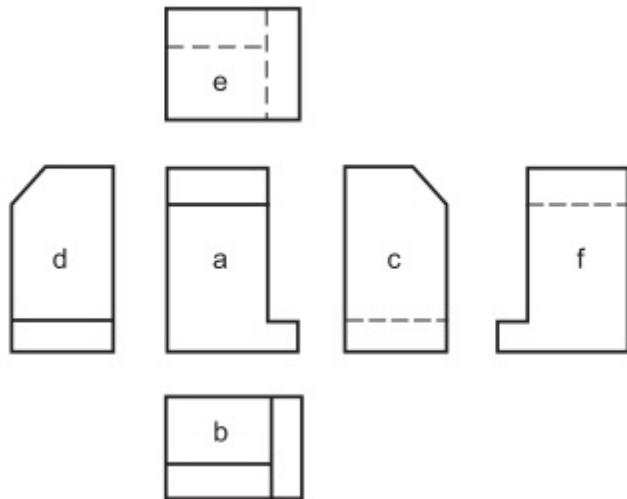


**Fig.6.10 Directions to obtain various views**

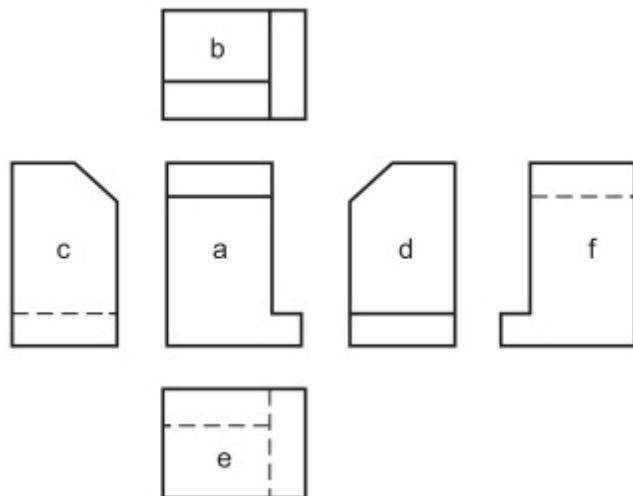
Figure 6.11 shows the relative positions of the above six views in first angle projection and Fig. 6.12, the same in third angle projection. In either method, it may be noted that the rear view may be placed either on the left or on the right, as may be found convenient.



1. A comparison of Figs. 6.11 and 6.12, reveals that in both the methods of projection, the views are identical in shape and detail. Only their location with respect to the front view is different.
2. Second and fourth angles of projections are not followed in practice. This is because, in these projections, both the front and top views will lie on one side of the reference line; causing confusion in visualizing the object.



**Fig.6.11 Relative positions of views - First angle projection**



**Fig.6.12 Relative positions of views - Third angle projection**

## 6.6 POSITION OF THE OBJECT

It is important to understand the significance of the position of the object relative to the planes of projection. To get useful information about the object in orthographic

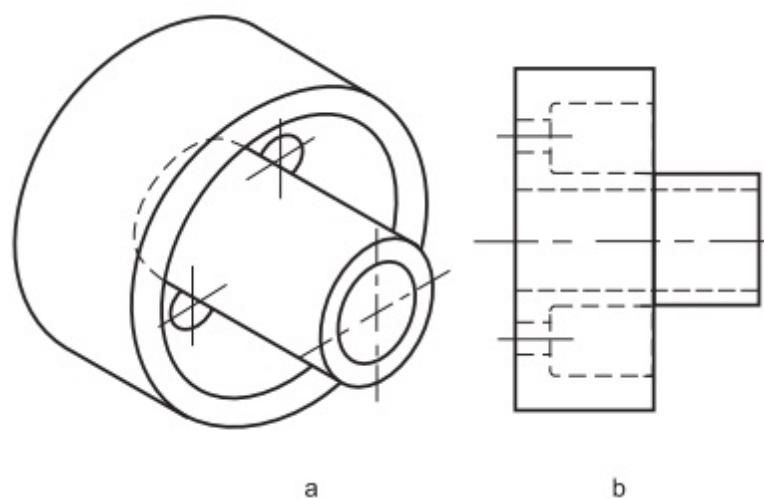
projections, the object may be imagined to be positioned properly because of the following facts:

1. Any line on an object will show its true length only when it is parallel to the plane of projection.
2. Any surface of an object will appear in its true shape only when it is parallel to the plane of projection.

In the light of the above, it is necessary that the object is imagined to be positioned such that, its principal surfaces are parallel to the planes of projection.

### 6.6.1 Hidden Lines

While obtaining the projection of an object on to any principal plane of projection, certain features of the object may not be visible. The invisible or hidden features are represented by short dashes of medium thickness. [Figure 6.13](#) shows the application of hidden lines in the projection of an object.



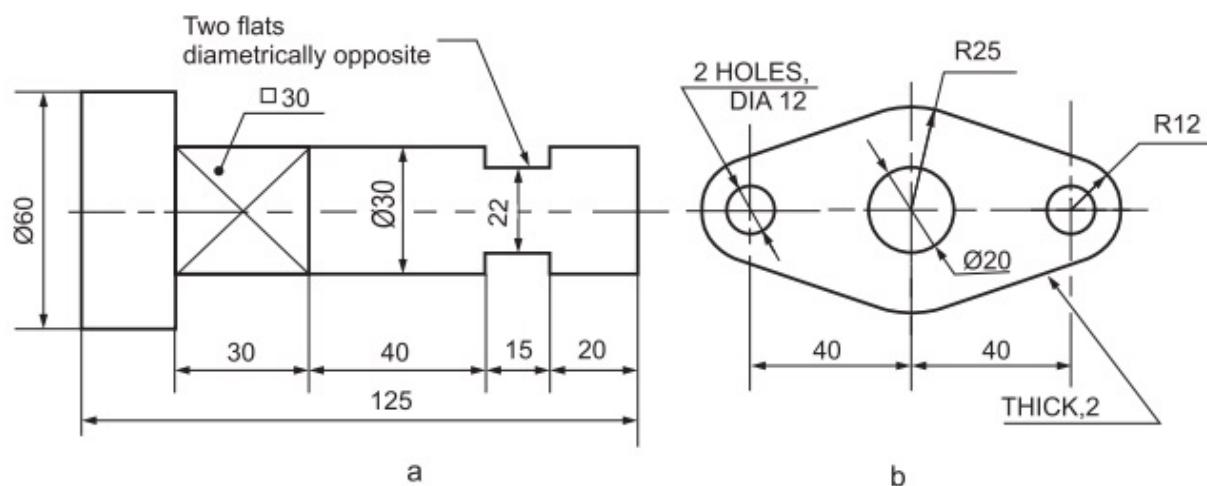
**Fig.6.13 Application of hidden lines**

## 6.7 SELECTION OF VIEWS

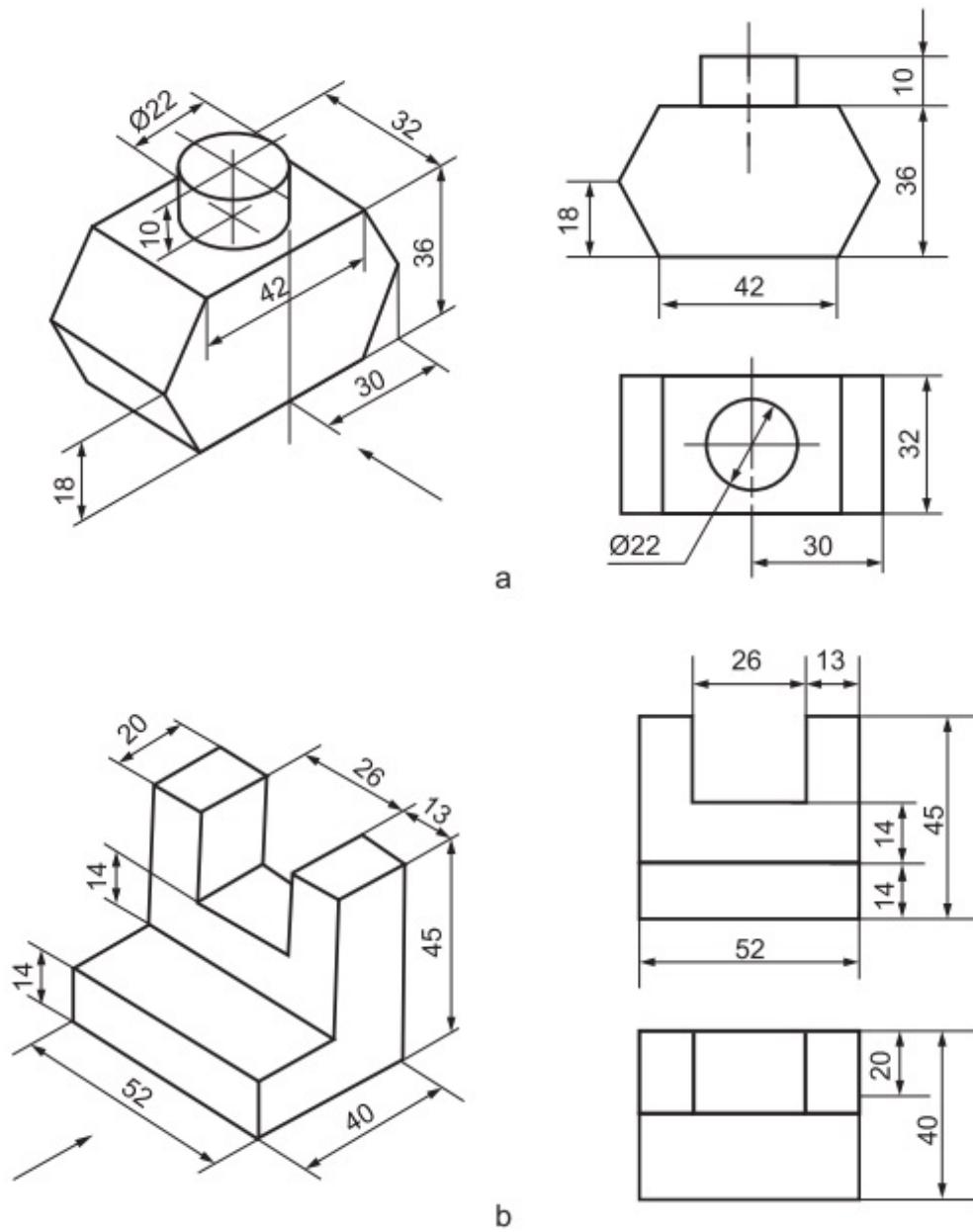
For describing an object completely through its orthographic projections, it is important to select a number of views. The number of views required to describe any object will depend upon the extent of complexity involved in it. The higher the symmetry, the lesser the number of views required.

### 6.7.1 One View Drawings

Some objects with cylindrical, square or hexagonal features or, plates of any size with any number of features in it may be represented by a single view. In such cases, the diameter of the cylinder, the side of the square, the side of the hexagon or the thickness of the plate may be expressed by a note or abbreviation. [Figure 6.14](#) shows some objects, which can be described by one view drawings.



**Fig.6.14 One view drawings**



**Fig.6.15 Two view drawings**

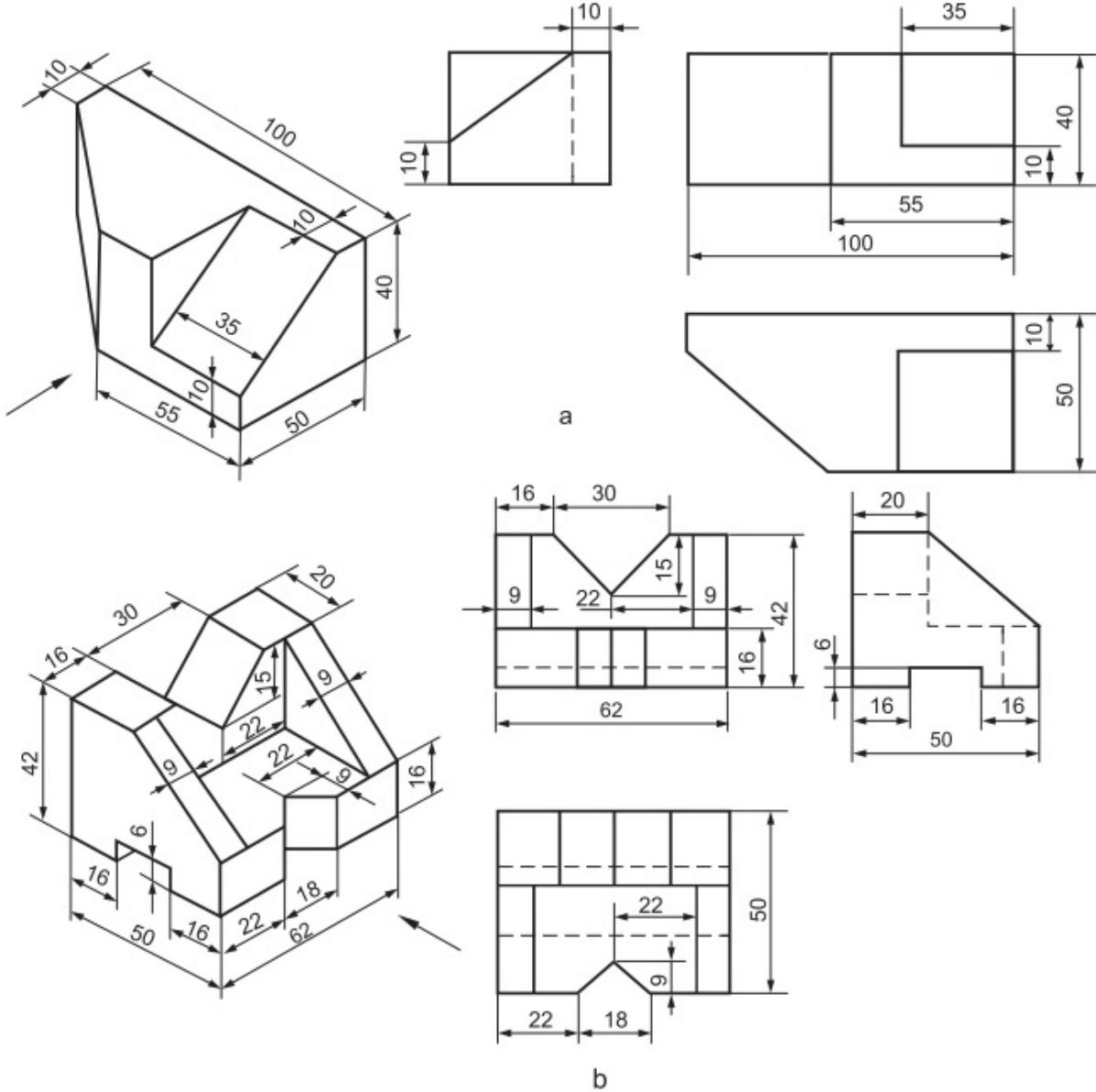
## 6.7.2 Two View Drawings

Some objects which are symmetrical about two axes may be represented completely by two views. Normally, the largest face showing most of the details of the object is selected for

drawing the front view. The shape of the object then determines whether the second view can be top view or side view. [Figure 6.15](#) shows the examples of two view drawings.

### 6.7.3 Three View Drawings

In general, most of the objects consisting of either a single component or an assembly of components are described with the help of three views. In such cases, the views normally selected are the front, top and left or right side views. [Figure 6.16](#) shows two objects and their three necessary views.



## **Fig.6.16 Three view drawings**

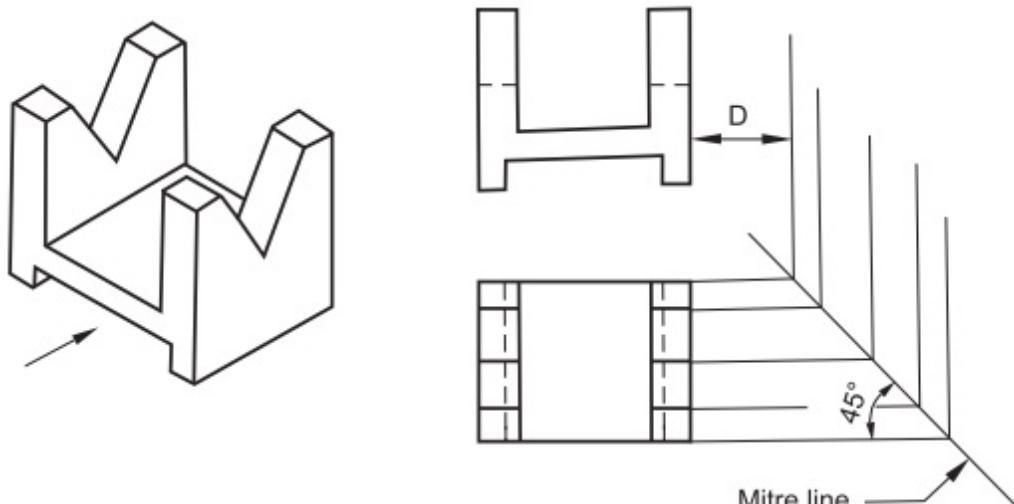
## **6.8 DEVELOPMENT OF MISSING VIEWS**

When two views of an object are given, the third view may be developed by the use of either a mitre line or arcs of circles.

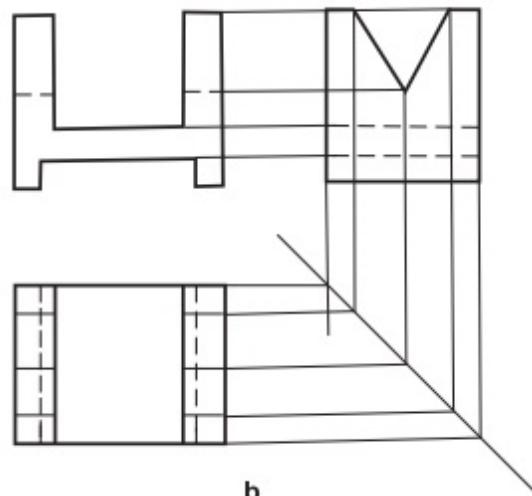
## **6.8.1 To Construct the Left Side View, from the Given Front and Top Views**

### ***Construction (Fig.6.17)***

1. Draw the given front and top views.
2. Draw the projection lines to the right of the top view.
3. Decide the distance D from the front view, at which the side view is to be drawn.
4. Construct a mitre line at  $45^\circ$  to the horizontal.
5. From the points of intersection between the mitre line and the above projection lines, draw vertical projection lines.



a



b

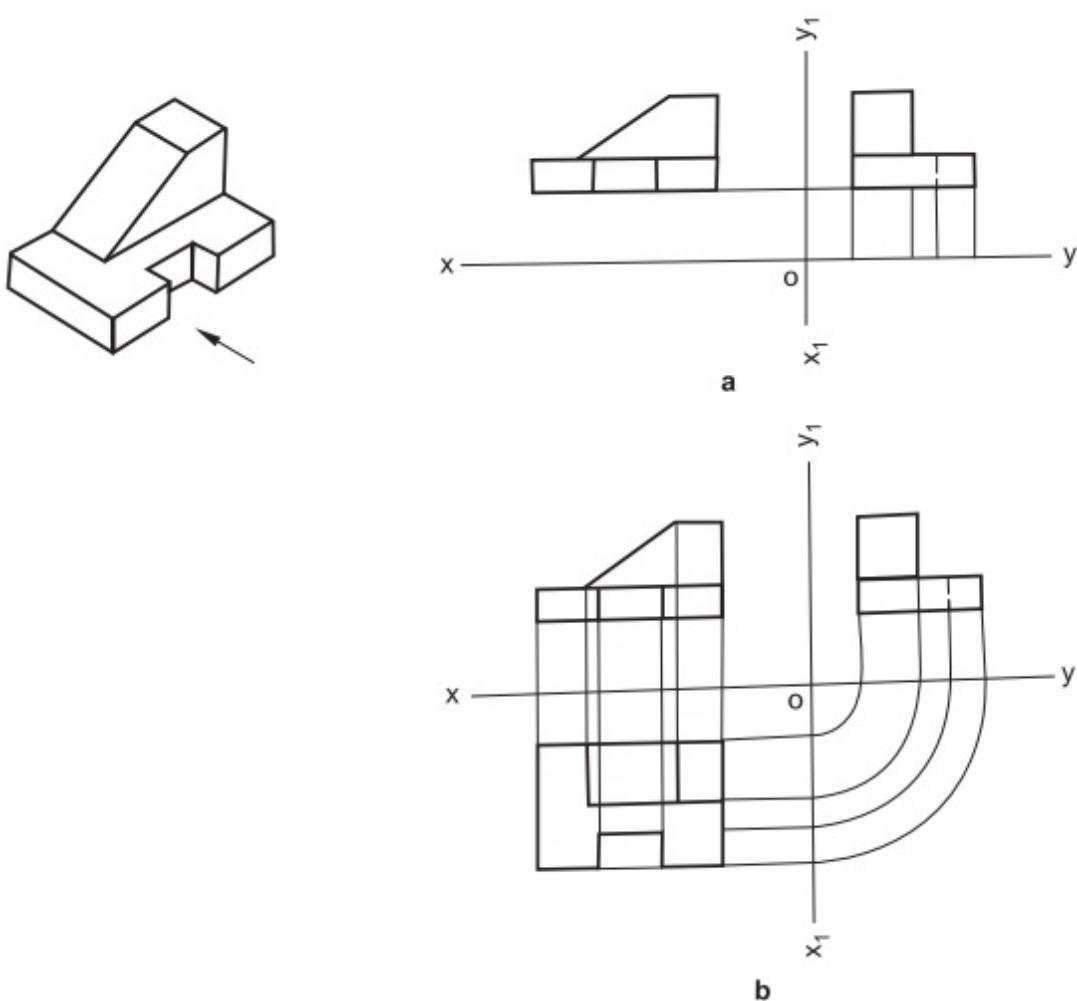
**Fig.6.17 Development of the side view**

6. Draw the horizontal projection lines from front view to intersect the above lines. The figure obtained by joining the points of intersection in the order is the required side view.

### 6.8.2 To Construct the Top View from the Given Front and Side Views

### ***Construction (Fig.6.18)***

1. Draw the given front and left side views.
2. Draw  $x_1 y_1$  perpendicular to  $xy$ , at any convenient location and mark o at the intersection point.
3. Draw vertical projection lines from the side view to intersect the  $xy$  line.
4. Transfer these intersection points to  $x_1y_1$ , by arcs of circles, taking o as the centre.
5. Draw horizontal projection lines from these points (on  $x_1y_1$ ).
6. Draw vertical projection lines from the front view to intersect the above lines.
7. Join the corresponding points of intersection between these projectors and obtain the



**Fig.6.18 Development of the top view**



These exercises are aimed at improving the practice in reading the views and developing the imagination of the student.

## 6.9 SPACING THE VIEWS

The views of a given object must be positioned on the drawing sheet so as to give a good and balanced appearance. Keeping in view, (i) number of views, (ii) scale and (iii) space between the views, the draughtsman should

decide about the placement of views on the drawing sheet. Sufficient space between the views must be provided to facilitate placement of dimensions, notes, etc., on the drawing without overcrowding.

## 6.10 EXAMPLES



For all the examples given, the following may be noted: Figure a. Isometric projection, Fig.b. Orthographic views in first angle projection.

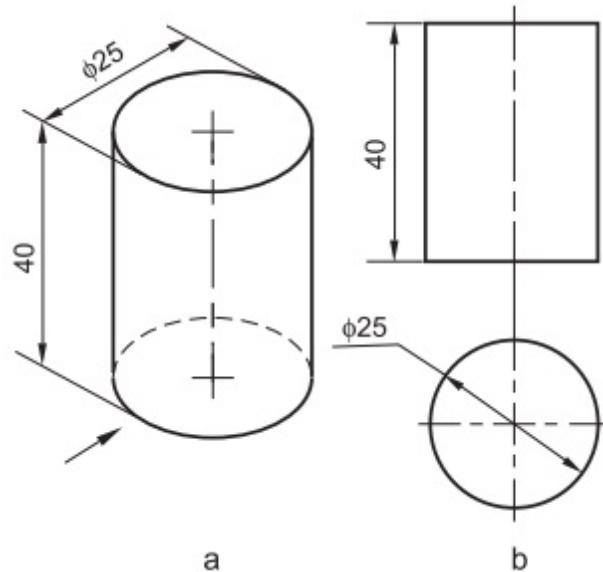
- 6.10.1 [Figure 6.19a](#) shows a cylinder. Draw its (i) front view and (ii) top view.
- 6.10.2 [Figure 6.20a](#) shows a cone. Draw its (i) front view and (ii) top view .
- 6.10.3 [Figure 6.21a](#) shows a square prism. Draw its (i) front view and (ii) top view.
- 6.10.4 [Figure 6.22a](#) shows a hexagonal prism. Draw its (i) front view and (ii) top view.
- 6.10.5 [Figure 6.23a](#) shows a pentagonal pyramid. Draw its (i) front view and (ii) top view.
- 6.10.6 [Figure 6.24a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.7 [Figures 6.25a and 6.26a](#) show certain machine blocks. Draw front and top views of the both.
- 6.10.8 [Figure 6.27a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.9 [Figures 6.28a and 6.29a](#) show certain machine blocks. Draw front, top and right side views of the both.
- 6.10.10 [Figure 6.30a](#) shows a machine block. Draw its (i)

front view, (ii) top view and (iii) right side view.

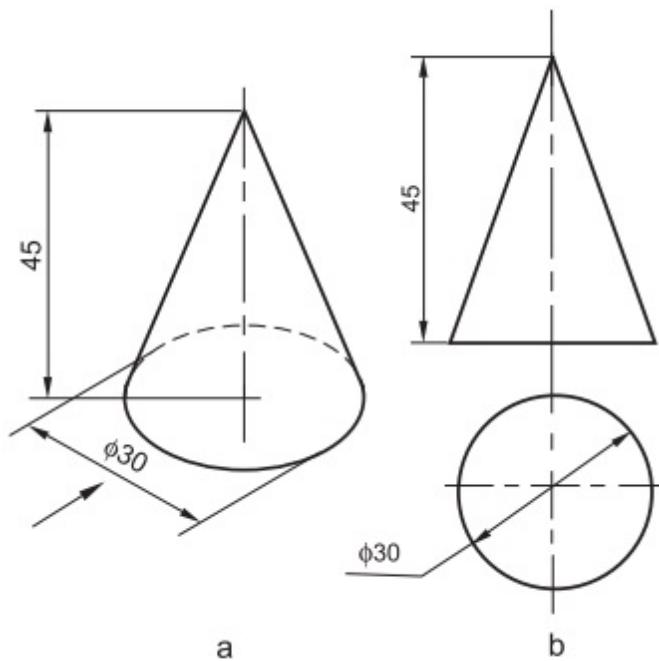
- 6.10.11 [Figure 6.31a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) both the side views.
- 6.10.12 [Figure 6.32a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) both the side views.
- 6.10.13 [Figure 6.33a](#) shows a bearing block. Draw its (i) front view and (ii) top view.
- 6.10.14 [Figure 6.34a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.15 [Figure 6.35a](#) shows a wall bracket. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.16 [Figure 6.36a](#) shows a guide block. Draw its (i) front view, (ii) top view and (iii) right side view.
- 6.10.17 Draw (i) front, (ii) top and (iii) both the side views of the object shown in [Fig. 6.37a](#).
- 6.10.18 [Figure 6.38a](#) shows the isometric projection of an object. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.19 [Figure 6.39a](#) shows a bracket. Draw its (i) front view, (ii) top view and (iii) left side view.
- 6.10.20 [Figure 6.40a](#) shows a machine block. Draw its (i) front view, (ii) top view and (iii) right side view.
- 6.10.21 [Figure 6.41](#) shows the orthographic views of some components with some line(s) omitted. Read the views and indicate the missing line(s).  
 Missing line(s) are indicated in the view shown in brackets.
- 6.10.22 [Figure 6.42](#) shows two orthographic views of some

machine components. Read the views and add the third view.

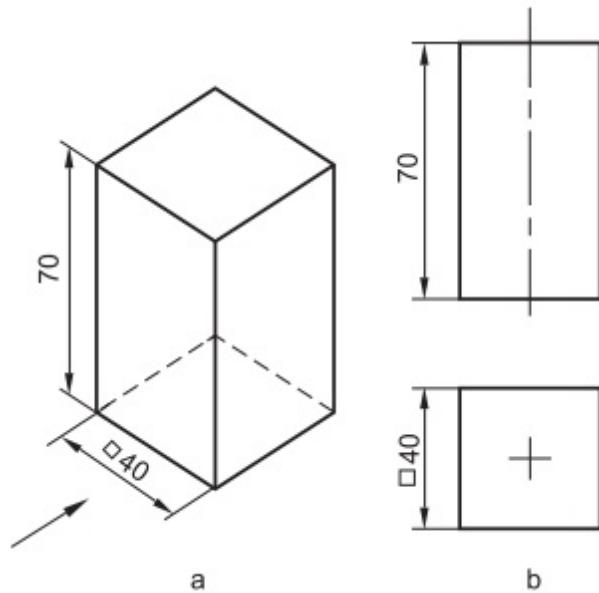
The third view is shown in brackets.



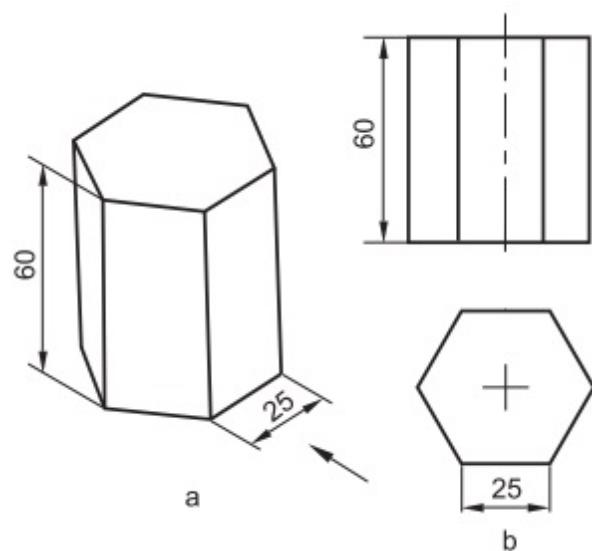
**Fig.6.19**



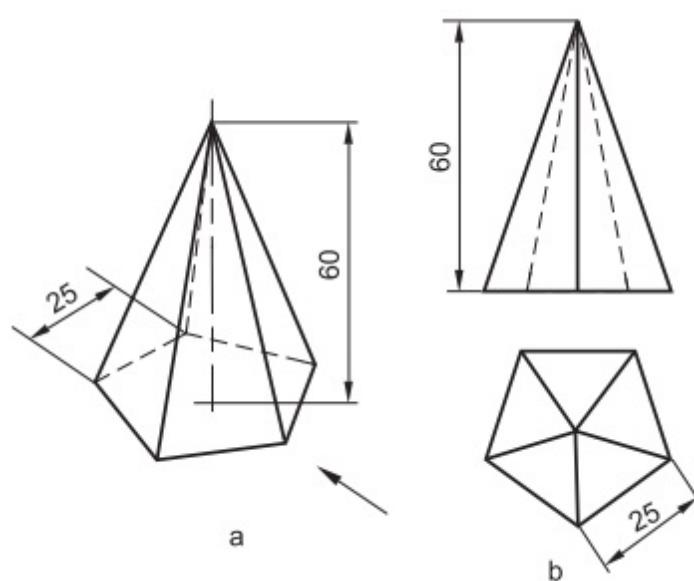
**Fig.6.20**



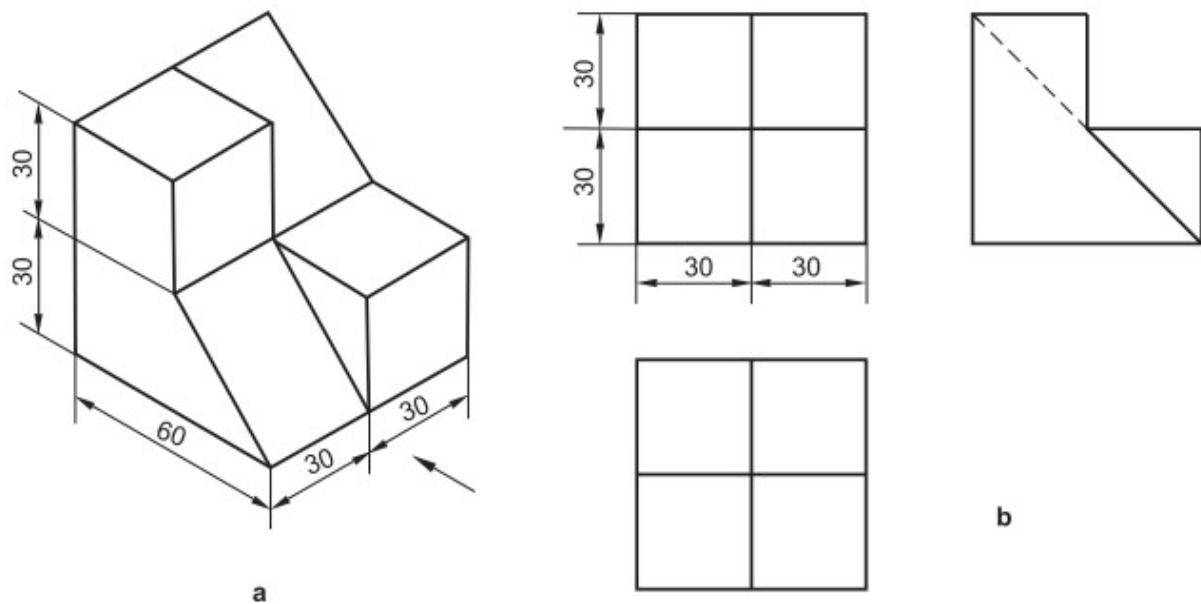
**Fig.6.21**



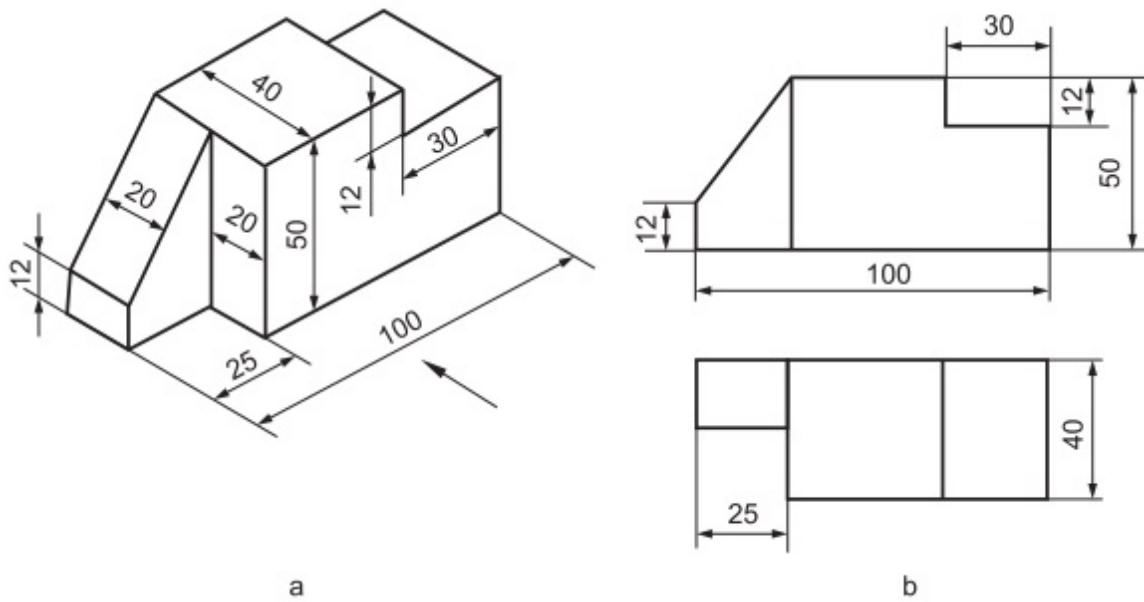
**Fig.6.22**



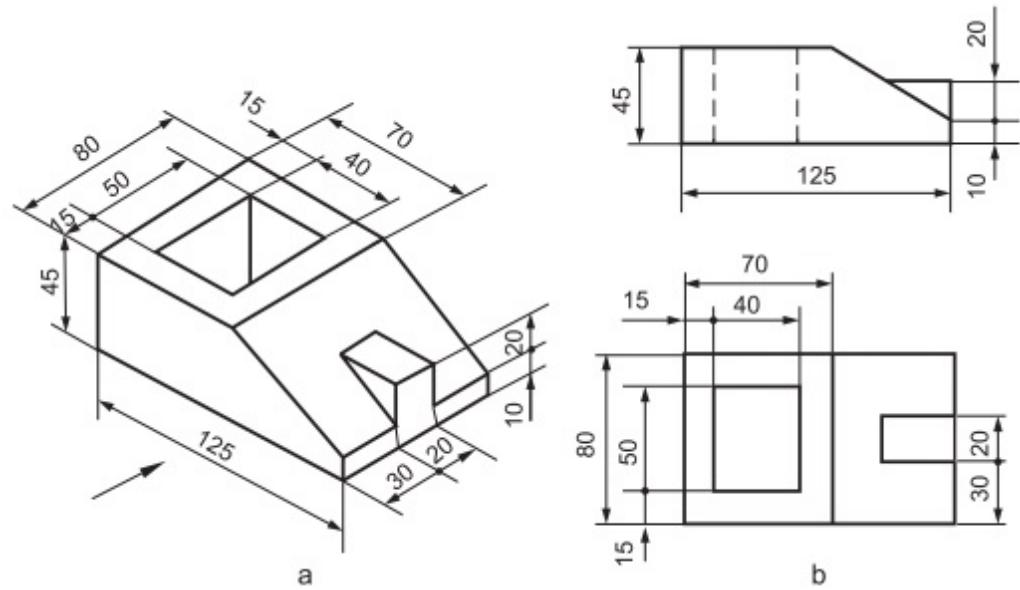
**Fig.6.23**



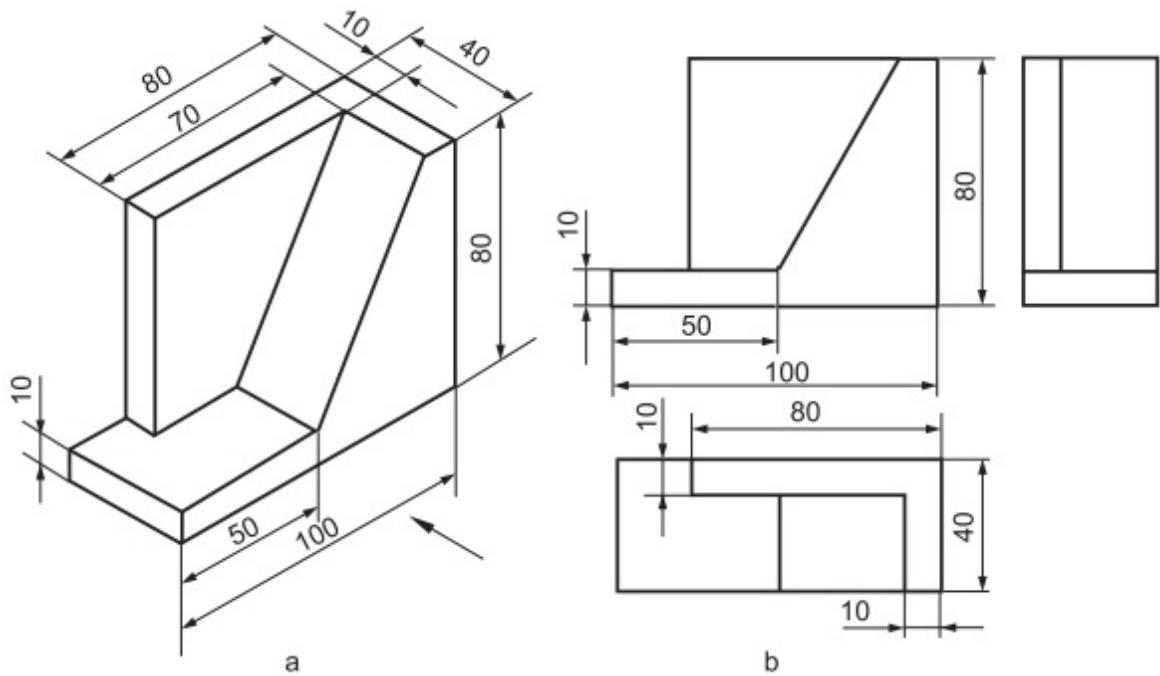
**Fig.6.24**



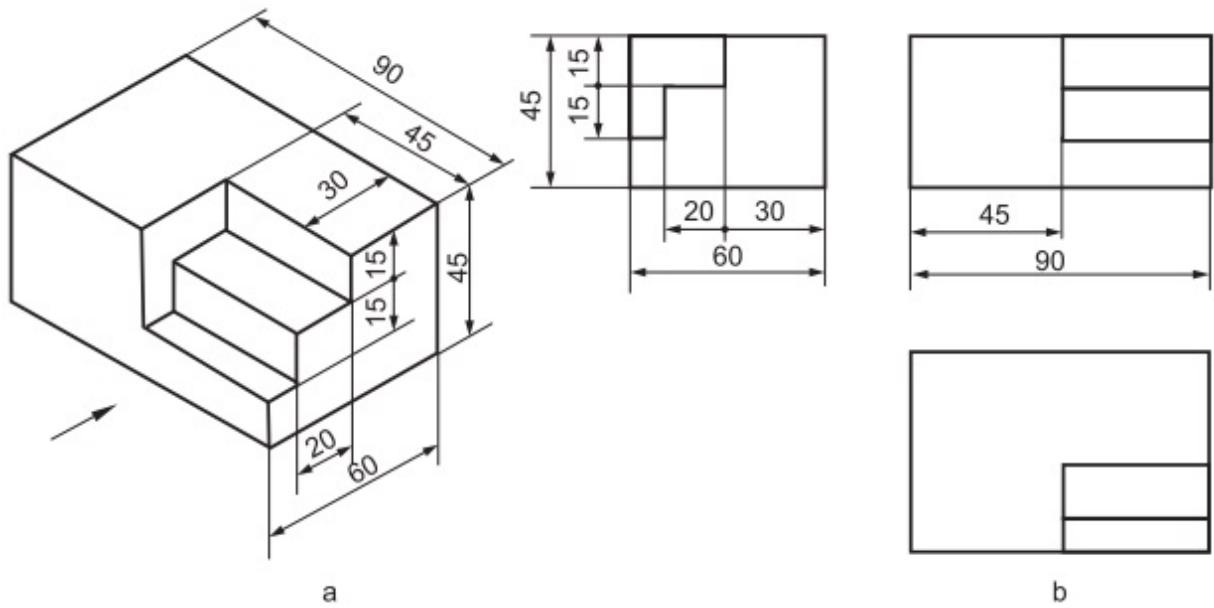
**Fig.6.25**



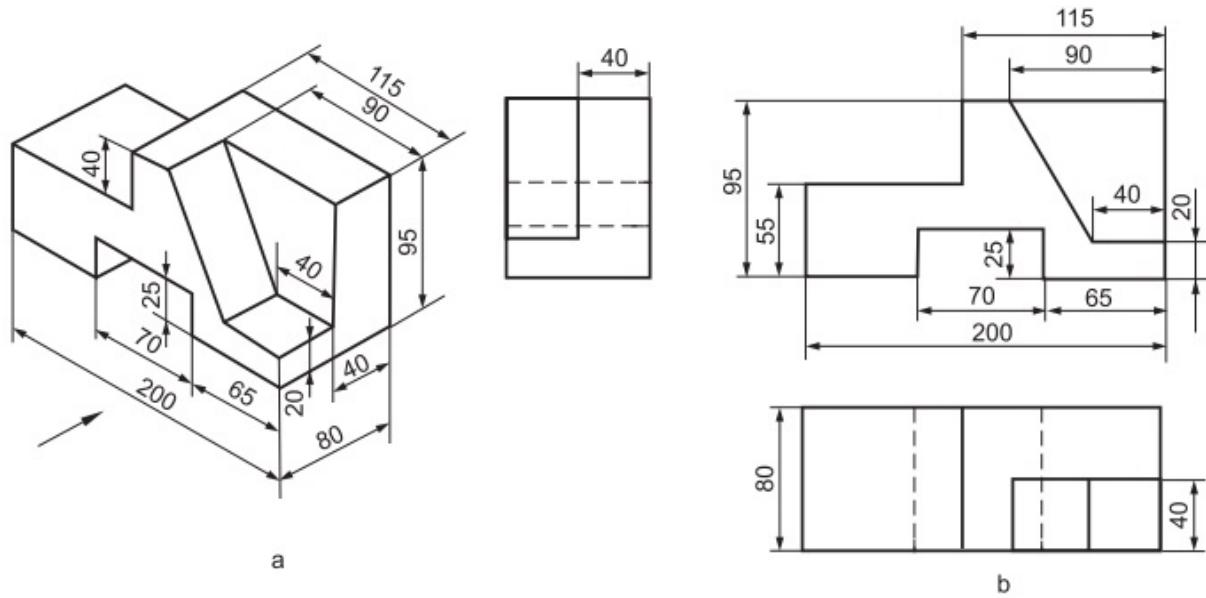
**Fig.6.26**



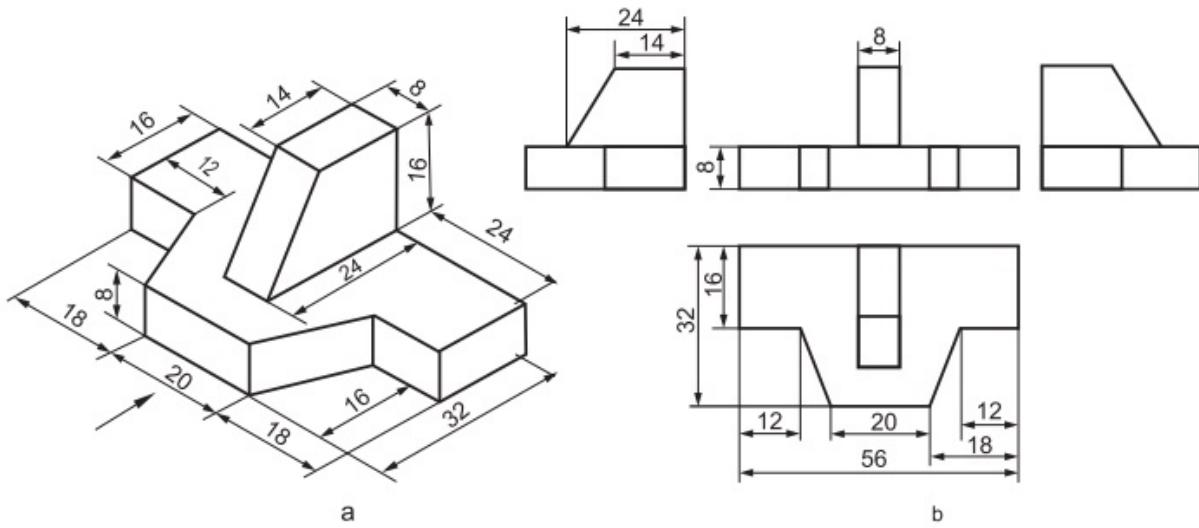
**Fig.6.27**



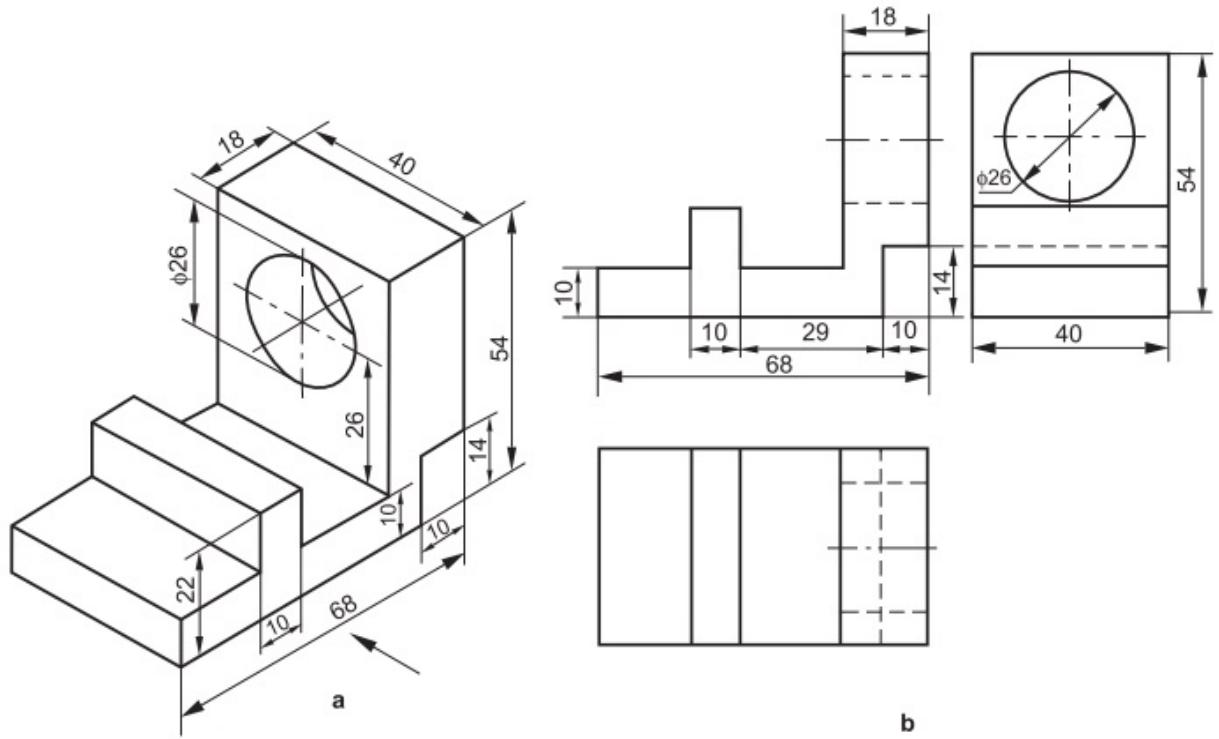
**Fig.6.28**



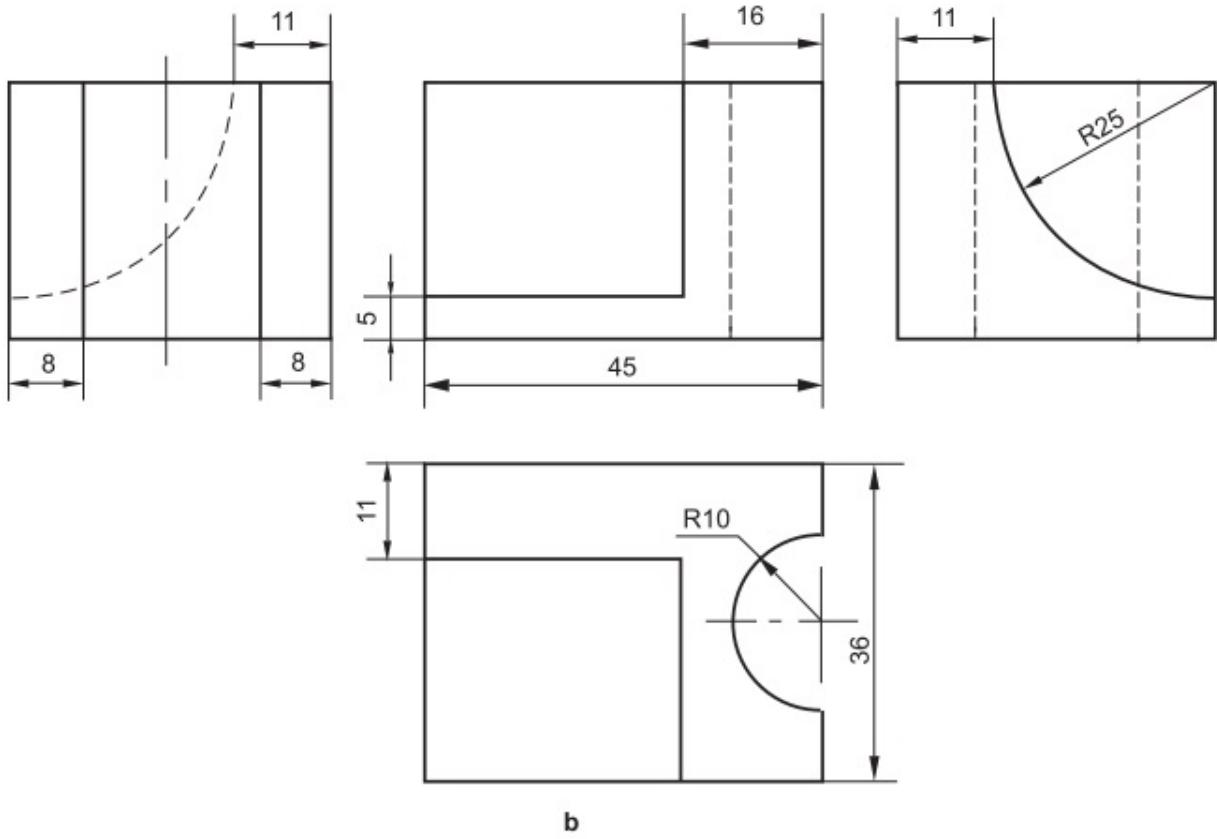
**Fig.6.29**



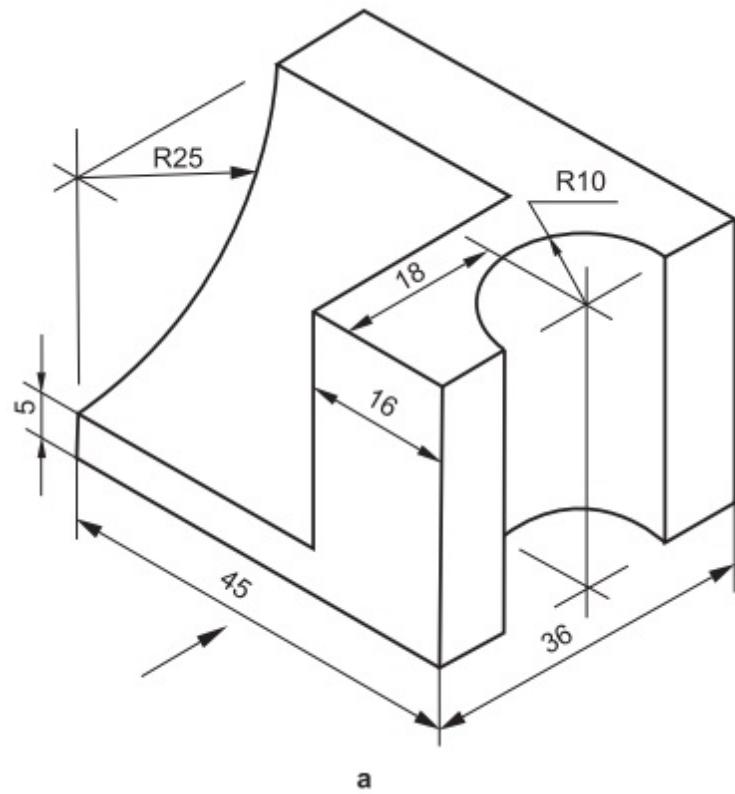
**Fig.6.30**



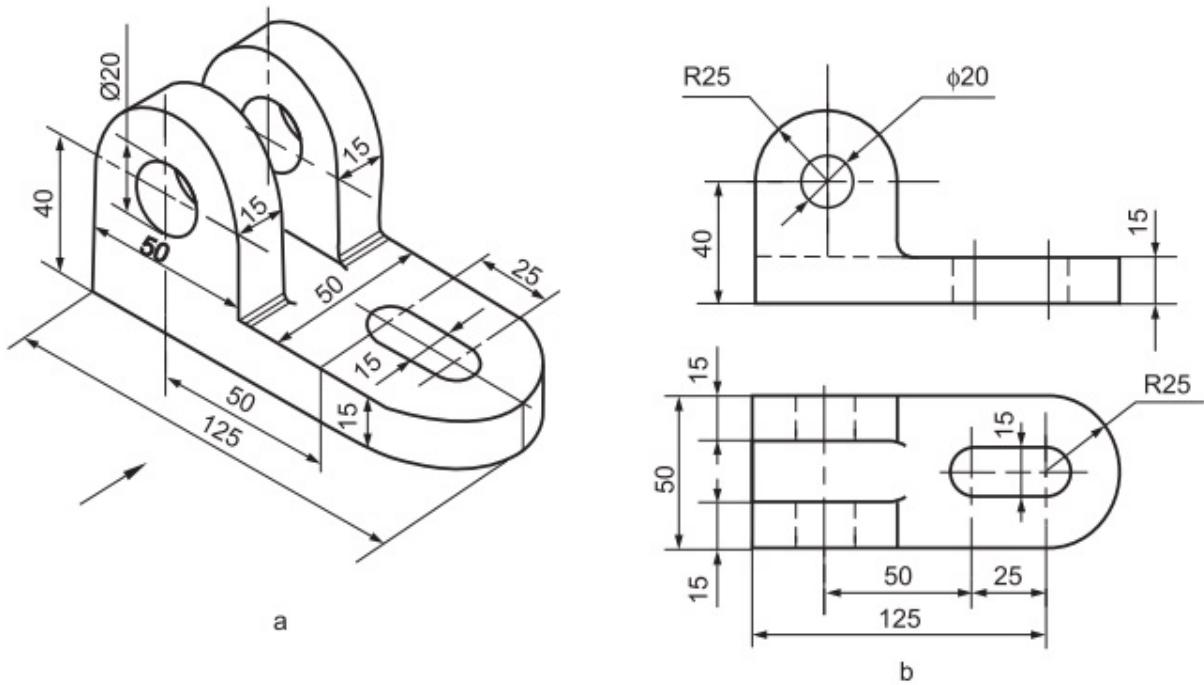
**Fig.6.31**



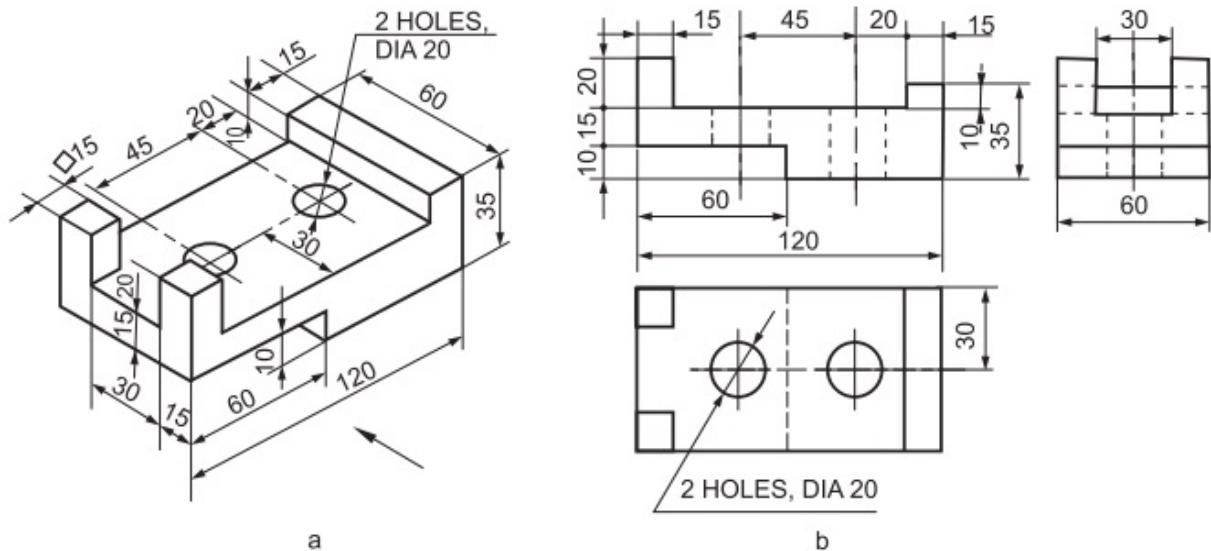
**Fig.6.32**



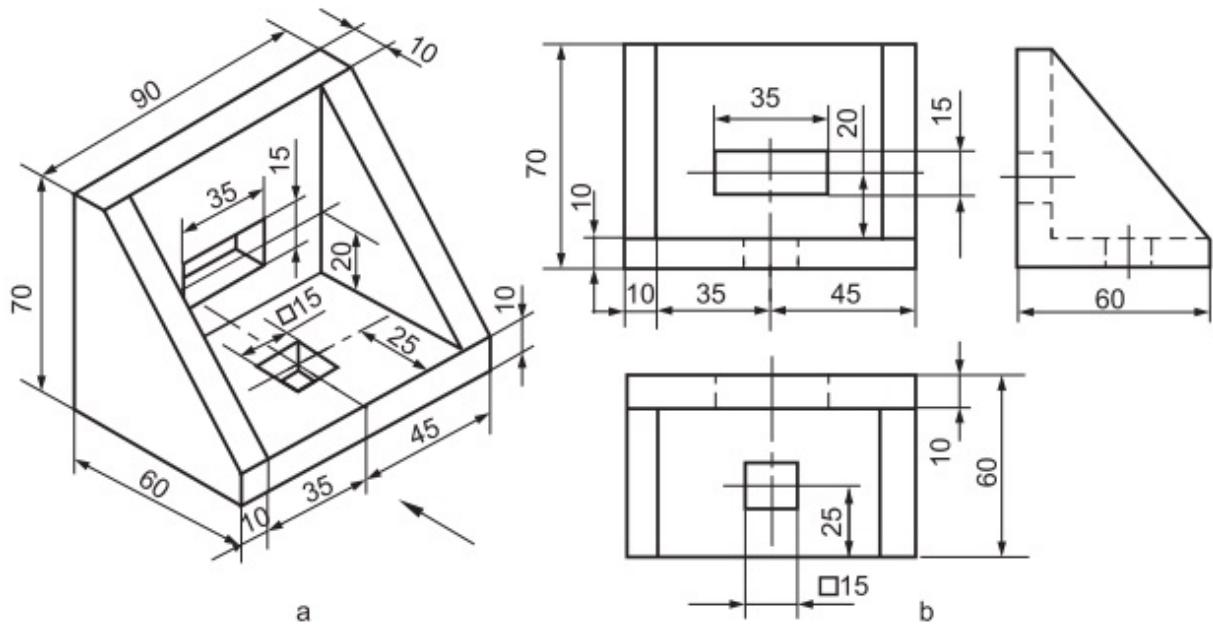
**Fig.6.32 (Contd.)**

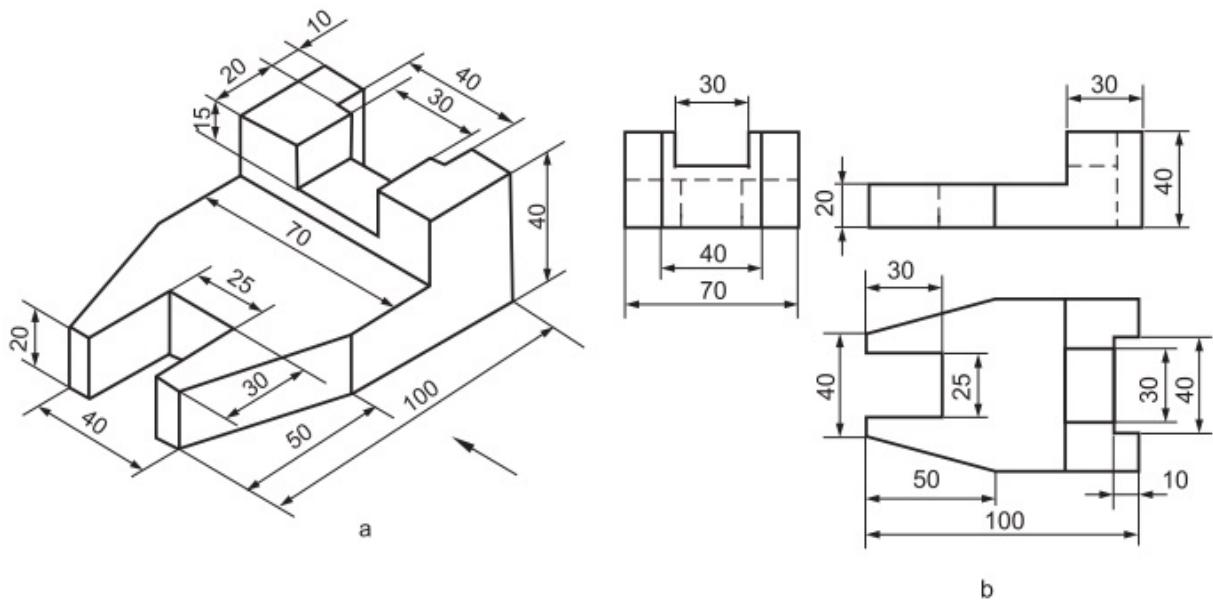


**Fig.6.33 Bearing block**

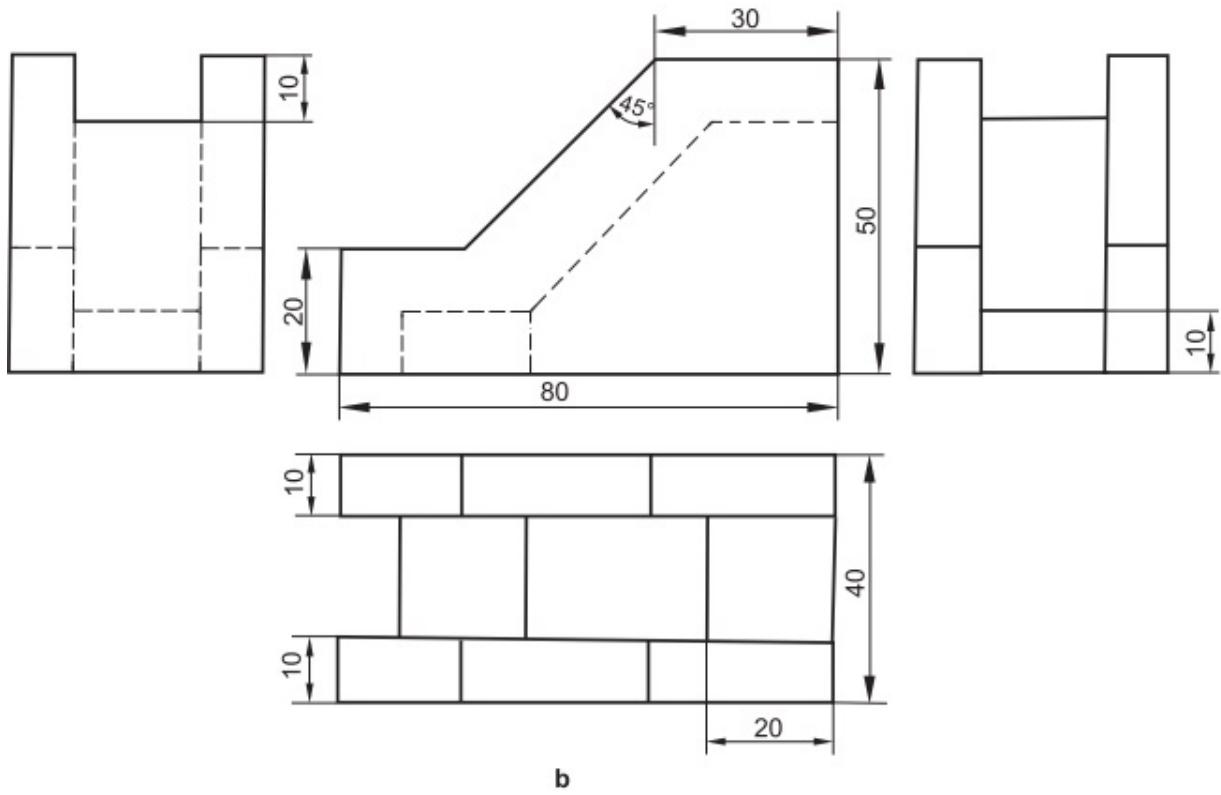


**Fig.6.34 Machine block**

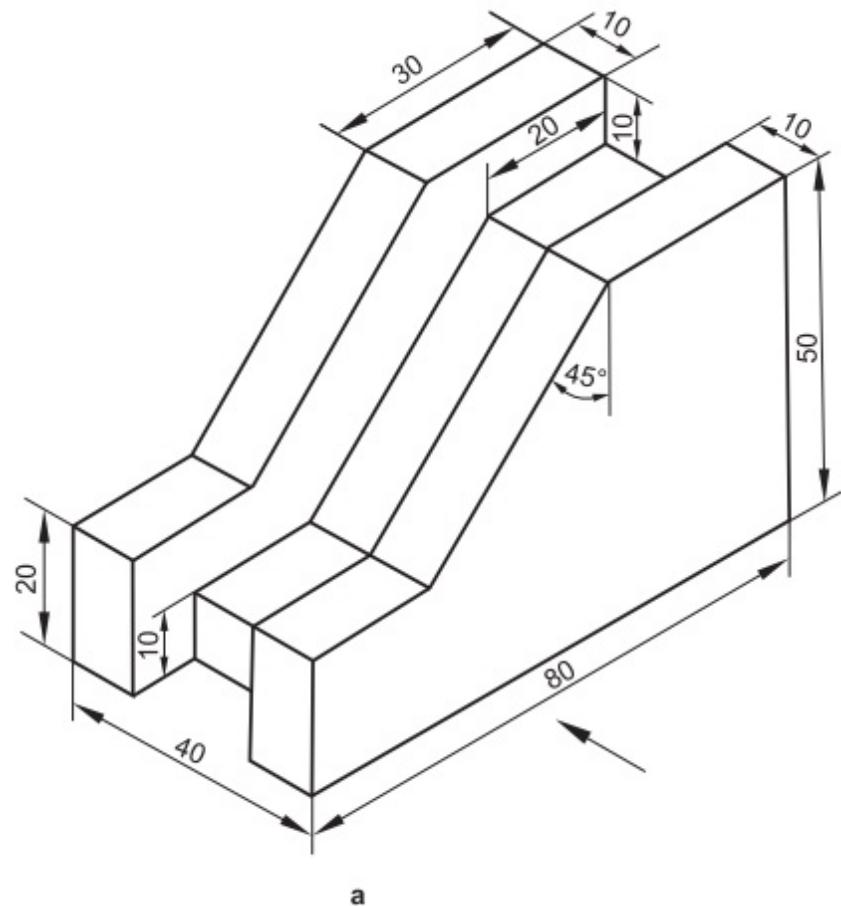




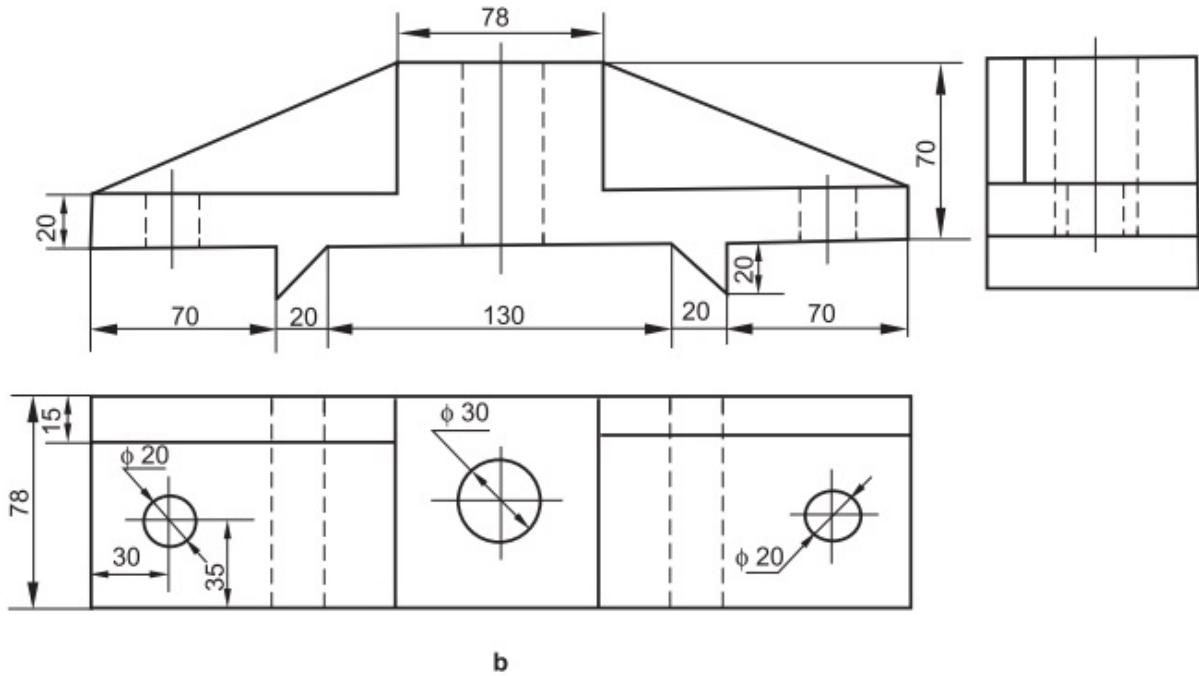
**Fig.6.36 Guide block**



**Fig.6.37**

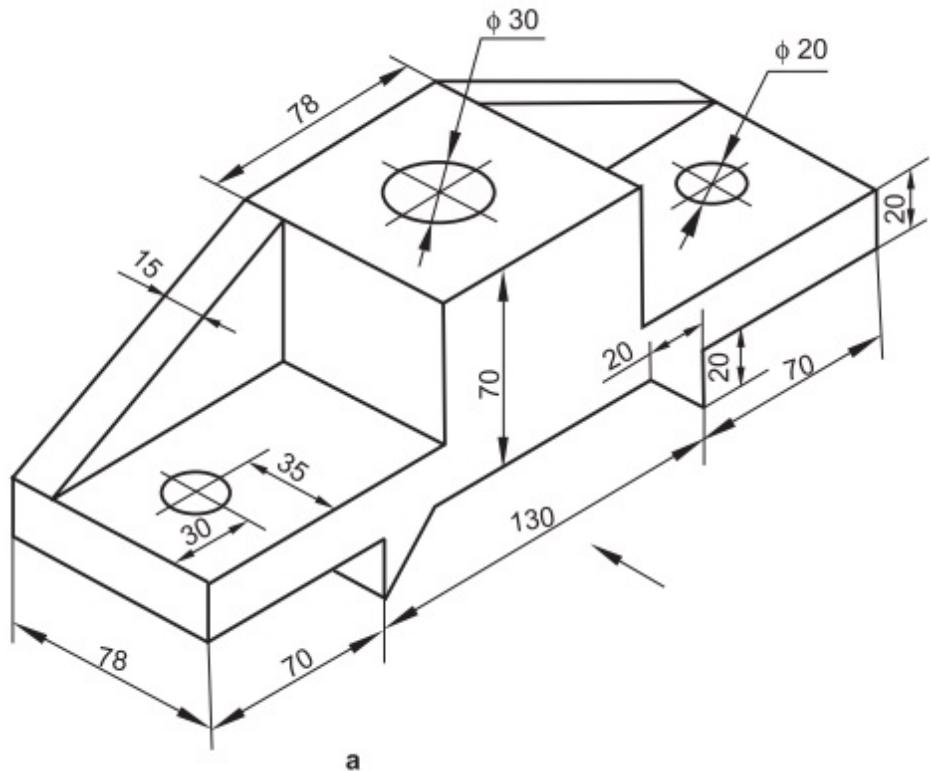


**Fig.6.37 (Contd.)**



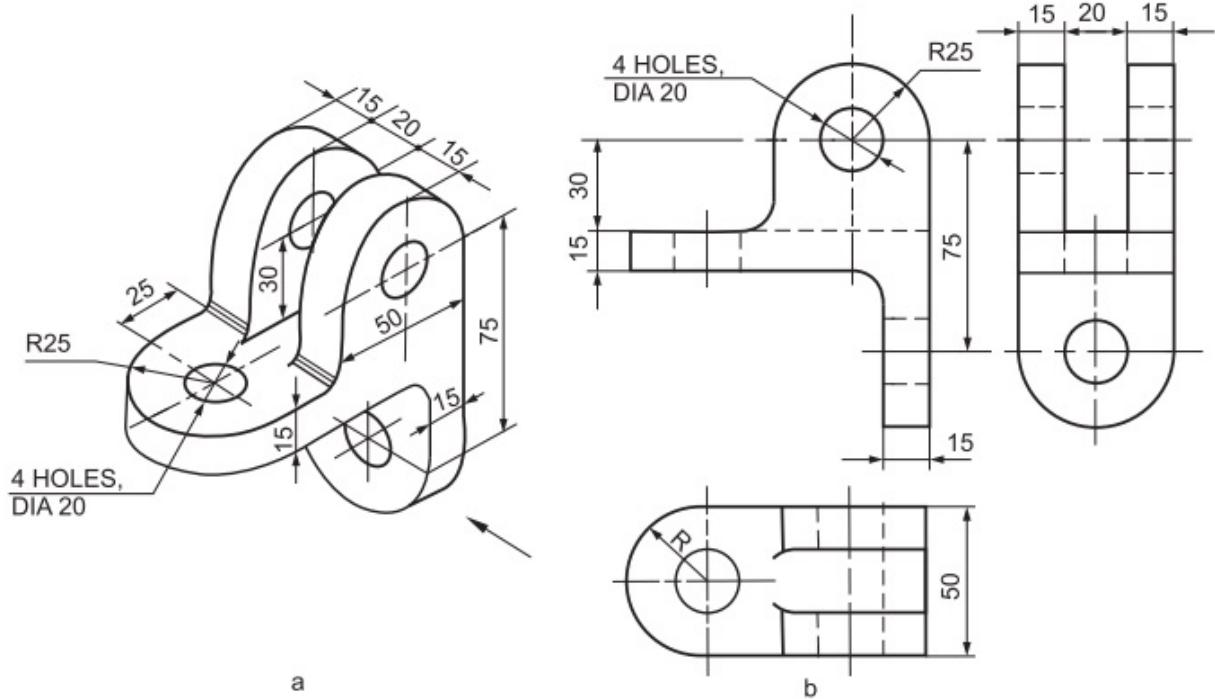
b

**Fig.6.38**

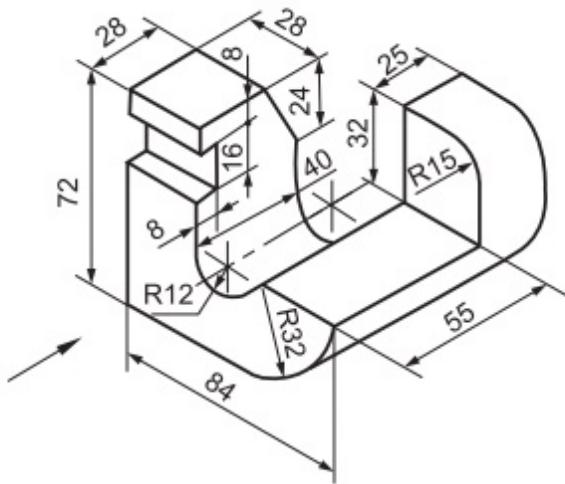


a

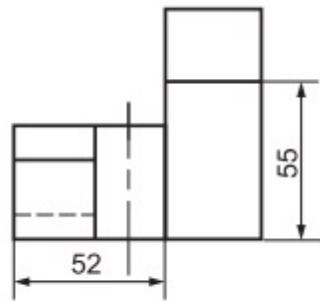
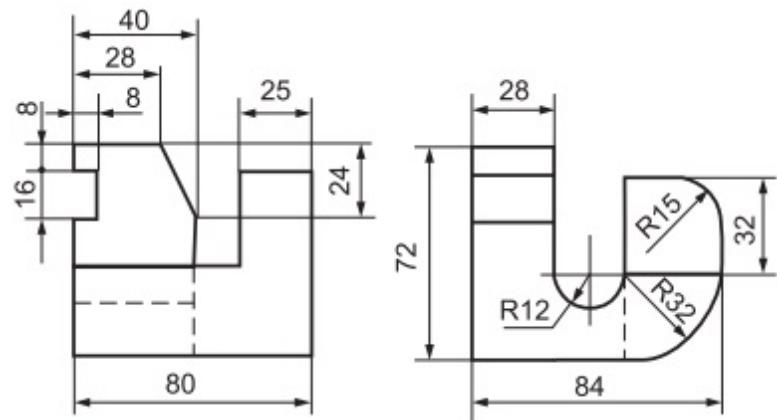
**Fig.6.38 (Contd.)**



**Fig.6.39 Bracket**

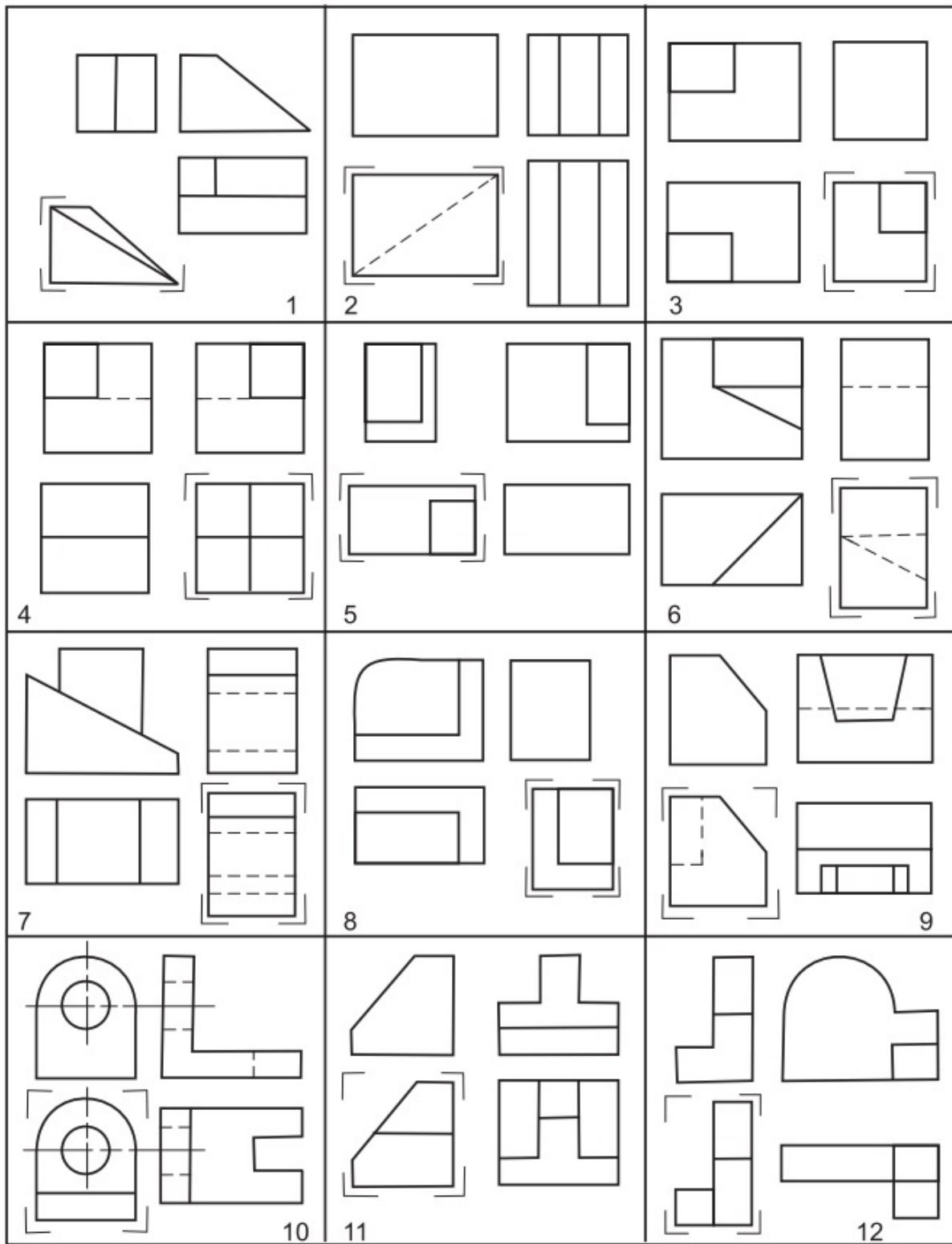


a

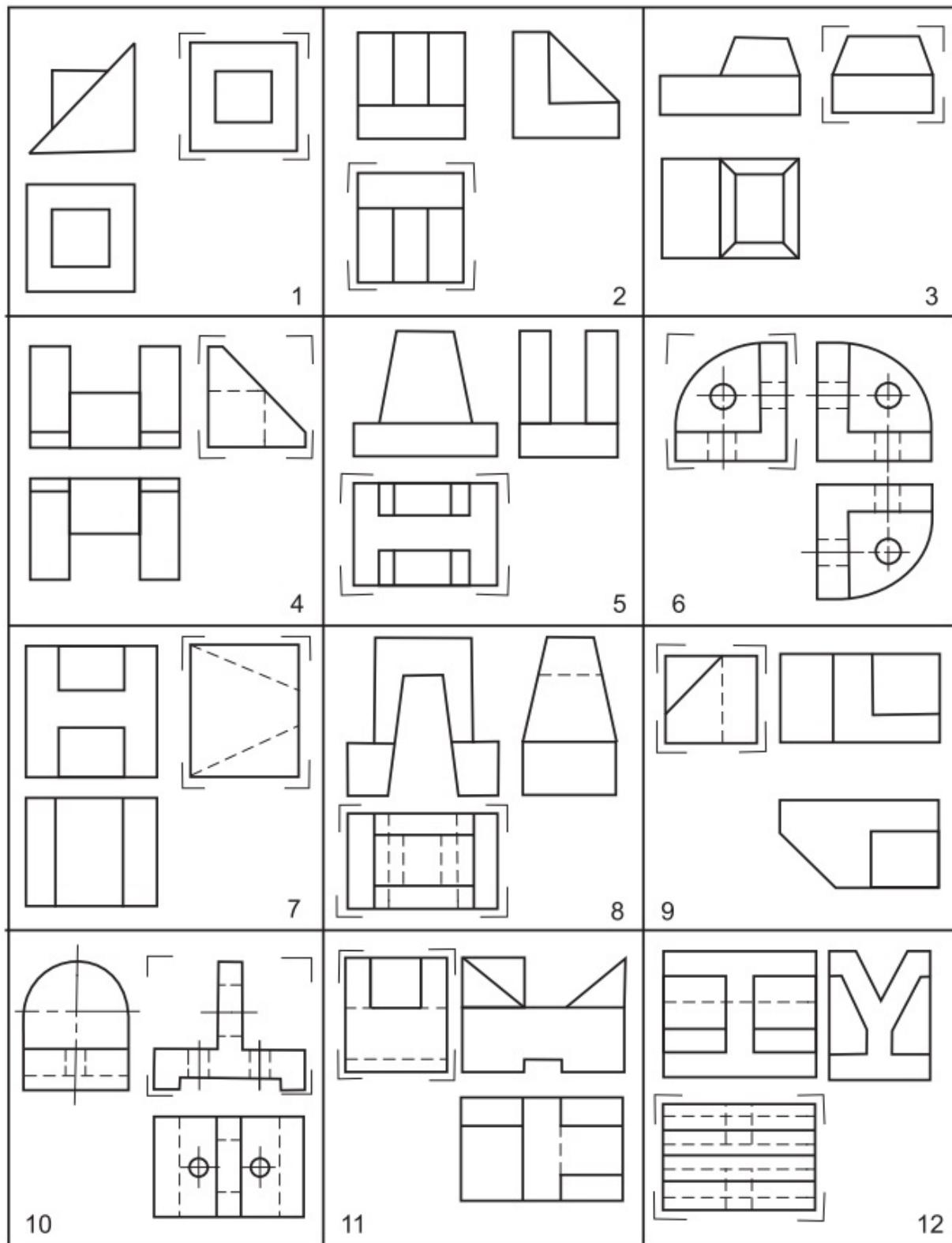


b

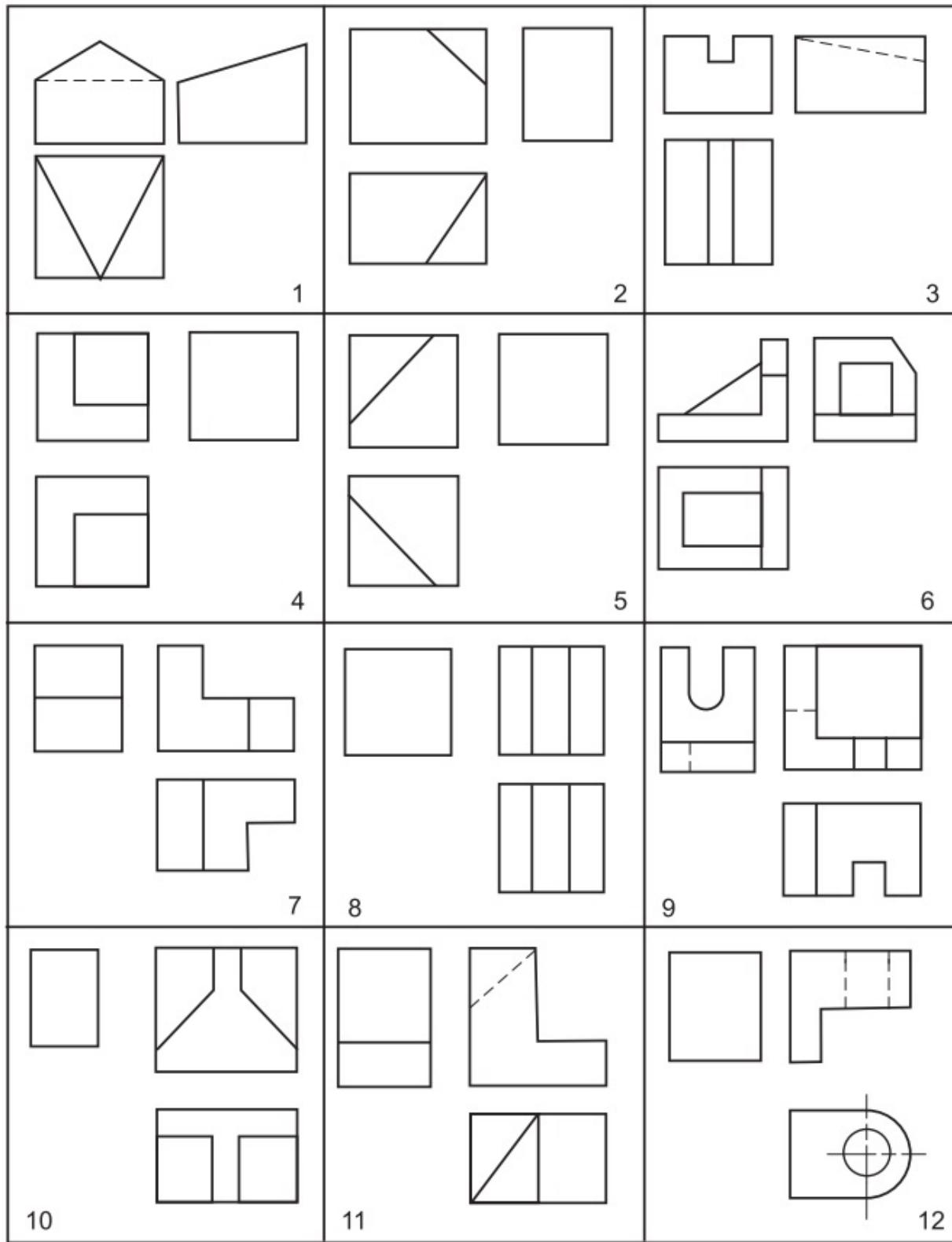
**Fig.6.40 Machine block**



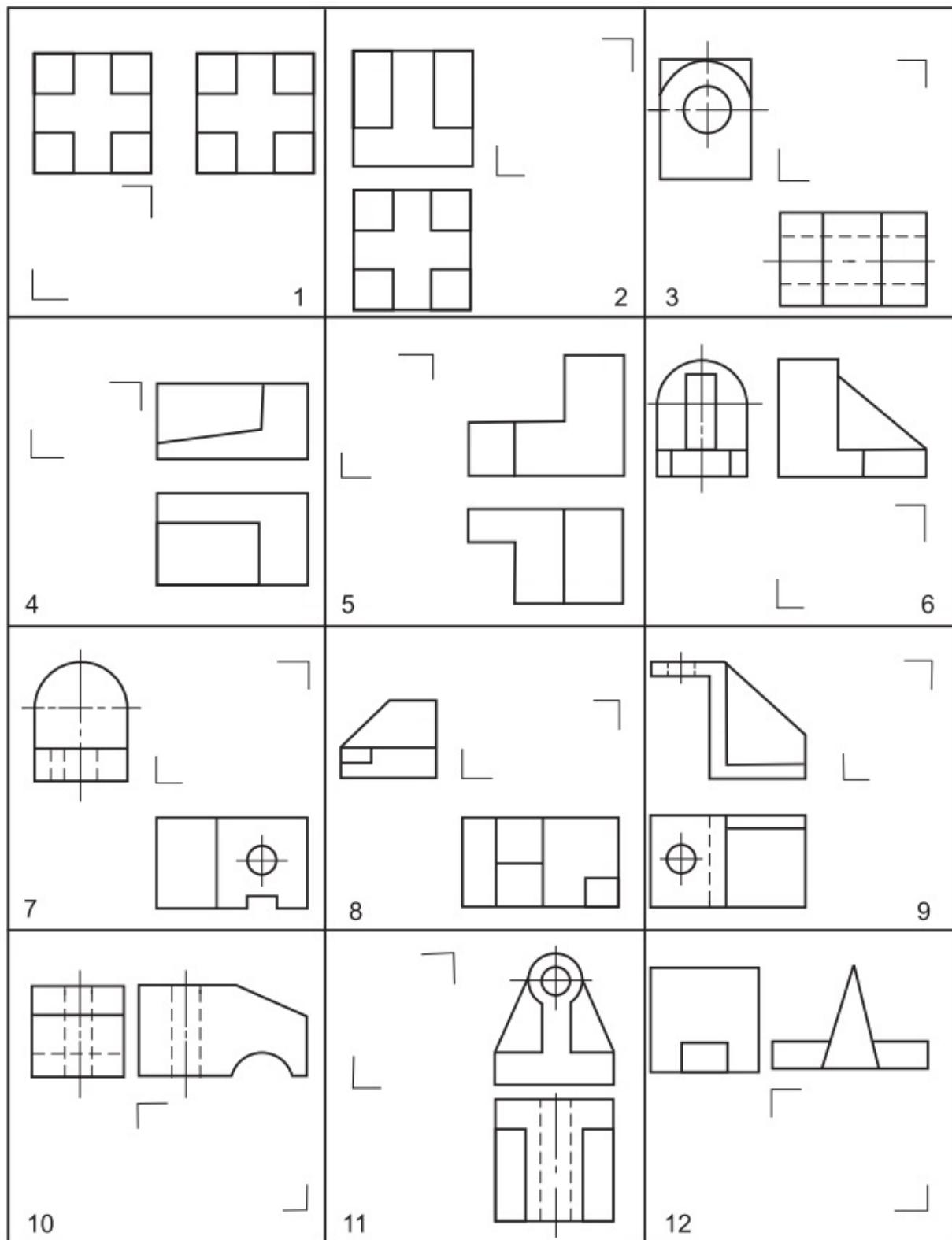
**Fig.6.41 Missing line examples**



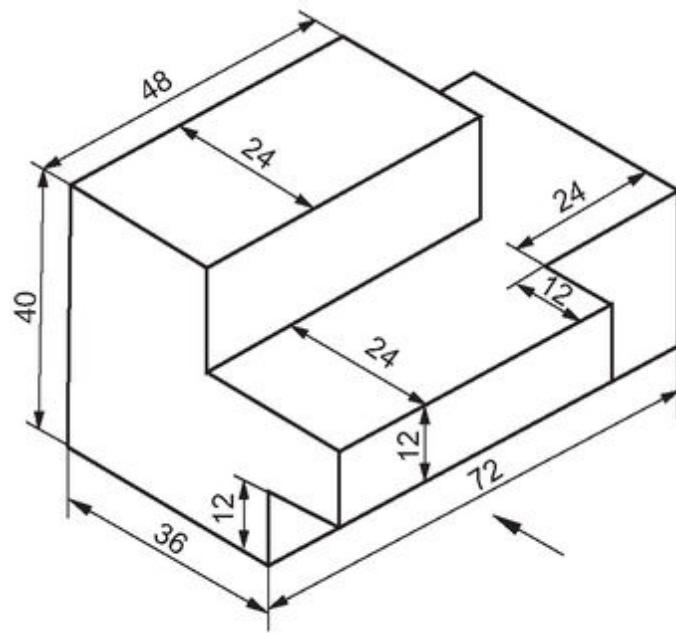
**Fig.6.42 Missing view examples**



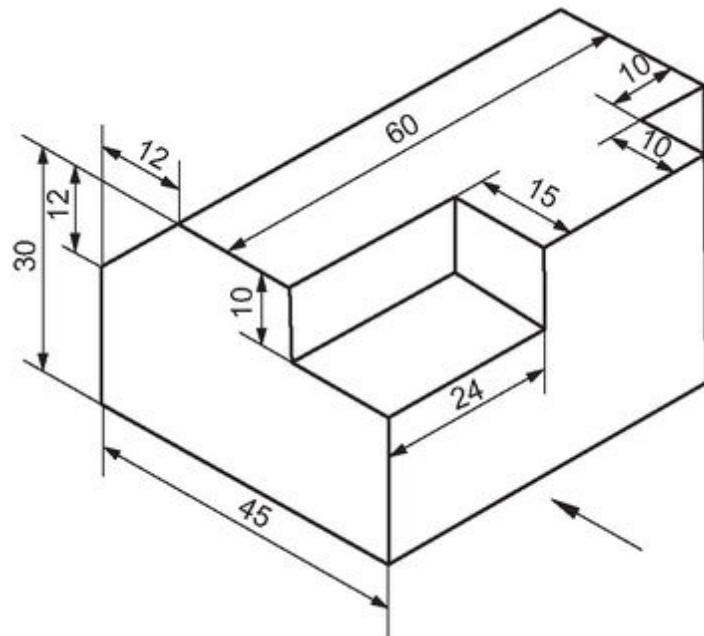
**Fig.6.43 Missing line exercises**



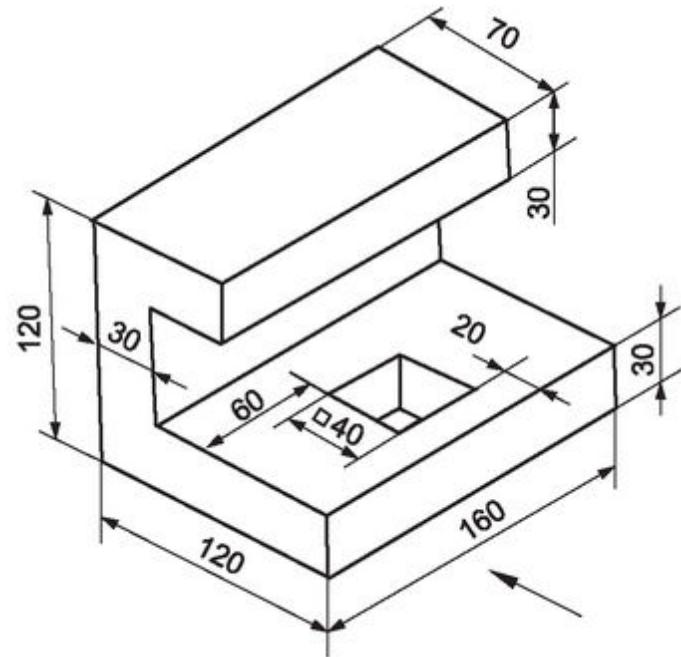
**Fig.6.44 Missing view exercises**



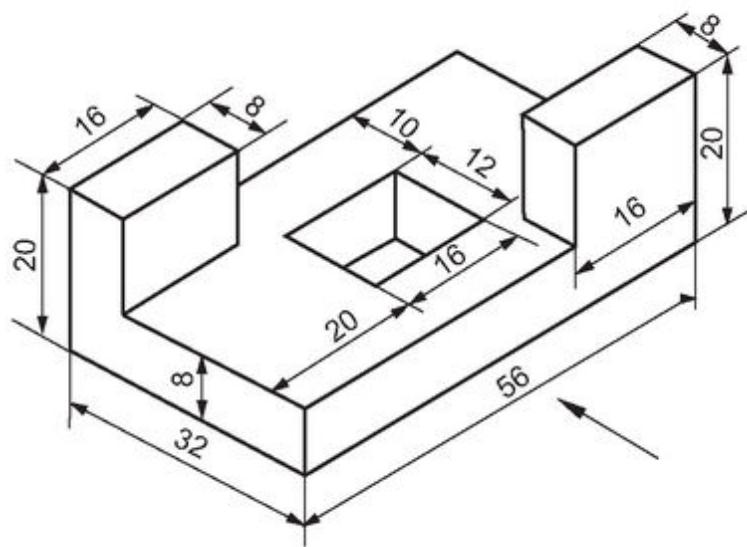
**Fig.6.45**



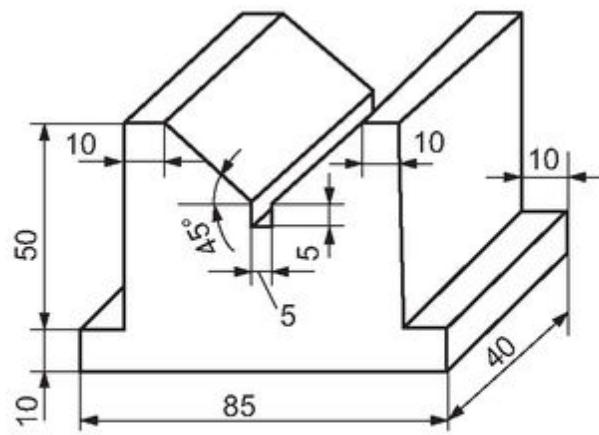
**Fig.6.46**



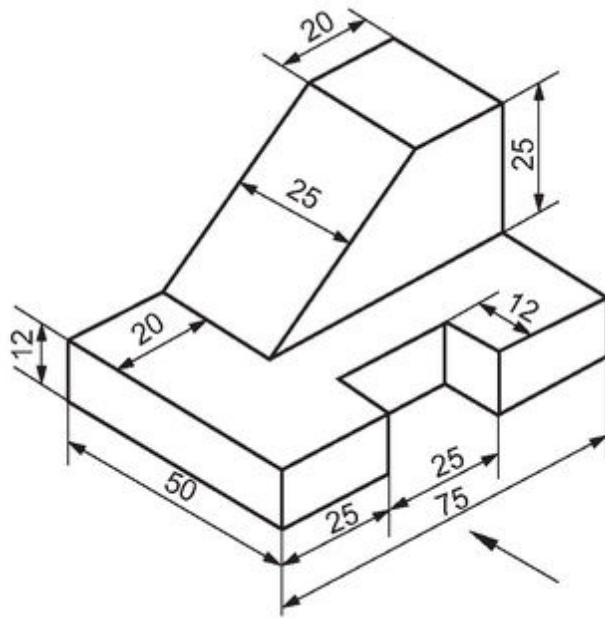
**Fig.6.47**



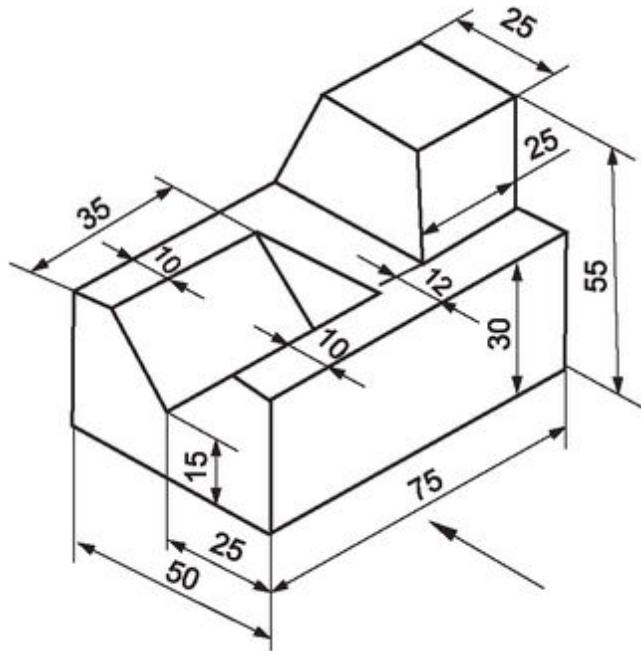
**Fig.6.48**



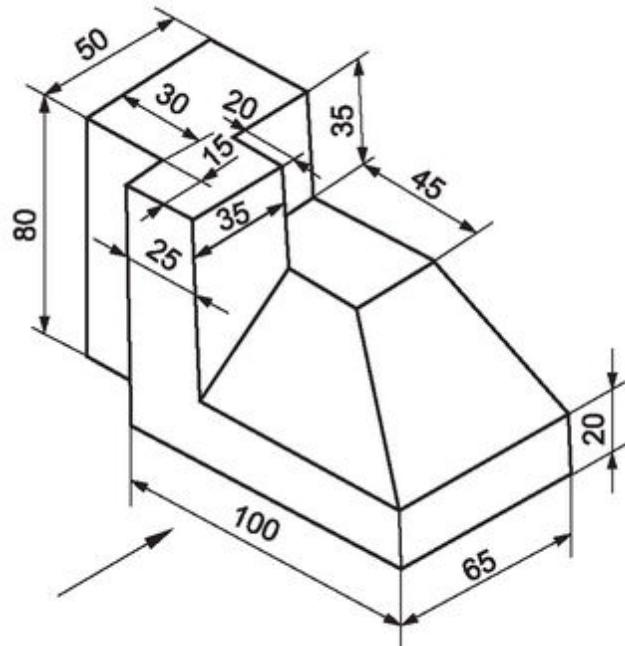
**Fig.6.49 V - block**



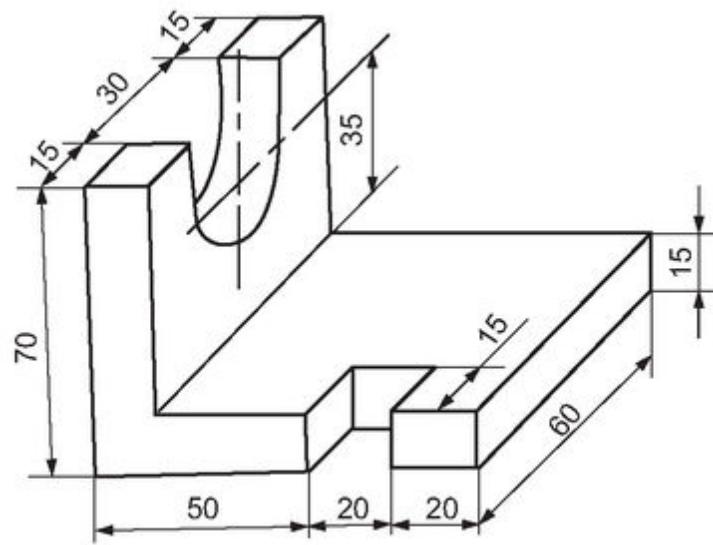
**Fig.6.50**



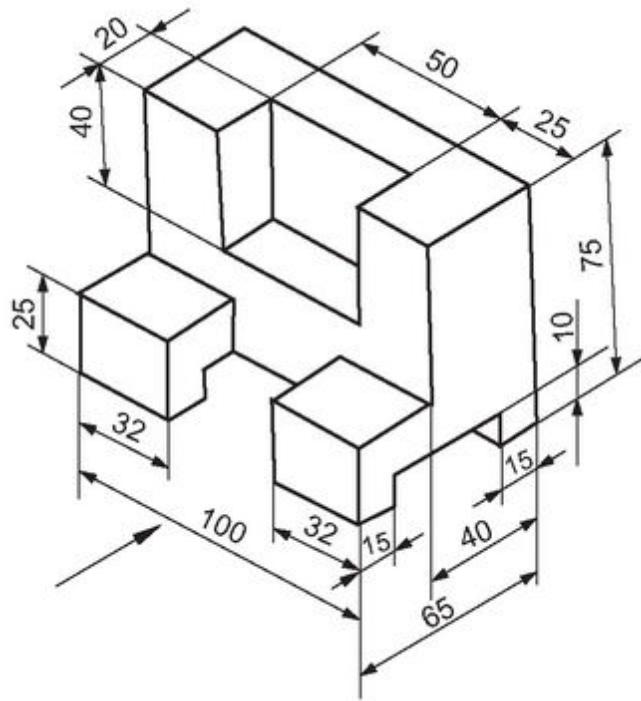
**Fig.6.51**



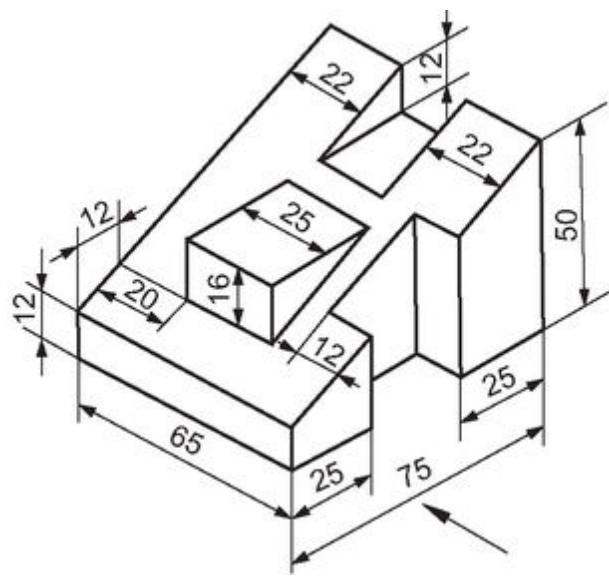
**Fig.6.52**



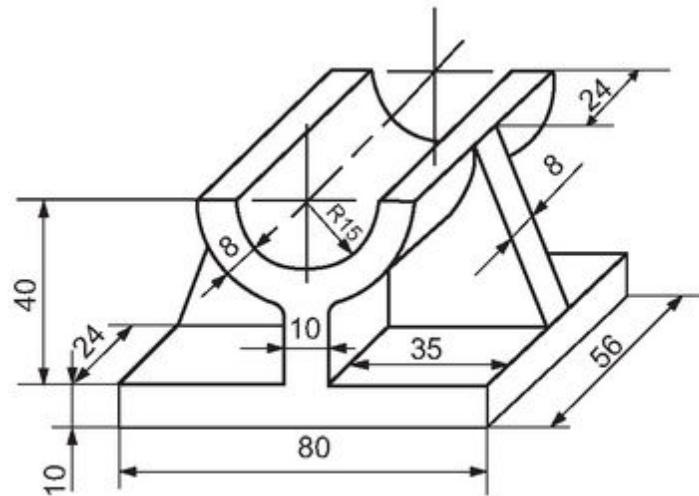
**Fig.6.53**



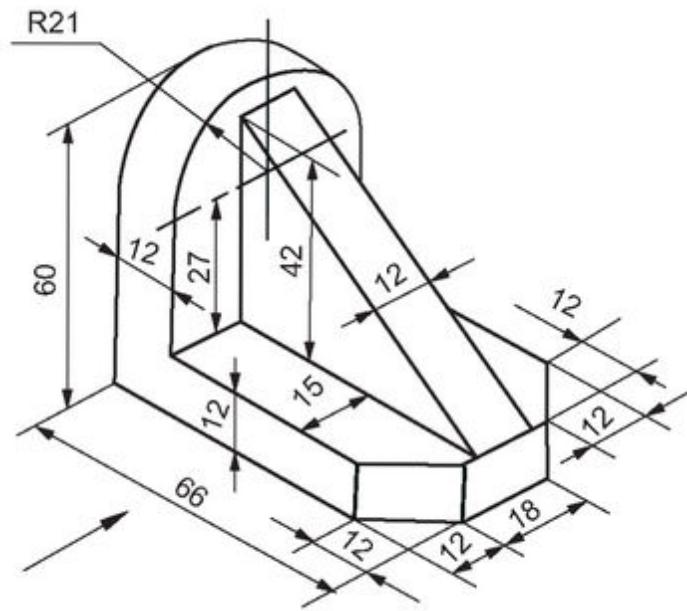
**Fig.6.54**



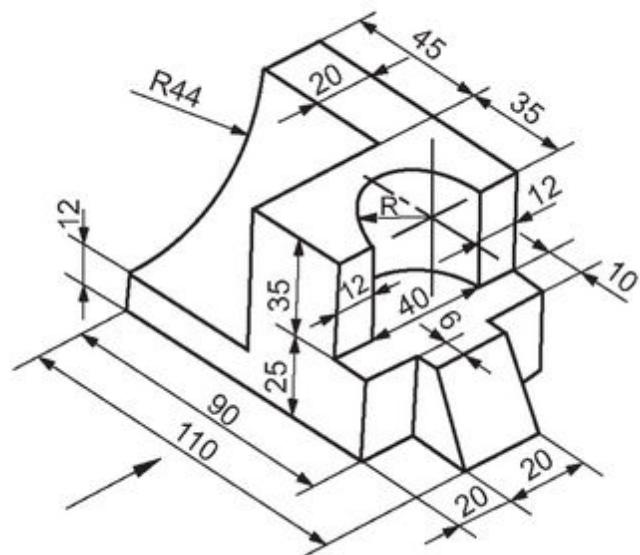
**Fig.6.55**



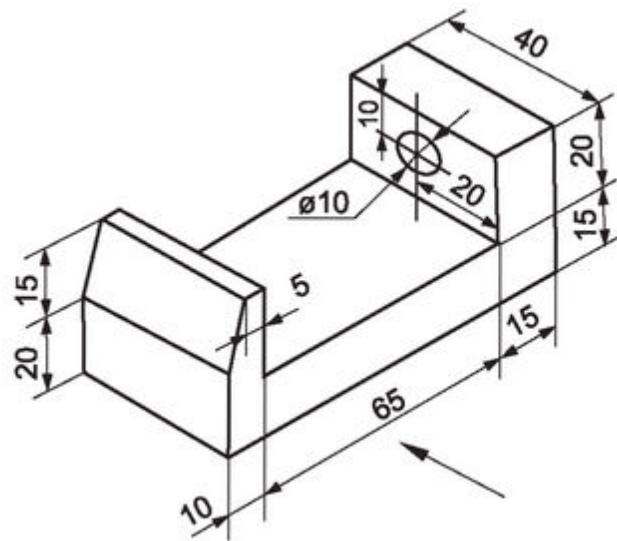
**Fig.6.56**



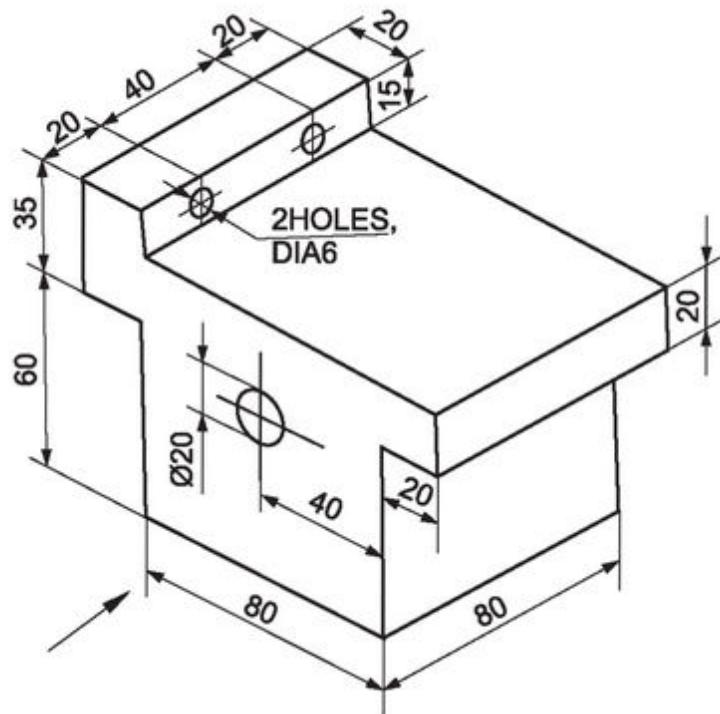
**Fig.6.57**



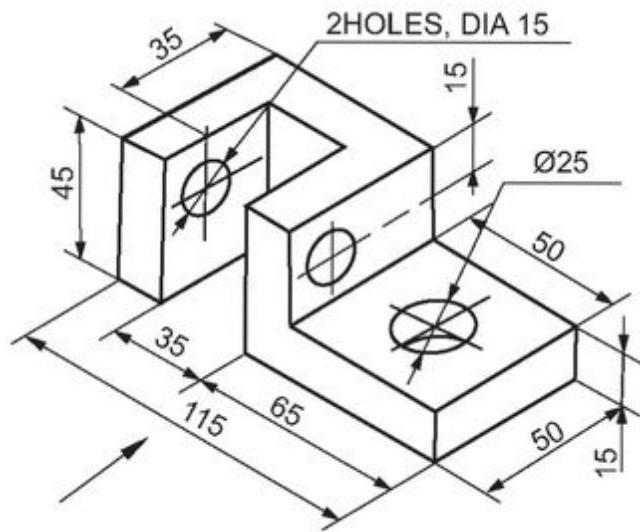
**Fig.6.58**



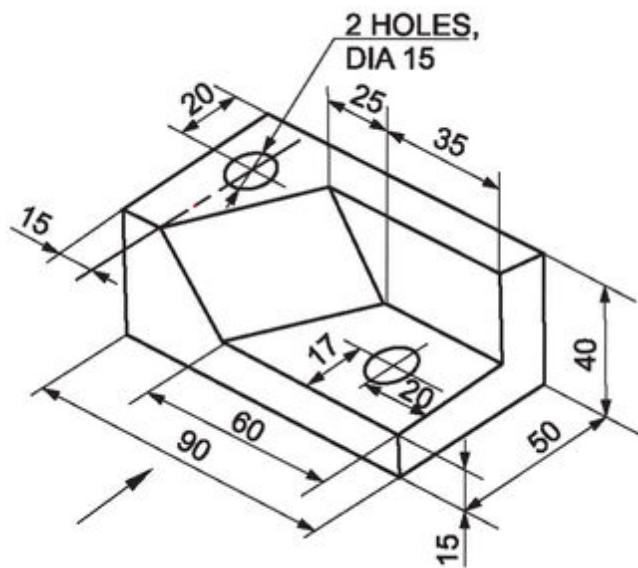
**Fig.6.59**



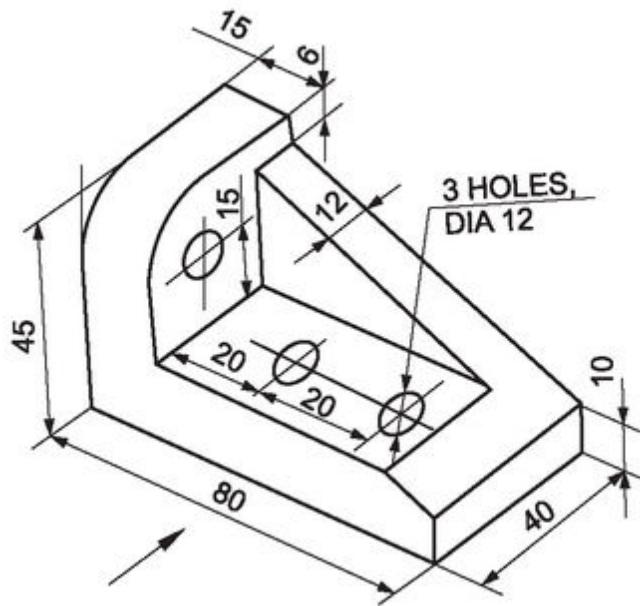
**Fig.6.60**



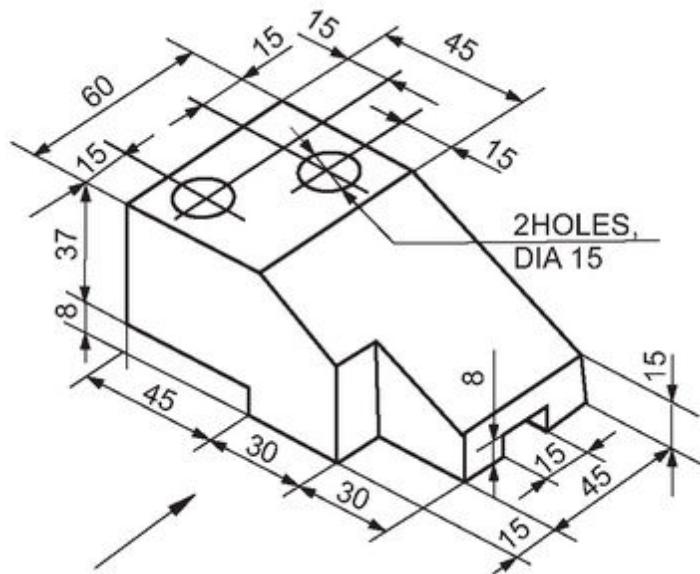
**Fig.6.61**



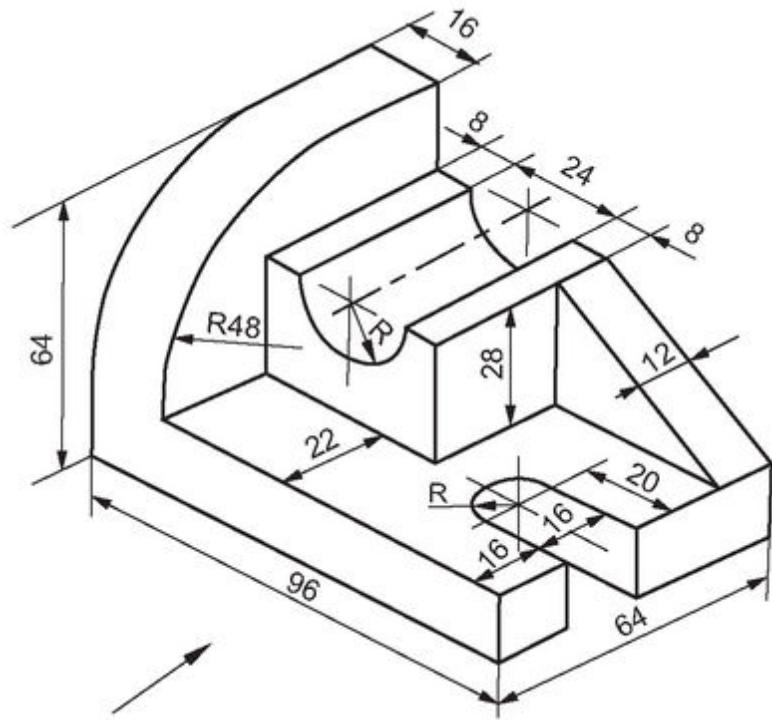
**Fig.6.62**



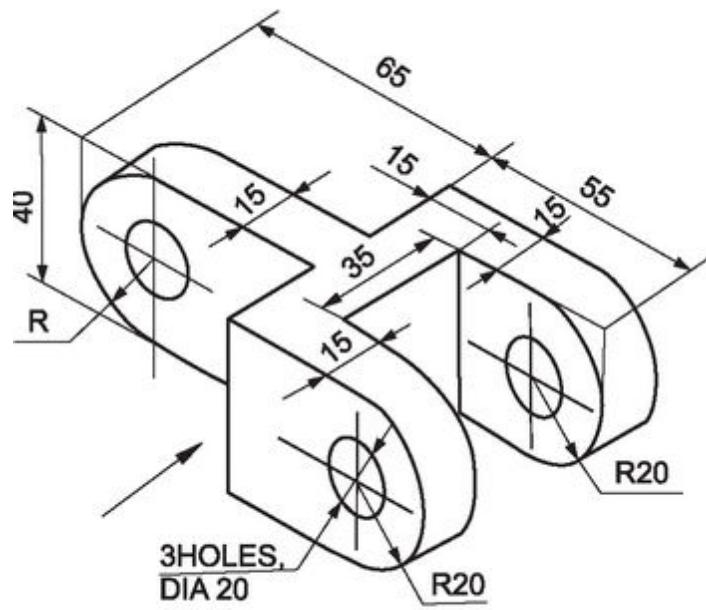
**Fig.6.63 Angle plate**



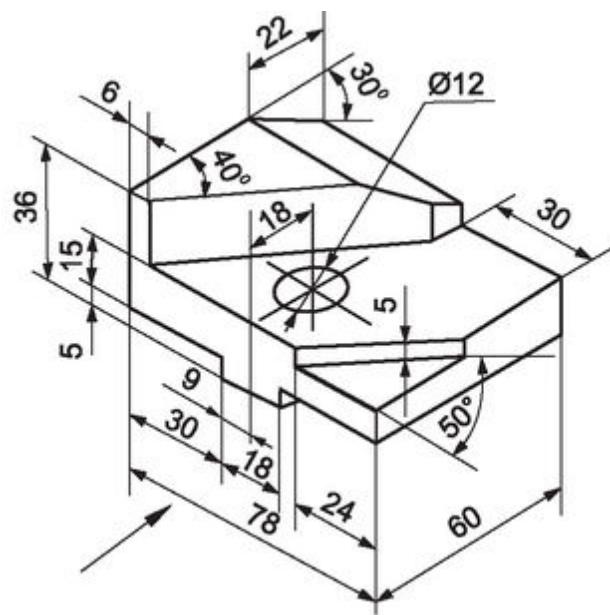
**Fig.6.64**



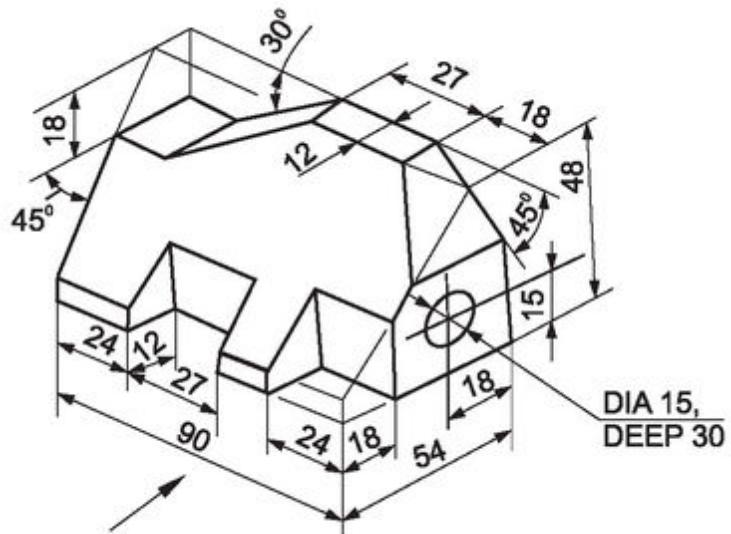
**Fig.6.65**



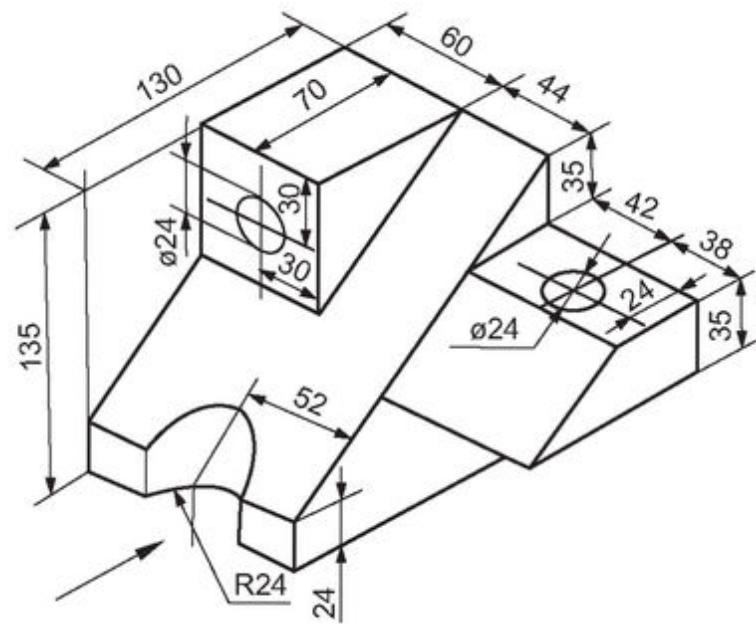
**Fig.6.66 Fork**



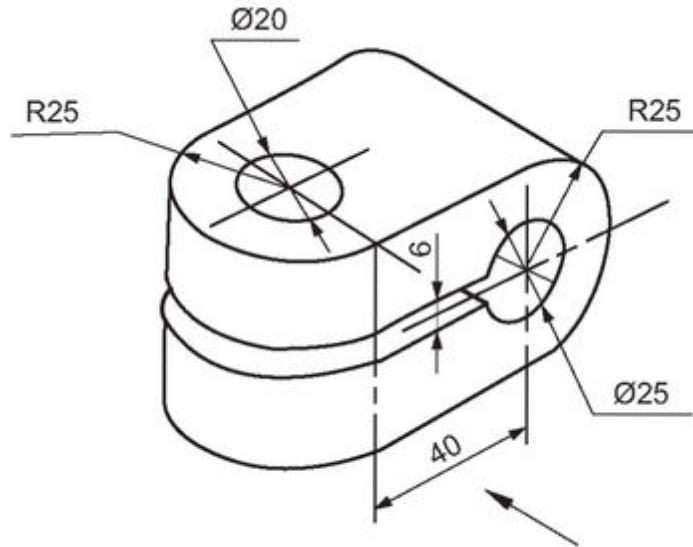
**Fig.6.67**



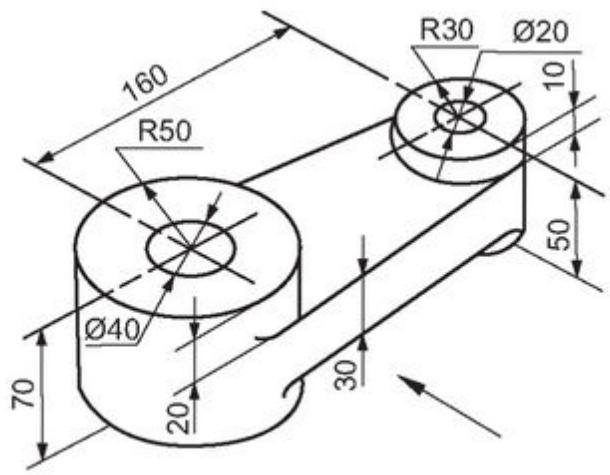
**Fig.6.68**



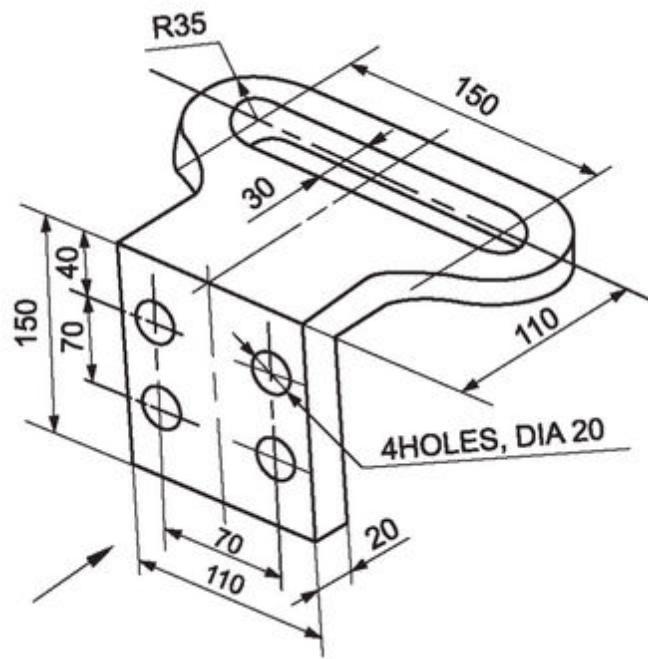
**Fig.6.69 Cross stop**



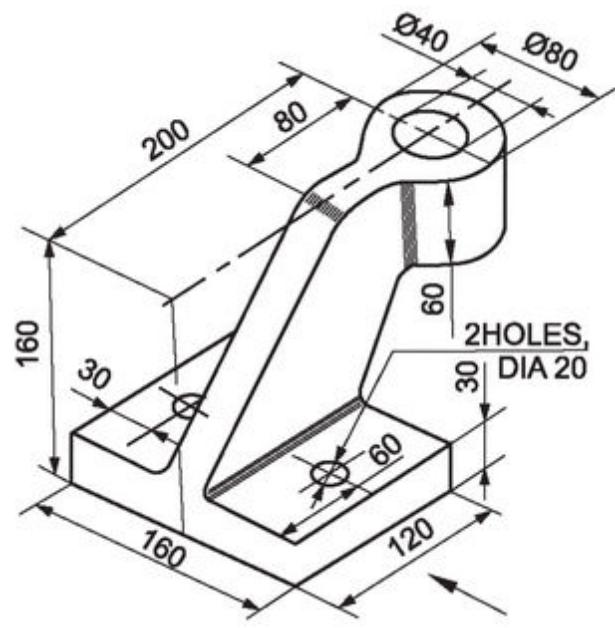
**Fig.6.70 Clamb**



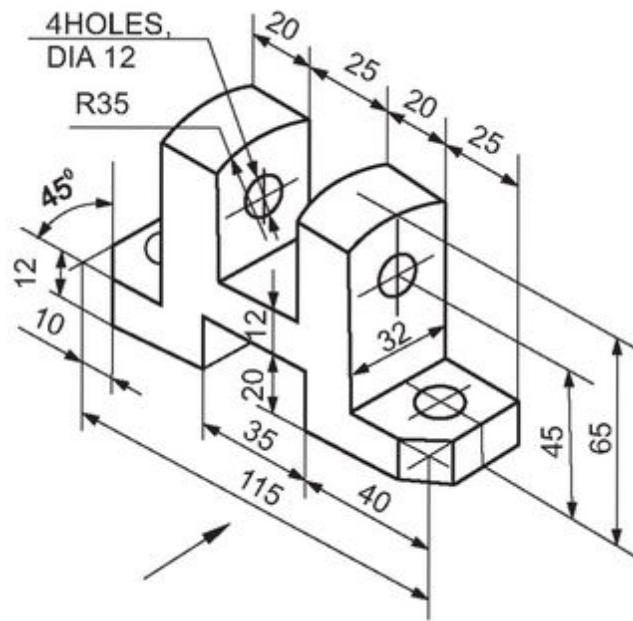
**Fig.6.71 Crank**



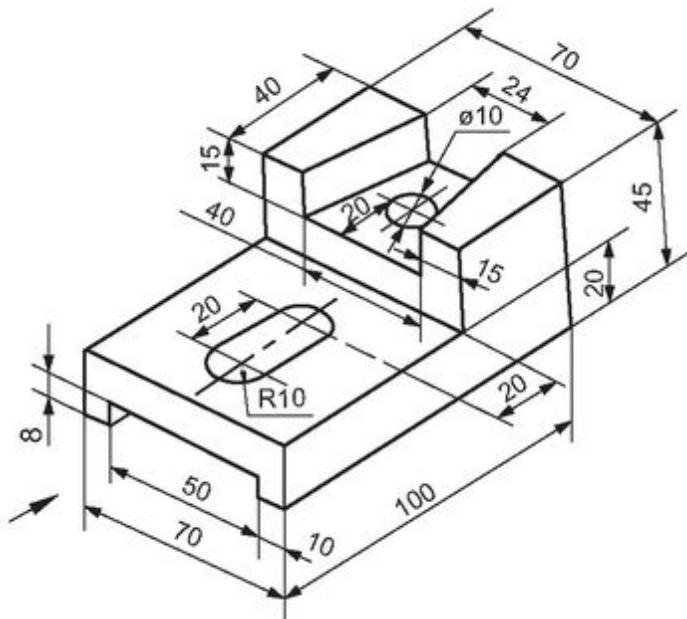
**Fig.6.72 Guide bracket**



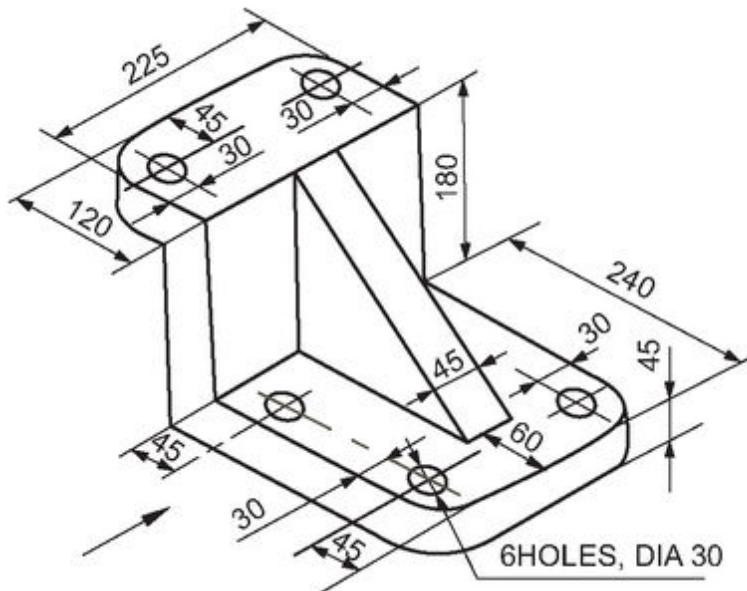
**Fig.6.73 Bearing**



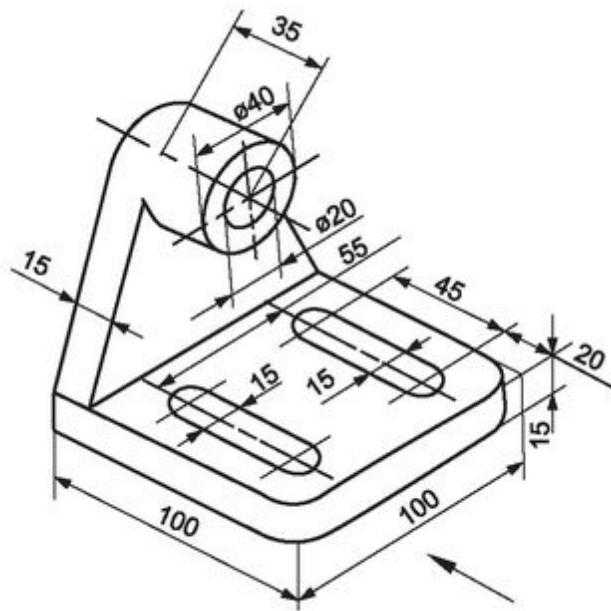
**Fig.6.74**



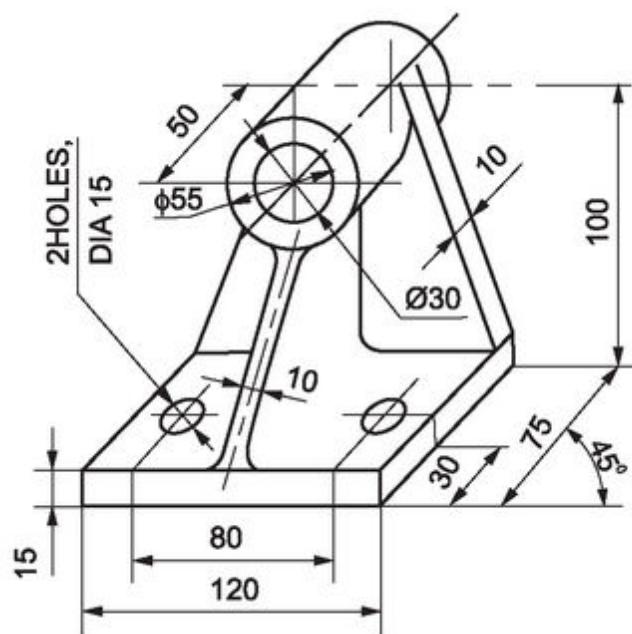
**Fig.6.75**



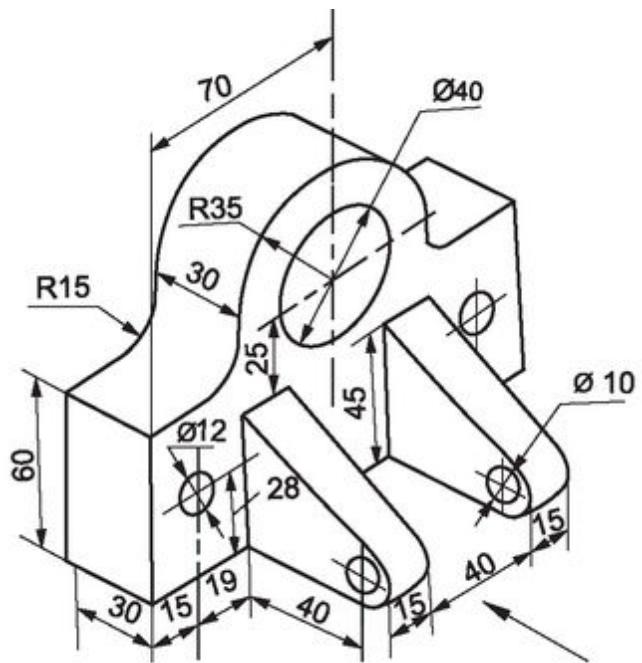
**Fig.6.76 Bracket**



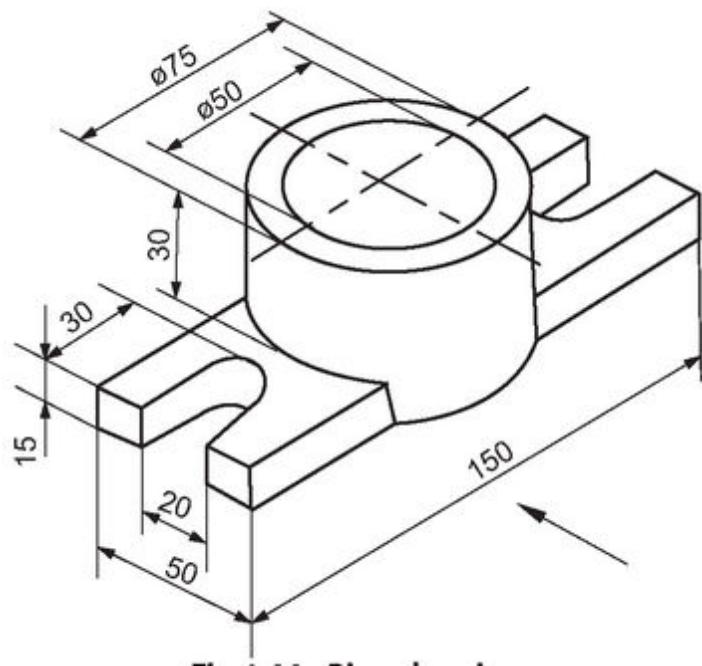
**Fig.6.77**



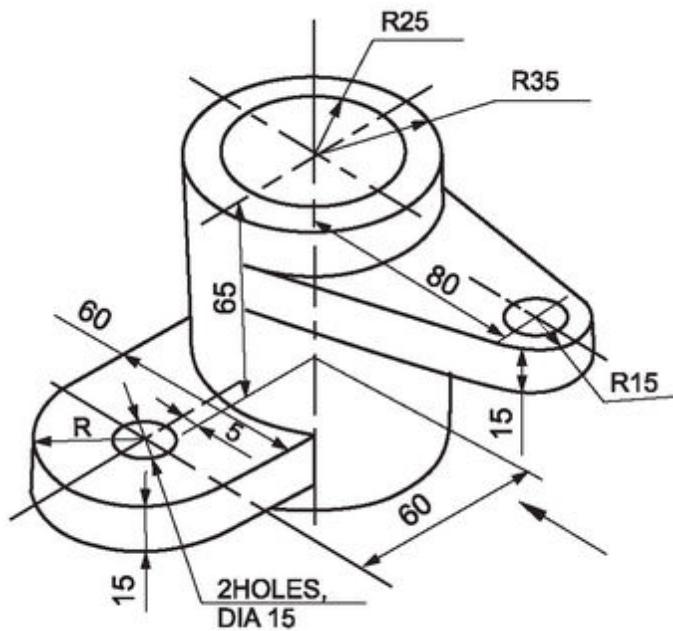
**Fig.6.78**



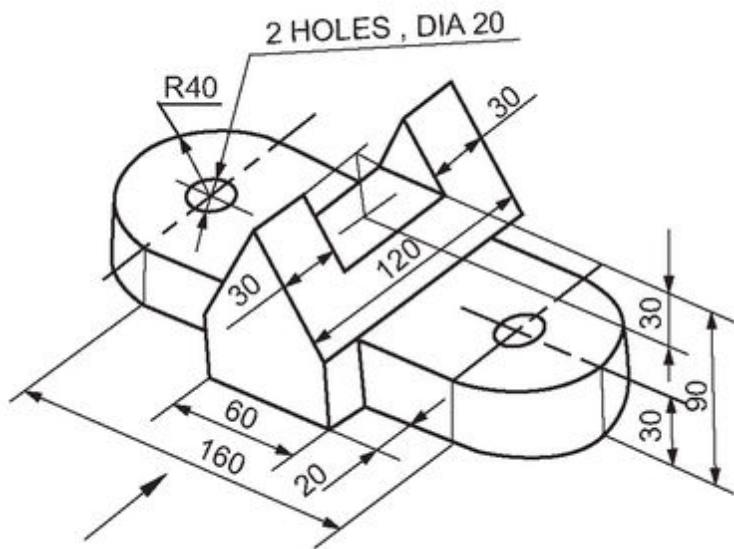
**Fig.6.79**



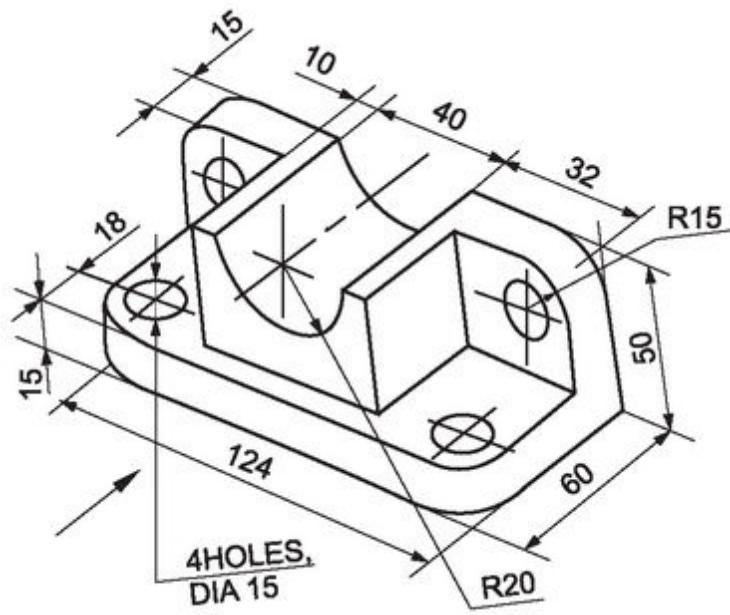
**Fig.6.80 Pivot bearing**



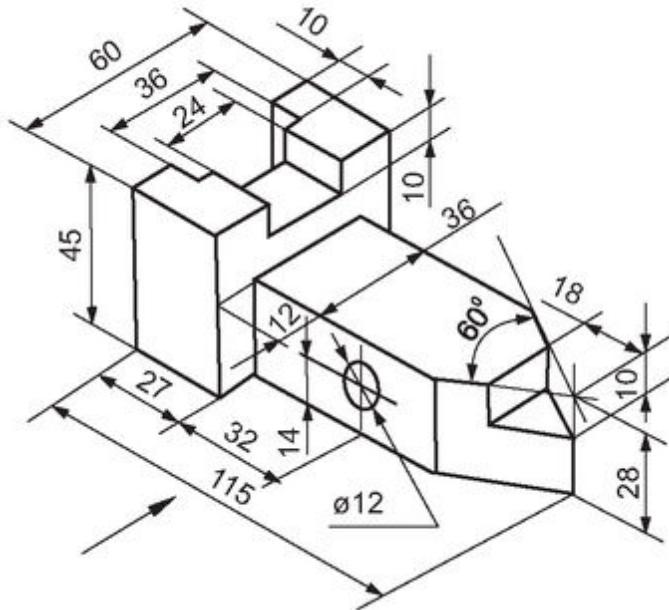
**Fig.6.81 Lever**



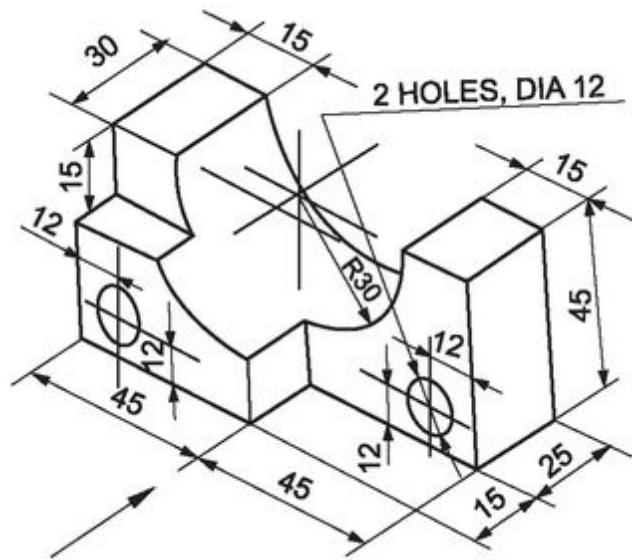
**Fig.6.82 Wedge block**



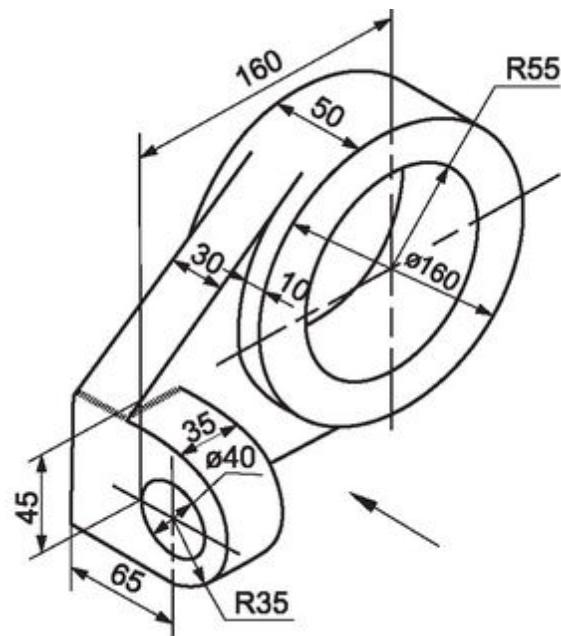
**Fig.6.83 Shaft support**



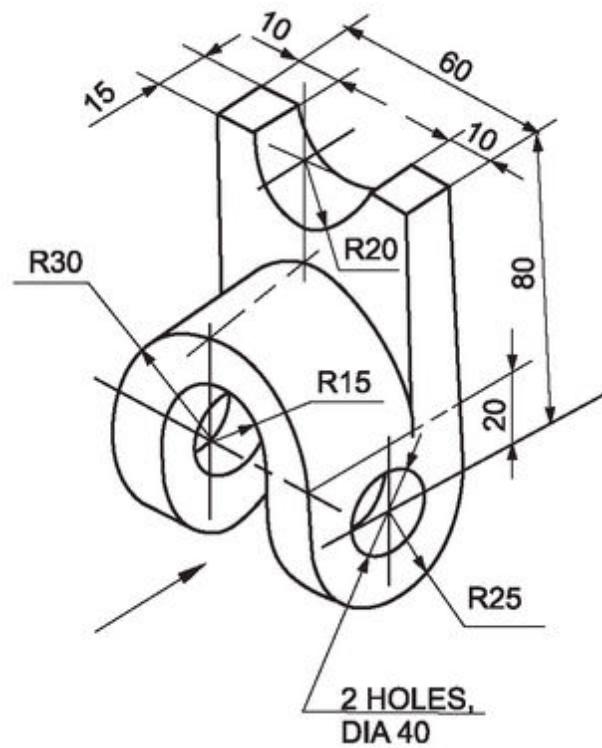
**Fig.6.84 Safety key**



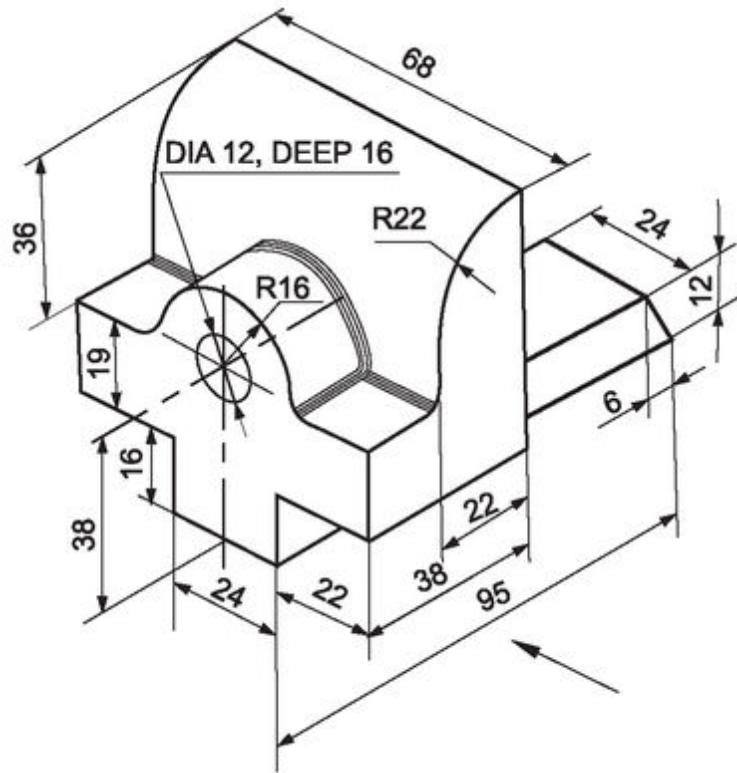
**Fig.6.85 Bearing brass**



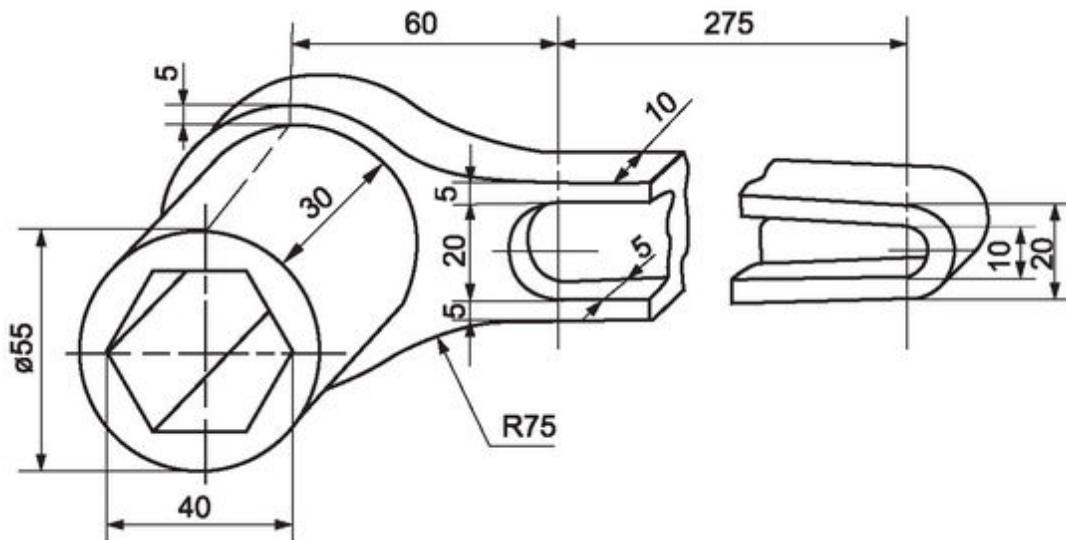
**Fig.6.86 Tube hanger**



**Fig.6.87**



**Fig.6.88 Vice jaw**



**Fig.6.89 Plug wrench**

## EXERCISES

- 6.1 Add the missing line(s) in the views shown in Fig. 6.43.
- 6.2 Supply the missing views in the orthographic projections shown in Fig. 6.44.
- 6.3 Draw the front and top views for the objects shown in Figs. 6.49, 6.59, 6.66, 6.71, 6.73, 6.76, 6.80, 6.83 and 6.89 .
- 6.4 Draw the front, top and right side views for the machine components shown in Figs. 6.47, 6.57, 6.58, 6.60, 6.61, 6.62, 6.63, 6.64, 6.67, 6.68, 6.74, 6.75, 6.77, 6.78, 6.79, 6.84 and 6.85.

- 6.5 Draw the front, top and left side views for the machine components shown in Figs. 6.51, 6.52, 6.54, 6.55, 6.56, 6.65, 6.69, 6.70, 6.72, 6.81, 6.82, 6.86 and 6.90.
- 6.6 Draw the front and right side views for the machine component shown in Fig. 6.53.
- 6.7 Draw the front and left side views for the machine component shown in Fig. 6.87.
- 6.8 Draw the front, top and both the side views for the objects shown in Figs. 6.45, 6.46, 6.48, 6.50.

## REVIEW QUESTIONS

- 6.1 What is a projection?
- 6.2 How the projection of an object is obtained?
- 6.3 What is meant by an orthographic projection?
- 6.4 Sketch the symbols used to represent (i) first angle projection and (ii) third angle projection.
- 6.5 Differentiate between the first and third angle projections.
- 6.6 Describe the method of obtaining the orthographic views of an object, giving an example.
- 6.7 For what type of objects, one-view drawings are sufficient? How the shape and size of such features are indicated?

## OBJECTIVE QUESTIONS

- 6.1 Name the two systems of projections, which are in vogue.
- 6.2 In orthographic projection, the \_\_\_\_ are perpendicular to the \_\_\_\_\_ of projection.
- 6.3 In — — projection, any view is so placed that it represents the side of the object nearer to it.
- 6.4 In — — projection, any view is so placed that it represents the side of the object away from it.
- 6.5 In the first angle projection, the object is positioned in-between the observer and the plane of projection.  
(True/False)
- 6.6 In the third angle projection, the object is positioned in-between the observer and the plane of projection  
(True/False)
- 6.7 A surface of an object appears in its true shape, when it is ----- to the plane of projection.
- 6.8 The front view of an object is obtained as a projection on the----- plane, by looking the object, \_\_\_\_\_ to its front surface.
- 6.9 The top view of an object is obtained as a projection on the ----- plane, by looking the object, normal to its ----- surface.
- 6.10 In first angle projection:
- The top view is ----- the front view, and
  - The right side view is to the ----- of the front view.
- 6.11 In third angle projection:
- The top view is ----- the front view, and

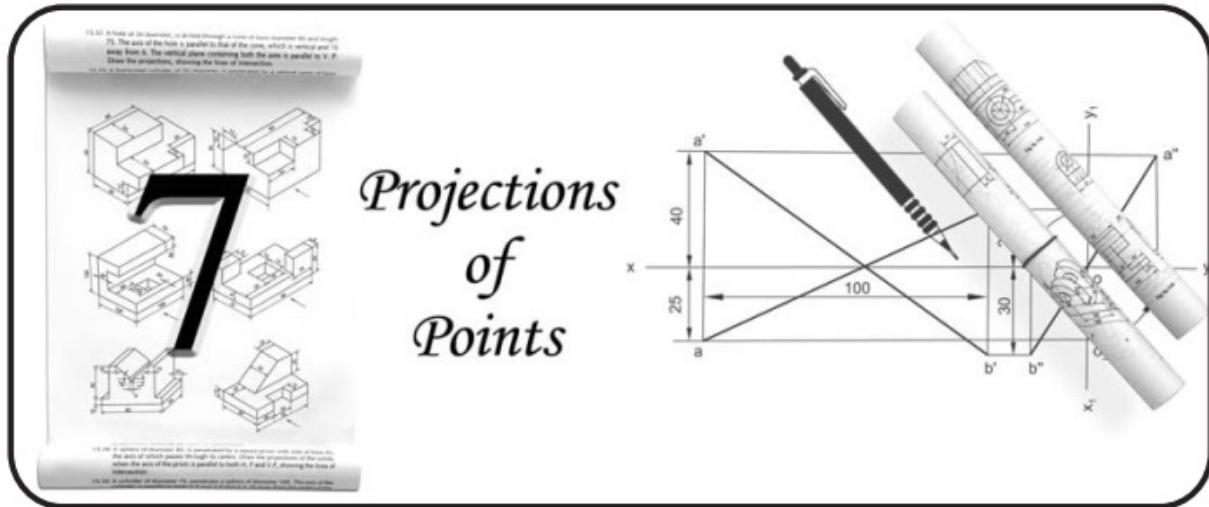
- (ii) The left side view is to the ----- of the ----- view.
- 6.12 The side view of an object is obtained as a projection on the ----- plane, by looking the object ----- to its ----- surface.
- 6.13 In both the methods of projection, the views are identical in shape and detail but only their location with respect to the front view is different.
- (True/False)
- 6.14 The selection of the number of views required for an object depends upon its ---.

## ANSWERS

- 6.1 First and third angle projections
- 6.2 projectors, plane
- 6.3 Third angle
- 6.4 First angle
- 6.5 True
- 6.6 False
- 6.7 parallel
- 6.8 vertical, normal
- 6.9 horizontal, top
- 6.10 (i) below, (ii) left
- 6.11 (i) above, (ii) left, front
- 6.12 profile, normal, side
- 6.13 True

6.14 complexity

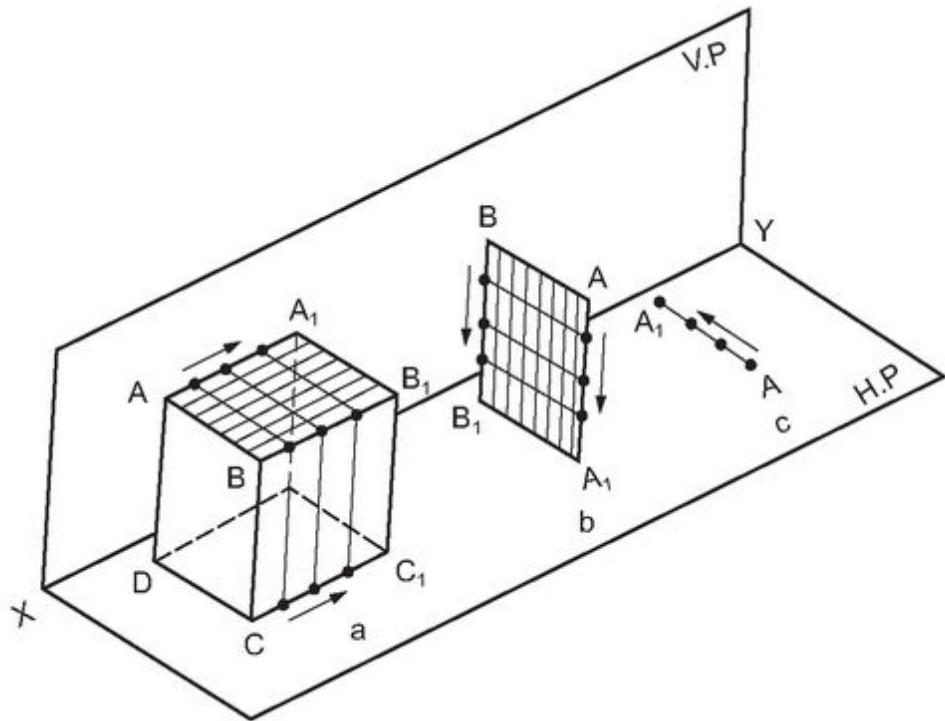
*OceanofPDF.com*



## 7.1 INTRODUCTION

Projections of points, lines and planes must be studied in order to understand the projections of solids, because it could be said that a solid consists of a number of planes, a plane consists of a number of lines and a line consists of a number of points.

A solid may be generated by a plane moving in space ([Fig.7.1a](#)), a plane may be generated by a straight line moving in space ([Fig.7.1b](#)) and a straight line in-turn, may be generated by a point moving in space ([Fig.7.1c](#)).



**Fig.7.1**

To understand the principles of orthographic projections of solids, it is necessary to study first the principles involved in the projections of the points, lines and planes. This chapter deals with the principles involved in the orthographic projections of points.

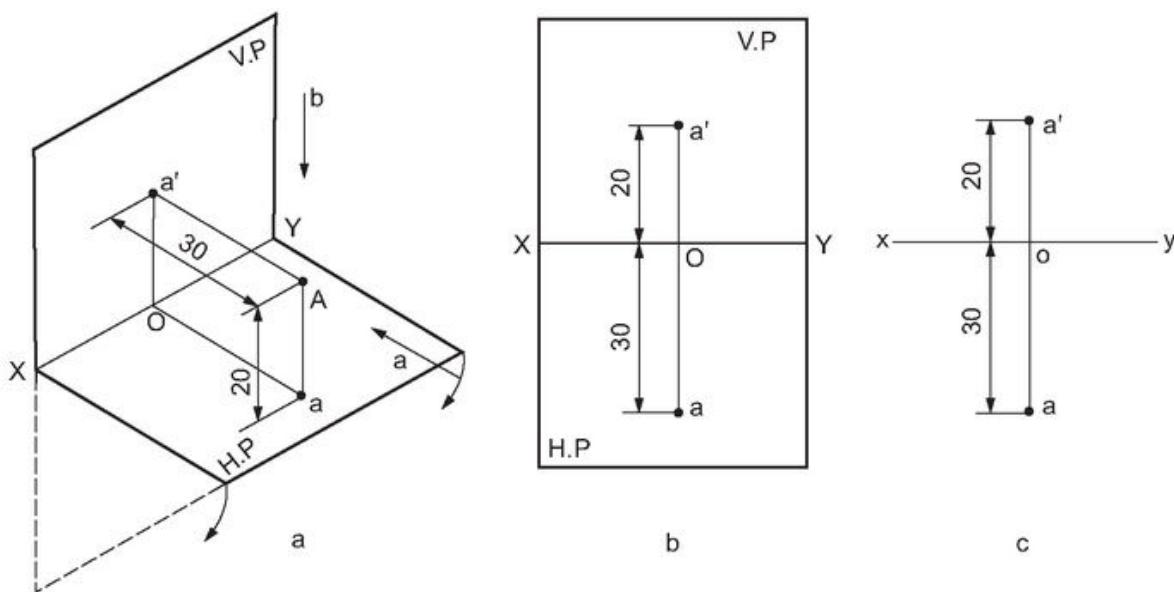
## 7.2 TWO VIEW PROJECTIONS OF POINTS

In space, one can identify three mutually perpendicular planes. A point in space may be thought of lying either in one of the four quadrants formed by the planes or on one of the three planes. This article deals with the two view projections of points in space.

## 7.2.1 Projections of a Point Situated in First Quadrant

**Problem 1** A point A is 20 above H.P and 30 in front of V.P. Draw the projections of the point.

Figure 7.2a shows the position of the point A in the first quadrant. When the point is viewed in the direction **a**, the front view  $a'$  is obtained at the intersection point between the ray of sight through A and V.P. When the point is viewed in the direction **b**, the top view  $a$  is obtained at the intersection point between the ray of sight and H.P.



**Fig.7.2 Projections of a point in first quadrant**

For presenting the views on a plane sheet, the H.P is rotated till it comes in-line with V.P. In general, as stated in the previous chapter, irrespective of the location of the point, the plane (s) must be rotated such that, the first quadrant always opens-out. Figure 7.2b shows the relative positions of the views or projections, as otherwise called,

along with the planes of projection. [Figure 7.2c](#) shows only the relative positions of the views, as it is customary not to show the planes of projection.

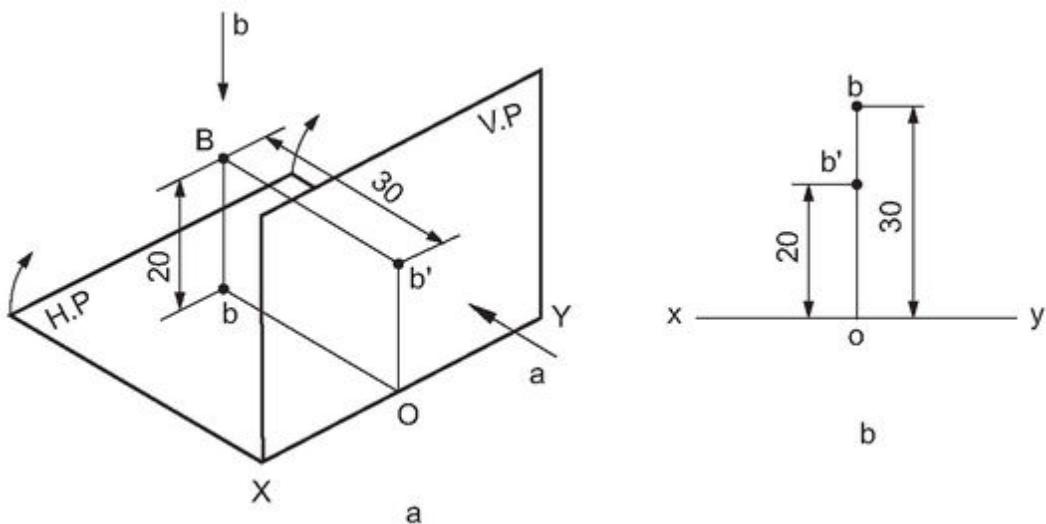
The following may be noted from the study of [Fig.7.2](#):

1. The line XY is the intersection line between H.P and V.P (refer [Figs.7.2a and b](#)). In [Fig. 7.2c](#), the line is represented by xy, which is known as the base or reference line. Actually, XY is the line about which the rotation of the plane (H.P) is made.
2. It is customary to use capital letters to specify the position of the points (objects) in space and lower case letters for their projections. As an example, for the point A in space, the front view, top view and side view are represented by  $a'$ ,  $a$  and  $a''$  respectively.
3. The ray of sight passing through the point A meets the corresponding plane of projection at right angle to it. This line is known as a projection line.
4. The line joining  $a$  and  $a'$ , intersects the reference line xy at right angle at o. This line is known as a projector.
5. The front view  $a'$  is above xy and top view  $a$  is below xy.
6. The distance  $a'o$  is equal to the distance of the point from H.P.
7. The distance  $ao$  is equal to the distance of the point from V.P.

## 7.2.2 Projections of a Point Situated in Second Quadrant

**Problem 2** A point B is 20 above H.P and 30 behind V.P. Draw its projections.

Figure 7.3a shows the position of the point B in the second quadrant. When the point is viewed in the direction **a** and assuming V.P to be transparent, the front view  $b'$  is obtained at the point of intersection between the ray of sight (towards the point) and V.P. Similarly, the top view  $b$  is obtained on H.P by viewing the point in the direction **b**.



**Fig.7.3 Projections of a point in second quadrant**

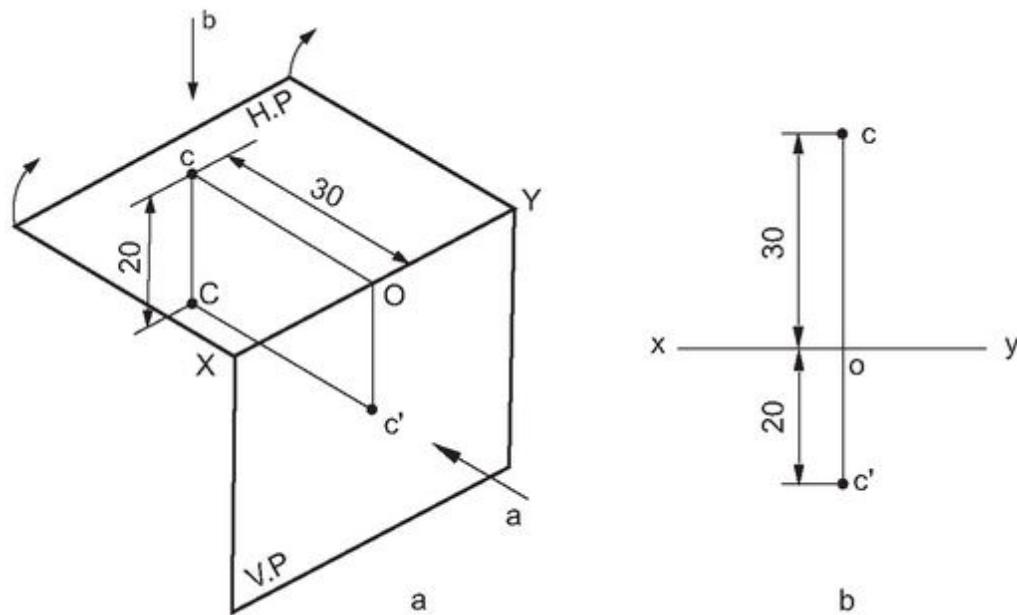
Figure 7.3b shows the relative positions of the views. These are obtained by rotating the H.P till it coincides with V.P. It may be noted that both the front and top views are above the reference line  $xy$ .

### 7.2.3 Projections of a Point Situated in Third Quadrant

**Problem 3** A point C is 20 below H.P and 30 behind V.P. Draw its projections.

Figure 7.4a shows the position of the point C in the third quadrant.  $c'$  and  $c$  are the front and top views obtained on V.P and H.P, by viewing the point in the directions **a** and **b** respectively. Here, it is assumed that V.P is transparent.

Figure 7.4b shows the relative positions of the views. These are obtained by rotating the H.P till it comes in-line with V.P. It may be noted that the front view  $c'$  is below  $xy$  and the top view  $c$  is above  $xy$ .



**Fig.7.4 Projections of a point in third quadrant**

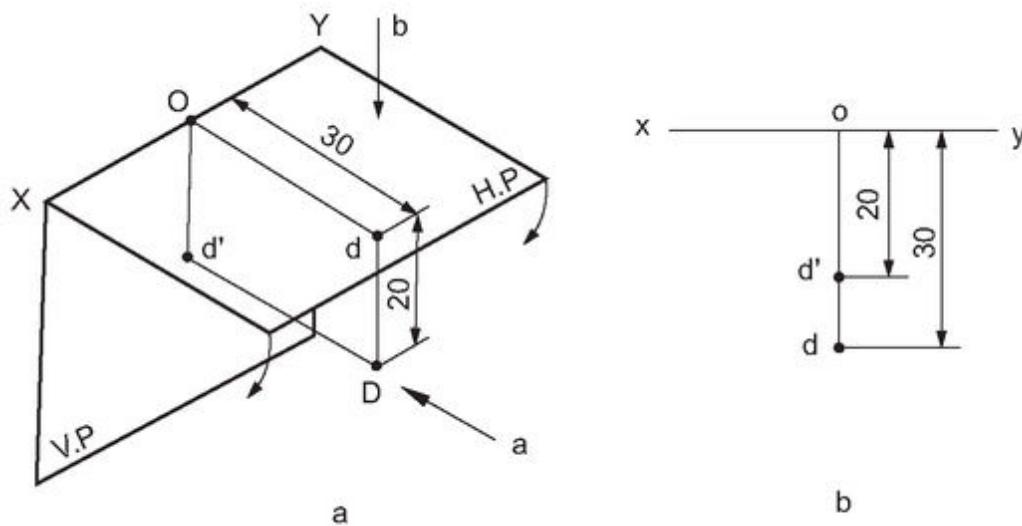
#### 7.2.4 Projections of a Point Situated in Fourth Quadrant

**Problem 4** A point D is 20 below H.P and 30 in front of V.P. Draw its projections.

Figure 7.5a shows the position of the point D in fourth quadrant.  $d'$  and  $d$  are the front and top views obtained on

V.P and H.P by viewing the point in the directions **a** and **b** respectively. Here, it is assumed that H.P is transparent.

[Figure 7.5b](#) shows the relative positions of the views. These are obtained by rotating H.P till it coincides with V.P. It may be noted that both the front and top views are situated below the reference line  $xy$ .



**Fig.7.5 Projections of a point in fourth quadrant**

### 7.2.5 Projections of a Point Situated on H.P and in Front of V.P

**Problem 5** A point  $E$  is on H.P and 30 in front of V.P. Draw its projections.

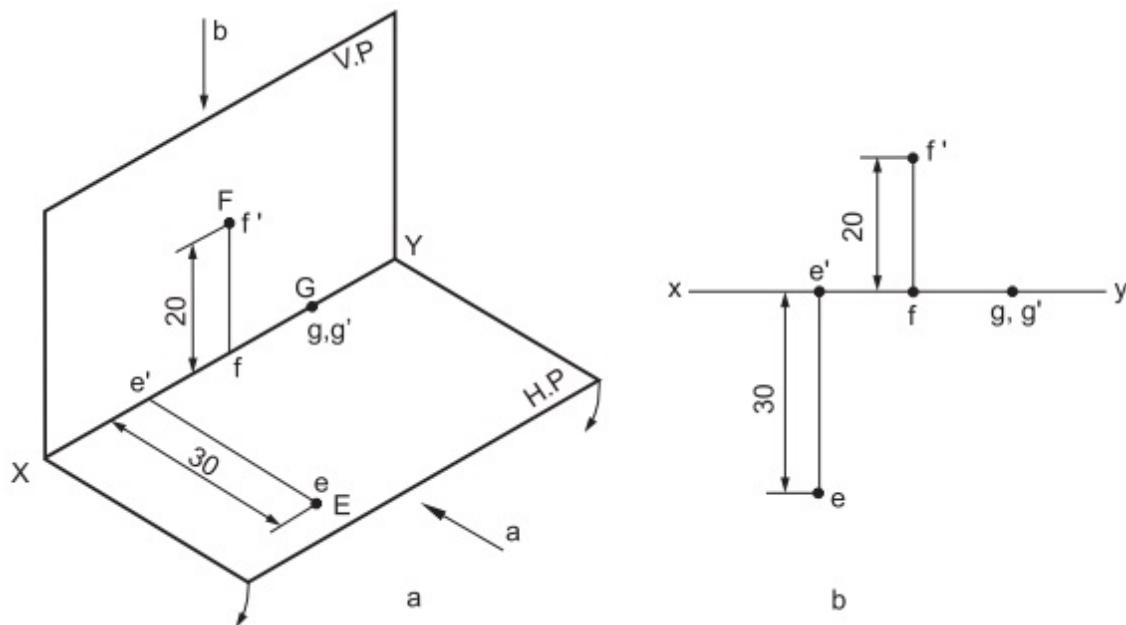
### 7.2.6 Projections of a Point Situated on V.P and Above H.P

**Problem 6** A point  $F$  is on V.P and 20 above H.P. Draw its projections.

### 7.2.7 Projections of a Point Situated on Both H.P and V.P

**Problem 7** A point  $G$  is lying on both H.P and V.P. Draw its projections.

All the above three cases are shown in Fig. 7.6a. Figure 7.6b shows the relative positions of the views for each case. The following may be noted from the Fig. 7.6b:



**Fig.7.6**

1. When a point lies on H.P, its front view will lie on  $xy$ .
2. When a point lies on V.P, its top view will lie on  $xy$ .
3. When a point lies both on H.P and V.P, its front and top views will lie on  $xy$ .

The student is advised to understand the principles



of projection for the cases when (i) a point lies on H.P but behind V.P and (ii) on V.P but below H.P.

## 7.3 THREE VIEW PROJECTIONS OF POINTS

In Chapter 6, it is mentioned that, when the two views of an object are not sufficient to describe the shape completely, then it is necessary to go for a third view, preferably a side view.

Generally, for obtaining projections of a solid, the solid is imagined to be either in the first or third quadrant. The projections thus obtained are respectively known as the first and third angle projections. It is not advisable to imagine the solid to be placed in the second and fourth quadrants because in both the cases the front and top views will lie on one side of xy. This is not convenient for understanding the views.

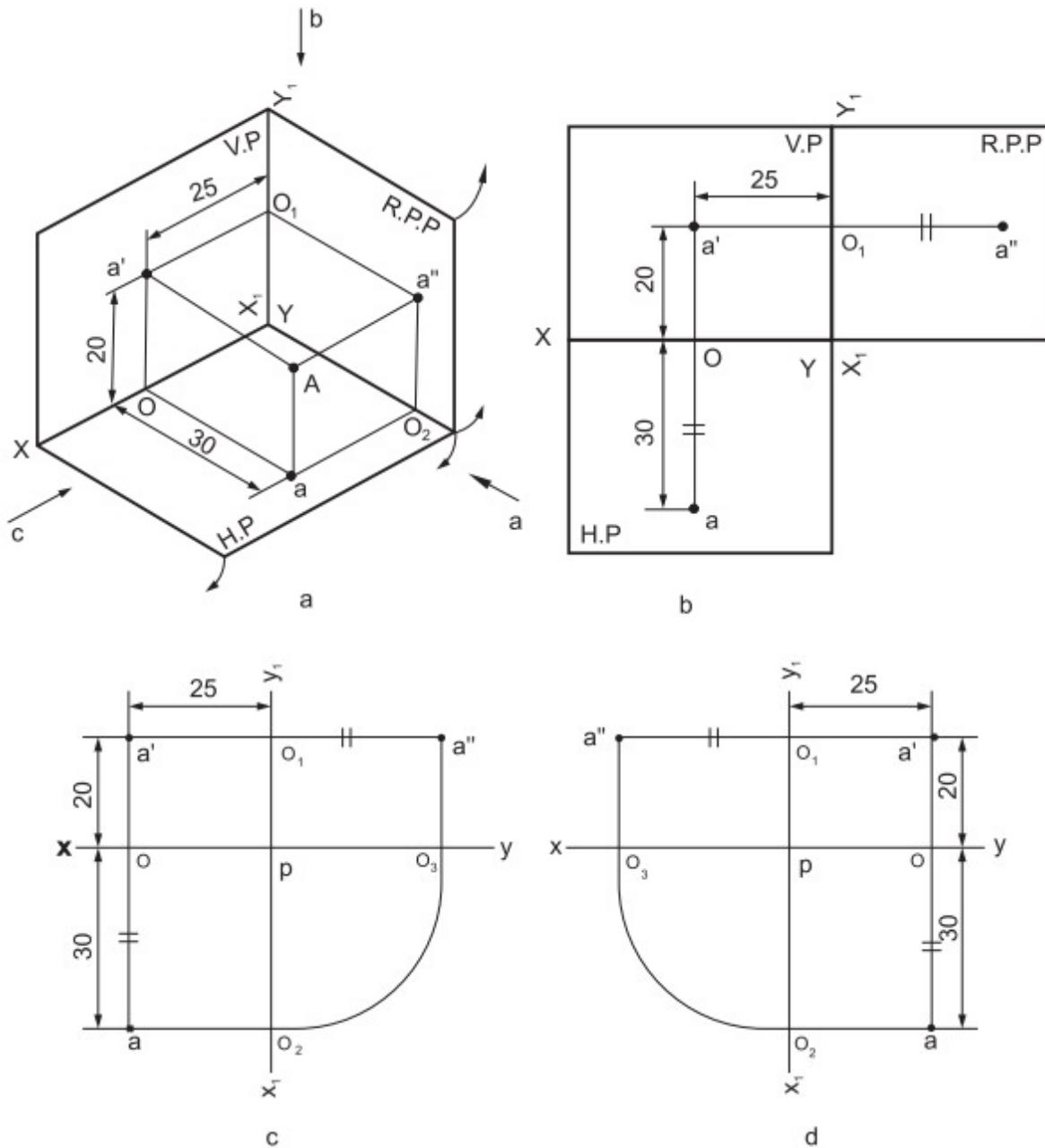
As a preliminary to the projections of solids, the treatment here is presented for the case when the point lies in the first quadrant only.

### 7.3.1 Projections of a Point Situated in First Quadrant

**Problem 8** A point A is 20 above H.P, 30 in front of V.P and 25 in front of P.P. Draw (i) front view, (ii) top view and (iii) left side view of the point.

Figure 7.7a shows the position of the point A in the first quadrant along with the right profile plane (R.P.P), which is

perpendicular to both H.P and V.P. The projection of the point A on the P.P is obtained by viewing the point in the direction **c**.



**Fig.7.7 Three view projections of a point**

The view obtained on the P.P, viz.,  $a''$  is known as the left side view because, the point is viewed from the left side and projected on to the right profile plane.

[Figure 7.7b](#) shows the relative positions of the views along with the three planes of projection. To obtain the views on a plane sheet, the P.P is rotated till it is in-line with V.P. [Figure 7.7c](#) shows the relative positions of the views, without showing the planes of projection. The line  $X_1 Y_1$  ([Fig. 7.7a](#)) is the line about which the rotation of P.P is made. It can be seen from [Fig. 7.7b](#), the distance  $a' o_1$  is equal to the distance of the point A from P.P, where  $O_1$  is the point on the line  $X_1 Y_1$ . The line passing through the points  $a'$ ,  $O_1$ ,  $a''$  is called the projector and is perpendicular to the line  $X_1 Y_1$ . It may be noted that in this case, the left side view comes to the right side of front view.

The right side view of point A is obtained by projecting on to the left profile plane (refer [Fig.7.7d](#)).



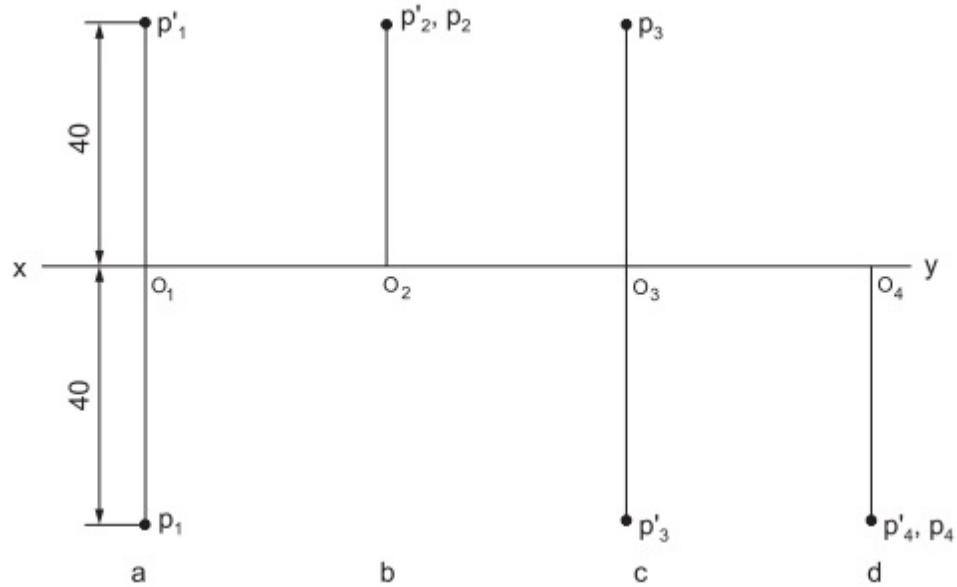
The side view may be obtained as follows ([Figs. 7.7c and d](#)):

1. Through the point  $a$ , draw a line parallel to  $xy$ , intersecting the line  $x_1 y_1$  at  $o_2$ .
2. With  $p$  as centre and radius  $po_2$ , draw an arc intersecting the line  $xy$  at  $o_3$ .
3. Locate the side view  $a''$  at the intersection between the projectors drawn from  $o_3$  and  $a'$ .

## 7.4 EXAMPLES

**Problem 9** A point P is 40 from both the principal planes of projection. Draw its projections.

There are four possible positions  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$ , as there are four quadrants in space.



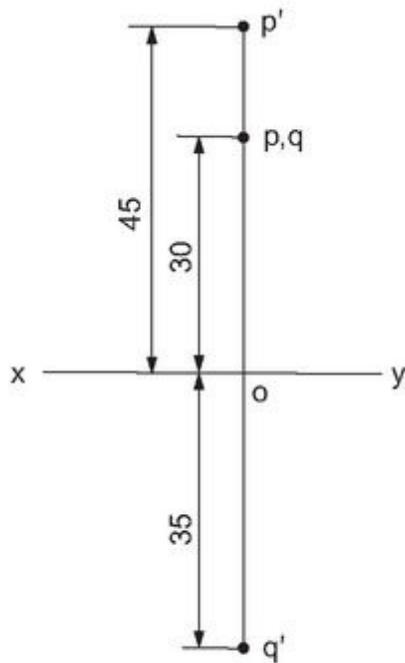
**Fig.7.8**

**Construction (Fig.7.8a)**

1. Draw the reference line  $xy$ .
2. Through any point  $o_1$  on it, draw a projector.
3. On the above projector, mark  $p'_1$  and  $p_1$  above and below  $xy$  respectively such that,  $o_1p'_1 = o_1p_1 = 40$ .  $p'_1$  and  $p_1$  are the projections of the point  $P_1$  assuming it to be situated in the first quadrant. [Figure 7.8b to d](#) show the projections of the points  $P_2$ ,  $P_3$ , and  $P_4$ , assuming the points to be in the second, third and fourth quadrants respectively.

**Problem 10** A point 30 above  $xy$  line is the top view of two points  $P$  and  $Q$ . The front view of  $P$  is 45 above the H.P while that of the point  $Q$  is 35 below the H.P. Draw the projections of the points and state their positions with reference to the principal planes and the quadrants in which they lie.

**Construction (Fig.7.9)**



**Fig.7.9**

1. Draw the reference line  $xy$ .
2. Through any convenient point  $o$ , draw a projector and mark  $p$  and  $q$ , the top views of  $P$  and  $Q$ , 30 above  $xy$ .
3. Locate  $p'$ , 45 above  $xy$ ; the front view of  $P$ .
4. Locate  $q'$ , 35 below  $xy$ ; the front view of  $Q$ .

The following may be concluded from the Fig.7.9: The point  $P$  lies 45 above H.P and 30 behind V.P and hence lies in II quadrant. The point  $Q$  lies 35 below H.P and 30 behind V.P and hence lies in III quadrant.

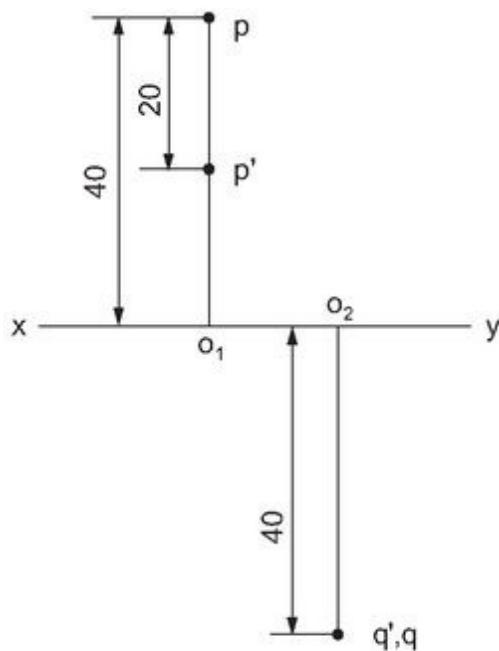
**Problem 11** State the quadrants with the help of drawing, in which the following points are situated:

- i. A point  $P$ ; its top view is 40 above  $xy$ ; the front view 20 below the top view.

ii. A point Q; its projections coincide with each other at 40 below xy.

### **Construction (Fig.7.10)**

1. Draw the reference line xy.
2. Draw a projector at point  $o_1$  and locate the top view  $p$ , 40 above xy and front view  $p'$ , 20 below p; representing the views of P.  
P is located in II quadrant because both of its views are above the xy line.



**Fig.7.10**

3. Draw a projector at point  $o_2$  and locate the top and front views of Q, 40 below the xy line. Q is located in IV<sup>th</sup> quadrant because both of its views are below the xy line.

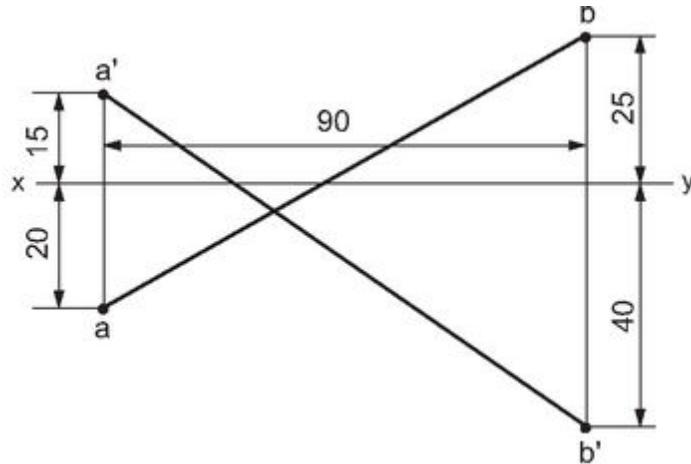
**Problem 12** A point A is 15 above H.P and 20 in front of V.P. Another point B is 25 behind V.P and 40 below H.P.

*Draw the projections of A and B, keeping the distance between the projectors equal to 90. Draw straight lines, joining (i) the top views and (ii) the front views.*

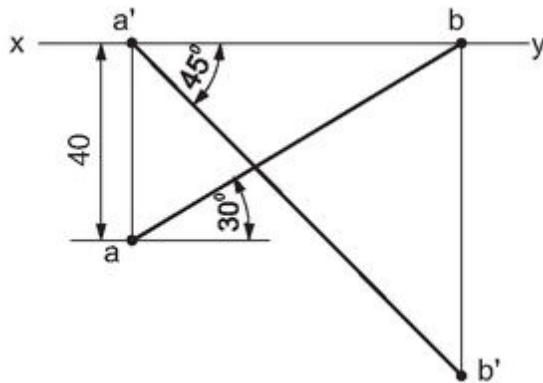
**Construction (Fig. 7.11)**

1. Draw two projectors, 90 apart, on the reference line  $xy$ .
2. Locate the front and top views of the points A and B, on the above projectors.
3. Join  $a'$ ,  $b'$  and  $a, b$ .  
 $a'b'$  and  $ab$  are the required straight lines.

**Problem 13** *The point A is on H.P and 40 in front of V.P. Another point B is on V.P and below H.P. The line joining their front views makes an angle of  $45^\circ$  with  $xy$ , while the line joining their top views makes an angle of  $30^\circ$ . Find the distance of the point B from H.P.*



**Fig. 7.11**



**Fig.7.12**

**Construction (Fig. 7.12)**

1. Draw the reference line  $xy$ .
2. On any projector chosen arbitrarily, locate the projections of the point A. For the given location of the point B; the front view  $b'$  lies below  $xy$  and the top view  $b$  lies on  $xy$ .
3. Through  $a$ , the top view of the point A, draw a line at  $30^\circ$  intersecting the reference line  $xy$  at  $b$ .
4. Through  $b$ , draw a projector.
5. Through  $a'$ , the front view of the point A, draw a line at  $45^\circ$  intersecting the above line at  $b'$ .

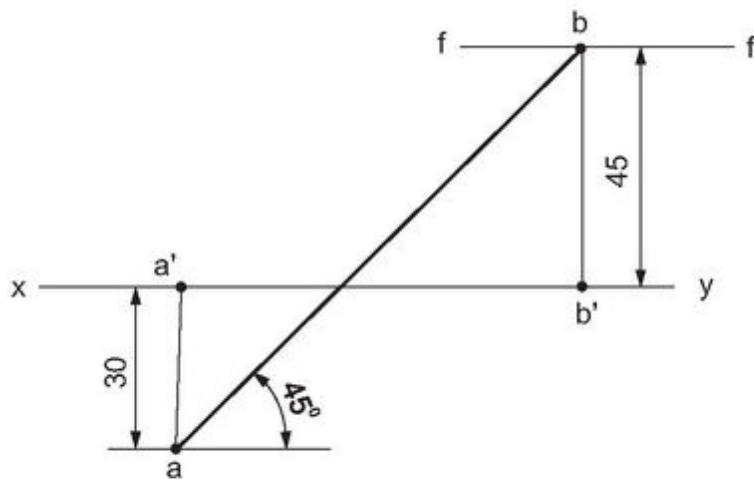
The length of the line  $bb'$  is the distance of the point B from H.P.

6. With centre  $p$  and radius  $po_3$ , draw an arc meeting the line  $x_1 y_1$  at  $o_2$ .
7. Through  $o_2$ , draw a horizontal projector meeting the above projector (step no. 2) at  $a$ , the top view of the point.

**Problem 14** Two points A and B are on H.P; the point A being 30 in front of V.P, while B is 45 behind V.P. The line

joining their top views makes an angle of  $45^\circ$  with  $xy$ . Find the horizontal distance between the two points.

**Construction (Fig. 7.13)**



**Fig 7.13**

1. Draw the reference line  $xy$ .
2. On any projector chosen arbitrarily, locate the projections of the point A. For the given location of the point B, the front view  $b'$  lies on  $xy$  and the top view  $b$  lies above  $xy$ .
3. Draw a line  $t-t$ , parallel to and 45 above  $xy$ .
4. Through  $a$ , the top view of the point A, draw a line at  $45^\circ$  intersecting the above line,  $t-t$  at  $b$ .
5. Through  $b$ , draw a projector meeting the line  $xy$  at  $b'$ .

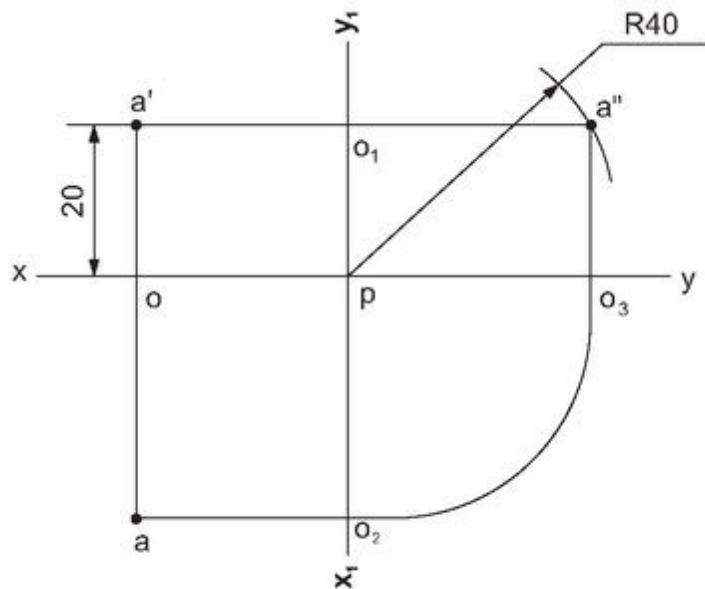
The length of the line  $a'b'$  is the horizontal distance between the two points A and B.

**Problem 15** A point A is 20 above H.P and in the first quadrant. Its shortest distance from the reference line XY is

40. Draw the projections of the point and determine its distance from V.P.

**Construction (Fig. 7.14)**

1. Draw the reference lines  $xy$  and  $x_1 y_1$ , intersecting at  $p$ .
2. Draw a projector through any convenient point  $o$  on  $xy$  and locate the front view  $a'$  of the point, 20 above  $xy$ .
3. Through  $a'$ , draw a horizontal projector.
4. With centre  $p$  and radius 40, draw an arc intersecting the above projector at  $a''$ , the left side view of the point.
5. Through  $a''$ , draw a projector meeting the line  $xy$  at  $o_3$ .
6. With centre  $p$  and radius  $po_3$ , draw an arc meeting the line  $x_1 y_1$  at  $o_2$ .
7. Through  $o_2$ , draw a horizontal projector meeting the above projector (step no. 2) at  $a$ , the top view of the point.



**Fig 7.14**



oa is the distance of the point from V.P.

op is the distance of the point from the profile plane.

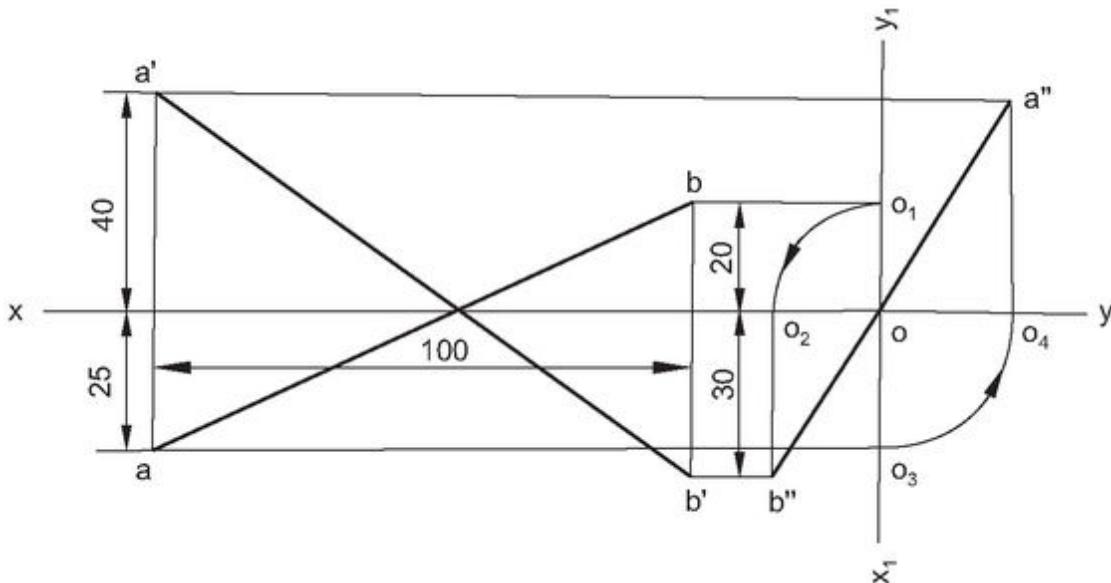
**Problem 16** A point A is 40 above H.P and 25 in front of V.P. Another point B is 20 behind V.P and 30 below H.P. The horizontal distance between the points is 100. Draw the three projections of the points and join their front views, top views and side views.

**Construction (Fig.7.15 )**

1. Draw two projectors, 100 apart, on the reference line xy.
2. Locate the front and top views of the points A and B on the above projectors.

Point A is in first quadrant and point B is in third quadrant. Keeping this in view, obtain the side views of the points as shown.

3. Join  $a'$ ,  $b'$ ;  $a$ ,  $b$ ; and  $a''$ ,  $b''$ .



**Fig.7.15**

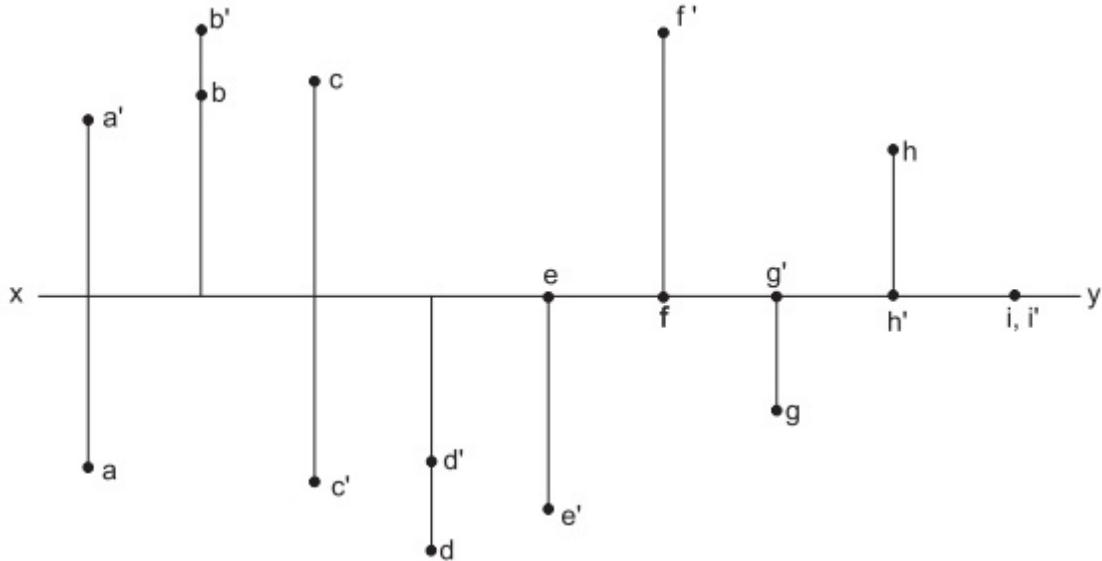
## **EXERCISES**

- 7.1 Draw the projections of the following points, keeping the distance between the projectors as 25 on the same reference line:
- A -25 above H.P and 45 in front of V.P
  - B -35 above H.P and 50 behind V.P
  - C -40 below H.P and 30 behind V.P
  - D -30 below H.P and 40 in front of V.P
  - E -50 above H.P and on V.P
  - F -45 below H.P and on V.P
  - G -on H.P and 35 in front of V.P
  - H -on H.P and 25 behind V.P
  - I - on both H.P and V.P
- 7.2 State the quadrants in which the following points are located:
- A -front and top views are above xy
  - B -front view below xy and top view above xy
  - C -front view above xy and top view below xy
  - D -front and top views are below xy
- 7.3 Mention the relative positions of the projections of the following points with respect to xy:
- A -in the fourth quadrant
  - B -in the second quadrant

C -in the third quadrant

D -in the first quadrant

- 7.4 Indicate the positions of the points shown in Fig. 7.16; with respect to the planes of projection.



**Fig.7.16**

- 7.5 A point at 25 above the reference line XY is the front view of two points A and B. The top view of A is 40 behind V.P and the top view of B is 50 in front of V.P. Draw the projections of the points and state their positions relative to the planes of projection and the quadrants in which they lie.
- 7.6 For the points A, B, C and D in Problems 7.1, add the left side views.
- 7.7 A point A is 40 above H.P and 25 in front of V.P. Another point B is 20 behind V.P and 30 below H.P. The horizontal distance between the points is 100. Draw the three projections of the points A and B and join their front views, top views and left side views.

- 7.8 Draw the projections of a point B, lying in first quadrant such that, its shortest distance from the reference line XY is 50 and it is equi-distant from H.P and V.P.

The point is 30 from P.P. Draw the projections of the point and determine its distances from H.P and V.P.

- 7.9 Two points P and Q are in the H.P. The point P is 30 in front of V.P and Q is behind the V.P. The distance between their projectors is 80 and line joining their top views makes an angle of  $40^\circ$  with xy. Find the distance of the point Q from the V.P

- 7.10 Two pegs fixed on a wall are 4.5m apart. The distance between the pegs measured parallel to the floor is 3.6 m. If one peg is 1.5m above the floor, find the height of the second peg and the inclination of the line joining the two pegs, with the floor.

- 7.12 A point P is 20 below the H.P and lies in the third quadrant. Its shortest distance from XY is 40. Draw its projections.

## REVIEW QUESTIONS

- 7.1 How the following are generated in space: (i) Solid, (ii) plane and (iii) straight line?
- 7.2 What is a projector?
- 7.3 Why second and fourth angles of projections are not followed in practice?
- 7.4 How the side view of a point is obtained from its front and top views?

- 7.5 Given the orthographic projections of a point, how its location in space may be determined?
- 7.6 What is the difference between the left and right side views?

## OBJECTIVE QUESTIONS

- 7.1 A straight line is generated as the \_\_\_\_\_ of a moving point.
- 7.2 The \_\_\_\_\_ view of a point is obtained as the intersection point between the ray of sight and V.P.
- 7.3 The top view of a point is obtained as the intersection point between the ray of sight and \_\_\_\_\_.
- 7.4 To represent the projections on a paper, the planes must be rotated such that, \_\_\_\_\_ quadrant always opens out.
- 7.5 The projecting lines meet the plane of projection at an angle of  $90^\circ$  to it.

(True /False)

- 7.6 The line joining the projections of a point, intersects the line  $xy$  at an angle of  $90^\circ$ / other than  $90^\circ$ .
- 7.7 The position of a point in space may be determined from any one of its projections.

(True/False)

- 7.8 The distance of a point from H.P, is marked from  $xy$  to (a) top view, (b) front view, (c) side view.

( )

7.9 When a point lies on H.P, its front view will lie (a) on xy, (b) below xy, (c) above xy.

( )

7.10 When both the projections of a point lie below xy, the point is situated in (a) first quadrant, (b) second quadrant, (c) third quadrant, (d) fourth quadrant.

( )

7.11 When a point lies on V.P, its top view lies on xy.

(True/False)

7.12 When a point is above H.P, its front view is \_\_\_\_ xy.

7.13 When a point is \_\_\_\_\_ V.P, its top view is above xy.

7.14 When a point lies — — — —, its two views lie on xy.

## ANSWERS

7.1 Locus

7.2 Front

7.3 H.P

7.4 first

7.5 True

7.6 At an angle of  $90^\circ$

7.7 False

7.8 b

7.9 a

7.10 d

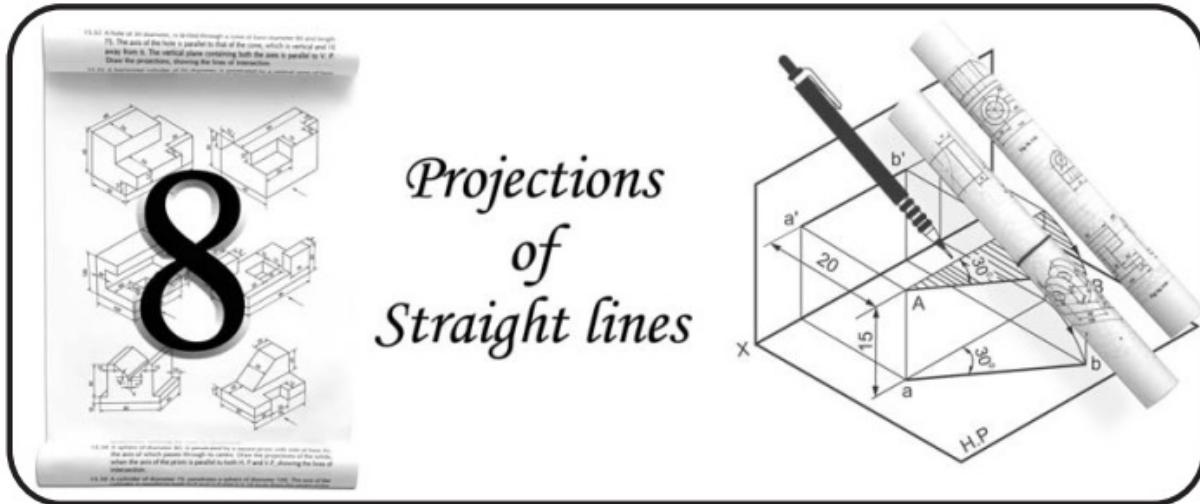
7.11 True

7.12 above

7.13 behind

7.14 on both HP and VP

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## 8.1 INTRODUCTION

In Chapter 7, it is stated that a straight line may be generated by a point moving in one direction. A straight line may also be defined as the shortest distance between two points. A straight line may be located in space either by specifying the location of the end points or by specifying the location of one end point and the direction.

## 8.2 TWO VIEW PROJECTIONS OF STRAIGHT LINES

The following are the possible positions of a straight line, with respect to the planes of projection:

1. Parallel to both the planes
2. Perpendicular to one plane
3. Inclined to one plane and parallel to the other
4. Inclined to both the planes

5. Contained by a plane, perpendicular to both the principal planes

## 8.2.1 Straight line Parallel to Both the Planes

When a line is parallel to any plane, its projection on that plane is a straight line of the same length. This is because, in orthographic projections, the line is imagined to be viewed from infinity. Hence, the rays of sight are parallel to each other. When they pass through the end points of the straight line, meet the plane of projection at two points, the distance between them is equal to the length of the projected line itself.

**Problem 1** A line AB of 50 long, is parallel to both H.P and V.P. The line is 40 above H.P and 30 in front of V.P. Draw the projections of the line.

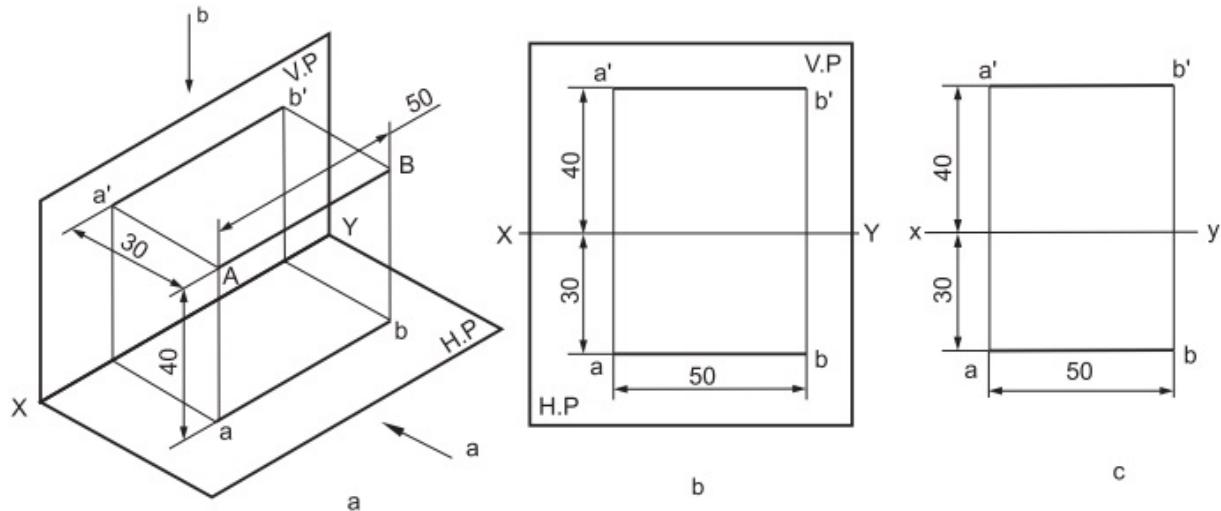
Figure 8.1a shows the position of the line AB in the first quadrant. The points a', b' on V.P and a, b on H.P are the front and top views of the ends A and B of the line AB. The lines a'b' and ab are the front and top views of the line AB respectively.

Figure 8.1b shows the relative positions of the views along with the planes, after rotating H.P till it is in-line with V.P. Figure 8.1c shows the relative positions of the views only.

### Construction (Fig.8.1c)

1. Draw the reference line xy and draw a projector at any convenient point on it.

2. Locate the projections of the end point A of the line a', at 40 above xy and a, at 30 below xy.
  3. Through a', and a, draw lines a'b' and ab, parallel to xy and of length 50.
  4. Join the points, b' and b by a projector.
- a' b' and ab are the projections of the line AB.



**Fig 8.1**

## 8.2.2 Straight Line Perpendicular to one Plane

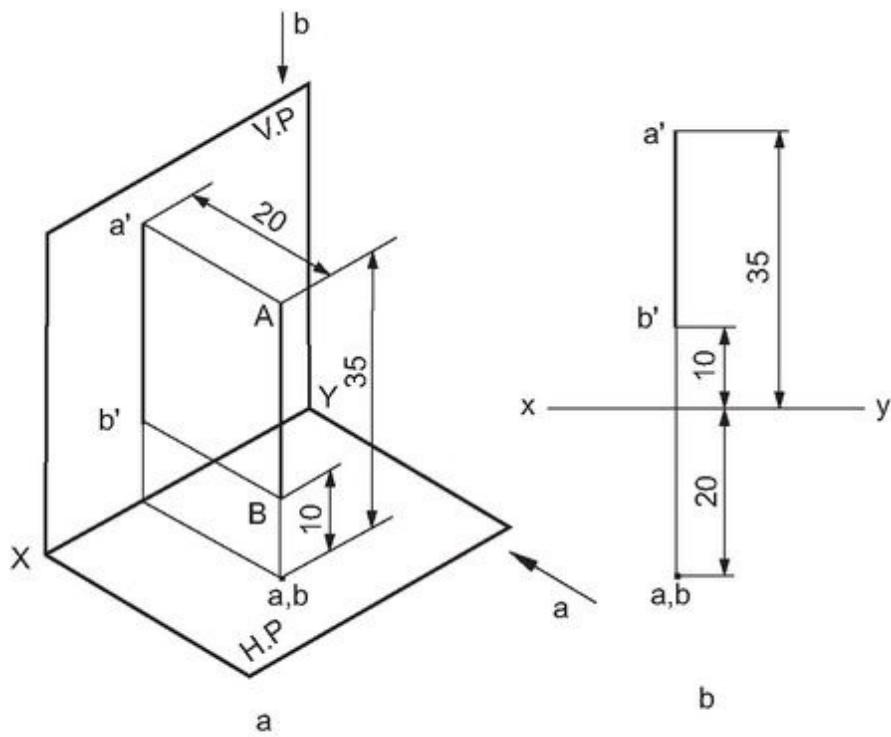
When a line is perpendicular to one of the planes, it is evident that it is parallel to the other plane. Further, when a line is parallel to a plane, the length of the projection on that plane is equal to the true length of the line.

**Problem 2** A line AB of 25 long, is perpendicular to H.P and parallel to V.P. The end points A and B of the line are 35 and 10 above H.P respectively. The line is 20 in front of V.P. Draw the projections of the line.

Figure 8.2a shows the position of the line AB in the first quadrant. As the line is parallel to V.P, the length of the front view is equal to the true length of the line and the top view appears as a point.

### **Construction (Fig.8.2b)**

1. Draw the front view  $a'b'$ , a line perpendicular to  $xy$  such that,  $a'$  and  $b'$  are 35 and 10 above  $xy$  respectively.
2. Locate the top view of the line ab; a point 20 below  $xy$ .



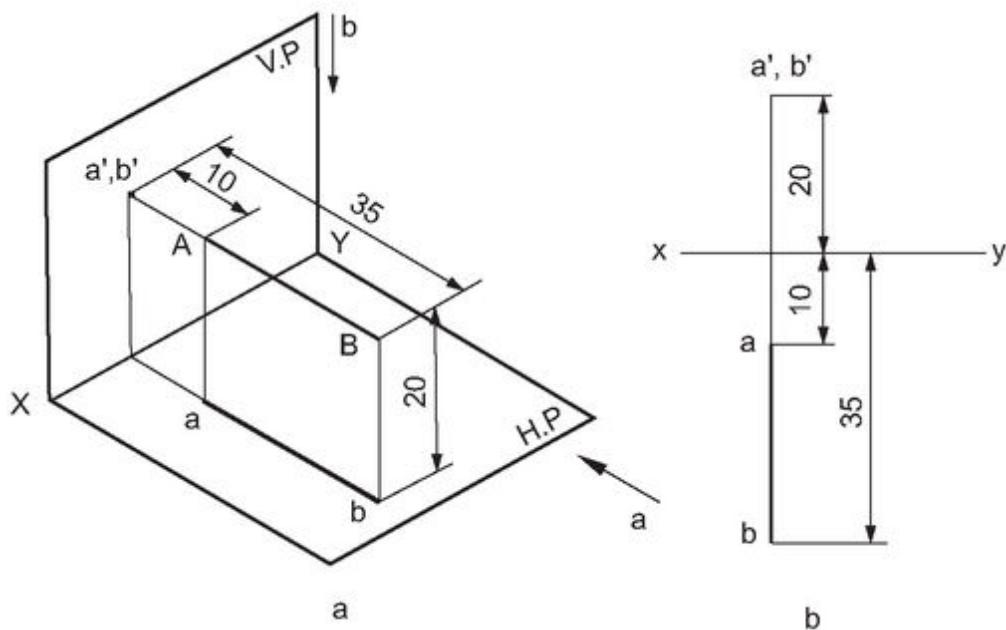
**Fig 8.2**

**Problem 3** A line AB of 25 long, is perpendicular to VP and parallel to HP. The end points A and B of the line are 10 and 35 in front of VP respectively. The line is 20 above HP. Draw its projections.

Figure 8.3a shows the position of the line AB in the first quadrant. As the line is parallel to H.P, the length of the top view is equal to the true length of the line and the front view appears as a point.

### **Construction (Fig.8.3b)**

1. Draw a line perpendicular to  $xy$  (the top view of the line) such that, a and b are 10 and 35 below  $xy$  respectively.
2. Locate the front view of the line  $a'$ ,  $b'$ ; a point 20 above  $xy$ .



**Fig.8.3**



In both the above cases, the projectors drawn, connecting both the views, intersect the reference line  $xy$  at right angle.

### **8.2.3 Straight Line Inclined to one Plane and Parallel to the Other**

The problems of this nature are normally solved in two stages. In the first stage, the line is assumed to be parallel to both the planes and projections are drawn. In the second stage, the line is rotated to make the required angle and the final views are obtained.

**Problem 4** A line AB is 30 long and inclined at  $30^\circ$  to H.P and parallel to V.P. The end A of the line is 15 above H.P and 20 in front of V.P. Draw the projections of the line.

Figure 8.4a shows the position of the line in the first quadrant, along with the views obtained by projection on H.P and V.P.

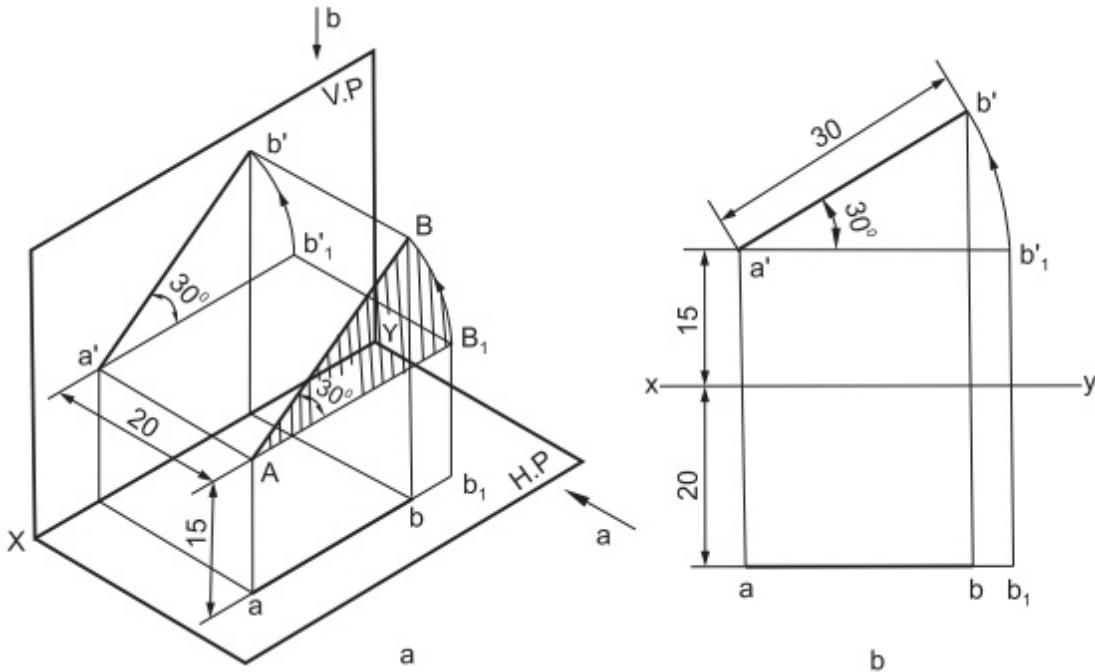
### **Construction (Fig.8.4b)**

**Stage I** Assume that the line is parallel to both H.P and V.P.

1. Draw the projections  $ab_1$  and  $a'b_1'$  of the line.

**Stage II** Rotate the line such that, it makes the given angle with H.P.

2. Rotate the line  $a'b_1'$  by  $30^\circ$ , to the position  $a'b'$ .
3. Drop a projector from  $b'$  till it meets the line  $ab_1$  at b.  
 $a'b'$  and  $ab$  are the required projections.



**Fig.8.4**



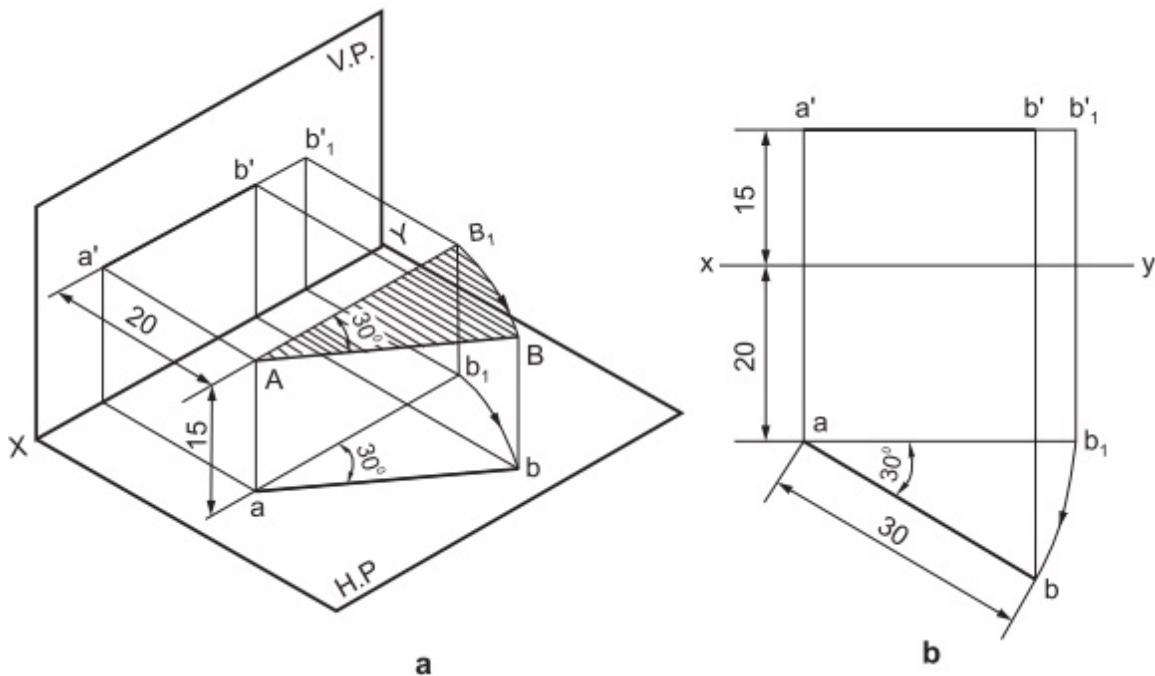
1. The distance 20 of the point B from V.P remains unchanged, irrespective of the angle of inclination with H.P. The actual position of b on  $ab_1$  depends upon the angle  $\theta$ . In other words, the line  $ab_1$  may be termed as the locus of the top view of the end point B.
2. It may be noted that the length  $a'b'$  is the true length of the given line AB and the inclination of the front view with xy is the true inclination of the line with H.P.

**Problem 5** A line AB is 30 long and inclined at  $30^\circ$  to VP and parallel to H.P. The end A of the line is 15 above H.P and 20 in front of V.P. Draw its projections.

Figure 8.5a shows the position of the line in the first quadrant, along with the views obtained by projection.

**Construction (Fig.8.5b)**

1. Draw the projections  $a'b_1'$  and  $ab_1$  for the line, assuming that it is parallel to both H.P and V.P.
2. Rotate the line  $ab_1$  by  $30^\circ$ , to the position  $ab$ .
3. Drop a projector from  $b$  till it meets the line  $a'b_1'$  at  $b'$ .  
 $a'b'$  and  $ab$  are the required projections.



**Fig.8.5**



1. The line  $a'b_1'$  is the locus of the front view of the end point B.
2. The length  $ab$  is the true length of the given line and the inclination of the top view with  $xy$  is the true inclination of the line with V.P.

## 8.2.4 Straight Line Inclined to Both the Planes

A line inclined to both H.P and V.P is called an oblique line. The methods discussed in the above two problems may be combined to draw the projections of oblique lines.

**Problem 6** A line AB of 100 length, is inclined at an angle of  $30^\circ$  to H.P and  $45^\circ$  to V.P. The point A is 15 above H.P and 20 in front of V.P. Draw the projections of the line.

### **Construction (Fig.8.6)**

**Stage I** Assume that the line is inclined at  $30^\circ$  to H.P and parallel to V.P

1. Draw the projections  $a'b_1'$  and  $ab_1$  of the line  $AB_1 = AB$  (Figs.8.6a,b).

Keeping the inclination  $30^\circ$  constant, rotate the line  $AB_1$  to  $AB$ , till it is inclined at  $45^\circ$  to V.P. This process of rotation does not change the length of the top view  $ab_1$  and the distance of the point  $B_1 (=B)$  from H.P. Hence, (i) the length of  $ab_1$  is the final length of the top view and (ii) the line f-f, parallel to xy and passing through  $b_1'$  is the locus of the front view of the end point B.

**Stage II** Assume that the line is inclined at  $45^\circ$  to V.P and parallel to H.P

2. Draw the projections  $ab_2$  and  $a'b_2'$  of the line  $AB_2 = AB$  (Figs.8.6c,d).

Extending the discussion on the preceding stage to the present one, the following may be concluded:

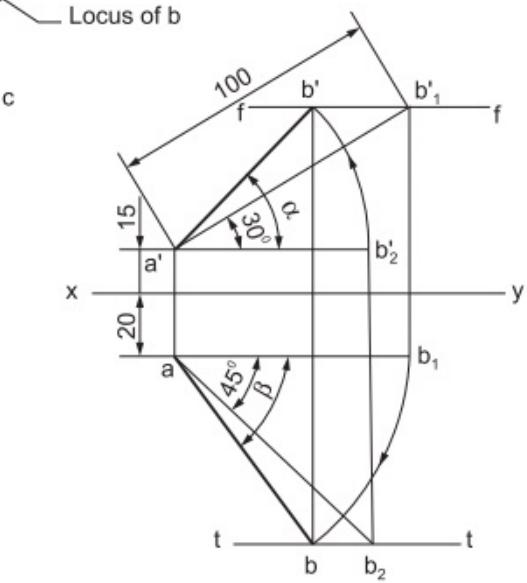
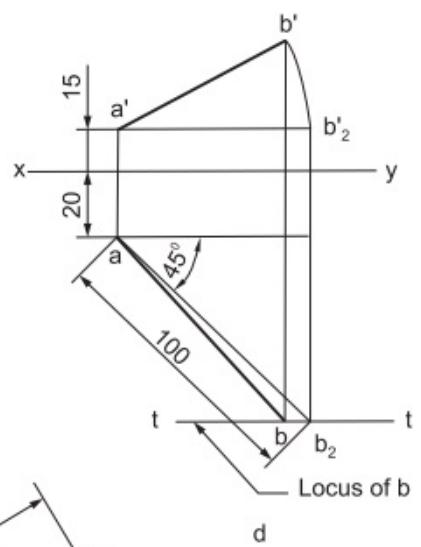
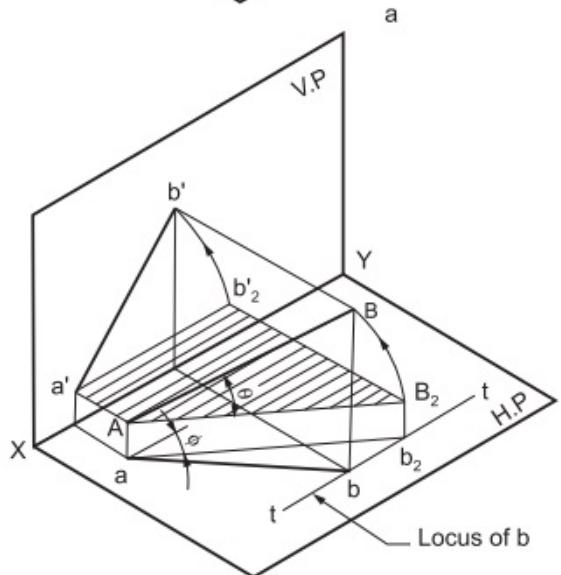
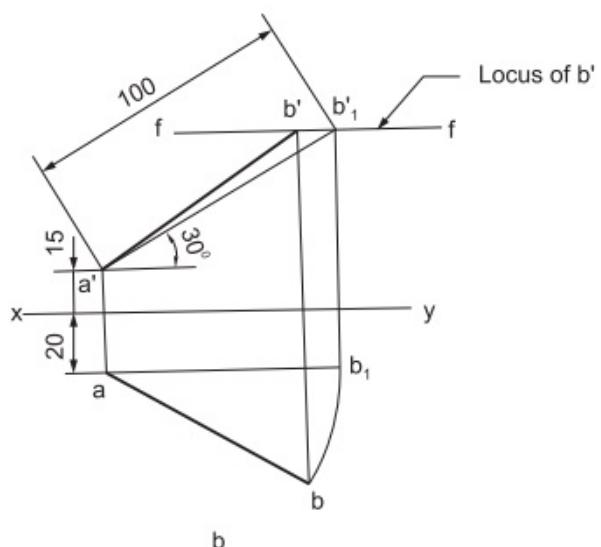
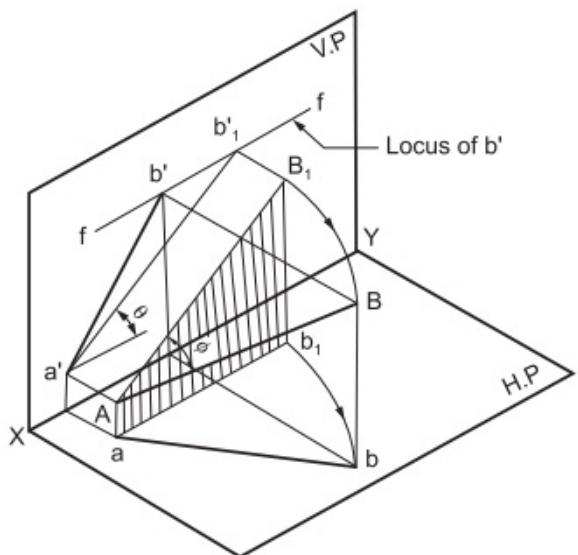
- (i) The length  $a'b_2'$  is the final length of the front view and (ii) The line t-t, parallel to xy and passing through  $b_2$  is the locus of the top view of the end point B.

### **Stage III Combine stages I and II**

3. Obtain the final projections by combining the results from stages I and II, as detailed below ([Fig.8.6e](#)):
  - (i) Draw the projections  $a'b_1'$  ( $=AB$ ) at  $30^\circ$  and  $ab_2$  ( $=AB$ ) at  $45^\circ$  with  $xy$ , after locating the projections of A.
  - (ii) Obtain the projections  $a'b_2'$  and  $ab_1$ , parallel to  $xy$ .
  - (iii) Draw the lines, f-f and t-t, the loci, parallel to  $xy$  and passing through  $b_1'$  and  $b_2$  respectively.
  - (iv) With centre  $a'$  and radius  $a'b_2'$ , draw an arc meeting f-f at  $b'$ .
  - (v) With centre  $a$  and radius  $ab_1$ , draw an arc meeting t-t at  $b$ .
  - (vi) Join  $a'$ ,  $b'$  and  $a$ ,  $b$ ; forming the required projections.

The following may be noted from [Fig.8.6e](#):

1. The points  $b'$  and  $b$  lie on a single projector.
2. The projections  $a'b'$  and  $ab$  make angles  $\alpha$  and  $\beta$  with  $xy$ , which are respectively greater than  $30^\circ$  ( $\theta$ ) and  $45^\circ$  ( $\phi$ ). The angles  $\alpha$  and  $\beta$  are known as the apparent angles.

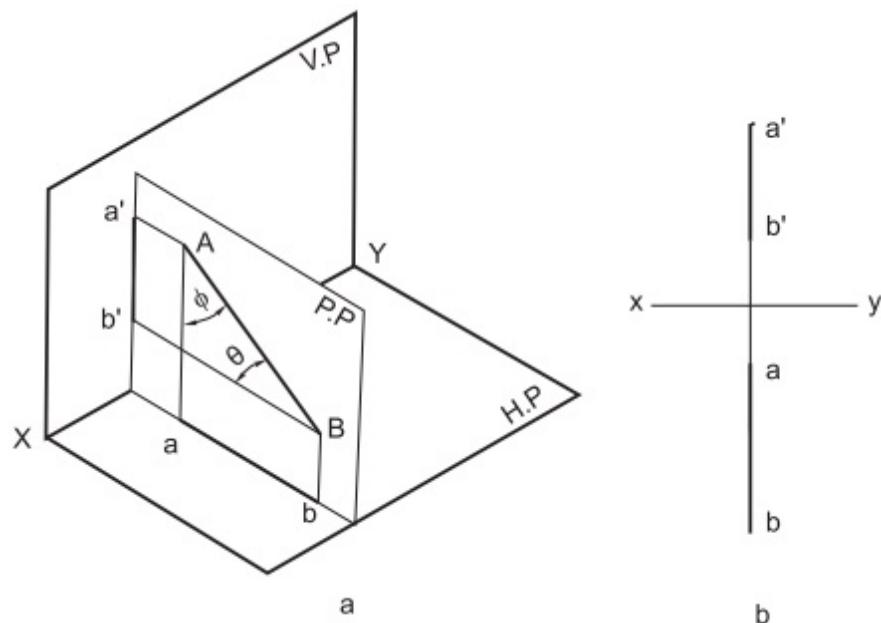


**Fig.8.6**

### **8.2.5 Straight Line Contained by a Plane (Profile Plane) Perpendicular to Both H.P and V.P**

When a line is contained by a plane which is perpendicular to both H.P and V.P; the sum of the inclinations  $\theta$  and  $\phi$  with H.P and V.P is equal to  $90^\circ$ .

Figure 8.7a shows the position of the line in the first quadrant along with the views obtained by projection on H.P and V.P. Figure 8.7b shows the relative positions of the front and top views of the line. It may be noted that both the front and top views, i.e.,  $a'b'$  and  $ab$  lie on a single projector and are shorter than the true length of the line.



**Fig.8.7**

## 8.2.6 True Length and True Inclinations

When projections of a line are given, its true length and true inclinations with H.P and V.P are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane. Following are the methods employed for the purpose:

**Method I** *Rotating line method* In this, each view is made parallel to the reference line and the other view is projected from it. This is the exact reversal of the procedure adopted in article 8.2.4.

**Method II** *Trapezoidal method* In this, the line is rotated about its projections till it lies in H.P or V.P.

**Method III** *Auxiliary plane method* In this, the views are projected on auxiliary planes, parallel to each view (Refer Ch.10).

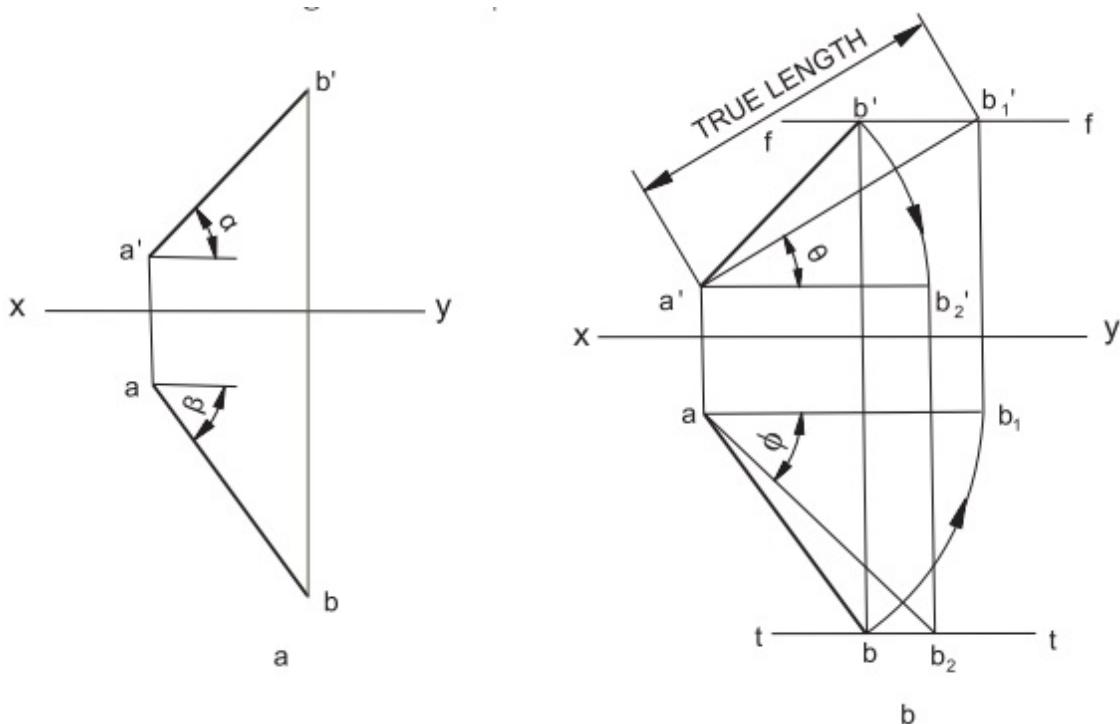
**Problem 7** *Figure 8.8a shows the projections of a line AB. Determine the true length of the line and its inclinations with H.P and V.P.*

**Method I**

**Construction (Fig.8.8b)**

1. Draw the given projections  $a'b'$  and  $ab$ .
2. Draw  $f-f$  and  $t-t$ , the loci passing through  $b'$  and  $b$  and parallel to  $xy$ .
3. Rotate  $a'b'$  to  $a'b_2'$ , parallel to  $xy$ .

4. Draw a projector through  $b_2'$  to meet the line t-t at  $b_2$ .
5. Rotate ab to  $ab_1$ , parallel to xy.
6. Draw a projector through  $b_1$  to meet the line f-f at  $b_1'$ .
7. Join  $a'$ ,  $b_1'$  and  $a$ ,  $b_2$ .
8. Measure and mark angles  $\theta$  and  $\phi$ .



**Fig.8.8**

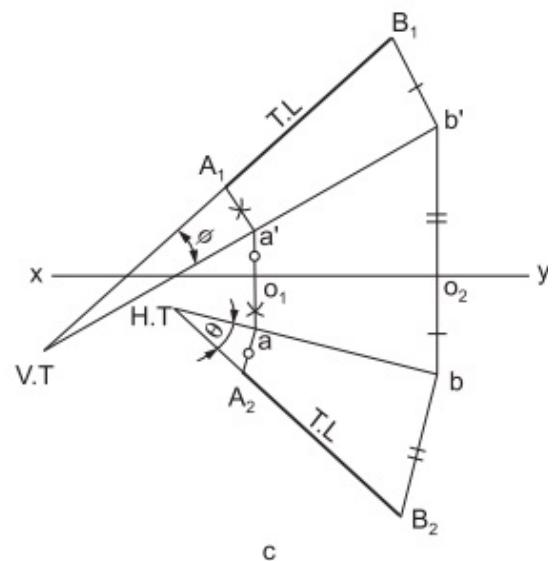
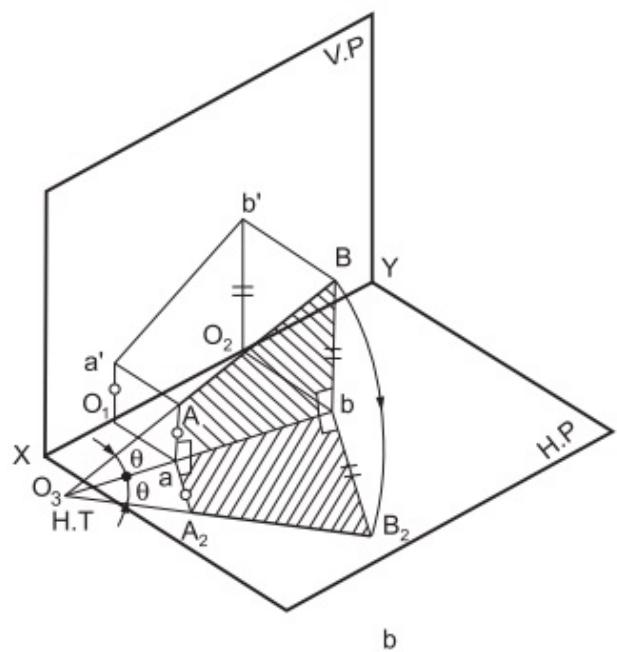
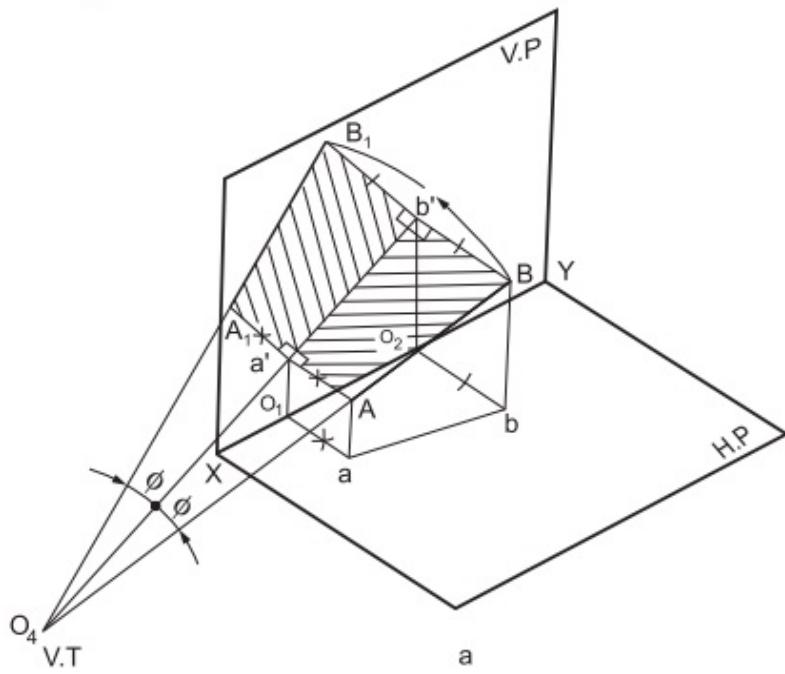
The length  $a'b_1'$  ( $= ab_2$ ) is the true length of the line and the angles  $\theta$  and  $\phi$  are the true inclinations of the line with H.P and V.P respectively.

### **Method II Trapezoidal method**

#### **Principle of the method (Figs.8.9 a, b)**

Figure 8.9a shows the position of the line AB in the first quadrant along with the projections  $a'b'$  and  $ab$  of the line. By geometry, the following may be observed:

1. In the trapezoid  $ABb'a'$ ,  $a'A$  and  $b'B$  are perpendicular to  $a'b'$  and are equal to  $ao_1$  and  $bo_2$  (the distances of  $a$  and  $b$  from  $xy$  in the top view).
2. The angle between  $AB$  and  $a'b'$  is the true angle of inclination of  $AB$  with V.P.  $\phi$ .
3. In the trapezoid  $ABba$ ,  $aA$  and  $bB$  are perpendicular to  $ab$  and are equal to  $ao_1$  and  $bo_2$  (the distances of  $a'$  and  $b'$  from  $XY$  in the front view).



**Fig.8.9**

The angle between AB and ab is the true angle of inclination of AB with H.P,  $\theta$

The true length of the line and the true inclinations with the H.P and V.P may be obtained as follows:

- (i) Rotate the trapezoid  $ABb'a'$  about  $a'b'$  till the line  $AB$  coincides with V.P at  $A_1 B_1$
- (ii) Rotate the trapezoid  $ABba$  about  $ab$  till the line  $AB$  coincides with H.P at  $A_2B_2$ .
- (iii) The lengths  $A_1B_1$  and  $A_2B_2$  are equal to  $AB$ , the true length of the given line.

The angles between the lines  $A_2B_2$ ,  $ab$  and  $A_1B_1$ ,  $a'b'$  are respectively equal to  $\theta$  and  $\phi$ , the true inclinations with H.P and V.P respectively.

### ***Construction (Fig.8.9c)***

1. Draw the given projections  $a'b'$ ,  $ab$  and locate  $o_1$ ,  $o_2$ .
2. Erect perpendiculars  $aA_2$  and  $bB_2$  to the line  $ab$ , equal to  $o_1a'$  and  $o_2b'$  respectively.
3. Join the points  $A_2$ ,  $B_2$  and measure the angle  $\theta$ .
4. Erect perpendiculars  $a'A_1$  and  $b'B_1$  to the line  $a'b'$ , equal to  $o_1a$  and  $o_2b$  respectively.
5. Join the points  $A_1$ ,  $B_1$  and measure the angle  $\phi$ .

$A_1 B_1$  and  $A_2B_2$  represent the true length of the given line, while the angles  $\theta$  and  $\phi$  are the true inclinations of the line with H.P and V.P.

## **8.3 TRACES OF A LINE**

The trace of a line is the point of intersection between the given line or its extension and the plane of projection. When the line (or its extension) meets H.P, the intersection point

is known as horizontal trace or H.T and when it meets V.P, it is known as vertical trace or V.T.

### 8.3.1 Straight Line Parallel to Both H.P and V.P

When a straight line is parallel to both H.P and V.P, the line will not meet the plane of projection, even when it is extended. Therefore, there are no traces for the line (Fig.8.10).

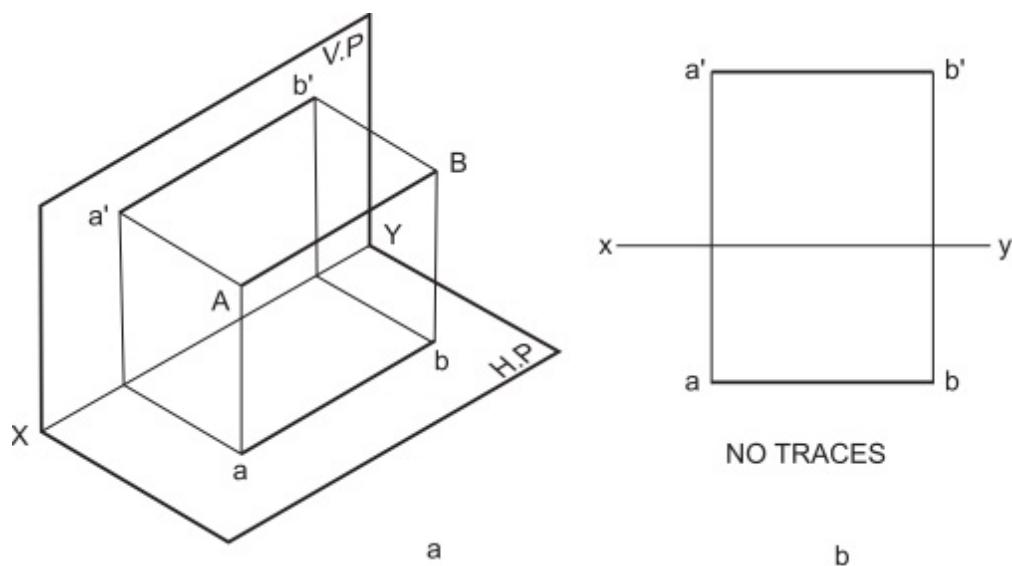
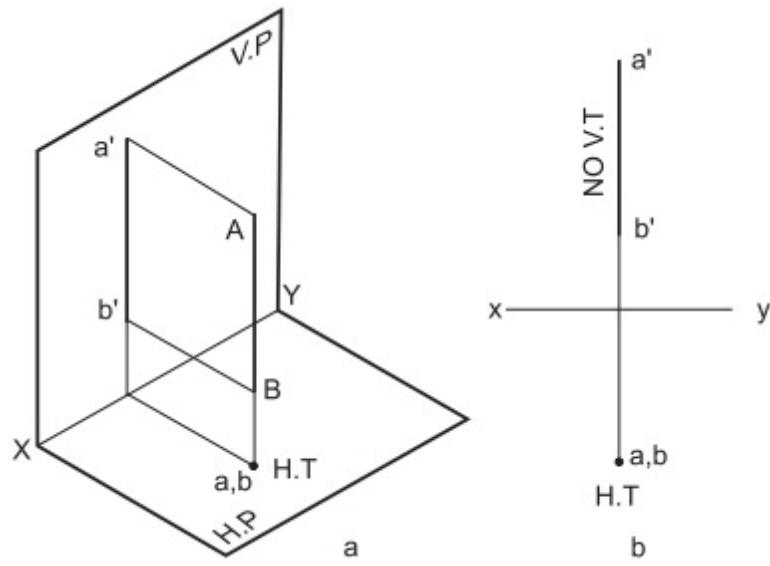


Fig 8.10

### 8.3.2 Straight Line Perpendicular to H.P and Parallel to V.P

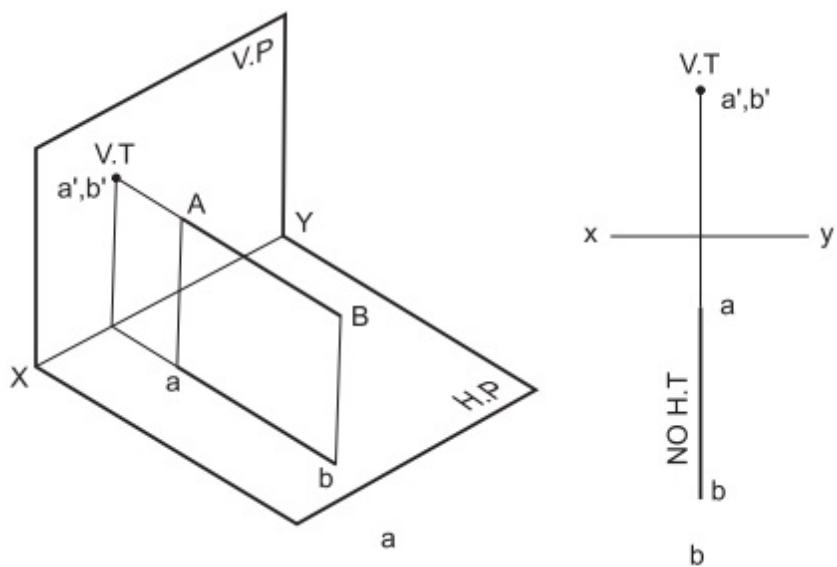
In this case, H.T of the line will coincide with the top view of the line and there is no V.T, as the line is parallel to V.P (Fig.8.11).



**Fig 8.11**

### 8.3.3 Straight Line Perpendicular to V.P and Parallel to H.P

In this case, V.T of the line will coincide with the front view of the line and there is no H.T, as the line is parallel to H.P (Fig.8.12).



**Fig.8.12**

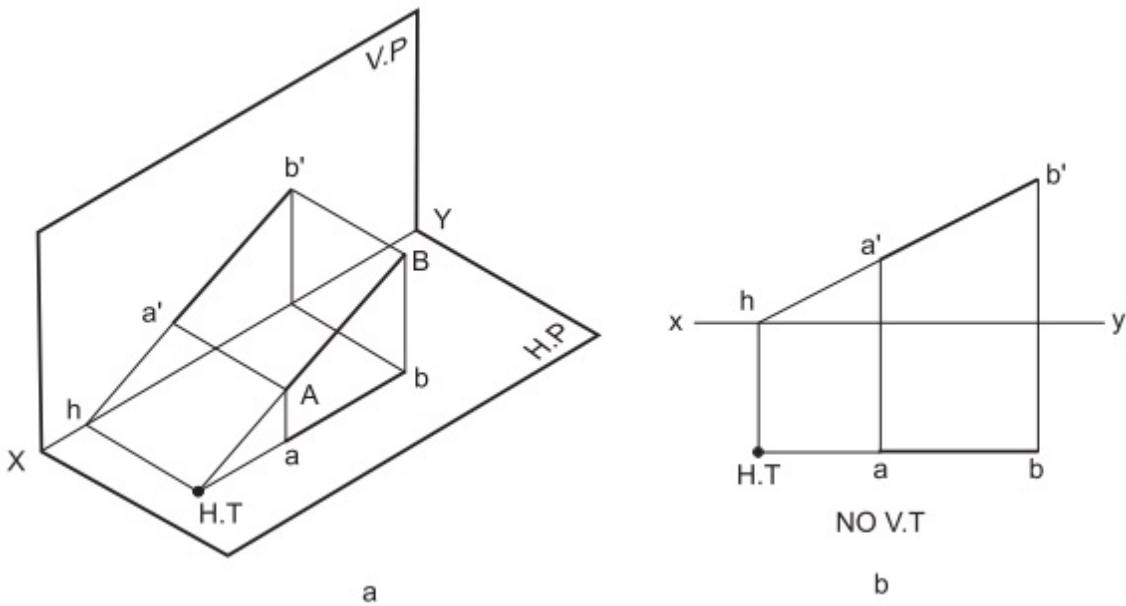
### **8.3.4 Straight Line Inclined to H.P and Parallel to V.P**

Referring to [Fig.8.13a](#), the line BA when extended will meet H.P at H.T. The following may be observed from the figure:

- (i) When  $b'a'$  is extended, it meets the reference line XY at h.
- (ii) The points h and H.T lie on a projector.
- (iii) The H.T lies on the line  $ba$  extended.
- (iv) There is no V.T, as the line is parallel to V.P.

#### ***Construction (Fig.8.13b)***

1. Draw the projections  $a'b'$  and  $ab$ .
2. Locate the point h at the intersection between the reference line  $xy$  and  $b'a'$  (or its extension).
3. Locate H.T at the intersection between a projector drawn from h and the top view  $ba$  (or its extension).



**Fig.8.13**

### 8.3.5 Straight Line Inclined to V.P and Parallel to H.P

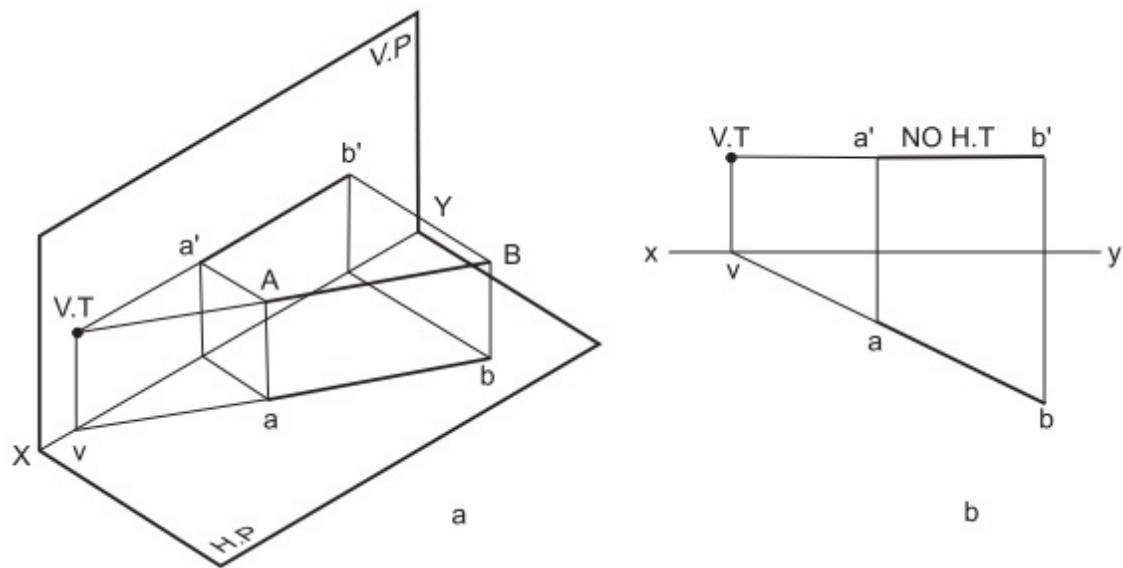
Figure 8.14a shows the position of a line AB and the location of V.T. The following may be observed from the figure:

- (i) When the top view  $ba$  is extended if necessary, it meets the reference line XY at v.
- (ii) The points v and V.T lie on a projector.
- (iii) The V.T lies on the line  $b' a'$  (or its extension).
- (iv) There is no H.T, as the line is parallel to H.P.

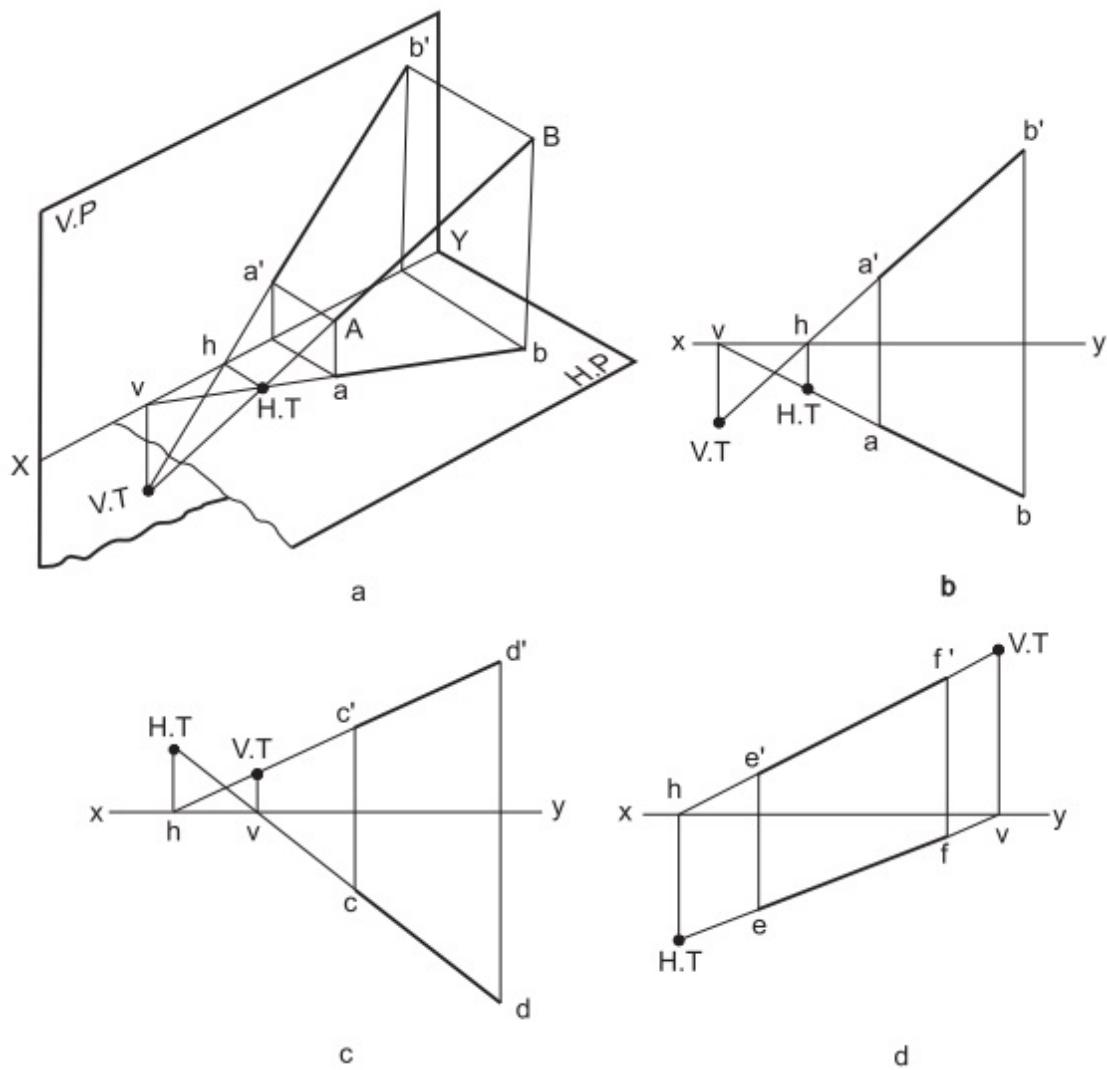
#### **Construction (Fig.8.14b)**

1. Draw the projections  $a' b'$  and  $ab$ .
2. Locate the point v at the intersection between the reference line  $xy$  and  $ba$  (or its extension).

3. Locate V.T at the intersection between a projector drawn from v and the front view  $b'a'$  (or its extension).



**Fig.8.14**



**Fig.8.15**

### 8.3.6 Straight Line Inclined to Both H.P and V.P

**Method I** Figure 8.15a shows the position of the line AB and its traces marked, using the methods presented in the sections 8.3.4 and 8.3.5.

#### **Construction (Fig.8.15b)**

1. Draw the projections  $a'b'$  and  $ab$ .

2. Extend the line  $b'a'$  to meet  $xy$  at  $h$ .
3. Draw the projector from  $h$  to intersect the line  $ba$  extended at H.T.
4. Extend the line  $ba$  to meet  $xy$  at  $v$ .
5. Draw a projector from  $v$  to intersect the line  $b'a'$  extended at V.T.



1. In general, once the position of the line is fixed, the position of the traces remains unchanged even if the length of the line is altered.
2. The relative positions of the traces H.T and V.T, relative to the reference line  $xy$  depends upon the orientation of the line with respect to H.P and V.P. [Figure 8.15c](#) shows the position of the views of a line  $CD$  and its traces marked and [Fig.8.15d](#), that of a line  $EF$ .

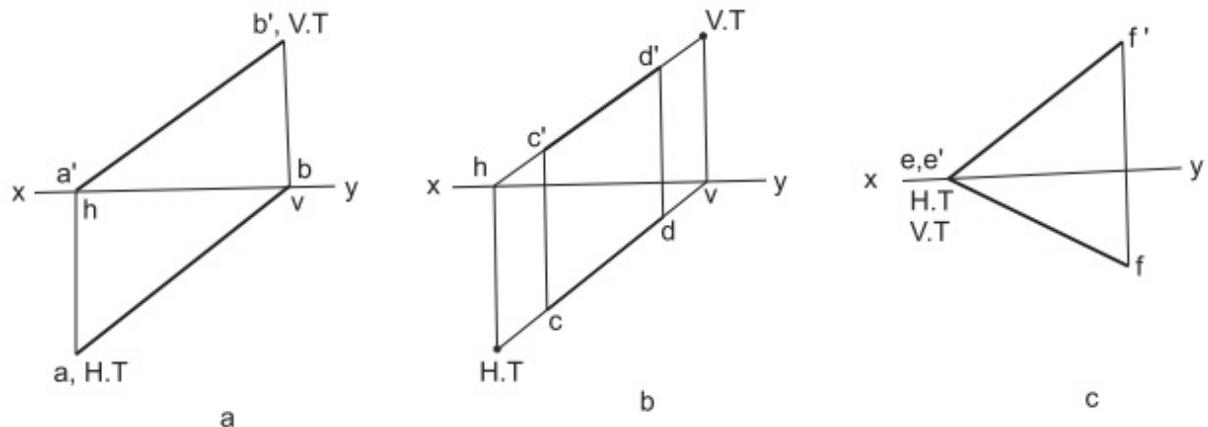
**Method II** It may be observed from [Figs.8.9a and b](#), that the rotation of the line  $AB$  about  $ab$  and  $a'b'$  has actually taken place about  $o_3$  and  $o_4$  respectively as centres. Also, it may be observed that the point  $o_3$  lies on H.P and  $o_4$  on V.P. As per the definition of the traces of a line, these points are respectively known as H.T and V.T.

### **Construction ([Fig.8.9c](#))**

1. Draw the projections of the line  $AB$ .
2. Draw the lines  $A_1B_1$  and  $A_2B_2$  as described in article 8.2.6 (method II).
3. Extend the lines  $b'a'$  and  $B_1A_1$  and locate the point of intersection, the V.T of the line.
4. Extend the lines  $ba$  and  $B_2A_2$  and locate the point of intersection, the H.T of the line.

### 8.3.6.1 Special Cases of Traces

**Case I** A line AB is inclined to both H.P and V.P and has its end A in H.P and end B



**Fig.8.16**

Figure 8.16a shows the projections of a line. From the figure, it is clear that H.T of the line coincides with a, the top view of A and V.T coincides with b', the front view of B.

**Case I Same as Case I, but the line AB is shortened from both its ends and occupying the position CD.**

Figure 8.16b shows the projections of the line along with the traces located. From the figure, it is clear that the traces of the line CD are still the same as those for Case I.

**Case III A line EF is inclined to both H.P and V.P and has its end E on both H.P and V.P.**

Figure 8.16c shows the projections of the line. From the figure, it is clear that H.T and V.T of the line coincide with e and e', the projections of the end point E.

Hence, it may be concluded that when a line has an end on a plane, its trace on that plane coincides with the projection of that end on that plane.

### 8.3.7 Straight Line Contained by a Profile Plane

When a line is contained by a profile plane, its projections lie on a single projector. Therefore, the method I of the preceding article cannot be used to locate the traces. However, the method II of the preceding article may be used as shown in Fig.8.17.

## 8.4 THREE VIEW PROJECTIONS OF STRAIGHT LINES

Projections of a straight line on the two principal planes of projection, viz., H.P and V.P, result in the top and front views respectively. The line also may be projected on to the profile plane. In the first angle projection, if a left profile plane is considered; the view obtained on it is known as the right side view. A left side view may be obtained by projecting the line on to a right profile plane.

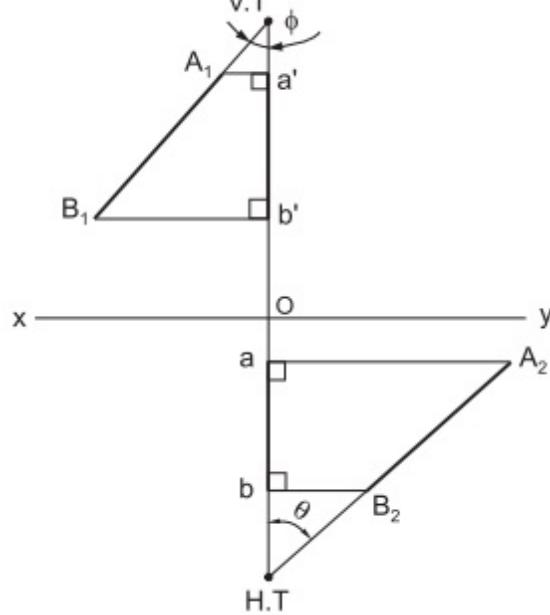
### 8.4.1 Straight Line Inclined to Both H.P and V.P

**Problem 8** A line AB of 100 length is inclined at an angle of  $30^\circ$  to H.P and  $45^\circ$  to V.P. The point A is 15 above H.P, 20 in

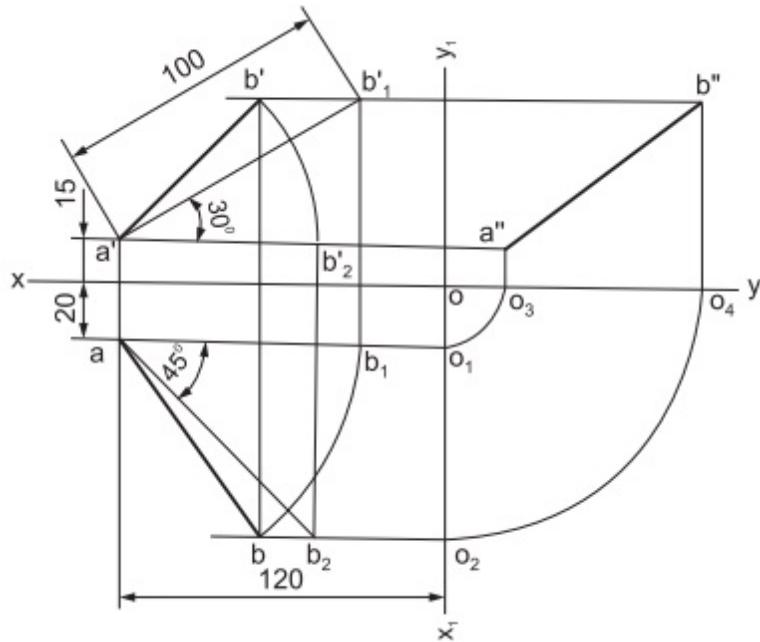
*front of V.P and 120 from right profile plane (RPP). Draw (i) front view, (ii) top view and (iii) left side view of the line AB.*

### **Construction ([Fig.8.18](#))**

1. Draw the front and top views  $a'b'$  and  $ab$  of the line AB, following the Construction: [Fig.8.6](#).
  2. Draw the line  $x_1y_1$  at right angle to  $xy$ , intersecting at o and at 120 from the projector containing  $a'$ ,  $a$ .
  3. Draw the projectors through  $a$  and  $b$ , meeting  $x_1y_1$  at  $o_1$  and  $o_2$ .
  4. With centre o and radii  $oo_1$  and  $oo_2$ , draw arcs meeting  $xy$  at  $o_3$  and  $o_4$ .
  5. Through  $o_3$  and  $o_4$ , draw projectors.
  6. Through  $a'$  and  $b'$ , draw projectors meeting the above projectors at  $a''$  and  $b''$  respectively.
  7. Join  $a'', b''$ .
- $a'b'$ ,  $ab$  and  $a''b''$  are the three views of the given line.



**Fig.8.17**



**Fig.8.18**

## 8.5 LOCATION OF A POINT ON A LINE

A line is defined as the locus of a moving point. In other words, it may be said that a line is composed of infinite number of points. It is often required to specify the location of specific points on lines. From the projections of straight lines, it is already observed that the end points of a line are located on the projectors, which are perpendicular to the line xy.

**Problem 9** *Figure 8.19a shows (i) front view, (ii) top view and (iii) right side view of a line AB and the front view o' of a point O on a'b'. Locate the top and right side views of the point O on the top and right side views of the line.*

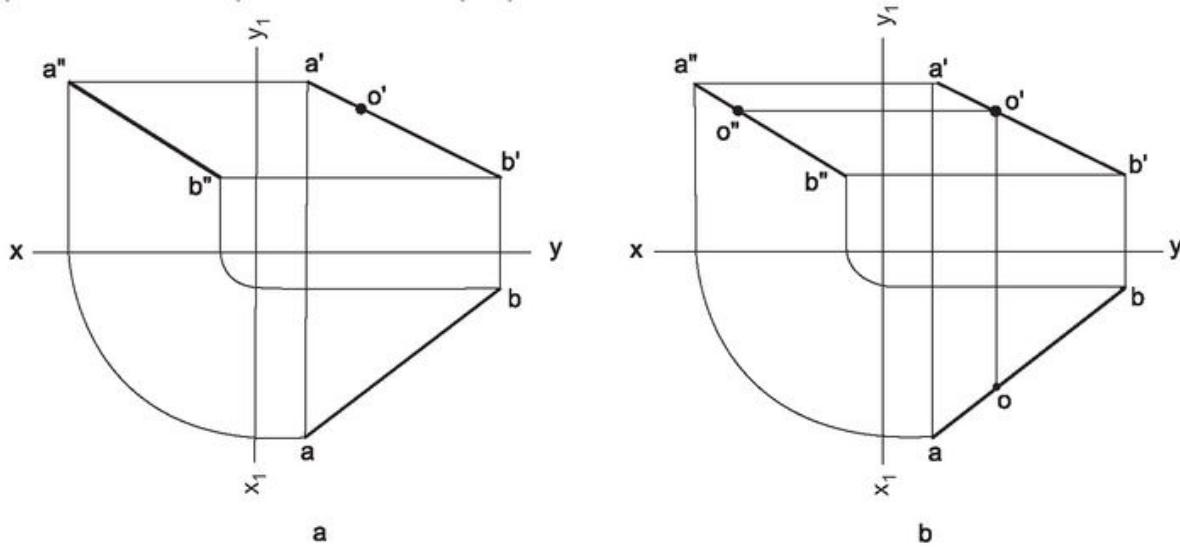
### **Construction (Fig.8.19b)**

1. Draw the three views of the line and locate the point  $o'$  on  $a' b'$ .
2. Draw the projectors through  $o'$  to meet  $ab$  at  $o$  and  $a'' b''$  at  $o''$ .

It may be noted that the points  $o'$ ,  $o$  and  $o''$  divide the lines  $a' b'$ ,  $ab$  and  $a'' b''$  respectively in the same ratio, i.e.,

$$\frac{a'o'}{a'b'} = \frac{ao}{ab} = \frac{a''o''}{a''b''}$$

From the above, it is evident that if a point is located at the middle of a line, it will appear at the mid-point in all the projections.



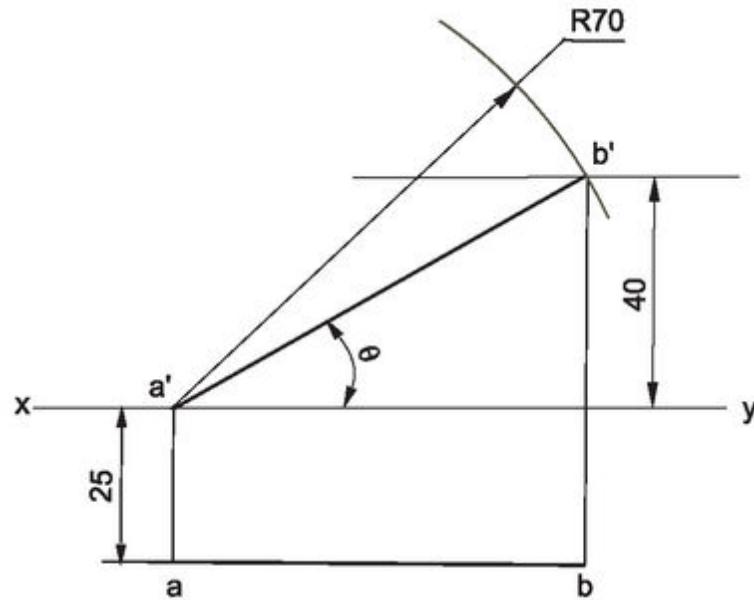
**Fig.8.19**

## **8.6 EXAMPLES**

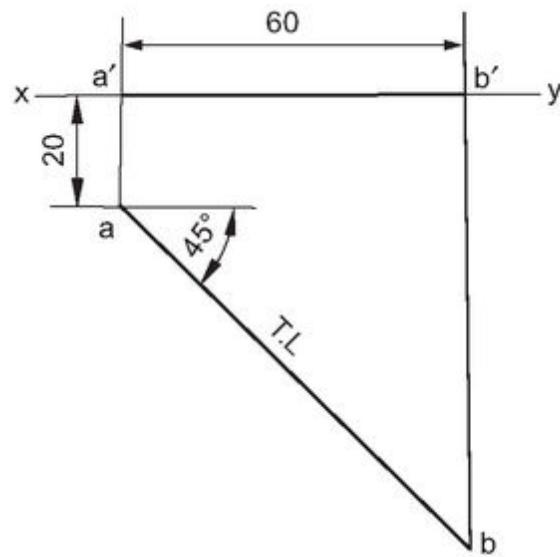
**Problem 10** A line  $AB$  of 70 long, is parallel to and 25 in front of V.P. Its one end is on H.P while the other is 40

above H.P. Draw the projections of the line and determine its inclination with H.P.

**Construction (Fig.8.20)**



**Fig.8.20**



**Fig.8.21**

1. Draw the reference line xy and locate the point a' on it.

2. Draw a line, parallel to and 40 above xy.
3. With centre  $a'$  and radius 70, draw an arc intersecting the above line at  $b'$ . Join  $a'$ ,  $b'$ .
4. Draw a line, parallel to and 25 below xy.
5. Locate the points  $a$  and  $b$  on the above line, by drawing projectors through  $a'$  and  $b'$ .

$a'$   $b'$  and ab are the projections of the line and the inclination  $\theta$  of the line  $a'$   $b'$  with xy is the inclination of the line with H.P.

**Problem 11** A line AB is on H.P and its one end A is 20 in front of V.P. The line makes an angle of  $45^\circ$  with V.P and its front view is 60 long. Draw the projections of the line and determine the true length.

### **Construction (Fig.8.21)**

1. Locate the projections  $a'$  and  $a$  of the end A of the line AB.
2. Draw  $a'$   $b'$  of 60 long and along xy.
3. Through  $a$ , draw a line at  $45^\circ$  with xy, meeting the projector through  $b'$  at  $b$ .

$a'$   $b'$  and ab are the projections of the line and the length ab represents the true length of the line.

**Problem 12** The front view of a 75 long line measures 55. The line is parallel to the H.P and one of its ends is in the V.P and 25 above the H.P. Draw the projections of the line and determine its inclination with V.P.

### **Construction (Fig.8.22)**

1. Locate the projections of one end of the line,  $a'$  and  $a$  with respect to xy.

2. Draw the front view of the line  $a' b'$ , parallel to  $xy$  and of length 55.
3. Draw a projector through  $b'$  to locate the top view of B on it.
4. With a as centre and 75 radius, draw an arc intersecting the above projector at b.
5. Join a, b; forming the top view of the line.

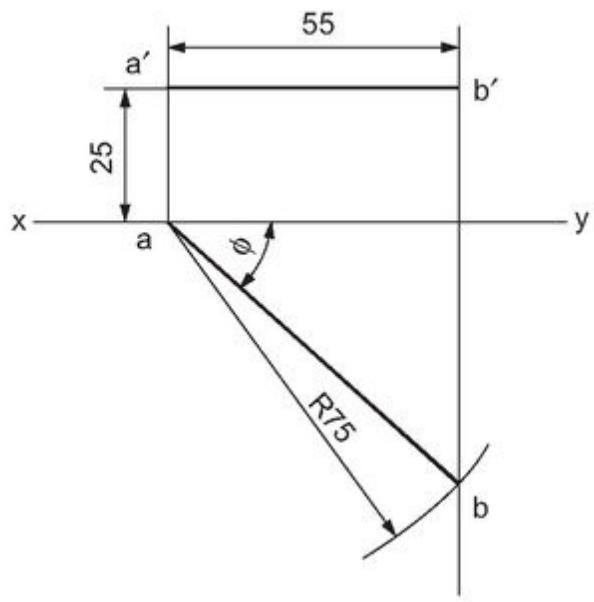
The inclination of ab with  $xy$  is the angle of inclination of the line AB with V.P ( $\phi$ ).

**Problem 13** An electric switch (A) and bulb (B), fixed on a wall are 5m apart. The distance between them, measured parallel to the floor is 4 metres. If the switch is 1.5 metres above the floor, find the height of the bulb and the inclination of the line joining the switch and bulb, with the floor.

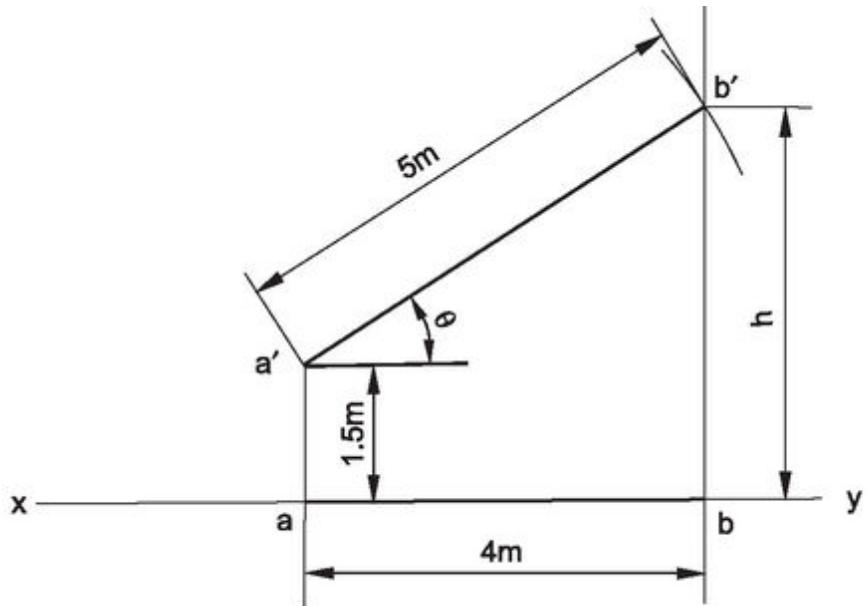
### **Construction (Fig.8.23)**

1. Locate the projections  $a'$  and  $a$  with respect to  $xy$ , choosing a suitable scale.
2. Locate  $b$  on  $xy$ , at 4 m from  $a$  and draw a projector through it.
3. With centre  $a'$  and radius 5m, draw an arc intersecting the above projector at  $b'$ .
4. Join  $a', b'$ .

The length  $b' b$  ( $= h$ ) represents the height of the bulb and the angle  $\theta$ , the inclination of the line joining the switch and bulb with the floor.



**Fig.8.22**



**Fig.8.23**



1. The switch and the bulb are considered as points.
2. The wall is treated as V.P and the floor as H.P.

**Problem 14** A line measuring 80 long has one of its ends 60 above H.P and 20 in front of V.P. The other end is 15 above H.P and in front of V.P. The front view of the line is 60 long. Draw the top view.

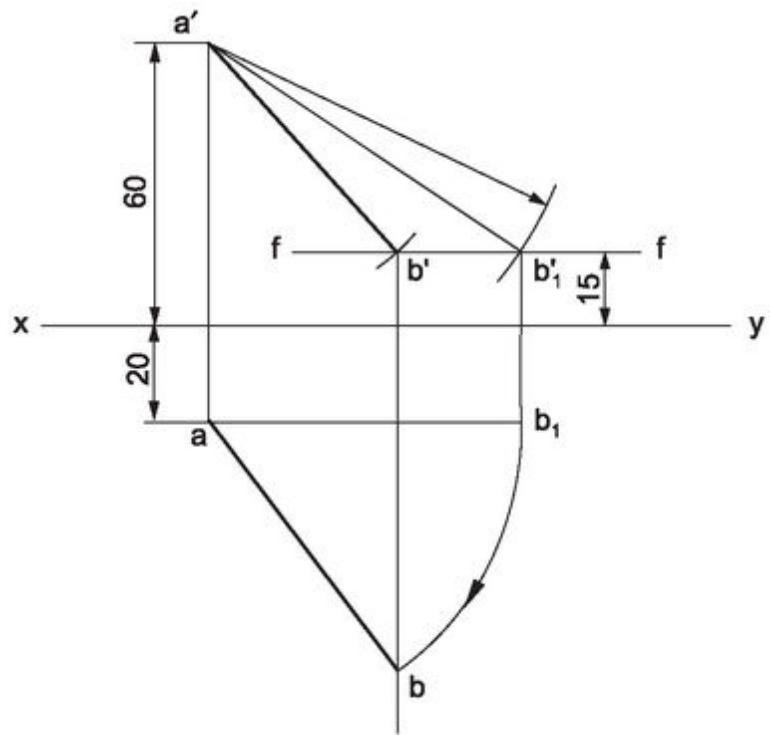
**Construction (Fig.8.24)**

1. Draw the line  $xy$  and locate the projections of the end A of the line AB.
2. Draw  $f-f$ , 15 above  $xy$ ; representing the locus of front view of the end B.
3. With centre  $a'$  and radius 80, draw an arc; intersecting  $f-f$  at  $b_1'$ .
4. Drop a projector from  $b_1'$ , to meet the horizontal line through  $a$  at  $b_1$ .  $ab_1$  is the length of the top view.
5. With centre  $a'$  and radius 60 (front view length), draw an arc intersecting  $f-f$  at  $b'$ .
6. Draw a projector through  $b'$ .
7. With centre  $a$  and radius  $ab_1$ , draw an arc intersecting the above projector at  $b$ .

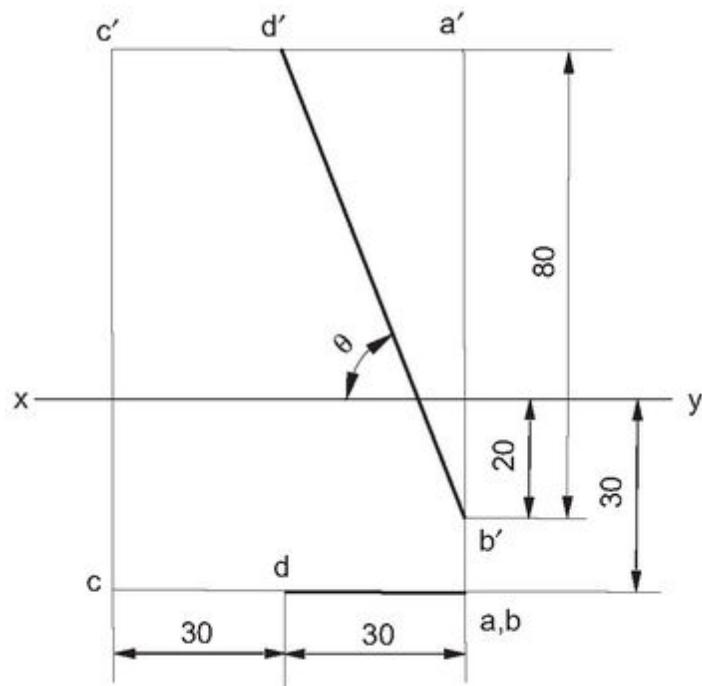
Join  $a, b$ ; representing the top view of the line.

**Problem 15** A line AB, which is perpendicular to H.P and 80 long, has its end B, 20 below H.P and 30 in front of V.P. Another line AC, which is 60 long, is parallel to both H.P and V.P. The mid-point D of the line AC is joined to B. Draw the projections and determine the inclination of the line BD with H.P.

**Construction (Fig.8.25)**



**Fig.8.24**



**Fig.8.25**

1. Draw the front and top views of the line AB, i.e.,  $a'$   $b'$  and  $ab$ .
2. Draw the front and top views of the line AC, i.e.,  $a'$   $c'$  and  $ac$ , parallel to  $xy$  and locate the projections of the mid-point,  $d'$  and  $d$ .
3. Join  $d'$ ,  $b'$  and measure the angle  $\theta$ .

$\theta$  is the inclination of the line BD with H.P.

**Problem 16** Two oranges A and B on a tree are respectively at 1m and 2 m above the ground and 0.3 m and 1.5 m from a 0.35 m thick wall but on opposite sides of the wall. The distance between the oranges measured along the ground and parallel to the wall is 3 m. Determine the true distance between the oranges.

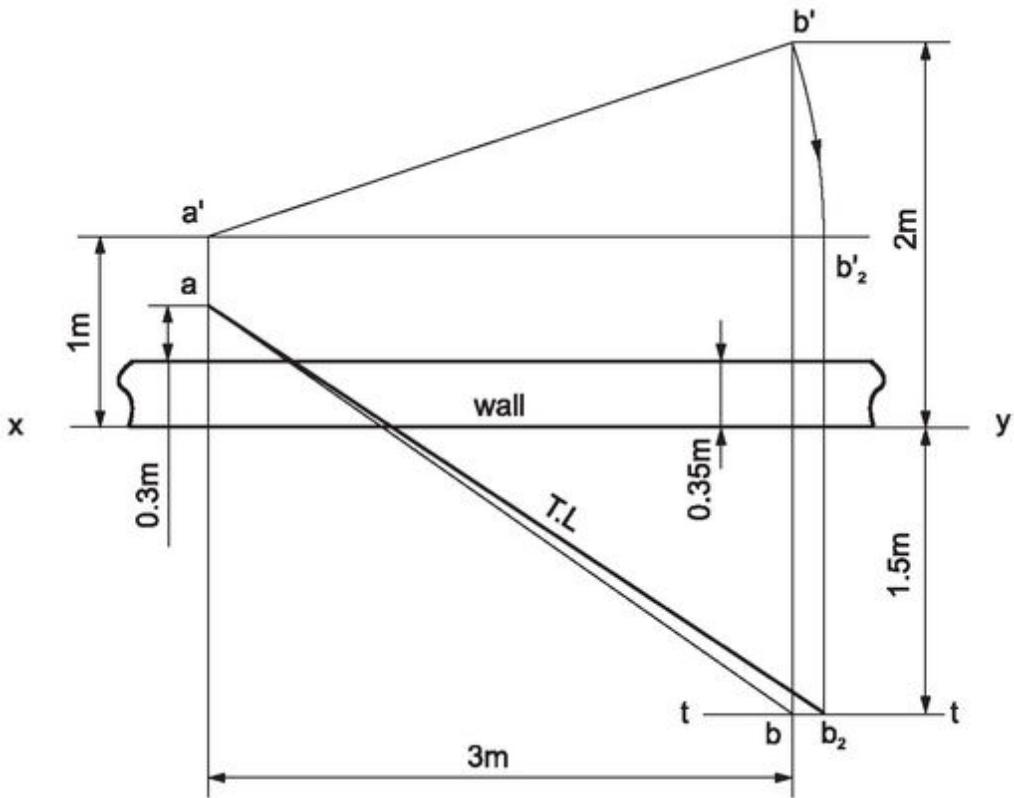
### **Construction (Fig.8.26)**

1. Locate the projections  $a'$  and  $b'$  at the given heights from the ground ( $xy$ ), keeping the distance between them along  $xy$  as 3m.
2. Represent the wall (thickness) and locate  $a$  and  $b$  on either side of it and at the given distances 0.3m and 1.5m respectively.
3. Rotate  $a'$   $b'$  to  $a'b_2'$ , parallel to  $xy$  and obtain  $b_2$  on t-t, the locus of top view of the point B.
4. Join  $a$ ,  $b_2$ .

The distance  $ab_2$  represents the true distance between the oranges.



The wall and orange A are assumed to be in the second quadrant and the orange B in the first quadrant.



**Fig.8.26**

**Problem 17** A line  $AB$  of 70 long, has its end  $A$ , 20 above H.P and 15 in front of V.P. The line is inclined at  $30^\circ$  to H.P and  $60^\circ$  to V.P. Draw its projections.

### **Method I**

#### **Construction (Fig.8.27a)**

1. Locate the projections of the end  $A$  of the line  $AB$ .
2. Draw the projections  $a'$   $b_1'$  and  $ab_1$  of the line, assuming it to be inclined to H.P by  $30^\circ$  and parallel to V.P.  $ab_1$  represents the top view length.
3. Draw the projections  $ab_2$  and  $a'$   $b_2'$  of the line, assuming it to be inclined to V.P by  $60^\circ$  and parallel to H.P.  $a'$   $b_2'$  represents the front view length.

4. Draw  $f - f$  and  $t - t$ , through  $b_1'$  and  $b_2$  respectively, representing the loci of front and top views of the end B of the line.
5. With  $a'$  as centre and  $ab_2'$  as radius, draw an arc meeting the line  $f - f$  at  $b'$ .
6. With  $a$  as centre and  $ab_1$  as radius, draw an arc meeting the line  $t - t$  at  $b$ .
7. Join  $a', b'$  and  $a, b$  forming the required projections.



If the construction is correct, the front and top views of the line should lie on the same projector.

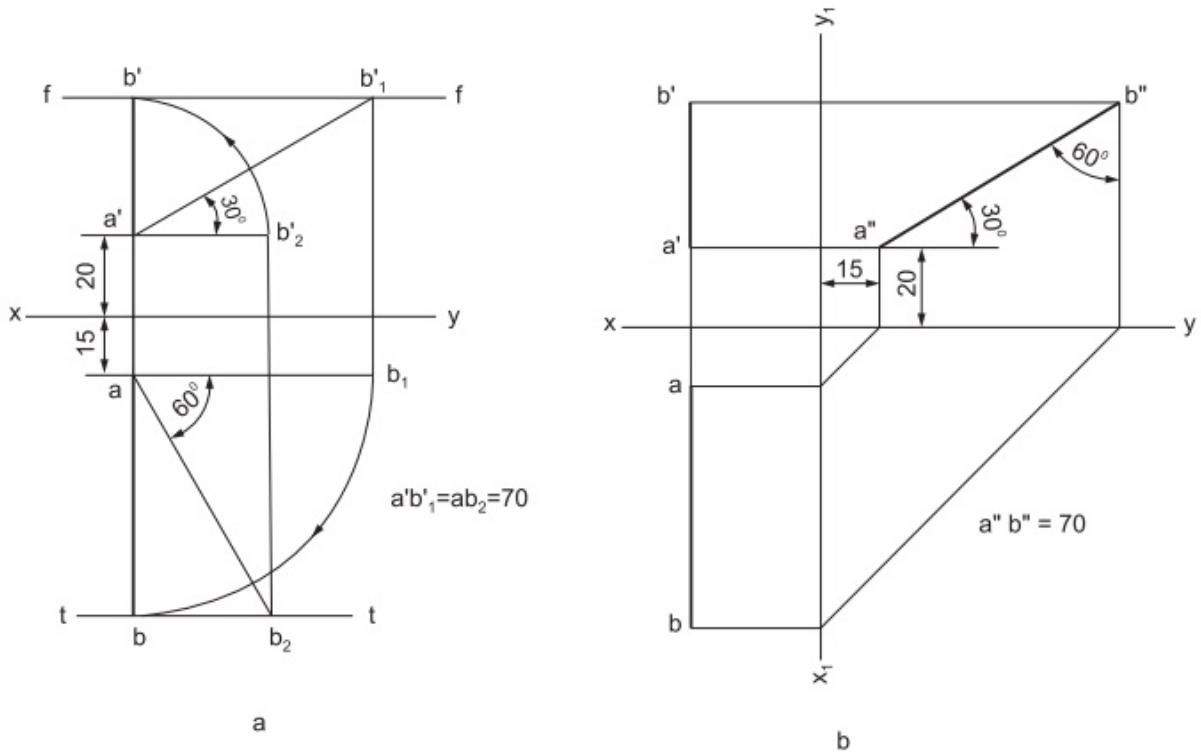
## **Method II**

### **Construction (*Fig.8.27b*)**

1. Draw the reference lines  $xy$  and  $x_1y_1$ .
2. Locate the end view  $a''$  of the end A of the line AB.
3. Draw the end view  $a'' b''$  of 70 length (true length of the line), making  $30^\circ$  with  $xy$  (H.P) and  $60^\circ$  with  $x_1y_1$  (V.P).
4. Obtain the front and top views  $a', b'$  and  $ab$ , by projection.



This method could be used only when  $\theta + \phi = 90^\circ$ .



**Fig.8.27**

**Problem 18** Three edges  $oa$ ,  $ob$  and  $oc$ ; 30, 50 and 65 long, each making  $120^\circ$  with the other two, form the top view of the slant edges of an irregular triangular pyramid; the edge  $oa$  being parallel to  $xy$ . The corners of the base  $A, B$  and  $C$  of the pyramid are on H.P while the apex  $O$  of it is 100 above it. Draw the front view and determine the true lengths and inclinations of the slant edges with H.P.

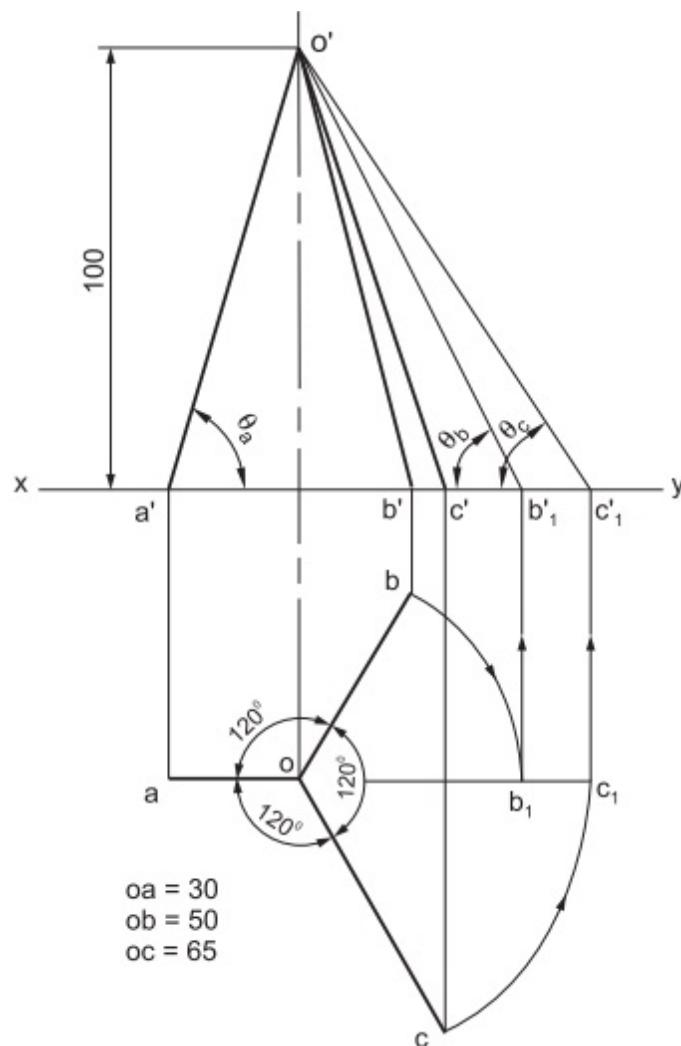
### **Construction (Fig.8.28)**

1. Draw the reference line  $xy$ .
2. Draw the top view of the slant edges  $oa$ ,  $ob$  and  $oc$ ; keeping  $oa$  parallel to  $xy$  and  $ob$  and  $oc$  making  $120^\circ$  with  $oa$ .
3. Draw the front views of the slant edges,  $o' a'$ ,  $o' b'$  and  $o' c'$ ; locating the front view of the apex  $o'$  at 100 above  $xy$ .

The length of  $o' a'$  represents the true length of the edge OA and the angle  $\theta_a$ , the true inclination of the edge OA with H.P, as the top view oa is parallel to xy.

4. Rotate ob to  $ob_1$ , parallel to xy.
5. Through  $b_1$ , draw a projector meeting xy at  $b_1'$ . The length  $o' b_1'$  represents the true length of the edge OB and the inclination  $\theta_b$ , the true inclination of the edge OB with H.P.
6. Rotate oc to  $oc_1$ , parallel to xy.
7. Through  $c_1$ , draw a projector meeting xy at  $c_1'$ .

The length  $o' c_1'$  represents the true length of the edge OC and the inclination  $\theta_c$ , the true inclination of the edge OC with H.P.



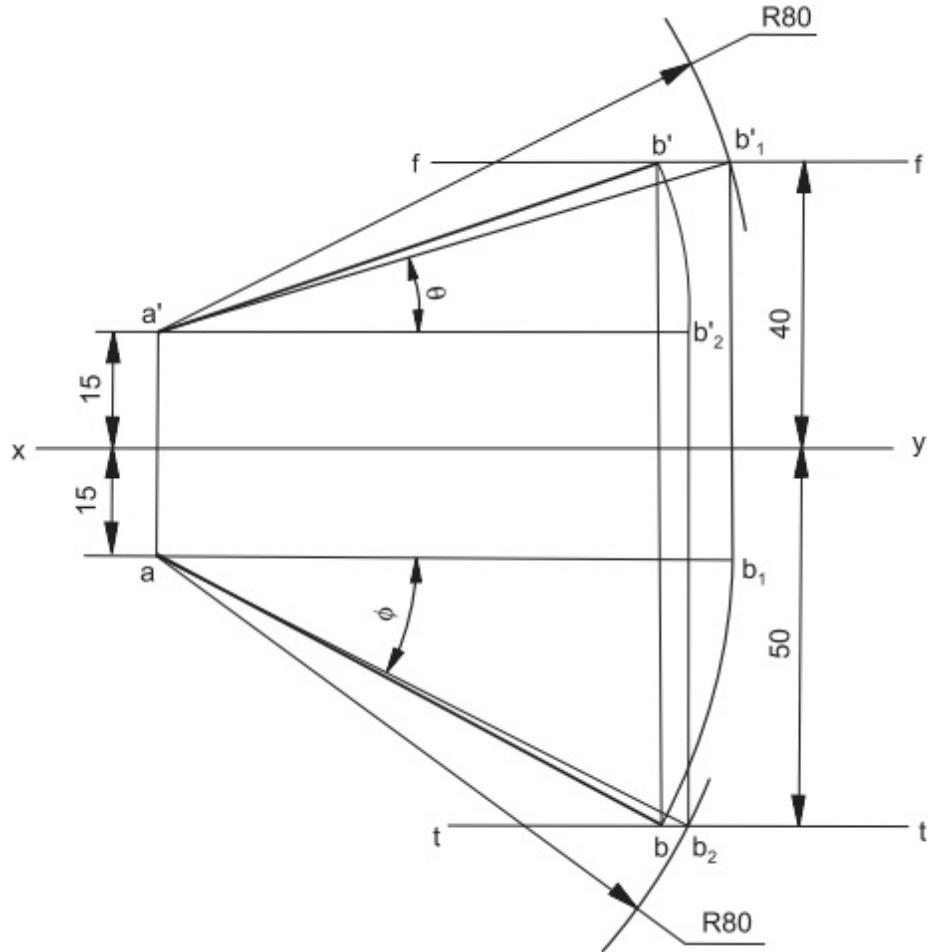
**Fig.8.28**

**Problem 19** A line AB of 80 long, has its end A, 15 from both H.P and V.P. The other end B is 40 above H.P and 50 in front of V.P. Draw the projections of the line and determine the inclinations of the line with H.P and V.P.

**Construction (Fig.8.29)**

1. Locate the projections  $a'$  and  $a$  of the end A of the line AB.
2. Draw the loci f-f and t-t of the front and top views of the end B respectively.

3. With centre  $a'$  and radius 80, draw an arc intersecting  $f-f$  at  $b'_1$ .
4. Join  $a'$ ,  $b'_1$  and measure its inclination  $\theta$  with  $xy$ .
5. With centre  $a$  and radius 80, draw an arc intersecting  $t-t$  at  $b_2$ .
6. Join  $a$ ,  $b_2$  and measure its inclination  $\phi$  with  $xy$ .
7. Obtain the projections  $a'$   $b'$  and  $ab$  of the line AB, following the principle of Construction: [Fig.8.6e](#).  
 $\theta$  and  $\phi$  are the inclinations of the line AB with H.P and V. P respectively.

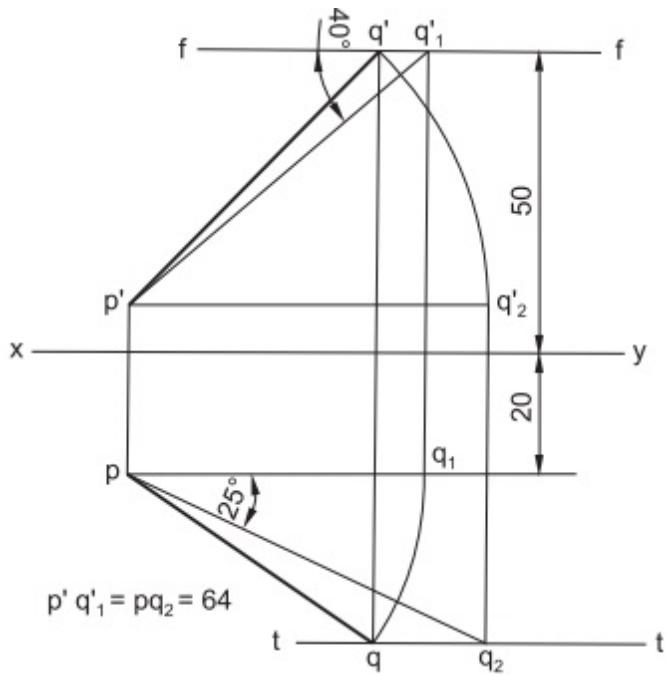


**Fig.8.29**

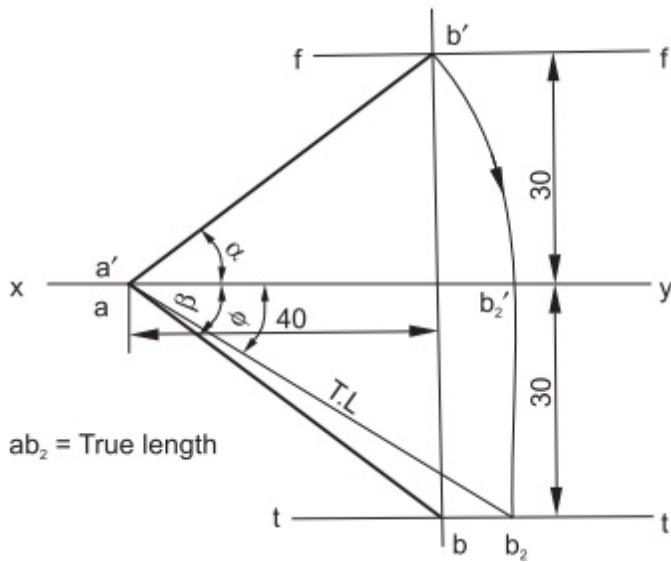
**Problem 20** A line  $PQ$  of 64 long has one of its extremities 20 in front of V.P and the other 50 above H.P. The line is inclined at  $40^\circ$  to H.P and  $25^\circ$  to V.P. Draw its projections.

**Construction (Fig.8.30)**

1. Draw a line  $xy$  and draw the locus of  $q'$ , 50 above and locus of  $p$ , 20 below  $xy$  respectively.
  2. At any convenient point on the locus of  $q'$ , choose a point  $q'_1$  and draw a line of 64 length inclined at  $40^\circ$  to  $xy$  and mark  $p'$ .
  3. Project and obtain  $p$  and  $q_1$  on the locus of  $p$ .  $pq_1$  represents the top view length.
  4. Draw a line  $pq_2$  of 64 long at  $25^\circ$  to  $xy$  and draw  $t - t$ , the locus of top view of the point Q.
  5. Project and obtain  $q'_2$  such that,  $p'q'_2$  (parallel to  $xy$ ) represents the length of the front view of the line  $PQ$ .
  6. With  $p'$  as centre and radius  $p'q'_2$ , draw an arc and locate  $q'$  on the locus of  $q'$  and with  $p$  as centre and  $pq_1$  as radius, draw an arc intersecting  $t - t$  at  $q$ .
- $p'q'$  is the front view and  $pq$  is the top view of the line  $PQ$ .



**Fig.8.30**



**Fig.8.31**

**Problem 21** The H.T and V.T and the end A of a line coincide and lie on xy. The distance between the top and front views of the end B of the line is 60. The line is equally inclined to the VP and the HP. The distance between the

*projectors as measured parallel to xy is 40. Draw the projections and find the true length of the line.*

**Construction (Fig.8.31)**

1. Draw the reference line  $xy$  and locate the front and top views  $a'$  and  $a$  of the end A at any convenient point on it.
2. Draw the loci of the front and top views  $f-f$  and  $t-t$ , of the end B at 30 from  $xy$ .
3. Draw a projector at 40 from  $a'(a)$ , intersecting  $f-f$  at  $b'$  and  $t-t$  at  $b$ .
4. Join  $a'$ ,  $b'$  and  $a$ ,  $b$  forming the required projections of the given line.
5. Rotate  $a'$   $b'$  to  $a' b_2'$ , coinciding with  $xy$ .
6. Draw a projector through  $b_2'$  to meet the locus  $t-t$  at  $b_2$ .
7. Join  $a$ ,  $b_2$  the length of which is equal to the true length of the line AB.



As the inclinations  $\theta$  and  $\phi$  of the line AB with H.P and V.P are equal; the front and top views of the end B are symmetrically placed with respect to  $xy$ ; i.e., at 30 from  $xy$ .

**Problem 22** A line AB of 90 long, is inclined at  $45^\circ$  to H.P and its top view makes an angle of  $60^\circ$  with  $xy$ . The end A is on H.P and 12 in front of V.P. Draw its projections and find its inclination with V.P.

**Construction (Fig.8.32)**

1. Draw the reference line  $xy$  and locate the projections of the end A of the line AB.

2. Draw the line  $a'b_1'$  of length 90 and making  $45^\circ$  with  $xy$ .
3. Through  $b_1'$ , draw a projector meeting the horizontal line from  $a$  at  $b_1$ .
4. Through  $a$ , draw a line at  $60^\circ$  to  $xy$ .
5. Rotate  $ab_1$  about  $a$  till it meets the above line at  $b$ .
6. Through  $b_1'$ , draw the line  $f-f$  representing the locus of the front view of the end  $B$ .
7. Through  $b$ , draw a projector meeting  $f-f$  at  $b'$ .
8. Join  $a', b'$ .
9. Erect perpendiculars  $a'A$  and  $b'B$  to the line  $a'b'$ , equal to the distances of  $a$  and  $b$  from  $xy$  respectively.
10. Join  $A, B$  and measure the angle  $\phi$ .

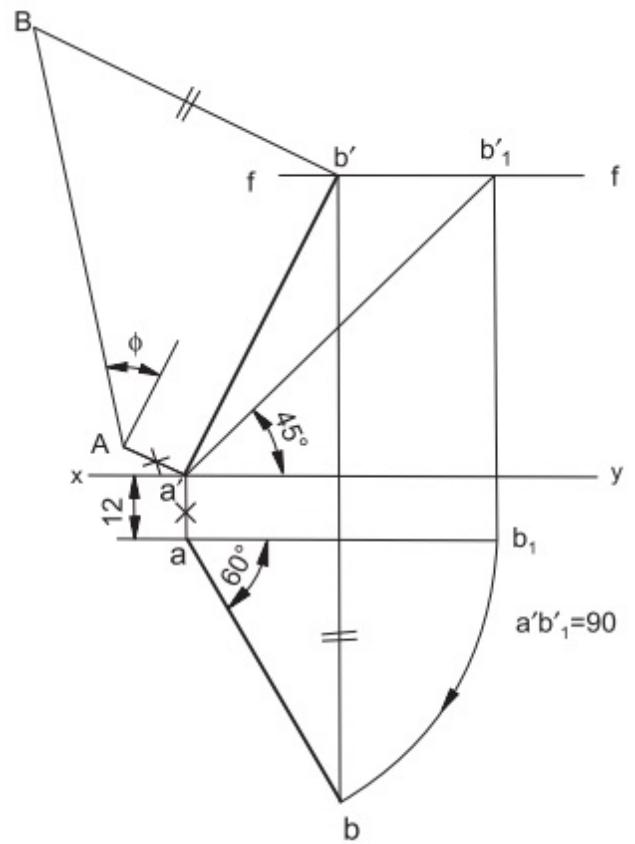
The angle  $\phi$  represents the inclination of the line  $AB$  with V.P.

**Problem 23** *Figure 8.33a shows the projections of a line  $AB$ . Determine its true length, inclinations with H.P and V.P and locate the traces.*

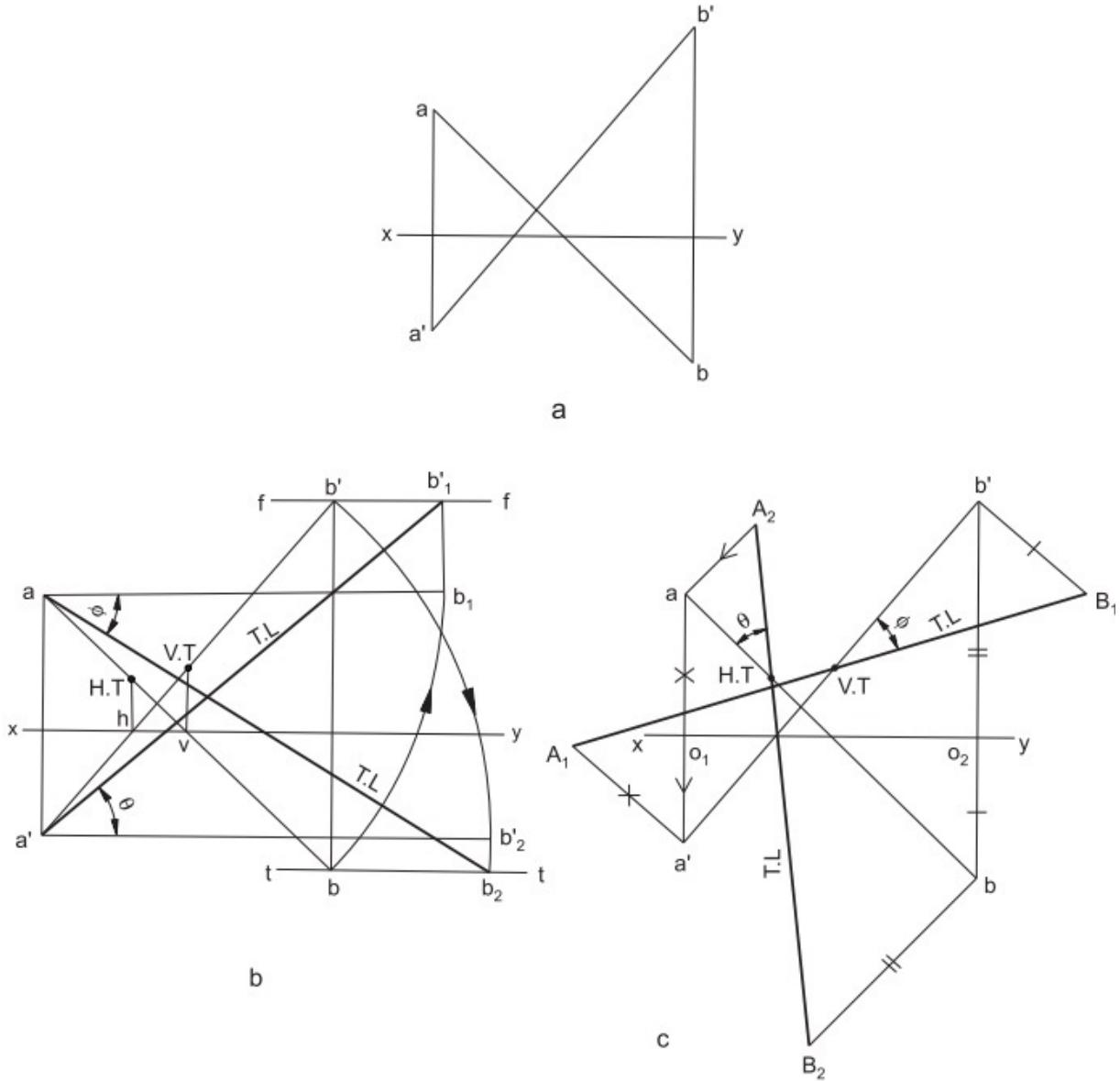
### **Method I**

#### **Construction ([Fig.8.33b](#))**

1. Draw the given projections  $a'b'$  and  $ab$ .
2. Determine the true length of the line and its inclinations with H.P and V.P, following the principle of Construction: [Fig.8.8b](#).
3. Locate the traces of the line, following the principle of Construction: [Fig.8.14b](#).



**Fig.8.32**



**Fig.8.33**

### **Method II**

#### **Construction (Fig.8.33c)**

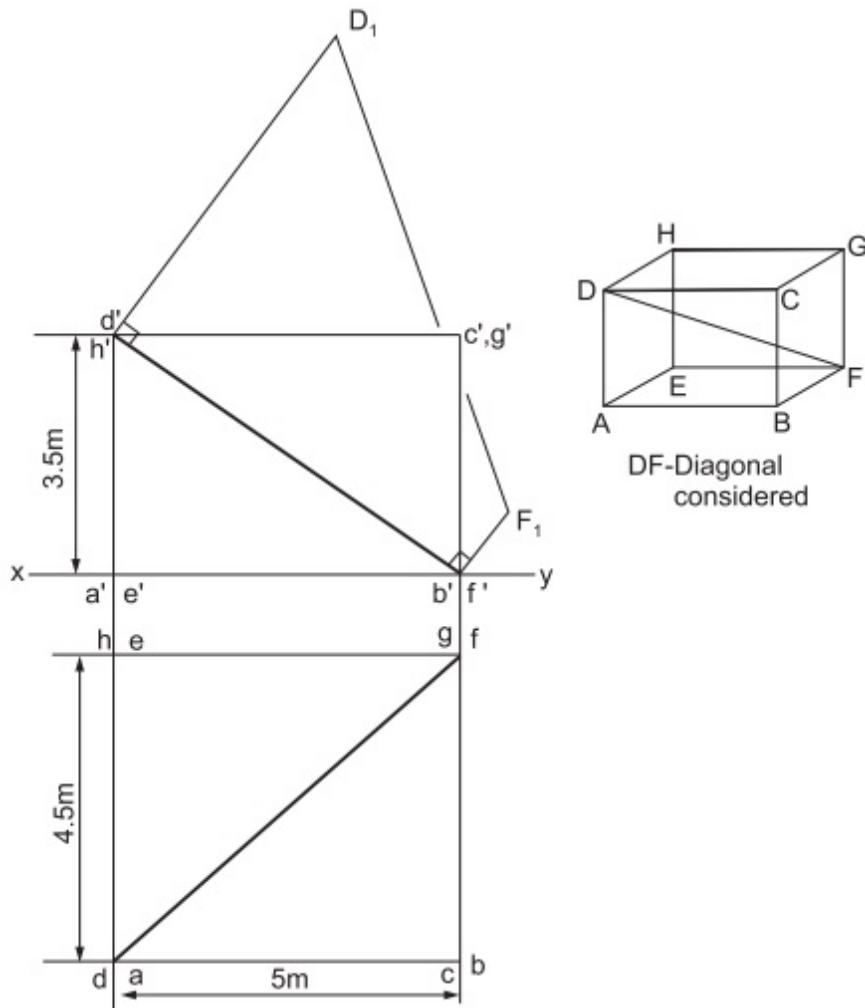
1. Draw the given projections  $a'b'$  and  $ab$ .
2. Determine the true length of the line and its inclinations with H.P and V.P and locate the traces, following the principle of Construction: [Fig.8.9c](#).



As the ends of the lines  $a'b'$  and  $ab$  are on either side of  $xy$ , the perpendiculars should be drawn on opposite sides to obtain the true length.

**Problem 24** A room is  $5 \times 4.5 \times 3.5$  m high. Determine the distance between the top corner and the bottom corner, diagonally opposite to it, by drawing the projections of the line joining the two corners.

**Construction (Fig.8.34)**



**Fig.8.34**

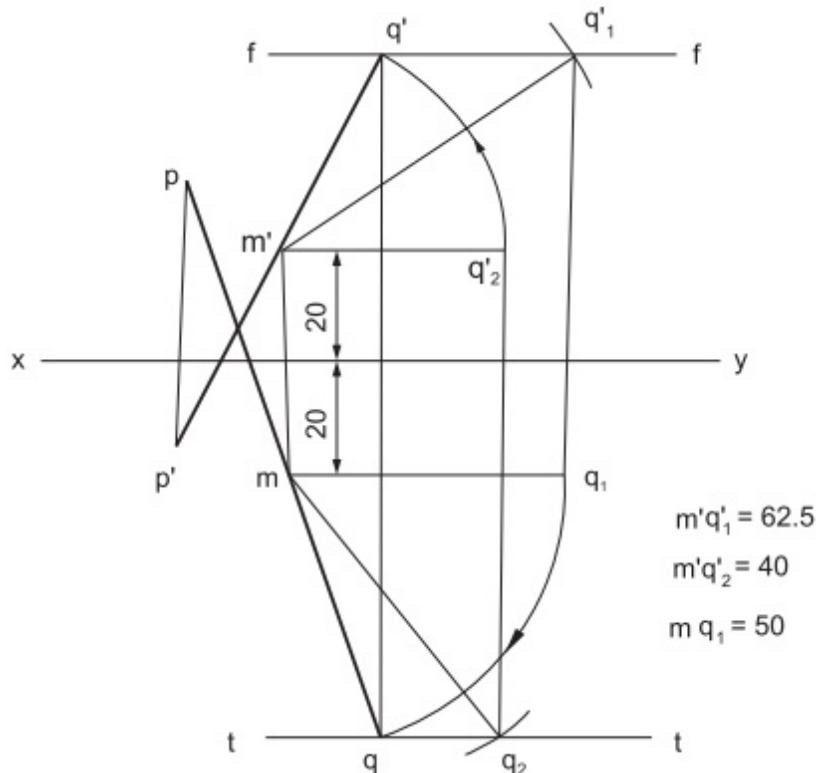
1. Draw the front and top views of the room with respect to the reference line  $xy$ .

2. Considering the diagonal DF, joining the top corner D and bottom corner F which are diagonally opposite to each other; join  $d'$ ,  $f'$  and  $d$ ,  $f$ ; thus completing the front and top views of the diagonal DF.
3. Erect perpendiculars  $d'D_1$  and  $fF_1$  to the line  $d'f'$ , equal to the distances of  $d$  and  $f$  from  $xy$  respectively.
4. Join  $D_1$ ,  $F_1$ .

The length  $D_1F_1$  represents the required distance.

**Problem 25** *The front view of a 125 long line PQ measures 80 and its top view measures 100. Its end Q and the mid-point M are in the first quadrant; M being 20 from both the planes. Draw the projections of the line PQ.*

**Construction (Fig.8.35)**



**Fig.8.35**

1. Mark the projections  $m'$  and  $m$  of the mid-point  $M$ .
2. Through  $m'$ , draw a line  $m'q_2'$  of length 40 (half of front view length) and parallel to  $xy$ .
3. Draw a projector through  $q_2'$ .
4. With centre  $m$  and radius 62.5 (1/2 of true length), draw an arc intersecting the above projector at  $q_2$ .
5. Draw  $t-t$  parallel to  $xy$  passing through  $q_2$ ; representing the locus of top view of end  $Q$ .
6. Draw  $mq_1 = 50$  (1/2 of top view length) parallel to  $xy$  and draw a projector through  $q_1$ .
7. With centre  $m'$  and radius 62.5 (1/2 of true length) draw an arc intersecting the above projector at  $q_1'$ .
8. Draw  $f-f$  parallel to  $xy$  and passing through  $q_1'$ ; representing the locus of the front view of end  $Q$ .
9. With centre  $m$  and radius  $mq_1$ , draw an arc intersecting  $t-t$  at  $q$ .
10. With centre  $m'$  and radius  $m'q_2'$ , draw an arc intersecting  $f-f$  at  $q'$ .

$q$  and  $q'$  lie on a single projector and  $m'q'$  and  $mq$  represent the projections of the line  $MQ$  (1/2 of  $PQ$ ).

11. Extend  $q'm'$  to  $p'$  such that,  $m'p' = m'q'$ .
12. Extend  $qm$  to  $p$  such that,  $mp = mq$ .

$p'q'$  and  $pq$  are the required projections of the line  $PQ$ .

**Problem 26** A room is  $6\text{ m} \times 5\text{ m} \times 3.5\text{ m}$  high. An electric bulb is above the centre of the longer wall and  $1\text{ m}$  below the ceiling and  $0.35\text{ m}$  away from the wall. The switch for the light is  $1.25\text{ m}$  above the floor, on the centre of an adjacent

wall. Determine graphically, the shortest distance between the bulb and switch.

HINT The longer wall may be taken as V.P, the floor as H.P and side wall as P.P.

### **Construction ([Fig.8.36](#))**

1. Draw the front and top views of the room.
2. Locate the projections of the bulb (B), b' and b and switch (S), s and s' in the above views, satisfying the given conditions.
3. Obtain the two views of the line, joining the bulb and switch.
4. Rotate the line sb to the position sb<sub>1</sub>, parallel to xy.
5. Draw f-f parallel to xy, passing through b', the locus of the front view of B.
6. From b<sub>1</sub>, draw a projector meeting f-f at b<sub>1</sub>'.
7. Join s', b<sub>1</sub>'.

The length of the line s'b<sub>1</sub>' represents the shortest distance (true length) between the switch and bulb.

**Problem 27** A line of 100 long, makes an angle of 35° with H.P and 45° with V.P. Its mid-point is 20 above H.P and 15 in front of VP. Draw the projections of the line.

### **Construction ([Fig.8.37](#))**

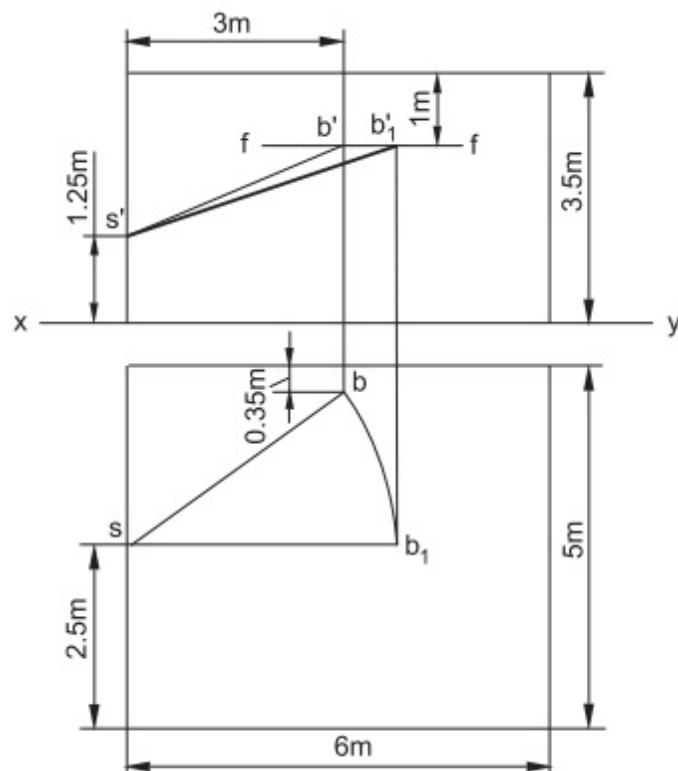
1. Mark the projections m' and m of the mid-point M.
2. Draw the projections m'b' and mb of the part of the line MB (=1/2AB, following the steps given under Construction: [Fig.8.6e](#)).
3. Extend b'm' to a' such that, m'a' = m'b'.

4. Extend  $bm$  to a such that,  $ma = mb$ .

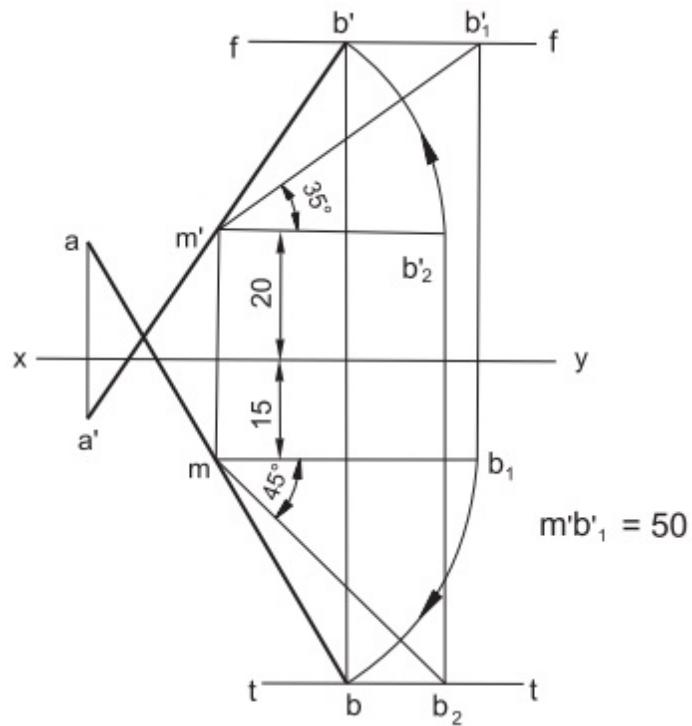
$a'b'$  and  $ab$  are required projections of the line.



The construction is said to be correct if  $a$  and  $a'$  lie on the same projector.



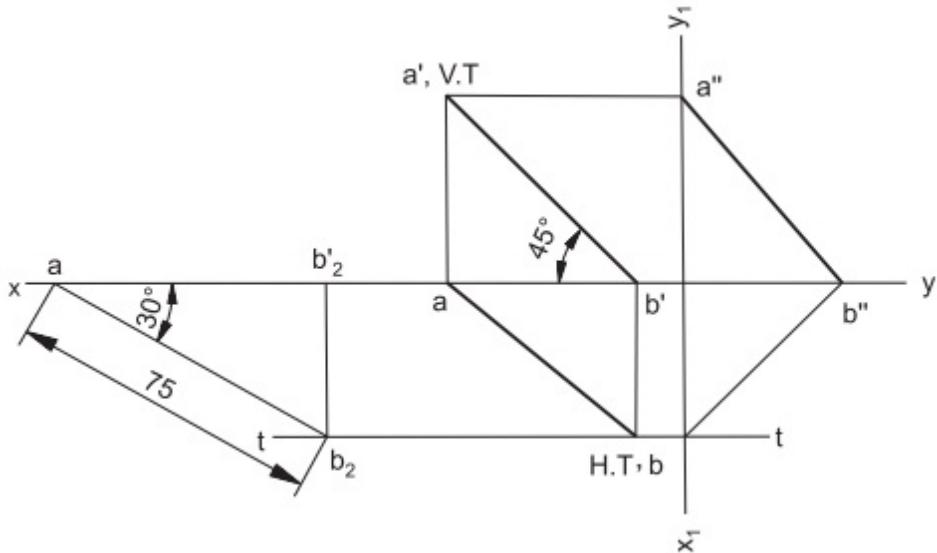
**Fig.8.36**



**Fig.8.37**

**Problem 28** A straight line  $AB$  of 75 long, has the end  $A$  on V.P and the end  $B$  on H.P. The line is inclined at  $30^\circ$  to V.P and its front view makes an angle of  $45^\circ$  with  $xy$ . Draw the projections of the line and add the left side view and locate the traces.

**Construction (Fig.8.38)**



**Fig.8.38**

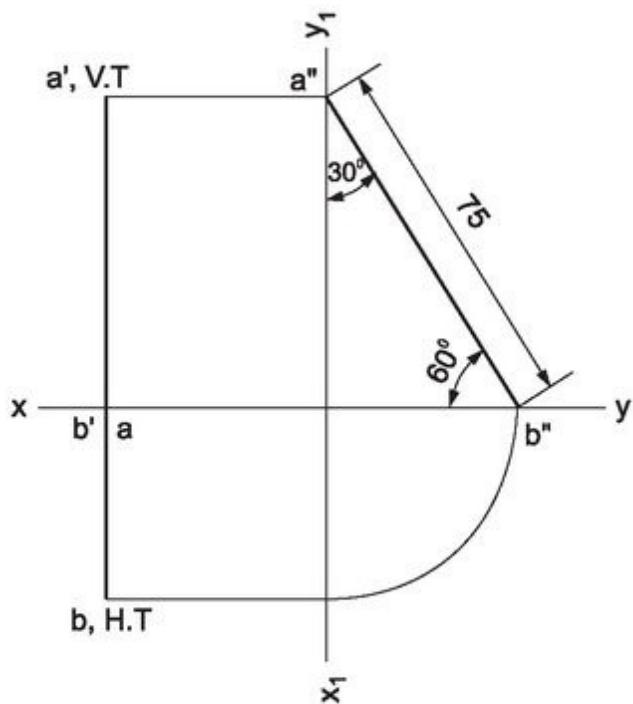
1. Locate  $a$  on  $xy$  and draw  $ab_2$  of 75 long, making  $30^\circ$  with  $xy$ .
2. Draw a projector through  $b_2$  to meet  $xy$  at  $b_2'$ .  
 $ab_2'$  is the length of the front view.
3. Draw  $t-t$  through  $b_2$  and parallel to  $xy$ , representing the locus of top view of the end  $B$ .
4. Through any convenient point  $b'$  on  $xy$ , draw  $b'a'$  of length equal to  $ab_2'$  and at  $45^\circ$  with  $xy$ .
5. Draw a projector through  $a'$ , meeting  $xy$  at  $a$ .
6. Through  $b'$ , draw a projector meeting  $t-t$  at  $b$ .
7. Join  $a, b$ .  
 $a'b'$  and  $ab$  are the required projections.  
The point  $a'$  and  $b$  represent respectively V.T and H.T of the line.
8. Add the left side view, following the principle of Construction: [Fig.8.18](#).

**Problem 29** A line  $AB$  of 75 long, is inclined at an angle of  $30^\circ$  with V.P and lies in a plane perpendicular to both H.P and V.P. Its end  $A$  is in V.P and the end  $B$  is in H.P. Draw the projections of the line  $AB$  and locate its traces.

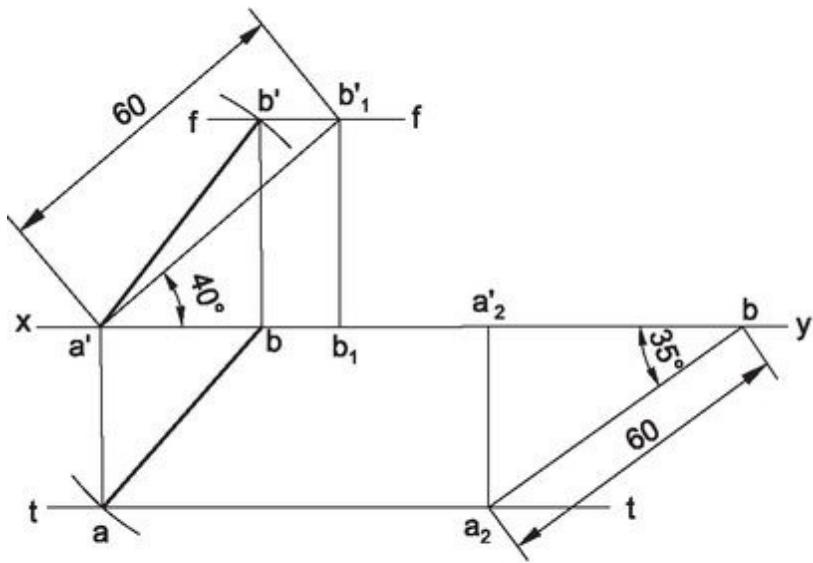


As the line lies in a plane perpendicular to both H.P and V.P,  $\theta + \phi = 90^\circ$ . Hence, the side view should be drawn first and it represents the true length of the line.

**Construction (Fig.8.39)**



**Fig.8.39**



**Fig.8.40**

1. Locate  $b''$  at any point on  $xy$  and draw a line  $a'' b''$ , equal to 75 long and at an angle  $60^\circ$  with  $xy$ .
  2. Draw  $x_1 y_1$  through  $a''$  and perpendicular to  $xy$ .
  3. Draw the projections  $a'b'$  and  $ab$  of the line at any convenient distance from  $x_1y_1$ , by projection.
- $a'$  represents V.T and  $b$ , the H.T.

**Problem 30** *The distance between the traces of a line is 60 and the line is inclined to H.P and V.P at  $40^\circ$  and  $35^\circ$  respectively. Draw the projections of the line.*

**HINT** Assume that the length of the line is equal to the distance between the traces. Then, one end of the line is on H.P and the other on V.P.

**Construction (Fig.8.40)**

1. Through any point  $a'$  on  $xy$ , draw the line  $a' b_1'$  of length 60 and at an angle of  $40^\circ$  with  $xy$ .  
The projected length  $a'b_1'$  represents the length of the top view.
2. Through  $b_1'$ , draw the line  $f-f$ , parallel to  $xy$ .
3. Through any other convenient point  $b$  on  $xy$ , draw the line  $ba_2$  of length 60 and at an angle of  $35^\circ$  with  $xy$ .  
The projected length  $ba_2'$  represents the length of the front view.
4. Through  $a_2$ , draw the line  $t-t$ , parallel to  $xy$ .
5. With  $a'$  as centre and radius equal to  $ba_2'$ , draw an arc intersecting  $f-f$  at  $b'$ .
6. Join  $a', b'$ .
7. Obtain  $b$  on  $xy$  by projection.
8. With  $b$  as centre and radius equal to  $a'b_1'$ , draw an arc intersecting  $t-t$  at  $a$ .



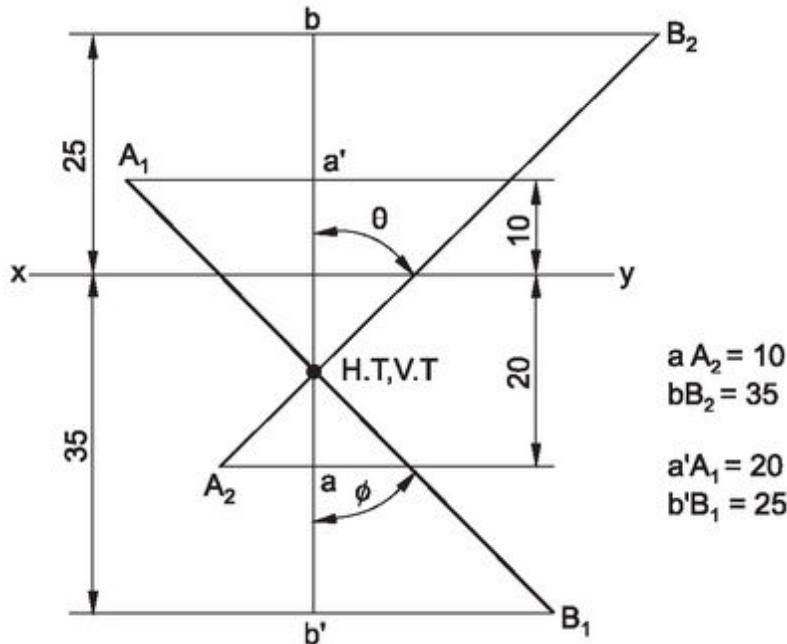
The construction is said to be correct when  $a'$  and  $a$  lie on the same projector.

**Problem 31** *The projections of a line AB are on the same projector. A is 10 above H.P and 20 in front of V.P. B is 35 below H.P and 25 behind V.P. Draw the projections of the line AB and determine its true length, inclinations with H.P and V.P and locate its traces.*

### **Construction ([Fig.8.41](#))**

1. Draw a projector through any point on  $xy$  and locate the projections  $a'$ ,  $a$  and  $b'$ ,  $b$  of end points A and B of the line AB.

2. Determine the true length of the line, its inclinations with H.P and V.P, by following the procedure given under Construction: Fig.8.9c.

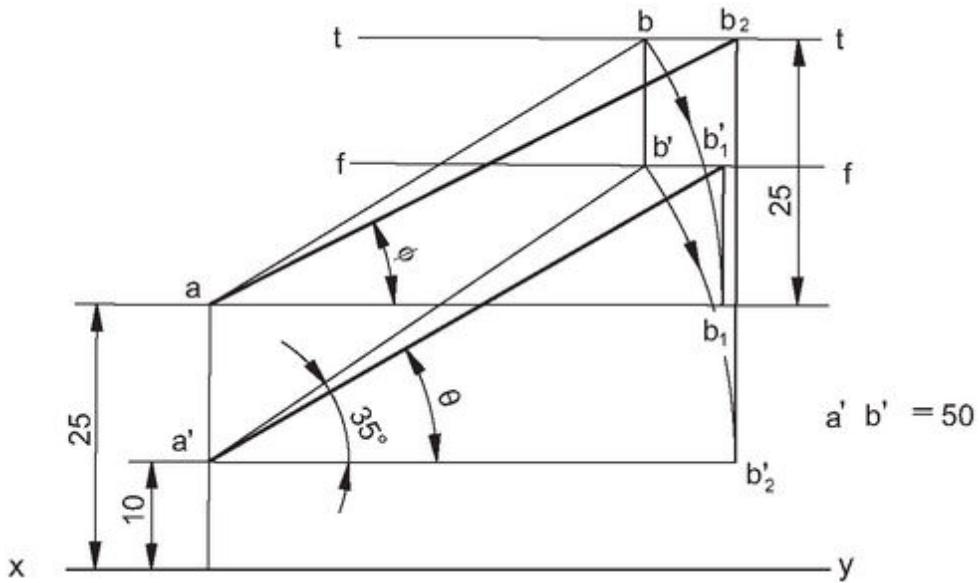


**Fig.8.41**

- (i)  $A_1B_1 = A_2 B_2$  represents the true length of the line.
- (ii) The angle between  $A_2B_2$  and  $ab$  represents  $\theta$  and the angle between  $A_1B_1$  and  $a'b'$  represents  $\phi$ , the true angles of inclination with H.P and V.P respectively.
- (iii) The points of intersection between  $ab$  and  $A_2B_2$  and between  $a'b'$  and  $A_1B_1$  represent H.T and V.T respectively.

**Problem 32** The front view of a line AB is 50 long and it makes an angle of  $35^\circ$  with  $xy$ . The point A lies 10 above H.P and 25 behind V.P. The difference between the distances of A and B from V.P is 25. The line AB is in second quadrant. Draw the projections of the line, determine its true length and inclinations with H.P and V.P.

### **Construction (Fig.8.42)**



**Fig.8.42**

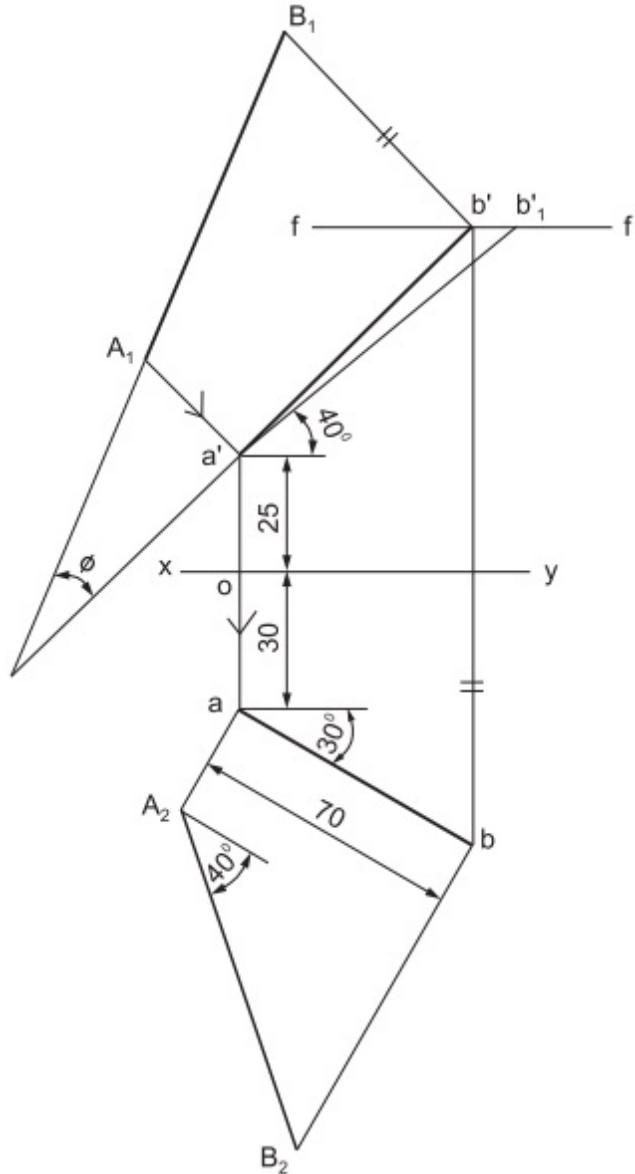
1. Locate the projections  $a'$  and  $a$  of the end A.
2. Draw  $a'b'$  of length 50 and at an angle of  $35^\circ$  with  $xy$  and draw the locus  $f-f$  through  $b'$ .
3. Draw the locus  $t-t$ , parallel to and  $50 (= 25 + 25)$  above  $xy$ .
4. Locate  $b$  on  $t-t$  and join  $a, b$ , forming the top view of the line.
5. Determine the true length of the line, following the procedure, similar to Construction: [Fig.8.8b](#).

$ab_2 = a'b'_1$  represents the true length of the line. The angles  $\theta$  and  $\phi$  between  $a'b'_1$  and  $xy$  and  $ab_2$  and  $xy$  represent respectively the true inclinations of the line with H.P and V.P.

**Problem 33** A line AB is inclined at  $40^\circ$  to H.P. Its one end A is 25 above H.P and 30 in front of V.P. The top view of the line is 70 and is inclined at  $30^\circ$  to  $xy$ . Draw the projections

of the line and determine its true length and its inclination with V.P.

### **Construction (Fig.8.43)**



**Fig.8.43**

1. Locate the projections of A with respect to xy and locate the point of intersection o between xy and the projector a'a.
2. Draw the top view ab of 70 long, making 30° with xy.

3. Through a and b, draw perpendiculars to ab.
4. Mark  $A_2$  such that,  $aA_2 = oa'$ .
5. Draw a line through  $A_2$ , making  $40^\circ$  with ab, meeting the perpendicular through b at  $B_2$ .
6. Through  $a'$ , draw a line  $a'b'_1 (= A_2B_2)$  at  $40^\circ$  with xy.
7. Draw f-f through  $b'_1$  and parallel to xy.
8. Draw a projector through b to meet f-f at  $b'$ .
9. Join  $a', b'$ .
10. Following the procedure, similar to the one given under Construction: [Fig.8.9c](#), determine the true inclination  $\phi$  with V.P.

$A_2B_2 (= A_1 B_1)$  is the true length of the given line.



A method similar to the one given under Construction: [Fig.8.8b](#) may also be followed to solve the problem.

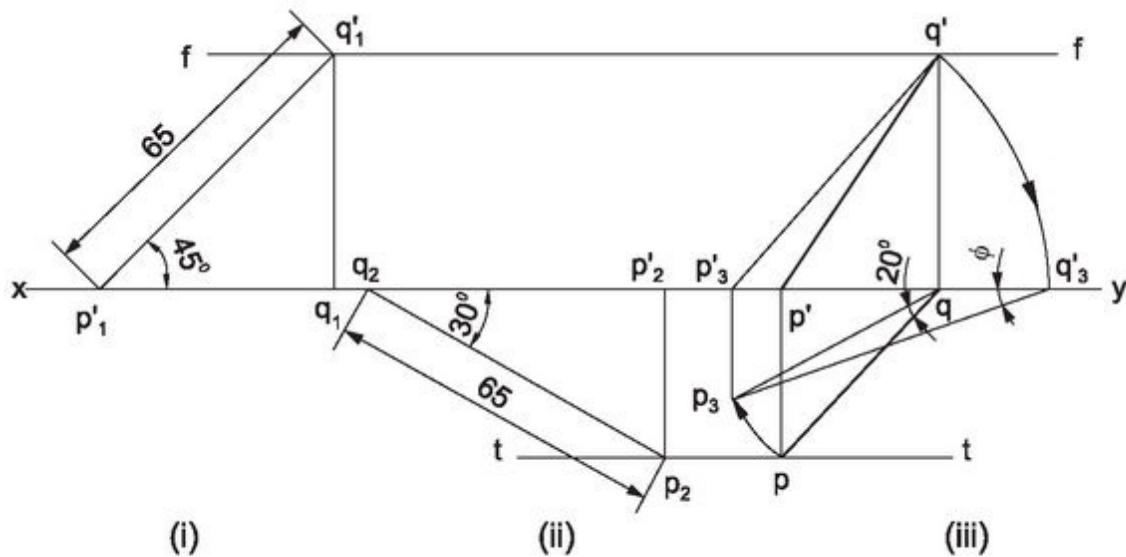
**Problem 34** *Draw the projections of a line PQ of 65 long, which is inclined at  $45^\circ$  to H.P and  $30^\circ$  to V.P. P is on H.P and Q is on V.P. It is rotated about Q such that, the top view of the line is rotated through  $20^\circ$  from the previous position, towards V.P. Determine the inclination of the line with V.P after rotation.*

#### **Construction ([Fig.8.44](#))**

1. From a point  $p'_1$  on xy, draw  $p'_1q'_1 (=65)$  at  $45^\circ$  and determine the length of the top view  $p'_1 q_1$  by projection (Fig. i).
2. From a point  $q_2$  on xy, draw  $q_2 p_2 (=65)$  at  $30^\circ$  and determine the length of the front view  $q_2 p_2'$  by projection (Fig. ii).

3. Draw the lines f-f and t-t, representing the paths of points Q and P in front and top views respectively.
4. Draw the projections of the line  $p'q'$  and  $pq$ , by combining the steps (i) and (ii) (Fig. iii).
5. Rotate  $qp$  to  $qp_3$  by an angle  $20^\circ$  towards xy and locate  $p_3'$  by projection.
6. Considering the projections  $p_3' q'$  and  $p_3 q$ , determine the true inclinations of the line with V.P, i.e.,  $\phi$  as follows:
  - (i) Rotate  $p_3' q'$  parallel to xy to  $p_3' q_3'$ , coinciding with xy.
  - (ii) Join  $p_3, q_3'$ .

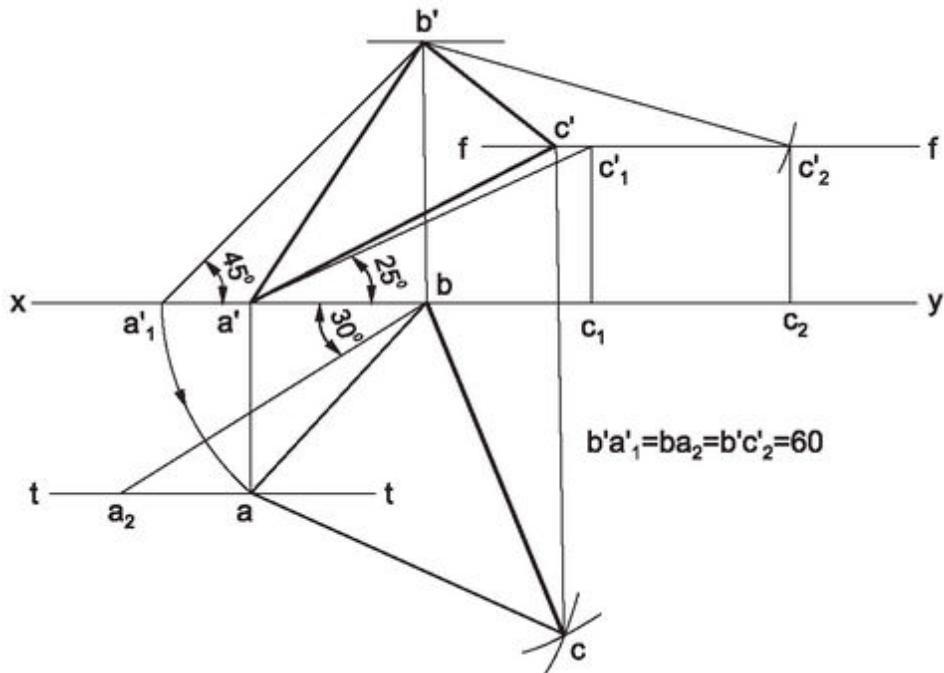
The inclination of the line  $q_3'p_3$  with xy, i.e.,  $\phi$  is true inclination of the line PQ with V.P.



**Fig.8.44**

**Problem 35** An equilateral triangle ABC of side 60, lies with A on H.P and B on V.P and with side AB, making  $45^\circ$  and  $30^\circ$  respectively with H.P and V.P. The side CA makes an angle of  $25^\circ$  with H.P. Draw the projections of the triangle ABC.

**Construction (Fig.8.45)**



**Fig.8.45**

1. Draw the projections  $ab'$  and  $ab$  of the line AB, satisfying the given conditions.
2. Draw  $a'c'_1$  ( $= AC$ ) at  $25^\circ$  to  $xy$  and obtain the length of the top view of AC ( $= a'c'_1$ ).
3. Draw  $f-f$ , parallel to  $xy$  and passing through  $c'_1$  (path of C in front view).
4. With  $b'$  as centre and radius 60, draw an arc intersecting  $f-f$  at  $c'_2$ .

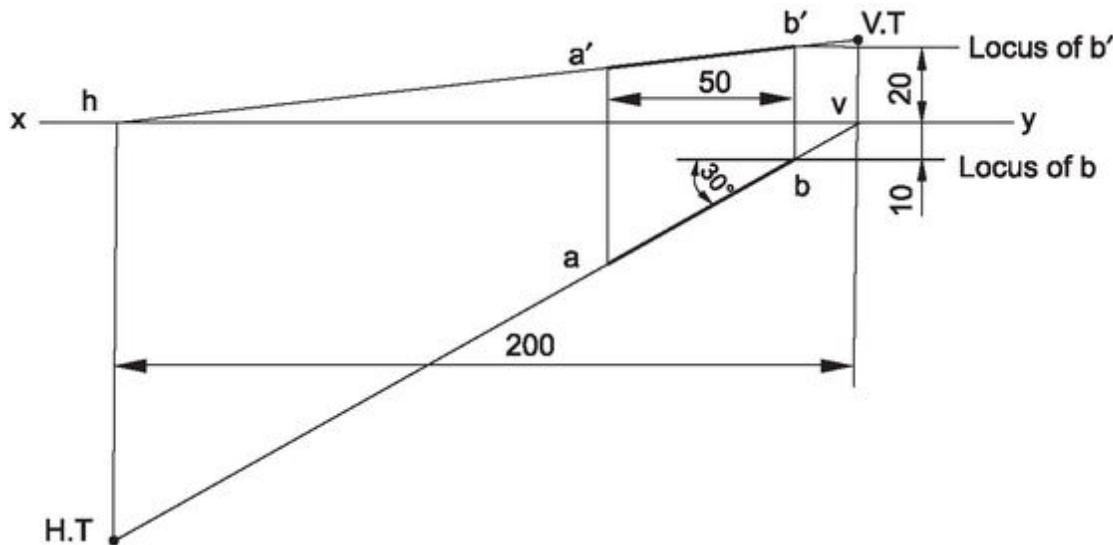
The projected length  $bc_2$  is the length of the top view of BC.

5. With b as centre and radius  $bc_2$ , draw an arc.
6. With a as centre and radius  $a'c_1$ , draw an arc intersecting the above arc at c.
7. Join a, b and c and complete the top view.
8. Draw a projector through c, meeting f-f at  $c'$ .

The figure formed by joining the points  $a'$ ,  $b'$  and  $c'$  is the front view.

**Problem 36** The H.T and V.T of the straight line AB is below and above xy respectively. The distance between the H.T and V.T measured parallel to xy is 200. The end B of the line is nearer to the V.P than the end A. The top view of the line makes  $30^\circ$  to xy. The end B is 10 from VP and 20 from the H.P. The distance between the end projectors of the line measures 50 parallel to xy. Draw the projections of the line.

**Construction (Fig.8.46)**



**Fig.8.46**

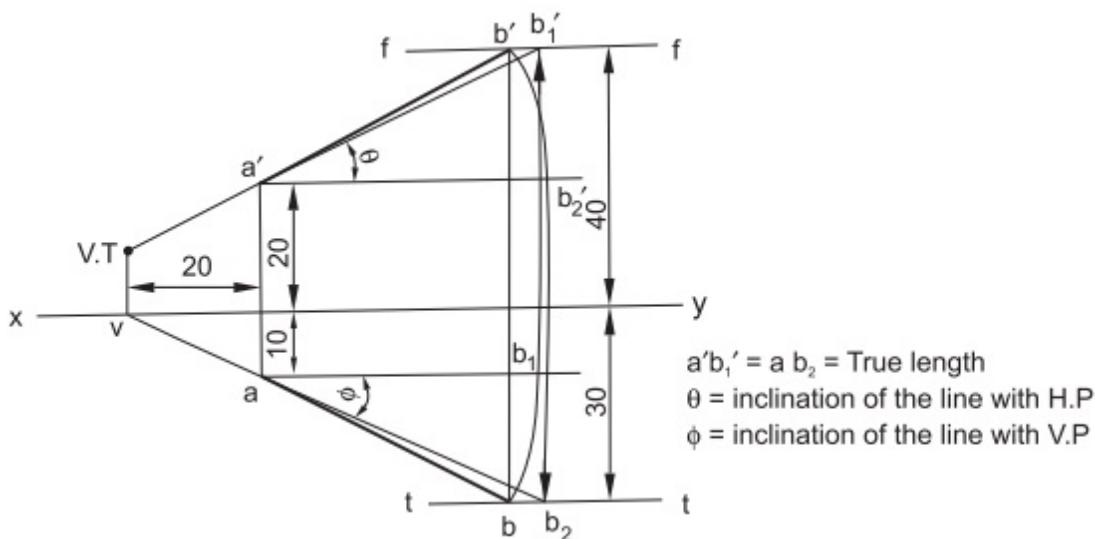
1. Draw lines 10 below and 20 above xy and parallel to xy, representing the loci of b and b'.
  2. Draw two projectors on xy separated by 200; representing the projectors through H.T and V.T. Locate h and v on xy (front and top views of H.T and V.T).
  3. Draw a line at  $30^\circ$  to xy from v to meet the projector through h at H.T and crossing the locus of b at b.
  4. Draw a projector through b and locate b' on the locus of b'.
  5. Join hb' and extend to locate V.T on the projector through v.
  6. Draw a projector at 50 from the projector through b and locate a and a' on this.
- a' b' and a b represent the projections of the line AB.

**Problem 37** *The end A of a straight line AB is 10 from the V.P and 20 from the H.P. The end B is 30 from the V.P and 40 from H.P. The VT of the line is 20 from the end A as measured parallel to xy. Draw the projections and find the true length and the inclinations of the line.*

### **Construction ([Fig.8.47](#))**

1. Draw the reference line xy and on any chosen projector, locate the projections of the end A of the line AB.
2. Draw a projector at 20 from the above projector and locate v on it on xy.
3. Draw f-f, the locus of the front view of the end B, parallel to and 40 above xy.

4. Draw  $t-t$ , the locus of the top view of the end B, parallel to and 30 below  $xy$ .
5. Join  $v$ ,  $a$  and extend, meeting  $t-t$  at  $b$ .
6. Through  $b$ , draw a projector, meeting  $f-f$  at  $b'$ .
7.  $a' b'$  and  $ab$  are the projections of the line AB. By rotating line method, determine the true length and true inclinations of the line with H.P and V.P as shown.



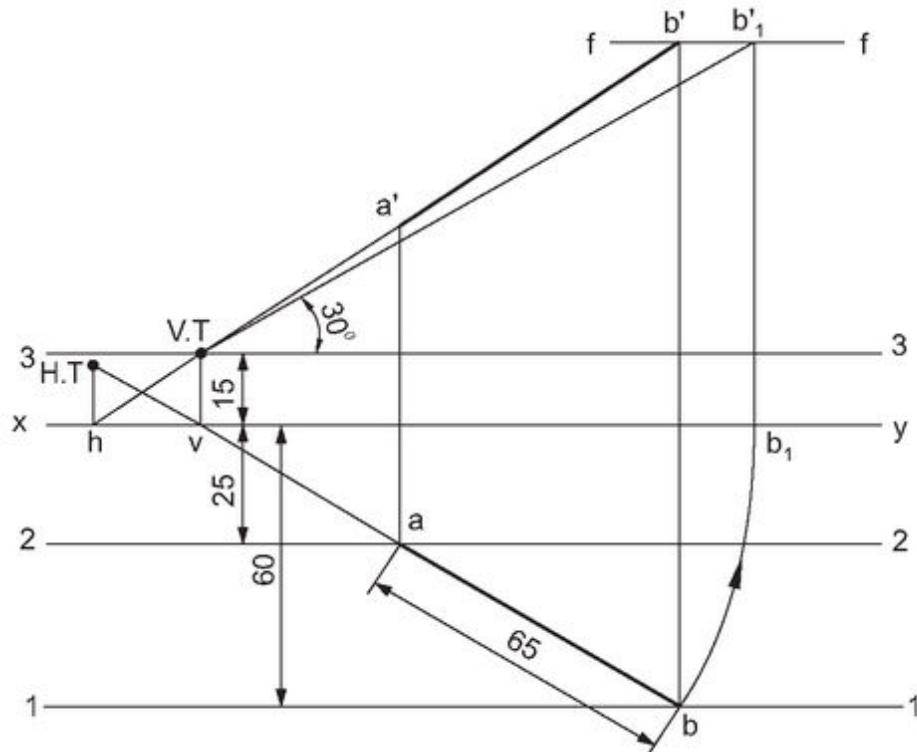
**Fig.8.47**

**Problem 38** A line AB, which is inclined at  $30^\circ$  to H.P, has its ends A and B, at 25 and 60 in front of V.P respectively. The length of the top view is 65 and its V.T is 15 above H.P. Draw the projections of the line and locate its H.T.

**Construction (Fig.8.48)**

1. Draw the lines 1-1, 2-2 and 3-3, parallel to  $xy$  and to contain  $b$ ,  $a$  and V.T.
2. Locate  $a$  at any convenient point on 2-2.
3. With centre  $a$  and radius 65, draw an arc meeting 1-1 at  $b$ .

4. Join b, a and extend to meet xy at v.
  5. Through v, draw a projector meeting 3-3 at V.T.
  6. Rotate vb to  $vb_1$ , parallel to xy.
  7. Draw a line through V.T at  $30^\circ$  to xy, meeting the projector through  $b_1$  at  $b'_1$ .
  8. Draw f-f through  $b'_1$  and parallel to xy.
  9. Draw a projector through b, meeting f-f at  $b'$ .
  10. Join  $b'$ , V.T and extend, meeting xy at h.
  11. Draw a projector through h, meeting ba extended at H.T.
  12. Draw a projector through a, meeting  $b'$ -V.T at  $a'$ .  
 $a'b'$  and ab are the required projections.



**Fig.8.48**

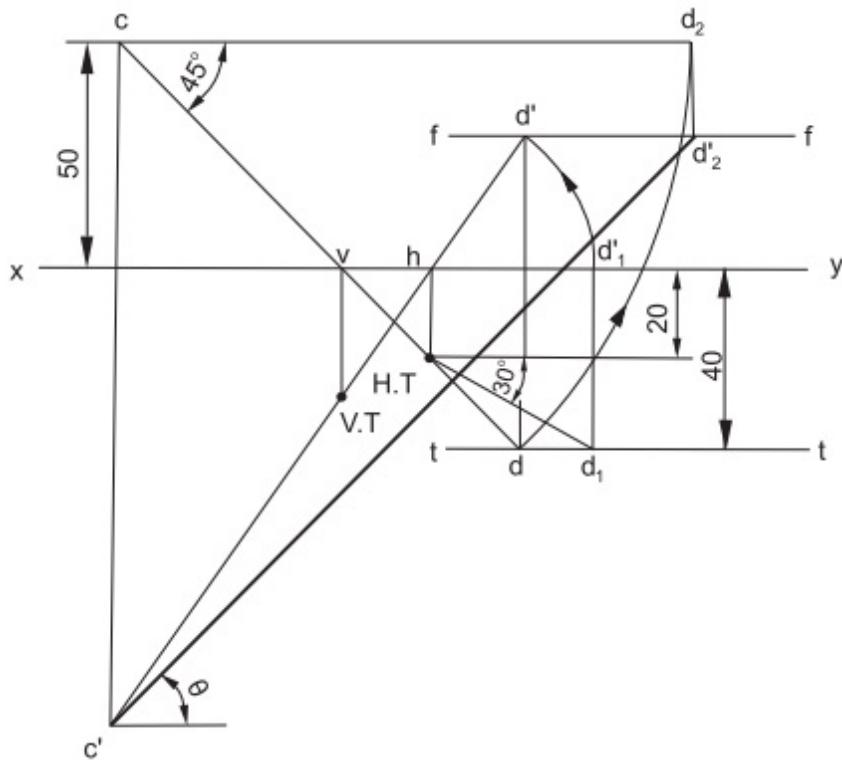
**Problem 39** The end C of a line CD is in third quadrant and is 50 from V.P. The end D is in first quadrant and is 40 from V.P. The top view of the line is inclined at  $45^\circ$  to xy and H.T of the line is 20 below xy. The line CD is inclined at  $30^\circ$  to V.P. Draw the projections of the line and determine (i) true length, (ii) inclination of the line with H.P and (iii) the location of V.T from H.P, stating whether it is below or above H.P.

### **Construction (Fig.8.49)**

1. Through any convenient point on xy, draw a projector and locate c, the top view of the end C, at 50 above xy.
2. Through c, draw the line cd, inclined at  $45^\circ$  with xy; d being 40 below xy.
3. Through d, draw the locus t-t.
4. Locate H.T on cd such that, it is 20 below xy.
5. Draw a projector through H.T, meeting xy at h.
6. Considering the part of the line H.T-D, obtain its projections hd' and H.T-d.
7. Draw the locus f-f, through d' and parallel to xy.
8. Join d', h and extend, meeting the projector through c at c'.
- cd and c'd' are the projections of the line.
9. Rotate cd about c to cd<sub>2</sub>, parallel to xy.
10. Draw a projector through d<sub>2</sub>, meeting f-f at d'<sub>2</sub>'.
- c'd'<sub>2</sub>' represents the true length of the line and its inclination  $\theta$  with xy represents the true inclination with H.P.
11. Locate v, the point of intersection between xy and cd.

12. Draw a projector through v, meeting c'd' at V.T.

V.T is below H.P.



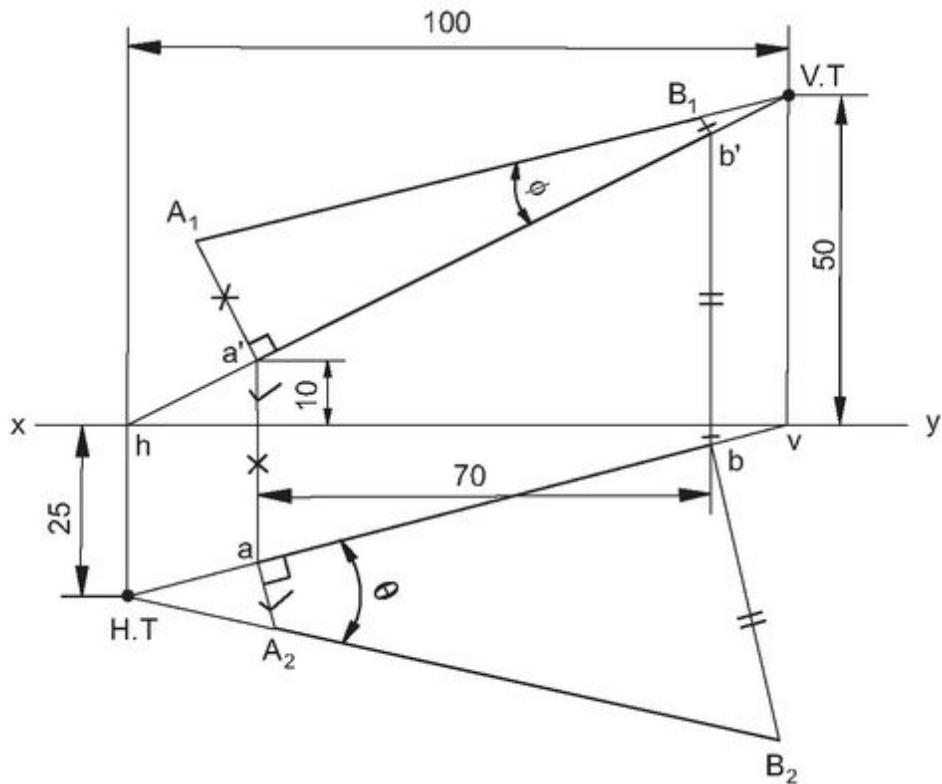
**Fig.8.49**

**Problem 40** The projections drawn from H.T and VT of a straight line AB are 100 apart, while those drawn from its ends are 70 apart. The H.T is 25 in front of VP, the VT is 50 above H.P and the end A is 10 above H.P. Draw the projections of the line and determine its true length and inclinations with the principal planes.

### **Construction (Fig.8.50)**

1. Draw the projectors, which are 100 apart and locate H.T and V.T and then h and v.
2. Join h, V.T and v, H.T.
3. Locate a' on the line h-V.T and at 10 above xy.
4. Draw a projector through a' and locate a on v-H.T.

5. Draw another projector at 70 from  $a'$ , intersecting h-V.T at  $b'$  and v-H.T at  $b$ .  
 $a'b'$  and  $ab$  are the projections of the line.
6. Determine the true length and true inclinations of the line, following the method of Construction: [Fig.8.9c](#).



**Fig.8.50**

**Problem 41** A line  $AB$  of 70 long, has its end  $A$  at 10 above H.P and 15 in front of V.P. Its front view and top view measure 50 and 60 respectively. Draw the projections of the line and determine its inclinations with H.P and V.P.

**Construction ([Fig.8.51](#))**

1. Draw the reference line  $xy$  and locate the projections  $a'$ ,  $a$  of the end  $A$ .

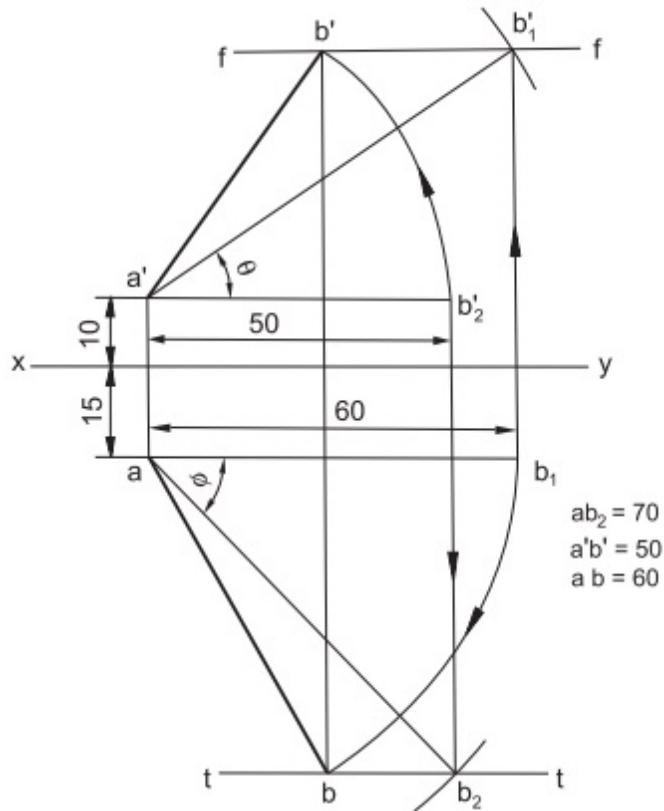
2. Draw  $a'b_2'$  ( $=50$ ), parallel to  $xy$ , representing the length of the front view.
3. With centre  $a$  and radius 70 (true length), draw an arc intersecting the projector through  $b_2'$  at  $b_2$ .
4. Join  $a, b_2$ .

Inclination  $\phi$  of the line  $ab_2$  with  $xy$ , represents the true inclination of the line with V.P.

5. Draw  $ab_1$  ( $=60$ ), parallel to  $xy$ , representing the length of the top view.
6. With centre  $a'$  and radius 70 (true length), draw an arc intersecting the projector through  $b_1$  at  $b_1'$ .

The inclination  $\theta$  of the line  $a'b_1'$  with  $xy$  represents the true inclination of the line with H.P.

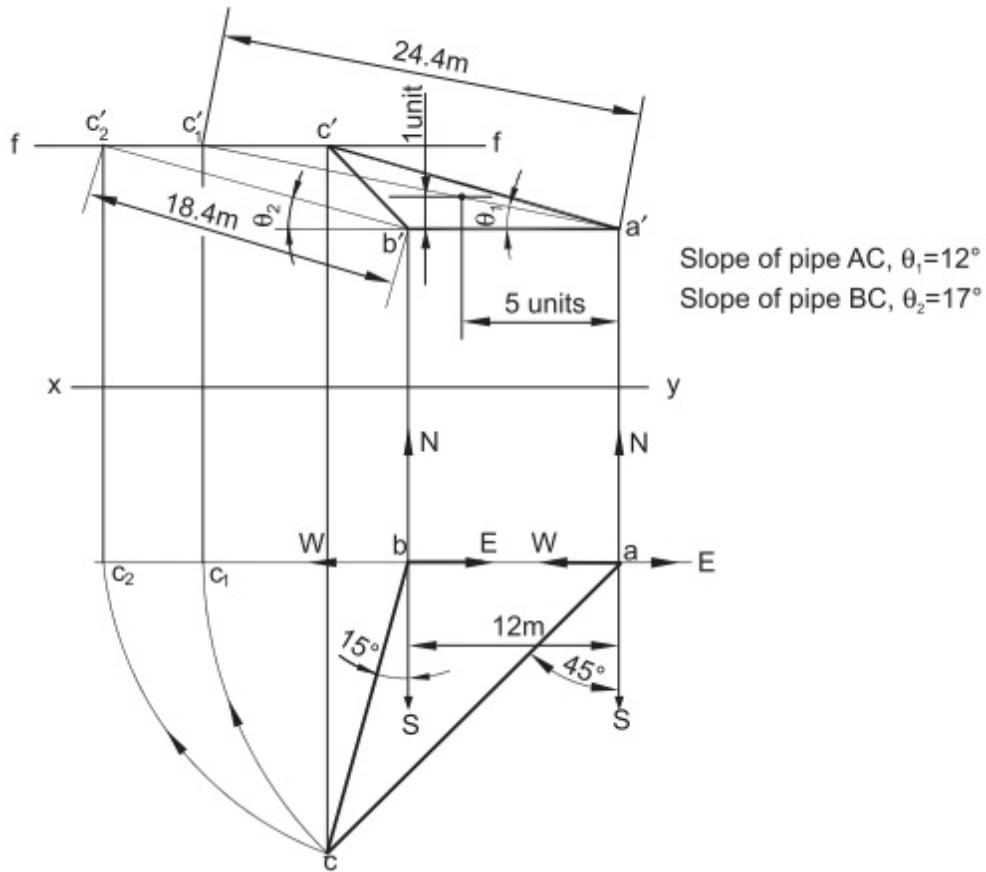
7. Through  $b_1'$ , draw the line  $f-f$ ; representing the locus of front view of B.
8. Through  $b_2$ , draw the line  $t-t$  representing the locus of top view of B.
9. With centre  $a'$  and radius  $a'b_2'$ , draw an arc intersecting  $f-f$  at  $b'$ .
10. Join  $a', b'$ ; representing the front view of the line.
11. With centre  $a$  and radius  $ab_1$ , draw an arc intersecting  $t-t$  at  $b$ .
12. Join  $a, b$ ; representing the top view of the line.



**Fig.8.51**

**Problem 42** A pipe line from a point A running due South West, has an upward gradient of 1 in 5. Another point B on the level of A is 12m due West of A. Pipe line through B running  $15^\circ$  West of South, meets the pipe line from A at C. Find the slope and true length of each pipe.

**Construction (Fig.8.52)**



**Fig.8.52**

1. Draw the reference line  $xy$  and draw a line parallel to  $xy$  and below it at any convenient distance.
2. Locate  $a$ , the top view of the pipe end A on the line drawn below  $xy$ .
3. Through  $a$ , mark East, West, South and North directions.
4. Through  $a$ , draw a projector and locate  $a'$  on it at any convenient height above  $xy$ .
5. Locate  $b$  at 12m from  $a$  on the western side of  $a$ .
6. Through  $b$ , draw a projector meeting the horizontal line through  $a'$  at  $b'$ .

7. Through a, draw a line due South West, i.e., at  $45^\circ$  downwards from the vertical.
8. Through b, draw a line at  $15^\circ$  West of South, i.e., at  $15^\circ$  downwards from the vertical; meeting the above line at c; the point at which the pipes AC and BC meet in the top view.
9. Rotate ac to  $ac_1$ , parallel to xy.
10. Through  $a'$ , draw a line having an upward gradient of 1 in 5.
11. Through  $c_1$ , draw a projector meeting the above line at  $c_1'$ .
12. Through  $c_1'$ , draw f-f, parallel to xy, representing the locus of front view of the pipe end C.
13. Join  $a', c_1'$ .
14. Rotate bc to  $bc_2$ , parallel to xy.
15. Through c draw a projector meeting f-f at  $c_2'$ .
16. Join  $b', c_2'$ .
17. Through c, draw a projector meeting f-f at  $c'$ .

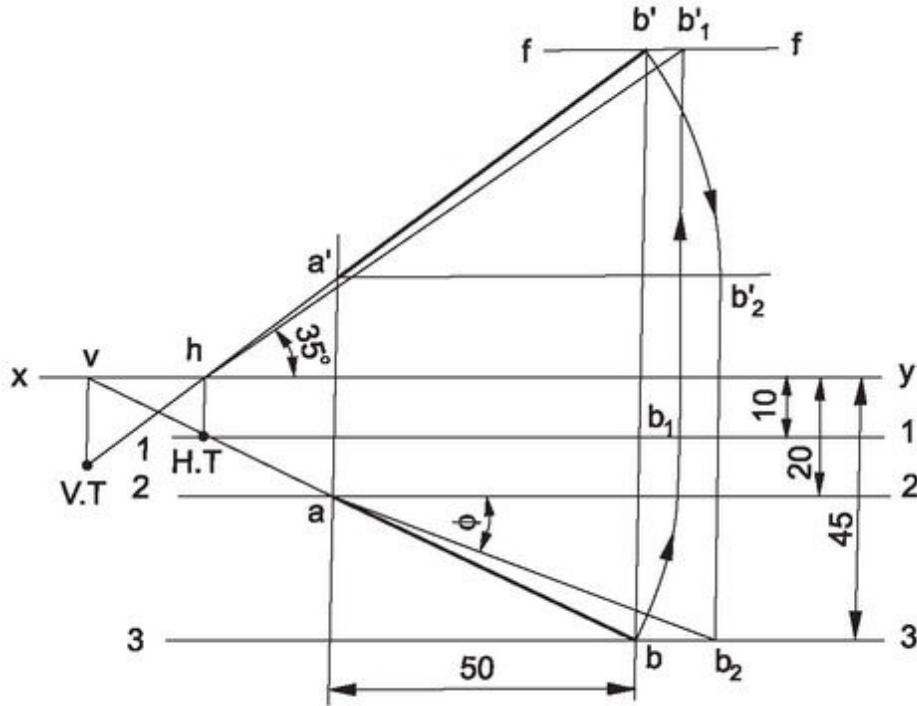
Join  $b', c'$  and  $a', c'$  forming the front view of the piping ABC.

The lengths of the lines  $a'c_1'$  and  $b'c_2'$  represent the true lengths of the pipes AC and BC.

**Problem 43** A line AB has its ends A and B, 20 and 45 in front of V.P respectively. The end projectors of the line are 50 apart. The H.T of the line is 10 in front of V.P. The line AB is inclined at  $35^\circ$  to H.P. Draw the projections of the line and determine the true length of the line and locate its V.T.

*Find the distance of V.T of the line from H.P and inclination of the line with V.P.*

**Construction (Fig.8.53)**



**Fig.8.53**

1. Draw the lines 1-1, 2-2 and 3-3 at 10, 20 and 45 below xy respectively, to contain H.T, a and b.
2. Draw two projectors, which are 50 apart, to contain the projections of the ends A and B.
3. Locate a and b and then join a, b; forming the top view of the line.
4. Extend ba, intersecting the line 1-1 at H.T and xy at v.
5. Draw a projector through H.T, meeting xy at h.
6. Through h, draw a line at 35° to xy.
7. Rotate H.T - b to H.T-b<sub>1</sub>, parallel to xy.

- Through  $b_1$ , draw a projector meeting the line through  
 8.  $h$  at  $b_1'$ .
9. Through  $b_1'$ , draw  $f-f$ , the locus of the front view of B.
10. Through  $b$ , draw a projector meeting  $f-f$  at  $b'$ .
11. Join  $h, b'$ .
12. Locate  $a'$ , the point of intersection between  $hb'$  and the projector through a.
13. Rotate  $a'b'$  to  $a'b_2'$ .
14. Through  $b_2'$ , draw a projector meeting 3-3 at  $b_2$ .
15. Join  $a, b_2$ , obtaining the true length of the line.
16. The inclination  $\phi$  of the line  $ab_2$  with  $xy$  is the true inclination of the line with V.P.
17. Through  $v$ , draw a projector meeting the line  $b'h$  extended at V.T.

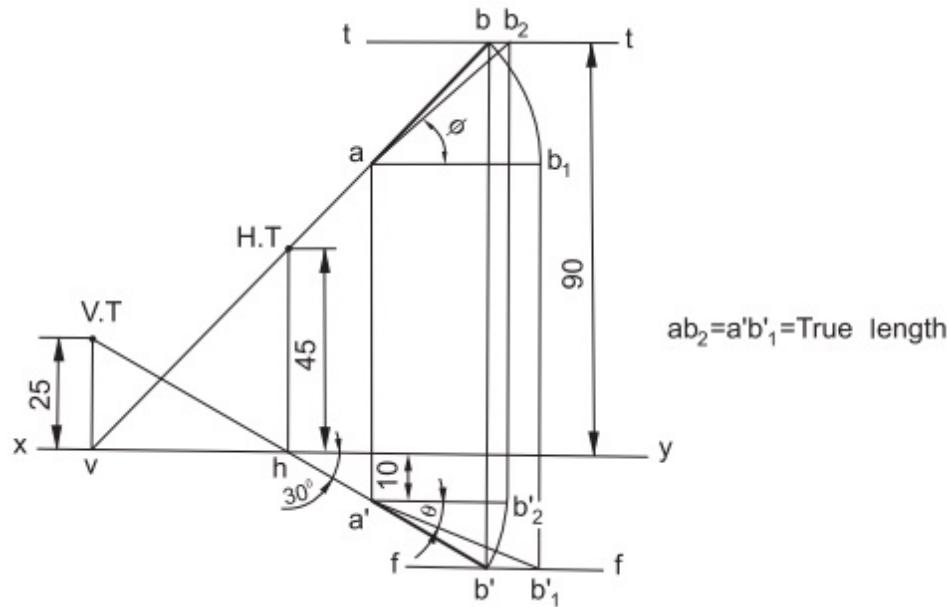
The length  $v-V.T$  is the distance of V.T from H.P.

**Problem 44** *The front view of a line makes an angle of  $30^\circ$  with  $xy$ . The H.T of the line is 45 behind V.P, while its V.T is 25 above H.P. One end of the line is 10 below H.P and the other end is 90 behind V.P. Draw the projections of the line and determine (i) its true length and (ii) its inclinations with H.P and V.P.*

### **Construction ([Fig.8.54](#))**

- At any point  $h$  on  $xy$ , draw a line at  $30^\circ$ , on which the front view of the line AB is to be located.
- Extend the line and locate V.T such that, it is 25 above  $xy$ . Locate  $v$ .

3. On the projector through  $h$ , locate H.T such that, it is 45 above xy.
4. Locate  $a'$  on V.T-h extension such that, it is 10 below xy.
5. Draw the locus of top view of B, t-t, parallel to xy and 90 above it.
6. Join v-H.T and extend, intersecting the above line at  $b$ .
7. Draw a projector through  $a'$ , meeting the line v-H.T at  $a$ .
8. Locate  $b'$  on the projector through  $b$  (intersection point between the projector through  $b$  and V.T-h extension).  
 $a'b'$  and  $ab$  are the projections of the line.
9. Determine the true angles of inclination and the length of the line AB, following Construction: [Fig.8.8b.](#)



**Fig.8.54**

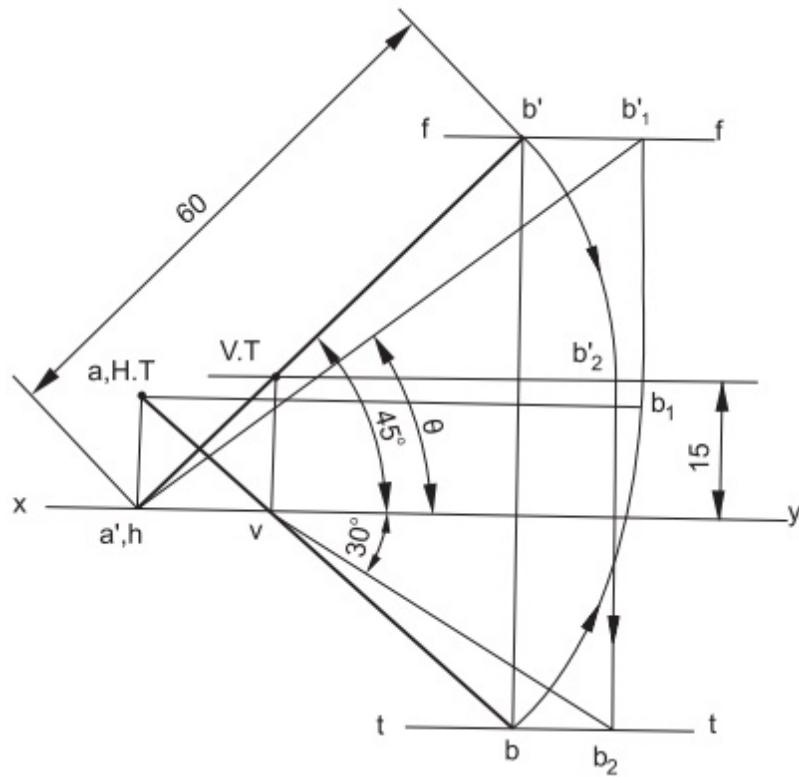
**Problem 45** The front view of a line AB measures 60 and makes an angle of  $45^\circ$  with xy. A is in H.P and VT of the line is 15 above H.P. The line is inclined at  $30^\circ$  to V.P. Draw the

*projections of AB and determine its true length and inclination with H.P. Also locate its H.T.*

### **Construction (*Fig.8.55*)**

1. Draw the reference line  $xy$  and locate  $a'$  at any convenient point on it.
2. Through  $a'$ , draw a line  $a'b'$  of length 60 and at  $45^\circ$  to  $xy$ . Through  $b'$ , draw the line  $f-f$ , the locus of the front view of B.
3. Draw a line parallel to  $xy$  and 15 above it, intersecting  $a'b'$  at V.T. Draw a projector from V.T, meeting  $xy$  at v.
4. Rotate the line V.T- $b'$  about V.T, to the position V.T- $b_2'$ , parallel to  $xy$ .
5. Draw a projector through  $b_2'$ .
6. Through v, draw a line at  $30^\circ$  to  $xy$ , meeting the above projector at  $b_2$ .
7. Through  $b_2$ , draw the line  $t-t$ , the locus of top view of B.
8. Draw a projector through  $b'$ , meeting  $t-t$  at b.
9. Join  $b$ ,  $v$  and extend, meeting the projector through  $a'$  at a.
10. Rotate  $ab$  about a, to the position  $ab_1$ , parallel to  $xy$ .
11. Through b draw a projector meeting  $f-f$  at  $b_1'$ .
12. Join  $a'$ ,  $b_1'$ .

The line  $a'b_1'$  represents the true length of the line and its inclination  $\theta$  with  $xy$  represents the true inclination with H.P. The point **a** represents the H.T of the line.



**Fig.8.55**

**Problem 46** Three guy wires  $AB$ ,  $CD$  and  $EF$  are tied at  $A, C$  and  $E$  on a 15 m long vertical post at heights of 14m, 12m and 10m respectively from the ground. The lower ends of the wires are tied to hooks at points  $B, D$  and  $F$  at the ground level. If the points  $B, D$  and  $F$  lie at the corners of an equilateral triangle of 9m side, and if the post is situated at the centre of the triangle, determine the length of each rope and its inclination with the ground. Assume thickness of the post and the wires to be equal to that of a line.

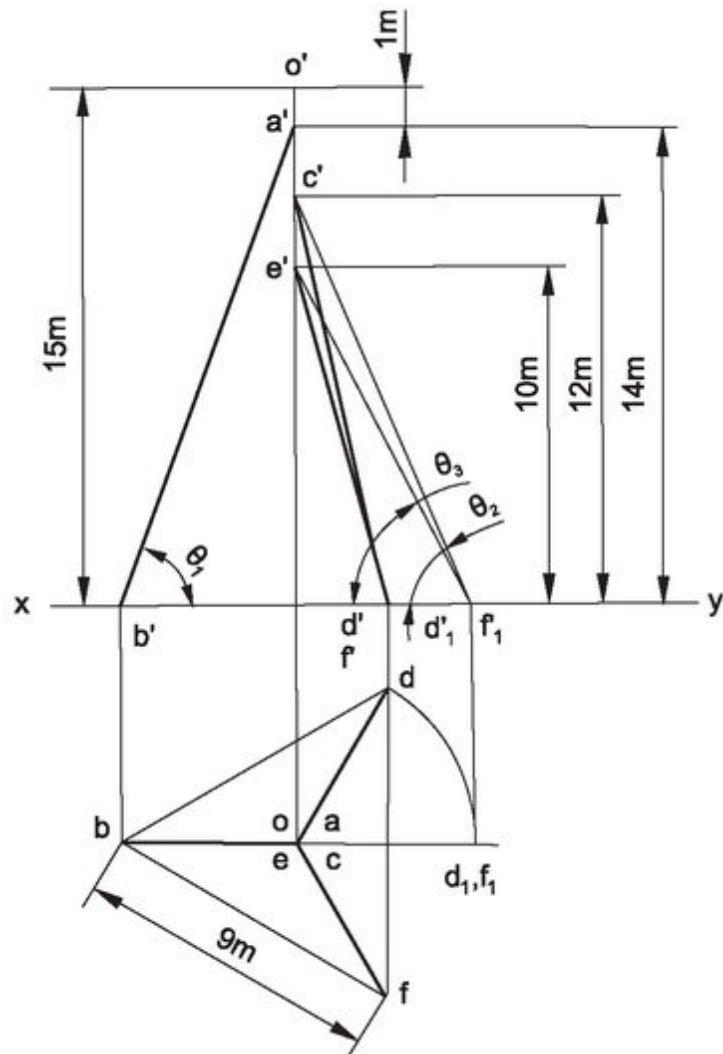
**Construction (Fig.8.56)**

- 1 Draw the equilateral triangle  $bdf$  of 9 m side and locate its centre  $o$ .

The points  $o$  and  $b, d, f$  represent the top views of the post and hooks.

2. Draw front view of the post and locate the ends of the wires,  $a'$ ,  $c'$  and  $e'$  on it.
3. Draw the projections of the wires  $a'b'$ ,  $ab$ ;  $c'd'$ ,  $cd$  and  $e'f'$ ,  $ef$ .
4. Determine the true lengths and inclinations of the guy wires with H.P, by following the principle of Construction: Fig.8.8b.

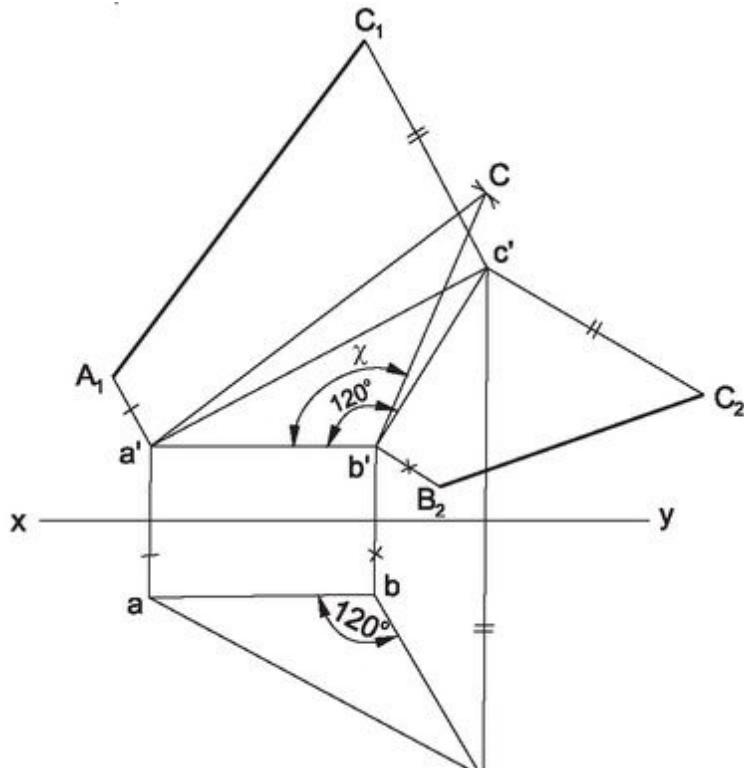
$a'b'$ ,  $c'd'$  and  $e'f'$ , represent the true lengths and  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  represent the true inclinations of guy wires AB, CD and EF with the ground respectively.



**Fig.8.56**

**Problem 47** The front and top views of two intersecting lines AB and BC have an included angle of  $120^\circ$  between them. AB is parallel to both H.P and V.P. Determine the angle between the lines.

**Construction (Fig.8.57)**



**Fig.8.57**

1. Draw the projections of the lines  $a'b'$ ,  $b'c'$  and  $ab$ ,  $bc$ , taking convenient lengths for the lines such that, the angle between  $a'b'-b'c'$  and  $ab-bc$  is  $120^\circ$  and  $a'b'$ ,  $ab$  are parallel to  $xy$ .
2. Join  $a'$ ,  $c'$  and  $a$ ,  $c$ .
3. Determine the true lengths  $A_1 C_1$  and  $B_2 C_2$  of AC and BC, following the procedure under Construction: [Fig.8.9c.](#)

4. Construct the triangle  $a'b'C$  such that,  $a'C = A_1C_1$  and  $b'C = B_2C_2$ .

$\angle a'b'C$  is the true angle between the lines AB and BC.

**Problem 48** A transmission line laid along a level ground from a power station at  $N60^\circ E$  to a sub-station is 3 km long. Another line from the sub-station laid to a village along an uphill, due east is 4 km long and has a slope of  $45^\circ$ . Determine the true length and slope of a proposed telephone line, connecting the power station and village.



Let A, B and C represent the power station, sub-station and village respectively.

### Construction (Fig.8.58)

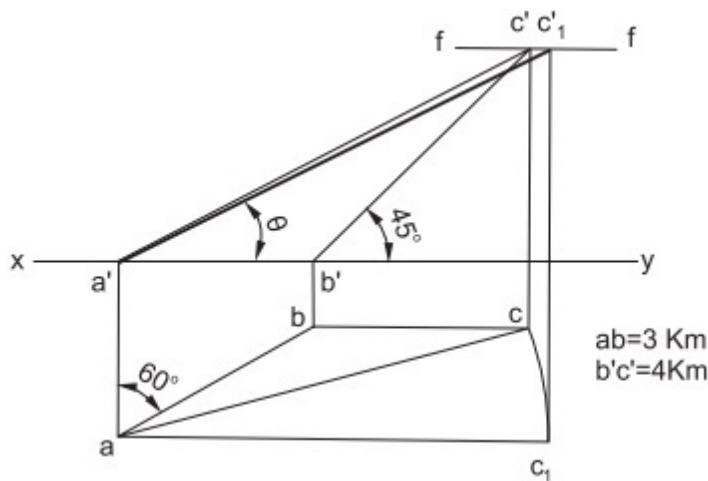


Fig.8.58

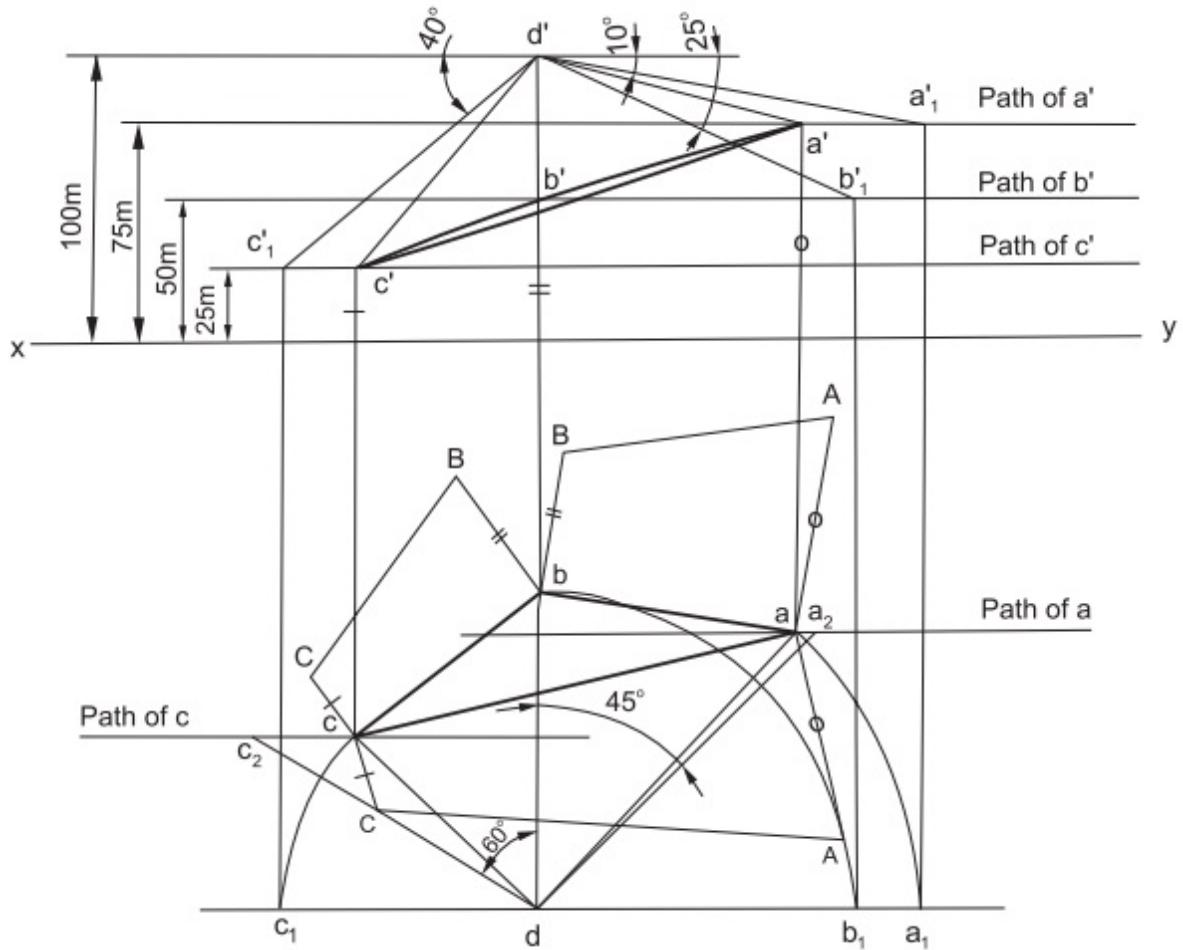
1. Draw  $ab$  to a suitable scale, representing 3 km, at  $N 60^\circ E$  and obtain  $a'b'$ , representing the level ground.
2. Draw  $b'c'$  of 4 km long and at  $45^\circ$  to  $xy$  and obtain  $bc$  by projection.
3. Join  $a'$ ,  $c'$  and  $a$ ,  $c$ , representing the projections of the telephone line.

Obtain the true length of AC and its slope, by following the method of Construction: [Fig.8.8b](#).

**Problem 49** Three stations A, B and C are connected with each other by a commuter train in a city. The station A is located at 75m, B at 50 m and C at 25 m above the mean sea level. The angles of depression of the stations A, B and C, observed from a tower D, located at 100m above MSL are  $10^\circ$ ,  $25^\circ$  and  $40^\circ$  respectively. A is at N  $45^\circ E$ , B is due North and C is at N  $60^\circ W$  of the tower D. Find the lengths of the connecting track.

**HINT** The angles of depression represent the true inclinations of the lines with H.P. The orientations of the stations represent the true inclinations with V.P.

**Construction ([Fig.8.59](#))**



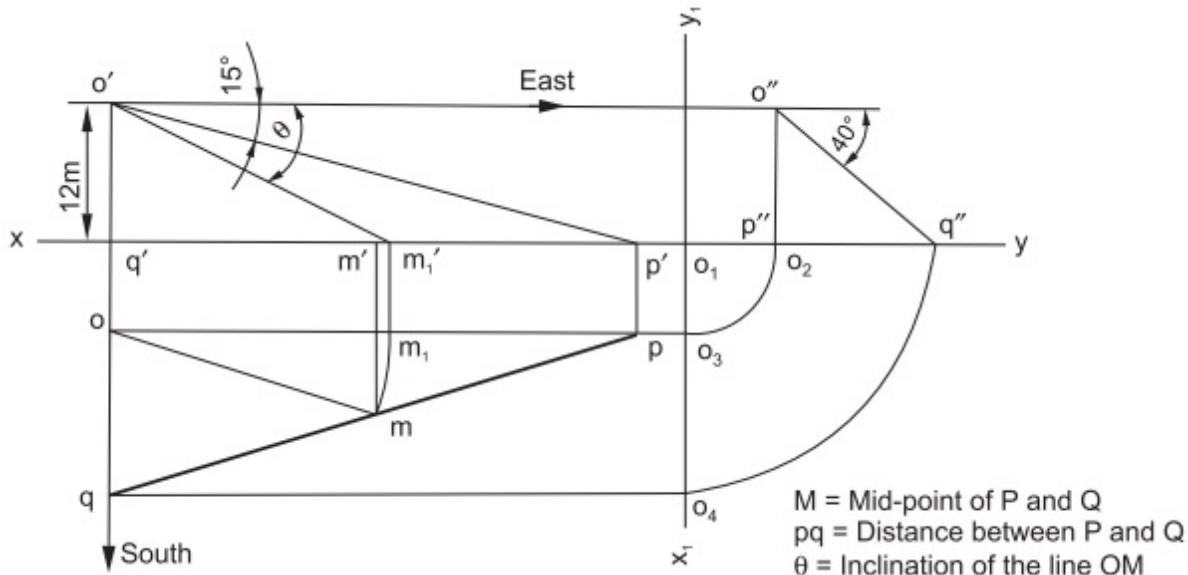
**Fig.8.59**

1. Locate the paths of stations A, B and C in the front view at 75 m, 50 m and 25 m above xy.
2. Locate  $d'$ , the front view of the tower, which is 100 m above xy. Draw the lines,  $d'a$ ,  $d'b_1'$  and  $d'c_1'$  by using (true) angles of depression.  
The lengths of the lines,  $d'a_1'$ ,  $d'b_1'$  and  $d'c_1'$  represent the true distances of the stations A, B and C from D.
3. Draw the lines  $da_2$  ( $= d'a_1'$ ) and  $dc_2$  ( $= d'c_1'$ ), following the given orientations of the stations A and C with respect to the tower D.

4. Locate the paths of A and C in the top view.
  5. Draw the projections  $d'a'$ ,  $da$ ;  $d'b'$ ,  $db$ ; and  $d'c'$ ,  $dc$  of the lines DA, DB and DC, following the principles of Construction: [Fig.8.8b](#).
  6. Join  $a'$ ,  $b'$ ,  $c'$  and  $a$ ,  $b$ ,  $c$ .
  7. Determine the true lengths of the sides AB, BC and CA, representing the true distances of the connecting track, following the principles of Construction: [Fig.8.9c](#).

**Problem 50** An observer at the top of a tower 12m high observes the angles of depression of two objects P and Q on the ground to be  $15^\circ$  and  $40^\circ$ , the direction of P being East and the direction of Q being South. Find the distance between P and Q. What is the inclination of a line to the ground which connects the mid-point of P and Q to the top of the tower.

## **Construction (Fig.8.60)**



**Fig.8.60**

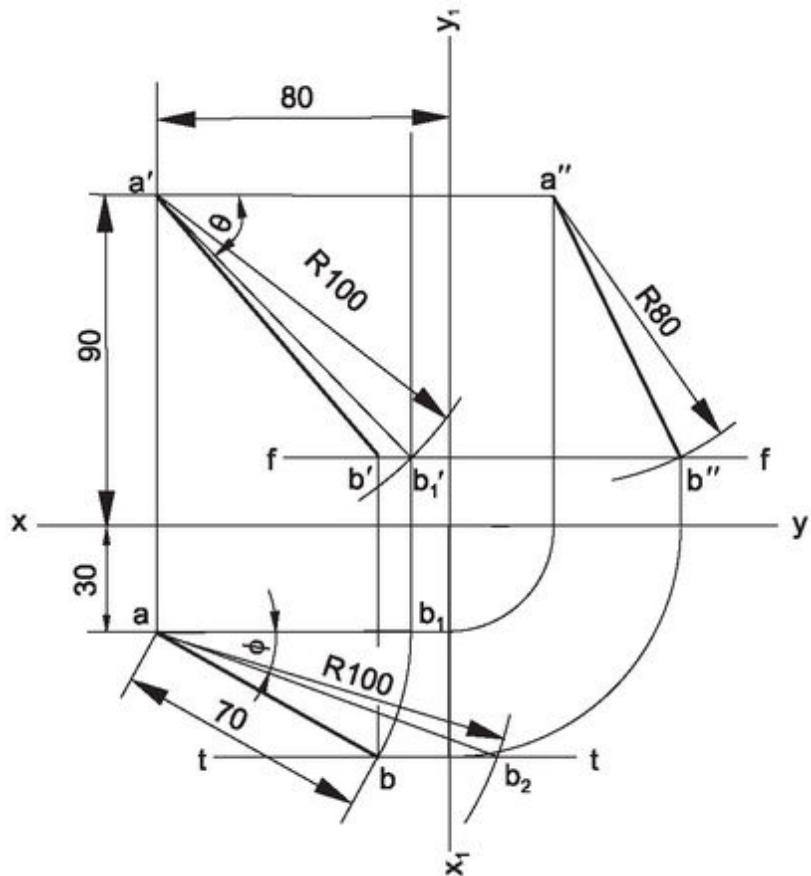
1. Draw the reference lines  $xy$  and  $x_1y_1$  and on the chosen projectors, locate the front view  $o'$  and side view  $o''$  of the tower and 12m above  $xy$ .
2. Through  $o'$ , draw a line at  $15^\circ$  East to meet  $xy$  and locate the front view  $p'$  of the object P on  $xy$ .
3. Through  $o''$ , draw a line at  $40^\circ$  South and locate the end view of the object Q, i.e.,  $q''$  where it meets the reference line  $xy$ . Locate  $p''$ , the side view of P on  $xy$ .
4. Locate  $q'$ , the front view of the object Q on the projector through  $o'$  and lying on  $xy$ .

Obtain the top view  $pq$  from front and side views.

5. Locate  $m'$  and  $m$ , the projections of the mid-point M lying on PQ.
6. Rotate  $om$  to  $om_1$  parallel to  $xy$ .
7. Through  $m_1$ , draw a projector meeting  $xy$  at  $m_1'$ .
8. The inclinations of the line  $o'm_1'$ , is the required inclination of the line to the ground which connects the mid-point of PQ to the top of the tower.

**Problem 51** *The end A of the line AB of 100 long, is 90 above H.P, 80 from P.P and 30 in front of V.P. The length of the top view is 70 and the projected length of the line on P.P is 80. Draw the projections of the line and determine the true inclinations of the line with H.P and V.P.*

**Construction (Fig.8.61)**



**Fig.8.61**

1. Locate the projections  $a'$  and  $a$  with respect to  $xy$ , choosing a suitable scale.
2. Draw  $x_1y_1$  at 80 from the projector connecting  $a'$  and  $a$  and perpendicular to  $xy$ .
3. Draw  $ab_1$  ( $=70$ ), parallel to  $xy$ , representing the length of the top view.
4. With  $a'$  as centre and radius 100 (true length of AB), draw an arc intersecting the projector through  $b_1$  at  $b_1'$ .
5. Draw  $f-f$ , the locus of front view of B, through  $b_1'$  and parallel to  $xy$ .

6. Obtain the side view  $a''$  from  $a$  and  $a'$ .
7. With  $a''$  as centre and 80 as radius, draw an arc intersecting  $f-f$  at  $b''$ .
8. Draw the locus  $t-t$  (locus of top view of B) by projecting from  $b''$ .
9. With  $a$  as centre and radius  $ab_1$ , draw an arc intersecting  $t-t$  at  $b$ .
10. Draw a projector through  $b$ , meeting  $f-f$  at  $b'$ .
11. Join  $a', b'; a, b;$  and  $a'', b''$  forming the three views of the line AB.

The inclination of the line  $a'b_1'$  with  $xy$  is the true inclination of the line AB with H.P ( $\theta$ ).

12. With  $a$  as centre and radius 100, draw an arc intersecting  $t-t$  at  $b_2$ .

The inclination of the line  $ab_2$  with  $xy$  is the true inclination of the line AB with VP ( $\phi$ ).

## EXERCISES

- 8.1 Draw the projections of a straight line AB of 60 long, in the following positions:
  - (a) Parallel to both H.P and V.P and 20 from each.
  - (b) Parallel to and 20 above the H.P and on V.P.
  - (c) Parallel to and 30 in front of V.P and on H.P.
  - (d) Perpendicular to H.P, 30 in front of V.P and one end 25 above H.P.

- (e) Perpendicular to H.P, 30 in front of V.P and one end on V.P.
  - (f) Perpendicular to H.P and in V.P and one end on H.P.
  - (g) Inclined at  $30^\circ$  to V.P, in H.P and one end on V.P.
  - (h) Inclined at  $45^\circ$  to H.P, one end 20 above H.P and parallel to and 30 in front of V.P.
  - (i) Inclined at  $60^\circ$  to V.P, one end 20 in front of V.P and parallel to and 25 above H.P.
- 8.2 A line of 100 long is parallel to and 30 above H.P. Its two ends are 25 and 50 in front of V.P respectively. Find its inclination with V.P.
- 8.3 A line of 90 long is parallel to and 20 in front of V.P. Its one end is in H.P while the other end is 40 above H.P. Find its inclination with H.P.
- 8.4 The top view of a line, which is 75 long, measures 50. The line is in V.P; its one end being 25 above H.P. Draw its projections.
- 8.5 The front view of a line, which is inclined at  $30^\circ$  to V.P, is 65 long. Draw the projections of the line when it is parallel to and 30 above H.P; its one end being 30 in front of V.P.
- 8.6 Draw the projections of a straight line AB, 70 long when it is inclined at  $45^\circ$  to H.P with one end 20 above H.P and parallel to and 30 in front of V.P.
- 8.7 (a) What is the true length of a line whose top view measures 90 and whose inclination to H.P is  $45^\circ$ ?
- (b) What is the true length of a line whose front view measures 160 and whose inclination to V.P is  $40^\circ$ ?

- Two points A and B are on V.P and are 100 apart.
- 8.8 The horizontal distance between the points is 70. If the point A is 25 above H.P, find the height of the point B above H.P and the inclination of the line joining A and B with H.P.
- 8.9 End A of a line AB is 30 above H.P and 5 in front of V.P and end B is 10 above H.P and 25 in front of V.P. The distance between the end projectors is 40. Draw the projections of the line.
- 8.10 A line MN of 50 length, is parallel to V.P and inclined at  $45^\circ$  to H.P. The end M is 20 above H.P and 15 in front of V.P. Draw the projections of the line and find its traces.
- 8.11 Two points A and B are in H.P. The point A is 30 in front of V.P; while B is behind V.P. The distance between their projectors is 75 and the line joining their top views makes an angle of  $45^\circ$  with xy. Find the distance of the point B from V.P.
- 8.12 Two points A and B are on H.P; the point A being 30 in front of V.P, while B is 45 behind V.P. The line joining their top views makes an angle of  $45^\circ$  with xy. Draw the projections of the line and find the horizontal distance between two points.
- 8.13 A line AB is 75 long. A is 50 in front of V.P and 15 above H.P. B is 15 in front of V.P and is above H.P. Top view of AB is 50 long. Draw and measure the front view. Find the true inclinations.
- 8.14 A divider opened at  $45^\circ$  is placed on H.P such that, both the ends are on H.P and equi-distant from V.P and the hinged end is 60 above H.P. If the distance between the ends is 75, draw the projections and determine the true lengths of the legs of the divider.

Also, determine the inclinations of the legs with the planes of projection.

- 8.15 A line AB of 70 long is inclined at  $45^\circ$  to H.P and  $30^\circ$  to V.P. Its end A is on H.P and 25 in front of V.P. Draw its projections.
- 8.16 A line CD of 60 long has its end C in H.P and 12 behind V.P. The line is inclined at  $45^\circ$  to H.P and  $30^\circ$  to V.P. Draw its projections.
- 8.17 A line PQ of 100 long is inclined at  $45^\circ$  to H.P and  $30^\circ$  to V.P. Its end P is 30 above H.P and 20 behind V.P; whereas the other end Q is in the fourth quadrant. Draw the projections of the line and locate its traces.
- 8.18 A line CD of 100 long is inclined at  $30^\circ$  to H.P and  $45^\circ$  to V.P. Its mid-point is on V.P and 20 above H.P. Draw its projections and determine its traces.
- 8.19 A line AB of 65 long has its end A, 25 above H.P and 20 in front of V.P. The end B is 40 above H.P and 50 in front of V.P. Draw its projections and find its inclinations with H.P and V.P. Determine its traces.
- 8.20 A line of 100 long makes an angle of  $35^\circ$  to H.P and  $45^\circ$  to V.P. Its mid-point is 20 above H.P and 15 in front of V.P. Draw the projections of the line.
- 8.21 The end A of a line AB is 20 above H.P and 25 in front of V.P. The other end is 50 above H.P and 65 in front of V.P. The distance between the end projectors of the line is 70. Draw the projections of the line, determine its true length and inclinations with H.P and V.P.
- 8.22 Draw the projections of a line AB, 90 long; its mid-point M being 40 above H.P and 50 in front of V.P.

The end A is 10 above H.P and 20 in front of V.P. Determine the true inclinations with the reference planes.

- 8.23 The top view of a 75 long line AB, measures 65; while the length of its front view is 50. Its one end A is in H.P and 12 in front of the V.P. Draw the projections of the line AB and determine its inclinations with H.P and V.P.
- 8.24 The distance between the end projectors of a line is 60. One end is 15 above H.P and 50 in front of V.P. The other end is 60 above H.P and 10 in front of V.P. Draw the projections and find the true length of the line.
- 8.25 The end A of a line AB is in H.P and 25 behind V.P. The end B is in V.P and 50 above H.P. The distance between the end projectors is 75. Draw the projections of AB and determine its true length, traces and inclinations with the two planes.
- 8.26 The mid-point of a line of 80 long, is 25 above H.P and 30 in front of V.P. The line is inclined at  $30^\circ$  to H.P and  $40^\circ$  with V.P. Draw the projections of the line.
- 8.27 The front view of a 125 long line PQ, measures 75 and its top view measures 100. Its end Q and the mid-point M are in the first quadrant; M being 20 from both the planes. Draw the projections of the line PQ.
- 8.28 The end A of a line AB is 25 in front of V.P and 20 above H.P. The line is inclined at  $30^\circ$  to H.P and its top view, measuring 60, is inclined at  $45^\circ$  to xy. Draw the projections of the line, determine true length and inclination with V.P.

- 8.29 The top view of a line AB, 75 long measures 50. The end A is 40 in front of V.P and 15 below H.P. B is 15 in front of V.P and is above H.P. Draw the projections of the line and determine the distance of B from H.P and also the inclinations of the line AB with both the planes. Show also the traces.
- 8.30 A straight line DE measuring 160 long, has its V.T 100 below H.P and H.T is 120 behind V.P. The projectors through the traces are 150 apart. If the point D is 40 above H.P, draw the two projections of DE and find its inclinations with the two principal planes. Use a scale of 1:2.
- 8.31 The projectors through the traces of a line are 125 apart and those of the line 65 apart. One end of the line is 5 above H.P. If the top and front views make  $30^\circ$  and  $28^\circ$  respectively with xy, draw the projections. Determine the inclinations and true length of the line. State the quadrant in which the ends lie.
- 8.32 The projectors drawn from H.T and V.T of a straight line AB are 80 apart while those drawn from its ends are 50 apart. The H.T is 35 behind V.P and V.T is 55 below H.P and end A is 10 below H.P. Draw the projections of the line and determine its true length and inclinations with the principal planes.
- 8.33 The front view of a line is 65 long and is inclined at  $40^\circ$  to xy. The end A is on H.P and 15 in front of V.P. The end B is 40 in front of V.P. Draw the projections of the line, determine its true length and inclinations with H.P and V.P.
- 8.34 The front and top views of a line of length 90, measures 75 and 60 respectively. Its one end A is 20

in front of V.P and 25 above H.P. Draw the projections of the line and determine its inclinations with H.P and V.P.

- 8.35 The front view of a line makes an angle of  $30^\circ$  with xy. The H.T of the line is 45 in front of V.P while its V.T is 30 above H.P. One end of the line is 10 below H.P and the other end is 100 behind V.P. Draw the projections of the line and determine (i) its true length and (ii) its inclinations with both the planes.
- 8.36 The projectors through the ends of a line PQ are 80 apart. The end P is 25 above H.P and point Q is in the third quadrant. The line is 120 long and its front view makes an angle of  $30^\circ$  with xy. The V.T of the line is in H.P. Draw the projections of the line and determine its inclinations with H.P and V.P. Also, find the true length of that portion of the line which is in the first quadrant.
- 8.37 The end C of a line CD is in the third quadrant and is 50 from V.P. The end D is in the first quadrant and is 40 from V.P. The top view of the line is inclined at  $45^\circ$  to xy and H.T of the line is 20 below xy. The line CD is inclined at  $30^\circ$  to V.P. Draw the projections of the line and determine (i) true length of the line, (ii) inclination of the line with H.P and (iii) the location of V.T from H.P, stating whether it is above or below H.P.
- 8.38 A line AB of 75 long makes an angle of  $30^\circ$  with V.P and lies in a plane perpendicular to both V.P and H.P. Its end A is on H.P and the end B on V.P. Draw the projections of the line AB and locate its traces.
- 8.39 The following data refers to a straight line PQ:  
Length of the top view is 60. Length of the front view

is 65. Distance between the end projectors is 45. The end P of the line is 30 above H.P and 35 in front of V.P. Draw the top and front views of the line and determine the true length and inclinations with the principal planes of projection.

8.40 A line of 75 long is inclined to H.P and V.P at  $40^\circ$  and  $35^\circ$  respectively. The end A is on H.P whereas the end B is on V.P. Draw its projections.

8.41 A straight line AB of 75 long, has the end A on V.P and the end B on H.P. The line is inclined at  $30^\circ$  to V.P and its front view makes an angle of  $45^\circ$  with xy. Draw the projections of the line and locate its traces.

8.42 A line PQ of 100 long is inclined  $45^\circ$  to H.P and  $30^\circ$  to V.P. Its end P is in the first quadrant and Q is in the third quadrant. A point R on PQ, 40 from P, lies on both H.P and V.P. Draw the projections of the line PQ and determine its traces.

8.43 A line is inclined at  $40^\circ$  to H.P. Its one end A is 25 above H.P and 30 in front of V.P. The top view of the line is 70 and is inclined at  $30^\circ$  to xy. Draw the projections of the line AB and determine its true length and its inclination with V.P.

8.44 The projections of a line measure 80 in the top view and 70 in the front view. The mid-point of the line is 45 behind V.P and 35 below H.P. One end is 10 behind V.P and nearer to it. The other end is nearer to H.P. Draw the projections of the line. Find the true length and true inclinations.

8.45 The projectors drawn from H.T and V.T of a line AB are 75 apart and the projectors drawn from its ends are 45 apart. The H.T is 30 behind V.P and V.T is 50 below H.P. The end B is 10 below H.P. Draw the

projections of AB and determine its true length and inclinations with the reference planes.

- 8.46 The end A of a line AB is on H.P and 25 behind V.P. The end B is on V.P and 50 above H.P. The distance between the end projectors is 80. Draw the projections of AB and determine its true length, traces and inclinations with the two planes.
- 8.47 The length of the top view of a line is 40 and the length of the front view is 50. The top view is inclined at  $30^\circ$  to xy. Draw the projections of the line, assuming that its one end is situated on H.P and 25 in front of V.P. Determine the true length and inclinations of the line with H.P and V.P.
- 8.48 A straight line PQ is in third quadrant. Its top view is 50 and makes an angle of  $30^\circ$  with xy. The point Q is 10 from V.P and 25 from H.P. The difference between the distances of P and Q from H.P is 45. Draw the projections and determine its (i) true length and (ii) inclinations with H.P and V.P. Also, locate the traces.
- 8.49 Draw an isosceles triangle abc of base 40 and altitude 75, with a in xy and ab inclined at  $45^\circ$  to xy. The figure is the top view of a triangle, whose corners A, B and C are respectively 75, 25 and 50 above H.P. Determine the true shape of the triangle and inclinations of all the sides with the planes of projection.
- 8.50 A triangle PQR rests on a corner R on H.P. The corner P is 10 above H.P and 20 behind V.P. The corner Q is 30 above H.P and 30 behind V.P. The distance between the projectors of P and Q is 45. The true lengths of PQ and QR are 40 and 50

respectively. Draw the projections of the triangle and determine its true shape.

- 8.51 The front view  $a'b'$  of a line AB is 40 long and it makes an angle of  $35^\circ$  with xy. The point A lies 20 below H.P and 35 in front of V.P. The difference between the distances of A and B from V.P is 15. The line AB is in the fourth quadrant. Draw the projections of the line. Determine its true length and inclinations with H.P and V.P.
- 8.52 A line MN of 90 long is inclined at  $45^\circ$  to V.P and  $30^\circ$  to H.P. Its end M is in the second quadrant and the end N is in the fourth quadrant. A point P on MN, which is 60 from M, lies on both H.P and V.P. Draw the projections of the line MN and find its traces.
- 8.53 Two mangoes on a tree are respectively 12 m and 3 m above the ground and 1.5 m and 2.5 m from the central plane of a wall, but on opposite sides of the wall. The distance between the mangoes measured along the ground and parallel to the wall is 2.5 m. Determine the true distance between the mangoes and the angles of inclination of the line joining the mangoes (i) with the ground and (ii) with the wall.
- 8.54 A line MN of 90 long has one end M at 50 from both the planes and end N, 20 from both the planes. Draw the front and top views of the line and find its inclinations with H.P and V.P. Also, locate a point P lying on XY such that,  $PN = PM$ .
- 8.55 A line AB is in third quadrant. Its ends A and B are 25 and 75 behind V.P respectively. The distance between the end projectors is 100. The line is inclined at  $30^\circ$  to H.P and its H.T is 15 below xy.

Draw the projections of AB and determine its true length and V.T.

8.56 The front view of a line AB, measures 55 and is inclined at  $45^\circ$  to xy. Its one end A is 20 above H.P. The H.T of the line is 15 in front of V.P. The line is inclined at  $30^\circ$  to H.P. Draw the projections of the line and determine its true length, inclination with V.P and locate its V.T.

8.57 A straight line PQ of length 160, has its V.T at 100 below H.P and H.T, 120 behind V.P. The projectors through the traces are 150 apart. If the point P is 40 above H.P, draw the projections and determine the inclinations.

8.58 The end projectors of a straight line AB are 80 apart. The H.T and V.T of the line, coincide with each other on xy, at a point 25 from the projector of the end A and situated in-between the end projectors of the line. The end A is 20 above H.P and B is 65 behind V.P. Draw the projections of the line and determine the true length and inclinations of the line.

8.59 A line AB of 75 long has its end A on H.P and in front of V.P. The other end B is on V.P. The line is inclined at  $60^\circ$  to H.P and  $30^\circ$  to V.P. Draw its projections.

8.60 The front view of a line AB, measures 65 and makes an angle of  $45^\circ$  with xy. A is on H.P and V.T of the line is 15 above H.P. The line is inclined at  $30^\circ$  to V.P. Draw the projections of the line and find its true length and inclination with H.P. Also, locate its H.T.

8.61 An electric transmission line laid along an uphill from a hydroelectric power station, due West to a sub-station, is 2 km long and has a slope of  $30^\circ$ . Another line from the sub-station, at  $W45^\circ N$  to a

village is 4 km long, but is laid on a level ground. Determine the length and slope of a proposed telephone line, joining the power station and the village.

- 8.62 A line PQ of 75 long makes an angle of  $60^\circ$  with H.P and is on an auxiliary vertical plane, which makes an angle of  $45^\circ$  with V.P. If the end A is 15 from both the planes, determine the true inclination with V.P and also locate its traces.
- 8.63 A line MN of 75 long is on an auxiliary inclined plane, which makes an angle of  $45^\circ$  with H.P. The front view of the line measures 60 and one end is 10 from both the planes. Draw the projections and determine the true inclinations of the line with the planes.
- 8.64 The projections of a line AB are on the same projector and A is 10 above H.P and 20 behind V.P. B is 40 above H.P and 50 behind V.P. Draw the projections of the line AB and determine its true length, inclinations with H.P and V.P and locate its traces.
- 8.65 The ends of a line PQ are on the same projector. The end P is 30 below H.P and 12 behind V.P. The end Q is 55 above H.P and 45 in front of V.P. Determine the true length and traces of PQ and its inclinations with the two planes.
- 8.66 A line KL of 75 long makes an angle of  $30^\circ$  with V.P and is on the profile plane. One end of the line is on H.P while the other end is on V.P. Draw its projections and also locate the traces.
- 8.67 A line AB of 80 length, has its end A on H.P and 10 in front of V.P. The end B is in the third quadrant. The

line is inclined at  $30^\circ$  to H.P and  $60^\circ$  to V.P. Draw the projections of the line AB.

8.68 A line PQ is to be established with a bearing of N  $60^\circ$  E and with a  $60^\circ$  gradient from P. The distance from P to Q is 200 metres. Draw the front and top views of PQ (Assume any position for the point P).

8.69 A chimney of a boiler is 30m high and 1.5m in diameter. It is supported by three guy wires appearing at  $120^\circ$  to each other in the top view. The ends of the wires are pegged to the ground at distances 4m, 6m and 8m from the centre of the chimney. The other ends of the wires are connected to the chimney at 5m from the top. Find the lengths of the wires.

8.70 A line AB of 80 length, is in the second quadrant, with the end A in the H.P and the end B in the V.P. The line is inclined at  $30^\circ$  with H.P and  $45^\circ$  to V.P. Draw the projections of AB.

8.71 The top view of a line AB is 80 and front view is 70 long. The mid-point of the line is 40 in front of V.P and 30 above H.P. One end of the line is 10 in front of V.P. Draw the projections of the line and determine the true length of it.

8.72 A line PQ inclined at  $40^\circ$  to V.P has its ends 60 and 20 above H.P. The length of its front view is 75 and its vertical trace is 10 above H.P. Determine the true length of PQ, its inclination with H.P, and its H.T.

8.73 The H.T and V.T and the end A of the line AB coincide and lie on XY. The distance between the top and front views of the end B is 60. The line is equally inclined to H.P and V.P. The distance between the end projectors as measured parallel to XY is 40.

Draw the projections and find the true length of the line.

- 8.74 The H.T and V.T of a straight line AB is below and above xy respectively. The distance between the traces as measured parallel to xy is 200. The end B of the line nearer to V.P than the end A. The top view of the line makes  $30^\circ$  to xy. The end B is 10 from V.P and 20 from H.P. The distance between the end projectors of the line measures 50 parallel to xy. Draw the projections of the line.

## REVIEW QUESTIONS

- 8.1 Define a straight line.
- 8.2 How a straight line can be located in space?
- 8.3 What is a profile plane?
- 8.4 List the possible positions of a straight line with respect to the planes of projections.
- 8.5 What are the relative positions of the projections of a line contained by a profile plane?
- 8.6 What is meant by the trace of a line?
- 8.7 How the traces of a line are determined from its projections?
- 8.8 How the side view of a line is obtained from its projections?
- 8.9 How a given point is located on the projections of a line?
- 8.10 What are the characteristics of projections of a straight line which is parallel to both H.P and V.P?

## OBJECTIVE QUESTIONS

- 8.1 A straight line is defined as the \_\_\_\_\_ distance between two points.
- 8.2 The projection of a line on a plane parallel to it appears in its true length.  
(True/False)
- 8.3 When both the views of a line coincide with  $xy$ , the line is lying (a) on H.P, (b) on V.P, (c) both on H.P and V.P.  
( )
- 8.4 When a line is perpendicular to one of the planes, it is \_\_\_\_\_ to the other plane.
- 8.5 When a line is perpendicular to H.P, its front view is perpendicular /parallel to  $xy$ .
- 8.6 When a line is perpendicular to V.P, its \_\_\_\_\_ is a point.
- 8.7 When a line is inclined to \_\_\_\_\_ and parallel to \_\_\_\_\_, its front view represents the true length of the line.
- 8.8 When a line is inclined to V.P and parallel to H.P, its front view is \_\_\_\_\_ to  $xy$ .
- 8.9 When a line is inclined to V.P and parallel to H.P, the inclination of the top view with  $xy$  represents \_\_\_\_\_.
- 8.10 When a line is inclined to a plane, its projection on that plane is a line shorter than its true length.

(True /False)

8.11 The projections of an oblique line make angles with xy which are equal to/ less than / greater than the true inclinations of the line with the planes of projection.

8.12 When a line is contained by a plane, its projection on that plane is a point/equal to its true length.

8.13 A profile plane is perpendicular to H.P and inclined to V.P.

(True /False)

8.14 When a line is contained by a profile plane, the sum of the angles of inclination with H.P and V.P is (a) equal to  $90^\circ$ , (b) less than  $90^\circ$ , (c) greater than  $90^\circ$

( )

8.15 The H.T and V.T of a line will always lie on a single projector. (True/False)

8.16 When a line is parallel to both H.P and V.P; it has (a) only H.T, (b) only V.T, (c) both H.T and V.T, (d) no H.T and V.T.

( )

8.17 When a line is perpendicular to H.P, its \_\_\_\_\_ trace will coincide with \_\_\_\_\_ view of the line.

8.18 When a line is positioned in the first quadrant, its H.T and V.T must always be below and above xy respectively.

(True/False)

8.19 When a line is contained by a profile plane, both H.T and V.T are present/ absent.

8.20 The trace of a line is a line / point.

8.21 The edge view of a line is a \_\_\_\_\_.

8.22 When a line is parallel to both H.P and V.P, its side view is a point/line.

## ANSWERS

8.1 shortest

8.2 True

8.3 c

8.4 parallel

8.5 Perpendicular

8.6 front view

8.7 H.P, V.P

8.8 parallel

8.9 true inclination

8.10 True

8.11 Greater than

8.12 Equal to its true length

8.13 False

8.14 a

8.15 False

8.16 d

8.17 horizontal, top

8.18 False

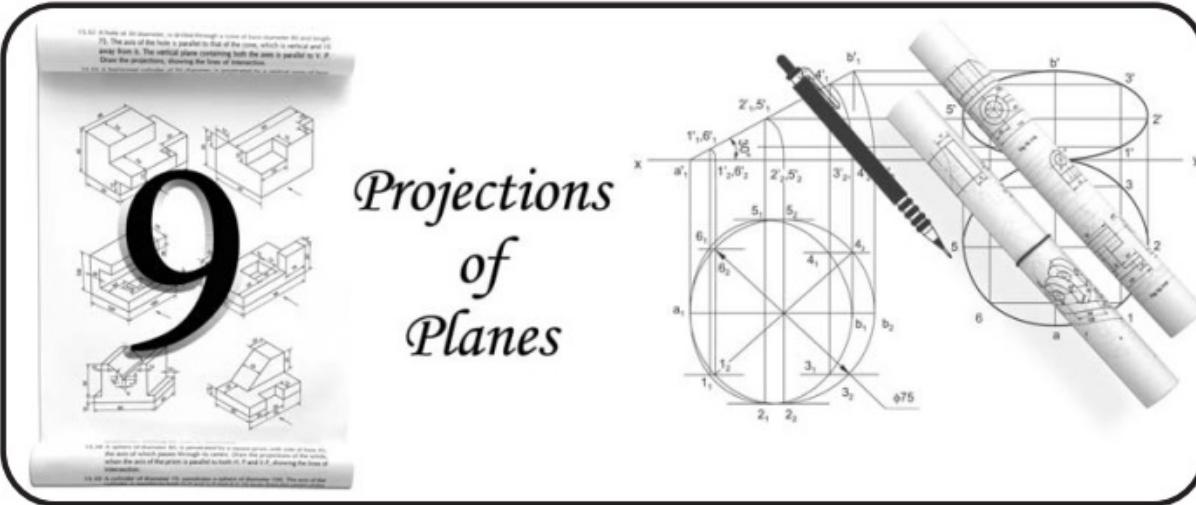
8.19 Present

8.20 Point

8.21 point

8.22 Point

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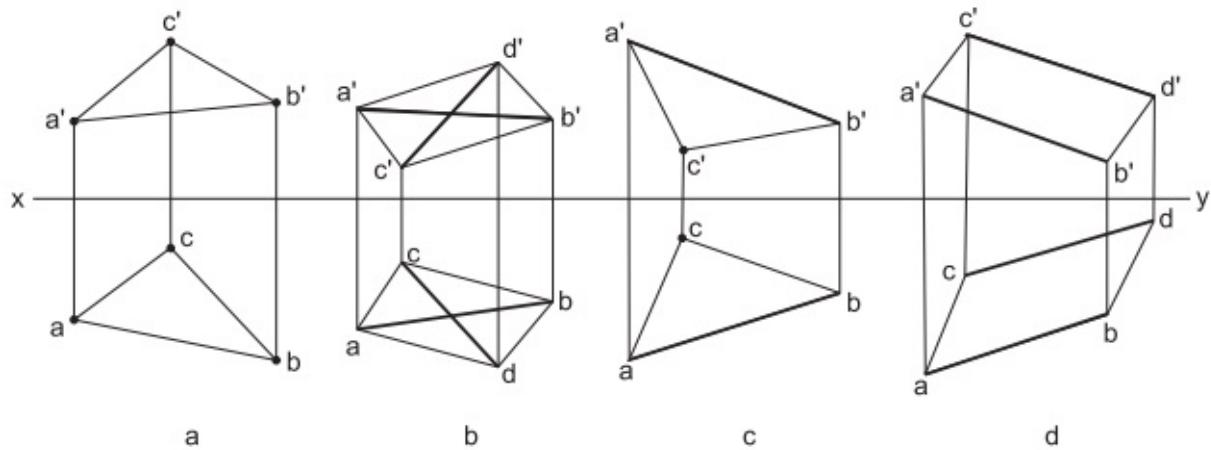
## 9.1 INTRODUCTION

In structural design, it is necessary to know the representation of plane surfaces of different shapes in all the possible orientations. The treatment presented here is based on the principles dealt with, in the preceding two chapters.

Plane surfaces have two dimensions, viz., length and breadth; the third dimension, the thickness being zero. Plane surfaces may be considered of infinite sizes. However, for convenience, segments of planes only are considered in the treatment presented here. Planes are represented in space by the following:

- a- three points, not on a straight line.
- b- two intersecting lines.
- c- a line and a point.
- d- two parallel lines.

The front and top views for the above cases are shown in Fig.9.1.



**Fig.9.1**

## 9.2 TWO VIEW PROJECTIONS

The following are the possible orientations of the planes, with respect to the principal planes of projection:

1. Plane parallel to one of the principal planes and perpendicular to the other.
2. Plane inclined to one of the principal planes and perpendicular to the other.
3. Plane perpendicular to both the principal planes.
4. Plane inclined to both the principal planes.

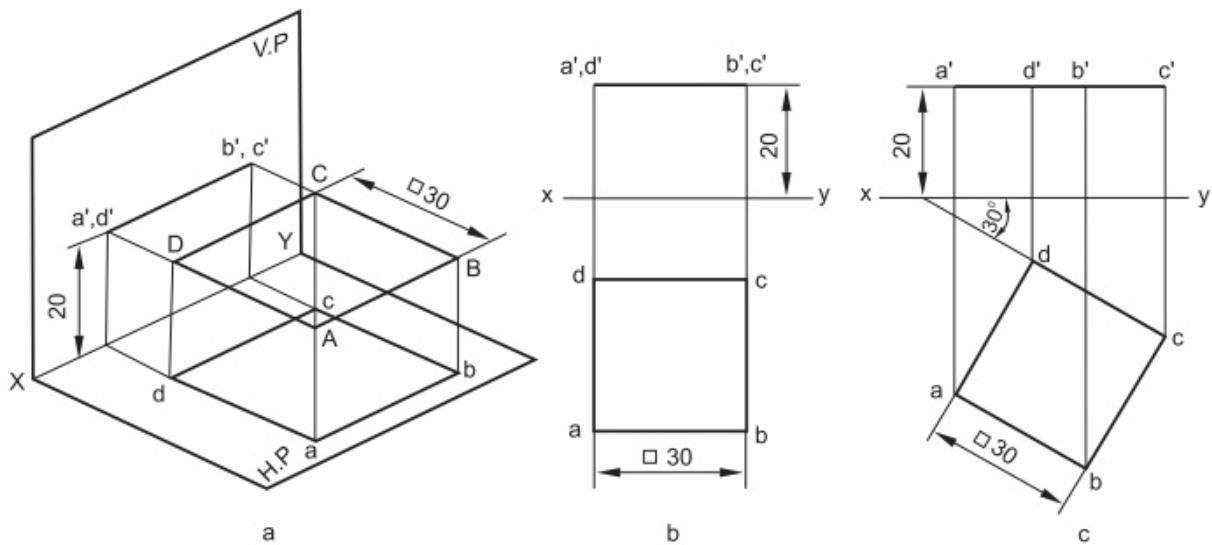
### 9.2.1 Plane Parallel to one of the Principal Planes and Perpendicular to the other

#### 9.2.1.1 *Plane Parallel to H.P and Perpendicular to V.P*

**Problem 1** A square plane ABCD of side 30, is parallel to H.P and 20 away from it. Draw the projections of the plane, when two of its sides are (i) parallel to V.P and (ii) inclined at  $30^\circ$  to V.P.

Figure 9.2a shows the first quadrant with the plane ABCD such that, two of its sides are parallel to V.P. As the plane is parallel to H.P, its projection on H.P, viz., the top view reveals the true shape of the plane. The front view appears as a straight line, parallel to xy.

### Construction (Fig.9.2b)



**Fig.9.2**

1. Draw a square abcd of side 30 such that, one of its sides, say dc (ab) is parallel to xy.
  2. Draw projectors from the points d and c.
  3. Locate the point a' (d') at 20 above xy.
  4. Draw a line through a' and parallel to xy, intersecting the projector through c at b' (c').
- a'b'c'd' and abcd are the required projections.

[Figure 9.2c](#) shows the projections of the plane, when two of its sides are inclined at  $30^\circ$  to V.P.



When the inclination  $\phi$  of the side  $dc$  with  $xy$  is  $45^\circ$ , then all the sides of the plane are said to be equally inclined to V.P.

### 9.2.1.2 **Plane Parallel to V.P and Perpendicular to H.P**

**Problem 2** An equilateral triangular plane  $ABC$  of side 40, has its plane parallel to V.P and 20 away from it. Draw the projections of the plane when one of its sides is (i) perpendicular to H.P, (ii) parallel to H.P and (iii) inclined to H.P at an angle of  $45^\circ$ .

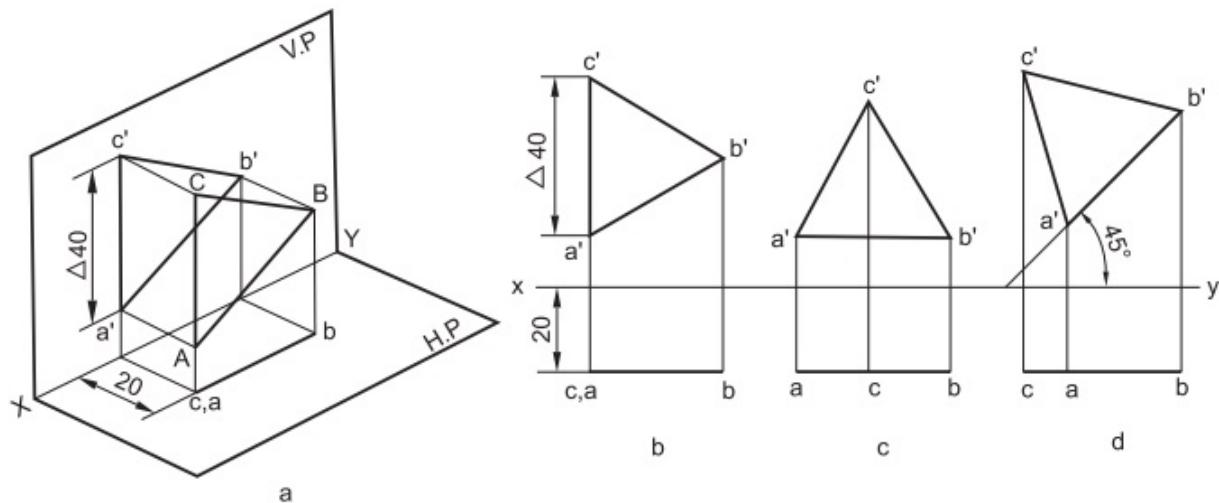
[Figure 9.3a](#) shows the first quadrant with the plane  $ABC$  such that, one of its sides is perpendicular to H.P. As the plane is parallel to V.P, its projection on V.P, i.e., the front view appears in its true shape. The top view appears as a straight line, parallel to  $xy$ .

#### **Construction ([Fig.9.3b](#))**

1. Draw an equilateral triangle  $a'b'c'$  of side 40 such that, one side, say  $a'c'$  is perpendicular to  $xy$ .
  2. Draw projectors from the points  $a'$  and  $b'$ .
  3. Locate the point  $c$  (a) at 20 below  $xy$ .
  4. Draw a line through  $c$  and parallel to  $xy$ , intersecting the projector through  $b'$  at  $b$ .
- $a'b'c'$  and  $abc$  are the required projections.

[Figure 9.3c](#) shows the projections of the plane, when one of its sides  $AB$  is parallel to H.P. [Figure 9.3d](#) shows the

projections of the plane, when one of its sides AB is inclined at  $45^\circ$  to H.P.



**Fig.9.3**

- ☞ The symbol  $\square$  preceding the dimension, represents the side of a square. Similarly, the symbol  $\Delta$  is used to represent the side of an equilateral triangle.

## 9.2.2 Plane Inclined to one of the Principal Planes and Perpendicular to the other

### 9.2.2.1 *Plane Inclined to H.P and Perpendicular to V.P*

The problems of this nature, as explained in the preceding chapter; are normally solved in two stages. In the first stage, the plane is assumed to be parallel to that plane to which it is actually inclined and projections are drawn. In the second stage, the plane is rotated till it makes the

required angle with the plane and then the final views are obtained.

**Problem 3** Draw the projections of a regular pentagon of 25 side with its surface making an angle of  $45^\circ$  with H.P. One of the sides of the pentagon is parallel to H.P and 15 away from it.

Figure 9.4a shows the first quadrant with the plane in it, depicting the two stages of obtaining the projections.

### **Construction (Fig.9.4b)**

**Stage I** Assume that the plane is parallel to H.P and perpendicular to V.P.

1. Draw the top view  $abc_1d_1e_1$ , at any convenient location below  $xy$ , keeping the side  $ab$  perpendicular to  $xy$ .

The plane appears in its true shape in the top view, as it is parallel to H.P.

2. Draw projectors through  $a$ ,  $e_1$  and  $d_1$ .
3. Locate the point  $b'$  ( $a'$ ) at 15 above  $xy$ , on the projector through  $a$ .
4. Draw a line through  $b'$  and parallel to  $xy$ , intersecting the above projectors at  $c'_1(e'_1)$  and  $d'_1$  respectively, forming the front view.

The front view appears as a line, parallel to  $xy$ , as the plane is parallel to H.P and perpendicular to V.P.

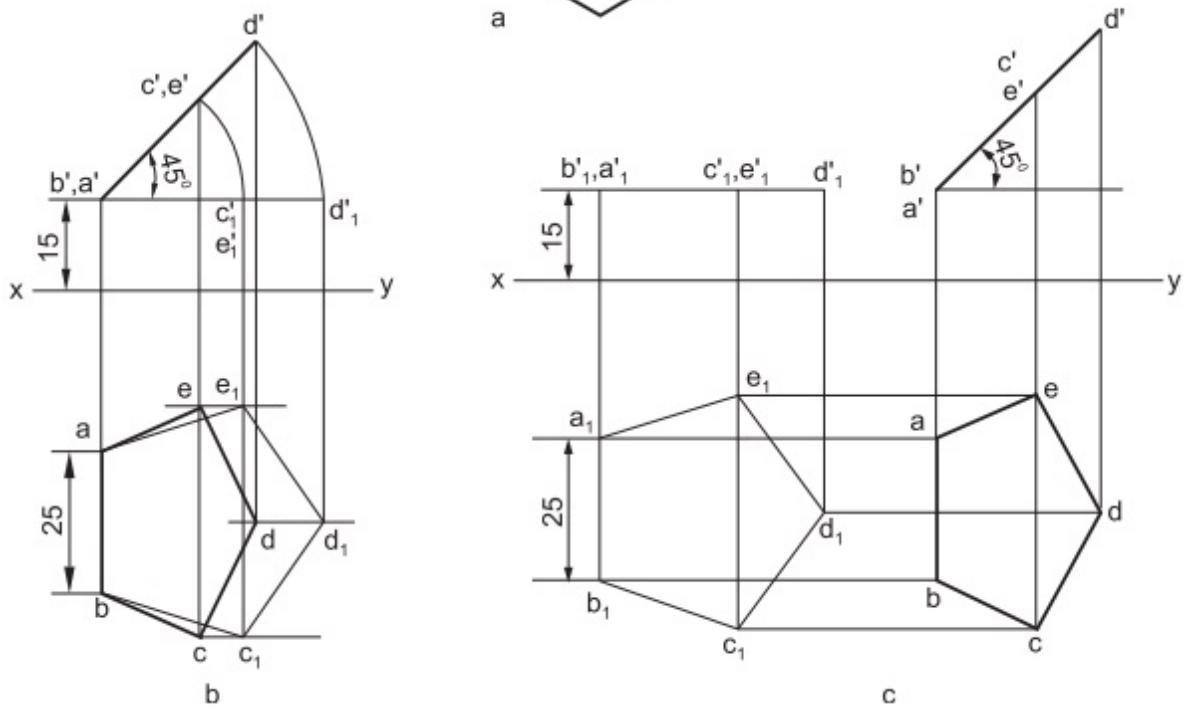
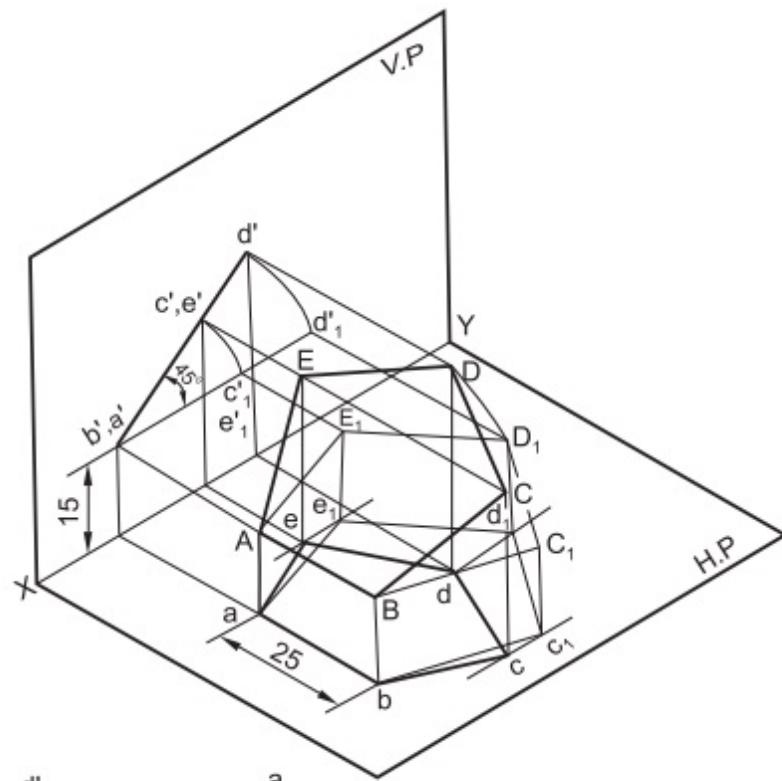
**Stage II** Rotate the plane till it makes the given angle  $45^\circ$  with H.P.

5. With  $b'$  ( $a'$ ) as centre, rotate the front view through  $45^\circ$  to  $xy$ , to the position  $b'(a')$   $c'(e')$   $d'$ , forming the final front view of the plane.

6. Through  $c_1$ ,  $d_1$  and  $e_1$  draw lines parallel to  $xy$ , representing the loci of the top views of the corners C, D and E of the plane.
7. Through  $c'$  ( $e'$ ) and  $d'$ , draw projectors meeting the above loci at c, e and d.
8. Join b, c; c, d; d, e and e, a, forming the final top view of the plane.

[Figure 9.4c](#) shows the method of drawing the projections, in which the construction pertaining to stage II is shown separately. While drawing the projections of planes, the following may be observed:

- (i) If a plane has an edge parallel to H.P, that edge should be kept perpendicular to V.P. Similarly, if the edge of a plane is parallel to V.P, that edge should be kept perpendicular to H.P.
- (ii) If a plane has a corner on H.P, the sides containing that corner should be equally inclined to V.P whereas, if the corner is on V.P, the sides containing the corner should be kept equally inclined to H.P.



**Fig.9.4**

**9.2.2 2 Plane Inclined to V.P and Perpendicular to H.P**

**Problem 4** A regular hexagonal plane of 30 side has a corner at 20 from V.P and 50 from H.P. Its surface is inclined at  $45^\circ$  to V.P and perpendicular to H.P. Draw the projections of the plane.

Figure 9.5a shows the first quadrant with the plane in it, depicting the two stages of obtaining the projections.

### **Construction (Fig.9.5b)**

**Stage I** Assume that the plane is parallel to V.P and perpendicular to H.P.

1. Draw the front view  $a'b_1' c_1' d_1' e_1' f_1'$  such that, the corner  $a'$  is 50 above xy.

The plane appears in its true shape in the front view, as it is parallel to V.P.

2. Draw projectors through  $a'$ ,  $b_1'$ ,  $c_1'$  and  $d_1'$ .
3. Locate the point  $a$  at 20 below xy.
4. Draw a line through  $a$  and parallel to xy, intersecting the above projectors at  $f_1$  ( $b_1$ ),  $e_1$  ( $c_1$ ) and  $d_1$  respectively; forming the top view.

The top view appears as a line parallel to xy, as the plane is parallel to V.P and perpendicular to H.P.

**Stage II** Rotate the plane till it makes the given angle  $45^\circ$  with V.P.

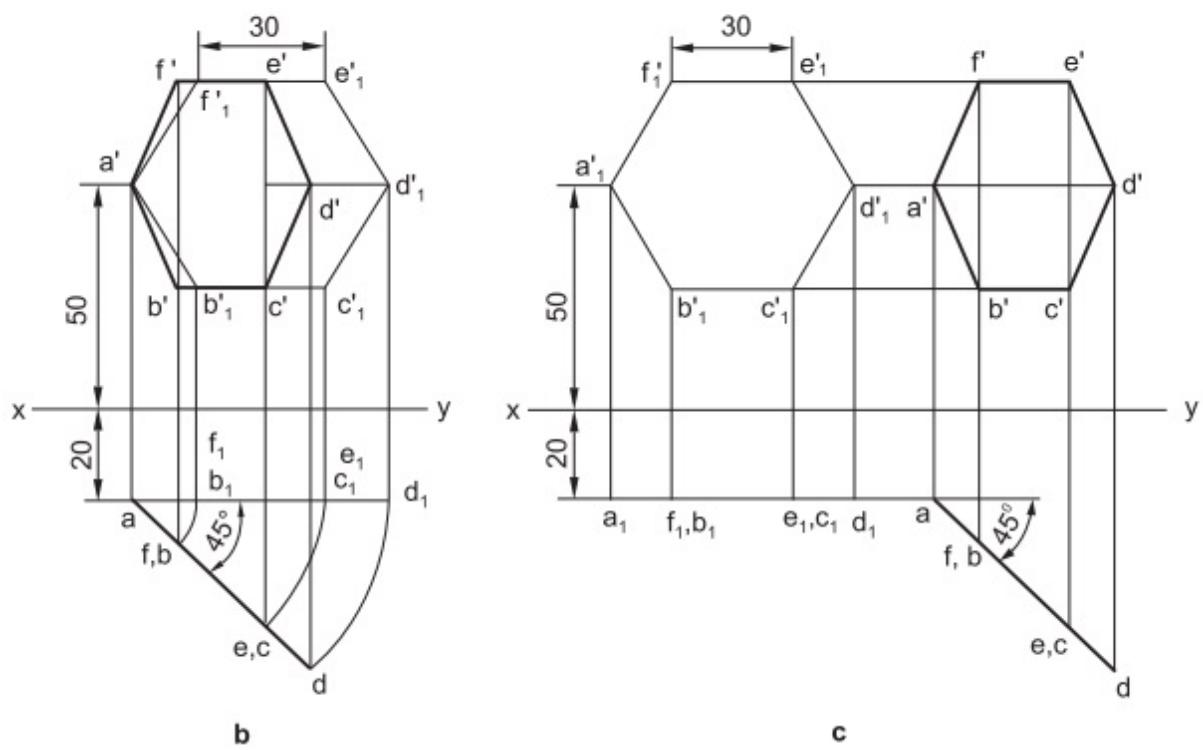
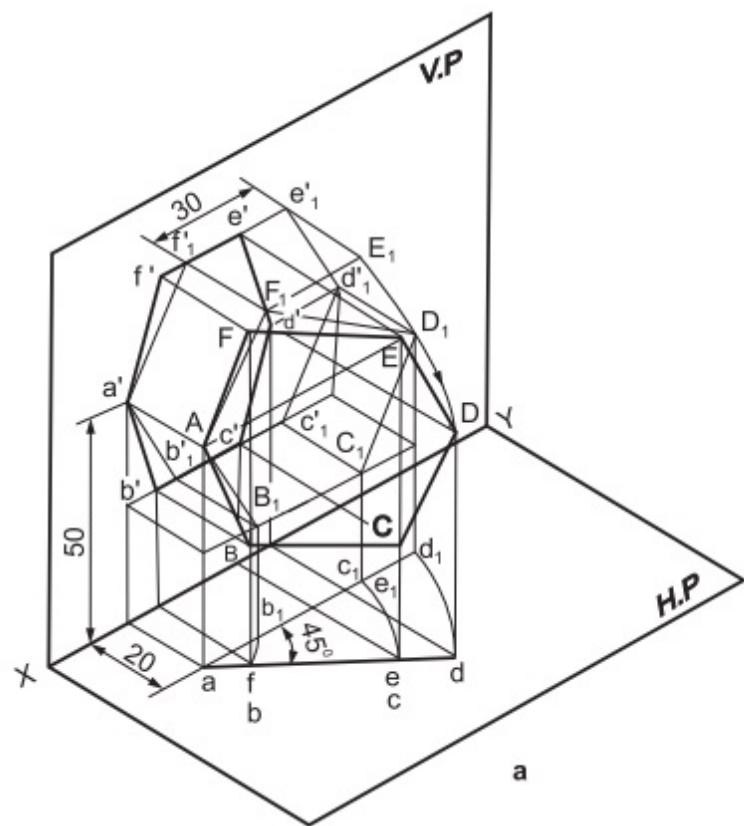


As the plane is to be rotated about the corner A; to make it inclined to V.P, the sides containing the corner A, i.e., AB and AF should be equally inclined to H.P.

5. With  $a$  as centre, rotate the top view through  $45^\circ$  to xy, to the position  $a f$  ( $b$ )  $e$  ( $c$ )  $d$ ; forming the final top view of the plane.

6. Through  $b_1'$  ( $c_1'$ ),  $d_1'$  and  $e_1'$  ( $f_1'$ ), draw lines parallel to  $xy$ , representing the loci of the front views of the corners B (C), D and E (F) of the plane.
7. Through  $f$  ( $b$ ),  $e$  ( $c$ ) and  $d$ , draw projectors, meeting the above loci at  $f'$ ,  $b'$ ,  $e'$ ,  $c'$  and  $d'$ .
8. Join  $a'$ ,  $b'$ ;  $b'$ ,  $c'$ ;  $c'$ ,  $d'$ ;  $d'$ ,  $e'$ ;  $e'$ ,  $f$ ; and  $f$ ,  $a'$ , forming the final front view of the plane.

Figure 9.5c shows the method of drawing the projections, in which the construction with respect to stage II is shown separately.



**Fig.9.5**

### 9.2.3 Plane Perpendicular to Both H.P and V.P

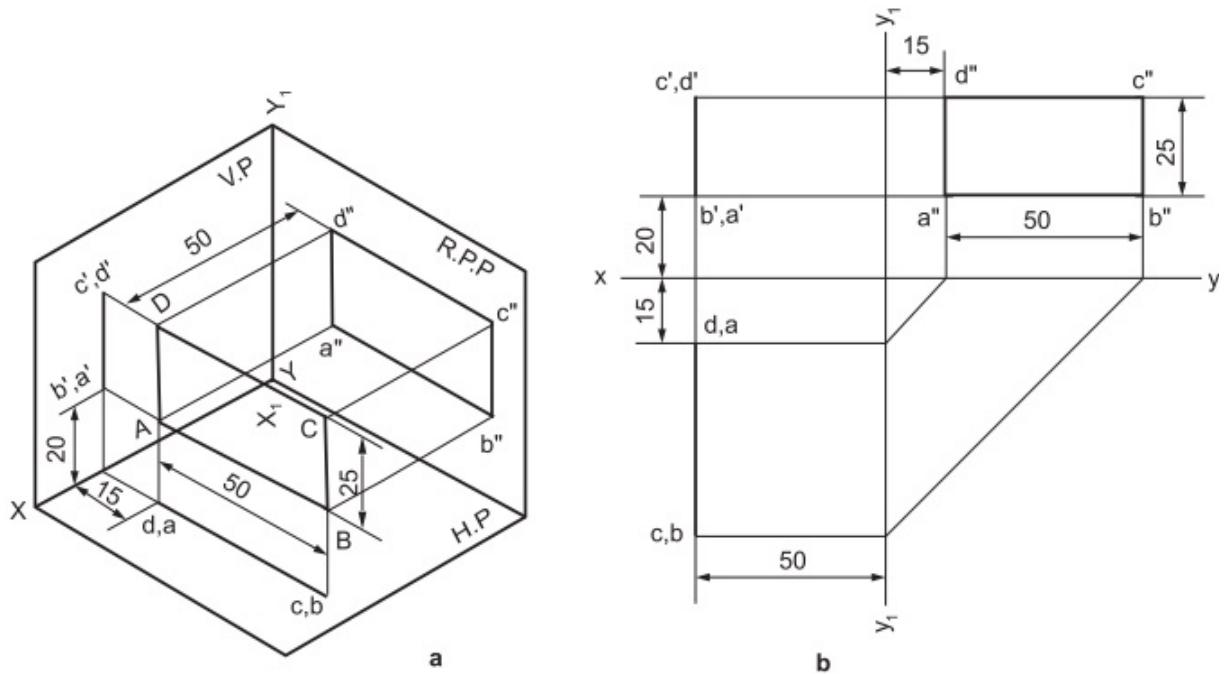
When a plane is positioned such that, it is perpendicular to both the principal planes, then it is said to be lying parallel to the profile plane. In this case, the side view appears in its true shape and both the front and top views appear as straight lines on a single projector.

**Problem 5** A rectangular plane of  $50 \times 25$  size is perpendicular to both H.P and V.P. The longer edges are parallel to H.P and the nearest one is 20 above it. The shorter edge, nearer to V.P is 15 from it. The plane is 50 from the profile plane. Draw the projections of the plane.

*Figure 9.6a shows the first quadrant, with the rectangular plane positioned in it. The figure also shows the method of obtaining the three views on the respective planes.*

#### **Construction (Fig.9.6b)**

1. Draw the reference lines  $xy$  and  $x_1 y_1$ .
2. Draw the side view  $a'' b'' c'' d''$  such that, the longer edge  $a'' b''$  is 20 above  $xy$  and the shorter edge  $a'' d''$  is 15 from  $x_1 y_1$ .
3. Draw a projector at 50 from  $x_1 y_1$ .
4. Obtain the front and top views of the plane, from the side view, by projection.



**Fig.9.6**

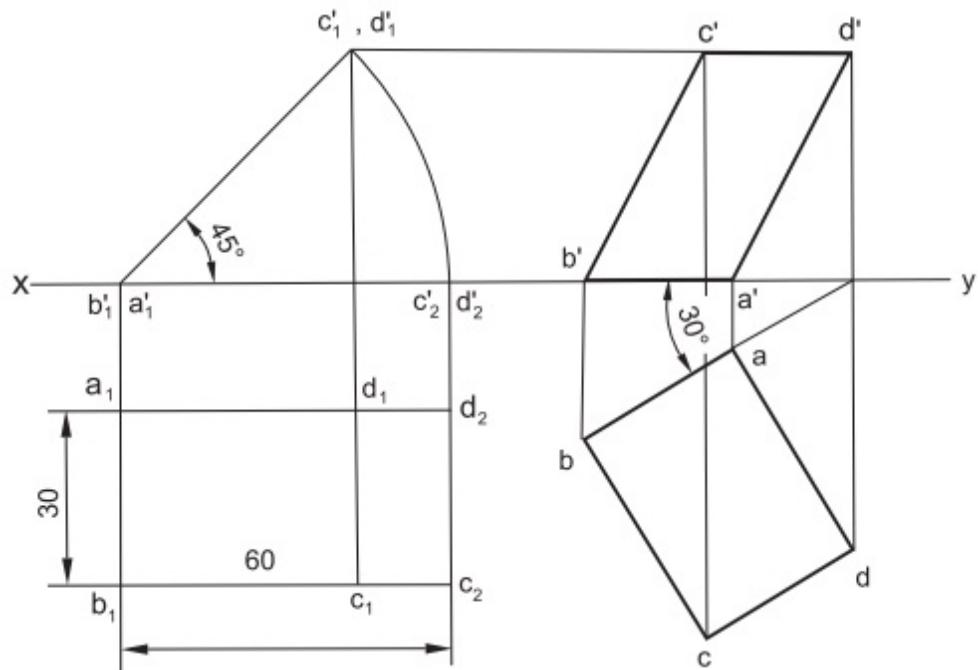
#### 9.2.4 Oblique Plane (Plane Inclined to Both H.P and V.P)

The projections of an oblique plane are obtained in three stages. While drawing the projections of oblique planes, the following may be observed:

1. The surface of the plane should be considered parallel to the principal plane, to which it is actually inclined.
2. The edge should be considered perpendicular to the principal plane, to which it is inclined.
3. The surface of the plane is tilted through the required angle in the II stage, and
4. The edge is tilted through the required angle in the III stage, for obtaining the final projections.

**Problem 6** A rectangular plane of size  $60 \times 30$  has its shorter side on H.P and inclined at  $30^\circ$  to V.P. Draw the projections of the plane, if its surface is inclined at  $45^\circ$  to H.P.

**Construction (Fig.9.7)**



**Fig.9.7**

**Stage I** Assume that the plane is lying in H.P and the shorter edge of it is perpendicular to V.P.

1. Draw the top view  $a_1 b_1 c_2 d_2$ ; representing the true shape of the plane, at any convenient position below xy.
2. Project the front view  $b_1'$  ( $a_1'$ )  $c_2'$  ( $d_2'$ ), which is a straight line, coinciding with xy.

**Stage II** Tilt the plane such that, it makes an angle of  $45^\circ$  with H.P.

3. Rotate the front view about  $b_1'$  ( $a_1'$ ), to the position  $b_1'$  ( $a_1'$ )  $c_1'$  ( $d_1'$ ) such that, it makes  $45^\circ$  with  $xy$ .
4. Obtain the top view  $a_1 b_1 c_1 d_1$ , by projection.

**Stage III** Rotate the plane such that, its shorter edge  $AB$  makes  $30^\circ$  with V.P.

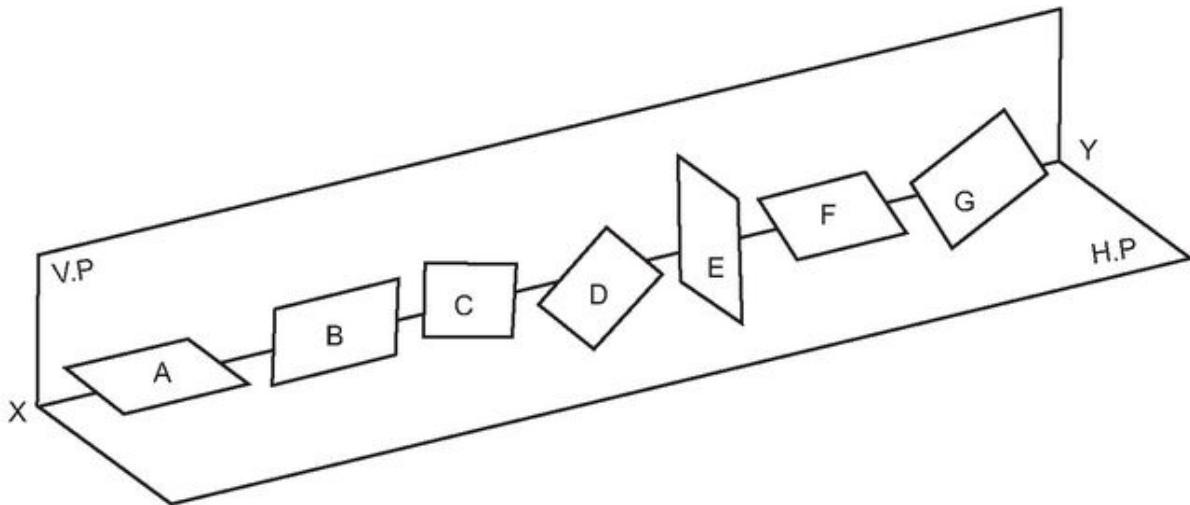
5. Redraw the top view (abcd) such that, the side  $ab$  makes  $30^\circ$  with  $xy$ .  
abcd is the final top view of the plane.
6. Obtain the final front view  $a' b' c' d'$ , by projection.



When the plane is tilted such that its shorter edge makes the given angle with V.P, the shape of the top view and the distances of the corners of the plane from H.P are not altered.

## 9.3 TRACES OF PLANES

A plane extended if necessary, will meet the principal planes of projection along the lines known as traces. The intersection of a plane or its extension with H.P is called the horizontal trace, H.T and with V.P, the vertical trace, V.T. Normally, planes are represented by their traces.



**Fig.9.8**

Figure 9.8 shows the quadrant with the following types of planes situated in it:

- A - Plane is parallel to H.P and perpendicular to V.P. It has only the V.T.
- B - Plane is parallel to V.P and perpendicular to H.P. It has only the H.T.
- C - Plane is perpendicular to H.P and inclined to V.P by an angle  $\phi$ . Its V.T is perpendicular to xy and its H.T is inclined to xy by an angle  $\phi$ .
- D - Plane is perpendicular to V.P and inclined to H.P by an angle  $\theta$ . Its H.T is perpendicular to xy and its V.T is inclined to xy by an angle  $\theta$ .
- E - Plane is perpendicular to both H.P and V.P. It has both H.T and V.T, at right angle to xy.
- F - Plane is inclined to both H.P and V.P, but parallel to xy. It has both H.T and V.T, which are parallel to xy.
- G - Plane is inclined to both H.P and V.P but not parallel to xy. It has both H.T and V.T, which are inclined to xy.

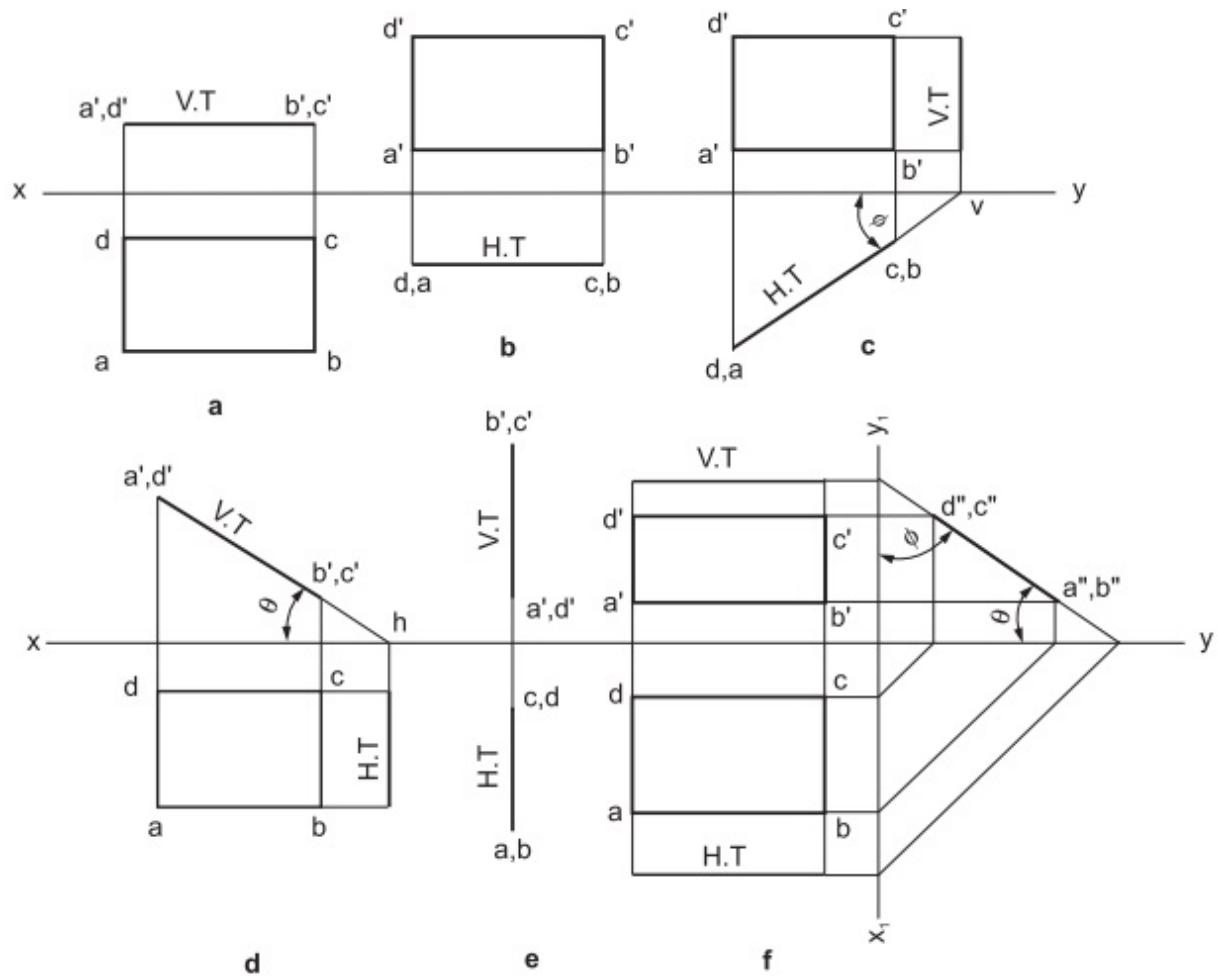
[Figure 9.9](#) shows the relative positions of the views and the traces for the planes A to F and [Fig.9.10](#) shows the same for the plane G. The method of obtaining the traces is similar to that of the lines.

**Problem 7** [Figure 9.10](#) shows the projections of an oblique plane ABCD of type G. Locate the traces of the plane.

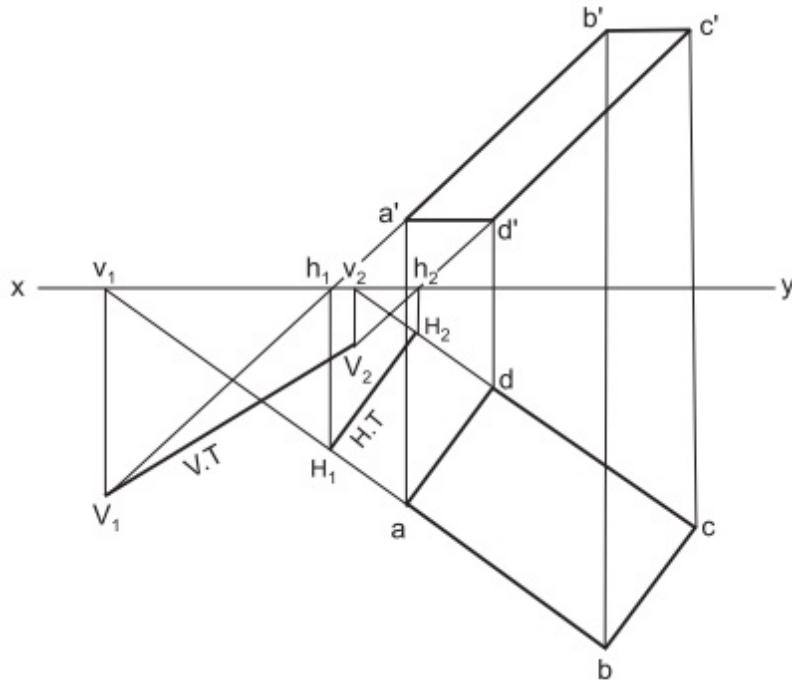
**Construction ([Fig.9.10](#))**

1. Draw the given projections of the plane.
2. Extend  $b'a'$  and  $c'd'$  to meet  $xy$  at  $h_1$  and  $h_2$  respectively.
3. Draw projectors through  $h_1$  and  $h_2$ .
4. Extend  $ba$  and  $cd$  to meet the above projectors at  $H_1$  and  $H_2$  respectively.
5. Join  $H_1, H_2$ .
6. Extend  $ba$  and  $cd$  to intersect  $xy$  at  $v_1$  and  $v_2$  respectively.
7. Draw projectors through  $v_1$  and  $v_2$ .
8. Extend  $b'a'$  and  $c'd'$  to intersect the above projectors at  $V_1$  and  $V_2$  respectively.
9. Join  $V_1, V_2$ .

$H_1 H_2$  and  $V_1 V_2$  are the required traces.



**Fig.9.9**



**Fig.9.10**

## 9.4 EXAMPLES

**Problem 8** A rectangle  $ABCD$  of size  $40 \times 25$ , has the corner  $A$ , 10 above H.P and 15 in front of V.P. All the sides of the rectangle are equally inclined to H.P and parallel to V.P. Draw its projections and locate its traces.

**Construction (Fig.9.11)**

1. Draw the front view  $a'b'c'd'$  (rectangle of size  $40 \times 25$ ), with the corner  $a'$  at 10 above  $xy$  and all the sides are inclined at  $45^\circ$  to  $xy$ .
2. Draw the projectors through the corners of the front view and obtain the top view, a line, parallel to  $xy$  and 15 below it.

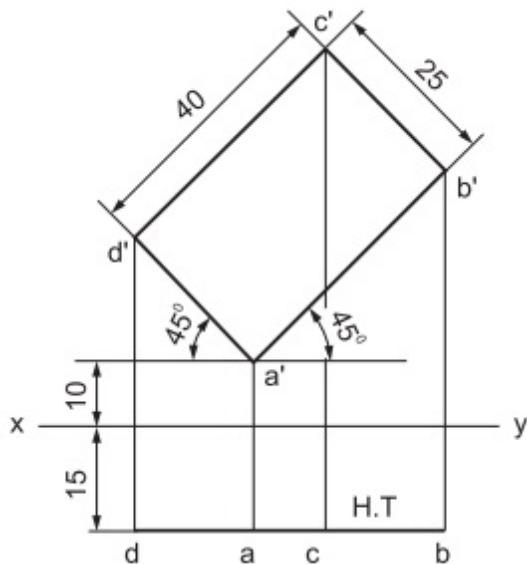
For the given plane, there is no V.T. The top view is its H.T and also called the edge view of the plane.

**Problem 9** A regular pentagon of 25 side is parallel to H.P and perpendicular to V.P. The plane is 15 above H.P and an edge of it lies on V.P. Draw the projections and show its traces.

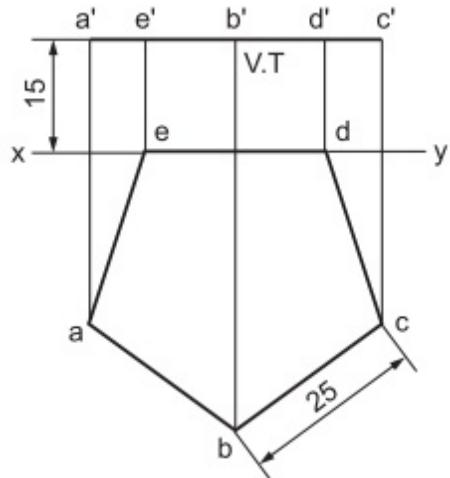
**Construction (Fig.9.12)**

1. Draw the top view abcde (pentagon of side 25) so that, an edge (de) coincides with xy.
2. Draw the projectors through the corners of the top view and obtain the front view, a line, parallel to xy and 15 above it.

For the given plane, there is no H.T. The front view is its V.T and also called the edge view of the plane.



**Fig.9.11**

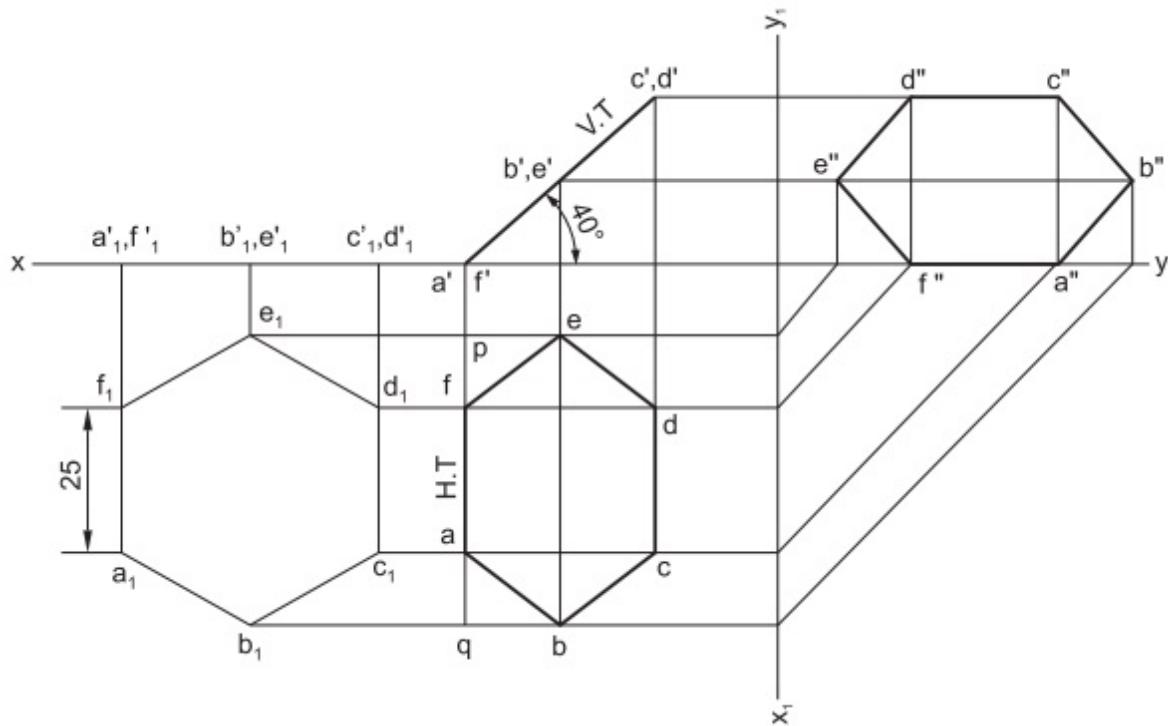


**Fig.9.12**

**Problem 10** A regular hexagon of 25 side has its one edge on H.P. The surface of the plane is perpendicular to VP and inclined at  $40^\circ$  to H.P. Draw the three views of the plane and locate the traces.

**Construction (Fig.9.13)**

1. Draw the projections of the plane, assuming it to be on H.P and one edge perpendicular to V.P.
2. Redraw the front view such that, it makes  $40^\circ$  with xy and one end of it, a' (f') lies on xy (final front view).
3. Obtain the final top view, by projection.



**Fig.9.13**

4. Draw the reference line  $x_1 y_1$ , perpendicular to  $xy$  and obtain the side view, by projection.

The side view obtained is the left side view of the plane. The VT coincides with the front view and the line pq represents the HT of the plane.

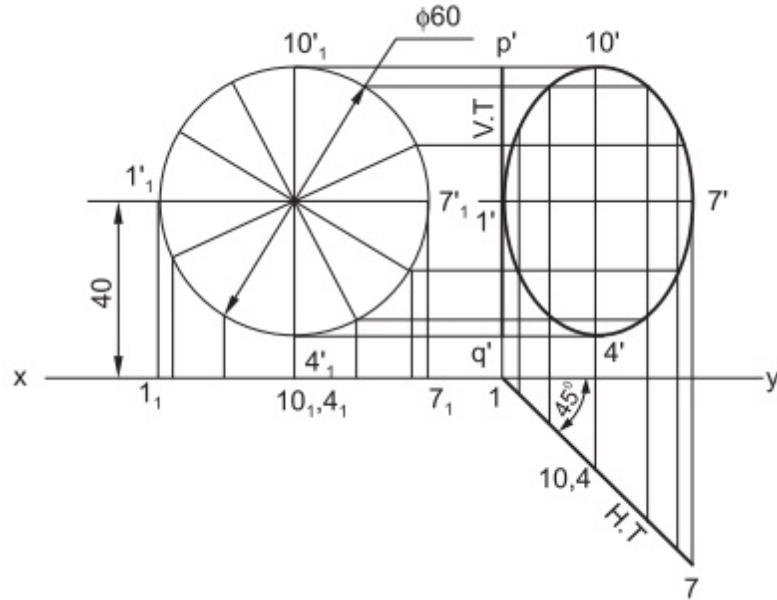
**Problem 11** *Draw the projections of a circle of 60 diameter resting on VP on a point on the circumference. The plane is inclined at  $45^\circ$  to VP and perpendicular to HP. The centre of the plane is 40 above HP. Also, locate its traces.*

### **Construction (Fig.9.14)**

1. Draw the projections of the circle, assuming it to be lying on VP and with its centre at 40 above HP.
2. Divide the circumference of the circle (front view) into some equal parts, say 12.
3. Transfer the division points on to the top view.

- Redraw the top view such that, it makes an angle of  $45^\circ$  with  $xy$  and one end of it lies on  $xy$ . This forms the final top view.
- Obtain the final front view, by projection.

The H.T coincides with the top view and the line  $p'q'$  represents the V.T of the plane.



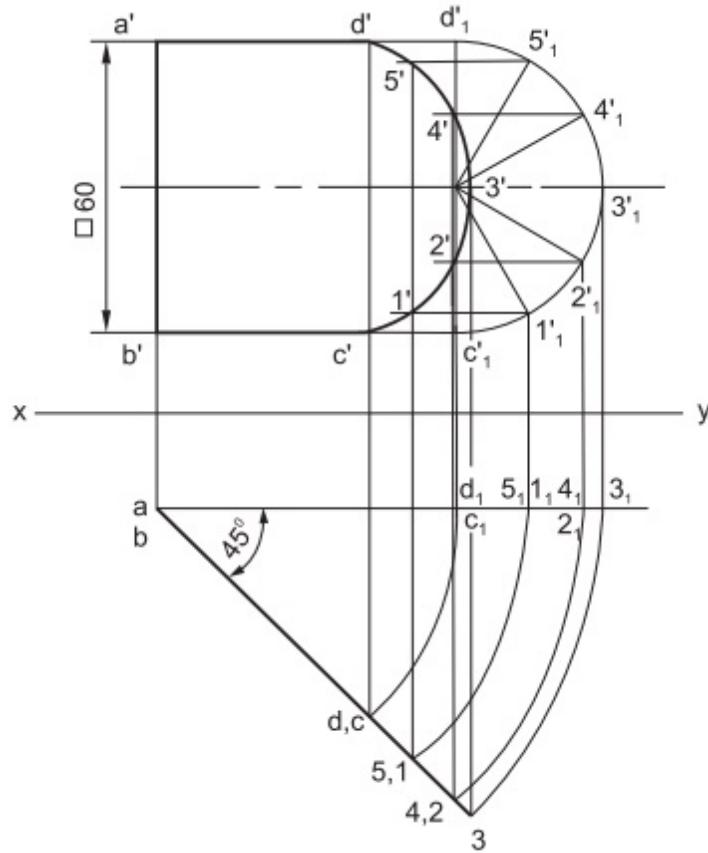
**Fig.9.14**

**Problem 12** A composite plane  $ABCD$  consists of a square of 60 side, with an additional semi-circle constructed on  $CD$  as diameter. Draw the projections of the plane, when the side  $AB$  is vertical and the plane makes an angle of  $45^\circ$  with V.P.

### **Construction (Fig.9.15)**

- Draw the projections of the plane, assuming it to be parallel to V.P and perpendicular to H.P; keeping the edge  $AB$  vertical.
- Rotate the top view till it makes  $45^\circ$  with  $xy$ . This is the final top view.

3. Obtain the final front view, by projection.



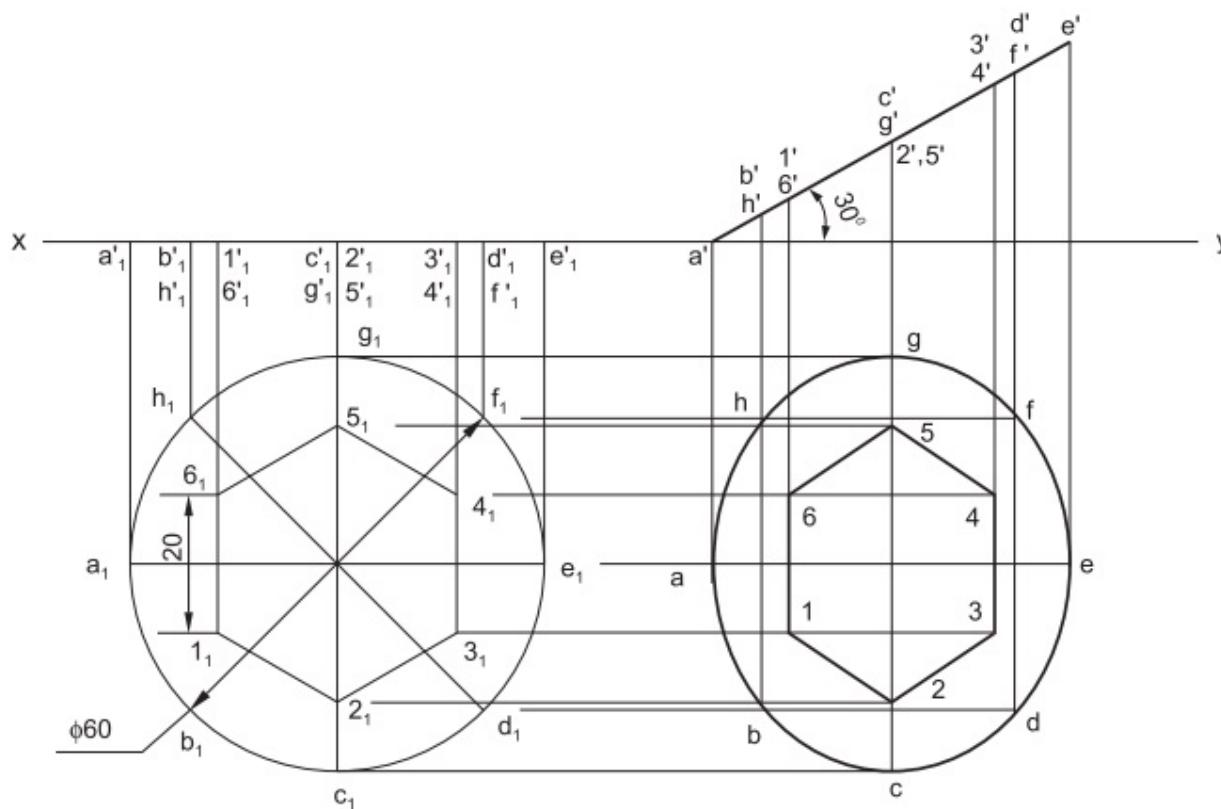
**Fig.9.15**

**Problem 13** A circular plate of 60 diameter, has a hexagonal hole of 20 side, centrally punched. Draw the projections of the plate, resting on H.P on a point, with its surface inclined at 30° to H.P. Any two parallel sides of the hexagonal hole are perpendicular to V.P. Draw the projections of the plate.

**Construction (Fig.9.16)**

1. Draw the projections of the plate with a hexagonal hole; assuming it to be resting on H.P such that, two parallel sides of the hexagonal hole are perpendicular to V.P.

2. Divide the circumference of the circle (top view) into some equal parts, say 8.
3. Transfer the division points and the points pertaining to the corners of the hexagon, to the front view.
4. Redraw the front view such that, it makes an angle of  $30^\circ$  with  $xy$  and one end of it lies on  $xy$ .
5. Obtain the final front view, by projection.



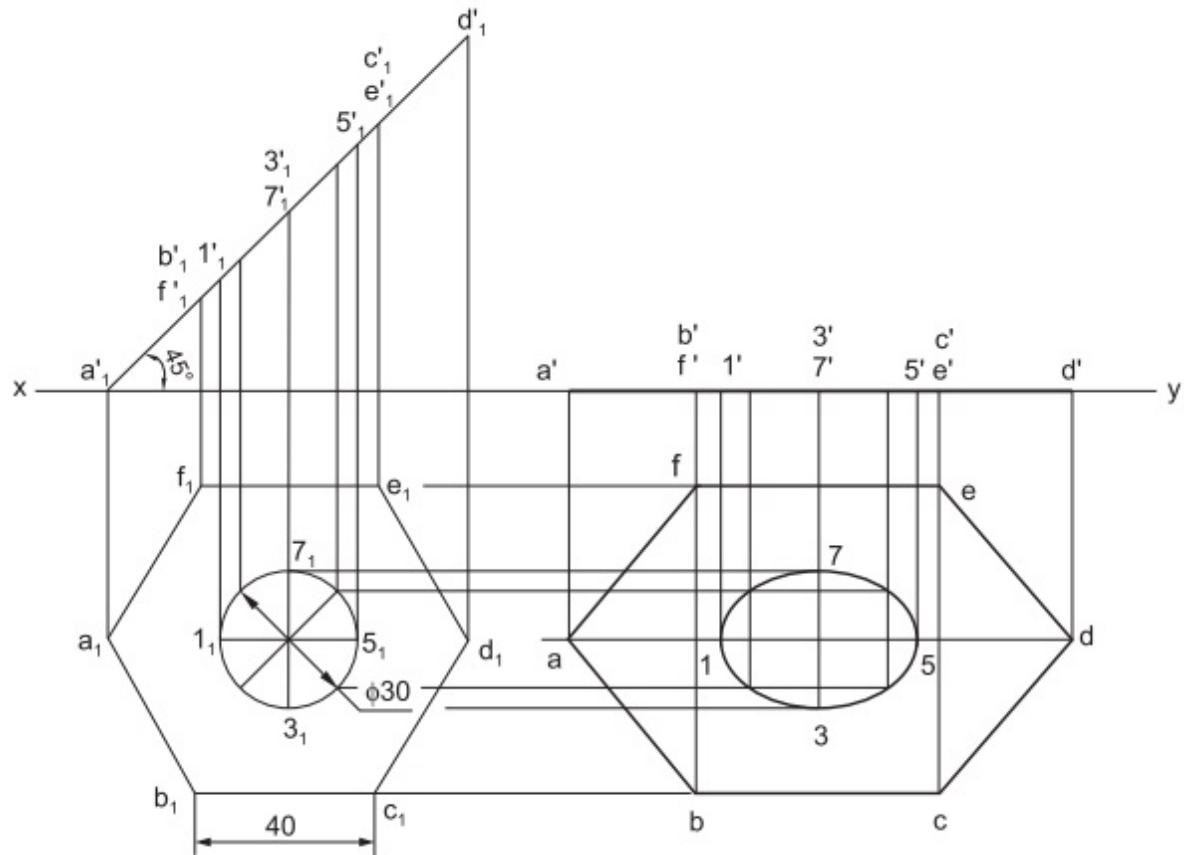
**Fig.9.16**

**Problem 14** The top view of a plane object is a regular hexagon of side 40, with a central hole of 30 diameter and with two sides of the hexagon parallel to  $xy$ , when the surface of the object is inclined at  $45^\circ$  to H.P and with a corner on H.P. Determine the true shape of the object.

**Construction (Fig.9.17)**

1. Draw the top view of the object, a hexagon of side 40 with a central hole of diameter 30 such that, two sides of the hexagon are parallel to xy.
2. Draw the front view, which is an edge view of the plane such that, it is inclined at  $45^\circ$  with xy and one end of it is on xy.
3. Divide the circle into, say 8 equal parts and locate the corresponding points in the front view.
4. Redraw the front view such that, it coincides with xy.
5. Obtain the true shape of the plane, by projection.

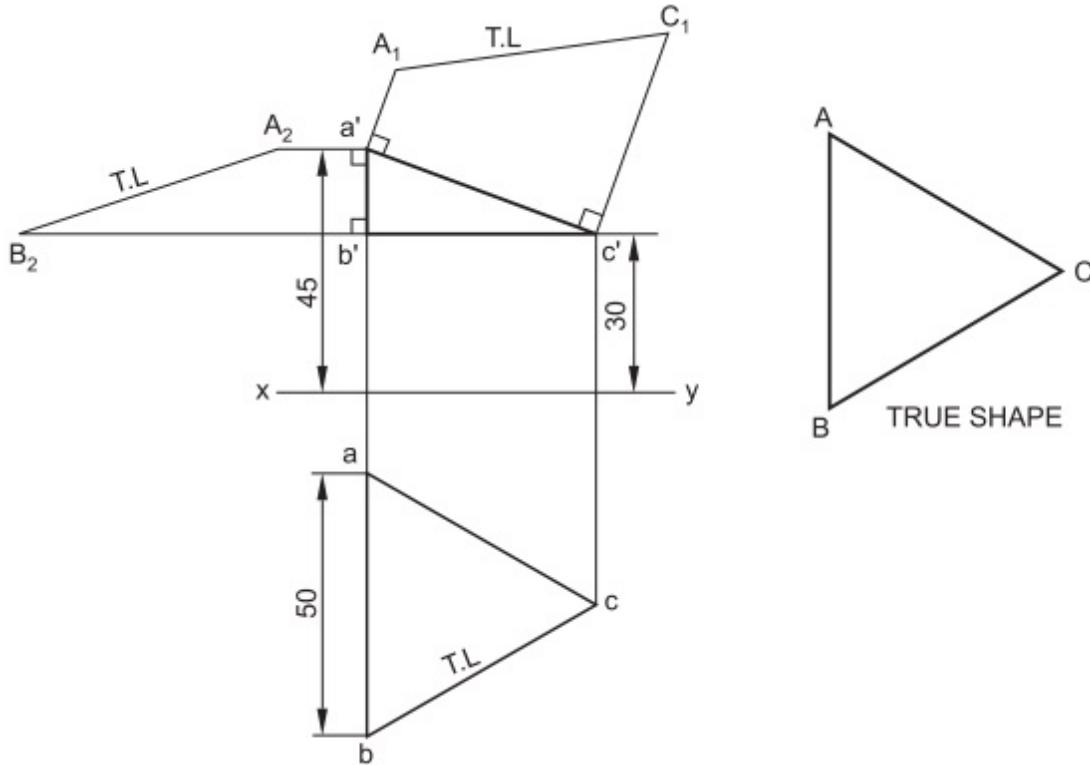
**Problem 15** *Construct an equilateral triangle abc of side 50 with ab perpendicular to xy. abc represents the top view of triangle ABC. Points B and C are 30 above H.P while point A is 45 above H.P. Draw the front view and find the true shape of the triangle ABC.*



**Fig.9.17**

### ***Construction (Fig.9.18)***

1. Draw the equilateral triangle abc, keeping AB perpendicular to xy; representing the top view of the triangle ABC.



**Fig.9.18**

2. Project and obtain the front view  $a'b'c'$  such that, the points  $b', c'$  are at 30 above xy and  $a'$  at 45 above xy.
3. The length of the top view  $bc$  represents the true length of the edge BC of the triangle ABC; since the front view  $b'c'$  is parallel to xy.
4. Determine the true lengths of the edges AB ( $=A_2 B_2$ ) and AC ( $=A_1 C_1$ ) by using the trapezoidal method (refer Construction: [Fig. 8.9c](#)).
5. Using the true lengths of the edges, draw the true shape of the triangle ABC, as shown.

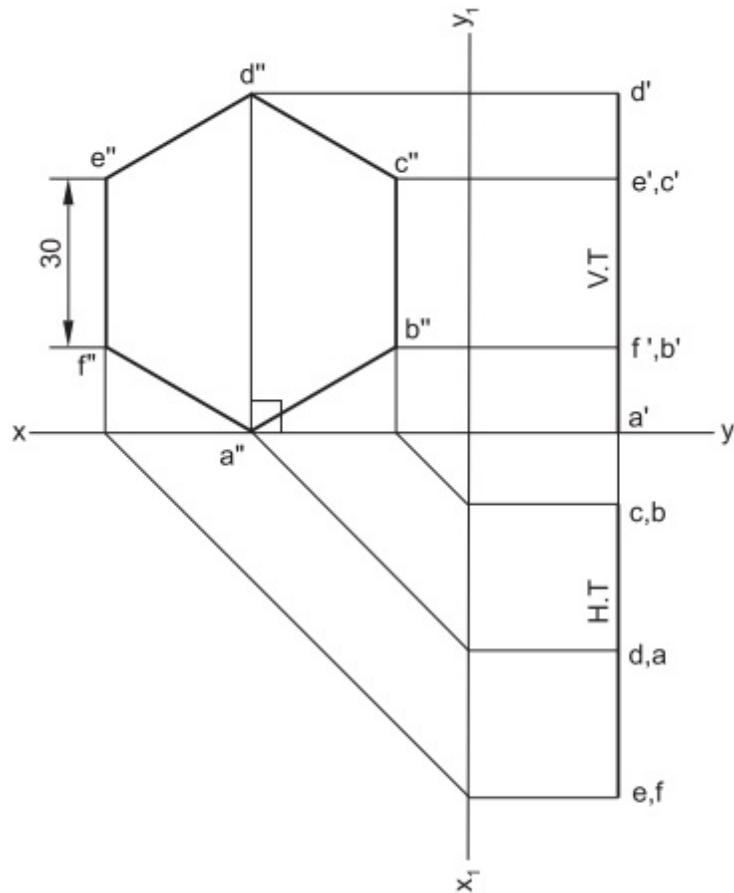
**Problem 16** A hexagon of 30 side has one of its corners on H.P. The plane of the hexagon is perpendicular to both H.P and V.P. The longest diagonal passing through the corner on H.P is perpendicular to H.P. Draw its projections.

### **Construction (Fig.9.19)**

1. Draw the side view of the hexagon, keeping the corner A on H.P and the diagonal AD, perpendicular to it.
2. Draw the projectors from the side view and obtain the front view such that, it is perpendicular to xy.
3. Obtain the top view, by projection.



The H.T coincides with the top view and the V.T, with the front view.



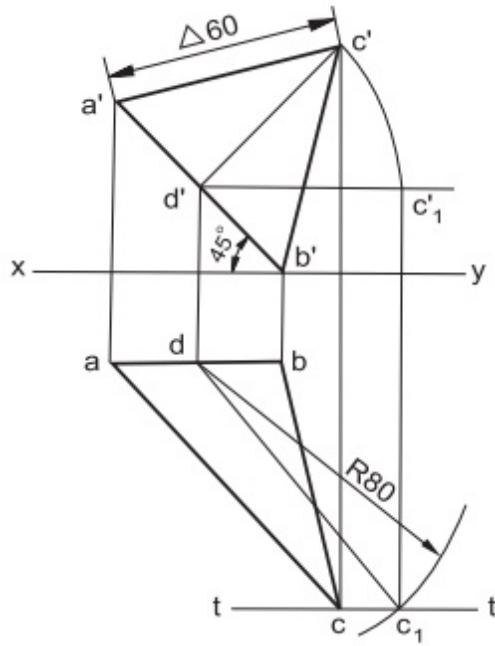
**Fig.9.19**

**Problem 17** The front view of a plane object is an equilateral triangle of side 60, with one side inclined at  $45^\circ$

*to xy. The true shape of the plane is an isosceles triangle of base 60 and altitude 80. Draw the projections of the plane.*

**Construction ([Fig.9.20](#))**

1. Draw the front view  $a'b'c'$ , an equilateral triangle of side 60, with the side (base)  $a'b'$  making an angle of  $45^\circ$  with  $xy$ .
  2. Project  $a'b'$  and obtain  $ab$ , the top view of the base, parallel to  $xy$ , as  $a'b'$  represents the true length of the base.
  3. Rotate the altitude  $d'c'$  about  $d'$ , to the position  $d'c_1'$ , parallel to  $xy$ .
  4. Draw a projector through  $c_1'$ .
  5. With centre  $d$  and radius 80 (true length of the altitude), draw an arc intersecting the above projector at  $c_1$ .
  6. Through  $c_1$ , draw  $t-t$ , parallel to  $xy$ , representing the locus of top view of C.
  7. Draw a projector through  $c'$ , meeting  $t-t$  at  $c$ .
  8. Join  $a, c$  and  $c, b$ .
- $a'b'c'$  and  $abc$  are the required projections.



**Fig.9.20**

**Problem 18** Draw the projections of a rhombus, having diagonals 120 and 60 long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at  $30^\circ$  to H.P.

### **Construction (Fig.9.21)**

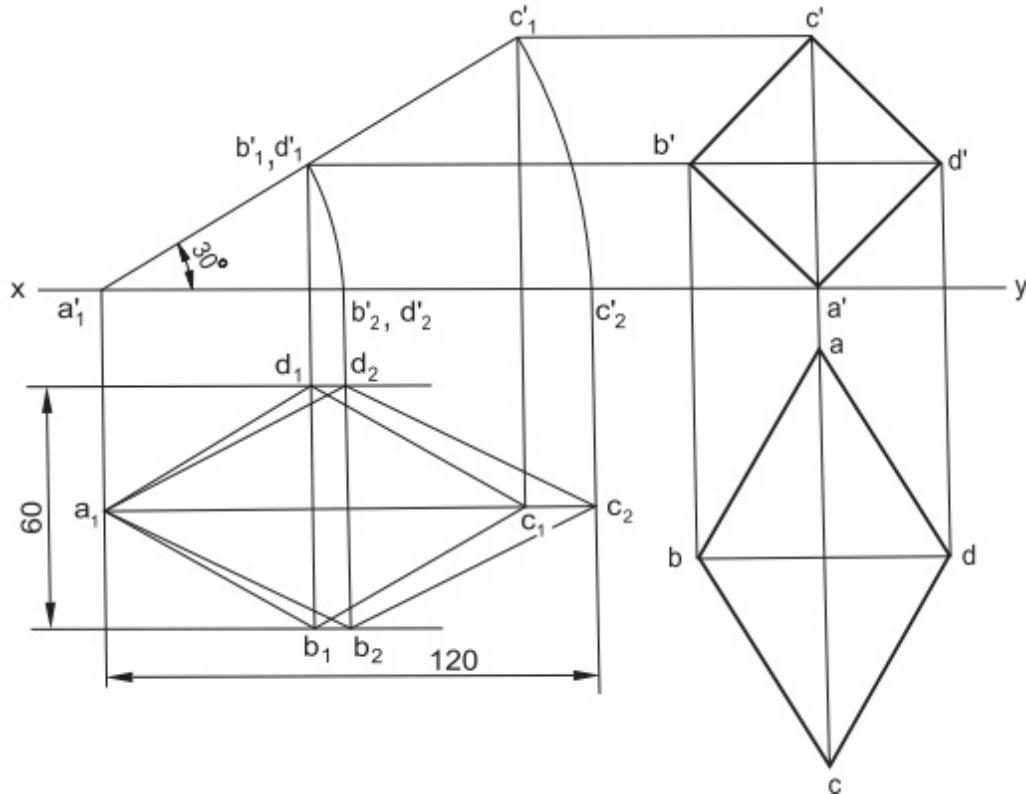
**Stage I** Assume that the plane is on H.P, with the smaller diagonal perpendicular to V.P.

1. Draw the top view (true shape of the rhombus) such that, the smaller diagonal is perpendicular to xy.
2. Obtain the front view (edge view) of the plane, coinciding with xy.

**Stage II** Rotate the plane such that, the surface (longer diagonal) makes  $30^\circ$  with H.P.

3. Rotate the front view about  $a_1'$  such that, the longer diagonal makes  $30^\circ$  with xy.
4. Obtain the top view by projection.

- Redraw the top view such that, the smaller diagonal  $bd$  is parallel to V.P (xy). This is the final top view.
- Obtain the final front view, by projection.

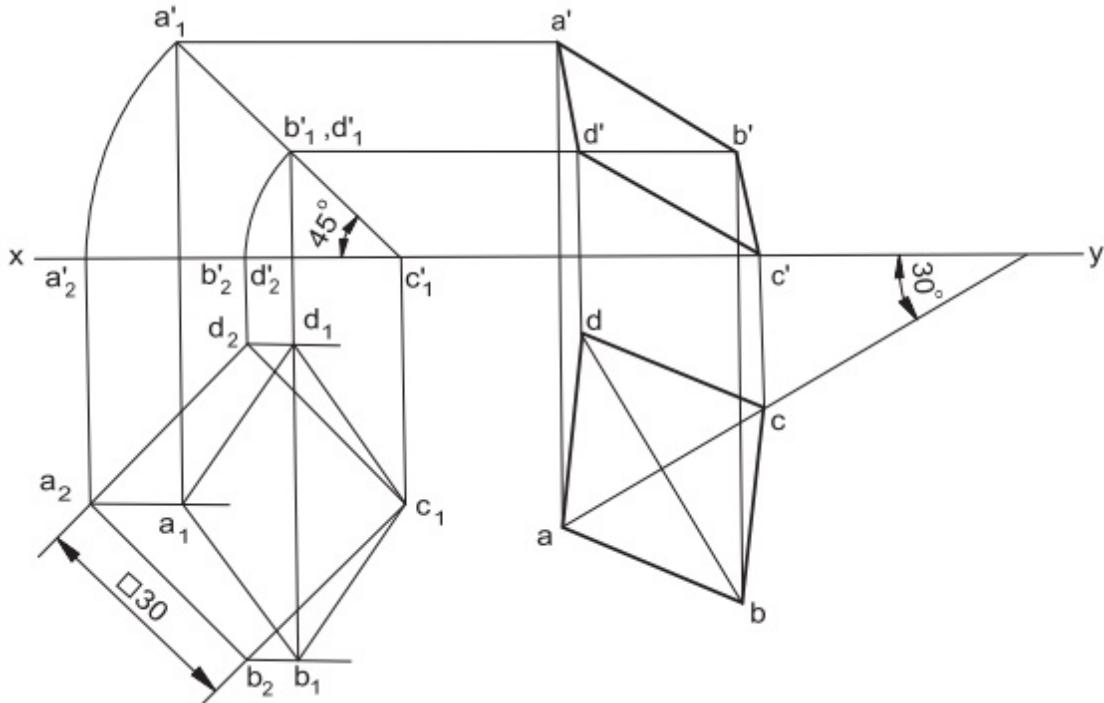


**Fig.9.21**

**Problem 19** A square lamina  $ABCD$  of 30 side, rests on the corner  $C$  such that, the diagonal  $AC$  appears at  $30^\circ$  to V.P in the top view. The two sides  $BC$  and  $CD$ , containing the corner  $C$  make equal inclinations with H.P. The surface of the lamina makes  $45^\circ$  with H.P. Draw its projections.

**Construction (Fig.9.22)**

- Draw the projections of the square lamina  $ABCD$ , assuming it to be lying on H.P and the sides are equally inclined to V.P.



**Fig.9.22**

2. Rotate the front view about  $c_1'$  through  $45^\circ$ .
3. Obtain the second top view, by projection.
4. Redraw the above view such that, ca makes  $30^\circ$  with xy. This is the final top view.
5. Obtain the final front view, by projection.

**Problem 20** A thin  $30^\circ$  -  $60^\circ$  set-square has its longest edge (diagonal) on H.P and inclined at  $30^\circ$  to V.P. Its surface makes an angle of  $45^\circ$  with H.P. Draw the projections, choosing suitable size for the set-square.

**Construction (Fig.9.23)**

**Stage I** Assume that the set-square is on H.P, with its diagonal perpendicular to V.P.

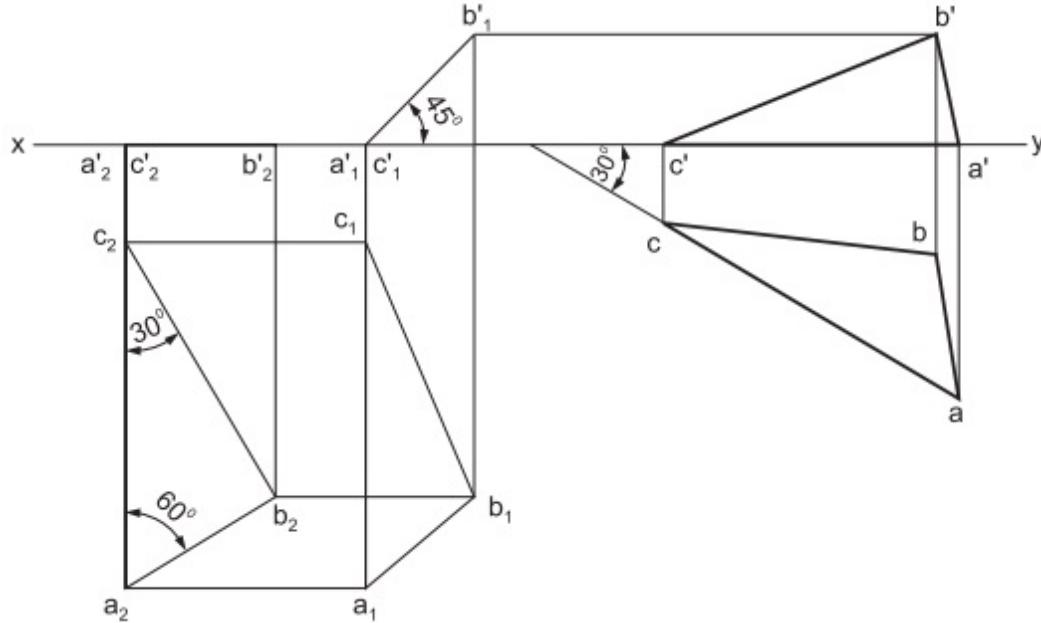
1. Draw the projections of the set-square, choosing suitable size.

**Stage II** Rotate the plane such that, the surface makes  $45^\circ$  with H.P.

2. Redraw the front view such that, it is inclined at  $45^\circ$  with xy; keeping one end  $a'_1 (c'_1)$  on xy.
3. Obtain the top view, by projection.

**Stage III** Rotate the plane such that, its diagonal makes an angle of  $30^\circ$  with V.P.

4. Redraw the top view such that, the diagonal ac makes  $30^\circ$  with xy. This forms the final top view.
5. Obtain the final front view, by projection.



**Fig.9.23**

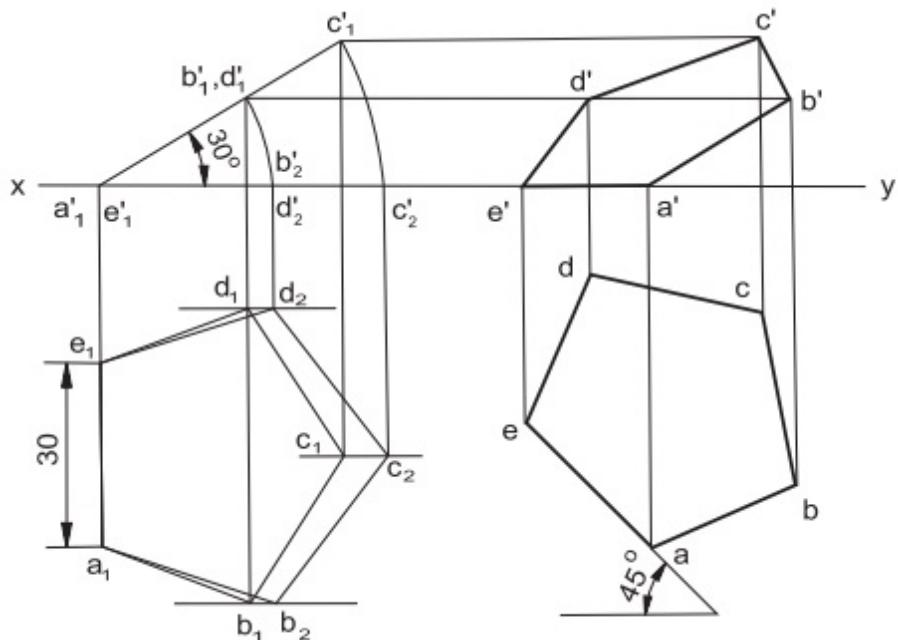
**Problem 21** A regular pentagon of 30 side is resting on one of its edges on H.P, which is inclined at  $45^\circ$  to V.P. Its surface is inclined at  $30^\circ$  to H.P. Draw its projections.

**Construction (Fig.9.24)**

1. Draw the projections of the pentagon, assuming it to be lying on H.P, with one of its edges AE perpendicular

to V.P.

2. Rotate the front view, about  $a_1'$  ( $e_1$ ), till it makes  $30^\circ$  with  $xy$ .
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the edge  $ae$  makes an angle of  $45^\circ$  with  $xy$ . This is the final top view.
5. Obtain the final front view, by projection.

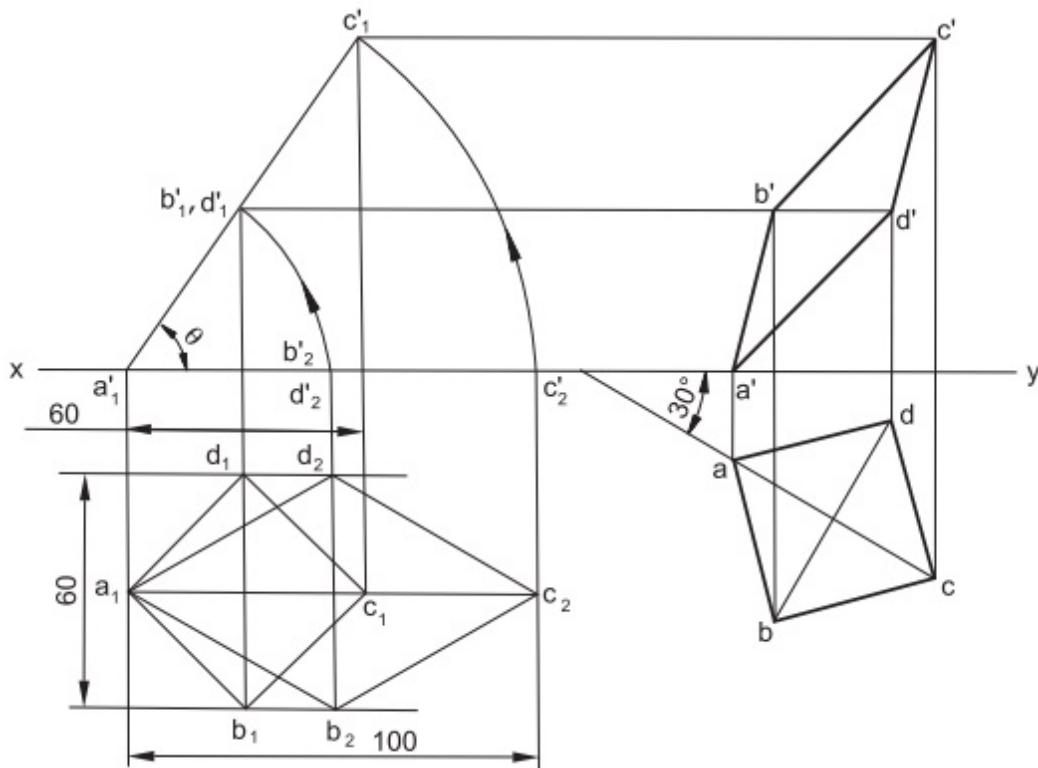


**Fig.9.24**

**Problem 22** A rhombus has its diagonals 100 and 60 long. Draw the projections of the rhombus, when it is so placed that its top view appears to be a square of diagonal 60 long and the vertical plane through the longer diagonal makes  $30^\circ$  with V.P.

**Construction (Fig.9.25)**

1. Draw the projections of the rhombus, assuming it to be lying on H.P with the shorter diagonal perpendicular to V.P.
2. Rotate the front view about  $a_1'$  such that, the projected length of the longer diagonal in the top view is 60 and then complete the second top view.
3. Redraw the above top view such that, the top view of the longer diagonal  $ac$  makes  $30^\circ$  with  $xy$ . This is the final top view.
4. Obtain the final front view, by projection.

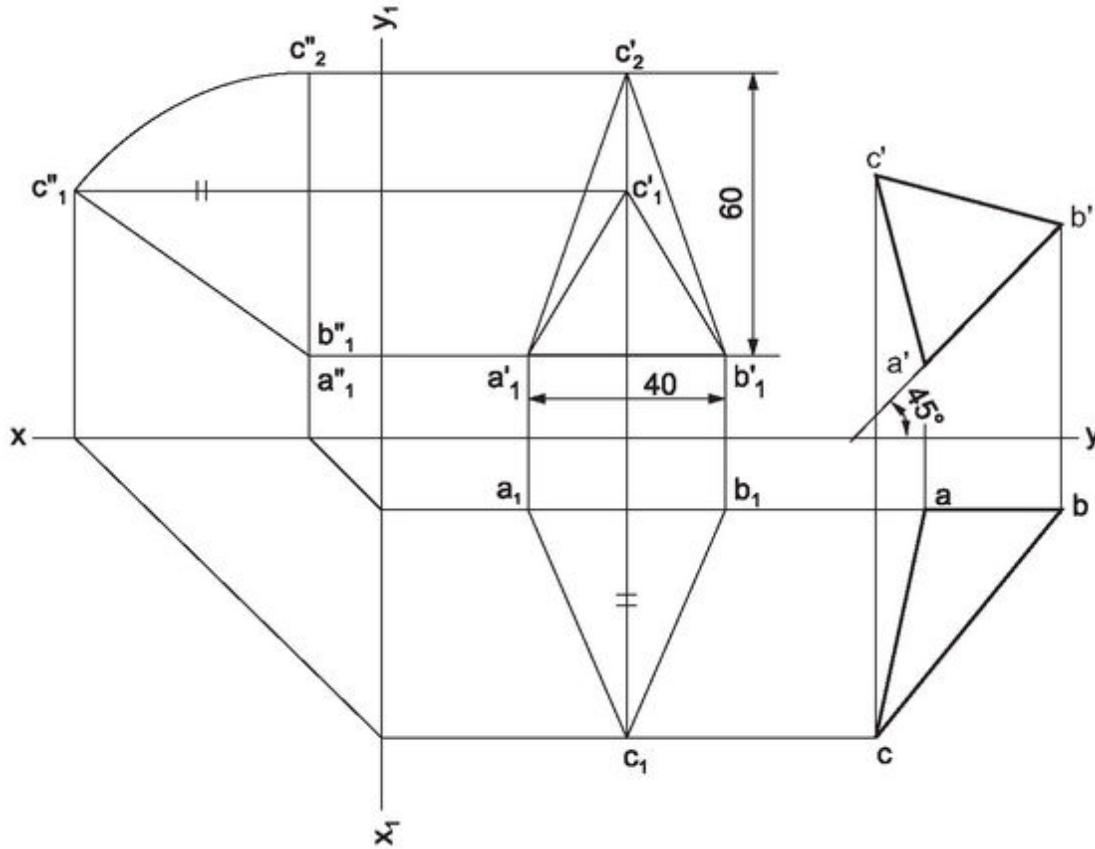


**Fig.9.25**

**Problem 23** A plate is of the shape of an isosceles triangle of base 40 and altitude 60. Draw the projections of the plate, when it is placed such that, the front view appears as

an equilateral triangle of side 40 and one of the edges of the plate make  $45^\circ$  with H.P.

### **Construction (Fig.9.26)**



**Fig.9.26**

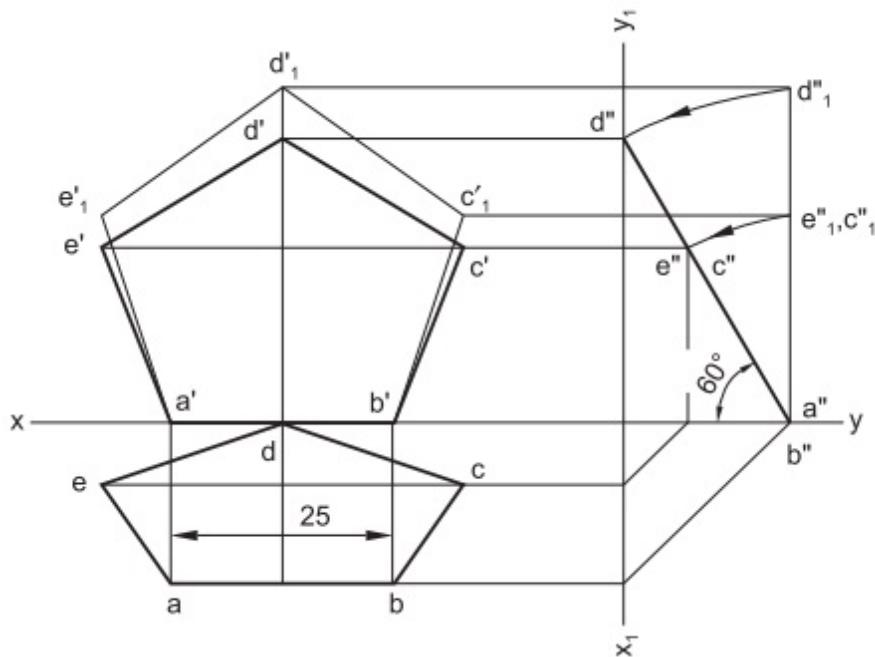
1. Draw the front view of the plate, assuming it to be parallel to V.P, with the base parallel to H.P.
2. Obtain the side view.
3. Rotate the side view about its end  $b_1''$  ( $a_1''$ ), till the front view becomes an equilateral triangle of side 40.
4. Obtain the top view, by projection.
5. Redraw the above front view such that, the edge  $a' b'$  makes an angle of  $45^\circ$  with  $xy$ . This is the final front view.

6. Obtain the final top view, by projection.

**Problem 24** A regular pentagonal lamina of 25 side, is resting on H.P on one of its sides, while the opposite corner touches V.P. Draw the projections of the lamina, when it makes an angle of  $60^\circ$  with H.P.

**Construction (Fig.9.27)**

1. Draw the front and side views of the plane, assuming it to be parallel to V.P and resting on an edge on H.P.
2. Tilt the side view about  $a''$  ( $b''$ ), till it makes  $60^\circ$  with  $xy$ , ensuring that the point  $d''$  touches  $x_1 y_1$ .
3. Obtain the final front and top views, by projecting from the side view.



**Fig.9.27**

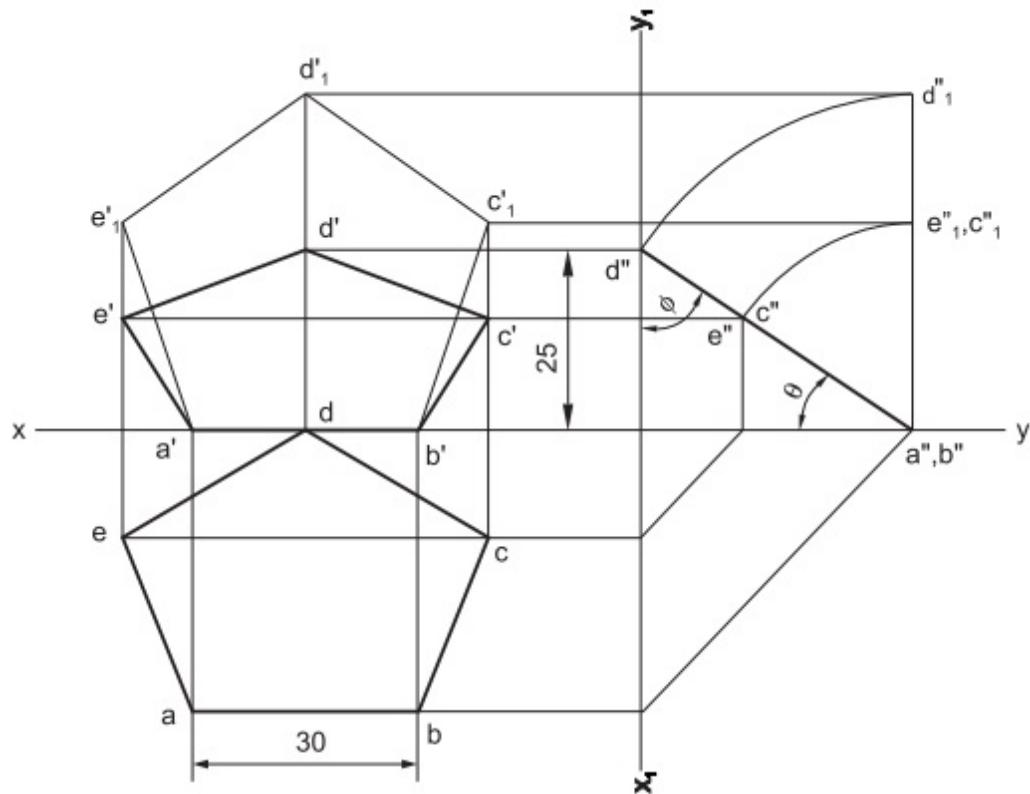
**Problem 25** A pentagonal plane of side 30 is resting on an edge on H.P and parallel to V.P. The corner, opposite to the resting edge, is on V.P and 25 above H.P. Draw the projections of the plane and determine its inclinations with

*the planes of projection and also determine the distance between the resting edge and V.P.*

### **Construction (*Fig.9.28*)**

1. Draw the front and side views of the plane, assuming it to be parallel to V.P and resting on an edge on H.P.
2. Rotate the side view about  $a''$  ( $b''$ ), till the point  $d_1''$  moves to the position  $d''$ , which is 25 above  $xy$ . This is the final end view of the plane.
3. Draw the reference line  $x_1y_1$ , passing through  $d''$  and perpendicular to  $xy$ .
4. Obtain the final front and top views, by projection from the side view.

The distance of  $a''$  ( $b''$ ) from  $x_1y_1$  is the distance between the resting edge and V.P.



### **Fig.9.28**

**Problem 26** *ABCDE is a regular pentagonal plate of 40 side and has its corner A on H.P. The plate is inclined to H.P such that, the top view lengths of edges AB and AE are each 35. The side CD is parallel to both the reference planes. Draw the projections of the plate and find its inclination with H.P.*

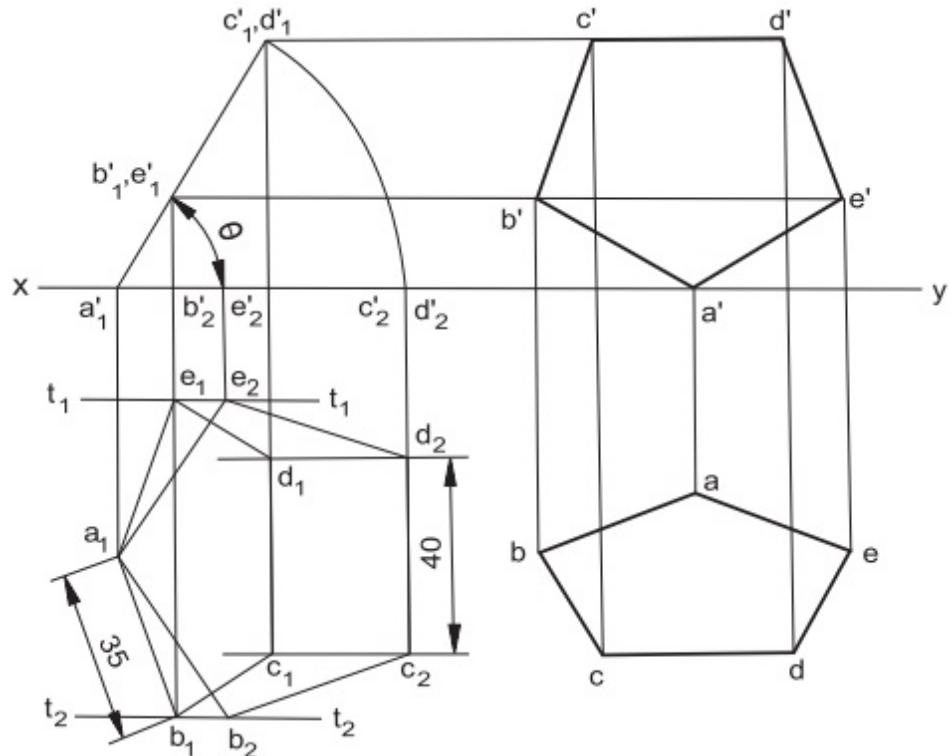
#### **Construction (Fig.9.29)**

1. Draw the projections of the pentagonal plate, assuming it to be lying on H.P; with the sides AB and AE equally inclined to V.P.
2. Draw the lines  $t_1 - t_1$  and  $t_2 - t_2$  which are parallel to xy and passing through  $e_2$  and  $b_2$  respectively.  $t_1 - t_1$  and  $t_2 - t_2$  represent the loci of top views of the edges E and B respectively.
3. Draw the second top views of the edges, i.e.,  $a_1e_1$  and  $a_1b_1$  such that, they are of lengths 35 and the points  $e_1$  and  $b_1$  lie on  $t_1 - t_1$  and  $t_2 - t_2$  respectively.
4. Draw a projector passing through  $b_1$  and  $e_1$ .
5. With  $a'_1$  as centre and radius  $a'_1b'_2$  ( $a'_1e'_2$ ), draw an arc intersecting the above projector at  $b'_1(e'_1)$ .
6. Join  $a'_1, b'_1(e'_1)$  and extend.
7. With  $a'_1$  as centre and radius  $a'_1c'_2$  ( $a'_1d'_2$ ), draw an arc intersecting the above extended line at  $c'_1(d'_1)$ ; forming the second front view.
8. Complete the second top view, by projection.
9. Redraw the above top view such that, the top view cd of the edge CD is parallel to xy. This is the final top

view.

- Obtain the final front view, by projection.

Angle  $\theta$  represents the inclination of the plate with H.P.



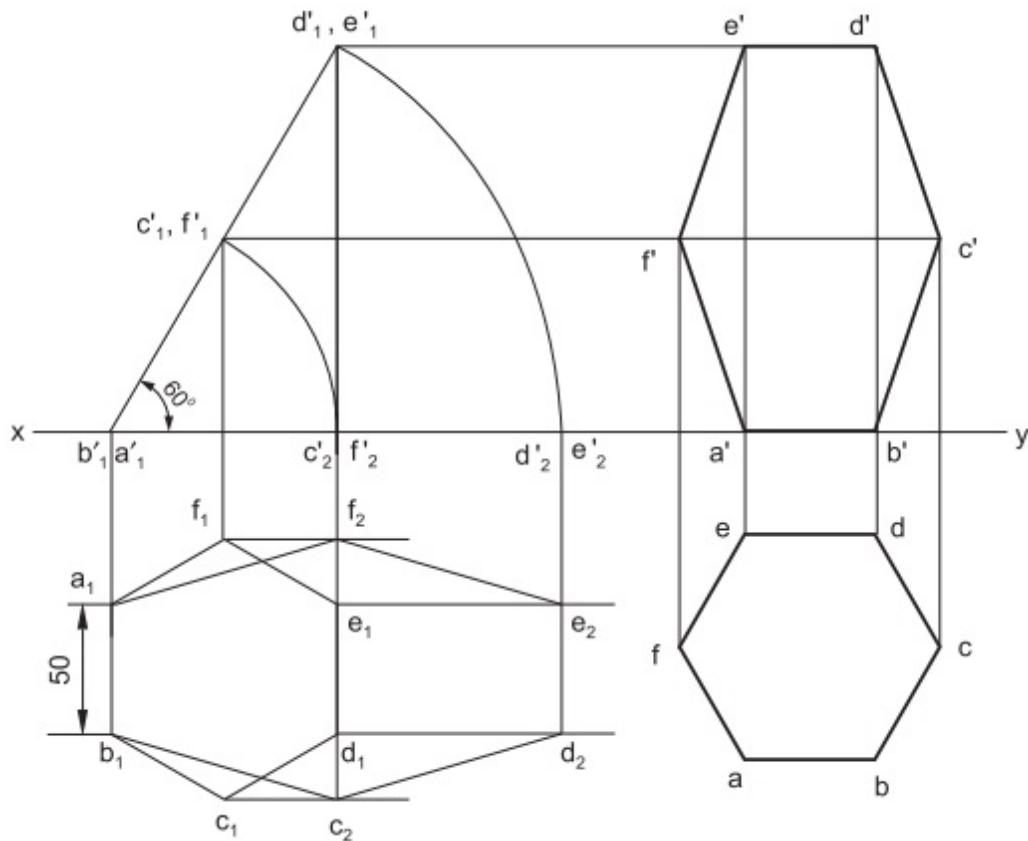
**Fig.9.29**

**Problem 27** Top view of a plate, the surface of which is perpendicular to V.P and inclined at  $60^\circ$  to H.P is a regular hexagon of side 50, with an edge perpendicular to xy. (i) Find the true shape of the plate and (ii) draw the projections of the plate, when the edge whose top view was perpendicular to xy earlier, becomes parallel to V.P; while the surface of the plate is still inclined at  $60^\circ$  to H.P.

### **Construction (Fig.9.30)**

- Draw the top view of the plate; a hexagon of side 50 such that, a side of it is perpendicular to xy.

2. Draw the front view which is an edge view of the plate such that, it is inclined at  $60^\circ$  with xy and one end of it is on xy.
3. Rotate the above front view till it coincides with xy.
4. Obtain the true shape of the plate  $a_1b_1c_2d_2e_2f_2a_1$ , by projection.
5. Redraw the initial top view such that, the top view ab of the edge AB is parallel to xy. This is the final top view.
6. Obtain the final front view, by projection.



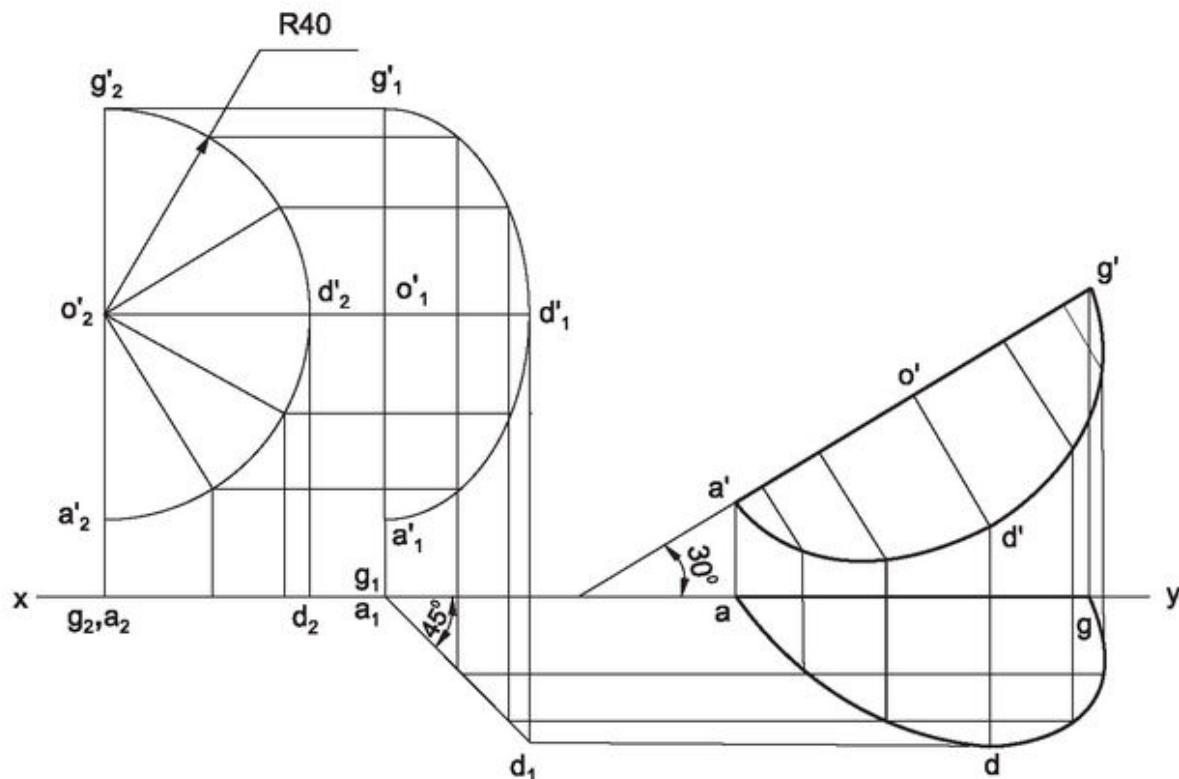
**Fig.9.30**

**Problem 28** A semi-circular plate of 80 diameter, has its straight edge on V.P and inclined at  $30^\circ$  to H.P, while the

surface of the plate is inclined at  $45^\circ$  to V.P. Draw the projections of the plate.

### **Construction (Fig.9.31)**

1. Draw the projections of the plate, assuming it to be lying on V.P. with the straight edge perpendicular to H.P.
2. Redraw the top view such that, it makes  $45^\circ$  with xy and one end of it  $g_1$  ( $a_1$ ), coincides with xy.
3. Obtain the second front view, by projection.
4. Redraw the above front view such that, the straight edge makes  $30^\circ$  with xy. This is the final front view.
5. Obtain the final top view, by projection.

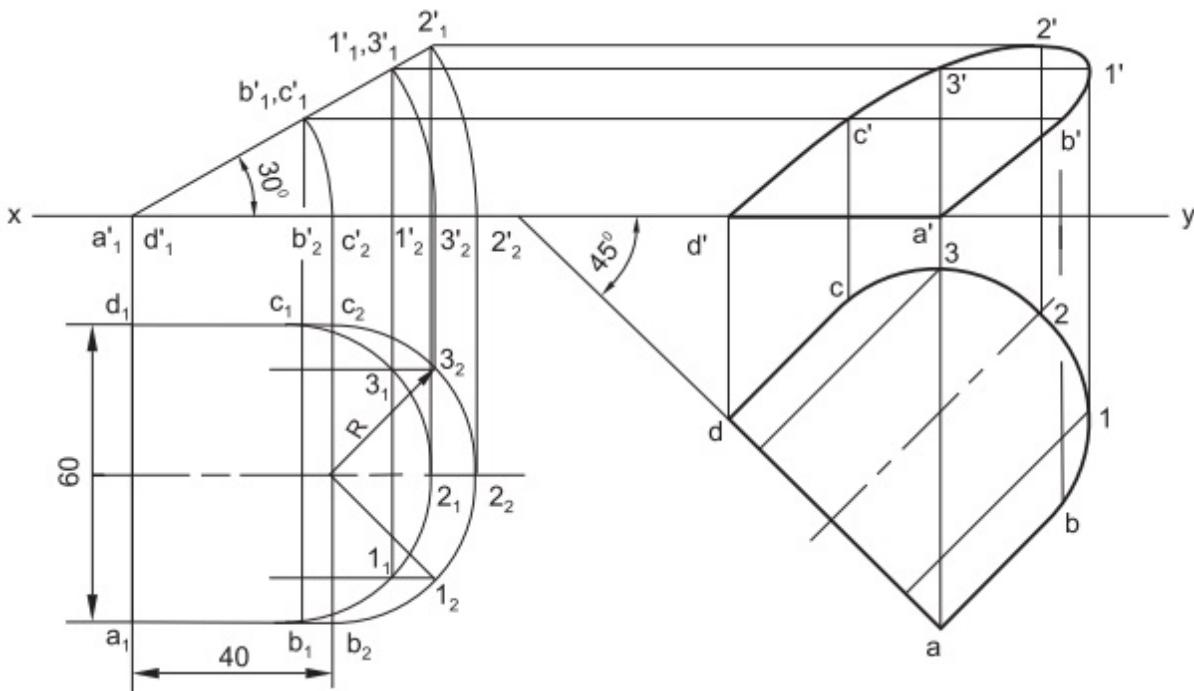


**Fig.9.31**

**Problem 29** A composite plate of negligible thickness is made up of a rectangle  $60 \times 40$  and a semi-circle on one of its longer sides. Draw its projections when the longer side is parallel to the H.P and inclined at  $45^\circ$  to V.P. The surface of the plate makes  $30^\circ$  angle with H.P.

**Construction (Fig.9.32)**

1. Draw the projections of the composite plate, assuming it to be lying on H.P, with the longer edge perpendicular to V.P.



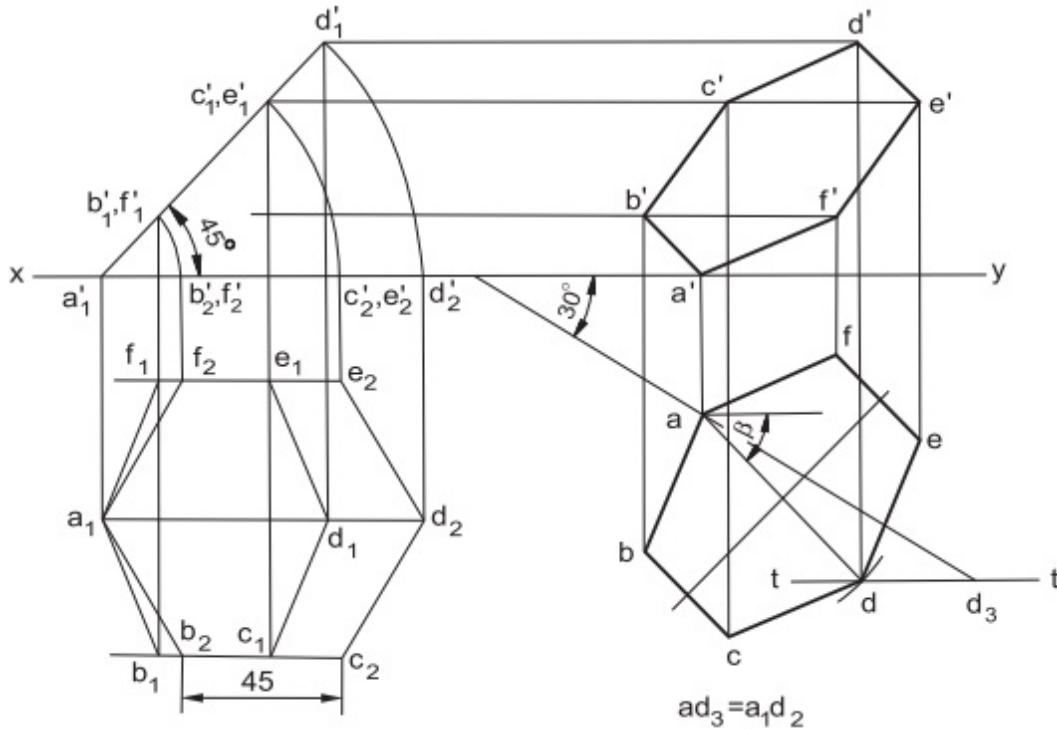
**Fig.9.32**

2. Rotate the front view about  $a_1'(d_1')$ , till it makes  $30^\circ$  with xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the longer edge makes  $45^\circ$  with xy. This is the final top view.
5. Obtain the final front view by projection.

**Problem 30** A regular hexagonal plane of 45 side has a corner on H.P, and its surface is inclined at  $45^\circ$  to H.P. Draw the projections, when the diagonal through the corner, which is on H.P, makes  $30^\circ$  with V.P.

**Construction (Fig.9.33)**

1. Draw the projections of the plane, assuming it lying on H.P with the diagonal AD parallel to V.P.
2. Rotate the front view about the corner  $a_1'$ , till it makes  $45^\circ$  with xy.
3. Obtain the second top view, by projection.
4. Determine the apparent angle of inclination  $\beta$ , which the diagonal AD makes with V.P.



**Fig.9.33**



The diagonal AD is inclined at an angle of  $45^\circ$  with H.P and when it also makes an angle  $\phi$  ( $30^\circ$ ) with

V.P, the top view of AD will be inclined at an apparent angle  $\beta$  with xy, which is larger than  $\phi$ .

*To determine the apparent angle of inclination*

- (i) Draw a line making  $\phi$  ( $30^\circ$ ) with xy, from any convenient point on xy.
- (ii) Mark the true length of the diagonal AD ( $=ad_3$ ) along the line and draw the locus t-t of  $d_3$ , parallel to xy.
- (iii) With a as centre and top view length of AD ( $=a_1d_1$ ) as radius, draw an arc intersecting the locus at d.
- (iv) Join a, d.

The angle ad makes with xy, is the apparent angle  $\beta$ .

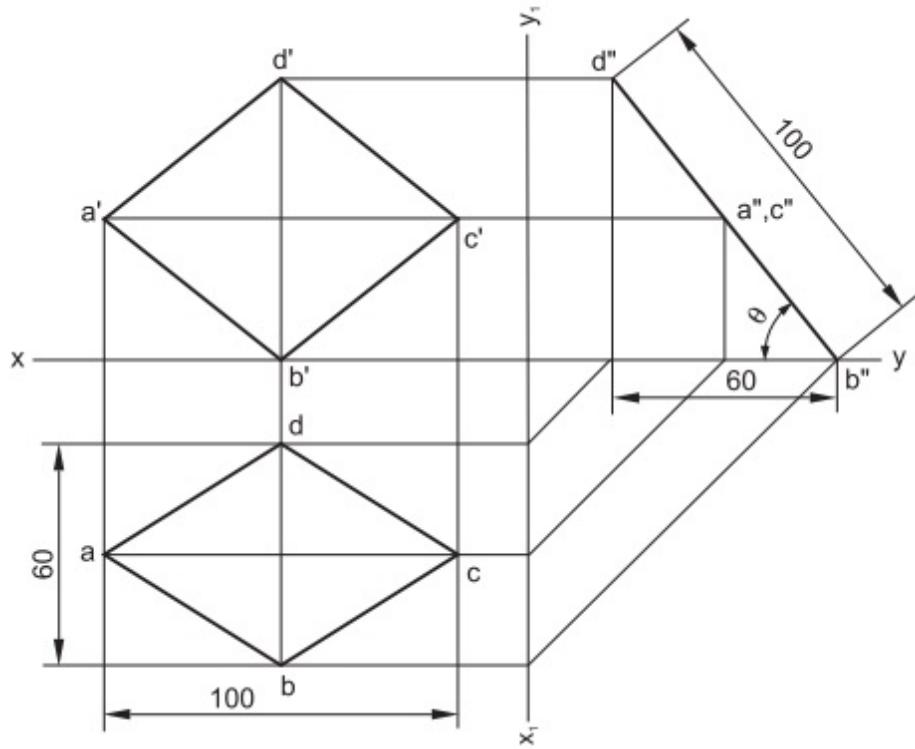
5. Redraw the above top view such that, the diagonal a d makes p with xy.
6. Obtain the final front view, by projection.

**Problem 31** *Draw a rhombus of diagonals 100 and 60 long, with the longer diagonal horizontal. The figure is the top view of a square lamina of 100 long diagonal, with a corner on H.P. Draw its front view and determine the angle, its surface makes with H.P.*

### ***Construction (Fig.9.34)***

1. Assuming that the lamina is perpendicular to P.P, with a corner on H.P, draw the side view such that, the projected length of its diagonal on H.P is 60 long.
2. Obtain front and top views, by projecting from the side view.

The inclination of the side view with xy, i.e.,  $\theta$ , is the true angle of inclination of the surface with H.P.



**Fig.9.34**

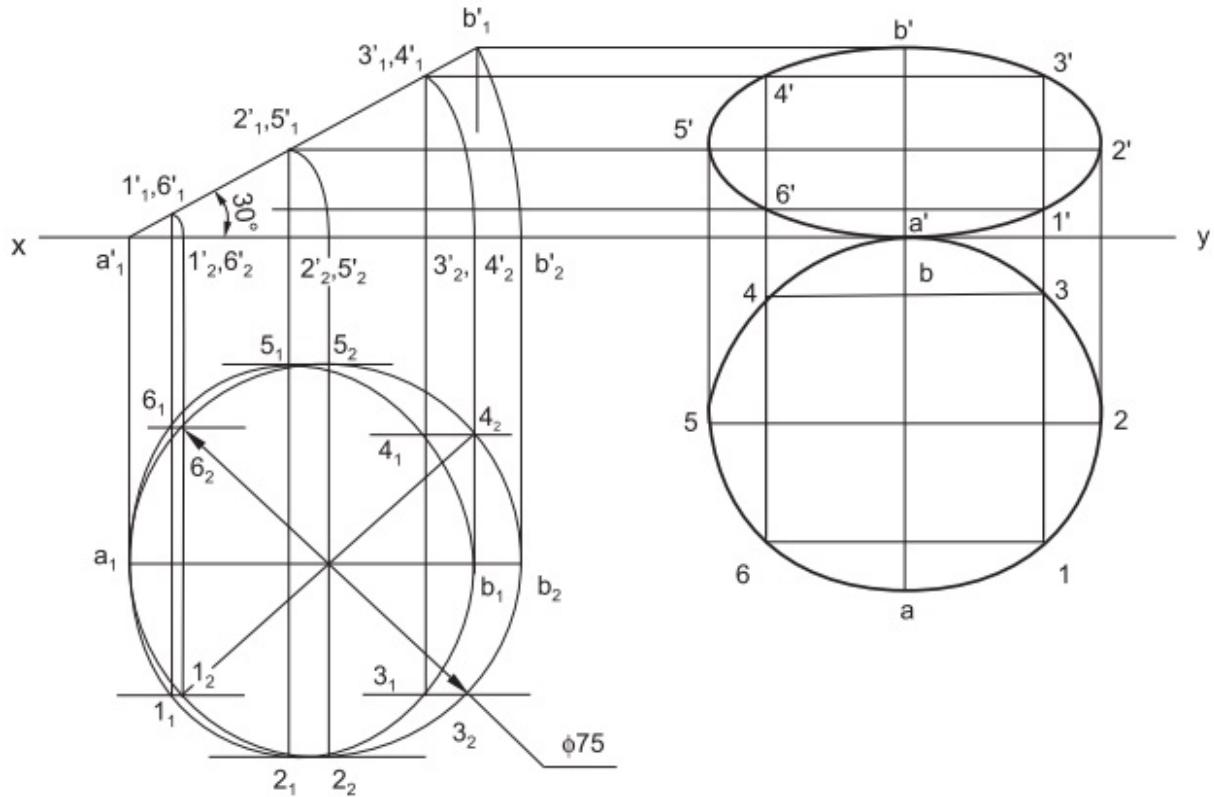
**Problem 32** Draw the projections of a circle of 75 diameter having the end A of the diameter AB in H.P, and the end B in V.P and the surface inclined at  $30^\circ$  to H.P and  $60^\circ$  to V.P.

**Construction (Fig.9.35)**

1. Draw the projections of the circle, assuming it to be lying on H.P and with the diameter AB, parallel to V.P.
2. Divide the circumference of the circle (top view) into some equal parts, say 8.
3. Transfer the division points on to the front view.
4. Rotate the above front view about  $a_1'$  till it makes  $30^\circ$  with  $xy$ .
5. Obtain the (second) top view, by projection.
6. Redraw the above top view such that, the top view of the diameter, ab is perpendicular to  $xy$  and b coincides

with  $xy$ . This is the final top view.

- Obtain the final front view, by projection.



**Fig.9.35**

**Problem 33** A circle of 40 diameter, is resting on H.P on a point, with its surface inclined at  $30^\circ$  to H.P. Draw the projections of the circle, when (i) the top view of the diameter, through the resting point, makes an angle of  $45^\circ$  with  $xy$  and (ii) the diameter passing through the resting point makes an angle of  $45^\circ$  with V.P.

### **Construction (Fig.9.36)**

- Draw the projections of the circle, assuming it to be lying on H.P.
- Redraw the front view, making  $30^\circ$  with  $xy$  and one end of it is lying on  $xy$ .

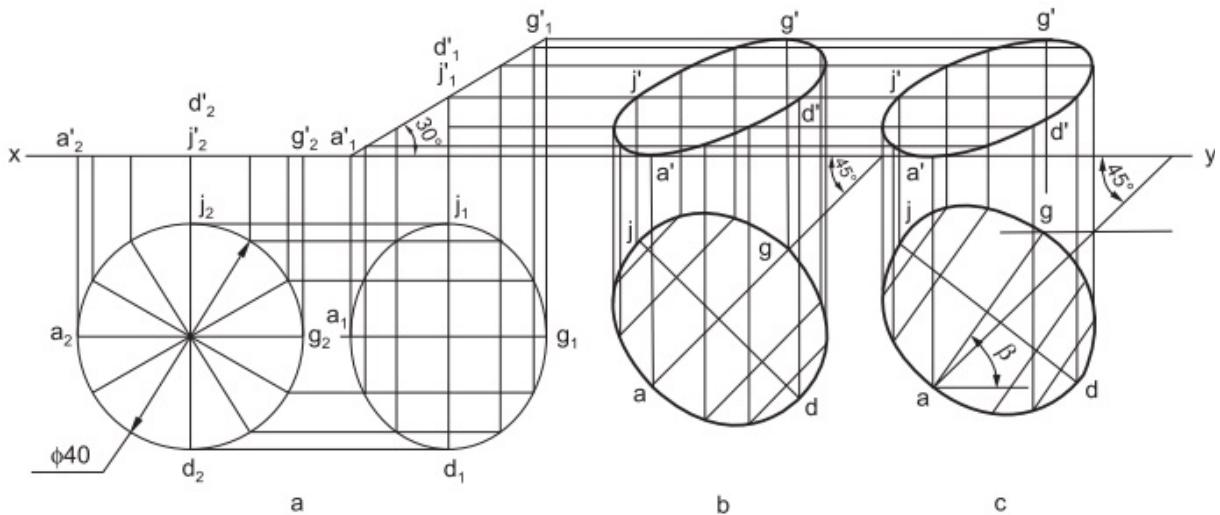
3. Obtain the second top view, by projection (Fig. 9.36a).

### **Case I**

4. Redraw the second top view such that, the top view of the diameter  $a_1g_1$  ( $=ag$ ) makes  $45^\circ$  with  $xy$ . This is the final top view.
5. Obtain the final front view, by projection (Fig. 9.36b).

### **Case II**

6. Determine the apparent angle of inclination  $p$  which the diameter AB makes with V.P.
7. Redraw the second top view such that, the top view of the diameter, i.e.,  $a_1 g_1$  ( $=ag$ ) makes  $\beta$  with  $xy$ . This is the final top view.
8. Obtain the final front view, by projection (Fig. 9.36c).

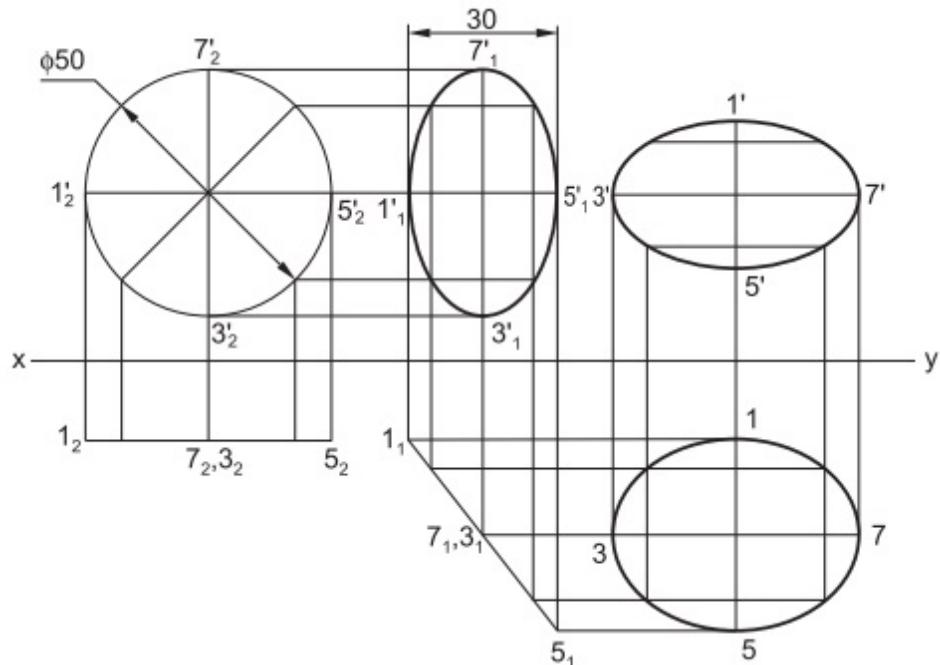


**Fig.9.36**

**Problem 34** A circular plate of 50 diameter, appears as an ellipse in the front view, having its major axis 50 long and minor axis 30 long. Draw the top view, when the major axis of the ellipse is horizontal.

**Construction (Fig.9.37)**

1. Draw the projections of the plate, assuming it to be parallel to V.P.
2. Locate  $1_1'$  and  $5_1'$  on the horizontal line through  $1_2'(5_2')$  such that,  $1_1'5_1' = 30$ , the minor axis of the ellipse.
3. Draw a projector through  $5_1'$ .
4. Locate the top view  $1_1$ , by projection.
5. With  $1_1$  as centre and radius  $1_25_2$ , draw an arc; intersecting the projector through  $5_1'$  at  $5_1$ .
6. Join  $1_1, 5_1$  forming the second top view. Locate the points  $2_1, 3_1$ , etc., on this view.
7. Obtain the second front view, by projection. Obviously, the shape of this view is an ellipse with major axis equal to 50.
8. Redraw the final front view such that, the major axis  $3'-7'$  is horizontal (parallel to xy).
9. Obtain the final top view, by projection.



**Fig.9.37**

## EXERCISES

- 9.1 A square lamina of 40 side is perpendicular to H.P. One of its sides is 20 above H.P and 15 in front of V.P. Draw its projections.
- 9.2 An equilateral triangular lamina of side 30 is parallel to H.P and perpendicular to V.P. One of its sides is 20 in front of V.P and 30 above H.P. Draw its projections.
- 9.3 A pentagonal plate of 35 side is perpendicular to V.P and parallel to H.P. One of its edges is perpendicular to V.P. Draw its projections.
- 9.4 An equilateral triangle of 50 side, has its plane parallel to H.P and 30 away from it. Draw the projections when one of its sides is (i) perpendicular

to V.P, (ii) parallel to V.P and (iii) inclined to V.P at an angle of  $45^\circ$ . Locate its traces.

- 9.5 A triangular lamina of 50 side is standing on one of its sides, which is inclined at  $45^\circ$  to V.P, and the surface of the lamina is making an angle of  $30^\circ$  to H.P. Draw the projections.
- 9.6 An isosceles triangle ABC of base 60 and altitude 75, has its base AC in H.P and inclined at  $30^\circ$  to V.P. The corners A and B are in the V.P. Draw its projections.
- 9.7 A rectangle ABCD of size  $60 \times 40$ , has a corner on H.P and 20 away from V.P. All the sides of the rectangle are equally inclined to H.P and parallel to V.P. Draw its projections and locate its traces.
- 9.8 A thin rectangular plate of  $60 \times 40$  size, has its shorter edge on H.P and inclined at  $30^\circ$  to V.P. Draw the projections of the plate, when its top view is a square of 40 side.
- 9.9 A thin rectangle ABCD of size  $60 \times 40$  has a corner on V.P and 20 away from H.P. The front view of the plate is a square of 40 side. The smaller edge of the plate through the corner (which is on V.P) is inclined at  $30^\circ$  to H.P and parallel to V.P. Draw the projections of the plate.
- 9.10 A square of 30 side has one side on H.P. Its plane is inclined at  $60^\circ$  to H.P and perpendicular to V.P. Draw its projections and locate its traces.
- 9.11 A square lamina ABCD of 30 side, rests on one of its corners on the ground. Its plane is inclined at  $35^\circ$  with H.P and diagonal DB inclined at  $65^\circ$  to V.P and parallel to H.P. Draw its projections.

- 9.12 A square ABCD of 50 side, has its corner A in the H.P, its diagonal AC inclined at  $30^\circ$  to the H.P and the diagonal BD inclined at  $45^\circ$  to the V.P and parallel to H.P. Draw its projections.
- 9.13 A pentagon of 30 side has one corner on H.P. Its plane is inclined at  $65^\circ$  to V.P and parallel to H.P. Draw its projections.
- 9.14 A square lamina ABCD of 30 side, rests on one of its corners on the ground. Its plane is inclined at  $35^\circ$  with H.P and diagonal DB inclined at  $65^\circ$  to V.P and parallel to H.P. Draw its projections.
- 9.15 Draw the projections of a regular pentagon of 40 side having its surface inclined at  $30^\circ$  to V.P and the side on which it rests on V.P, makes an angle of  $60^\circ$  with H.P.
- 9.16 A regular pentagon of length of 30 side has one of its corners on V.P and its surface is inclined at  $60^\circ$  to V.P. The edge, opposite to the corner on V.P, makes an angle of  $45^\circ$  with H.P. Draw the projections of the plane.
- 9.17 Draw the projections of a regular hexagon of 25 side having one of its sides in H.P and inclined at  $60^\circ$  to V.P and its surface making an angle of  $45^\circ$  with H.P.
- 9.18 A circular plate is parallel to H.P. Its radius is 30 and the centre is 50 above H.P and 40 in front of V.P. Draw its projections.
- 9.19 Draw the projections of a circle of diameter 50 and having its plane vertical and inclined at  $30^\circ$  to V.P. Its centre is 30 above H.P and 20 in front of V.P.
- 9.20 Draw the projections of a regular pentagon of 50 side having its surface inclined at  $45^\circ$  with H.P. A

side of the pentagon lies on H.P and inclined at  $30^\circ$  to V.P.

9.21 A circular plate of negligible thickness and 60 diameter appears as an ellipse in the top view, having its major axis 60 and minor axis 30. Draw its projections and find the inclination of the plate with H.P.

9.22 A circular plate of 60 diameter has a hexagonal hole of 20 side, centrally punched. Draw the projections of the plate, resting on H.P on a point, with its surface inclined at  $30^\circ$  to H.P. Any two parallel sides of the hexagonal hole are perpendicular to V.P. Draw the projections of the plate.

9.23 A circular plane of 60 diameter rests on V.P on a point A on its circumference. Its plane is inclined at  $45^\circ$  to V.P. Draw the projections of the plane when (i) the front view of the diameter AB makes  $30^\circ$  with H.P and (ii) the diameter AB itself makes  $30^\circ$  with H.P.

9.24 Draw the projections of a circle of 50 diameter resting in H.P on a point A on the circumference; its plane being inclined at  $45^\circ$  to H.P, and

(i) The top view of the diameter AB making  $30^\circ$  with the V.P, and

(ii) The diameter AB making  $30^\circ$  with the V.P.

9.25 Draw the projections of a circle of 60 diameter, resting on the ground on a point A on the circumference, when its plane inclined at  $45^\circ$  to H.P and the top view of the diameter AB making  $30^\circ$  with the V.P.

- 9.26 A thin semi-circular plate of 70 diameter, has its straight edge in H.P and inclined at  $45^\circ$  to V.P; while the surface of the plate is inclined at  $30^\circ$  to H.P. The end A of the diameter AB is nearer to the V.P and is at a distance of 25 from it. Draw the projections of the plate.
- 9.27 Draw the projections of a circle of 75 diameter having the end A of the diameter AB on H.P and the end B on V.P, and the surface inclined at  $30^\circ$  to H.P and  $60^\circ$  to V.P.
- 9.28 The top view of a plate, the surface of which is perpendicular to V.P and inclined at  $60^\circ$  to H.P is a circle of 60 diameter. Draw its three views.
- 9.29 Top view of a plate, the surface of which is perpendicular to V.P and inclined at  $60^\circ$  to H.P, is a regular pentagon of side 50, with one edge perpendicular to xy. (a) Find the true shape of the plate and (b) draw the projections of the plate, when the edge whose top view was perpendicular to xy earlier, becomes parallel to V.P; while the surface of the plate is still inclined at  $60^\circ$  to H.P.
- 9.30 A regular hexagonal plate of 35 side has one corner touching V.P and another opposite corner touching H.P. The plate is inclined at  $60^\circ$  to H.P and  $30^\circ$  to V.P. Draw the projections of the plate, neglecting the thickness of it.
- 9.31 A regular hexagon of 40 side has a corner on H.P. Its surface is inclined at  $45^\circ$  to H.P and the top view of the longest diagonal through the corner on which it rests, makes an angle of  $60^\circ$  with V.P. Draw its projections.

- 9.32 Draw the projections of a regular hexagon of 25 side, having one of its sides on H.P and inclined at  $60^\circ$  to V.P and its surface making an angle of  $45^\circ$  with H.P.
- 9.33 A regular hexagonal plane of 50 side, has a corner on V.P and its surface is inclined at  $45^\circ$  to V.P. Draw the projections when (a) the front view of the diagonal through the corner which is on V.P, makes  $30^\circ$  with H.P and (b) the diagonal itself makes  $30^\circ$  with H.P.
- 9.34 A regular hexagonal plane of 45 side has a corner on H.P, with its surface inclined at  $45^\circ$  to H.P. Draw its projections when (i) the top view of the diagonal through the resting corner makes  $30^\circ$  with V.P and (ii) the diagonal itself makes  $30^\circ$  with V.P.
- 9.35 Draw the projections of a rhombus having diagonals 120 and 50 long; the smaller diagonal being parallel to both the principal planes, while the other is inclined at  $30^\circ$  to H.P.
- 9.36 A thin  $45^\circ$ -  $45^\circ$  set-square, has its longest edge (250 long) on V.P and inclined at  $30^\circ$  to H.P. Its surface makes an angle of  $45^\circ$  with V.P. Draw its projections.
- 9.37 An equilateral triangular lamina of side 50 is perpendicular to both the planes. Draw its projections.
- 9.38 A circular plate of 50 diameter is perpendicular to both planes. Its centre is 60 above H.P and 50 in front of V.P. Draw the projections of the plate.
- 9.39 Draw the projections of a circle of 75 diameter having the end A of diameter AB in H.P, the end B in V.P and the surface inclined at  $30^\circ$  to H.P and  $60^\circ$  to V.P.

- 9.40 A regular pentagon of side 30 side is resting on one of its edges on H.P, which is inclined at  $45^\circ$  to V.P. Its surface is inclined at  $30^\circ$  to H.P. Draw its projections.
- 9.41 Draw the projections of a regular pentagon of 20 side with its surface making an angle of  $45^\circ$  with H.P. One of the sides of the pentagon is parallel to H.P and 15 away from it.
- 9.42 A thin  $30^\circ$ - $60^\circ$  set square has its longest edge in the V.P and inclined at  $30^\circ$  to H.P. Its surface makes an angle of  $45^\circ$  with the V.P. Draw its projections.
- 9.43 A hexagonal plane with distance between parallel sides 50, is resting on a side on H.P and the opposite side on V.P at a height of 30 from H.P. Draw the projections of the plane.

## REVIEW QUESTIONS

- 9.1 How do you specify a plane in space?
- 9.2 Name the possible orientations of the planes, with respect to the principal planes of projection.
- 9.3 What is a trace of a plane?
- 9.4 What is an oblique plane?
- 9.5 When the traces of an oblique plane will be parallel to xy?
- 9.6 What is an edge view of a plane?
- 9.7 When both the views of a plane are straight lines?
- 9.8 Explain the three stages of obtaining projections of an oblique plane.

## OBJECTIVE QUESTIONS

9.1 When a plane is perpendicular to a reference plane, its projection on the plane is a\_\_\_\_\_.

9.2 The traces of planes are straight lines/ points.

9.3 When a plane is perpendicular to both the reference planes, its traces are (a) inclined to xy, (b) perpendicular to xy, (c) parallel to xy.

( )

9.4 When a plane is lying parallel to a profile plane, the side view of it appears in its true shape.

(True/False)

9.5 One of the projections of an oblique plane is a straight line.

(True/False)

9.6 When a plane is inclined to the three principal planes, at least one of the projections will appear as a straight line.

(True/False)

9.7 When two planes intersect each other, their intersection is a straight line.

(True/False)

## ANSWERS

9.1 line

9.2 Straight lines

9.3 b

9.4 True

9.5 False

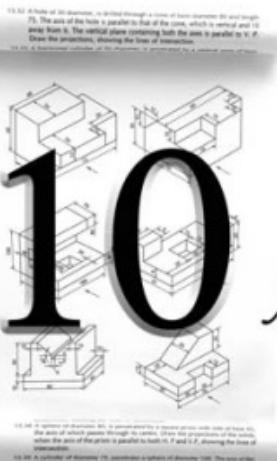
9.6 False

9.7 True

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# 10

## Auxiliary Projections



### 10.1 INTRODUCTION

Three views, viz., front, top and side views of an object are sometimes not sufficient to reveal complete information regarding the size and shape of the object. To serve the purpose, additional views known as auxiliary views, projected on auxiliary planes may be used. Auxiliary plane is a plane, perpendicular to one of the principal planes of projection and inclined to the other. Auxiliary views may also be used to determine the true length of a line and the true shape of plane surfaces.

### 10.2 TYPES OF AUXILIARY PLANES AND VIEWS

Two types of auxiliary planes, viz., (i) auxiliary vertical plane and (ii) auxiliary inclined plane are made use of, to obtain the auxiliary views.

## 10.2.1 Auxiliary Vertical Plane (A.V.P)

It is a plane perpendicular to H.P and inclined to V.P. The projection on to an A.V.P is called an auxiliary front view.

When the inclination of an A.V.P to V.P is  $90^\circ$ , it then becomes a profile plane. The projection on the profile plane gives the auxiliary front view, known as the side view of the object. Projections of points, lines and planes on the profile planes are already dealt under three view drawings.

## 10.2.2 Auxiliary Inclined Plane (A.I.P)

It is a plane inclined to H.P and perpendicular to V.P. The projection on an A.I.P is called an auxiliary top view.

For showing the relative positions of the views, the auxiliary plane should always be rotated about the plane to which it is perpendicular.

## 10.3 PROJECTIONS OF POINTS

### 10.3.1 Projections of a Point on an A.V.P

**Problem 1** A point A is 25 above H.P and 15 in front of V.P. Draw the front and top views of the point. Also, obtain the auxiliary front view of the point on a plane, which makes an angle of  $60^\circ$  with V.P and perpendicular to H.P.

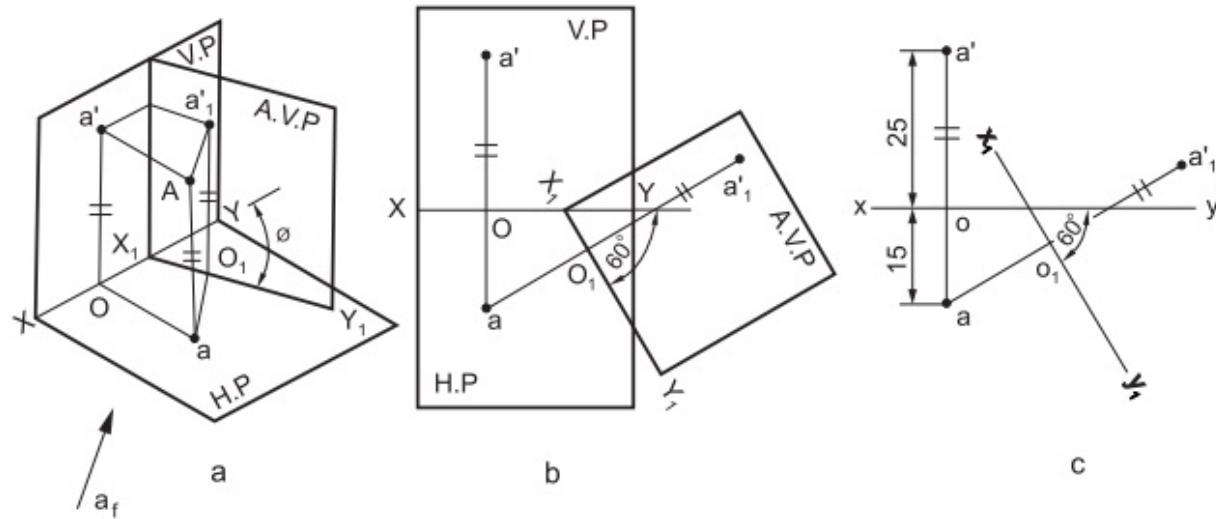
Figure 10.1a shows the quadrant with the point A marked in it. The auxiliary front view is obtained by viewing

the point in the direction  $a_f$  such that, the ray of sight passing through A, meets the A.V.P perpendicularly. [Figure 10.1b](#) shows the three planes opened out with the views marked.  $X_1 Y_1$  is the reference line between H.P and A.V.P. [Figure 10.1c](#) shows the relative positions of the views, with the planes removed.

From the geometry of the [Figs.10.1a and b](#), the following may be observed:

- The distance of the auxiliary front view from  $X_1 Y_1$  is equal to the distance of the front view from XY, which in-turn is the distance of the point A from H.P.
- The line  $X_1 Y_1$  is inclined to XY by an angle  $\phi$ , which is the true angle of inclination of A.V.P with V.P.
- The top view and auxiliary front view lie on a single projector, which is perpendicular to  $X_1 Y_1$ .

### ***Construction ([Fig.10.1c](#))***



**Fig.10.1**

- Draw the reference line xy and locate the front and top views of the point,  $a'$ ,  $a$  and mark o, the point of

intersection between the projector  $a'a$  and  $xy$ .

2. Draw the reference line  $x_1 y_1$ , at any convenient location, making an angle of  $60^\circ$  with  $xy$ .
3. Through  $a$ , draw a line perpendicular to  $x_1 y_1$ , intersecting it at  $o_1$ .
4. Locate the auxiliary front view  $a_1'$  such that,  $o_1 a_1' = oa'$ .

$a'$ ,  $a$  are the front and top views and  $a_1'$  is the auxiliary front view of the point.

It may be noted that there are four possible positions for the line  $x_1 y_1$  relative to  $xy$ .

### 10.3.2 Projections of a Point on an A.I.P

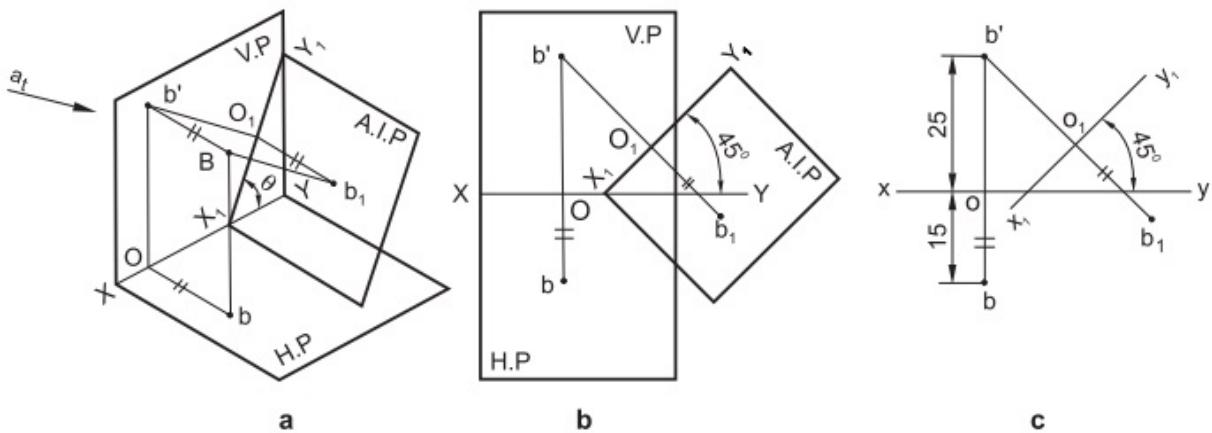
**Problem 2** A point  $B$  is 25 above H.P and 15 in front of V.P. Draw the front and top views of the point. Also, obtain the auxiliary top view of the point on a plane, which makes an angle of  $45^\circ$  with H.P and perpendicular to V.P.

Figure 10.2a shows the quadrant with the point  $B$  marked in it. The auxiliary top view is obtained by viewing the point in the direction  $a_t$  such that the ray of sight passing through  $B$ , meets A.I.P perpendicularly. Figure 10.2b shows the relative positions of the three planes after opening out. Here,  $X_1 Y_1$  is the reference line between V.P and A.I.P. Figure 10.2c shows the relative positions of the views, with the planes removed.

From geometry of the Figs.10.2a and b, the following may be observed:

- (i) The distance of the auxiliary top view from  $x_1 y_1$  is equal to the distance of the top view from XY, which in turn is the distance of the point B from V.P.
- (ii) The line  $x_1 y_1$  is inclined to xy by an angle  $\theta$ , equal to the true angle of inclination of A.I.P with H.P.
- (iii) The front view and auxiliary top view lie on a single projector, which is perpendicular to  $x_1 y_1$ .

**Construction (Fig.10.2c)**



**Fig.10.2**

1. Draw the reference line xy and locate the front and top views  $b'$ ,  $b$  of the point and mark o, the point of intersection between the projector  $b' b$  and  $xy$ .
2. Draw the reference line  $x_1 y_1$  at any convenient location, making an angle of  $45^\circ$  with  $xy$ .
3. Through  $b'$ , draw a line perpendicular to  $x_1 y_1$ , intersecting it at  $o_1$ .
4. Locate the auxiliary top view  $b_1$  such that,  $o_1 b_1 = ob$ .  $b'$ ,  $b$  are the front and top views and  $b_1$  is the auxiliary top view of the point.

## 10.4 PROJECTIONS OF STRAIGHT LINES

Projections of straight lines on the auxiliary planes may be used to obtain the following:

1. The true lengths of lines,
2. The point or edge views of lines, and
3. The conventional projections.

It is already understood that when a line is parallel to one of the principal planes, its projection on that plane reveals the true length and its other projection is parallel to the reference line. When a line is inclined to both the principal planes, none of the projections show its true length or are parallel to the reference line. So, to determine the true length of a line, an auxiliary plane is placed parallel to the line and its projection on that plane shows the true length of the line. This is called the normal view of a line.

If a line is perpendicular to any plane, its projection on that plane is a point and its projection on the other plane, which is perpendicular to the former plane, shows the true length. In other words, the projection of a line on a plane, which is perpendicular to the actual line, will be a point.

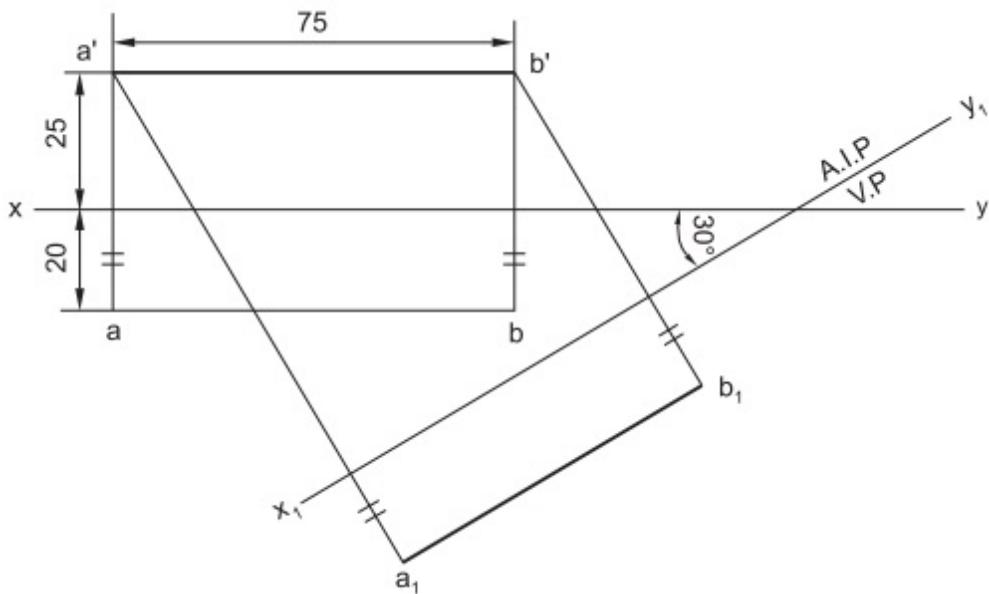
The auxiliary projection is a powerful tool; the advantages of which can only be appreciated in projections of solids and sections of solids. In this chapter, the principles and the just applications of it with respect to points, lines and planes are dealt; the knowledge of which is required later.

## 10.4.1 Projections of a Straight Line, Inclined to H.P and Parallel to V.P

**Problem 3** A straight line AB of 75 length, is inclined at  $30^\circ$  to H.P. The end A of the line is 25 above H.P and 20 in front of V.P. Draw the projections, by auxiliary plane method.

### Construction ([Fig.10.3](#))

1. Draw the projections  $a'$   $b'$  and  $ab$  of the line, assuming it to be parallel to both H.P and V.P and satisfying the distances of the point A from H.P and V.P.
  2. Draw the reference line  $x_1 y_1$ , corresponding to an A.I.P, making the angle of  $30^\circ$  with  $xy$ .
  3. Project the auxiliary top view  $a_1 b_1$  (refer Construction: [Fig.10.2c](#)).
- $a'b'$  and  $a_1 b_1$  are the required projections.

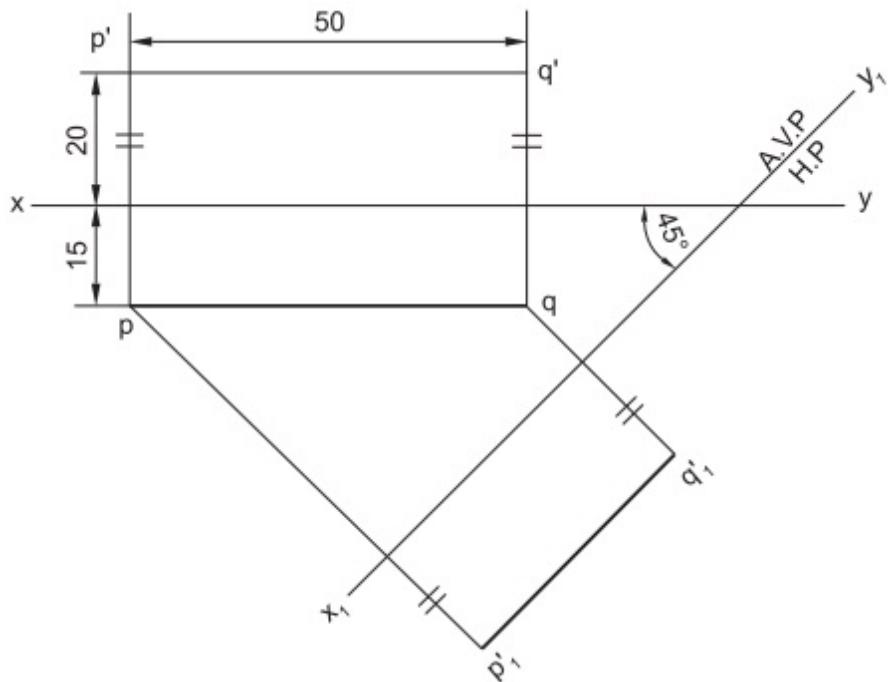


**Fig.10.3**

## 10.4.2 Projections of a Straight Line, Inclined to V.P and Parallel to H.P

**Problem 4** A straight line  $PQ$  of 50 length, is inclined at  $45^\circ$  to V.P. The end  $P$  of the line is 20 above H.P and 15 in front of V.P. Draw the projections, by auxiliary plane method.

**Construction (Fig.10.4)**



**Fig.10.4**

1. Draw the projections,  $p'q'$  and  $pq$  of the line, assuming it to be parallel to both H.P and V.P and satisfying the distances of point  $P$  from both the planes.
2. Draw the reference line  $x_1y_1$ , corresponding to an A.V.P, making the angle of  $45^\circ$  with  $xy$ .
3. Project the auxiliary front view  $p_1q_1'$  (refer Construction: Fig.10.1c).

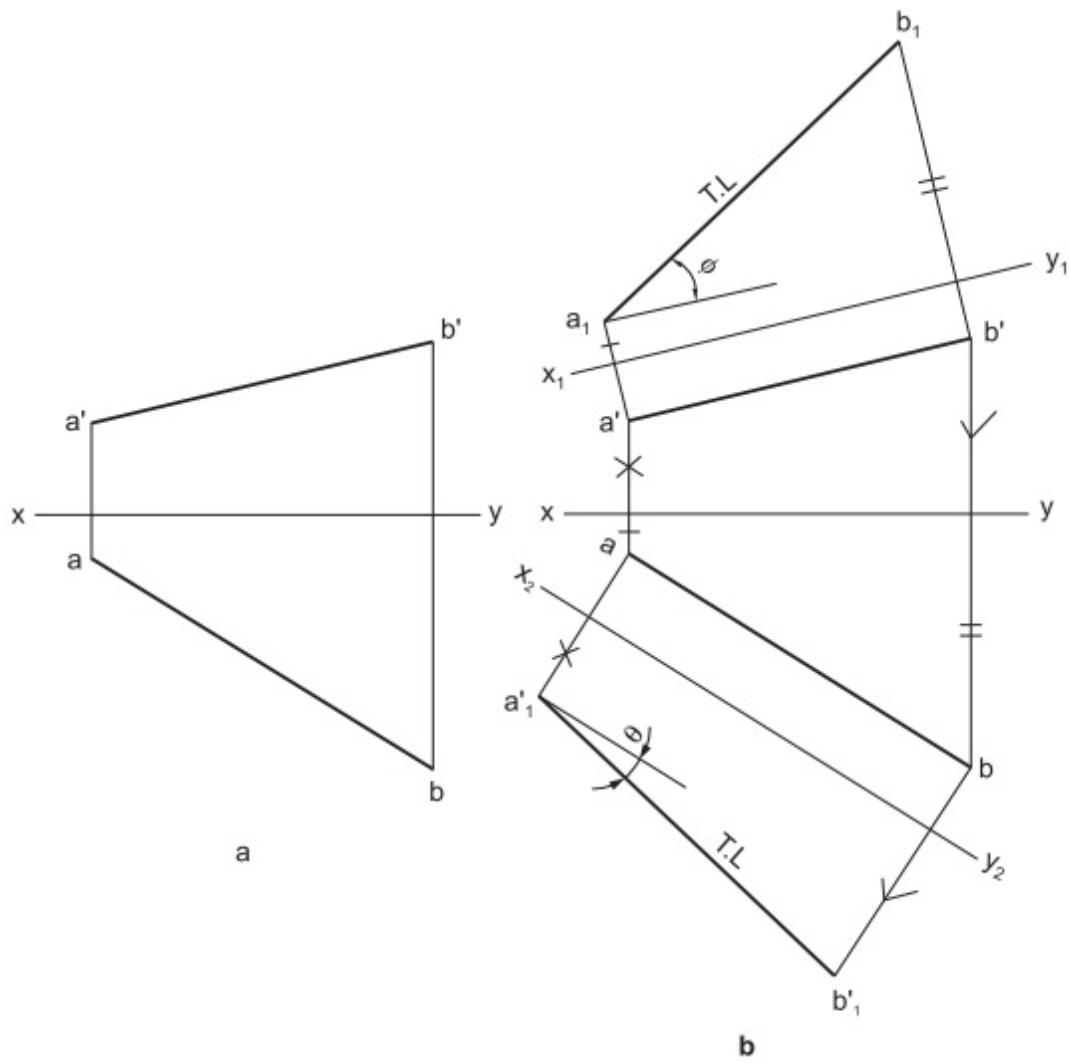
$p_1q_1$  and  $pq$  are the required projections.

### 10.4.3 True Length and True Inclinations

**Problem 5** *Figure 10.5a shows the projections of a straight line, inclined to both H.P and V.P. Determine the true length of the line and its true inclinations with H.P and V.P.*

The true length of a line and its inclinations with the planes of projection may be determined by making each of its projections, parallel to the reference line. For this, a new reference line, representing an auxiliary plane may be drawn, parallel to the projection concerned. The projection on this plane will reveal the true length and true inclination of the line with the other principal plane.

**Construction (Fig.10.5b)**



**Fig.10.5**

1. Draw the projections of the line.
2. Draw a reference line  $x_1 y_1$ , representing an A.I.P, parallel to  $a'b'$ .
3. Project the auxiliary top view  $a_1 b_1$ , which is the true length of the given line and its inclination with  $x_1 y_1$  is the true inclination  $\phi$  of the line with V.P.
4. Draw a reference line  $x_2 y_2$ , representing an A.V.P, parallel to  $ab$ .

5. Project the auxiliary front view  $a_1' b_1'$ , which is also the true length of the given line and its inclination  $\theta$  with  $x_2 y_2$  is the true inclination of the line with H.P.

#### 10.4.4 Successive Auxiliary Views

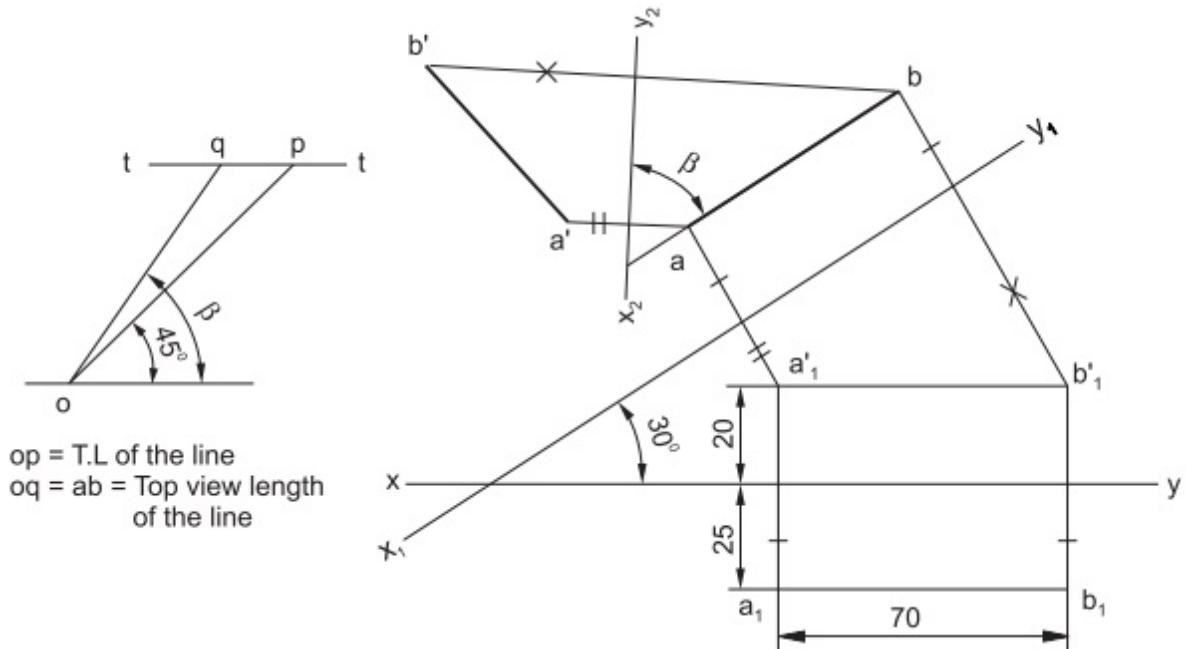
Successive auxiliary views are used to determine the true size and shape of the surfaces, inclined to the principal planes of projection. The auxiliary views projected on to either an A.I.P or A.V.P in the Ist stage are termed the primary auxiliary views. An auxiliary view obtained in the II stage is known as the second auxiliary view and it is adjacent to the primary auxiliary view. Thus, any number of successive auxiliary views may be obtained, to get the required projections.

The application of successive auxiliary views is found in the following articles:

#### 10.4.5 Projections of a Straight Line, Inclined to Both H.P and V.P

**Problem 6** A line AB of 70 length, has its end A, at 20 above the H.P and 25 in front of V.P. The line is inclined at  $30^\circ$  to H.P and  $45^\circ$  to V.P. Draw the projections, by the auxiliary plane method.

**Construction (Fig.10.6)**

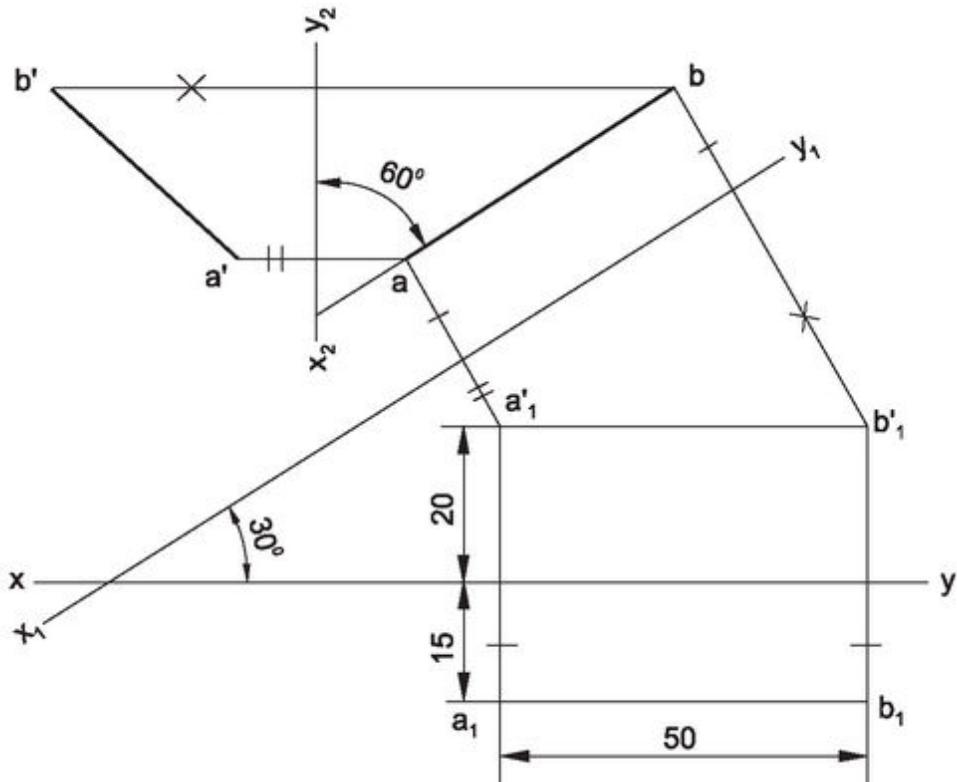


**Fig.10.6**

1. Draw the projections  $a'_1 b'_1$  and  $a_1 b_1$  of the line, assuming it to be parallel to both H.P and V.P and satisfying the distances of point A from H.P and V.P.
2. Draw the reference line  $x_1 y_1$ , corresponding to an A.I.P, making the angle of  $30^\circ$  with  $xy$ .
3. Project the auxiliary top view  $ab$ .
4. Draw the reference line  $x_2 y_2$ , corresponding to an A.V.P, making the apparent angle  $\beta$  with  $ab$  (refer construction for the method of determining  $\beta$ ).
5. Project the auxiliary front view  $a'b'$ .  
 $a'b'$  and  $ab$  are the required projections.

**Problem 7** A straight line AB of 50 length, is inclined at  $30^\circ$  to H.P and its top view makes an angle of  $60^\circ$  with  $xy$ . The end A of the line is 20 above H.P and 15 in front of V.P. Draw the projections of the line, by auxiliary plane method.

### **Construction (Fig.10.7)**



**Fig.10.7**

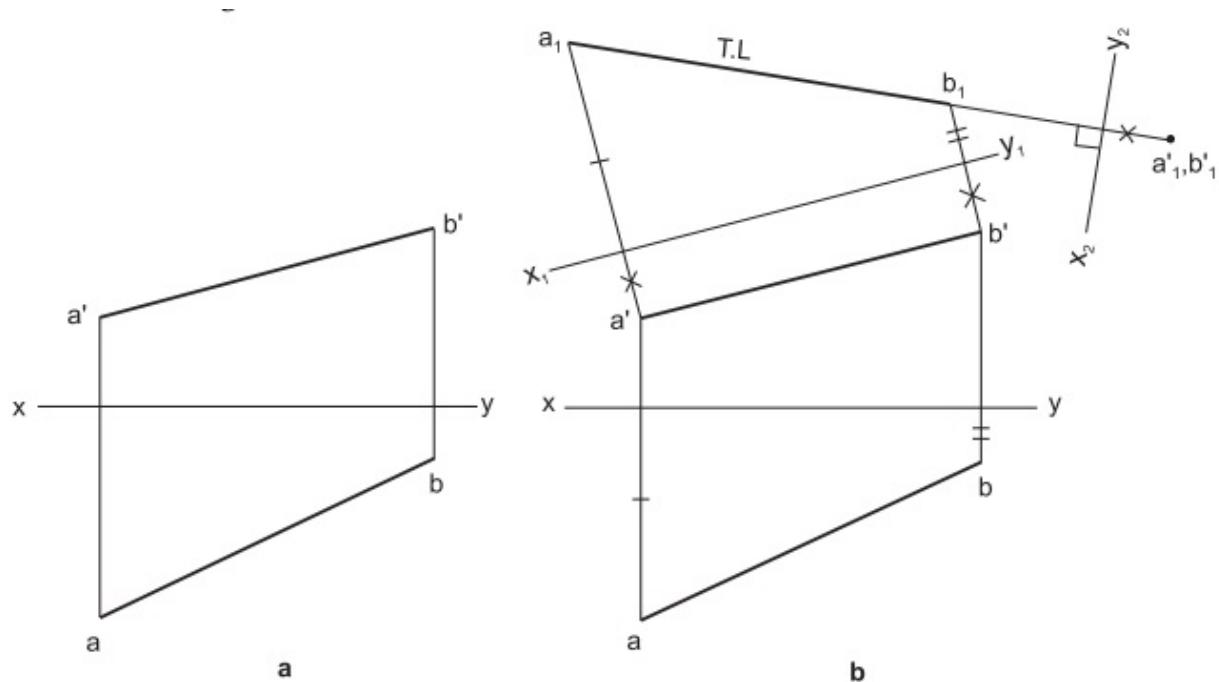
1. Draw the projections a<sub>1</sub>' b<sub>1</sub>' and a<sub>1</sub>b<sub>1</sub> of the line, assuming it to be parallel to both H.P and V.P and satisfying the given conditions.
  2. Draw the reference line x<sub>1</sub>y<sub>1</sub>, corresponding to an A.I.P, making the angle of 30° with xy.
  3. Project the auxiliary top view ab.
  4. Draw the reference line x<sub>2</sub>y<sub>2</sub>, corresponding to an A.V.P, making the angle of 60° with ab.
  5. Project the auxiliary front view a'b'.
- a'b' and ab are the required projections.

### 10.4.5.1 Point or Edge View of a Line

**Problem 8** *Figure 10.8a shows the projections of a straight line, inclined to both H.P and V.P. Determine the point or edge view of the line.*

It is observed that when a line is perpendicular to a principal plane, its projection on that plane is a point, while its projection on the other plane shows its true length. Thus, it may be stated that if an auxiliary plane is chosen perpendicular to the view of a line, representing its true length, then the auxiliary projection will be a point view.

**Construction (Fig.10.8b)**



**Fig.10.8**

1. Draw the given projections a'b' and ab of the line.
2. Draw the reference line x<sub>1</sub>y<sub>1</sub>, corresponding to any of the auxiliary planes, say an A.I.P, parallel to a'b'.

3. Project the auxiliary top view  $a_1 b_1$ ; representing the true length of the line.
4. Draw another reference line  $x_2 y_2$ , perpendicular to  $a_1 b_1$ , corresponding to the second auxiliary plane.
5. Project the second auxiliary view, which is the point or edge view of the given line.

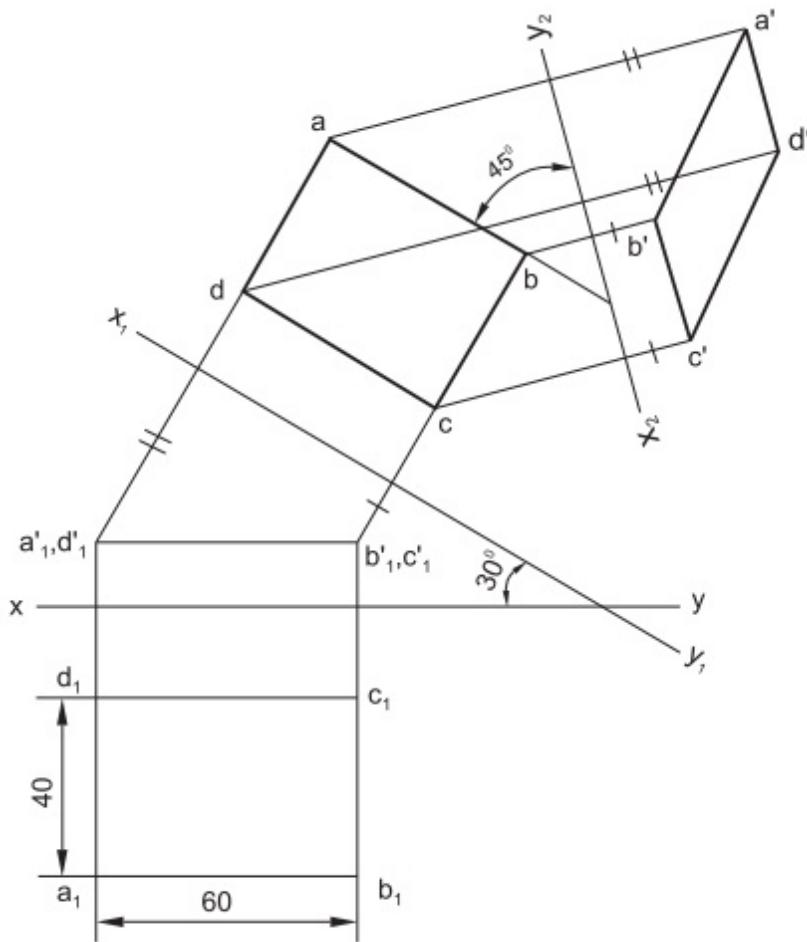
## 10.5 PROJECTIONS OF PLANES

Auxiliary projections of planes are used to obtain (i) the edge view of a plane and (ii) the true shape of a plane, in addition to the conventional projections.

When a plane is inclined to both the principal planes, none of its projections give the edge view or the true shape. The true shape (normal view) of the plane is obtained after obtaining its edge view. The edge view of a plane is (straight line) obtained by looking in the direction of the slope of the plane. Hence, the slope of the plane has to be determined first by considering an element parallel to  $xy$  in one of the projections. This element, when projected in the other view, gives its true length as well as the slope of the plane. By considering an auxiliary plane perpendicular to this element and projecting on to it, the edge view of the plane is obtained. The true shape of the plane is obtained by projecting the edge view on an auxiliary plane parallel to it.

**Problem 9** A rectangular plane  $ABCD$  of size  $60 \times 40$ , is inclined to H.P by an angle of  $30^\circ$ ; the longer edge of which is making an angle of  $45^\circ$  with V.P. Draw the projections, by auxiliary plane method.

**Construction (Fig.10.9)**

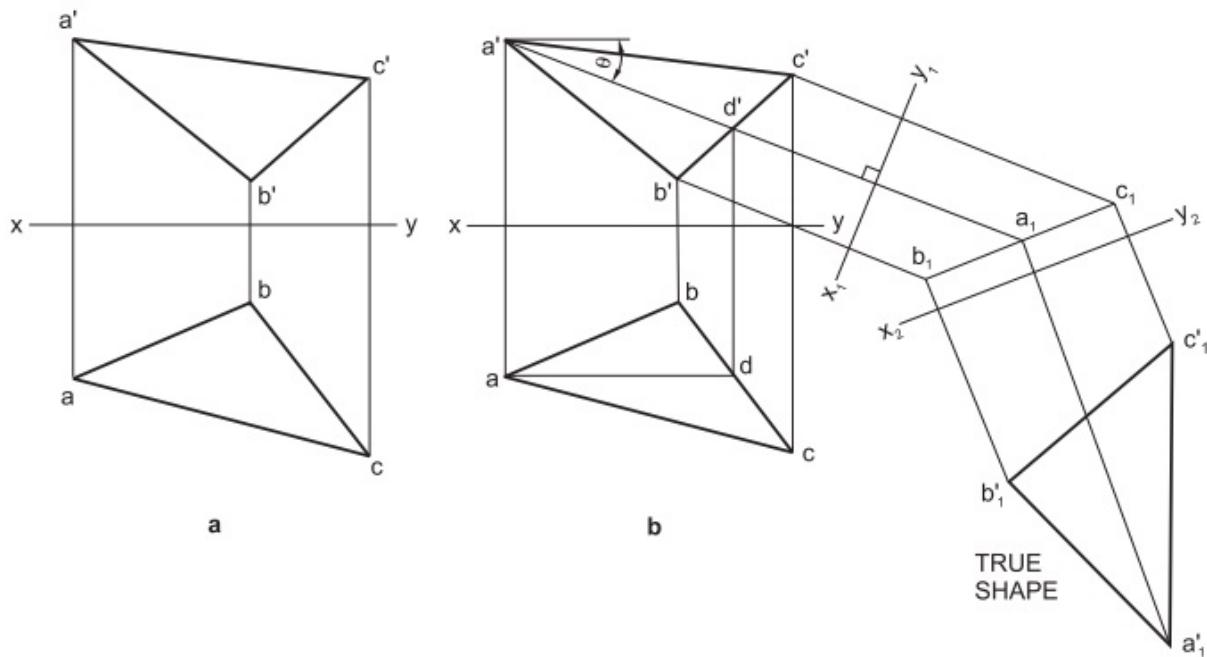


**Fig.10.9**

1. Draw the projections of the plane  $a'_1 (d'_1)$   $b'_1 (c'_1)$  and  $a_1 b_1 c_1 d_1$ , assuming it to be parallel to H.P, with its longer edge parallel to V.P.
2. Draw the reference line  $x_1 y_1$ , corresponding to an A.I.P, making the angle  $30^\circ$  with  $xy$ .
3. Project and obtain the auxiliary top view  $abcd$ .
4. Draw the reference line  $x_2 y_2$ , corresponding to an A.V.P, making the angle  $45^\circ$  with  $ab$  (longer edge).
5. Project the auxiliary front view  $a'b'c'd'$ .  
 $a'b'c'd'$  and  $abcd$  are the required projections.

**Problem 10** *Figure 10.10a shows the projections of a triangular plane, inclined to both H.P and V.P. Determine the edge and normal views of the plane.*

**Construction (Fig.10.10)**



**Fig.10.10**

1. Draw the given projections  $a'b'c'$  and  $abc$  of the plane.
2. Select an element  $ad$  in the top view, parallel to  $xy$ .
3. Obtain the front view  $a'd'$ , the true length of the line  $AD$  (angle  $\theta$  represents the inclination of the plane with H.P).
4. Draw a reference line  $x_1 y_1$ , perpendicular to  $a'd'$ .
5. Obtain the edge view  $a_1 b_1 c_1$  of the plane, by projection.
6. Draw a reference line  $x_2 y_2$ , parallel to  $a_1 b_1 c_1$ .
7. Project and obtain the normal view  $a_1' b_1' c_1'$ ; representing the true shape of the plane.



The above construction may also be used to determine the angle between two intersecting lines AB and AC; given their projections.

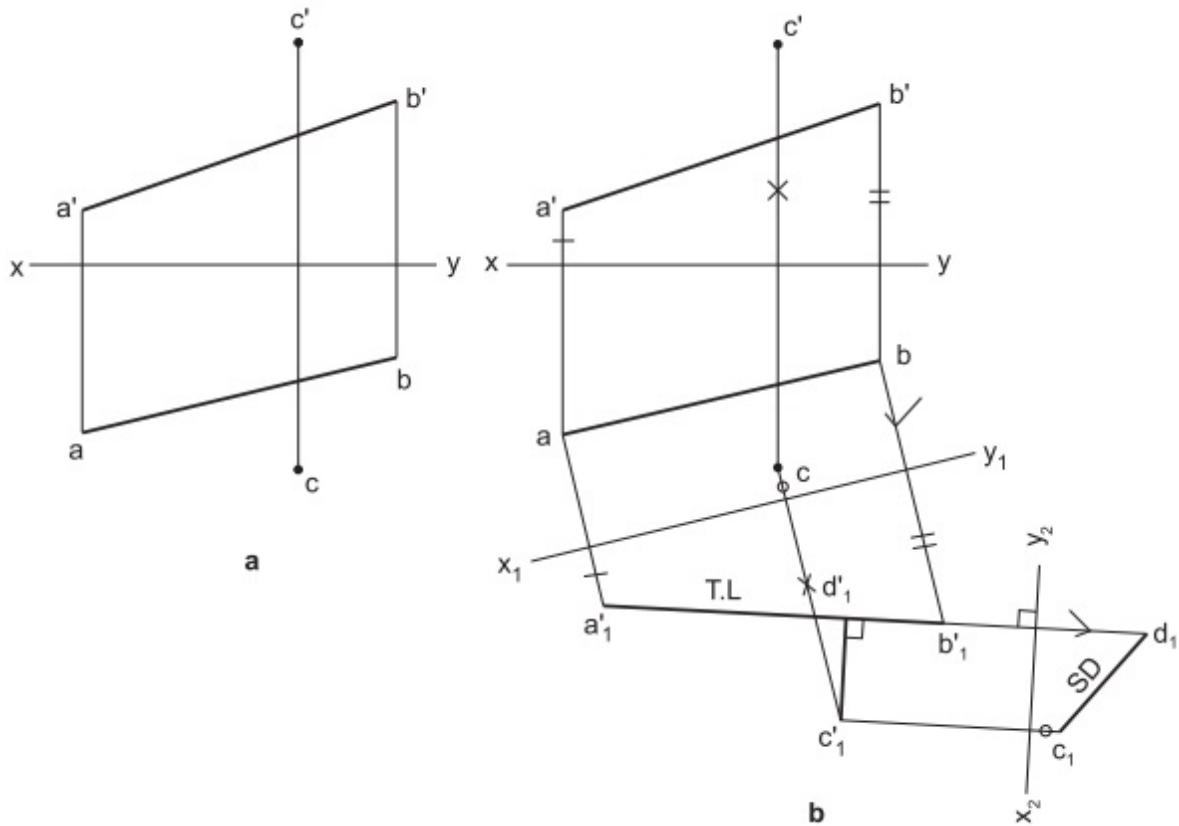
## 10.6 SPECIAL APPLICATIONS OF AUXILIARY PROJECTIONS

### 10.6.1 Distance From a Point to a Line

It is the shortest distance between the point and the line. It is measured along a perpendicular, from the point to the line.

**Problem 11** *Figure 10.11a shows the projections of a point C and the line AB. Determine the shortest distance from the point C to AB.*

**Construction (Fig.10.11b)**



**Fig.10.11**

1. Draw the given projections of the line AB and the point C.
2. Draw the reference line  $x_1 y_1$ , parallel to, say the top view of the line.
3. Project the auxiliary front views of the line  $a'_1 b'_1$  and the point  $c'_1$ . The length of the line  $a'_1 b'_1$  represents the true length of the line.
4. Through  $c'_1$ , drop a perpendicular  $c'_1 d'_1$  to  $a'_1 b'_1$ .
5. Draw the reference line  $x_2 y_2$ , parallel to  $c'_1 d'_1$ .
6. Project the normal view  $c_1 d_1$  of the line CD.

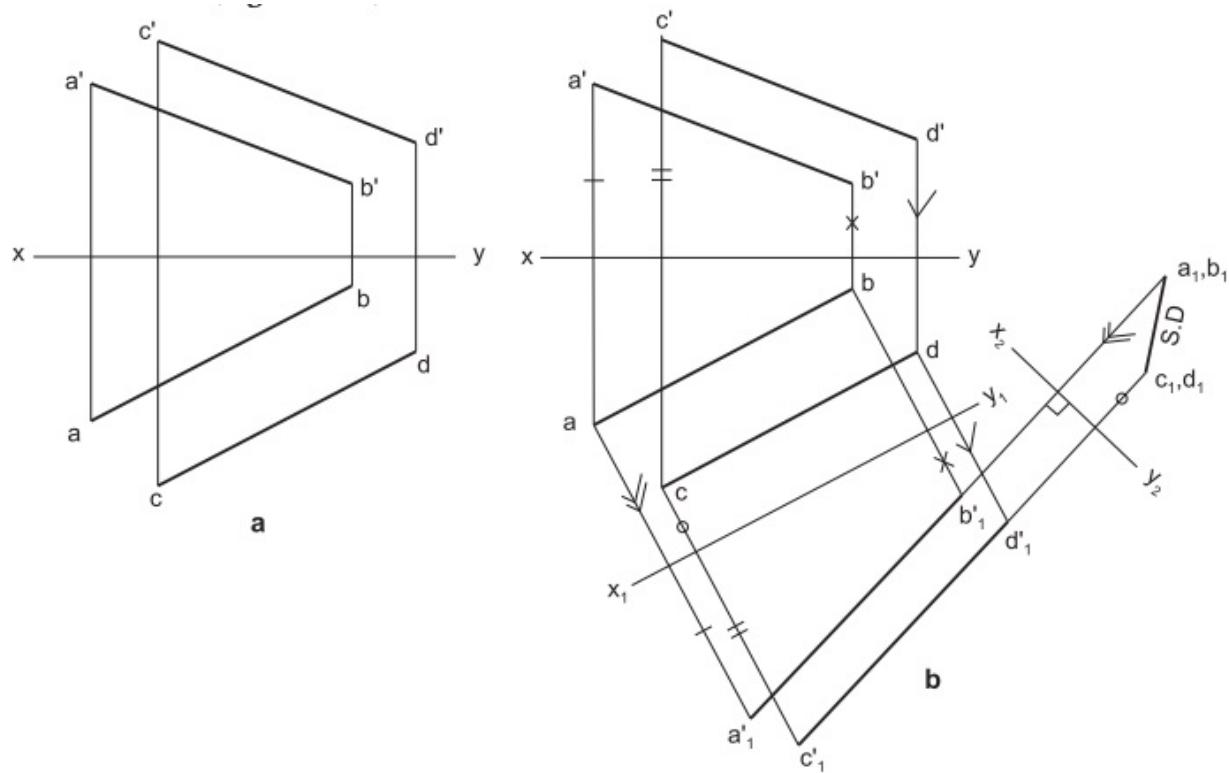
$c_1 d_1$  is the shortest distance (S.D) of the point C from the line AB.

## 10.6.2 Shortest Distance Between two Parallel Lines

Lines which have a common direction or inclination with the principal planes of projection are said to be parallel. Parallel lines appear parallel in all projections. Two views are required to determine the parallelism between the lines. The shortest distance between two parallel lines may be determined from their edge views.

**Problem 12** *Figure 10.12a shows the projections of two parallel lines AB and CD. Determine the shortest distance between the lines.*

**Construction (Fig.10.12b)**



**Fig.10.12**

1. Draw the given projections of the parallel lines AB and CD.
2. Draw the reference line  $x_1 y_1$ , parallel to, say the top views of the lines.
3. Project the normal views of the lines,  $a_1' b_1'$  and  $c_1' d_1'$ .
4. Draw the reference line  $x_2 y_2$ , perpendicular to the above normal views.
5. Project the edge views of the lines  $a_1, b_1$  and  $c_1, d_1$ .

The distance between the edge views is the required shortest distance between the lines.



If both the lines are contained by a single profile plane and are parallel, the distance between the lines, in the side view, represents the shortest distance.

### 10.6.3 Shortest Distance Between two Oblique (Skew) Lines

**Problem 13** *Figure 10.13a shows the projections of two lines AB and CD, which are inclined to both H.P and V.P. Determine the shortest distance between the lines.*

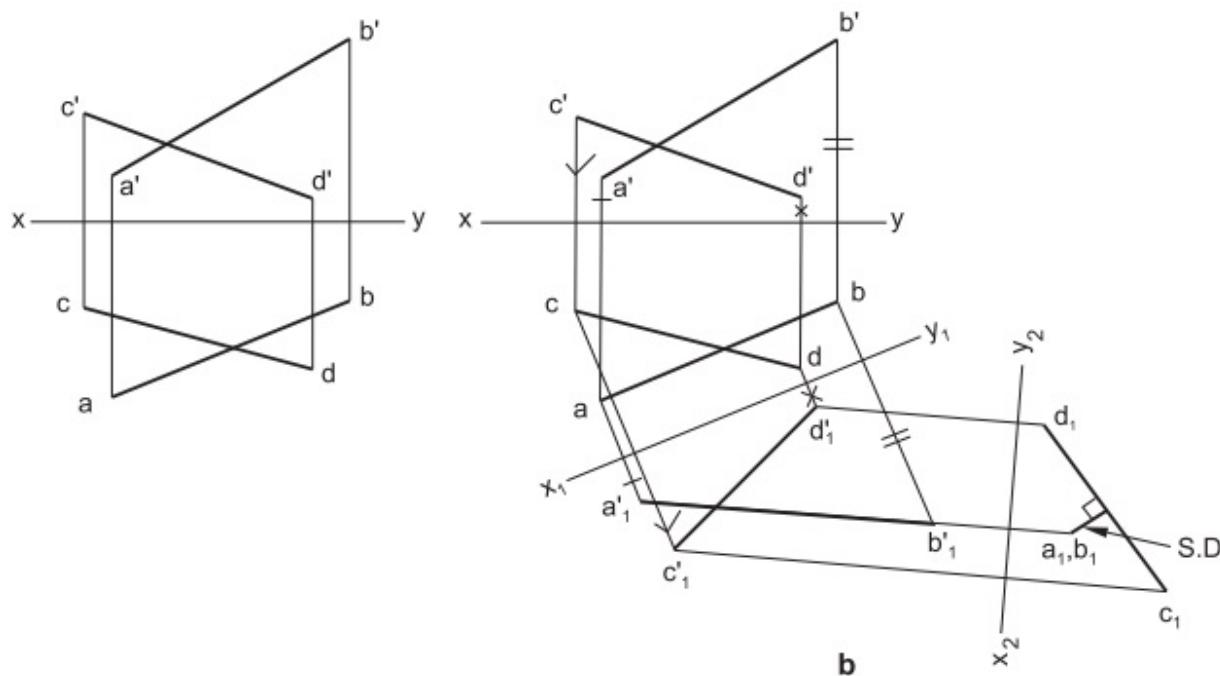
#### **Construction (Fig.10.13b)**

1. Draw the given projections of the lines AB and CD.
2. Draw the reference line  $x_1 y_1$ , say parallel to ab at any convenient distance from it.
3. Project the auxiliary front views (primary auxiliary views) of the lines,  $a_1' b_1'$  and  $c_1' d_1'$ .  $a_1' b_1'$  is the

normal view of the line AB; representing the true length of it.

4. Draw a reference line  $x_2 y_2$ , perpendicular to  $a'_1 b'_1$ , at any convenient distance from it.
5. Obtain the second auxiliary views  $a_1 (b_1)$  and  $c_1 d_1$ . The point  $a_1 (b_1)$  represents the edge view of the line AB.

The perpendicular distance from  $a_1 (b_1)$  to the line  $c_1 d_1$ , is the required shortest distance.



**Fig.10.13**

#### 10.6.4 Shortest Distance from a Point to a Plane

**Problem 14** *Figure 10.14a shows the projections of a plane ABC and a point P. Determine the shortest distance from*

*the point P to the plane ABC.*

**Construction (Fig.10.14b)**

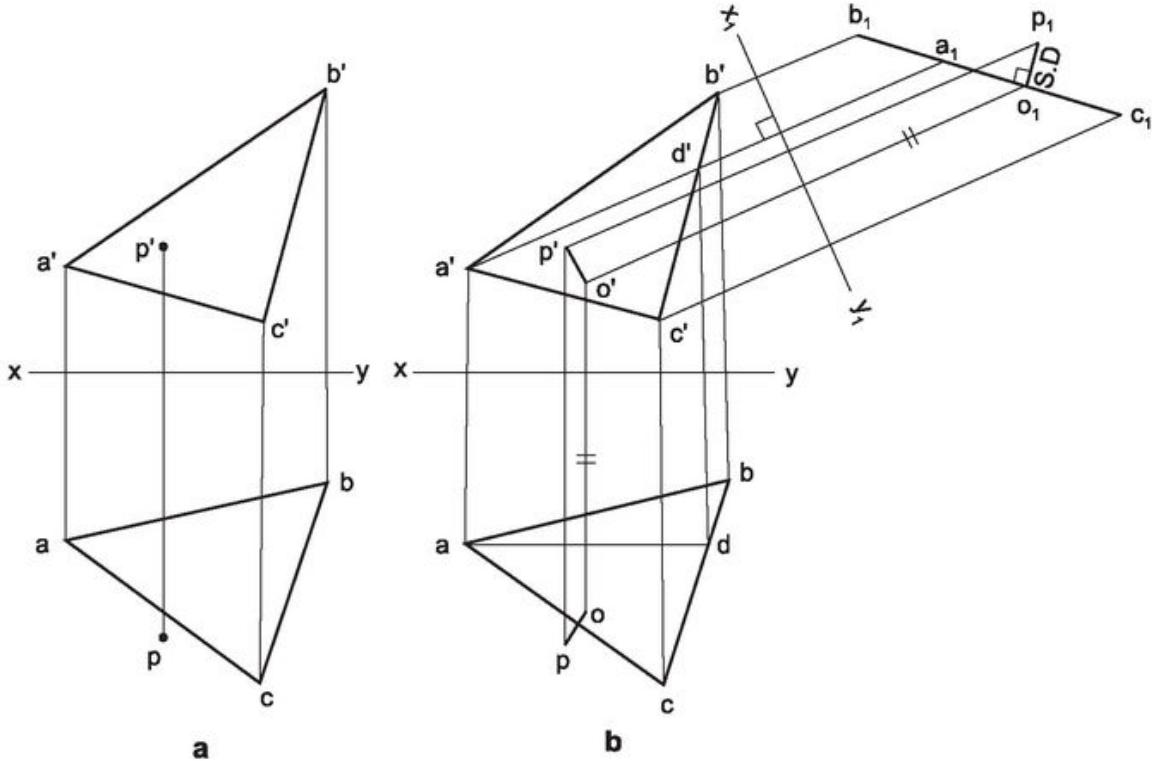
1. Draw the given projections of the plane ABC and the point P.
2. Obtain the edge view of the plane  $a_1 b_1 c_1$  and locate  $p_1$  with respect to it (refer Construction: Fig.10.11).
3. Draw a perpendicular from  $p_1$  to  $a_1 b_1 c_1$ , intersecting it at  $o_1$ .

The length  $p_1 o_1$  is the required shortest distance.

*To show the line PO with respect to the given projections of the plane:*

1. Draw a line through  $p'$ , parallel to  $x_1 y_1$ .
2. Through  $o_1$ , draw a line perpendicular to  $x_1 y_1$ , meeting the above line at  $o'$ .
3. Join  $p', o'$ .
4. Draw a projector through  $o'$ , perpendicular to  $xy$ .
5. Mark the point  $o$  on it such that, its distance from  $xy$  is the same as the distance of  $o_1$  from  $x_1 y_1$ .
6. Join  $p, o$ .

$p' o'$  and  $po$  are the projections of the line  $PO$ .

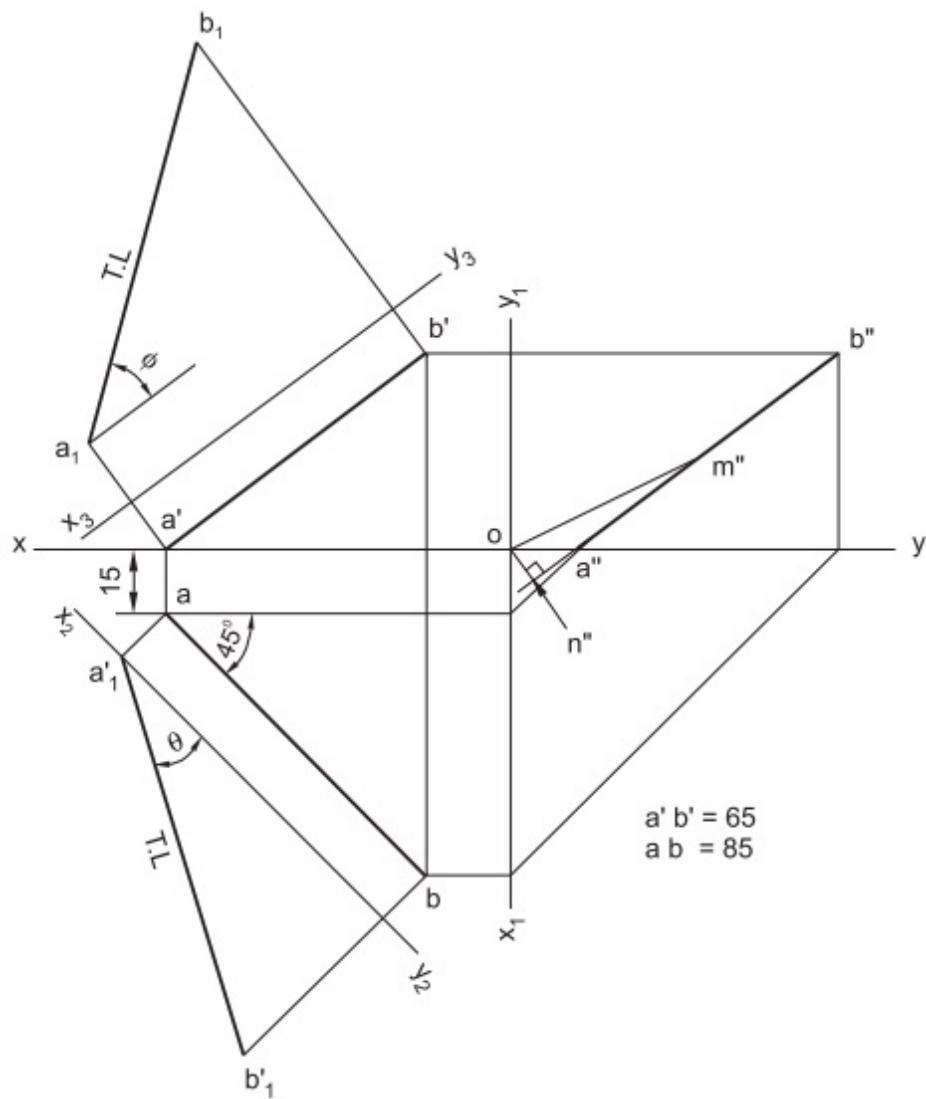


**Fig.10.14**

## 10.7 EXAMPLES

**Problem 15** *The top view of a line AB, inclined at  $45^\circ$  to xy, measures 85, while the length of the front view is 65. Its one end A is on H.P and 15 in front of V.P. Draw the projections of AB and determine its true length, inclinations with H.P and V.P. Find the distance of the mid-point of AB from xy and also the shortest distance of the line from xy.*

**Construction (Fig.10.15)**



**Fig.10.15**

1. Locate the projections a' and a of the point A, with respect to xy.
2. Draw the top view of the line ab, making 45° with xy and 85 long.
3. Locate b' on the projector through b and at 65 from a'. a' b' and ab are the projections of AB.
4. Draw a reference line x<sub>1</sub>y<sub>1</sub>, perpendicular to xy and intersecting it at o and obtain the side view a'' b'' of the

line, by projection.

5. Locate  $m''$ , the mid-point of  $a'' b''$  and draw the line  $n' o'$ , perpendicular to  $a'' b''$ .

$m''o'$  is the distance of the mid-point of AB from xy and  $n''o'$  is the shortest distance of the line from xy.

6. Draw a reference line  $x_2 y_2$ , parallel to the top view and obtain the auxiliary front view  $a_1 b_1$ .

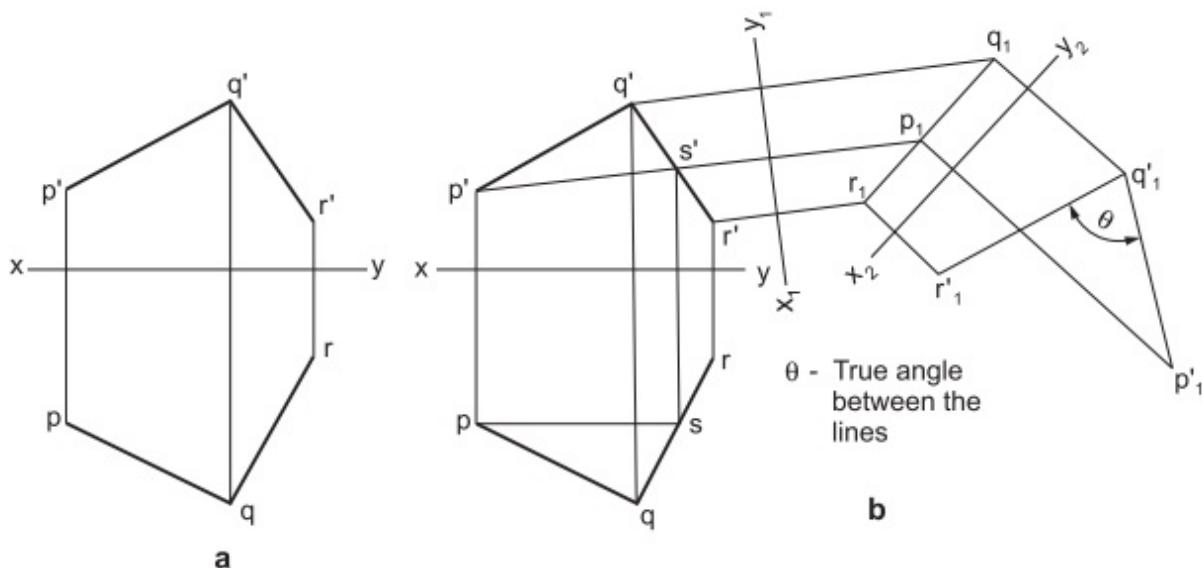
$a_1' b_1'$  is the true length of the line and its inclination  $\theta$  with  $x_2 y_2$  is the true angle of inclination with H.P.

7. Draw a reference line  $x_3 y_3$ , parallel to the front view and obtain the auxiliary top view  $a_1 b_1$ , by projection.

$a_1 b_1$  is the true length of the line and its inclination  $\phi$  with  $x_3 y_3$  is the true angle of inclination with V.P.

**Problem 16** *Figure 10.16a shows the projections of two intersecting lines. Determine the true angle between the lines.*

### **Construction (Fig.10.16b)**



**Fig.10.16**

1. Draw the projections of the given intersecting lines.
2. Select an element ps in the top view, parallel to xy.
3. Obtain the front view  $p's'$ , the true length of the element PS.
4. Draw a reference line  $x_1y_1$ , perpendicular to  $p's'$ .
5. Obtain the edge view  $p_1q_1r_1$  of the lines, by projection.
6. Draw a reference line  $x_2, y_2$ , parallel to  $p_1q_1r_1$ .
7. Project and obtain the normal views  $p_1' q_1'$ ,  $q_1' r_1'$  of the lines.

The angle  $\theta$  between the lines  $P_1'q_1'$  and  $q_1'r_1'$  is the true angle between the lines.

**Problem 17** Determine the shortest distance between the non-intersecting diagonals of any two adjacent faces of a cube of 50 edge.

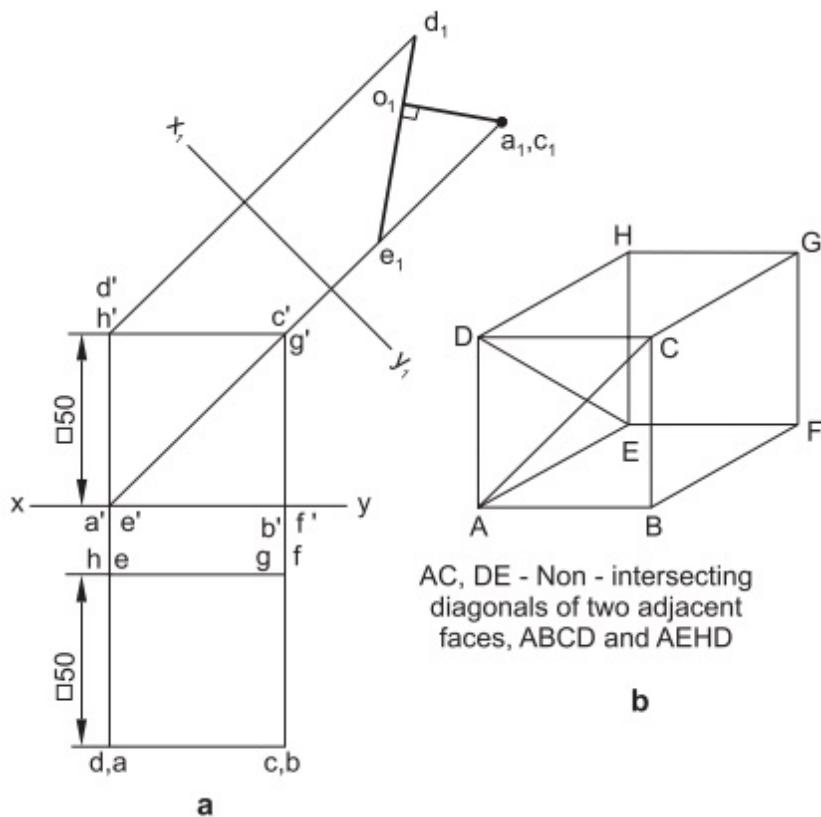
### **Construction ([Fig.10.17](#))**



[Figure 10.17](#) shows a cube with the corners marked. The adjacent faces considered are ABCD and ADHE. The non-intersecting diagonals considered on these two faces are: AC and DE.

1. Draw the projections of the cube.
2. Draw the reference line  $x_1y_1$ , perpendicular to  $a'c'$ , representing the true length of the diagonal AC. (This is because the top view, ac is parallel to xy).
3. Obtain the auxiliary top view of the diagonals AC and DE, by projection.
4. Drop a perpendicular from  $a_1(c_1)$  to the line  $d_1e_1$ , meeting it at  $o_1$ .

$o_1 a_1(c_1)$  is the shortest distance between the diagonals AC and DE.



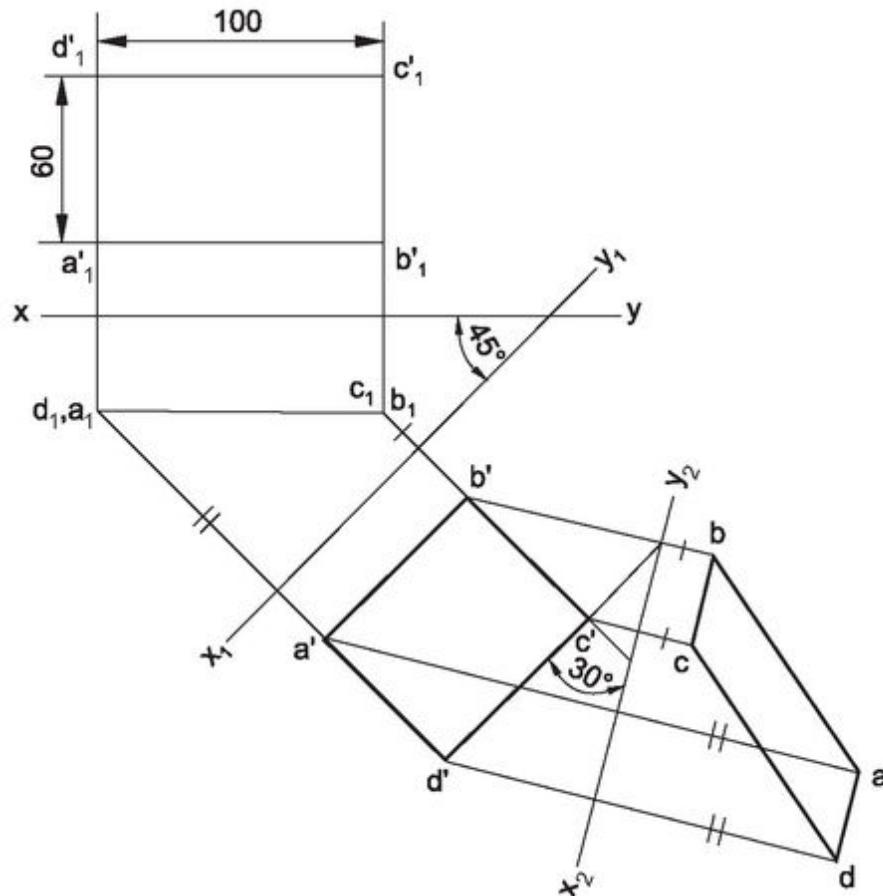
**Fig.10.17**

**Problem 18** A rectangular plane ABCD of size  $100 \times 60$ , is inclined to V.P by an angle of  $45^\circ$ ; longer edge of which is making an angle of  $30^\circ$  with H.P. Draw the projections, by auxiliary plane method.

**Construction (Fig.10.18)**

1. Draw the projections of the plane  $a_1'b_1'c_1'd_1'$  and  $a_1(d_1)b_1(c_1)$ , assuming it to be parallel to V.P, with its longer edge parallel to H.P.
2. Draw the reference line  $x_1 y_1$ , corresponding to an A.V.P making the angle  $45^\circ$  with  $xy$ .
3. Project and obtain the auxiliary front view  $a'b'c'd'$ .

4. Draw the reference line  $x_2y_2$ , corresponding to an A.I.P, making the angle  $30^\circ$  with  $c'd'$  (longer edge).
5. Project and obtain the auxiliary top view  $abcd$ .  
 $a'b'c'd'$  and  $abcd$  are the required projections.

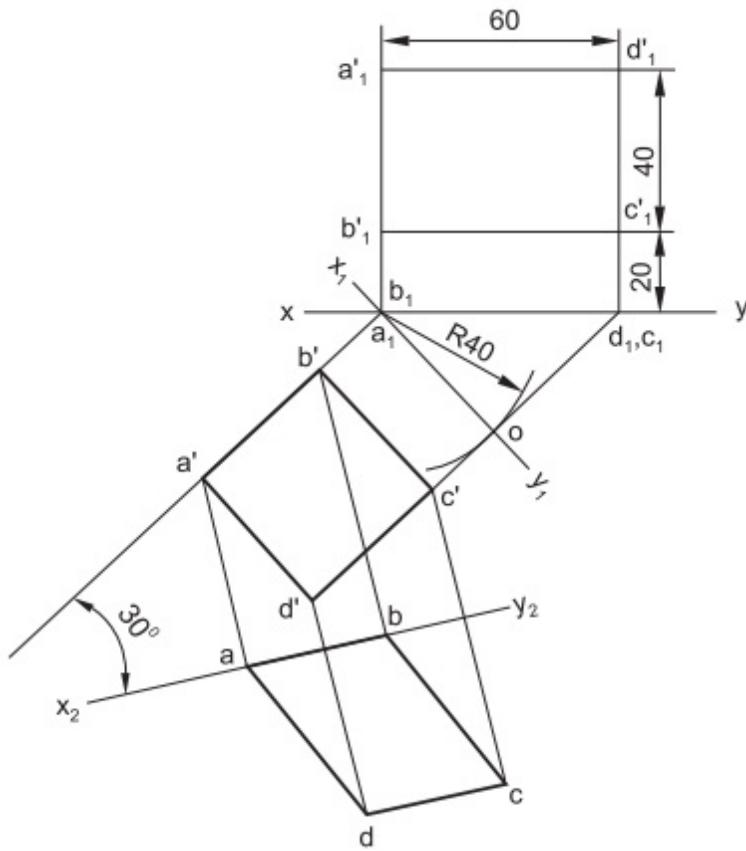


**Fig.10.18**

**Problem 19** A thin rectangular plate  $ABCD$  of size  $60 \times 40$ , has its smaller edge on V.P with its lower corner, 20 above H.P. The front view of the plate is a square of 40 side. The smaller edge of the plate, lying on V.P is inclined at  $30^\circ$  to H.P. Draw the projections.

**Construction (Fig.10.19)**

1. Draw the projections of the plate, assuming it to be lying on V.P with the smaller edge AB, perpendicular to H.P.
2. With  $a_1(b_1)$  as centre and radius 40, draw an arc.
3. Draw a line through  $d_1(c_1)$ , tangential to the above arc, touching it at o.
4. Join  $a_1(b_1)$ , o and extend forming the reference line  $x_1y_1$ .
5. Obtain the auxiliary front view  $a'b'c'd'$  which is a square of 40 side, by projection. This is the final front view.
6. Draw a reference line  $x_2y_2$ , at the angle of  $30^\circ$  with  $a'b'$ , the auxiliary front view of the smaller edge AB of the plate.
7. Obtain the (auxiliary) final top view abcd, by projection.



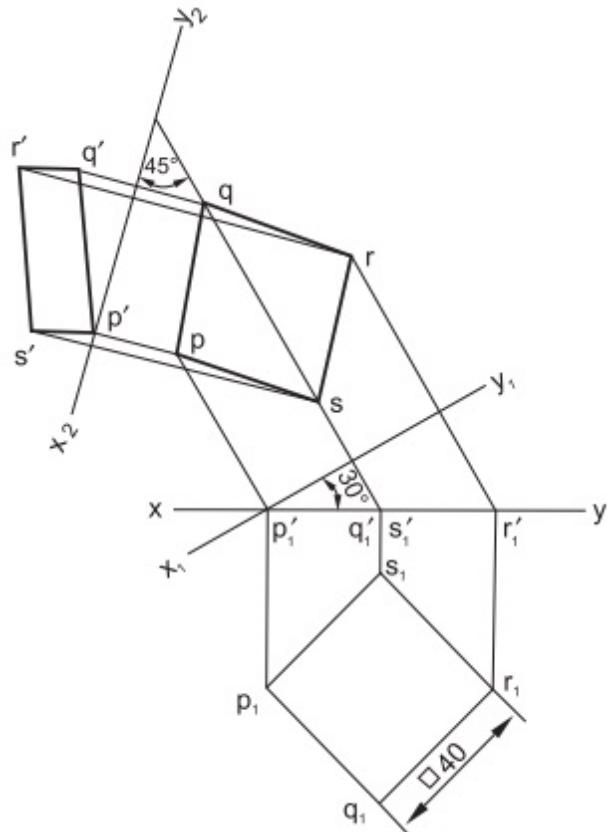
**Fig.10.19**

**Problem 20** A square  $PQRS$  of 40 side has its corner  $P$  in the H.P, its diagonal  $PR$  inclined at  $30^\circ$  to H.P and the diagonal  $QS$  inclined at  $45^\circ$  to V.P and parallel to the H.P. Draw its projections by auxiliary plane method.

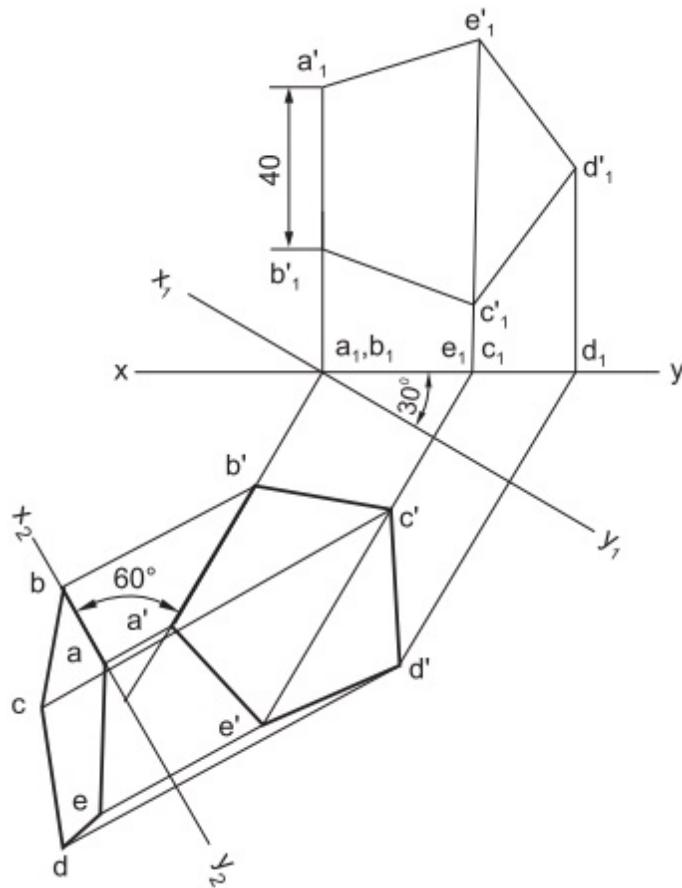
**Construction (Fig.10.20)**

1. Draw the projections of the square, assuming it to be lying on H.P and the sides equally inclined to V.P.
2. Draw a reference line  $x_1y_1$ , making the angle of  $30^\circ$  with  $xy$  (inclination of the diagonal  $PR$  with H.P) and passing through  $p_1'$ .
3. Obtain the auxiliary top view  $pqr_s$ , by projection.

4. Draw a reference line  $x_2y_2$ , at the angle of  $45^\circ$  with  $qs$  (the inclination of the diagonal  $QS$  with V.P.)
  5. Obtain the auxiliary front view  $p'q'r's'$ , by projection.
- $p'q'r's'$  and  $pqrss'$  are the required projections



**Fig.10.20**



**Fig.10.21**

**Problem 21** Draw the projections of a regular pentagon of 40 side, having its surface inclined at  $30^\circ$  to V.P and the side on which it rests on V.P, making an angle of  $60^\circ$  with H.P.

**Construction (Fig.10.21)**

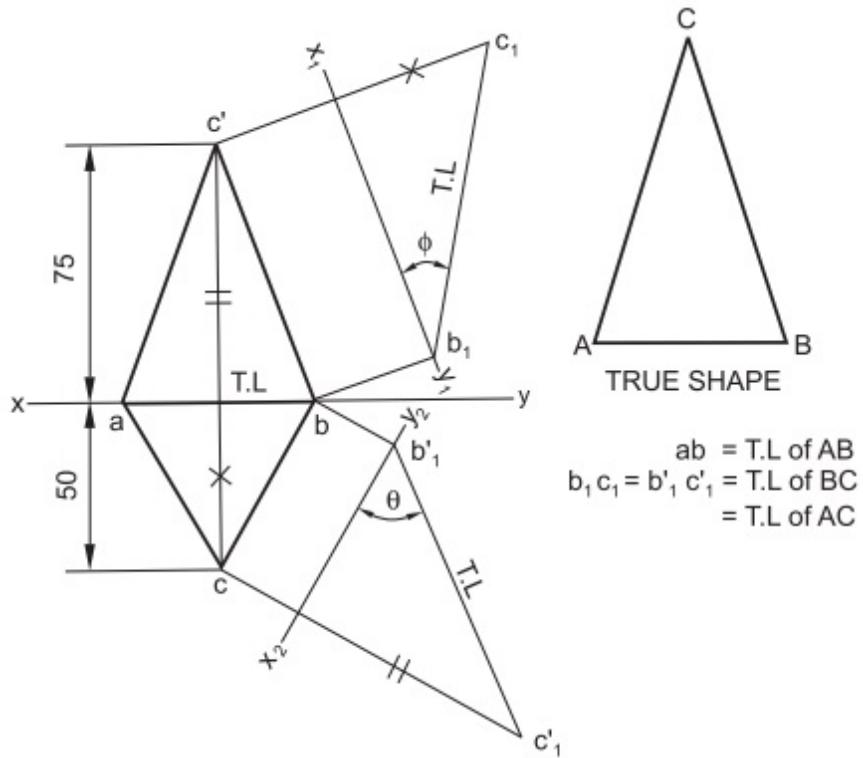
1. Draw the projections of the pentagon, assuming it to be lying on V.P. with the edge AB perpendicular to H.P.
2. Draw a reference line  $x_1 y_1$ , making the angle of  $30^\circ$  with  $xy$  and passing through  $a_1$  ( $b_1$ ).
3. Obtain the auxiliary front view  $a' b' c' d' e'$ , by projection.

4. Draw a reference line  $x_2 y_2$ , at the angle of  $60^\circ$  with  $a'b'$ .
5. Obtain the auxiliary top view  $abcde$ , by projection.  
 $a'b'c'd'e'$  and  $abcde$  are the required projections.

**Problem 22** *abc is an equilateral triangle of altitude 50, with ab in xy and c below it. abc' is an isosceles triangle of altitude 75 and c' above xy. Determine the true shape of the triangle ABC of which abc is the top view and abc' is the front view. Also, find the inclinations of BC with the principal planes.*

### **Construction (Fig.10.22)**

1. Draw the projections  $abc$  and  $abc'$  of the triangle ABC.  
The length of the view  $ab$  represents the true length of the edge AB, as its front and top views coincide with  $xy$ .
2. Draw a reference line  $x_1y_1$ , parallel to  $c'b'$ ; representing an A.I.P.
3. Project the auxiliary top view  $b_1c_1$ , which is the true length of the edge BC. Its inclination,  $\phi$  with  $x_1y_1$ , is the true inclination of the edge BC with V.P.
4. Draw a reference line  $x_2y_2$ , parallel to  $bc$ , representing an A.V.P.
5. Project the auxiliary front view  $b'_1c'_1$ , which is also the true length of the edge BC and its inclination  $\theta$  with  $x_2y_2$ , is the true inclination of the edge BC with H.P.
6. Using the true lengths of the edges AB, BC and CA, draw the true shape of the triangle, as shown.



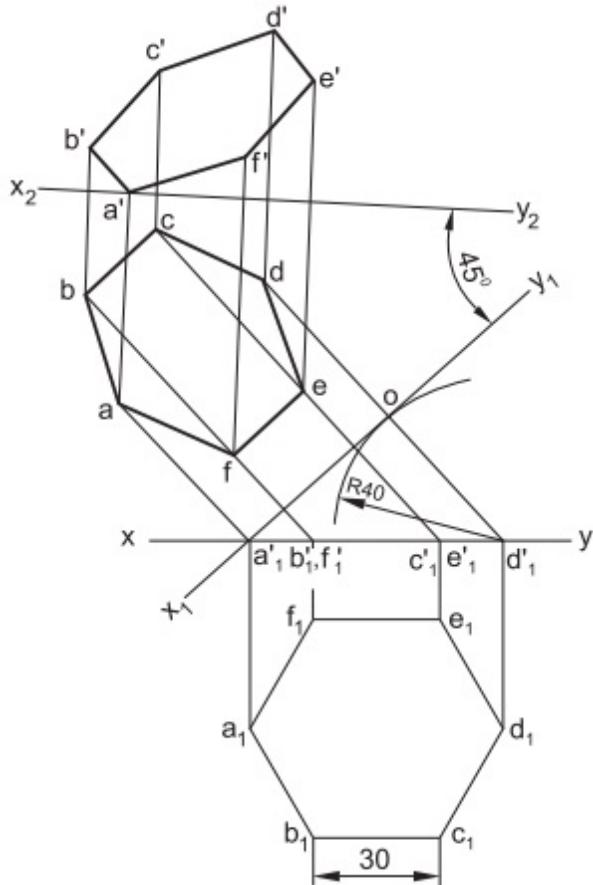
**Fig.10.22**

**Problem 23** A thin regular hexagonal plate of 30 side, is resting on a corner on H.P. The end of the longest diagonal through the corner is 40 above H.P. Draw the projections of the plate. Also, draw auxiliary front view on an A.V.P, inclined at  $45^\circ$  with V.P.

**Construction (Fig.10.23)**

1. Draw the projections of the hexagonal plate, assuming it to be lying on H.P, with an edge parallel to V.P.
2. With  $d'_1$  as centre and radius 40, draw an arc.
3. Draw a line through  $a'_1$ , tangential to the above arc, touching it at o.
4. Join  $a'_1$ , o and extend forming the reference line  $x_1y_1$ .
5. Obtain the (auxiliary) final top view, abcdefa, by projection.

6. Draw a reference line  $x_2y_2$ , at an angle of  $45^\circ$  with  $x_1y_1$ .
7. Obtain the (auxiliary) final front view, by projection.



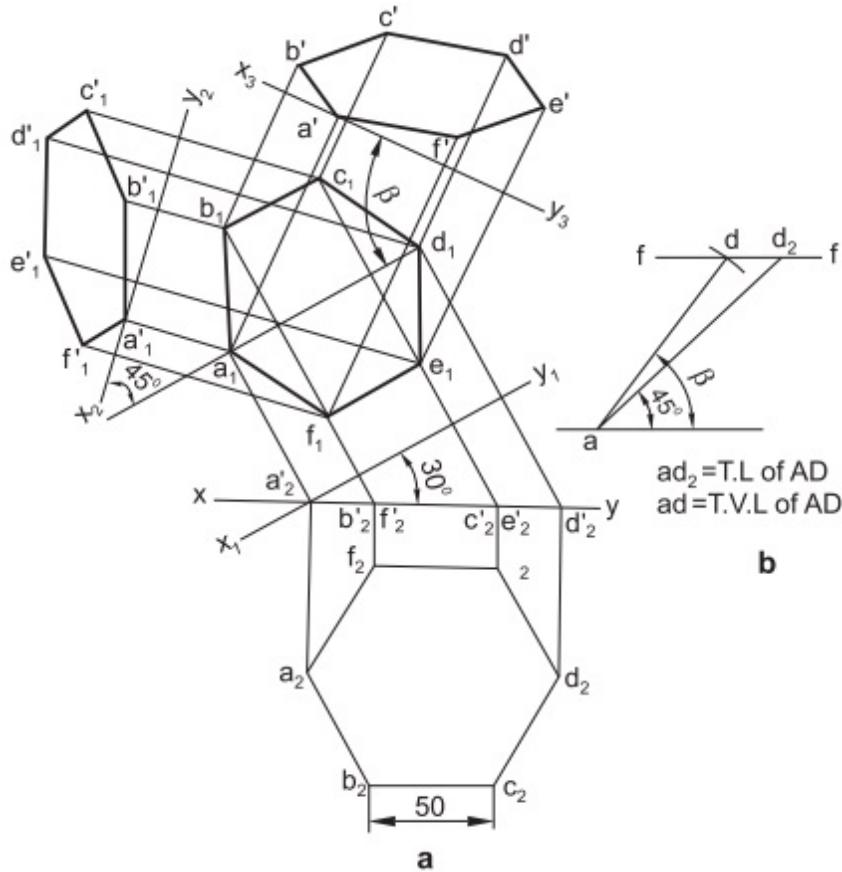
**Fig.10.23**

**Problem 24** A regular hexagon of 50 side has a corner on H.P. Its surface is inclined at  $30^\circ$  to H.P. Draw the projections of the hexagon, when (i) the top view of the diagonal through the corner which is on H.P makes  $45^\circ$  with V.P and (ii) the above diagonal itself makes  $45^\circ$  with V.P.

**Construction (Fig.10.24)**

1. Draw the projections of the hexagon, assuming it to be lying on H.P and with a side of it parallel to V.P.

2. Draw a reference line  $x_1y_1$ , corresponding to an A.I.P, making an angle of  $30^\circ$  and passing through  $a_2'$ , the front view of the corner, which is on H.P.
3. Obtain the (auxiliary) final top view, by projection.
4. Draw a reference line  $x_2y_2$ , corresponding to an A.V.P, and making an angle of  $45^\circ$  with the top view,  $a_1d_1$  of the diagonal through the corner, which is on H.P.
5. Obtain the (auxiliary) final front view  $a_1'b_1'c_1'd_1'e_1'f_1'a_1'$ , by projection.
6. Draw a reference line  $x_3y_3$ , corresponding to an A.V.P and making the apparent angle  $\beta$  with  $a_1d_1$  (refer [Fig.10.23b](#), for construction to determine  $\beta$ ).
7. Obtain the (auxiliary) final front view  $a'b'c'd'e'f'a'$ , by projection.



**Fig.10.24**

**Problem 25** An isosceles triangular lamina PQR, having the base PQ, 50 long and altitude 100, has its corners P, Q and R at 25, 60 and 90 respectively above H.P. Draw its projections.

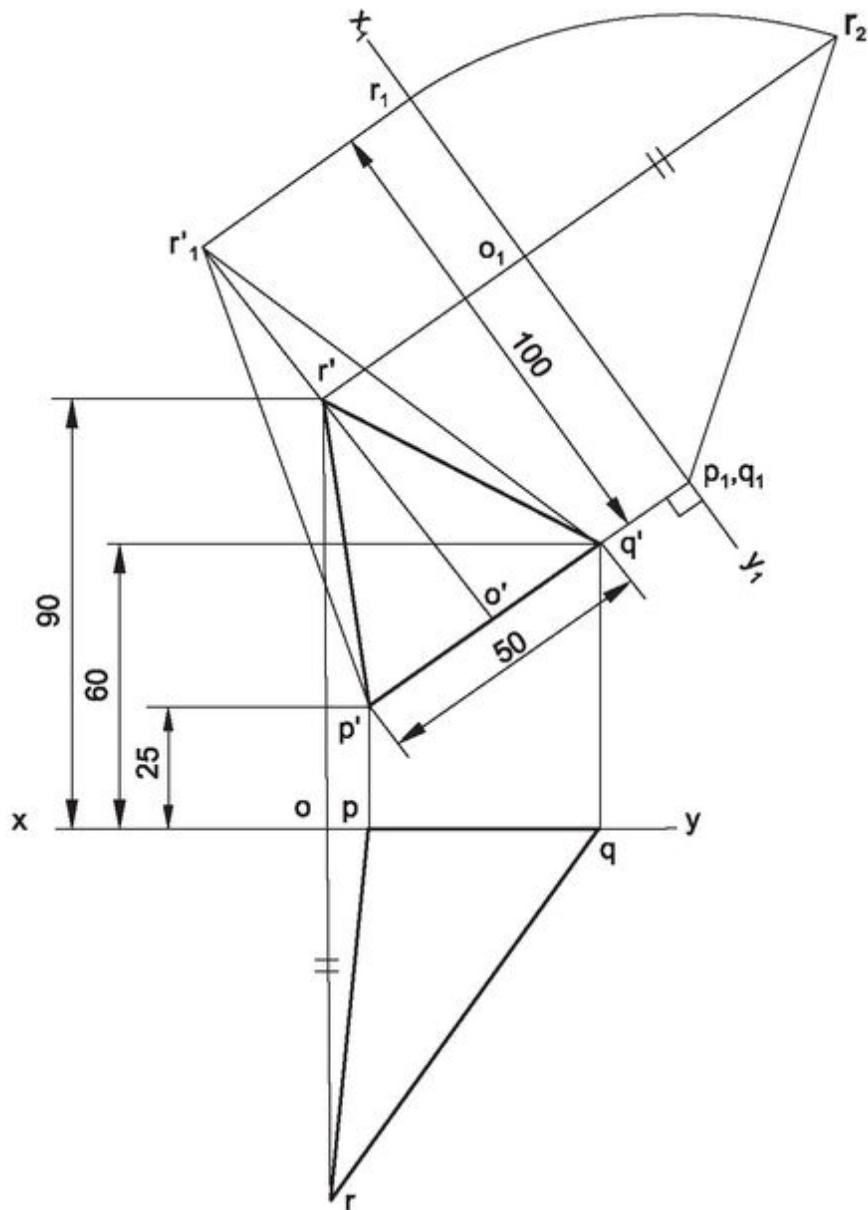
**Construction (Fig.10.25)**

**Stage I** Assume the plane to be in V.P such that, the corners P and Q are 25 and 60 above H.P respectively.

1. Draw the front view  $p' q' r_1'$ , the true shape of the plane such that, the corners  $p'$  and  $q'$  are 25 and 60 above  $xy$  respectively.
2. Locate p and q on  $xy$ .

**Stage II** Rotate the front view about  $p'q'$  such that,  $r'$  moves along the median  $o'r_1'$ , to the position  $r'$  which is 90 above  $xy$ .

3. Draw a reference line  $x_1 y_1$ , perpendicular to  $p'q'$ .
4. Obtain the edge view of the plane  $p_1 (q_1) r_1$ , by projection.
5. With  $p_1$  as centre and  $p_1 r_1$  as radius, draw an arc meeting the projector through  $r'$  at  $r_2$ .
6. Locate  $r$  on the projector through  $r'$  such that,  $or = o_1r_2$ .
7. Join  $p$ ,  $r$  and  $r$ ,  $q$ .  
 $p'q'r'$  and  $pqr$  are the required projections.



**Fig.10.25**

**Problem 26** An isosceles triangle ABC with base 40 and altitude 60 has its base AC in H.P and inclined at  $30^\circ$  to V.P. The corners A and B are on V.P. Draw its projections.

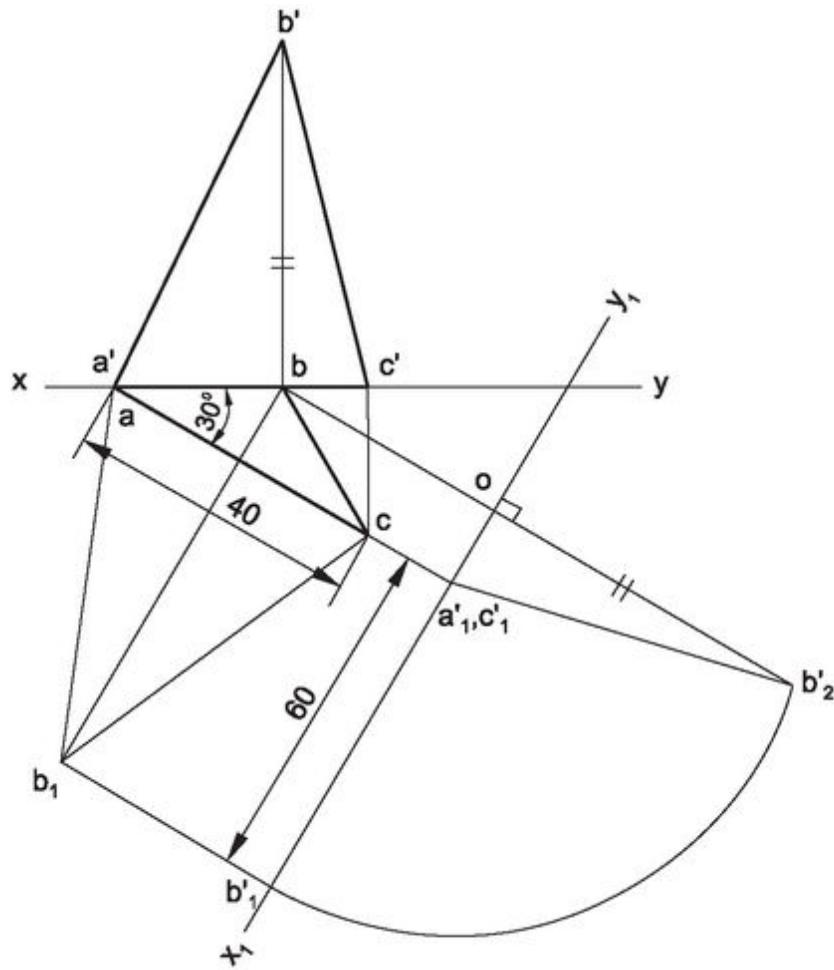
**Construction (Fig.10.26)**

**Stage I** Assume the plane to be in H.P and the corner A lying in both H.P and V.P.

1. Draw the top view  $ab_1c$ , the true shape of the plane such that,  $ac$  makes  $30^\circ$  with  $xy$ .
2. Locate  $a'$  and  $c'$  on  $xy$ .

**Stage II** Rotate the top view about  $ac$  such that,  $b_1$  moves along the median and touches  $xy$  at  $b$ .

3. Draw a reference line  $x_1 y_1$ , perpendicular to  $ac$ .
4. Obtain the edge view of the plane  $a_1' (c_1') b_1'$ , by projection.
5. With  $a_1'$  as centre and  $a_1' b_1'$  as radius, draw an arc to meet the projector through  $b$  at  $b_2'$ .
6. Locate  $b'$  on the projector through  $b$  such that,  $bb' = ob_2'$ .
7. Join  $a'$ ,  $b'$  and  $b'$ ,  $c'$ .  
 $a'b'c'$  and  $abc$  are the required projections.



**Fig.10.26**

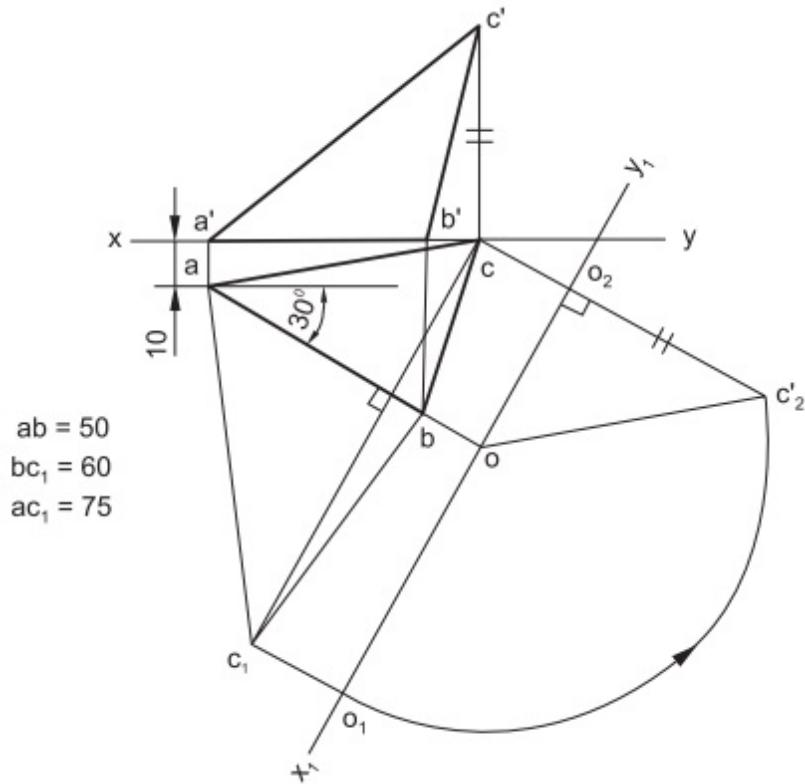
**Problem 27** A triangular card board ( $AB=50$ ,  $BC=60$ ,  $CA=75$ ) has its side  $AB$  resting on H.P and the vertex  $C$  touching V.P. The side  $AB$  is inclined to V.P at  $30^\circ$  and  $A$  is 10 in front of V.P. Draw the projections of the card board.

**Constructions (Fig.10.27)**

1. Draw the projections  $abc_1$  and  $a'b'c'$  of the card board  $ABC$ , assuming it to be lying on H.P; with the corner  $A$  at 10 from V.P and the side  $AB$  making  $30^\circ$  with V.P.
2. Through  $c_1$ , draw a line perpendicular to  $ab$  and extend to meet  $xy$  at  $c$ .

c is the top view of the corner C, as it lies on V.P.

3. Draw the reference line  $x_1y_1$ , parallel to  $c_1c$ .
4. Extend ab, till it meets  $x_1y_1$  at o. Through  $c_1$ , draw a projector, till it meets  $x_1y_1$  at  $o_1$ .
5. With o as centre and  $oo_1$  as radius, draw an arc, till it meets the projector through c at  $c'_2$ .  $o_2$  is the point of intersection between  $x_1y_1$  and  $cc'_2$ .
6. Through c, draw a projector and mark  $c'$  on it such that,  $cc' = o_2c_2'$ .
7. Join  $a'$ ,  $c'$  and  $b'$ ,  $c'$  forming the front view.
8. Join a, c and b, c forming the top view.

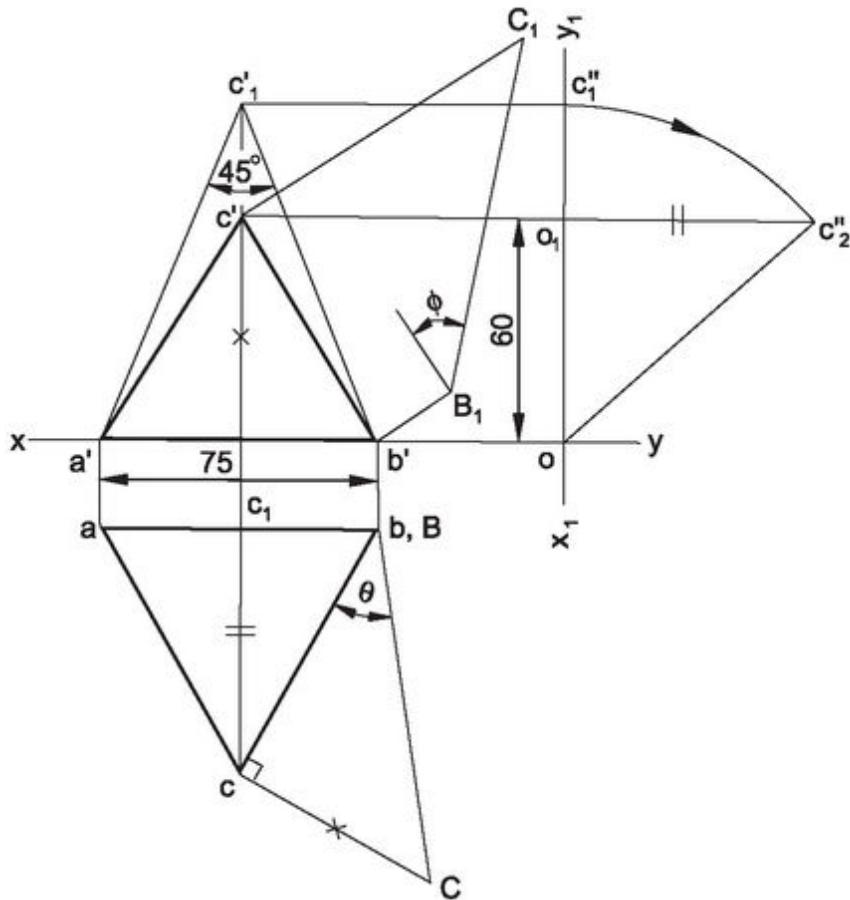


**Fig.10.27**

**Problem 28** A divider opened at  $45^\circ$  is placed on V.P such that, both the ends are in V.P and equi-distant from H.P and the hinged end is 60 in front of V.P. If the distance between the ends is 75, draw the projections and determine the true lengths of the legs of the divider. Also, determine the inclinations of the legs with the planes of projection.

**Construction (Fig.10.28)**

1. Draw the projections  $a'b'c'_1$  and  $abc_1$ , of the divider, in opened position and assuming it to be parallel to V.P; with its both the ends on H.P.
2. Locate the point  $c'$  on the projector through  $c'_1$  such that,  $c'$  is 60 above  $xy$ . Join  $a'b'$  and  $b'c'$ ; forming the final front view.
3. Draw a reference line  $x_1y_1$  and project  $c'_1$  and  $c'$  meeting  $x_1y_1$  at  $c''_1$  and  $o_1$  respectively. Let  $o$  be the point of intersection between  $xy$  and  $x_1y_1$ .
4. With  $o$  as centre and radius  $oc''_1$ , draw an arc meeting the projector through  $c'o_1$  extended at  $c''_2$ .
5. On the projector through  $c_1$ , locate  $c$  such that,  $c_1c = o_1c''_2$ .
6. Join  $a, c$  and  $b, c$  forming the final top view.
7. Determine the true lengths of the legs of the divider ( $=BC=B_1C_1$ ) and the inclinations  $\theta$  and  $\phi$  with H.P and V.P respectively, by trapezoidal method, as shown.

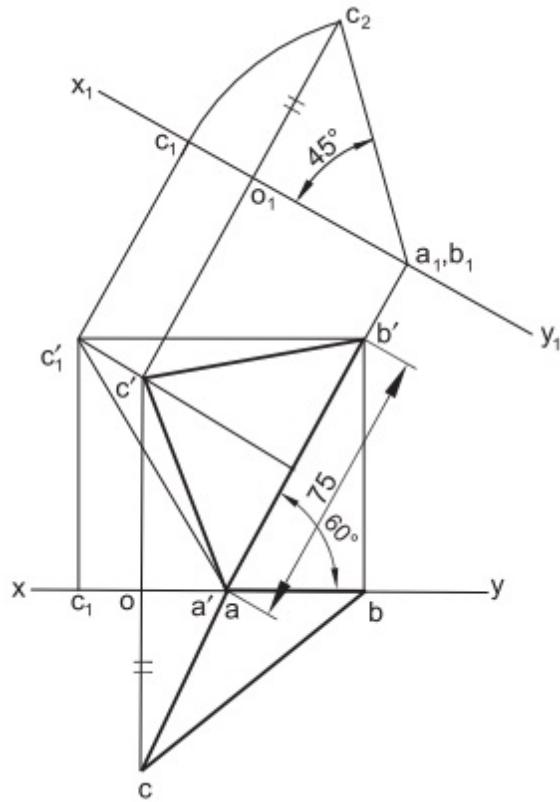


**Fig.10.28**

**Problem 29** An equilateral triangle  $ABC$  of side 75 long has its side  $AB$  on V.P and inclined at  $60^\circ$  to H.P. Its plane makes an angle  $45^\circ$  with V.P. Draw its projections.

**Construction (Fig.10.29)**

1. Draw the reference line  $xy$ . Assuming that the plane is lying on V.P with the side  $AB$  making  $60^\circ$  with H.P and the end  $A$  is on H.P, draw the front view  $a'b'c'_1$ .
2. Locate  $a$  and  $b$  on  $xy$  as  $AB$  lies on V.P.
3. Draw the reference line  $x_1y_1$ , corresponding to an AIP, and perpendicular to  $a'b'$ .

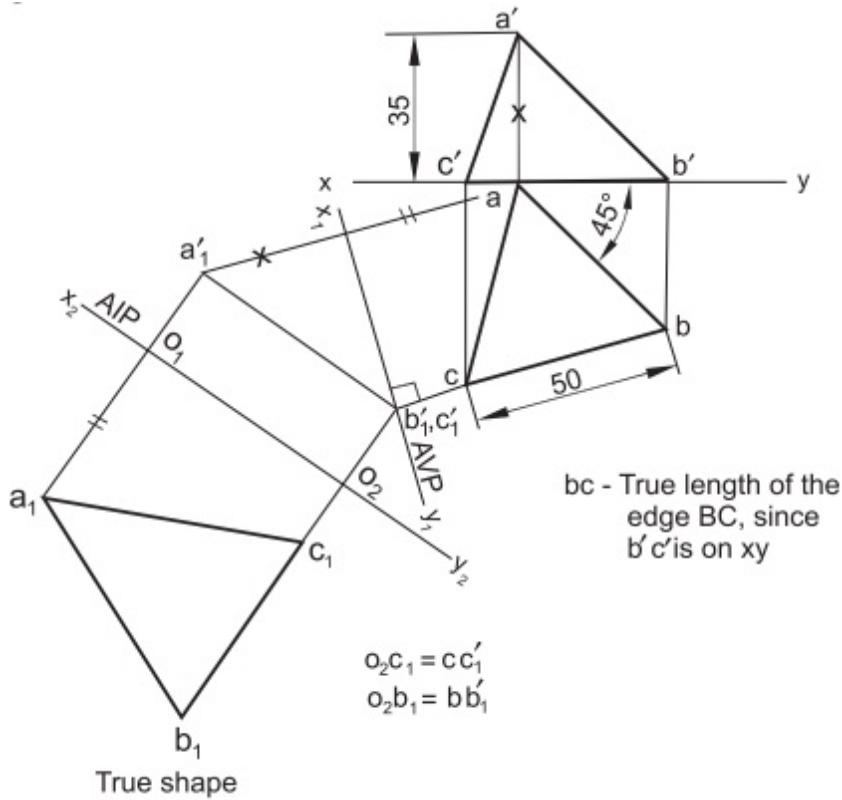


**Fig.10.29**

4. Project and obtain the edge view of the plane  $c_1a_1(b_1)$ , by projection.
5. Rotate the edge view about  $a_1(b_1)$  to  $a_1(b_1)c_2$  such that, it makes an angle  $45^\circ$  with  $x_1y_1$ .
6. Through  $c$  draw a projector meeting the median of the triangle through  $c'_1$  at  $c'$ .
7. Locate  $c$  on the projector through  $c'$  such that,  $oc = o_1c_2$ .
8. Join  $a', c'$  and  $b', c'$ .
9. Join  $a, b; b, c$  and  $a, c$ .  
 $a'b'c'$  and  $abc$  are the required projections.

**Problem 30** The top view  $abc$  of a triangle  $ABC$  is an equilateral triangle of side 50;  $ab$  being inclined at  $45^\circ$  to  $xy$ . The point  $A$  is on VP and 35 above H.P and the points  $B$  and  $C$  are on H.P. Draw the projections of the triangle and determine the true shape.

**Construction (Fig.10.30)**



**Fig.10.30**

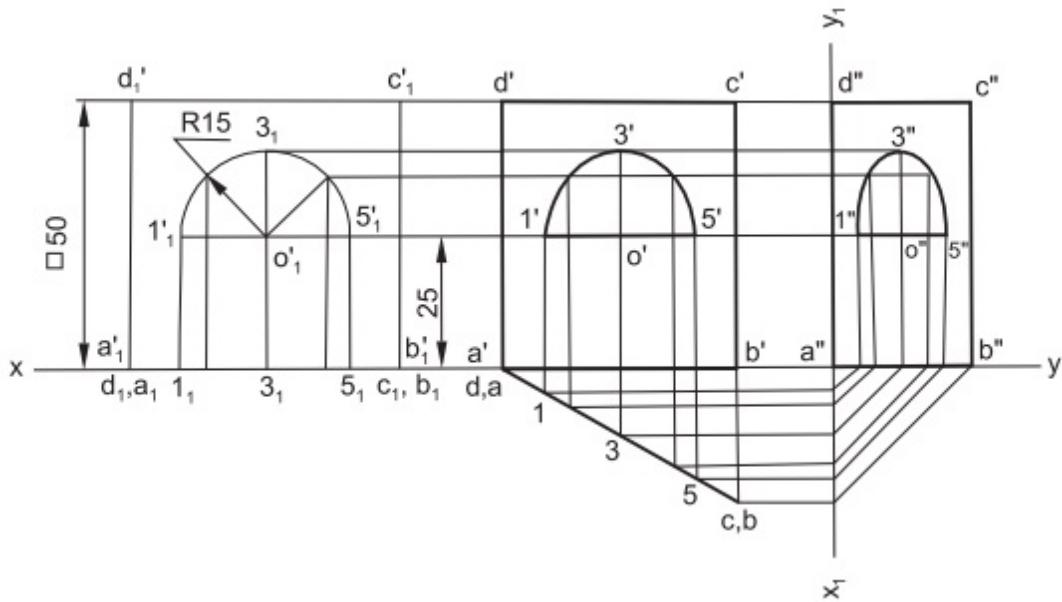
1. Draw the reference line  $xy$  and complete the top view  $abc$ , satisfying the given conditions.
2. Locate  $a'$  at 35 above  $xy$  and on the projector through  $a$  and complete the front view, satisfying the other given conditions.
3. Draw the reference line  $x_1y_1$ , corresponding to an AVP, and perpendicular to  $cb$  ( $cb$  represents the true length of the edge  $CB$  of the plane).

4. Project and obtain the edge view (auxiliary front view)  $a_1' b_1' (c_1')$  of the plane.
5. Draw the reference line  $x_2y_2$ , corresponding to an AIP, and parallel to the above edge view.
6. Project and obtain the normal view  $a_1b_1c_1$  (auxiliary top view), representing the true shape of the triangle.

**Problem 31** A square plate of side 50 has a semi-circular hole of 30 diameter. The centre of the straight edge of the semi-circle coincides with the centre of the plate. The straight edge of the hole is parallel to the side of the square. The plate is resting on H.P with one of its sides in V.P and the surface of the plate is inclined at  $30^\circ$  to V.P. Draw the three views of the plate, when the straight edge of the hole is parallel to H.P.

### **Construction ([Fig.10.31](#))**

1. Draw the front and top views of the plate, assuming it to be lying in V.P and one of its sides on H.P.
2. Redraw the top view such that, it is inclined at  $30^\circ$  with  $xy$ , keeping one end d (a) on  $xy$ . This is the final top view.
3. Obtain the final front view, by projection.
4. Draw the line  $x_1y_1$  at right angle to  $xy$ , forming the reference line between the V.P. and P.P.
5. Obtain the side view of the plate, by projecting front and top views.



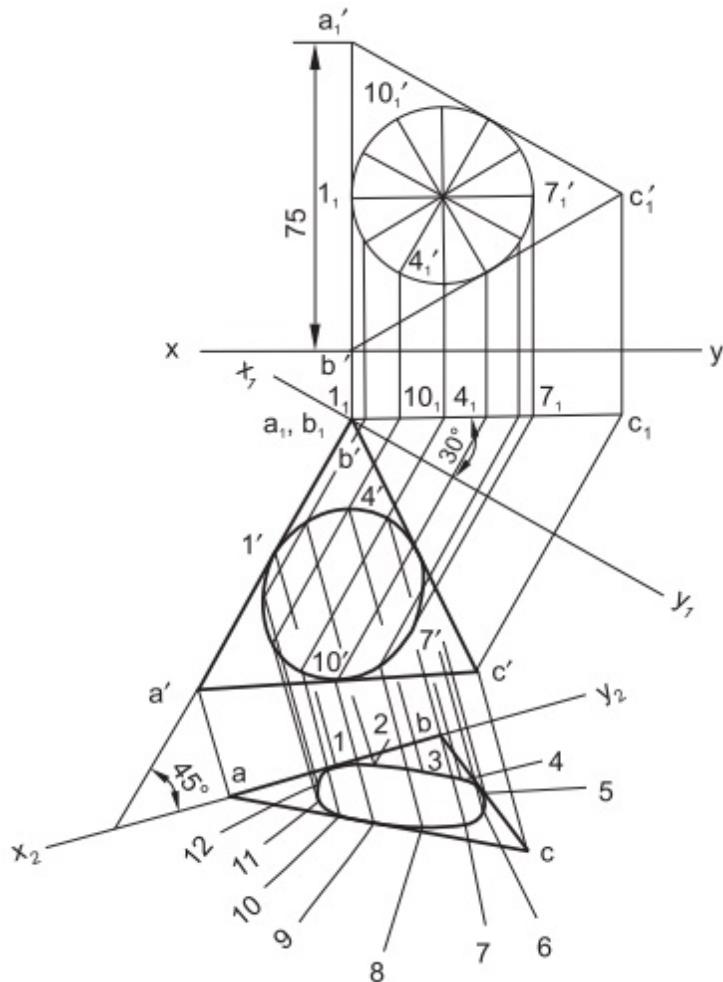
**Fig.10.31**

**Problem 32** Draw the equilateral triangle of 75 side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at  $30^\circ$  to the V.P and one of the sides of the triangle is inclined at  $45^\circ$  to the H.P. Use auxiliary plane method.

**Construction (Fig.10.32)**

1. Draw the projections of the triangular plane with a circle inscribed in it and with a side of it perpendicular to the H.P.
2. Divide the circle into an equal number of parts and obtain their top views, by projection.
3. Draw a reference line  $x_1y_1$ , making angle of  $30^\circ$  with  $xy$  and passing through  $a_1(b_1)$ .
4. Obtain the auxiliary front view, by projection.
5. Draw a reference line  $x_2y_2$  at an angle of  $45^\circ$  with  $a'b'$  (the inclination of the side AB with H.P).
6. Obtain the auxiliary top view by projection.

$a'b'c'$  and  $abc$  are the required projections of the triangle along with the projections of the circle that is inscribed in it.



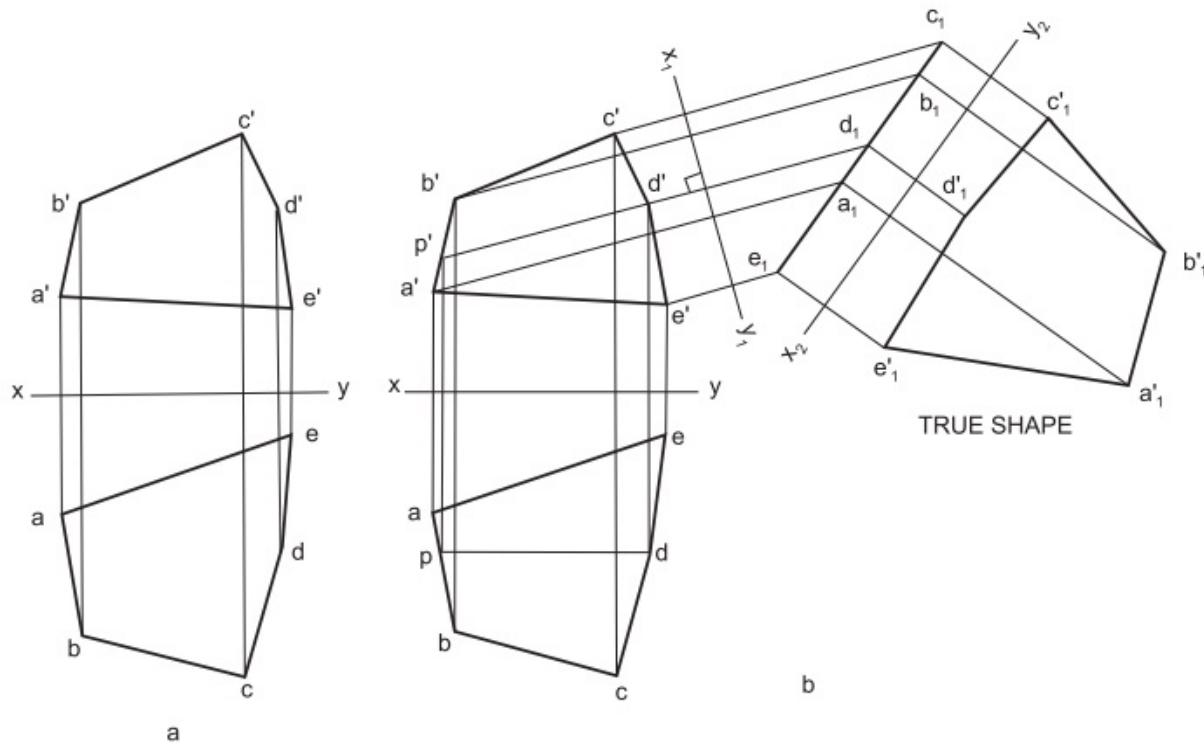
**Fig.10.32**

**Problem 33** *Figure 10.33a shows the projections of a plane figure ABCDE. Determine its true shape.*

**Construction (Fig.10.33b)**

1. Draw the given projections of the plane figure.
2. Select any line pd in the top view, which is parallel to xy.

3. Obtain the front view  $p'd'$ , the true length of the line PD.
4. Draw the reference line  $x_1 y_1$ , perpendicular to  $p'd'$ .
5. Project and obtain the edge view  $a_1 b_1 c_1 d_1 e_1$  of the plane.
6. Draw a reference line  $x_2 y_2$ , parallel to the above edge view.
7. Project and obtain the normal view  $a'_1 b'_1 c'_1 d'_1 e'_1$ , representing the true shape of the plane.

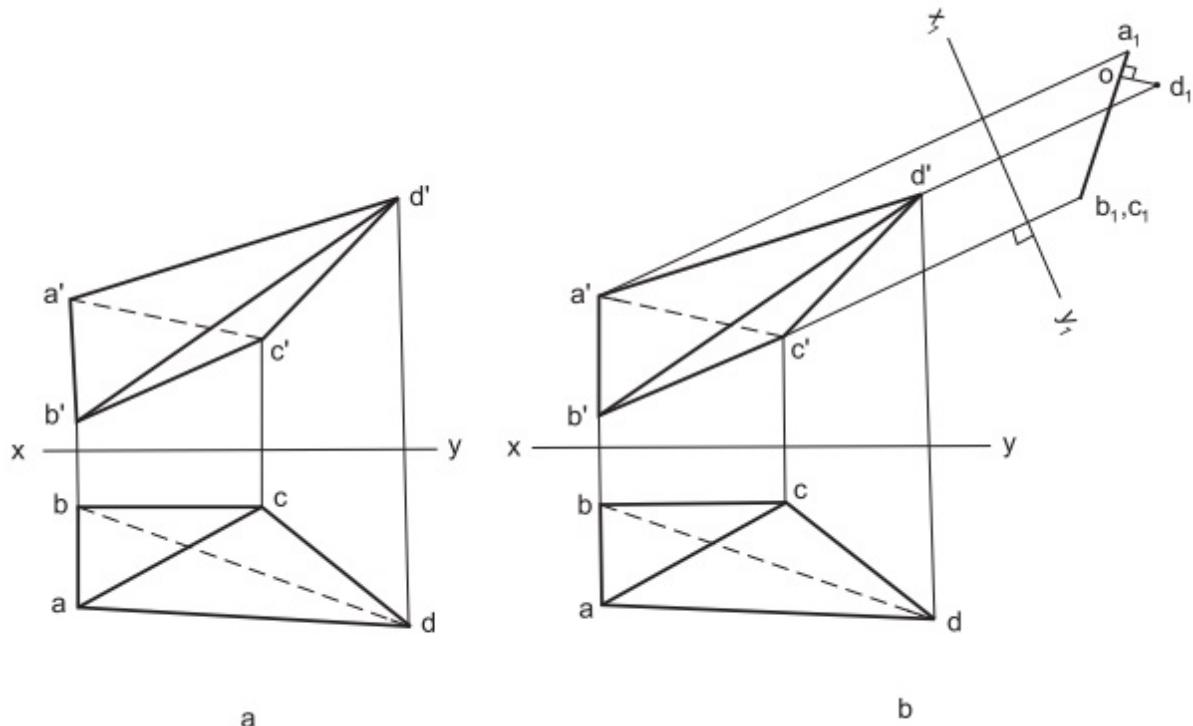


**Fig.10.33**

**Problem 34** *Figure 10.34a shows the projections of an irregular solid ABCD, with its base edge BC, parallel to V.P. Determine the distance between the point D and the base ABC of the solid.*

**Construction (Fig.10.34b)**

1. Draw the given projections of the solid ABCD.
2. Draw a reference line  $x_1 y_1$ , perpendicular to  $b'c'$ , representing the true length of the base edge BC (because the top view bc is parallel to xy).
3. Obtain the auxiliary top view of the corners of the solid, by projection.  
 $a_1b_1c_1$  represents the edge view of the base.
4. Drop a perpendicular from  $d_1$  to the line  $a_1b_1c_1$ , meeting it at o.  
 $od_1$  is the required distance.



**Fig.10.34**

## EXERCISES

10.1 A square ABCD of 40 side, has a corner A on H.P and 25 in front of V.P. All the sides of the square are equally inclined to H.P and parallel to V.P. Draw the projections of the plane and add:

- (i) auxiliary front view of the points A and C on an A.V.P, making an angle of  $30^\circ$  with V.P.
- (ii) auxiliary top view of the points B and D on an A.I.P, making an angle of  $45^\circ$  with H.P.
- (iii) auxiliary front view of the lines AB and CD on an A.V.P, making an angle of  $45^\circ$  with V.P.
- (iv) auxiliary top view of the lines AD and BC on an A.I.P, making an angle of  $60^\circ$  with H.P.

10.2 A line AB of 60 length has its end A on both H.P and V.P. It is inclined at  $30^\circ$  to H.P and  $45^\circ$  to V.P. Draw, its projections.

10.3 The top view of a line AB of 100 long, measures 85, while the length of the front view is 65. Its one end A is on H.P and 15 behind V.P. Draw the projections of AB and determine its inclinations with H.P and V.P. Find the distance of the mid-point of AB from xy.

10.4 The length of the top view of a straight line is 45 and the length of the front view is 60. The top view ab is inclined at  $30^\circ$  to xy. Draw the projections of the line AB, assuming the point A to be situated on H.P and 30 behind V.P. Find (i) the distance of the mid-point from H.P, (ii) the distance of the mid-point from xy and (iii) the shortest distance of AB from xy line.

10.5 An isosceles triangle ABC, of base 60 and altitude 40, has its base AC on H.P and inclined at  $30^\circ$  to V.P. The corners A and B are on V.P. Draw its projections.

- 10.6 Draw an isosceles triangle abc, of base ab equal to 40 and altitude 75 with a on xy and ab inclined at  $45^\circ$  to xy. The figure is the top view of a triangle, whose corners A, B and C are respectively 75, 25 and 50 above H.P. Determine the true shape of the triangle and the inclinations of AB with H.P and V.P.
- 10.7 Determine the true shape of the plane figure, the front view of which is a straight line, making an angle of  $30^\circ$  with xy and the top view, a hexagon of side 30, with one side making an angle of  $45^\circ$  with xy.
- 10.8 An equilateral triangle ABC of side 75 long, has its side AB on V.P and inclined at  $60^\circ$  to H.P. Its plane makes an angle of  $45^\circ$  with V.P. Draw its projections.
- 10.9 The top view of a triangular lamina is an equilateral triangle of side 40, with one side parallel to xy, while the front view is a line of 50 length. Determine the true shape of the triangle.
- 10.10 Top view of a plate, the surface of which is inclined at  $60^\circ$  to H.P and perpendicular to V.P, is a regular pentagon of side 50, with one edge perpendicular to xy.
- (a) Find the true shape of the plate, and
- (b) Draw the projections of the plate, when the edge whose top view was perpendicular to xy earlier, becomes parallel to V.P; while the surface of the plate is still at  $60^\circ$  to H.P.
- 10.11 The top view of a plane abcd, is a square of 50 side with ab making  $30^\circ$  to xy. The corners B,C and D are respectively 35, 85 and 50 above H.P, while A is on H.P. Find the true shape of the plane.

10.12 A circle of 75 diameter, has the end A of the diameter AB on H.P. Its surface is inclined at  $60^\circ$  to H.P and the top view of the diameter makes an angle of  $30^\circ$  with xy. Draw its projections.

10.13 A circular plane of diameter 40, is resting on H.P on a point. Its surface is inclined at  $30^\circ$  to H.P. Draw the projections of the circle when (i) the top view of the diameter through the resting point makes an angle of  $45^\circ$  with xy and (ii) the diameter passing through the resting point makes an angle of  $45^\circ$  with V.P.

10.14 Draw the projections of a regular pentagon of 25 side, having one of its sides resting on H.P and making an angle of  $60^\circ$  with V.P and its surface is making an angle of  $40^\circ$  with H.P.

10.15 A thin regular hexagonal plate of 30 side is resting on V.P on one of its edges, which makes an angle of  $45^\circ$  with H.P and the surface is inclined at  $30^\circ$  to V.P. Draw its projections.

10.16 The top view abc of a triangle ABC is an equilateral triangle of side 50; ab being inclined at  $45^\circ$  to xy. The point A is on V.P and 35 above H.P and the points B and C are on H.P. Draw the projections of the triangle and determine the true shape.

10.17 A regular pentagon of edge 30, is resting on H.P on one of its corners such that the surface makes an angle of  $60^\circ$  to H.P. The edge opposite to this corner makes an angle of  $45^\circ$  with V.P. Draw its projections.

10.18 The top view abcd of a plane is a square of 50 side, with ab making  $30^\circ$  with xy. The corners B,C,D are respectively 35, 80 and 50 above H.P, while A is on H.P. Find the true shape of the plane.

A thin composite plate consists of a square of side 70  
10.19 with an additional semicircle on CD as diameter.

The side AB is vertical and the surface of the plate makes an angle of  $45^\circ$  with V.P. Draw its projections. Project another auxiliary top view on an A.I.P, making an angle of  $30^\circ$  with the side AB.

## REVIEW QUESTIONS

- 10.1 What is an auxiliary plane?
- 10.2 When are auxiliary views preferred?
- 10.3 Define A.V.P. What type of view is obtained on it?
- 10.4 Define A.I.P. What type of view is obtained on it?
- 10.5 What are the applications of the auxiliary projections with respect to (i) straight lines and (ii) planes?
- 10.6 How is the true length of a line obtained by the auxiliary plane method?
- 10.7 What is an edge view of a line? How can it be obtained?
- 10.8 Differentiate between the primary and successive auxiliary views.
- 10.9 What is an edge view of a plane? How is it obtained?
- 10.10 How are normal views of a (i) line and (ii) plane obtained?

## OBJECTIVE QUESTIONS

- 10.1 To position the view, the auxiliary plane should be rotated about the plane to which it is \_\_\_\_\_.  
10.2 The distance of the auxiliary front view of a point from an A.V.P is equal to the distance of the point from (a) H.P, (b) V.P, (c) A.I.P.  
( )
- 10.3 The front view and the auxiliary front view of a point lie on a single projector.  
(True /False)
- 10.4 There are \_\_\_\_\_ possible positions at which the auxiliary views may be drawn.
- 10.5 The distance from a point to a line is the \_\_\_\_\_ distance.
- 10.6 The angle between two intersecting lines will lie in a plane containing the lines.  
(True /False)
- 10.7 The shortest distance from a point to a plane is seen in the \_\_\_\_\_ view of the plane.
- 10.8 The angle between two oblique planes is measured in a plane perpendicular / inclined to both the planes.
- 10.9 Two planes are said to be parallel when their \_\_\_\_\_ views are parallel.
- 10.10 The parallelism of two lines lying on P.P will be revealed only in \_\_\_\_\_ view.

## ANSWERS

10.1 perpendicular

10.2 a

10.3 False

10.4 four

10.5 shortest

10.6 True

10.7 edge

10.8 Perpendicular

10.9 edge

10.10 side

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# 11

## *Projections of Solids*



### 11.1 INTRODUCTION

A solid is a three dimensional object having length, breadth and thickness. In engineering practice, one often comes across solids bounded by simple or complex geometric surfaces. To represent a solid in orthographic projections, the number and types of views necessary will depend upon the type of solid and its orientation with respect to the principal planes of projection. The applications of auxiliary planes are also considered here.

### 11.2 TYPES OF SOLIDS

Solids may be classified as (i) polyhedra and (ii) solids of revolution.

#### 11.2.1 Polyhedra

A polyhedron is defined as a solid bounded by plane surfaces called faces. There are two types of polyhedra: (i) Regular and (ii) irregular or oblique polyhedra.

### 11.2.1.1 ***Regular Polyhedra***

A regular polyhedron is a solid bounded by plane surfaces, which are equal and regular. The examples are: (i) Tetrahedron, (ii) hexahedron, (iii) octahedron, (iv) dodecahedron and (v) icosahedron.

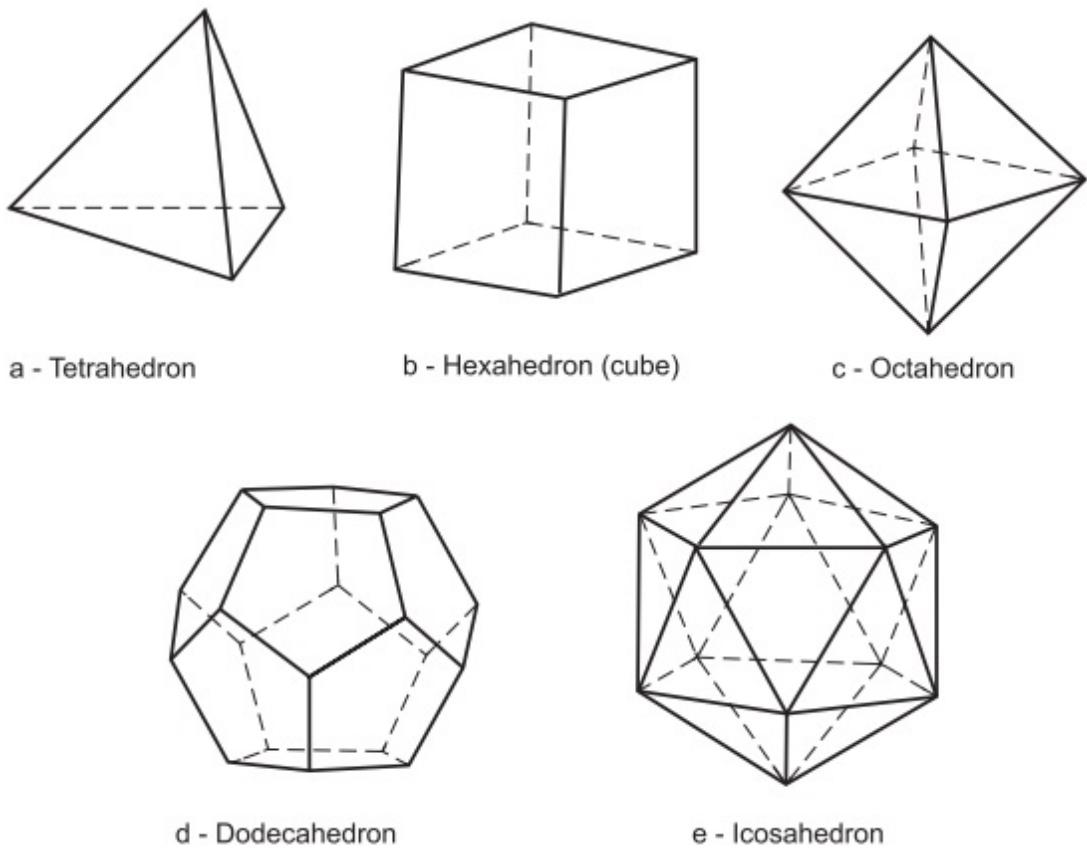
*Tetrahedron* It has four equal faces, each an equilateral triangle ([Fig.11.1a](#)).

*Hexahedron* or cube It has six equal faces, each a square ([Fig.11.1b](#)).

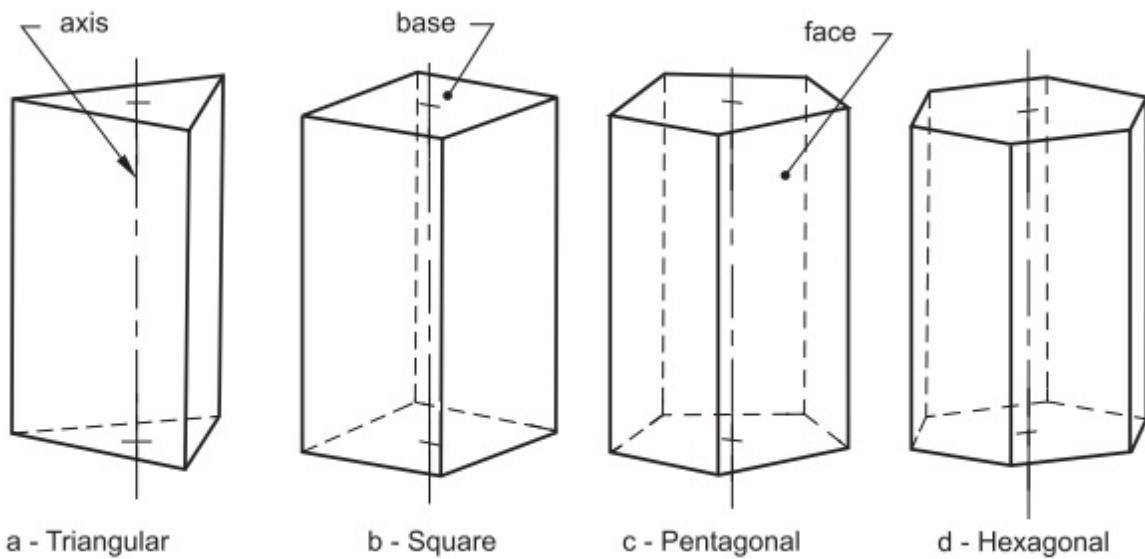
*Octahedron* It has eight equal faces, each an equilateral triangle ([Fig.11.1c](#)).

*Dodecahedron* It has twelve equal faces, each a regular pentagon ([Fig.11.1d](#)).

*Icosahedron* It has twenty equal faces, each an equilateral triangle ([Fig.11.1e](#)).



**Fig.11.1 Regular polyhedra**

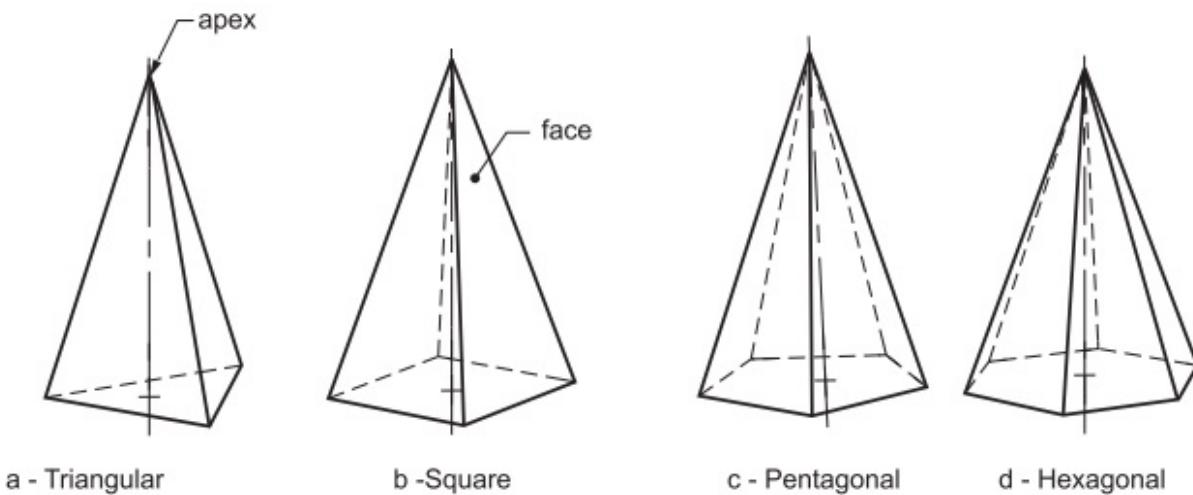


**Fig.11.2 Regular prisms**

There are two more categories of polyhedra, namely (i) prisms and (ii) pyramids.

**Prism** A prism is a polyhedron having two equal ends or bases, parallel to each other. The two bases are joined by faces, which are rectangles (Fig.11.2). The imaginary line joining the centres of the bases is called the axis of the solid.

**Pyramid** A pyramid is a polyhedron having one base and a number of isosceles triangular faces, meeting at a point called the vertex or apex (Fig.11.3). The imaginary line joining the centre of the base and apex is called the axis of the solid.



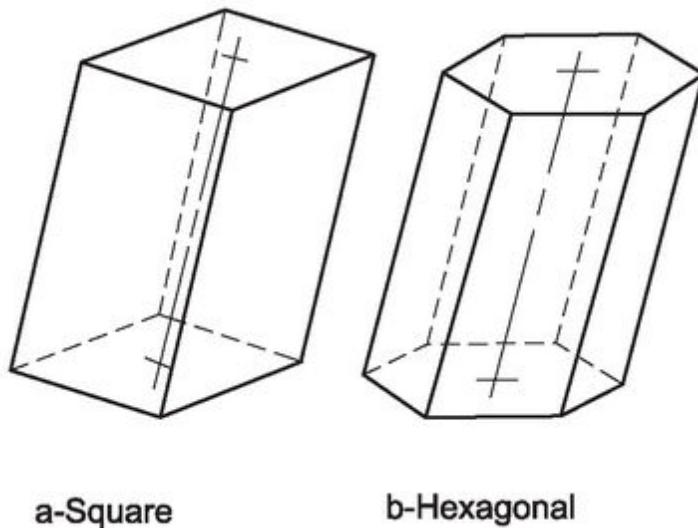
**Fig.11.3 Regular pyramids**

A prism or pyramid is said to be regular when the axis is perpendicular to the base. Both prisms and pyramids are named according to the shape of the base, viz., triangular prism/pyramid, pentagonal prism/pyramid and so on.

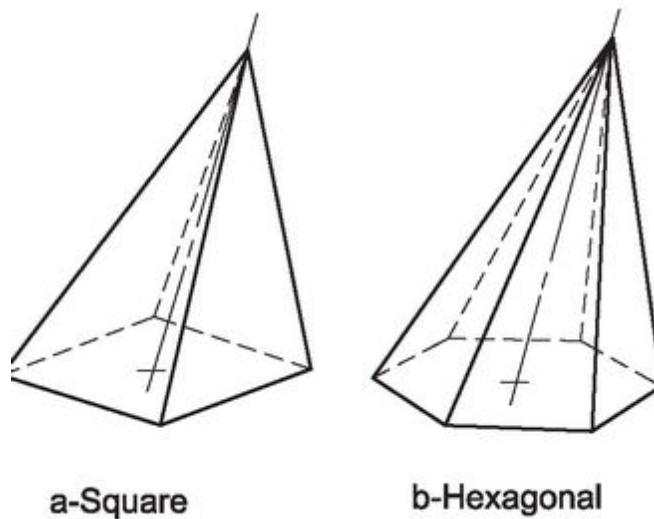
#### 11.2.1.2 *Irregular (Oblique) Polyhedra*

In case of irregular polyhedra, the axis is inclined to the base. The faces of an oblique prism (Fig.11.4) are parallelograms and faces of pyramids are triangles, which are not similar (Fig.11.5). However, the bases of oblique prisms are parallel, equal and similar.

In the case of oblique polyhedra, a section at right angle to the axis, will not produce a regular polygon.



**Fig.11.4 Oblique prisms**



**Fig.11.5 Oblique pyramids**

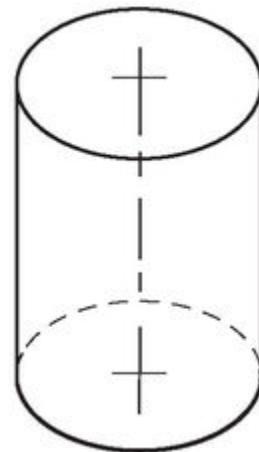
## 11.2.2 Solids of Revolution

Solids of revolution are obtained or generated by rotating a plane figure about one of its edges. Examples are: (i) Cylinder, (ii) cone and (iii) sphere. Solids of revolution, viz., cylinders and cones, may also be classified as: (i) Regular and (ii) oblique.

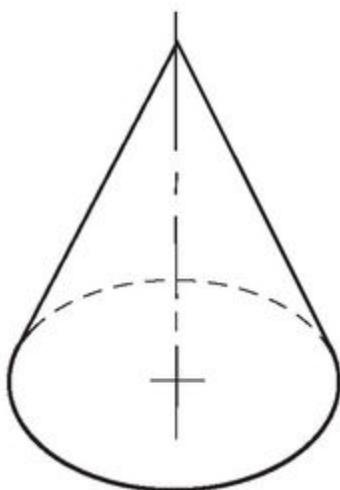
### 11.2.2.1 *Regular Solids of Revolution*

**Cylinder** A cylinder is generated by rotating a rectangle about one of its edges. The lateral surface is connected at its either end by two circular bases ([Fig.11.6](#)).

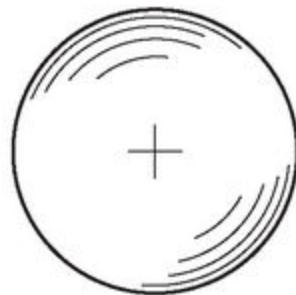
**Cone** A cone is generated by rotating a right angled triangle about one of its perpendicular sides. The lateral surface of the cone is connected by a circular base ([Fig.11.7](#)).



**Fig.11.6 Cylinder**



**Fig.11.7 Cone**



**Fig.11.8 Sphere**

A cylinder or a cone is said to be regular when the axis is perpendicular to the base. The lines drawn on the surface of a cylinder and parallel to the axis, are known as generators. The length of a generator is equal to the height of the cylinder. A line drawn from the vertex to any point on the base of a cone is also known as a generator, whose length is equal to the slant height of the cone.

**Sphere** A sphere is also a solid of revolution generated by rotating a semi-circle about its diameter ([Fig.11.8](#)). The mid-point of the diameter is the centre of the sphere. All points on the surface of a sphere are equi-distant from the centre.

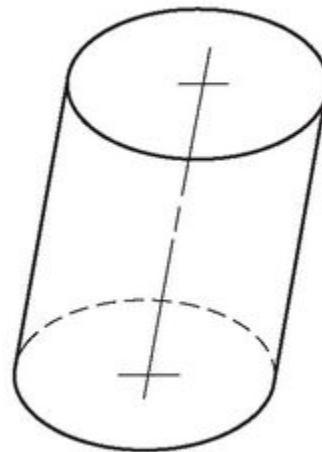
Cylinders and cones may also be termed as solids with single curved surfaces, whereas a sphere has a double curved surface.

### 11.2.2.2 *Oblique Solids*

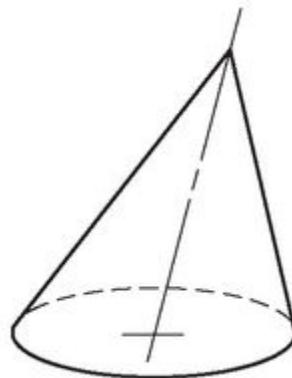
**Oblique cylinder (Fig.11.9)** The axis of the oblique cylinder is inclined to the bases. The lateral surface is connected by two equal and parallel circular bases. The generators of an oblique cylinder are of equal length. A section at right angle to the axis does not produce a circle.

**Oblique cone (Fig.11.10)** The axis of the oblique cone is inclined to the base. The base of the cone is a circle. The generators drawn on the lateral surface, connecting the base and apex are of unequal length. A section at right angle to the axis does not produce a circle.

In this chapter, mostly the projections of regular solids are considered. If nothing is specified about the type of solid, it must be assumed to be a regular one.



**Fig.11.9 Oblique cylinder**



**Fig.11.10 Oblique cone**

## 11.3 TWO-VIEW DRAWINGS

The position of a solid in space may be specified by the location of either the axis, edges, diagonals or surfaces, with the principal planes of projection. The following are some of the positions of solids:

- (i) Axis perpendicular to one of the principal planes,
- (ii) Axis inclined to one of the principal planes and parallel to the other and
- (iii) Axis inclined to both the principal planes.

### 11.3.1 Axis Perpendicular to one of the Principal Planes

When the axis of a solid is perpendicular to one of the principal planes, it is parallel to the other. Also, when the axis of a solid is perpendicular to any plane, the projection on that plane, will show the true shape and size of its base and the other projection will reveal the true length of the solid. So, when the axis is perpendicular to H.P, the top view must be drawn first and the front view is then

projected from it. When the axis is perpendicular to V.P, the front view must be drawn first and then the top view is projected from it.

**Problem 1** *Draw the projections of a cylinder of base 30 diameter and axis 50 long, when it is resting on H.P on one of its bases.*

**Construction (Fig.11.11)**

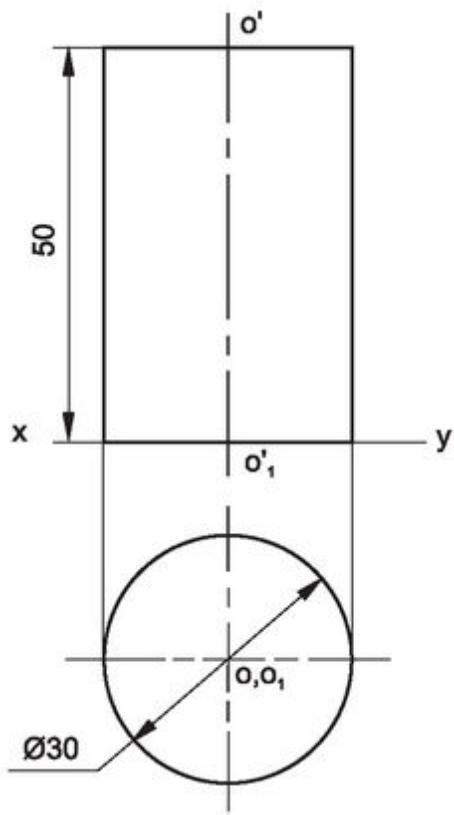
1. Draw a circle of 30 diameter, representing the top view of the cylinder.
2. Project the front view, which is a rectangle of height 50.

The bottom base in the front view, coincides with  $xy$ , as the solid is resting on H.P on one of its bases.

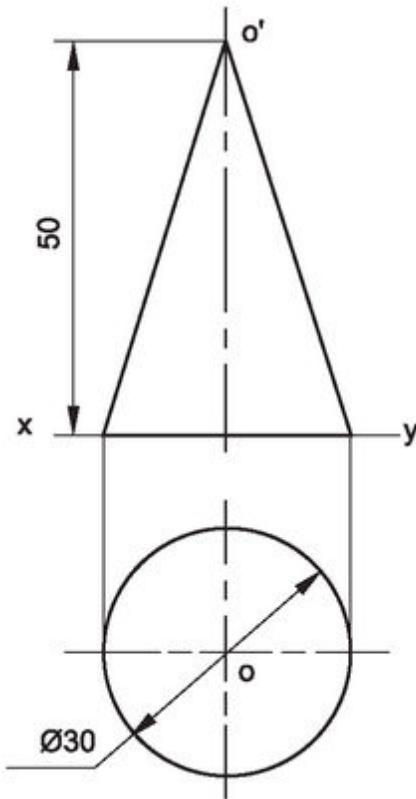
**Problem 2** *Draw the projections of a cone of base 30 diameter and axis 50 long, when it is resting on H.P on its base.*

**Construction (Fig.11.12)**

1. With centre  $o$ , draw a circle of 30 diameter; representing the top view of the cone.
2. Through  $o$ , draw a projector and locate the apex  $o'$ , at 50 above  $xy$ .
3. Obtain the front view which is a triangle, making the base coinciding with  $xy$ .



**Fig.11.11**



**Fig.11.12**

**Problem 3** A cube of 40 side, is resting with a face on H.P such that, the vertical faces are equally inclined to V.P. Draw, its projections.

**Construction ([Fig.11.13a](#))**

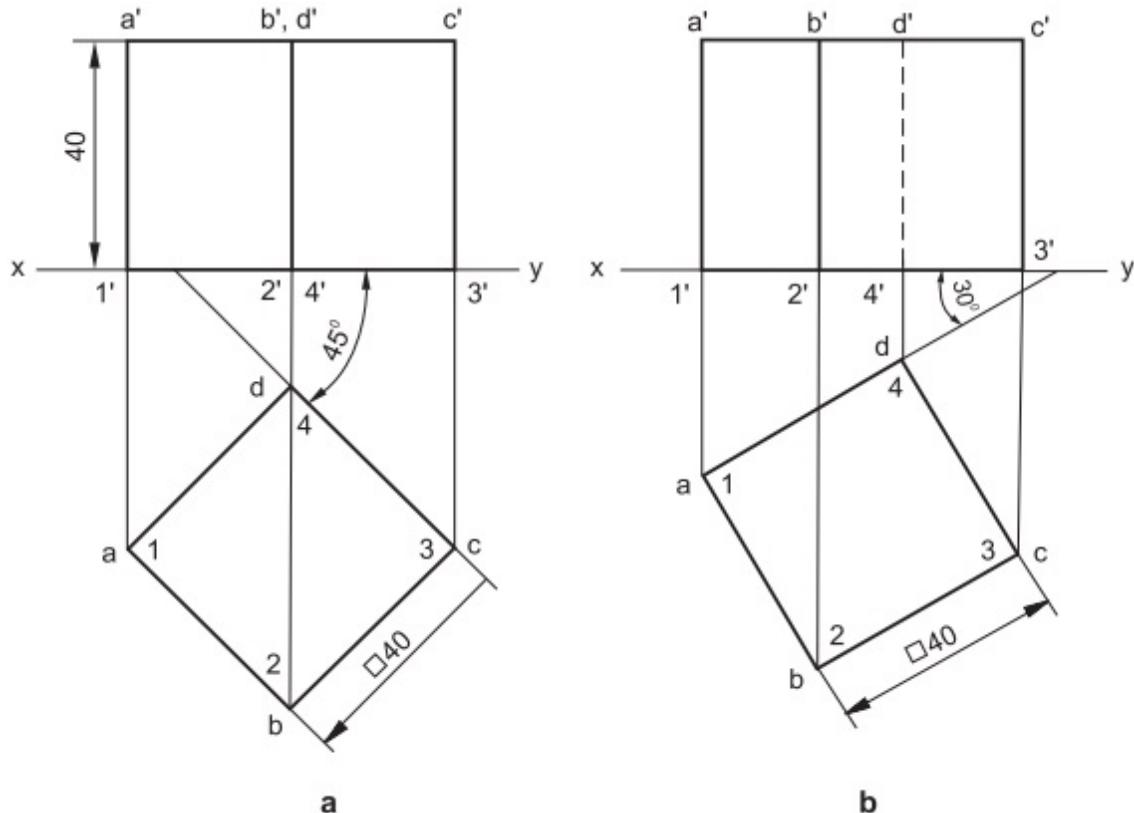
1. Draw a square abcd of side 40 and with its sides making  $45^\circ$  with xy. This represents the top view of the cube.
2. Project the front view, keeping the height equal to 40.

It may be noted from the front view that the front view d'4' of the invisible edge D4 coincides with the front view b'2' of the visible edge B2.

[Figure 11.13b](#) shows the projections of the cube, when one of its vertical faces is inclined at  $30^\circ$  to V.P. It may be

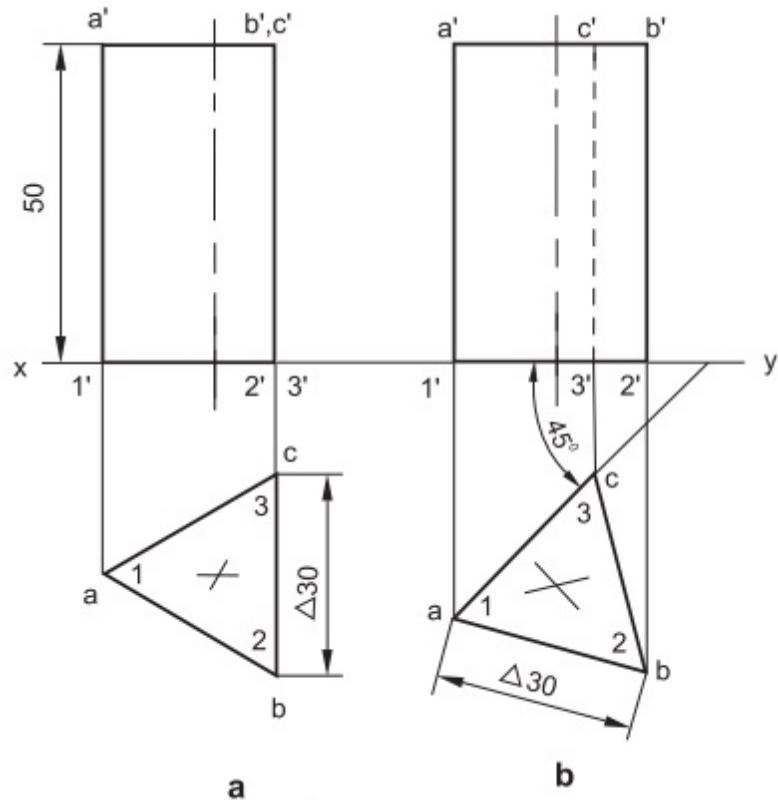
noted that the edge D4 in the front view ( $d'4'$ ) is invisible and hence, represented by dotted line.

**Problem 4** A triangular prism of base 30 side and axis 50 long, is resting on H.P on one of its bases, with a face perpendicular to V.P. Draw the projections of the solid.



**Fig.11.13**

**Construction (Fig.11.14a)**



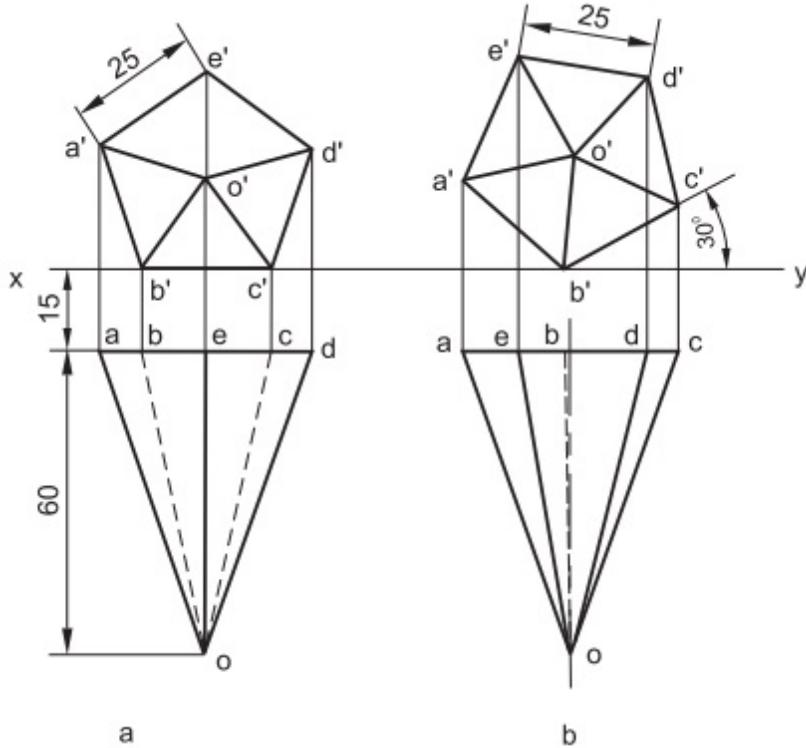
**Fig.11.14**

1. Draw the top view, an equilateral triangle of 30 side such that, one edge of it is perpendicular to xy.
2. Project the front view, keeping its height equal to 50 and the bottom edge coinciding with xy.

Figure 11.14b shows the projections of the above solid, when one of its faces is inclined at 45° with V.P.

**Problem 5** A pentagonal pyramid of base 25 side and axis 60 long, is resting on an edge of the base. Draw the projections of the pyramid, when its axis is perpendicular to V.P and the base is at 15 from V.P.

**Construction (Fig.11.15a)**



**Fig.11.15**

1. Draw the front view  $a'b'c'd'e'$ , a pentagon of side 25, coinciding with the side  $b'c'$  with  $xy$ .
2. Project the top view such that, the top view of the base  $abcde$  is at 15 from  $xy$  and the apex  $o$  is at 60 from  $abcde$ .



For the given position of the solid, when it is seen from above, the slant edges  $OB$  and  $OC$  are invisible. Hence, the lines,  $ob$  and  $oc$  are represented by dotted lines in the top view.

Figure 11.15b shows the projections of the pentagonal pyramid, when it is resting on H.P on a base corner, with an edge of the base containing that corner, making  $30^\circ$  with H.P.

### **11.3.2 Axis Inclined to one of the Principal Planes and Parallel to the other**

When the axis of the solid is inclined to any principal plane, the final projections are drawn in two stages. In the first stage, the axis of the solid is assumed to be perpendicular to the principal plane, to which it is actually inclined and the views are drawn. In the second stage, the position of one of the views is altered to suit the given condition and the other view is projected from it. This method is known as change of position method.

In the second stage, instead of reconstructing one of the views as mentioned above, an auxiliary plane is imagined satisfying the given condition and the other view is projected on it. This method is known as change of reference line method. This is advantageous compared to the former one, as this avoids redrawing one of the views in the second stage. This advantage may be appreciated with respect to the solids having curved surfaces or too many edges.

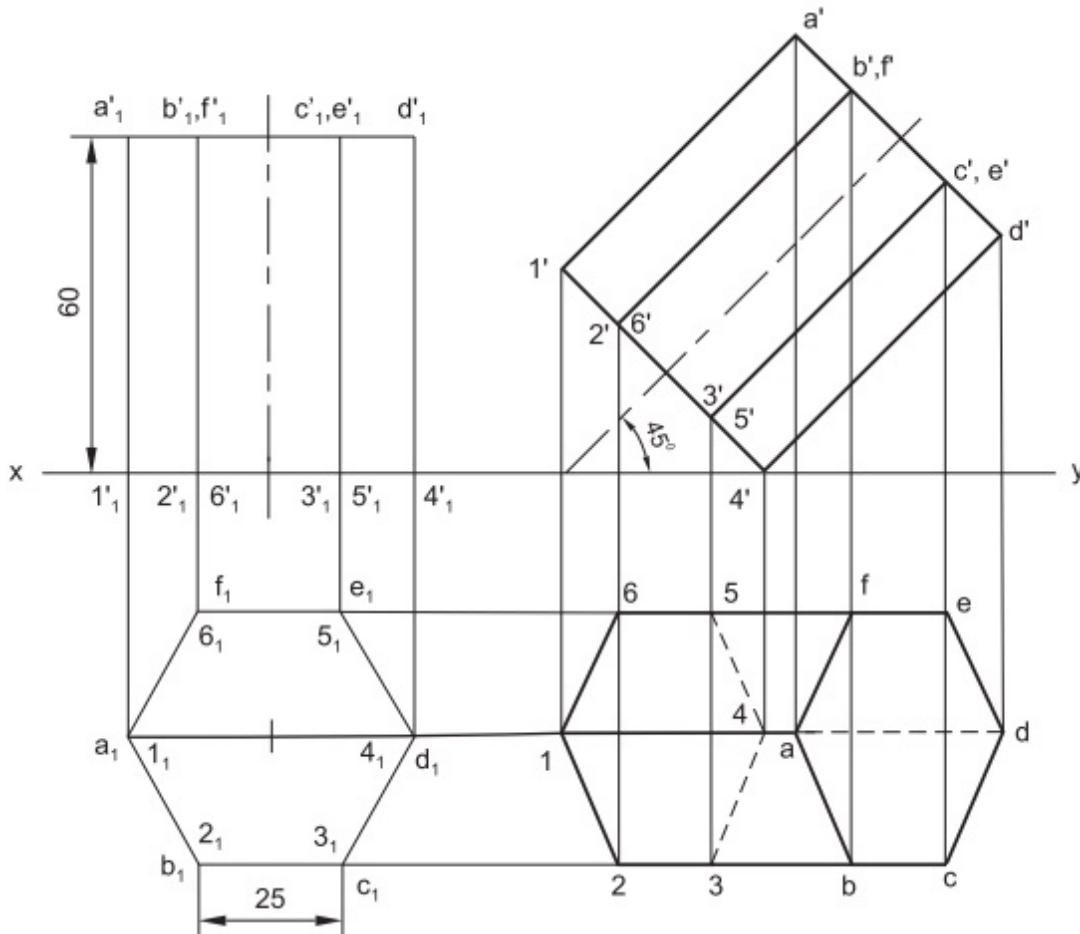
While drawing the projections of solids, the following must be observed:

1. If a solid has an edge of its base parallel to H.P or on H.P, that edge should be kept perpendicular to V.P. If the edge of the base is parallel to V.P or on V.P, that edge should be kept perpendicular to H.P.
2. If a solid has a corner of its base on H.P, the sides of the base containing that corner should be kept equally inclined to V.P; if the corner of the base is on V.P, the

sides of the base containing that corner should be kept equally inclined to H.P.

**Problem 6** Draw the projections of a hexagonal prism of base 25 side and axis 60 long, when it is resting on one of its corners of the base on H.P. The axis of the solid is inclined at  $45^\circ$  to H.P. Follow the change of position method.

**Construction (Fig.11.16)**



**Fig.11.16**

### **Stage I**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, keeping a side of the base

parallel to V.P.

The orientation chosen will permit the prism to be lying on one of its corners of the base on H.P, when it is tilted so as to make the axis inclined to H.P.

### **Stage II**

2. Redraw the front view such that, its axis makes  $45^\circ$  with xy. This forms the final front view.
3. Obtain the final top view, by projection and by following the rules of visibility.

For example, intersecting point between the horizontal projector through  $a_1$  and vertical projector through  $a'$  locates a in the final top view. Similarly, locate all the other points in the final top view.

In drawing the final views of the solids inclined to one or both the principal planes, the following rules of visibility and sequence may be observed:

1. Draw the lines of the edges of the visible base.  
 In the first angle projection, the base which is away from xy in one view will be fully visible in the other view.
2. Draw the lines corresponding to the longer edges of the solid.  
 The lines that pass through the visible base are invisible.
3. Draw the lines corresponding to the edges of the other base.  
 (i) It must be kept in mind that the lines corresponding to the boundary of the view must

be visible and hence must be represented by thick lines.

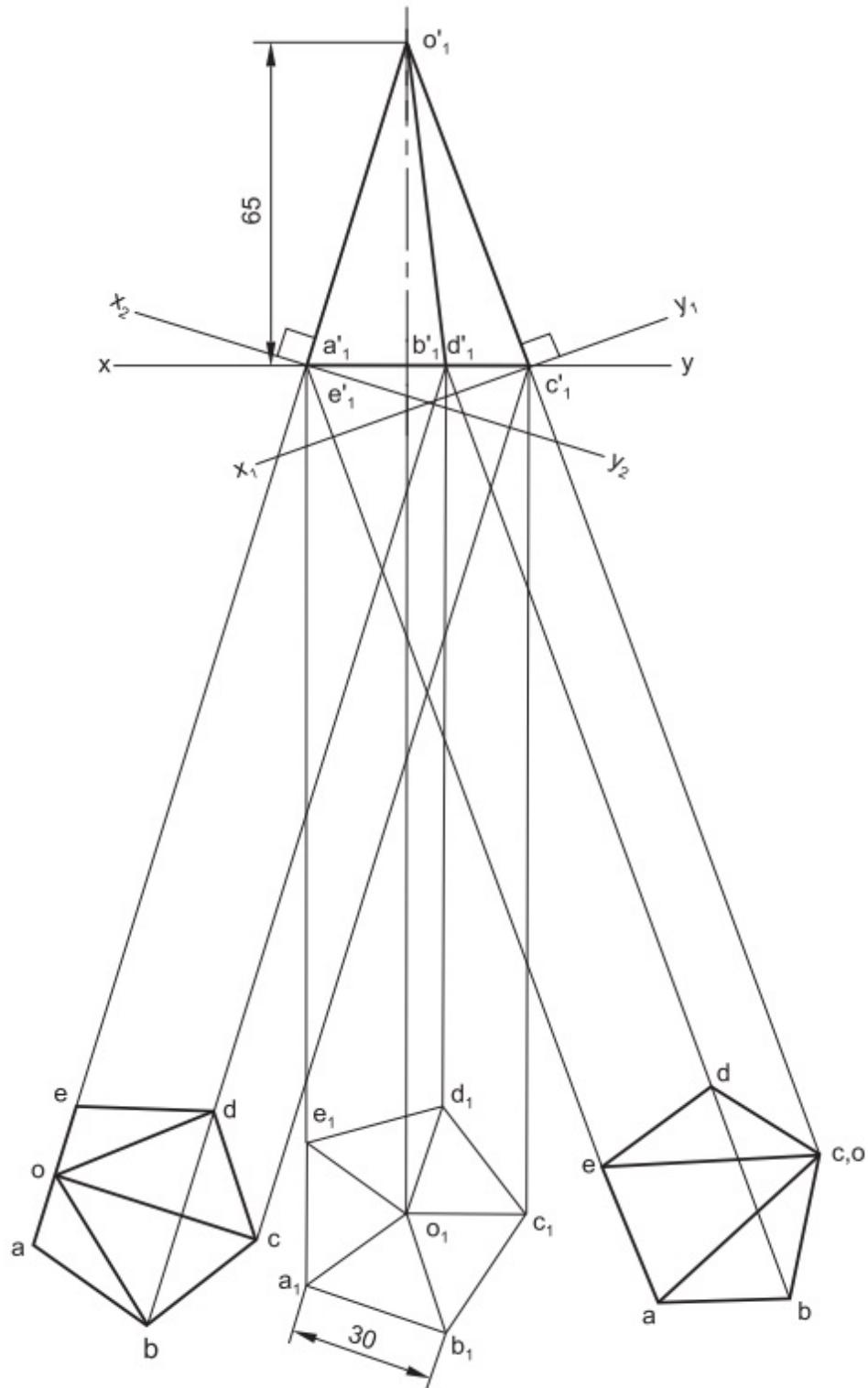
- (ii) When two lines representing the edges cross each other; one of them must be invisible and therefore, must be represented by dotted lines (refer Sec.19.3 for the rules of visibility).

**Problem 7** *Draw the projections of a pentagonal pyramid of side of base 30 and altitude 65, when (i) one of its sloping edges is vertical and (ii) one of its triangular faces is perpendicular to H.P. Follow the change of reference line (auxiliary plane) method.*

### **Construction ([Fig.11.17](#))**

#### **Stage I**

1. Draw the projections of the solid, assuming that it is resting on its base on H.P, keeping an edge of the base perpendicular to V.P.



**Fig.11.17**

**Stage II** One of its sloping edges vertical

2. Draw the reference line  $x_1y_1$ , representing auxiliary inclined plane; passing through  $c_1'$  in the front view and perpendicular to  $c_1'o'$ .
3. Draw projectors from all the points in the front view and perpendicular to  $x_1y_1$ .
4. On the above projectors, mark points, keeping the distance of each point from  $x_1y_1$  equal to its distance from  $xy$  in the top view and obtain the auxiliary top view, by joining the points.

The front view and the auxiliary top view with respect to  $x_1y_1$  are the required views.

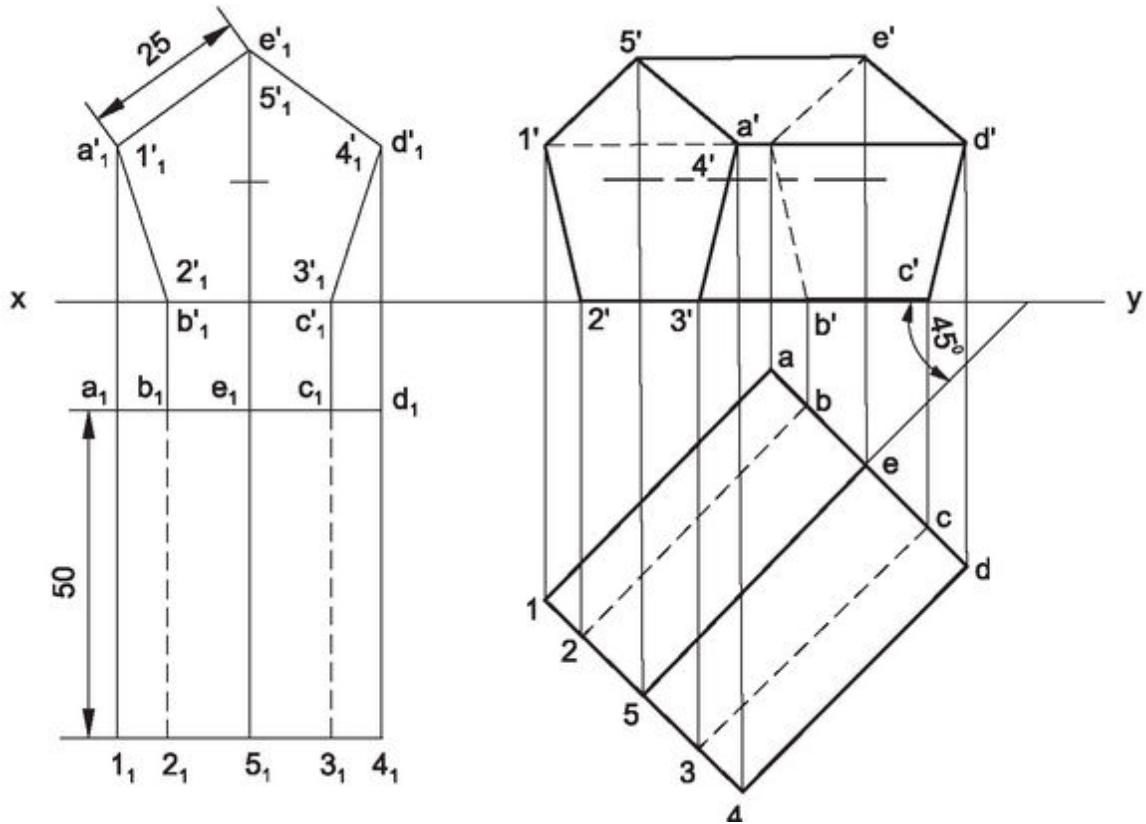
***Stage III One of its triangular faces perpendicular to H.P***

2. Draw the reference line  $x_2y_2$ , representing the auxiliary inclined plane; passing through  $a_1'$  ( $e_1'$ ) in the front view and perpendicular to  $a_1o_1'$ .
3. Obtain the auxiliary top view, by projection.

The front view and the auxiliary top view with respect to  $x_2y_2$  are the required views.

**Problem 8** *Draw the projections of a pentagonal prism of base 25 side and axis 50 long, when it is resting on one of its rectangular faces on H.P. The axis of the solid is inclined at 45° to V.P. Follow the change of position method.*

***Construction (Fig.11.18)***



**Fig.11.18**

### **Stage I**

1. Draw the projections of the solid, assuming that it is resting on one of its faces on H.P, with its axis perpendicular to V.P.

### **Stage II**

2. Redraw the top view such that, the axis makes  $45^\circ$  with xy, forming the final top view.
3. Obtain the final front view, by projection.

### **11.3.3 Axis Inclined to Both the Principal Planes**

A solid is said to be inclined to both the planes when, (i) the axis is inclined to both the planes or (ii) the axis is inclined to one plane and an edge of the base is inclined to the other. In all such cases, the final projections are obtained in three stages.

**Stage I** Assume that the axis is perpendicular to one of the planes and draw the projections.

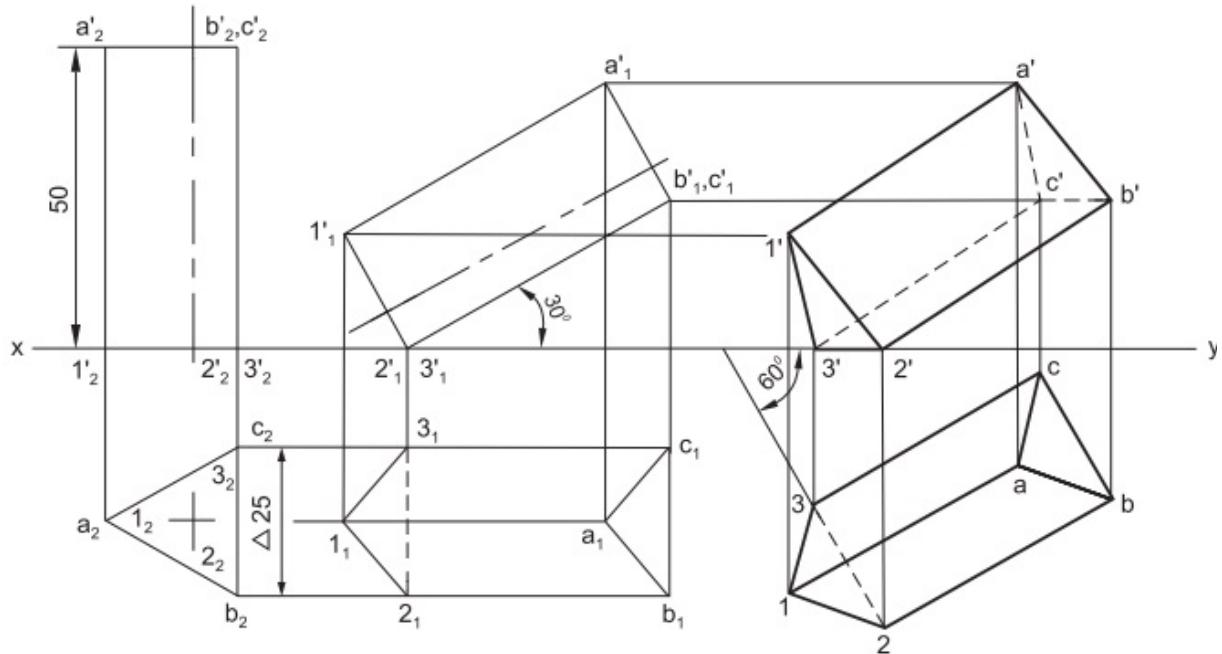
**Stage II** Redraw one of the views, by making the axis inclined to one of the planes and project the other view from it.

**Stage III** Redraw one of the views obtained in stage II, satisfying the remaining condition and project the other view from it.

Stages II and III may also be drawn by the use of auxiliary plane method.

**Problem 9** An equilateral triangular prism of side of base 25 and axis 50 long, is resting on an edge of its base on H.P. The face containing that edge is inclined at  $30^\circ$  to H.P. Draw the projections of the prism, when the edge on which the prism rests, is inclined at  $60^\circ$  with V.P. Follow the change of position method.

**Construction (Fig.11.19)**



**Fig.11.19**

**Stage I** Assume that the solid is resting on its base on H.P. with one edge perpendicular to V.P.

1. Draw the projections of the solid.

**Stage II** Tilt the solid about the edge, which is perpendicular to V.P such that, the face containing the edge makes  $30^\circ$  to H.P.

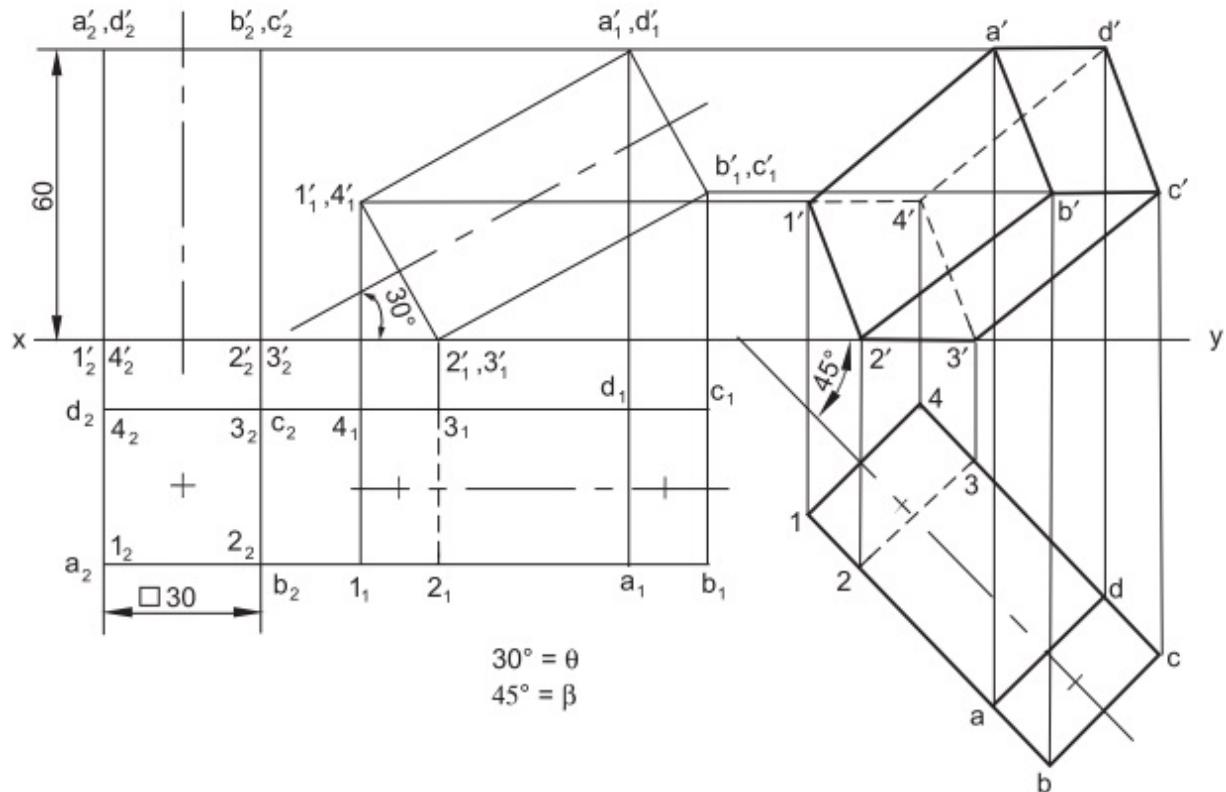
2. Redraw the front view such that, the front view of the face 23BC is inclined at  $30^\circ$  to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.

**Stage III** Rotate the solid, till the edge on which it rests is inclined at  $60^\circ$  to V.P.

4. Redraw the above top view such that, the top view of the edge 2-3 is inclined at  $60^\circ$  to xy. This forms the final top view.
5. Obtain the final front view, by projection.

**Problem 10** Draw the projections of a square prism, side of base 30 and axis 60 long, resting with one of the edges of its base on H.P. Its axis is inclined at  $30^\circ$  to H.P and the top view of the axis at  $45^\circ$  to xy line.

**Construction:** (**Fig.11.20**)



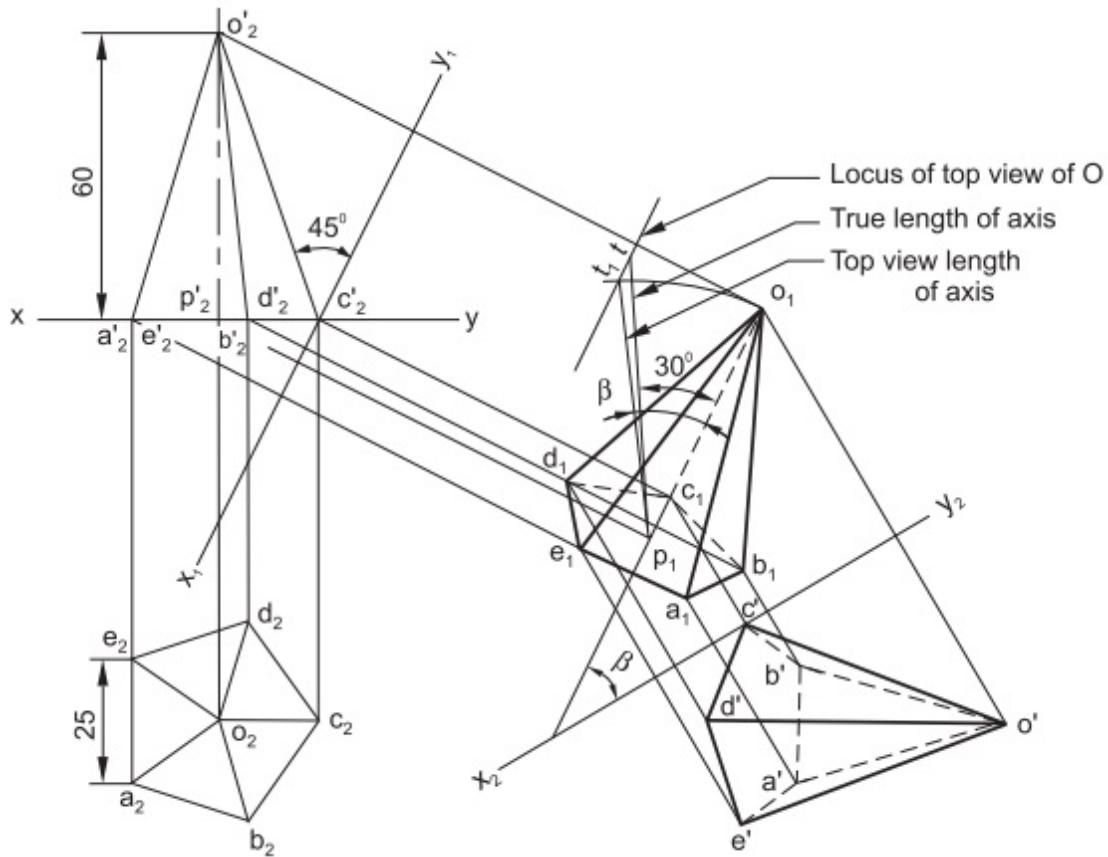
**Fig.11.20**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with one edge perpendicular to V.P.
2. Redraw the front view such that, the axis is inclined at  $30^\circ$  to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.
4. Redraw the above top view such that, the axis is inclined at  $45^\circ$  to xy. This forms the final top view.

5. Obtain the final front view, by projection.

**Problem 11** A pentagonal pyramid of edge of base 25 and height 60, is resting on a corner of its base on H.P and the slant edge containing that corner is inclined at  $45^\circ$  with H.P. Draw the projections of the solid, when its axis makes an angle of  $30^\circ$  with V.P. Follow the auxiliary plane method.

**Construction (Fig.11.21)**



**Fig.11.21**

**Stage I** Assume that the solid is resting on its base on H.P, with an edge of the base perpendicular to V.P (two of the adjacent base edges are equally inclined to V.P).

1. Draw the projections of the solid.

**Stage II** Tilt the solid till it rests on a corner of the base on H.P and the slant edge through the resting corner is inclined at  $45^\circ$  with H.P.

2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane, passing through  $c_2'$  and making an angle of  $45^\circ$  with  $c_2o_2'$ .
3. Obtain the auxiliary top view (final top view), by projection.

**Stage III** Rotate the solid till the axis of the solid makes  $30^\circ$  with V.P.

4. Determine the apparent angle of inclination  $\beta$ , the axis making with the reference line.
5. Draw the reference line  $x_2y_2$ , representing the auxiliary vertical plane, making the angle  $\beta$  with the axis in the top view.
6. Obtain the final front view, by projection.



To determine the apparent angle  $\beta$  (ref. auxiliary top view),

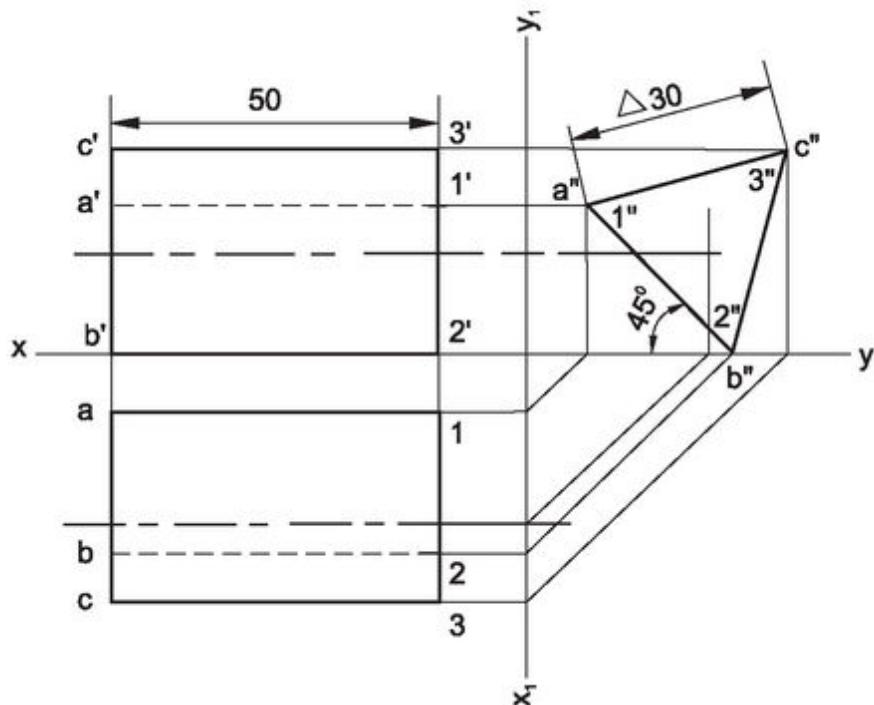
- (i) Through  $p_1$ , draw the line  $p_1t$ , equal to the true length of the axis of the solid and making an angle of  $30^\circ$  with  $p_1o_1$ .
- (ii) Through  $t$ , draw a line parallel to  $p_1o_1$ , representing the locus of the apex in the top view.
- (iii) With  $p_1$  as centre and  $p_1o_1$  as radius, draw an arc intersecting the above locus at  $t_1$ .  
 $\angle o_1p_1t_1$  is the apparent angle.

## 11.4 THREE - VIEW DRAWINGS

The number of views required to describe the shape of any object will depend upon the complexity of it. In some cases, three views are required to describe the shape of an object. To understand the principles of projection better, the students are advised to practice the addition of the side view to all the cases of projection of solids, dealt with under the preceding section. However, one case is considered here, where the solution starts with the side view.

**Problem 12** A triangular prism of side of base 30 and axis 50 long, is resting on a longer edge on H.P and a face containing that edge is inclined at  $45^\circ$  with H.P. Draw the projections of the solid, when its axis is parallel to both H.P and V.P.

**Construction (Fig.11.22)**



**Fig.11.22**

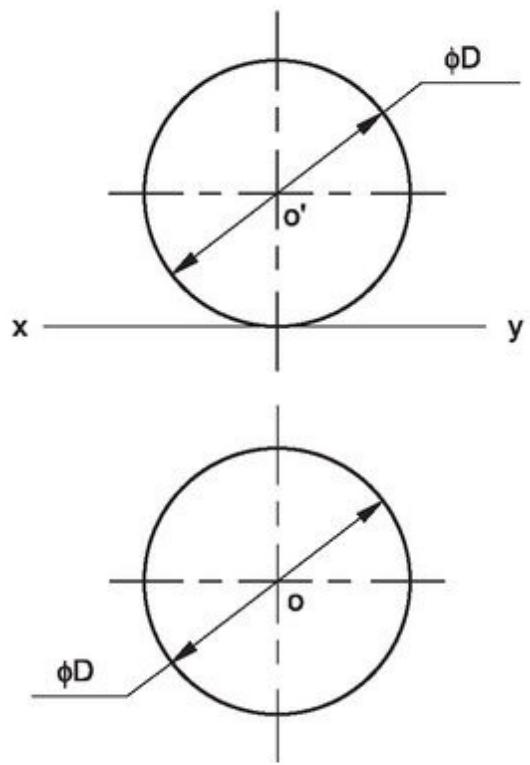
1. Draw the reference lines  $xy$  and  $x_1y_1$ .
2. Draw the side view  $a''b''c''$  such that, the corner  $b''(2'')$  is on  $xy$  and the side of the face  $1AB2$  is inclined at  $45^\circ$  with  $xy$ .
3. Obtain the front and top views, by projection.

## 11.5 PROJECTIONS OF SPHERES

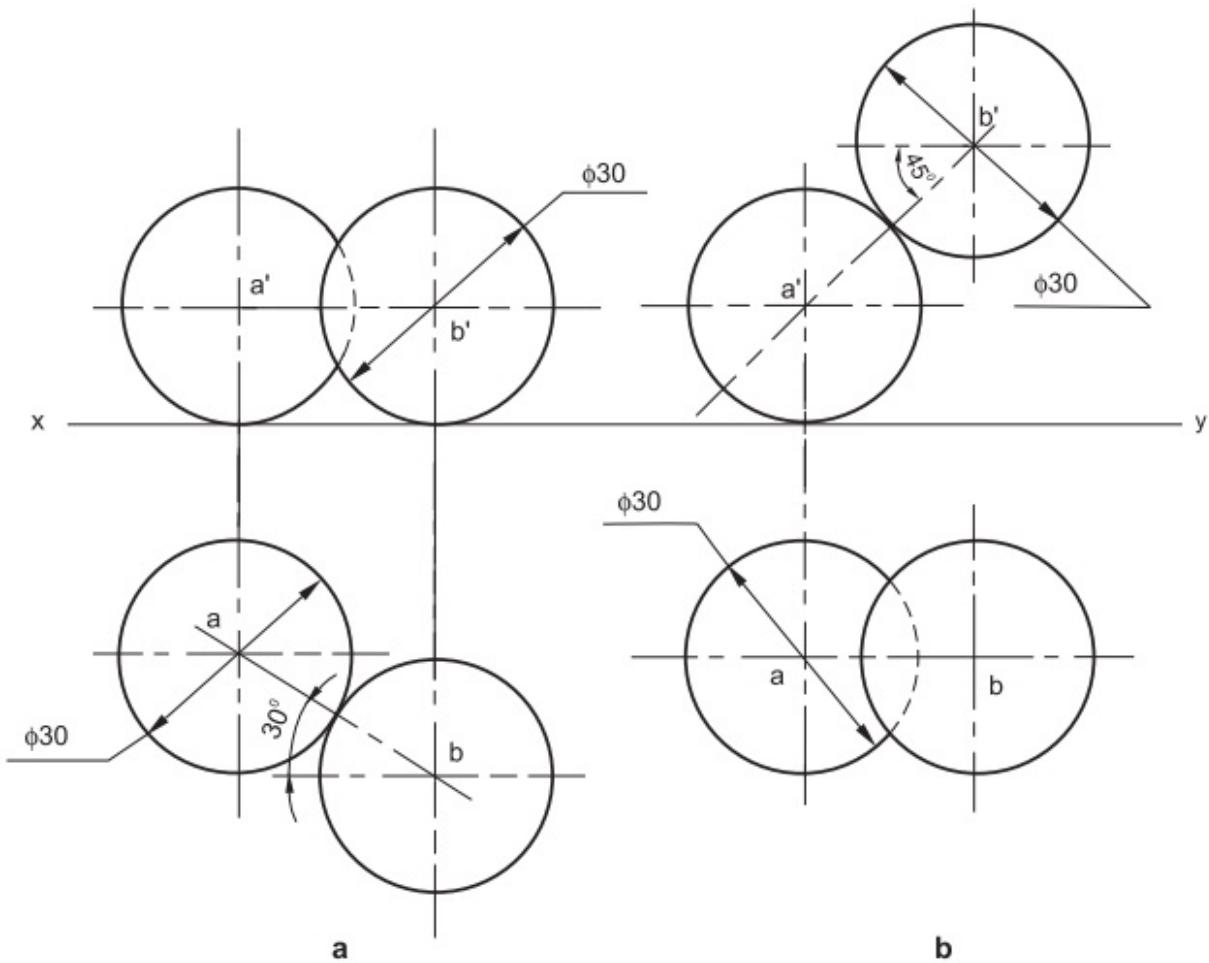
The projection of a sphere in any position and on any plane is always a circle of diameter equal to the diameter of the sphere itself. [Figure 11.23](#) shows the projections of a sphere, lying on H.P.

**Problem 13** *Draw the projections of two equal spheres of 15 radius, resting on H.P and in contact with each other so that the line joining their centres is inclined at (i)  $30^\circ$  to V.P and (ii)  $45^\circ$  to H.P.*

**Construction ([Fig.11.24a](#))**



**Fig.11.23**



**Fig.11.24**

1. With centers  $a'$  and  $a$  and radius 15, draw circles representing the projections of a sphere, say A.
2. Locate the centre  $b$  of the other sphere B in the top view such that, the line  $ab$  joining the centres, make an angle of  $30^\circ$  with  $xy$ .
3. Locate the centre  $b'$  in the front view, by projection.  
The line  $a'b'$  is parallel to  $xy$ , as the spheres are equal in size and are lying on H.P.
4. Draw the projections of the sphere B.
  - (i) The two circles, representing the top views

of the two spheres are fully visible.

- (ii) In the front view, sphere B is fully visible and sphere A is partially visible.

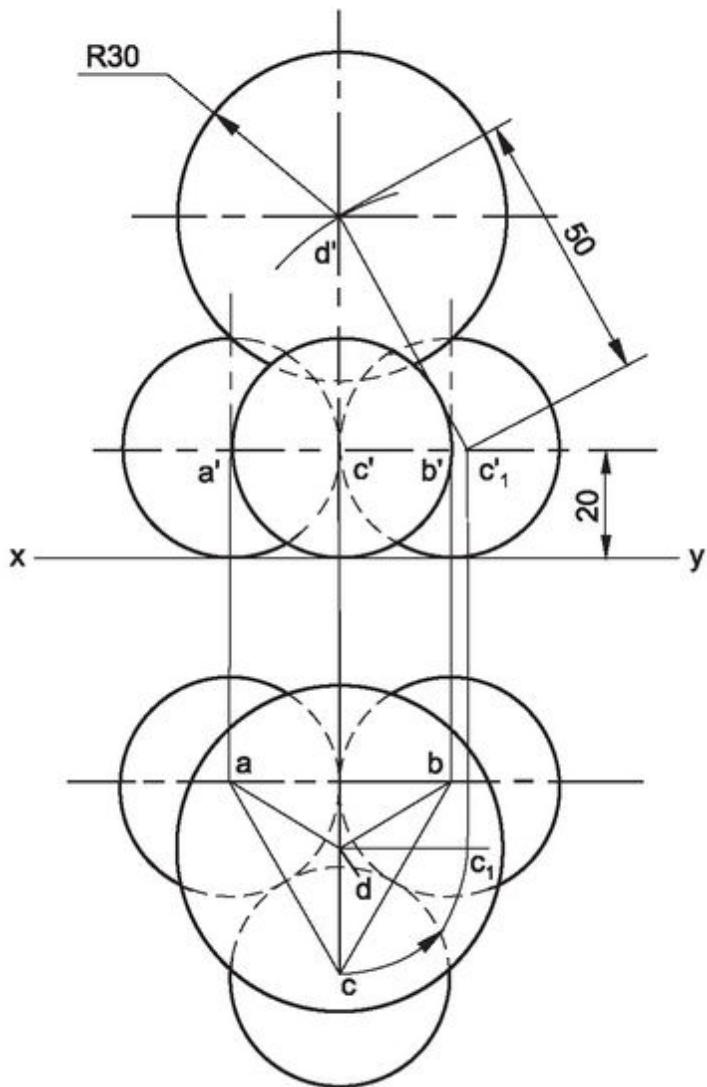
Figure 11.24b shows the projections of the spheres, when the line joining their centres is inclined at  $45^\circ$  to H.P.

**Problem 14** Three equal spheres of 40 diameter are lying on H.P such that, each touches the other two and the line joining the centres of two spheres is parallel to V.P. A fourth sphere of diameter 60 is placed on top of the three spheres. Draw the projections of the arrangement.

### **Construction (Fig.11.25)**

All the three spheres are equal in size and are resting on H.P. Hence the line joining the centres will be parallel to xy in the front view. In the top view, the centres will be lying at the corners of an equilateral triangle of side equal to 40 (twice the radius of the sphere).

1. In the top view, locate the centres of spheres a, b and c at the corners of an equilateral triangle abc of 40 side, with one side, say ab parallel to xy.
2. In the front view, locate the centres a', b' and c', by projection and on a line parallel to and 20 above xy.



**Fig.11.25**

3. With centres a, b, c, a', b' and c' and radius 20, draw circles, forming the projections.

When the fourth sphere is placed on top of the three spheres, its centre d will lie above the centre of the triangle abc, while a, b, c and d form the corners of a triangular pyramid.

*To locate d'*

- (i) Rotate dc parallel to xy to  $dc_1$ .
  - (ii) Through  $c_1$ , draw a projector meeting the line of centres in the front view at  $c_1'$ .
  - (iii) With  $c_1'$  as centre and radius  $50 = (20 + 30)$ , draw an arc intersecting the projector through d at  $d'$ .
4. With  $d'$  and d as centres and radius 30, draw circles.
  5. Follow the rules of visibility and complete the projections.

### 11.5.1 Projections of Unequal Spheres

When two unequal spheres are on H.P and in contact with each other, the length of the line joining the centres will be seen in its true length in the front view, if the line is parallel to V.P. In the top view, the length of the line will be shorter but remains constant, even when it is inclined to V.P.

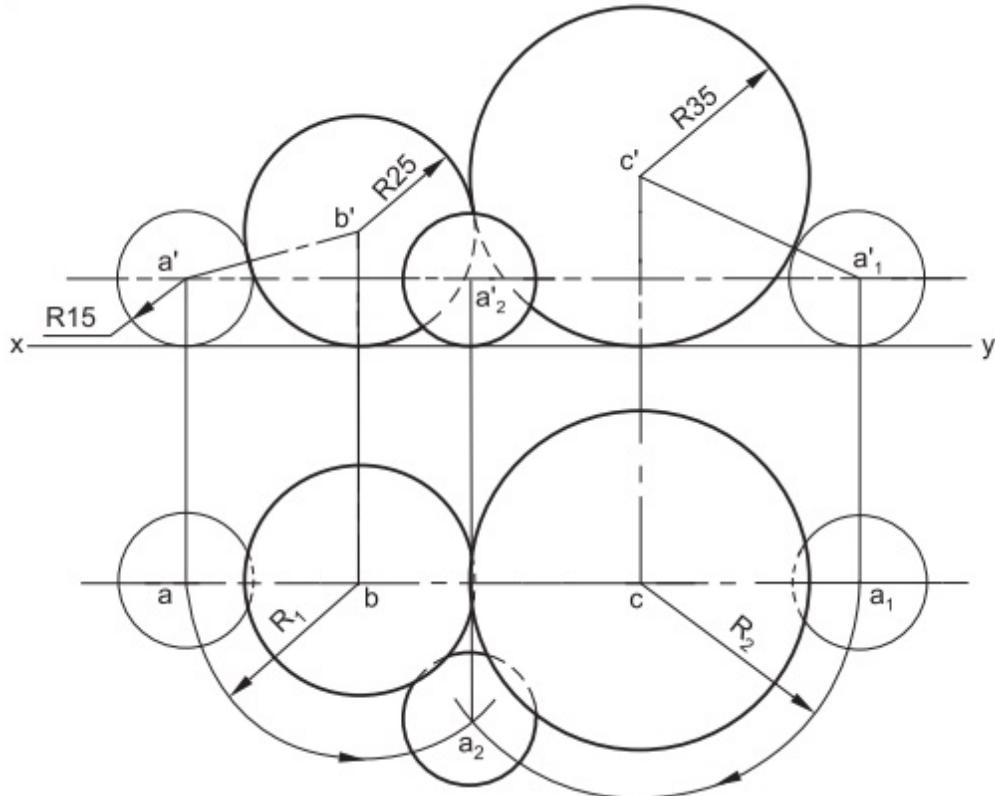
**Problem 15** *Three spheres A, B and C of 30, 50 and 70 diameters are placed on H.P such that, each one touches the other two. Draw the projections of the arrangement, when the line joining the centres of the spheres B and C is parallel to V.P.*

#### **Construction (Fig.11.26)**

1. Draw the projections of the spheres B and C, touching each other such that, the line joining their centres is parallel to V.P.
2. Draw the projections of the spheres A-B and C-A, touching each other, assuming that the lines joining their centres are parallel to V.P. Determine the top view lengths of ba and  $ca_1$ . The line joining  $a'$  and  $a_1'$  is

the locus of the centre of the sphere A in the front view.

3. With b as centre and radius  $ba$ , draw an arc.
4. With c as centre and radius  $ca_1$ , draw an arc intersecting the above arc at  $a_2$ .
5. With  $a_2$  as centre, draw the top view of the sphere A.
6. Draw a projector through  $a_2$ , meeting the locus at  $a'_2$ .  
 $a'_2$  is the centre of the sphere A in the front view.
7. With  $a'_2$  as centre, draw the front view of the sphere A.

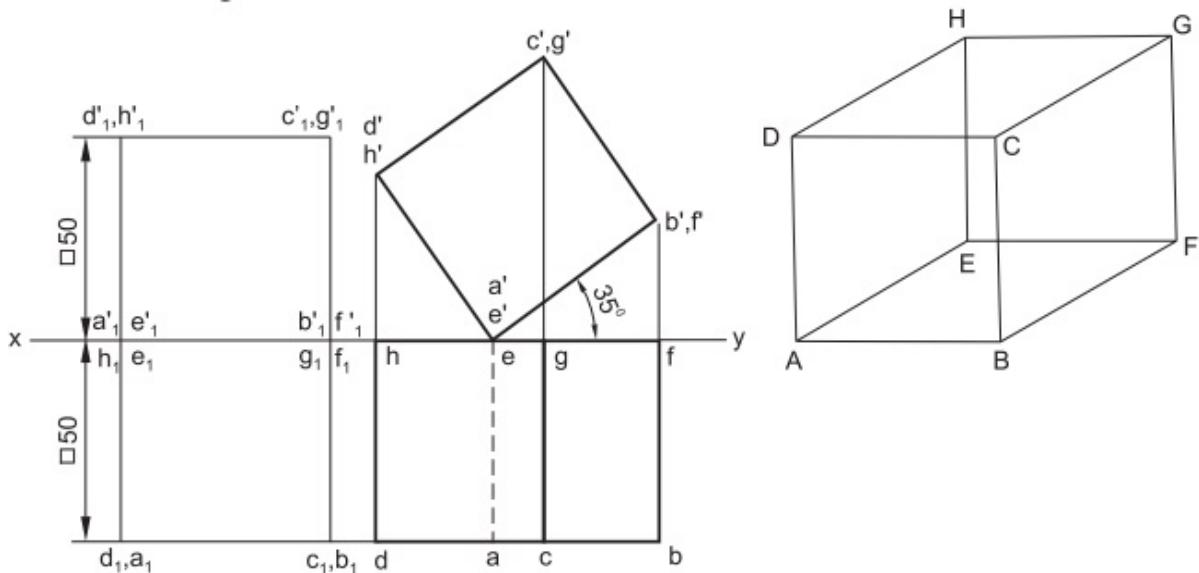


**Fig.11.26**

## 11.6 EXAMPLES

**Problem 16** A cube of 50 side, has one face on V.P and an adjacent face inclined at  $35^\circ$  to H.P; the lower edge of the later face being on H.P. Draw its projections.

**Construction (Fig.11.27)**



**Fig.11.27**

1. Draw the projections of the cube, assuming it to be lying on a face on V.P and the adjacent lower face on H.P.
2. Redraw the front view such that, the face lying on H.P is inclined at  $35^\circ$  to xy and the front view  $a'e'$  of the lower edge of the inclined face is on xy. This is the final front view.
3. Obtain the final top view, by projection.

**Problem 17** A triangular pyramid of base 30 side and axis 50 long, is resting on H.P on its base, with a face perpendicular to V.P. Draw the projections of the pyramid.

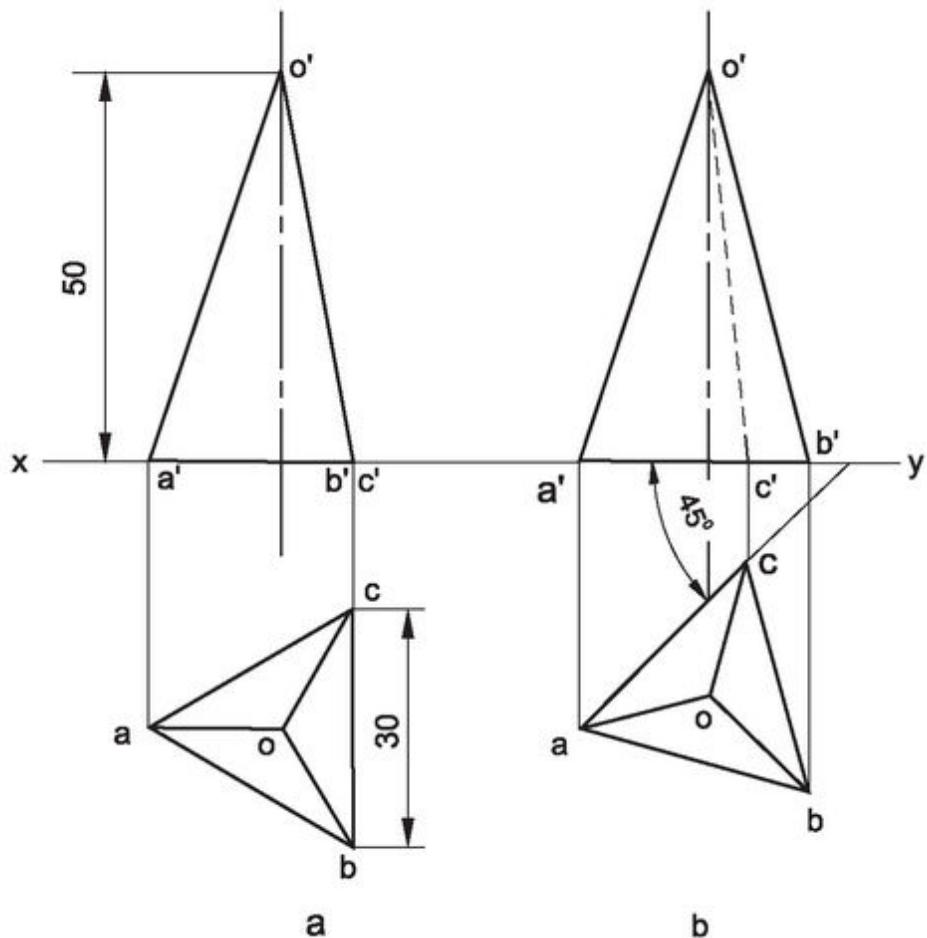
**Construction (Fig.11.28a)**

1. Draw the top view of the pyramid, keeping one edge of the base perpendicular to xy.

It may be noted that triangle abc represents the top view and the lines oa, ob and oc meeting at the centre of the triangle, represent the slant edges of the pyramid.

2. Obtain the front view such that, the base a'b'c' lies on xy and the apex o' is at 50 from xy.

**Figure 11.28b** shows the projections of the pyramid, when an edge of the base is inclined at  $45^\circ$  to V.P.



**Fig.11.28**

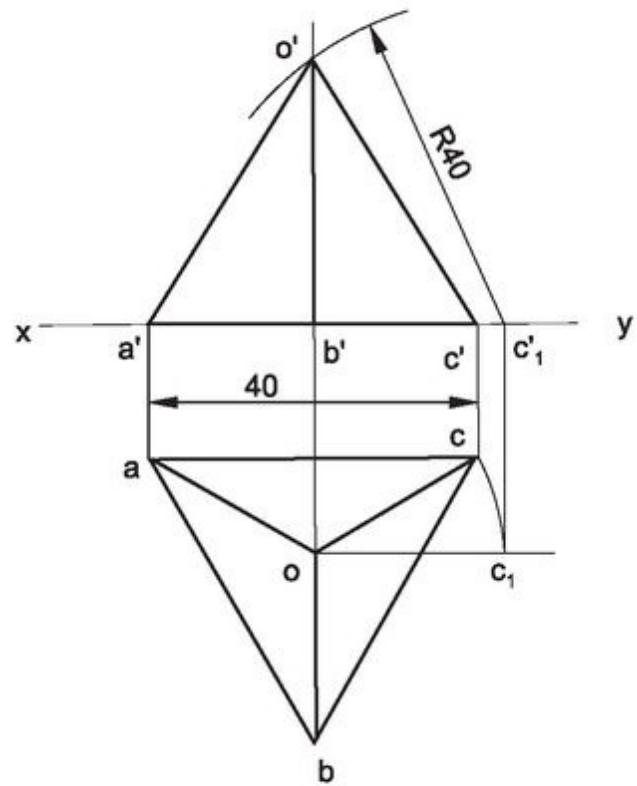
**Problem 18** A tetrahedron of 40 long edges, is resting on H.P on one of the faces, with an edge of that face parallel to V.P. Draw the projections of the solid.

**Construction (Fig.11.29)**

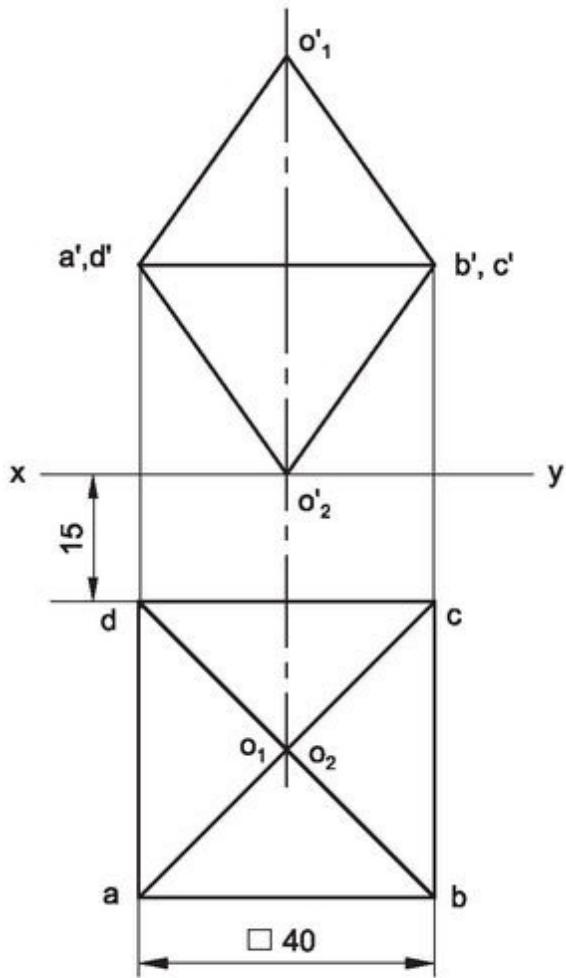
1. Draw an equilateral triangle abc of side 40, with one side, say ac parallel to xy. Locate its centre o and join it with the corners, forming the top view of the tetrahedron.
2. Project and obtain the front view  $a'b'c'$  of the face ABC, coinciding with xy.
3. Through o, draw a projector.

*To locate  $o'$  in the front view*

- (i) With o as centre, rotate, say oc to  $oc_1$ , parallel to xy.
- (ii) Project  $c_1$  to  $c'_1$  on xy.
- (iii) With  $c'_1$  as centre and radius 40, draw an arc intersecting the projector through o at  $o'$ .
- (iv) Join  $o', a'; o', b'$  and  $o', c'$ , forming the front view.



**Fig.11.29**



**Fig.11.30**

**Problem 19** An octahedron of side 40, is resting with one of its corners on H.P. The axis passing through this corner is vertical and one of its horizontal edges is parallel to and 15 in front of V.P. Draw the projections of the solid.

**Construction (Fig.11.30)**

1. Draw a square abcd of side 40, with one side, say cd parallel to and 15 below xy. Locate its centre  $o_1$  ( $o_2$ ) and join it with the corners, forming the top view of the solid.

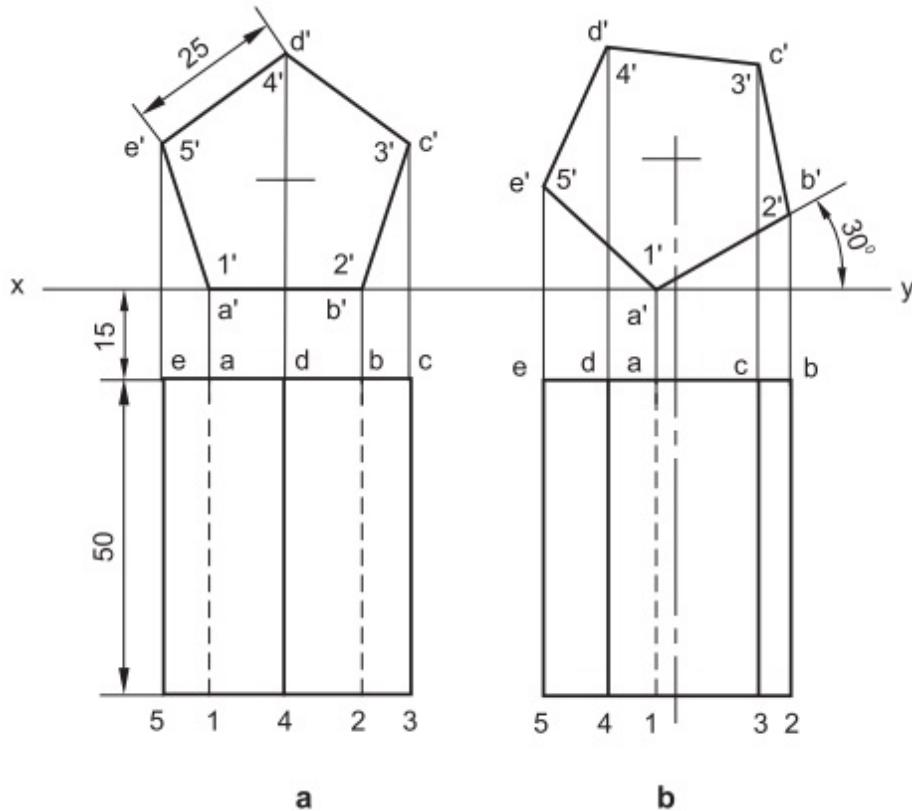
2. Through  $o_1$  ( $o_2$ ), draw a projector and locate  $o_2'$  on  $xy$ .

The lengths,  $ac$  and  $bd$  represent the true lengths of the diagonals  $AC$  and  $BD$ , as these diagonals are parallel to H.P. Hence,

3. Mark  $o_1'$  on the projector through  $o_1$  ( $o_2$ ) such that,  $o_1'o_2' = ac = bd$ .
4. Through the mid-point of  $o_1'o_2'$ , draw a horizontal line.
5. Locate the points  $a'$  ( $d'$ ) and  $b'$  ( $c'$ ), by projection.
6. Join  $a'$  ( $d'$ ) and  $b'$  ( $d'$ ) with  $o_1'$  and  $o_2'$ , forming the front view.

**Problem 20** A pentagonal prism of side of base 25 and axis 50 long, is resting on one of its faces on H.P. with the axis perpendicular to V.P and a base 15 away from V.P. Draw its projections.

**Construction (Fig.11.31a)**



**Fig.11.31**

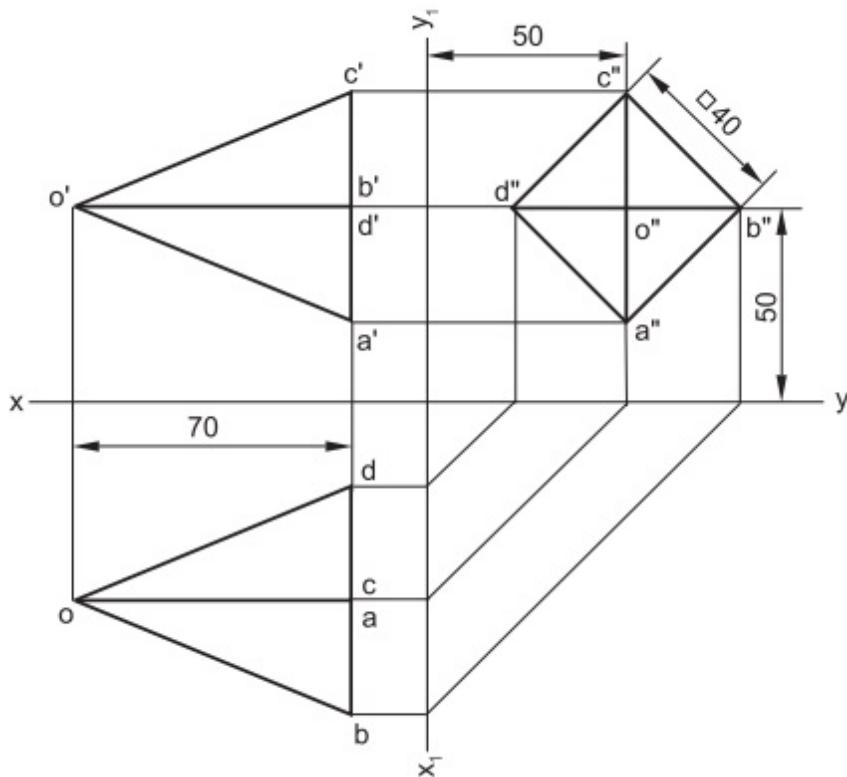
1. Draw the front view  $a'b'c'd'e'$ , a pentagon of side 25, keeping the edge  $a'b'$  on xy.
2. Project the top view such that, the base abcde is at 15 from xy and of length 50.

Figure 11.31b shows the projections of the prism, when it is resting on a lateral edge on H.P and a rectangular face containing that edge is inclined at  $30^\circ$  to H.P.

**Problem 21** A square pyramid of base 40 and altitude 70, lies with all the edges of the base equally inclined to H.P and the axis parallel to and 50 from both H.P and V.P. Draw its projections.

**Construction (Fig.11.32)**

1. Draw the side view of the pyramid such that, the base edges are equally inclined to H.P.
2. Draw the reference lines  $xy$  and  $x_1y_1$  such that, they are at 50 from the centre,  $o''$  of the side view.
3. Obtain the front and top views, by projection; keeping the length of the axis as 70.



**Fig.11.32**

**Problem 22** Draw the projections of a cylinder of 40 diameter and axis 60 long, when it is lying on H.P with its axis inclined at  $45^\circ$  to H.P and parallel to V.P. Follow the change of position method.

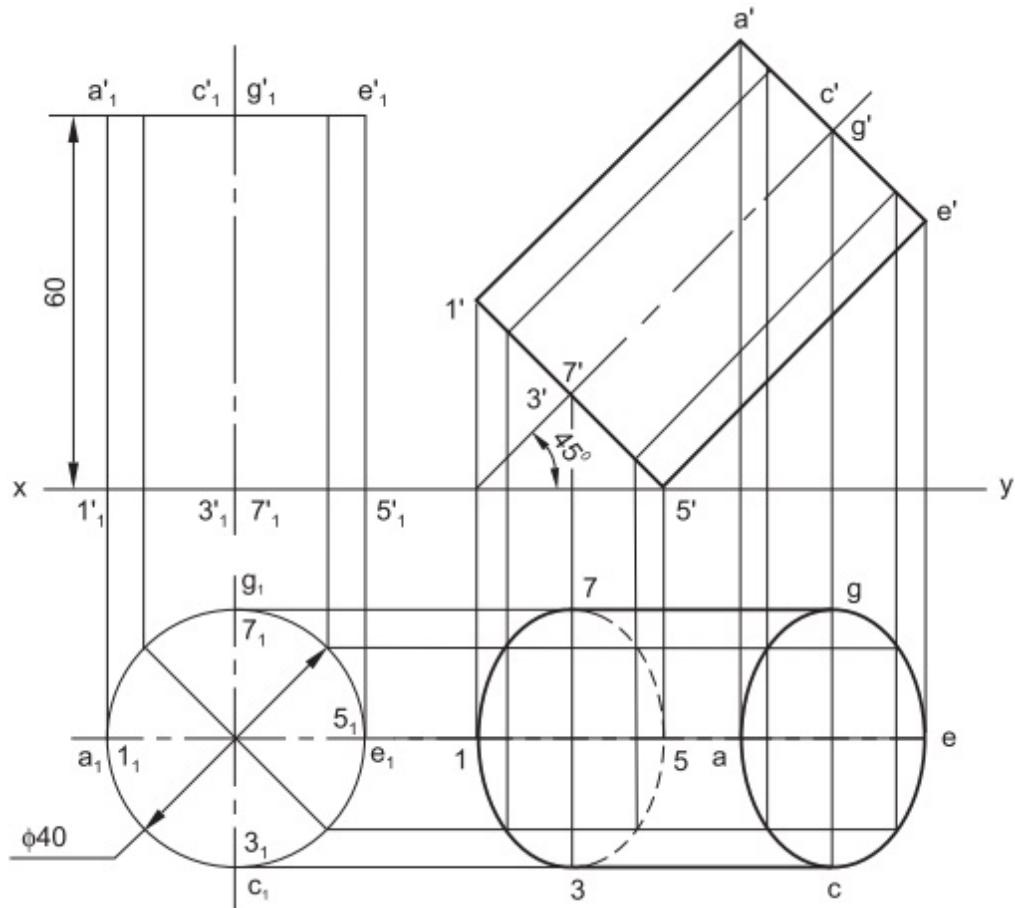
**Construction (Fig.11.33)**

1. Draw the projections of the cylinder, assuming that it is lying on its base on H.P.

2. Divide the circle (top view) into a number of equal parts, say 8 and draw the corresponding generators in the front view.
3. Redraw the front view such that, it rests on a point of its base on xy and the axis is inclined at  $45^\circ$  to xy.
4. Obtain the final top view, by projection.



- (i) In the final top view, the axis is parallel to xy.
- (ii) The students are advised to divide the circle into twelve equal parts, so that more number of points may be obtained to draw smooth curves in the final top view.
- (iii) The generators are only imaginary lines and so they must be represented by thin lines.



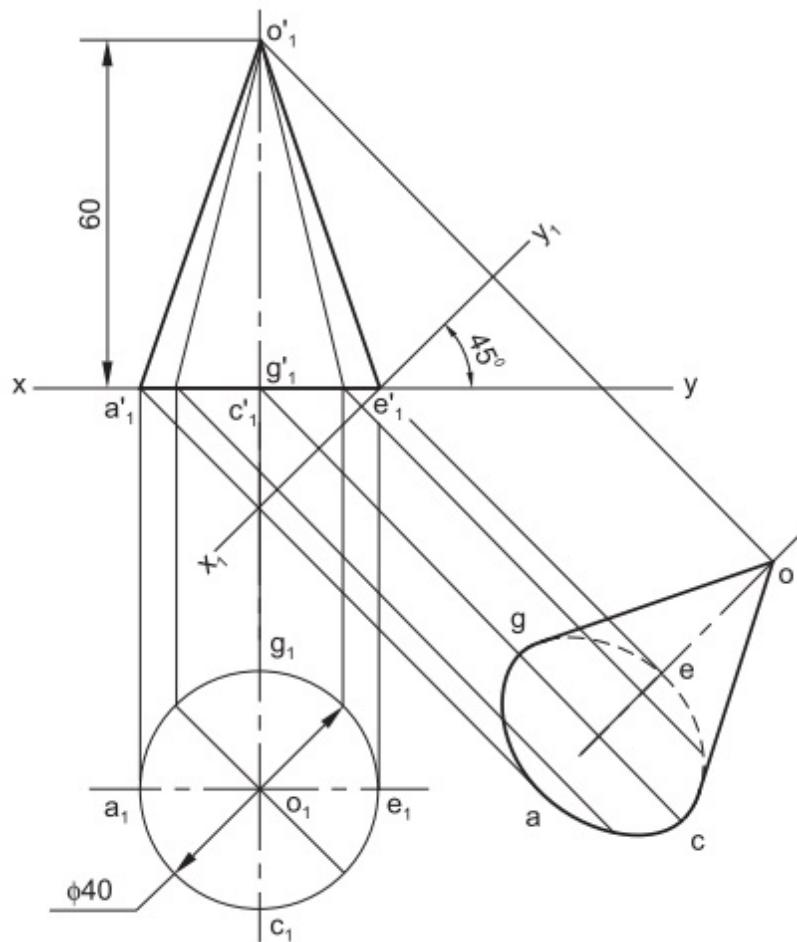
**Fig.11.33**

**Problem 23** Draw the projections of a cone of diameter of base 40 and axis 60 long, when it is lying on a point of the base on H.P. with its axis inclined at  $45^\circ$  to H.P and parallel to V.P. Follow the auxiliary plane method.

**Construction (Fig.11.34)**

1. Draw the projections of the cone, assuming that it is resting on its base on H.P.
2. Divide the circle (top view) into a number of equal parts and draw the corresponding generators in the front view.

- Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane and passing through the lowest point  $e'_1$  of the extreme generator, making an angle of  $45^\circ$  with the axis.
- Obtain the final (auxiliary) top view, by projection.

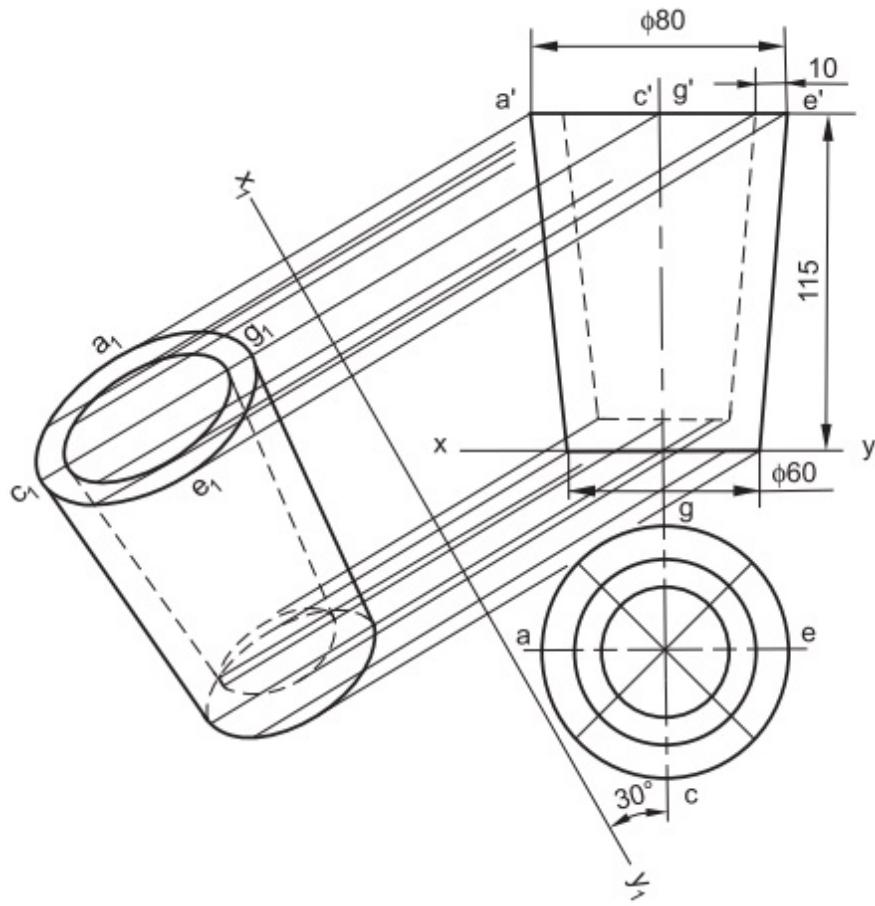


**Fig.11.34**

**Problem 24** A conical vessel is open at the top, 115 high; 60 diameter at the bottom and 80 diameter at the top (outside); thickness of the wall being 10. The vessel is kept on a table such that, the axis makes an angle of  $30^\circ$  with H.P. Draw its projections.

**Construction (Fig.11.35)**

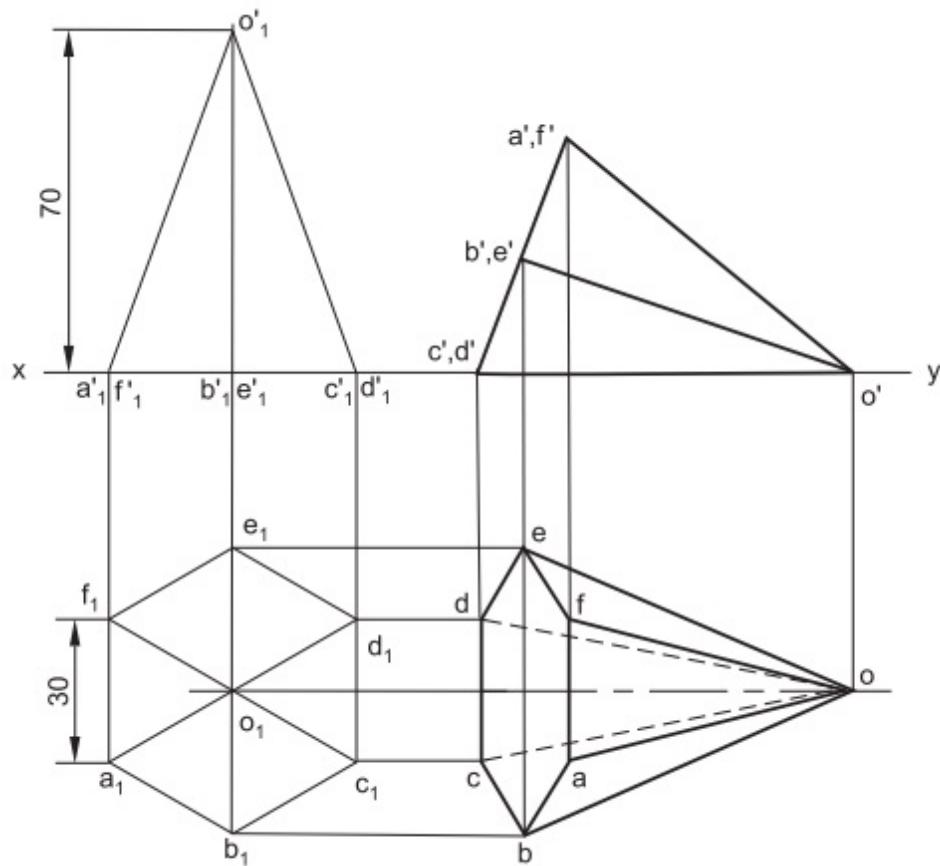
1. Draw the projections of the conical vessel, assuming it to be lying on H.P on its bottom base.
2. Divide the circles in the top view into a number of equal parts, say 8 and draw the corresponding generators in the front view (all the points are not named and all the generators are not shown for clarity sake).
3. Draw the reference line  $x_1y_1$ , corresponding to an A.I.P, making an angle of  $30^\circ$  with axis.
4. Obtain the final (auxiliary) top view, by projection.



**Fig.11.35**

Problem 25 Draw the projections of a hexagonal pyramid, with side of base 30 and axis 70 long, which is resting with

a slant face on H.P such that, the axis is parallel to V.P.  
Follow the change of position method.



**Fig.11.36**

### **Construction (Fig.11.36)**

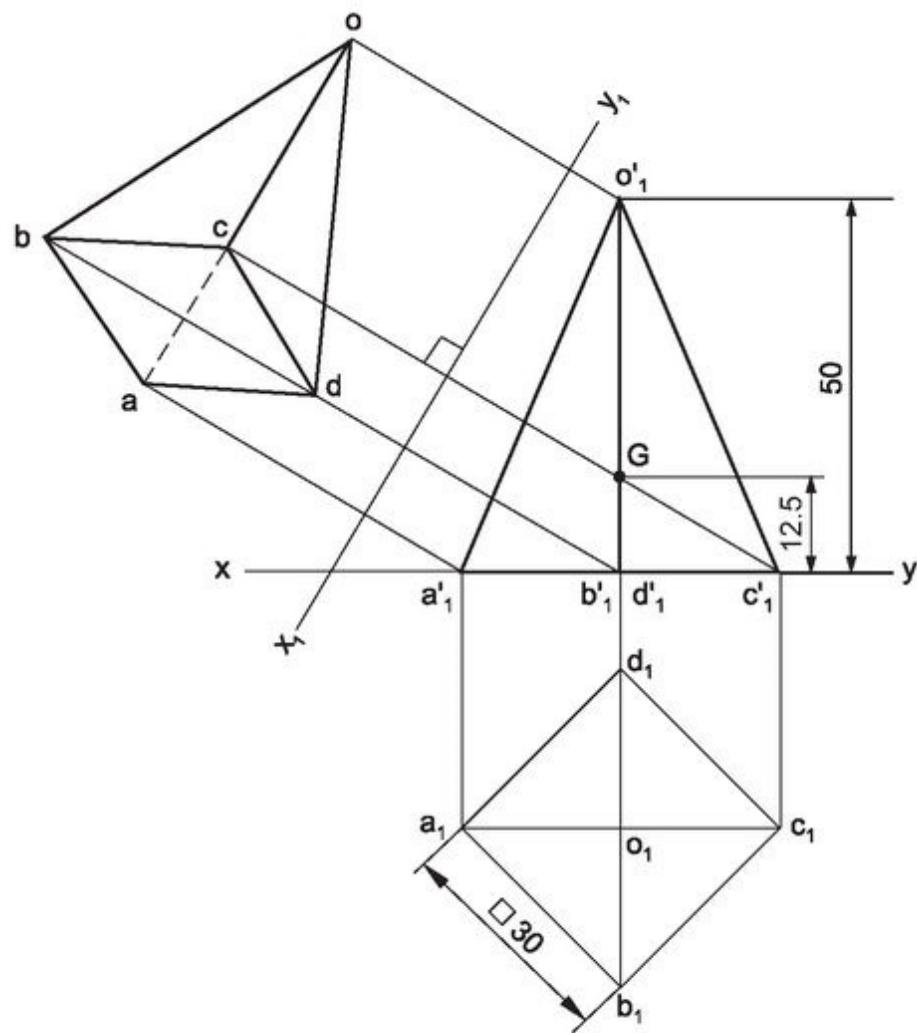
1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P, with two edges of the base perpendicular to V.P.
2. Redraw the front view such that, the line  $o'c'$  ( $d'$ ), representing front view of the slant face OCD, coincides with xy.
3. Obtain the final top view, by projection.

**Problem 26** A square pyramid of side of base 30 and axis 50 long, is freely suspended from a corner of its base. Draw

the projections. Follow the auxiliary plane method.

**HINT** When a solid is freely suspended from a corner, the imaginary line passing through that corner and the centre of gravity of the solid will be vertical. It is obvious that the centre of gravity of a prism and a cylinder will lie at the midpoint of the axis. However, for a pyramid and a cone, it lies at  $1/4^{\text{th}}$  of the length of the axis from the base.

**Construction (Fig.11.37)**



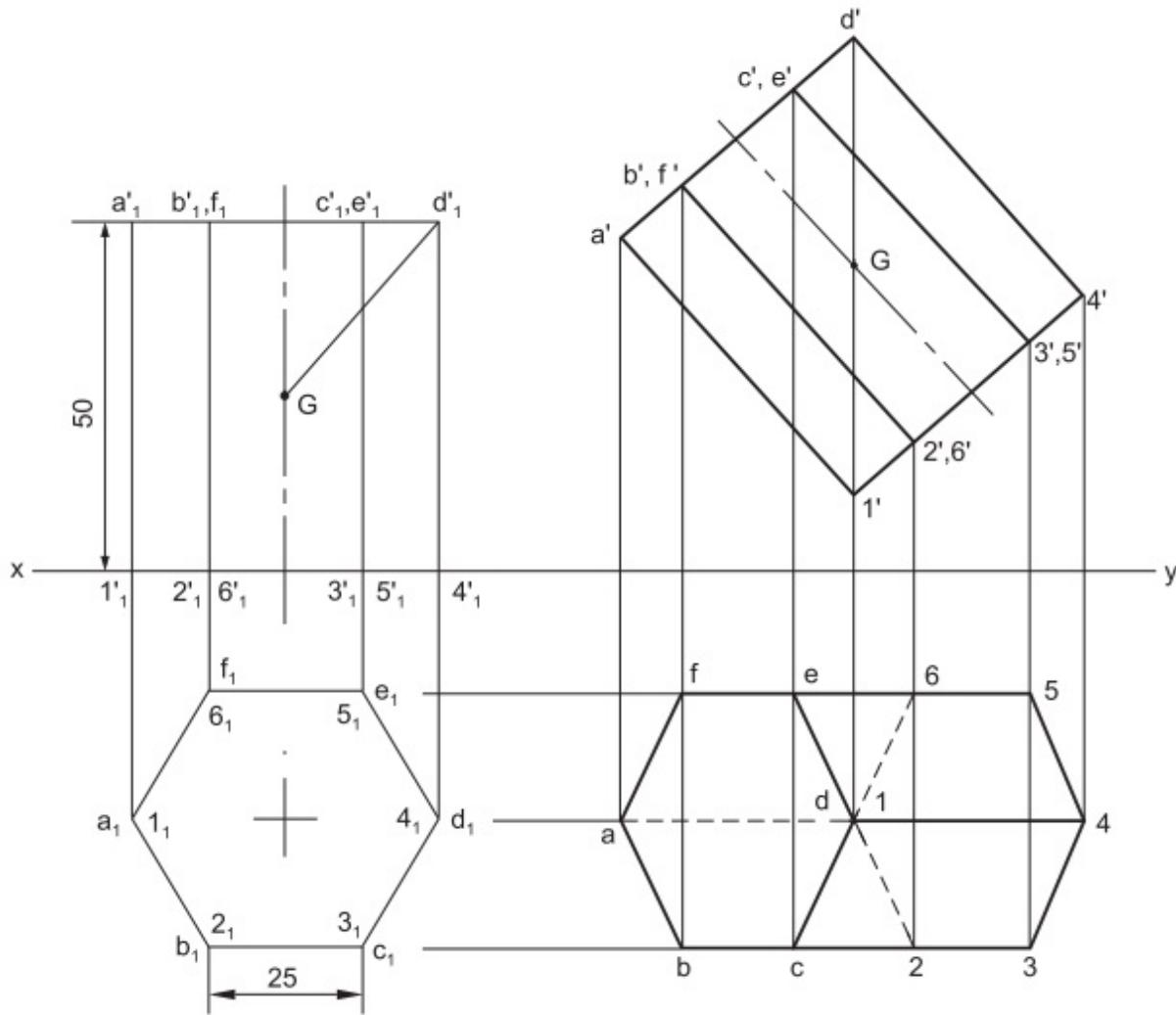
**Fig.11.37**

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P, with the base edges equally inclined to V.P.
2. Locate the centre of gravity G, on the axis at 12.5 from the base.
3. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane and perpendicular to  $c_1'G$ .
4. Obtain the final (auxiliary) top view, by projection.

**Problem 27** A hexagonal prism of side of base 25 and axis 60 long is freely suspended from a corner of the base. Draw the projections, by the change of position method.

**Construction (Fig. 11.38)**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with an edge of the base parallel to V.P.
2. Locate the centre of gravity G, at the mid-point of the axis.
3. Redraw the front view such that, the line passing through, say  $d_1'G$  is perpendicular to  $xy$ .
4. Obtain the final top view, by projection.



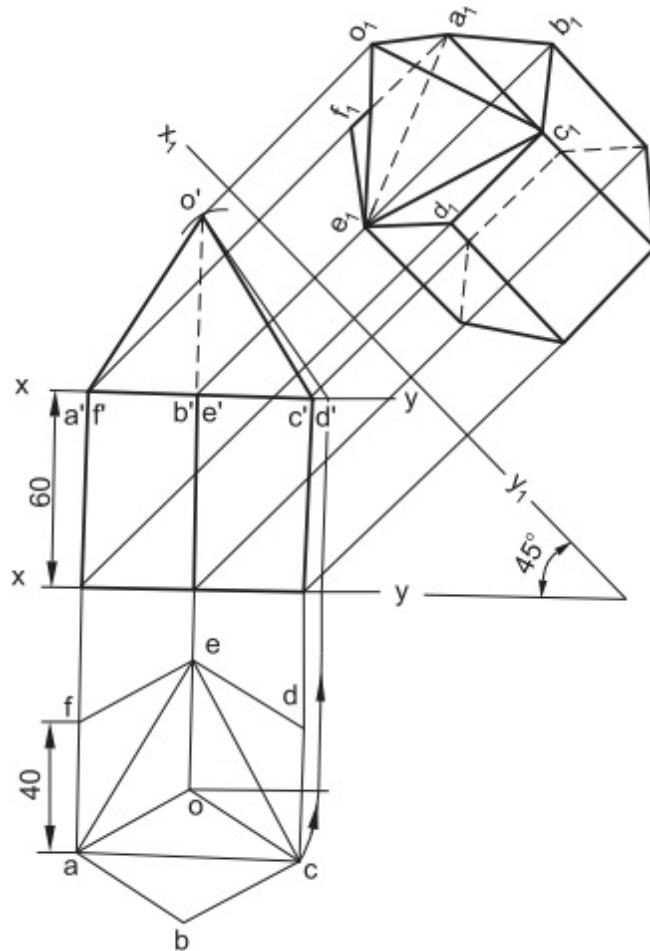
**Fig.11.38**

**Problem 28** A hexagonal prism with side of base 40 and axis 60 long, is resting on H.P on its base, with the vertical face perpendicular to V.P. A tetrahedron is placed on the prism such that, the corners of it coincide with the alternate corners of the prism. Draw an auxiliary top view, on an inclined plane making  $45^\circ$  with H.P.

**Construction (Fig.11.39)**

1. Draw the projections of the combination of solids, satisfying the given conditions.

2. Draw the reference line  $x_1y_1$ , representing an A.I.P and making  $45^\circ$  with  $xy$ .
3. Obtain the final (auxiliary) top view, by projection.

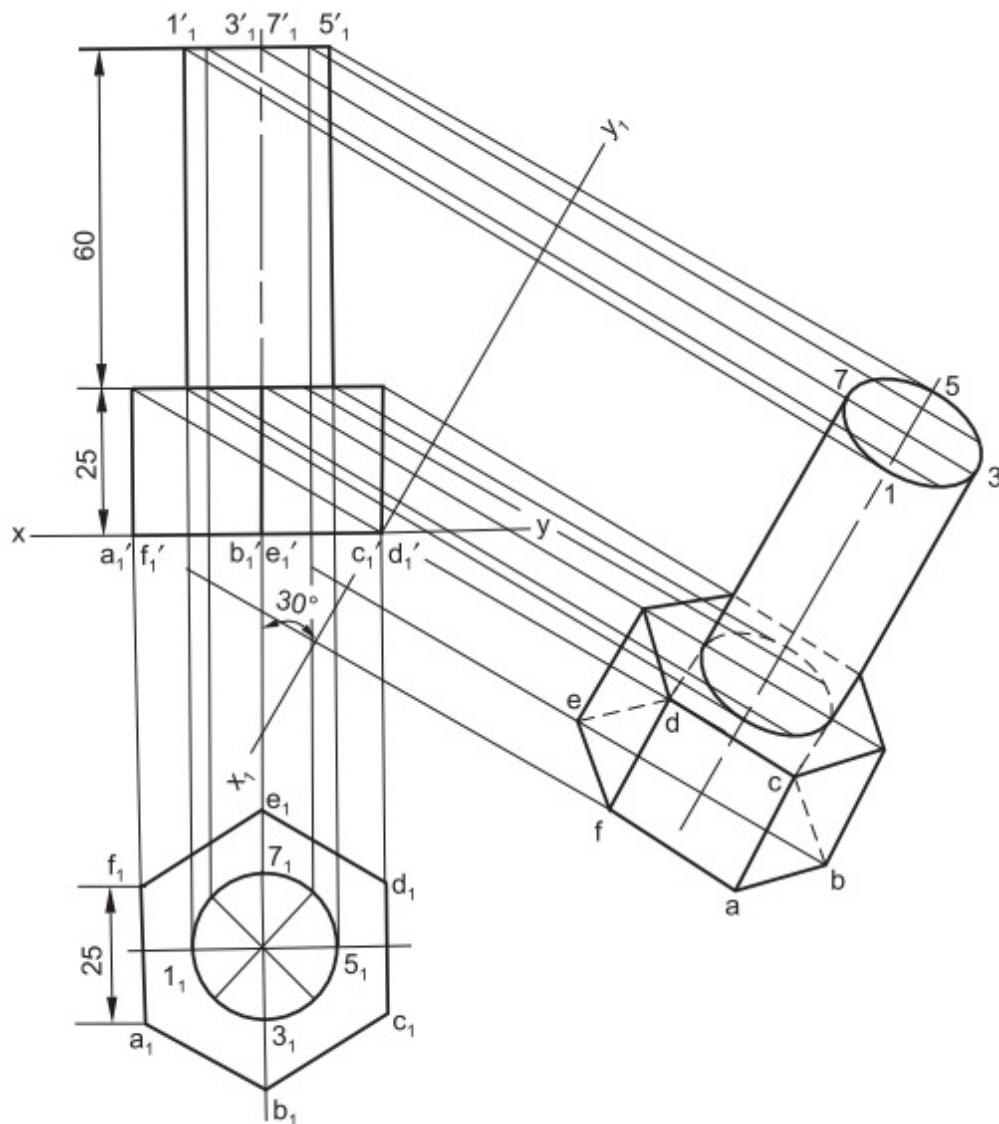


**Fig.11.39**

**Problem 29** A hexagonal headed rod of 25 diameter has its cylindrical portion 60 long. The side of hexagonal head is 25, having a thickness of 25. Draw the projections of the rod, when it is lying on H.P on one of its edges of the head and with its axis inclined at  $30^\circ$  to H.P. Follow the auxiliary plane method.

**Construction (Fig.11.40)**

1. Draw the projections of the rod, assuming that it is resting on hexagonal head on H.P and with a side of the base perpendicular to V.P.
2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane and passing through  $c_1'$  ( $d_1'$ ), making an angle of  $30^\circ$  with the axis.
3. Obtain the final (auxiliary) top view, by projection.

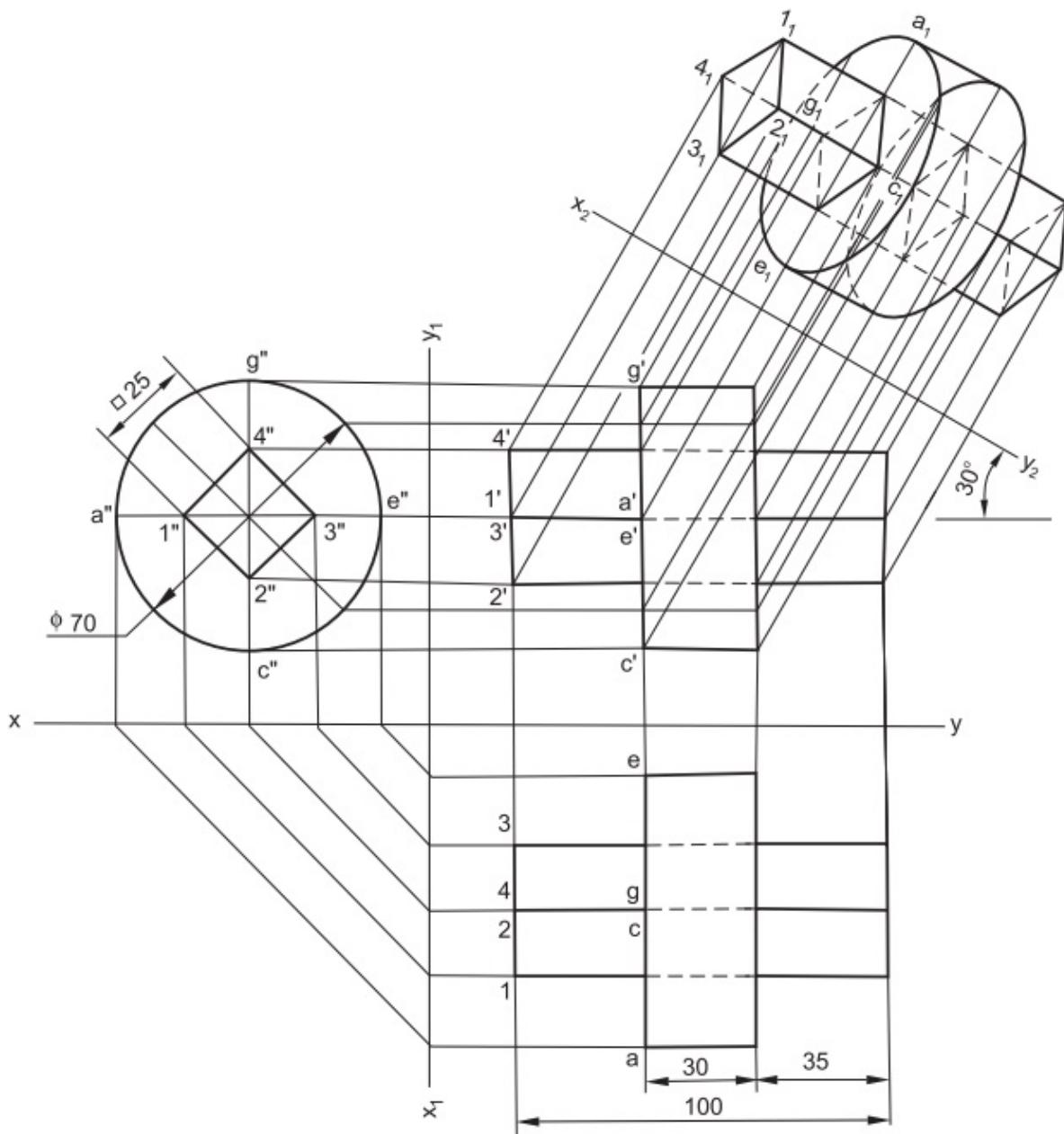


**Fig.11.40**

**Problem 30** A square rod, with side of base 25 and length 100, is penetrated centrally through a cylindrical disc of 70 diameter and 30 thick. The rod projects equally on either side of the cylindrical disc. Draw the projections of the combination of solids; when the axis of the rod is inclined at  $30^\circ$  to H.P and parallel to V.P. The faces of the square rod are equally inclined to H.P.

**Construction (Fig.11.41)**

1. Draw the three views of the combination of the solids, satisfying the given conditions and assuming that the common axis of the solids is parallel to both H.P and V.P.
2. Draw the reference line  $x_2y_2$ , representing an A.I.P and making an angle of  $30^\circ$  with the common axis in the front view.
3. Obtain the final (auxiliary) top view, by projection.

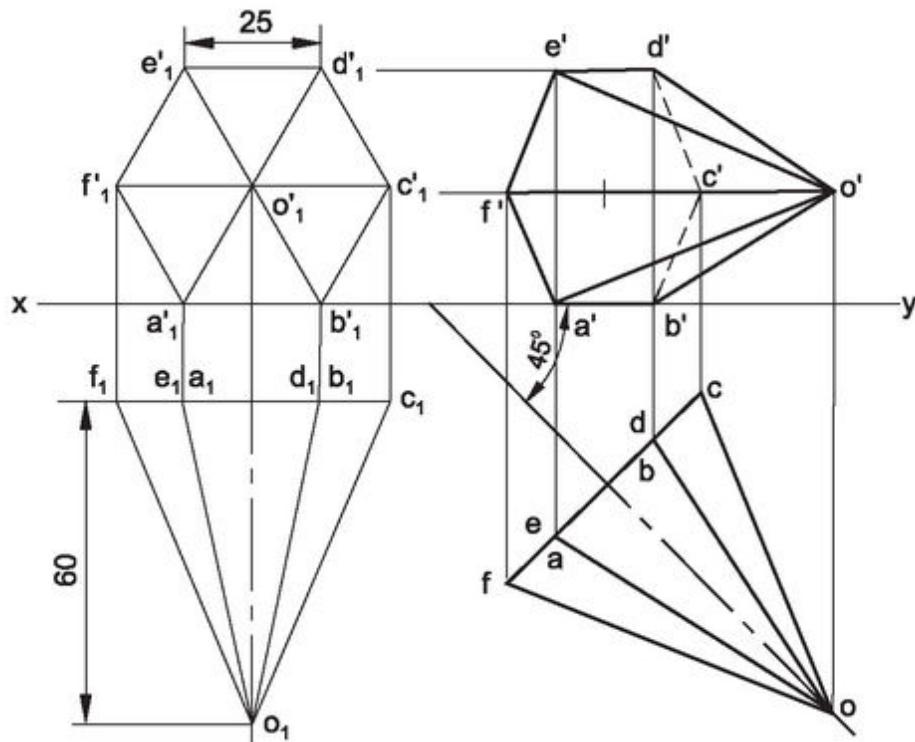


**Fig.11.41**

**Problem 31** A hexagonal pyramid of side of base 25 and axis 60 long, is resting on an edge of the base on H.P. Draw the projections of the solid, when the axis makes an angle of 45° with V.P and the base of the solid is nearer to V.P. Follow the change of position method.

**Construction (Fig.11.42)**

1. Draw the projections of the solid, assuming that its base is parallel to V.P and an edge of the base on H.P.
2. Redraw the top view such that, the axis makes an angle of  $45^\circ$  with xy.
3. Obtain the final front view, by projection.



**Fig.11.42**

**Problem 32** A cylindrical block of diameter 75 and 25 thick, has a hexagonal hole of 25 side, cut centrally through its flat faces. Draw the projections of the block, when its flat faces are vertical and inclined at  $30^\circ$  to V.P and a face of the hexagonal hole is parallel to H.P. Follow the auxiliary plane method.

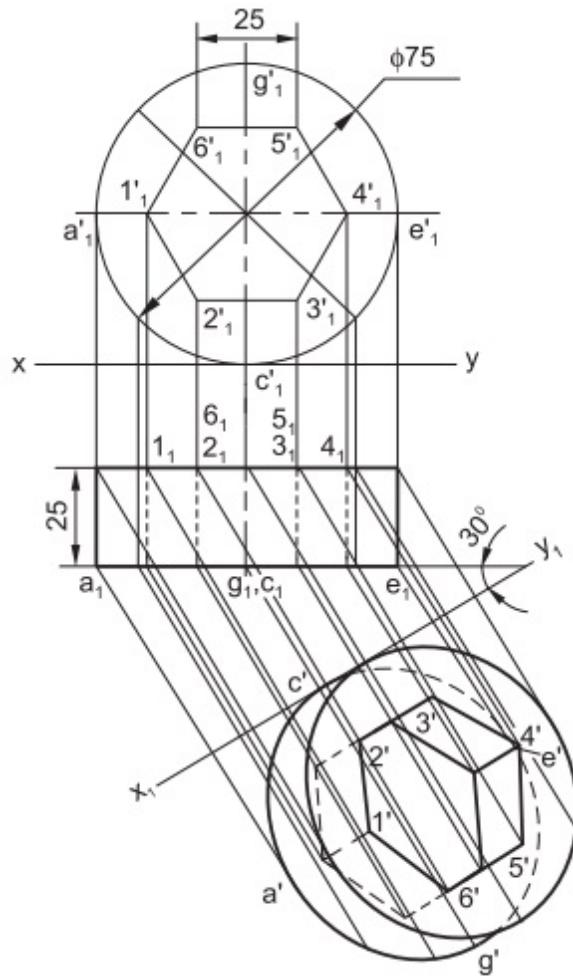
**Construction (Fig.11.43)**

1. Draw the projections of the block, assuming that it is resting on H.P, with its axis perpendicular to V.P and

an edge of the hexagonal hole is parallel to H.P.

2. Draw the reference line  $x_1y_1$ , representing the auxiliary vertical plane, making an angle of  $30^\circ$  with the flat faces of the block in the top view.
3. Obtain the final (auxiliary) front view, by projection.

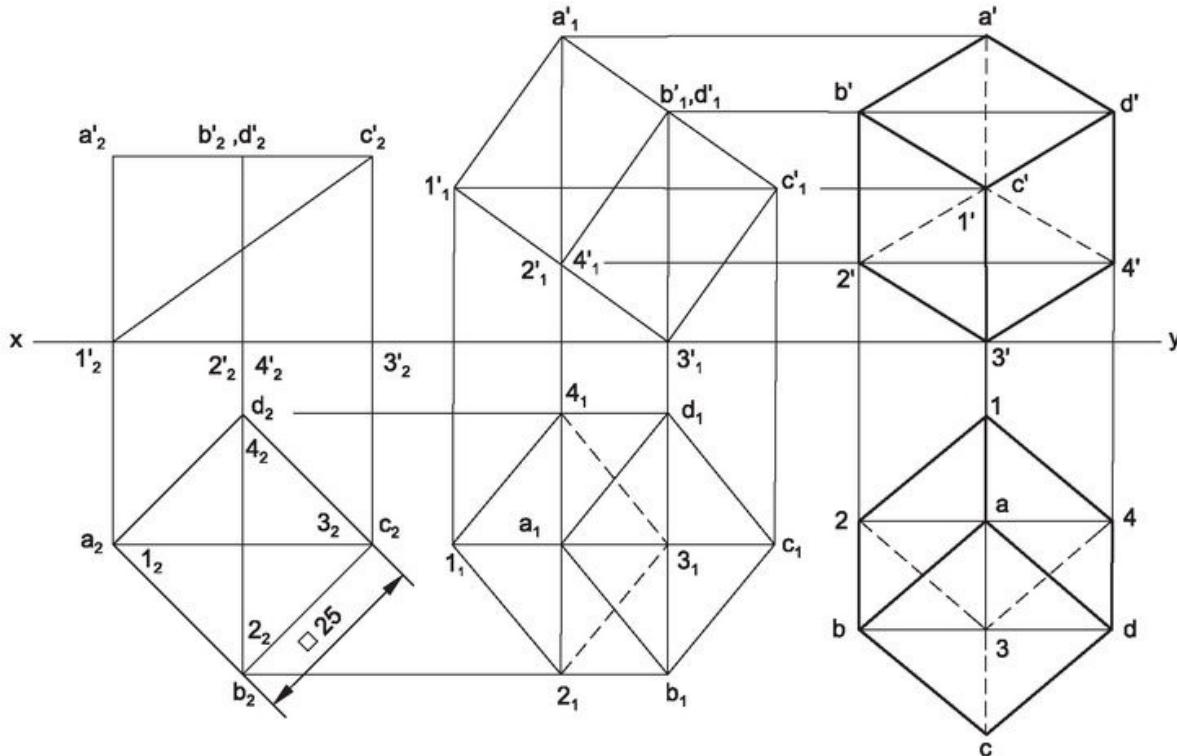
**Problem 33** *Draw the projections of a cube of side 25, resting on H.P on one of its corners, with a solid diagonal perpendicular to V.P. Follow the change of position method.*



**Fig.11.43**

**Construction (Fig.11.44)**

1. Draw the projections of the cube, assuming that it is resting on H.P on one of its faces, with the vertical faces equally inclined to V.P.
2. Locate the solid diagonal  $1_2'-c_2'$  in the front view.
3. Redraw the front view such that, the solid diagonal is parallel to xy.
4. Obtain the (second) top view, by projection.
5. Redraw the above top view such that, the solid diagonal is perpendicular to xy.
6. Obtain the final front view, by projection.

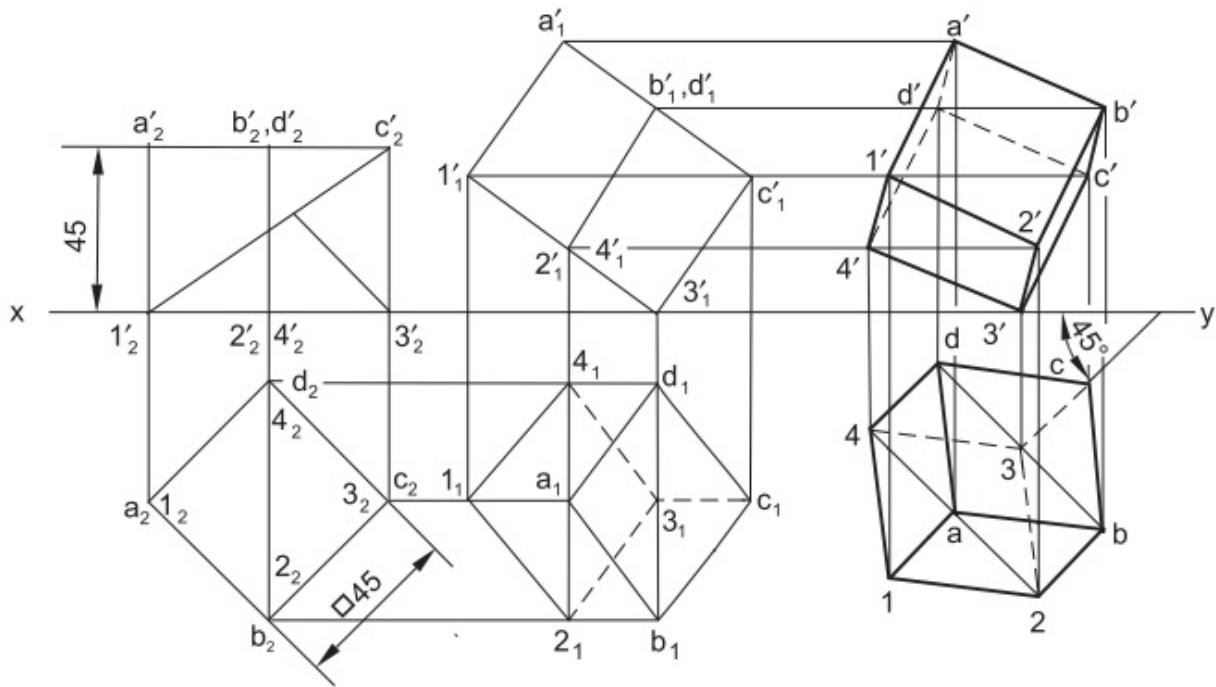


**Fig.11.44**

**Problem 34** One of the body diagonals of the cube of 45 edge, is parallel to H.P and inclined at  $45^\circ$  to V.P. Draw the projections of the cube.

### **Construction: (Fig.11.45)**

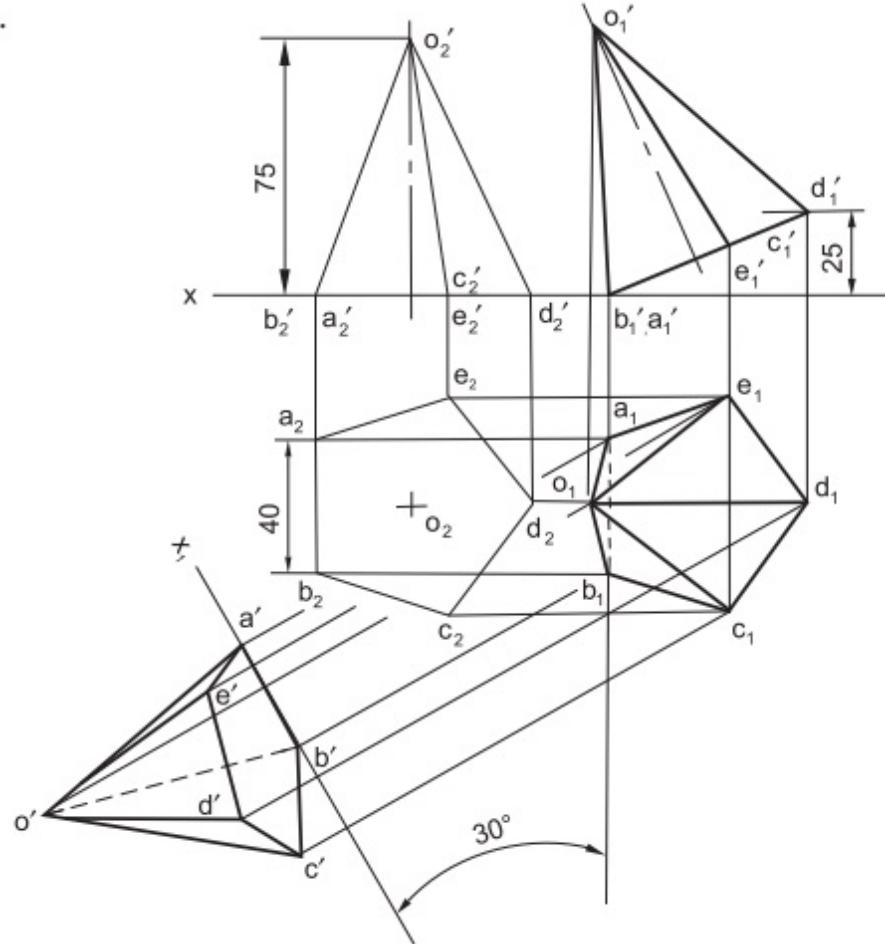
1. Draw the projections of the cube, assuming that it is resting on H.P on one of its faces, with the vertical faces equally inclined to V.P.
2. Locate the solid diagonal  $1'_2 - c'_2$  in the front view.
3. Redraw the front view such that, the solid diagonal is parallel to  $xy$ .
4. Obtain the (second) top view, by projection.
5. Redraw the above top view such that, the solid diagonal makes  $45^\circ$  with  $xy$ .
6. Obtain the final front view, by projection.



**Fig.11.45**

**Problem 35** A pentagonal pyramid of base 40 side and height 75, rests on an edge of the base on the ground so that the highest point in the base is 25 above the ground. Draw its projections when the axis is parallel to the V.P.

*Draw another front view on a reference line inclined at  $30^\circ$  to the edge on which it is resting, and show that the base is visible.*



**Fig.11.46**

### ***Construction (Fig.11.46 )***

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P such that, one of the base edges is perpendicular to V.P.
2. Redraw the front view such that, it rests on the base edge AB and the front view of the opposite corner (highest point in the base) is at 25 from xy.
3. Obtain the top view, by projection.

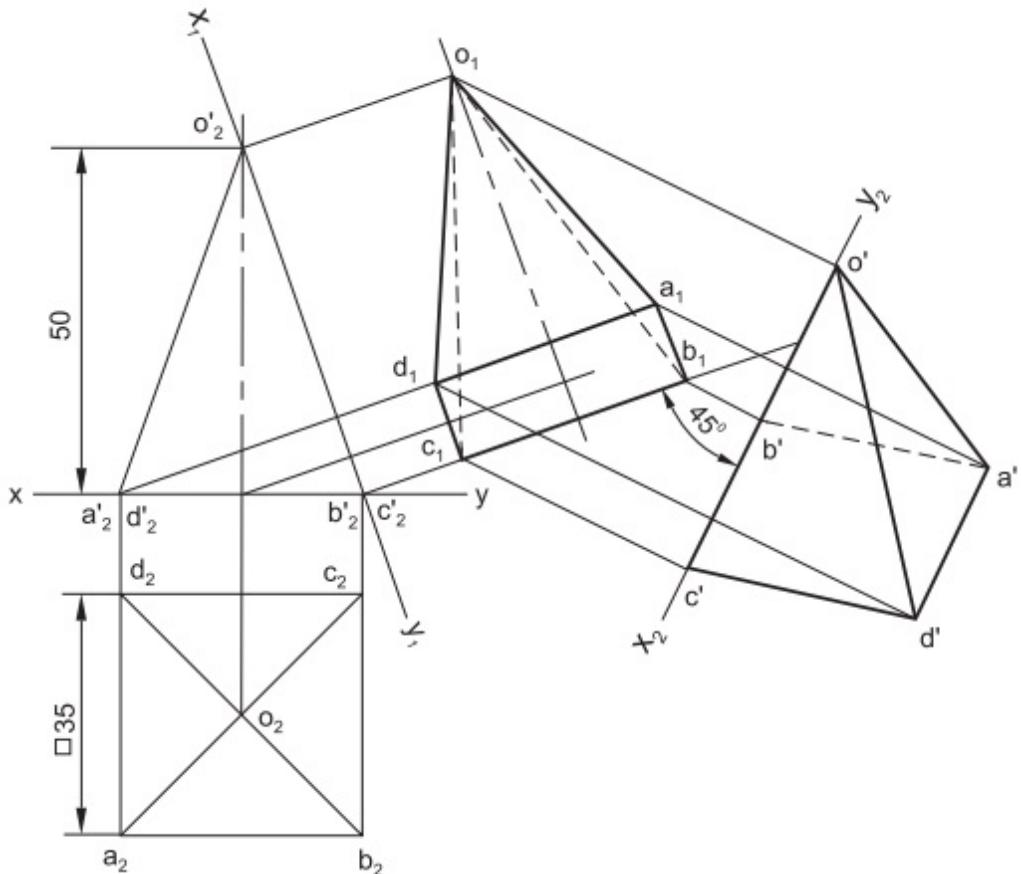
4. Draw the reference line  $x_1y_1$ , at the angle  $30^\circ$  with  $a_1b_1$  (the inclination of the edge on which the solid is resting).
5. Obtain the final front view of the pyramid, by projection.



The position of the reference line  $x_1y_1$  is chosen such that, the base of the solid is visible in the final front view.

**Problem 36** A square pyramid of base 35 side and axis 50 long, is resting on one of its triangular faces on H.P, with the edge of the base containing that face inclined at  $45^\circ$  to V.P. Draw the projections of the pyramid. Follow the auxiliary plane method.

**Construction (Fig.11.47)**

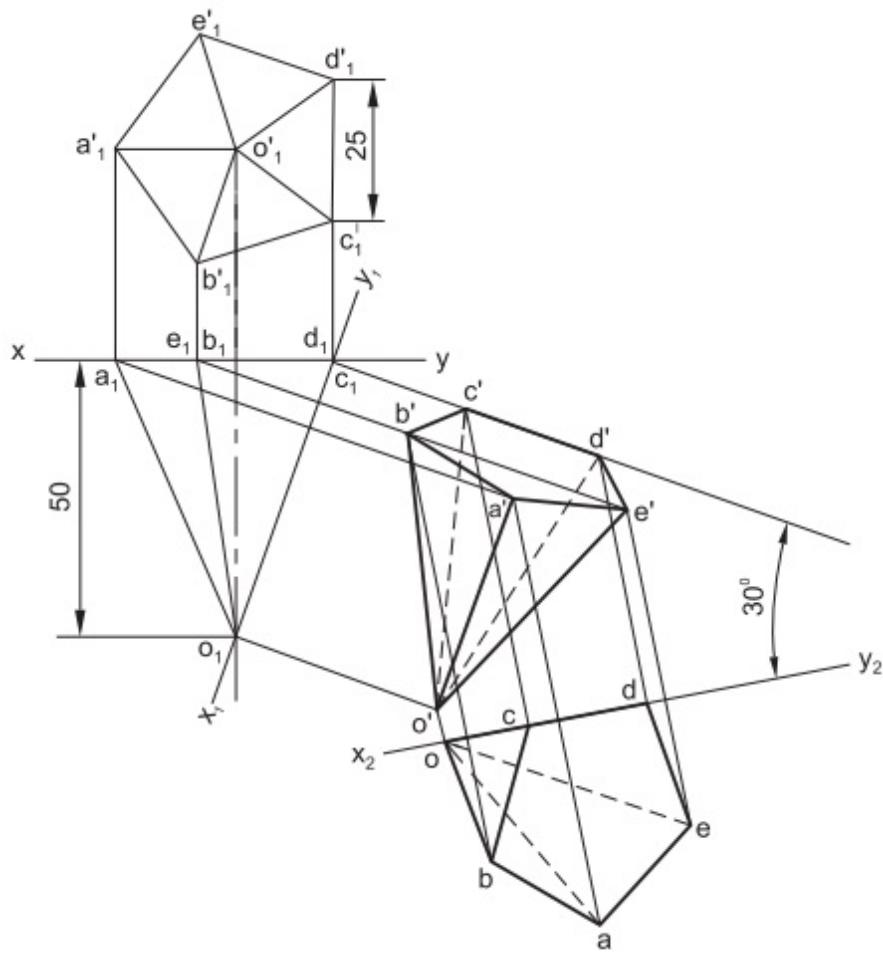


**Fig.11.47**

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P such that, an edge of its base is perpendicular to V.P.
2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane, passing through the line  $o_2' b_2'$  ( $c_2'$ ), the front view of the triangular face OBC.
3. Obtain the auxiliary top view, by projection.
4. Draw the reference line  $x_2y_2$ , representing the auxiliary vertical plane, making an angle of  $45^\circ$  with the edge  $b_1c_1$ , in the above top view.
5. Obtain the final front view, by projection.

**Problem 37** A pentagonal pyramid with side of base 25 and axis 50 long, has a triangular face on V.P and the edge of the base contained by that face is inclined at  $30^\circ$  to H.P. Draw the projections.

**Construction (Fig.11.48)**



**Fig.11.48**

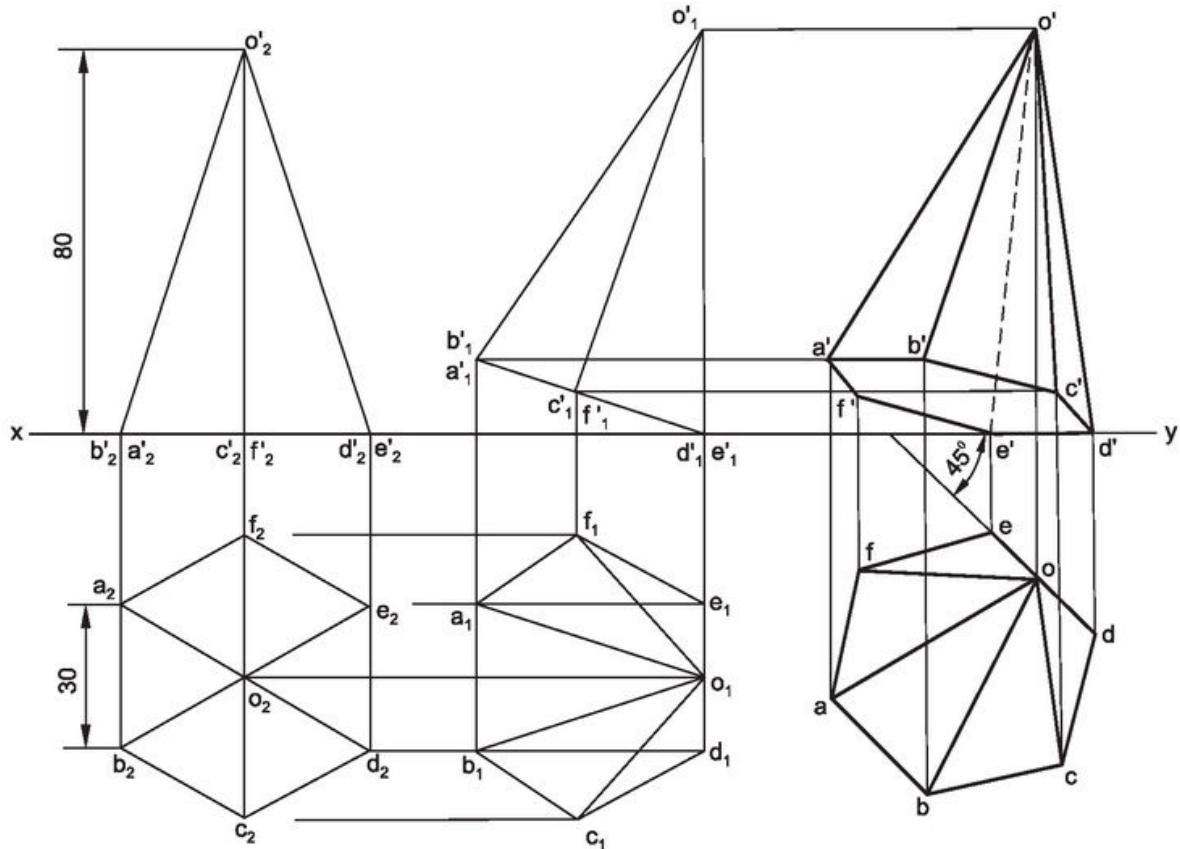
1. Draw the projections of the pyramid, assuming it to be lying with its base on V.P and an edge of the base perpendicular to H.P.
2. Draw the reference line  $x_1y_1$ , representing an A.V.P and passing through the top (edge) view of the face OCD.

3. Obtain the final (auxiliary) front view of the pyramid, by projection.
4. Draw the reference line  $x_2y_2$ , representing an A.I.P, making an angle of  $30^\circ$  with  $c'd'$ ; the edge of the base, contained by the face lying on V.P.
5. Obtain the final (auxiliary) top view of the pyramid, by projection.

**Problem 38** *A hexagonal pyramid with side of base 30 long and height 80, has one of its triangular faces perpendicular to H.P and inclined at  $45^\circ$  to V.P. The base-side of this triangular face is on H.P. Draw its projections.*

**Construction (Fig. 11.49)**

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P, with an edge of the base perpendicular to  $xy$ .
2. Redraw the front view such that, the front view ( $o_1'd_1'e_1'$ ) of the triangular face ODE is perpendicular to  $xy$ .
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the top view ( $ode$ ) of the triangular face ODE makes  $45^\circ$  with  $xy$ .
5. Obtain the final front view, by projection.



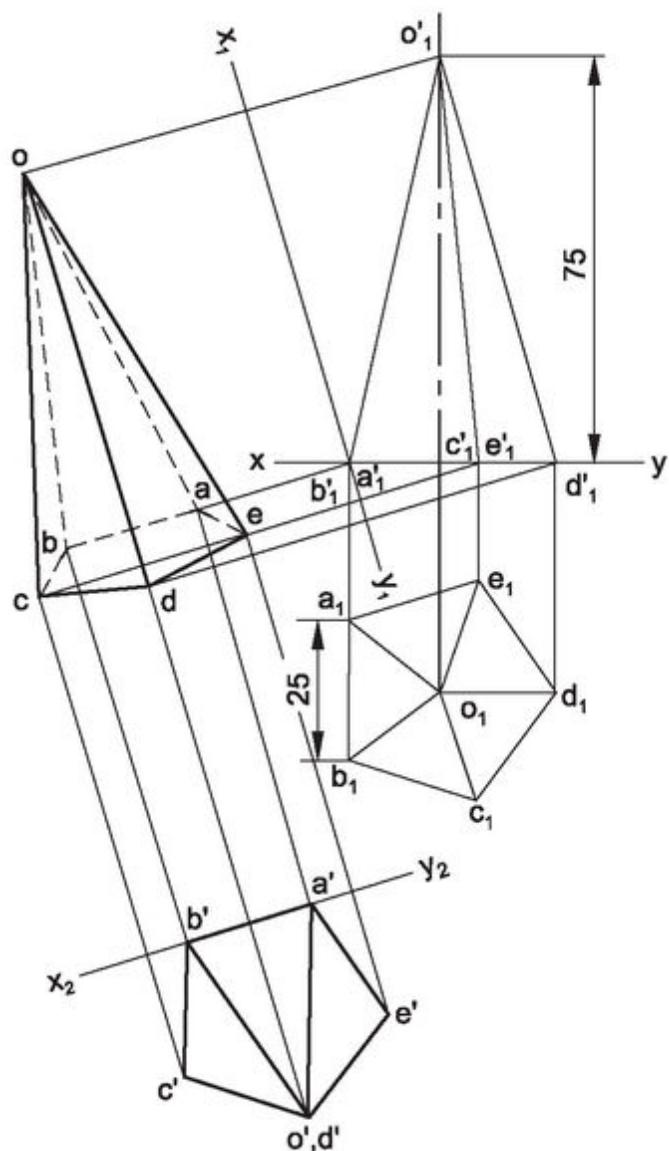
**Fig.11.49**

**Problem 39** A pentagonal pyramid, with side of base 25 and axis 75 long, rests on an edge of the base on H.P such that, the edge is parallel to V.P. The base is tilted such that, the top-most slant edge is parallel to H.P and perpendicular to V.P. Draw the projections of the solid.

**Construction (Fig.11.50)**

1. Draw the projections of the pyramid, assuming it to be lying with its base on H.P and an edge of the base (AB), perpendicular to V.P.
2. Draw the reference line  $x_1y_1$ , representing an A.I.P and passing through the front view  $a_1'$   $b_1'$  of the edge AB and parallel to the front view of top-most slant edge OD.

3. Obtain the final (auxiliary) top view of the pyramid, by projection.
4. Draw the reference line  $x_2y_2$ , representing an A.V.P and parallel to ab, as the edge AB is parallel to V.P (the edge OD is perpendicular to V.P).
5. Obtain the final (auxiliary) front view of the pyramid, by projection.

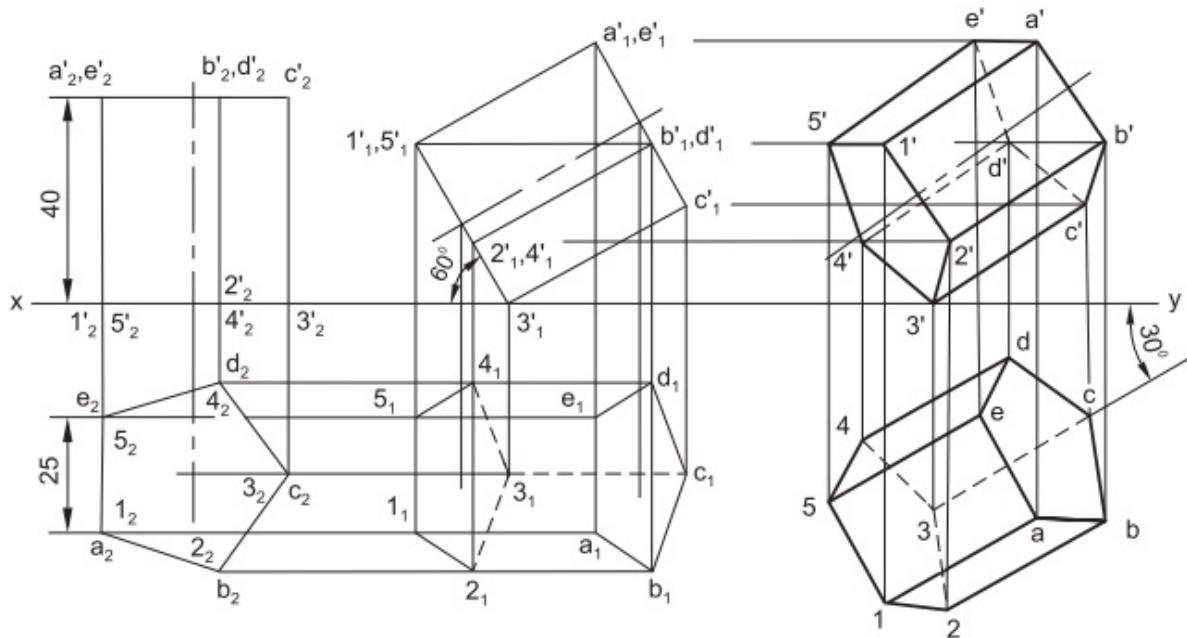


**Fig.11.50**

**Problem 40** A pentagonal prism of side of base 25 and axis 40 long, is resting on H.P on a corner of its base. Draw the projections of the prism, when the base is inclined at  $60^\circ$  to H.P and the axis appears to be inclined at  $30^\circ$  to V.P. Follow the change of position method.

**Construction (Fig.11.51)**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with two adjacent edges of the base equally inclined to V.P.
2. Redraw the front view such that, the corner  $31'$  lies on  $xy$  and the front view of the base 1-2-3-4-5, makes an angle of  $60^\circ$  with  $xy$ .
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, its axis makes an angle of  $30^\circ$  with  $xy$ .
5. Obtain the final front view, by projection.

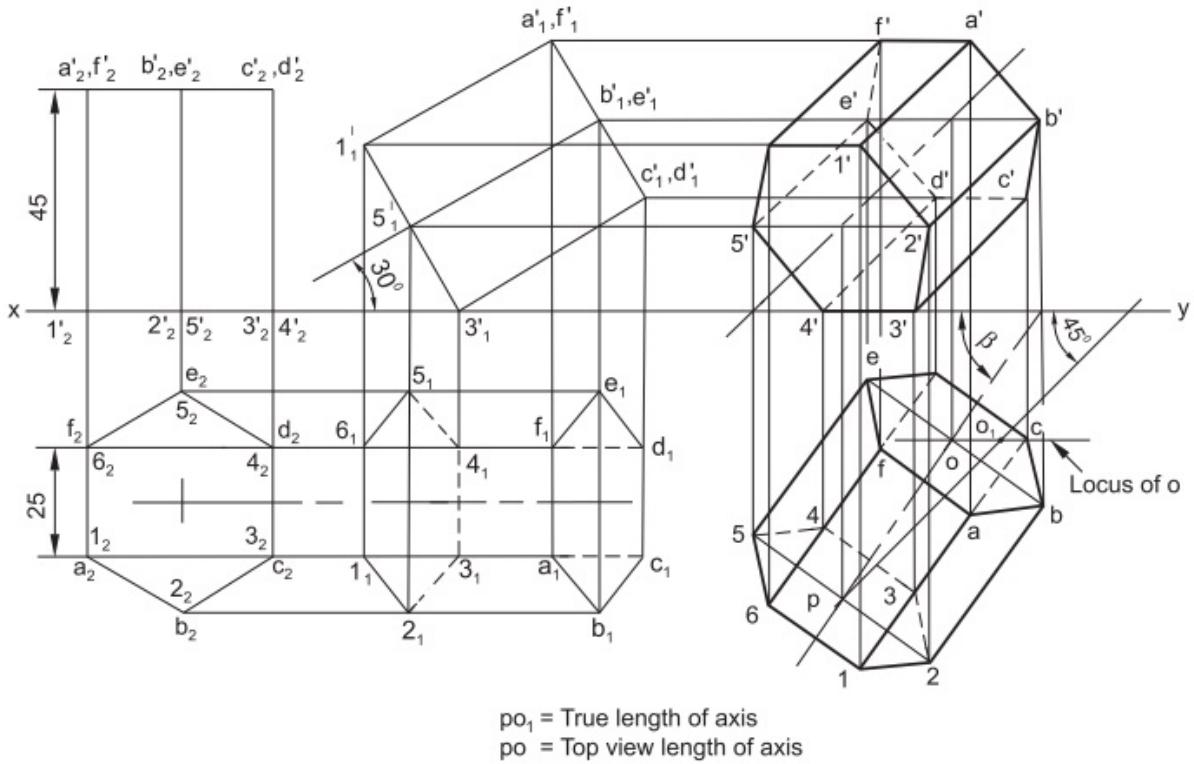


**Fig.11.51**

**Problem 41** A hexagonal prism of base 25 side and axis 45 long, is positioned with one of its base edges on H.P such that, the axis is inclined at  $30^\circ$  to H.P and  $45^\circ$  to V.P. Draw its projections. Follow the change of position method.

**Construction (Fig.11.52)**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P and with an edge of the base perpendicular to V.P.
2. Redraw the front view such that, the front view of the base edge 3-4, lies on xy and the axis makes an angle of  $30^\circ$  with xy.
3. Obtain the second top view, by projection.
4. Determine the apparent angle  $\beta$ , the inclination, the axis makes with xy in the final top view.
5. Redraw the top view such that, its axis makes the angle  $\beta$  with xy.
6. Obtain the final front view, by projection.



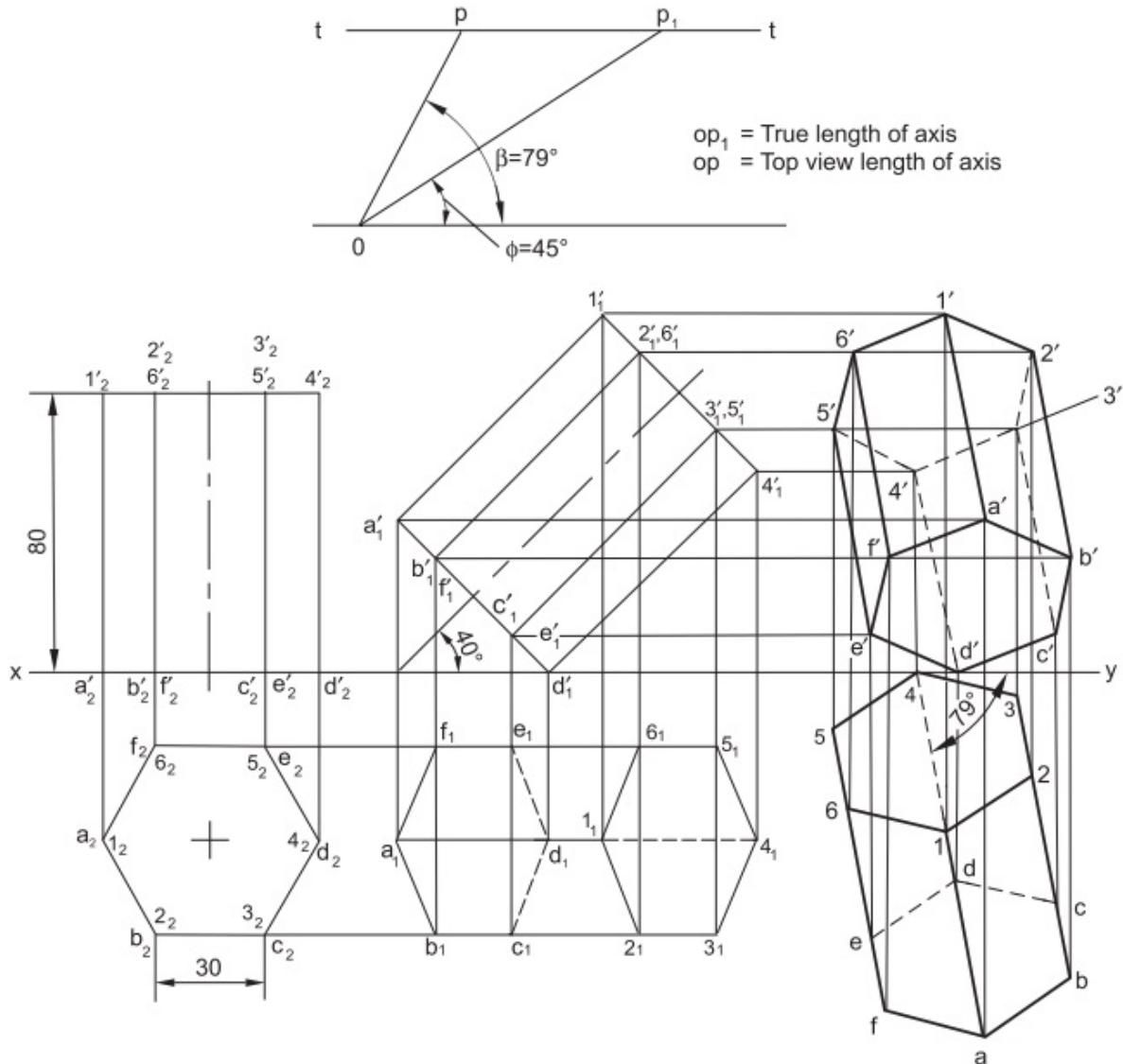
**Fig.11.52**

**Problem 42** One end of the longer edges of a regular hexagonal prism of side of base 30 and height 80, is on H.P and the other end of the same edge is on V.P. The axis of the solid makes  $40^\circ$  with H.P and  $45^\circ$  with V.P. Draw the projections of the solid.

**Construction:** ([Fig.11.53](#))

1. Draw the projections of the prism, assuming that it is resting on its base on H.P and with an edge of the base parallel to V.P.
2. Redraw the front view such that, the end of the longer edge (D-4), i.e.,  $d_1'$  lies on xy and the axis makes an angle of  $40^\circ$  with xy.
3. Obtain the second top view, by projection.

- Determine the apparent angle  $\beta$ , the axis makes with xy in the final top view.
- Redraw the second top view such that, its axis makes the angle  $\beta$  with xy.
- Obtain the final front view, by projection.



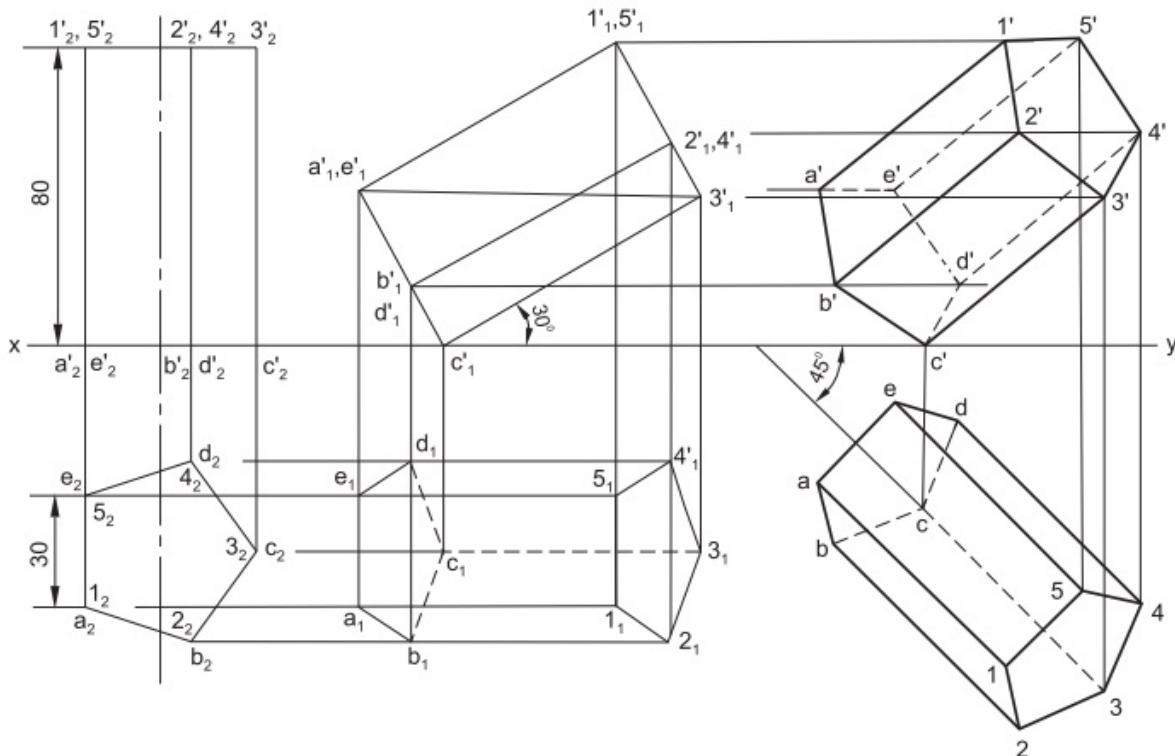
**Fig.11.53**

**Problem 43** A pentagonal prism is resting on one of the corners of its base on H.P. The longer edge containing that corner is inclined at  $30^\circ$  to H.P and the vertical plane

containing that edge is inclined at  $45^\circ$  to V.P. Draw the projections of the solid. Take the side of the base as 30 and length of the axis as 80.

### **Construction (Fig.11.54)**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with an edge of the base, perpendicular to V.P.
2. Redraw the front view such that, the corner  $c_1'$  lies on xy and the front view ( $c_1' 3_1'$ ) of the longer edge C-3 makes  $30^\circ$  with xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the vertical trace of the vertical plane, containing the longer edge c-3, makes  $45^\circ$  with xy. This is the final top view.
5. Obtain the final front view, by projection.



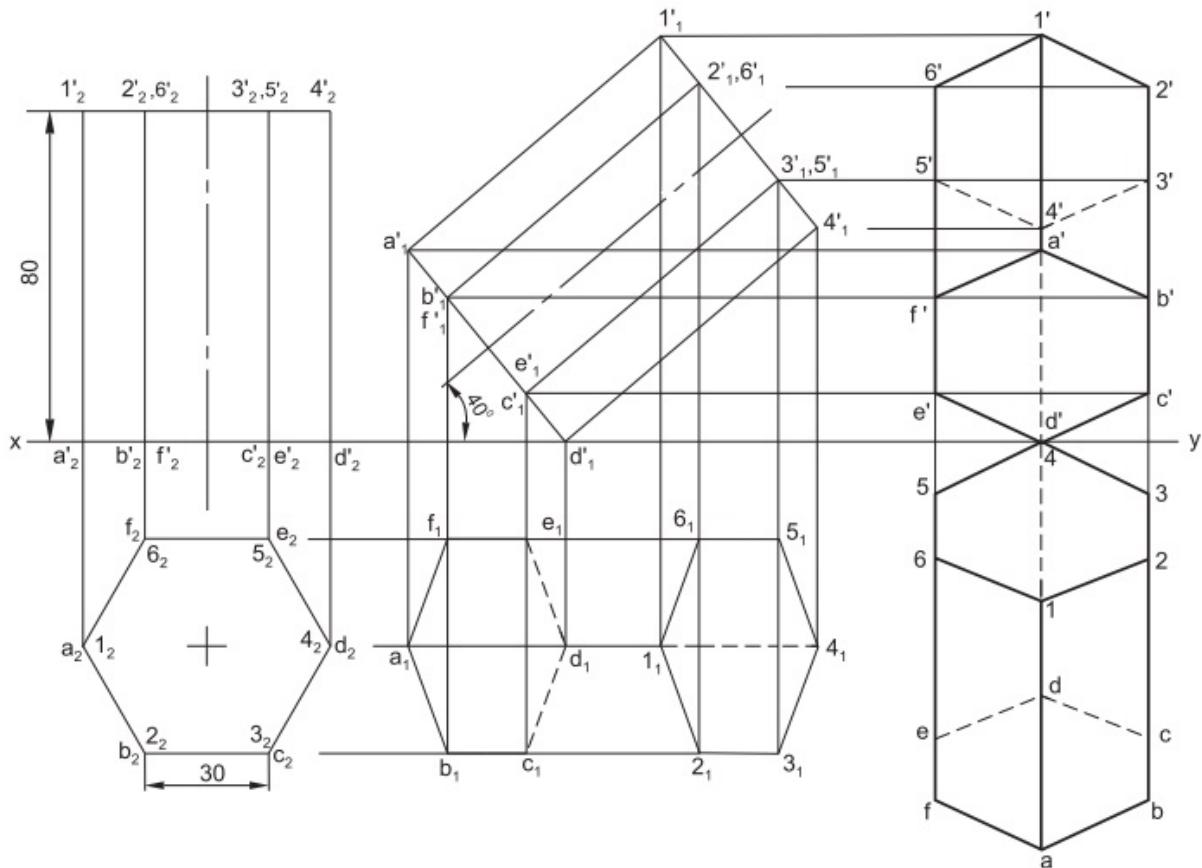
### **Fig.11.54**

**Problem 44** One end of a longer edge of a regular hexagonal prism of side of base 30 and height 80 is on V.P and the other end of the same edge is on the ground. The axis makes  $50^\circ$  to V.P and  $40^\circ$  to the ground. Draw its projections.

#### **Construction (Fig.11.55)**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with an edge of the base parallel to V.P.
2. Redraw the front view such that, one end of the longer edge D-4( $d_1'$ ) lies on xy and the axis makes  $40^\circ$  with xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the top view (4) of the other end of the longer edge D-4, lies on xy. This is the final top view.
5. Obtain the final front view, by projection.

**Problem 45** A regular square prism lies with its axis inclined at  $60^\circ$  to H.P and  $30^\circ$  to V.P. The prism is 60 long and has a face width of 25. The nearest corner is 10 away from V.P and the farthest one is 100 from H.P. Draw the projections of the solid.



**Fig.11.55**

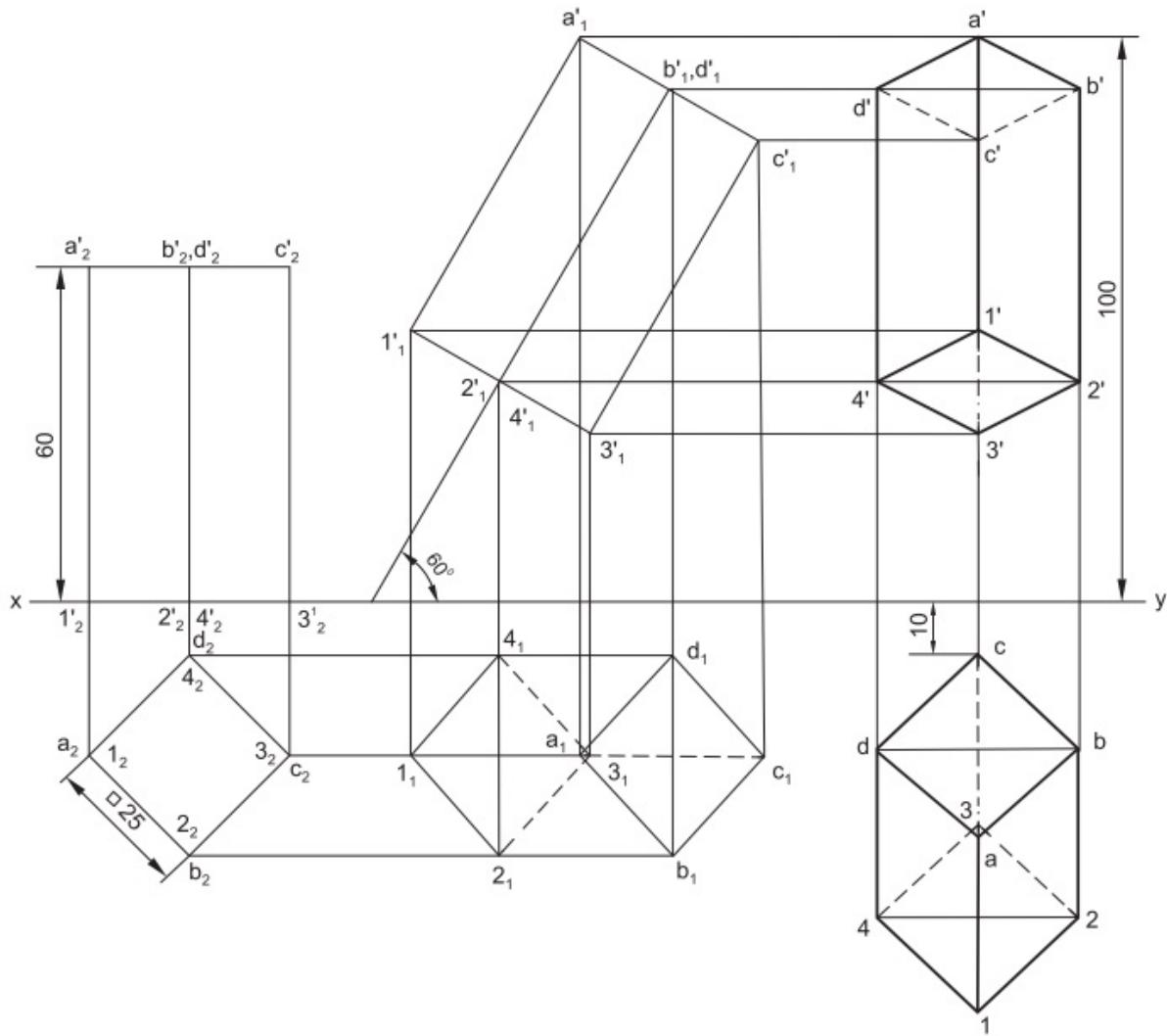
### ***Construction (Fig.11.56)***

1. Draw the projections of the solid, assuming it to be resting on its base on H.P, with the vertical faces equally inclined to V.P.
2. Redraw the front view such that, the axis makes an angle of  $60^\circ$  with xy and the farthest corner a' is 100 above xy.
3. Obtain the second top view, by projection.
4. Redraw the top view such that, the axis is perpendicular to xy and the nearest corner c is 10 away from xy. This is the final top view.
5. Obtain the final front view, by projection.



The axis of the prism is making an angle of  $60^\circ$  with the H.P. As  $\theta + \phi = 90^\circ$ , the front and top views of the axis should lie on a single projector.

**Problem 46** A hexagonal prism, with the edge of base 25 and axis 75 long, has a longer edge inclined at  $60^\circ$  to H.P and an edge of the base, adjacent to it, is inclined at  $20^\circ$  to H.P. Draw its projections.



**Fig.11.56**

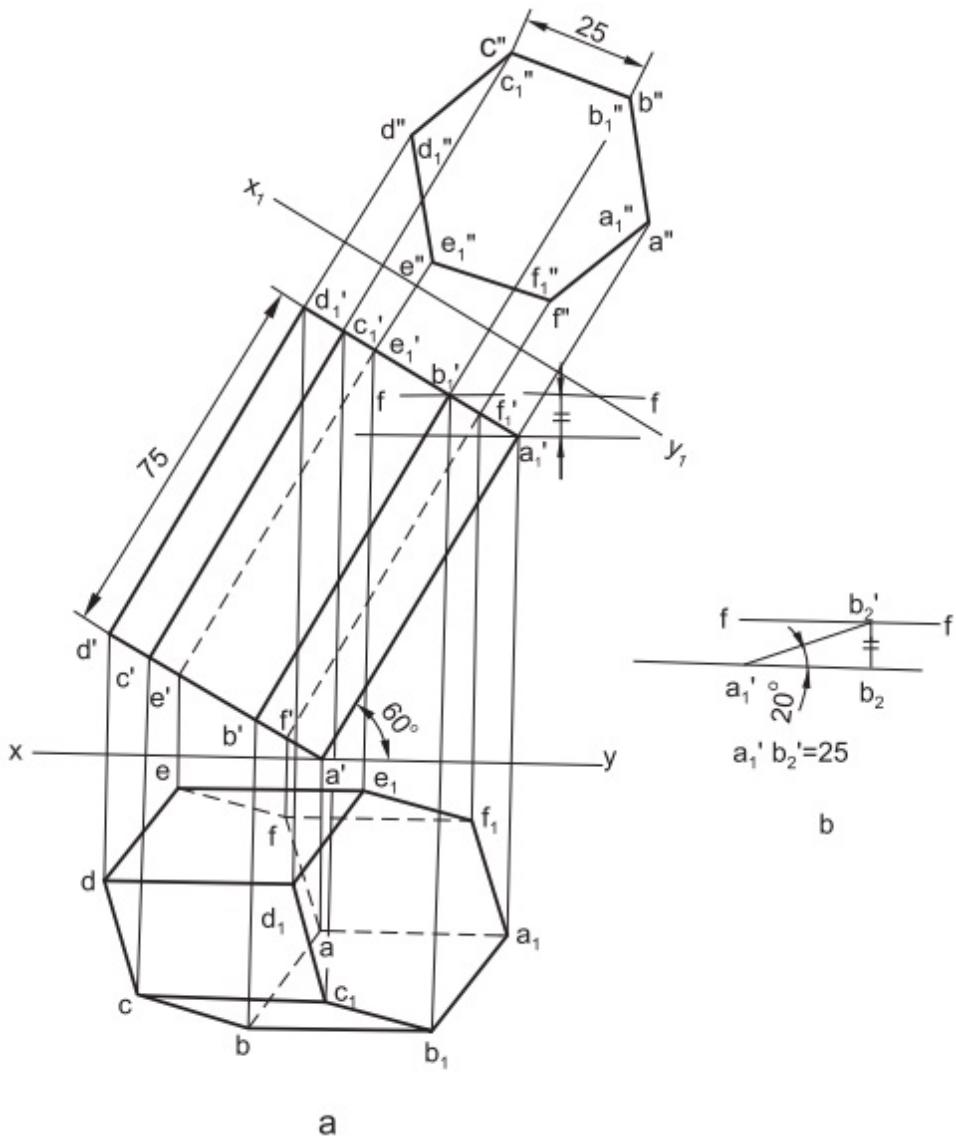
**Construction (Fig.11.57a)**

1. Draw the reference line  $xy$  and through any point  $a'$  on it, draw a line making  $60^\circ$  with  $xy$ . Mark  $a_1'$  on the above line such that,  $a' a_1' = 75$ , the length of the solid.
2. Through  $a_1'$  and  $f-f$ , draw two horizontal parallel lines keeping distance between them equal to  $b_2 b_2'$  (Refer Construction: [Fig.11.57b](#)).



The distance  $b_2 b_2'$  is equal to the height of the front view of the corner  $B_1$ , when the edge  $A_1 B_1$  (of the upper base) is making an angle of  $20^\circ$  with H.P. This edge is adjacent to the longer edge  $AA_1$ .

3. Draw a line through  $a_1'$  and perpendicular to  $a' a_1'$ , intersecting  $f-f$  at  $b_1'$ .
4. Draw the reference line  $x_1y_1$  parallel to  $a_1' b_1'$ .
5. Mark  $a_1''$  ( $a''$ ) at any convenient location, on the projector through  $a' a_1'$  extended.
6. Draw a projector through  $b_1'$  perpendicular to  $x_1y_1$
7. With  $a_1''$  as centre and 25 as radius, draw an arc intersecting the above projector at  $b_1''(b'')$ .
8. Complete the edge view of the prism, taking  $a_1'' b_1''$  as one of the edges of the base.
9. Obtain the front view and top view, by projection.



**Fig.11.57**

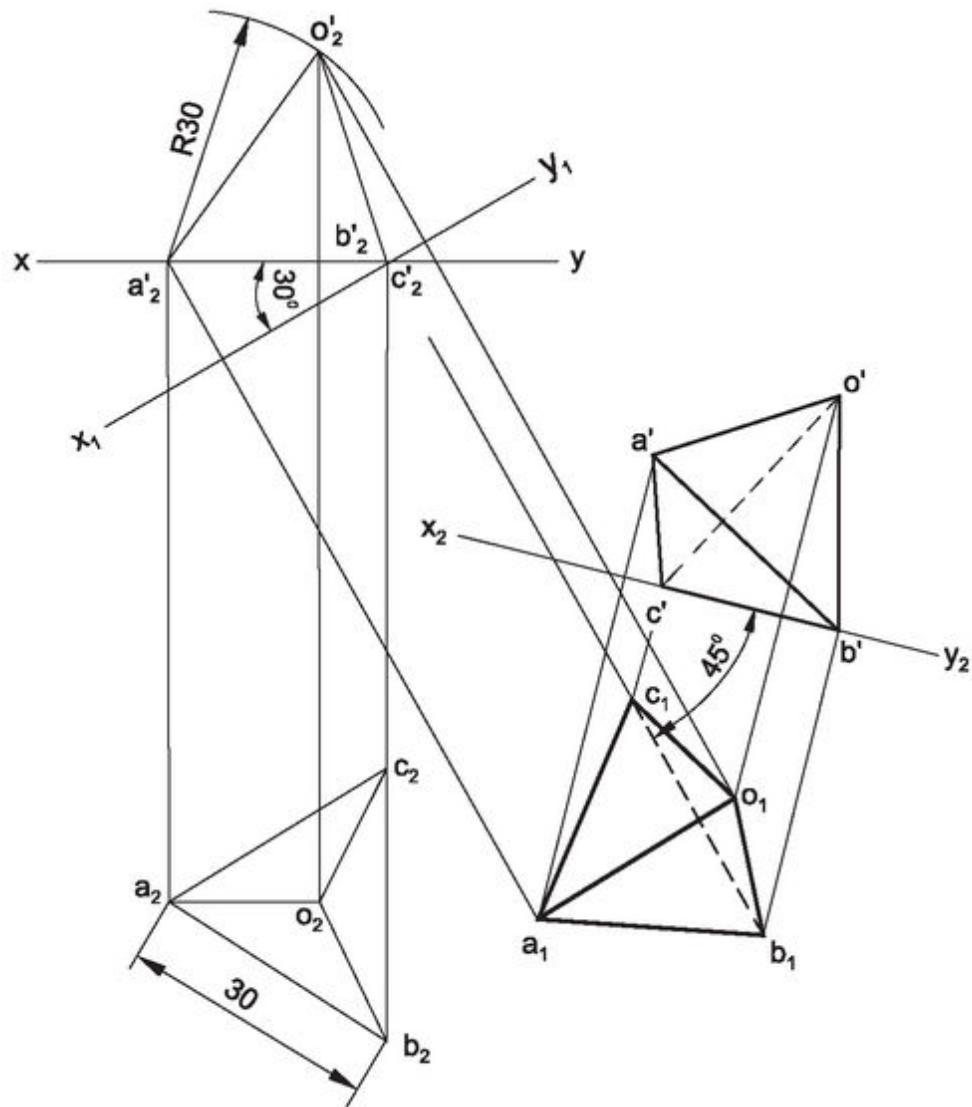
**Problem 47** A tetrahedron of 30 side is resting with one of its edges on H.P. The edge on which it rests is inclined at  $45^\circ$  to V.P and a face containing that edge is inclined at  $30^\circ$  to H.P. Draw the projections of the solid. Follow the auxiliary plane method.

**Construction (Fig.11.58)**

1. Draw the projection of the solid, assuming that it is resting on the face ABC on H.P, with edge BC

perpendicular to V.P.

2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane, passing through the front view  $b_2'c_2'$  of the edge BC and making an angle of  $30^\circ$  with the face ABC.
3. Obtain the (auxiliary) top view, by projection.
4. Draw the reference line  $x_2y_2$ , representing the auxiliary vertical plane and making an angle of  $45^\circ$  with  $b_1c_1$ , in the above top view.
5. Obtain the final front (auxiliary) view, by projection.



**Fig.11.58**

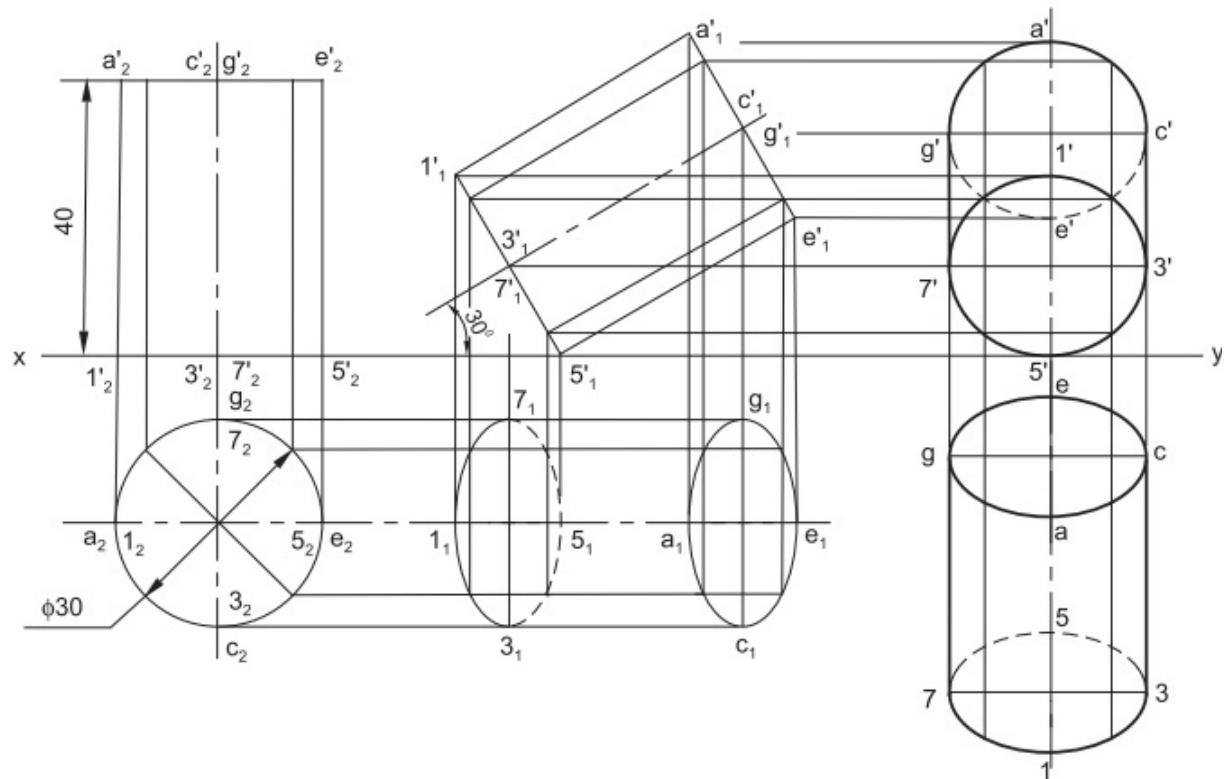
**Problem 48** Draw the projections of a cylinder of base 30 diameter and axis 40 long, which lies on H.P. on a point of its rim, with its axis inclined at 30° to H.P. The top view of the axis is perpendicular to V.P. Follow the change of position method.

**HINT** This is a case of projections of a solid, inclined to both H.P. and V.P. such that,  $\theta + \phi = 90^\circ$ . The inclination with H.P. is given as 30°. From the position of the axis in the top view, the apparent angle  $\beta = 90^\circ$ .

Obviously, the inclination of the axis with V.P, i.e.,  $\phi = 60^\circ$ .

### **Construction (Fig.11.59)**

1. Draw the projections of the cylinder, assuming that it is resting on a base on H.P.
2. Redraw the front view such that, it rests on a point of its base on xy and the axis makes  $30^\circ$  with xy.
3. Obtain the (second) top view, by projection.
4. Redraw the above top view such that, its axis appears to be perpendicular to xy. This forms the final top view.
5. Obtain the final front view, by projection.

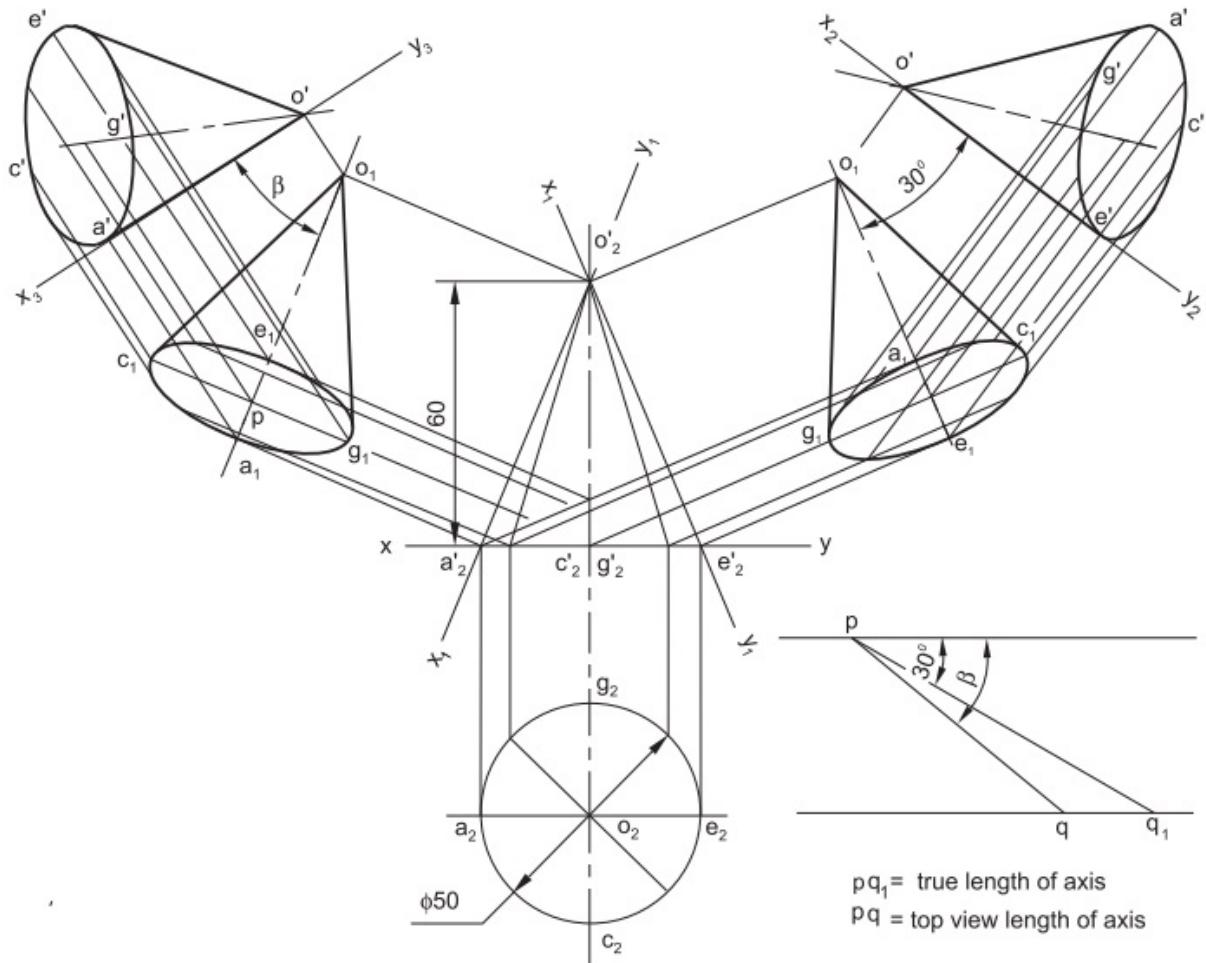


**Fig.11.59**

**Problem 49** Draw the projections of a cone of base 50 diameter and altitude 60; lying on one of its generators on H.P when (i) the top view of the axis makes an angle of  $30^\circ$  with xy and (ii) the axis makes an angle of  $30^\circ$  with V.P. Follow the auxiliary plane method.

**HINT** This is also a problem on projections of a solid, inclined to both H.P and V.P. When the cone lies on a generator on the H.P, the inclination of the axis with H.P is established. For the first part of the problem, the inclination  $30^\circ$  with xy refers to the apparent angle of inclination. For the second part, the inclination with V.P refers to the actual inclination of the axis with V.P.

**Construction (Fig.11.60)**



**Fig.11.60**

1. Draw the projections of the cone, assuming that it is resting on its base on H.P.
2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane and passing through an extreme generator in the front view.
3. Obtain the (auxiliary) top view, by projection.

### **Case I**

4. Draw the reference line  $x_2y_2$ , representing the auxiliary vertical plane and making an angle of 30° with the axis in the above top view.

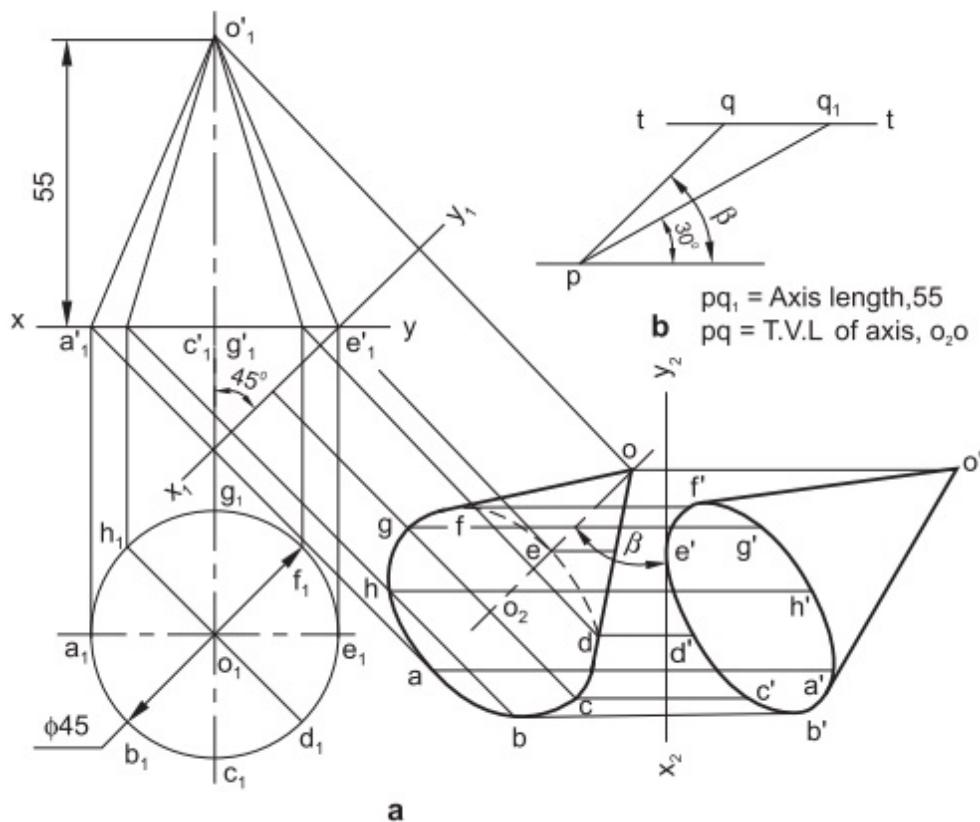
5. Obtain the final (auxiliary) front view, by projection.

### **Case II**

6. Determine the apparent angle of inclination  $\beta$ , which the axis makes with the reference line.
7. Draw the reference line  $x_3y_3$ , representing the auxiliary vertical plane and making the angle  $\beta$  with the axis.
8. Obtain the final (auxiliary) front view, by projection.

**Problem 50** A cone of base diameter 45 and axis 55 long, lies on a point on its base on H.P, with its axis inclined at  $30^\circ$  to V.P and  $45^\circ$  to H.P. Draw its projections.

### **Construction (Fig.11.61)**



**Fig.11.61**

1. Draw the projections of the cone, assuming that it is resting on its base on H.P.
2. Draw the reference line  $x_1y_1$ , representing the auxiliary inclined plane and passing through an extreme point of the base in the front view.
3. Obtain the final (auxiliary) top view, by projection.
4. Determine the apparent angle of inclination  $\beta$ , which the axis makes with the reference line (Fig.11.61b).
5. Draw the reference line  $x_2y_2$ , representing the auxiliary vertical plane and making the angle  $\beta$  with the axis of the final top view.
6. Obtain the final (auxiliary) front view, by projection.

**Problem 51** A right circular cone of base 50 diameter is situated such that, the axis appears to be perpendicular to XY both in H.P and V.P. The axis measures 60 in the V.P and 50 in the H.P. The apex is nearer to the V.P than the base. Neither the base nor the apex is touching the V.P or H.P. Draw the projections of the object.

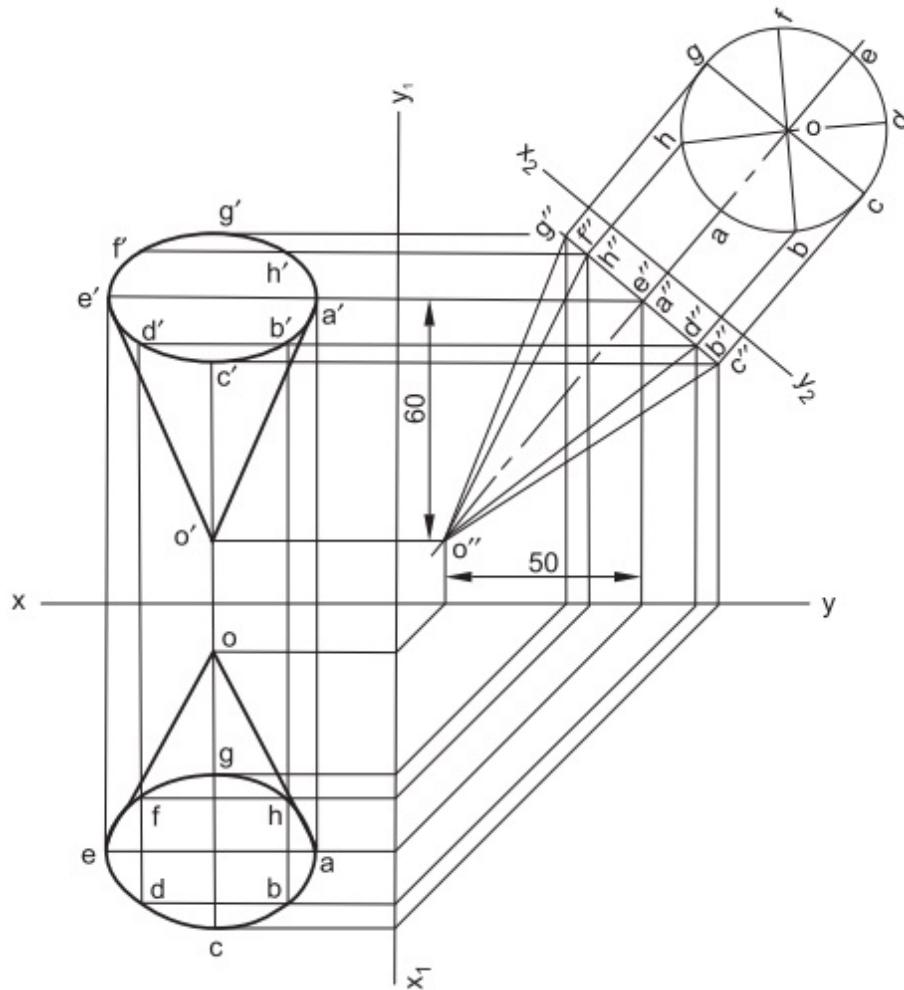
### **Construction (Fig.11.62)**



This is a case of projections of a solid, inclined to both the H.P and V.P such that,  $\theta + \phi = 90^\circ$ . As the inclinations are not given, the solution has to be started with the end view so that, the length of the axis is 60 in the V.P and 50 in the H.P.

1. Draw the reference lines  $xy$  and  $x_1y_1$  and the side view of the cone, satisfying the given conditions.
2. Draw the reference line  $x_2y_2$  and draw the base of the solid at suitable distance from  $x_2y_2$ .

3. Obtain the front and top views of the solid, ensuring that the axes in both the front and top views are perpendicular to xy.



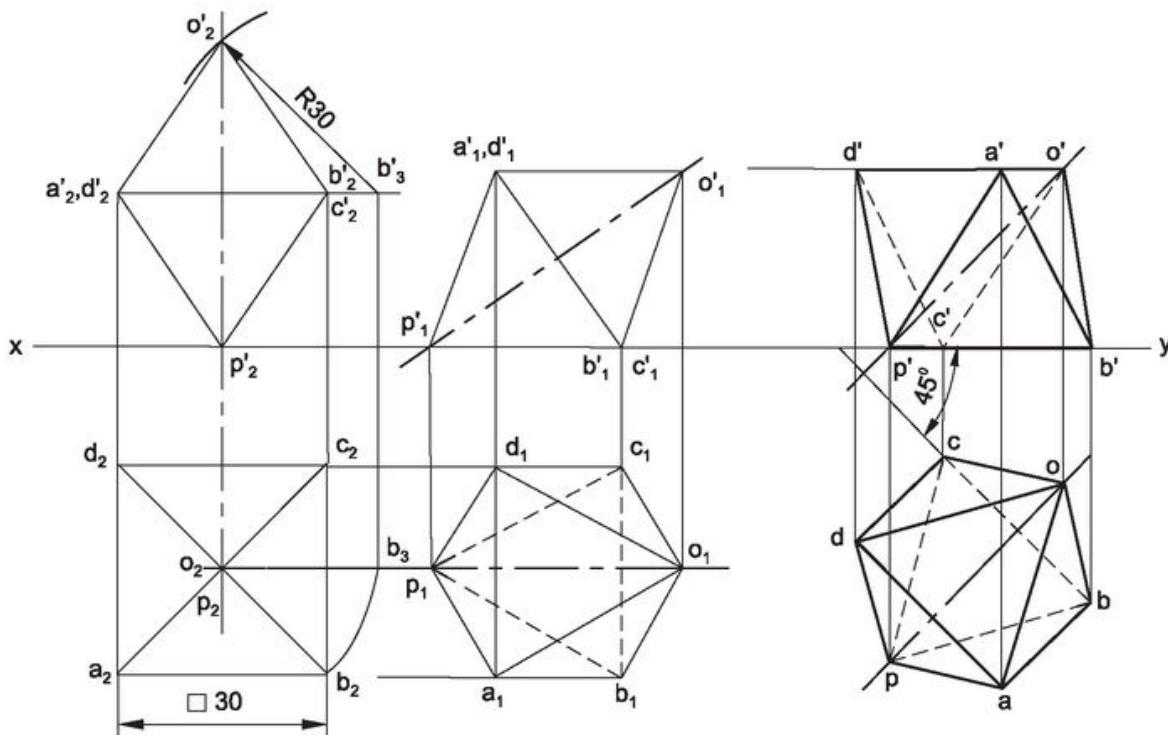
**Fig.11.62**

**Problem 52** An octahedron of 30 edge, lies with one of its faces on H.P and an edge of that face inclined at  $45^\circ$  to V.P. Draw its projections. Follow the change of position method.

**Construction (Fig.11.63)**

1. Draw the projections of the solid, assuming that it is resting on one of its corners on H.P such that, a horizontal edge is parallel to V.P.

2. Redraw the front view such that, the line  $p_1' b_1' (c_1)$ , representing one of its faces, lies on xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the line bc (an edge of the face on H.P) makes an angle of  $45^\circ$  with xy.
5. Obtain the final front view, by projection.



**Fig.11.63**

**Problem 53** Two spheres of diameters 40 and 20 are placed on H.P touching each other. Draw its projections when the line joining their centres in the top view appears to be inclined at  $40^\circ$  to xy line.

**Construction (Fig.11.64)**

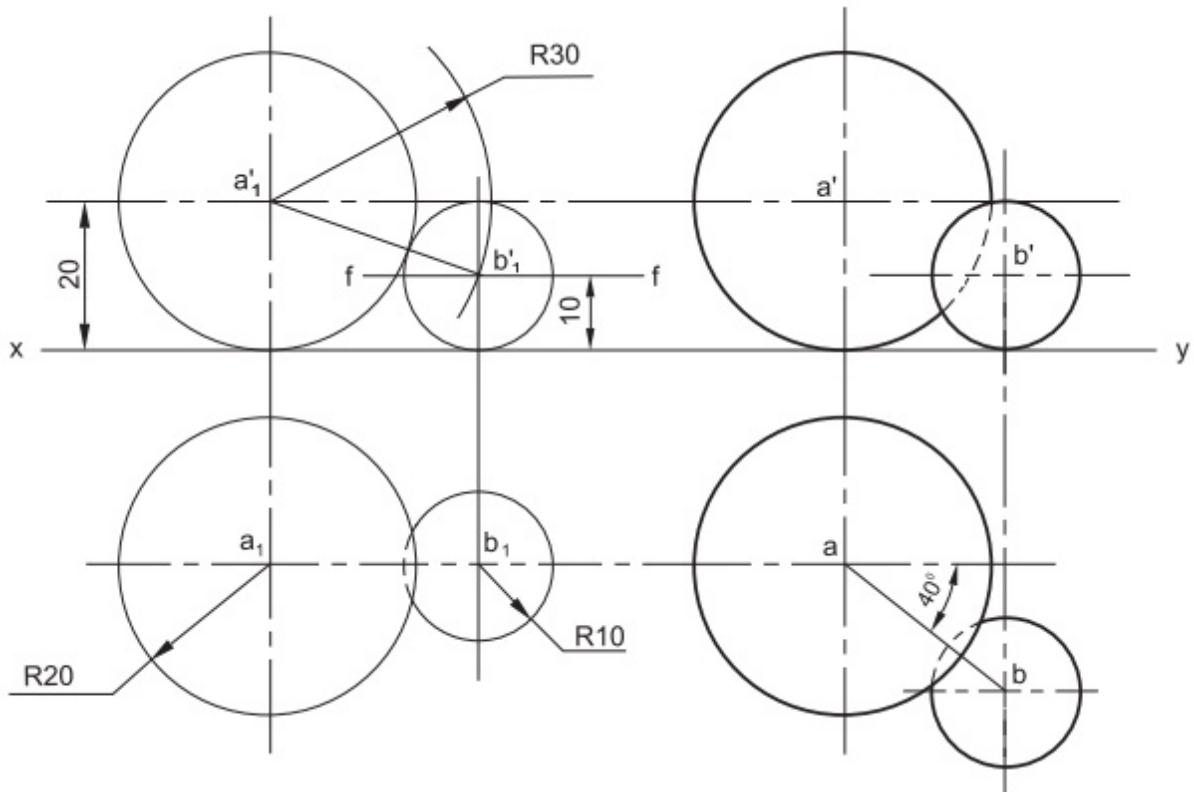
1. Draw the reference line xy.
2. With  $a_1'$  (the centre of the larger sphere which is at a height of 20 from xy) as centre, draw a circle

representing the front view of the larger sphere and draw an arc of radius 30(=20+10) to locate the centre of the smaller sphere.

3. Draw a line f-f, parallel to and 10 above xy; intersecting the above arc at  $b_1'$ .
4. With  $b_1'$  as centre and radius equal to 10, draw a circle; representing the front view of the smaller sphere.

This is the first front view of the spheres touching each other.

5. Complete the top view by projection and obtain the top view length of the line joining the centres of the spheres.
6. Redraw the top view such that, the line joining the centres of the spheres makes an angle  $40^\circ$  with xy. This is the final top view.
7. Obtain the final front view, by projection.



**Fig.11.64**

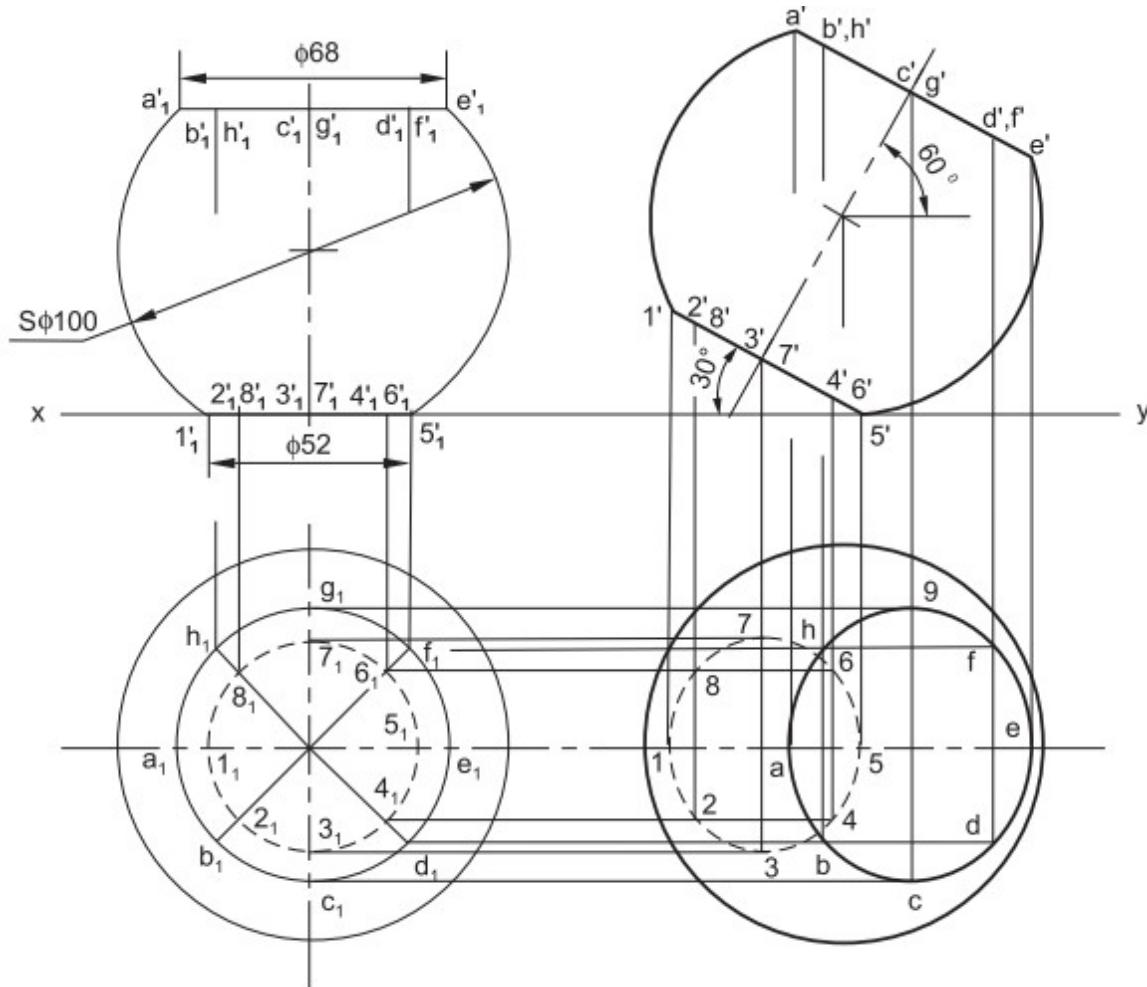
**Problem 54** An ashtray made up of a thin sheet of steel, is spherical in shape with flat, circular top of 68 diameter and bottom of 52 diameter and parallel to each other. The greatest diameter of it is 100. Draw the projections of the ashtray when its axis is parallel to the V.P and,

- (a) Makes an angle  $60^\circ$  with the H.P
- (b) Its base is inclined at  $30^\circ$  to H.P

**Construction (Fig.11.65)**

1. Draw the projections of the given spherical ashtray, satisfying the given dimensions.
2. Divide the circular top and bottom bases into 8 equal parts and number them as shown.

- Redraw the front view such that, the base makes an angle  $30^\circ$  with xy. This is the final front view.
- Obtain the final top view, by projection.



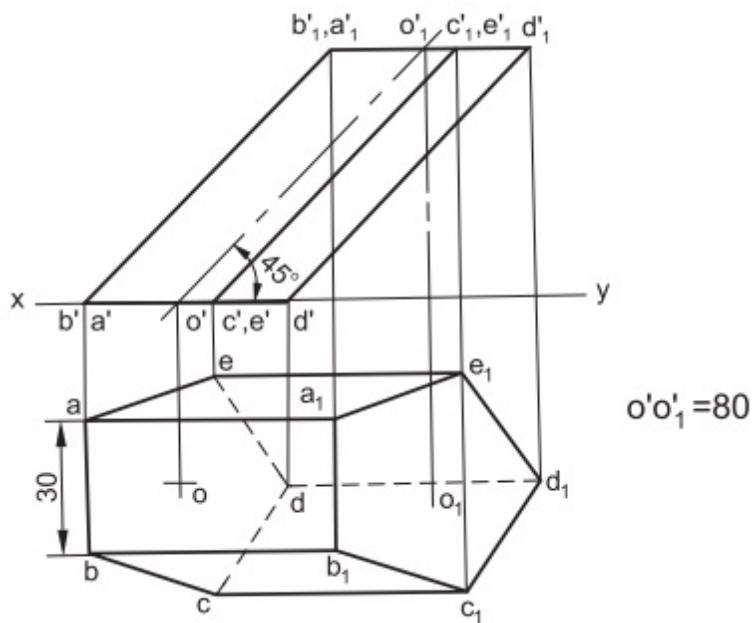
**Fig.11.65**

**Problem 55** An oblique pentagonal prism, with edge of base 30 and 80 long, has its base on H.P. The axis makes an angle of  $45^\circ$  with the base and parallel to V.P. Draw the projections, when one of its sides is perpendicular to V.P.

**Construction (Fig.11.66)**

- Draw the top view of the bottom base abcdea, keeping the edge ab perpendicular to xy.

2. Obtain the front view of the base, coinciding with  $xy$ , by projection.
3. Draw the axis  $o'o_1'$  of length 80, through  $o'$  and making an angle of  $45^\circ$  with  $xy$ .
4. Draw the top base in the front view, through  $o_1'$  and complete the front view of the oblique prism.
5. Complete the top view of the solid, by projection.



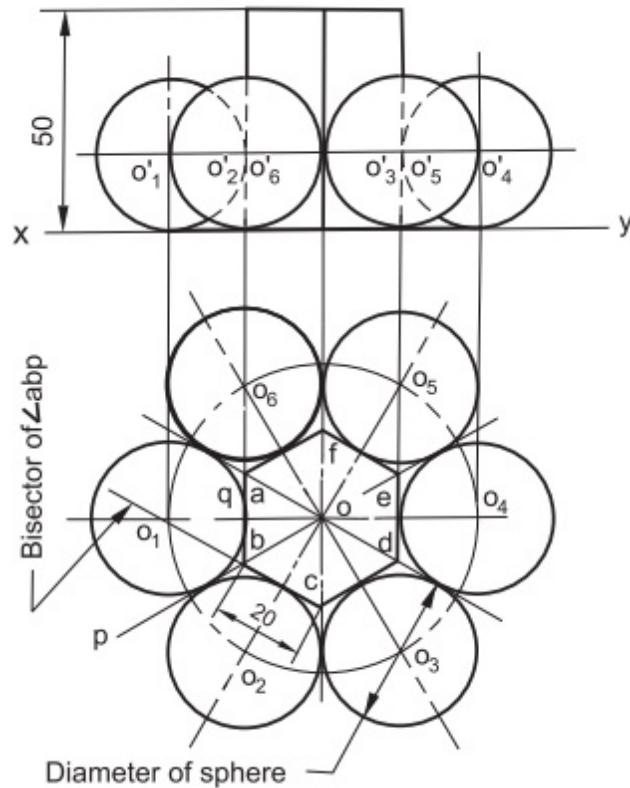
**Fig.11.66**

**Problem 56** A hexagonal prism of side of base 20 and axis 50 long, rests with its base on H.P and with one of its rectangular faces perpendicular to V.P. Six spheres are placed such that, each sphere touches the rectangular face of the prism and two other spheres. Draw the projections of the arrangement and find the diameter of the spheres.

**Construction (Fig.11.67)**

1. Draw the projections of the prism.

2. Locate the centres of the spheres in the top view, as follows:
  - (i) Join o,b and produce it to p.
  - (ii) Bisect  $\angle abp$ .
  - (iii) Through o, draw perpendicular bisector to ab, intersecting the above bisector at  $o_1$  and ab at q.
  - (iv) With o as centre and radius  $oo_1$ , draw a circle, known as the pitch circle of the spheres.
  - (v) Draw perpendicular bisectors of the sides bc, cd, etc., intersecting the pitch circle at  $o_2, o_3$ , etc., the centres of the spheres in the top view.
3. With centres  $o_1, o_2$ , etc., and radius  $o_1q$ , draw circles in the top view.  
Diameter of the shere is equal to  $2 \times o_1q$ .
4. Draw projectors through these centres and locate them in the front view at a height equal to the radius of the spheres and above the reference line xy.
5. With these centres in the front view and radius  $o_1q$ , draw the circles, completing the front view.



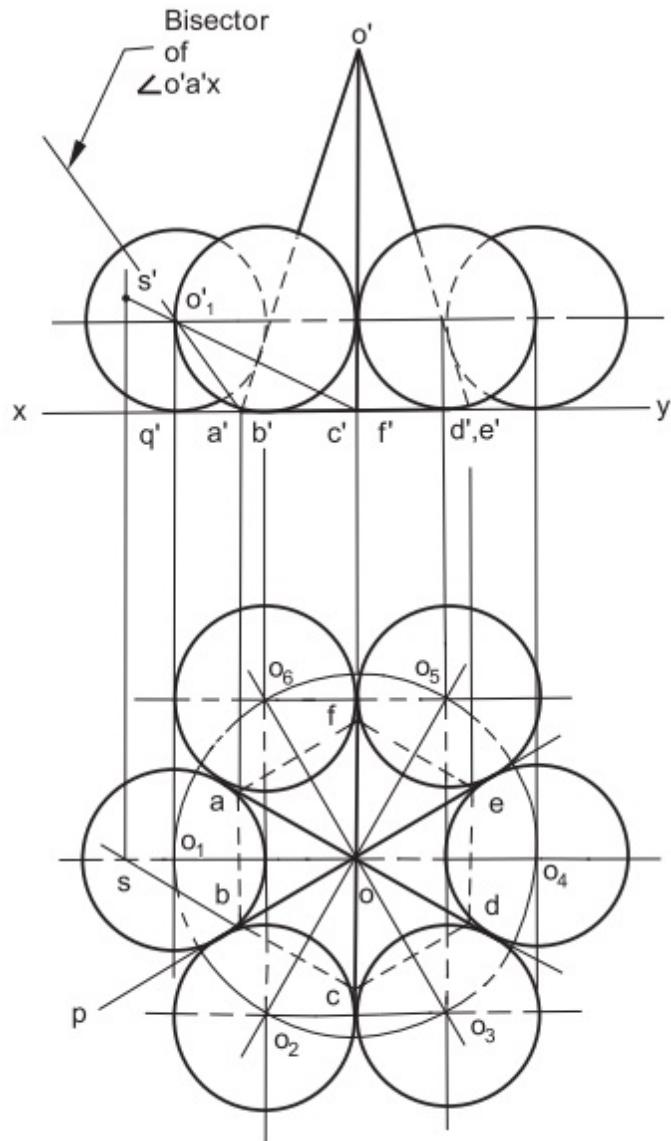
**Fig.11.67**

**Problem 57** A hexagonal pyramid of side of base 20 and axis 50 long, rests with its base on H.P and with an edge of its base perpendicular to V.P. Six equal spheres are placed such that, each one touches the triangular face of the pyramid and two other spheres. Draw the projections of the arrangement and find the diameter of the spheres.

**Construction (Fig.11.68)**

1. Draw the projections of the given pyramid.
2. Assuming the solid to be a prism, locate the centre of the imaginary sphere  $s$  and  $s'$ , as in the previous example.
3. Join  $s'$  to the centre of the base, which coincides with  $c'$ . The centre of the sphere touching the triangular face of the pyramid will have its centre on this line.

4. Draw the bisector of  $\angle o'a'x$ , meeting  $c's'$  at  $o_1'$ , the centre of one of the spheres in the front view.
5. Draw a projector through  $o_1'$ , meeting the line  $os$  at  $o_1$  and  $xy$  at  $q'$ .
6. With  $o_1$  and  $o_1'$  as centres and radius  $o_1'q'$ , draw circles, forming the projections of a sphere.  
Diameter of the sphere is  $2 \times o_1'q'$ .
7. With  $o$  as centre and radius  $oo_1$ , draw the pitch circle.
8. Locate the centres of the other spheres in the top view, by drawing perpendicular bisectors to  $bc$ ,  $cd$ , etc., intersecting the pitch circle at  $o_2$ ,  $o_3$ , etc., respectively.
9. Locate the centres of these spheres in the front view, by projection and complete the projections.



**Fig.11.68**

## EXERCISES

### General

- 11.1 Draw the projections of the following solids, situated in their respective positions, taking the side of the base equal to 40 and the length of the axis as 70:

- (a) A square prism with base on H.P and a side of base inclined at  $30^\circ$  to V.P.
- (b) A hexagonal pyramid with base on H.P and a side of the base parallel to and 25 from V.P.
- (c) A pentagonal prism with a rectangular face on H.P, axis perpendicular to V.P and one base on V.P.
- (d) A triangular pyramid with base on H.P, an edge of the base inclined at  $45^\circ$  to V.P and the apex at 40 from V.P.
- (e) A cube of edge 40, with a base on H.P and a face perpendicular to V.P. The nearest edge parallel to V.P is 5 in front of it.
- (f) A square pyramid with its base on V.P and one edge of the base inclined at  $30^\circ$  to H.P and a corner contained by that edge is on H.P.
- (g) A hexagonal pyramid with its base on V.P and an edge of the base inclined at  $20^\circ$  to H.P.

11.2 Draw the projections of a pentagonal pyramid, axis 60 long, base 30 side, having base on the ground and one of the edges of the base inclined at  $45^\circ$  to V.P.

11.3 A triangular prism, base 40 side and height 65, is resting on H.P on one of its rectangular faces, with the axis parallel to V.P. Draw its projections.

11.4 Draw the projections of the following solids, situated in their respective positions, taking the diameter of the base equal to 50 and the length of the axis as 70:

- (a) A cylinder, with its axis perpendicular to V.P and 40 above H.P and one end at 20 from V.P.
- (b) A cone, with its apex on H.P and 40 from V.P; the axis being perpendicular to H.P.

(c) A cylinder, resting with its base on H.P and axis 30 in front of V.P.

11.5 A tetrahedron of 75 long edges has one edge parallel to H.P and inclined at  $45^\circ$  to V.P while a face containing that edge is vertical. Draw its projections.

11.6 A tetrahedron of 40 side rests with one of its edges on H.P and inclined at  $45^\circ$  to V.P. The triangular face containing that edge is inclined at  $30^\circ$  to H.P. Draw the front and top views of the solid.

11.7 An octahedron of 30 edge lies with one of its faces on H.P and an edge of the face inclined at  $45^\circ$  to V.P. Draw its projections.

11.8 An octahedron of 30 side, has a face on V.P, with an edge of that face inclined at  $45^\circ$  to H.P. Draw the projections of the solid.

11.9 A tetrahedron of 85 edge has an edge on H.P and inclined at  $45^\circ$  to V.P, while the face containing that edge is vertical. Draw its projections.

11.10 A tetrahedron of edge 50, rests with one of its edges on H.P such that, the vertical plane containing that edge and the centre of gravity of the solid makes  $45^\circ$  with V.P. Draw the projections.

11.11 A 70 diameter spherical paper weight has a 45 diameter flat base. It is resting on H.P such that (a) its flat base is perpendicular to H.P and inclined at  $30^\circ$  to V.P, (b) its axis is parallel to H.P and inclined at  $60^\circ$  to V.P. Draw its projections.

11.12 Three spheres of diameter 70, 50 and 30 are resting on H.P touching each other. Draw its projections when the line joining the centres of two bigger

spheres is parallel to V.P. The smallest sphere is placed in front to the other spheres.

## Cubes

11.13 One of the body diagonals of a cube of 55 edge is vertical and the vertical plane passing through this body diagonal and one edge of the cube containing the bottom-most corner, makes  $30^\circ$  with V.P. Draw the projections.

11.14 One of the body diagonals of a cube is perpendicular to V.P. The edge of the cube is 50. Draw the projections.

11.15 A cube of 30 edge, rests on one of its corners on H.P such that, an edge containing this corner is inclined at  $60^\circ$  to H.P and parallel to V.P. The other two edges passing through that corner are equally inclined to H.P. Draw the projections of the cube.

11.16 One of the body diagonals of a cube of 45 edge, is parallel to H.P and inclined at  $45^\circ$  to V.P. Draw the front view and top view of the cube.

11.17 Draw the projections of a cube of 25 long, resting on V.P on one of its corners, with a solid diagonal perpendicular to H.P.

## Prisms

11.18 A square prism with side of base 30 and axis 45 long, lies on H.P such that, its axis is parallel to both H.P and V.P. Draw the front and top views of the prism, when

- (a) it lies with one of its rectangular faces on H.P, and
- (b) it lies with one of its longer edges on H.P.

11.19 An equilateral triangular prism of side of base 25 and axis 50 long, is resting on an edge of its base on H.P. The face containing that edge is inclined at  $30^\circ$  to H.P. Draw the projections of the prism, when the edge on which the prism rests, is inclined at  $60^\circ$  with V.P.

11.20 A hexagonal prism has one of its rectangular faces parallel to H.P. Its axis is perpendicular to V.P and 35 above the ground. Draw its projections when the nearer end is 20 in front of V.P. Side of base is 25 long and axis is 50 long.

11.21 A hexagonal prism of side of base 25 and axis 70 long, lies on (i) one of its rectangular faces on H.P and (ii) one of its longer edges on H.P such that, its axis is parallel to both H.P and V.P. Draw its projections.

11.22 A pentagonal prism of side of base 25 and axis 60 long, is lying on H.P on one of its faces, with the axis parallel to V.P. Draw its projections.

11.23 A regular pentagonal prism of side of base 40 and length 80, lies with one of its rectangular faces on H.P and its axis is inclined to V.P at  $40^\circ$ . Draw the projections of the prism.

11.24 A triangular prism with side of base 40 and height 65, is resting on a corner of its base on H.P, with a longer edge containing that corner, inclined at  $45^\circ$  to H.P. A vertical plane containing that edge and the axis is inclined at  $30^\circ$  to V.P. Draw its projections.

11.25 A square prism with base 40 side and height 65, has its axis inclined at  $45^\circ$  to H.P and has an edge of its base on the H.P and inclined at  $30^\circ$  to V.P. Draw its projections.

11.26A triangular prism of base side 45 and length of axis 75, has a corner in H.P, the face opposite to that corner makes  $50^\circ$  to H.P while the axis of the solid makes  $30^\circ$  to V.P. Obtain the two views of the solid.

11.27A pentagonal prism is resting on a corner of its base on H.P. The longer edge containing that corner is inclined at  $30^\circ$  to H.P and the vertical plane containing that edge is inclined at  $45^\circ$  to V.P. Draw the projections of the solid.

11.28A hexagonal prism, with side of base 30 and axis 70 long, is resting on one of its longer edges on H.P, with its axis inclined at  $40^\circ$  to V.P. A rectangular face containing the longer edge, on which the prism rests, is inclined at  $30^\circ$  to H.P. Draw the projections of the prism.

11.29A hexagonal prism with side of base 40 and height 55, stands on one of its bases on H.P and a vertical face parallel to V.P and 35 from it. A triangular pyramid stands with its base on the top surface of the prism such that, the corners of its base coincide with alternate corners of the top surface of the prism. Height of the pyramid is 65. Draw the projections and also obtain an auxiliary projection of the combination of the solids on a plane parallel to one of the triangular faces of the pyramid.

11.30A hexagonal prism with side of base 20 and axis 45 long, rests with a side of base on H.P and the base is inclined at  $45^\circ$  to H.P. Obtain the projections of the prism, when the top view of the axis is inclined at  $60^\circ$  to xy.

11.31A hexagonal prism with side of base 35 and axis 65 long, is resting on a corner of its base on H.P, with its

longer edge containing that corner, inclined at  $45^\circ$  to H.P and a vertical plane containing that edge and the axis, inclined at  $30^\circ$  to V.P. Draw its projections.

11.32 One end of a longer edge of a regular hexagonal prism of side of base 30 and height 80 is on the V.P and the other end of the same edge is on the ground. The axis makes  $30^\circ$  to V.P and  $40^\circ$  to the ground. Draw its projections.

11.33 A triangular prism, with edge of base 35 and 80 long, has a concentric hole of diameter 15. It is resting on one of its faces on H.P, with its axis parallel to both H.P and V.P. Draw its projections.

11.34 Draw the projections of a square prism with side of base 30 and axis 60 long, is resting with one of its edges of its base on H.P. Its axis is inclined at  $30^\circ$  to H.P and the top view of the axis is at  $45^\circ$  to xy line.

11.35 A pentagonal prism is resting on one of its corners of its base on H.P, the longer edge containing that corner is inclined at  $45^\circ$  to H.P. The axis of the prism makes an angle of  $30^\circ$  to V.P. Draw the projections.

11.36 Draw the projections of a 70 long regular hexagonal prism with base edges 30, to satisfy the following conditions: One of the longer edges is on the H.P, one of the base edges is parallel to V.P and the axis is inclined at  $60^\circ$  to V.P.

11.37 A regular square prism lies with its axis inclined at  $60^\circ$  to H.P and  $30^\circ$  to V.P. The prism is 60 long and has a face width of 25. The nearest corner is 10 away from V.P and the farthest shorter edge is 100 from H.P. Draw the projections of the solid.

## Pyramids

11.38 A square pyramid with base 40 side axis 65 long, has its base in the V.P. One edge of the base is inclined at  $30^\circ$  to H.P and a corner contained by that edge is on H.P. Draw its projections.

11.39 Draw the projections of a pentagonal pyramid with base 30 edge and axis 50 long, having its base on the H.P and an edge of the base parallel to V.P. Also, draw its side view.

11.40 Draw the projections of a regular hexagonal pyramid, with side of base 30 and axis 70 long, which is resting with a slant face on H.P such that, the axis is parallel to V.P.

11.41 A square pyramid with base 40 side and axis 90 long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of  $45^\circ$  with V.P. Draw its projections.

11.42 Draw the projections of a square pyramid of base 40 side and axis 70 long, when the solid lies with one of its slant edges on H.P and the vertical plane passing through that slant edge and axis makes  $30^\circ$  with V.P.

11.43 A pentagonal pyramid, with side of base 30 and axis 75 long, has an edge of the base on H.P and inclined at  $30^\circ$  to V.P. The triangular face containing that edge makes an angle of  $45^\circ$  with V.P. Draw the projections of the solid.

11.44 A hexagonal pyramid with side of base 25 and axis 60 long, has an edge of its base on V.P. Its axis is inclined at  $30^\circ$  to V.P and parallel to H.P. Draw the projections of the solid.

11.45 A pentagonal pyramid with side of base 25 and axis 70 long, has a corner of the base on H.P. Its axis is

inclined at  $30^\circ$  to H.P and parallel to V.P. Draw the projections of the solid.

11.46A regular pentagonal pyramid with base 30 side and height 80, rests on one edge of its base on the ground so that the highest point in the base is 30 above the ground. Draw its projections, when the axis is parallel to V.P.

11.47A regular pentagonal pyramid with a side of its base 30 and height 80, rests on an edge of the base on H.P. The base is tilted until its apex is 50 above the level of the edge of the base on which it rests. Draw the projections of the pyramid when the edge on which it rests is parallel to V.P and the apex of the pyramid points towards the V.P.

11.48A pentagonal pyramid with side of base 30 and height 75, has one of its triangular faces perpendicular to H.P and inclined at  $45^\circ$  to V.P. Draw the projections of the solid, when the base side of the triangular face is on H.P.

11.49A pentagonal pyramid with side of base 25 and axis 60 long, has a triangular face on H.P and the edge of the base contained by that face makes an angle of  $30^\circ$  with V.P. Draw its projections.

11.50A hexagonal pyramid with side of base 25 and axis 55 long, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to H.P and inclined at  $45^\circ$  to V.P. Draw its projections, when the apex is nearer the V.P than the base.

11.51A square pyramid with side of base 40 and axis 50 long, is resting on a triangular face on H.P such that,

a slant edge containing that face is parallel to V.P.  
Draw the projections of the solid.

11.52A hexagonal pyramid with side of base 30 and axis 75 long, has an edge of the base on V.P and inclined at  $30^\circ$  to H.P. The triangular face containing the edge makes an angle of  $45^\circ$  with V.P. Draw the projections of the solid.

11.53 Hexagonal pyramid with side of base 30 and axis 50 long, rests with one of the corners of its base on H.P. Its axis is inclined at  $35^\circ$  to H.P and  $45^\circ$  to V.P. Draw its projections.

11.54A hexagonal pyramid with side of the base 25 long and height 70, has one of its triangular faces perpendicular to H.P and inclined at  $45^\circ$  to V.P. The base side of this triangular face is parallel to H.P. Draw its projections.

11.55 Draw the projections of a tetrahedron of side 60, when it is resting on H.P on one of its faces, with an edge of its face (i) inclined at  $45^\circ$  to V.P, (ii) parallel to V.P and (iii) inclined at  $60^\circ$  to V.P.

11.56 Draw the projections of an octahedron of side 30, with one of its faces parallel to H.P and a side of that face inclined at  $20^\circ$  to V.P.

11.57 Draw the projections of an octahedron of side 40, when one of its axes is vertical and one of the horizontal edges is making an angle of  $30^\circ$  with V.P.

11.58 A square pyramid of base edge 30 and altitude 40, has one of its slant faces in the V.P and the edge of the base contained by the face is inclined at  $45^\circ$  to H.P. Draw the projections of the pyramid, when the vertex is in H.P.

11.59 A square pyramid with side of base 45 and axis 60, rests with one of its base diagonals inclined at  $60^\circ$  to H.P and the other diagonal parallel to both H.P and V.P. Draw the projections.

11.60 Five equal spheres are resting on the ground, each touching a face of a vertical pentagonal prism and adjacent spheres. Find the diameter of the spheres and draw the projections when a side of the base of the prism is perpendicular to V.P.

11.61 A pentagonal pyramid with base 35 and height 70, rests on an edge of its base on H.P. The highest point in the base is 25 above H.P. Draw its projections, when the axis is parallel to V.P. Draw another front view on a reference line, inclined at  $45^\circ$  to the edge on which it is resting, so that the base is visible.

11.62 A square pyramid with side of base 40 and height 80, is suspended freely from a point on a slant edge at distance of 20 from its apex. The top view of the axis of the pyramid is inclined at  $30^\circ$  to xy line. Draw the projections.

## Cylinders

11.63 Draw the projections of a cylinder of diameter 75 and length 100, lying on the ground with its axis inclined at  $40^\circ$  to V.P and parallel to the ground.

11.64 A cylinder of diameter 60 and axis 70 long, is having its axis inclined at  $45^\circ$  to V.P and  $30^\circ$  to H.P. Draw its projections.

11.65 Draw the projections of a cylinder of diameter 60 and 90 long, lying on V.P on one of its generators, with its axis inclined at  $60^\circ$  to H.P.

11.66A right circular cylinder with diameter of base 50 and length of axis 70, rests on H.P on its base rim such that, its axis is inclined at  $45^\circ$  to H.P and the top view of the axis is inclined at  $60^\circ$  to V.P. Draw its projections.

11.67A cylinder with base 35 diameter and axis 60 long, lies with one of its generators on H.P such that, its axis is parallel to V.P. Draw its projections.

11.68A hollow cylinder of outside diameter 64, thickness of wall 14, height 66, is resting on H.P. Its axis is inclined at  $45^\circ$  to V.P and parallel to H.P. Draw its front view and top view.

11.69A cylindrical boiler is 2m in diameter and has a cylindrical dome with 0.8m diameter and 0.6m high. The axis of the dome intersects the axis of the boiler. Draw three views of the arrangement. Also, develop the surface of the dome. Take a scale of 1cm = 0.2m.

## **Cones**

11.70Draw the projections of a cone with base 75 diameter and axis 100 long, lying on H.P on one of its generators, with the axis parallel to V.P.

11.71Draw the projections of a cone of base 60 diameter and axis 70 long, resting on a point of rim of the base on H.P, with a generator perpendicular to H.P. The plane containing the axis and the generator is parallel to V.P. Draw the projections of the cone.

11.72Draw the projections of a cone of 100 height and 75 diameter, resting on one of its generators on H.P and its axis lies in a vertical plane, inclined at  $30^\circ$  to V.P.

11.73Three cones, each of base 50 diameter and axis 80 long, are resting on their bases on H.P such that,

each touching the other two. A sphere of 45 diameter is placed centrally between the cones. Determine the height of the centre of the sphere from H.P.

11.74A cone of base 50 diameter and axis 80 long, has one of its generators on V.P and inclined at  $30^\circ$  to H.P. Draw the three views of the cone.

11.75Draw the projections of a cone with base 45 diameter and axis 60 long, when it is resting on the ground on a point of its base circle, with the axis making an angle of  $30^\circ$  with H.P and  $45^\circ$  with V.P.

11.76A thin lamp shade is in the form of a cone, the ends of which are 80 and 160 diameter and the vertical height is 150. It rests on H.P, on a point of its larger end, with the axis inclined at  $40^\circ$  to H.P and  $30^\circ$  to V.P. The smaller end of the shade is nearer to V.P. Draw the projections of the shade.

11.77Draw three views of a cone with base 50 diameter and axis 75 long, having one of its generators in V.P and inclined at  $30^\circ$  to H.P; the apex being in H.P.

11.78A right circular cone with diameter of base 50 and height 65, lies on one of its elements in H.P such that, the element is inclined to V.P at  $30^\circ$ . Draw its projections.

11.79A right circular cone of base diameter 50 long, is freely suspended from a point on the periphery of the base. Draw its projections when the axis is parallel to V.P.

## Oblique Solids

11.80An oblique pentagonal prism, with edge of base 30 and 80 long, has its base on H.P. The axis makes an

angle of  $45^\circ$  with the base and parallel to V.P. Draw the projections, when one of its sides is perpendicular to V.P.

11.81 An oblique hexagonal pyramid, with edge of base 25 and 65 long, has its base parallel to V.P and an edge of the base perpendicular to H.P. The axis makes  $60^\circ$  with the base and parallel to H.P. Draw the projections.

## REVIEW QUESTIONS

- 11.1 What is the minimum number of views required to convey the three dimensions of a solid?
- 11.2 When are the auxiliary views of a solid necessary?
- 11.3 Define the term, polyhedron.
- 11.4 Define the terms, (i) prism and (ii) pyramid.
- 11.5 What is a solid of revolution? Mention the examples.
- 11.6 How are a cylinder and a cone generated?
- 11.7 Differentiate between right and oblique solids?
- 11.8 Describe (i) change of position method and (ii) change of reference line method, with respect to projection of solids.

## OBJECTIVE QUESTIONS

- 11.1 The base of an oblique hexagonal prism is not a regular hexagon.

(True/False)

11.2 The base of an oblique cylinder is an ellipse.

(True /False)

11.3 When the axis of a solid is perpendicular to H.P, its \_\_\_\_\_ view reveals the true shape of the base.

11.4 When the axis of a solid is perpendicular to H.P, its relation with V.P is (a) parallel, (b) perpendicular, (c) inclined.

( )

11.5 When the axis of a solid is inclined to H.P, it may be parallel to V.P.

(True/False)

11.6 If a solid rests on an edge of its base on V.P, it must be kept perpendicular to H.P.

(True /False)

11.7 If a solid rests on a corner of its base on H.P, the sides of the base containing that corner should be kept equally inclined to V.P.

(True /False)

11.8 In a projection of a solid, when two lines representing the edges cross each other, one of the edges must be invisible.

(True /False)

11.9 When the axis of the solid is parallel to both H.P and V.P, \_\_\_\_\_ view reveals the true shape of the base.

## ANSWERS

11.1 False

11.2 False

11.3 top

11.4 a

11.5 True

11.6 True

11.7 True

11.8 True

11.9 side

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# 12

## *Sections of Solids*



### 12.1 INTRODUCTION

The conventional orthographic views, if selected and drawn properly, may reveal sufficient information about the shape and size of the object. However, the conventional views may consist of too many hidden lines for complicated objects, which make the interpretation difficult. To overcome this, it is customary to imagine the object, cut by a section plane. The portion of the object between the observer and the section plane is assumed to be removed. The projection of the remaining solid is known as a sectional view. The actual sectioned portion of the view is shown by cross-hatched lines.

## **12.2 POSITION OF SECTION PLANES**

The shape of the section obtained or revealed will depend upon the orientation of the solid and the section plane, with respect to the principal planes of projection. The following are some of the positions of the section planes:

1. Section plane parallel to H.P
2. Section plane parallel to V.P
3. Section plane inclined to H.P and perpendicular to V.P
4. Section plane inclined to V.P and perpendicular to H.P
5. Section plane perpendicular to both H.P and V.P
6. Section plane inclined to both H.P and V.P

Section planes are usually represented by their traces. The types of solids that are dealt here are: (i) Polyhedra and (ii) solids of revolution.

### **12.2.1 True Shape of a Section**

The projection of the section on a plane parallel to the section plane will appear in its true shape of the section. Thus, when the section plane is parallel to H.P, the true shape

of the section will be seen in the sectional top view. When it is parallel to V.P, the true shape of the section will appear in the sectional front view.

When the section plane is inclined, the section has to be projected on an auxiliary plane, parallel to the cutting

plane, to obtain its true shape. However, when the section plane is perpendicular to both H.P and V.P, the sectional side view shows the true shape of the section.

## 12.2.2 Sections

When the section plane is inclined to H.P and perpendicular to V.P, the section in the front view coincides with the V.T of the section plane. The section in the top view is known as apparent section and does not represent the true shape. Similarly, when the section plane is inclined to V.P and perpendicular to H.P, the section in the top view coincides with the H.T of the section plane and the front view shows the apparent section of the solid. The apparent section may be obtained by,

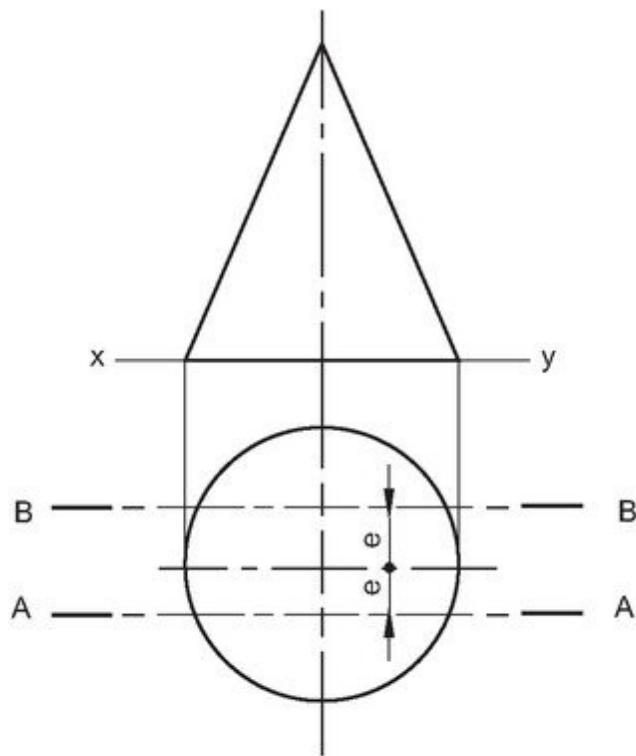
1. Locating the intersection points between the trace of the cutting plane and either the sides or generators of solid, and
2. Locating these points in the other view and joining them by straight lines or curved lines in the correct sequence.

## 12.3 SECTIONS OF POLYHEDRA AND SOLIDS OF REVOLUTION

When a section plane passes through any polyhedron, the intersection of the section plane with the surfaces of the solid consists of a number of straight lines. Hence, the sectioned portion in a projected view is a plane figure bounded by straight lines.

When a section plane passes through the lateral surface of the solids of revolution, the intersection is a smooth curve. If the section plane passes through the base, that portion of the boundary will be a straight line.

Figure 12.1 shows the method of indicating the trace of a cutting plane on the projection of a solid. In general, the cutting plane is represented by a chain dotted line. However, it should be noted that only the longer dash of the chain dotted line should cross the boundary of the projection; followed by a short dash and finally terminated by a thick longer dash.



**Fig.12.1**

Further, referring Fig. 12.1, given the off-set ( $e$ ) of the cutting plane from the axis of the solid, it may be seen that there are two possible positions A-A and B-B for its location. However, it is customary to select the position such that,

only a minor portion of the solid is removed. Thus, referring Fig.12.1, the cutting plane along the line A-A is preferred.

### 12.3.1 Section Plane Parallel to H.P

When a section plane parallel to H.P passes through any polyhedron resting on H.P, the sectioned portion will appear in its true shape in the top view. In the front view, it will appear as a straight line, parallel to xy and coincides with V.T of the section plane. When a section plane parallel to H.P, passes through any solid of revolution, resting on a base on H.P, the section is a circle and it also represents the true shape of the section.



When a cone or pyramid is cut by a section plane, parallel to the base; the retained portion of the solid is called the frustum.

**Problem 1** A cube of 40 edge, is resting on H.P on one of its edges, with a face parallel to V.P. One of the faces containing the resting edge is inclined at  $30^\circ$  to H.P. The solid is cut by a section plane, parallel to H.P and 10 above the axis. Draw the projections of the remaining solid.

**HINT** As the section plane is parallel to H.P, it is perpendicular to V.P. Hence, the section plane is represented by its trace (V.T) in the front view.

#### Construction (Fig.12.2)

1. Draw the projections of the cube, satisfying the given conditions.
2. Draw the V.T of the section plane in the front view, at a height of 10 above o'.

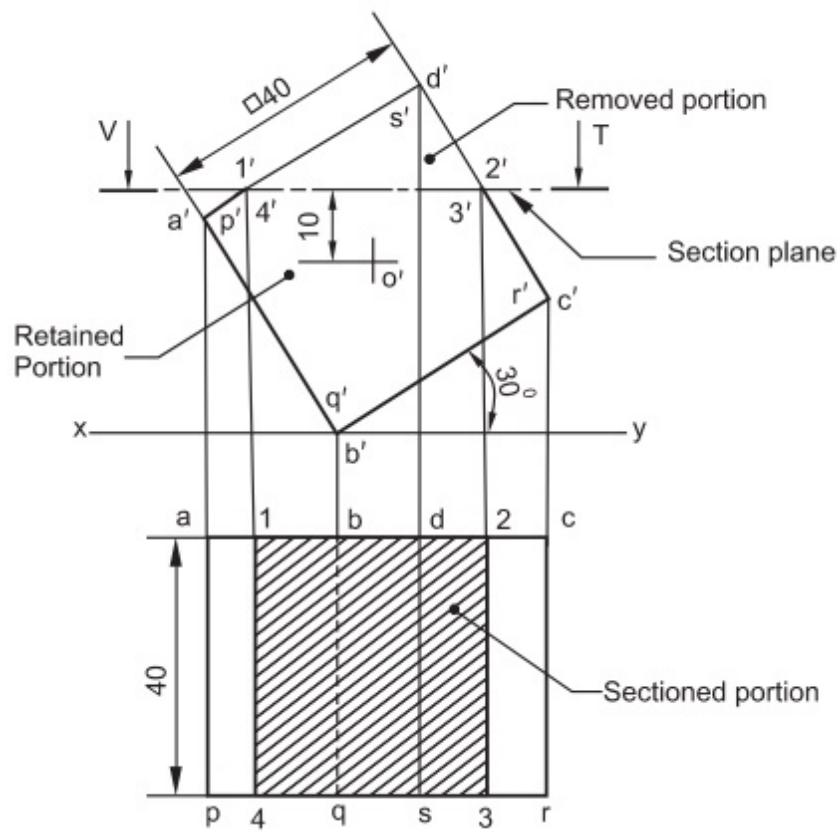
3. Locate the intersection points  $1'$ ,  $2'$ ,  $3'$  and  $4'$ , between the V.T and edges of the cube.
4. Project and locate these points, on the corresponding edges in the top view.
5. Join the points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.



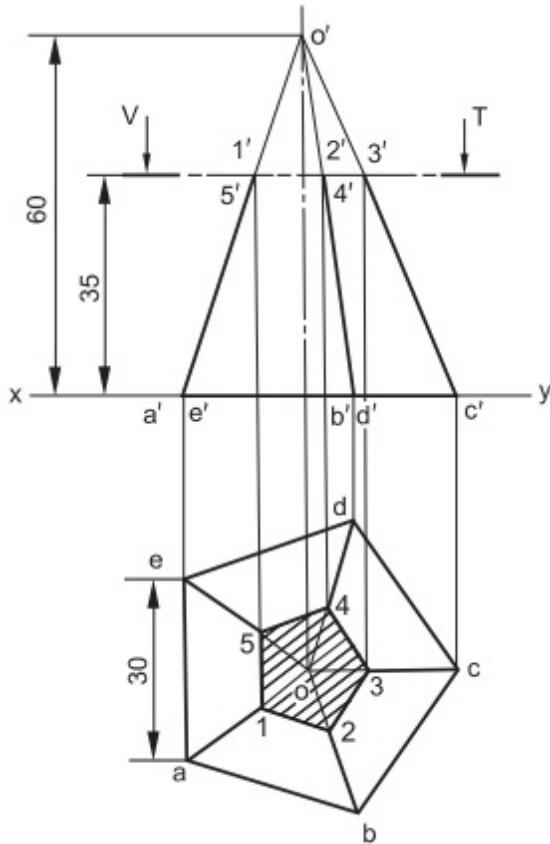
1. The points  $1'$ ,  $2'$ ,  $3'$  and  $4'$  are on the edges  $a'd'$ ,  $d'c'$ ,  $r's'$  and  $s'p'$  respectively.
2. The shape and size of the sectioned portion will depend upon the position of the section plane.
3. The section also represents the true shape.

**Problem 2** A pentagonal pyramid, with side of base 30 and axis 60 long, is resting with its base on H.P and one of the edges of its base is perpendicular to V.P. It is cut by a section plane, parallel to H.P and passing through the axis at a point 35 above the base. Draw the projections of the remaining solid.

**Construction (Fig.12.3)**



**Fig.12.2**



**Fig.12.3**

1. Draw the projections of the pyramid, keeping one edge of the base perpendicular to V.P.
2. Draw the V.T of section plane, at a height of 35 above xy.
3. Locate the intersection points  $1'$ ,  $2'$ , etc., between the trace and slant edges of the pyramid.
4. Project and locate the corresponding points in the top view, on the respective slant edges.
5. Join these points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.

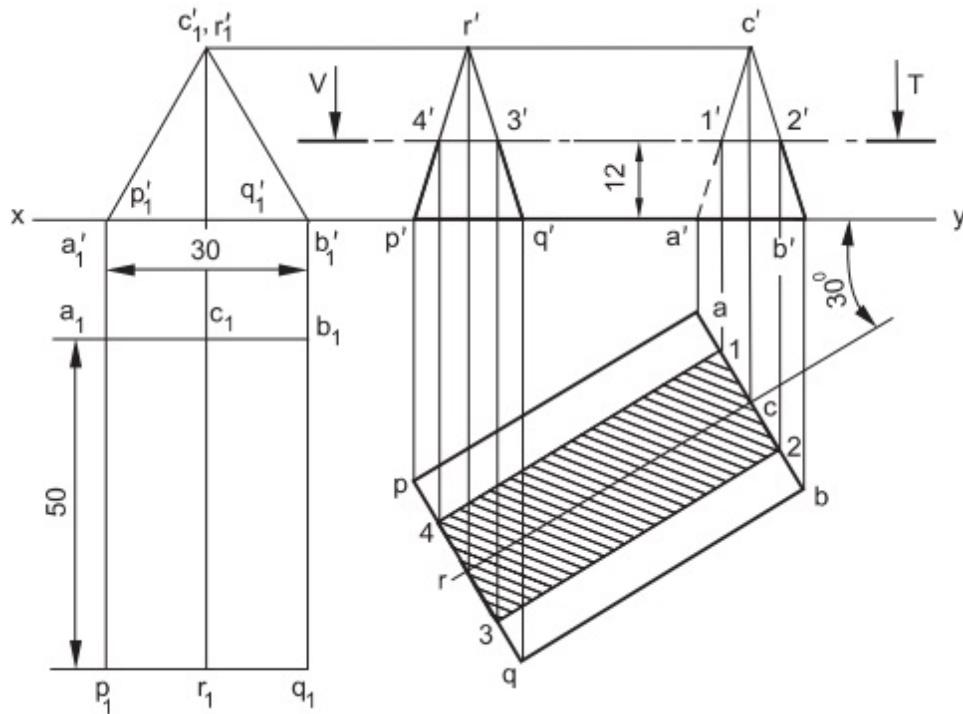
As the section plane is parallel to the base of the pyramid, the sectioned portion will appear in its



true shape as a pentagon, the size of which depends upon the location of the cutting plane.

**Problem 3** A triangular prism of base 30 side and axis 50 long, is lying on H.P on one of its rectangular faces, with its axis inclined at  $30^\circ$  to V.P. It is cut by a section plane, parallel to H.P and at a distance of 12 above H.P. Draw the front and sectional top view.

**Construction (Fig.12.4)**



**Fig.12.4**

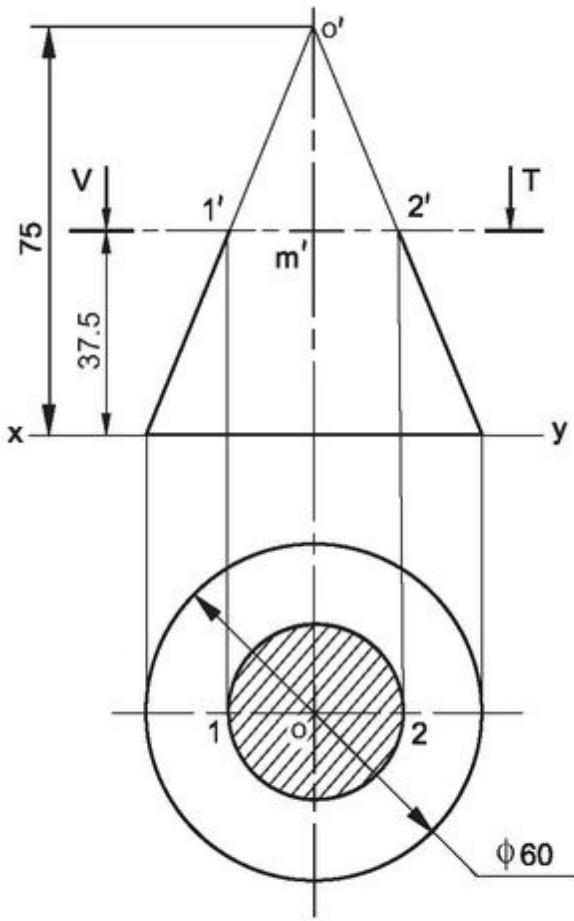
1. Draw the projections of the prism, satisfying the given conditions.
2. Locate the V.T of section plane, at a height of 12 from xy.
3. Locate the intersection points 1', 2', 3'and 4', between the V.T and edges of the solid a'c', c'b', q'r' and r'p' respectively.

4. Project and locate these points on the corresponding edges in the top view.
5. Join these points in the order by straight lines and complete the sectional top view by crosshatching the sectioned portion.

**Problem 4** A cone with base 60 diameter and *axis 75 long, is resting on its base on H.P. It is cut by a section plane parallel to H.P and passing through the mid-point of the axis. Draw the projections of the cut solid.*

**Construction (*Fig.12.5*)**

1. Draw the projections of the cone.
2. Draw the V.T of section plane, passing through the mid-point m' of the axis.

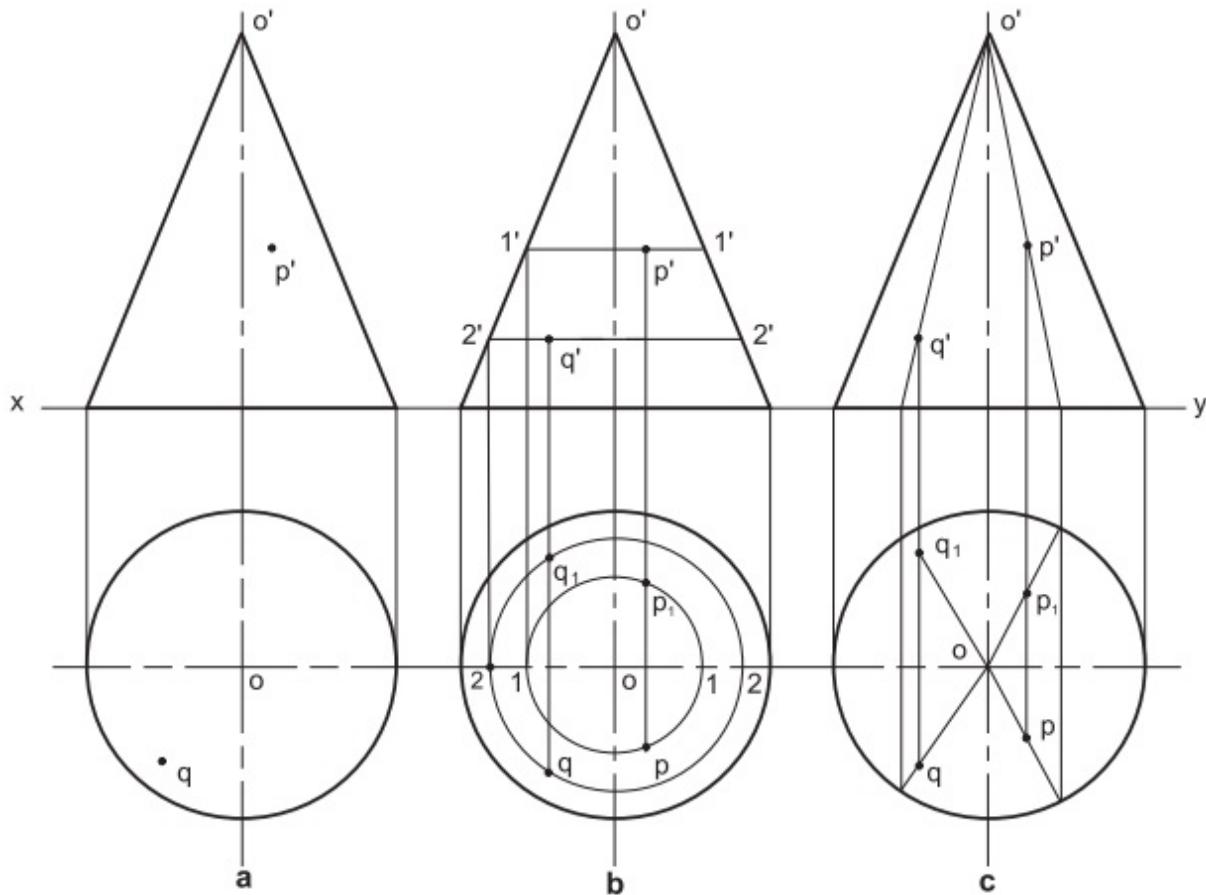


**Fig.12.5**

3. Locate the intersection points  $1'$  and  $2'$ , between the V.T and extreme generators of the cone.
4. With centre  $o$  and diameter  $1'-2'$ , draw a circle in the top view and cross-hatch it; completing the sectional top view.

**Problem 5** *Figure 12.6a shows the projections of a cone, with the front view  $p'$  of a point P and top view  $q$  of a point Q. Locate the top view of the point P and front view of the point Q.*

**Construction (Figs.12.6b and c)**



**Fig.12.6**

To locate the top view of  $Q$

**Method I**

1. Through  $p'$ , draw a line  $1'-1'$ , parallel to the base.
2. With centre  $o$  and diameter equal to  $1'-1'$ , draw a circle in the top view.
3. Through  $p'$ , draw a projector intersecting the circle at  $p$  and  $p_1$ .

$p$  is the top view of the visible point of  $p'$  and  $p_1$  is the top view of another point, lying on the rear side of the cone and coinciding with  $p'$  in the front view.

**Method II**

1. Through  $p'$ , draw a generator.
2. Project and obtain the corresponding division lines in the top view.
3. Through  $p'$ , draw a projector meeting the above lines at  $p_1$  and  $p$ .

*To locate the front view of Q*

### **Method I**

1. With centre  $o$  and radius  $oq$ , draw a circle meeting the horizontal axis at  $2$ .
2. Through  $2$ , draw a projector meeting the extreme generator at  $2'$ .
3. Through  $2'$ , draw the line  $2'-2'$ , parallel to the base.
4. Through  $q$ , draw a projector meeting  $2'-2'$  at  $q'$ , which is the required front view of the point Q.

### **Method II**

1. Join  $o, q$  and extend, forming the division line in the top view.
2. Draw the corresponding generator in the front view.
3. Through  $q$ , draw a projector meeting the above generator at  $q'$ .

## **12.3.2 Section Plane Parallel to V.P**

When a section plane passing through a solid is parallel to V.P, the sectioned portion will appear in its true shape in the front view. In the top view, it will appear as a straight line, parallel to  $xy$  and coincides with its H.T. When the section plane passes through a solid of revolution, the section

produced depends upon the type of solid. In the case of a cylinder, the section is a rectangle; in the case of a cone, when the section plane passes through the apex, it is a triangle; otherwise, it is a hyperbola and in the case of a sphere, it is a circle.

**Problem 6** A cylinder of 40 diameter and axis 55 long, stands vertically with its base on H.P. It is cut by a section plane, parallel to V.P and passes at a distance of 10 from the axis. Draw the projections of the retained solid.

**Construction (Fig.12.7)**

1. Draw the projections of the cylinder.
2. Draw the H.T of section plane, which lies at a distance of 10 from the axis.
3. Locate the points of intersection 1, 2, 3 and 4, between the H.T and bases of the cylinder (points 1 and 2 lie on the top base and 3 and 4 on the bottom base).
4. Project and locate the points 1', 2', 3' and 4', on the corresponding bases in the front view.
5. Join these points in the order by straight lines and complete the sectional front view, by cross-hatching the sectioned portion.

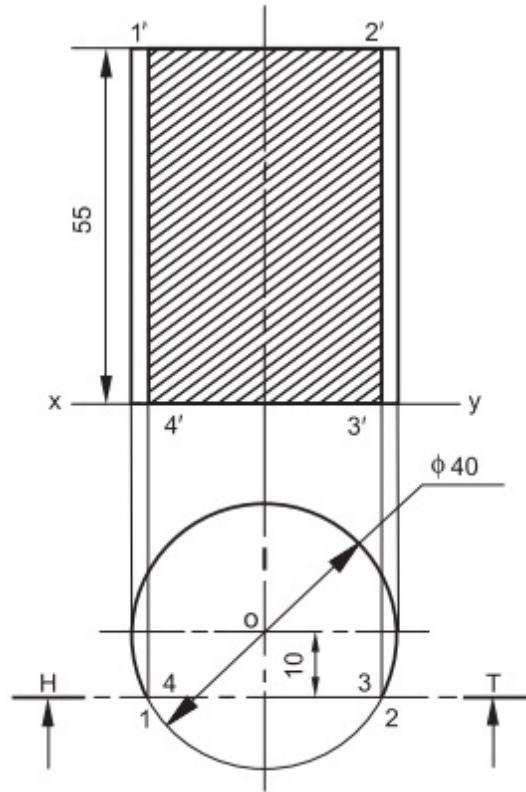


When a cylinder is cut by a section plane, parallel to its axis, the sectioned portion is a rectangle, the length being equal to the length of the axis but the width depends upon the position of the cutting plane from the axis.

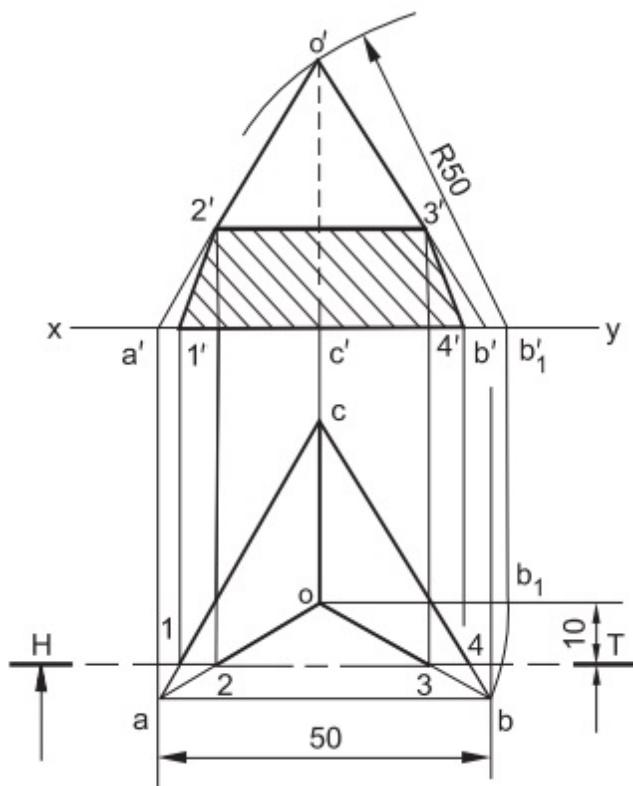
**Problem 7** A tetrahedron of side 50, is resting on H.P on one of its faces, with an edge of it parallel to V.P and away from it. A section plane cuts it, parallel to V.P and at a

*distance of 10 from the apex. Draw the projections of the retained solid.*

**Construction (Fig.12.8)**



**Fig.12.7**



**Fig.12.8**

1. Draw the projections of the tetrahedron.
2. Draw the H.T of section plane, at a distance of 10 from O.
3. Locate the points of intersection 1, 2, 3, 4, between the H.T and edges of the solid in a sequence.

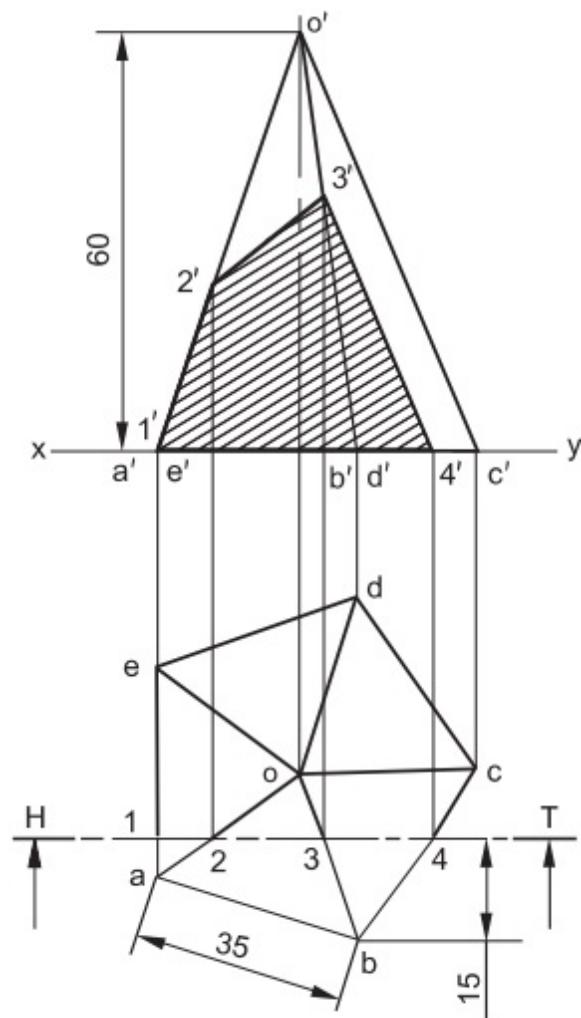
For the given position of the solid, points 1 and 4 are on the edges of the base AC and BC respectively and the points 2 and 3 are on the slant edges OA and OB respectively.

4. Project and locate these points on the corresponding edges in the front view.
5. Join the points in the order by straight lines and complete the sectional front view, by cross-hatching

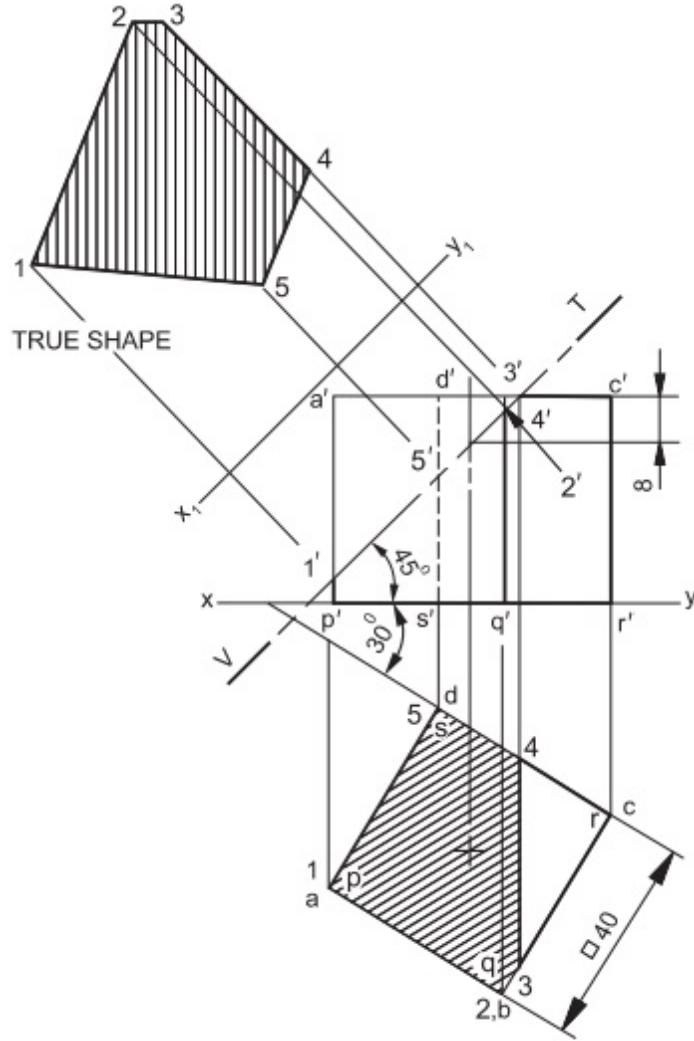
the sectioned portion.

**Problem 8** A pentagonal pyramid of side of base 35 and axis 60 long, stands with its base on H.P such that, one of the base edges is perpendicular to V.P. A section plane parallel to V.P, cuts the solid at a distance of 15 from the corner of the base which is nearer to the observer. Draw the top and sectional front views of the cut solid.

**Construction (Fig.12.9)**



**Fig.12.9**



**Fig.12.10**

1. Draw the projections of the pyramid.
2. Draw the H.T of section plane, at a distance of 15 from b.
3. Repeat steps 3 to 5 of Construction: Fig.12.8 and complete the sectional front view.

### 12.3.3 Section Plane Inclined to H.P and Perpendicular to V.P

When a section plane passing through a solid is inclined to H.P and perpendicular to V.P, its V.T is inclined to the reference line  $xy$ . The sectioned portion in the top view does not reveal the true shape. In all such cases, the true shape of the section may be obtained on an auxiliary plane (A.I.P), parallel to the section plane.

In general, the shape and size of the true shape of the section depends upon the orientation of the solid and the position of the section plane. The position of the section plane will be specified by the inclination of it, with the principal planes of projection and the point in the solid through which it passes.



When a solid is cut by a section plane, inclined to the base, the retained portion is called the truncated solid.

**Problem 9** A cube of side 40, is resting on H.P on one of its faces, with a vertical face inclined at  $30^\circ$  to V.P. It is cut by a section plane inclined at  $45^\circ$  to H.P and passing through the axis at 8 from the top surface. Draw the projections of the solid and also show the true shape of the section.

### **Construction ([Fig.12.10](#))**

1. Draw the projections of the cube.
2. Draw the V.T of section plane, inclined at  $45^\circ$  to  $xy$  and passing through a point at 8 from the top end of the axis.
3. Locate the points of intersection  $1'$ ,  $2'$ ,  $3'$  and  $4'$ , between the vertical trace and the edges of the cube.
4. Repeat steps 4 and 5 of Construction: [Fig.12.2](#) and complete the sectional top view.

**To obtain the true shape of the section:**

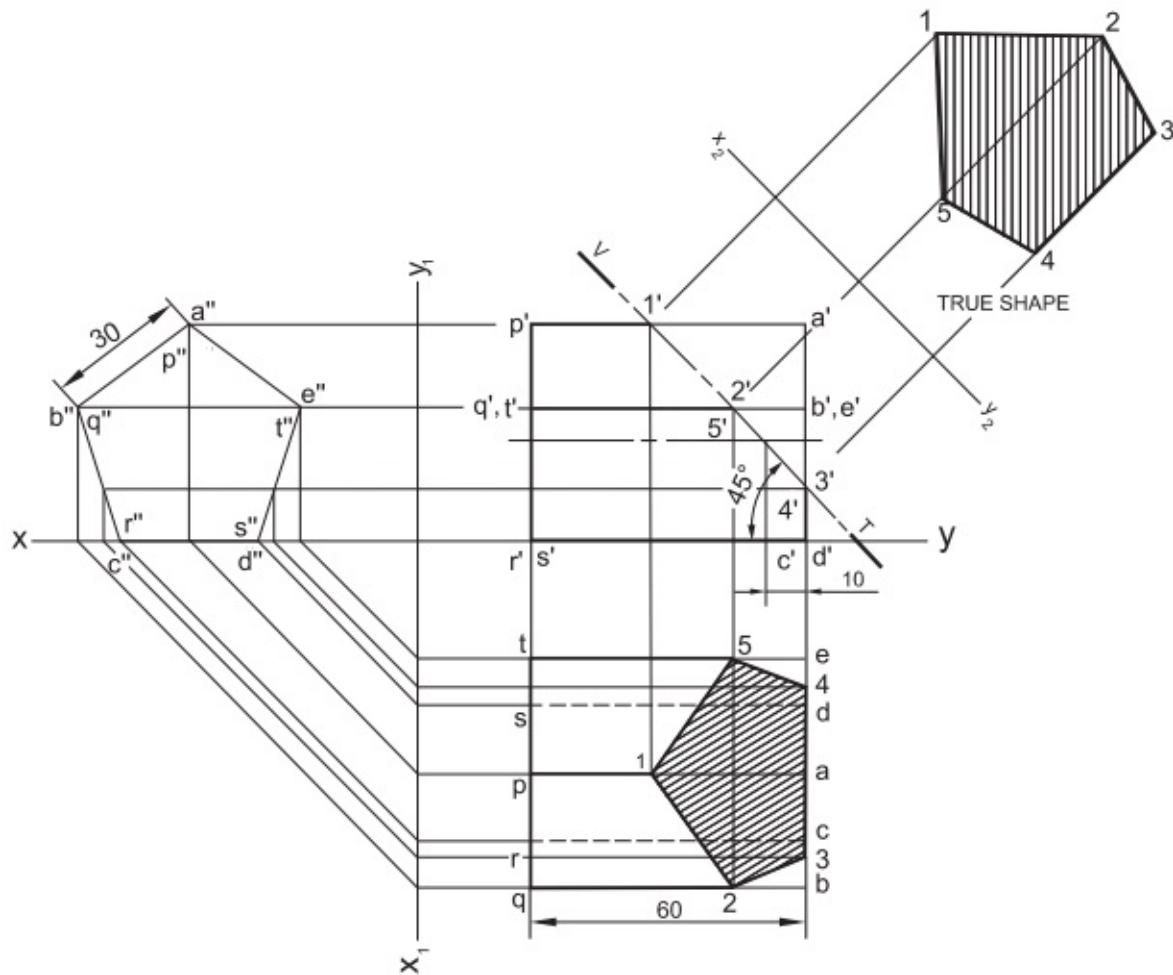
5. Draw the reference line  $x_1y_1$ , parallel to the V.T of section plane.
6. Project the points  $1'$ ,  $2'$ ,  $3'$  and  $4'$ , through  $x_1y_1$  and obtain the true shape of the section.

**Problem 10** A pentagonal prism of edge of base 30 and axis 60 long, is resting on one of its faces on H.P. The axis of the prism is parallel to both H.P and V.P. It is cut by a section plane inclined at  $45^\circ$  to H.P and passing through the axis at 10 from one base. Draw the projections and show the true shape of the section.

### **Construction ([Fig.12.11](#))**

1. Draw the projections of the prism.
2. Draw the V.T of section plane, inclined at  $45^\circ$  to xy and passing through the axis at 10 from one of the bases.
3. Locate the points of intersection  $1'$ ,  $2'$ , etc., between the V.T and edges of the prism.
4. Repeat steps 4 to 6 of Construction: [Fig.12.10](#) suitably and obtain the sectional top view and true shape of the section.

**Problem 11** A hexagonal pyramid of side of base 30 and axis 60 long, is resting on its base on H.P, with an edge of the base perpendicular to V.P. It is cut by a section plane, inclined at  $30^\circ$  to H.P and passing through the axis at 20 from the base. Draw the three views of the solid and obtain the true shape of the section.



**Fig.12.11**

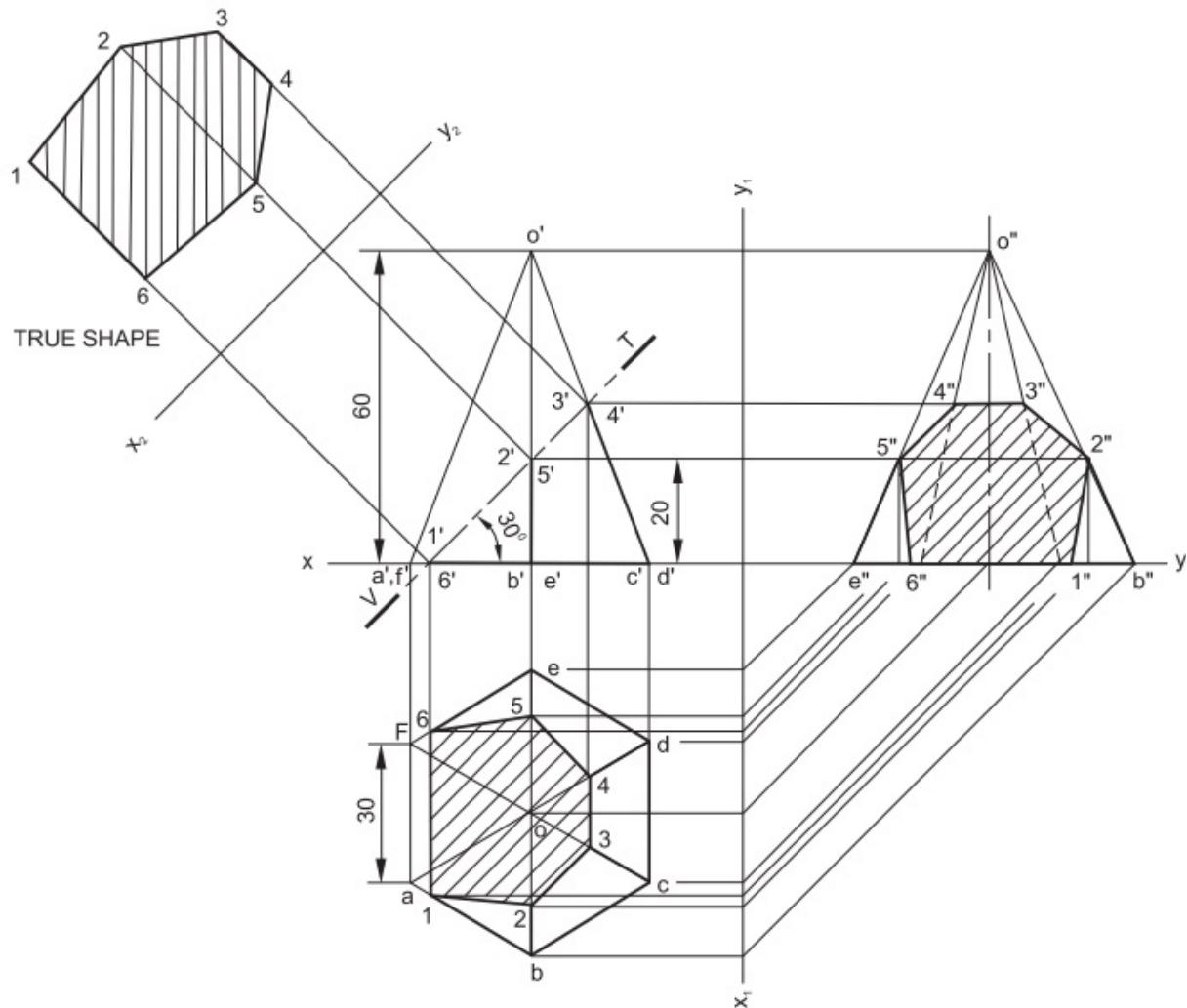
### **Construction (Fig.12.12)**

1. Draw the projections of the pyramid.
2. Draw the V.T of section plane, inclined at  $30^\circ$  to xy and passing through a point on the axis at 20 from the base.
3. Locate the points of intersection 1', 2', etc., between the V.T and edges of the pyramid.

It may be noted that the points 1' and 6' lie on the edges of the base, whereas the remaining points lie on the slant edges of the solid.

4. Repeat steps 4 to 6 of Construction: Fig.12.10 suitably and obtain the sectional top view and the true shape of the section.
5. Project the front and sectional top views and obtain the sectional left side view.

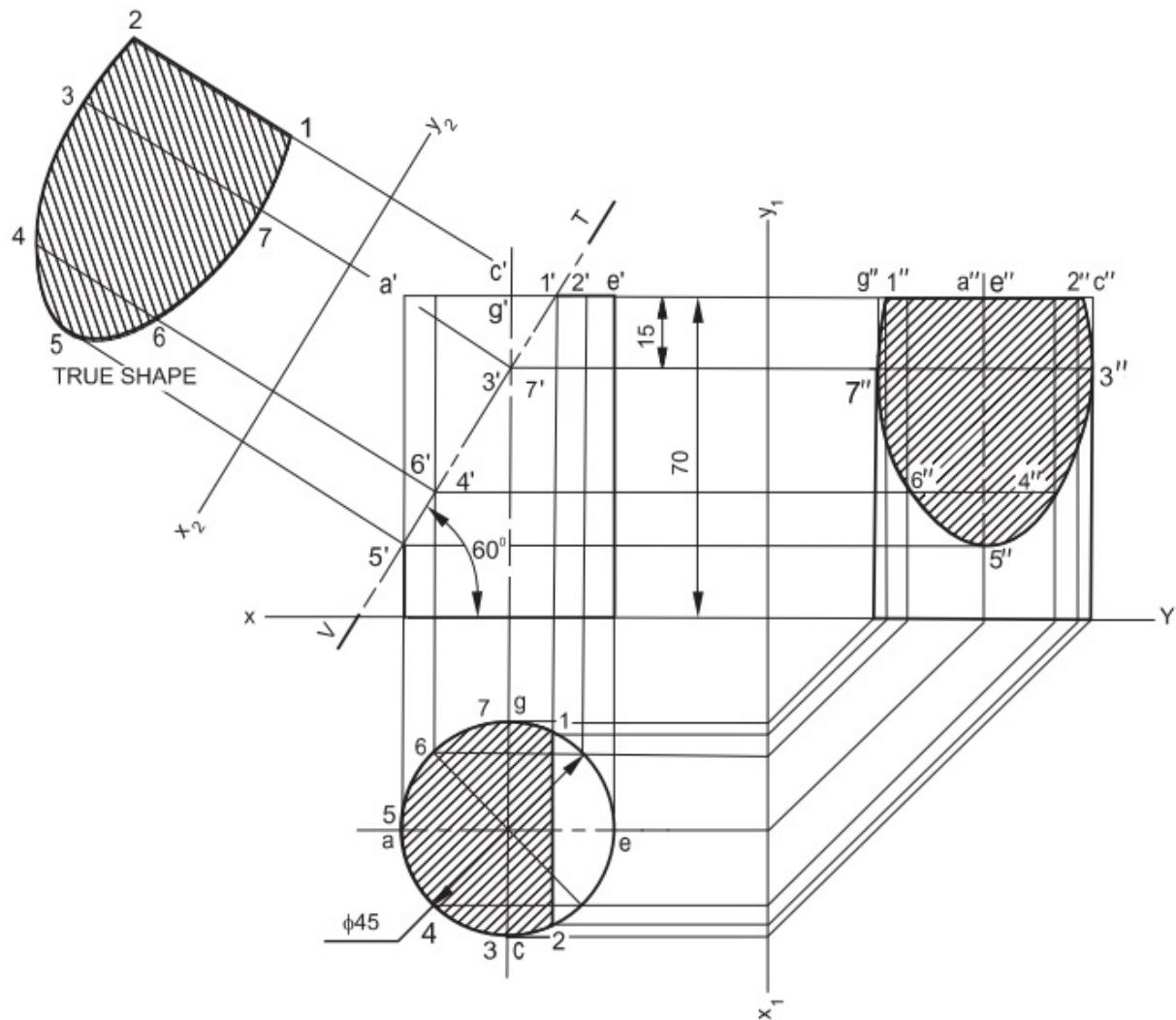
**Problem 12** A cylinder of 45 diameter and 70 long, is resting on one of its bases on H.P. It is cut by a section plane, inclined at  $60^\circ$  with H.P and passing through a point on the axis at 15 from one end. Draw the three views of the solid and also obtain the true shape of the section.



**Fig.12.12**

### ***Construction (Fig.12.13)***

1. Draw the projections of the cylinder.
2. Divide the base (top view) into a number of equal parts, say 8 and draw the corresponding generators in the front view.
3. Draw the V.T of section plane, inclined at  $60^\circ$  to xy and passing through a point on the axis at 15 from its top end.
4. Locate the points of intersection 1', 2', etc., between the V.T and base and generators of the cylinder.
5. Project and locate the corresponding points 1, 2, etc., in the top view.
6. Join the points in the order and complete the sectional top view, by cross-hatching the sectioned portion.
7. Obtain the true shape of the section and sectional left side view, by suitably following the principle of Construction: [\*\*Fig.12.12.\*\*](#)



**Fig.12.13**



1. The boundary of the intersection is a straight line, when the section plane passes through the base.
2. The remaining part of the boundary, corresponding to the plane, passing through the curved surface of the solid is a curve.

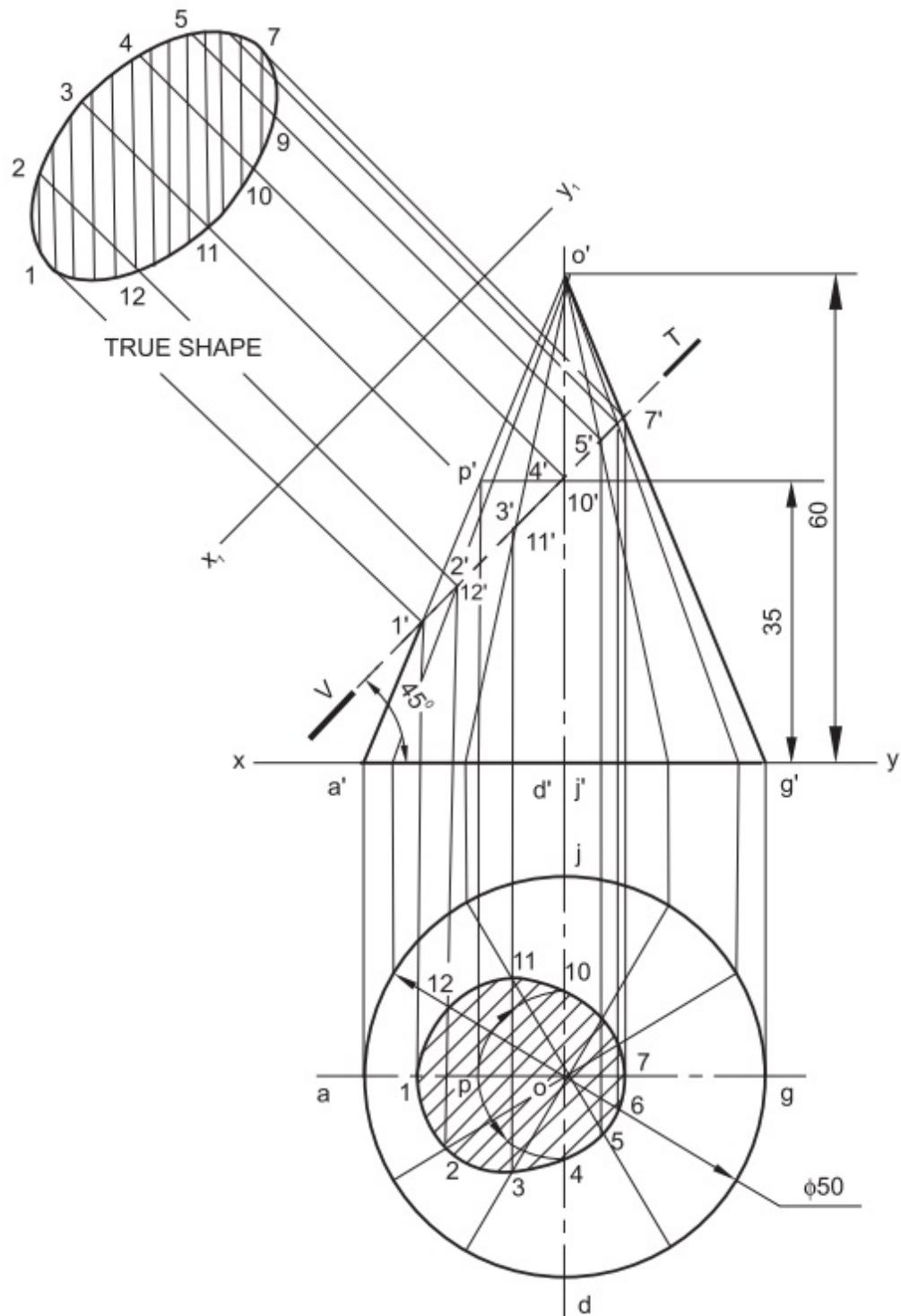
**Problem 13** A cone with diameter of base 50 and axis 60 long, is resting on its base on H.P. It is cut by a section plane inclined at  $45^\circ$  to H.P and passing through the axis at a point 35 above H.P. Draw the projections of the cut solid.

### ***Construction (Fig.12.14)***

1. Draw the projections of the cone.
2. Draw the V.T of section plane, inclined at  $45^\circ$  to xy and passing through a point on the axis at 35 from the base.

The two methods to locate the intersection points in the top view are: (i) Generator method and (ii) cutting plane method.

### ***Generator method (Fig.12.14a)***



**Fig.12.14a**

1. Divide the circle (top view) into say, 12 equal parts.
2. Locate the corresponding generators in the front view.

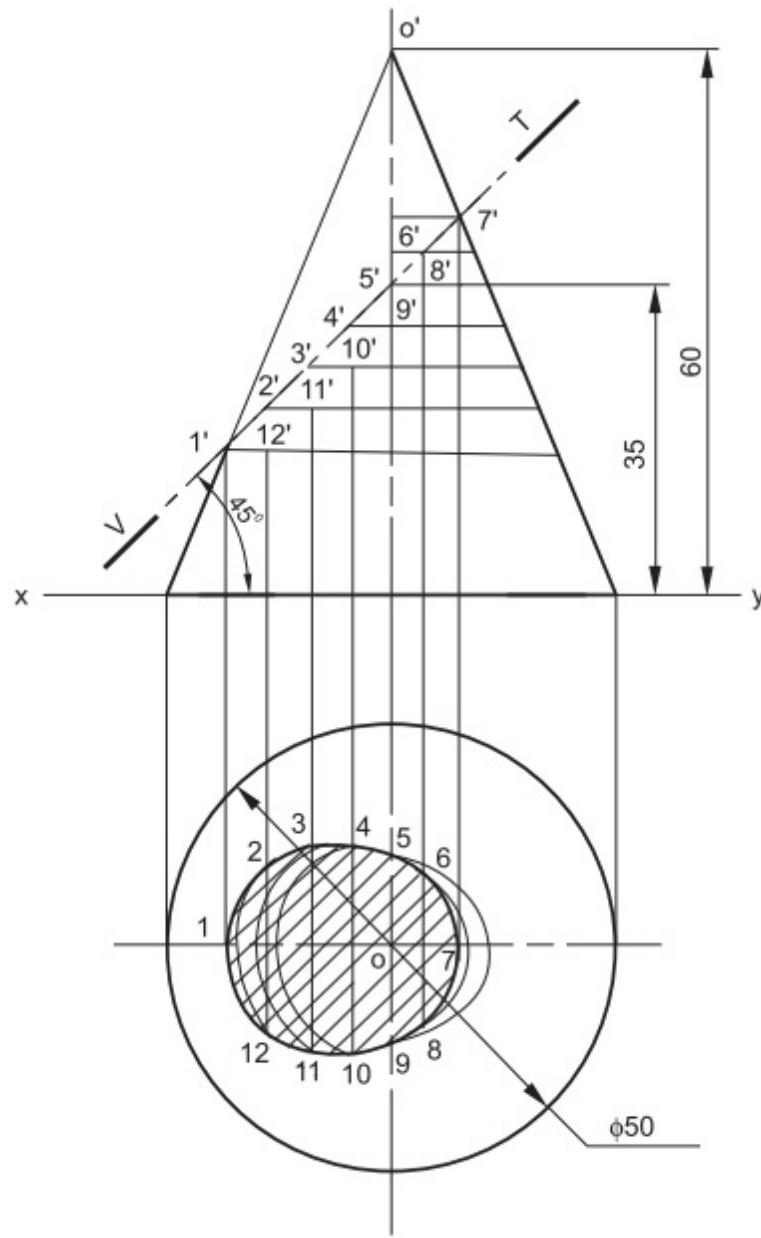
3. Locate the points of intersection 1', 2', etc., between the V.T and generators.
4. Obtain the corresponding points in the top view, by projection.
5. Join these points by a smooth curve and obtain the sectional top view.
6. Obtain the true shape of the section by projecting on an A.I.P parallel to the V.T.



*To locate the points 4 and 10 in the top view,*

- (i) Through 4' (10'), draw a line parallel to the base meeting the extreme generator at p'.
- (ii) Through p', draw a projector meeting the horizontal line ag in the top view at p.
- (iii) With centre o and radius op, draw an arc meeting the division lines oj and od at 4 and 10 respectively.

***Cutting plane method ([Fig.12.14b](#))***



**Fig.12.14b**

1. Locate the points  $1'$  and  $7'$  at which the V.T cuts the extreme generators.
2. Select a number of horizontal cutting planes, say 5 between  $1'$  and  $7'$ .
3. Locate the points of intersection between the V.T and cutting planes  $2', 3', 4', \dots$

4. Draw the circles, produced by these section planes in the top view and transfer the above intersection points on to the respective circles.
5. Join these points in the order by a smooth curve and obtain sectional top view.



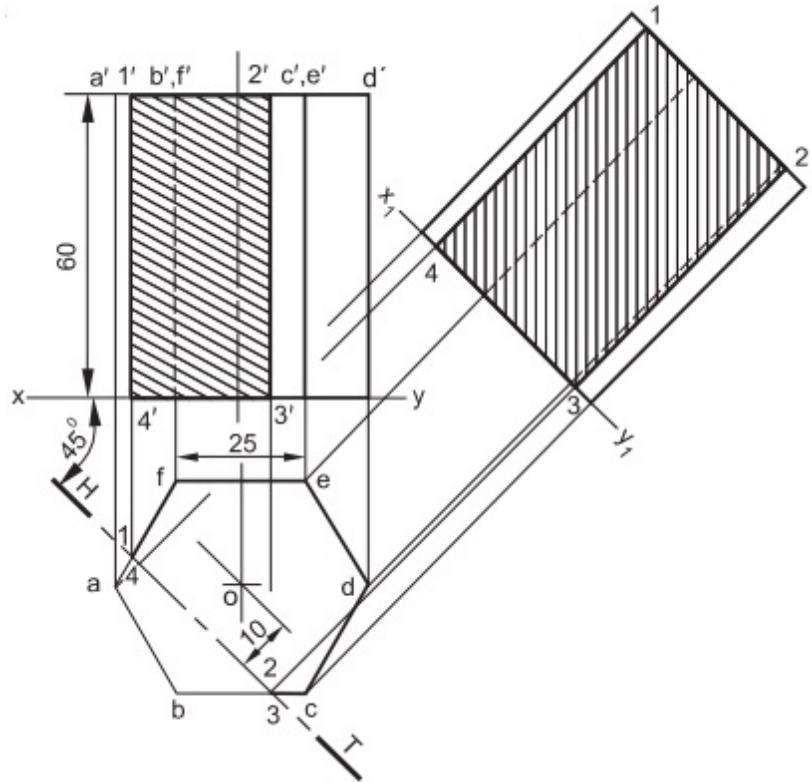
When a cone, resting on its base on H.P, is cut by a section plane, inclined to H.P such that, it passes through the extreme generators; the true shape of the section is an ellipse. If the length of the major axis is given; the V.T of section plane can be drawn through the front view of the cone such that, the distance between the points of intersection with the extreme generators is equal to the length of the major axis. Obviously, given the length of the major axis, there will be number of possible positions for location of the V.T.

#### 12.3.4 Section Plane Inclined to V.P and Perpendicular to H.P

When a section plane passing through the solid is inclined to V.P and perpendicular to H.P, its H.T is inclined to xy. The true shape of the section may be obtained on an A.V.P, parallel to the given section plane.

**Problem 14** *A hexagonal prism of side of base 25 and axis 60 long, is resting on its base on H.P such that, an edge of the base is parallel to V.P. It is cut by a section plane, inclined at  $45^\circ$  to V.P and 10 away from the axis. Draw the projections of the solid. Also obtain an auxiliary front view, showing the true shape of the section.*

**Construction (Fig.12.15)**

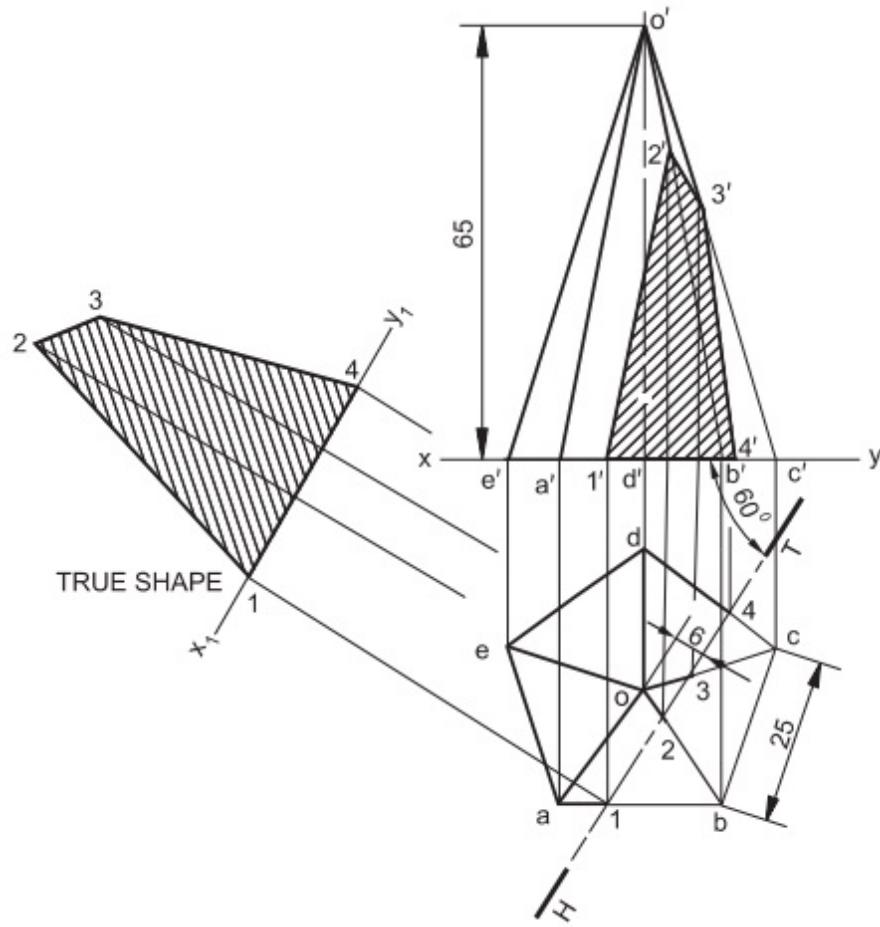


**Fig.12.15**

1. Draw the projections of the prism.
2. Draw the H.T of the section plane, inclined at  $45^\circ$  to xy and 10 away from axis.
3. Locate the points of intersection 1, 2, 3 and 4 between the H.T and edges of the bases.
4. Project and locate the corresponding points 1', 2', 3' and 4' in the front view.
5. Join the points in the order by straight lines and obtain the section.
6. Draw the reference line  $x_1y_1$ , parallel to the H.T of section plane and obtain the auxiliary front view for the retained portion of the solid, which also shows the true shape of the section.

**Problem 15** A pentagonal pyramid with edge of base 25 and axis 65 long, is resting on H.P on its base with an edge nearer to the observer, parallel to V.P. It is cut by a section plane, inclined at  $60^\circ$  to V.P and at a distance of 6 from the axis. Draw the projections and obtain the true shape of the section.

**Construction (Fig.12.16)**



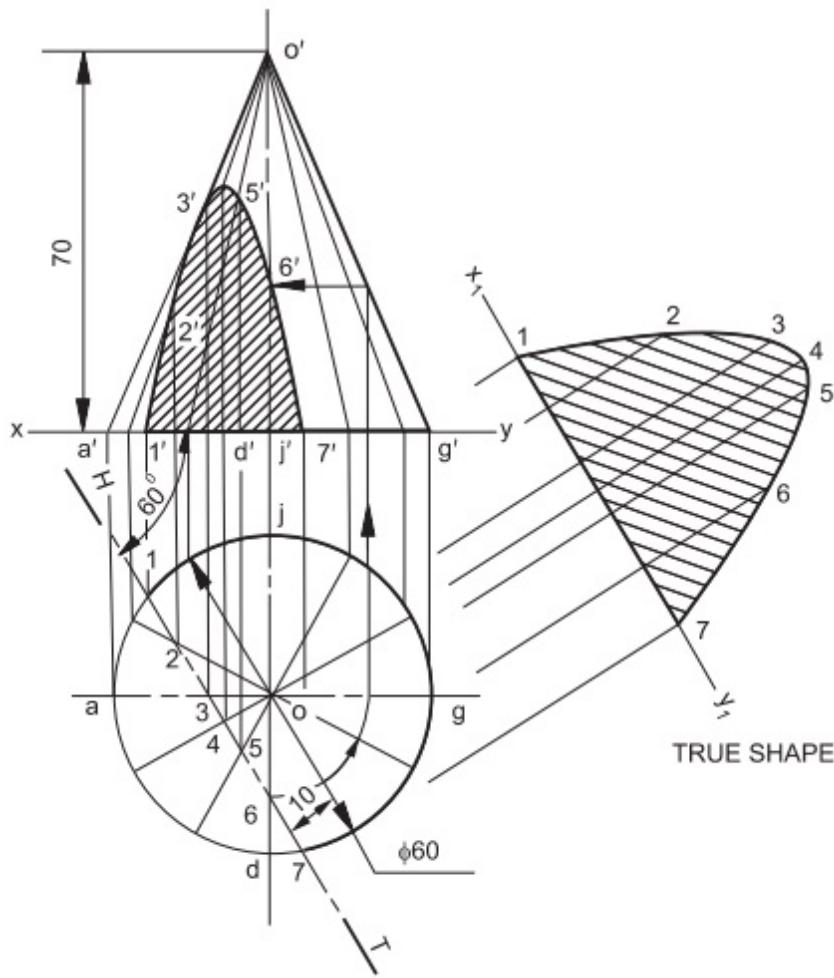
**Fig.12.16**

1. Draw the projections of the pyramid.
2. Draw the H.T of section plane, inclined at  $60^\circ$  to xy and 6 away from the axis.

3. Locate the points of intersection between the H.T and base (1 and 4) and the slant edges (2 and 3) of the pyramid.
4. Project and obtain the corresponding points in the front view.
5. Join these points in the order by straight lines and obtain the section.
6. Obtain the true shape of the section, considering an auxiliary vertical plane, parallel to the H.T of section plane.

**Problem 16** A cone of base 60 diameter and axis 70 long, is resting on its base on H.P. It is cut by a section plane, inclined at  $60^\circ$  to V.P and 10 away from its axis. Draw the projections of the cut solid and obtain the true shape of the section.

**Construction (Fig.12.17)**



**Fig.12.17**

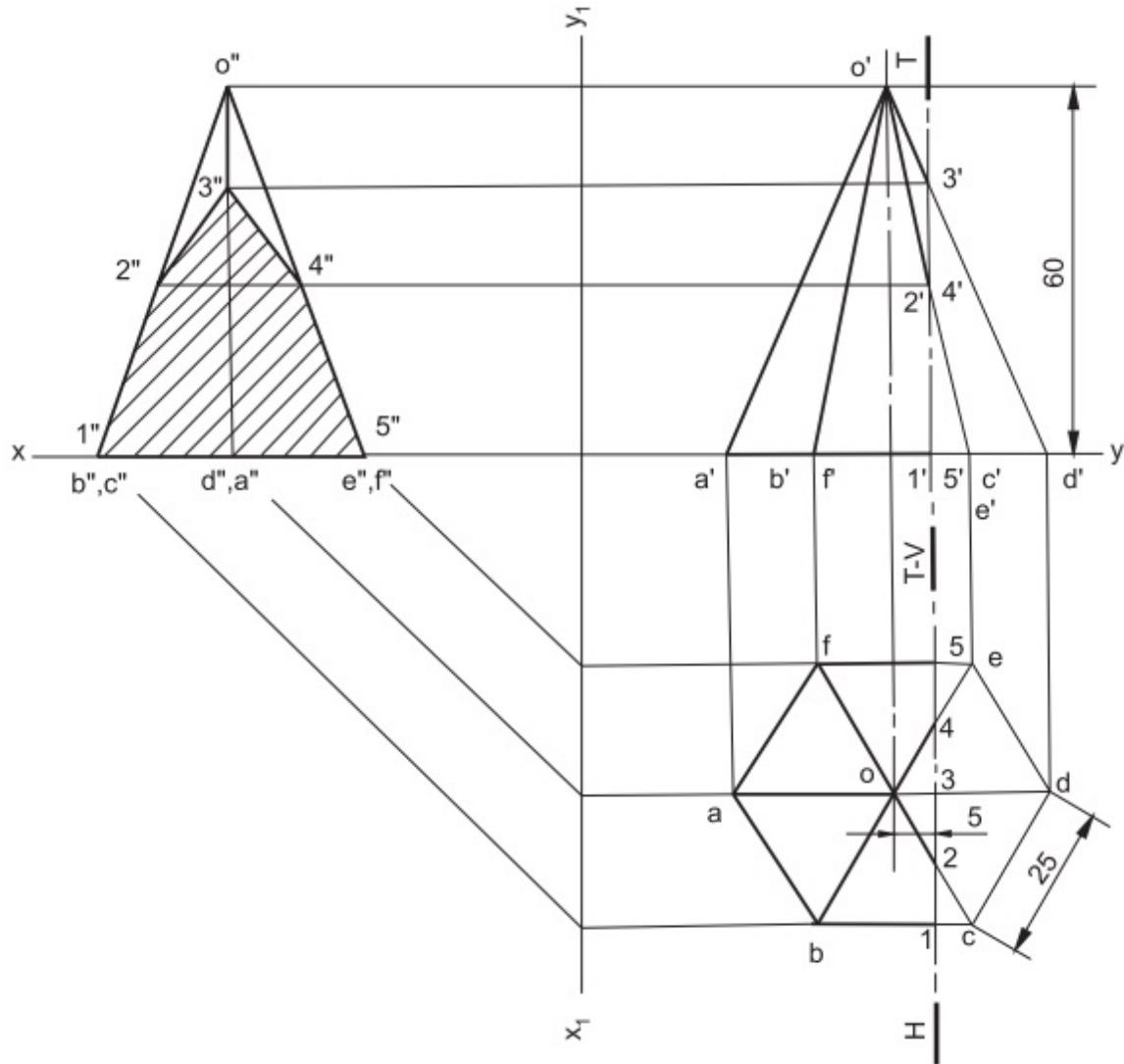
1. Draw the projections of the cone.
2. Draw the H.T of section plane, inclined at  $60^\circ$  to xy and 10 away from axis.
3. Locate the points of intersection between the H.T and base of the cone (1and 7) and the generators (2, 3, — 6).
4. Repeat steps 4 to 6 of Construction: [Fig.12.16](#) suitably, and obtain the sectional front view and the true shape of the section.

## 12.3.5 Section Plane Perpendicular to Both H.P and V.P

When a section plane, perpendicular to both H.P and V.P, passes through a solid resting on H.P, the section appears as a straight line, coinciding with the traces of the section plane both in front and top views. However, in the side view, the section appears in its true shape.

**Problem 17** *A hexagonal pyramid of side of base 25 and axis 60 long, is resting on its base on H.P, with an edge of the base parallel to V.P. A section plane perpendicular to both H.P and V.P cuts the solid, 5 away from the axis. Draw the sectional side view of the cut solid.*

**Construction** ([Fig.12.18](#))



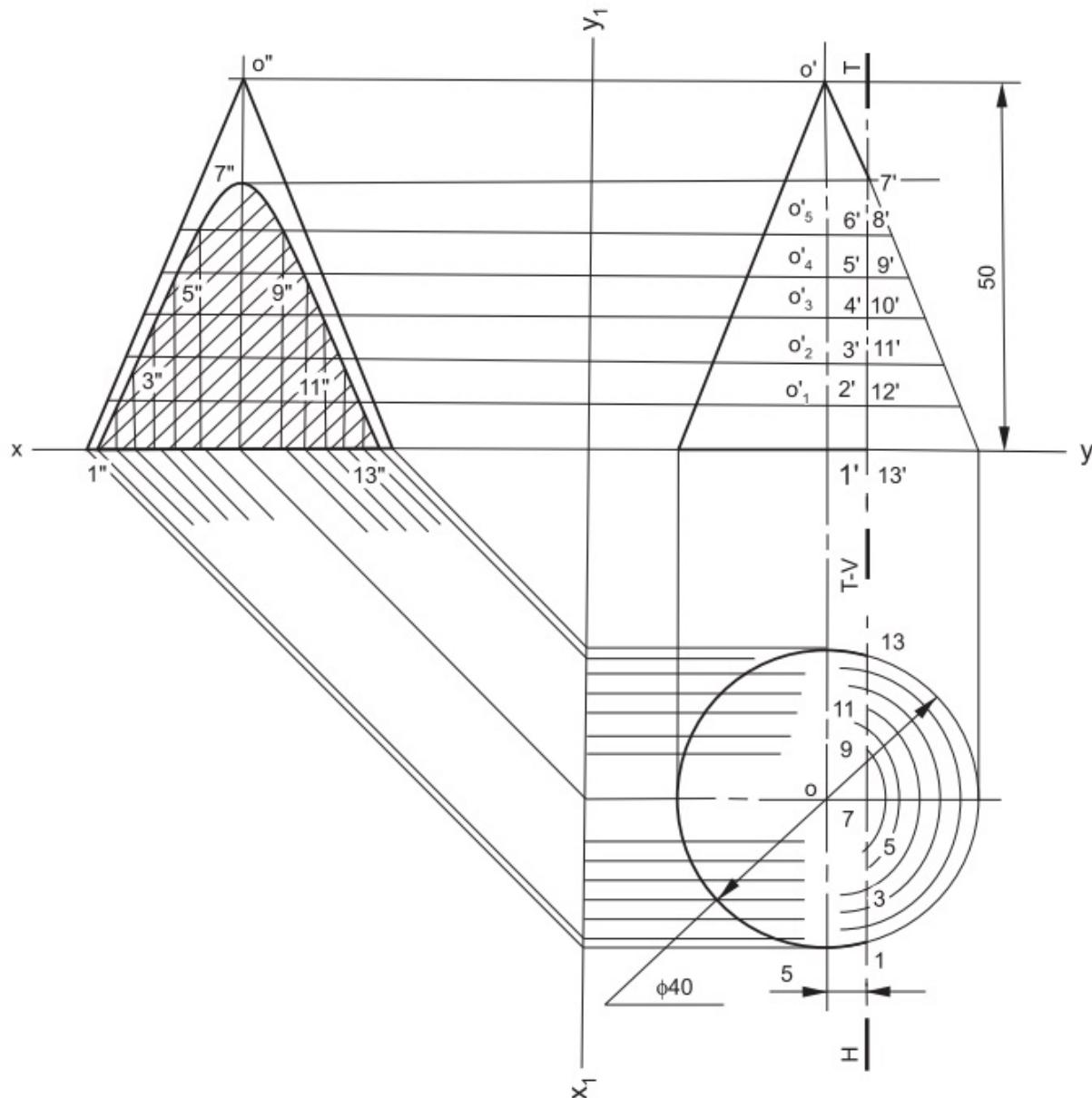
**Fig.12.18**

1. Draw the three views of the pyramid, satisfying the given conditions.
2. Draw the H.T and V.T of section plane, perpendicular to xy and 5 away from the axis.
3. Locate the points of intersection between the traces and base and slant edges of the pyramid, in both the projections.
4. Project and locate the corresponding points in the left side view.

- Join the points in the order by straight lines and obtain the section.

**Problem 18** A cone of base 40 diameter and axis 50 long, is resting on its base on H.P. It is cut by a section plane perpendicular to both H.P and V.P and 5 away from the axis. Draw the sectional side view of the cut solid.

### ***Construction (Fig.12.19)***



**Fig.12.19**

1. Draw the three views of the cone.
2. Draw both the traces of the section plane, perpendicular to xy and 5 away from the axis.
3. Identify a number of cutting planes, perpendicular to the axis, between 1' and 7' (need not be equi-distant).
4. Draw the circular sections produced in the top view by the cutting planes and locate the intersection points with the H.T of section plane.
5. Project and obtain these intersection points in the right side view.
6. Join the points by a smooth curve and complete the section by cross-hatching.

### 12.3.6 Section Plane Inclined to Both H.P and V.P

When a section plane is inclined to both H.P and V.P and cuts the solid, both front and top views contain sectioned portions and none of them reveal the true shape of the section.

**Problem 19** A cube with a side of edge 30, is resting on H.P on one of its faces and with a face parallel to V.P. Draw the projections of the cube when sectioned by a plane inclined to both H.P and V.P.

Figure 12.20a shows a cube resting on H.P and cut by an oblique section plane marked. The section plane cuts the edges of the cub at P, Q and R.

Figure 12.20b shows the projections of the un-sectioned solid with the traces of the section plane marked. Let the section plane pass through the point Q of the edge AB.

### **Construction ([Fig. 12.20c](#))**

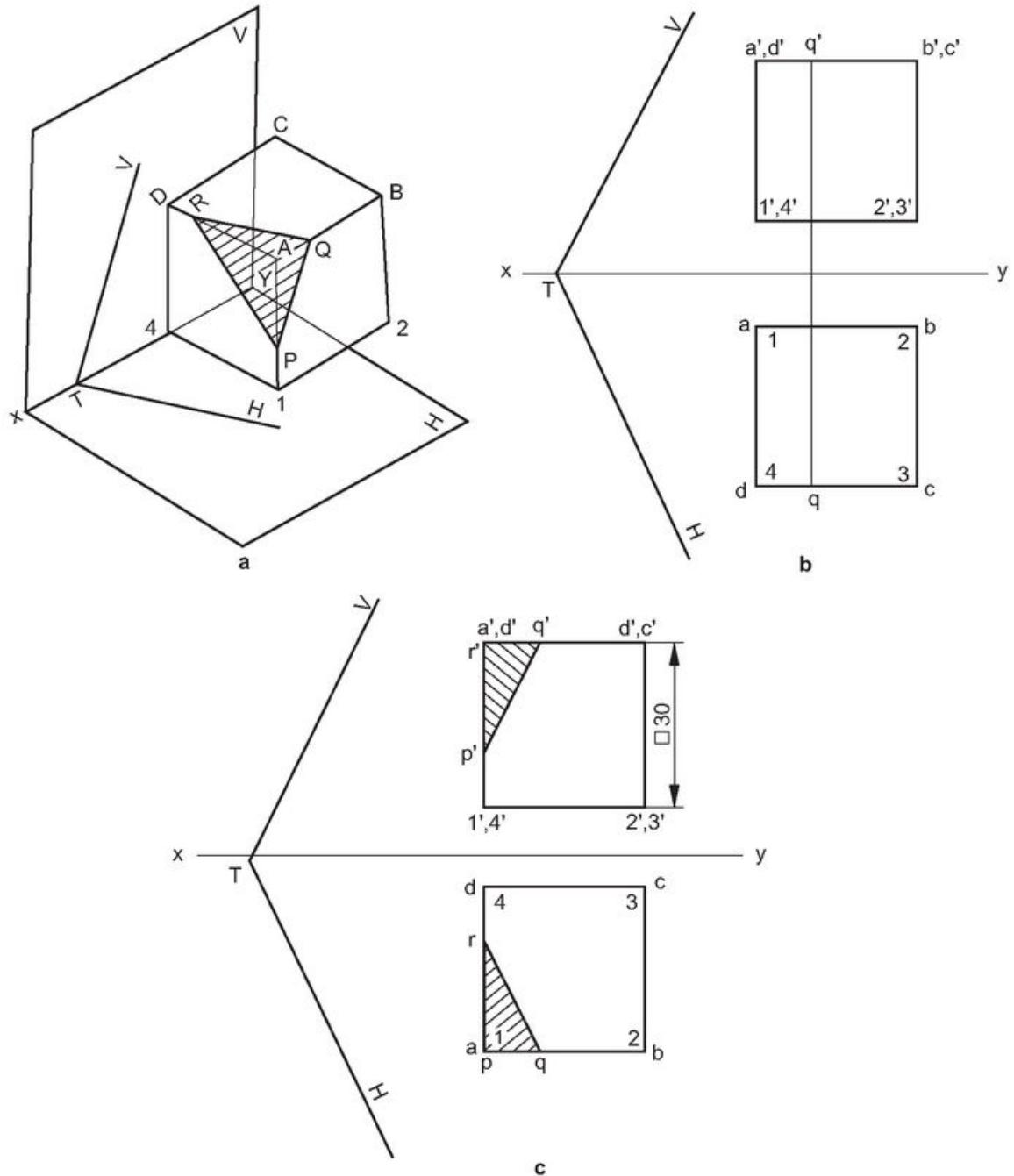
1. Draw the projections of the cube, satisfying the given conditions.
2. Draw the traces of the oblique section plane.
3. Locate the front and top views of the point Q.
4. Through  $q'$ , draw a line parallel to V.T intersecting the edge  $1'a'$  at  $p'$ .
5. Through  $q$  draw a line parallel to H.T intersecting the edge  $ad$  at  $r$ .
6. Join  $p'$ ,  $q'$  and  $q$ ,  $r$  and obtain the sectioned portions  $pqr$  and  $p'q'r'$ .



The lines V.T;  $p'q'$  and H.T;  $qr$  are parallel to each other because they are situated in parallel planes.

In general, the problem dealing with a solid cut by an oblique plane may be solved by first converting the oblique plane into an inclined plane.

[Figure 12.21](#) shows first quadrant with an oblique plane and its traces, H.T and V.T marked. If an auxiliary vertical plane is considered with its horizontal trace,  $x_1y_1$  perpendicular to the above H.T, then the oblique plane becomes an inclined plane with respect to the A.V.P.

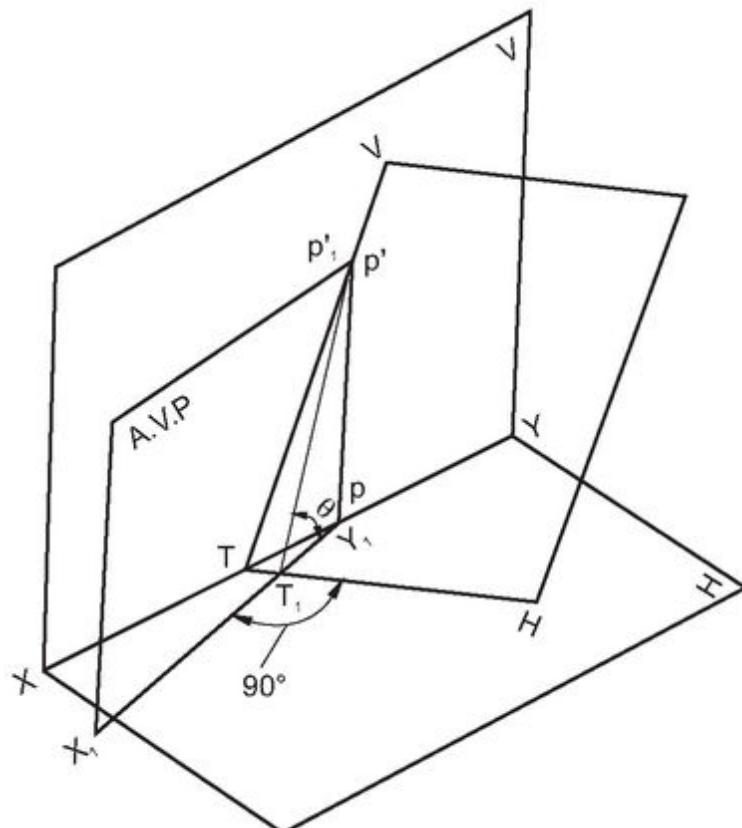


**Fig.12.20**

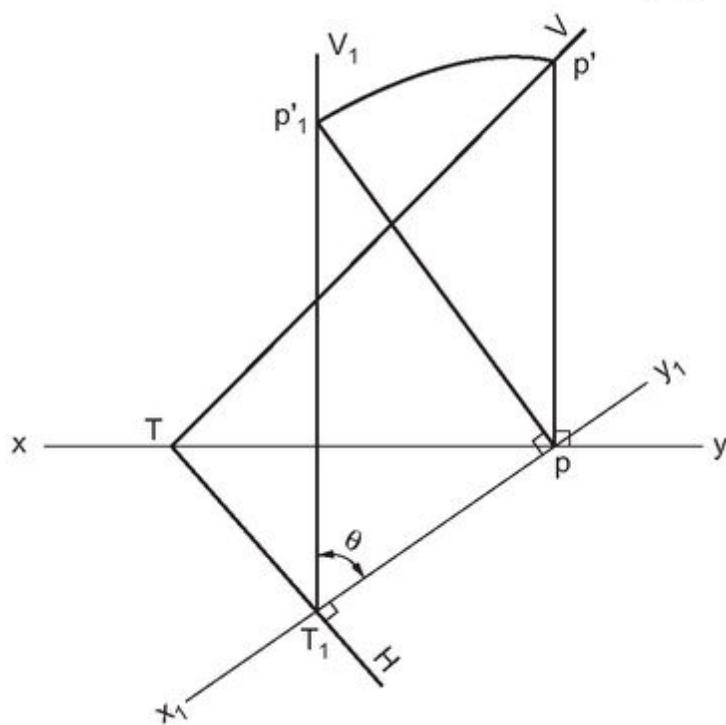
**Problem 20** Convert an oblique plane into an inclined plane

**Construction (Fig.12.22)**

1. Draw the given traces of the oblique plane.
2. Draw  $x_1y_1$  perpendicular to H.T at  $T_1$  on it to meet xy at p.
3. Erect perpendiculars to xy and  $x_1y_1$  at p.
4. Locate the intersection point  $p'$  between the V.T and the perpendicular to xy.
5. Locate  $p'_1$  on the perpendicular to  $x_1y_1$  such that,  $pp'_1 = pp'$ .
6. Join  $T_1, p'_1$  and extend to, say  $V_1$ .
7. HT<sub>1</sub> and V<sub>1</sub>T<sub>1</sub> are the traces of the inclined (A.V.P) plane with respect to x y



**Fig.12.21**



**Fig.12.22**



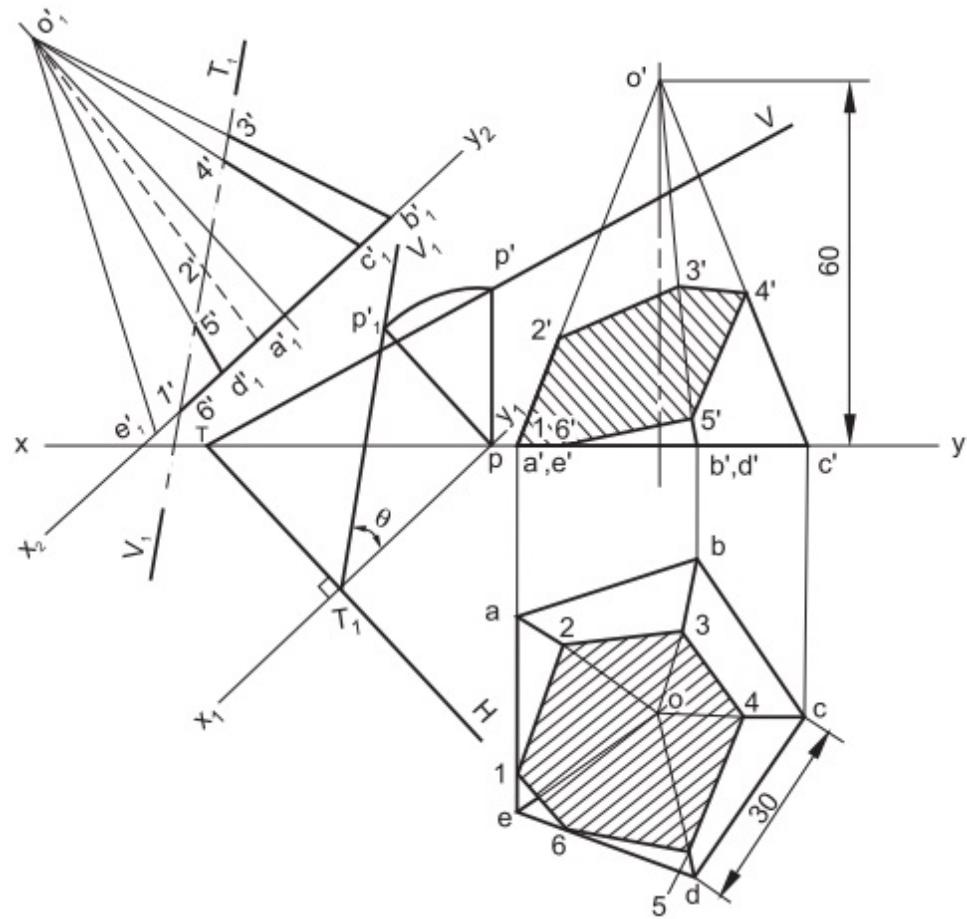
It may be seen that  $\theta$  between  $V_1T_1$  and  $x_1y_1$  is the true angle of inclination of the given oblique plane with the H.P, since  $p'_1T_1$  is contained by both the oblique plane and A.V.P.

**Problem 21** A pentagonal pyramid of side of base 30 and axis 60, is resting on H.P on its base with an edge of the base perpendicular to V.P. It is cut by an oblique section plane, given the traces of it. Draw the projections of the solid.

### ***Construction (Fig.12.23)***

1. Draw the projections of the solid.
2. Draw the given traces H.T and V.T.
3. Obtain  $V_1T_1$  by following the steps given under Construction: [Fig.12.22](#).

4. Draw the auxiliary projection of the pyramid, on  $x_2y_2$  parallel to  $x_1y_1$ .
5. Locate  $V_1T_1$  with respect to  $x_2y_2$ .
6. Draw the sectioned portion 123456 of the pyramid in the top view by projecting from the auxiliary front view.
7. Obtain the sectioned portion 1'2'3'4'5'6' in the front view, by projection.



**Fig.12.23**

**Problem 22** A cone of base 60 diameter and axis 75, is resting on H.P on its base and cut by an oblique plane, given the traces of it. Draw the projections of the cone.

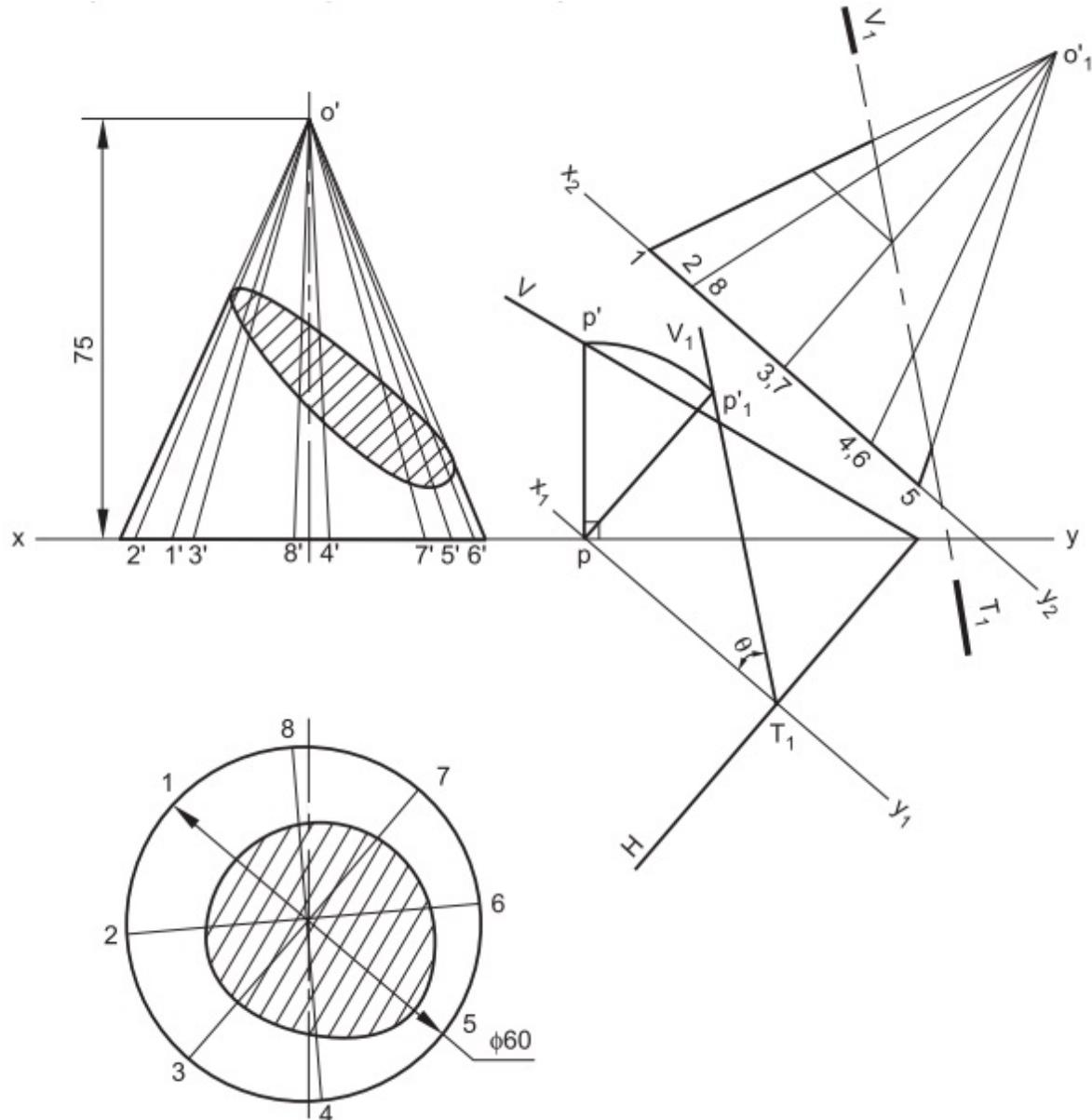
The problem may be solved by following the steps given under Construction: [Fig.12.23](#) (refer [Fig.12.24](#)).

## 12.4 EXAMPLES

**Problem 23** A hollow cylinder with outside diameter 50, axis 75 long and 10 thick, has its axis inclined to H.P at  $45^\circ$  and parallel to V.P. It is cut by a section plane, inclined to H.P at  $45^\circ$  and perpendicular to V.P and passing through the mid-point of the axis. Draw the auxiliary top view, showing true shape of the section.

**Construction ([Fig.12.25](#))**

1. Draw the 3 views of the hollow cylinder, satisfying the given conditions.
2. Draw the V.T of section plane, at  $45^\circ$  to xy and passing through the mid-point of the axis.
3. Locate the points of intersection 1', 2', 3', etc., between the V.T and generators of the cylinder in the front view.
4. Project and locate the points 1, 2, 3, etc., on the corresponding generators in the auxiliary top view.
5. Join these points in the order, by smooth curves and complete the auxillary top view, by cross-hatching the sectioned portion.

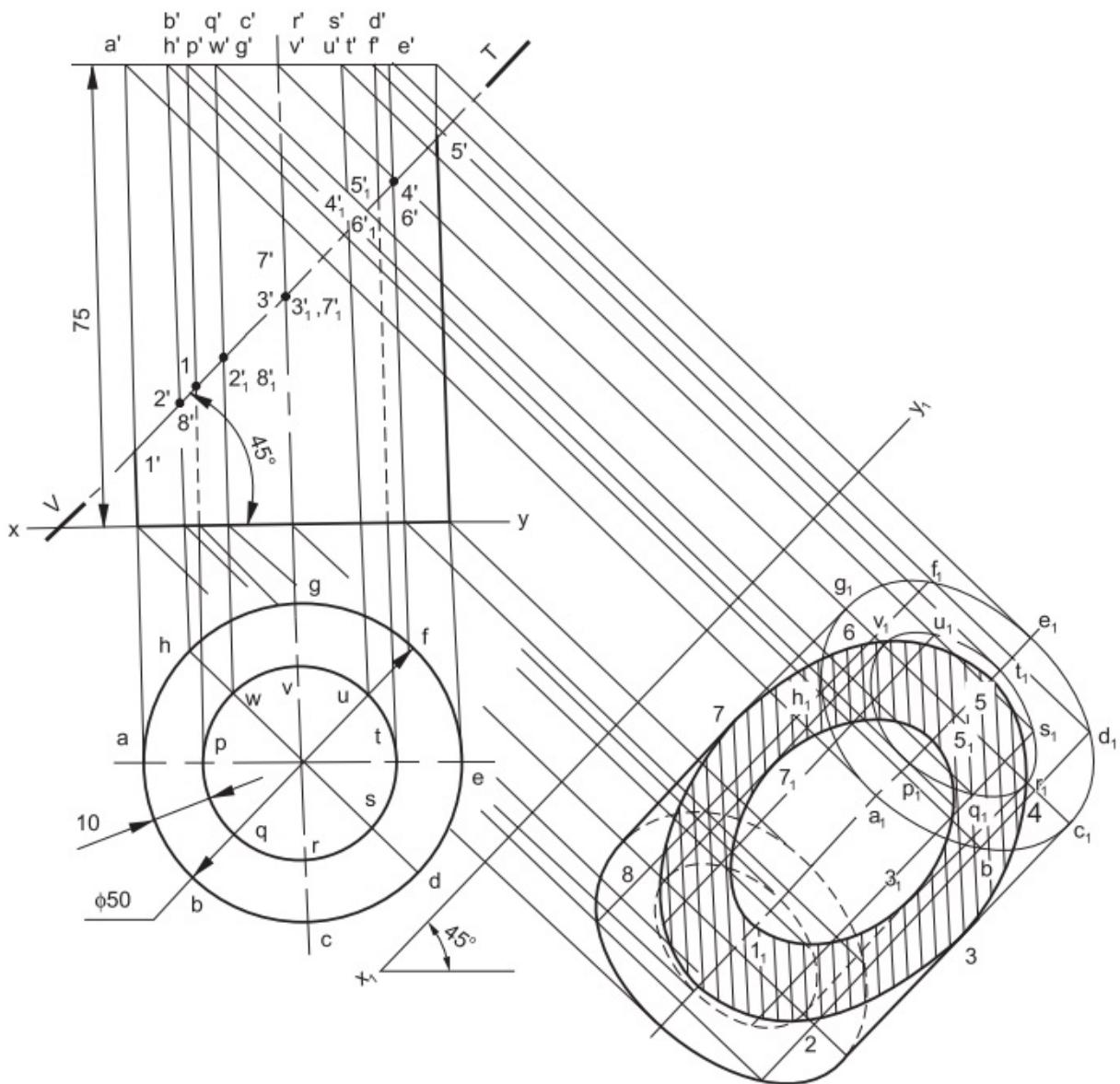


**Fig.12.24**

**Problem 24** A hexagonal prism of side of base 30 and 70 height, has a concentric tapered circular hole. The diameter of the hole at the top is 25 and at the bottom 45. The solid is resting on its base on H.P with an edge of the base parallel to V.P. The V.T of the section plane is inclined at  $60^\circ$  to H.P and bisects the axis. Draw the projections and obtain the true shape of the section.

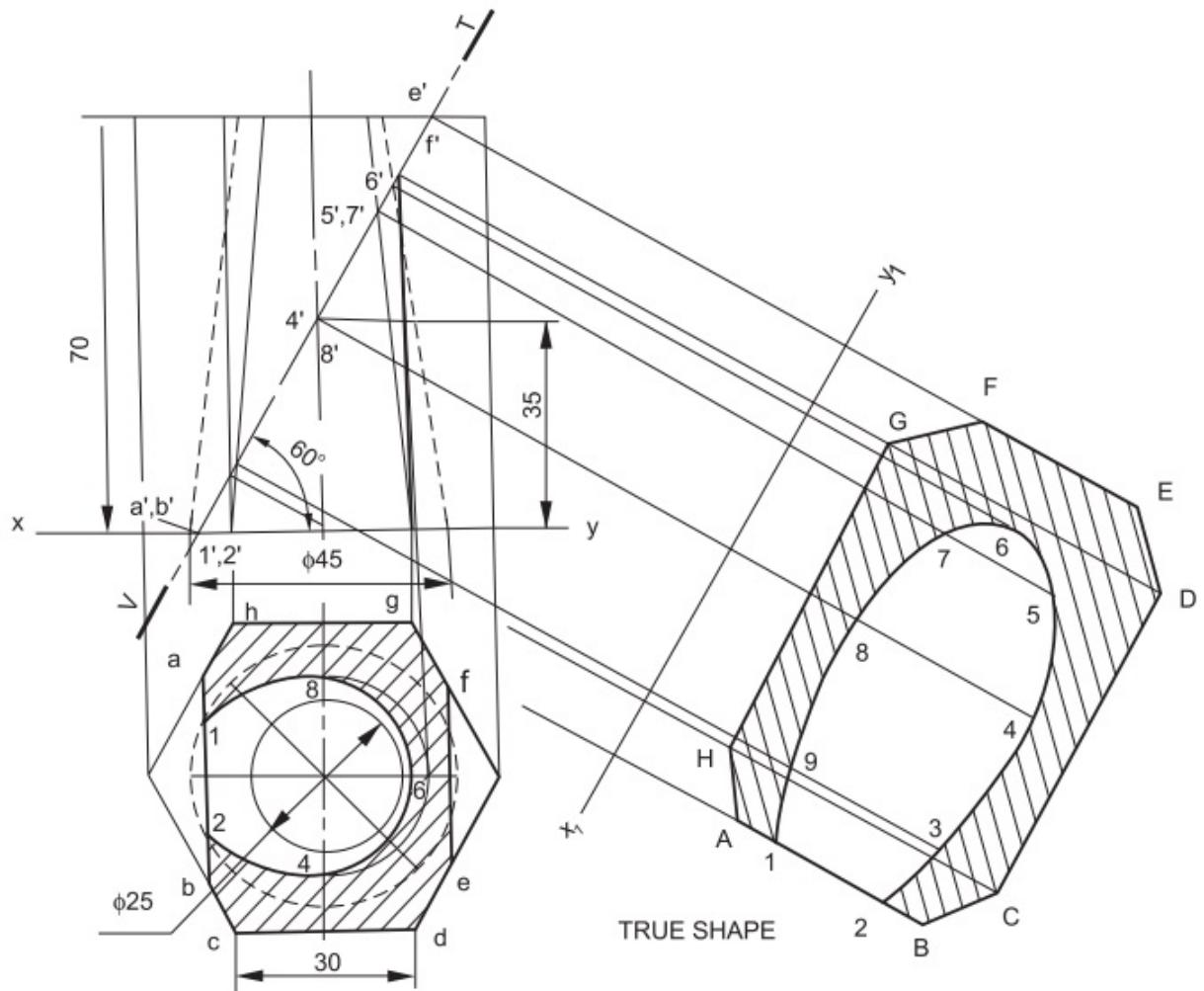
### ***Construction (Fig.12.26)***

1. Draw the projection of the prism, satisfying the given conditions.
2. Draw the V.T of section plane such that, it passes through the mid-point of the axis, making an angle  $60^\circ$  with xy.
3. Locate the points of intersection  $1'$ ,  $2'$ ,  $3'$ , etc., between the V.T and edges of the bases of the prism and generators of the tapered hole.
4. Project and locate the above points in the top view and complete the sectional top view.
5. Draw the reference line  $x_1y_1$ , parallel to the V.T of section plane and obtain the true shape of the section, by projection.



**Fig.12.25**

**Problem 25** A cube of edge 60 stands vertically on H.P such that, its vertical faces are equally inclined to V.P. A section plane, perpendicular to V.P and inclined to H.P cuts the solid in such a way that the true shape of the section is an equilateral triangle of 60 side. Draw the projections and true shape of the section and determine the inclination of the section plane with H.P.



**Fig.12.26**

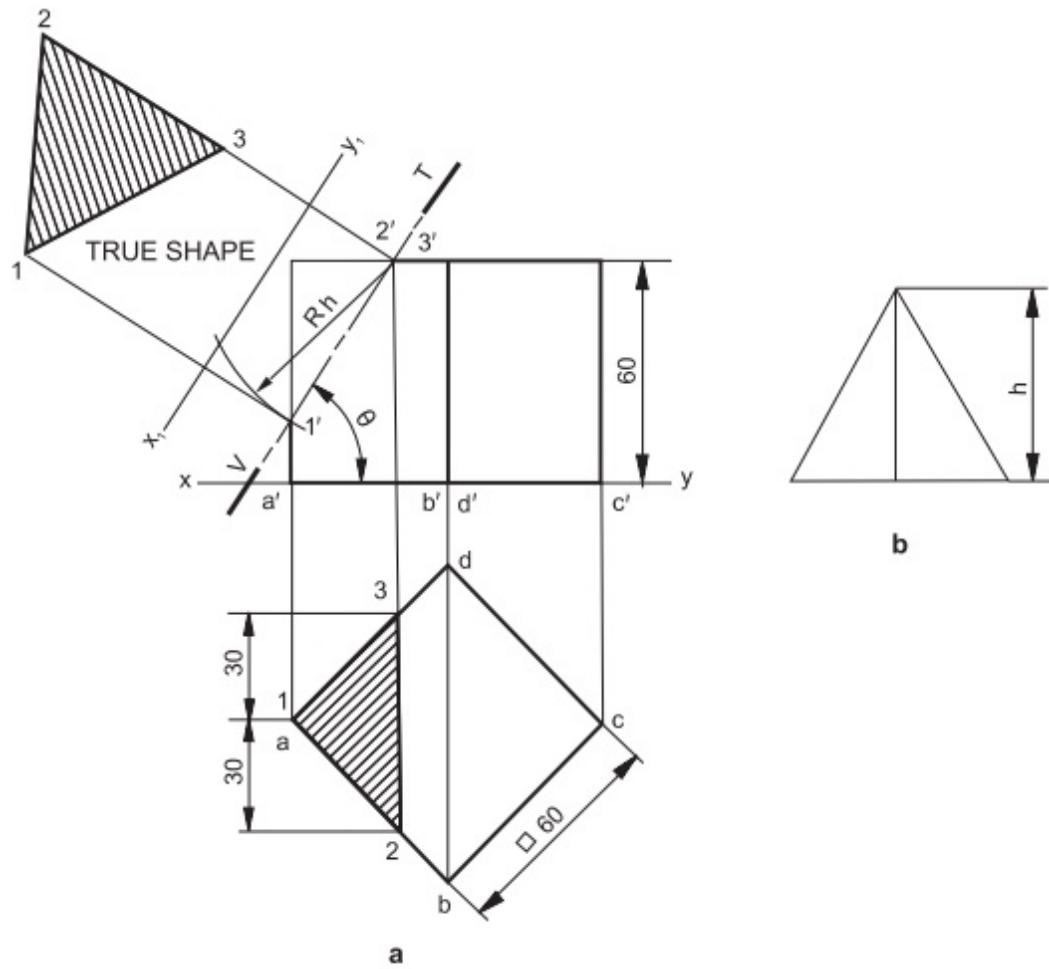
### ***Construction (Fig.12.27)***

1. Draw the projections of the cube.
2. Locate the points 2 and 3 on ab and ad in the top view such that, the line 2-3 is parallel to the diagonal bd and of length 60, the side of the equilateral triangle.
3. Through 2 (3), draw a projector meeting the top face of the cube in the front view at 2'(3').
4. With 2'(3') as centre and radius equal to the altitude of the equilateral triangle h (refer Fig.12. 27b), draw an arc meeting the extreme vertical edge at the left at 1'.

5. Draw the V.T of section plane passing through 1' and 2' (3').

The inclination of the V.T with xy is equal to the inclination of the section plane with H.P.

6. Cross-hatch the sectioned portion in the top view and complete the sectional top view.  
 7. Draw the reference line  $x_1y_1$ , parallel to the V.T of section plane and obtain the true shape of the section, by projection.



**Fig.12.27**

**Problem 26** A cube of 50 edge, rests on one face on H.P with its vertical faces equally inclined to V.P. It is cut by a section plane, perpendicular to V.P, producing a large rhombus. Draw the projections, true shape of the section and determine the inclination of the section plane with H.P.

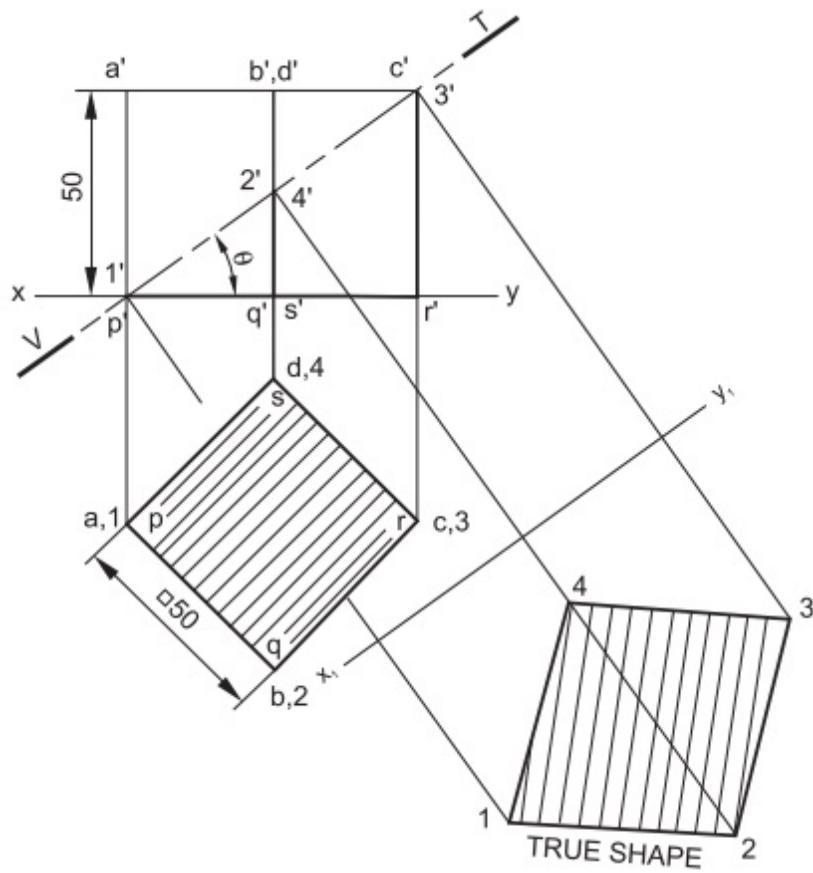
**Construction (Fig.12.28)**

1. Draw the projections of the cube, satisfying the given conditions.



When the V.T of a section plane is inclined to xy and passes through the extreme vertical edges of the front view, the true shape produced is a rhombus. So, to produce a largest possible rhombus, the section plane should pass through the diagonally opposite corners of the front view.

2. Draw the V.T of section plane such that, it passes through the diagonally opposite corners,  $p'$  and  $c'$  in the front view.
3. Locate the points of intersection  $1'$ ,  $2'$ ,  $3'$  and  $4'$  between the V.T and edges of the cube.
4. Cross-hatch the complete top view, as the entire area comes under sectioned zone.
5. Measure the angle  $\theta$  which is the inclination of the section plane with H.P.
6. Draw the reference line  $x_1y_1$ , parallel to the V.T of section plane and obtain the true shape of the section, by projection.



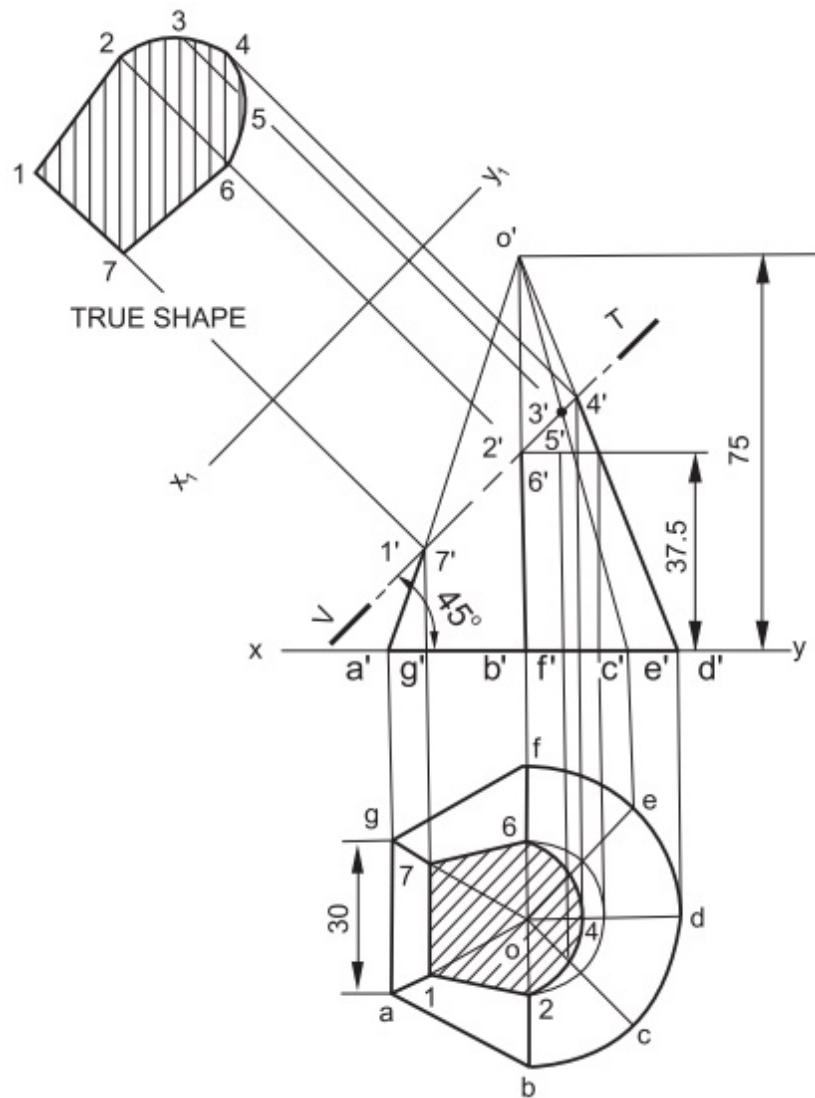
**Fig.12.28**

**Problem 27** A combination of a solid is composed of a half cone and a half hexagonal pyramid. The side of the hexagon is 30 and the length of the solid is 75. The solid is resting on its base on H.P with its common diagonal of the base, perpendicular to V.P. It is cut by a section plane, bisecting the axis. The V.T of the section plane is inclined at  $45^\circ$  to H.P. Draw the projections and true shape of the section.

**Construction (Fig.12.29)**

1. Draw the projections of the combined solid, satisfying the given conditions.
2. Draw the V.T of section plane, making  $45^\circ$  with xy and passing through the midpoint of the axis.

3. Locate the points of intersection  $1'$ ,  $2'$ ,  $3'$ , etc., between the V.T and slant edges of the half hexagonal pyramid and the generators of the half cone.
4. Project and obtain the corresponding points in the top view.
5. Join the points suitably in the order and complete the sectional top view, by crosshatching the sectioned portion.
6. Obtain the true shape of the section, by projecting the section on an A.I.P, parallel to the V.T.



### **Fig.12.29**

**Problem 28** A hexagonal prism of side of base 30 and length of axis 75, is resting on a corner of its base on H.P, with the longer edge containing that corner, inclined to H.P at  $30^\circ$ . It is cut by a section plane parallel to H.P and passing through the mid-point of the axis. Draw the front and sectional top views of the solid.

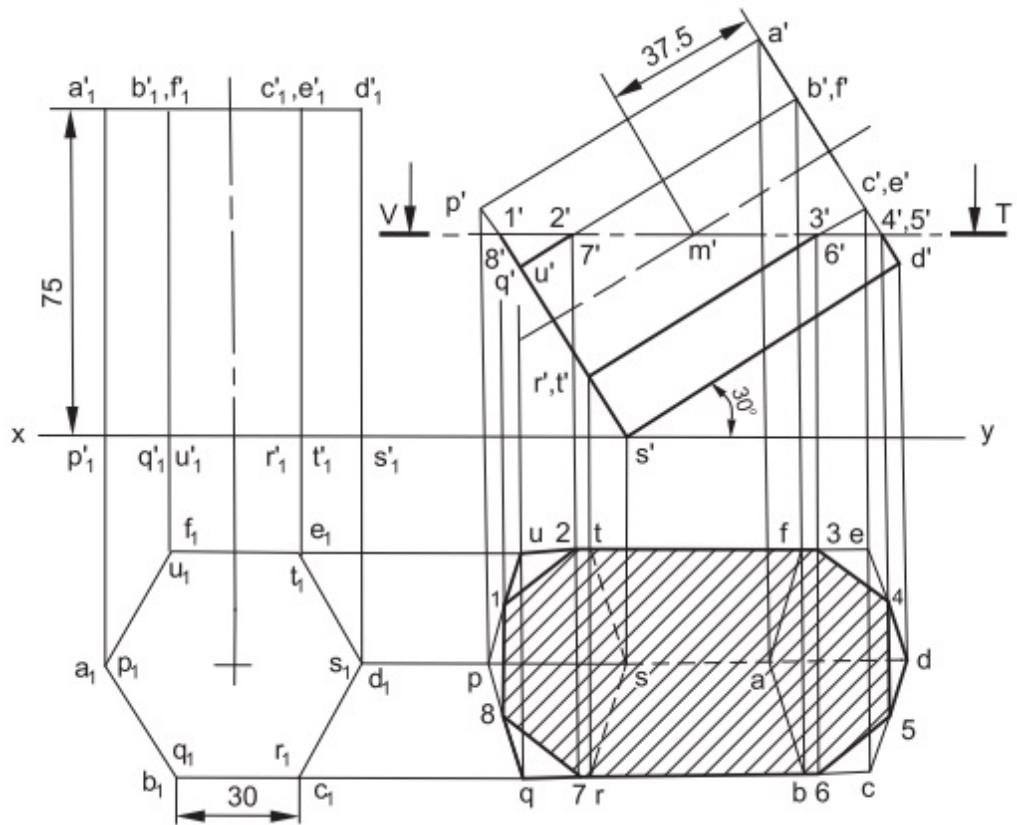
#### **Construction (Fig.12.30)**

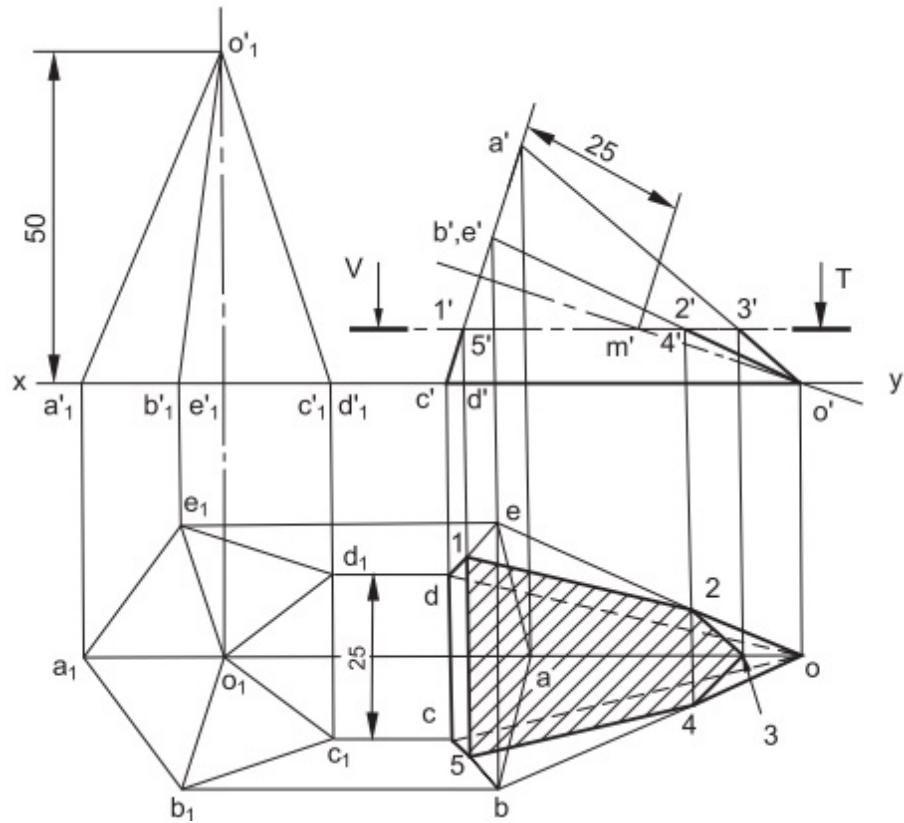
1. Draw the projections of the prism, satisfying the given conditions.
2. Draw the V.T of section plane, parallel to xy and passing through the mid-point m' of the axis.
3. Repeat steps 3 to 5 of Construction: Fig.12. 2 suitably and obtain the sectional top view.

**Problem 29** A pentagonal pyramid of side of base 25 and 50 height, rests on a triangular face on H.P, with its axis parallel to V.P. It is cut by a horizontal section plane, bisecting the axis. Draw the projections of the retained solid.

#### **Construction (Fig.12.31)**

1. Draw the projections of the pyramid, satisfying the given conditions.
2. Repeat steps 2 and 3 of Construction: Fig.12.30 suitably and obtain the sectional top view.

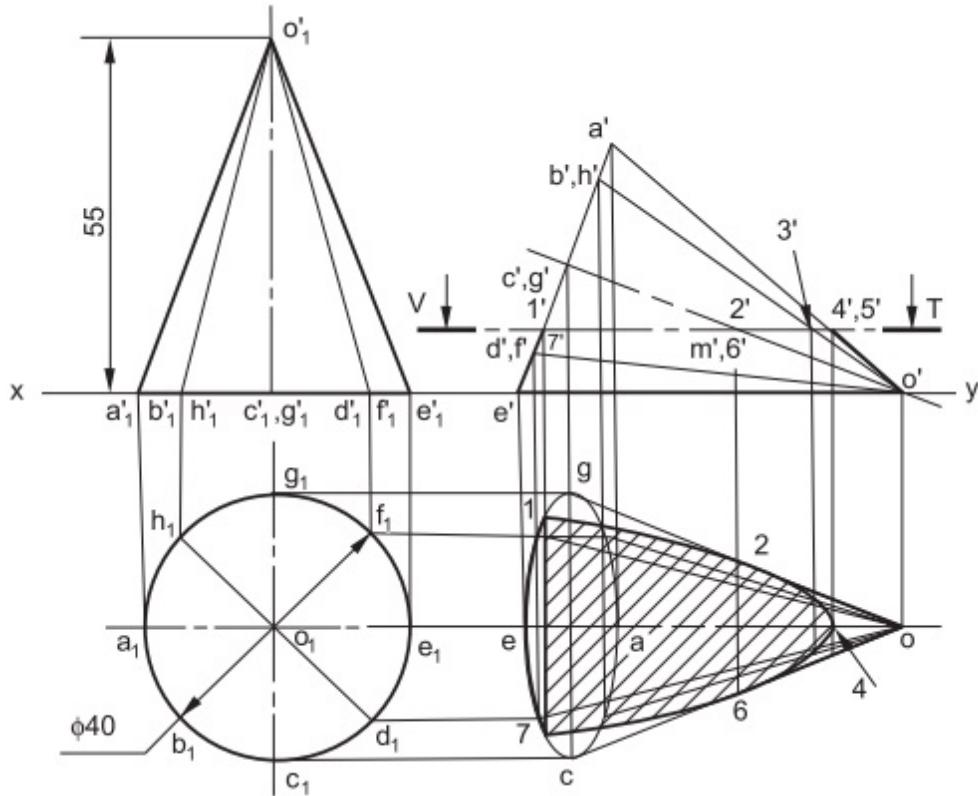




**Fig.12.31**

**Problem 30** A cone of base 40 diameter and axis 55 long, lies on one of its generators on H.P, with its axis parallel to V.P. A horizontal section plane, bisects the axis of the cone. Draw the projections of the retained portion of the solid.

**Construction (Fig.12.32)**



**Fig.12.32**

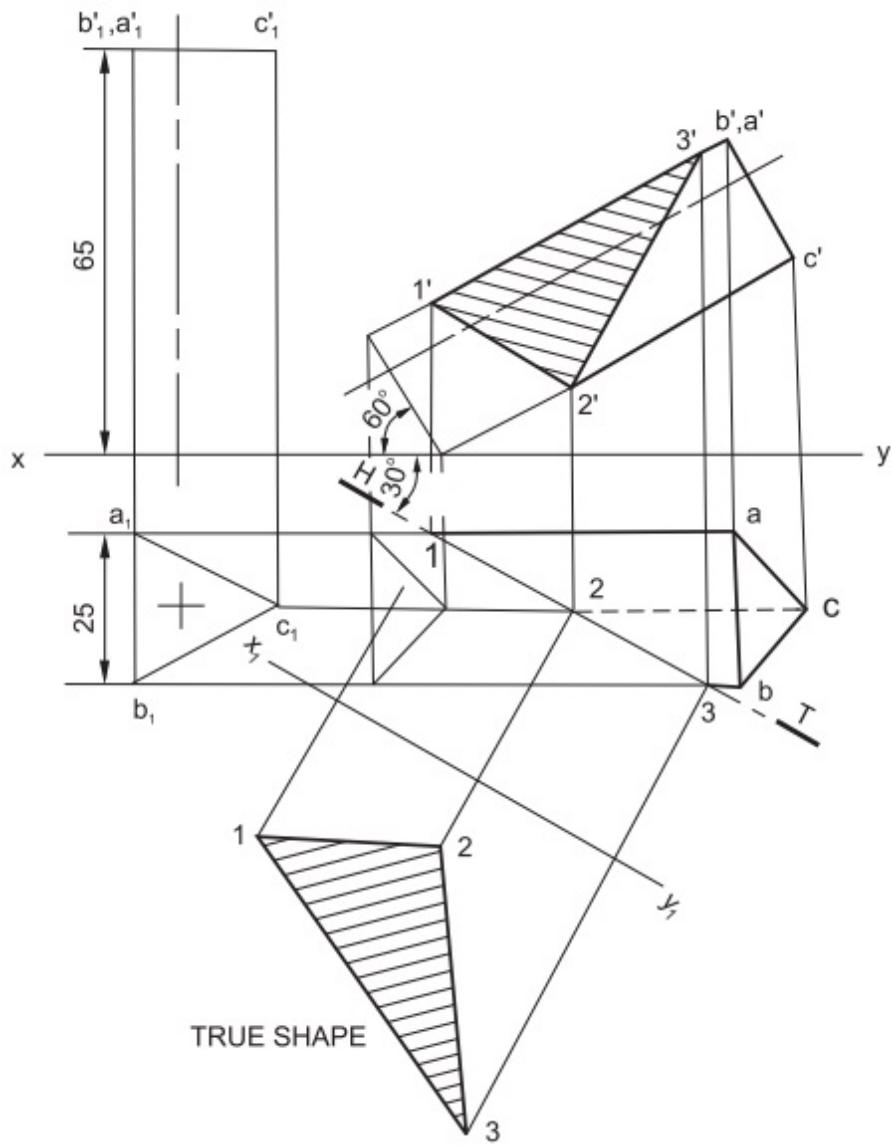
1. Draw the projections of the cone, satisfying the given conditions.
2. Repeat steps 2 and 3 of Construction: Fig.12.30 suitably and obtain the sectional top view.

**Problem 31** A triangular prism of side of base 25 and 65 long, rests on a corner of the base on H.P, with a base inclined at  $60^\circ$  to H.P and a rectangular face perpendicular to V.P. A section plane perpendicular to H.P and inclined at  $30^\circ$  to V.P, bisects the axis of the prism. Draw the projections and determine the true shape of the section.

**Construction (Fig.12.33)**

1. Draw the projections of the prism, satisfying the given conditions.

2. Draw the H.T of section plane, inclined at  $30^\circ$  to xy and passing through the midpoint of the axis.
3. Locate the points of intersection 1, 2 and 3 between the H.T and longer edges of the solid.
4. Project and locate the points  $1'$ ,  $2'$  and  $3'$  on the corresponding edges in the front view.
5. Join these points in the order, by straight lines and complete the sectional front view, by cross-hatching the sectioned portion.
6. Draw the reference line  $x_1y_1$ , parallel to the H.T of section plane and obtain the true shape of the section, by projection.



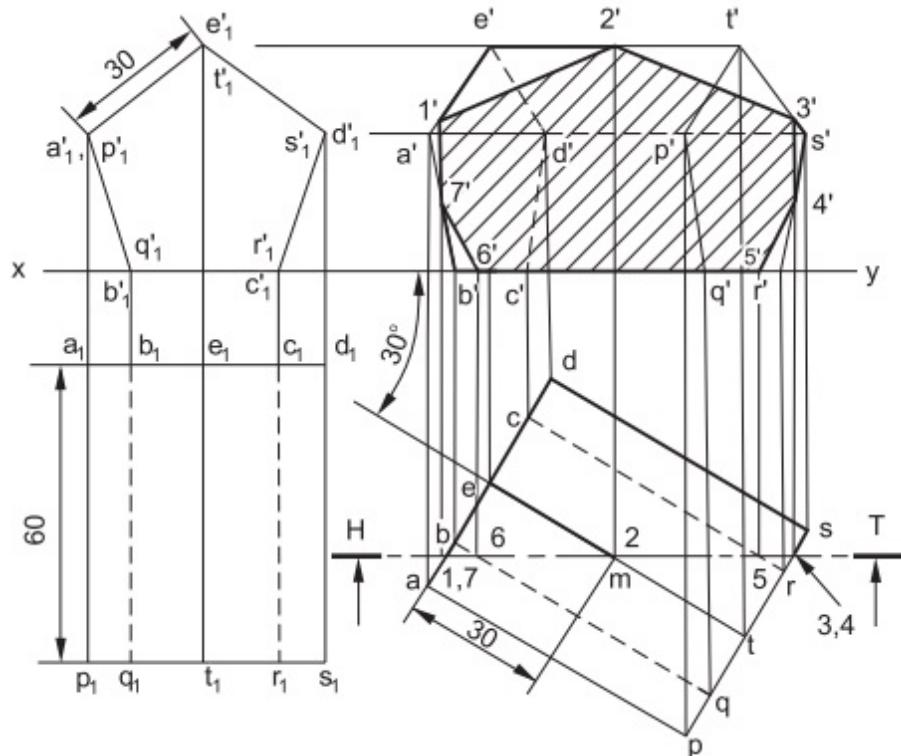
**Fig.12.33**

**Problem 32** A pentagonal prism of side of base 30 and axis 60 long, rests with one of its rectangular faces on H.P. with its axis inclined at  $30^\circ$  to V.P. A section plane parallel to V.P. cuts the solid into two halves. Draw the projections of the solid.

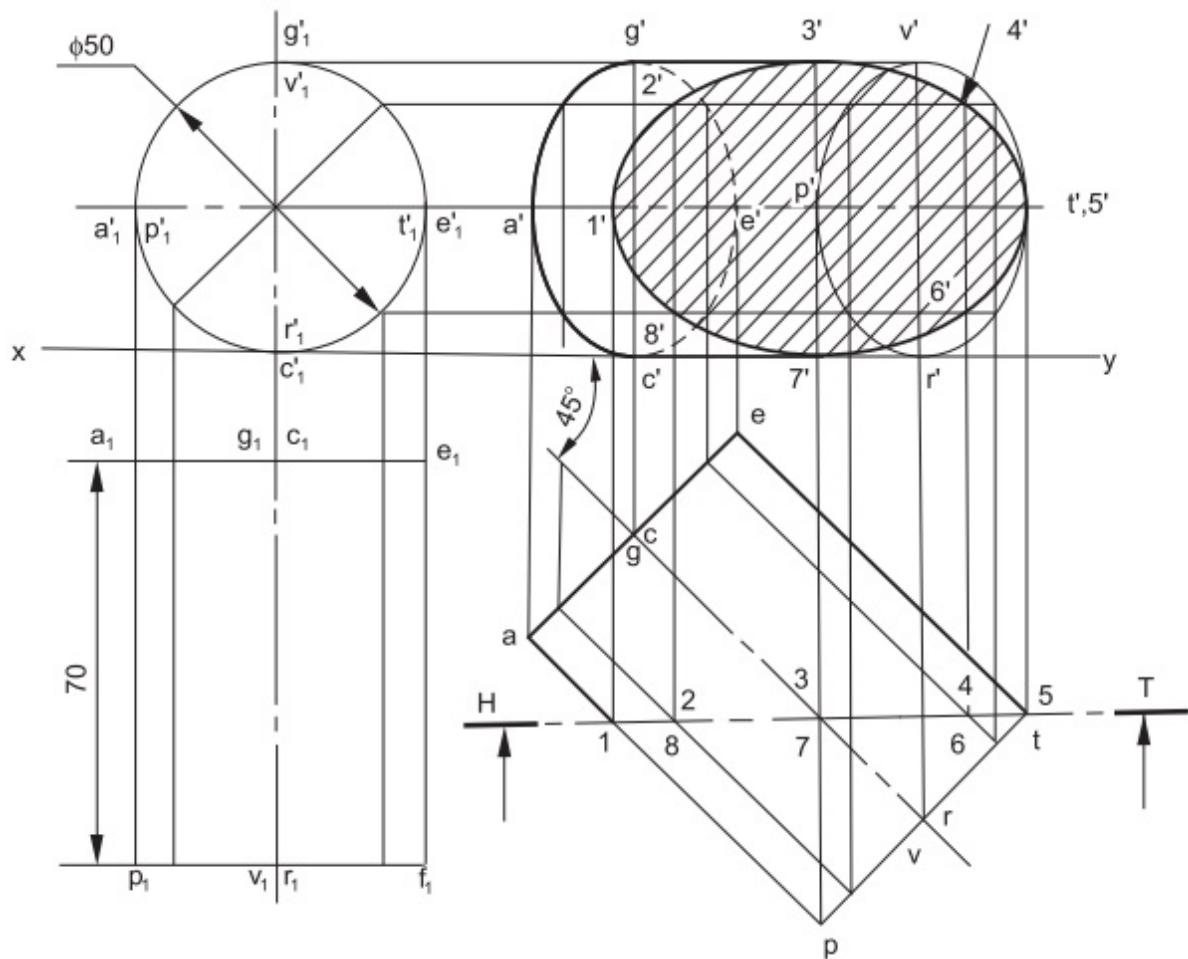
**Construction (Fig.12.34)**

1. Draw the projections of the prism, satisfying the given conditions.

2. Draw the H.T of section plane, parallel to xy and passing through the mid-point of the axis.
3. Locate the points of intersection 1, 2, 3, etc., between the H.T and bases and lateral edges of the prism.
4. Project and locate the points 1', 2', 3', etc., on the corresponding edges in the front view.
5. Join these points in the order, by straight lines and complete the sectional front view, by cross-hatching the sectioned portion.



**Fig.12.34**



**Fig.12.35**

**Problem 33** A cylinder of 50 diameter and axis 70 long, lies on H.P on one of its generators such that, the axis is inclined at  $45^\circ$  to V.P. A section plane parallel to V.P, passes through the farthest point of the visible base from the observer. Draw the projections of the cut solid.

**Construction (Fig.12.35)**

1. Draw the projections of the cylinder, satisfying the given conditions.
2. Draw the H.T of section plane, parallel to xy and passing through the farthest point of the visible base.

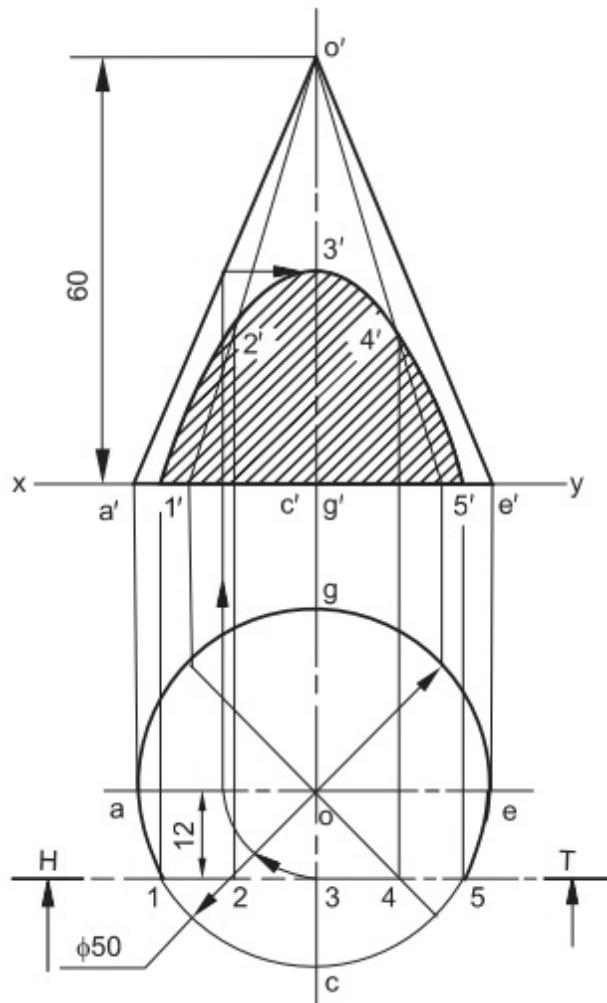
3. Locate the points of intersection 1, 2, etc., between the H.T and generators.
4. Repeat the steps 4 and 5 of Construction: [Fig.12.16](#) suitably and obtain the sectional front view.



The sectioned portion obtained in the front view is in its true shape and it is an ellipse.

**Problem 34** A cone of base 50 diameter and axis 60 long, stands with its base on H.P. A section plane parallel to V.P, cuts the solid at 12 from the axis. Draw the projections of the cut solid.

**Construction ([Fig.12.36](#))**



### **Fig.12.36**

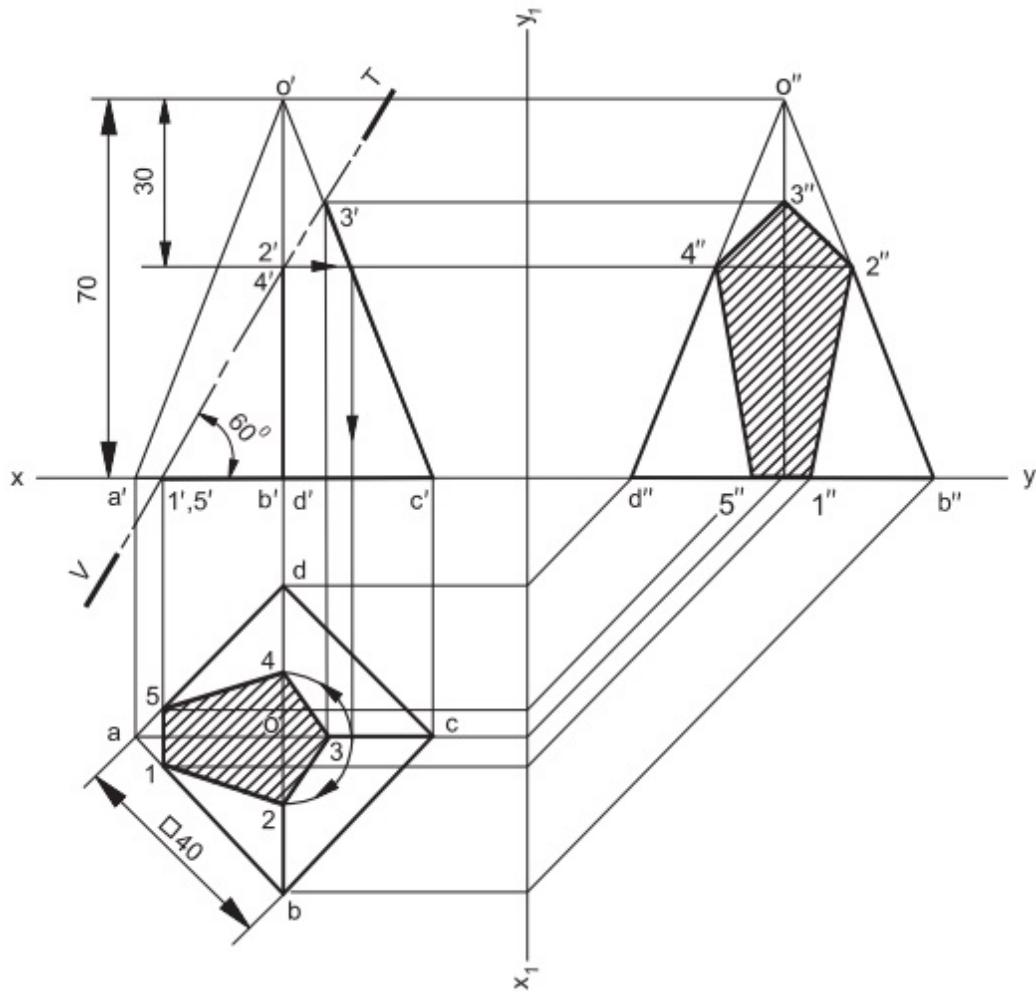
1. Draw the projections of the cone.
2. Draw the H.T of section plane, parallel to xy and at a distance of 12 from the axis.
3. Repeat steps 4 and 5 of Construction: [Fig.12.16](#) suitably and obtain the sectional front view.



The sectioned portion obtained in the front view is in its true shape and the name of the curve is hyperbola.

**Problem 35** A square pyramid of base 40 side and axis 70 long, rests with its base on H.P with all the edges of the base equally inclined to V.P. It is cut by a section plane inclined at  $60^\circ$  to H.P and passing through a point on the axis at 30 from the apex. Draw the three views of the cut solid.

**Construction ([Fig.12.37](#))**



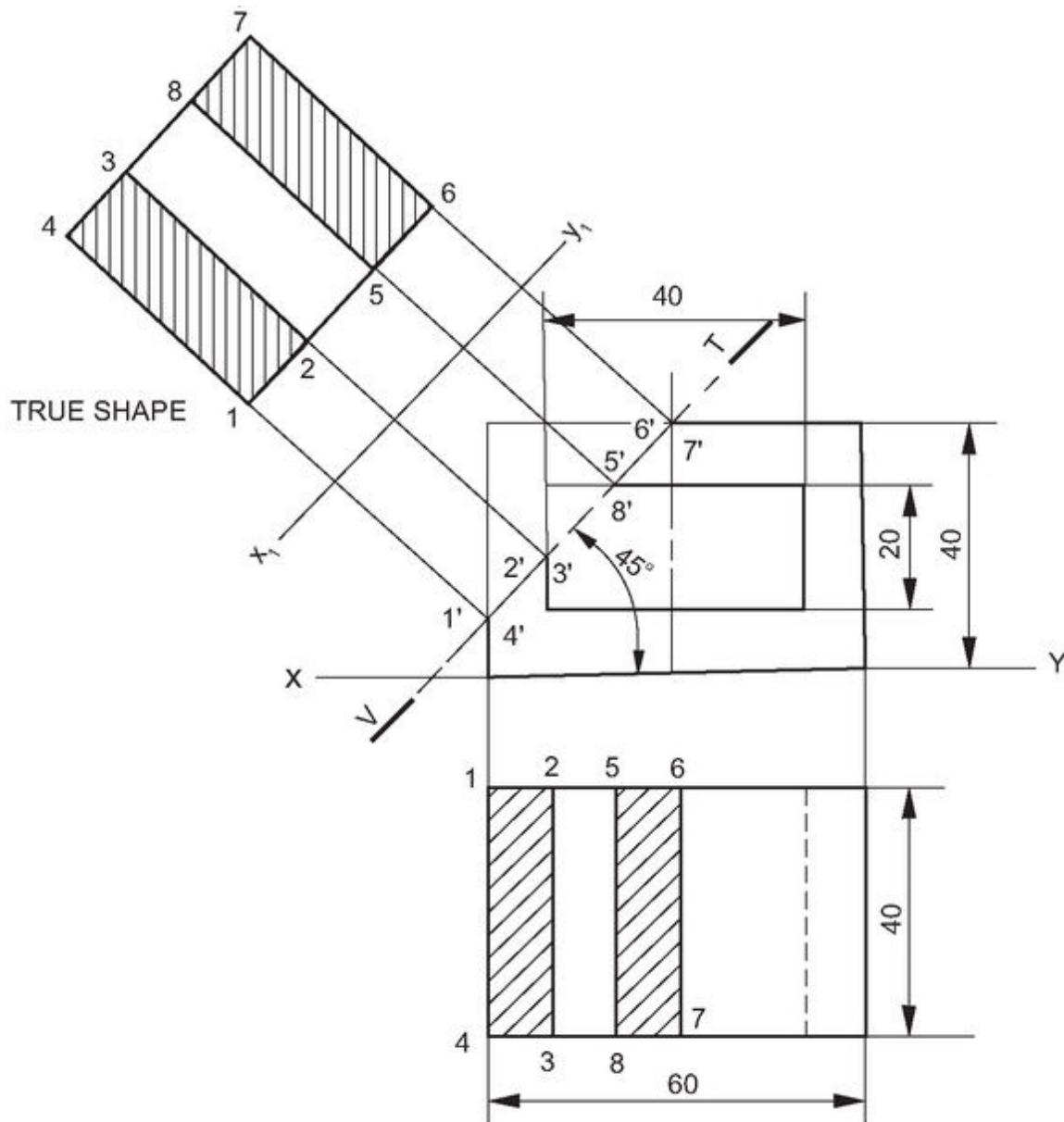
**Fig.12.37**

1. Draw the projections of the pyramid.
2. Draw the V.T of section plane, passing through the axis, at a distance of 30 from the apex and inclined at  $60^\circ$  to xy.
3. Repeat steps 3 to 5 Construction: [Fig.12.3](#) and obtain the sectional top view.
4. Project the front and top views and obtain the sectional left side view.

**Problem 36** A rectangular block,  $60 \times 40 \times 40$ , has a rectangular hole  $40 \times 20$ , cut centrally through it. It rests

on its base on H.P. A section plane inclined at  $45^\circ$  to H.P. passes through the top end of the axis. Draw its front view, sectional top view and the true shape of the section.

**Construction (Fig.12.38)**



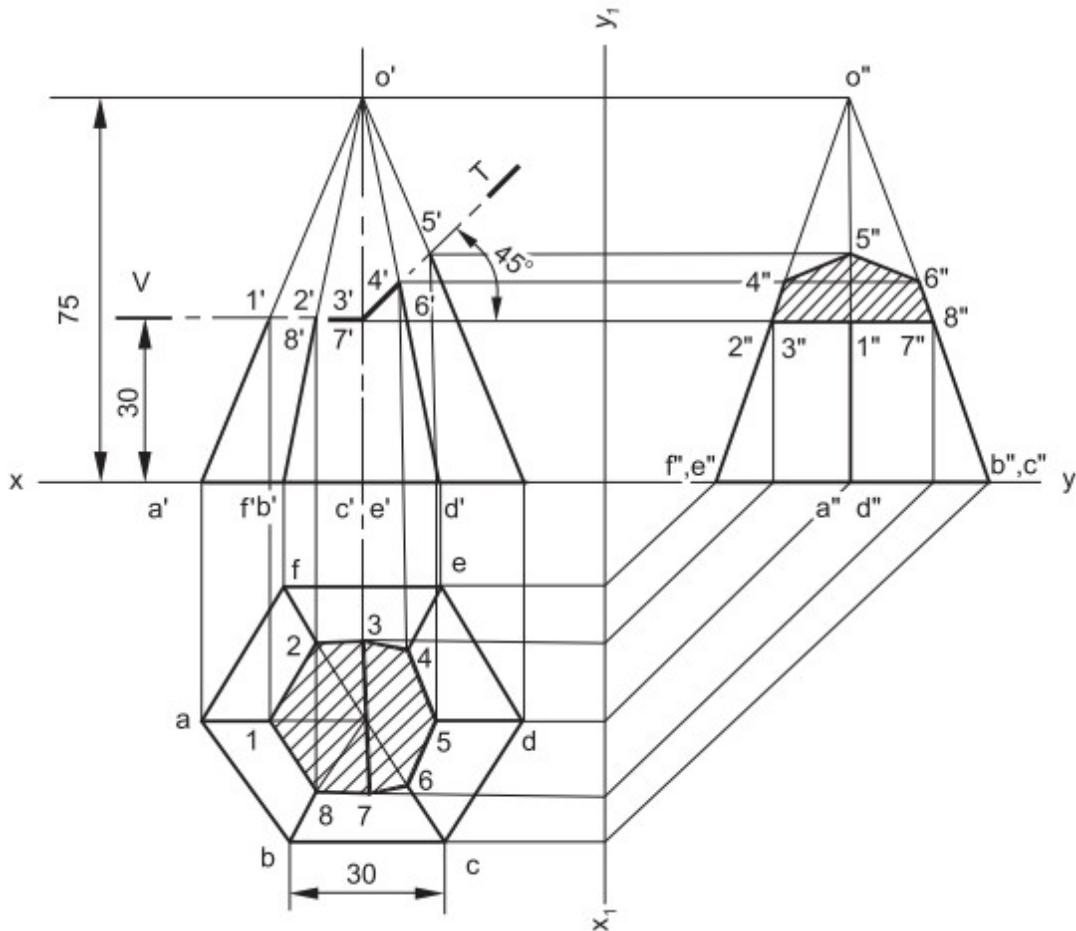
**Fig.12.38**

1. Draw the projections of the given rectangular block.

2. Draw the V.T of section plane, passing through the top end of the axis and inclined at  $45^\circ$  to xy.
3. Locate the points of intersection 1', 2', etc., between the V.T and both the external and internal edges.
4. Project and obtain the corresponding points in the top view.
5. Join the points in the order and complete the sectional top view, by cross-hatching the sectioned portions.
6. Obtain the true shape of the section, by projecting the section on an A.I.P, parallel to the V.T.

**Problem 37** *A hexagonal pyramid of side of base 30 and axis 75 long, is resting on its base on H.P with an edge of the base parallel to V.P. It is cut by two section planes, meeting the axis at a height of 30 from the base. One of the section planes is parallel to H.P and the other is inclined to H.P at  $45^\circ$  and leans upwards. Draw the three views of the solid.*

**Construction (Fig.12.39)**



**Fig.12.39**

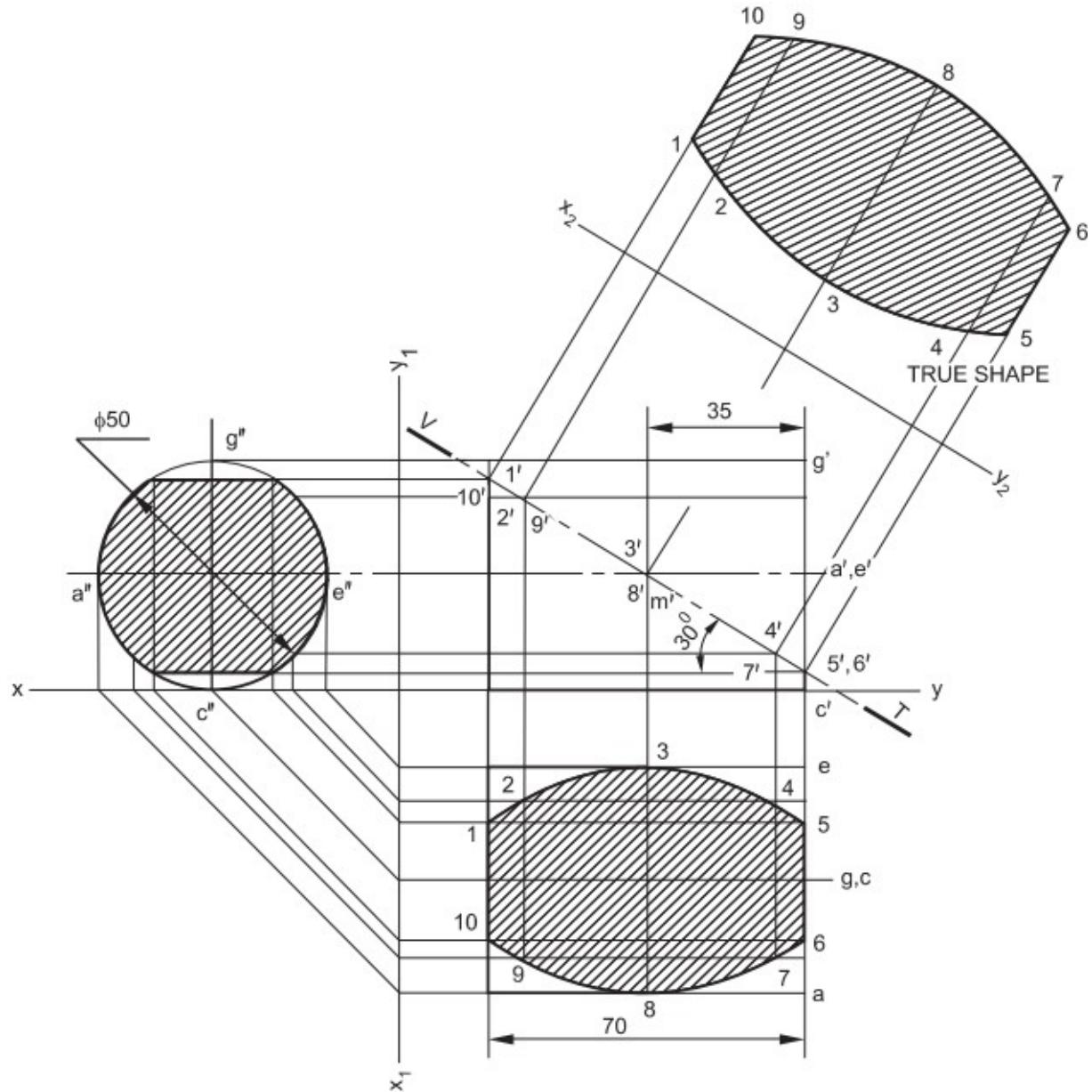
1. Draw the projections of the pyramid.
2. Draw the V.T of section planes, one parallel to xy and the other inclined at  $45^\circ$  to xy, meeting at a point on the axis at 30 above the base.
3. Repeat steps 3 to 5 of Construction: [Fig.12.2](#) and obtain the sectional top and left side views.



The two section planes meet along the line 7-3, separating the two zones of the sectional top view. As the two section planes are at an angle, the two zones of the section in the top view are cross-hatched by off-set lines.

**Problem 38** A cylinder of 50 diameter and 70 long, is lying on H.P on one of its generators such that, the axis is parallel to both H.P and V.P. A section plane inclined at  $30^\circ$  to H.P passes through the mid-point of the axis. Draw the projections of the solid and show the true shape of the section.

**Construction (Fig.12.40)**



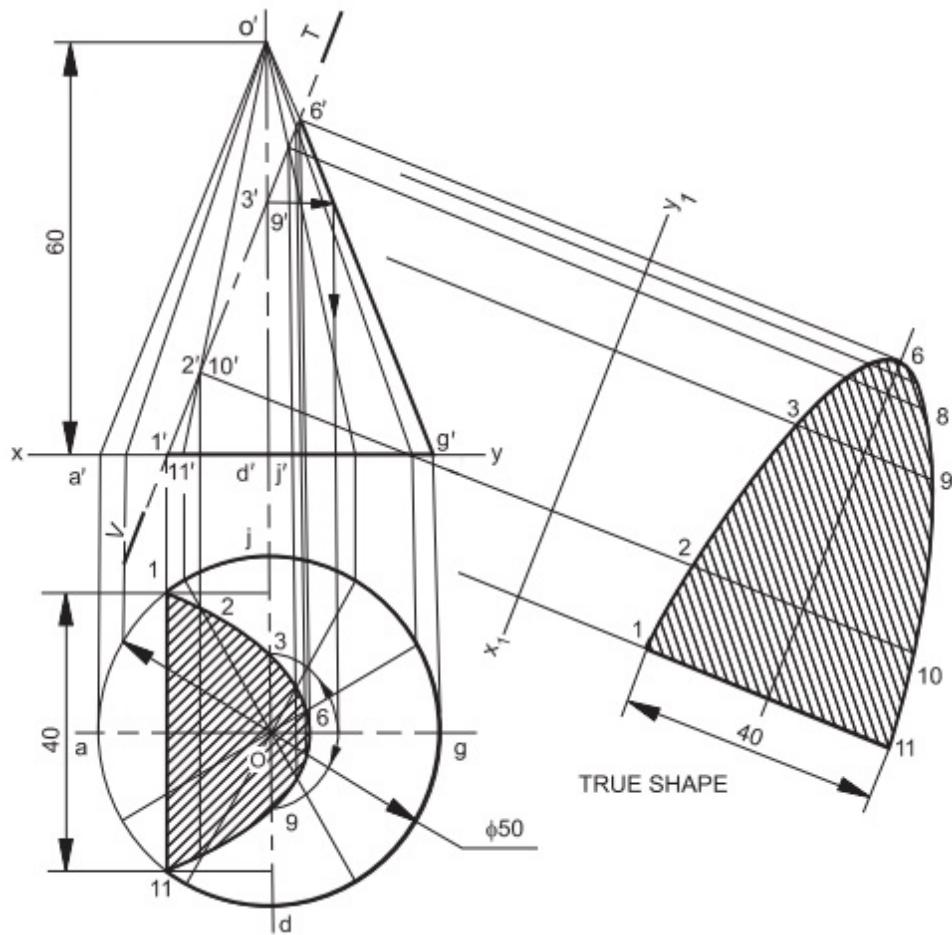
**Fig.12.40**

1. Draw the projections of the cylinder, along with the generators.
2. Draw the V.T of section plane, passing through the mid-point of the axis and making  $30^\circ$  with xy.
3. Locate the points of intersection 1', 2', etc., between the V.T and bases and generators of the solid.
4. Project and locate the corresponding points 1, 2, etc., in the top view.
5. Join the points in the order and obtain the sectional top view.
6. Obtain the true shape of the section on an A.I.P, parallel to the V.T.

**Problem 39** *A cone of base 50 diameter and 60 height, is resting on its base on H.P. It is cut by a section plane such that, the true shape produced is a parabola of base 40. Locate the V.T of the section plane and draw the projections of the solid.*

**HINT** When a cone is cut by a section plane, parallel to an extreme generator, the section produced is a parabola, the size of which depends upon the position of the section plane.

**Construction ([Fig.12.41](#))**

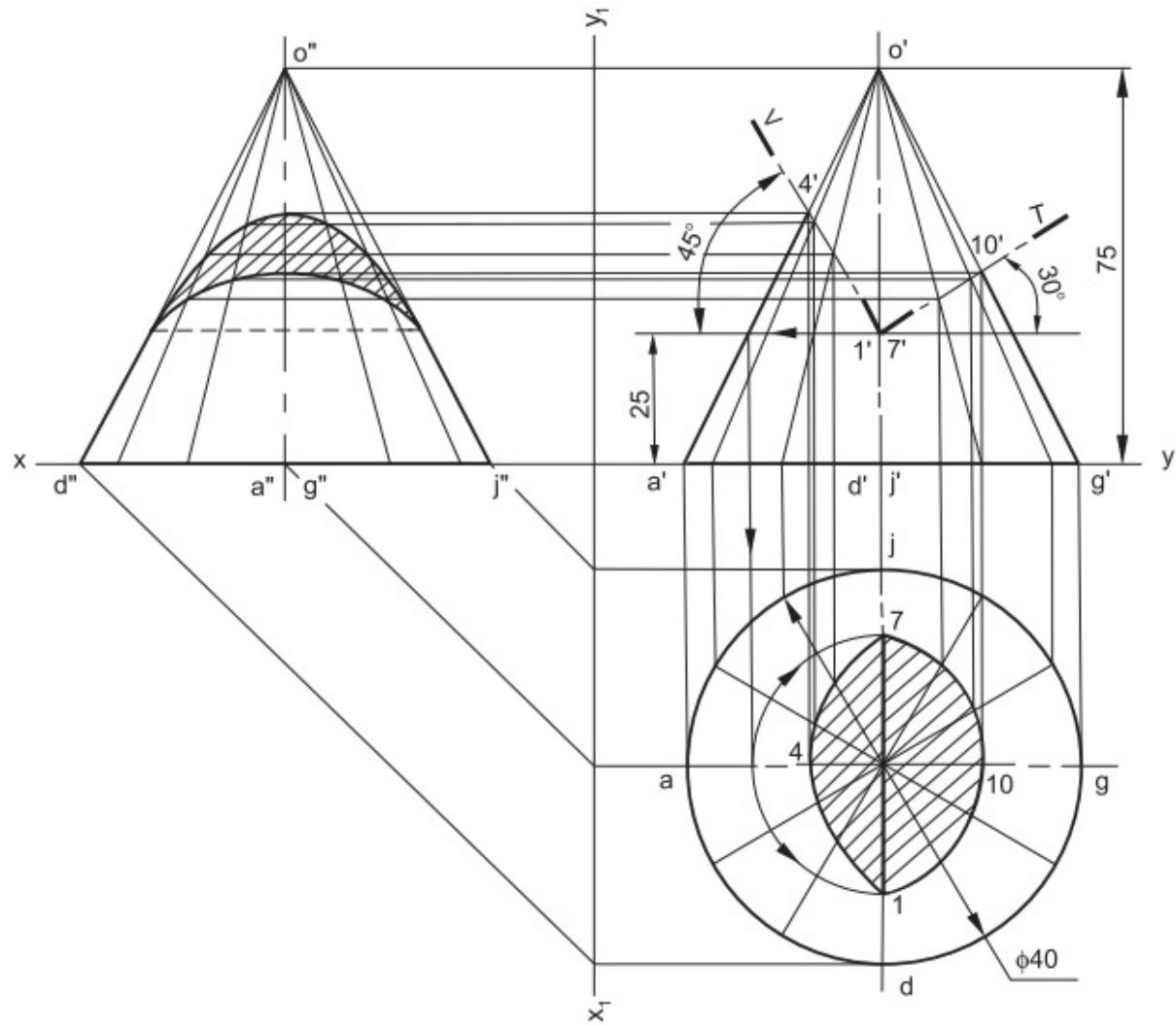


**Fig.12.41**

1. Draw the projections of the cone.
2. In the top view, locate the chord 1-11 of length 40 and perpendicular to xy.
3. Through 1 (11), draw a projector, meeting xy at 1' (11').
4. Through 1', draw the V.T. parallel to the extreme generator.
5. Repeat steps 3 to 6 of Construction: Fig.12.14a suitably and obtain sectional top view and the true shape of the section.

**Problem 40** A cone of base 75 diameter and axis 75 long, is resting on its base on H.P. The cone is cut by two inclined section planes, intersecting the axis at 25 above the base. The planes make  $30^\circ$  and  $45^\circ$  with the base. Draw the three views of the solid.

**Construction (Fig.12.42)**



**Fig.12.42**

1. Draw the projections of the cone and locate the generators.

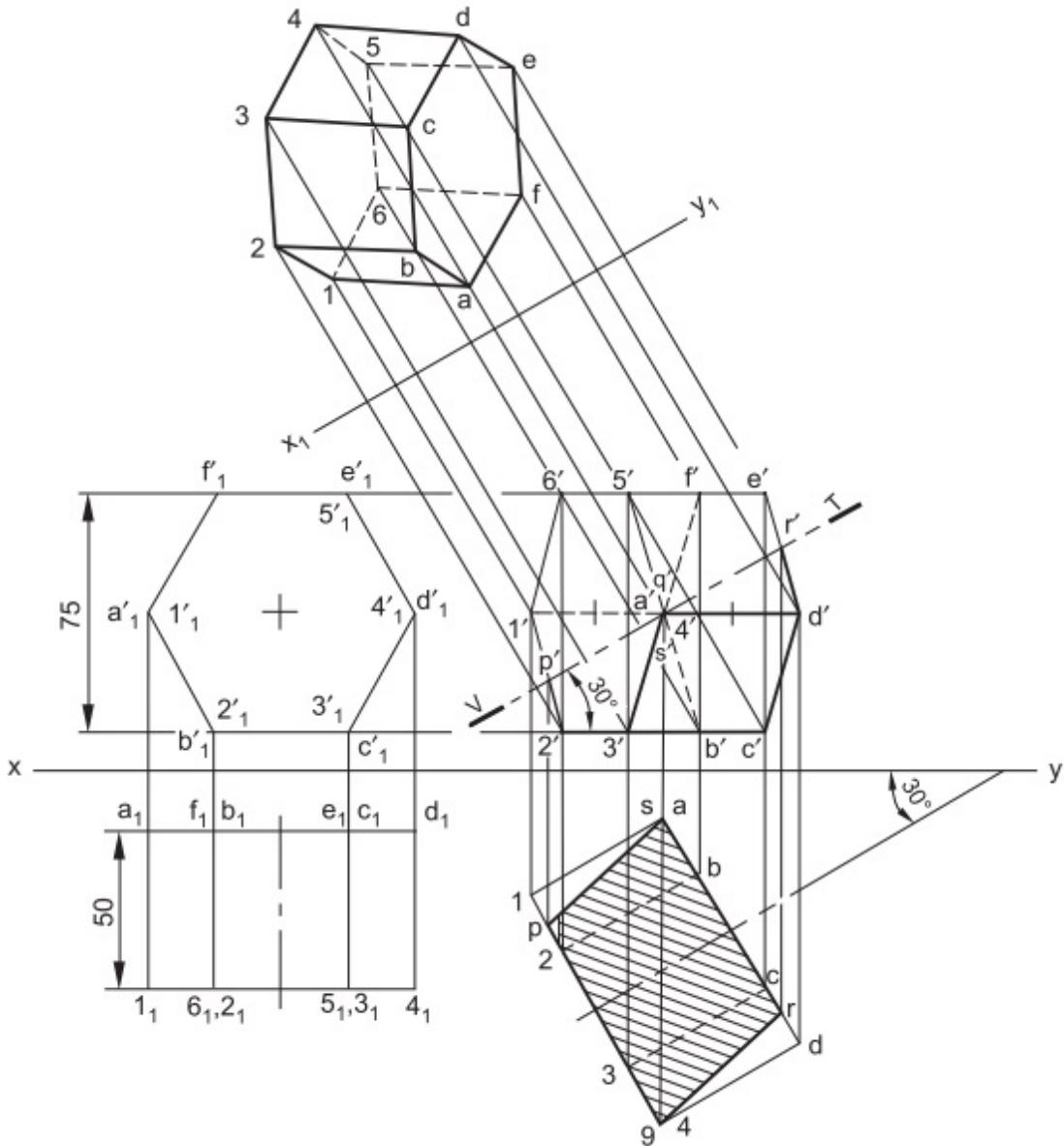
2. Draw the V.T of two section planes, one at an angle of  $30^\circ$  and the other at  $45^\circ$  with xy and at a point on the axis, 25 above xy.
3. Locate the points of intersection 1', 2', etc., between the V.T and generators.
4. Project and locate the corresponding points 1, 2, etc., in the top view.
5. Join the points in the order and obtain the sectional top view.
6. Project the front and sectional top views and obtain the sectional right side view.



The sectioned boundary in the top view consists of two curves, corresponding to the two parts of the intersecting V.T, meeting sharply at 1 and 7. Hence, the line 1-7 appears as a thick line and two portions of the section are represented by off-set cross-hatching lines.

**Problem 41** *The distance between the opposite parallel faces of a 50 thick hexagonal block is 75. The block has one of its rectangular faces parallel to H.P; and its axis makes an angle of  $30^\circ$  with V.P. It is cut by a section plane making an angle of  $30^\circ$  with H.P; normal to V.P and bisecting the axis. Draw its sectional top view and another top view on a plane parallel to the section.*

**Construction (Fig.12.43)**



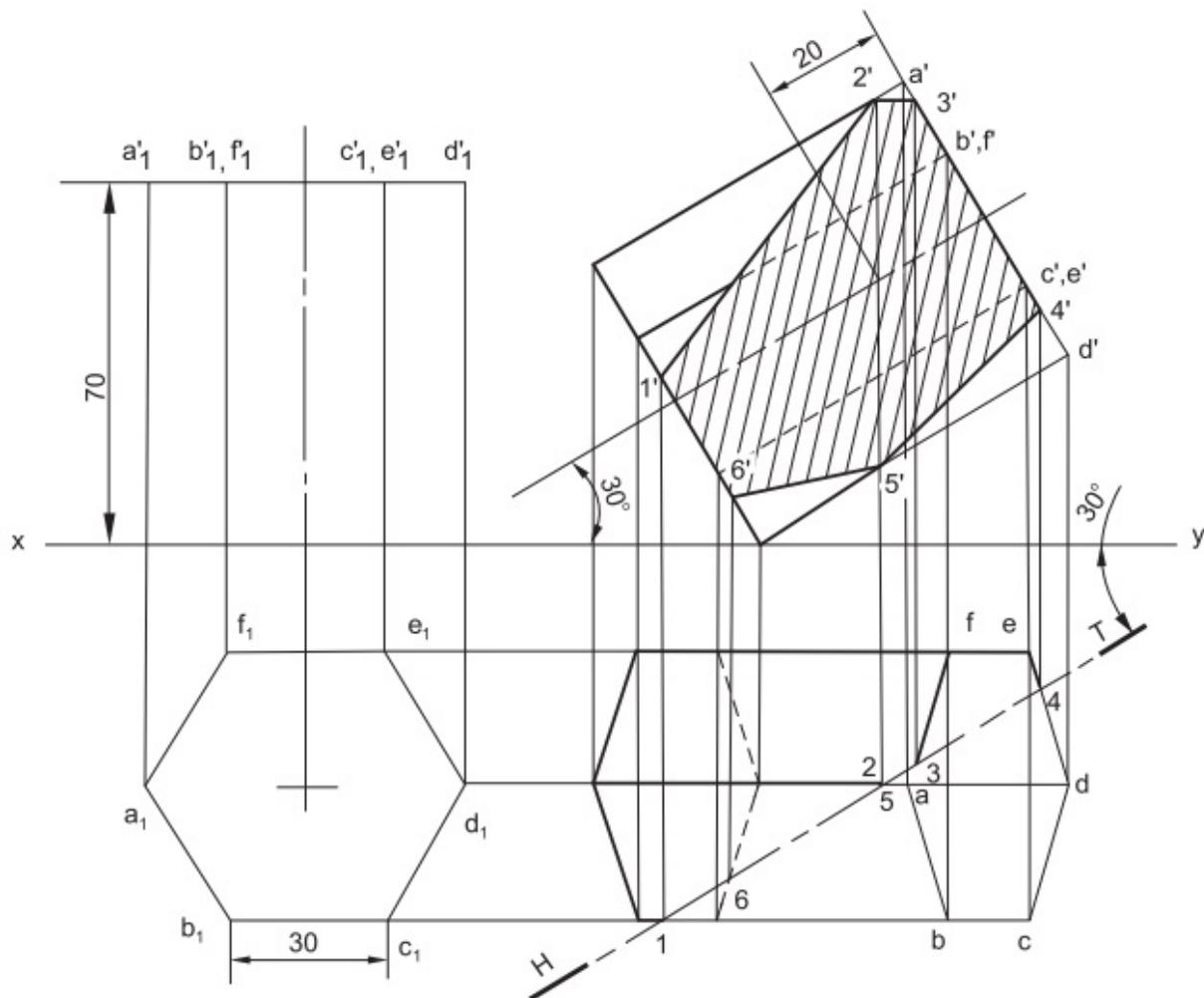
**Fig.12.43**

1. Draw the projections of the hexagonal block, satisfying the given conditions such that, the axis makes an angle  $30^\circ$  with xy (second top view).
2. Draw the V.T of the section plane, inclined at  $30^\circ$  to H.P and passing through the mid-point of the axis.
3. Locate the points of intersection p', q', r' and s' between the V.T and edges of the block.

4. Project and obtain the corresponding points in the top view.
5. Join the points by straight lines and complete the sectional top view, by crosshatching the sectioned portion.
6. Draw the reference line  $x_1y_1$  parallel to the section plane.
7. Obtain the auxiliary top view of the block, by projection.

**Problem 42** A hexagonal prism with side of base 30 and axis 70 long, rests on a corner of its base on H.P, with the axis inclined to H.P at  $30^\circ$  and parallel to V.P. It is cut by a vertical section plane, inclined at  $30^\circ$  to V.P and passing through the axis at 20 from one end. Draw the projections of the solid.

**Construction (Fig.12.44)**



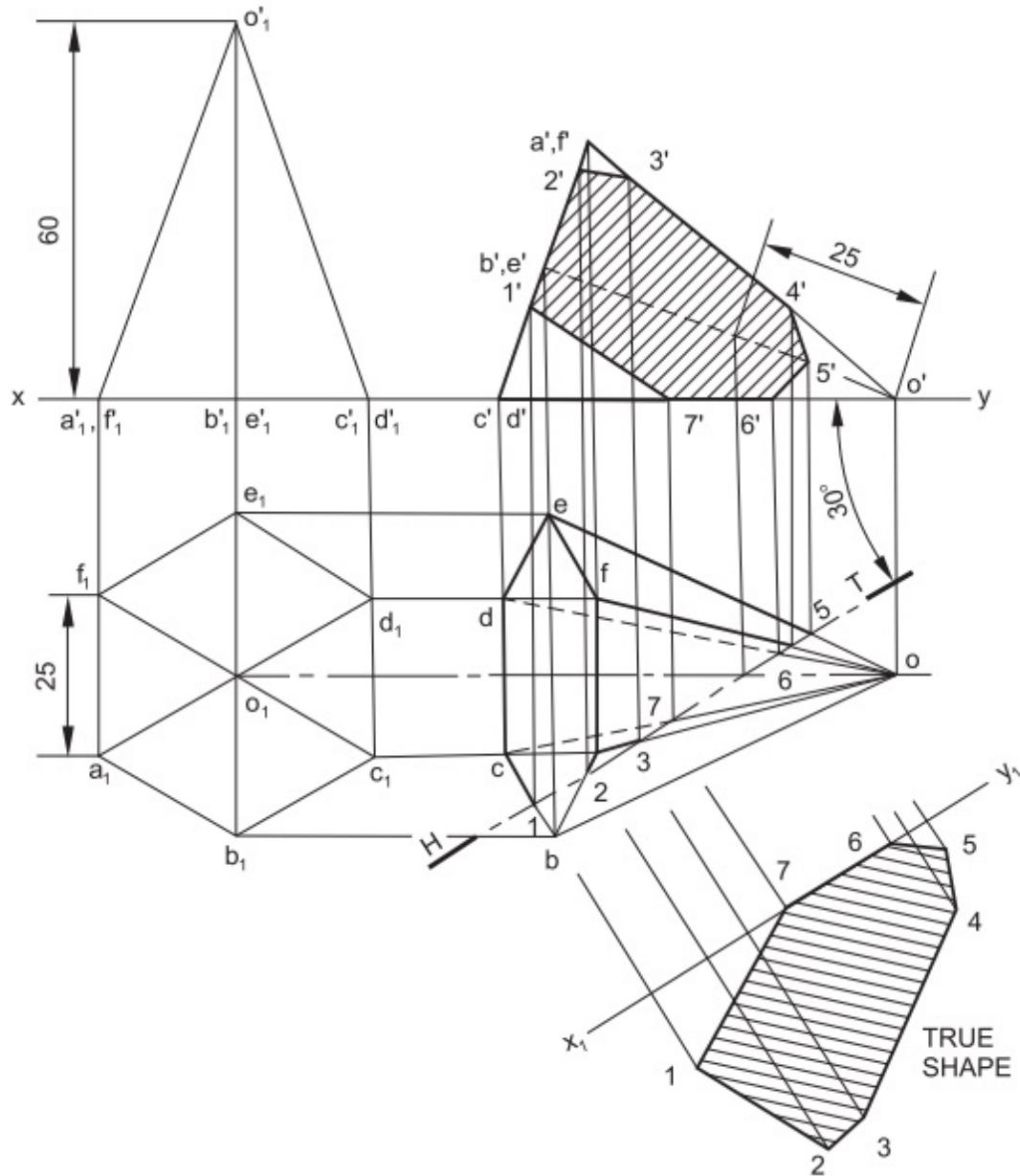
**Fig.12.44**

1. Draw the projections of the prism.
2. Draw the H.T of section plane, passing through a point on the axis at 20 from one end and inclined at  $30^\circ$  to xy.
3. Repeat steps 3 to 5 of Construction: [Fig.12.15](#) suitably and obtain the sectional front view.

**Problem 43** A hexagonal pyramid of side of base 25 and axis 60 long, rests on a triangular face on H.P. with its axis parallel to V.P. A section plane inclined at  $30^\circ$  to V.P intersects the axis at 25 from the apex of the pyramid.

*Draw the projections of the cut solid and show the true shape of the section.*

**Construction (Fig.12.45)**



**Fig.12.45**

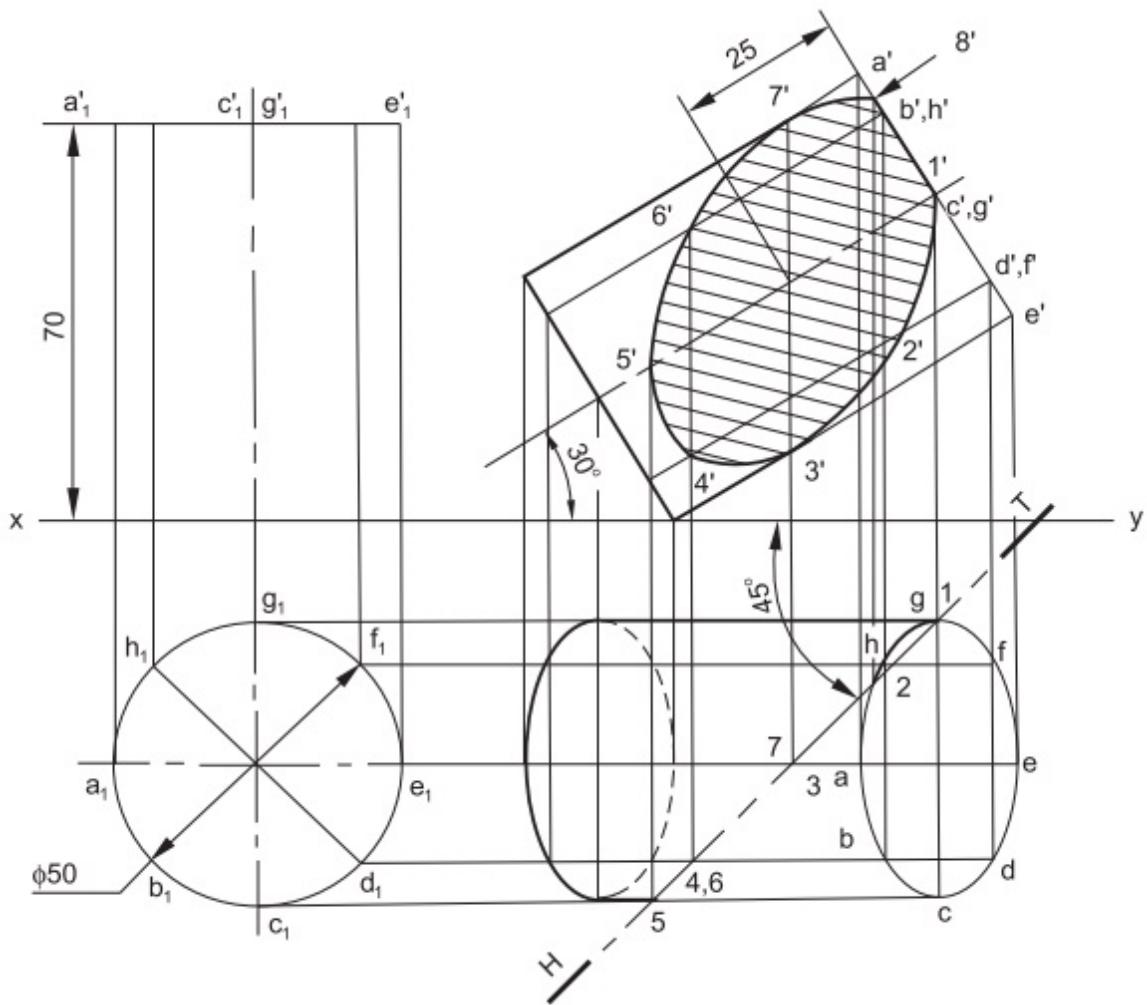
1. Draw the projections of the pyramid, satisfying the given conditions.

2. Draw the H.T of section plane, passing through a point on the axis at 25 from the apex and inclined at  $30^\circ$  to xy.
3. Repeat steps 3 to 6 of Construction: [Fig.12.15](#) suitably and obtain the sectional front view and the true shape of the section.

**Problem 44** A cylinder of 50 diameter and 70 long, is resting on H.P with its axis inclined at  $30^\circ$  to H.P and parallel to V.P. A section plane inclined at  $45^\circ$  to V.P, passes through the axis at 25 from one end of it. Draw the projections of the cut solid.

**Construction ([Fig.12.46](#))**

1. Draw the projections of the cylinder and locate the generators.
2. Draw the H.T of section plane, passing through the axis, at 25 from one end of it and inclined at  $45^\circ$  to xy.
3. Repeat steps 4 to 6 of Construction: [Fig.12.16](#) suitably and obtain the sectional front view.



**Fig.12.46**

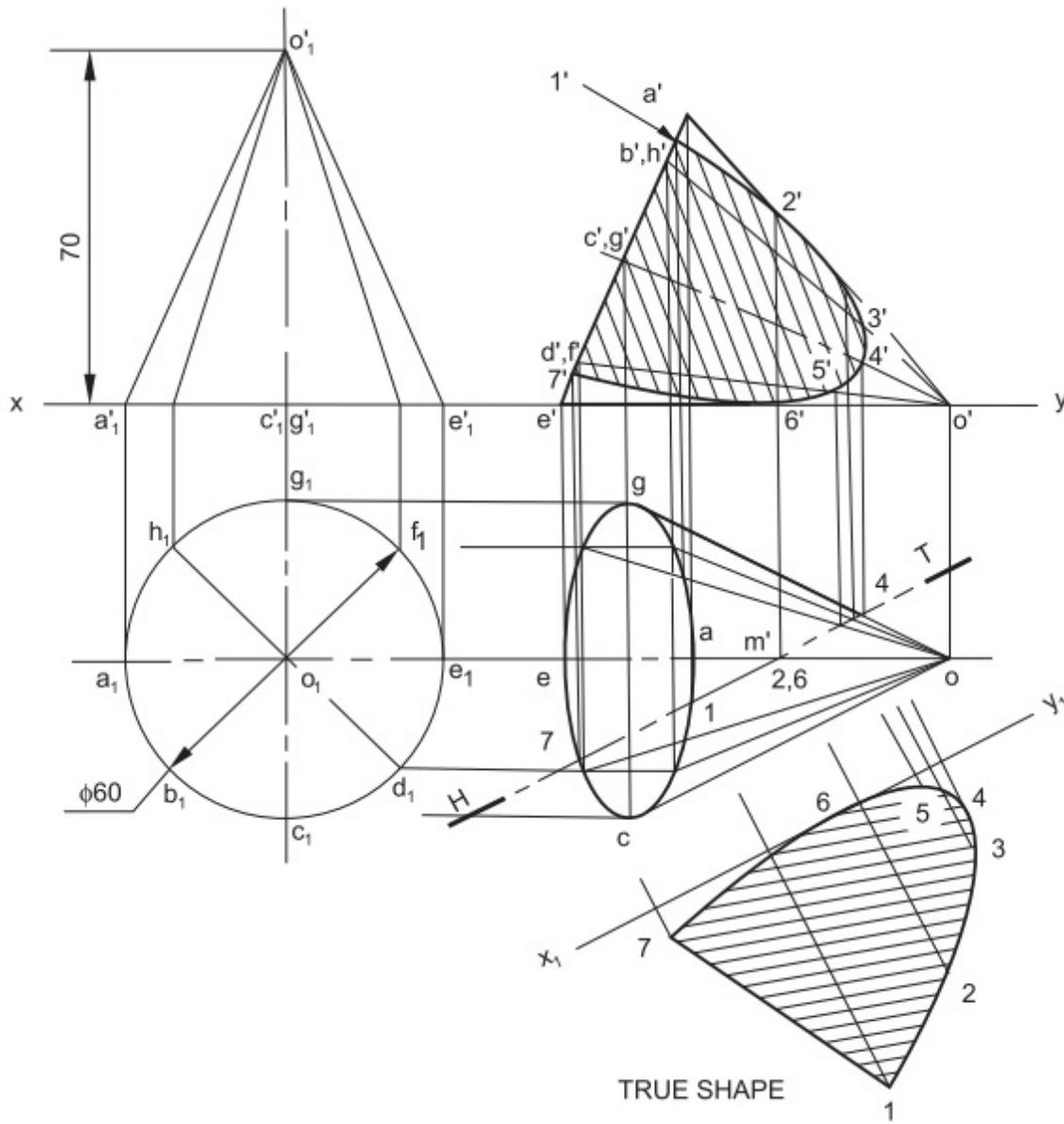
**Problem 45** A cone of base 60 diameter and axis 70 long, is lying on one of its generators on H.P. with its axis parallel to V.P. A vertical section plane, parallel to the extreme generator, bisects the axis. Draw the projections of the cut solid and show the true shape of the section.

**Construction (Fig.12.47)**

1. Draw the projections of the cone along with the generators.
2. Draw the H.T. of section plane, passing through the mid-point of the axis and parallel to the extreme

generator in the top view such that, only a minor portion of the solid is cut and removed.

3. Repeat steps 4 to 6 of Construction: Fig.12.16 suitably and obtain the sectional front view and the true shape of the section.



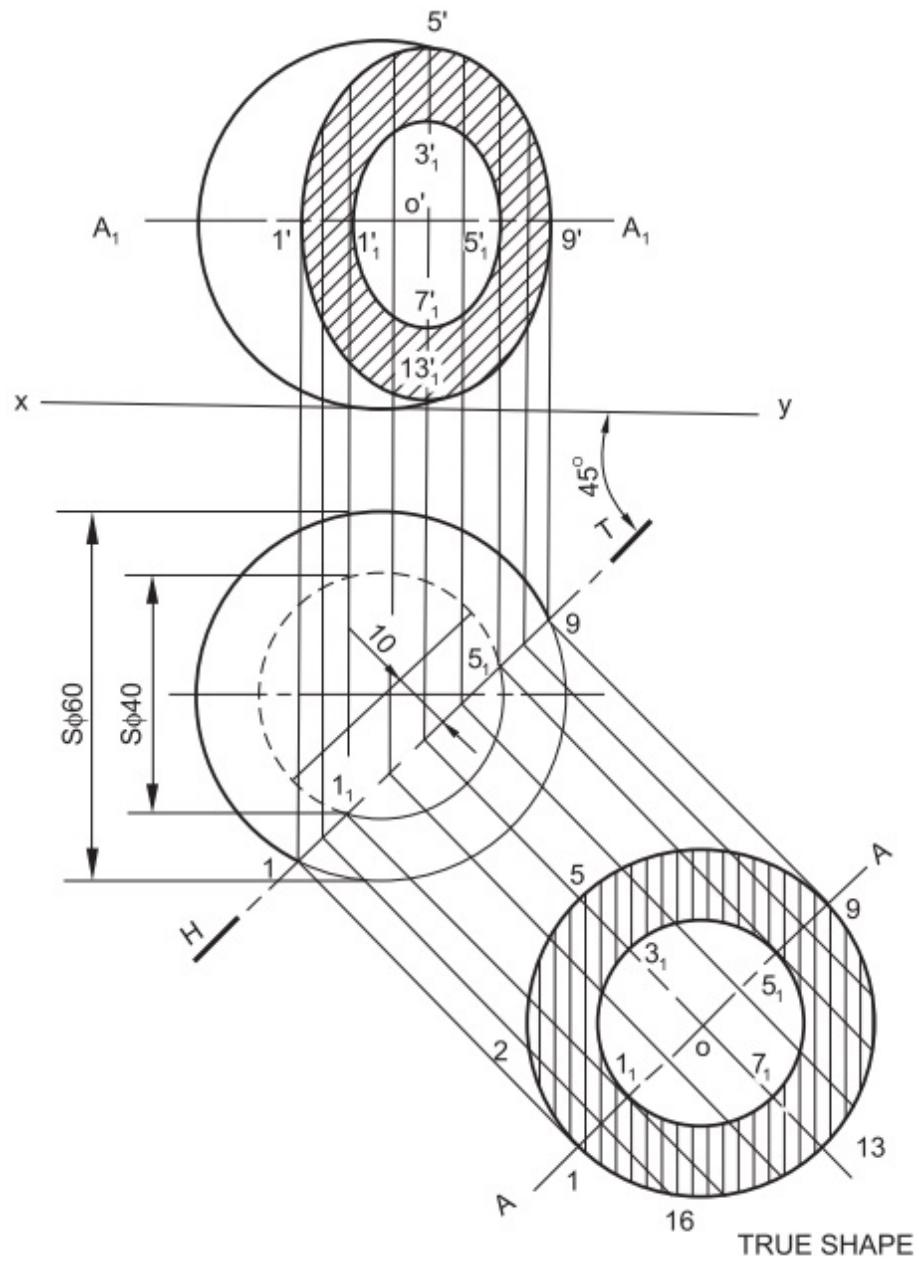
**Fig.12.47**

**Problem 46** A hollow sphere of 40 internal diameter and 60 external diameter, is cut by a section plane, inclined at

*45° to VP and at a distance of 10 from its centre. Draw the projections of the sphere.*

**Construction (Fig.12.48)**

1. Draw the projections of the hollow sphere.
2. Draw the H.T of section plane, inclined at 45° to xy and at 10 from the centre.
3. Draw the true shape of the section, consisting of two concentric circles with 1<sub>1</sub>-5<sub>1</sub> and 1-9 as diameters.
4. Transfer the points, which are symmetrically placed about A-A on the true shape, to the front view, which are symmetric to A<sub>1</sub>-A<sub>1</sub>.
5. Join the points in the order by smooth curves and complete the sectional front view.



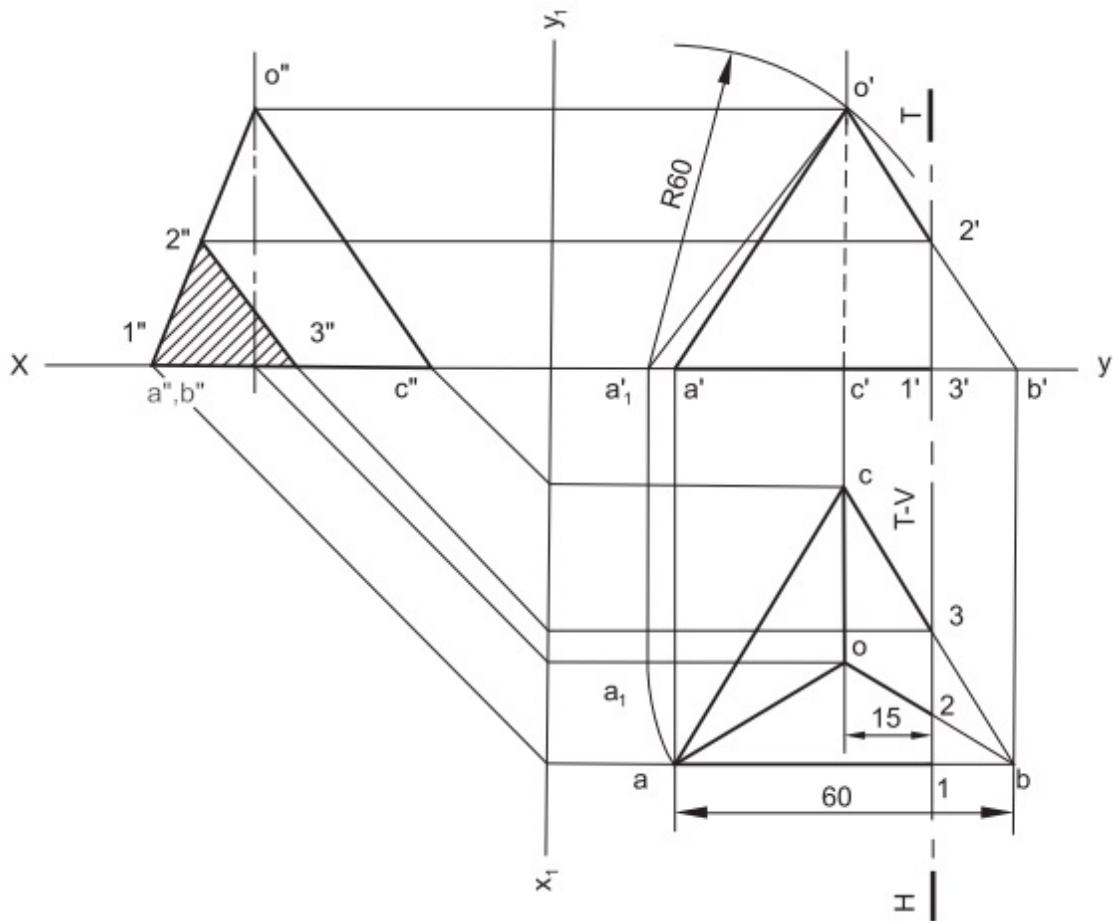
**Fig.12.48**

**Problem 47** A tetrahedron of side 60 is resting on H.P on one of its faces, with an edge of it parallel to V.P. It is cut by a section plane, perpendicular to both H.P and V.P and 15 away from the axis of the solid. Draw the sectional side view.

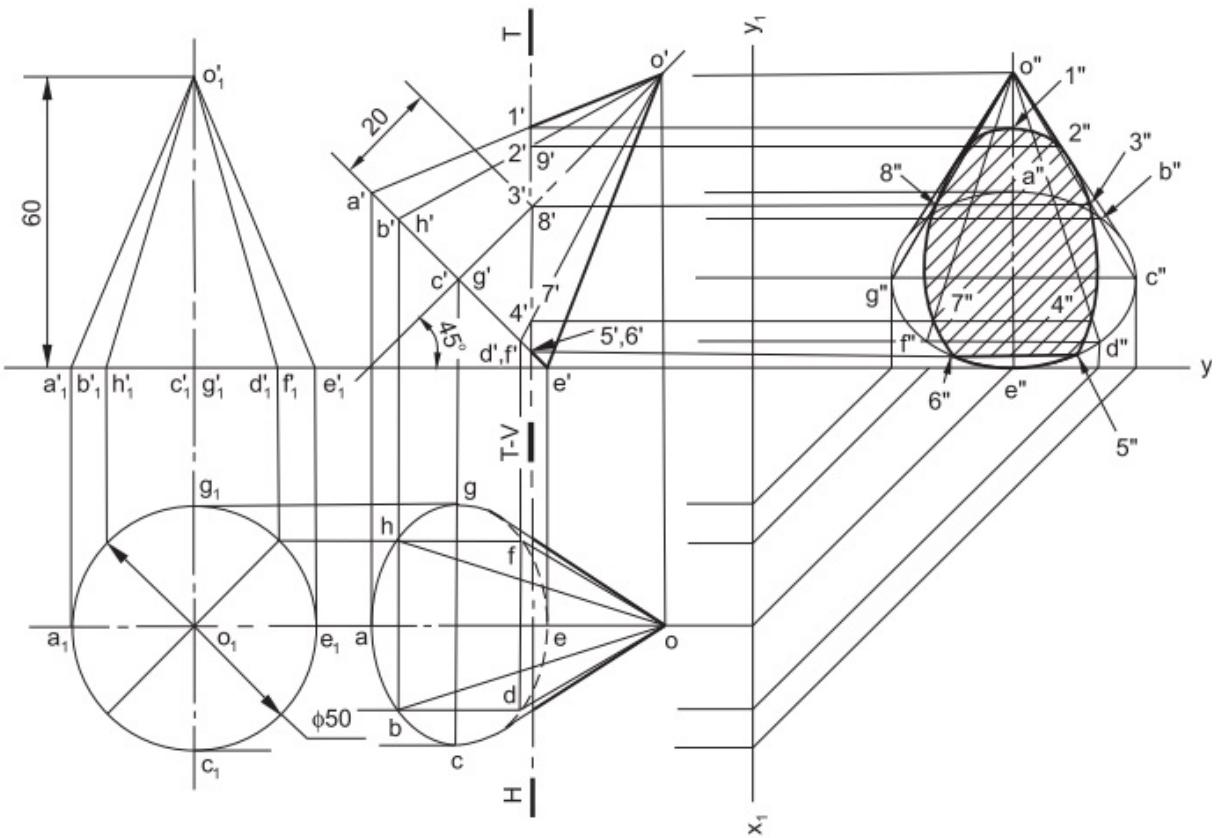
**Construction (Fig.12.49)**

1. Draw the three views of the tetrahedron.
2. Draw the H.T and V.T of section plane, perpendicular to xy and 15 from the axis.
3. Follow the Construction: Fig.12.18 and obtain the sectional right side view.

**Problem 48** A cone of base 50 diameter and axis 60 long, lies on H.P with its axis inclined at  $45^\circ$  to H.P and parallel to V.P. A section plane perpendicular to both H.P and V.P passes through the axis of the solid at 20 from the centre of the base. Draw the three views of the solid.



**Fig.12.49**



**Fig.12.50**

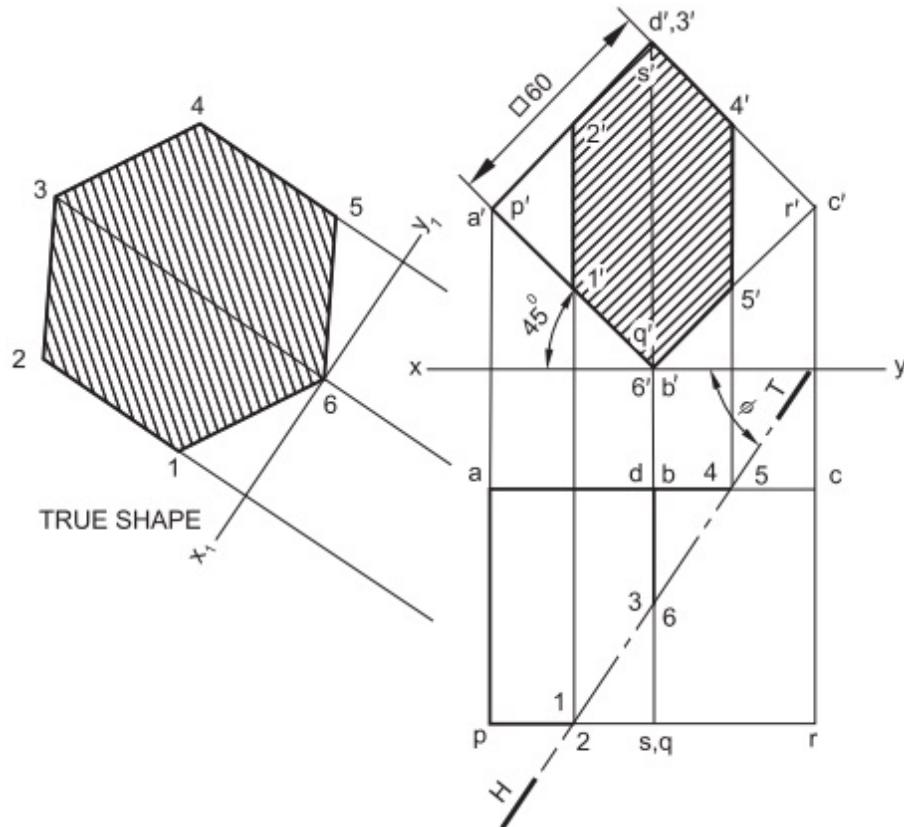
**Construction (Fig.12.50)**

1. Draw the three views of the cone along with the generators, satisfying the given conditions.
2. Draw the H.T and V.T of section plane, perpendicular to xy and 20 from the centre of the base.
3. Follow the steps 3 to 5 of Construction: Fig.12.18 suitably and obtain the sectional left side view.

**Problem 49** A cube of 60 long edges, is resting on one of its edges on H.P such that, a face containing that edge is inclined at  $45^\circ$  to H.P. It is cut by a section plane, perpendicular to H.P such that, the true shape of the section produced is a regular hexagon. Determine the inclination of the cutting plane with V.P and draw the

*projections of the solid and show the true shape of the section.*

### **Construction (Fig.12.51)**



**Fig.12.51**

1. Draw the projections of the cube.

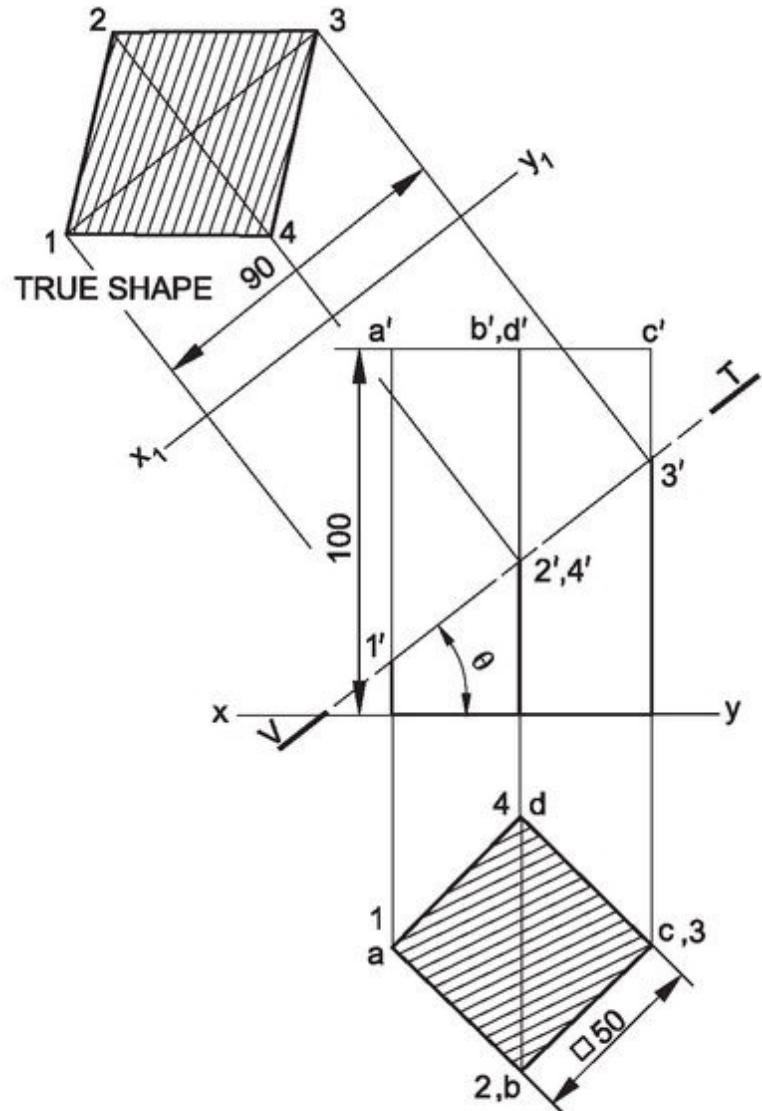
**HINT** (i) A section plane perpendicular to H.P and inclined to V.P, is to be assumed passing through the mid-point of the axis and both the bases, to produce a hexagonal section.

(ii) The distance between the corners of a hexagon is twice the side of the hexagon. Hence, the section plane should pass through the mid-point of the edges bc and pq.

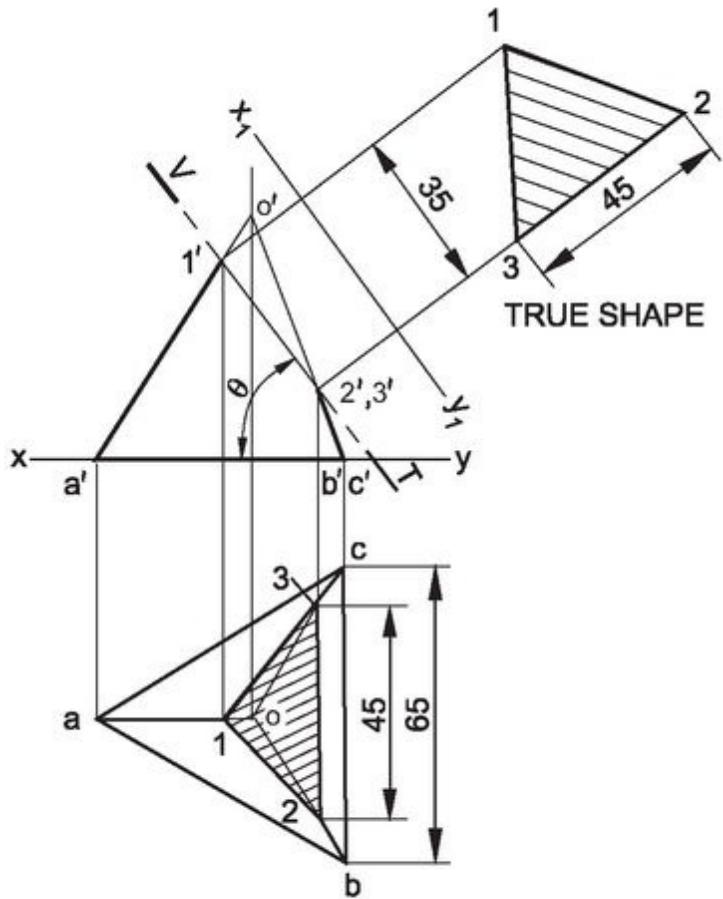
- Locate the H.T of the section plane such that, it passes
2. through the mid-points of bc and pq in the top view. The angle  $\phi$  is the inclination of the section plane with V.P.
  3. Project the points of intersection between the H.T and the edges of the cube and obtain the sectional front view.
  4. Consider a reference line  $x_1y_1$ , parallel to the H.T of the section plane and obtain the true shape of the section, by projection.

**Problem 50** A square prism of base 50 side and axis 100 long, stands with its base on H.P such that, all the faces are equally inclined to V.P. It is cut by a section plane, perpendicular to V.P such that, the true shape of the section is a rhombus of longer diagonal 90. Find the inclination of the section plane with H.P. Draw the projections of the solid.

**Construction (Fig.12.52)**



**Fig.12.52**



**Fig.12.53**

1. Draw the projections of the prism.

**HINT** For the true shape of the section to be a rhombus, the section plane should pass through the extreme lateral edges in the front view.

2. Locate the V.T of the section plane such that, it passes through the extreme lateral edges of the prism in the front view and the length of the intercept is equal to 90. The angle  $\theta$  is the inclination of the section plane with H.P.

3. Consider a reference line  $x_1y_1$ , parallel to the V.T of the section plane and obtain the true shape of the section, by projection.

**Problem 51** A tetrahedron of 65 long edges, is lying on one of its faces, with an edge perpendicular to V.P. It is cut by a section plane, which is perpendicular to V.P such that, the true shape of the section is an isosceles triangle of base 45 and altitude 35. Find the inclination of the section plane with H.P and draw the projections of the solid.

### **Construction (Fig.12.53)**

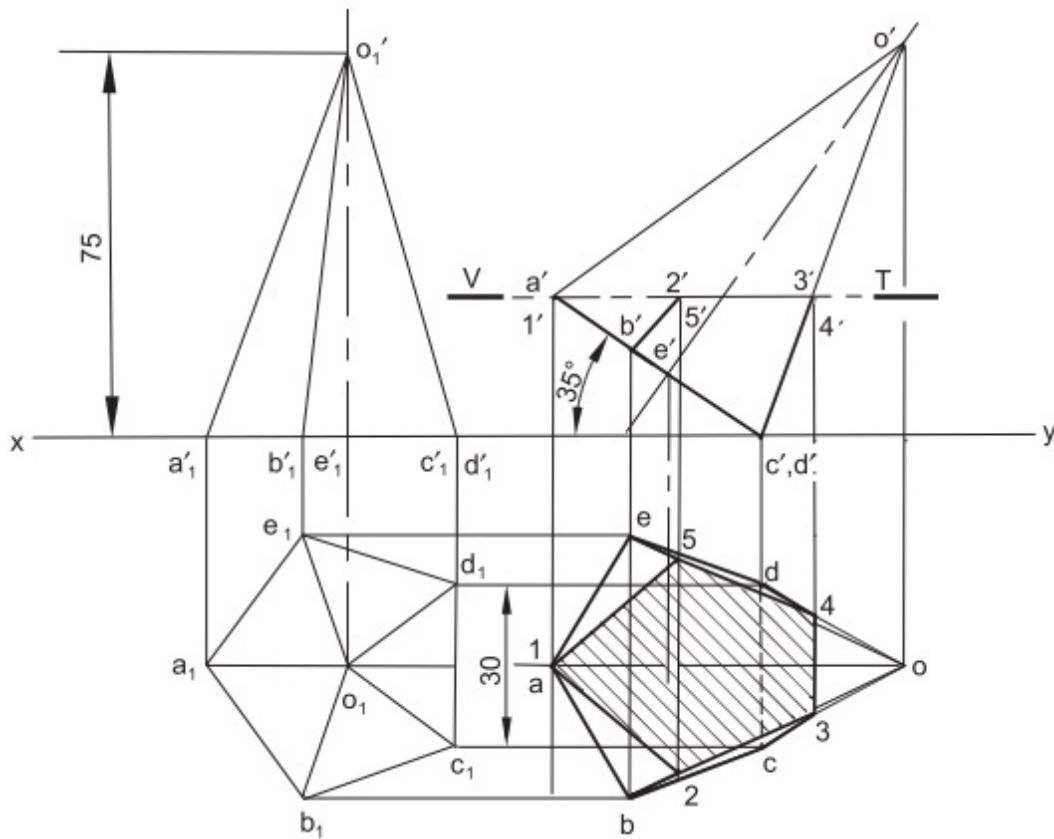
1. Draw the projections of the tetrahedron.  
To locate the section plane:
2. Locate the points 2 and 3 on oc and ob such that, the line 2-3 is parallel to cb and of length 45.
3. Locate 3' (2') in the front view.
4. With centre 3' (2') and radius 35, draw an arc intersecting o'a' at 1'.
5. Draw the V.T of section plane, passing through 1' and 3' (2'). The angle  $\theta$  is the inclination of the section plane with H.P.
6. Consider a reference plane  $x_1y_1$ , parallel to the V.T of the section plane and obtain the true shape of the section, by projection.

**Problem 52** A pentagonal pyramid, with side of base 30 and altitude 75, rests on one of its edges of the base on H.P, with the base making an angle of  $35^\circ$  with H.P. The pyramid is cut by a section plane, parallel to H.P and passing through that corner of the base, farthest from H.P. Draw the sectional top view of the truncated pyramid.

### **Construction: (Fig.12.54)**

1. Draw the projections of the pyramid, satisfying the given conditions.
2. Draw the V.T of the section plane, parallel to xy and passing through the corner of the base, farthest from xy.
3. Locate the intersection points 1', 2', etc., between the V.T and edges of the pyramid.
4. Project and locate these points, on the corresponding edges in the top view.
5. Join the points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.

Here, the sectioned portion in the top view also represents the true shape of the section.

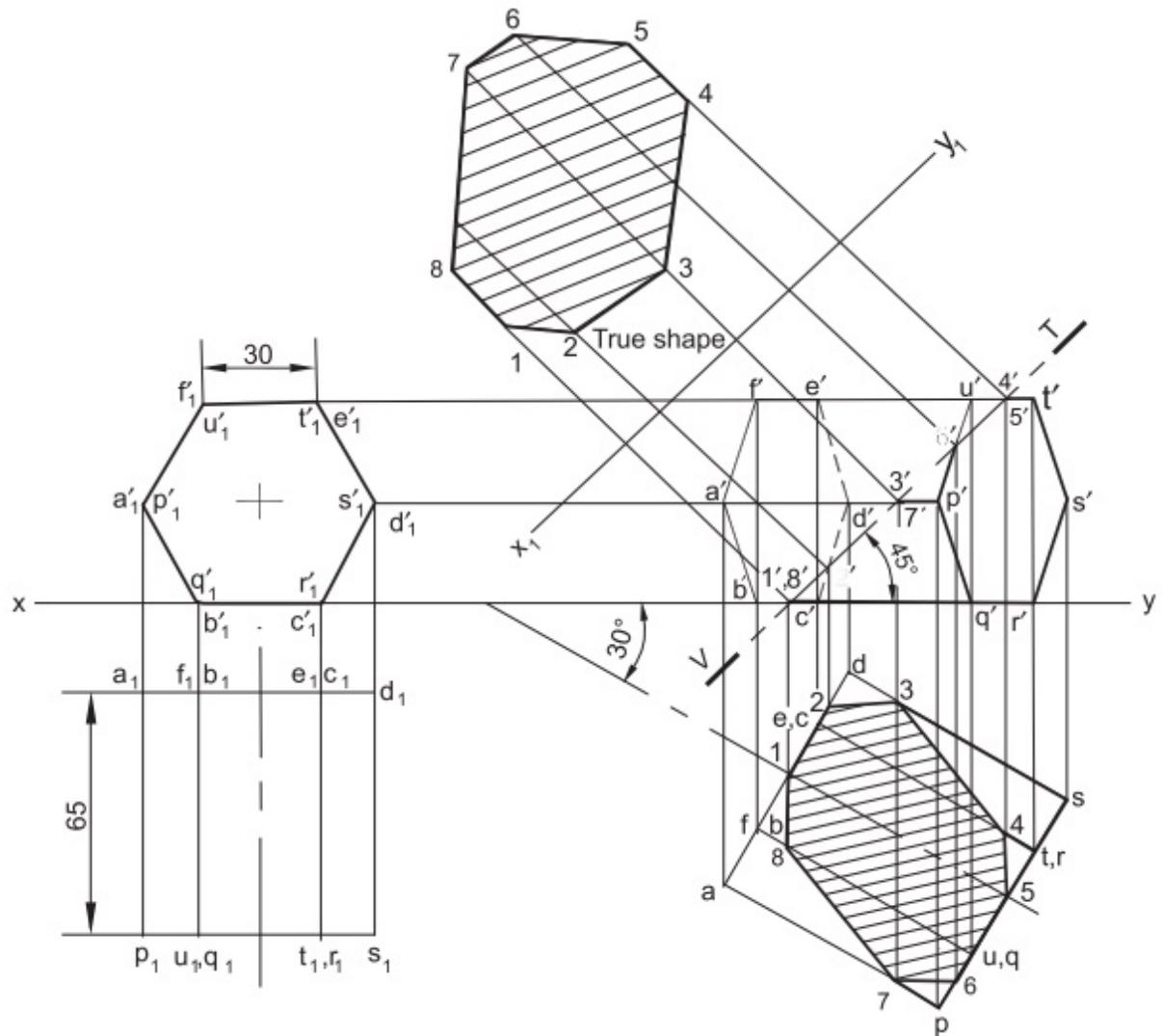


### **Fig.12.54**

**Problem 53** A hexagonal prism of side of base 30 and axis 65 long, is resting on one of its rectangular faces on H.P with the axis making an angle of  $30^\circ$  to V.P. It is cut by a section plane, perpendicular to V.P inclined at  $45^\circ$  to H.P and passing through the mid-point of the axis. Draw the sectional top view and true shape of the section.

#### **Construction (Fig.12.55)**

1. Draw the projections of the prism, assuming that the axis of it is perpendicular to V.P and lies on H.P on one of its faces.
2. Redraw the top view such that, the axis makes  $30^\circ$  to xy.
3. Obtain the final front view, by projection.
4. Draw the V.T of the cutting plane, passing through the mid-point of the axis and making  $45^\circ$  with xy.
5. Locate the points of intersection  $1'$ ,  $2'$ , — — —  $6'$  between the V.T and the base edges and lateral edges of the prism.
6. Project and locate the corresponding points  $1$ ,  $2$ , — — —  $6$  in the top view.
7. Join the points in the order and obtain the sectional top view.
8. Obtain the true shape of the section on an AIP, parallel to the V.T.



**Fig.12.55**

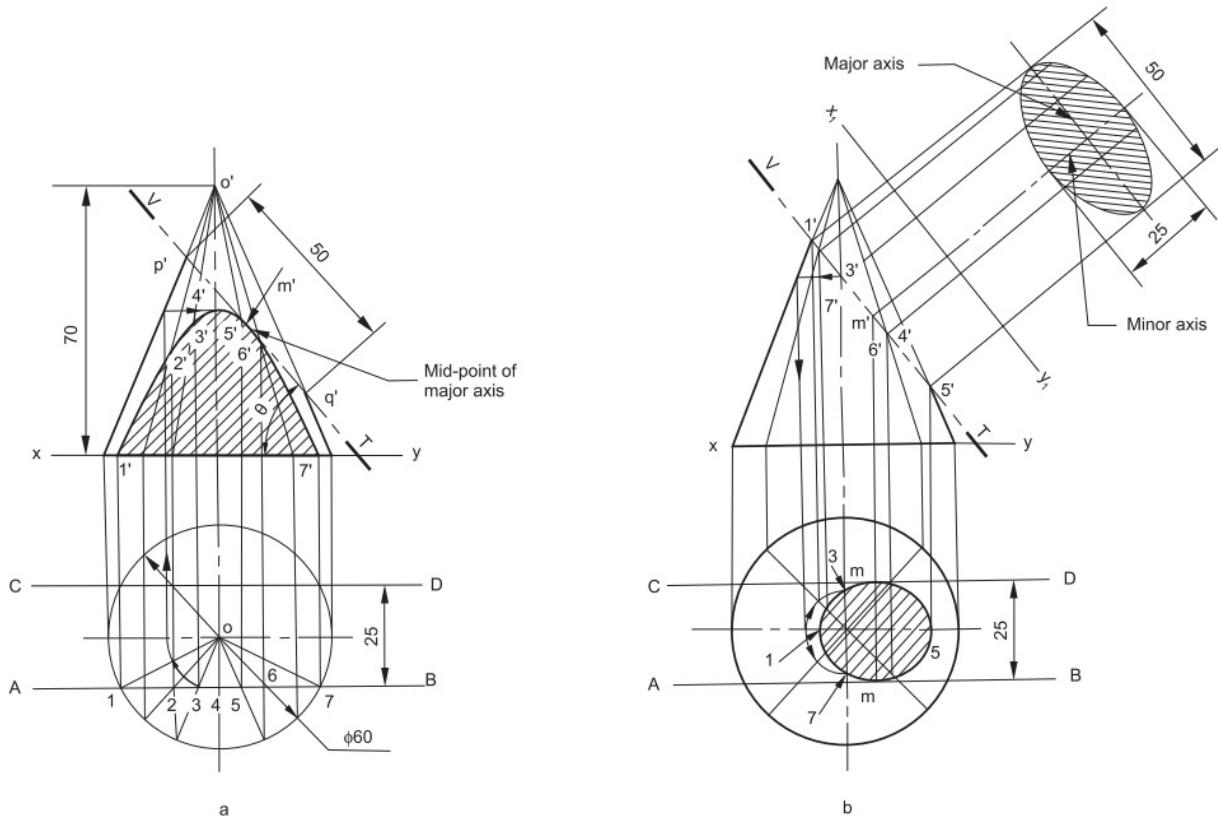
**Problem 54** A cone of base 60 diameter and axis 70 long, is resting on its base on H.P. It is cut by a section plane such that, the true shape of the section is an ellipse of major axis 50 and minor axis 25. Determine the inclination of the section plane with H.P and draw the projections of the solid.

**Construction (Fig.12.56a)**

**HINT** When an inclined section plane passes through the extreme generators of a cone, the conic section

produced is an ellipse. The length of the section plane, intercepted between the extreme generators is equal to the major axis of the ellipse. The minor axis will be equal to the distance between two section points in the top view that are projected from the mid-point of the section plane. Hence, the section plane should be selected in such a way that the required major and minor axes are obtained in the true shape of the section.

1. Draw the projections of the cone.
  2. In the top view, draw two lines AB and CD, which are 25 apart and equi-distant from the centre line and parallel to xy.



**Fig.12.56**

3. Draw the sectioned surface in the front view, corresponding to the section planes AB and CD (ref. Construction: [Fig.12.36](#)). Only one hyperbola is seen in the front view.
4. By trial, locate the mid-point  $m'$  (of the major axis) on the hyperbola, such that when a tangent is drawn at  $m'$  to the hyperbola, the intercept of the tangent between the extreme generators of the cone will be of 50 length.

This tangent line represents the required section plane and its inclination with  $xy$  is the inclination  $\theta$  with H.P.

5. Follow Construction: [Fig.12.14a](#) and obtain the true shape of the section ([Fig.12.56b](#)).

## EXERCISES

### Cubes

12.1 A cube of 50 edge, is resting on H.P with a vertical face inclined at  $30^\circ$  to V.P. It is cut by a section plane parallel to V.P and 10 away from the axis. Draw the sectional front view.

12.2 A cube of 40 edge rests with an edge on H.P and parallel to V.P. One of the surfaces containing that edge is inclined at  $30^\circ$  to H.P. A section plane perpendicular to V.P and inclined at  $45^\circ$  to H.P bisects the axis of the cube. Draw the projections of the cube.

12.3 A cube of edge 50 is resting on V.P, with a horizontal face inclined at  $30^\circ$  to H.P. It is cut by a section plane, perpendicular to H.P, inclined at  $45^\circ$  to V.P

and passing through a point on the axis at 30 in front of V.P. Draw the sectional front view.

12.4 A cube of edge 50 is resting on H.P on one of its faces, with a vertical face inclined at  $30^\circ$  to V.P. It is cut by a section plane, perpendicular to V.P and inclined at  $45^\circ$  to H.P. The section plane intersects the axis at 45 from the base. Draw the projections.

## Prisms

12.5 A triangular prism of base 40 side and axis 75 long, is lying on H.P on one of its rectangular faces, with its axis inclined at  $30^\circ$  to V.P. It is cut by a horizontal section plane, at a distance of 20 above H.P. Draw its front and sectional top views.

12.6 A triangular prism of side of base 60 and 90 high, standing with its axis at  $60^\circ$  to V.P, is cut by a section plane such that, the true shape is an isosceles triangle of sides 60, 75 and 75. Draw the sectional top view and front view.

12.7 A triangular prism of side of base 60 and axis 80 long, is resting on H.P with a side of the base perpendicular to V.P. A section plane cuts the prism such that, the true shape is a trapezoid of parallel sides 25 and 60, with a distance between them 45. Find the inclination of the section plane with H.P.

12.8 A hollow square prism, with side of base 50 (outside), axis 75 and 10 thick, is resting on H.P on one of its faces, with the axis inclined at  $60^\circ$  with V.P. A section plane parallel to V.P cuts the solid, passing through a point on the axis at 25 from one end of the base. Draw the sectional front view.

12.9 A square prism, with edge of base 45 and axis 90 long, has its axis parallel to both H.P and V.P. The lateral surfaces are equally inclined to H.P. It is cut by a vertical section plane, inclined at  $60^\circ$  to V.P and passing through the axis at 65 from one end. Draw the projections and determine the true shape of the section.

12.10 A hexagonal prism of side of base 20 and altitude 60, has a coaxial hole of diameter 20 and rests on its base on H.P. It is cut by a section plane, passing through the mid-point of the axis and making  $40^\circ$  with it. Determine the true shape of the section.

12.11 A hexagonal prism, with side of base 25 and height 65, has a face on H.P and the axis is parallel to V.P. It is cut by a section plane, perpendicular to H.P, inclined at  $45^\circ$  to V.P and passing through a point on the axis, 20 from one of its ends. Draw the projections.

12.12 A hexagonal prism of side of base 30 and axis 65 long, is resting on one of its rectangular faces on H.P, with the axis making an angle of  $30^\circ$  to V.P. It is cut by a section plane, perpendicular to V.P, inclined at  $45^\circ$  to H.P and passing through the mid-point of the axis. Draw the sectional top view and determine the true shape of the section.

12.13 A hexagonal prism, with side of base 40 and height 80, is resting on one of its corners on H.P, with longer edge containing that corner inclined at  $60^\circ$  to H.P and a rectangular face parallel to V.P. A horizontal section plane cuts the prism into two equal halves. (i) Draw the front view and sectional top view and (ii) draw another top view on an

auxiliary inclined plane which makes an angle of  $45^\circ$  with H.P.

12.14 A hexagonal prism, with side of base 30 and axis 60 long, is resting on an edge of the base on H.P; its axis being inclined at  $60^\circ$  to H.P and parallel to V.P. A section plane inclined at  $45^\circ$  to V.P and perpendicular to H. P passes through a point on the axis at 20 from its top end. Draw the sectional front view, profile view showing the section and determine the true shape of the section.

12.15 A hexagonal prism, with side of base 35 and axis 80 long, is resting on one of its corners on V.P, with a longer edge containing that corner, inclined at  $60^\circ$  to V.P and a rectangular face parallel to H.P. A vertical section plane passing through the mid-point of the axis and parallel to the bases cuts the prism. Draw the top view and sectional front view of the cut prism.

## Pyramids

12.16 A tetrahedron of 60 side, is resting on H.P on one of its faces, with an edge of it perpendicular to V. P. It is cut by a section plane, perpendicular to V.P and inclined to H. P by an angle of  $45^\circ$  and passing through a point on the axis at 30 from the base. Draw the sectional top view and determine the true shape of the section.

12.17 A tetrahedron of 65 long edges is lying on H.P on one of its faces, with an edge perpendicular to V.P. It is cut by a section plane, which is perpendicular to V.P such that, the true shape of the section is an isosceles triangle of base 40 and altitude 40. Find the inclination of the section plane with H.P and

draw the front view, sectional top view and determine the true shape of the section.

12.18A triangular pyramid, with side of base 60 and axis 80 long, stands with its base on H.P such that, an edge of the base is perpendicular to V.P. It is cut by a section plane perpendicular to V.P such that, the true shape of the section is an isosceles triangle of 40 base and altitude 45. Find the inclination of the section plane with H.P. Draw the projections and determine the true shape of the section.

12.19A square pyramid of 40 side of base and 75 height, rests on one corner on H.P such that, the longer edge passing through that corner is perpendicular to H.P; the axis being parallel to V.P. It is cut by a horizontal section plane, passing through the mid-point of the axis. Draw the projections and determine the true shape of the section.

12.20A square pyramid of side of base 40 and axis 80 long, is resting on H.P on one of its slant edges such that, the sides of the base are equally inclined to H.P; its axis being parallel to V.P. It is cut by a section plane, parallel to the axis and 10 from it. Draw the projections of the retained solid.

12.21A square pyramid of base 30 and 50 high, rests on a square block of 20 high, with an edge of the base parallel to V.P. The corners of the base of the pyramid coincide with the mid-points of the sides of the block. A section plane perpendicular to V.P and inclined at  $60^\circ$  to H.P bisects the axis of the pyramid. Draw the projections and determine true shape of the section.

12.22 A square pyramid, with side of base 50 and axis 70 long, is resting on its base, with one edge perpendicular to V.P. It is cut by an inclined section plane such that, the true shape of the section is a trapezium, whose parallel sides measure 40 and 20. Draw the projections of the solid and determine the true shape of the section.

12.23 A pentagonal pyramid, with side of base 35 and altitude 80, rests with its base on H.P and with one edge of the base parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at  $45^\circ$  to H.P. The plane intersects the axis of the pyramid at a height of 60 from the base. Draw the projections of the pyramid.

12.24 A pentagonal pyramid, with side of base 25 and axis 60 long, rests on a corner of the base on H.P and with its axis parallel to both H.P and V.P. It is cut by a vertical section plane, inclined at  $30^\circ$  to V.P and passing through a point on the axis at 25 from apex. Draw the projections of the retained portion of the pyramid.

12.25 A pentagonal pyramid, with side of base 30 and altitude 75, rests on one of its edges of the base on H.P, with the base making an angle of  $35^\circ$  with H.P. The pyramid is cut by a section plane, parallel to H.P and passing through that corner of the base, farthest from H.P. Draw the sectional top view of the frustum of the pyramid.

12.26 A pentagonal pyramid, with base 30 side and axis 80 long, is resting on H.P on its base, with the nearest edge of the base parallel to V.P. It is cut by two section planes, perpendicular to V.P and on either side of the axis and passing through its mid-point.

One plane is parallel to H.P while the other is at  $45^\circ$ , receding towards the apex. Draw the projections and determine the true shape of the section.

12.27A pentagonal pyramid, with side of base 25 and axis 75 long, is lying on H.P on one of its triangular faces and with its axis parallel to V.P. A vertical section plane, inclined at  $30^\circ$  to V.P, passes through the centre of the base and cuts the pyramid; the apex being retained. Draw the projections and a front view on an auxiliary plane, parallel to the section plane.

12.28A hexagonal pyramid of side of base 30 and axis 75 long, stands vertically on H.P with two edges of the base perpendicular to V.P. The pyramid is cut by an inclined section plane at  $30^\circ$  to H.P, intersecting the axis at the middle. Draw the projections of the solid and determine the true shape of the section.

12.29A hexagonal pyramid of base 30 and axis 65 long, is resting on one of its triangular faces on H.P and with its axis parallel to V.P. A horizontal section plane bisects the axis of the solid. Draw the projections.

12.30A hexagonal pyramid, with side of base 30 and axis 65 long, is resting on its base on H.P, with two edges parallel to V.P. It is cut by a section plane, inclined at  $45^\circ$  to H.P and passing through a point on the axis at 25 above the base. Draw the three views of the solid.

12.31A hexagonal pyramid, with side of base 25 and axis 70 long, rests with an edge of the base on H.P. The axis of the pyramid is parallel to H.P and inclined at  $30^\circ$  to V.P. A vertical section plane inclined at  $45^\circ$  to V.P, passes through the centre of the base of the

pyramid. Draw the projections of the retained portion of the solid.

12.32A hexagonal pyramid of base 30 side and axis 70 long, is resting on H.P with two edges parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at  $55^\circ$  to H.P and passing through the axis at 30 above the base. Draw the projections and determine the true shape of the section.

12.33A hexagonal pyramid, with edge of base 40 and axis 85 long, is lying on H.P on one of its triangular faces, with the axis parallel to V.P. A vertical section plane, the H.T of which makes an angle of  $45^\circ$  with the reference line, passes through the centre of the base and cuts the pyramid; the apex being retained. Draw the top view, sectional front view and true shape of the section.

## Cylinders

12.34A cylinder of diameter 50 and axis 70 long, is resting on H.P on one of its bases. It is cut by a section plane parallel to V.P and 10 away from the axis. Draw the sectional front view.

12.35A horizontal cylinder with diameter 30 and axis 60, rests centrally on the top of a frustum of a cone having base diameter 45, diameter at top 25 and 45 height. The two solids are cut by a section plane, which is parallel to V.P and situated at a distance of 10 from the axis of the cone and making  $45^\circ$  with the axis of the cylinder. Draw the projections of the cut solids.

12.36A cylinder of base 50 diameter and 60 long, has its axis parallel to both H.P and V.P. It is cut by a vertical section plane, inclined at  $30^\circ$  to V.P such

that, the axis is cut at a point 25 from one end. Draw the projections and determine the true shape of the section.

12.37A cylinder of base 80 diameter and 100 long, lies on H.P with its axis parallel to both H.P and V.P. It is cut by a vertical plane, which intersects the axis of the cylinder at an angle of  $30^\circ$  at its mid-point. Draw the sectional front view and determine the true shape of the section.

12.38A cylinder of base 60 diameter and 70 long, is resting on its base on H.P. It is cut by two section planes, on either side of the axis and meeting at its top point. The V.T's of the cutting planes make  $30^\circ$  to the axis. Draw the projections and also auxiliary views showing the true shapes of the sections.

12.39Draw the projections of a truncated cylinder such that, the section produced is an ellipse with 50 and 80 as its minor and major axes respectively. The smallest generator on the cylinder is 20 long.

12.40A right circular cylinder of base 35 diameter and 55 long, rests on its circular rim such that, the axis makes an angle of  $45^\circ$  with H.P and parallel to V.P. It is cut by a vertical section plane, parallel to V.P and at a distance of 10 from the axis. Draw the sectional front view.

12.41A cylinder of diameter and length, each 60, has a hexagonal hole through it; the side of the hexagon being 20. The axis of the hole coincides with the axis of the cylinder, which is horizontal. One of the sides of the hexagonal hole is vertical. The solid is cut by a vertical plane whose H.T coincides with one diagonal

of the square, which is the top view of the cylinder. Show the true shape of the section.

12.42A cylinder of base 50 diameter and axis 75 long, has a concentric square hole of 30 side. The cylinder is resting on H.P with the vertical faces of the hole equally inclined to V.P. It is cut by two section planes on either side of the axis, making  $30^\circ$  and  $45^\circ$  to the axis. The two planes meet at the top end of the axis. Draw the projections and determine the true shape of the sections produced.

## **Cones**

12.43A cone of base diameter 90 and height 110, rests on one of its generators on H.P, with the axis parallel to V.P. It is cut by a horizontal section plane, passing through the mid-point of the axis. Draw the true shape of the section.

12.44A cone of base 40 diameter and axis 60 long, is lying on one of its generators on H.P, with the axis parallel to V.P. A horizontal section plane bisects the axis of the solid. Draw the projections.

12.45A cone of base 90 diameter and altitude 110, rests with its base on H.P. Draw its front view, sectional top view, sectional side view and the true shape of the section, when (a) it is cut by a section plane, perpendicular to V.P, inclined at  $45^\circ$  to H.P and cutting the axis at a point 50 from the apex and (b) it is cut by a section plane, parallel to and 20 away from one of its generators.

12.46A cone of base 60 diameter and 75 height, stands vertically on H.P. A section plane, perpendicular to V.P and inclined to H.P, cuts the cone in such a way that the true shape of the section is an ellipse of

major axis 45 long. Find the inclination of the section plane with H.P and draw the front view, sectional top view and determine the true shape of the section.

12.47A cone of base 60 diameter and 75 height, is resting on its base on H.P. The V.T of the section plane is parallel to the extreme generator and 10 away from it. Draw the projections and determine the true shape of the section.

12.48A right circular cone of base diameter 50 and altitude 50, stands with its base on H.P. A vertical section plane, inclined at  $45^\circ$  to V.P, cuts the cone at 6 from the axis. Determine the apparent and true shapes of the sections.

12.49A cone of base 75 diameter and axis 75 long, has its axis parallel to H.P and inclined at  $45^\circ$  to V.P. A vertical section plane cuts the cone through the midpoint of the axis. Draw the sectional front view, top view and an auxiliary front view, on a plane parallel to the axis.

12.50A cone of base 65 diameter and axis 75 long, rests on H.P on a point on the circumference of the base. The axis of the cone makes  $60^\circ$  with H.P. It is cut by a section plane, inclined at  $30^\circ$  to V.P and passing through the mid-point of the axis. Draw the sectional front view and determine the true shape of the section.

12.51A frustum of a cone, with top end of 50 diameter and base 80 diameter and 75 height, has a coaxial circular hole of 35 diameter. It is cut by a section plane such that, the V.T of the section plane is inclined at  $45^\circ$  and passes through the midpoint of

the axis. Draw the projections and determine the true shape of the section.

12.52A frustum of a cone with 75 diameter at the base and 50 at the top and axis 70 long, has an axial hole of 25 diameter. It is resting on one of its generators on H.P with its axis parallel to V.P. The solid is cut by a section plane, inclined at  $30^\circ$  to V.P, perpendicular to H.P and passing through a point on the axis at 25 from the base. Draw its sectional front view and determine the true shape of the section.

## REVIEW QUESTIONS

- 12.1 What is a sectional view?
- 12.2 What is the purpose of sectioning a solid?
- 12.3 List out the possible positions of section planes.
- 12.4 Differentiate between a section and a sectional view.
- 12.5 What is meant by frustum of a solid?
- 12.6 What is a truncated solid?
- 12.7 How is the true shape of a section obtained?
- 12.8 Mention the position of the section plane, to produce the following true shapes of the sections, when it cuts the cone: (a) Circle, (b) ellipse, (c) parabola, (d) hyperbola and (e) triangle.

## OBJECTIVE QUESTIONS

- 12.1 In drawing a sectional view of a solid, the part of the object between the section plane and \_\_\_\_\_ is

assumed to be removed.

12.2 Section planes are represented by their traces / projections.

12.3 The true shape of the section is revealed in the top view, when the section plane, cutting a solid is parallel to \_\_\_\_\_.

12.4 When a section plane cutting a solid is parallel to V.P, the true shape of the section is revealed in the \_\_\_\_\_ view.

12.5 When a sphere is cut by a section plane, the true shape of the section is (a) ellipse, (b) circle and (c) parabola.

( )

12.6 The true shape of the section is an \_\_\_\_\_, when a cylinder is cut by a section plane, inclined to the axis.

12.7 The true shape of the section produced is a \_\_\_\_\_, when a cone is cut by a plane, parallel to the axis and passing through the apex; and a \_\_\_\_\_ when it is not passing through the apex.

12.8 The intersection between a polyhedron and a section plane, consists of (a) straight lines, (b) curved lines and (c) both straight and curved lines.

( )

12.9 The intersection between a cone and a section plane passing through its base and away from the apex, consists of \_\_\_\_\_.

12.10 The intersection between a section plane and the lateral surface of any \_\_\_\_\_ is a smooth curve.

# **ANSWERS**

- 12.1 observer
- 12.2 traces
- 12.3 H.P
- 12.4 front
- 12.5 b
- 12.6 ellipse
- 12.7 triangle, hyperbola
- 12.8 a
- 12.9 straight and curved portions
- 12.10 solid of revolution



## *Helical Surfaces*



### 13.1 INTRODUCTION

Helix is a curve, generated by a point moving on the surface of a cylinder or cone in a circular direction. The point moves at a constant angular velocity and with a simultaneous uniform rate of advance in axial direction; the ratio of the two movements being constant. The amount of the axial advance for one revolution is called the pitch or lead of the helix.

Threaded elements containing helical surfaces may be classified under two groups, depending upon their use.

1. Fasteners, for holding various parts in a structure, viz., bolts, studs, cap screws, machine screws and set-screws.
2. Elements, which transform the rotary motion into translatory motion, viz., screw conveyor, lifting screw, wheel puller, lead screw, cylindrical cam, propeller blade, helical chute, etc.

### 13.2 SINGLE HELIX

Single helix is a curve, generated by a single moving point on the surface of a cylinder or cone in a circumferential direction.

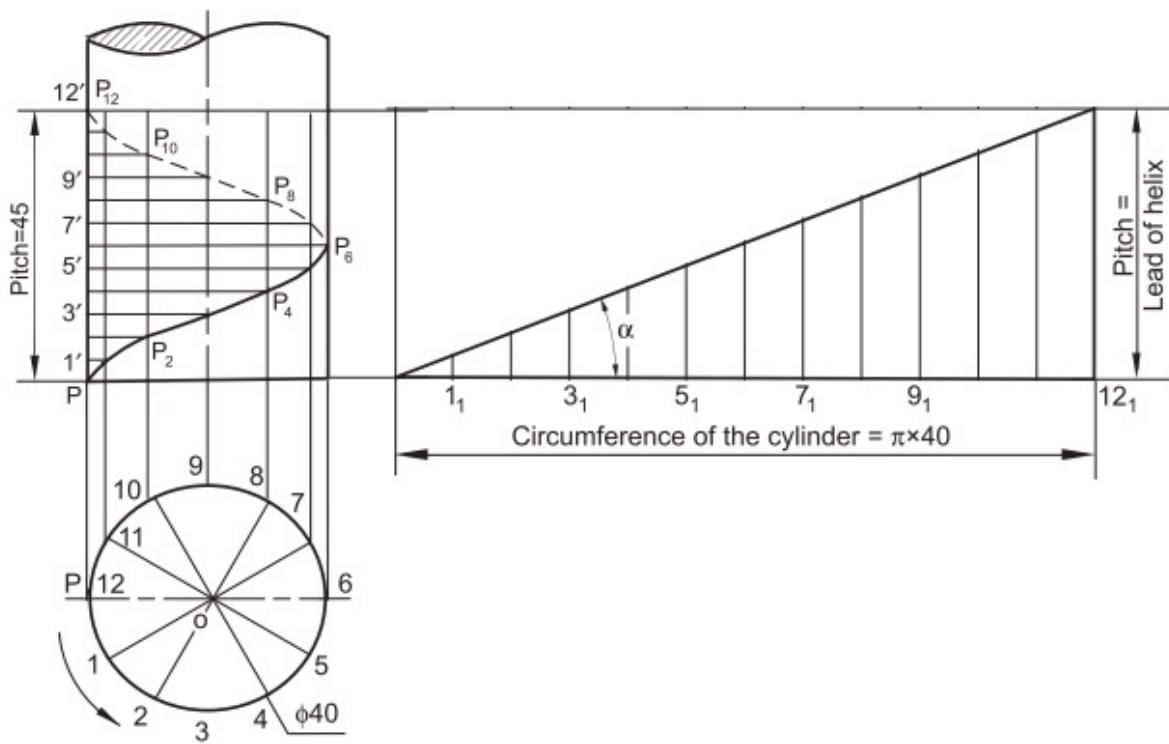
**Problem 1** *Draw a helix of pitch equal to 45, upon a cylinder of 40 diameter and develop the surface of the cylinder along with the helix. Assume the starting point P to be on the left extreme horizontal centre line in the top view.*

**HINT** Assume the generating point P to move upwards and in anti-clockwise direction.

### **Construction ([Fig. 13.1](#))**

1. Draw the projections of the cylinder.
2. Divide the base and pitch of the helix into the same number of equal parts, say 12.
3. Draw the generators in the front view, corresponding to the division points in the top view of the cylinder.
4. Mark  $P_1$ , the position of the point P at the intersection point between generator 1 and the first division point of the pitch (as the point moves by  $1/12^{\text{th}}$  of revolution from P to 1 around the cylinder, it advances along the axis of the cylinder by  $1/12^{\text{th}}$  of the pitch and occupies the position  $P_1$ ).
5. Locate the other points  $P_2, P_3$ , etc., in a similar manner.

A smooth curve through the points, P,  $P_1$ ,  $P_2$ , etc., is the required helix.



**Fig.13.1 Right hand helix on a cylinder**



- (i) The helix can be a left hand or right hand one. The helix obtained in Fig. 13.1 is a right hand one. If nothing is specified about the type of the helix, it is the usual practice to consider it as a right hand one.
- (ii) The helix corresponding to one revolution of the moving point is known as one convolution of the helix.
- (iii) The portion of the curve joining  $P_6$ ,  $P_7$ ,  $\dots$ ,  $P_{12}$  is invisible as it lies on the rear side of the cylinder.
- (iv) The distance, the generating point advances along the axis, for one convolution of the helix, is called the lead. For single helix, the lead and pitch are equal.

Figure 13.1 also shows the development of the cylinder along with the helix. The angle  $\alpha$  is known as the helix angle. The relation between the helix angle and the lead of the helix is given by,

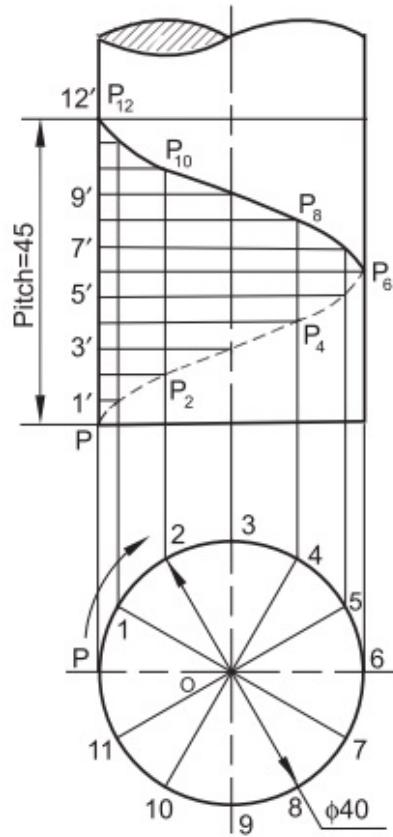
$$\tan\alpha = \frac{\text{lead of the helix}}{\text{circumference of the cylinder}}$$

Figure 13.2 shows a left hand helix on a cylinder of 40 diameter with 45 pitch. Here, the generating point P moves upwards and in clock-wise direction.

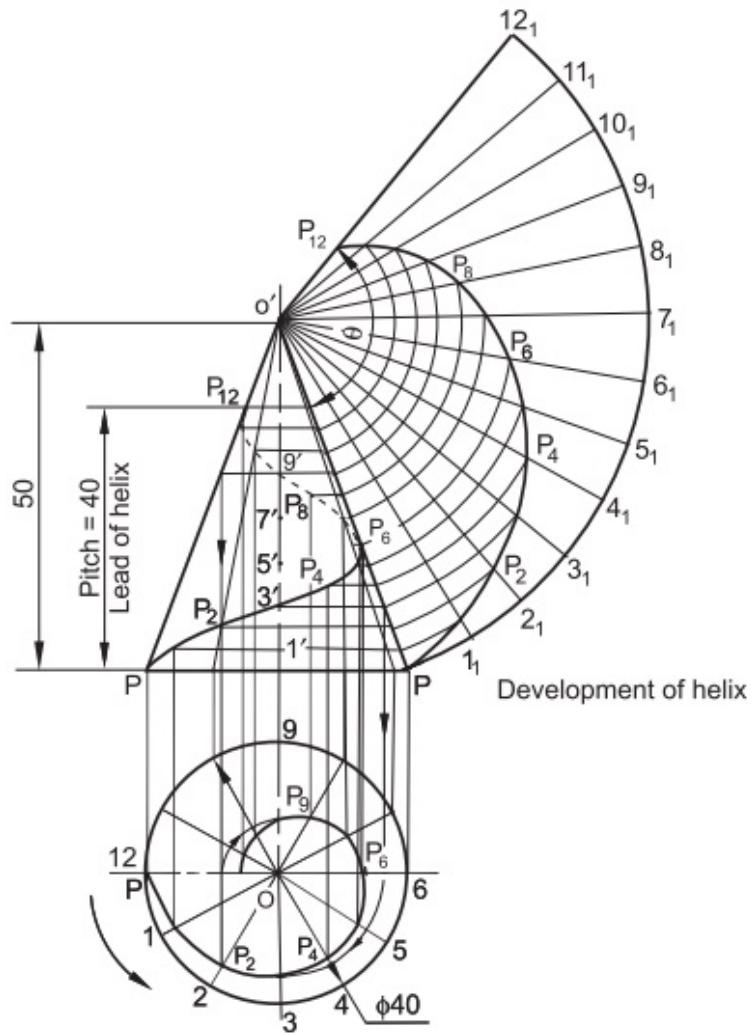
**Problem 2** *Draw a helix of one convolution upon a cone, with diameter of base 40, axis 50 long and pitch 40. Assume the starting point P to be on the left extreme horizontal centre line in the top view. Also, develop the surface of the cone and show the helix on it.*

**HINT** The pitch of the conical helix is measured parallel to the axis of the cone.

### **Construction (Fig.13.3)**



**Fig.13.2 Left hand helix on a cylinder**



**Fig.13.3 Right hand helix on a cone**

1. Draw the projections of the cone.
2. Divide the pitch (=lead) along the axis of the cone and the base of the cone into the same number of equal parts, say 12.
3. Draw the generators in the front view, corresponding to the division points in the top view.
4. Locate the points of intersection  $P_1, P_2, \dots$ , between the division lines of the pitch and generators.

5. Join the points, P, P<sub>1</sub>, P<sub>2</sub>, etc., by a smooth curve, forming the required conical helix. The part of the curve between the points P<sub>6</sub> to P<sub>12</sub> is invisible, as the points lie on the rear side of the cone.
6. Obtain the top view of the conical helix, by projection (It may be observed that the helix is an Archimedian spiral).

*To draw the development of the conical helix:*

The development of a cone is a sector of a circle of radius equal to the slant height (R) of the cone and the length of the arc is equal to the circumference of the base circle of the cone. Let r be the radius of the base circle and  $\theta$ , the angle subtended by the arc P-12<sub>1</sub>. Then,

$$\theta = 360 \times \frac{r}{R}$$

- (i) With centre o' and radius o'P, draw an arc P-12<sub>1</sub> to subtend an angle  $\theta$  at o'.
- (ii) Divide the arc P-12<sub>1</sub> into 12 equal parts and name the division points as 1<sub>1</sub>, 2<sub>1</sub>, etc.
- (iii) Join the above division points to o', forming the generators in the development.
- (iv) Transfer the points P, P<sub>1</sub>, P<sub>2</sub>, etc., from the front view, to the corresponding generators in the development.
- (v) Join these points by a smooth curve, forming the required development of the conical helix.

### 13.3 DOUBLE HELIX

If two points, whose initial positions are at the opposite ends of the base circle diameter, perform helical motion on the

surface of a cylinder in the same direction, then a double helix is obtained.

**Problem 3** *Draw a double helix on a cylinder of 40 diameter and lead equal to 60.*

**Construction ([Fig.13.4](#))**

1. Draw the projections of the cylinder.
2. Divide the lead of the helix and the base of the cylinder into the same number of equal parts, say 12.
3. Locate the generating points P and Q, diametrically opposite on the base of the cylinder.
4. Construct the two helices, following Construction: [Fig.13.1](#).



In the case of a double helix, the lead of the generating point is double the pitch.

## 13.4 THREAD FORMS

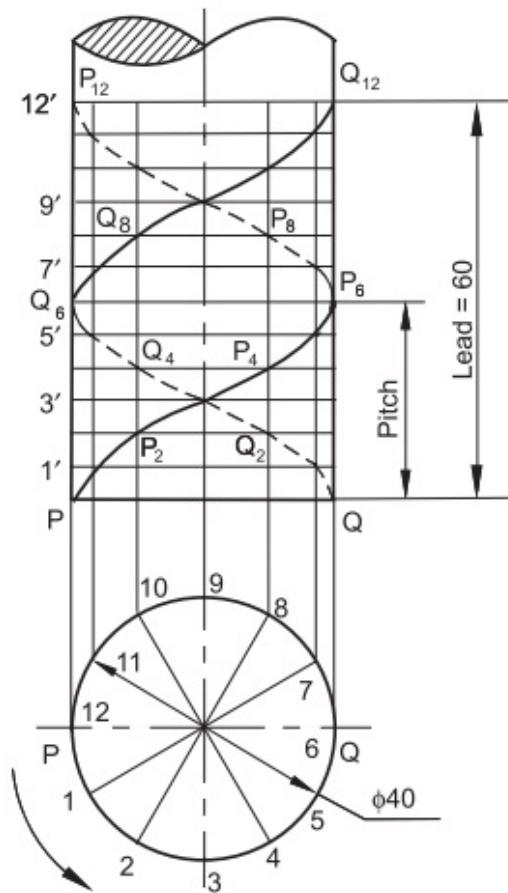
Screw threads are used on fasteners, on devices for making adjustments and for transmission of power and motion. For these purposes, a number of thread forms are in use. However, the treatment here is limited to only two forms, viz., sharp V-thread and square thread.

### 13.4.1 Sharp V-thread

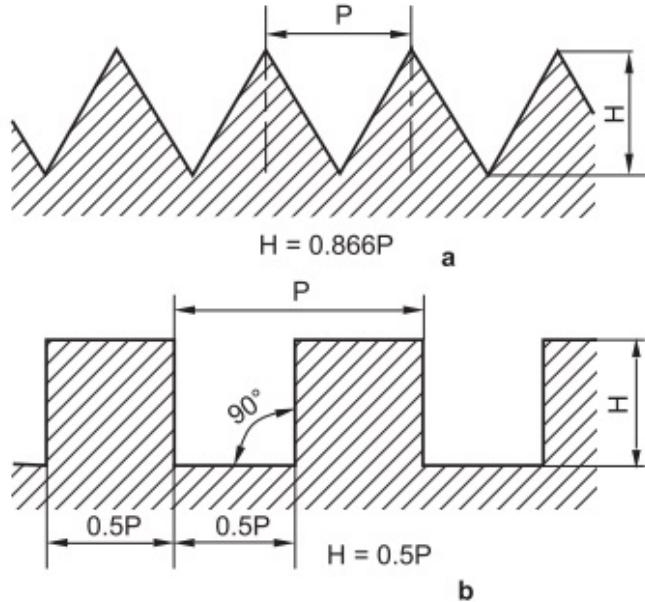
It has larger area of the flank and provides more friction ([Fig.13.5a](#)). It is used for brass pipe work and for effective positioning.

## 13.4.2 Square Thread

This is an ideal thread profile for power transmission (Fig.13.5b). In this, as the flank of the thread is at right angle to the axis; higher pressures can be transmitted, without damaging the nut.



**Fig.13.4 Double helix on a cylinder**



**Fig.13.5 V and square thread forms**

**Problem 4** Draw five turns of single start triangular or sharp V-thread, with 120 (nominal) diameter, 25 pitch and  $60^\circ$  as the angle of the thread.



- (i) The base of the profile triangle will be equal to the pitch of the screw.
- (ii) In screw threads, pitch is measured from a point on the thread to the corresponding point on the adjacent thread and parallel to the axis. The axial advance of the thread per revolution, known as lead is equal to the pitch in single start, is twice the pitch in double start thread and so on.

### **Construction ([Fig.13.6](#))**

1. Draw a semi-circle of 120 diameter, representing the outer diameter of the thread in the top view.
2. Determine the core or root diameter of the thread, from the front view, as follows:
  - (i) Mark A'B' equal to the given pitch 25.

(ii) Construct equilateral triangle A'B'C', known as profile triangle with A'B' as the base. The altitude of the triangle is the depth of the thread.

(iii) Determine the core diameter from,

$$\text{Core diameter} = \text{outer diameter} - 2 \times \text{depth of the thread}$$

3. Draw a semi-circle, representing the core diameter in the top view.
4. Divide the two semi-circles into equal parts, say 6 and name the division points. Draw the generators in the front view, through the above division points.
5. Draw the helices starting from A', B'; corresponding to the outer diameter, following Construction: [Fig.13.1](#).
6. Draw the helix, starting from C'; corresponding to the core diameter, following Construction: [Fig.13.1](#).

Following the rules of visibility, draw the visible parts of the thread, by thick lines.

**Problem 5** *Draw two and half turns of right hand single start square thread, with 80 outer (nominal) diameter and 30 pitch.*

**HINT** For a square thread of pitch P, the side of the square section of the thread is  $P/2$ , i.e., the depth and thickness of the thread is equal to  $P/2$ . Thus, the core (root) diameter of the thread is equal to  $D-P$ , where D is the outer diameter.

### ***Construction ([Fig.13.7](#))***

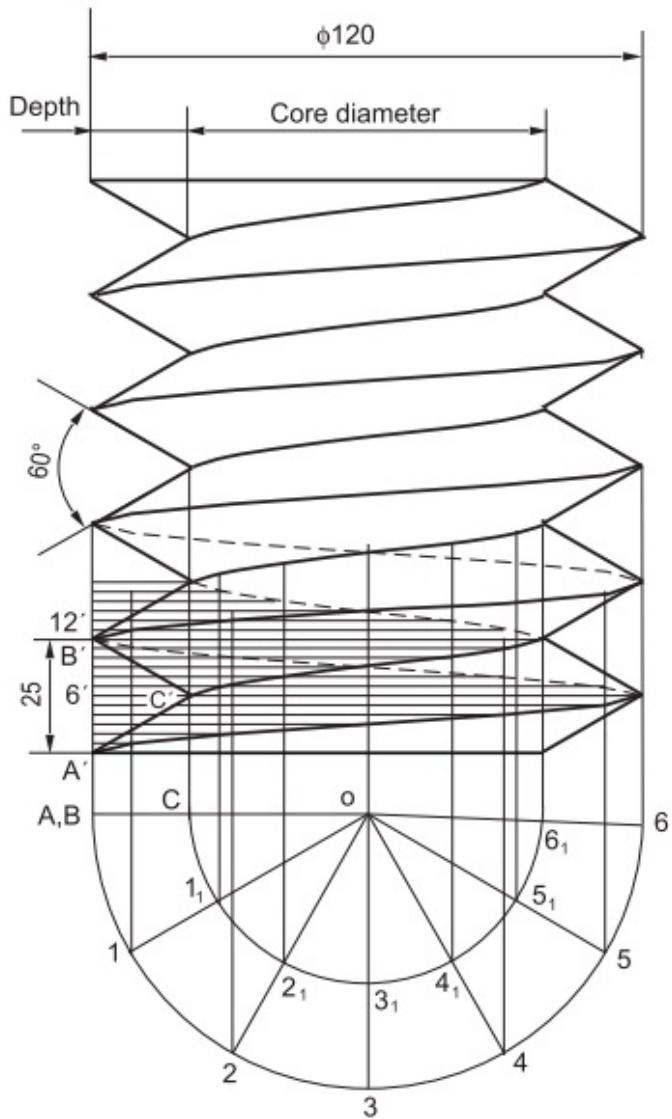
1. Draw two semi-circles, corresponding to the outer and core diameters of the thread in the top view.
2. Divide the semi-circles into equal parts, say 6 and name the division points.

3. Draw the generators in the front view, through the above division points.
4. Draw the square A'B'C'D' in the front view, say at the bottom left hand corner.
5. Draw the helices, starting from A', D'; corresponding to the outer diameter, following Construction: [Fig.13.1](#). Draw the helices, starting from B', C'; corresponding to the core diameter, following Construction: [Fig.13.1](#).

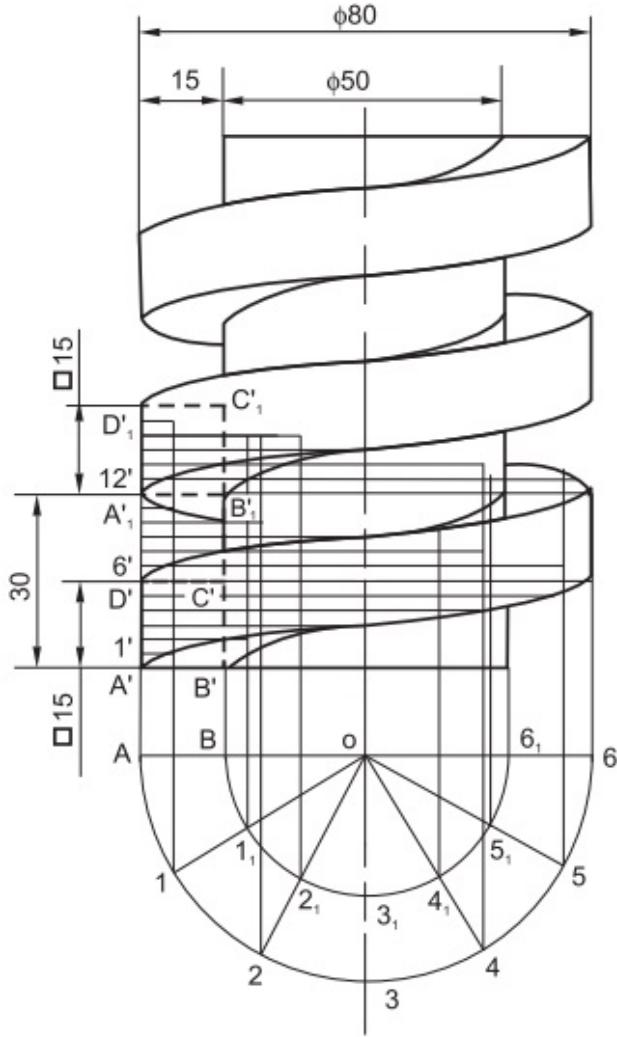
Following the rules of visibility, draw the visible parts of the helices, by thick lines.



The invisible parts of the helices are usually not shown, to avoid confusion.



**Fig.13.6 Single start V-thread**



**Fig.13.7 Single start square thread**

## 13.5 HELICAL SPRINGS

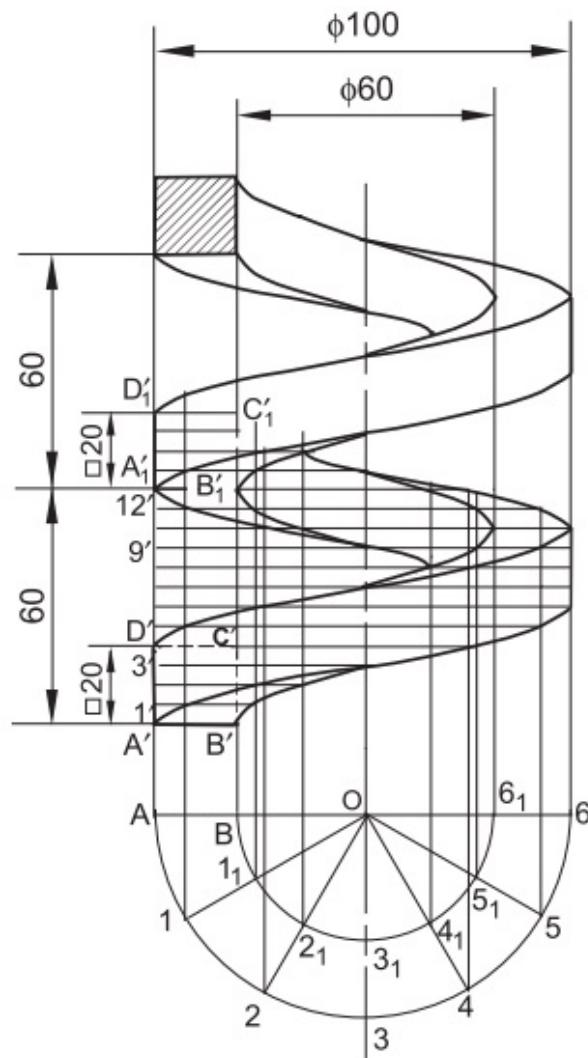
Helical springs have square, rectangular or circular cross-section. Square and rectangular helical springs have four corners. The outer two corners of the section may be assumed to move around a cylindrical surface with the diameter equal to that of the outer diameter of the spring. The inner two corners of the section are assumed to move on the surface of the cylinder having a diameter equal to the

inner diameter of the spring; the pitch of each corner being the same.

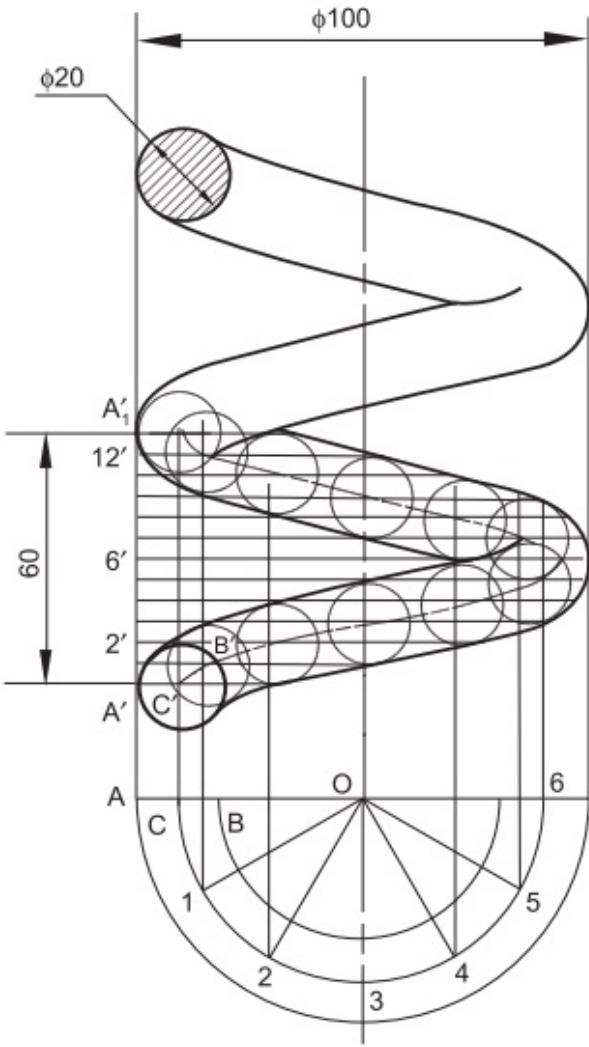
In the case of circular helical springs, the centre of the cross-section traces a helical curve.

**Problem 6** Draw two turns of a helical spring of square section of 20 side. The outer diameter of the spring is 100 and pitch 60.

**Construction (Fig.13.8)**



**Fig.13.8 Square section spring**



**Fig.13.9 Circular section spring**

1. Draw two semi-circles, corresponding to outer and inner diameters of the spring (Inner diameter of the spring =  $100 - 2 \times 20 = 60$ ).
2. Divide the semi-circles into a number of equal parts, say 6 and name the division points.
3. Draw the projectors through the above division points.
4. Draw the square A'B'C'D', representing the cross-section of the spring in the front view.

5. Trace the helices from A', D'; corresponding to the outer diameter of the spring.
6. Trace the helices from B', C'; corresponding to the inner diameter of the spring.
7. Following the rules of visibility, show only the visible portions of the spring.



The inner cylindrical portion should not be shown here, as there is no core material for the spring.

**Problem 7** Draw two turns of a helical spring of circular section of 20 diameter. The outer diameter of the spring is 100 and pitch 60.

**HINT** Inner diameter of the spring = outer diameter -  $2 \times$  diameter of the section = 60 mm

$$\text{Mean diameter of the spring} = \frac{\text{outer diameter} + \text{inner diameter}}{2} = 80 \text{ mm}$$

### **Construction ([Fig.13.9](#))**

1. Construct a helix, corresponding to the mean diameter of the spring (ref. Construction: [Fig.13.1](#)).
2. Choosing number of points as centres on the helix, draw circles of diameter 20.
3. Draw the outer profile of the spring, tangential to these circles.



Regarding the outer profile to be shown, attention should be paid at the turning points of the spring.

**Problem 8** Draw a conical spring, consisting of three turns (coils) of circular section of 20 diameter and pitch 50. The outer diameter of the larger end is 120 and the smaller end is 60.

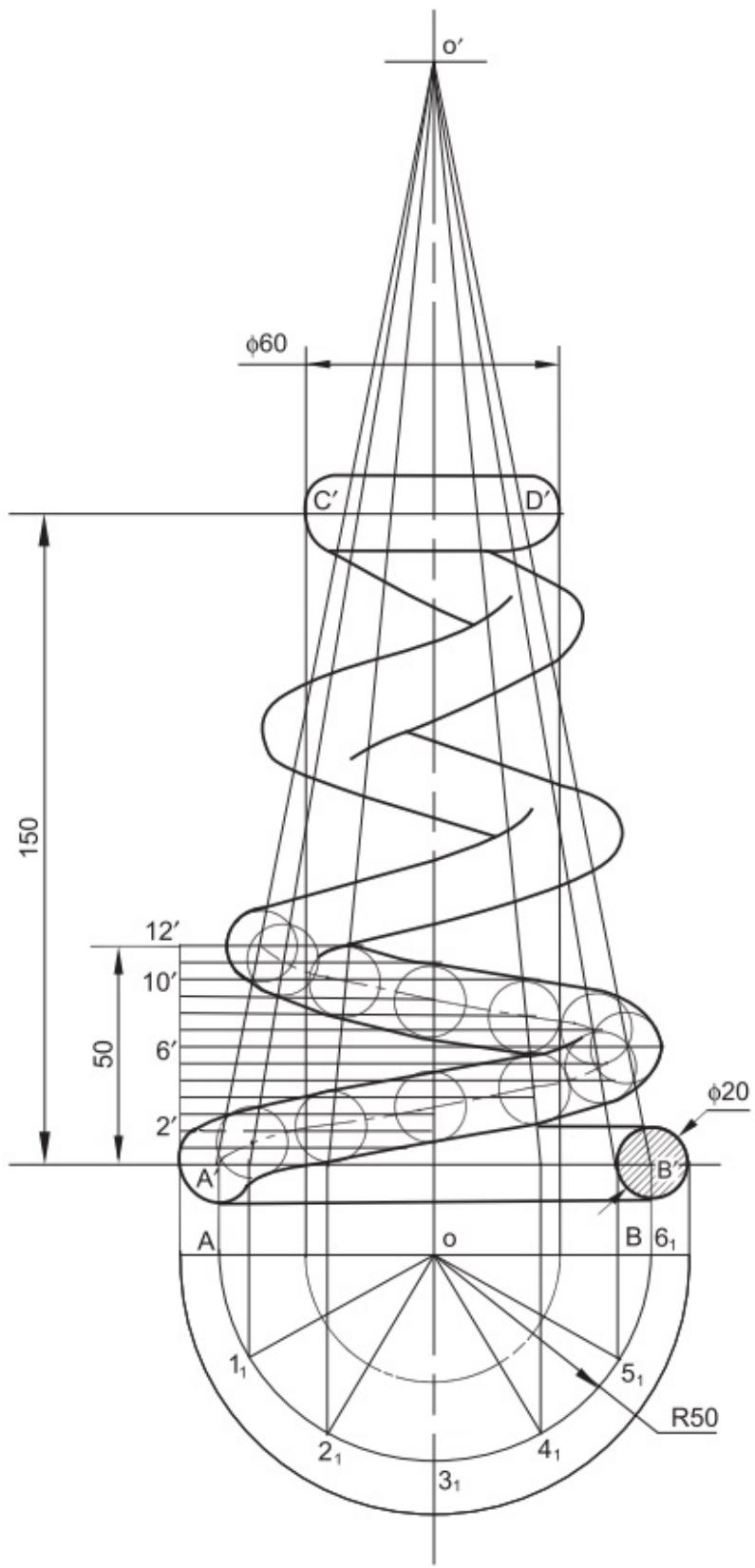
Mean diameter of the spring at the larger end = outer diameter at the larger end -  $2 \times$  radius of the circular section  
=  $120 - 2 \times 10 = 100$  mm

Mean diameter of the spring at the smaller end = outer diameter at the smaller end -  $2 \times$  radius of the circular section =  $60 - 2 \times 10 = 40$  mm

Mean height of the spring =  $3 \times$  pitch = 150 mm

### ***Construction (Fig.13.10)***

1. Draw a semi-circle of 100 diameter, representing the mean diameter of the spring at the larger end in the top view.
2. Divide the semi-circle into equal parts, say 6 and name the division points.



### **Fig.13.10 Conical spring**

3. Project the horizontal (mean) diameter AB and obtain the front view A'B'.
4. Through o, draw a projector representing the axis of the spring.
5. Draw a line C'D' (mean diameter) in the front view, at a height of 150 above A'B' and parallel to it and symmetrically placed with respect to the spring axis.
6. Join A', C' and B', D' and extend, intersecting at o', the apex of the imaginary cone.
7. Locate the generators in the front view, corresponding to the division points in the step 2.
8. Starting from A', trace the helix around the imaginary cone for three turns, ending at C'.
9. Choosing number of points as centres on the helix, draw circles of diameter 20.
10. Draw the outer profile of the spring, tangential to these circles.
11. Draw the horizontal spring seats at the bottom and top, as shown.

## **EXERCISES**

13. 1 Draw the helix of one and half convolution around a cylinder of 70 diameter and pitch 50. Develop the surface of the cylinder and determine the helix angle.
- 13.2 Draw a helix of pitch 50 around a cylinder of 75 diameter. Develop the surface of the cylinder and determine the helix angle. Assume the starting point to

be on the vertical centre line at the bottom in the top view.

- 13.3 Draw one convolution of right hand and left hand helices on a cylinder of 40 diameter and 75 lead.
- 13.4 Draw the projections of a helix having helix angle  $35^\circ$ , on a cylinder of 70 diameter.
- 13.5 Draw a helix of one convolution upon a cone of diameter of the base 70 and axis 100 long, having a pitch equal to 60. Take apex as the starting point for the curve.
- 13.6 Draw two helices of pitch 50 upon a cylinder of 70 diameter. Assume the starting points to be at the ends of the vertical diameter in the top view.
- 13.7 A point P starting from the base of a cone, reaches the apex while moving around the axis through two complete turns. Assuming the movement of P towards the apex (measured parallel to the axis) to be uniform with its movement around the axis, draw the projections and the development of the surface of the cone, showing the path of P in each. Diameter of the base of the cone is 75 and axis 100 long.
- 13.8 Project two complete turns of a V-thread, having outer diameter 120, pitch 30 and thread angle  $60^\circ$ .
- 13.9 Project two complete turns of a double start V-thread, having outer diameter 120, pitch 30 and thread angle  $60^\circ$ .
- 13.10 Project two complete turns of a square thread, having outer diameter 120 and pitch 40.
- 13.11 A screw conveyor of 20 base diameter and 100 outer diameter has a helical blade of 40x10, welded on its surface. The pitch is 60. Project two complete turns of the screw with the blade.

- 13.12 Draw two turns of a square threaded screw, with outside diameter 20 and pitch 2.5.
- 13.13 Draw two turns of a left hand double start square thread, having outer diameter 100 and lead 75.
- 13.14 On a cylindrical shaft of diameter 80, a helical square groove for  $1\frac{1}{2}$  convolutions is cut. The groove has a cross-section of  $15 \times 15$  with pitch 60. Draw the projections of the shaft.
- 13.15 A spiral spring is made of a wire of rectangular cross-section  $25 \times 20$ . Draw two complete turns of the spring. Take outer diameter as 100, inner diameter as 50 and pitch equal to 50.
- 13.16 Draw two complete turns of a left hand helical spring of outer diameter 110 and 40 pitch, made of 15 square section wire.
- 13.17 Project one complete turn of a helical spring of outer diameter 80 and pitch 60; the cross-section of the wire being a circle of 20 diameter.
- 13.18 Draw two complete turns of a right hand helical spring of 75 mean diameter, 50 pitch and made from 15 diameter wire.
- 13.19 Draw the projections of a conical spring of one convolution, having flat surfaces at both the ends. Outer diameter at the bottom is 80 and at the top 45. The wire diameter is 10 and pitch is 65.
- 13.20 A right circular vertical cylinder of 60 height and diameter 45 rotates uniformly. A plotter pen tip moves vertically at uniform speed on the surface of the cylinder from the bottom to the top, so that it moves through a distance of 60 while the cylinder completes one rotation. Draw the line marked on the cylinder in the front view and measure the true length of it.

## **REVIEW QUESTIONS**

- 13.1 What is a helix?
- 13.2 Classify the threaded elements, containing helical surfaces.
- 13.3 Differentiate between a helix and a helical surface.
- 13.4 What is meant by helix angle?
- 13.5 What is the relation between the helix angle and lead of the helix?
- 13.6 Sketch left hand and right hand helices on a cylinder.
- 13.7 What is meant by a convolution of a helix?
- 13.8 What is a double helix?
- 13.9 Define pitch of a thread.
- 13.10 Differentiate between the pitch of a single start and double start thread.
- 13.11 What is the relation between the pitch and lead of a double start thread?
- 13.12 What is the difference between a square thread and a spring with square cross-section?

## **OBJECTIVE QUESTIONS**

- 13.1 The ratio of the circumferential and axial motions of a point, tracing a helix is constant.  
(True / False)
- 13.2 A helix can be left hand or right hand.  
(True / False)

13.3 The pitch of a conical helix is measured parallel to the axis of the cone.

(True/ False)

13.4 The projection of a conical helix in the top view is \_\_\_\_\_.

13.5 Square threads are used in brass pipe work.

(True / False)

## ANSWERS

13.1 True

13.2 True

13.3 True

13.4 Archmedian spiral

13.5 False

# 14

## *Development of Surfaces*



### 14.1 INTRODUCTION

The complete surface of an object, laid-out on a plane is called the development of the surface or flat pattern of the object. The development of geometrical surfaces is important in the fabrication of not only small, simple shapes made of thin sheet metal, but also, sophisticated pieces of hardware such as space capsules. In actual practice, bend allowances have to be made in the layout of the pattern to ensure proper fabrication. These allowances depend upon the degree of bend, thickness of the metal and type of metal being used. However, these allowances are not considered in presenting the subject here, as it falls outside the scope of the book.

In making the development of a geometrical surface, the opening should be determined first. Every line used in making the development must represent the true length of that line on the actual surface.

Developments are made possible, with the application of basic graphic and geometric principles, in conjunction with

mathematics. Since, different shapes must be joined together in many instances, principles of intersections are closely related to developments. If a development problem is resolved into basic geometric elements, the solution will be simpler.

## 14.2 CLASSIFICATION OF OBJECTS

In general, objects are bounded by geometric surfaces. These may be classified as,

1. Solids bounded by plane surfaces - cube, prism, pyramid, etc.
2. Solids bounded by single curved surfaces - cylinder, cone, etc.
3. Solids bounded by double curved surfaces - sphere, paraboloid, etc.
4. Solids bounded by warped surfaces - ellipsoid, hyperboloid, etc.

The first two types of solids can be developed accurately, whereas the last two can only be developed approximately, by dividing them into a number of parts.

## 14.3 METHODS OF DEVELOPMENT

Solids bounded by plane surfaces and single curved surfaces can be developed by: (i) Parallel line development method, based on stretch-out line principle; used for prisms and cylinders, (ii) radial line development method, making use of true length of slant edge or generator; used for pyramids and cones, (iii) triangulation method normally

used for developing the transition pieces - connecting ducts, pipes, openings and similar objects with various sizes and shapes and (iv) approximate method, used to develop the objects with double curved or warped surfaces such as sphere, paraboloid, ellipsoid, hyperboloid, etc.

Only the lateral surfaces are generally developed and shown as presented here, omitting the bases or ends of solids.

### 14.3.1 Parallel Line Development

The surfaces of right prisms, cylinders and also oblique prisms and cylinders may be developed by this method.

**Problem 1** A square prism of side of base 40 and axis 80 long, is resting on its base on H.P such that, a rectangular face of it is parallel to V.P. Draw the development of the prism.

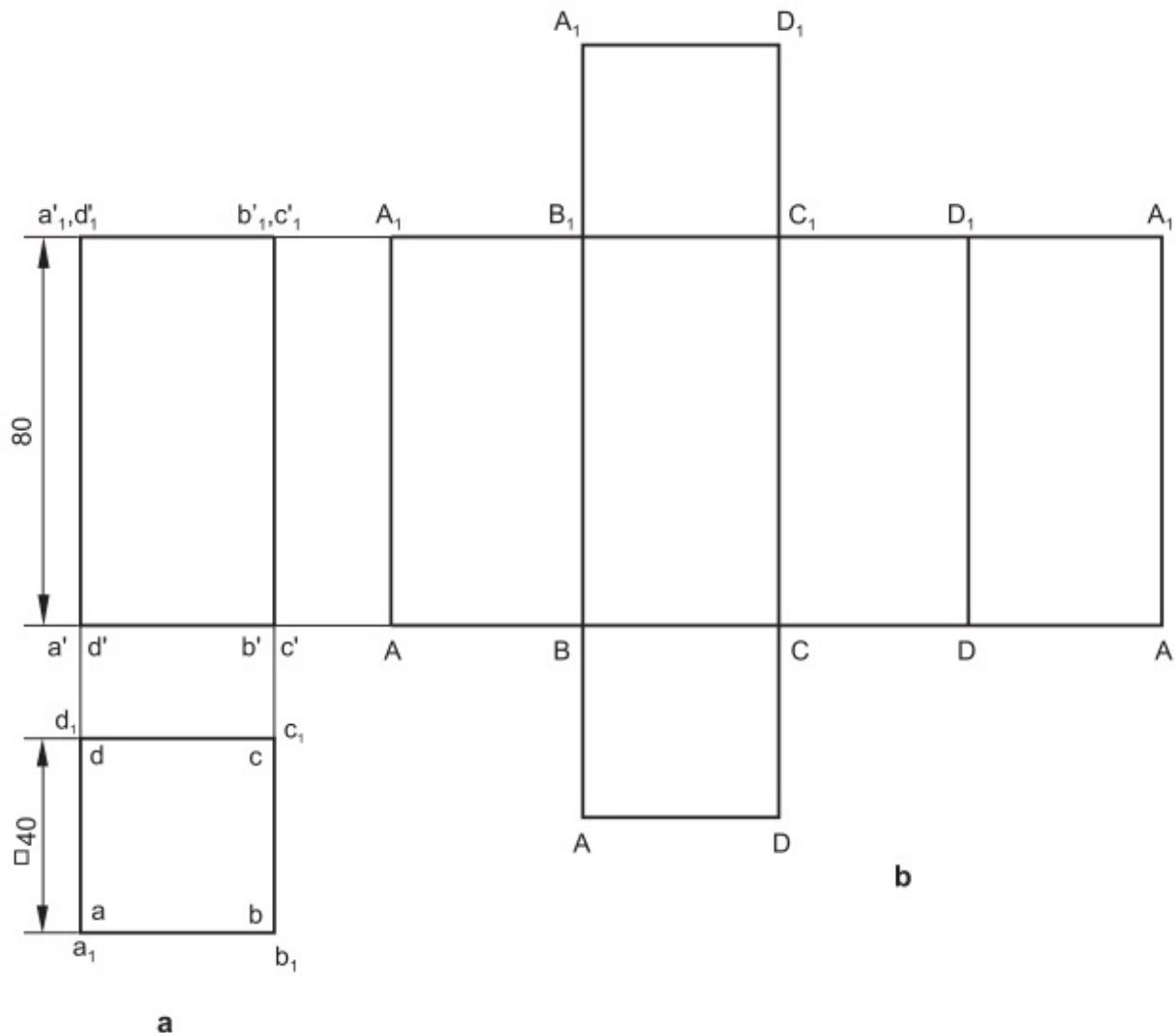
#### **Construction (Fig. 14.1)**

1. Draw the projections of the prism.
  2. Draw the stretch-out line AA and mark - off the sides of the base along this line in succession, i.e., AB, BC, CD and DA.
  3. Erect perpendiculars through these points and mark the edges  $AA_1$ ,  $BB_1$ , etc.
  4. Add the bases ABCD and  $A_1B_1C_1D_1$  suitably.
-  (i) Stretch-out line is drawn in-line with the base in the front view, to complete the development quickly.  
(ii) Generally, the lateral surfaces of the solids are

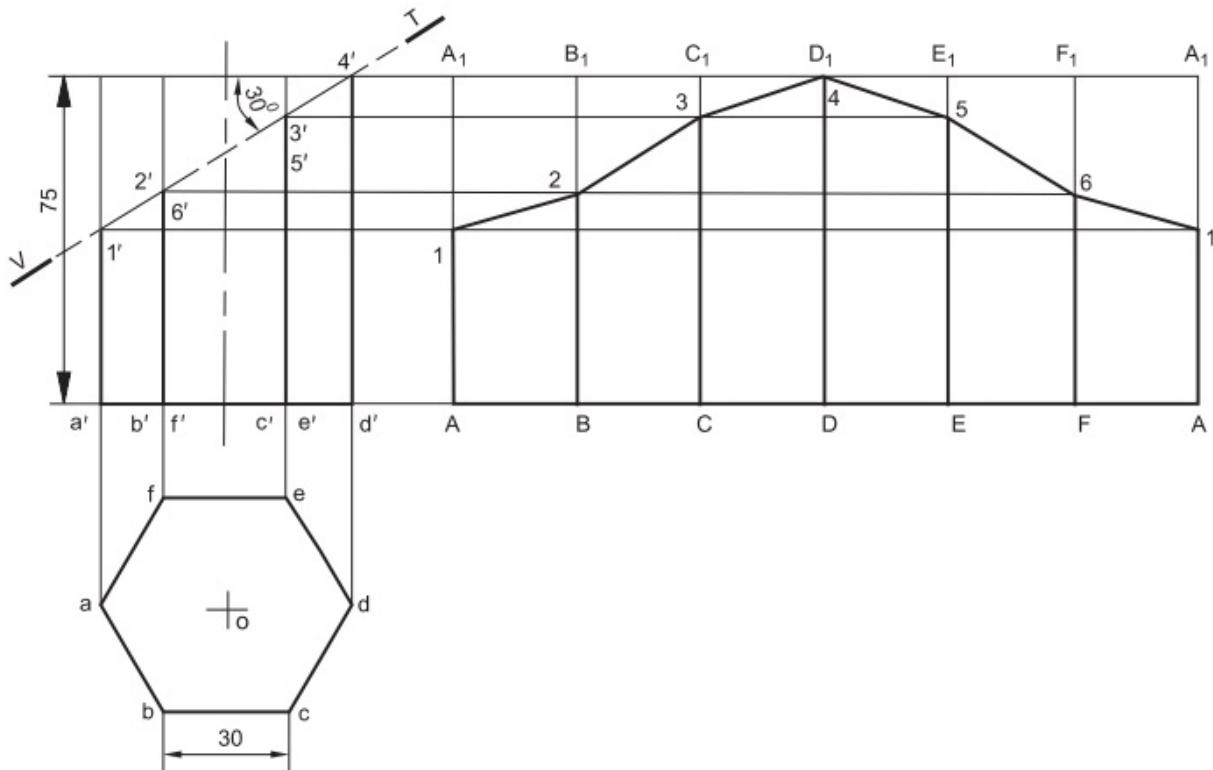
developed and the bases are omitted.

- (iii) All the lines on the development should represent the true lengths.

**Problem 2** A hexagonal prism of side of base 30 and axis 75 long, is resting on its base on H.P such that, a rectangular face is parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at  $30^\circ$  to H.P. The section plane is passing through the top end of an extreme lateral edge of the prism. Draw the development of the lateral surface of the cut prism.



**Fig 14.1**



**Fig 14.2**

### ***Construction (Fig.14.2)***

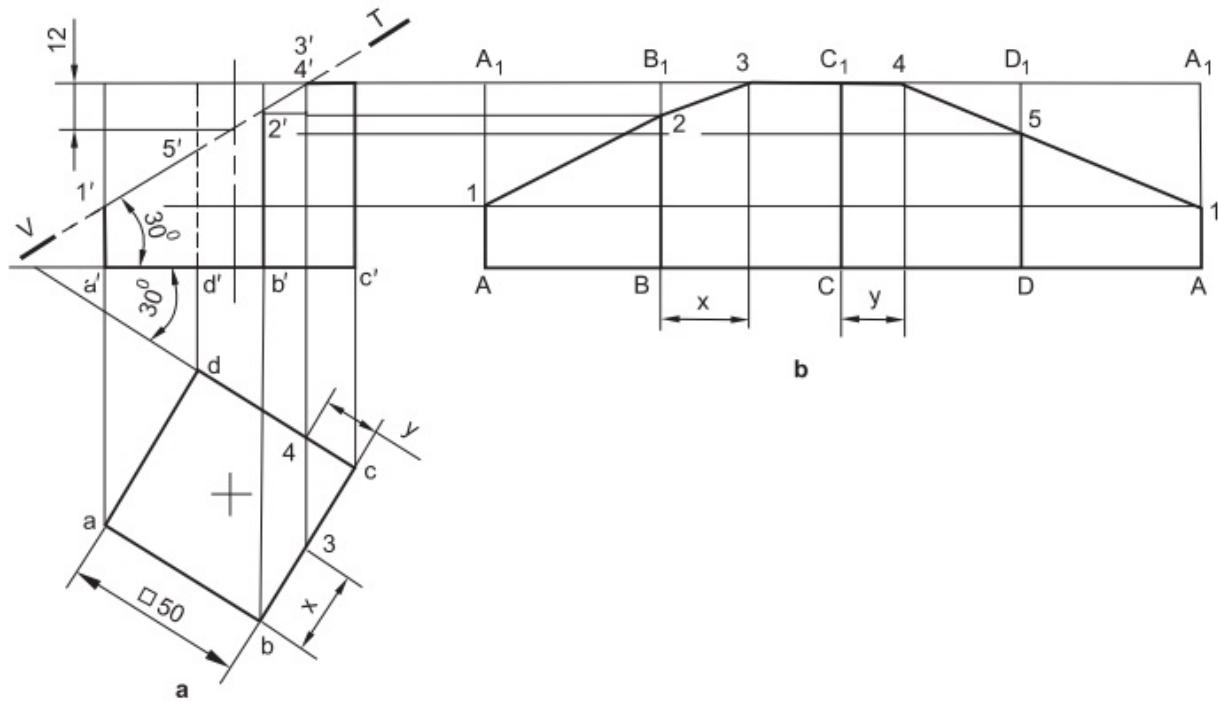
1. Draw the projections of the prism.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA<sub>1</sub> - A<sub>1</sub>A of the complete prism, following the stretchout line principle.
4. Locate the points of intersection 1', 2', etc., between the V.T and edges of the prism.
5. Draw horizontal lines through 1', 2', etc., and obtain 1, 2, etc., on the corresponding edges in the development.
6. Join the points 1, 2, etc., by straight lines and darken the sides, corresponding to the retained portion of the solid.



The surface of the solid is cut open (for the development) at the shortest edge/length.

**Problem 3** A cube of 50 edge, is resting on a face on H.P such that, a vertical face is inclined at  $30^\circ$  to V.P. It is cut by a section plane perpendicular to V.P and inclined to H.P at  $30^\circ$  and passing through a point at 12 from the top end of the axis. Develop the lateral surface of the lower portion of the cube.

**Construction (Fig.14.3)**



**Fig.14.3**

1. Draw the projections of the cube.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA<sub>1</sub> - A<sub>1</sub>A of the complete cube, following the stretchout line principle.

Repeat steps 4 to 6 of Construction: Fig.14.2 and  
4. obtain the development of the cut solid.



1. The points  $3'$  and  $4'$  are on the top surface of the cube. To locate the corresponding points in the development:
  - (i) Draw a projector through  $3'$  ( $4'$ ), meeting  $bc$  at  $3$  and  $cd$  at  $4$  in the top view.
  - (ii) Mark the points  $3$  and  $4$  in the development such that,  $B_13 = b3 = x$  and  $C_14 = c4 = y$ .
2. The sectioned portion in the top view is not cross-hatched as it is made use of only for locating certain points in the development.

**Problem 4** A cylinder of diameter of base 40 and axis 55 long, is resting on its base on H.P. It is cut by a section plane, perpendicular to V.P and inclined at  $45^\circ$  to H.P. The section plane is passing through the top end of an extreme generator of the cylinder. Draw the development of the lateral surface of the cut cylinder.

### **Construction (Fig.14.4)**

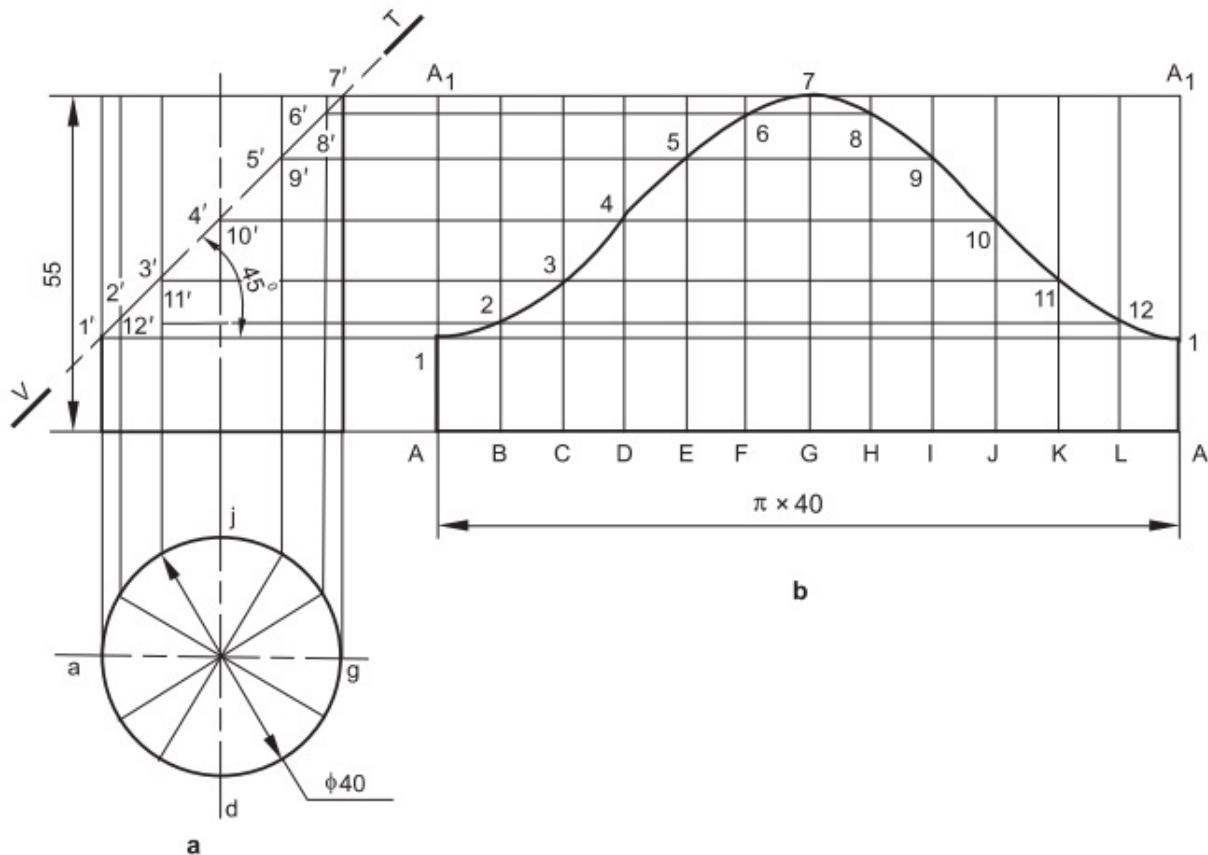
1. Draw the projections of the cylinder.
2. Divide the circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the V.T of section plane, satisfying the given conditions.
4. Draw the stretch-out line  $AA$ , equal to the circumference of the base of the cylinder.
5. Divide the stretch-out line  $AA$ , into the same number of equal parts as that of the base circle/ set-off chord

lengths by a divider and locate the generators through the division points B, C, D, etc.

6. Locate the points of intersection 1', 2', etc., between the V.T and generators.
7. Transfer these intersection points to the corresponding generators in the development, by projection.
8. Join the points 1, 2, etc., by a smooth curve and obtain the development.



- (i) AA<sub>1</sub> - A<sub>1</sub>A represents the development of the complete cylinder.
- (ii) Only one half of the development may be shown when a solid is symmetric about an axis.
- (iii) The generators should not be drawn thick, since they do not represent the folding edges.



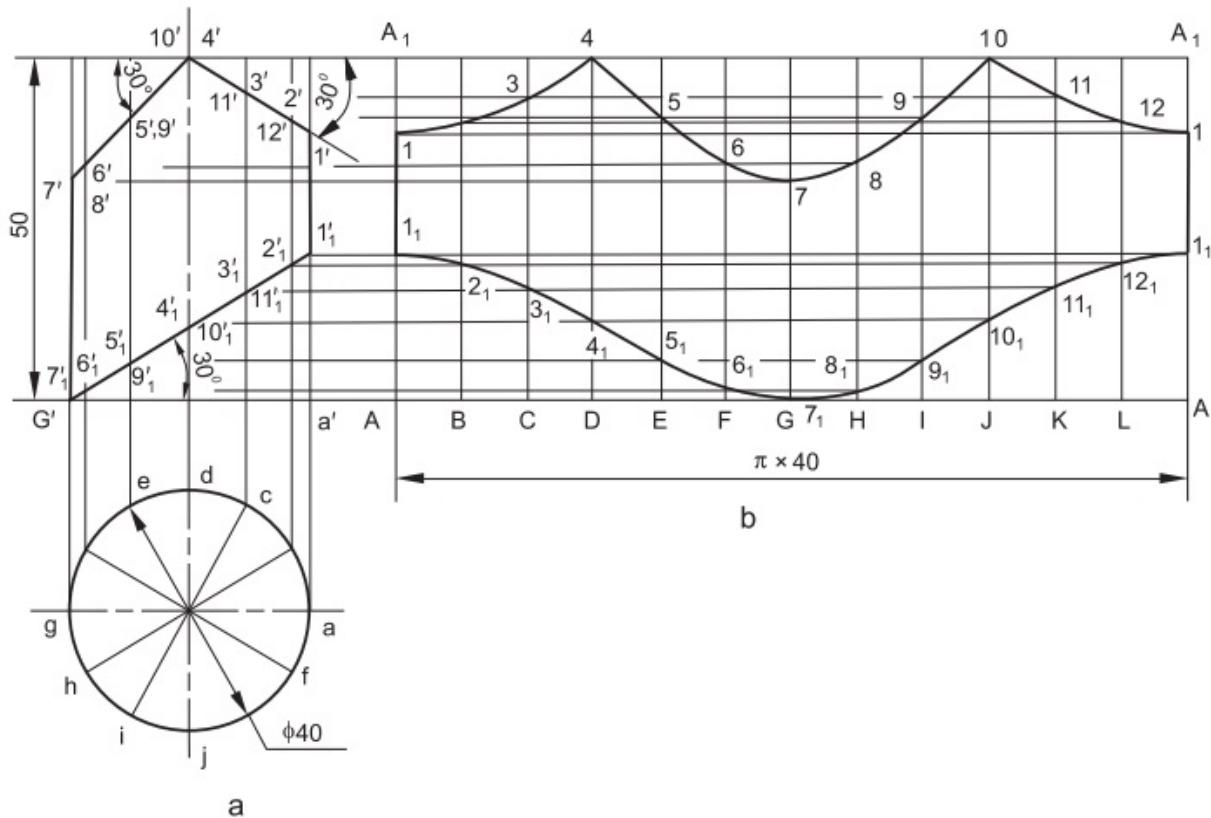
**Fig.14.4**

**Problem 5** *Figure 14.5a shows the projections of a cut cylinder. Draw the development of the lateral surface of the cut cylinder.*

**Construction (Fig.14.5)**

1. Draw the given projections of the cut cylinder.
2. Divide the base circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the stretch-out line AA, equal to the circumference of the base and complete the development of the complete cylinder.
4. Locate the generators in the development.

5. Locate the points of intersection between the cut edges and generators.
6. Repeat steps 7 and 8 of Construction: [Fig.14.4](#) suitably and obtain the development of the cut cylinder.



**Fig.14.5**

### 14.3.1.1 *Oblique Prism*

In an oblique prism, the axis will be inclined to the bases, which are regular polyhedra and parallel to each other. As the prisms are developed by parallel line development method, the lateral edges in the development must be drawn parallel to the inclined axis.

**Problem 6** An oblique hexagonal prism of side of base 30 and axis 90 long and inclined at  $45^\circ$  to the base, is resting on its base on H.P. It is cut by a section plane,

*perpendicular to H.P and V.P and passing through the top end of the axis. Draw the development of the cut prism.*

### **Construction ([Fig.14.6](#))**

1. Draw the projections of the given oblique prism.

*To draw the development of the complete prism:*

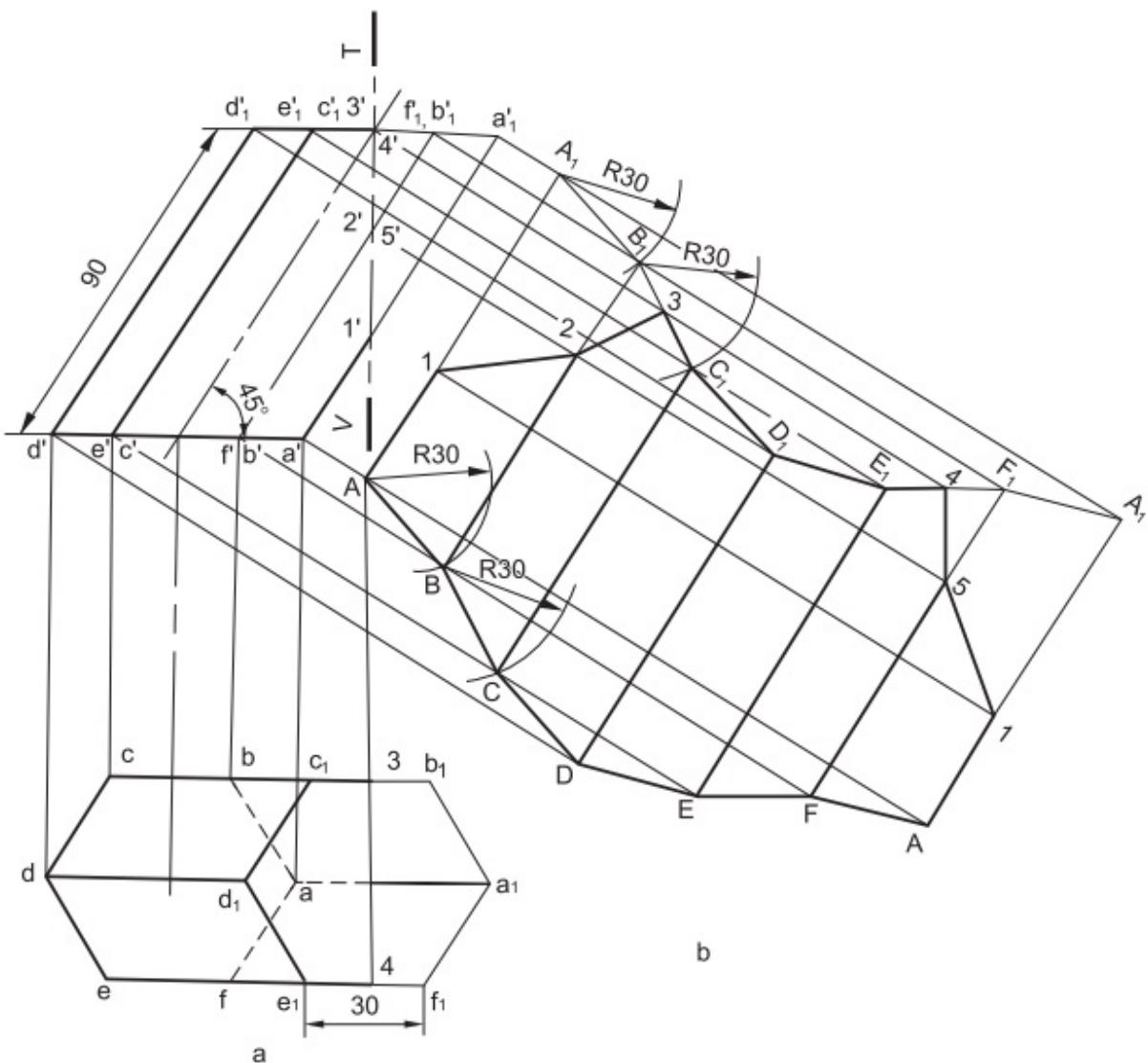
2. Draw perpendicular lines, at the ends of the lateral edges, representing the stretchout lines.
3. Assuming that the solid is cut open along the edge  $a'a_1'$ , draw a line  $AA_1$ , parallel to  $a'a_1'$ .
4. With centres A and  $A_1$  and radius equal to the side of the base, draw arcs intersecting the perpendiculars through  $b'$  and  $b_1'$  at B and  $B_1$  respectively.
5. Join  $A_1 B_1$ ,  $B_1 B$  and  $BA$



- (i)  $AA_1B_1B$  forms the development, corresponding to one lateral face of the solid.
  - (ii) The line  $B_1B$  is a folding edge on the development.
6. Repeat steps 4 and 5 suitably and obtain the development of the remaining lateral surfaces.

*To obtain the development of the cut prism:*

7. Draw the V.T of section plane, satisfying the given conditions.
8. Locate the points of intersection  $1'$ ,  $2'$ , etc., between the V.T and edges of the prism.



**Fig.14.6**

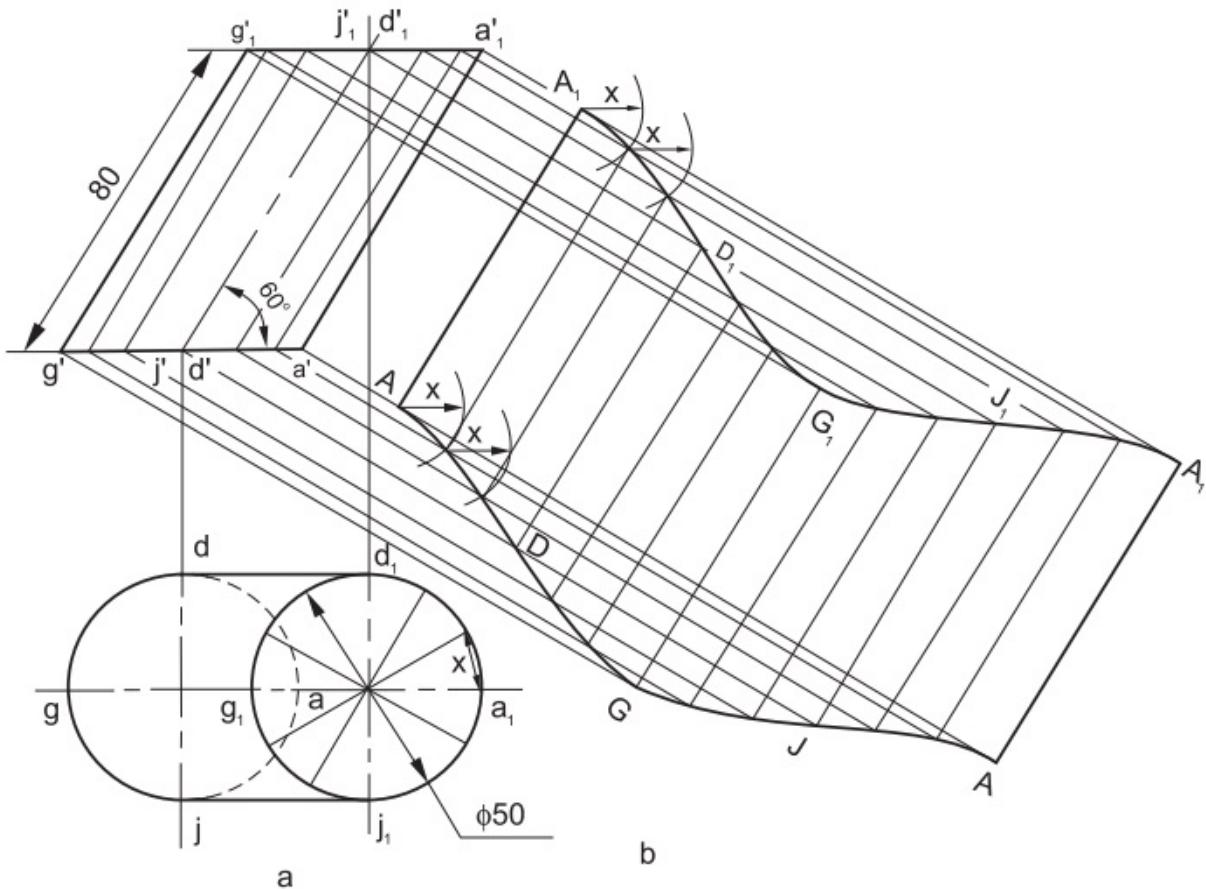
9. Through 1', 2', etc., draw lines parallel to the stretch-out lines and obtain 1, 2, etc., on the corresponding edges in the development.
10. Join these points by straight lines and darken the edges, corresponding to the retained portion of the prism.

#### 14.3.1.2 *Oblique Cylinder*

An oblique cylinder is similar to an oblique prism, in which the axis is inclined to the bases. The bases are circular and parallel to each other. The development of oblique cylinders are also obtained by the parallel line development method.

**Problem 7** Draw the development of an oblique cylinder of base 50 diameter, axis 80 long and inclined at  $60^\circ$  to the base.

**Construction (Fig.14.7)**



**Fig.14.7**

1. Draw the projections of the given oblique cylinder.
2. Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in the front view.
3. Draw perpendiculars at the ends of the generators.

4. Select the starting line  $AA_1$ , parallel to  $a'a_1'$ , for drawing the development.
5. With A and  $A_1$  as centres and radius equal to  $\times (1/12^{\text{th}}$  circumference of the base), draw arcs intersecting the perpendiculars through  $b'$  and  $b'_1$  at B and  $B_1$ .
6. Repeat step 5 and obtain other points C, D, etc., and  $C_1, D_1$ , etc.

A smooth curve passing through the above points is the required development.

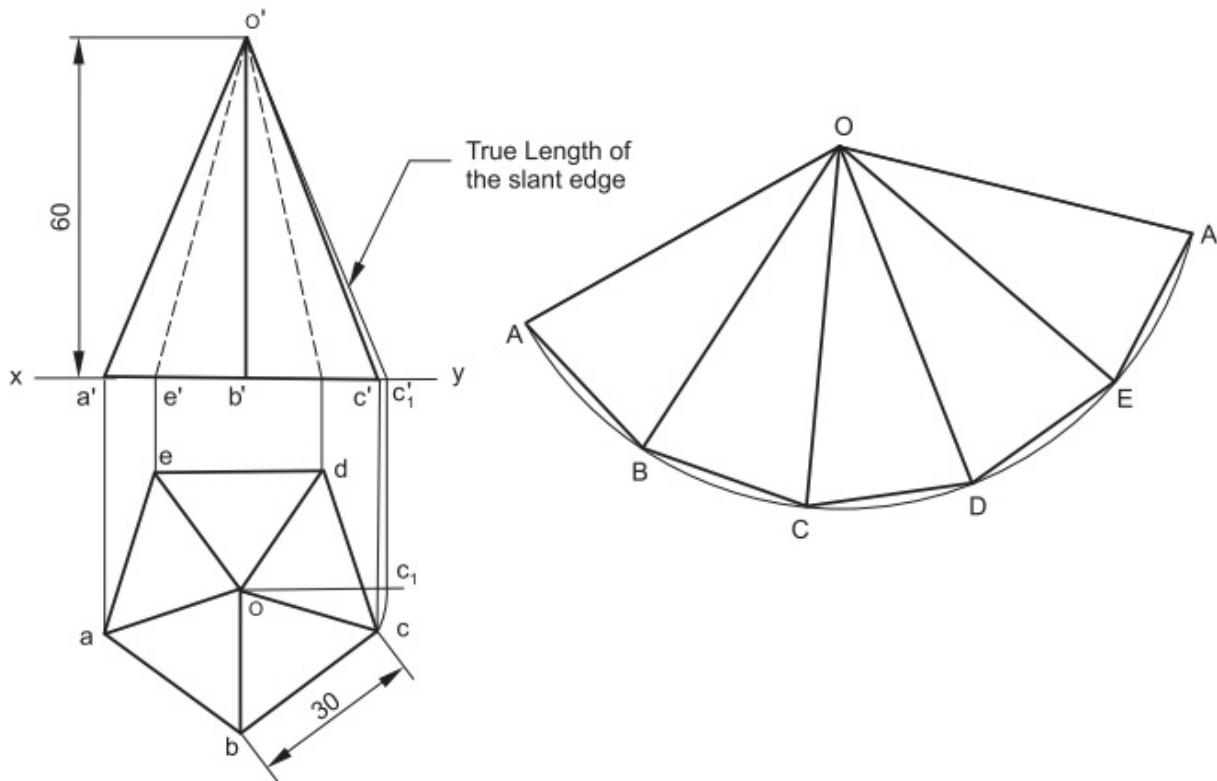
### 14.3.2 Radial Line Development

The lateral surfaces of right and oblique pyramids and cones may be developed by this method.

**Problem 8** *A pentagonal pyramid of side of base 30 and axis 60 long, is resting on its base on H.P. with an edge of the base parallel to V.P. Draw the development of the lateral surface of the pyramid.*

- HINT**
- (i) The development of a pyramid consists of a number of equal isosceles triangles. The base of the triangle is equal to the edge of the base and the sides equal to the slant height of the pyramid respectively.
  - (ii) The true length of the slant edge may be measured from the front view, by making its top view parallel to xy.

**Construction (Fig. 14.8)**



**Fig.14.8**

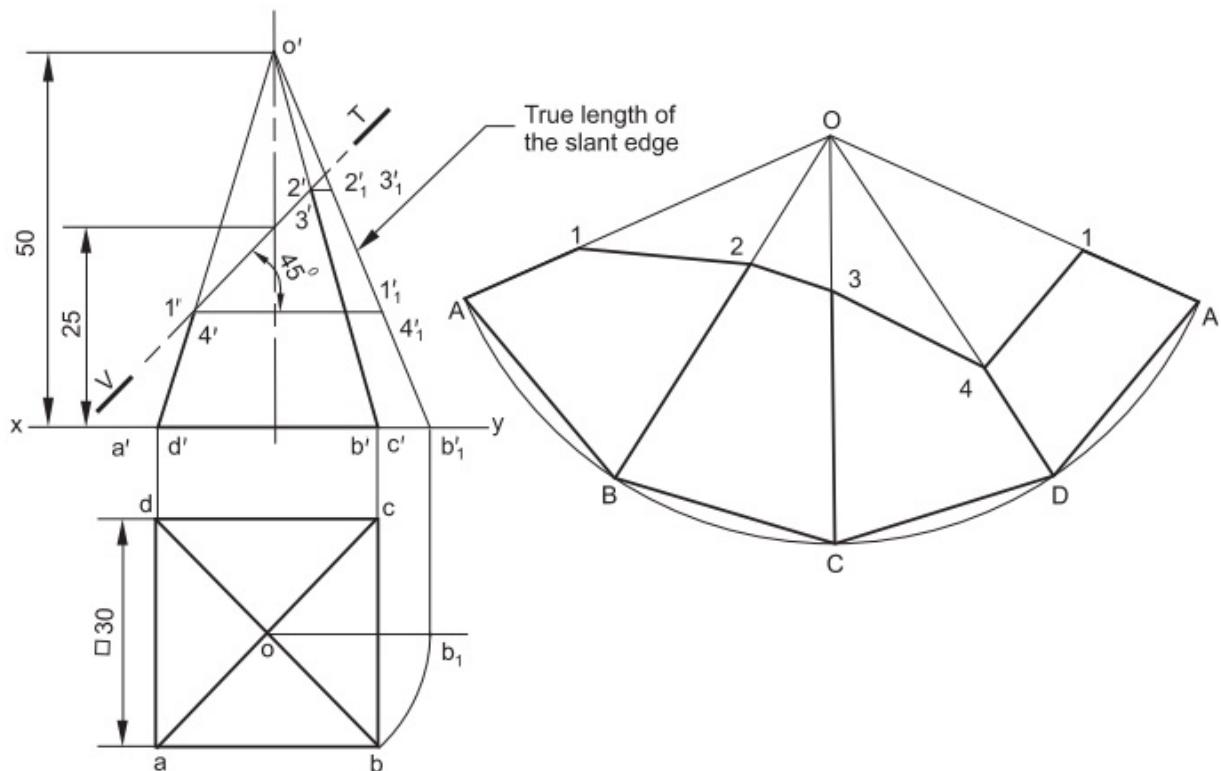
1. Draw the projections of the pentagonal pyramid.
2. Determine the true length of the slant edge OC of the pyramid as shown.
3. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
4. With radius equal to the side of the base, step-off five divisions on the above arc.
5. Join the above division points in the order and also with the centre of the arc.

The figure thus formed is the development of the lateral surface of the pyramid.

**Problem 9** A square pyramid with side of base 30 and axis 50 long, is resting on its base on H.P with an edge of the base parallel to V.P. It is cut by a section plane,

*perpendicular to VP and inclined at  $45^\circ$  to H.P. The section plane is passing through the mid-point of the axis. Draw the development of the surface of the cut pyramid.*

**Construction (Fig.14.9)**



**Fig.14.9**

1. Draw the projections of the given pyramid.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Locate the points of intersection 1', 2', etc., between the V.T and slant edges of the pyramid.
4. Determine the true length of the slant edge of the pyramid.
5. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of the circle.

- Follow steps 4 and 5 of Construction: [Fig.14.8](#) and
6. obtain the full development of the pyramid.
  7. Obtain the true lengths of the slant edges of the cut pyramid, by projecting on to the true length line.
  8. Transfer the above true lengths to the corresponding edges in the development.
  9. Join the points 1, 2, etc., by straight lines and obtain the development of the cut pyramid.

**Problem 10** *A cone of diameter of base 50 and axis 60 long, is resting on its base on H.P. Draw the projections of the cone and show on it, the shortest path traced by a point, starting from a point on the circumference of the base of the cone, moving around it and reaching the same point.*

**HINT** The solution to this problem makes use of development of lateral surface of the cone. The development of the lateral surface of the cone is a sector of a circle, the radius and length of the arc of which are equal to the slant height and circumference of the base circle of the cone respectively.

### **Construction ([Fig.14.10](#))**

1. Draw the projections of the cone.
2. Divide the circle (top view) into, say 8 equal parts and locate the corresponding generators in the front view.
3. With any point O as centre and radius equal to the true length of the slant height, draw a sector of a circle, representing the development of the cone. The length of the arc should be equal to the circumference of the base circle. This may be determined by two ways.

The angle subtended by the arc at O is given by,

$$\theta = 360 \times \frac{\text{radius of the base circle}}{\text{slant height of the cone}}$$

Divide the arc into 8 equal parts and draw the corresponding generators on it.

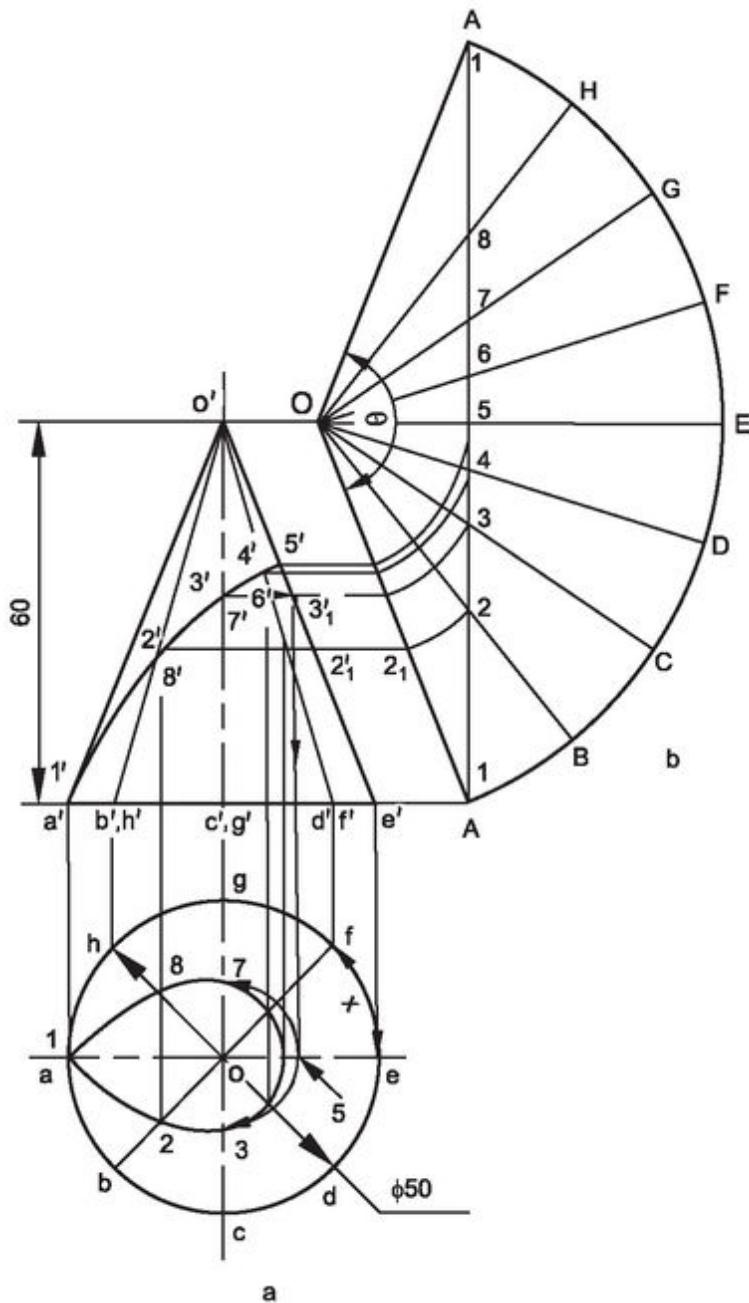
OR

Along the arc, starting from any point, say A, step-off with a divider, 8 equal divisions, each equal to the arc length of one division ( $\times$ ) and locate the corresponding generators on it.

4. Join A - A on the development, representing the shortest path, intersecting the generators at 1, 2, etc.
5. Transfer the above points on to both the projections.

To transfer the point, say 2 on the development to the front and top views:

- (i) Transfer the true length O<sub>2</sub> to the extreme generator o'e', in the front view.
  - (ii) Transfer the above point to the generator o'b', by drawing a horizontal line.
  - (iii) Locate the top view of the above point by projection, on the corresponding generator.
6. Join the points 1', 2', etc., and 1, 2, etc., by smooth curves, representing the paths traced by the point in the front and top views respectively.

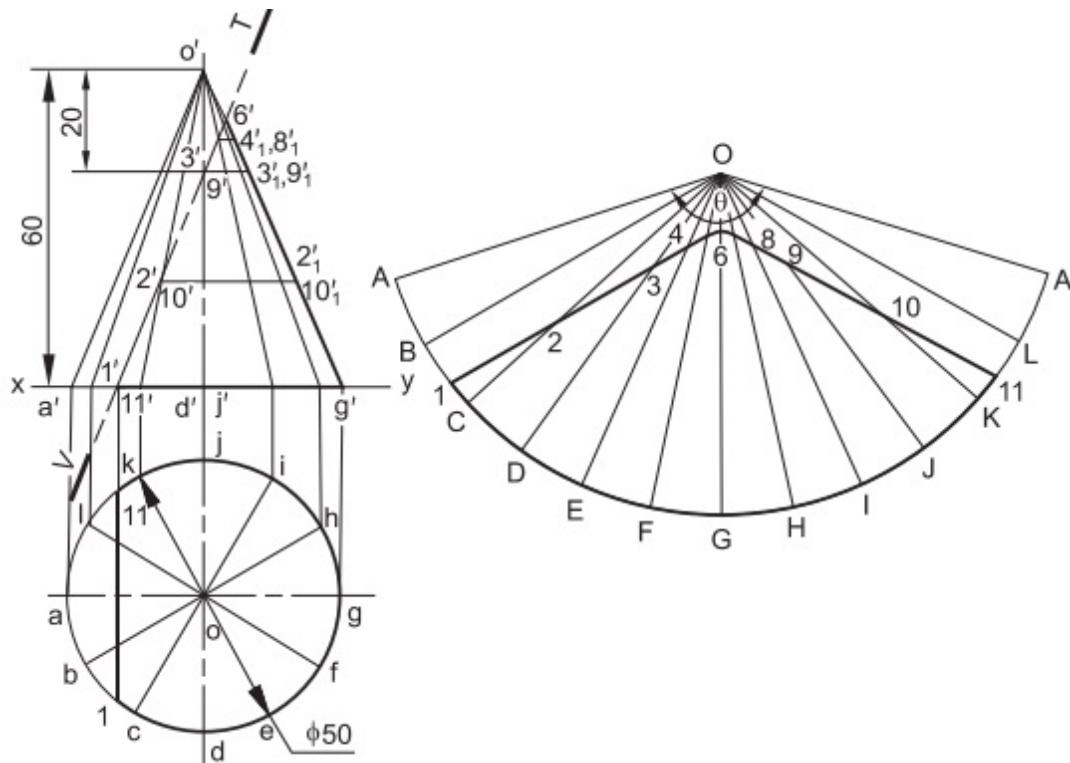


**Fig.14.10**

**Problem 11** A cone of base 50 diameter and axis 60 long, is resting on its base on H.P. It is cut by a section plane perpendicular to V.P and parallel to an extreme generator and passing through a point on the axis at a distance of 20 from the apex. Draw the development of the retained solid.

### **Construction (Fig.14.11)**

1. Draw the projections of the cone.
2. Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in the front view.
3. Draw the development of the complete cone, following Construction: [Fig.14.10](#).
4. Draw the V.T of section plane, satisfying the given conditions.
5. Locate the points of intersection between the V.T and generators and base of the cone.
6. Determine the true lengths of  $o'2'$ ,  $o'3'$ , etc., by drawing horizontal lines to the extreme generator.
7. Transfer these true lengths to the development.
8. Join the points 1, 2, etc., by a smooth curve and obtain the required development.

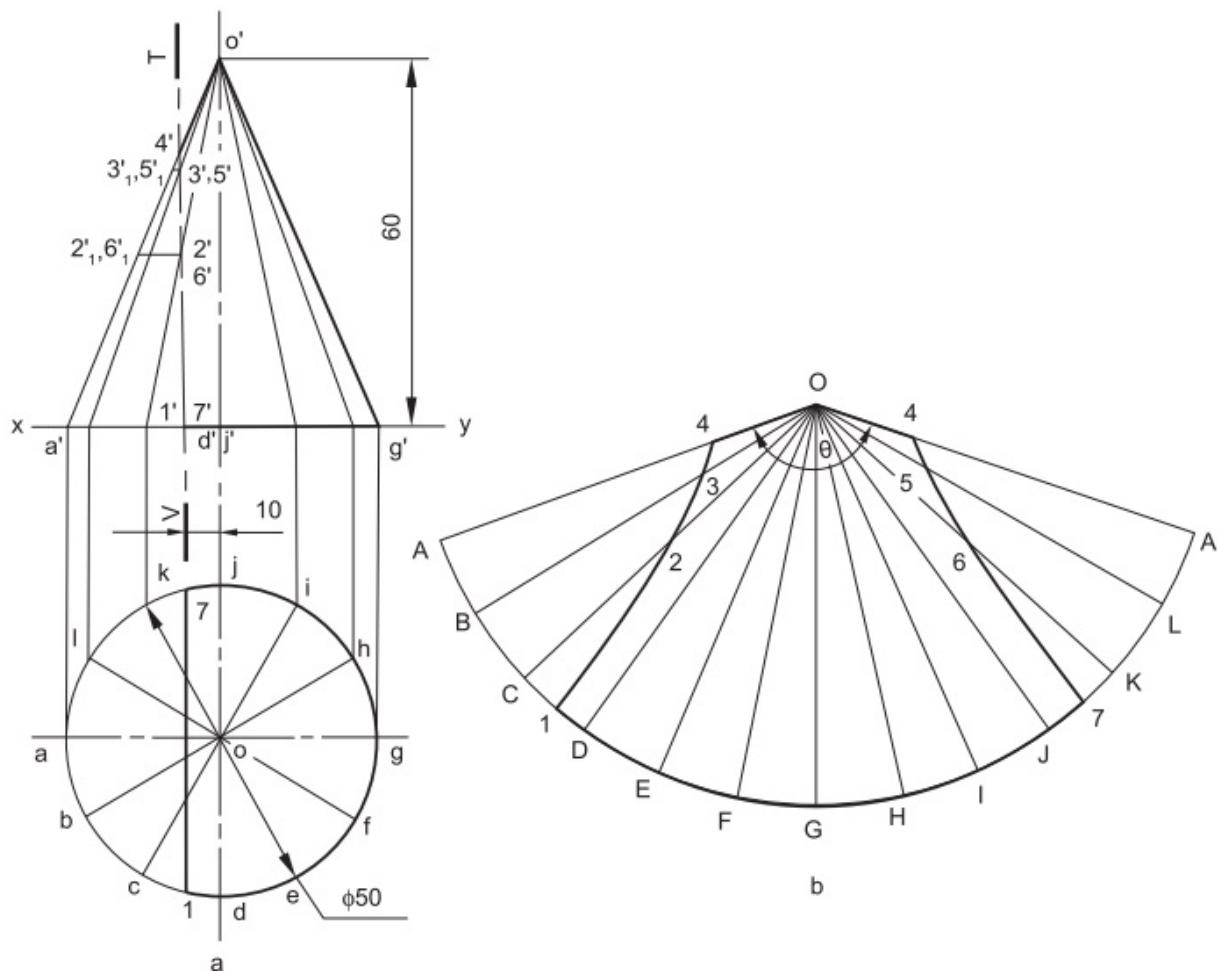


**Fig.14.11**

**Problem 12** A cone of base 50 diameter and axis 60 long, is resting on its base on H.P. A section plane perpendicular to V.P. cuts the cone at a distance of 10 from the axis. Draw the development of the cut solid.

**Construction (Fig.14.12)**

Draw the development, following the method of Construction: Fig.14.11.



**Fig.14.12**

**14.3.2.1 Oblique Pyramid**

The lateral surfaces of an oblique pyramid are in the form of triangular faces. However, unlike a right pyramid, each one of the slant edges will be of different length. The edges of the base will appear in true lengths in the top view, when the base is parallel to H.P.

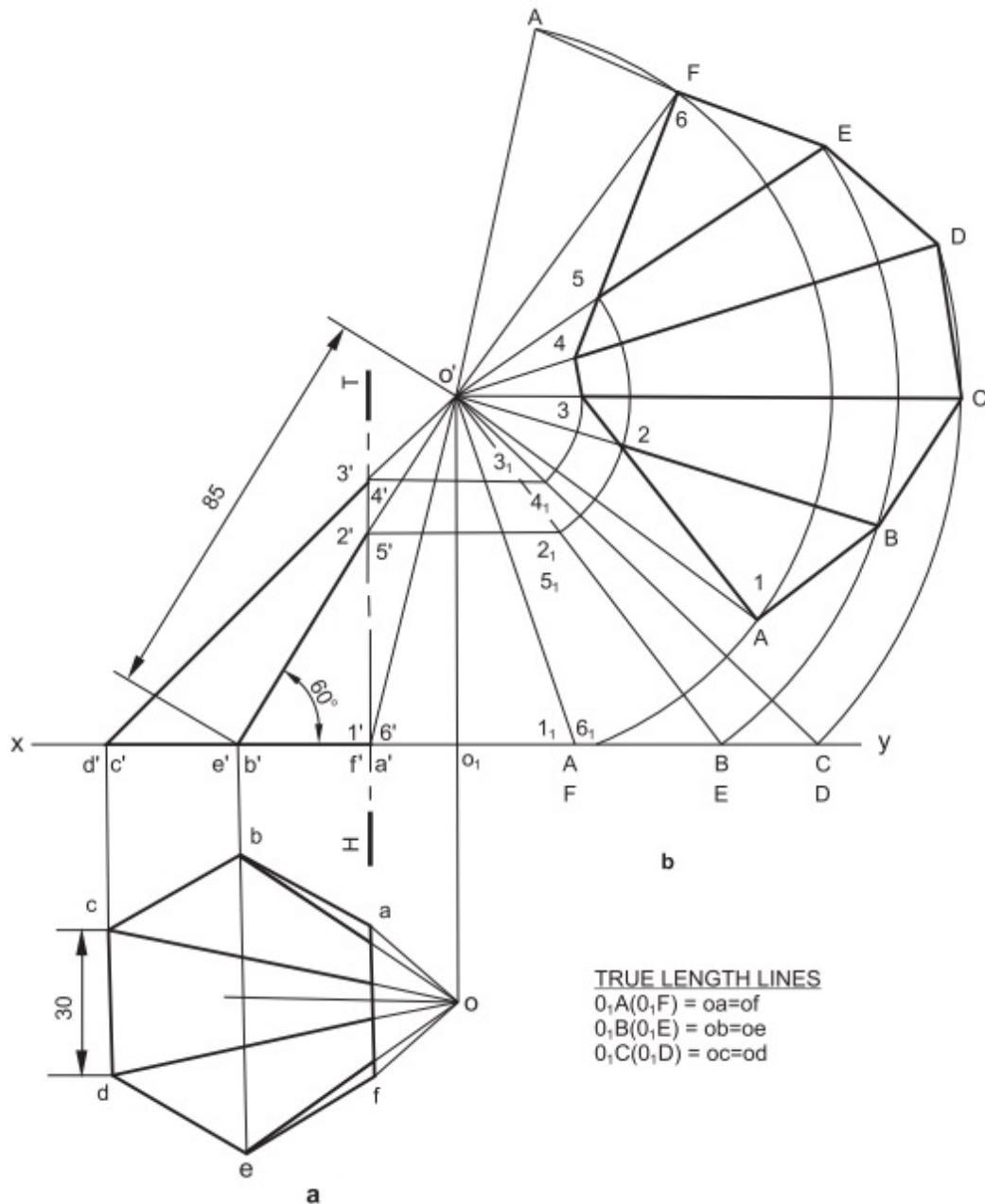
The true lengths of slant edges must be determined to draw the development. By using the principle of rotation, a separate true length diagram is to be drawn first to determine the true lengths of the slant edges.

**Problem 13** *An oblique hexagonal pyramid with side of base 30 and axis 85 long, is resting on its base on H.P, with an edge of the base perpendicular to V.P. The axis of the pyramid is inclined at  $60^\circ$  to the base. It is cut by a section plane, perpendicular to both H.P and V.P and passing through the inner extreme base edge of the pyramid. Draw the development of the cut pyramid.*

### **Construction (Fig.14.13)**

1. Draw the projections of the given pyramid.
2. Locate the point of intersection  $o_1$  between  $xy$  and the projector  $oo'$ . Mark the top view lengths of the slant edges along  $xy$  from  $o_1$ . For example  $o_1A = oa$ . Join  $o'$ ,  $A$ , representing the true length of the slant edge  $OA$ . Similarly, determine the true lengths of all the slant edges.
3. Draw the development of the total solid, by making use of the true lengths of the slant and base edges.
4. Draw the V.T of section plane, satisfying the given conditions.
5. Locate the points of intersection between the V.T and slant edges of the solid.

6. Locate these points on the true length lines by drawing horizontal lines, meeting the respective true length lines.



**Fig.14.13**

7. Transfer these points on to the development.

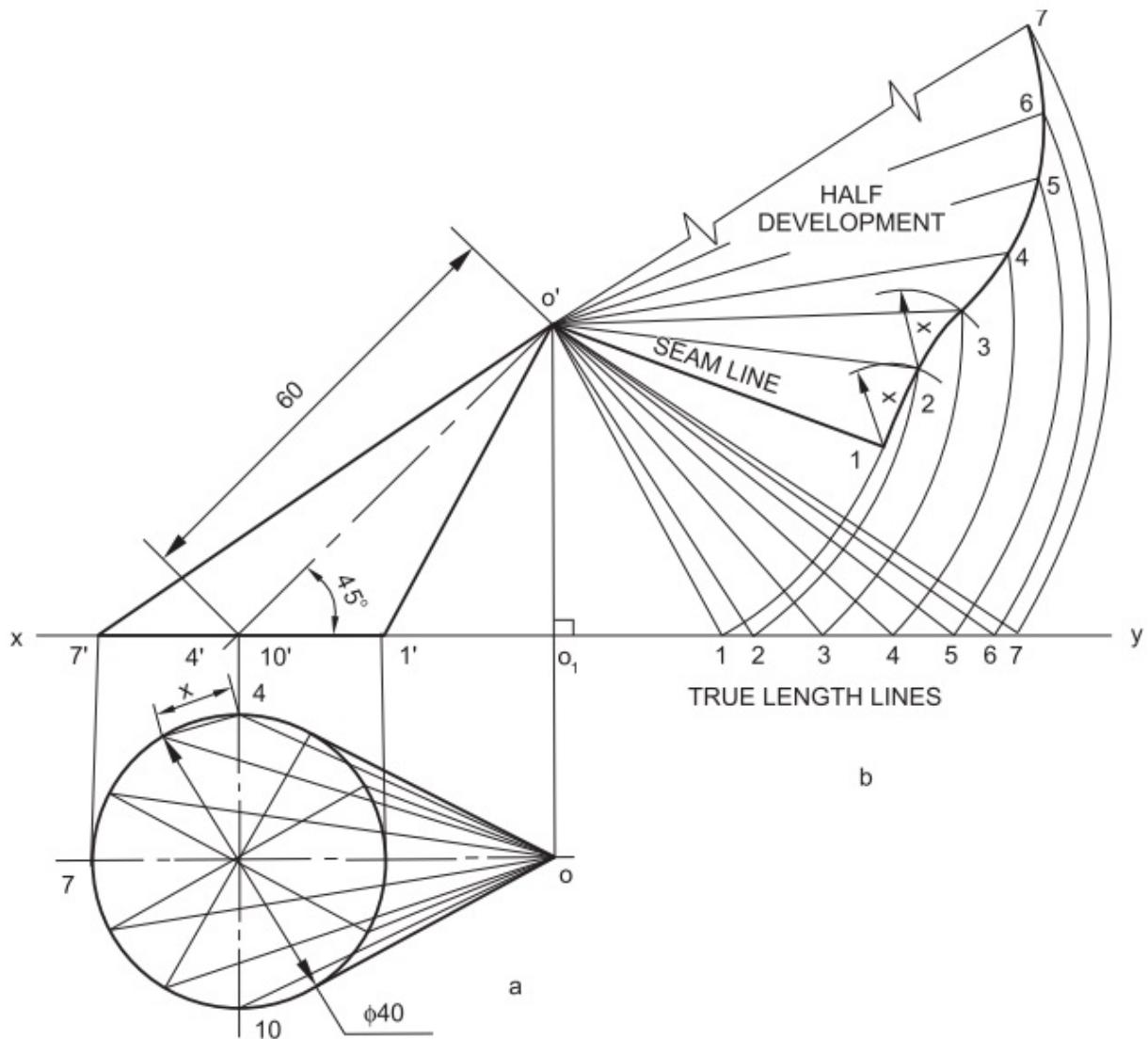
- Join the points 1, 2, etc., by straight lines and darken the edges, corresponding to the retained portion of the cut pyramid.

### 14.3.2.2 ***Oblique Cone***

The development of an oblique cone is obtained by the method similar to the one followed for oblique pyramid. The true lengths of the generators are determined from the true length diagram. The distance between two consecutive division points on the circular base may be taken as approximately equal to the chord length of the arc between the points.

**Problem 14** *An oblique cone of base 40 diameter and axis 60 long, is resting on its base on H.P. The axis is inclined at  $45^\circ$  to the base. Draw the development of the cone.*

**Construction (Fig.14.14)**



**Fig.14.14**

1. Draw the projections of the given oblique cone.
2. Divide the circle (top view) into, say 12 equal parts.
3. Join these points to the apex, forming the generators of the cone.
4. Determine the true lengths of the generators, following Construction: [Fig.14.13](#).
5. Develop the lateral surface, starting with the shortest element o'1.

6. Locate the next element  $O'2$  such that, the distance  $1-2$  is equal to the chord length  $x$ .
7. Repeat the procedure and complete the development.



One half of the development only is shown, as the other half is symmetric about the generator  $O7$ .

### 14.3.3 Triangulation Method of Development

This is an approximate method of developing a non-developable surface. This method assumes that the surfaces are made of a large number of triangular strips with very short bases that make up the non-developable surfaces. These triangles when laid out in their true size, with their common edges joined, produce an approximate development required for most practical purposes.

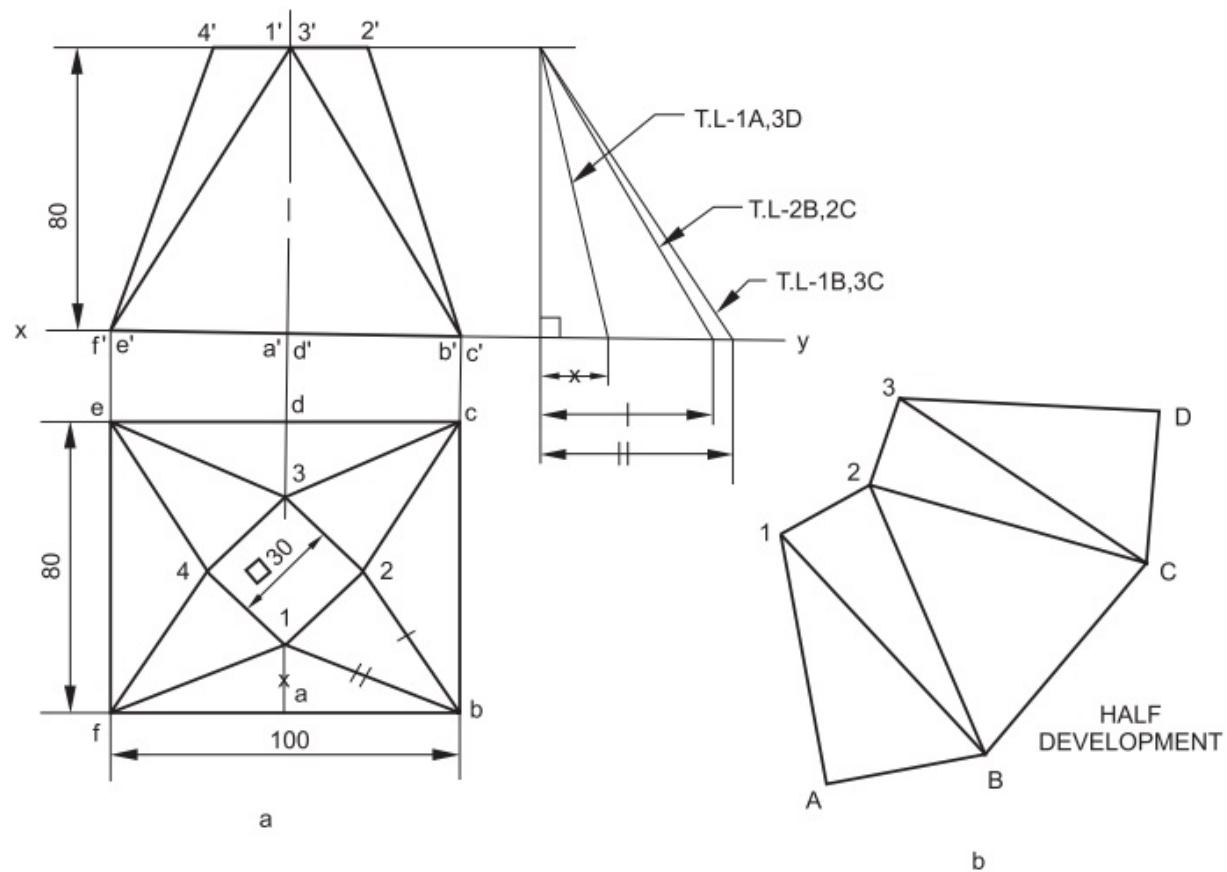
The triangulation method is used for developing the lateral surfaces of transition pieces and reducers. A transition piece is used to connect pipes or openings of different shapes of cross-section; whereas a reducer connects openings of same shape, but of different sizes. To develop warped surfaces also, this method is used. When making a development pattern that is symmetric about an axis of symmetry, one-half of the development is all that is necessary. In such instances, the development should be marked as one-half of the development and must begin on the axis of symmetry. Similarly, one-fourth development may suffice when there are two axes of symmetry.

**Problem 15** A transition piece of 80 long, connects a square opening of 30 side with coaxial rectangular opening of  $100 \times 80$ . The transition piece is lying on its rectangular

base on H.P. The edges of the square hole are equally inclined to V.P. whereas the longer edges of the rectangular hole are parallel to V.P. Draw the development of the transition piece.

### **Construction (Fig.14.15)**

1. Draw the projections of the transition piece.
2. Considering the axis of symmetry ad in the top view, identify the elements 1a, 1b, 2b, 2c, 3c and 3d.
3. Construct the true length diagram and determine the true lengths of the above elements.
4. Starting from 1A, construct half of the development of the transition piece, by making use of the true lengths of the above elements.



**Fig.14.15**

**Problem 16** Draw the development of a transition piece that connects a circular section of 25 diameter with a square section of side 35. The length of the transition piece is 35 and both the sections are coaxial. The transition piece is resting on its square base on H.P.

**Construction (Fig.14.16)**

1. Draw the projections of the given transition piece.
2. Considering horizontal axis of symmetry ae in the top view, identify the elements 1a, 1b, 2b, 3b, 4b, 4c, etc.
3. Construct the true length diagram and determine the true lengths of the above elements.

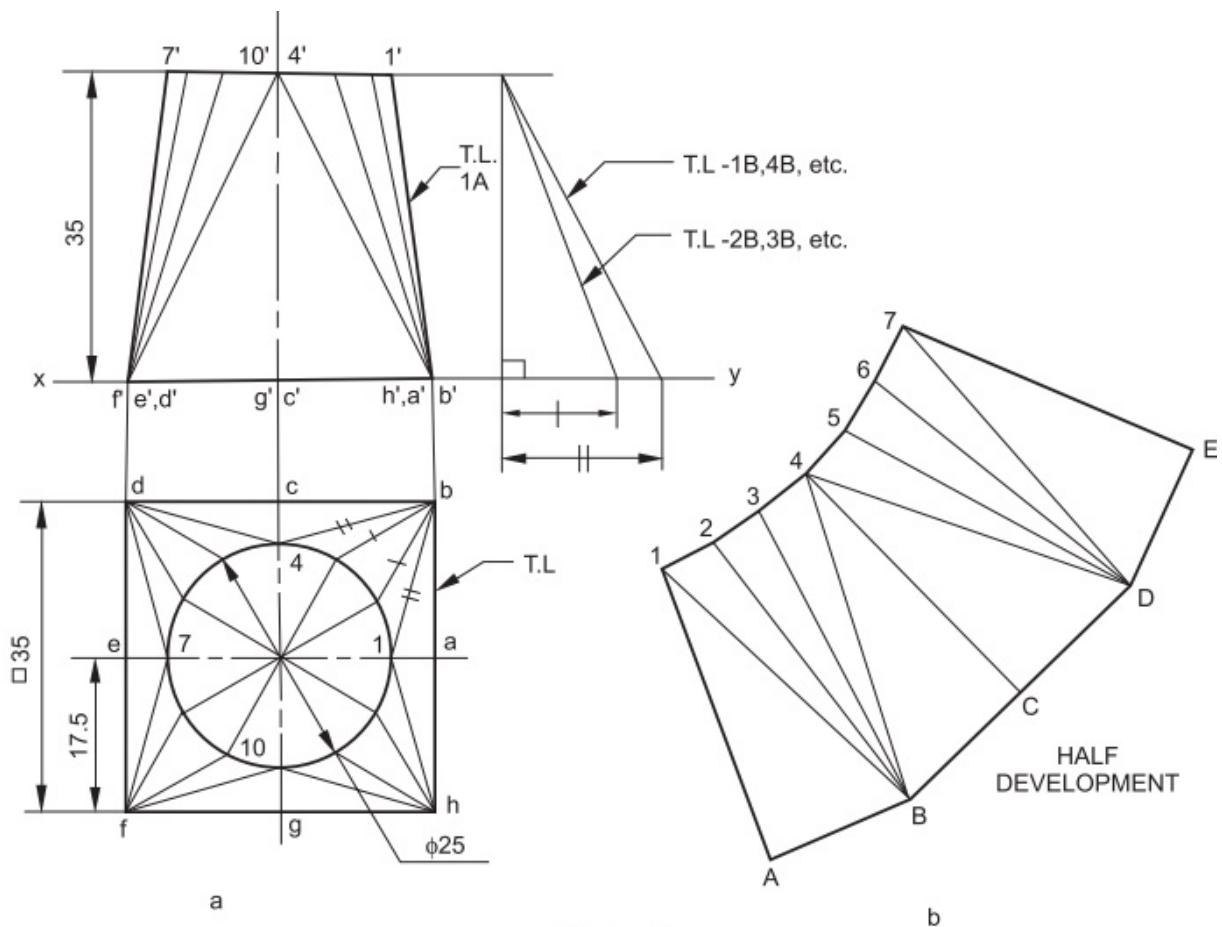
**HINT** (i) True length of arc 1-2 is approximately equal to the chord length 1-2.

(ii) The length ab represents the true length of half of the base edge.

4. Starting from 1A, construct half of the development of the transition piece, by making use of the true lengths of the above elements.



The points 1, 2, 3 etc., are joined by a smooth curve and A, B, C, D and E are joined by straight lines.



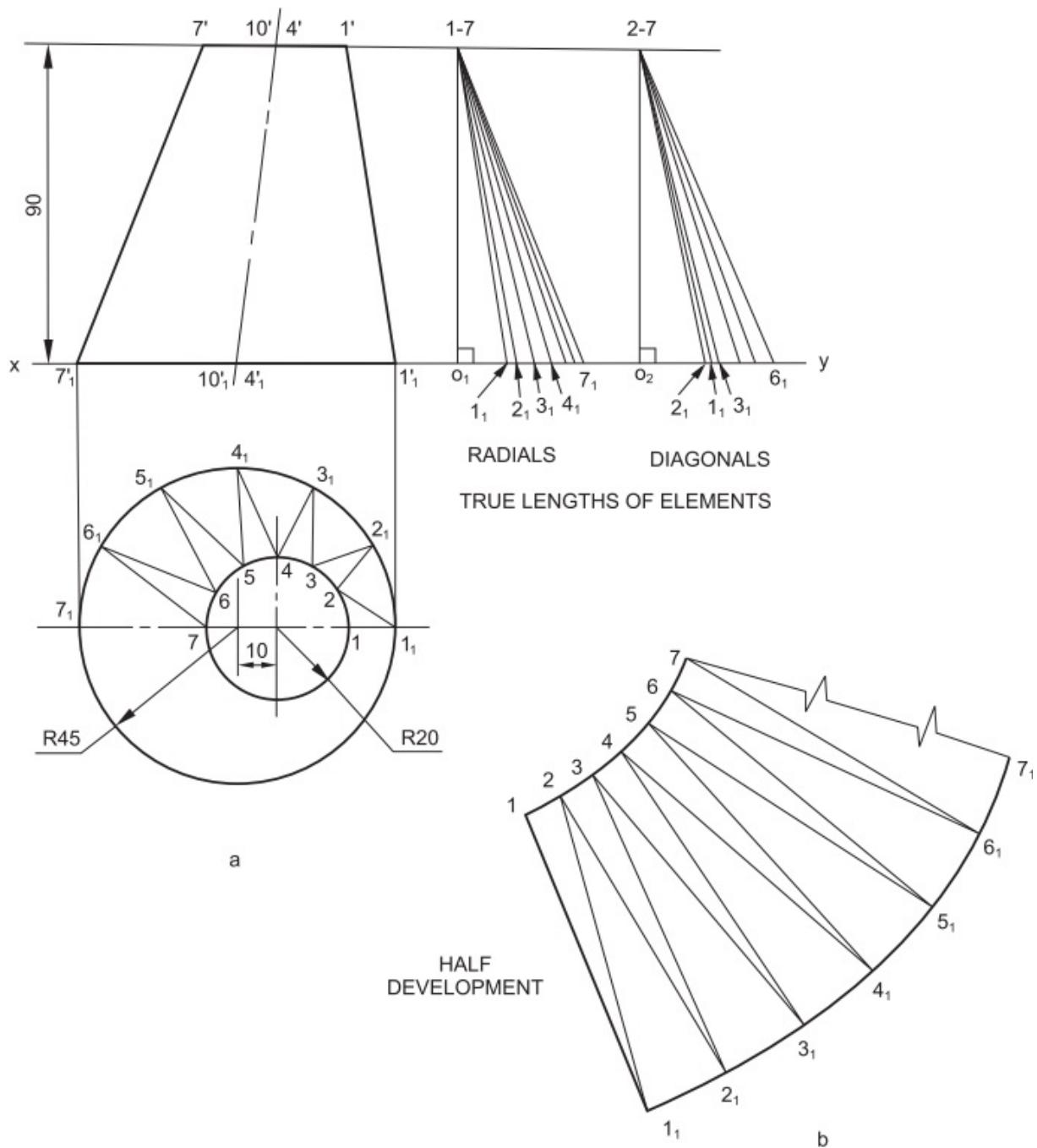
**Fig.14.16**

**Problem 17** Draw the development of the lateral surface of a conical off-set reducer piece required to connect two circular openings of 40 and 90 diameters. The height of the reducer is 90 and the off-set is 10. The reducer is resting on its larger base on H.P.

**Construction (Fig.14.17)**

1. Draw the projections of the given reducer.
2. Considering horizontal axis of symmetry in the top view, divide both the semicircles into equal parts, say 6 and identify the elements as: Radials-1-1<sub>1</sub>, 2-2<sub>1</sub>, 3-3<sub>1</sub>, 4-4<sub>1</sub>, etc., and Diagonals-2-1<sub>1</sub>, 3-2<sub>1</sub>, 4-3<sub>1</sub>, etc.

3. Construct the true length diagram and determine the true lengths of the above elements (Mark the top view lengths of radials from  $O_1$  and that of the diagonals from  $O_2$ ).
4. Starting from 1-1<sub>1</sub> lay the triangles, with their common edges joined.
5. Join the ends of these triangles by smooth curves and obtain half of the development as shown.



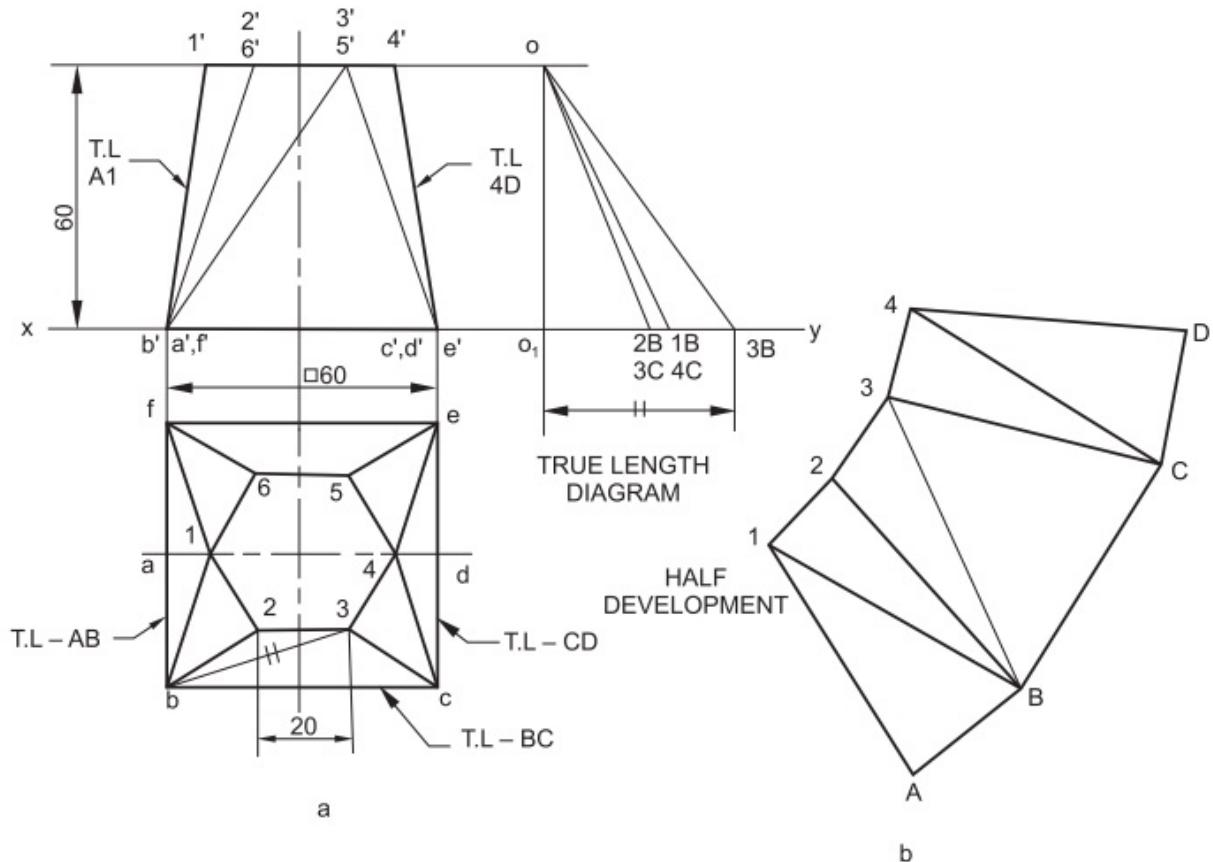
**Fig.14.17**

**Problem 18** Develop the lateral surface of a transition piece, connecting co-axial hexagonal and square holes, which are 60 apart. The side of the square is 60 and that of the hexagon is 20. The transition piece is resting on its

*square base on H.P; one side of both the bases being parallel to V.P.*

### **Construction (Fig.14.18)**

1. Draw the projections of the given transition piece.
2. Considering the horizontal axis of symmetry ad in the top view, identify the elements 1a, 1b, 2b, 3b, 3c, 4c and 4d.
3. Construct the true length diagram and determine the true lengths of the above elements.
4. Starting from 1-A, lay the triangles in their true size, with their common edges joined.
5. Join the ends of these triangles by straight lines and obtain half of the development as shown.



**Fig.14.18**

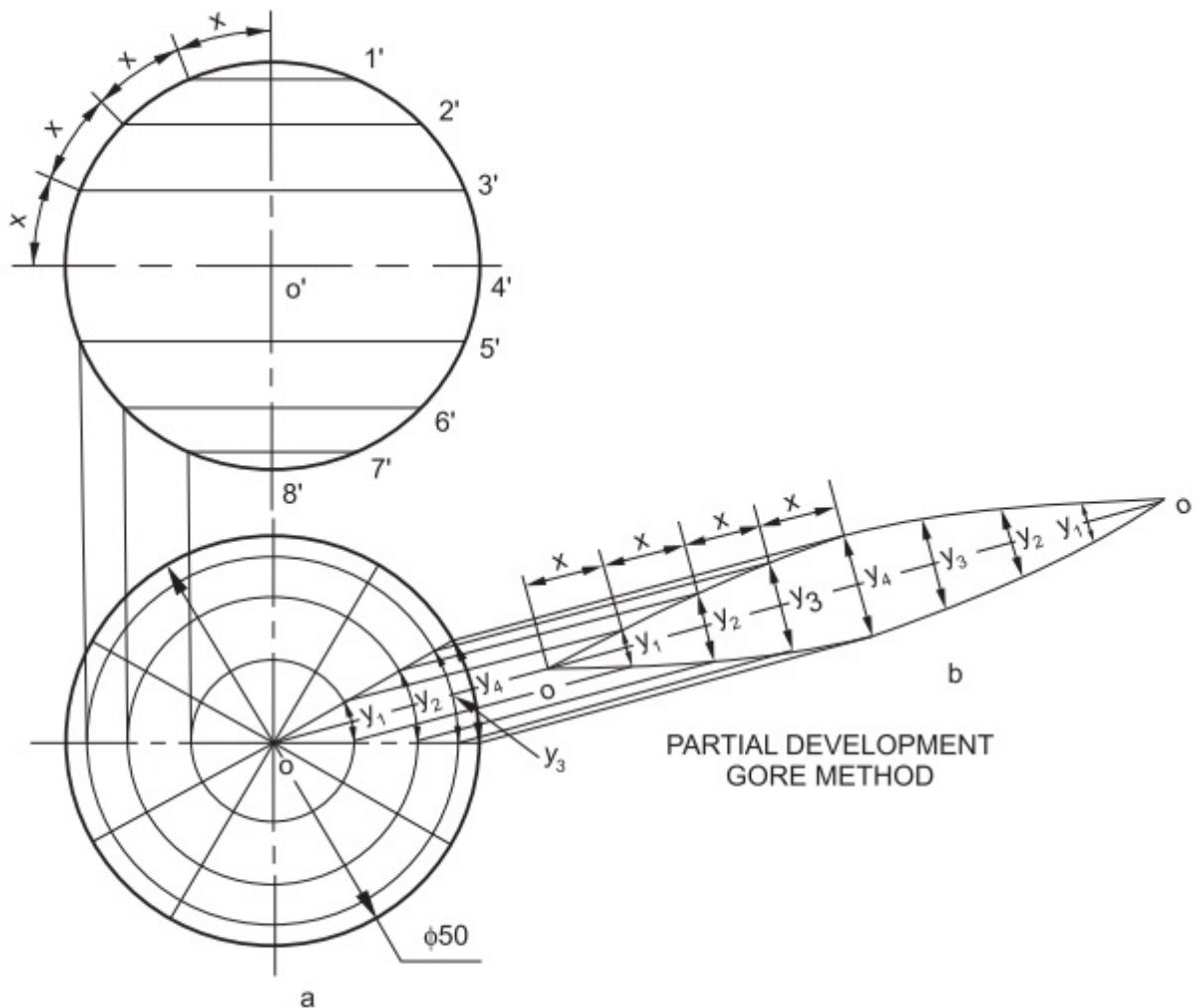
## **14.4 DEVELOPMENT OF SPHERICAL SURFACES**

A sphere has a double curved surface and hence, it cannot be developed as a single piece. However, a sphere may be developed approximately by dividing its surface into a number of segments. Developing the surface of the sphere by dividing it into a number of cylindrical segments, is called “poly-cylindrical or Gore or Lune method”. A Gore or Lune is the portion between two planes, which contain the axis of the sphere. Developing the surface of a sphere by dividing it into a number of conical segments is called “Poly-conic or Zone” method. A zone is a portion of the sphere, enclosed between two planes perpendicular to the axis.

**Problem 19** *Draw the development of a sphere of 50 diameter.*

**Construction (Fig.14.19)** *Gore (Lune) method*

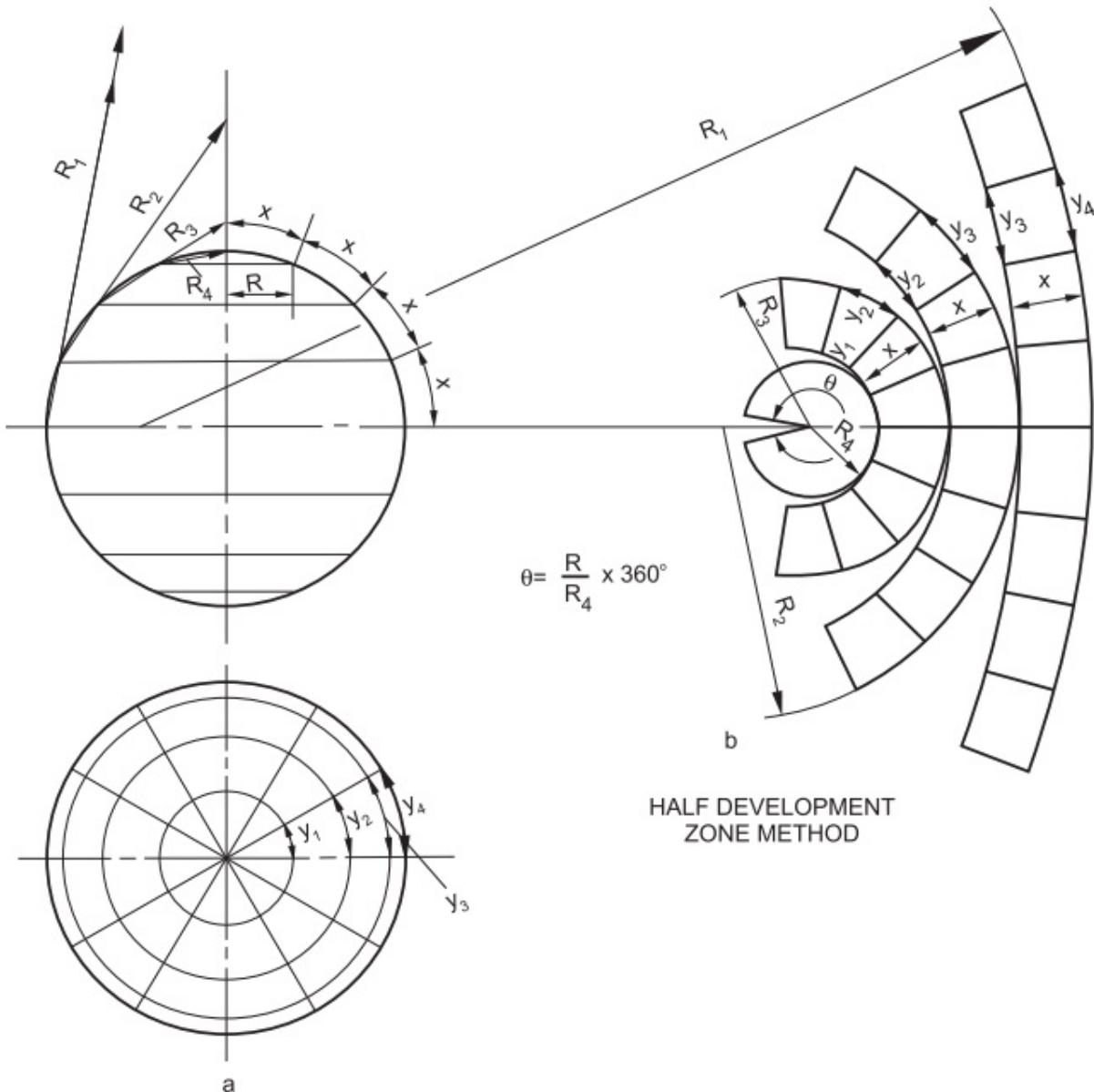
1. Draw the projections of the sphere.
2. Draw six equally spaced imaginary vertical cutting planes passing through the pole (centre) in the top view.
3. Draw a series of parallel lines on the front view, so that they establish equal arcs  $\times$  along the circumference. These lines divide the sphere into a number of slices, say 8 as shown.
4. Project the true size of the Gore, from the top view, keeping its length equal to  $8\times$ .



**Fig.14.19**

- (i) Drawing one section (Gore or Lune) is sufficient, as this may be used as a pattern for the remaining parts of the surface.
- (ii) In the present case, the sphere consists of 12 such parts when fully developed.

**Construction (Fig.14.20)** Zone method



**Fig.14.20**

1. Draw the projections of the sphere.
2. Draw a series of parallel lines on the front view, so that they establish arcs  $x$  along the circumference. These lines divide the sphere into a number of slices, say 8 as shown.

The slices are assumed to represent a series of truncated cones for which, one parallel serves as a



base of the cone and the other as the truncated top.

3. Determine the slant lengths of the cones, corresponding to the truncated portions, i.e.,  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$ .
4. Draw the development of the truncated cones, with radii equal to the slant lengths, the width being  $x$ .

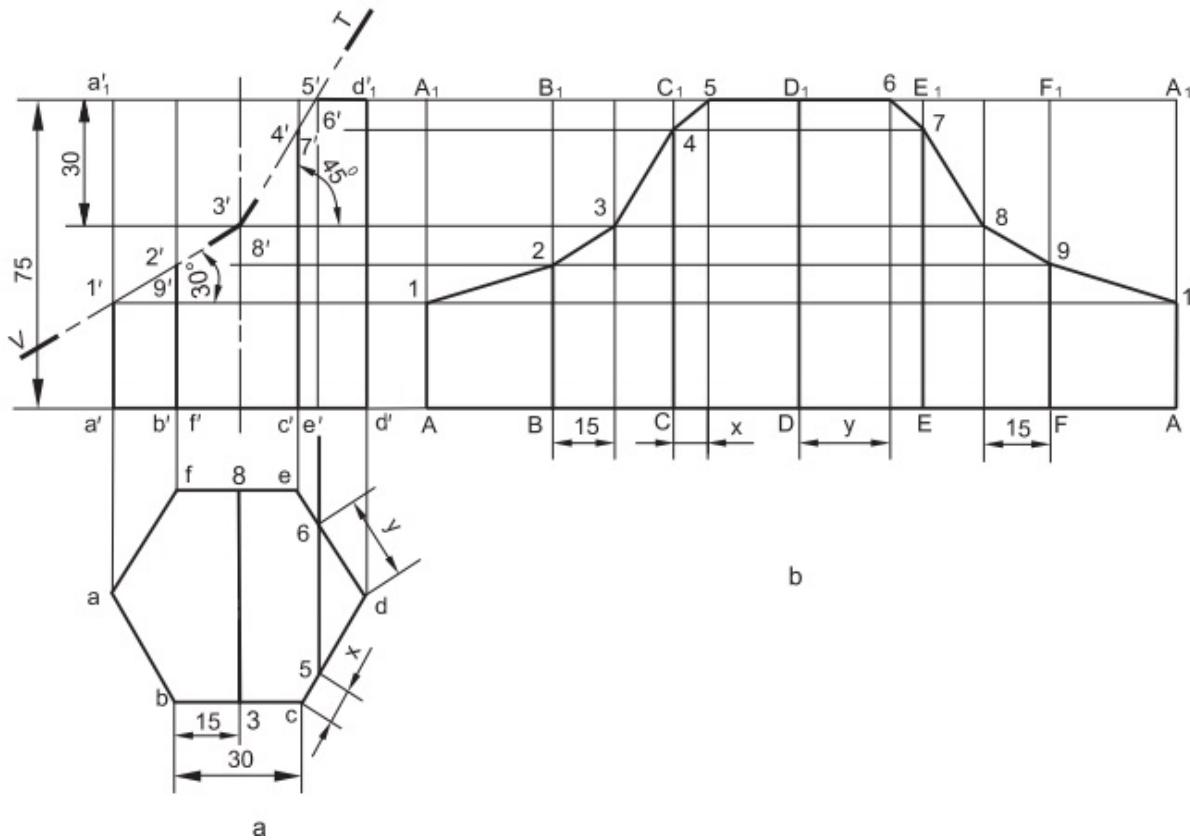
[Figure 14.20](#) shows the development of one-half of the sphere.

## 14.5 EXAMPLES

**Problem 20** [Figure 14.21a](#) shows the projections of a hexagonal prism, cut by two section planes as indicated. Draw the development of the retained portion of the prism.

### **Construction ([Fig. 14.21b](#))**

1. Draw the development  $AA_1-A_1A$  of the complete prism, following the stretchout line principle.
2. Locate the points of intersection  $1'$ ,  $2'$ , etc., between the V.T of the cutting plane and edges of the prism.
3. Transfer the intersection points to the development by horizontal projectors, except  $5'$  and  $6'$ , to the corresponding edges in the development.
4. Locate the points  $5$  and  $6$  such that  $C_1 5 = c5 = x$  and  $D_1 6 = d6 = y$ .
5. Join these points by straight lines and obtain the development of the cut prism, by darkening the edges.



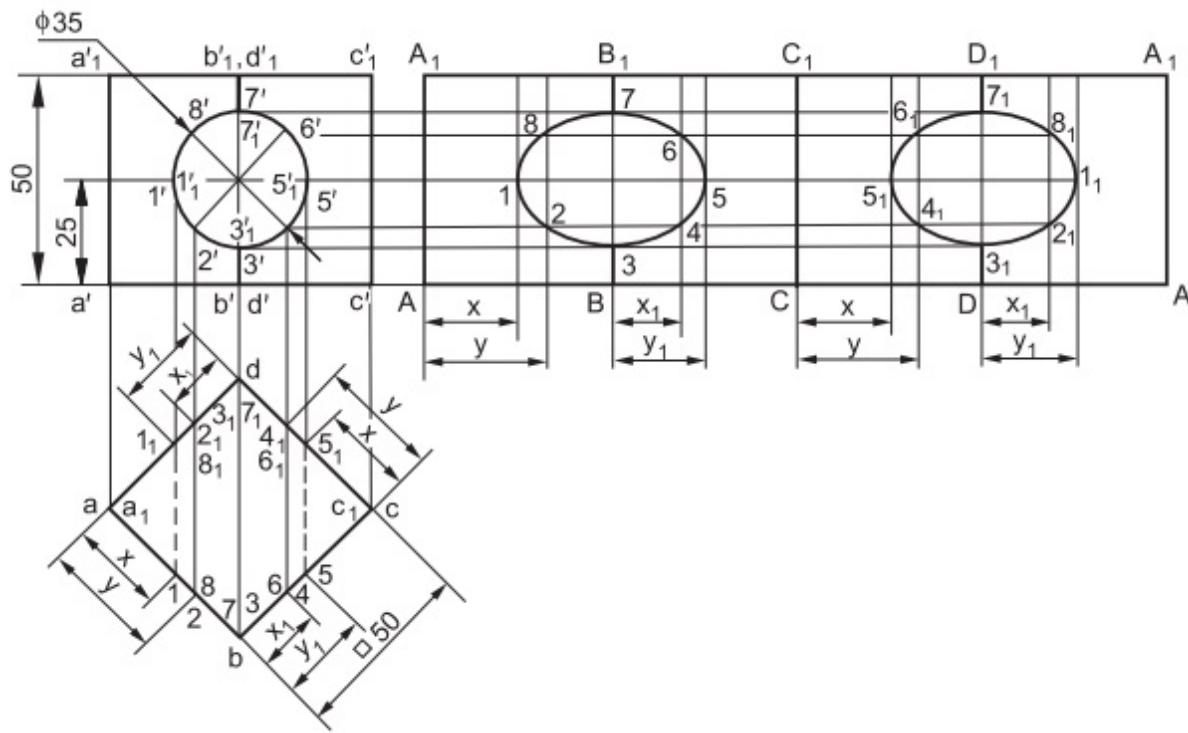
**Fig.14.21**

**Problem 21** A cube of 50 edge, stands on one of its faces on H.P with the vertical faces equally inclined to V.P. A hole of 35 diameter is drilled centrally through the cube such that, the axis of the hole is perpendicular to V.P. Draw the development of the cube.

**Construction (Fig.14.22)**

1. Draw the projections of the cube.
2. Draw the development AA<sub>1</sub> - A<sub>1</sub>A of the cube, following the stretch-out line principle.
3. Divide the circle in the front view, into equal number of parts, say 8 and obtain the corresponding generators in the top view.

4. Locate the points of intersection between the above generators and edges of the cube in the top view.
5. Draw horizontal projectors from  $1'$  ( $1_1'$ ),  $2'$  ( $2_1'$ ), etc., and locate the points  $1, 2, 3$ , etc., and  $1_1, 2_1, 3_1$ , etc., on the corresponding generators in the development.
6. Join the points  $1, 2, 3$ , etc., and  $1_1, 2_1, 3_1$ , etc., by smooth curves, representing the two openings of the hole in the development. Darken the edges corresponding to the retained portion of the cube.



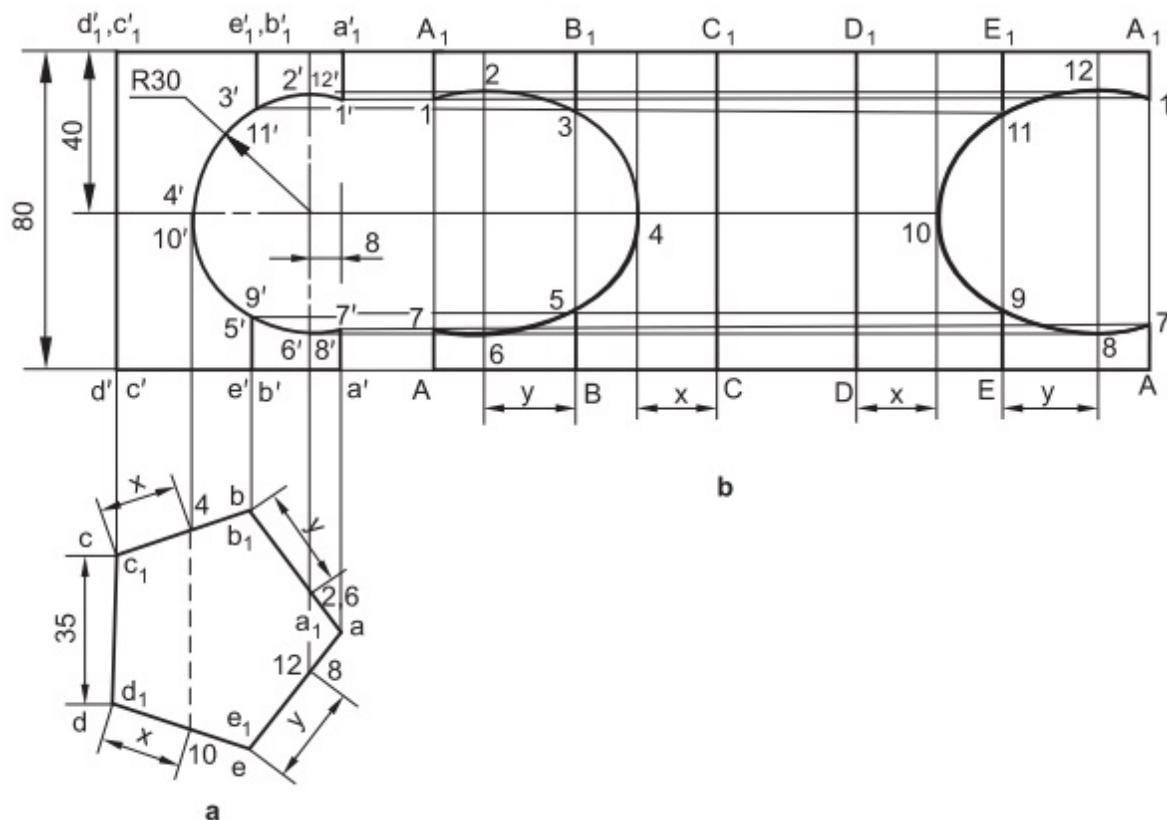
**Fig.14.22**

**Problem 22** *Figure 14.23a shows the projections of a pentagonal prism, with a partial hole as indicated. Draw the development of the prism.*

**Construction (Fig.14.23b)**

1. Draw the development  $AA_1 - A_1A$  of the complete prism, following the stretchout line principle.
2. Locate the points of intersection between the hole and edges of the prism.
3. Transfer the above points on to the development, by projection.
4. To get a good curve, consider some more generators, passing through the critical points of the hole, such as  $2' (12')$ ,  $4' (10')$  and  $6' (8')$ .
5. Locate the points  $2 (12)$ ,  $4 (10)$  and  $6 (8)$  on the development, measuring the distances from the edges in the top view.

A smooth curve through the points 1, 2, 3, etc., is the shape of the hole in the development.

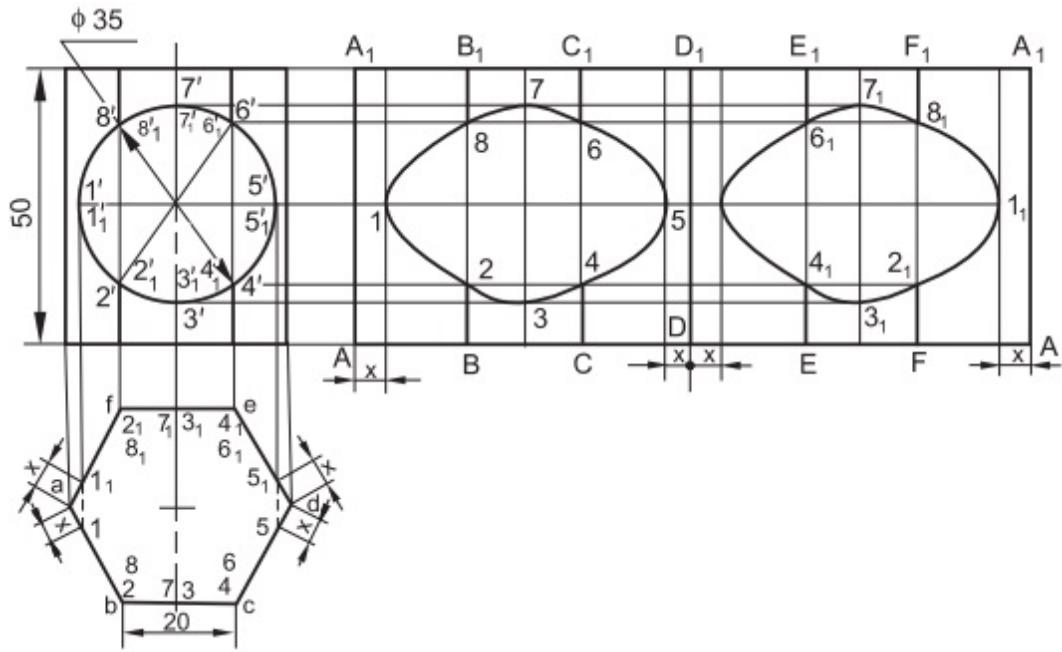


**Fig.14.23**

**Problem 23** A hexagonal prism of 20 side of base and 50 height, rests on a base on H.P. with a vertical face parallel to V.P. A circular hole of 35 diameter, is drilled through the prism such that, the axis of the hole bisects the axis of the prism and is perpendicular to V.P. Draw the development of the prism.

**Construction (Fig.14.24)**

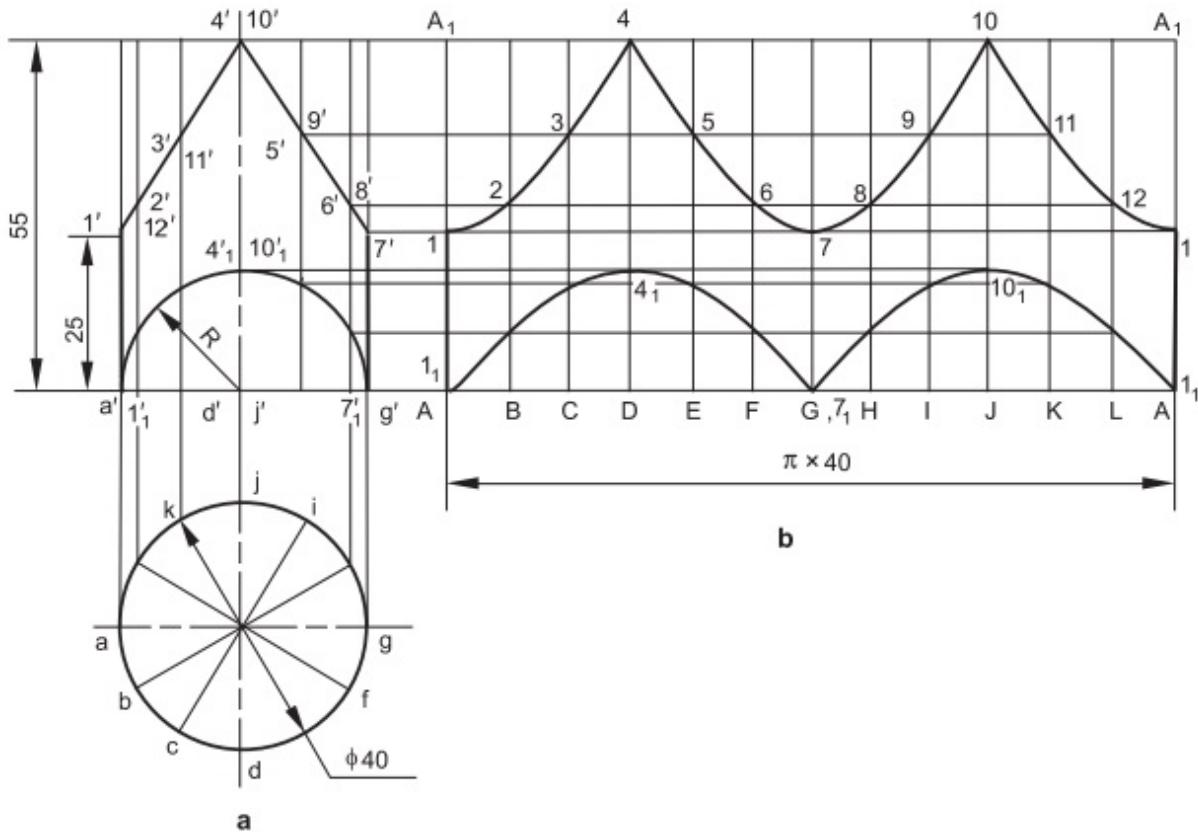
1. Draw the projections of the prism with the hole.
2. Draw the development AA<sub>1</sub>-A<sub>1</sub>A of the complete prism, following the stretchout line principle.
3. Divide the circle in the front view into a number of parts such that, certain points lie on the longer-edges of the prism. Also, locate the transition point 1', 5' and 1'\_1, 5'\_1, and obtain the corresponding points in the top view.
4. Draw horizontal lines from 1' (1'\_1), 2'(2'\_1), etc., and locate the points 1, 2, 3, etc., and 1<sub>1</sub>, 2<sub>1</sub>, 3<sub>1</sub>, etc., on the corresponding edges (lines) in the development.
5. Join the points 1, 2, 3, etc., and 1<sub>1</sub>, 2<sub>1</sub>, 3<sub>1</sub>, etc., by smooth curves, representing the two openings of the hole in the development.
6. Darken the edges corresponding to the retained portion of the prism and complete the development.



**Fig.14.24**

**Problem 24** Draw the development of the lateral surface of the cut cylinder, shown in Fig.14.25a.

**Construction (Fig.14.25b)**

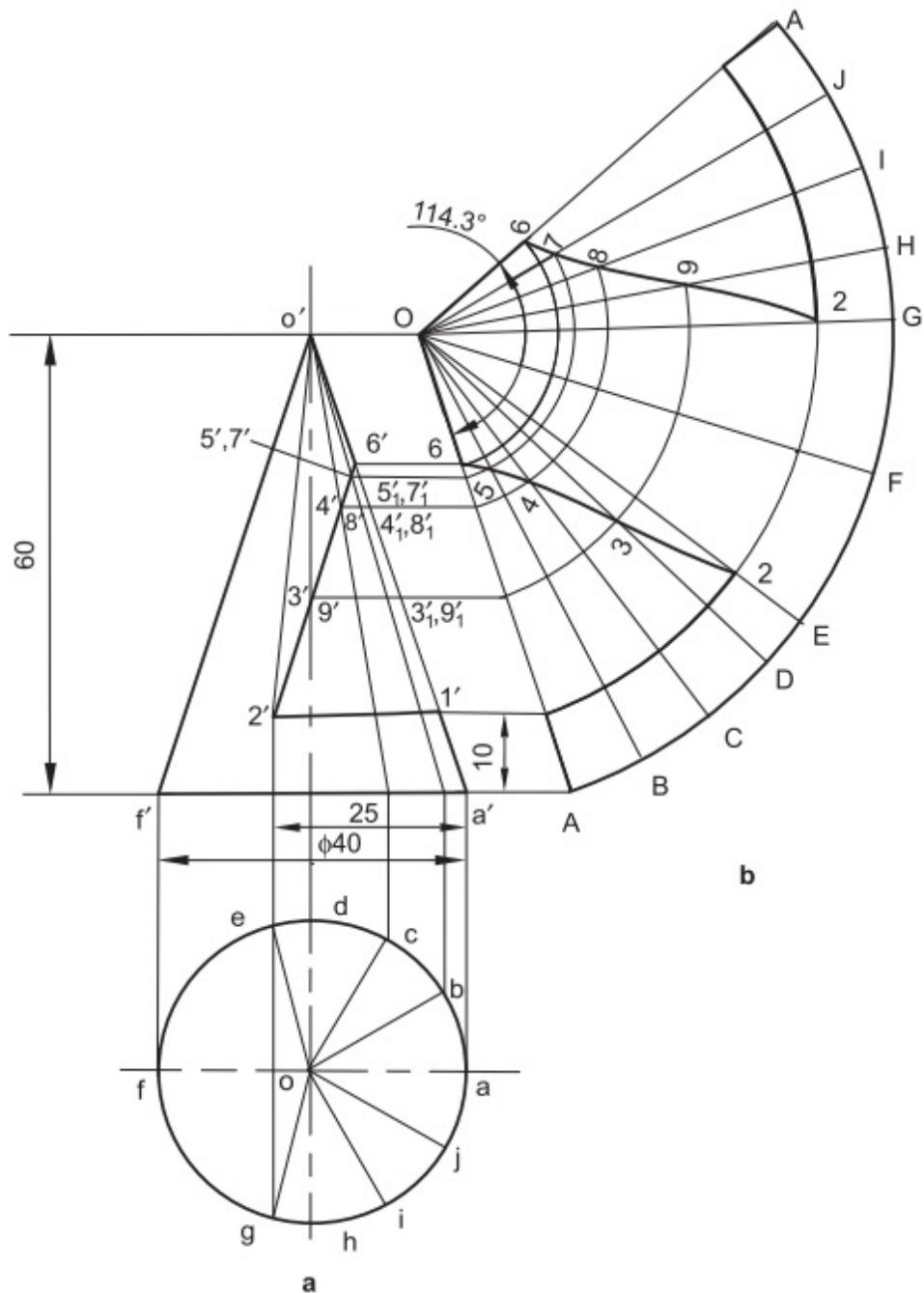


**Fig.14.25**

Follow the principle of Construction: [Fig.14.11](#) suitably and obtain the development as shown in [Fig.14.25b](#).

**Problem 25** Draw the development of the lateral surface of the retained portion of the cone, shown in [Fig.14.26a](#).

**Construction ([Fig.14.26b](#))**



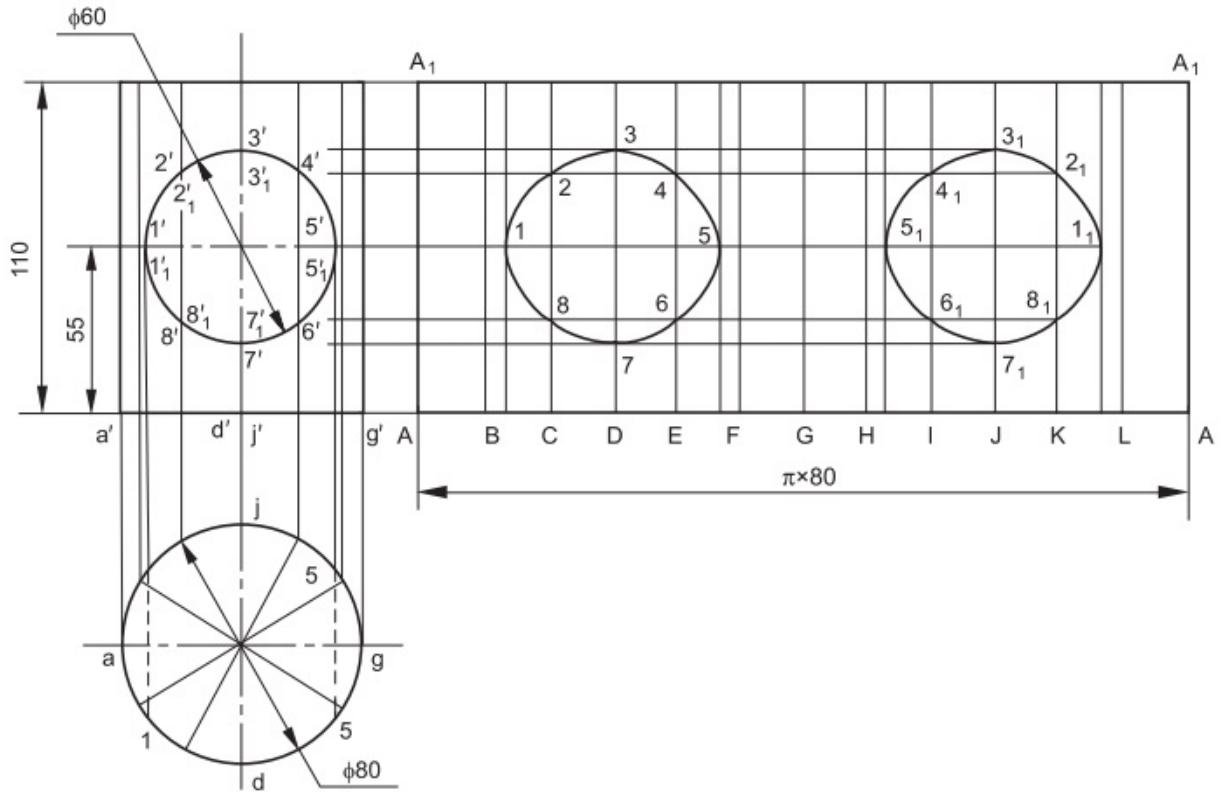
**Fig.14.26**

1. Draw the projections of the cone, indicating the cut portion in the front view.
2. Locate a number of generators in the top view which will cover the cut portion in the front view.

3. Project and obtain the corresponding generators in the front view.
4. Draw the development of the complete cone, following Construction: [Fig.14.21](#).
5. Locate the points of intersection between the cut portion and the generators,  $1'$ ,  $2'$ ,  $\dots$ ,  $9'$  in the front view.
6. Determine the true lengths of  $o'2'$ ,  $o'3'$ , etc., by drawing horizontal lines to the extreme generator.
7. Transfer these true lengths on to the corresponding generators on the development.
8. Join the points  $1$ ,  $2$ , etc., by smooth curves as shown and obtain the required development of the cut cone.

**Problem 26** A cylinder of base 80 diameter and axis 110 long, is resting on its base on H.P. It has a circular hole of 60 diameter, drilled centrally through such that, the axis of the hole is perpendicular to V.P and bisects the axis of the cylinder at right angle. Develop the lateral surface of the cylinder.

**Construction** ([Fig.14.27](#))



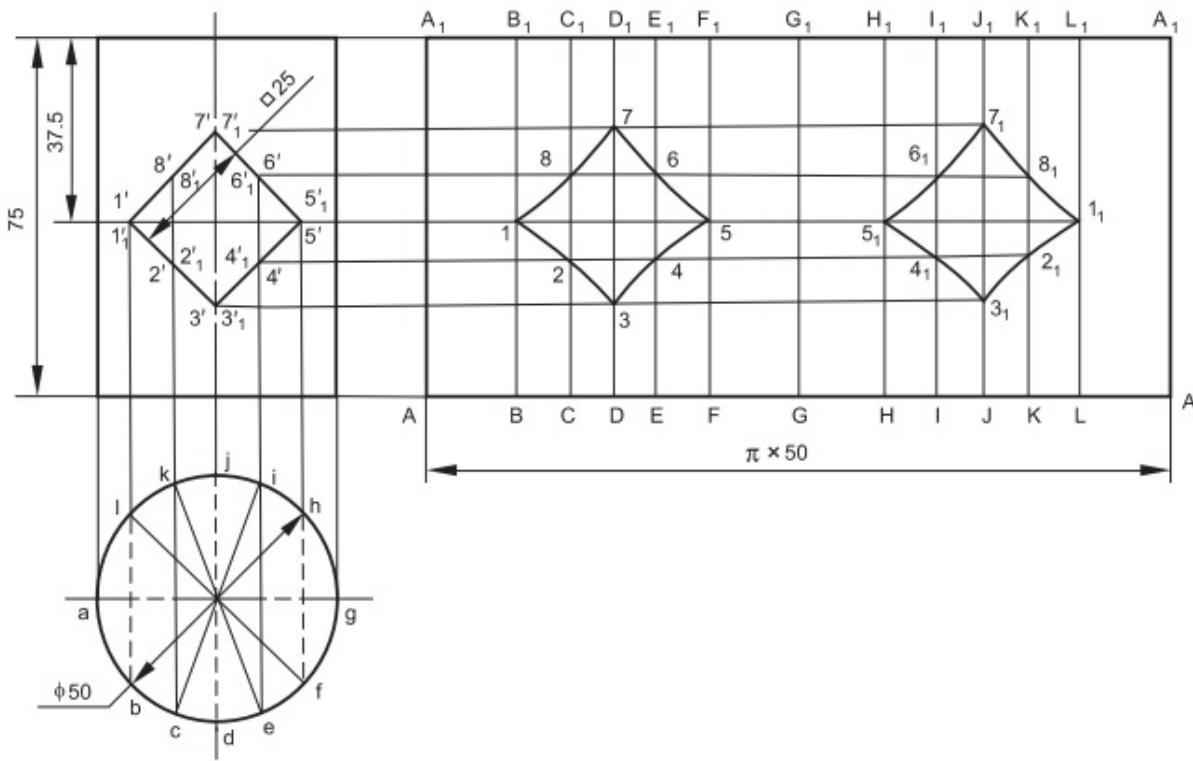
**Fig.14.27**

1. Draw the projections of the cylinder, with the hole through it.
2. Divide the circle (top view of the cylinder) into a number of equal parts, say 12 and draw the corresponding generators in the front view.
3. Obtain the complete development  $AA_1-A_1A$  of the cylinder and draw the generators on it.
4. Determine the points of intersection  $1'$ ,  $2'$ , etc., and  $1_1'$ ,  $2_1'$ , etc., between the hole and generators in the front view.
5. Transfer these points on to the development by projection, including the transition points  $1'$  ( $1_1'$ ) and  $5'$  ( $5_1'$ ).

6. Join the points 1, 2, etc., and  $1_1$ ,  $2_1$ , etc., by smooth curves and obtain the two openings in the development.

**Problem 27** Draw the development of a cylinder of 50 diameter and 75 height, containing a square hole of 25 side. The sides of the hole are equally inclined to the base and the axis of the hole bisects the axis of the cylinder.

**Construction (Fig.14.28)**



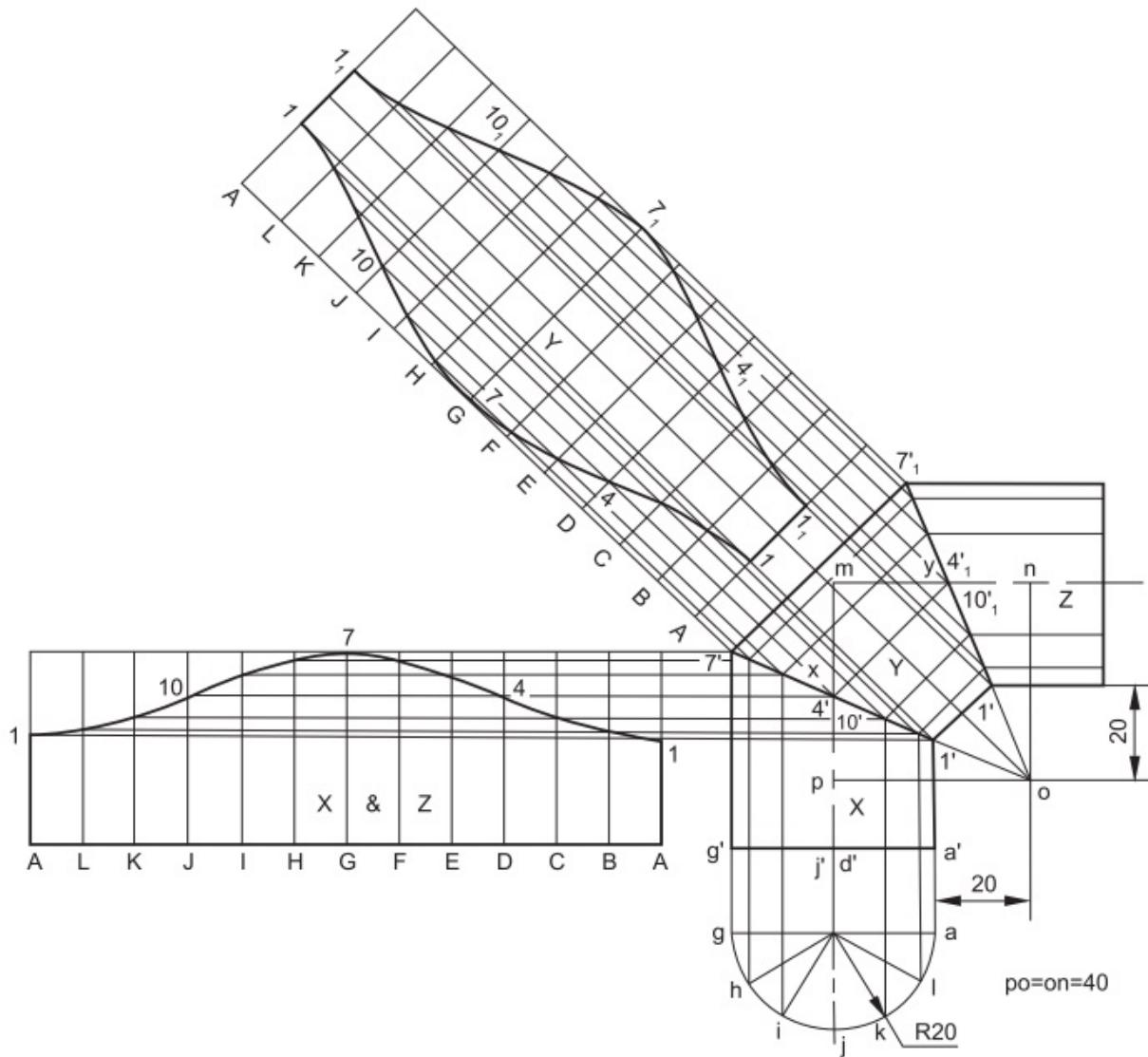
**Fig.14.28**

1. Draw the projections of the cylinder, with the hole.
2. Draw number of generators in the front view, passing through the chosen points  $1'$  ( $1_1'$ ),  $2$  ( $2_1$ ), etc., on the edges of the hole.
3. Locate the corresponding generators in the top view.

4. Obtain the full development  $AA_1 - A_1A$  of the complete cylinder and draw the chosen generators on it.
5. Transfer the points  $1', 2', 3'$ , etc., and  $1_1', 2_1', 3_1'$ , etc., to the development, by projection.
6. Join the points  $1, 2, 3$ , etc., and  $1_1, 2_1$ , etc., suitably and obtain the two openings in the development.

**Problem 28** *Develop the lateral surfaces of the three-piece pipe elbow, shown in Fig.14.28. Take the diameter of the pipe as 40.*

**Construction (Fig.14.29)**



**Fig.14.29**

1. Draw the projections of the given elbow.

*To draw the front view:*

- (i) Draw a square pmno of side equal to the diameter of the pipe.
- (ii) Bisect  $\angle pom$  and  $\angle mon$ , intersecting pm at x and mn at y. Then, xy is the axis of the middle piece Y.

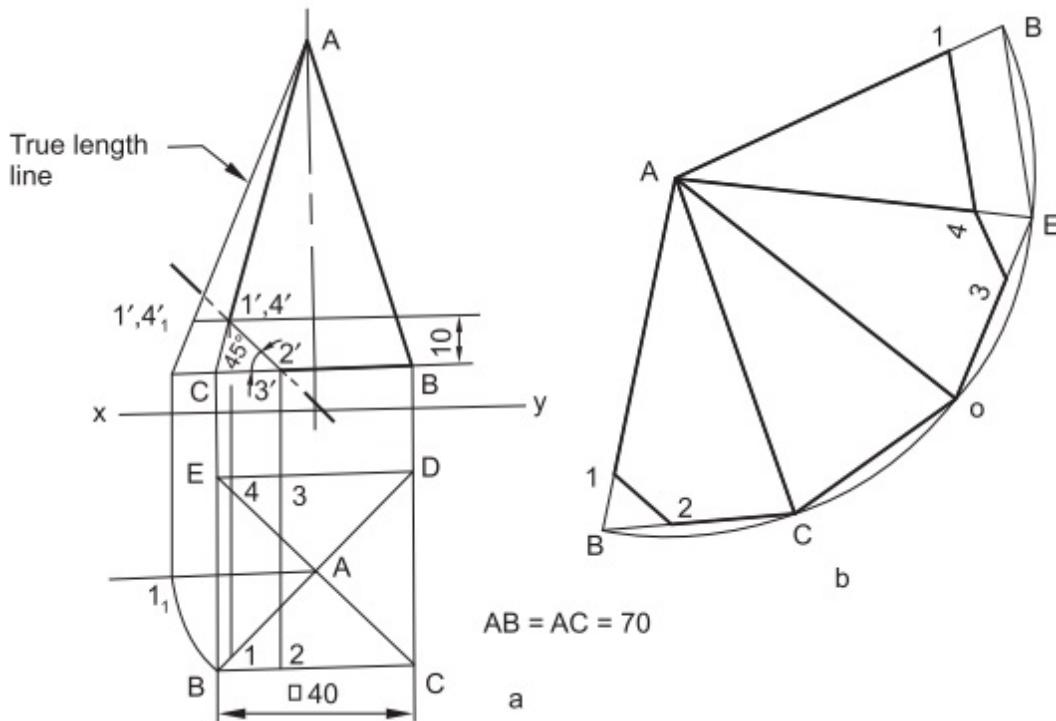
**HINT** 1. Parts X and Z are similar and are truncated cylinders. Part Y is a cylinder truncated at both the

ends.

2. Draw the development of the pieces X (Z) and Y, by using the principle of stretch-out line.

**Problem 29** The front view of a square pyramid is an isosceles triangle ABC with BC parallel to the ground line and  $BC = 40$ ,  $AB = AC = 70$ . The section plane appears as a straight line inclined at  $45^\circ$  to the base BC. The edge view of the section plane intersects AC at a height of 10 above BC. The edge view of the section plane leans towards the left. Draw the development of the surface of the object.

**Construction (Fig.14.30)**



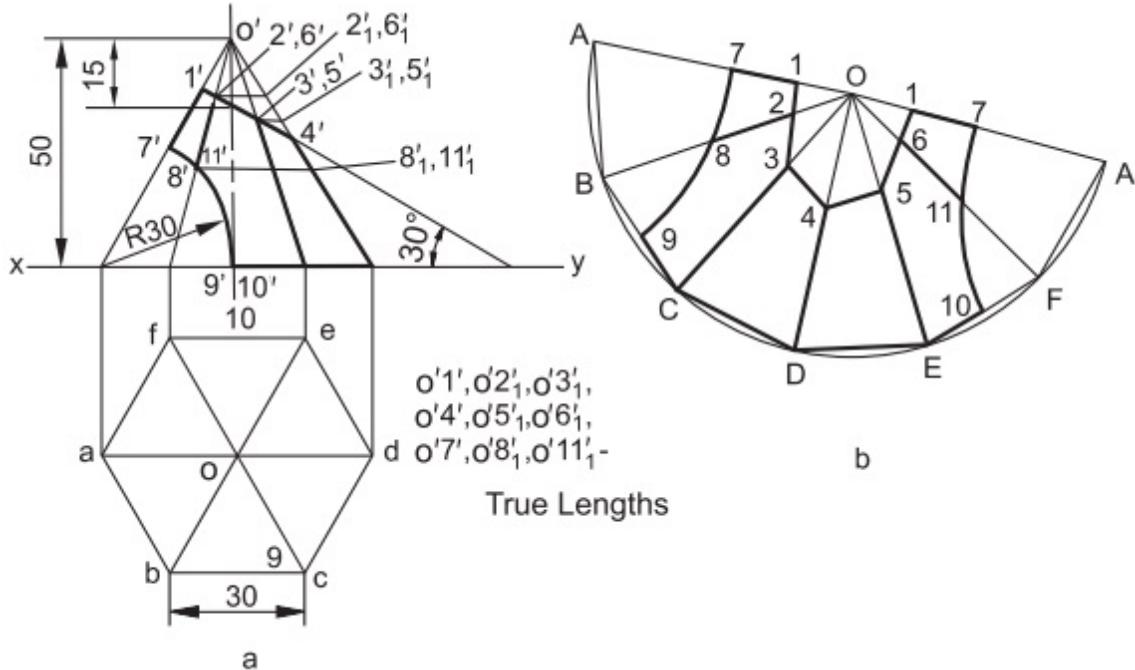
**Fig.14.30**

1. Draw the projections of the pyramid, satisfying the given conditions.

2. Draw the V.T of the section plane, satisfying the given conditions.
3. Locate the points of intersection 1', 2', 3' and 4' between the V.T and the edges of the pyramid.
4. Determine the true length of the slant edge of the pyramid.
5. With A as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
6. Follow steps 4 and 5 of Construction: [Fig.14.8](#) and obtain the full development of the pyramid.
7. Obtain the true lengths of the slant edges of the cut pyramid, by projecting on to the true length line.
8. Transfer the above true lengths as well as the points 2 and 3 to the corresponding edges in the development.
9. Join the points 1, 2, etc., by straight lines and obtain the development of the cut pyramid.

**Problem 30** *Draw the development of the hexagonal pyramid, shown in [Fig.14.31a](#).*

**Construction ([Fig.14.31](#))**



**Fig.14.31**

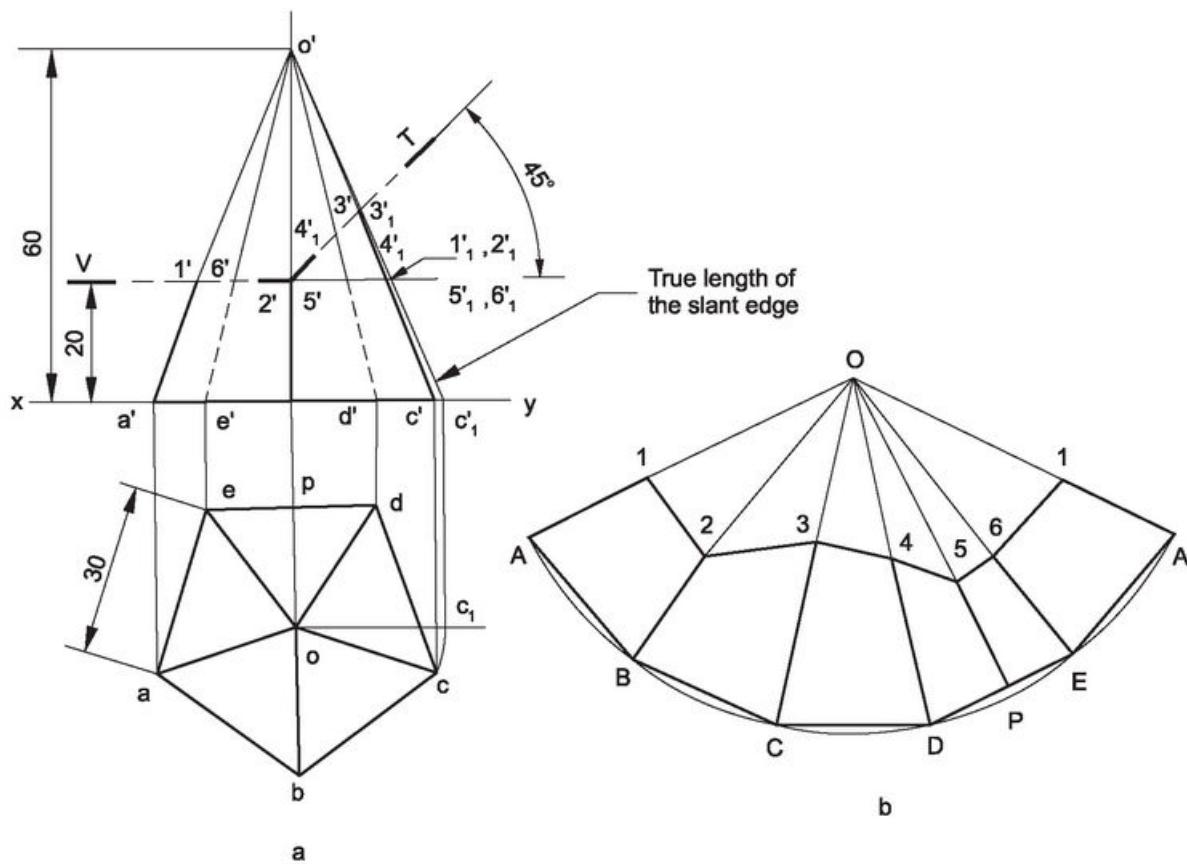
1. Draw the given projections of the cut hexagonal pyramid.
2. Locate the points of intersection 1', 2', 3', etc., between the cut sections and the edges of the pyramid.
3. Determine the true distances of the points 2', 3', 5', 6', 8' and 11' from o' in the front view by projecting on to the true length line. o'1', o'7' and o'4' are the true distances, as the points 1', 7' and 4' lie on the true length lines. The points 9 and 10 lie at the mid-points of the edges BC and EF respectively.
4. With any point o as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
5. Follow steps 4 and 5 of Construction: [Fig.14.8](#) suitably and obtain the full development of the pyramid.
6. Using the true lengths, transfer the above points to the corresponding edges in the development.

7. Join the points in the order and obtain the development of the cut pyramid.

**Problem 31** Draw the development of the lateral surface of the cut pentagonal pyramid, shown in Fig.14. 32a.

**Construction (Fig.14.32b)**

Follow Construction: Fig.14.9 and obtain the development as shown in Fig.14.32b.

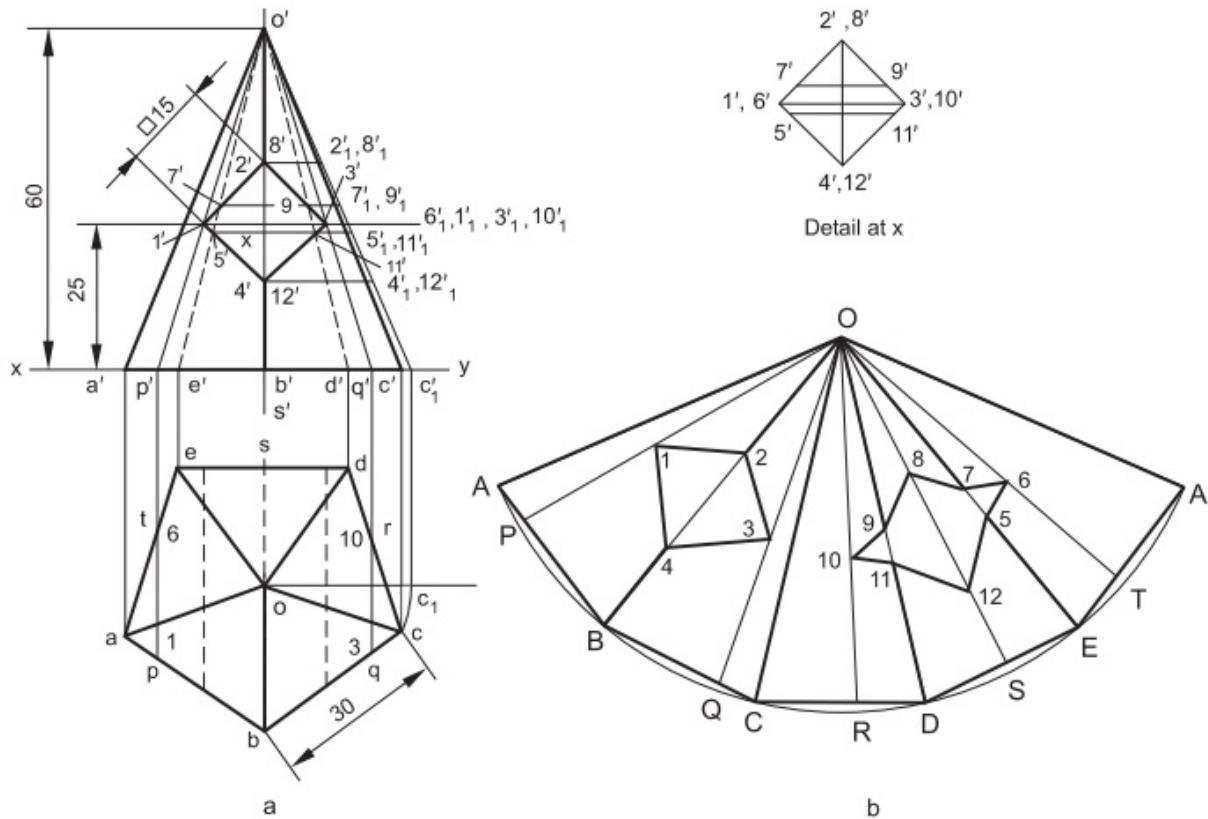


**Fig.14.32**

**Problem 32** Figure 14.33a shows the projections of a pentagonal pyramid with a square hole through it. Draw the development of the pyramid.

**Construction (Fig.14.33)**

- Locate the points of intersection between the edges of
- the hole and the slant edges of the pyramid in the front view.
  - Draw the full development of the pyramid, following the radial line development method.
  - Transfer the above points on to the corresponding lines on the development (ref. Construction: [Fig.14.9](#)).
  - Draw the generators through the critical points  $1'$  ( $6'$ ) and  $3'$  ( $10'$ ).
  - Transfer the above points, after locating the corresponding generators on the development.
  - Join the points in the order by straight lines, obtaining the two openings in the development.

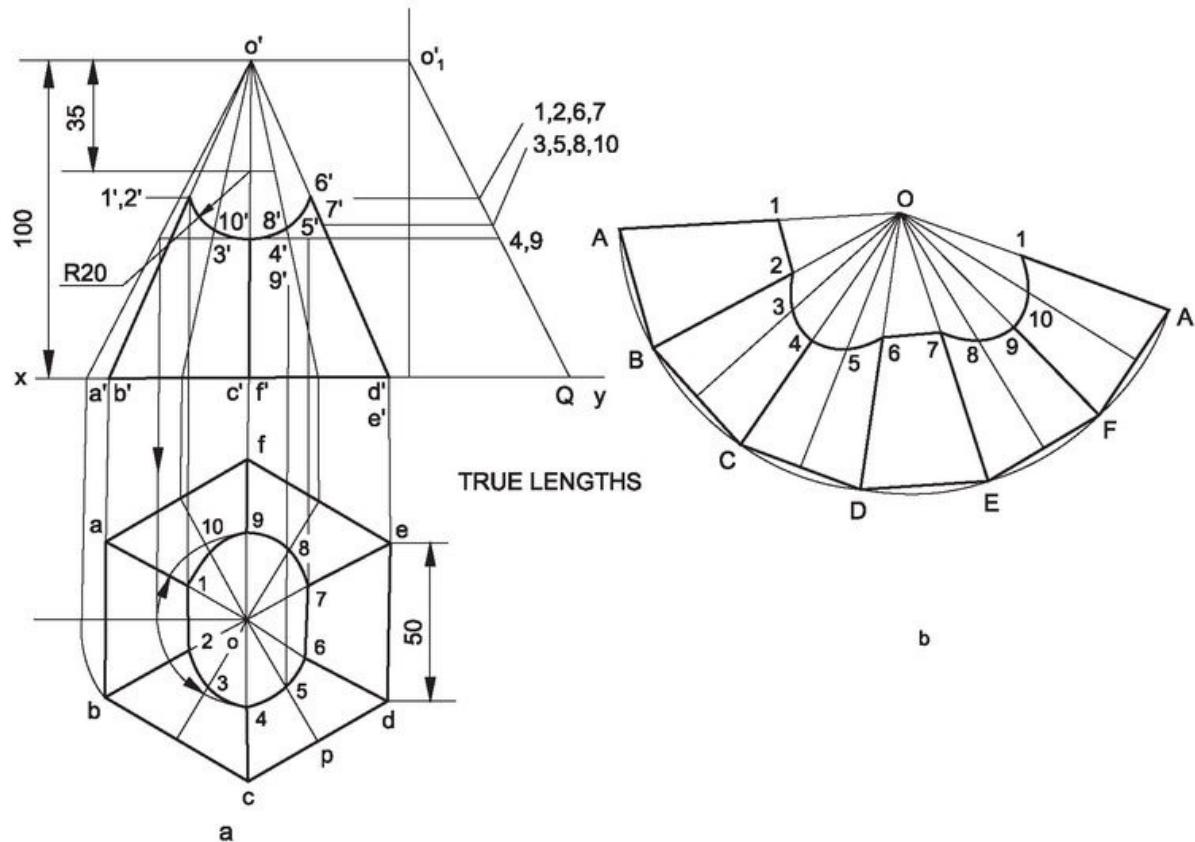


**Fig.14.33**

**Problem 33** A hexagonal pyramid of side of base 50 and axis 100 long, stands on its base on H.P with one of its base edges perpendicular to V.P. A hole of 40 diameter is drilled through the solid. The axis of the hole is perpendicular to V.P and intersects the axis of the pyramid, at a point 35 below the apex. Draw the development of the lateral surface of the solid.

**Construction (Fig.14.34)**

1. Draw the projections of the pyramid with the slot formed after drilling the hole.
2. Locate the points of intersection 1', 2', etc., between the slant edges and the surface of the slot.
3. Locate the above points in the top view and join suitably, as shown.
4. Determine the true lengths of the slant edges o'1', o'2', etc., of the retained solid, by drawing horizontal lines from 1', 2', etc., to the true length line o<sub>1</sub>d<sub>1</sub>.
5. Draw the development of the complete pyramid, following the radial line development method.
6. Locate the points 1, 2, etc., on the corresponding slant edges in the development, using the true lengths from step 4.
7. Join the points in the order suitably and complete the development of the retained solid.



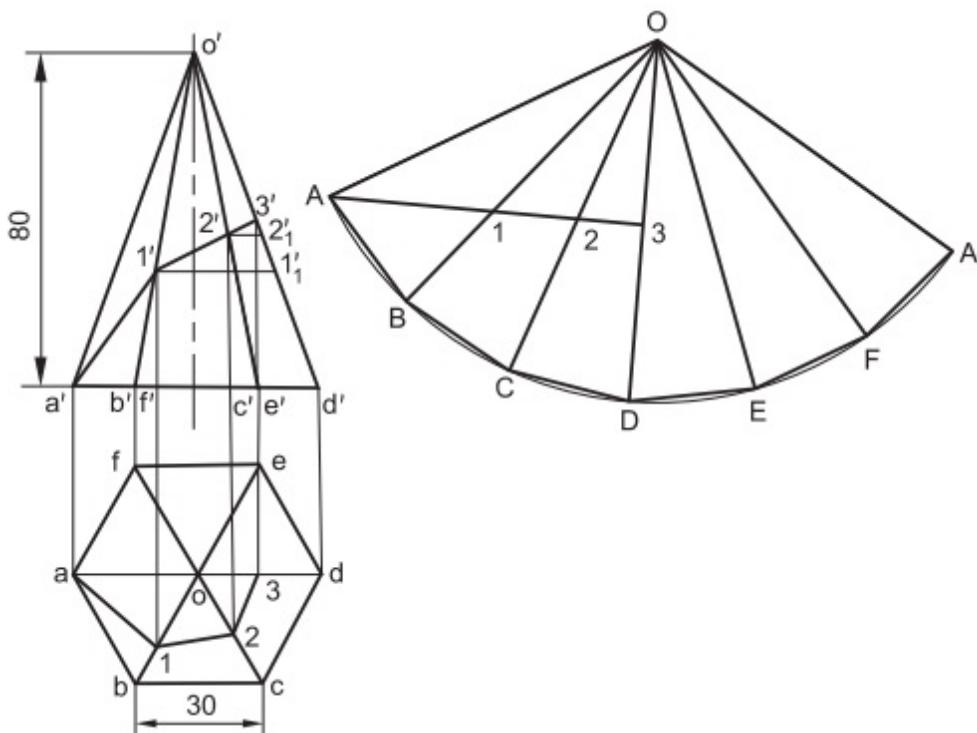
**Fig.14.34**

**Problem 34** A hexagonal pyramid of side of base 30 and length of axis 80, rests on its base on H.P. An ant, initially situated at the left extreme corner of the base, moves on the surface of the pyramid and reaches the mid-point on the right extreme edge (slant edge). Find the shortest path of travel of the ant. Show the path in front and top views.

**Construction (Fig.14.35)**

1. Draw the projections of the pyramid and its development.
2. Draw the path of the ant A-3 in the development; the point 3 being the mid-point of OD.
3. Locate the points 1 and 2 in the development, on the edges OB and OC.

4. Transfer the points 1, 2 and 3 on to both the projections ( $o'3' = O_3$ ,  $o'2_1' = O_2$  and  $o'1_1' = O_1$ ).
5. Join the points in the order by straight lines, in both the projections.



**Fig.14.35**

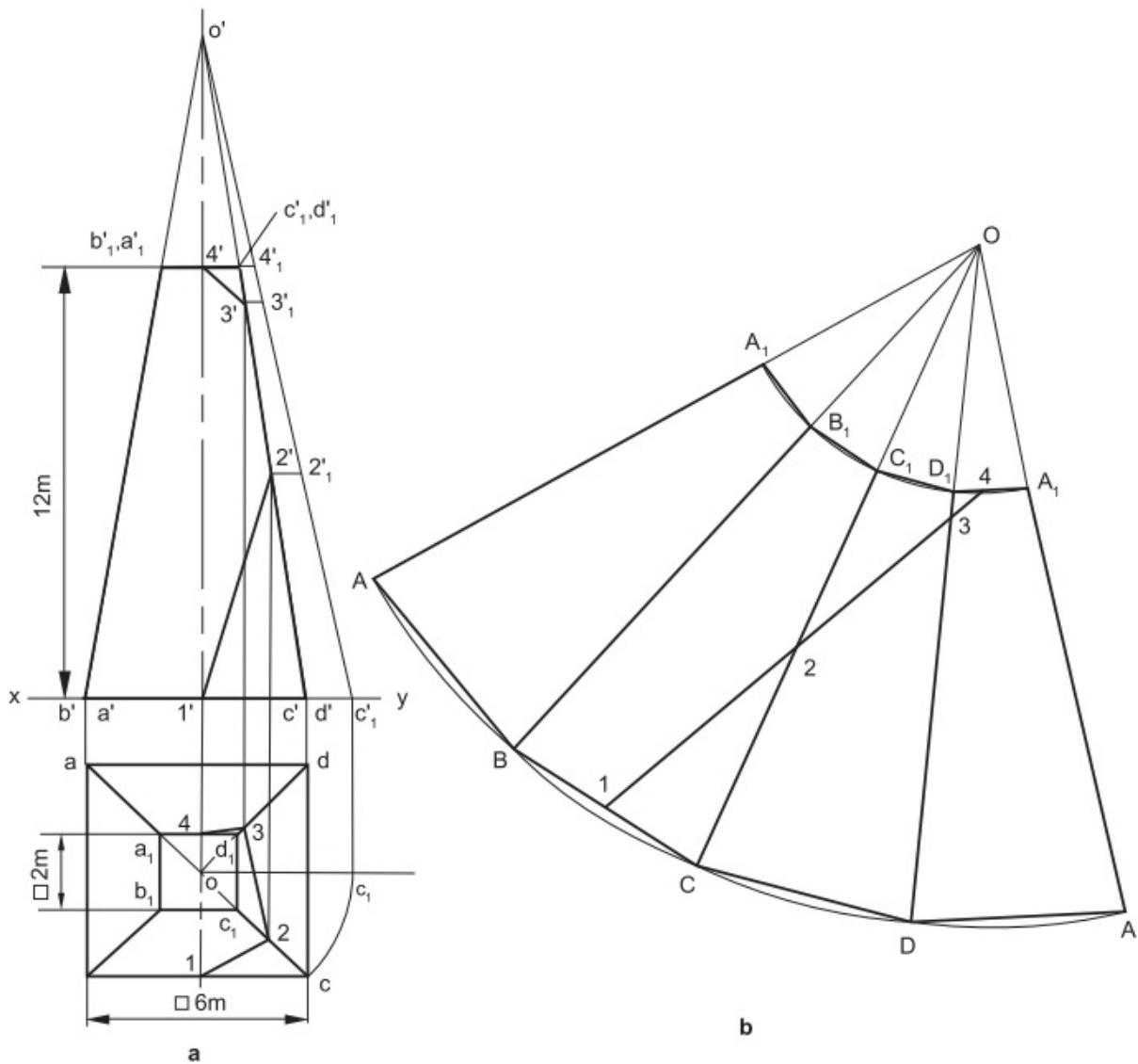
**Problem 35** A flume in the form of a frustum of a pyramid, has the base 6 m square, top 2m square and altitude 12 m. A lightening conductor is taken from the mid-point of one of the top edges to the mid-point of the opposite edge in the base. Determine the shortest length and draw the projections of its path.

**Construction (Fig.14.36)**

1. Draw the projections of the frustum of the pyramid and its development.
2. Locate the position of the conductor in the development such that, its ends 1 and 4 lie at the mid-

points of the bottom (BC) and top ( $d_1 A_1$ ) edges of any two alternate faces.

3. Locate the points 2 and 3 on the edges  $C-C_1$  and  $D-D_1$  respectively, in the development.
4. Transfer the points 1, 2, 3 and 4 on to both the projections ( $o-2 = o'2'_1$ ,  $o-3 = o'3'_1$ ).
5. Join the points in the order by straight lines, in both the projections.

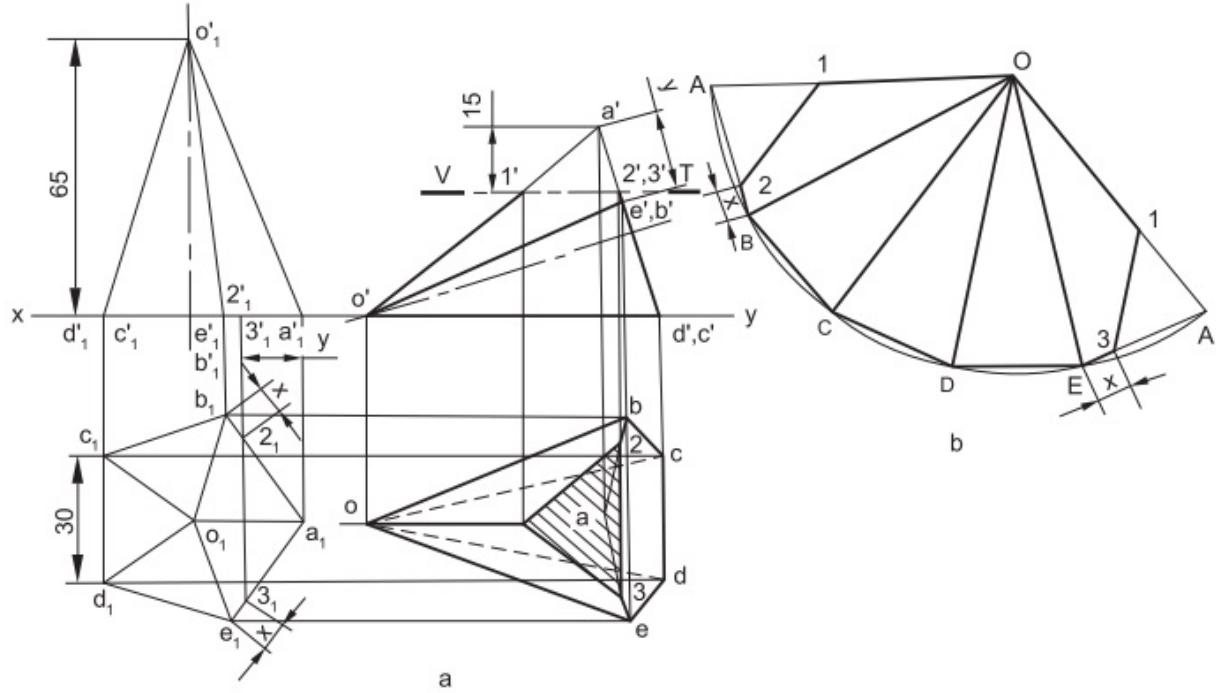


**Fig.14.36**

**Problem 36** A pentagonal pyramid with side of base 30 and length of axis 65, rests on one of its triangular faces on H.P. A horizontal section plane passes through the solid at 15 from the upper corner of the pyramid. Draw the projections and development of the remaining solid.

**Construction (Fig.14.37)**

1. Draw the projections of the pyramid, assuming it to be lying on its base on H.P, with one edge of the base, perpendicular to V.P.
2. Redraw the front view such that, the front view of the face  $o'd'c'$  coincides with  $xy$ .
3. Obtain the top view, by projection.
4. Locate the V.T of section plane, which is parallel to  $xy$  and at 15 from  $a'$ .
5. Locate the points of intersection  $1'$ ,  $2'$  and  $3'$  between the V.T and edges of the pyramid.
6. Project and locate the points 1, 2 and 3 on the corresponding edges in the top view and complete the sectional top view.
7. Draw the development of the remaining solid, as shown in Fig.14.37b ( $O_1 = o'1'$ ).



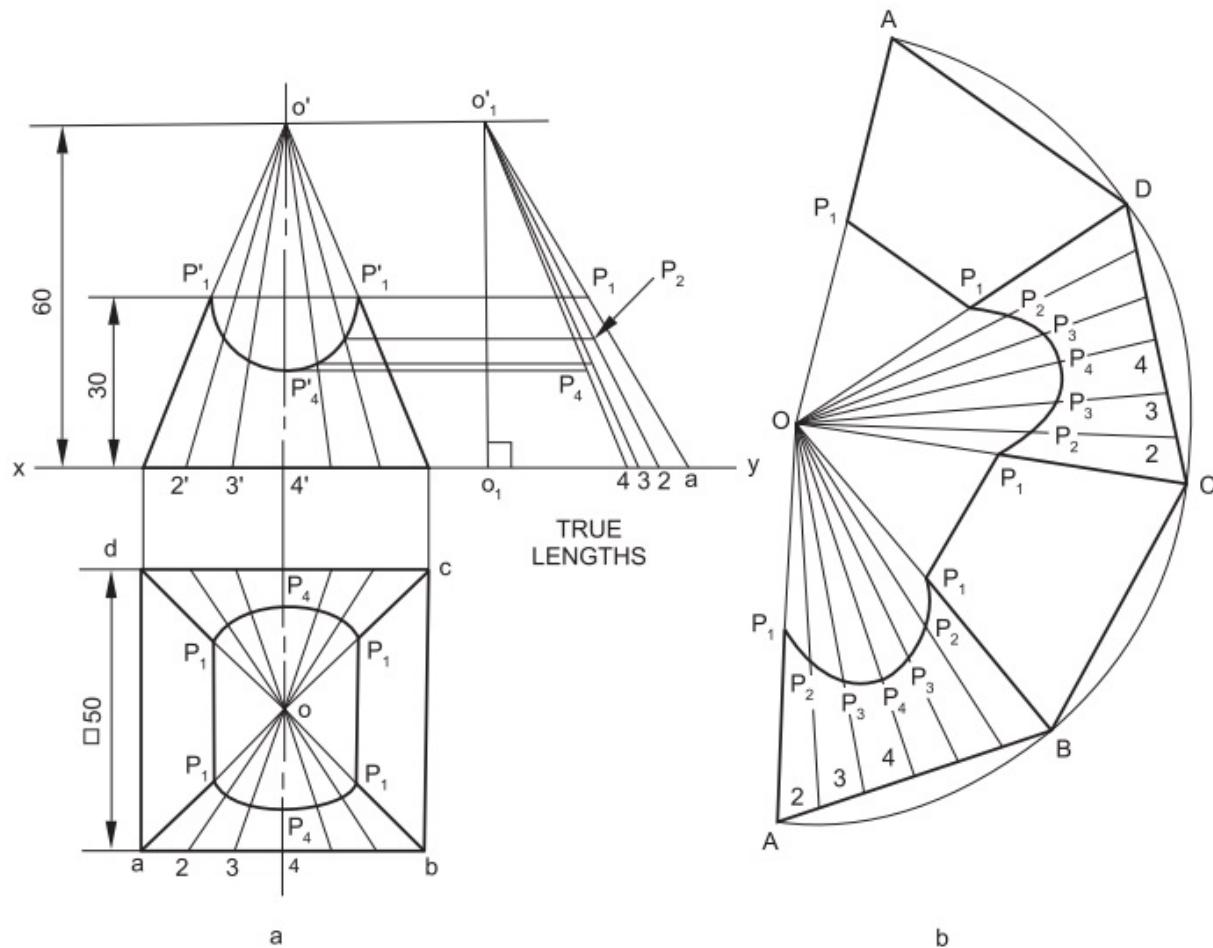
**Fig.14.37**

**Problem 37** *Figure 14.38a shows two views of a frustum of a square pyramid, with a semi-circular slot through it. Draw the development of the lateral surface of the solid.*

**Construction (Fig.14.38b)**

1. Locate a number of generators in both the views.
2. Locate the points of intersection  $p_1'$ ,  $p_2'$ , etc., between the generators and the surface of the slot in the front view.
3. Locate the above points in the top view, by projection and join the points by smooth curve.
4. Construct the true length diagram and determine the true lengths of the generators.
5. Draw horizontal lines from  $p_1'$ ,  $p_2$ , etc., to the true length lines and determine the true distances of these points from  $o_1'$ .

6. Draw the development of the complete pyramid, following the radial line development method.
7. Locate the generators in the development.
8. Locate the points  $P_1$ ,  $P_2$ , etc., in the development, on the corresponding generators, using the information from step 5.
9. Join the points related to the slot, by smooth curves and represent the other two top edges by straight lines.



**Fig.14.38**

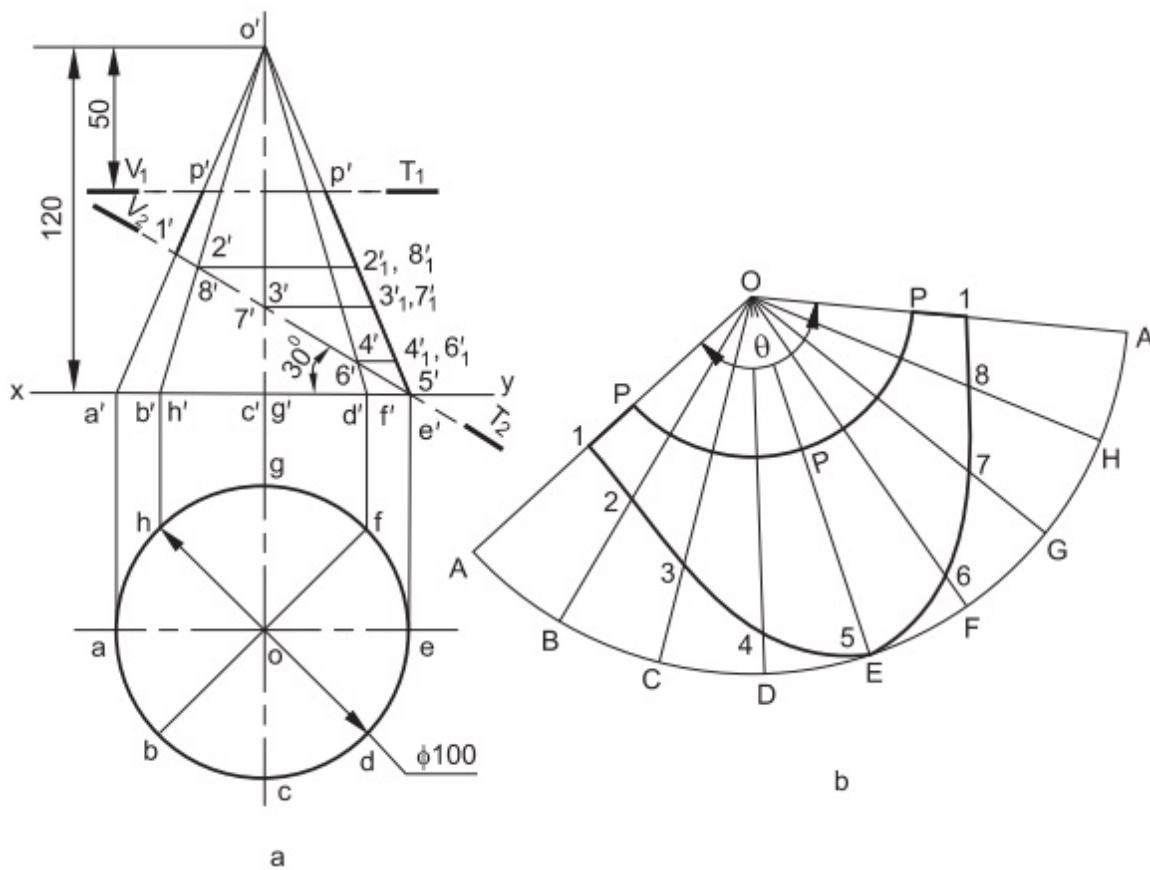
**Problem 38** *Figure 14.39a shows the projections of a cone, cut by two section planes, as indicated by the traces. Draw*

the development of the cut cone.

### **Construction (Fig.14.39)**

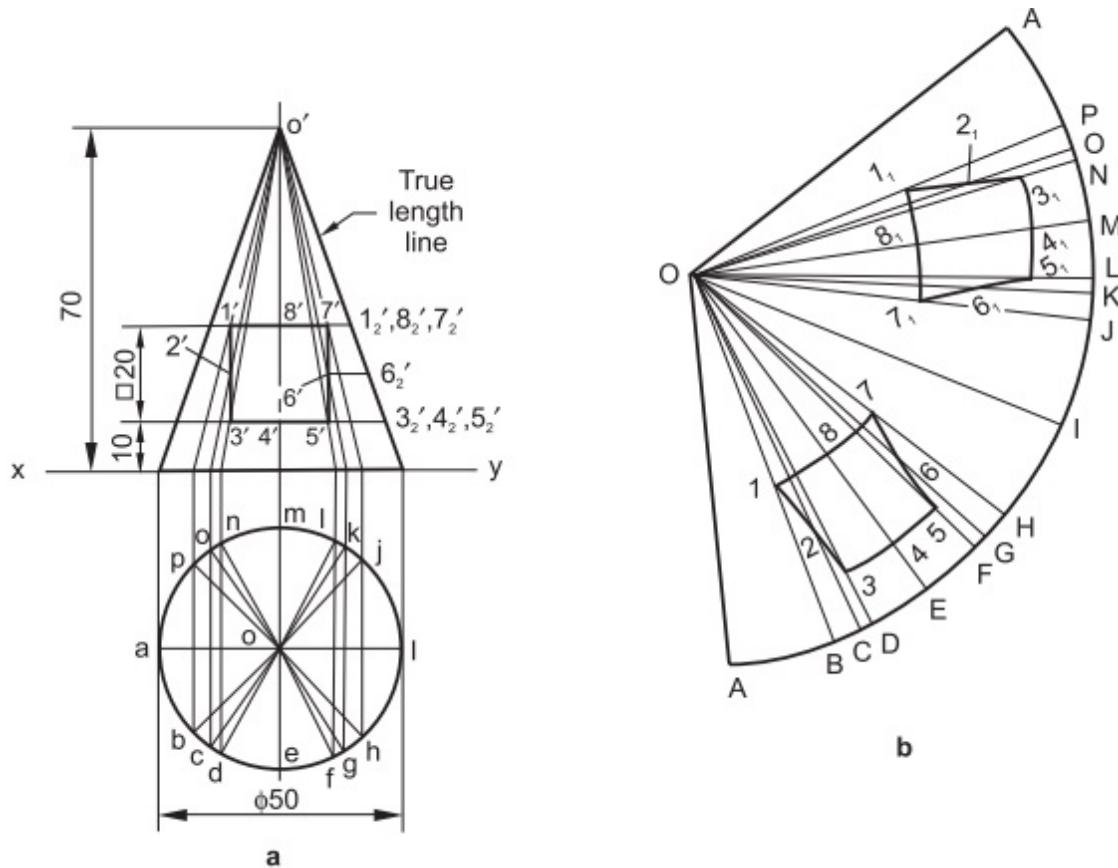
Follow the principle of Construction: Fig.14. 25 and obtain the development as shown in Fig.14.39b.

**Problem 39** A cone of base diameter 50 and height 70, is resting on its base on the ground. A square hole of 20 side passes through the object. The axis of the hole and the cone intersect at right angle to each other. One of the edges of the hole is parallel to and 10 above the base. Draw the development of the surface of the object.



**Fig.14.39**

### **Construction (Fig.14.40)**



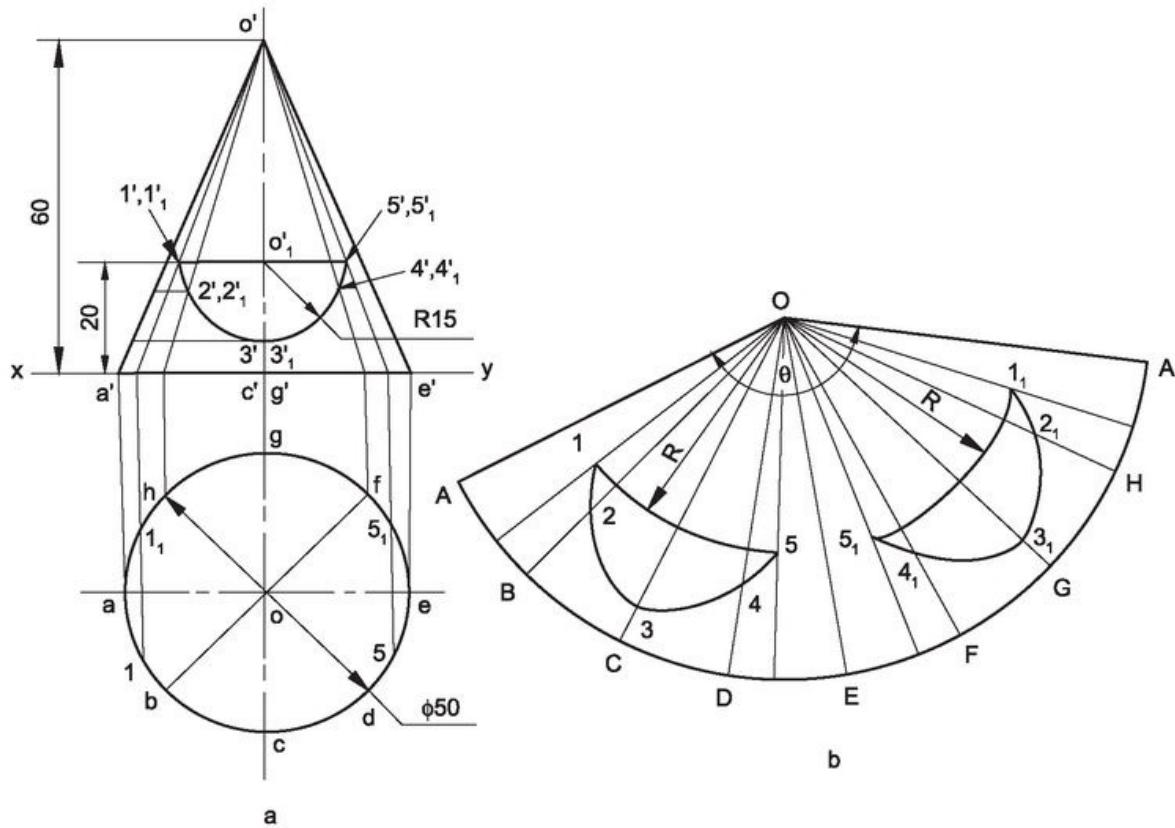
**Fig.14.40**

1. Draw the projections of the cone, with the square hole through it.
2. Locate a number of points 1', 2', 3', etc., on the edges of the hole in the front view and draw generators passing through them.
3. Locate these generators in the top view.
4. Draw horizontal lines from 1', 2', 3', etc., to the true length line and determine the distances of these points from o'.
5. Draw the development of the cone, by radial line development method.

6. Locate the generators in the development which are present in the top view.
7. Locate the points 1, 2, 3, etc., and  $1_1, 2_1, 3_1$ , etc., in the development, on the corresponding generators, using the information from step 4.
8. Join the points by smooth curves and obtain the two openings in the development.

**Problem 40** A right circular cone of base 50 diameter and axis 60 long, is resting on its base on H.P. A semi-circular hole of radius 15 is cut through the cone such that, the axis of the hole is perpendicular to VP and intersecting the axis of the cone at 20 above the base. The flat surface of the hole is parallel to H.P. Draw the development of the lateral surface of the cone.

**Construction (Fig.14.41)**



**Fig.14.41**

1. Draw the projections of the cone, with the hole through it.
2. Divide the circle (top view) into equal parts, say 8 and obtain the corresponding generators in the front view.
3. Locate the points of intersection 2' ( $2'_1$ ), 3' ( $3'_1$ ), etc., between the generators and the hole, including the transition points 1' ( $1'_1$ ) and 5' ( $5'_1$ ).
4. Draw the development of the cone, by radial line development method.
5. Transfer the above points on to the development, after determining the true distances of the points from the apex.

- Join these points by smooth curves and obtain the two openings in the development.



As the top surface of the hole is parallel to the base, the points 1, 5 and  $1_1, 5_1$  are connected by circular arcs with centre O and radius equal to O-1.

**Problem 41** In a semi-circular plate of 120 diameter, a largest circular hole is made. The plate is folded to form a cone. Draw the two views of the cone.

### Construction (Fig. 14.42)



The slant height s of the cone, formed after folding the plate is 60 and the radius r of the base of the cone is calculated from,

$$\theta = 180^\circ = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

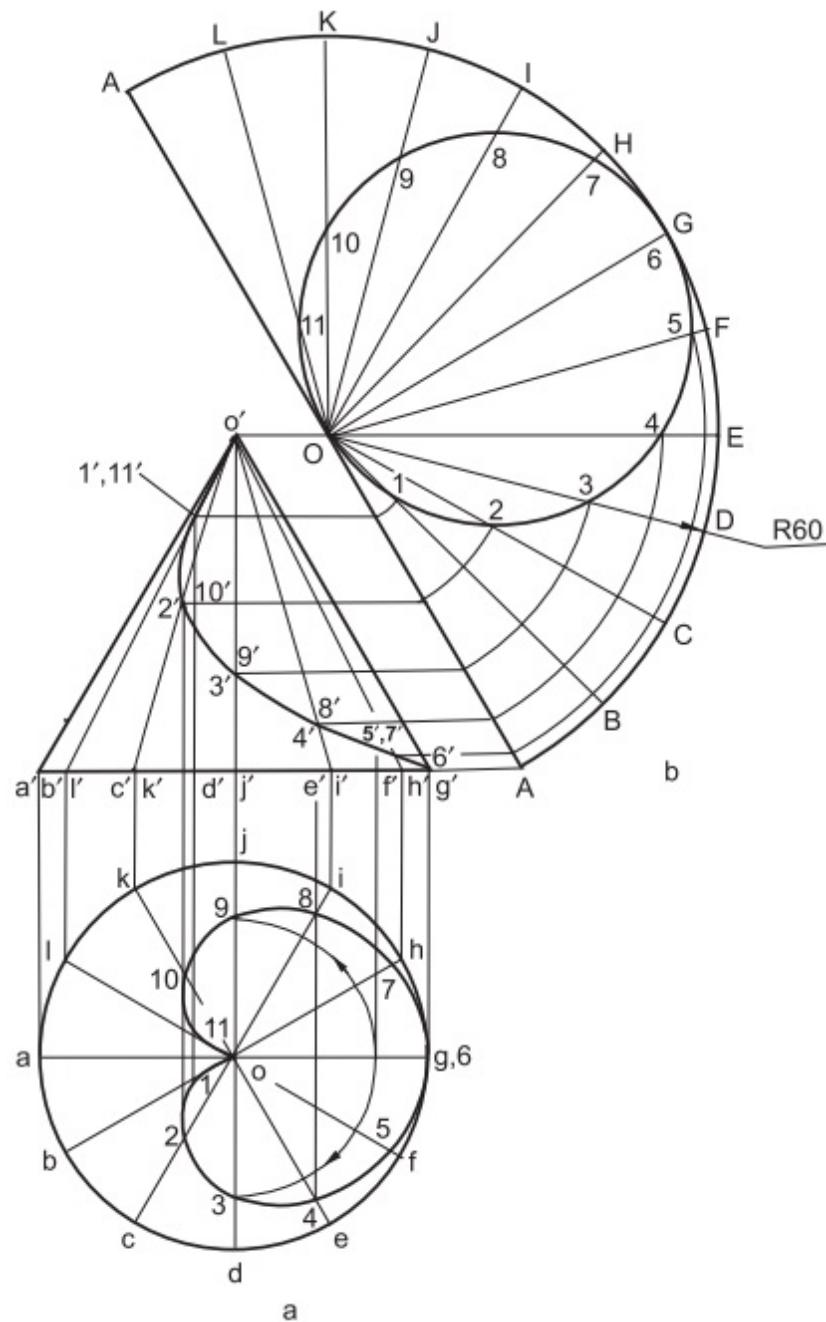
$$= 360^\circ \times \frac{r}{s} = \frac{360^\circ \times r}{60}$$

$$\therefore r = 30\text{mm}$$

- Draw the projections of the cone, with the radius of the base 30 and slant height 60.
- Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in both the views.
- Draw the given semi-circular plate with the hole; representing the development of the cut cone.
- Locate the generators on the development.
- Locate the points of intersection between the generators and the circular hole.

6. Transfer the above points to both the projections (Refer Construction: [Fig. 14.10](#)).

Join the points in the order by smooth curves, forming the projections of the cone with the cut.



**Fig.14.42**

**Problem 42** In a semi-circular plate of 120 diameter, a largest hole is made. The plate is folded to form a cone. Draw the two views of the cone, when the hole is "square" in form.



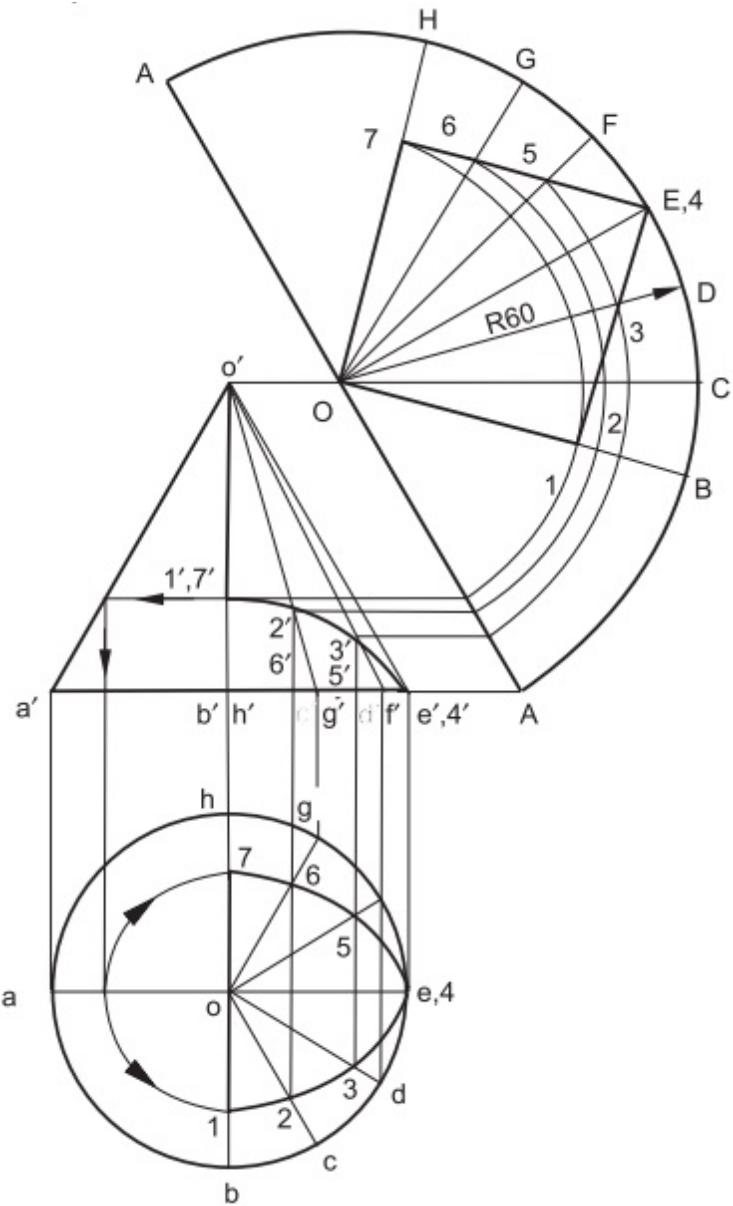
The slant height,  $s$  of the cone, formed after folding the plate is 60 and the radius,  $r$  of the base is obtained from,

$$\theta = 180^\circ = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

$$= 360^\circ \times \frac{r}{s} = \frac{360^\circ \times r}{60}$$

$$\therefore r = 30 \text{ mm}$$

**Construction (Fig.14.43)**



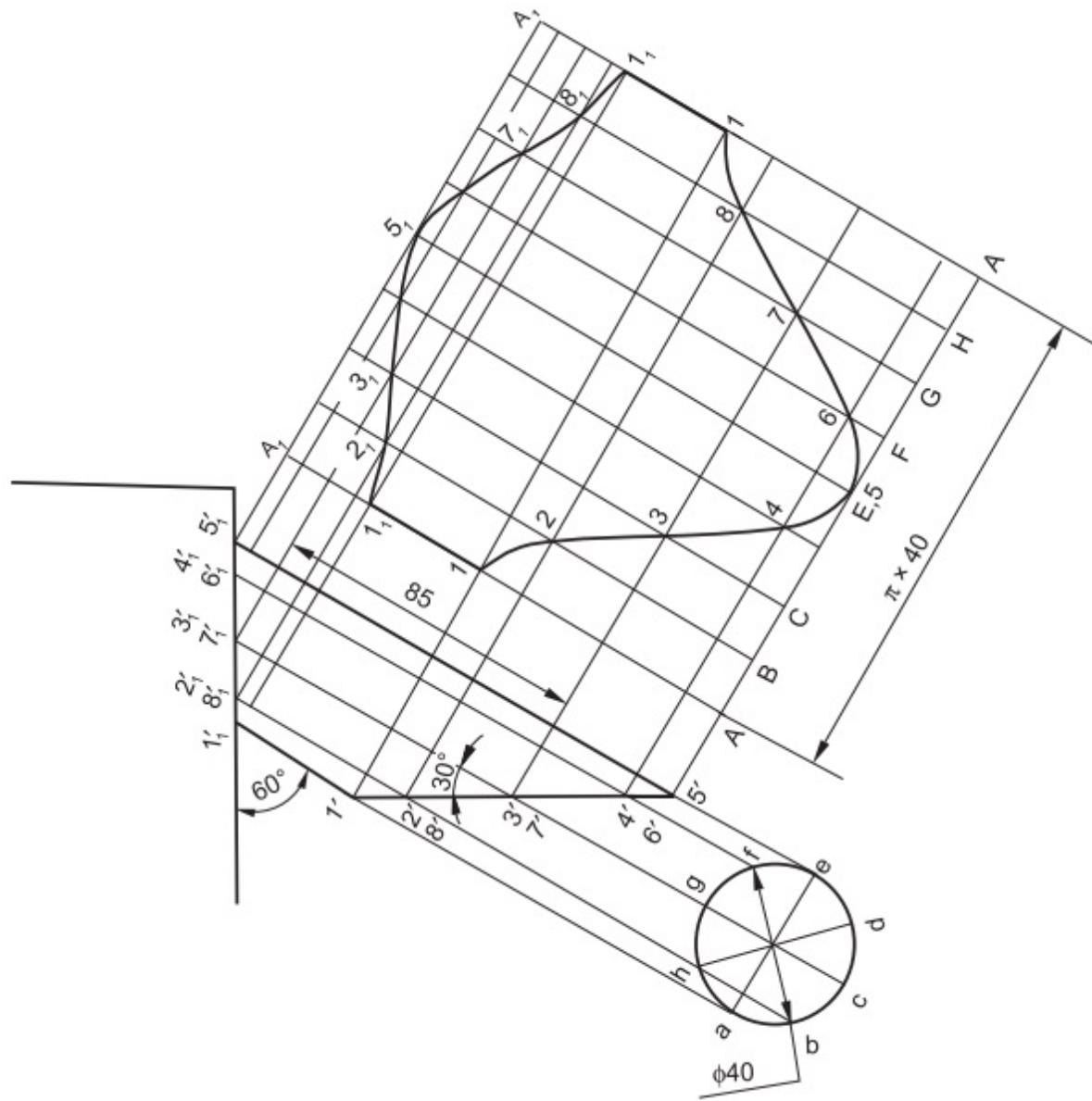
**Fig.14.43**

1. Draw the projections of the cone, with the radius of the base 30 and slant height 60.
2. Draw the given semi-circular plate with the hole; representing the development of the cone.
3. Draw a number of generators, passing through the hole in the development.

4. Locate the above generators; first in the top view and then in the front view.
5. Locate the points of intersection 1, 2, — — — 7, between the edges of the hole and the generators.
6. Transfer the above points to both the projections (Refer Construction: [Fig.14.10](#)).
7. Join the points in the order suitably, forming the projections of the cone with the cut.

**Problem 43** A pipe of 40 diameter and 85 long (along the axis) is welded to the vertical side of a tank. Show the development of the pipe if it makes an angle  $60^\circ$  with the side to which it is welded. The other end of the pipe makes an angle of  $30^\circ$  with its own axis. Neglect the thickness of the pipe.

**Construction ([Fig.14.44](#))**



**Fig.14.44**

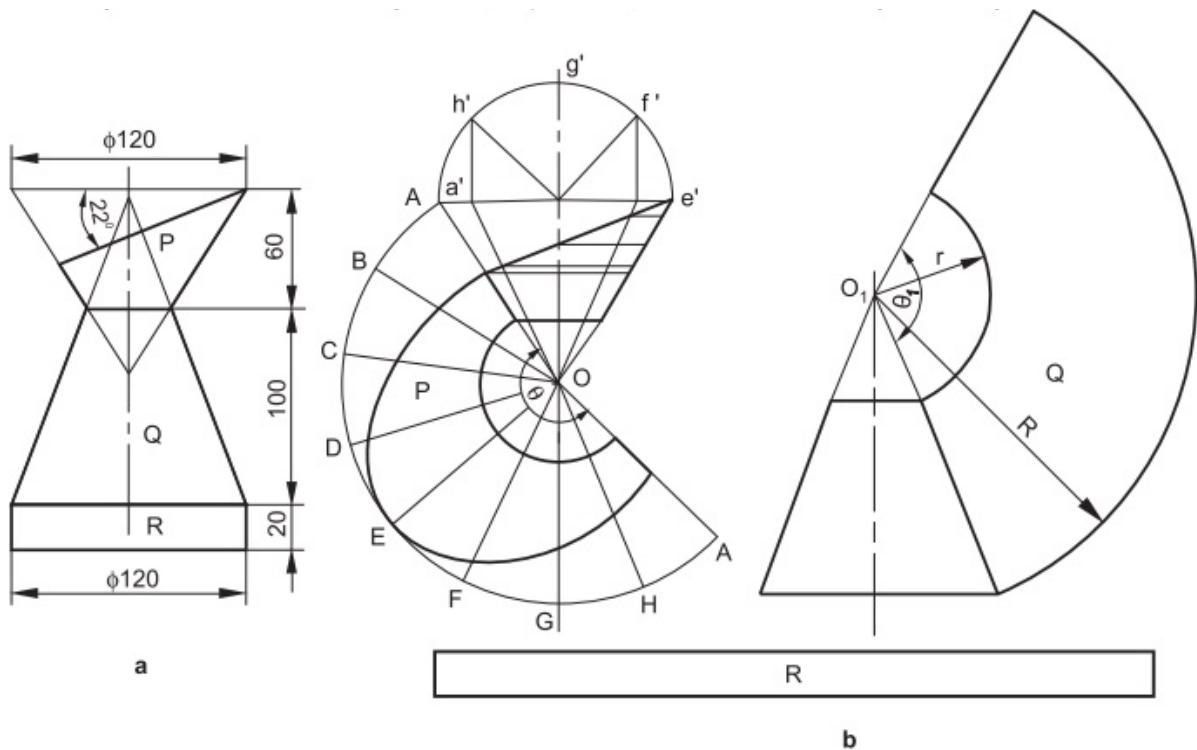
1. Draw the front view of the pipe and the tank assembly, satisfying the given conditions.
2. Draw the edge view of the pipe (circle) and divide it into 8 equal parts and locate the corresponding generators in the front view of the pipe.
3. Draw the stretch-out line A-A, equal to the circumference of the pipe and complete the

development of the complete uncut pipe.

4. Locate the generators on the development.
5. Locate the points of intersection between the generators and the cut portions of the pipe ends, 1', 2', etc., and 1<sub>1</sub>', 2<sub>1</sub>', etc.
6. Transfer these intersection points on to the corresponding generators in the development by projection.
7. Join the points 1, 2, etc., and 1<sub>1</sub>, 2<sub>1</sub>, etc., by smooth curves and obtain the development.

**Problem 44** *Draw the development of the lateral surface of the oil can, shown in Fig. 14.45a.*

The oil can consists of three parts P, Q and R; P and Q being parts of a cone and R, a cylindrical part. [Figure 14.45b](#) shows the construction for obtaining the developments of the three parts P, Q and R; which is self explanatory.



**Fig.14.45**

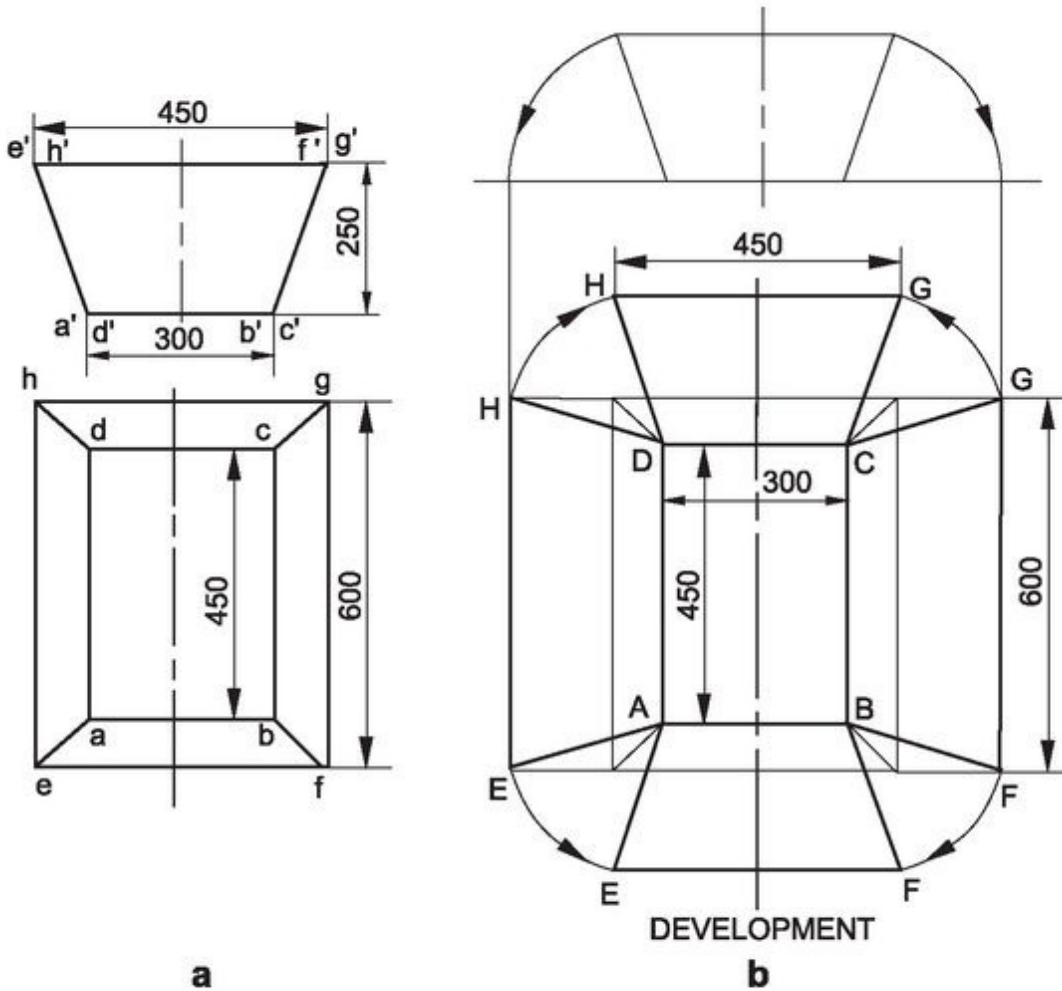
**Problem 45** Draw the development of the tray; the orthographic views of which are given in [Fig.14.46a](#).

**Construction ([Fig.14.46b](#))**



- (i) The base edges AB and CD and top edges EF and GH are parallel to both H.P and V.P. Hence, the lengths of the edges  $a' b'$  ( $=ab$ ),  $c' d'$  ( $=cd$ ) and  $e' f'$  ( $=ef$ ) and  $g' h'$  ( $=gh$ ) represent the true lengths.
  - (ii) All the corner edges AE, BF, CG and DH are inclined to both H.P and V.P and are of the same length. Hence, it is sufficient to determine the true length of any one edge.
1. Determine the true length of the edge, say AE by the rotation method.

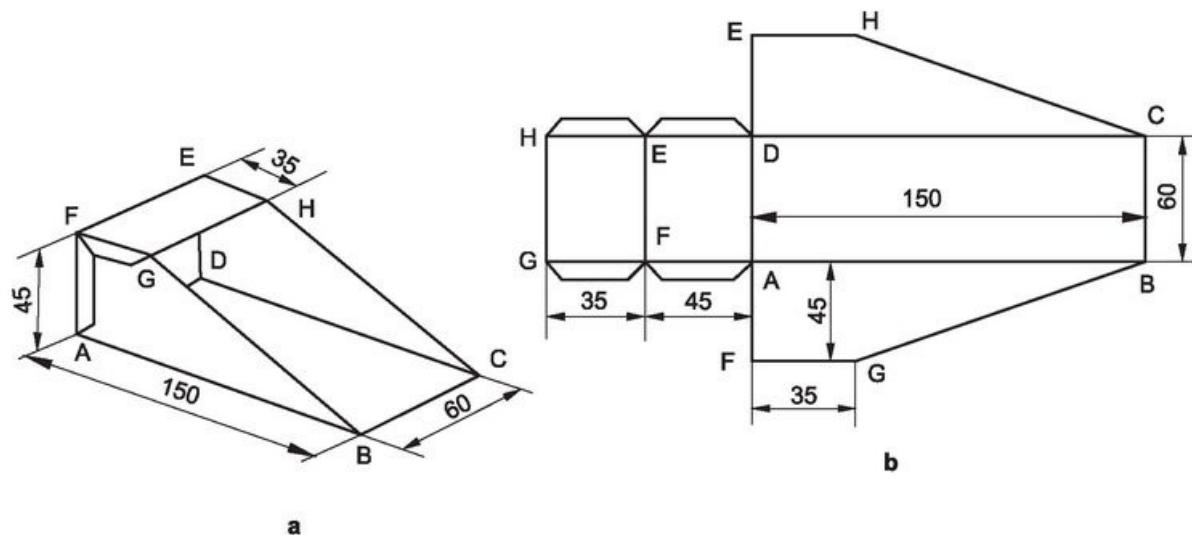
2. Complete the development as shown in Fig.14.46b, by making use of the true lengths.



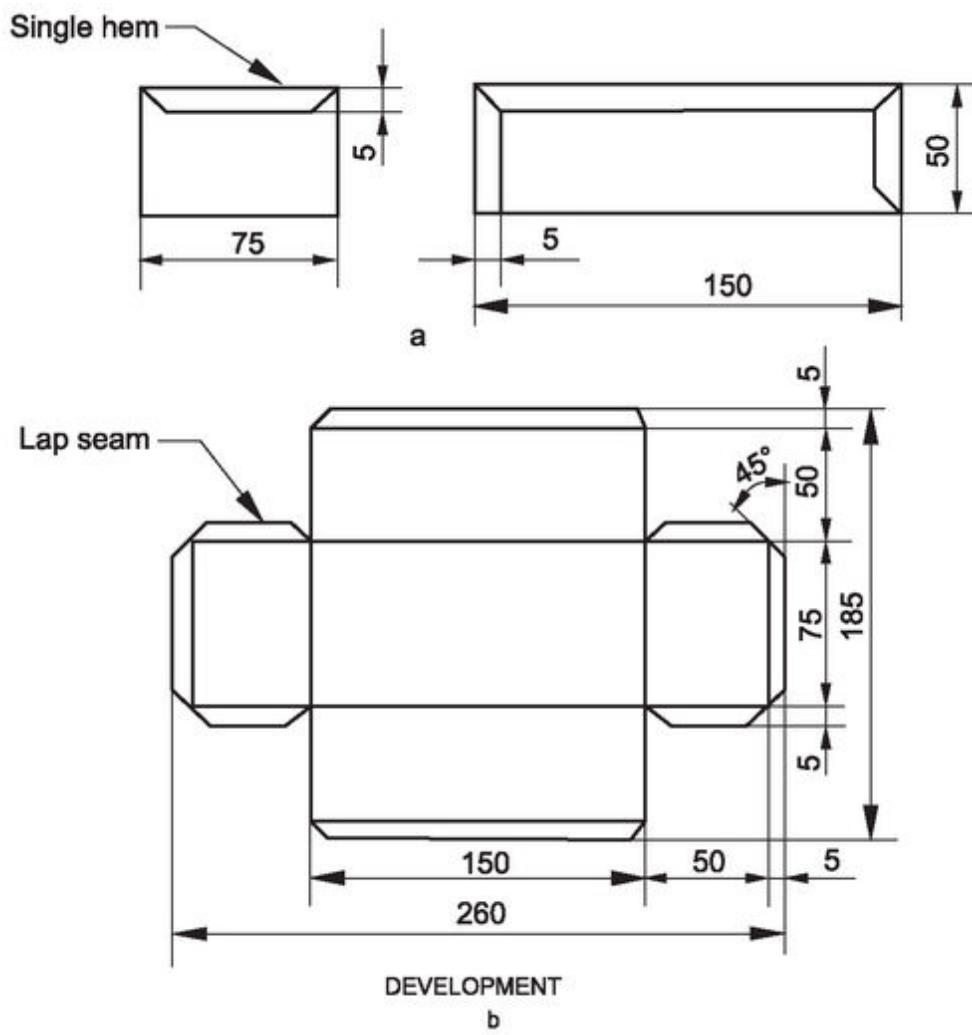
**Fig.14.46 Tray**

**Problem** Figures 14.47a to 14.51a show the orthographic views of rectangular scoop, tool's tray, chute, round scoop and transition piece respectively.

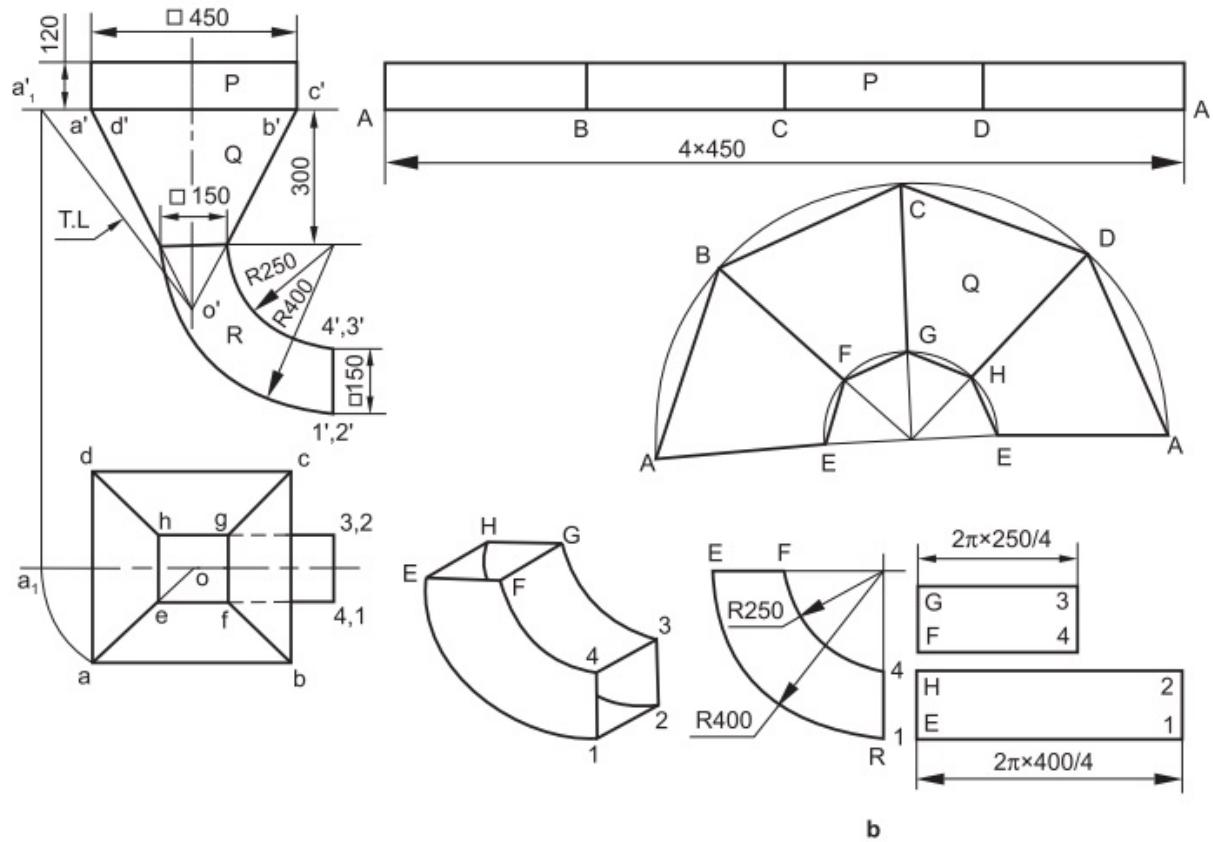
Figures 14.47b to 14.51b show the developments of the same; the constructions of which are self explanatory.



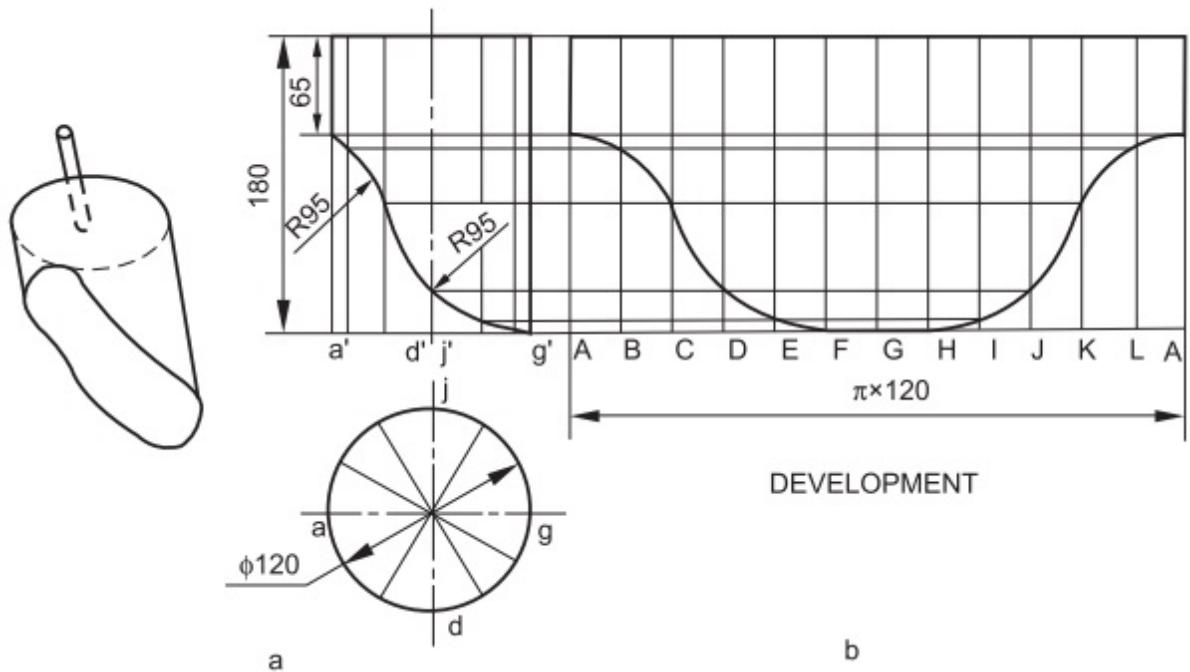
**Fig.14.47 Rectangular scoop**



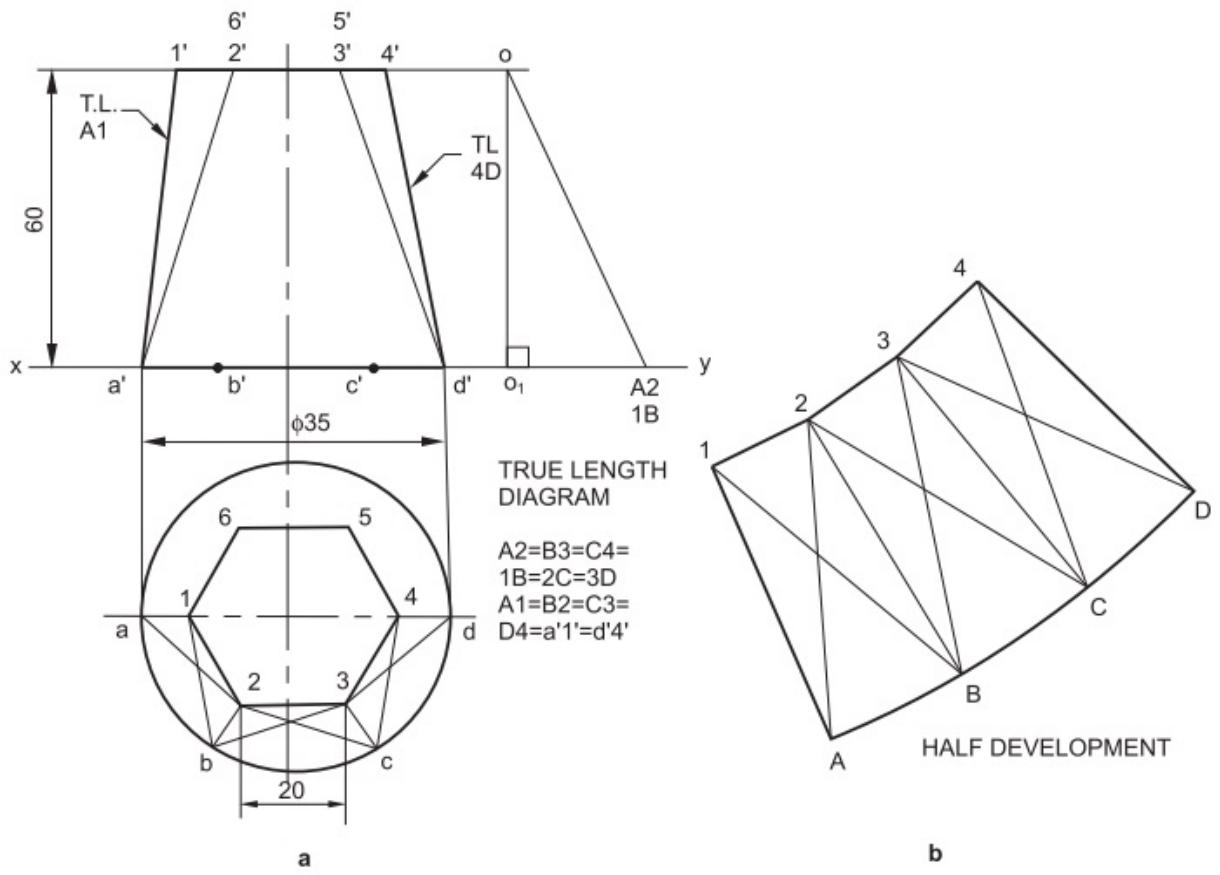
**Fig.14.48 Tool's tray**



**Fig.14.49 Chute**



**Fig.14.50 Round scoop**



**Fig.14.51 Transition piece**

## EXERCISES

### Pyramids

- 14.1 Draw the development of the frustum of a hexagonal pyramid of side of base 35 at the bottom and 15 at the top; the height of the frustum being 50.
- 14.2 A hexagonal pyramid of base 40 side and height 75, stands with its base on the ground such that, two of the base edges are parallel to V.P. It is cut by a section plane perpendicular to V.P and inclined at  $60^\circ$  to H.P and bisecting its axis. Draw the development of the surface of the cut solid.
- 14.3 A hexagonal pyramid with side of base 30 and height 80, has one of its triangular faces perpendicular to H.P and its base side on H.P. The axis of the solid is parallel to V.P. It is cut by a plane parallel to H.P and passing through a point on the axis 40 from the apex. Draw the development of the lateral surface of the solid.
- 14.4 A pentagonal pyramid of base 40 side and height 75, stands with its base on H.P such that, one of the base edges is parallel to V.P. It is cut by a section plane perpendicular to V.P, inclined at  $60^\circ$  to H.P and bisecting its axis. Draw the development of the surface of the pyramid.
- 14.5 A pentagonal pyramid with side of base 30 and height 55, stands with its base on H.P and an edge of the base is parallel to V.P. It is cut by a plane perpendicular to V.P, inclined at  $40^\circ$  to H.P and

passing through a point on the axis, 32 above the base. Draw the sectional top view and develop the lateral surface of the truncated pyramid.

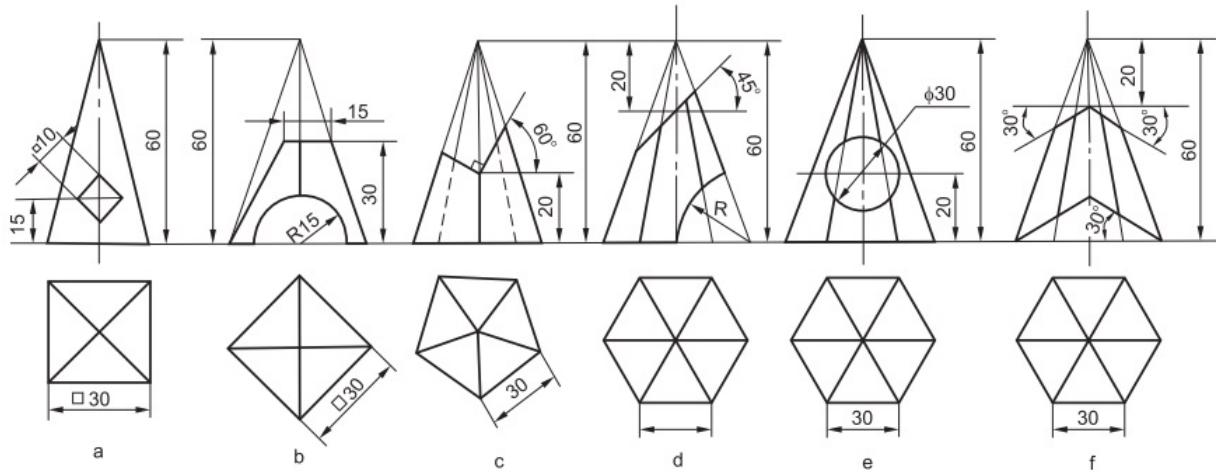
14.6 A square pyramid of side of base 40, altitude 50, resting with its base on H.P is cut by a section plane at  $45^\circ$  to the axis and intersecting it at a point, 30 from the base. Draw the development of the truncated pyramid.

14.7 Draw the developments of the lateral surfaces of the retained portions of the various pyramids, shown in Fig.14.52.

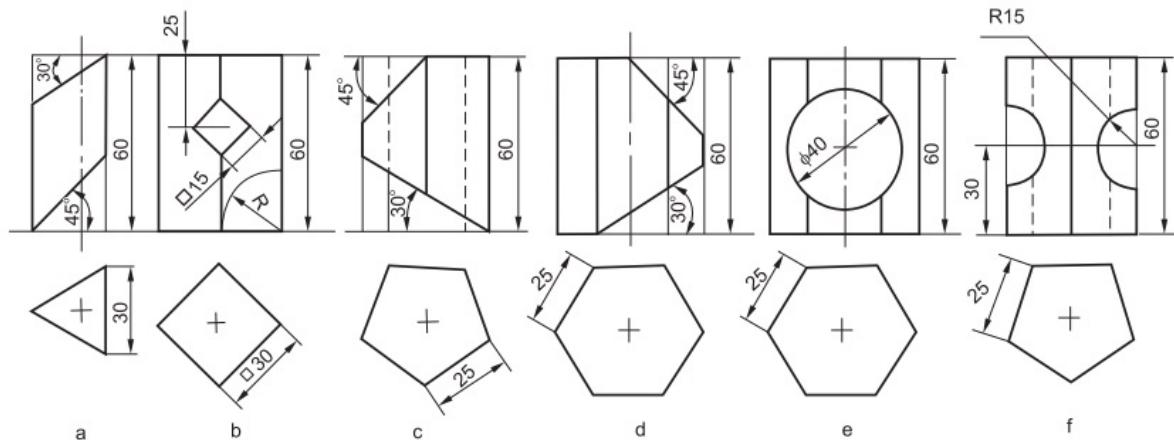
## Prisms

14.8 A pentagonal prism of side of base 40 and 80 high, is cut by a section plane, inclined at  $30^\circ$  to H.P, cuts the axis at 20 from the bottom, when a vertical face of the solid is perpendicular to V.P. Draw the development of the remaining solid.

14.9 A pentagonal prism with 20 side of base and 45 height, stands vertically on its base, with two of its rectangular faces equally inclined to V.P. The V.T of the cutting plane is inclined at  $45^\circ$  to the axis of the prism and passes through the left corner of the top base of the prism. Develop the lateral surface of the lower portion of the prism.



**Fig.14.52 Transition piece**



**Fig.14.53 Transition piece**

14.10A pentagonal prism of side of base 25 and axis 40 long, is resting on H.P on a corner of a base, with its axis inclined at  $60^\circ$  to H.P and parallel to V.P. It is cut by a plane perpendicular to V.P and inclined at  $45^\circ$  to H.P and passing through a point at 15 from the top base. Draw the development of the lateral surface of the solid.

14.11A vertical hexagonal prism of 25 side of base and 60 height, has one of the rectangular faces parallel to V.P. A circular hole of 40 diameter is drilled through

the prism such that, the axis of the hole bisects the axis of the prism at right angle and perpendicular to V.P. Draw the development of the lateral surface of the prism, showing the true shape of the hole in it.

14.12A hexagonal prism with edge of base 20 and axis 50 long, rests with its base on H.P such that, one of its rectangular faces is parallel to V.P. It is cut by a plane perpendicular to V.P, inclined at  $45^\circ$  to H.P and passing through the right corner of the top face of the truncated prism. Draw the sectional top view and develop the lateral surface of the truncated prism.

14.13Draw the developments of the lateral surfaces of the portions of the various prisms that are shown in [Fig.14.53](#).

14.14A solid is in the form of a square prism of side of base 30 upto a height of 50 and thereafter tapers into a frustum of a square pyramid, whose top surface is a square of 15 side. The total height of the solid is 70. Draw the development of the lateral surface of the solid.

## Cylinders

14.15A cylinder of diameter of base 60 and altitude 80, stands on its base. It is cut into two halves by a plane perpendicular to V.P and inclined at  $30^\circ$  to H.P. Draw the development of the lower half.

14.16Draw the sectional top view and the development of the lateral surface of the truncated cylinder of 45 base diameter and 55 long which rests with its base on H.P. It is cut by a plane perpendicular to V.P, inclined at  $60^\circ$  to H.P and passing through a point on the axis at 15 from the apex.

A cylinder of 45 diameter and 70 long, is resting on its base on H.P. It is cut by a section plane, inclined at  $60^\circ$  with H.P and passing through the axis at 15 from the top end. Draw the development of the retained portion of the solid.

14.18A cylinder of 40 diameter and axis 60 long, is lying on H.P with its axis inclined at  $45^\circ$  to H.P and parallel to V.P. It is cut by a section plane parallel to H.P and passing through the mid point of the axis. Draw the development of the bottom half of the cylinder.

14.19A cylinder of base 30 diameter and axis 40 long, lies on H.P on a point on its rim, with its axis inclined at  $30^\circ$  to H.P and parallel to V.P. It is cut by a plane inclined at  $45^\circ$  to H.P and passing through a point on the axis at 10 from the top base. Draw the development of the lower half of the solid.

14.20Draw the development of a cylinder of 50 diameter and 75 height, containing a square hole of 25 side. The sides of the hole are equally inclined to the base and the axis of the hole bisects the axis of the cylinder.

14.21Draw the development of a right circular cylinder of diameter 50 and 60 height, having a circular hole of 30 diameter drilled centrally through it such that, the axes of the hole and cylinder are mutually perpendicular to each other.

14.22Draw the developments of the lateral surfaces of the various cylinders, represented in Fig.14.54.

## **Cones**

14.23 A cone of 60 diameter and 70 height, is cut by a section plane such that, the plane passes through the mid-point of the axis and tangential to the base circle. Draw the development of the lateral surface of the bottom part of the cone.

14.24 In a semi-circular plate of 120 diameter, a largest hole is made. The plate is folded to form a cone. Draw the two views of the cone, when the hole is (i) circular, (ii) square and (iii) an equilateral triangle.

14.25 A right circular cone of 75 diameter and length of axis 100, rests on its base on H.P. A point P initially situated at the extreme right end of the base, moves around the surface of the cone and finally comes back to the starting point. Find the length of the shortest path, the point P will take, in covering the distance along the surface of the cone. Also, show the path in front and top views.

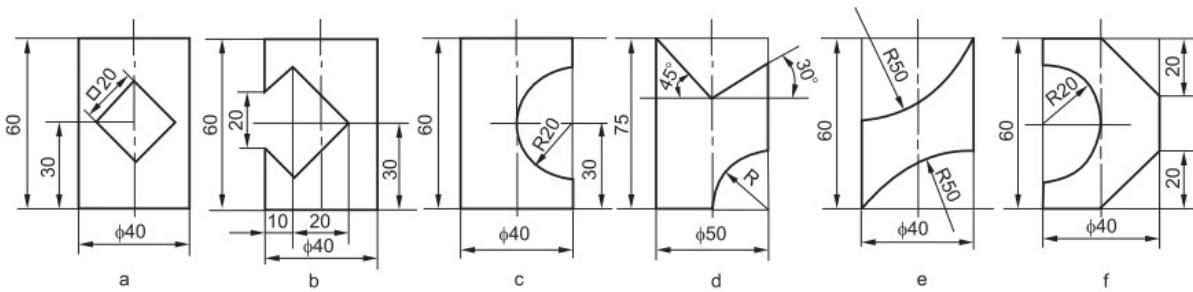
14.26 Draw the projections of a cone, resting on its base on H.P and show the shortest path by which a point P, starting from a point on the circumference of the base and moving around the cone, will return to the same point. Diameter of the base of the cone is 60 and axis 75 long.

14.27 A vertical cone of diameter of base 40 and height 50, is cut by a section plane, perpendicular to V.P and inclined at  $30^\circ$  to H.P, so as to bisect the axis. Draw the development of the truncated portion of the cone.

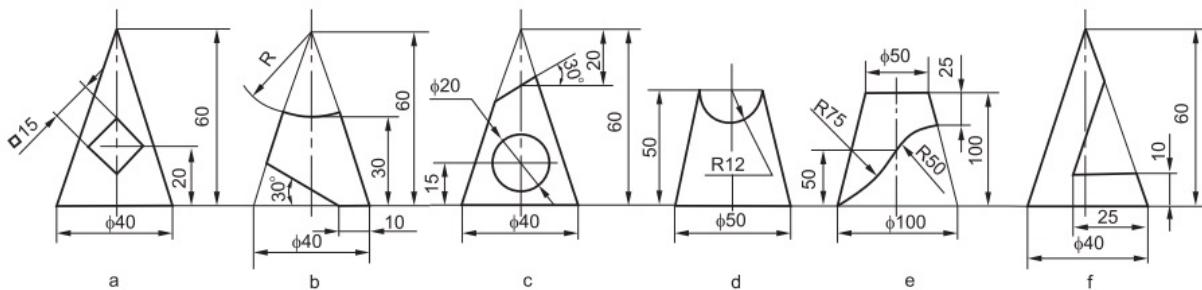
14.28 A cone with base diameter 54 and height 72, rests with its base on H.P. A section plane perpendicular to H.P and inclined at  $25^\circ$  to V.P cuts the cone at a distance of 13.5 from the axis. Draw the sectional

front view and develop the lateral surface of the remaining portion of the cone.

- 14.29A cone of base diameter 60 and axis 70 long, has the axis parallel to V.P and inclined at  $45^\circ$  to H.P. A section plane parallel to V.P cuts the cone at 8 away from the axis. Draw the development of the larger portion of the sectioned solid.



**Fig.14.54 Transition piece**

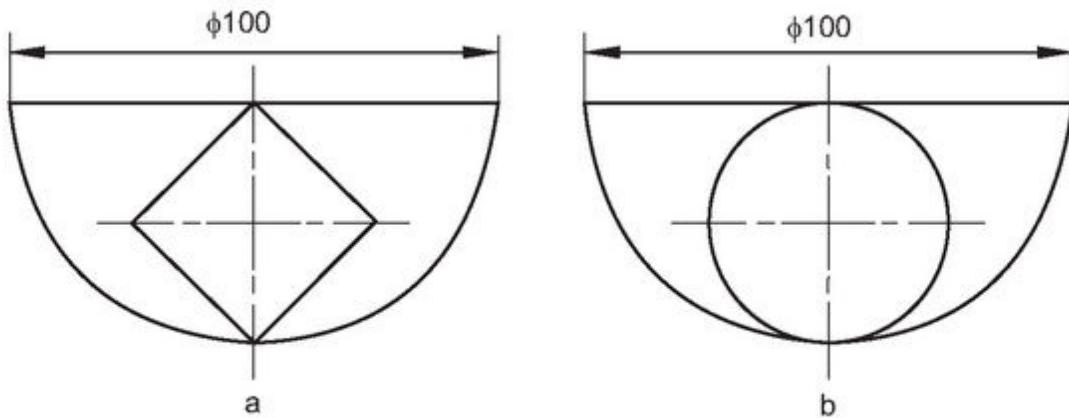


**Fig.14.55 Transition piece**

- 14.30A cone of diameter of base 80, and height 80, stands vertically with its base on the ground. A semi-circular hole of radius 25 is cut through the cone. The axis of the hole is perpendicular to V.P and parallel to H.P and intersects the axis at 30 above the base. The flat surface of the hole contains the axis of the cone and perpendicular to V.P. Draw the complete development of the cone.

14.31 Draw the developments of the lateral surfaces of various cones that are shown in Fig.14.55.

14.32 Figure 14.56 shows the developments of conical surfaces with slots. Draw the projections of the solids.



**Fig.14.56 Transition piece**

14.33 Draw the development of the transition piece, connecting a 30 diameter pipe with a square pipe of 50 side; the length of the piece being 50.

14.34 Draw the development of an oblique cylinder, the bases of which are parallel and 70 apart. The diameter of the base is 40 and the axis is inclined at  $60^\circ$  to the base.

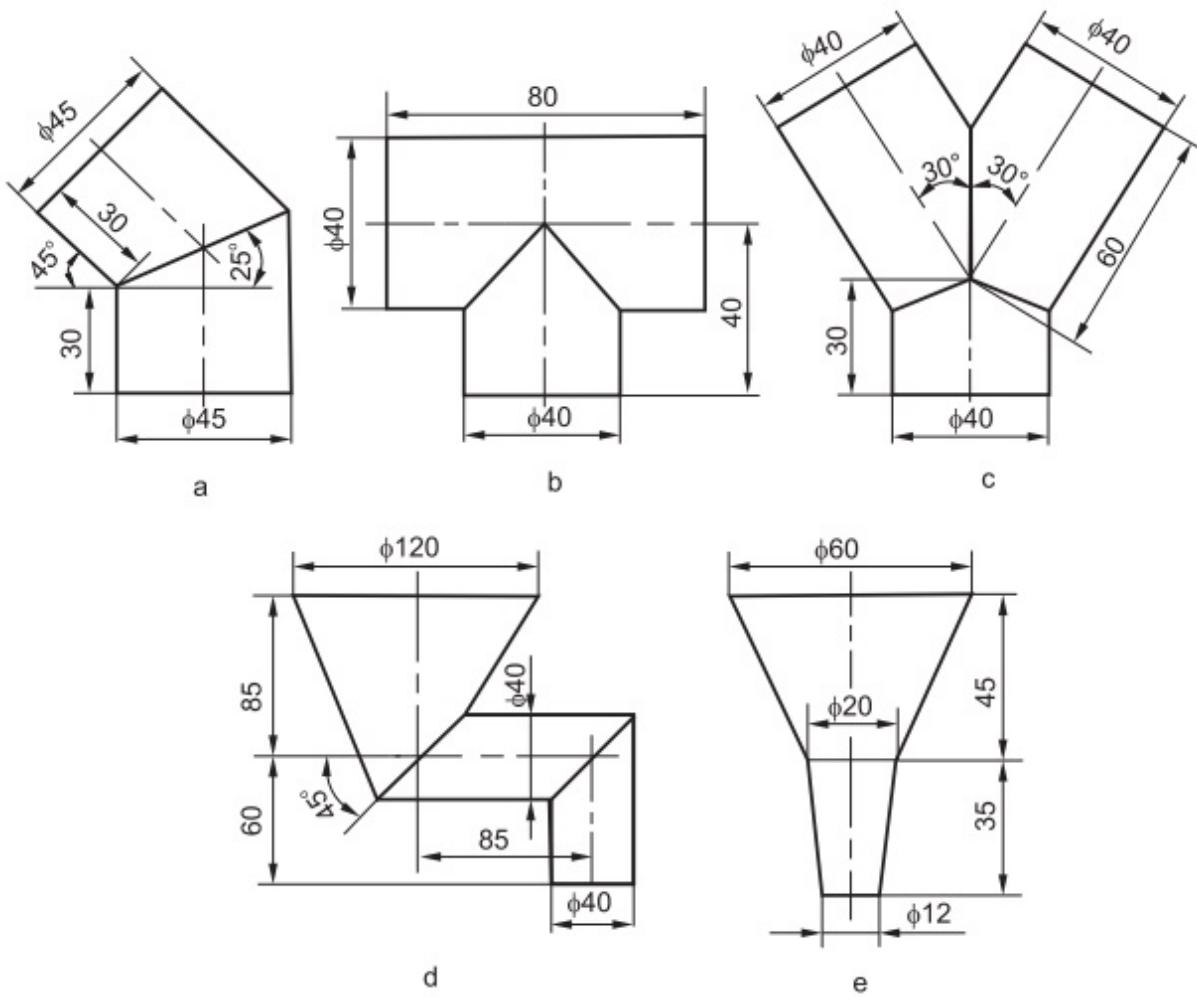
14.35 An oblique cylinder of base 60 diameter, has an axis 80 long and inclined at  $45^\circ$  to the base. It rests with its base on H.P and axis is parallel to V.P. It is cut by a vertical section plane, which is perpendicular to V.P and bisects the axis. Draw the development of the surface of the remaining solid.

14.36 An oblique cone of base 60 diameter and axis 75 long, rests on its base on H.P. The axis of the cone is

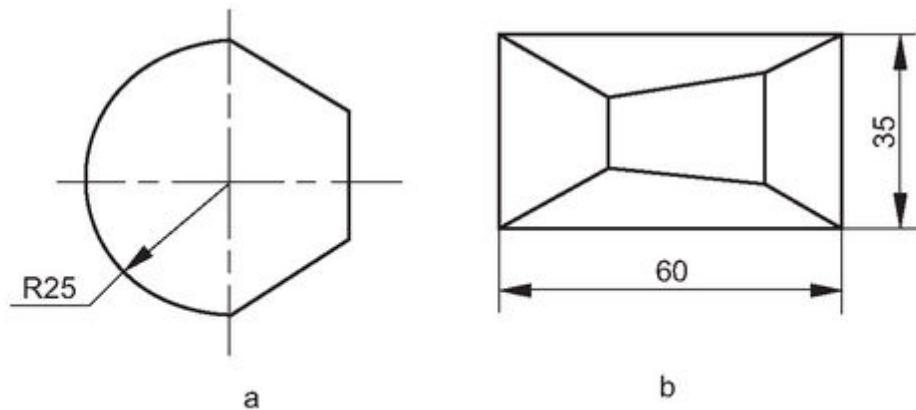
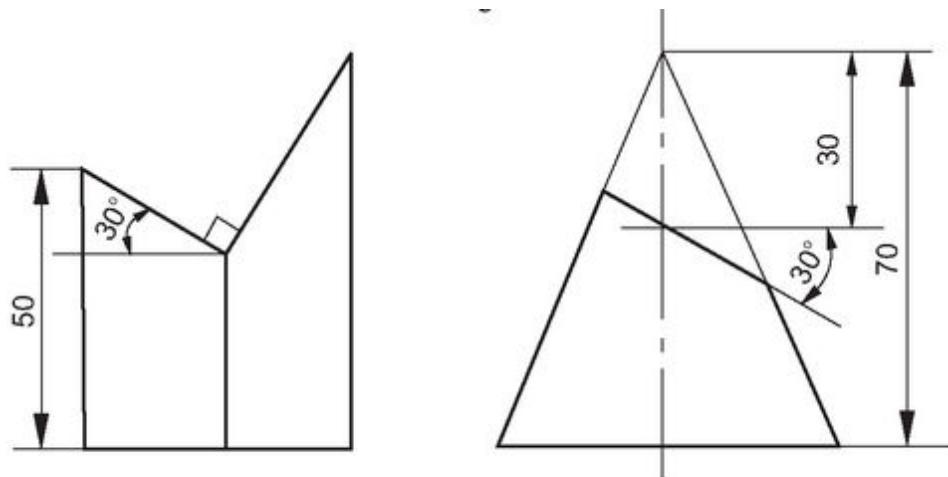
inclined at  $60^\circ$  to the base. It is cut by a horizontal section plane passing through the axis at 55 from the base. Draw the development of the remaining solid.

14.37 Draw the development of an oblique hexagonal pyramid of base 30 side and axis 110 long. Its axis is inclined at  $45^\circ$  to the base.

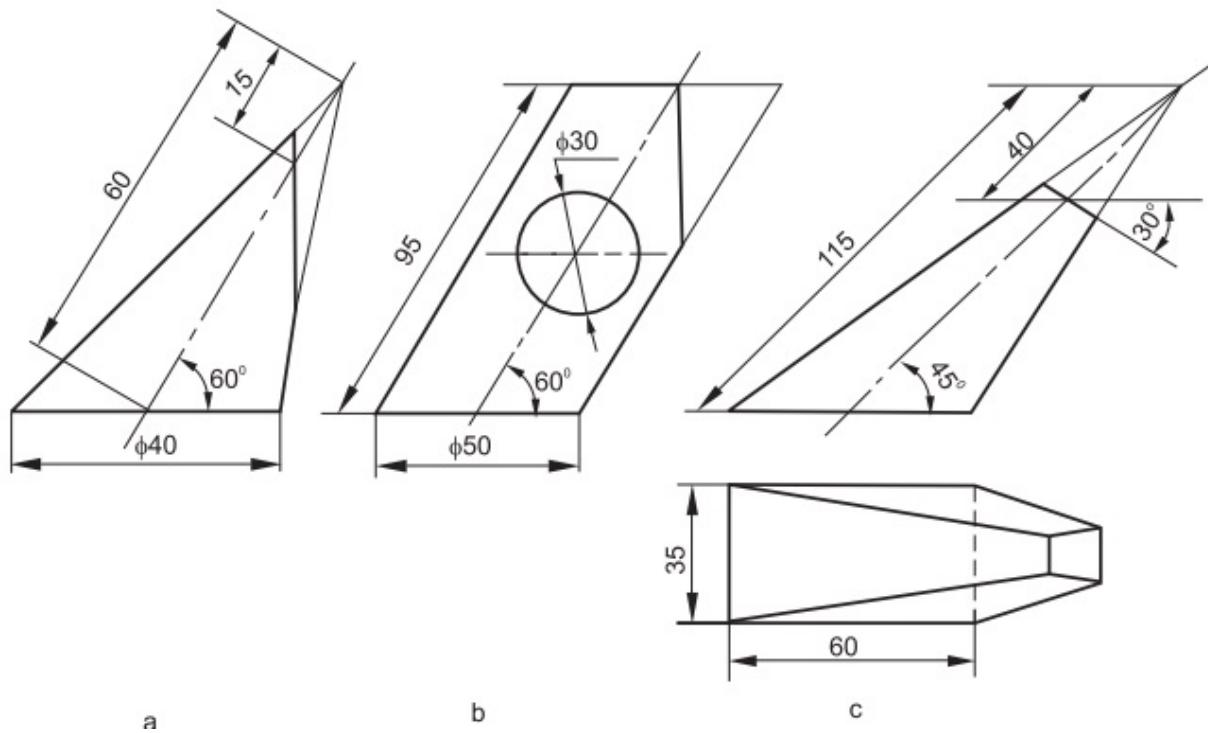
14.38 The apparent angles of inclination of the axis of an oblique pentagonal pyramid with H.P and V.P are  $45^\circ$  and  $25^\circ$  respectively. The pyramid is resting on its base on H.P, with an edge parallel to V.P. The edge of the base is 25 and the vertical height of the pyramid is 60. Draw the development of the lateral surface of the pyramid.



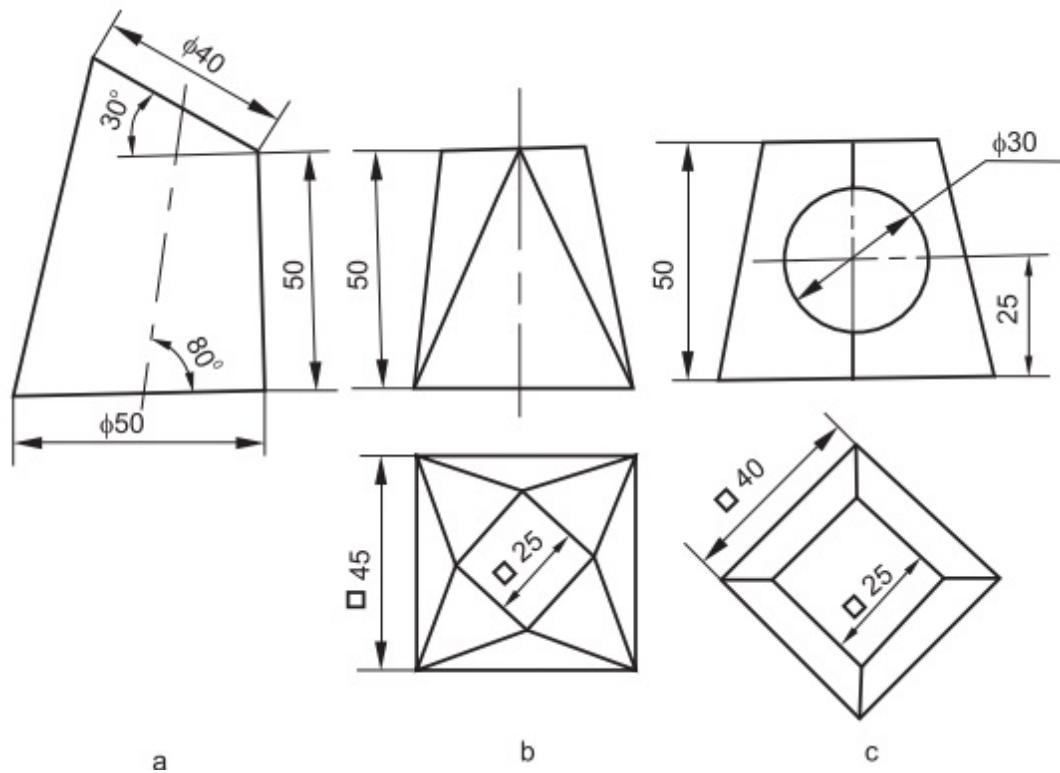
**Fig.14.57**



**Fig.14.58**



**Fig.14.59**



**Fig.14.60**

14.39 A hexagonal oblique prism lies on its base on H.P. with an edge parallel to V.P. The edge of the base is 30 and axis 70 long, which is inclined at  $60^\circ$  to H.P. A section plane perpendicular to V.P and inclined at  $60^\circ$  to H.P, bisects the axis and the section plane leans opposite to the axis. Draw the development of the surface of the lower portion of the prism.

14.40 [Figures 14.57 to 14.60](#) show certain objects. Draw the developments of the lateral surface of each.

## REVIEW QUESTIONS

- 14.1 What is meant by the development of a solid? What is its purpose?
- 14.2 What is the purpose of bend allowance in the layout of the development of lateral surface of an object?
- 14.3 On what do the bend allowances depend?
- 14.4 Classify the solids, based on the nature of lateral surfaces.
- 14.5 Why the surface of the solid is usually cut open at the shortest edge, during the layout for development?
- 14.6 Name the solids bounded by (i) single curved surfaces and (ii) double curved surfaces.
- 14.7 Explain briefly, the different methods of development.
- 14.8 What is meant by the stretch-out line of a solid. For what type of solids, can stretch-out lines be drawn?
- 14.9 What is the method followed in developing oblique prisms and cylinders?

14.10 Differentiate between a transition piece and a reducer.

14.11 What is a Gore?

## OBJECTIVE QUESTIONS

14.1 Every line on a development must be equal to the true length of that line on the actual surface.

(True /False)

14.2 Single curved surfaces can be accurately developed.

(True /False)

14.3 The surfaces of oblique prisms may be developed by radial line development method.

(True /False)

14.4 The lateral surfaces of oblique pyramids may be developed by parallel line development method.

(True/False)

14.5 For what type of solids, may the triangulation method of development be followed?

14.6 Name different methods used for developing spherical surfaces.

## ANSWERS

14.1 True

14.2 True

14.3 False

14.4 False

14.5 Transition pieces

14.6 Gore and zone methods

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# 15

## *Intersection of Surfaces*



### 15.1 INTRODUCTION

The term, “intersection of surfaces”, in general, is used when two curved surfaces meet together, such as cones and cylinders. And the term, “inter-penetration” of solids is used when solids having plane surfaces intersect each other. However, in practice, the term, “intersection of surfaces”, is used when the intersecting solids have either plane or curved surfaces. The knowledge of intersections is used to draft the junctions of two pipe lines, boilers, tanks, machine castings and structures of ships, automobiles, aircrafts, etc.

### 15.2 LINE OF INTERSECTION

When two surfaces intersect, the line of intersection is a line or curve, along which all the elements of one surface pierce the other. The line of intersection may be straight or curved, depending upon the nature of intersecting surfaces. In preparing orthographic projections, it is necessary to represent the line or curve of intersection between various

surfaces of a wide variety of objects. In the simple case, the intersection of two planes is a straight line. In finding the lines of intersection between various other surfaces, a number of points, common to both the surfaces are first determined and then are joined in the correct order.

Two plane surfaces such as faces of prisms and pyramids, intersect along straight lines. The line of intersection between two curved surfaces such as lateral surfaces of cylinders and cones or between a plane surface and a curved surface is a curve.

Determination of the line of intersection between two solids is more important, as their developments into flat patterns is otherwise not possible. The term development, refers to the flat pattern or template from which the required surfaces can be formed, viz., boiler with a dome, a manhole in the boiler, etc.

In view of the above important applications of intersection of surfaces, the subject content is presented under three general groups:

**Group I** Solids composed of plane surfaces, viz., intersection of prisms, pyramids or a prism and a pyramid.

**Group II** Solids composed of surfaces which are curved, viz., intersection of cylinders, cones or a cylinder and a cone.

**Group III** Solids composed of plane and curved surfaces, viz., intersection of a cylinder and a prism, a cylinder and a pyramid and a cone and a prism.

Problems of first group are solved, by locating the points through which, the edges of each pierce the other. These

points are called vertices. Normally, the edge view of one of the solids is best made use of for the purpose. Location of the line of intersection in the second group may be obtained by drawing elements on the lateral surfaces of both the solids and determining the points of intersection between them. A curve passing through these points will be the required line of intersection.

## 15.3 INTERSECTION BETWEEN PRISMS

The prisms have plane lateral surfaces. Hence, the line of intersection between two plane surfaces is obtained by locating the points at which the edges of one prism intersect the edges or faces of the other prism and then joining the points by straight lines.

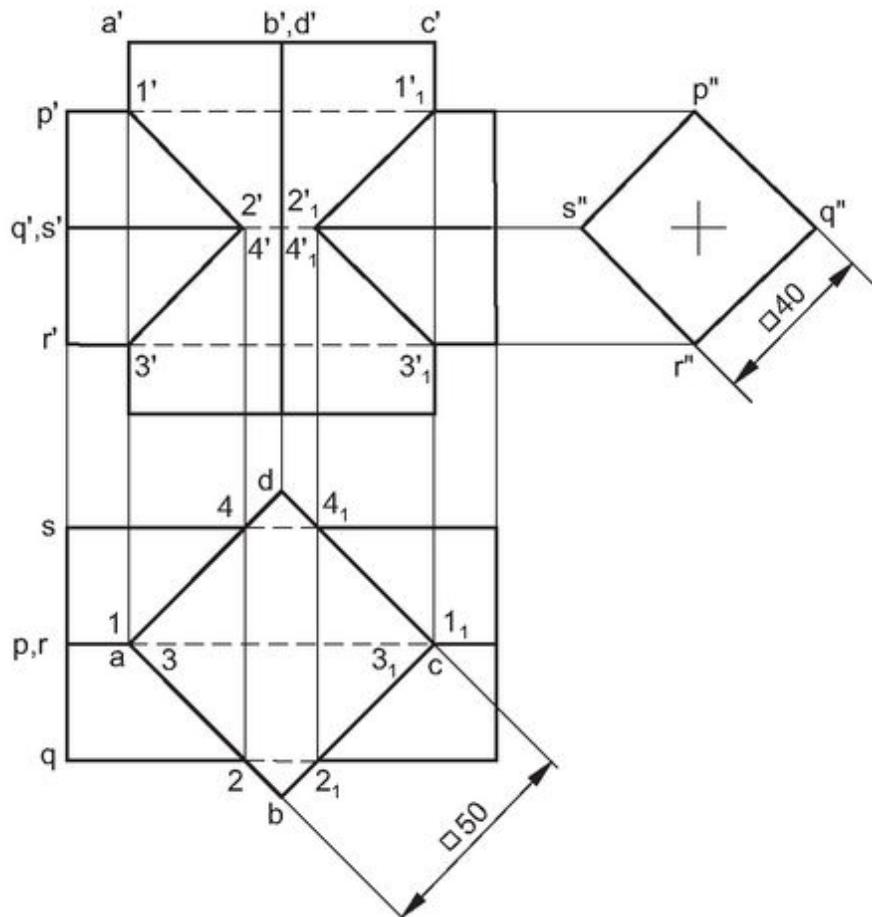
**Problem 1** A vertical square prism of base 50 side, is penetrated by a horizontal square prism of base 40 side such that, the axes intersect. The axis of the horizontal prism is parallel to VP and the faces of both the prisms are equally inclined to VP. Draw the projections of the two prisms, showing the lines of intersection.

### **Construction (Fig.15.1)**

1. Draw the projections of the prisms, satisfying the given conditions.
  2. In the edge view of the vertical prism, locate the points 1, 2, 3 and 4, where the edges of the horizontal prism, pierce the vertical prism.
  3. Project the above points and obtain the corresponding points  $1'$ ,  $2'$ ,  $3'$  and  $4'$ , in the front view.
  4. Join the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $1'$  by straight lines, forming the line of intersection on the left side.
  5. Follow the steps 2 to 4 and obtain the line of intersection on the right side.
- (i) The lines 1-2, 1-4;  $1_1-2_1$ ,  $1_1-4_1$  correspond to the lines of intersection in the top view.



- (ii) The points of intersection should be numbered correctly in the edge view (top view, in the present case), to get the correct line of intersection.
- (iii) The rear portions of the line of intersection, which are invisible, coincide with the visible portions of the line of intersection.
- (iv) In the front and top views, certain portions of the edges  $1'-1_1'$ ,  $2'-2_1'$ ,  $3'-3_1'$  are invisible and hence are shown by dotted lines.
- (v) Lengths of the solids may be assumed suitably, as the problem is basically on the determination of the line of intersection between the solids.

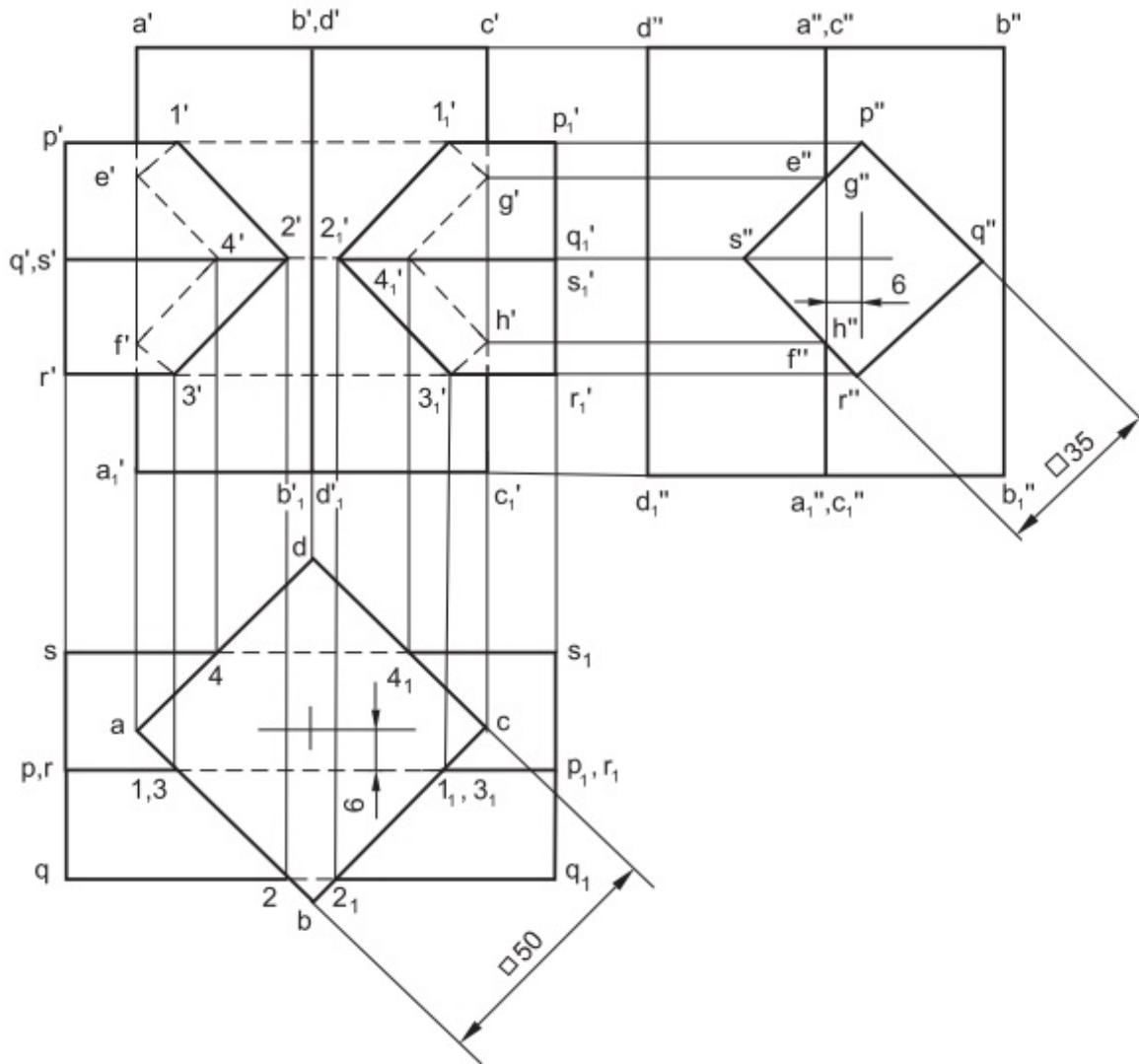


### **Fig.15.1**

**Problem 2** A vertical square prism of base 50 side, is penetrated by a horizontal square prism of base 35 side such that, the axes are 6 apart. The axis of the horizontal prism is parallel to V.P and the faces of both the prisms are equally inclined to V.P. Draw the projections of the two prisms, showing the lines of intersection.

#### **Construction (Fig.15.2)**

1. Draw the three views of the prisms.
2. In the edge view of the vertical prism, locate the points 1, 2, 3 and 4, where the edges of the horizontal prism, pierce the vertical one.
3. Project the above points and obtain the corresponding points 1', 2', 3' and 4', in the front view.
4. In the edge view of the horizontal prism, locate the points, e'' and f'', through which the faces PS and SR of the horizontal prism, pierce the edge AA<sub>1</sub> of the vertical prism.
5. Project the above points and obtain the corresponding points e' and f in the front view.
6. Following the rules of visibility, join the points 1', 2', 3', f, 4', e' and 1', by straight lines, forming the line of intersection on the left side.
7. Follow the steps 2 to 6 and obtain the line of intersection on the right side.



**Fig.15.2**



If the side view cannot represent the edge view, auxiliary view for the solution may be made use of.

### 15.3.1 Rules of Visibility

The visibility of a line in any view is always determined with reference to the adjacent views.

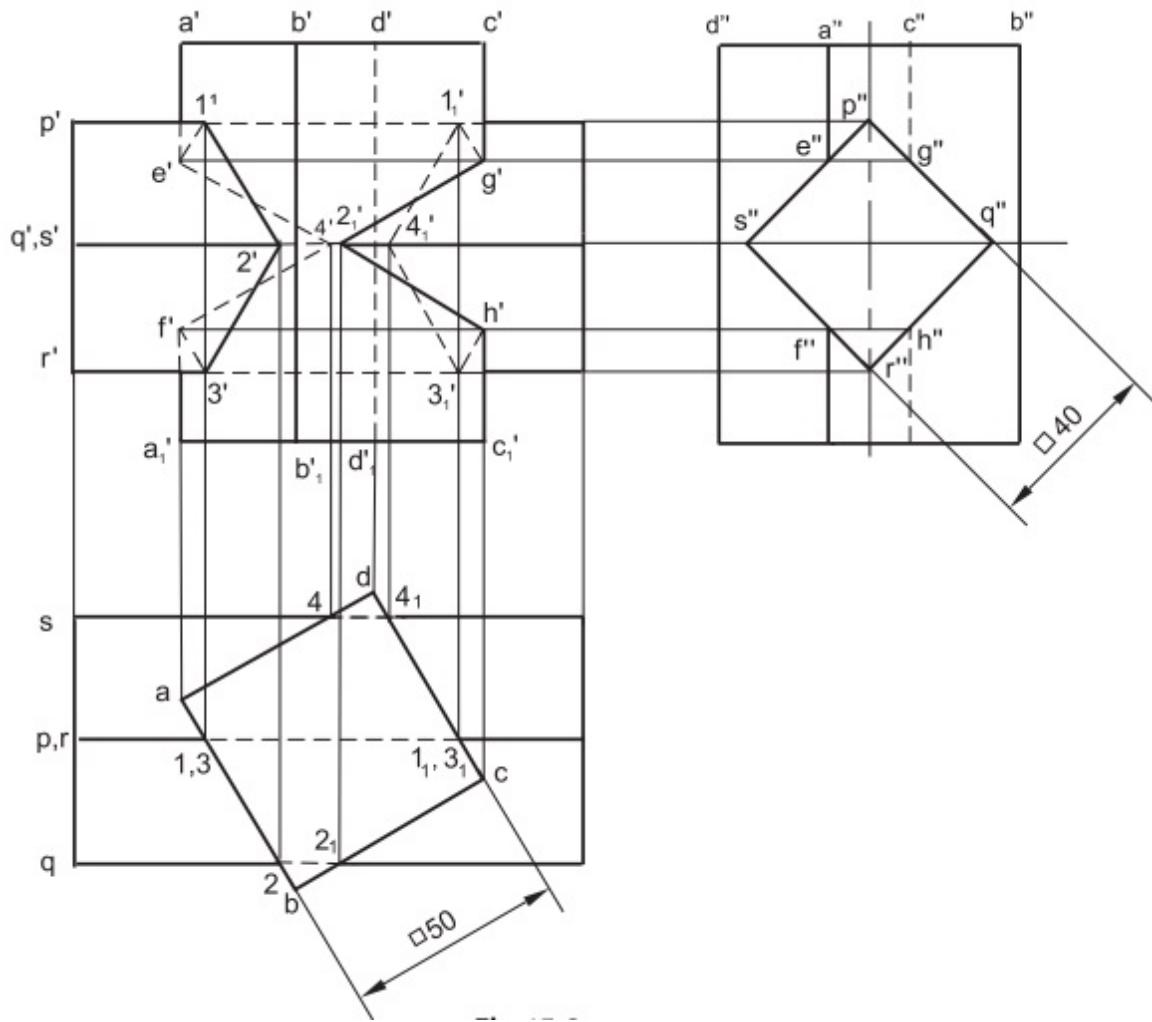
In Fig.15.2, the edges P-P<sub>1</sub> and R-R<sub>1</sub> are in front of the edges A-A<sub>1</sub> and C-C<sub>1</sub> and enter the vertical prism at 1 and 3 and emerge at 1<sub>1</sub> and 3<sub>1</sub> respectively. Between these two points, they are invisible. Edge S-S<sub>1</sub> enters the vertical prism at point 4 and emerges at 4<sub>1</sub>, but both the points are invisible in the front view, since they lie on the rear side of the vertical prism. The edges A-A<sub>1</sub> and C-C<sub>1</sub> meet the horizontal prism at e', f', g' and h' and all these points are invisible since they lie on the rear side of the horizontal prism.

The following principles may be observed while deciding the visibility of lines:

1. If a point lies on a visible edge or elements of both the intersecting surfaces, then it is visible.
2. A line of intersection, connecting two visible points is visible. If the line connects one visible and one invisible point, then it is completely invisible.
3. A line of intersection changes from invisible to visible or vice versa, only on crossing visible elements of one of the intersecting surfaces.

**Problem 3** A square prism of side of base 50, is resting on its base on H.P, with a face of it inclined at 30° to V.P. It is penetrated by another square prism, with side of base 40 and faces of which are equally inclined to V.P. The axes of the two prisms are intersecting each other at right angle. Draw the projections of two prisms, showing the lines of intersection.

Follow the principles of Construction: Fig.15.2 and obtain the lines of intersection, as shown in Fig.15.3.



**Fig.15.3**

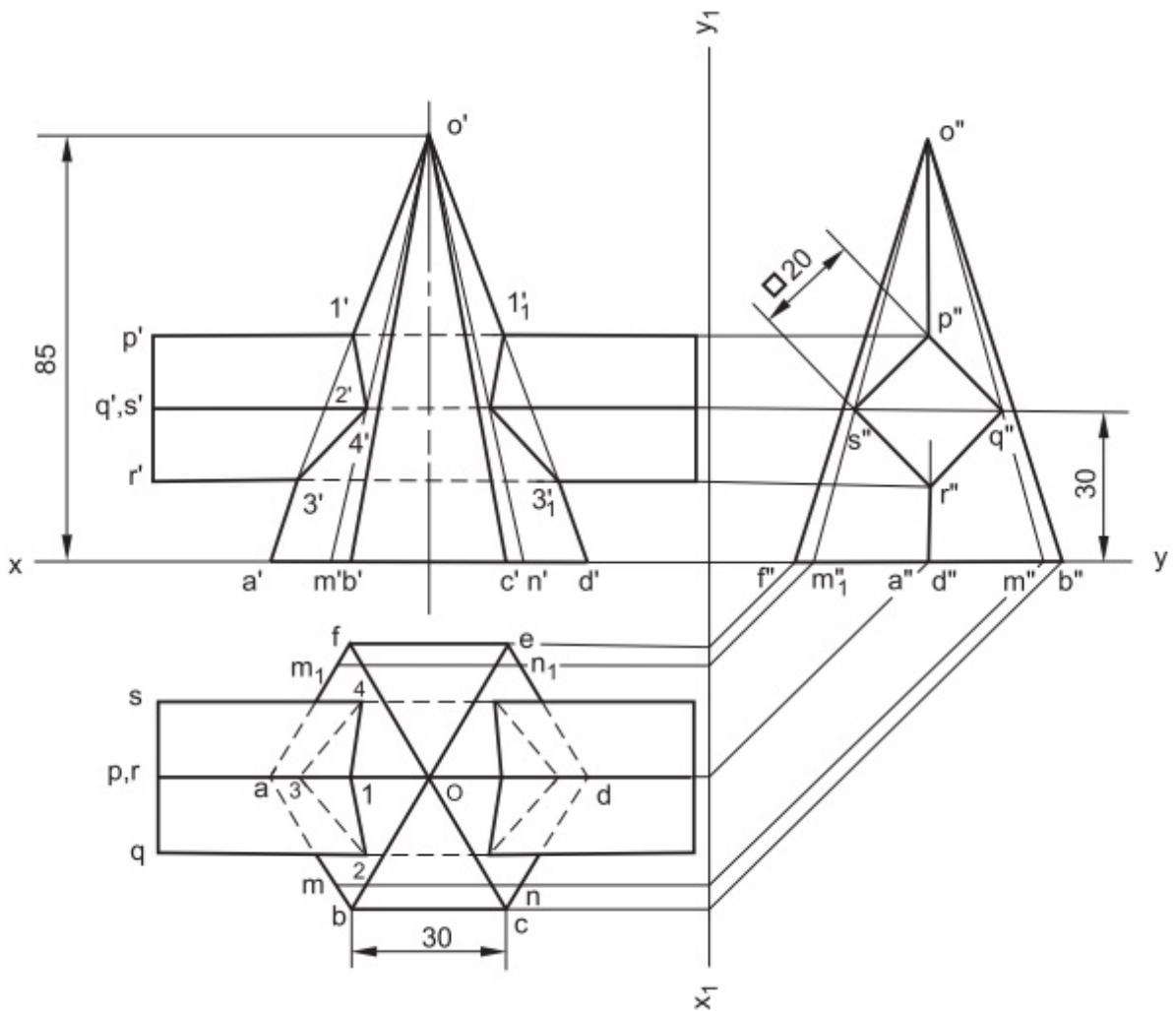
## 15.4 INTERSECTION BETWEEN PRISM AND PYRAMID & PYRAMID AND PYRAMID

The prisms have rectangular plane lateral surfaces, with lateral edges perpendicular to the base. However, the pyramids have isosceles triangular plane lateral surfaces, with lateral slant edges meeting at a point, called the apex.

The method of obtaining the line of intersection between a prism and a pyramid or pyramid and pyramid is similar to that followed for obtaining the line of intersection between two prisms.

**Problem 4** A hexagonal pyramid with side of base 30 and altitude 85, is resting on its base on H.P with an edge of base parallel to V.P. It is penetrated by a horizontal square prism of side of base 20 such that, the axes intersect at right angle. The base edges of the prism are equally inclined to H.P and its axis is 30 above H.P. Draw the projections of the solids, showing the line of intersection.

**Construction (Fig.15.4)**



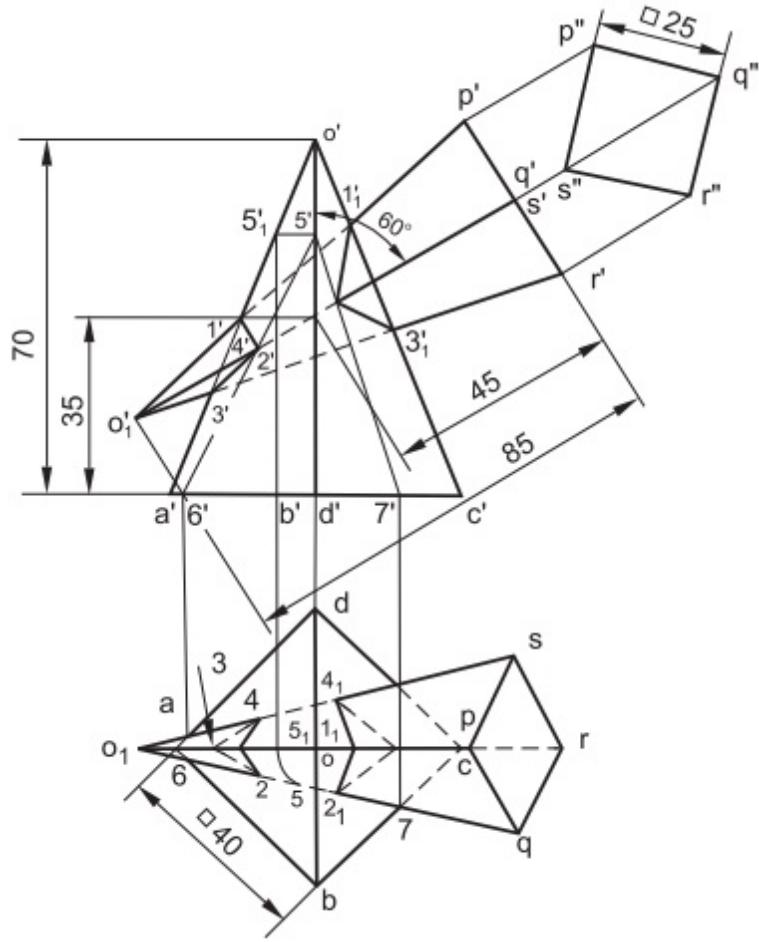
**Fig.15.4**

1. Draw the three views of the solids.
2. Locate the points of intersection 1' and 3' in the front view, between o' a' and the horizontal edges passing through p' and r' respectively.
3. In the side view, identify the generators, o'' m''\_1 and o' 'm'', passing through s'' and q''.
4. Project and obtain the corresponding generators in the front view.

5. Locate the points of intersection  $2'$  and  $4'$ , between the generator  $o'm'$  and the horizontal edges passing through  $q'$  and  $s'$  respectively, in the front view.
6. Project and obtain the corresponding points 2 and 4 in the top view.
7. Following the rules of visibility, join the points  $1', 2', 3'$  and  $4'$  and 1, 2, 3 and 4 by straight lines and obtain the lines of intersection in the front and top views, on the left side.
8. Follow the steps 2 to 7 and obtain the lines of intersection on the right side.

**Problem 5** A square pyramid with side of base 40 and altitude 70, resting on its base on H.P. is penetrated by another square pyramid of base 25 side and altitude 85 such that, the axes intersect at  $60^\circ$  with each other. The point of intersection is 35 and 45 from the bases of the vertical and inclined solids respectively. The sides of the bases of the two solids are equally inclined to V.P. Draw the projections of the two solids, showing the line of intersection.

**Construction (Fig.15.5)**



**Fig.15.5**

1. Draw the projections of the pyramids.
2. Locate the points of intersection  $1'$  and  $3'$  in the front view, between  $o'a'$  of the vertical pyramid and  $o_1'p'$  and  $o_1'r'$  of the penetrating pyramid respectively.

To locate the point of intersection  $2'$  on  $O_1Q$ , imagine a vertical section plane passing through  $o_1q$  (top view), intersecting the base of the vertical solid at 6 and the edge OB at 5.

*To locate the point  $2'$  in the front view:*

3. Locate the points  $5'$  and  $6'$ , by projection.

4. Locate the points of intersection 2' between 5'-6' and  $o_1'q'$ .
5. Obtain the corresponding point 2 in the top view, by projection and also locate 4 on  $o_1s$  by symmetry.
6. Following the rules of visibility, join the points 1', 2', 3' and 4' and 1, 2, 3 and 4 by straight lines and obtain the lines of intersection in the front and top views, on the left side.
7. Follow the steps 2 to 6 and obtain the lines of intersection on the right side.

## 15.5 INTERSECTION BETWEEN CYLINDERS

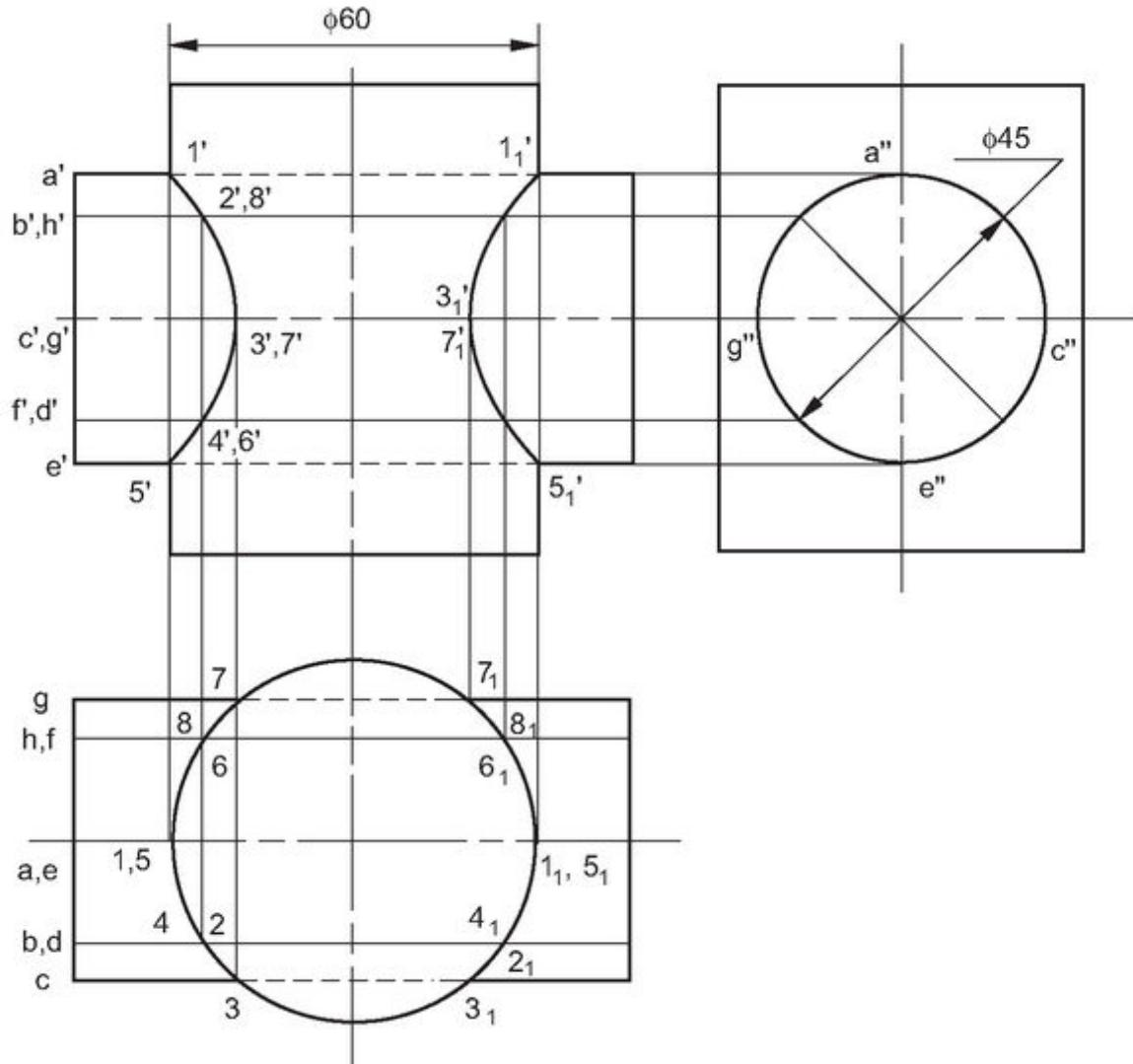
Cylinders have curved surfaces and hence, the line of intersection will be a curve. Points on the line of intersection may be located by either the generator method or cutting plane method. For plotting an accurate curve, certain points, known as critical, key or transition points, at which the curve changes its direction, must also be located. These are the points at which extreme generators of each cylinder, pierce the surface of the other.

**Problem 6** *A vertical cylinder of 60 diameter, is penetrated by another cylinder of 45 diameter. The axes of the two cylinders are intersecting at right angle. Draw the projections of the two cylinders, showing the lines (curves) of intersection.*

### **Construction (Fig.15.6)**

1. Draw the three views of the cylinders.
2. Divide the circle (side view of the horizontal cylinder) into some number of equal parts, say 8 and mark the division points.
3. Draw the generators in the front and top views, corresponding to the above division points.
4. Locate the points of intersection 1, 2, 3, etc., between the generators and the edge view of the vertical cylinder.
5. Project and obtain the corresponding points 1', 2', 3', etc., in the front view.

6. Join the points  $1'$ ,  $2'$ ,  $3'$ , etc., by a smooth curve and obtain the line of intersection on the left side.
7. Follow the steps 4 to 6 and obtain the line of intersection on the right side.



**Fig.15.6**

-  (i) The points  $1', 2', 5'$  lie on the front (visible) portion of the cylinders and hence, are joined by thick curve.
- (ii) The points,  $6', 7', 8'$  lie on the rear (invisible) portion of the cylinders, but coincide with the

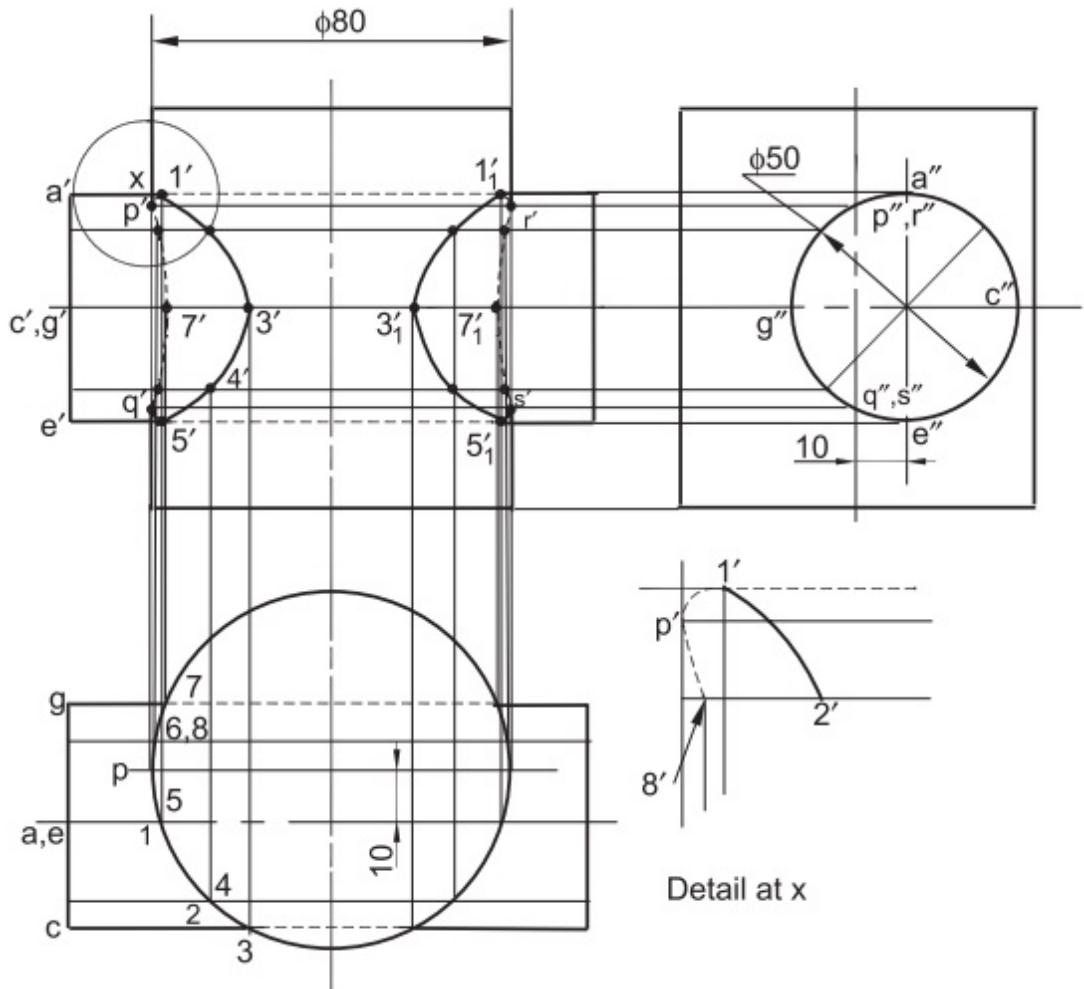
visible points 4', 3', 2' respectively.

**Problem 7** A vertical cylinder of 80 diameter, is penetrated by another cylinder of 50 diameter. The axis of the penetrating cylinder is parallel to both H.P and V.P and is 10 away from that of the vertical cylinder. Draw the projections, showing the lines (curves) of intersection.

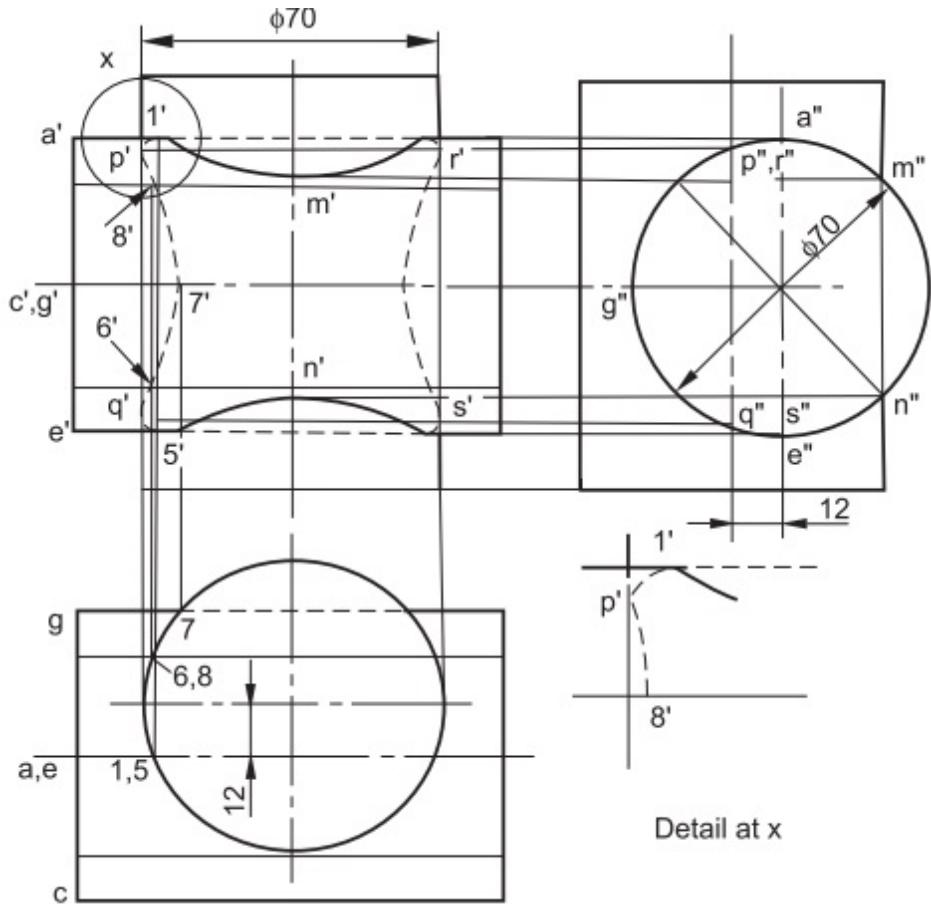
### **Construction ([Fig.15.7](#))**

Follow the principle of Construction: [Fig.15.6](#) and obtain the lines of intersection. However, because of the off-set between the axes, the key points p', q', r' and s' must be considered, while obtaining the line of intersection. Further, because of the off-set, the line of intersection consists of both visible and invisible portions.

**Problem 8** A vertical cylinder of 70 diameter, is penetrated by a horizontal cylinder of the same size. The axis of the penetrating cylinder is 12 away from the axis of the vertical cylinder. Draw the projections of the cylinders, showing the lines (curves) of intersection.



**Fig.15.7**



**Fig.15.8**

**HINT** As the cylinders are of equal size and their axes are off-set, a portion of the penetrating cylinder will be outside the vertical cylinder. Follow the principle of Construction: [Fig.15.7](#) and obtain the lines of intersection as shown in [Fig.15.8](#).

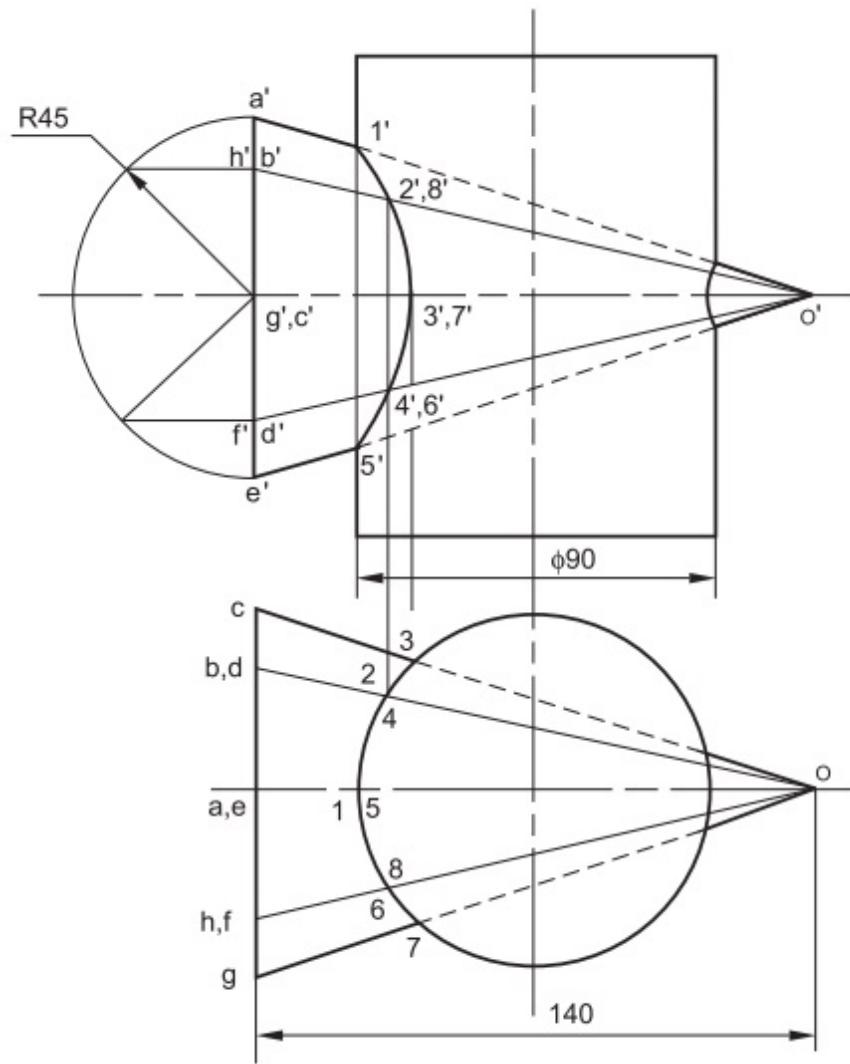
## 15.6 INTERSECTION BETWEEN CONE AND CYLINDER & CONE AND CONE

The method of obtaining the line of intersection between cylinder and cone and cone and cone is similar to that followed for obtaining the line of intersection between cylinder and cylinder.

**Problem 9** *A vertical cylinder of base 90 diameter, is penetrated by a cone of base diameter 90 and axis 140 long. The axes of the two solids bisect each other at right angle. Draw the projections of the two solids, showing the lines of intersection.*

### **Construction (Fig.15.9)**

1. Draw the projections of the two solids.
2. Divide the base circle of the cone into equal parts, say 8 and draw the corresponding generators in both the views.



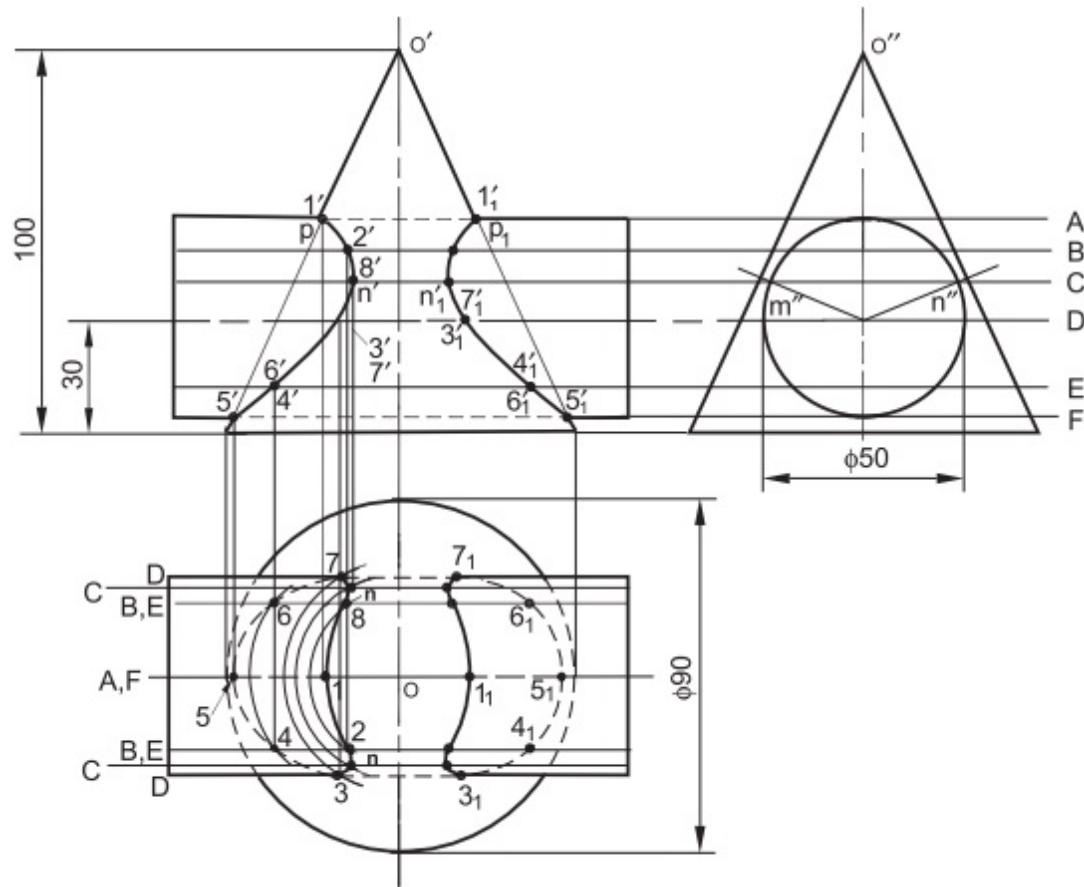
**Fig.15.9**

3. Locate the points of intersection 1, 2, 3, etc., between the above generators and the base circle of the cylinder, in the top view.
4. Obtain the corresponding points  $1'$ ,  $2'$ ,  $3'$ , etc., in the front view, by projection.
5. Join the points by smooth curve and obtain the line of intersection on left side.
6. Follow the steps 3 to 5 and obtain the line of intersection on the right side.

**Problem 10** A vertical cone with diameter of base 90 and axis 100 long, is penetrated by a horizontal cylinder of 50 diameter. The axis of the cylinder intersects the axis of the cone at a point 30 from the base. Draw the projections of the solids, showing the lines of intersection.

**Construction (Fig. 15.10)** Cutting plane method

1. Draw the three views of the solids.
2. Choose a number of cutting planes A, B, C, etc., at different levels and parallel to the base of the cone such that, they produce rectangles in the cylinder and circles in the cone.



**Fig.15.10**

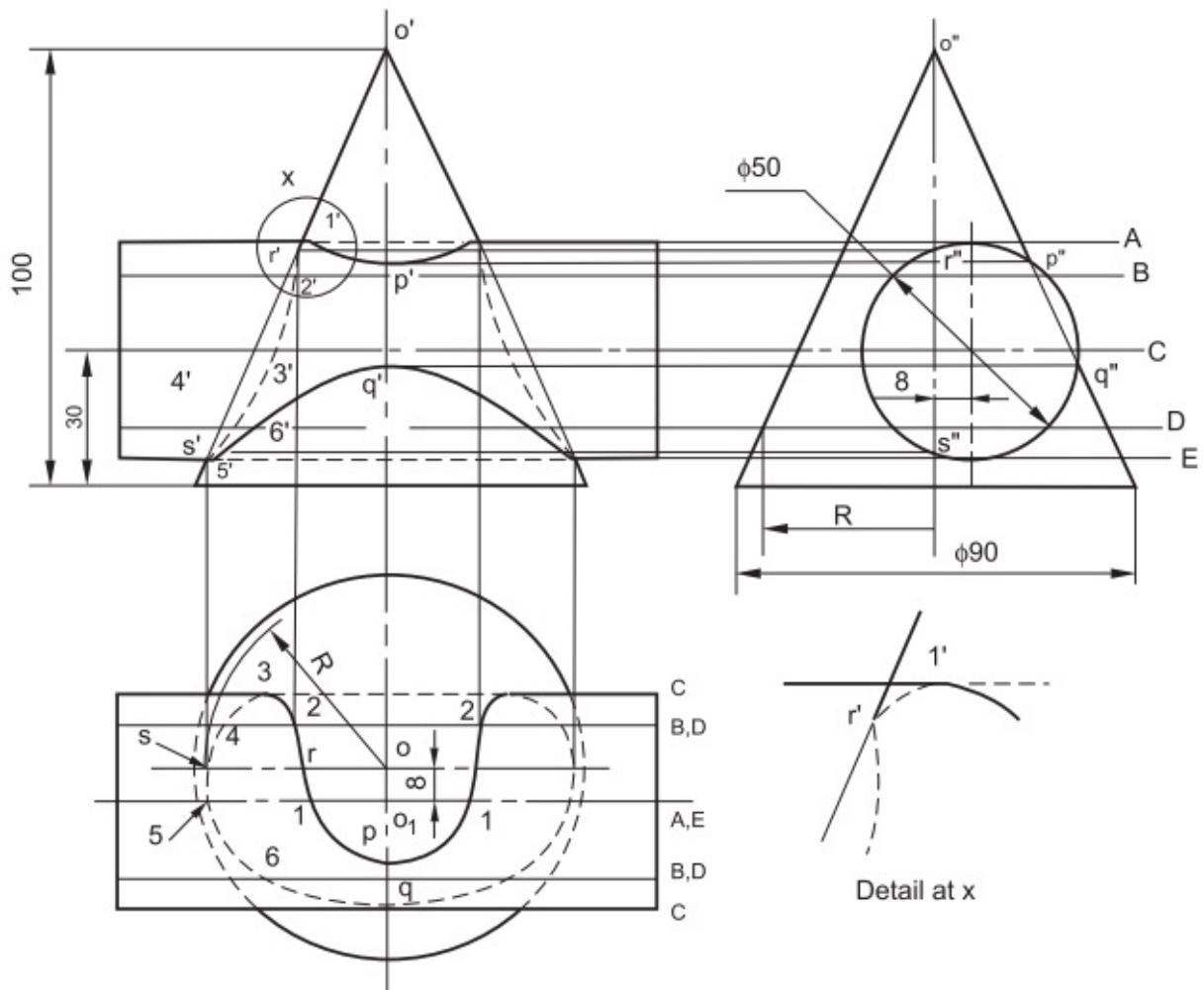
3. Locate the points of intersection in the top view, between the above rectangles and circular arcs.
4. Project these points to the front view.
5. Join the points in the order, in both the views and obtain the required lines of intersection.



- (i)  $m''$  and  $n''$  in the side view are the key points at which the curve changes / X its direction. These are the points at which, lines drawn from the centre of the circle and perpendicular to the extreme generators of the cone, intersect the circle.
- (ii) To locate the points, corresponding to the plane, say B:
  - (a) With centre o and diameter p-p<sub>1</sub>, draw an arc intersecting the rectangle through B-B, at 2-2 in the top view.
  - (b) Through 2, draw a projector, to meet the projector through B at 2', in the front view.

**Problem 11** A vertical cone with base 90 diameter and axis 100 long, is penetrated by a horizontal cylinder of 50 diameter. The axis of the cylinder is 30 above the base of the cone and is 8 away from the axis of the cone. Draw the projections of the solids, showing the lines of intersection.

The axis of the penetrating cylinder is displaced such that, a portion of the cylinder remains outside the cone.

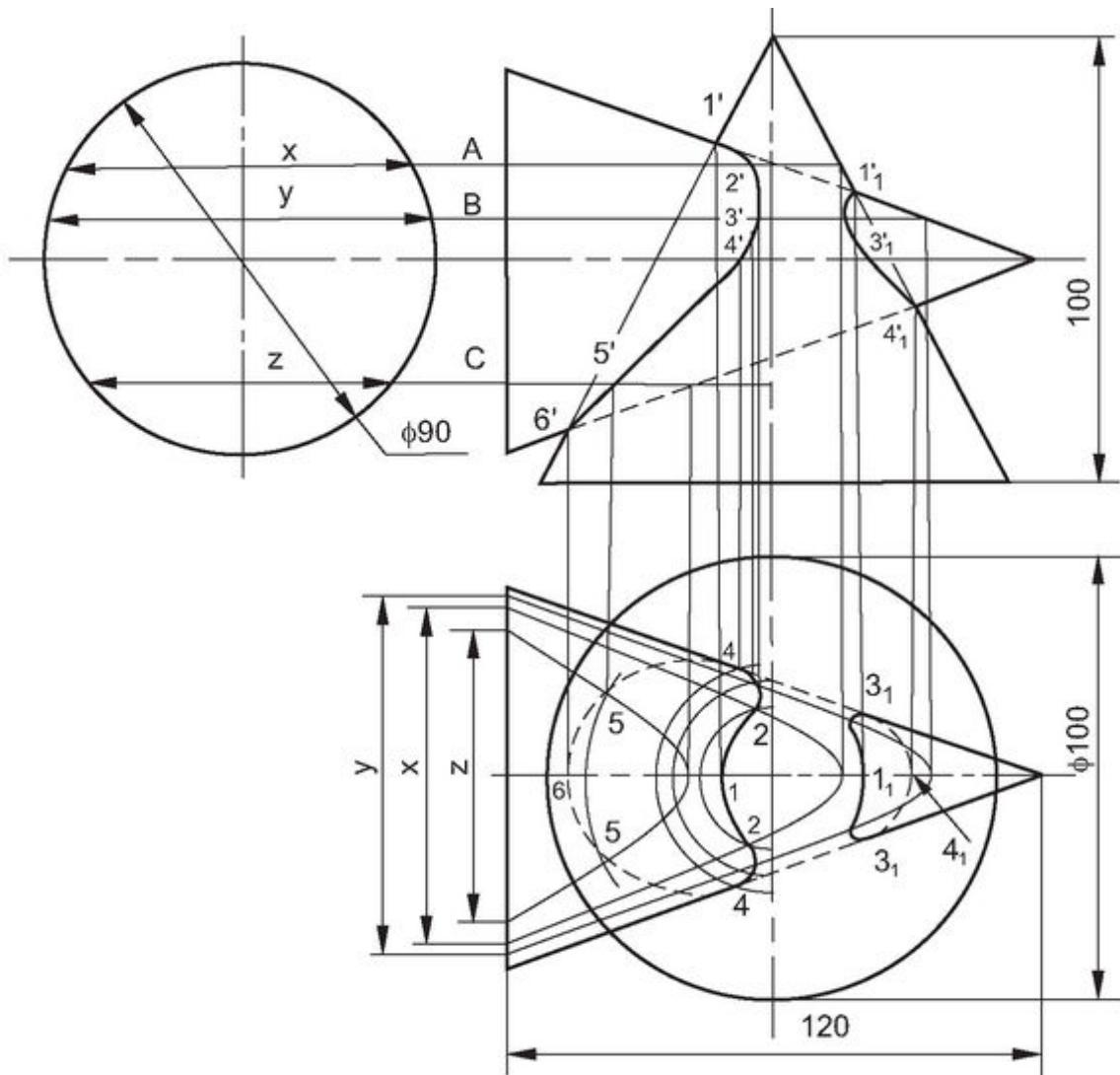


**Fig.15.11**

Follow the principles of Construction: Fig.15.10 and obtain the lines of intersection in the front and top views (Fig.15.11), keeping in mind that the curve changes its direction at key points  $p'$ ,  $q'$  and  $p$ ,  $q$ . The line of intersection is a single continuous curve.

**Problem 12** A vertical cone with diameter of base 100 and axis 100 long, is penetrated by another cone of base 90 diameter and axis 120 long. The axes of the two cones bisect each other at right angle. Draw the projections of the cones, showing the lines of intersection.

**Construction (Fig.15.12)**



**Fig.15.12**

1. Draw the projections of the two cones.
2. Consider horizontal section planes A, B and C, between 1' and 6' in the front view, producing circles in the vertical cone and hyperbolas in the horizontal cone. x, y and z are the bases of the hyperbolas produced by the section planes A, B and C respectively.
3. Draw the hyperbolas and circular arcs in the top view, corresponding to the above section planes and locate

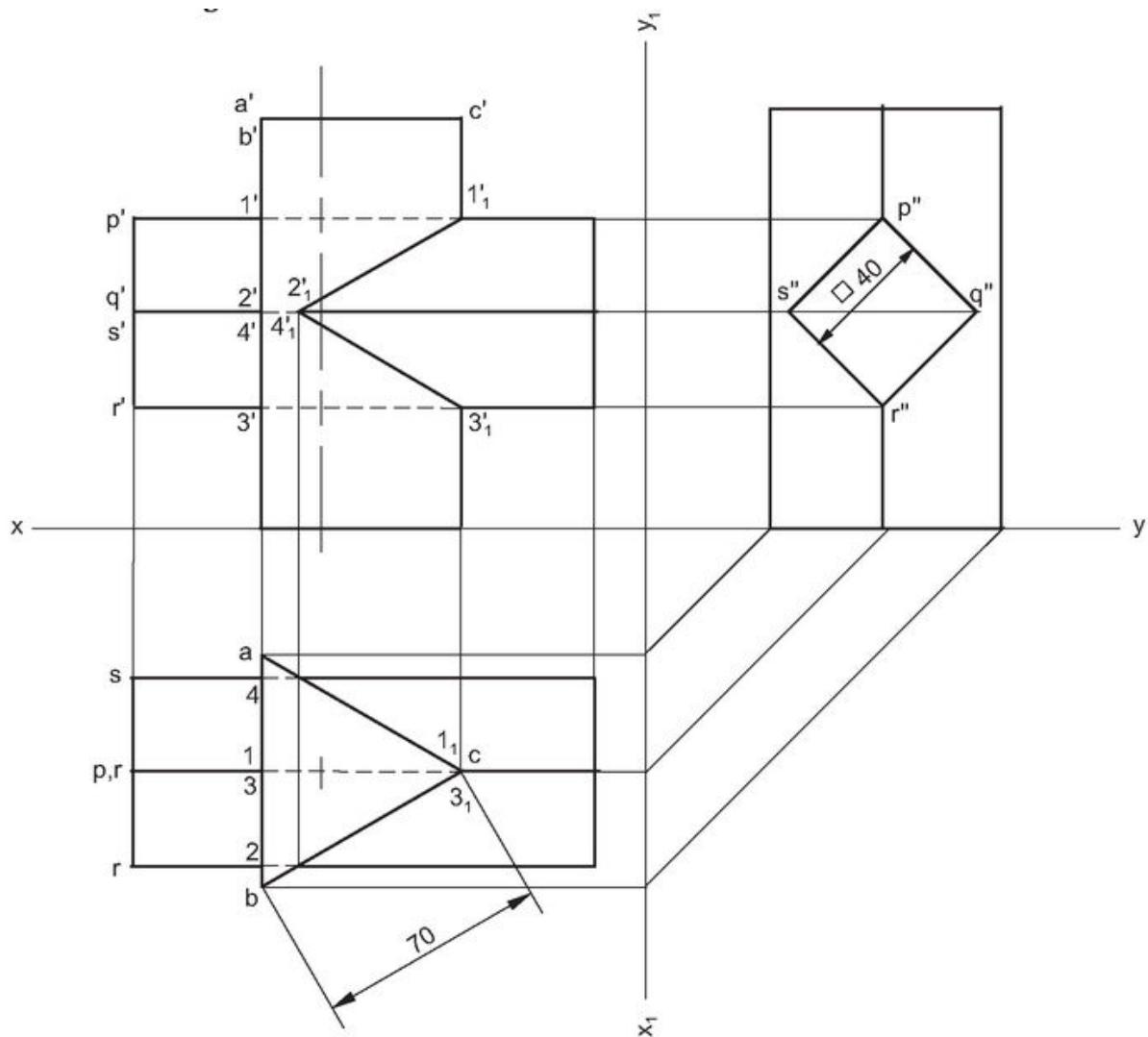
the points of intersection 1, 2-2, 3-3, etc. (refer Construction: [Fig.12.19](#), for drawing the hyperbolas).

4. Project these points on to the corresponding section planes in the front view.
5. Join the points in both the views and obtain the lines of intersection.

## 15.7 EXAMPLES

Problem 13 A triangular prism with side of base 70, is resting on one of its bases on H.P and with a face perpendicular to V.P. It is penetrated by a horizontal square prism of base 40 side. The axes of the two solids intersect each other and a plane passing through the axes of the solids is parallel to V.P. Draw the lines of intersection, when the faces of the square prism are equally inclined to H.P.

***Construction ([Fig.15.13](#))***



**Fig.15.13**

1. Draw the three views of the prisms.
2. In the top view, locate the points  $1, 2, 3, 4$  and  $1_1, 2_1, 3_1, 4_1$  where the edges of the horizontal prism pierce the vertical prism.
3. Project the above points and obtain the corresponding point  $1', 2', 3', 4'$  and  $1_1', 2_1', 3_1', 4_1'$  in the front view.

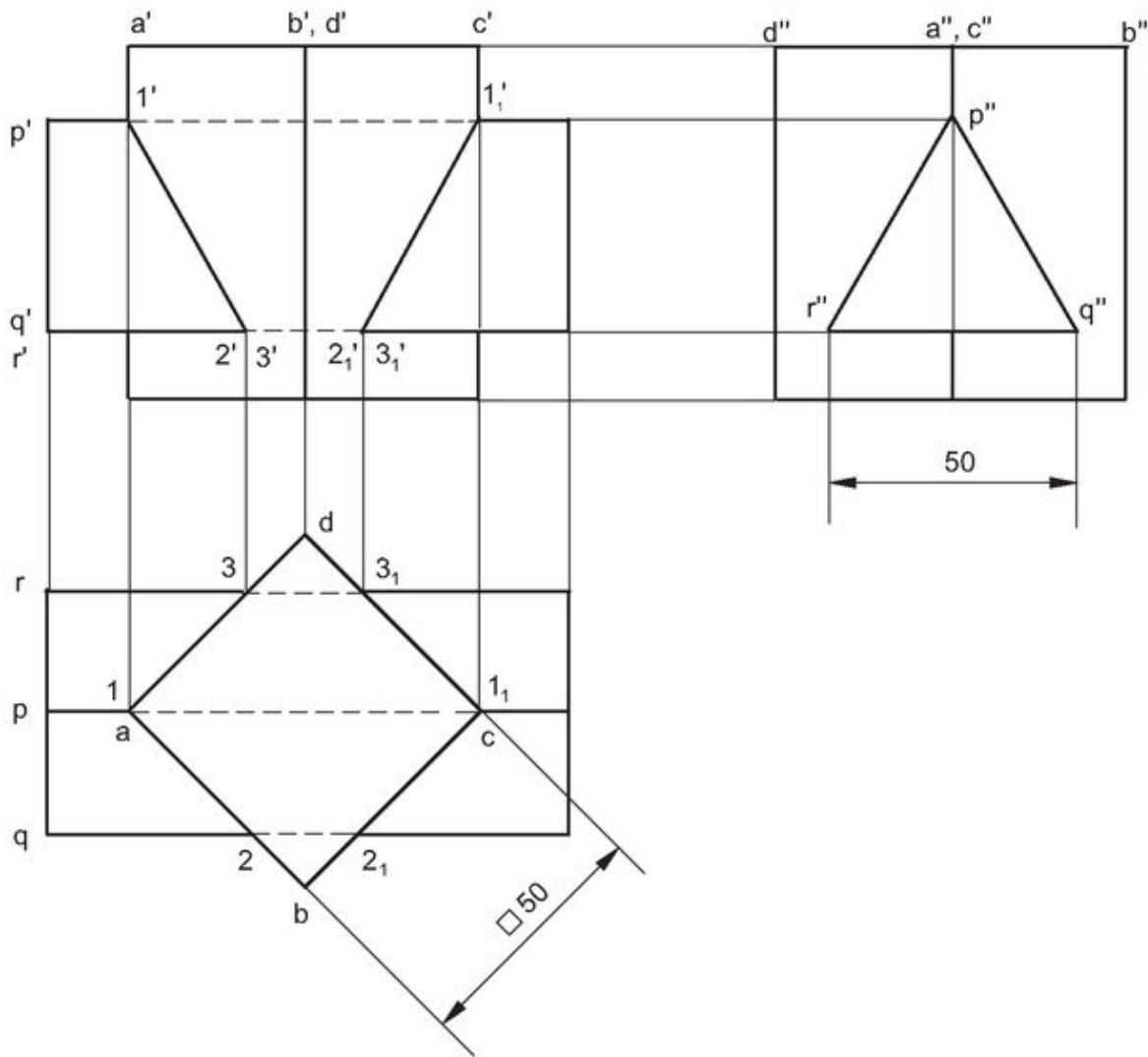
The points  $1', 2', 3'$  and  $4'$  lie on the left vertical face of vertical prism and hence lie along the visible vertical line in the front view.

- Join the points  $1_1'$ ,  $2_1'$ ,  $3_1'$  and  $4_1'$  by the straight lines, forming the line of intersection on the right side.

It may be noted that the visible and invisible lines of intersection are overlapping on the right side, as the axes of the solids intersect without off-set.

**Problem 14** A square prism, with side of base 50, is resting on its base on H.P. It is penetrated by another triangular prism of side of base 50 such that, the axes intersect each other at right angle. If the faces of the square prism are equally inclined to V.P, draw the projections of the solids, showing the lines of intersection.

Follow the principles of Construction: [Fig.15.1](#) and obtain the lines of intersection, as shown in [Fig.15.14](#).



**Fig.15.14**

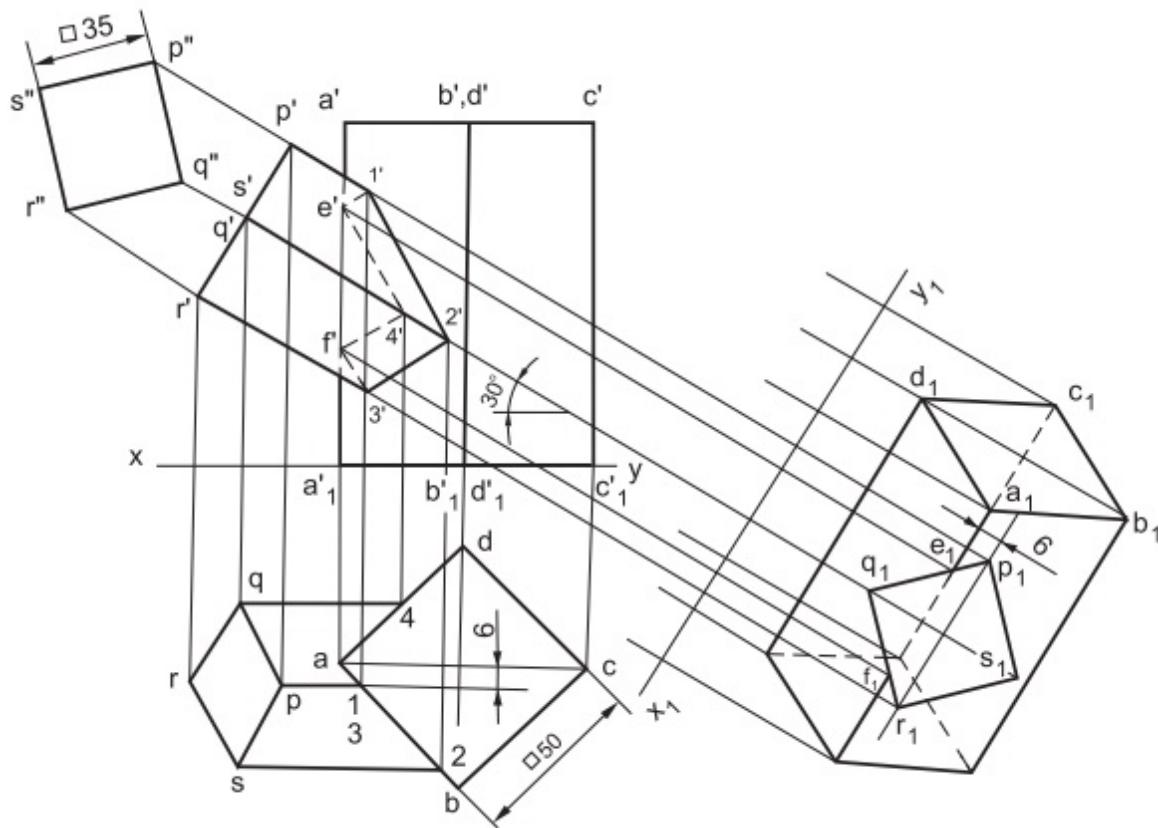
Problem 15 A vertical square pipe of 50 side, has a similar branch of 30 side. The axes of the pipes intersect at  $45^\circ$  with each other. The faces of both the pipes are equally inclined to V.P. Draw the projections of the pipes, showing the lines of intersection.

***Construction (Fig.15.15)***

1. Draw the projections of the vertical pipe and its branch, satisfying the given conditions.

- In the edge view of the vertical prism, locate the points 1, 2, 3 and 4, where the edges of the branch pipe, meet the vertical pipe.
- Repeat steps 3 and 4 of Construction: Fig.15.1 and obtain the line of intersection in the front view.

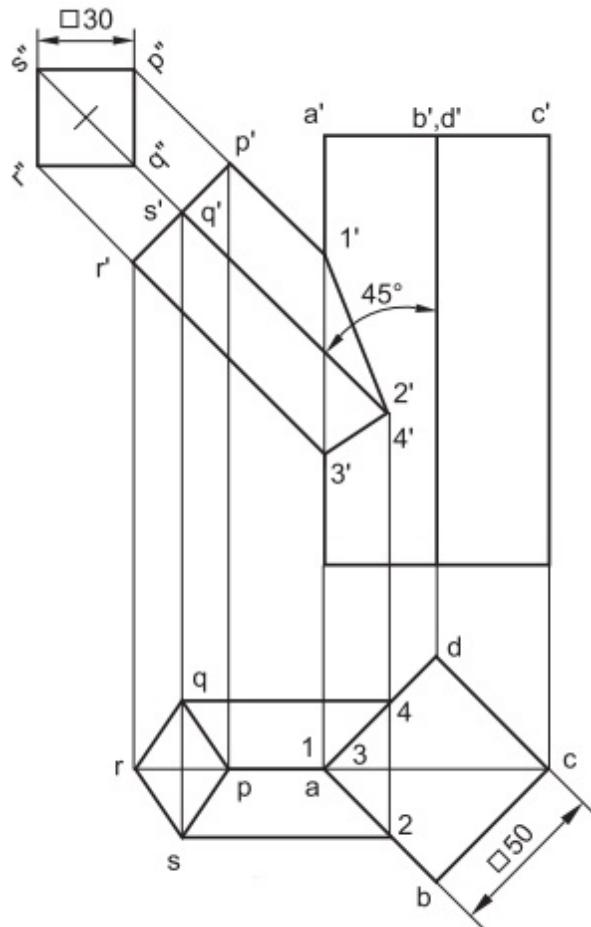
**Problem 16** A square prism with side of base 50, is resting on its base on H.P. It is met by another square prism, with side of base 35 and the axis of which is parallel to V.P and inclined at  $30^\circ$  to H.P. The axes of the two prisms are 6 apart and the faces of both the prisms are equally inclined to V.P. Draw the projections of the two prisms, showing the lines of intersection.



**Fig.15.16**

**Construction (Fig.15.16)**

1. Draw the projections of the arrangement, including an auxiliary view, showing the edge view of the inclined prism.



**Fig.15.17**

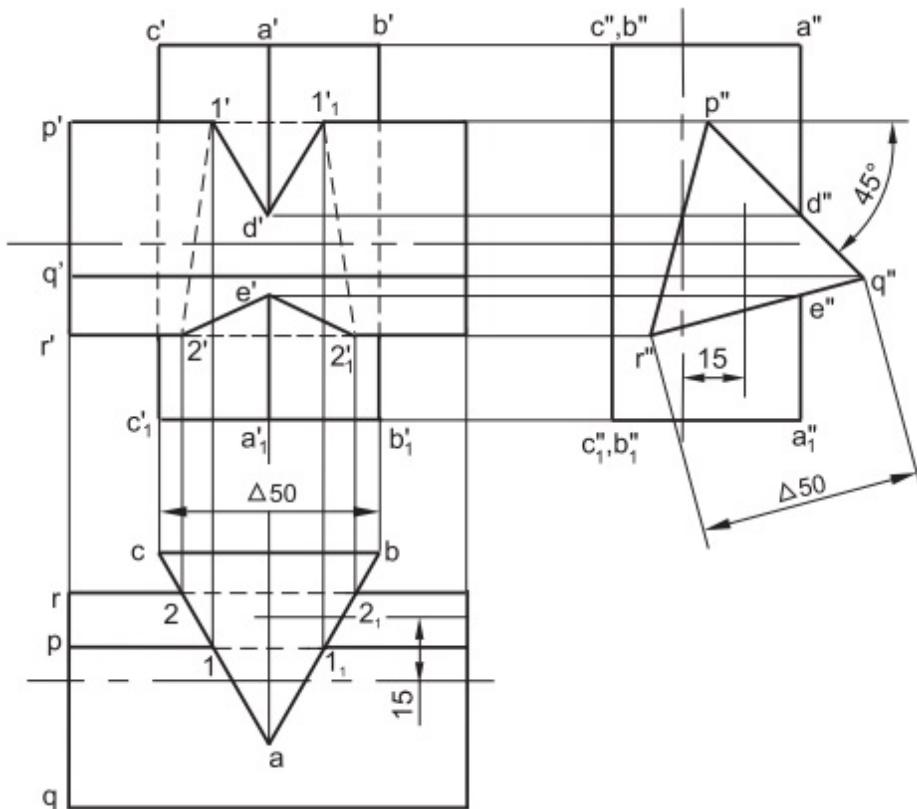
2. Locate the points of intersection 1, 2, 3 and 4, between the edges of the inclined prism and the edge view of the vertical prism.
3. Locate the points e' and f', at which the edge A-A<sub>1</sub> of the vertical prism is cut; by projecting the points e<sub>1</sub> and f<sub>1</sub> from the auxiliary top view.
4. Following the rules of visibility, join the points 1', 2', 3', f', 4', e' and 1', by straight lines and obtain the line

of intersection.

**Problem 17** A horizontal triangular prism, with side of base 50, with the top rectangular face inclined at  $45^\circ$  with H.P. penetrates a vertical triangular prism, with side of base 50, having a face parallel to V.P. The axis of the horizontal prism is 15 in front of that of the vertical prism. Draw the projections of the two prisms, showing the lines of intersection.

**HINT** As the prisms are of the same size and their axes are off-set, and also because of the orientation of the horizontal prism, a part of the penetrating prism will be out-side the vertical prism.

### Construction ([Fig.15.17](#))

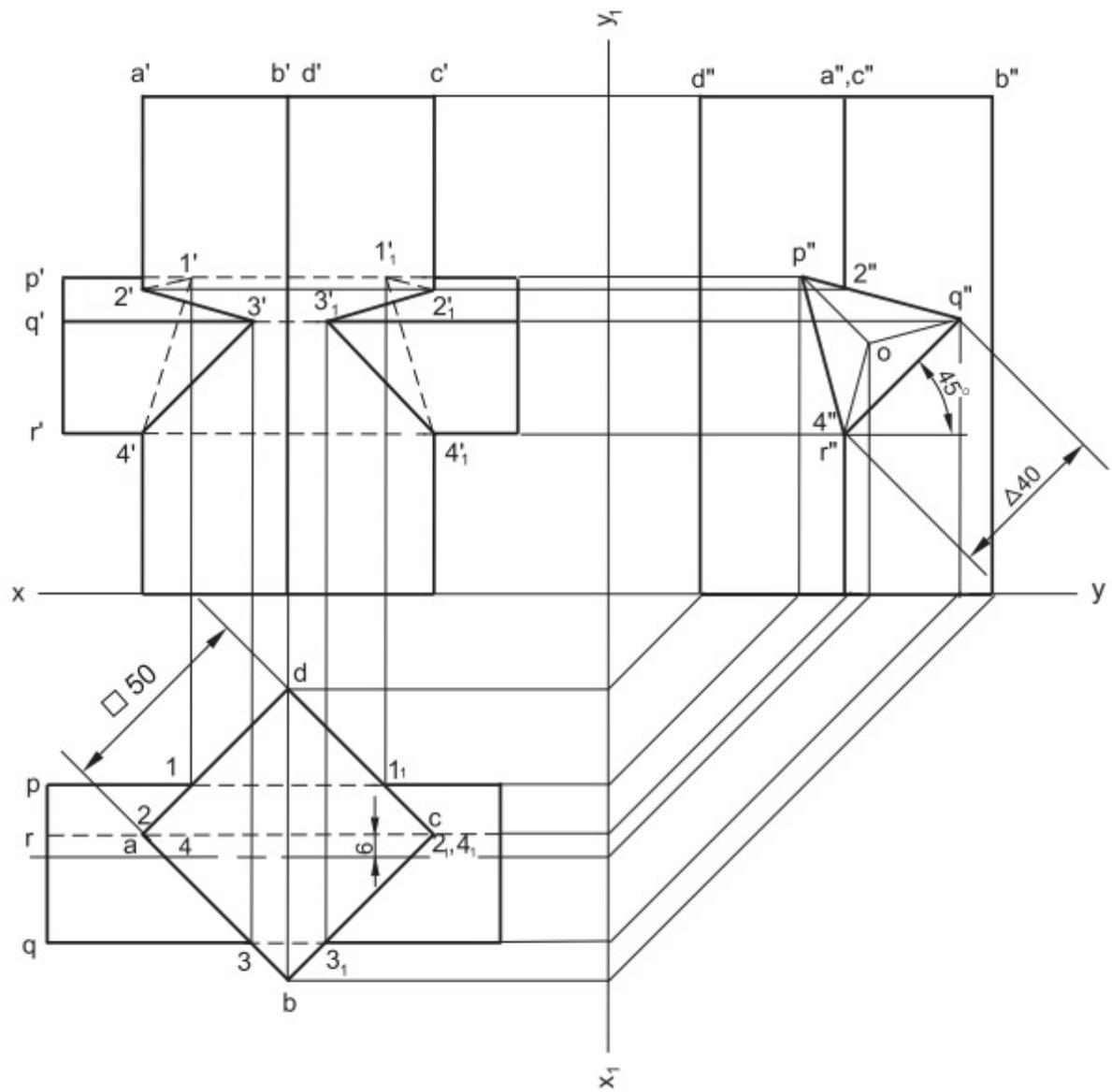


**Fig.15.17**

1. Draw the three views of the prisms.
2. In the top view, locate the points of intersection 1, 2 and  $1_1$ ,  $2_1$ , where the edges of the horizontal prism, pierce and emerge the vertical one.
3. Obtain the points  $1'$ ,  $2'$  and  $1'_1$ ,  $2'_1$  in the front view, by projection.
4. In the side view, locate the points  $d''$ ,  $e''$  on the edge A- $A_1$  of the vertical prism, through which the horizontal prism penetrates. Obtain the points  $d'$ ,  $e'$  in the front view, by projection.
5. Following the rules of visibility, join the points  $1'$ ,  $2'$ ,  $e'$ ,  $2'_1$ ,  $1'_1$ ,  $d'$  and  $1'$ , by straight lines and obtain the line of intersection.

**Problem 18** A square prism with side of base 50, is resting on its base on H.P. It is penetrated by a horizontal triangular prism of base 40 side such that, the axes are offset by 6. The faces of the square prism are equally inclined to V.P, while a face of the triangular prism is inclined at  $45^\circ$  with H.P. Draw the projections of the two solids, showing the lines of intersection.

**Construction (Fig.15.18)**



**Fig.15.18**

1. Draw the three views of the prisms.
2. In the edge view of the square prism, locate the points 1 and 3, where the triangular prism pierces the vertical one.
3. Obtain the points 1' and 3' in the front view, by projection.

4. In the side view, locate the points  $2''$  and  $4''$ , through which the horizontal prism pierces the vertical edge A-A<sub>1</sub>.
5. Obtain the points  $2'$  and  $4'$  in the front view, by projection.
6. Join the points in the order, by straight lines and obtain the line of intersection on the left side.
7. Repeat the steps 2 to 6 and obtain the line of intersection on the right side.

**Problem 19** A hexagonal prism with side of base 40, is resting on one of its bases on H.P with a face parallel to V.P. The prism is penetrated by a square prism of side of base 40. The axis of the square prism is parallel to V.P and inclined at  $30^\circ$  to H.P with its faces equally inclined to H.P. Draw the projections, showing the lines of intersection, when

- (i) The axes of both the solids are intersecting,
- (ii) The axis of the square prism is off-set by 8 from the axis of the hexagonal prism, and
- (iii) A face of the hexagonal prism is perpendicular to V.P and the axis of the square prism is off-set by 8 from the axis of the hexagonal prism.

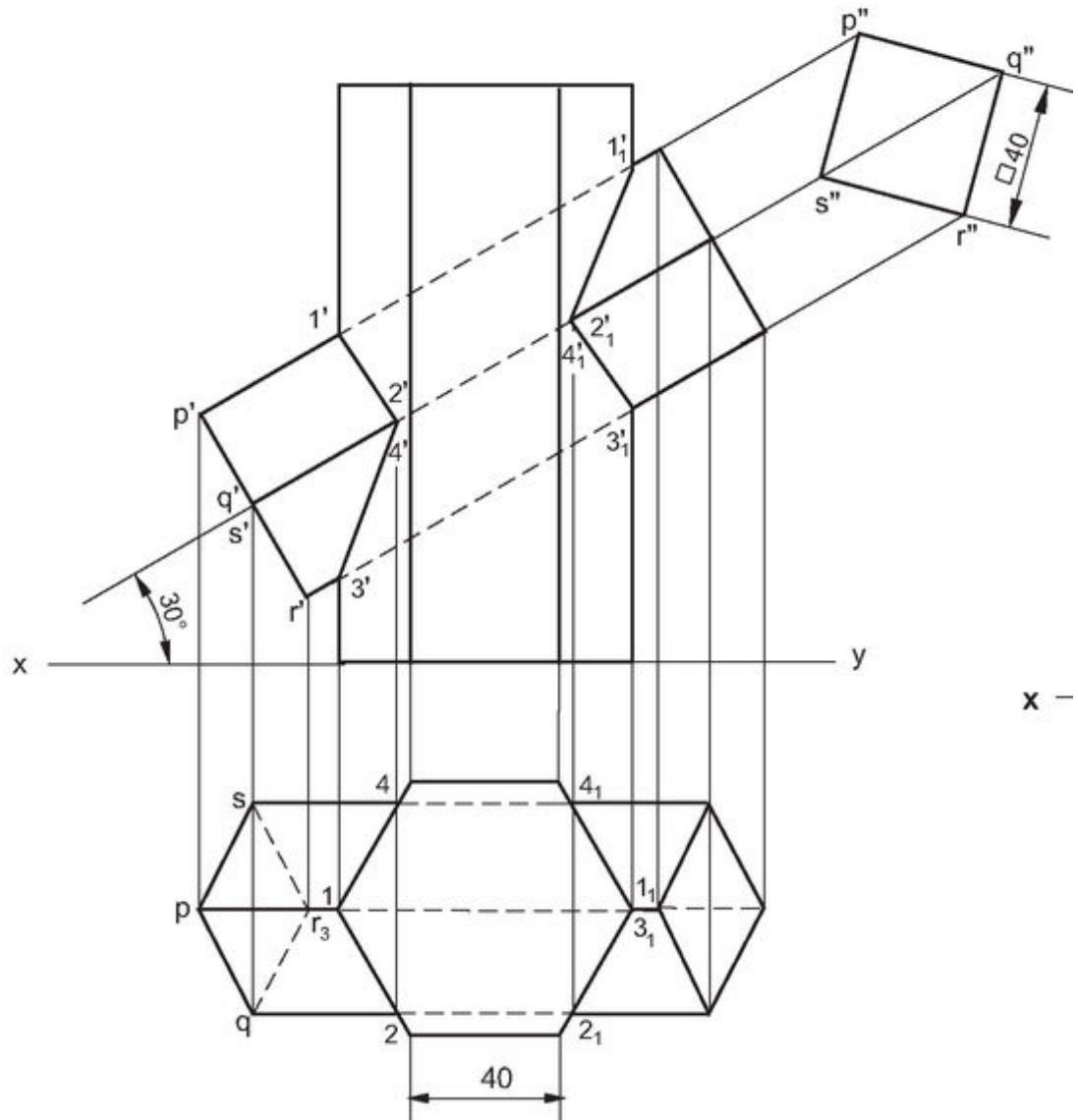
### **Construction (*Fig.15.19a*)**

1. Draw the projections of the two prisms, including the edge view of the penetrating prism.
2. Locate the points of the intersection  $1,2,3,4$  and  $1_1,2_1,3_1,4_1$ , between the edges of the inclined prism and the edge (top) view of the vertical prism.

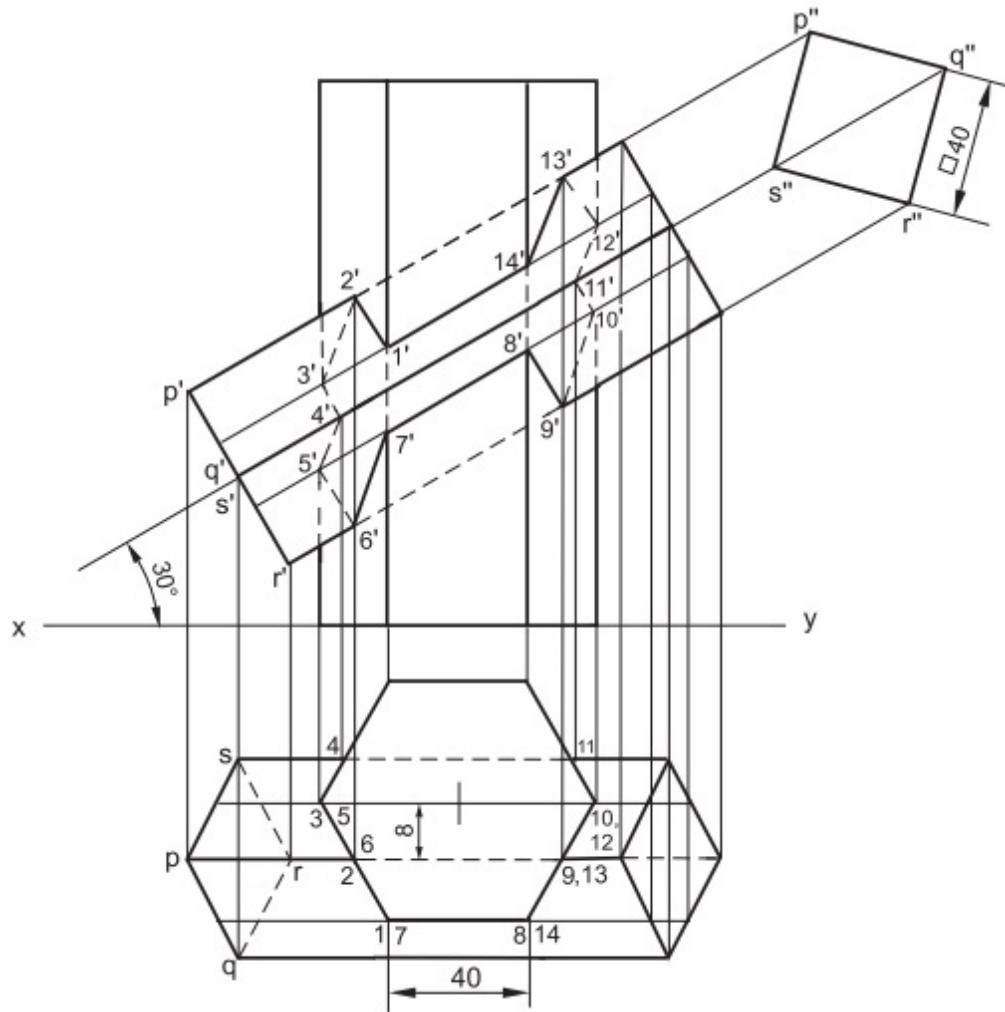
3. Obtain the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $1_1'$ ,  $2_1'$ ,  $3_1'$ ,  $4_1'$  on the corresponding edges in the front view, by projection.
4. Join the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $1_1'$ ,  $2_1'$ ,  $3_1'$ ,  $4_1'$  by straight lines, forming the lines of intersection on the left and right sides.

It may be noted that as the axes of the solids intersect, the visible and the invisible lines of intersection are overlapping.

[Figure 15.19b](#) shows the details of the construction for obtaining the lines of intersection when the axes are off-set by 8. Because of the off-set, the invisible portions of the lines of intersection are separated and so are represented by the dotted lines.

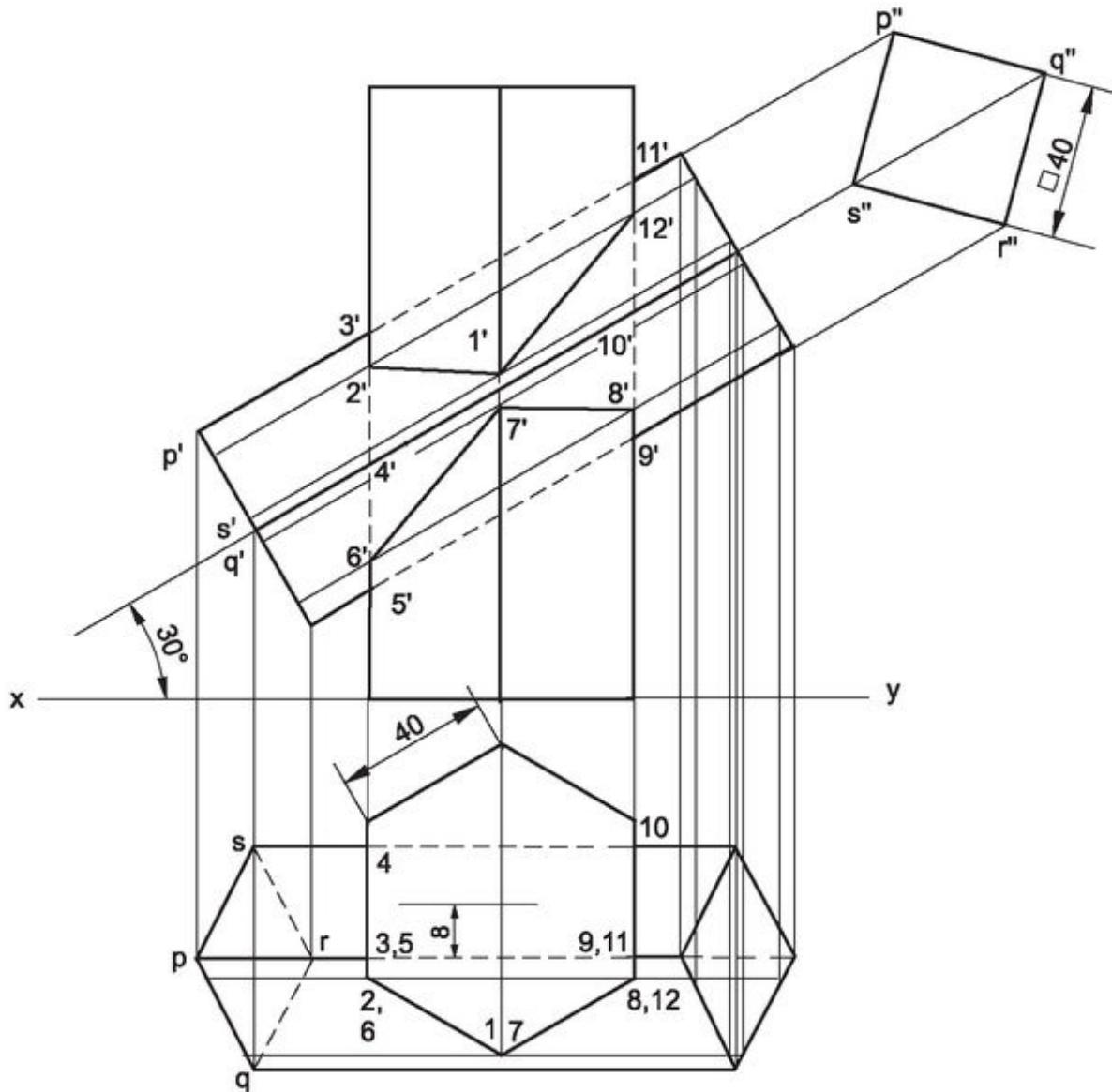


**Fig.15.19a**



**Fig.15.19b**

Figure 15.19c shows the details of the construction for obtaining the lines of intersection when the axes are off-set by 8 and the two opposite faces of the hexagonal prism are perpendicular to V.P.



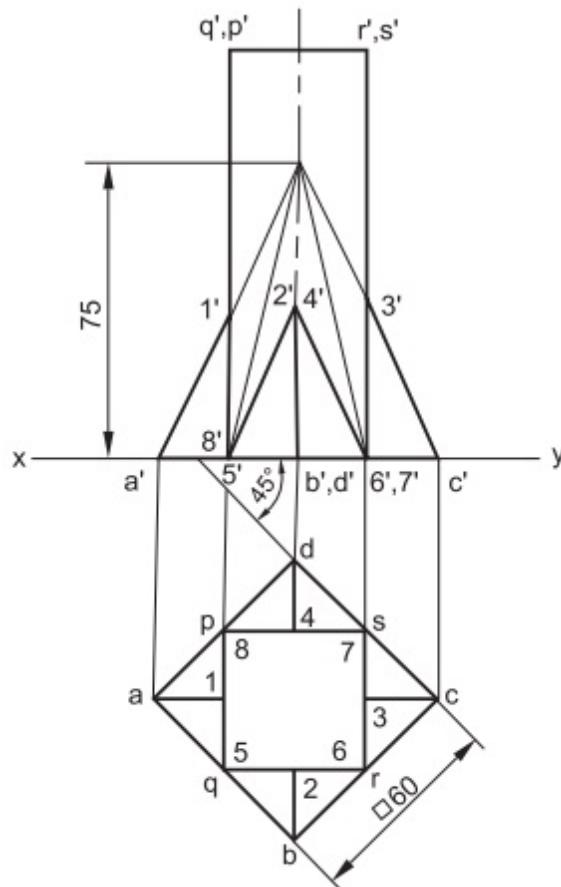
**Fig.15.21c**

**Problem 20** A square pyramid with side of base 60 and axis 75 long, is resting on its base on H.P, with the sides of the base equally inclined to V.P. A vertical square prism meets the pyramid such that, the axes coincide, while a face of it is parallel to V.P. The diagonal of the base of the prism is 60. Draw the projections, showing the lines of intersection.

**Construction (Fig.15.20)**

1. Draw the projections of the two solids, satisfying the given conditions.
2. In the top view, locate the points of intersection 1,2,3 and 4 where the faces of the square prism are touching the slant edges of the square pyramid.
3. Obtain the front views of the above points, viz., 1', 2', 3' and 4', by projection.
4. Join the above points in the order by straight lines and obtain the line of intersection in the front view.

It may be noted that in the present case, the lines of intersection are straight lines and the visible and invisible portions are overlapping.



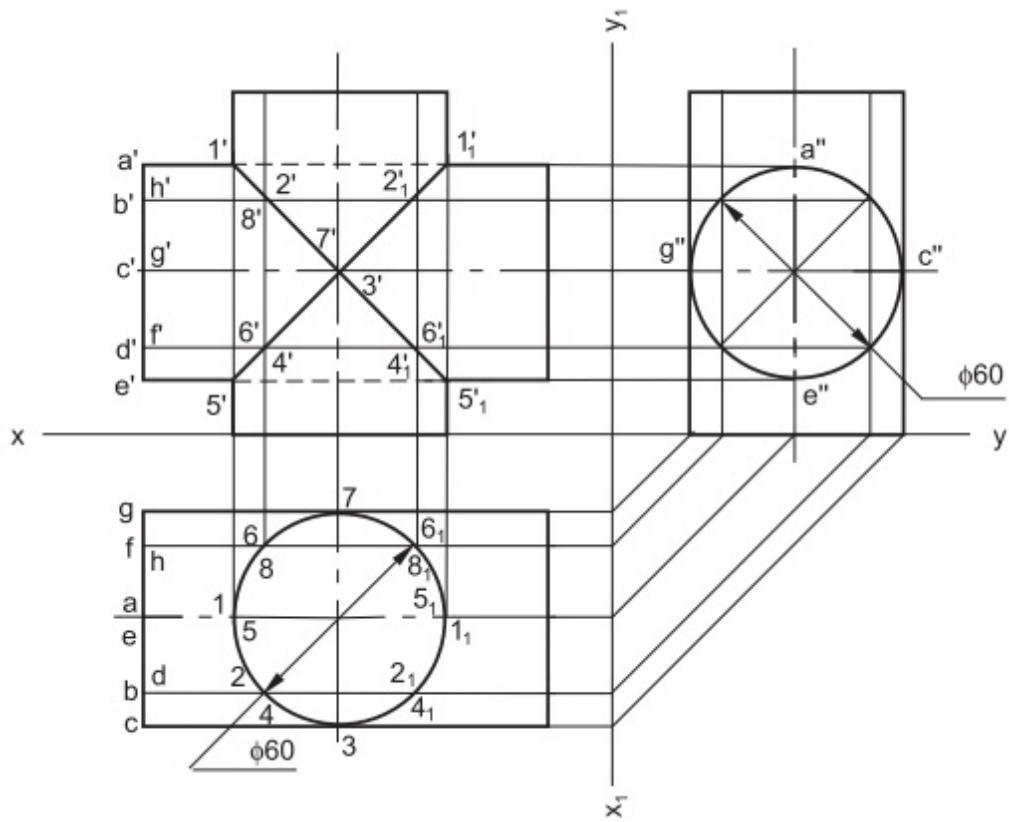
**Fig.15.20**

**Problem 21** A vertical cylinder of 60 diameter is penetrated by another cylinder of 60 diameter. The axes of the two cylinders are intersecting at right angle. Draw the projections of the two cylinders, showing the lines of intersection.

**Construction (Fig.15.21)**

1. Draw the three views of both the cylinders.
2. Divide the circle (side or the edge view of the horizontal cylinder) into a number of equal parts, say 8 and mark the division points.
3. Locate the generators in the front and the top views, corresponding to the above division points.
4. Locate the points of penetration and emergence 1,2,3, etc.,  $1_1, 2_1, 3_1$ , etc., of the generators with the vertical cylinder in the top (edge) view.
5. Project and obtain the points  $1'$ ,  $2'$ ,  $3'$ , etc.,  $1'_1$ ,  $2'_1$ ,  $3'_1$ , etc., in the front view, on the corresponding generators.
6. Join the points in the order and obtain the lines of intersection.

It may be noted that in the present case, i.e., when both the cylinders are of the same size; the lines of intersection are straight lines.



**Fig.15.21**

**Problem 22** A vertical cylinder of 80 diameter is penetrated by another cylinder of 50 diameter. The axis of the penetrating cylinder is inclined at  $30^\circ$  with H.P and parallel to V.P. Draw the projections of the cylinders, showing the lines of intersection, when the axes of two cylinders, intersect each other.

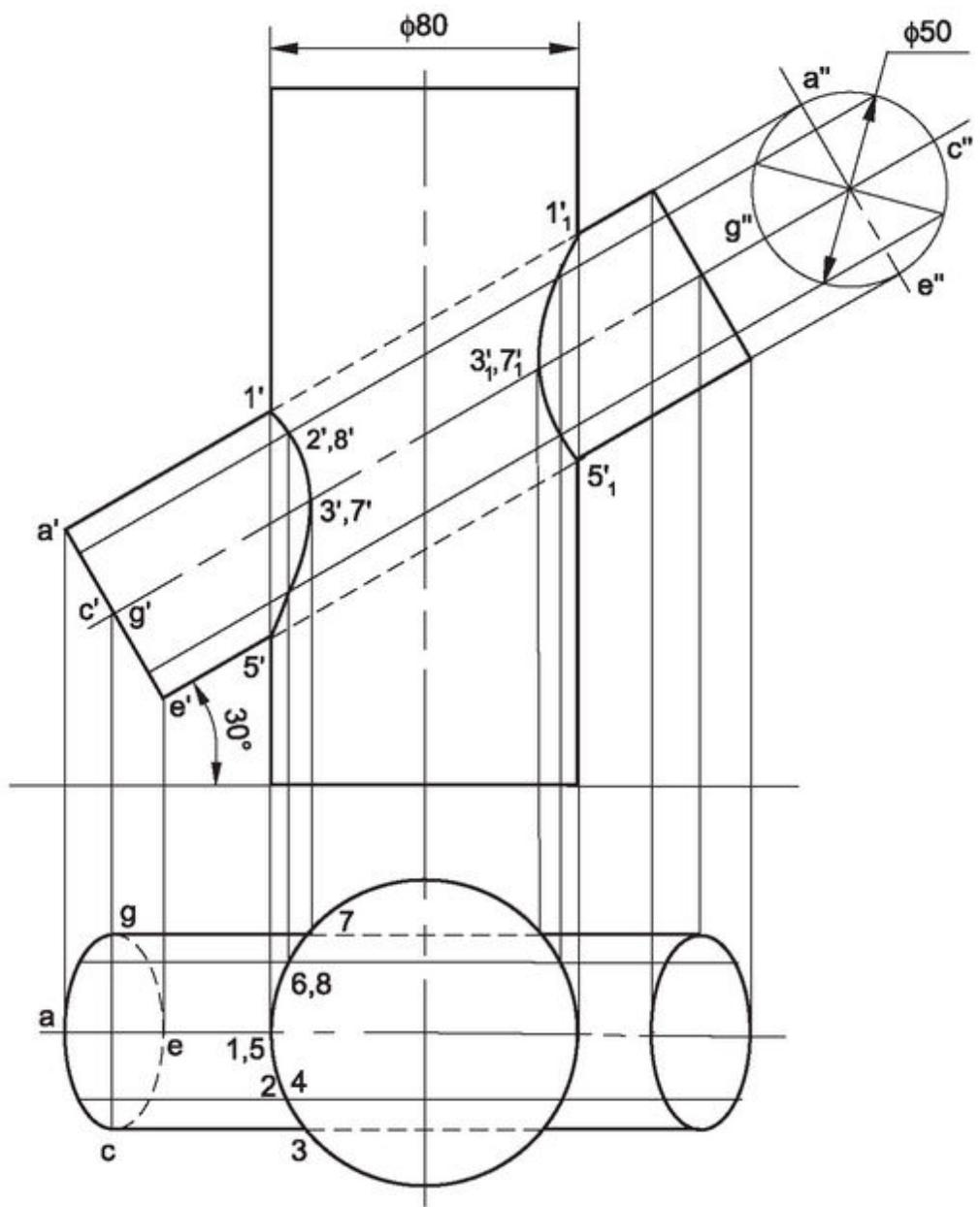
**Construction (Fig.15.22)**

1. Draw the projections of the two cylinders, including the edge view of the penetrating cylinder.
2. Divide the edge view of the penetrating cylinder into some equal parts, say 8 and draw the corresponding generators in the front and top views.
3. Follow the steps 4 to 7 of Construction: Fig.15. 6 and obtain the lines of intersection.

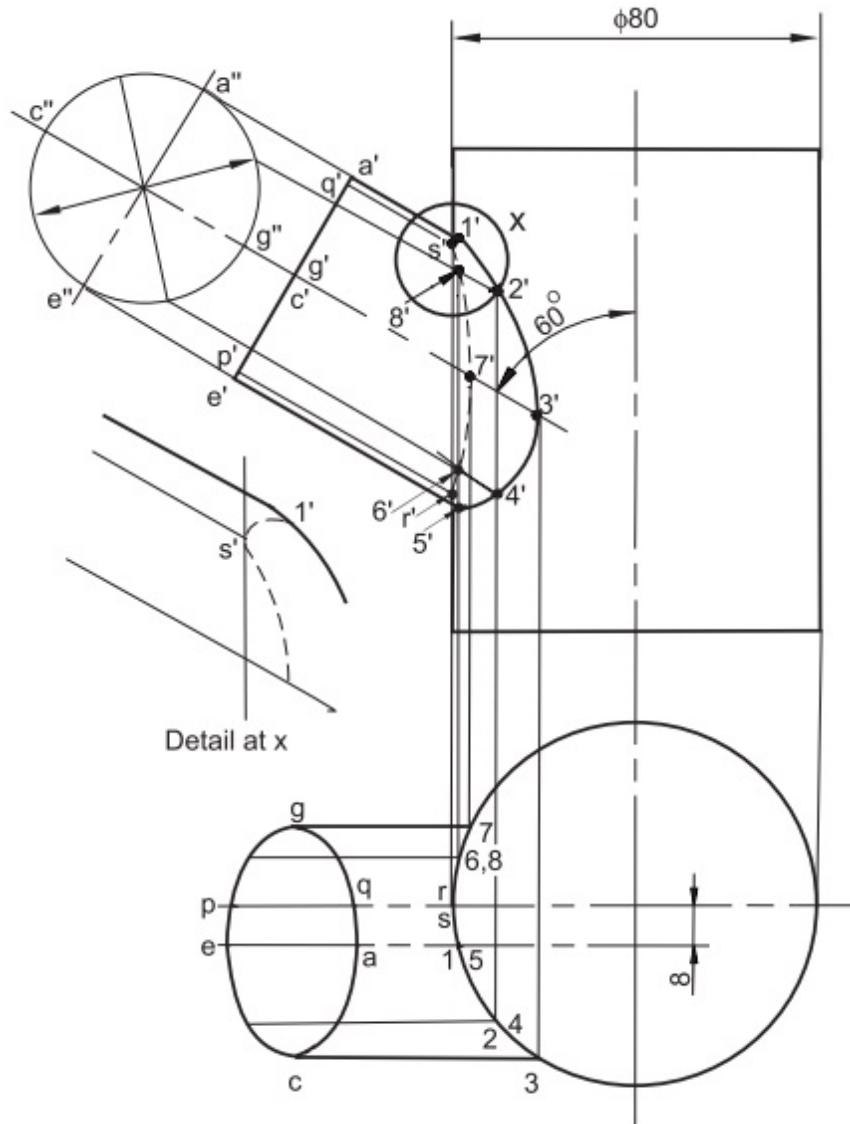
**Problem 23** A vertical pipe of 80 diameter, has a branch of 50 diameter. The axis of the branch is inclined at  $60^\circ$  with that of the main pipe and 8 away from it. Draw the projections of the pipes, showing the line of intersection.

**Construction (Fig.15.23)**

1. Draw the projections of the vertical and branch pipes, including the edge view of the branch pipe.
2. Divide the edge view of the branch pipe into some equal parts, say 8 and draw the corresponding generators in the front and top views.
3. Follow the steps 4 and 5 of Construction: Fig.15. 6 and obtain the points 1, 2, 3, etc., and 1', 2', 3', etc.



**Fig.15.22**

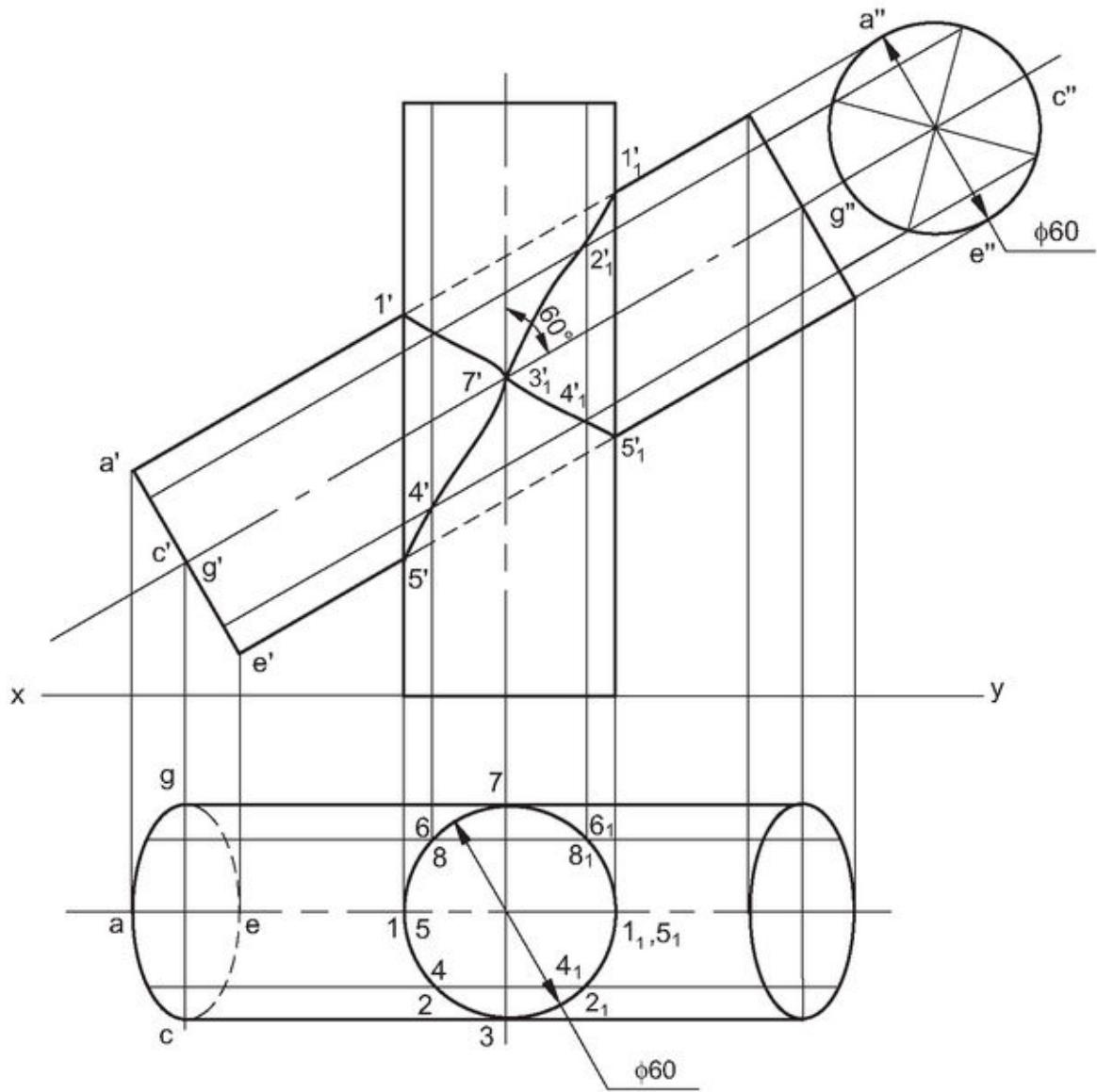


**Fig.15.23**

4. Considering the generators through p and q in the top view, locate the transition points r' and s' in the front view, by projection.
5. Following the rules of visibility, join the points in the order and obtain the line of intersection.

**Problem 24** A vertical cylinder of 60 diameter is penetrated by another cylinder of the same size. Draw the line of intersection, when the axes intersect at 60°.

### **Construction (Fig.15.24)**



**Fig.15.24**

1. Draw the projections of the two cylinders, including the edge view of the penetrating cylinder.
2. Divide the edge view of the penetrating cylinder into a number of equal parts, say 8 and mark the division points.

3. Draw the generators in the front and top views, corresponding to the above division points.
4. Locate the points of intersection 1, 2, 3, etc.,  $1_1$ ,  $2_1$ ,  $3_1$ , etc., between the generators and the top (edge) view of vertical cylinder.
5. Project and obtain the corresponding points  $1'$ ,  $2'$ ,  $3'$ , etc., in the front view.
6. Join the points in the order, by smooth curves and obtain the lines of intersection as shown.

In the present case, though both the cylinders are of the same size; the lines of intersection appear as smooth curves, because of the obliquity between the axes.

**Problem 25** *A cylinder of 60 diameter stands on its base on the ground. It is penetrated centrally by a cylinder of 30 diameter whose axis is parallel to H.P but inclined at an angle of  $45^\circ$  to V.P. Draw the projections, showing the curves of intersection.*

### **Construction ([Fig.15.25](#))**

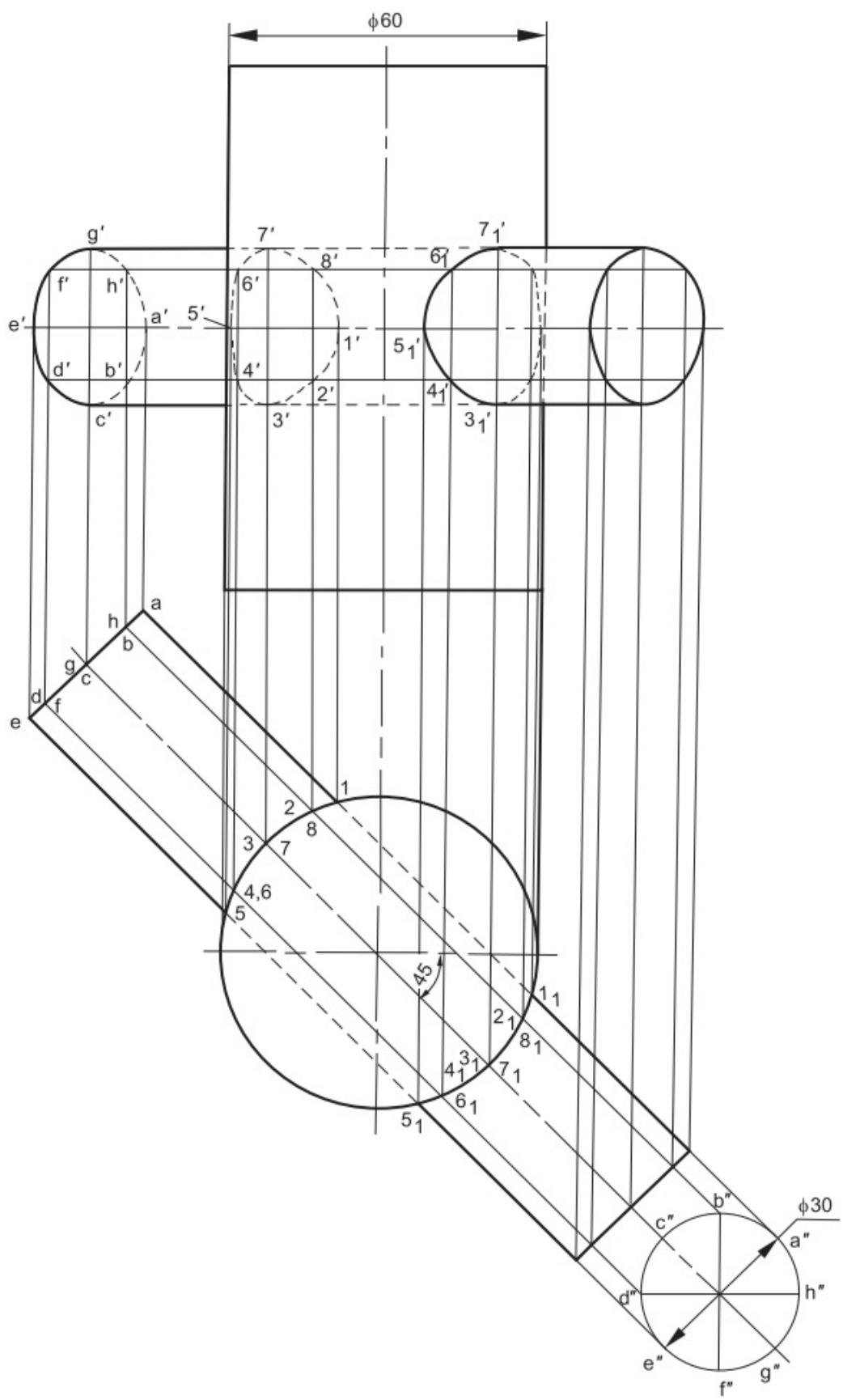
1. Draw the projections of both the cylinders and the edge view of the penetrating cylinder.
2. Divide the edge view of the penetrating cylinder into some equal parts, say 8 and draw the corresponding generators in the top and front views.
3. Locate the points of intersection 1, 2, 3, etc., and  $1_1$ ,  $2_1$ ,  $3_1$ , etc., between the generators and the edge view of the vertical cylinder.
4. Project and obtain the corresponding points  $1'$ ,  $2'$ ,  $3'$ , etc. and  $1_1'$ ,  $2_1'$ ,  $3_1'$ , etc., in the front view.

- Join the points  $1'$ ,  $2'$ ,  $3'$ , etc. and  $1_1'$ ,  $2_1'$ ,  $3_1'$ , etc., by smooth curves and obtain the lines of intersection on both right and left sides of the front view.

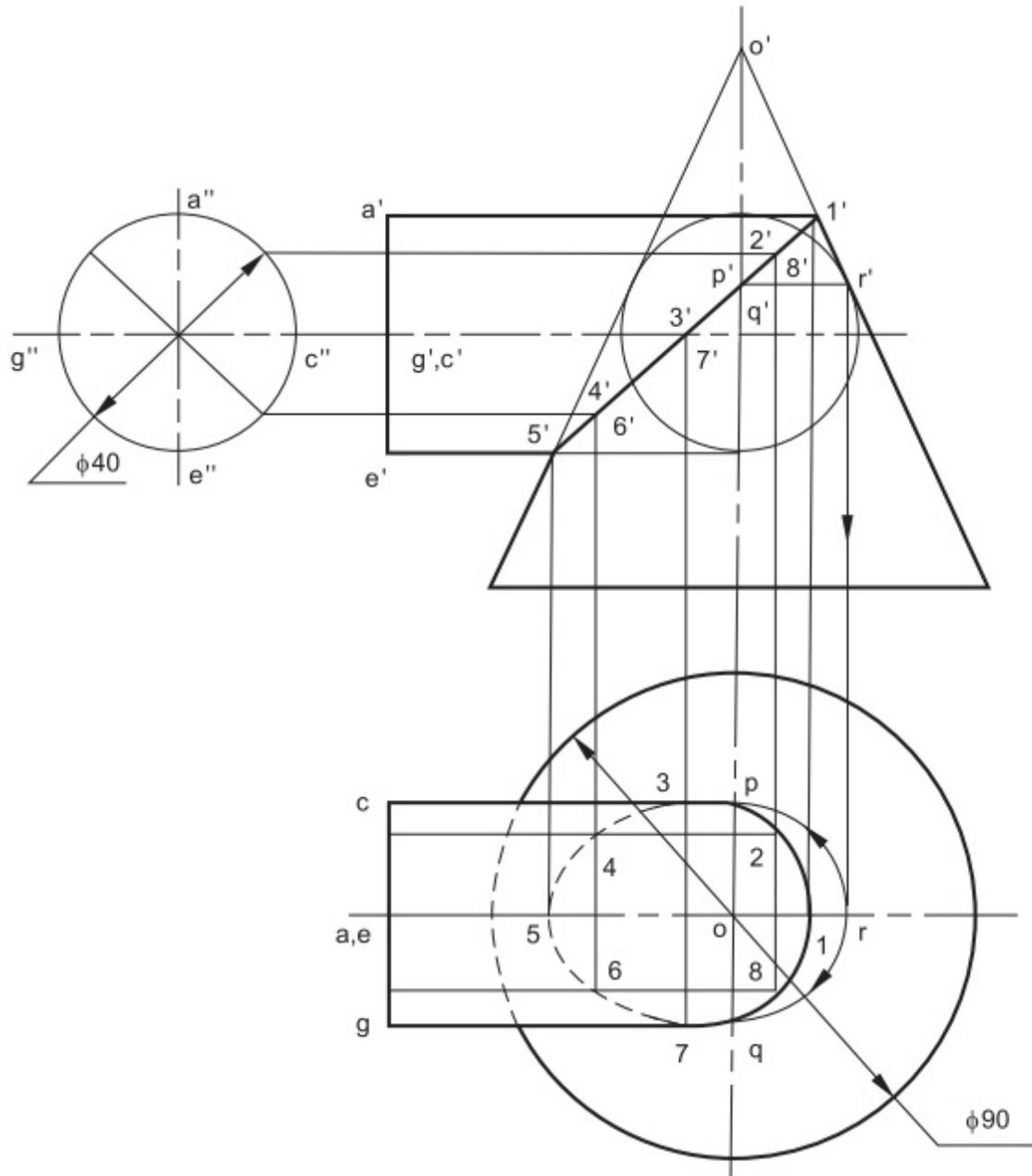
**Problem 26** A funnel is made of a cylindrical pipe and a conical part, both enveloping a common sphere of 40 diameter, with their axes intersecting at right angle. The diameter at the mouth of the funnel is 90. Draw the projections of the funnel, showing the line of intersection, when it is placed with its mouth on H.P.

### **Construction (Fig.15.26)**

- Draw the projections of the arrangement, satisfying the given conditions, including the edge view of the cylindrical pipe.
- Divide the edge view into some equal parts and obtain the corresponding generators in the front and top views.
- Locate the points of intersection  $1'$ ,  $2'$ ,  $3'$ , etc., in the front view, between the generators and the line of intersection  $1'-5'$ . Also, locate the transition points  $p'$  and  $q'$  between the axis of the conical part and the line of intersection.
- Obtain the points corresponding to the above, viz., 1, 2, 3, etc., and p and q in the top view, by projection.
- Join the points in the order, and obtain the line of intersection.



**Fig.15.25**

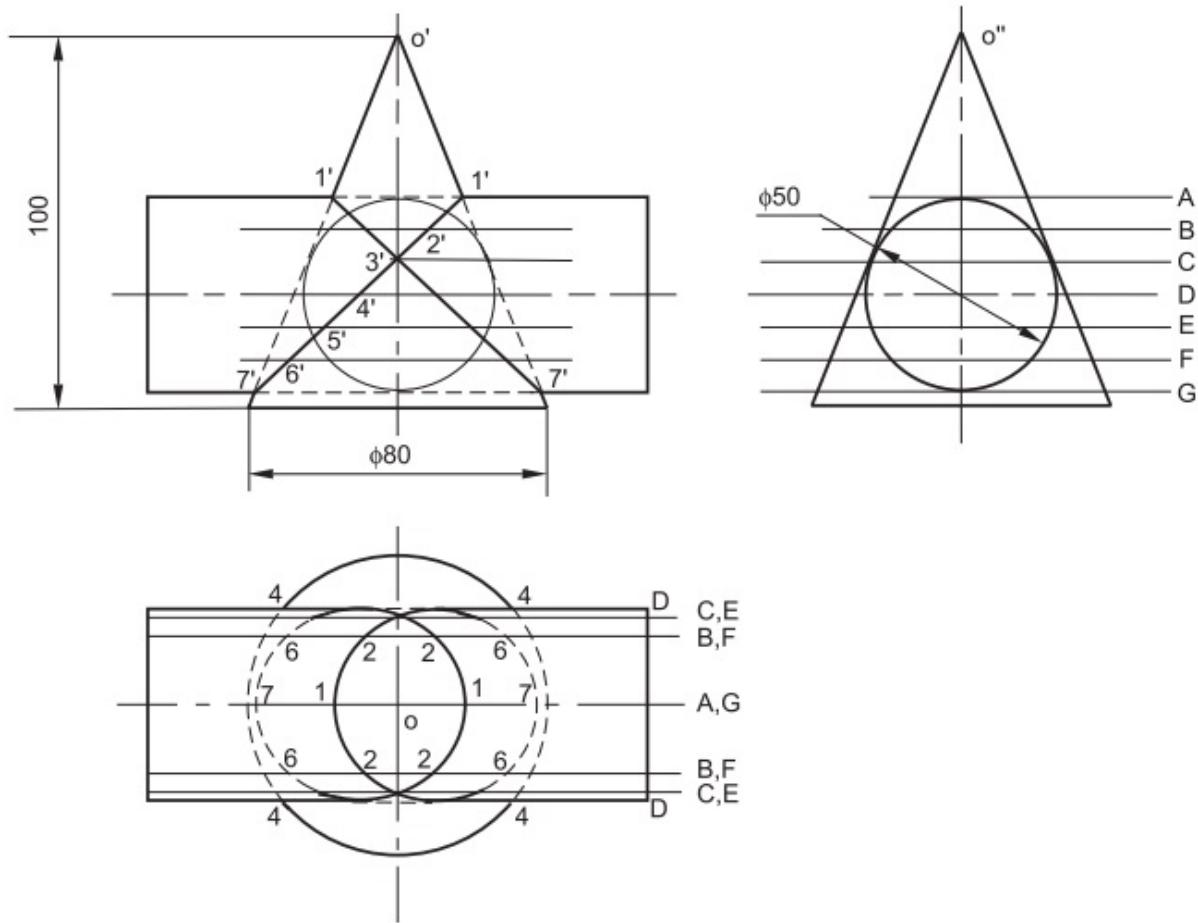


**Fig.15.26**

**Problem 27** A vertical cone with base 80 diameter and axis 100 long, is penetrated by a horizontal cylinder of 50 diameter, in such a way that both the solids envelop an imaginary common sphere and their axes intersect each

other. Draw the projections of the solids, showing the line of intersection.

Following the principles of Construction: Fig.15.12, obtain the lines of intersection, as shown in Fig.15.27.



**Fig.15.27**

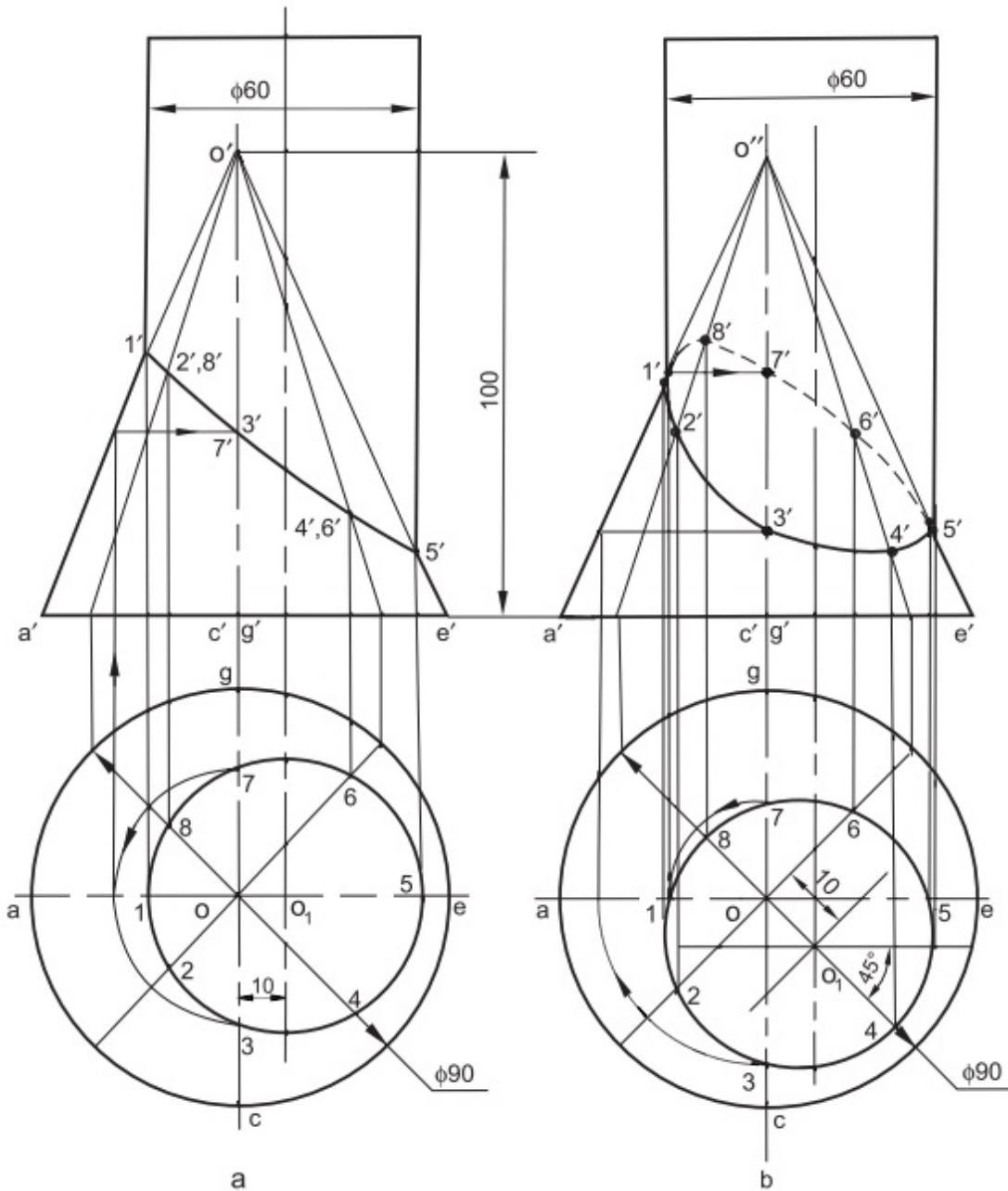
**Problem 28** A vertical cone with diameter of base 90 and axis 100 long, is penetrated by a cylinder of 60 diameter, the axis of which is parallel to and 10 away from that of the cone. Draw the projections, showing the lines of intersection, when the plane containing the two axes is (i) parallel to V.P and (ii) inclined at  $45^\circ$  to V. P.

**Construction (Fig.15.28a)**

- Draw the projections of the two solids, satisfying the
1. given conditions.
  2. Divide the base circle (top view) of the cone into some equal parts, say 8 and draw the corresponding generators in the front view.
  3. Locate the points of intersection 1, 2, 3, etc., between the generators and the edge view of the cylinder.
  4. Project and obtain the corresponding points 1', 2', 3', etc., in the front view.
  5. Join the points in the order, by a smooth curve and obtain the lines of intersection.



- (i) The edge view of the cylinder represents the line of intersection in the top view.
- (ii) In the front view, the visible and invisible parts of the line of intersection coincide.



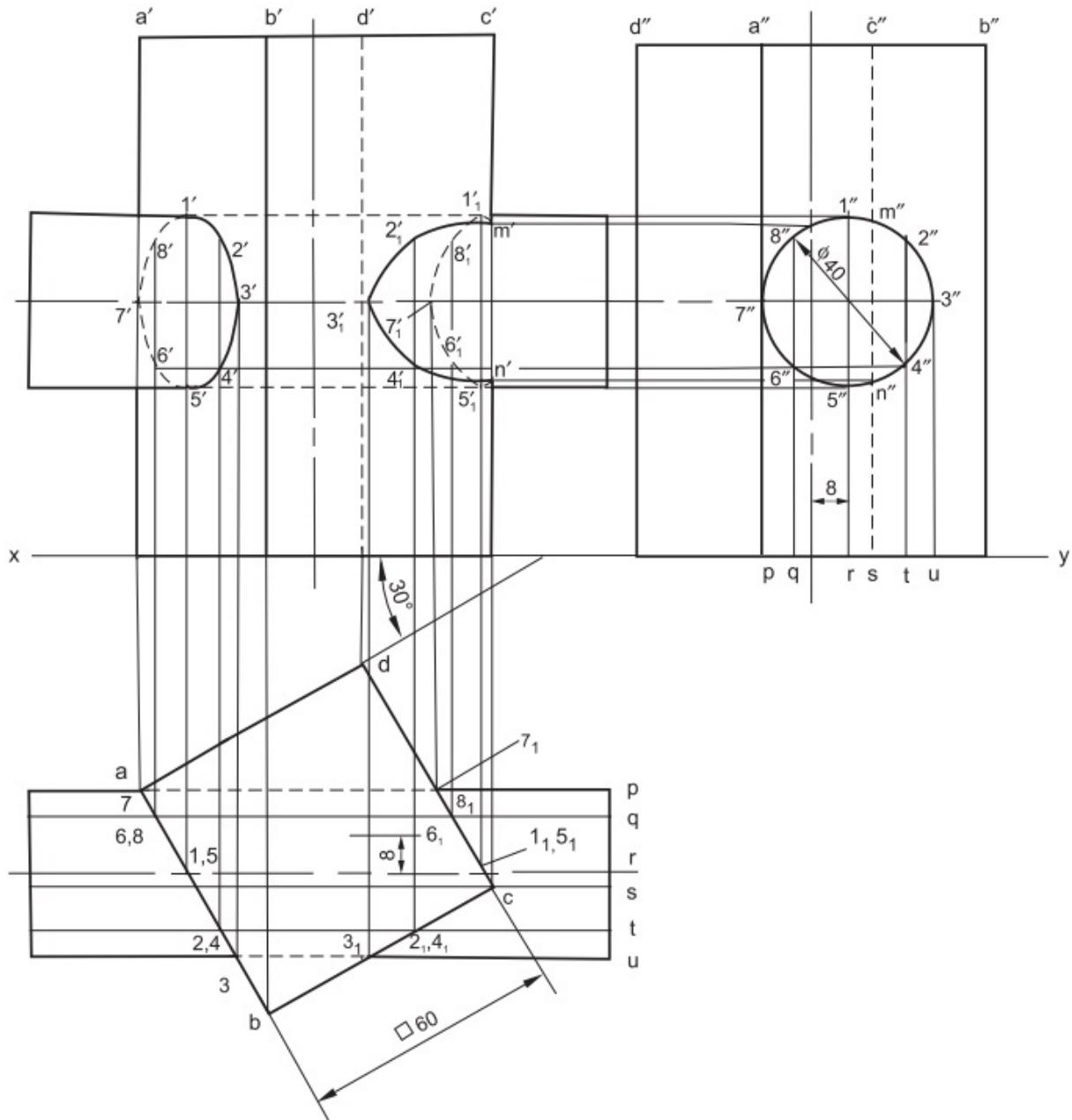
**Fig.15.28**

Figure 15.28b shows the method of obtaining the line of intersection, for the case when the plane containing the two axes is inclined at  $45^\circ$  to V.P. It may be noted that the line of intersection is a closed curve and the visible and invisible portions are separated.

**Problem 29** A vertical square prism with side of base 60, is resting on its base on H.P such that, two of its opposite faces are making  $30^\circ$  with V.P. A cylinder of diameter 40, penetrates the prism, the axis of which is parallel to both H.P and V.P and 8 away from that of the prism. Draw the projections of the solids, showing the lines of intersection.

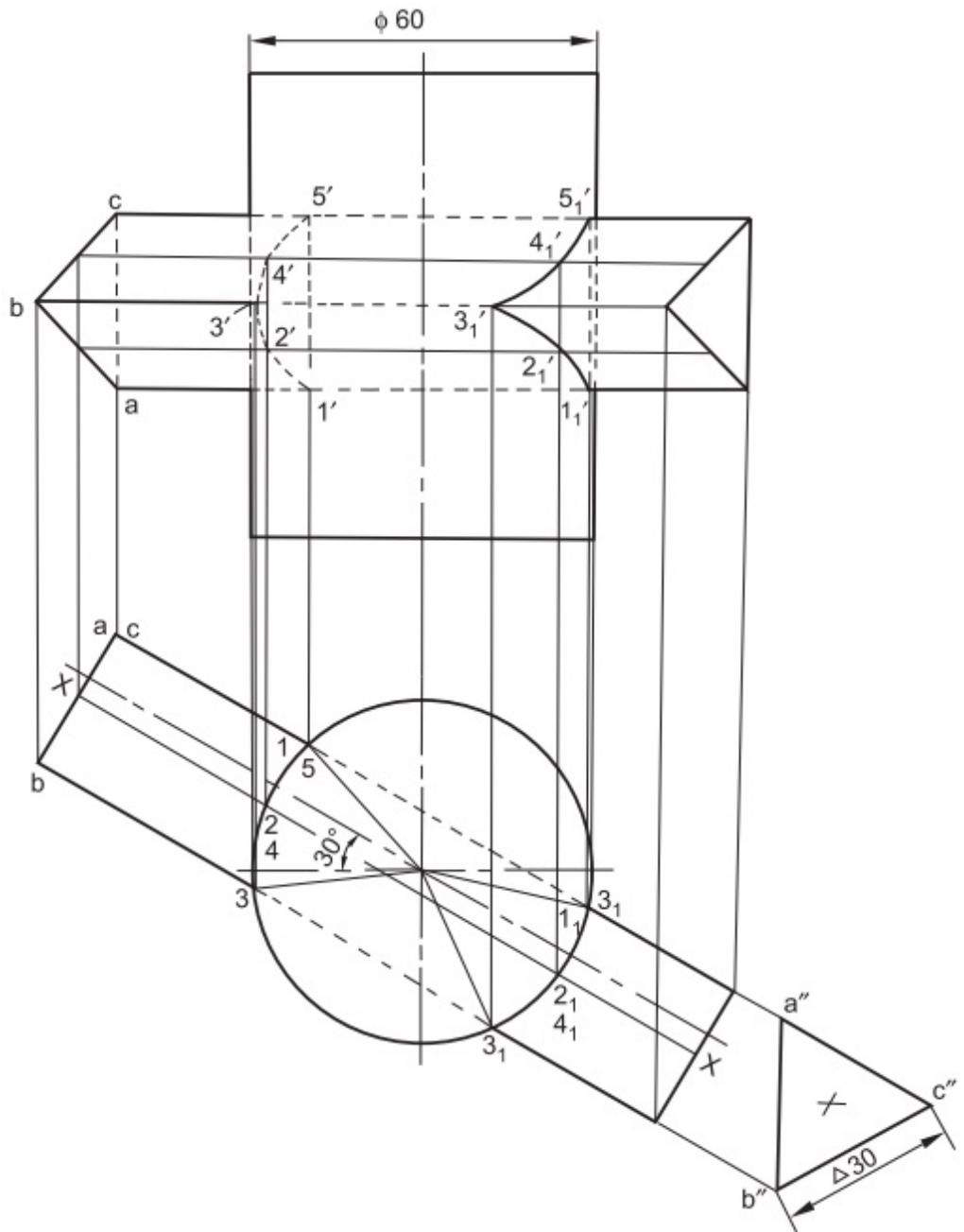
**Construction (Fig.15.29)**

1. Draw the three views of the combination of the solids, satisfying the given conditions.
2. Consider a number of vertical section planes p, q, r, etc., through the edge view of the cylinder and locate the points of intersection.
3. Locate the section planes in the top view and the corresponding points of intersection on the lateral surfaces of the prism.
4. Obtain the points  $1'$ ,  $2'$ , etc., and  $1_1'$ ,  $2_1'$ , etc., in the front view, by projection, including the transition points  $1'$ ,  $5'$  on the section plane r and  $m'$  and  $n'$  on the section plane s.
5. Join the points by smooth curves and obtain the lines (curves) of intersection on the left and right sides.



**Fig.15.29**

**Problem 30** A triangular prism of edge of base 30, has its axis parallel to the H.P and inclined at  $30^\circ$  to the V.P. This prism interpenetrates a vertical cylinder of base diameter 60. The axes of the objects intersect each other and one of the faces on the prism is perpendicular to the H.P. Draw the curves of intersection.



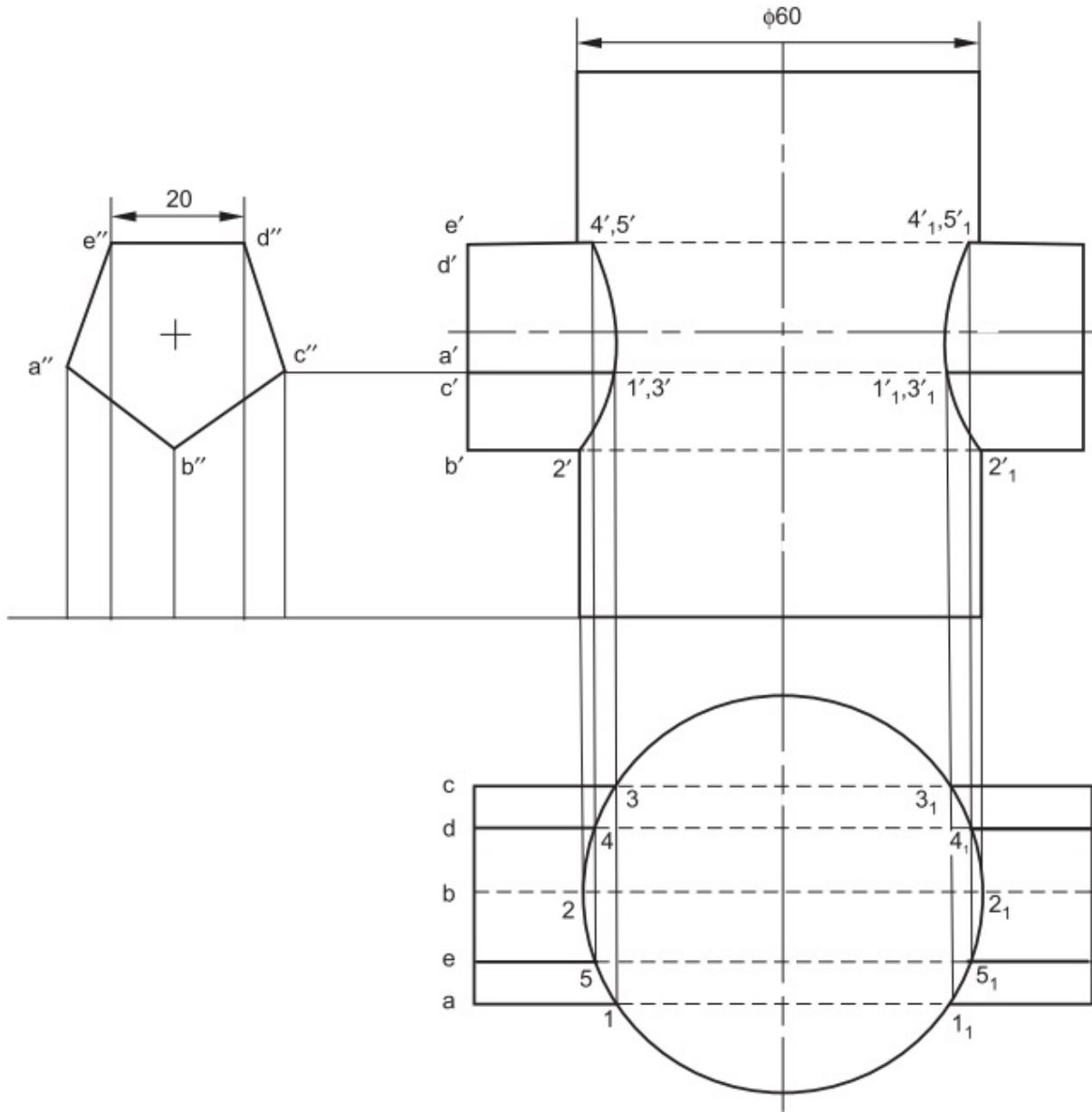
**Fig.15.30**

**Construction (Fig.15.30)**

1. Draw the projections of the two solids, satisfying the given conditions and the edge view of the penetrating prism.

2. Locate generators in the edge view of the vertical cylinder meeting the lateral edges of the penetrating prism.
3. Locate the points of intersection  $1, 3, 5$  and  $1_1, 3_1, 5_1$  between them.
4. Locate generators  $x-x$  on the lateral faces of the prism, and the points of intersection  $2, 4$  and  $2_1, 4_1$  between generators  $x-x$  and the edge view of the cylinder.
5. Project and locate the corresponding points  $1', 2', 3', 4', 5'$  and  $1_1', 2_1', 3_1', 4_1', 5_1'$  in the front view.
6. Join these points suitably and obtain the lines of intersection on both sides in the front view.

**Problem 31** A pentagonal prism of edge of base 20, has one of its longer edges on H.P and the face opposite to this edge parallel to H.P. The prism penetrates a vertical cylinder of base diameter 60 such that, the axes of both intersect each other and parallel to the V.P. Draw the curves of intersection.



**Fig.15.31**

***Construction (Fig.15.31)***

1. Draw the projections of the two solids, satisfying the given conditions and also the edge view of the penetrating prism.
2. Locate the points of intersection 1, 2, 3, etc., and 1', 2', 3', etc., between the lateral edges of the prism and the

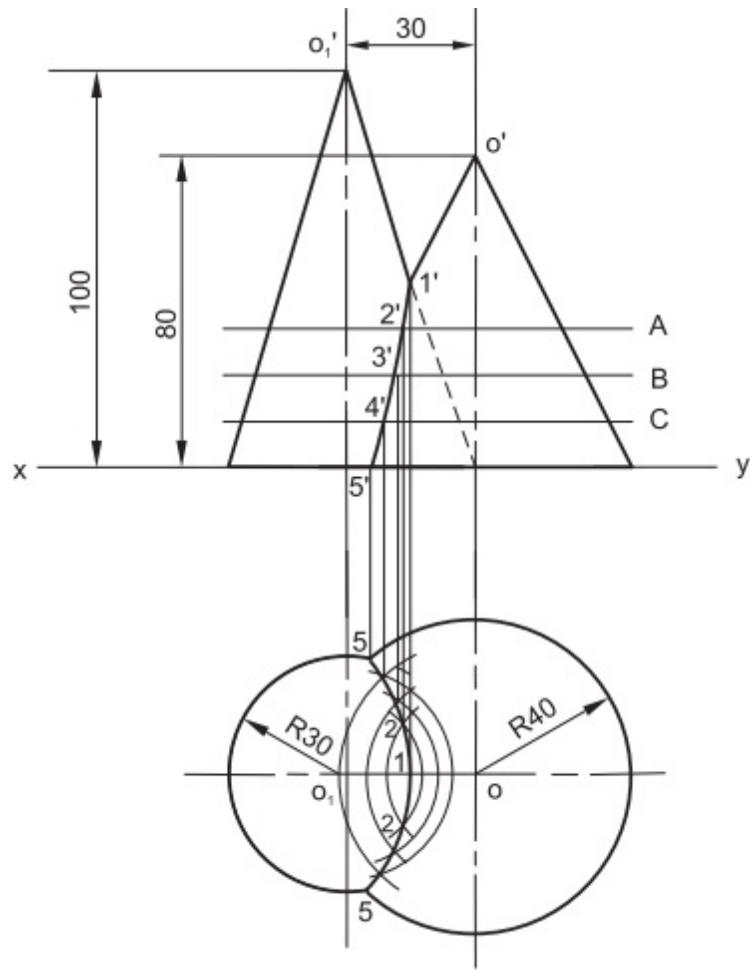
edge view of the vertical cylinder.

3. Project and obtain the corresponding points  $1'$ ,  $2'$ ,  $3'$ , etc., and  $1_1'$ ,  $2_1'$ ,  $3_1'$ , etc., in the front view.
4. Join the points  $1'$ ,  $2'$ ,  $3'$ , etc., and  $1_1'$ ,  $2_1'$ ,  $3_1'$ , etc., by smooth curves and obtain the lines of intersection on the left and right sides of the front view.

**Problem 32** A cone with base 80 diameter and axis 80 long, is penetrated by another cone of base 60 diameter and axis 100 long. The cones are resting on the bases on H.P and a plane containing the axes is parallel to V.P. The distance between the axes is 30. Draw the projections of the cones, showing the lines of intersection.

**Construction (Fig.15.32)**

1. Draw the projections of the cones.



**Fig.15.32**

2. Consider a number of section planes, say A, B and C, parallel to H. P and passing through both the cones, producing circles of different diameters.
3. With centres o and \$o\_1\$, draw circles (arcs), produced by the corresponding section plane.
4. Locate the points of intersection between the circular arcs and obtain the corresponding points in the front view, by projection.
5. Join the points by a smooth curve, obtaining the required lines of intersection.

**Problem 33** A vertical cone with diameter of base 100 and axis 100 long, is penetrated by a vertical square prism with edge of the base 60. The axis of the prism coincides with that of the cone. Draw the projections showing the lines of intersection, when two opposite base edges of the prism are parallel to V.P.

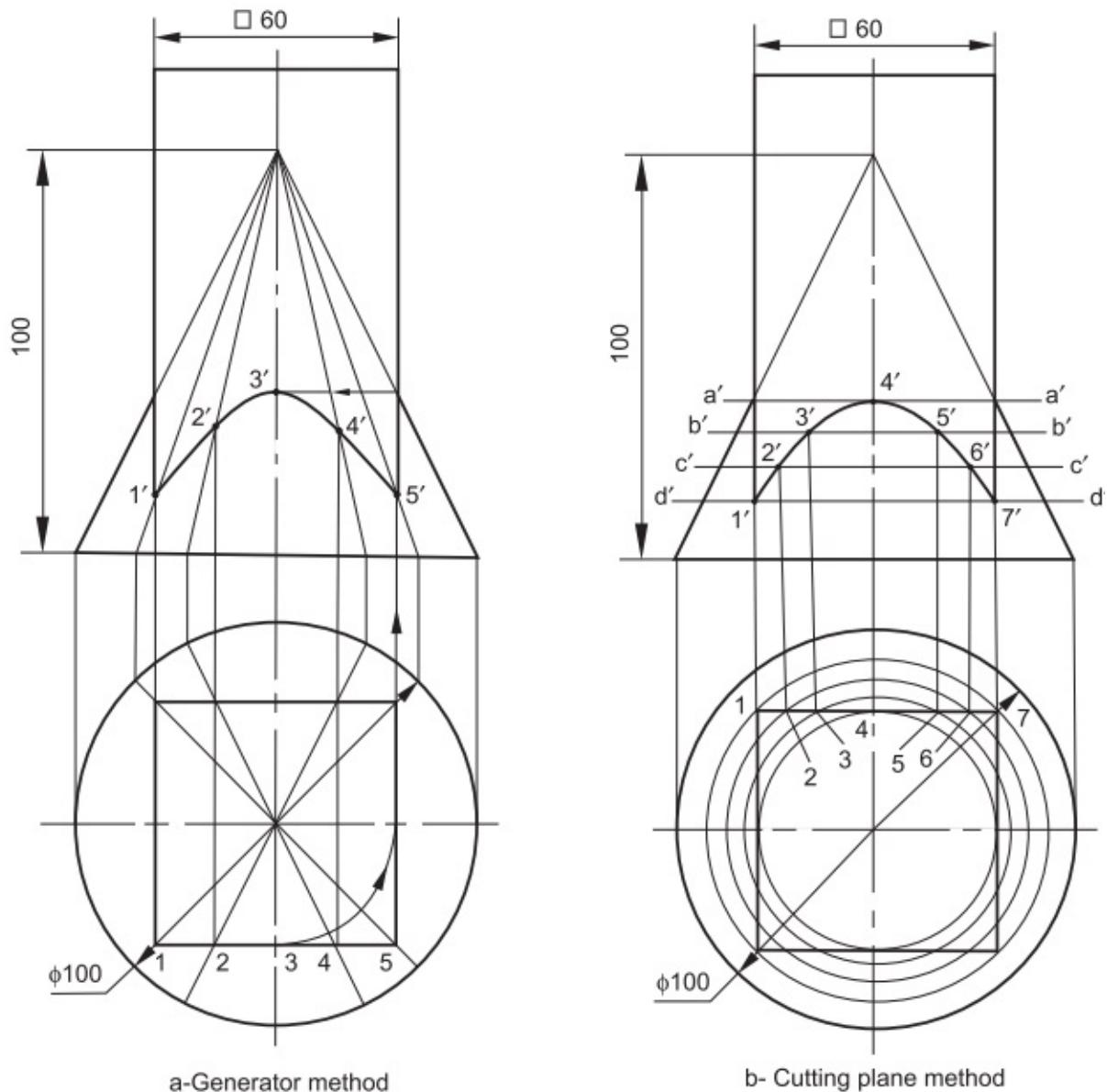
**Construction (Fig.15.33a ) Generator method**

1. Draw the projections of the two solids satisfying the given conditions.
2. Identify a number of generators in the top view of the cone, passing through the base edges of the square, parallel to V.P.
3. Locate the corresponding generators in the front view of the cone.
4. Locate the points of intersection 1, 2, etc., between the generators and edges of the prism in the top view.
5. Project and obtain the corresponding points 1', 2', etc., in the front view.
6. Join the points in the order by a smooth curve and obtain the line of intersection.

**Construction (Fig.15.33 b) Cutting plane method**

1. Draw the projections of the solids, satisfying the given conditions.
2. Draw a number of circles in the top view passing through the edges of the prism; choosing a number of suitable locations.
3. Locate the points of intersection 1, 2, 3, etc., between the above circles and the front edge of the prism.

4. Locate the cutting planes  $a'-a'$ ,  $b'-b'$ , etc., (sectioning the cone) corresponding to the circles drawn in the top view.
5. Project the points 1, 2, 3, etc., and obtain  $1'$ ,  $2'$ ,  $3'$ , etc., on the above cutting planes in the front view.
6. Join the points in the order, by a smooth curve and obtain the line of intersection.



**Fig.15.33**

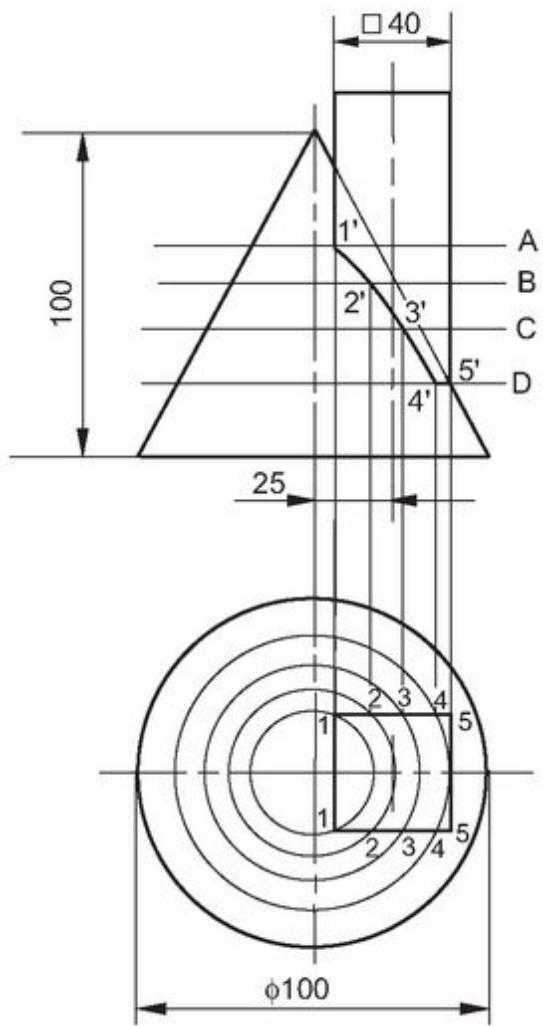


1. In the front view, the visible and invisible parts of the line of intersection coincide.
2. The lines of intersection on the left and right sides of the solids are not visible as the vertical faces of the prism are in edge view.

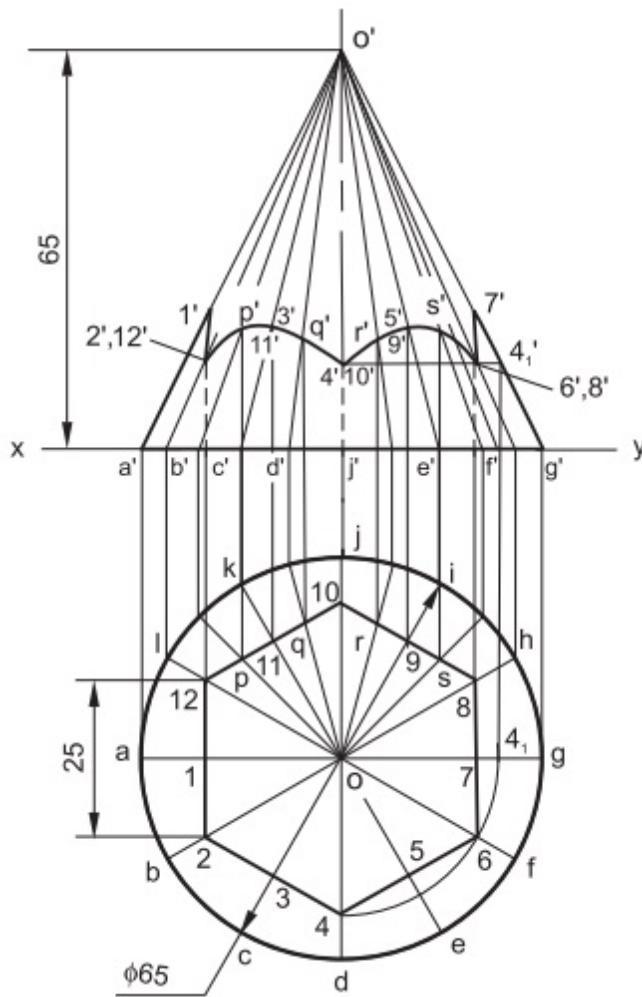
**Problem 34** A cone with base 100 diameter and axis 100 long, is met by a square prism, with the edge of base 40. The axis of the prism is parallel to and 25 away from that of the cone. Draw the projections, showing the lines of intersection, when the plane containing the axes of the two solids is parallel to V.P.

**Construction (Fig.15.34)**

1. Draw the projections of the two solids, satisfying the given conditions.
2. Locate number of cutting planes A,B,C and D, in the front view.



**Fig.15.34**



**Fig.15.35**

3. Draw the corresponding circles produced in the cone. Obviously, cutting planes A and D are critical. The circle corresponding to the plane a should pass through the points 1,1 and the circle corresponding to the plane D should be tangential to the corresponding cutting planes.
4. Locate the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $5'$  in the front view, by projection, on the corresponding cutting planes.
5. Join the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $5'$  by a smooth curve, forming the required lines of intersection in the front

view.

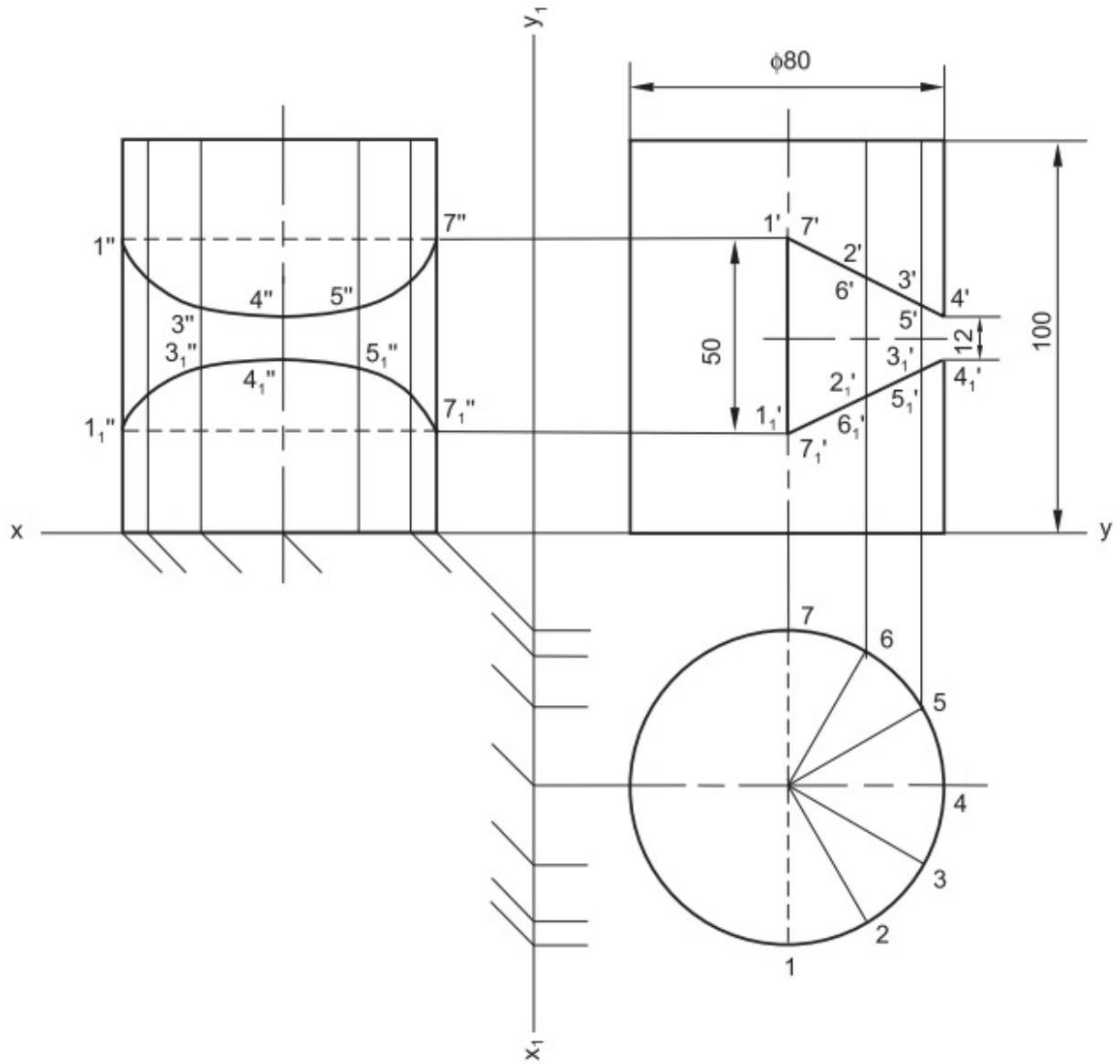
**Problem 35** A right circular cone with diameter of base 65 and height 65, rests on its base on H.P. A concentric hexagonal through hole of side 25, is made in the cone such that, the two vertical faces of the hole are perpendicular to V.P. Draw the projections of the cone.

**Construction (Fig.15.35)**

1. Draw the two views of the cone with the concentric hexagonal hole, satisfying the given conditions.
2. Divide the circle in the top view into a number of equal parts, say 12 and draw the corresponding generators in both the views.
3. Locate the points of intersection 1, 2, 3, etc., between the generators and the edges of the hexagon, in the top view.
4. Project and obtain the corresponding points 1', 2', 3', etc., in the front view.
5. Join the points in the order as shown, obtaining the lines of intersection between the cone and faces of the hexagonal hole.

**Problem 36** Figure 15.36 shows the projections of a cylinder, with a partial triangular slot. Add the right side view.

**Construction (Fig.15.36)**



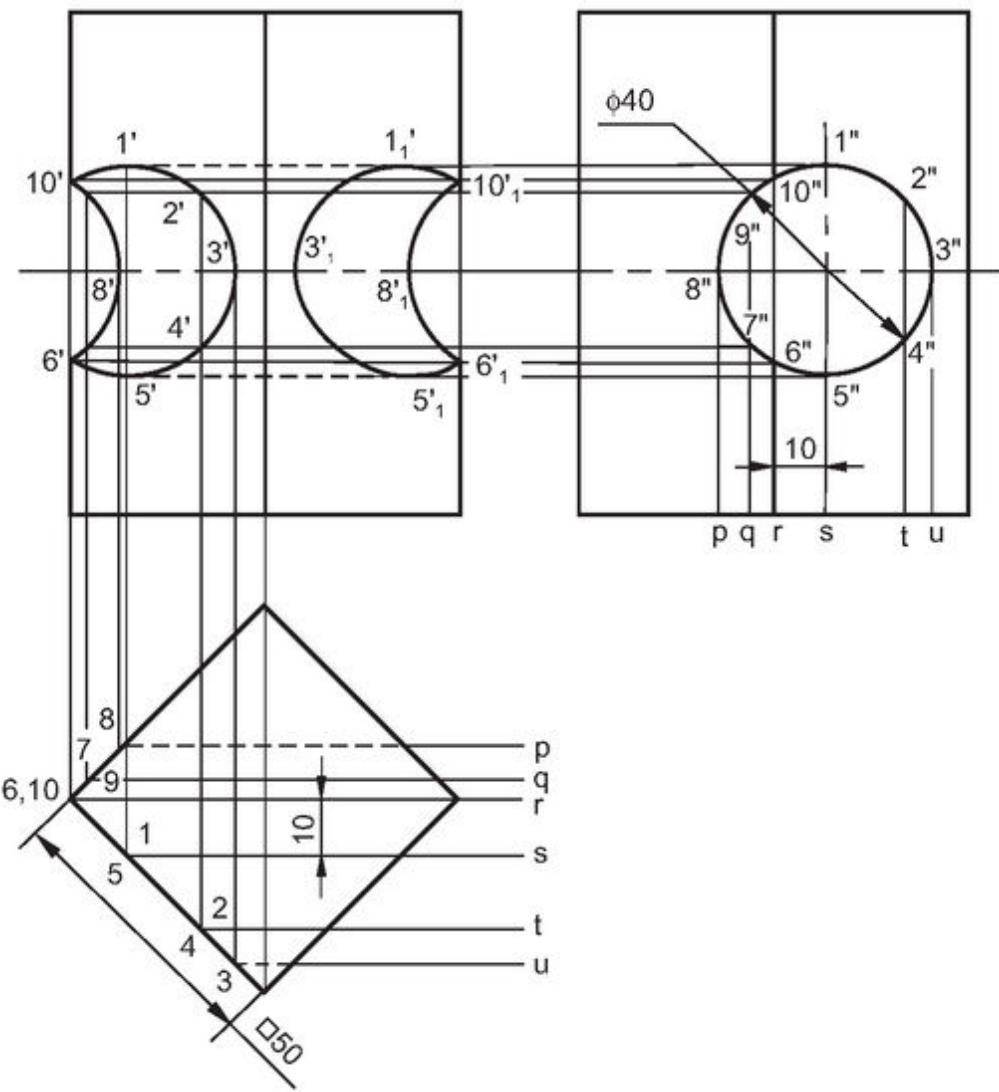
**Fig.15.36**

1. Draw the three views of the cylinder, containing the slot.
2. Locate a number of generators passing through the slot, in all the three views.
3. Locate the points of intersection 1', 2', 3', etc., and 1<sub>1</sub>', 2<sub>1</sub>', 3<sub>1</sub>', etc., between the generators and the edges of the slot, in the front view.

4. Obtain the corresponding points in the right side view, by projection.
5. Join the points in the order by smooth curves, obtaining the required lines of intersection.

**Problem 37** A vertical square prism with side of base 50, is resting on its base on H. P such that, its faces are equally inclined to V. P. A circular hole of diameter 40, is cut through the prism, the axis of which is parallel to both H. P and V. P and 10 away from that of the prism. Draw the projections of the prism.

**Construction** ([Fig.15.37](#))



**Fig.15.37**

1. Draw the three views of the prism, with a circular hole through it.
2. Consider a number of vertical section planes p, q, r, etc., through the edge view of the hole and locate the points of intersection.
3. Locate the section planes in the top view and the corresponding points of intersection on the lateral surfaces of the prism.

4. Obtain the points  $1'$ ,  $2'$ , etc., in the front view, by projection, including the transition points 6 and 10, that lie on the section plane r.
5. Join the points by smooth curve and obtain the line (curve) of intersection on the left side.
6. Repeat the steps 2 to 5 and obtain the line of intersection on the right side.



The complete portion of the intersection curve is visible in the front view as the material is removed from the solid in the form of a hole.

**Problem 38** A vertical cylinder of 60 diameter has a square hole of 30 side, cut through it. The axis of the hole is horizontal and 8 away from the axis of the cylinder. The edges of the square hole are equally inclined to H. P. Draw the projections of the cylinder.

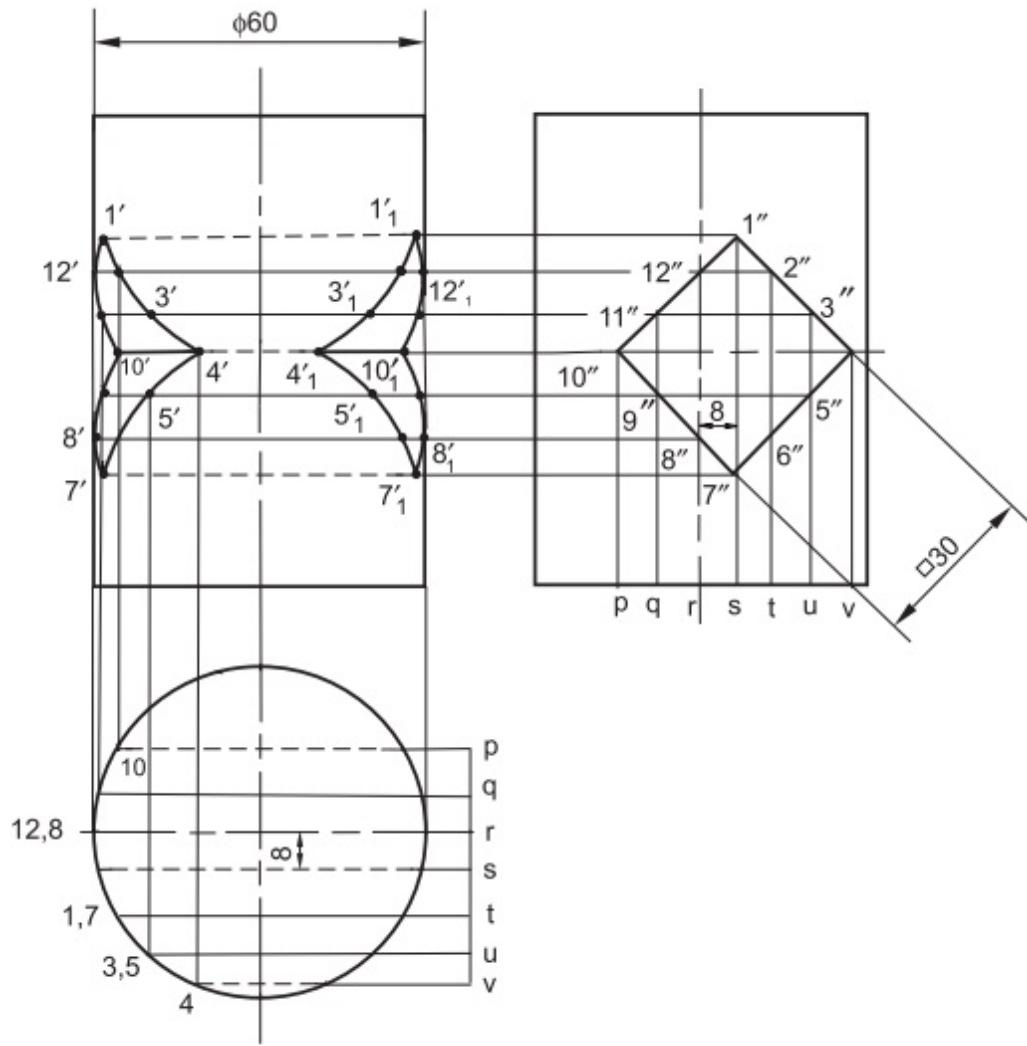
Follow the principles of Construction: Fig.15.37 and obtain the lines (curves) of intersection, as shown in Fig.15.38.

**Problem 39** A tetrahedron of side 70, is resting on one of its faces on H. P such that, one edge of that face is parallel to V. P. A concentric circular hole of 40 diameter, is drilled through the solid. Draw the projections of the solid.

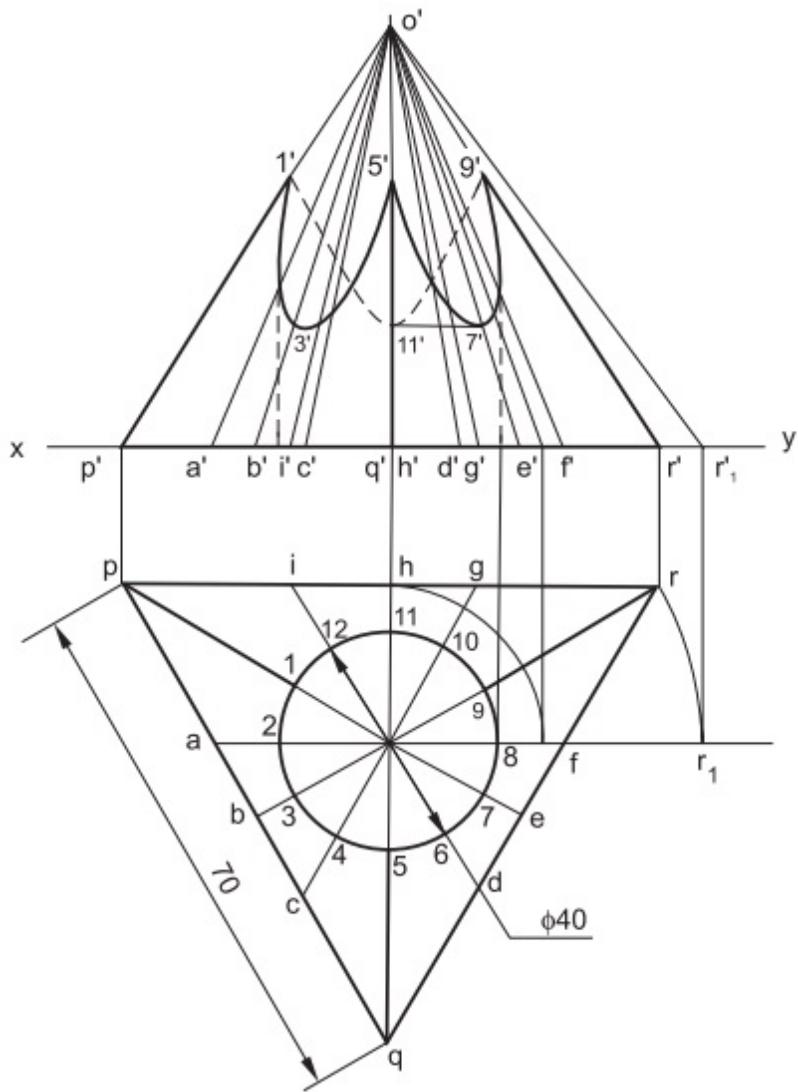
### **Construction (Fig.15.39)**

1. Draw the two views of the tetrahedron with the concentric hole, satisfying the given conditions.
2. Draw the generator ob and consider other generators oa and oc, symmetrical to ob in the top view, on the face OPQ.
3. Obtain the corresponding generators in the front view.

4. Locate the points of intersection 1, 2, 3, 4 and 5 between the generators and the circle, in the top view.
5. Obtain the points  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  and  $5'$ , by projection.
6. On a horizontal line from  $1'$ , locate the point  $5'$ , on the edge  $o'q'$  and  $9'$  on the edge  $o'r'$ .
7. Locate the points of intersection, lying on the other two faces, in a manner similar to the above.
8. Join the points in the order by smooth curves, obtaining the lines (curves) of intersection, in the front view.



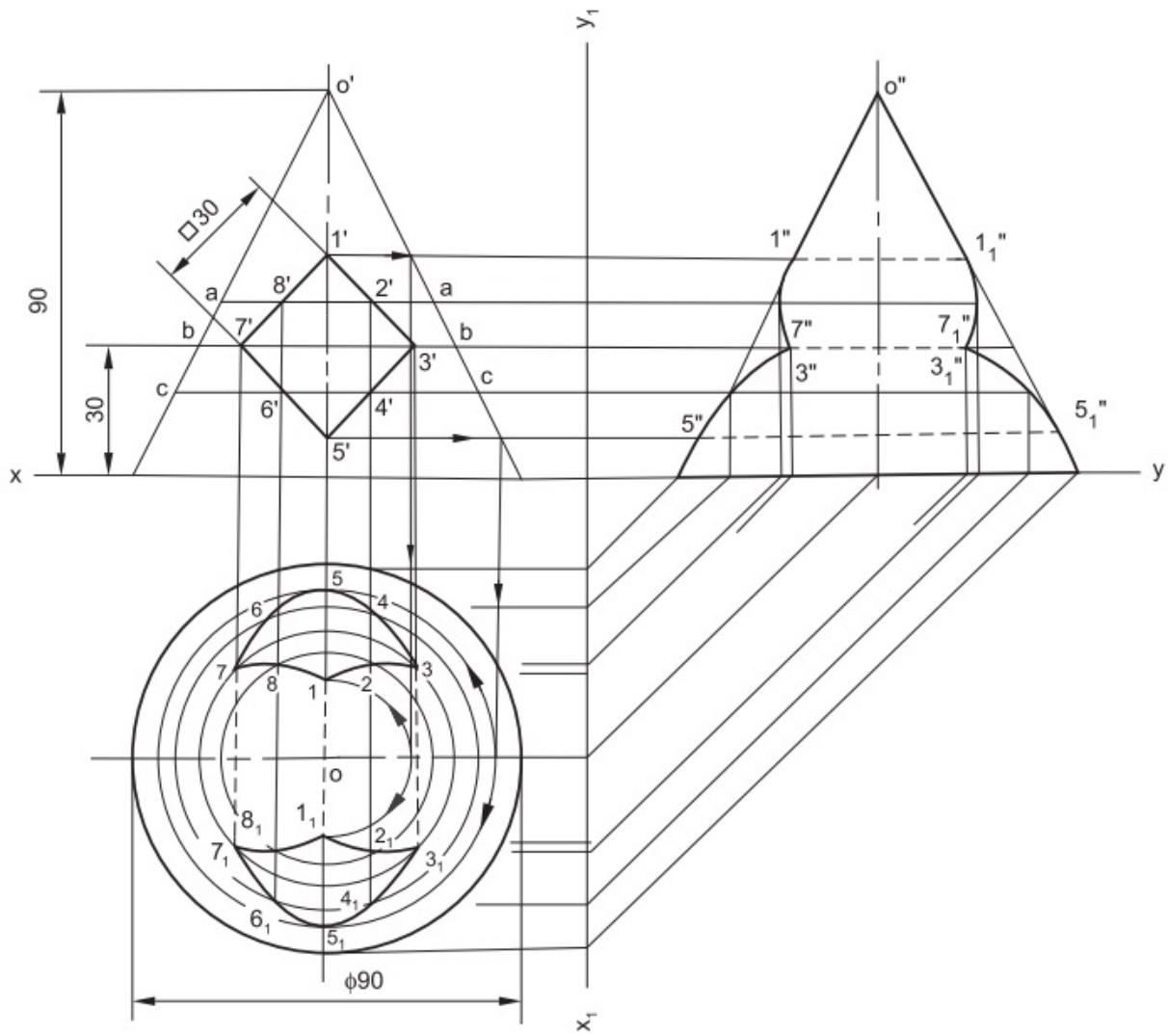
**Fig.15.38**



**Fig.15.39**

**Problem 40** A cone with diameter of base 90 and axis 90 long, is resting on its base on H. P. A square hole of side 30, is cut through the cone such that, the sides of the hole are equally inclined to H. P. The axis of the hole is perpendicular to V. P and intersects the axis of the cone at right angle and at a height of 30 from the base of the cone. Draw the three views of the cone.

**Construction (Fig.15.40)**

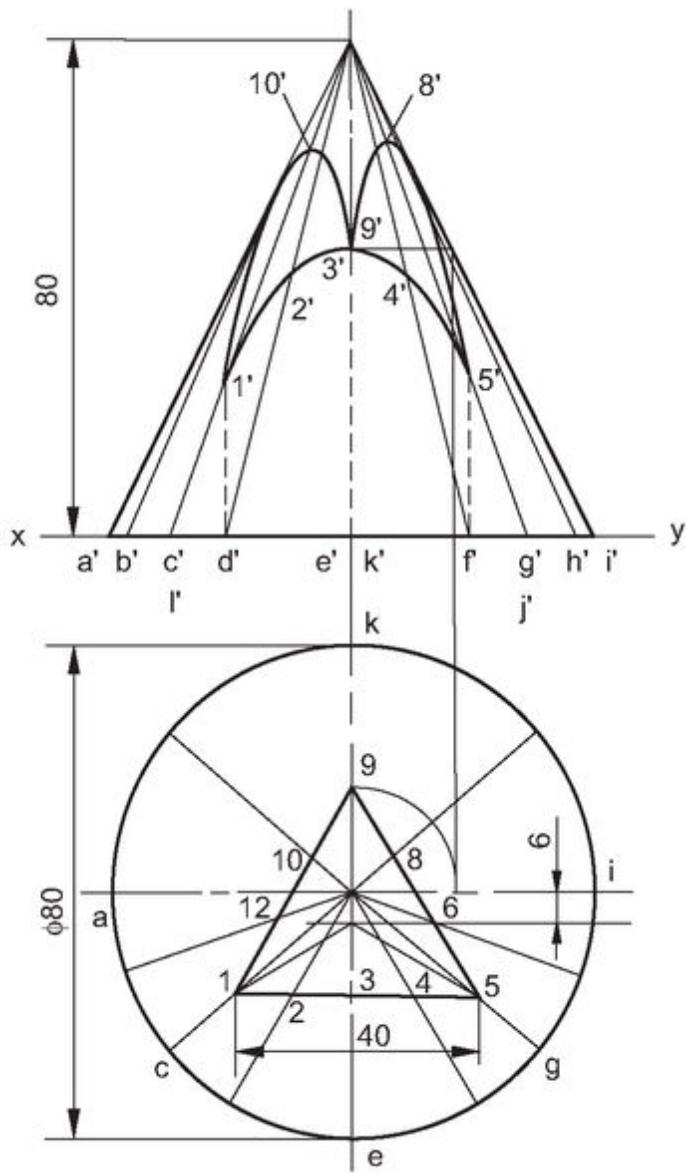


**Fig.15.40**

1. Draw the three views of the cone, with the hole through it, in the front view.
2. Consider a number of horizontal section planes passing through the hole. The section planes produce circular sections in the cone.
3. Locate the points of intersection 1', 2', 3', etc., between the edges of the hole and the section planes.
4. Obtain the points 1, 2, 3, etc., and 1<sub>1</sub>, 2<sub>1</sub>, 3<sub>1</sub>, etc., by projection.

*To locate the points 2 and 8:*

- (i) With centre o and diameter a-a, draw a circle.
  - (ii) Through  $2'$  ( $2_1'$ ) and  $8'$  ( $8_1'$ ), draw projectors intersecting the above circle at  $2$ ,  $2_1$  and  $8$ ,  $8_1$ .
5. Join the points in the order by smooth curves and obtain the two openings in the top view.
  6. Obtain the points  $1''$ ,  $2''$ ,  $3''$ , etc., and  $1_1''$ ,  $2_1''$ ,  $3_1''$ , etc., in the side view, by projection.
  7. Join the above points in the order and obtain the lines (curves) of intersection in the side view.



**Fig.15.41**

**1Problem 41** A cone of base 80 diameter and axis 80 long, has an equilateral triangular hole of 40 side, cut through it. The axis of the hole is parallel to that of the cone and 6 away from it. Draw the projections of the cone, showing the lines of intersection, when a vertical plane containing the two axes is perpendicular to V.P and a face of the hole is parallel to V.P.

Following the principles of Construction: Fig.15.39 suitably, obtain the lines of intersection, as shown in Fig.15.41.

## EXERCISES

- 15.1 Two equal triangular prisms, whose axes intersect each other at right angle, have 40 base side and 100 long. The vertical prism has one edge of its base perpendicular to V. P. The horizontal prism has one of its rectangular faces vertical, making an angle of  $30^\circ$  with V. P. Draw the projections, showing the line of intersection.
- 15.2 A vertical square prism of base 60 side, is penetrated by a horizontal square prism of base 40 side so that their axes intersect. The axis of the horizontal prism is parallel to V. P and the faces of both the prisms are equally inclined to V. P. Draw the projections of the solids, showing the lines of intersection.
- 15.3 Solve problem 15.2, when one face of the vertical prism is making  $60^\circ$  with V. P.
- 15.4 Solve problem 15.2, when the axes of both the prisms are 10 apart.
- 15.5 Solve problem 15.2, when the axes of both the prisms are 25 apart.
- 15.6 A vertical square prism of side of base 60, is penetrated by a horizontal triangular prism of 40 side. The axes are 5 apart. One rectangular face of the vertical prism is inclined at  $60^\circ$  to V. P, while that of the horizontal prism is parallel to V. P. Draw the projections, showing the lines of intersection

- 15.7 A square prism of 40 edge of base, rests vertically with its base on H. P such that, a vertical face is inclined at  $60^\circ$  to V.P. This vertical prism is penetrated by a horizontal square prism, whose rectangular faces make equal inclinations with both H.P and V.P. The axis of the horizontal prism is 10 away from that of vertical prism. If the horizontal prism is of the same size as that of the vertical one, draw the projections and show the lines of intersection.
- 15.8 A square prism of 50 edge of base, is standing on H.P, with its axis vertical and the front left rectangular face is inclined at  $30^\circ$  to V. P. This prism is completely penetrated by a horizontal square prism of 40 edge of base such that, its axis is parallel to V.P and intersects the axis of the vertical prism. A face of the horizontal prism is inclined at  $30^\circ$  to H. P. Draw the projections, showing the lines of intersection.
- 15.9 One end of a rectangular brass bar of  $130 \times 70$ , is turned into a cylindrical bar of 60 diameter, with a fillet radius of 30. The other end of the rod is flat, leaving 25 thick bar. Draw the projections and show the intersection curve.
- 15.10 A hexagonal prism, with side of base 30, is resting on one of its bases on H. P with a face parallel to V. P. The prism contains a square hole of 20 side. The axis of the hole is parallel to V. P and inclined at  $30^\circ$  to H. P, intersecting the axis of the prism. The faces of the hole are equally inclined to V. P. Draw the lines of intersection.
- 15.11 A hexagonal prism, with edge of base 35 and axis vertical, pierces a square prism of side of base 40.

The axis of the square prism is inclined at  $30^\circ$  to H. P and parallel to V. P. The distance between the axes is 15. Draw the projections of the solids, showing the lines of intersection, when a vertical surface of the hexagonal prism is perpendicular to V. P and the lateral surfaces of the square prism are equally inclined to V. P.

15.12A triangular pyramid with side of base 80 and axis 100 long, is resting on its base on H. P; with one edge parallel to V.P. It is met by a vertical triangular prism of base 40 side such that, the axes coincide and a face of the prism is inclined at  $30^\circ$  to V.P. Draw the lines of intersection.

15.13A triangular pyramid with side of base 60 and axis 100 long, is resting on its base on H. P; with a side parallel V. P. It is penetrated by a horizontal triangular prism of base 60 side and its axis is perpendicular to V.P, while one of its faces is parallel to H.P. The axes of the two solids intersect at 50 from the base of the pyramid. Draw the lines of intersection.

15.14A square prism with side of base 60, is resting on one of its bases on H.P; with a face inclined at  $60^\circ$  to V.P. It is penetrated by a horizontal square pyramid of base 50 side and axis 120 long such that, the axes intersect. The axis of the pyramid is inclined at  $30^\circ$  to V. P, while the intersection point between the axes is 55 from the base of the pyramid. Draw the lines of intersection.

15.15A vertical square prism with side of base 60, has one of its vertical faces inclined at  $30^\circ$  to V. P. It is completely penetrated by a cylinder of 40 diameter, the axis of which is parallel to both H. P and V. P and

is 8 away from the axis of the prism. Draw the projections, showing the lines of intersection.

15.16A square pyramid with edge of base 40 and height 80, is resting on H. P such that, all of its base edges are equally inclined to V. P. A horizontal cylinder of 30 diameter, meets the pyramid on one side such that, the axes of both the solids intersect each other at a height of 45 from the base of the pyramid. The axis of the cylinder is parallel to V.P. Draw the projections of the solids, showing the lines of intersection.

15.17A vertical square prism of base 50 side, has a face inclined at  $30^\circ$  to V. P. It has a circular hole of 65 diameter, drilled through. The axis of the hole is parallel to both H. P and V.P and is 5 away from the axis of the prism. Draw the projections, showing the lines of intersection.

15.18A vertical cylinder of 60 diameter, is met by another cylinder of the same size. Draw the line of intersection, when the axes intersect at  $60^\circ$ .

15.19A cone of base diameter 60 and axis 90 long, is resting on H. P on its base. A horizontal triangular prism with its edge of base 25, penetrates the cone such that, the axes of both the solids intersect each other at a height of 40 from the base of the cone. The axis of the prism is parallel to V. P and one of its rectangular faces makes  $30^\circ$  with H. P. Draw the projections of the solids, showing the lines of intersection.

15.20A vertical cylinder of 60 diameter, is penetrated by a horizontal square prism of base 40 side, the axis of which is parallel to V. P and 10 away from the axis of

the cylinder. A face of the prism makes an angle of  $30^\circ$  with H.P. Draw the projections of the solids, showing the lines of intersection.

15.21A cylinder of 75 diameter, stands on its base on H. P. It is penetrated by another horizontal cylinder of 50 diameter, whose axis is inclined at  $30^\circ$  to V. P. Draw the projections of the solids, showing the lines of intersection, when the axes intersect each other.

15.22A vertical cylinder of 45 diameter, is penetrated by another cylinder of 35 diameter, with its axis inclined at  $30^\circ$  to H. P and parallel to V. P. The axes of the cylinders are 5 apart. Draw the lines of intersection.

15.23A vertical cylinder of 60 diameter, is penetrated by a horizontal square prism of 35 side. The axes of the two solids intersect each other. A rectangular face of the prism is inclined at  $60^\circ$  to V.P. Draw the lines of intersection.

15.24Draw the intersection curve, if in the above problem, the axes are 15 apart.

15.25A cylinder of 75 diameter, standing on its base on H. P, is completely penetrated by another cylinder of 55 diameter, with their axes intersecting at right angle. Draw the projections, showing the lines of intersection, assuming that the axis of the smaller cylinder is parallel to V. P.

15.26A vertical cylinder of 70 diameter, is penetrated by a horizontal cylinder of the same size. The axis of the horizontal cylinder is parallel to both H. P and V. P and is 10 away from the axis of the vertical cylinder. Draw the projections, showing the lines of intersection.

A cylindrical pipe of 30 diameter, has a branch of the same size. The axis of the branch intersects the axis of the main pipe at an angle of  $45^\circ$ . Draw the projections, when the two axes lie in a plane parallel to V.P and the axis of the main pipe is vertical.

15.28A vertical cone of 80 diameter and axis 100 long, is penetrated by a horizontal cylinder of 60 diameter and 90 long such that, its axis is 5 behind the axis of the cone, at a height of 40 above its base. Show the lines of intersection, when the axes of both the solids are parallel to V. P.

15.29A cylinder of 60 diameter, stands vertically on its base. It is pierced by a horizontal square prism of 35 side of base such that, the axes of two solids intersect each other at right angle. A face of the prism is inclined at  $60^\circ$  to H.P and  $30^\circ$  to V.P. Draw the projections, showing the lines of intersection.

15.30A cylinder of 60 diameter, rests with its base on H. P. It is penetrated by a horizontal square prism of 40 side of base such that, the distance between the two axes is 10. The faces of the prism are equally inclined to V.P. The axis of the prism is in front of the axis of the cylinder. Draw the projections, showing the lines of intersection.

15.31A cylinder of 50 diameter, branches - off another cylinder of 75 diameter. The axis of the smaller cylinder is vertical and that of the other is horizontal. The distance between the two axes is 10. Draw the three views of the cylinders, showing the lines of intersection.

15.32A hole of 30 diameter, is drilled through a cone of base diameter 80 and length 75. The axis of the hole

is parallel to that of the cone, which is vertical and 10 away from it. The vertical plane containing both the axes is parallel to V. P. Draw the projections, showing the lines of intersection.

15.33A horizontal cylinder of 50 diameter, is penetrated by a vertical cone of base 90 diameter and height 110. The axes of the solids intersect each other at a point 35 from the apex of the cone. Draw the projections, showing the lines (curves) of intersection.

15.34A vertical cone of base diameter 100 and height 120, is penetrated by another horizontal cone of base diameter 60 and axis 100 long. The axis of the penetrating cone is parallel to V. P, 40 above the base and 10 from the axis of the vertical cone. Draw the lines of intersection in both the views, when the axes of solids bisect each other in the front view.

15.35A vertical cylinder of 50 diameter, is penetrated by a horizontal cone of base diameter 50 and axis 75 long. The axes meet at a height of 40 from the base of the cone. Draw the views, showing the lines of intersection.

15.36A vertical cone of base diameter 50 and height 80, is penetrated by a horizontal cylinder of 30 diameter. The axes of the solids meet at a height of 20 from the base of the cone. Draw the lines of intersection.

15.37A right circular cone of base diameter 90, is resting on its base on H. P. It is penetrated by a cylinder of 50 diameter, the axis of which is parallel to H. P and inclined at  $30^\circ$  to V. P. The axis of the horizontal cylinder is 40 above the base of the cone. The axes

are separated by a distance of 10. Draw the projections, showing the lines of intersection.

15.38A sphere of diameter 80, is penetrated by a square prism with side of base 45, the axis of which passes through its centre. Draw the projections of the solids, when the axis of the prism is parallel to both H. P and V.P, showing the lines of intersection.

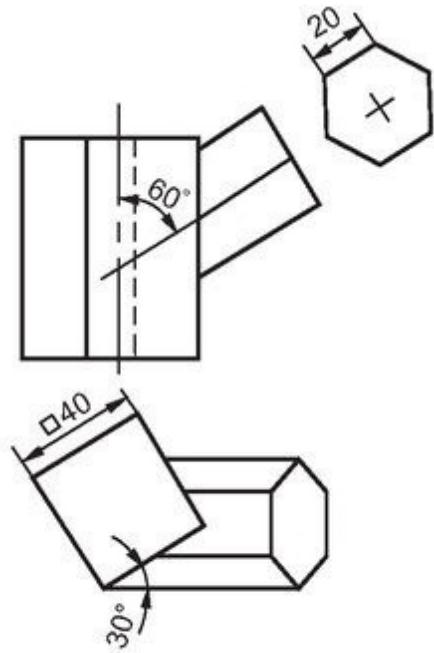
15.39A cylinder of diameter 75, penetrates a sphere of diameter 100. The axis of the cylinder is parallel to both H.P and V.P and it is 10 away from the centre of the sphere. The plane containing the axis of the cylinder and the centre of the sphere, is inclined at  $45^\circ$  to H.P. Draw the projections of the solids, showing the lines of intersection.

15.40The handle of a hand tool is hexagonal in cross-section, terminating into a hemi-sphere at one end. The other end is connected to a solid of revolution.

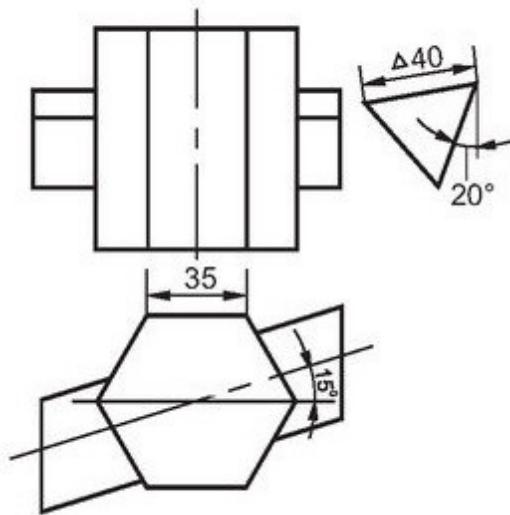
The other end is made cylindrical of diameter 25, joining with the hexagonal portion with a fillet radius of 65. The hexagonal cross-section has a size of 45 across flats and the total length of the handle is 110. Draw the three views of the handle, looking from the cylindrical end.

15.41A pipe of 25 diameter, is joined to another vertical pipe of 50 diameter, at an angle of  $45^\circ$ . The axes are 12.5 apart. Draw the projections of the pipes, showing the lines of intersection.

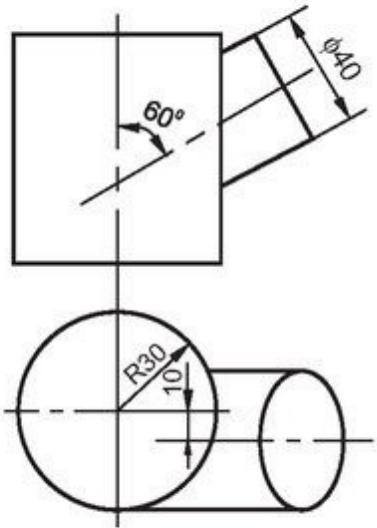
15.42**Figures 15.42 to 15.47** show the projections of certain interpenetrating solids. Draw the lines of intersection for each case.



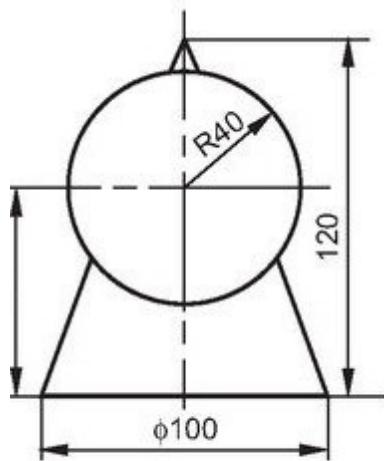
**Fig.15.42**



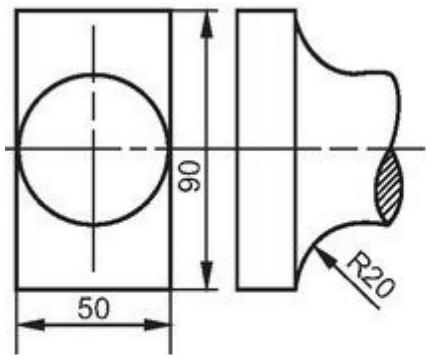
**Fig.15.43**



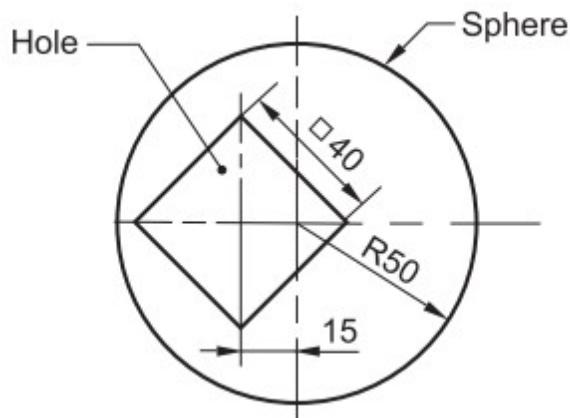
**Fig.15.44**



**Fig.15.45**



**Fig.15.46**



**Fig.15.47**

## REVIEW QUESTIONS

- 15.1 Differentiate between intersection of surfaces and interpenetration of solids.
- 15.2 List out the applications of intersection of surfaces.
- 15.3 Discuss the method of obtaining the line of intersection between (i) solids composed of plane

surfaces and (ii) solids consisting of curved surfaces.

15.4 List out the rules of visibility.

15.5 Define the term, “line of intersection”.

15.6 Differentiate between cutting plane method and generator method of obtaining the lines of intersection.

## OBJECTIVE QUESTIONS

15.1 The intersection between a solid body and a straight line is a \_\_\_\_\_.

15.2 The edge view of the lateral surface of a cylinder is a circle.

(True/False)

15.3 The intersection between a solid resting on H. P and a plane inclined to H.P and perpendicular to V.P is a \_\_\_\_\_ in the front view.

15.4 The line of intersection between cylinders is a \_\_\_\_\_.

15.5 The line of intersection between a prism and a pyramid consists of \_\_\_\_\_

15. 6The line of intersection between two cylinders or cylinder and a prism consists of \_\_\_\_\_.

## ANSWERS

15.1 point

15.2 True

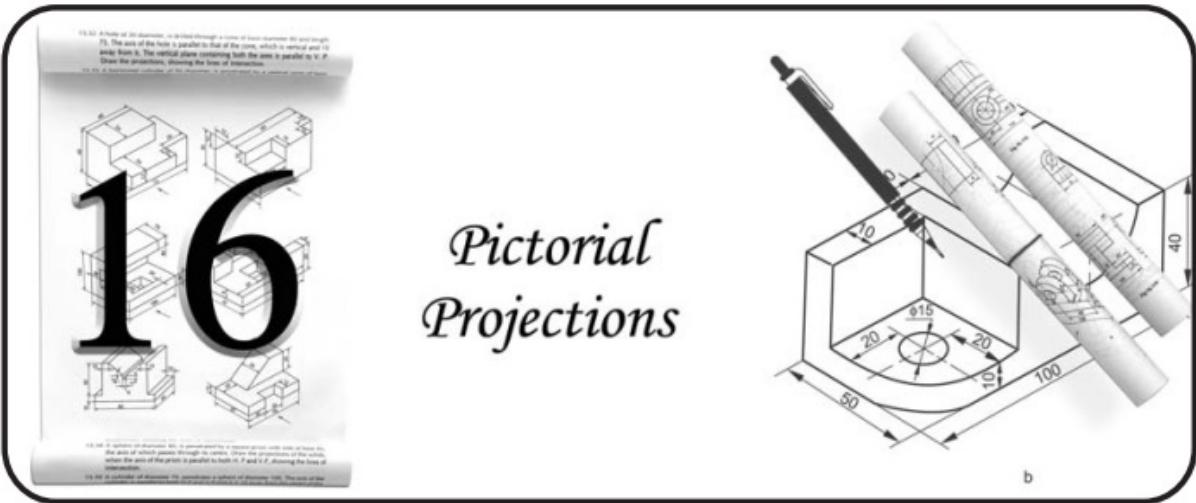
15.3 straight line

15.4 curve

15.5 straight lines

15.6 curved lines

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## 16.1 INTRODUCTION

Interpretation of the shape of an object from a multi-view drawing is difficult, without the knowledge of the principles of orthographic projections. Pictorial drawings are used to convey specific information to persons who cannot visualize an object from its views. Pictorial drawings are mainly used to show complicated structures such as aircraft, rocket shell, etc. Pictorial drawings in the form of exploded views are used in the maintenance catalogues and manuals. These are also used for patent drawings, furniture designs and structural details, which would be difficult to visualize.

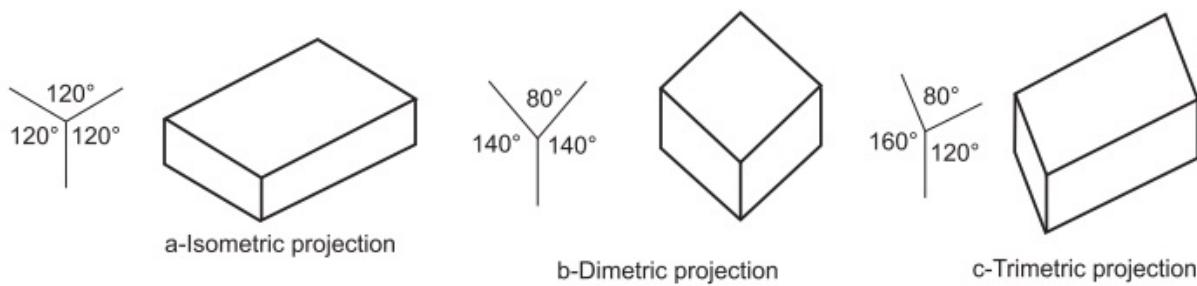
These drawings suffer from the limitations such as, distorted appearance, execution times being unduly long, difficult to dimension, etc.

## 16.2 CLASSIFICATION OF PICTORIAL PROJECTIONS

There are three types of pictorial projections: Axonometric, oblique and perspective projections.

## 16.2.1 Axonometric Projection

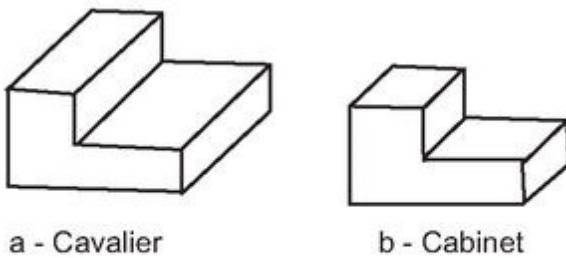
An axonometric projection of an object is one, in which all the three faces of an object are inclined to the plane of projection. Axonometric projections are further classified into three types: Isometric, dimetric and trimetric projections (Fig.16.1).



**Fig.16.1 Axonometric projections**

In isometric projection, the three principal faces and axes of an object are equally inclined to the plane of projection (Fig.16.1a). In dimetric projection, two of the principal faces and axes of the object are equally inclined to the plane of projection (Fig.16.1b). In trimetric projection, all the three faces and axes of the object, make different angles with the plane of projection (Fig.16.1c). However, isometric projection is the most popular form of axonometric projection, which is dealt here in detail.

## 16.2.2 Oblique Projection

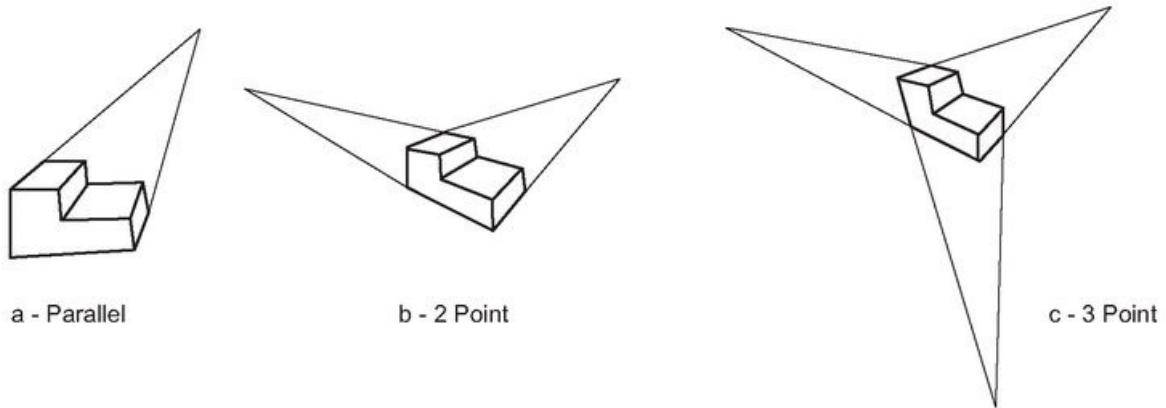


**Fig.16.2 Oblique projections**

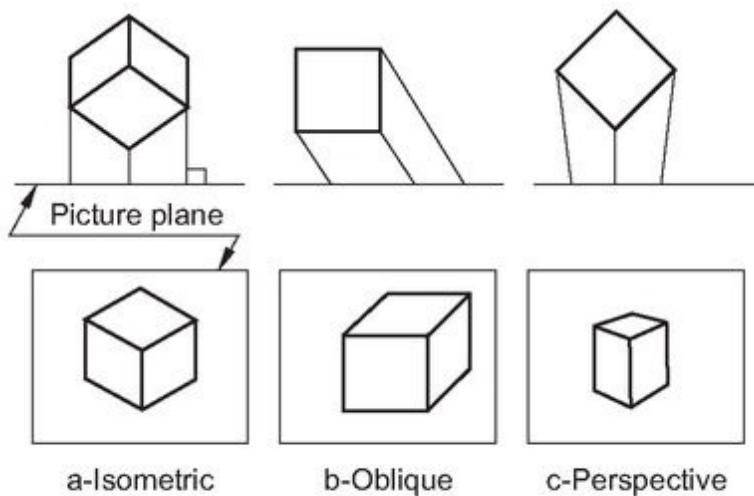
This is also a three dimensional projection, obtained on a plane of projection. In this, the projectors are parallel to each other, but are oblique to the plane of projection. Any surface of the object, parallel to the plane of projection, will appear in its true size and shape. [Figure 16.2](#) shows the two types of oblique projections; cavalier and cabinet, based on the width of the object used on the drawing, in terms of its true width.

### 16.2.3 Perspective Projection

This is the most realistic projection. In this, the projectors will converge towards the viewer's eye, making different angles with the picture plane.



**Fig.16.3 Perspective projections**



**Fig.16.4 Types of pictorial projections**

Figure 16.3 shows the three types of perspective projections.

Figure 16.4 shows the differences between isometric, oblique and perspective projections. It may be noted that in isometric projection, the parallel projectors are perpendicular to the picture plane.

## 16.3 ISOMETRIC PROJECTION

Isometric projection is a pictorial projection of an object and it is a single view, in which all the three dimensions of an object are revealed. Isometric projection gives a clear picture of an object and hence it is helpful even to a layman, for proper understanding of an object. Isometric projection is used by engineers for the preparation of rough sketches on the site, to convey ideas. These projections are also used by the design engineers, in the design and development of new or complicated parts, the shape of which is difficult to understand from the multi-view drawings.

### 16.3.1 Principles of Isometric Projection

The principles of isometric projection may be understood from the isometric projection of a cube. The isometric projection of a cube is obtained, when the line of sight is parallel to its solid diagonal. If a cube is resting on one of its corners on H.P, with its solid diagonal perpendicular to V.P, the front view of the cube is its isometric projection (refer Construction: [Fig.11.44](#)).

Isometric means equal measurement, i.e., each of the three planes of the cube is equally fore-shortened. This projection is more appropriate in the case of small objects, but larger objects may appear to be unnatural. Perspective projection is used to represent larger objects.

In isometric projection, the three planes will be equally fore-shortened and the axes are equally spaced at  $120^\circ$  apart. The axes OX, OY and OZ are called isometric axes (ref. [Fig.16.5a](#)). Lines parallel to these axes are called

isometric lines and planes parallel to the faces of the cube in the isometric projection, are called isometric planes.

### 16.3.2 Isometric Scale

In isometric projection, the magnitudes of dimensions are reduced; therefore, a special scale is required to draw the projections. Referring Fig.16.5a, construct a square with YX as diagonal, representing the true shape of the top surface. From the figure, it is evident that the lines YB and YC are inclined at  $30^\circ$  and  $45^\circ$  to YA respectively. From the geometry of the figure,

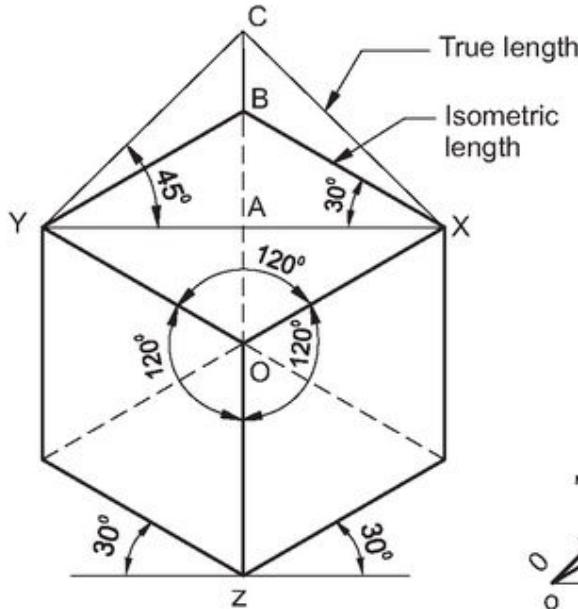
$$\frac{YB}{YC} = \frac{YB}{YA} \times \frac{YA}{YC} = \frac{\cos 45^\circ}{\cos 30^\circ} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$YB = \left( \frac{\sqrt{2}}{\sqrt{3}} \right) YC$$

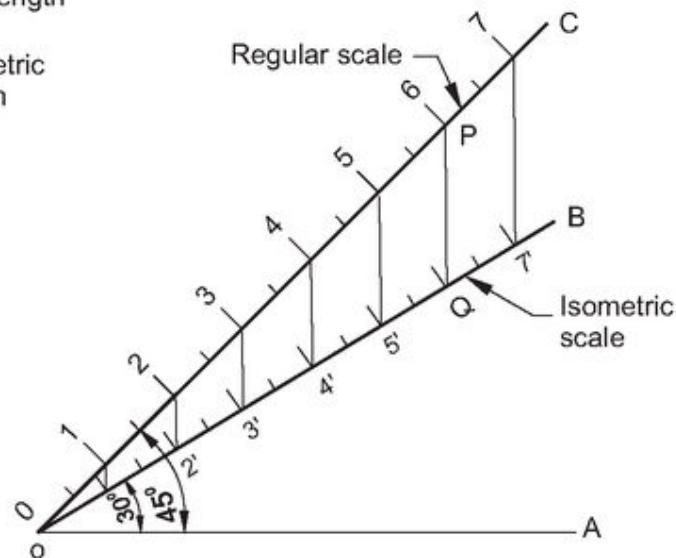
Hence,

Thus, all the dimensions in the isometric projection are  $\frac{\sqrt{2}}{\sqrt{3}}$  times the true size or approximately 82% of the true size. To draw an isometric projection, the magnitudes of all the dimensions are to be reduced to this proportion. A convenient method of reducing the dimensions is to use an isometric scale.

**Construction (Fig.16.5b)**



a-Isometric projection of a cube



b-Construction of isometric scale

**Fig.16.5**

1. Draw a horizontal line OA.
2. Draw lines OB and OC, making  $30^\circ$  and  $45^\circ$  with OA respectively.
3. Construct a regular scale to full size, along the line OC.
4. From the division points on the regular scale, draw perpendiculars to OA, meeting OB.
5. Mark the corresponding points on the line OB, obtaining the isometric scale.

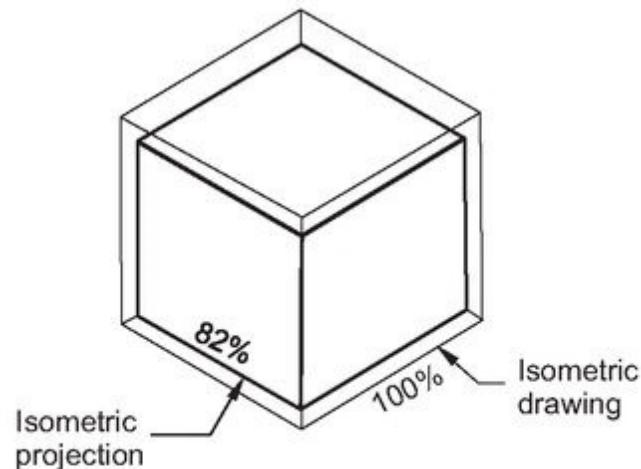
To obtain the isometric length of any dimension, say 60mm, draw a vertical line from P, the 60mm division point, meeting the isometric scale OB at Q. The length OQ represents the isometric length of the dimension, 60mm.

### 16.3.3 Isometric (View) Drawing

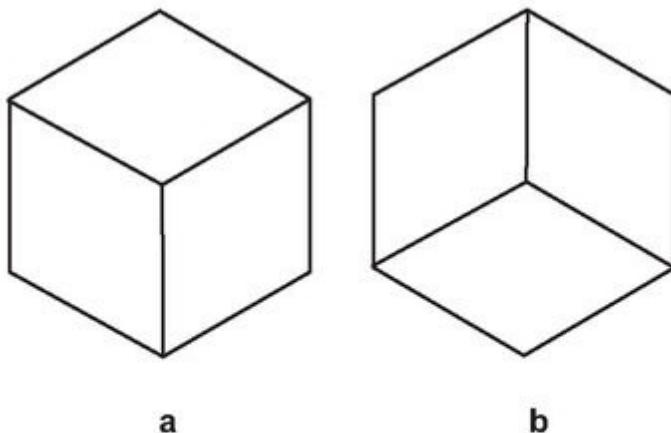
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An isometric view or drawing is similar in form to isometric projection, but actual measurements are transferred from the object to the axes; consequently, the isometric view or drawing will be larger, as shown in Fig.16.6.

It is customary to make an isometric view or drawing rather than isometric projection because, it is much easier to execute, as the sizes need not be reduced to the isometric projection each time. Also, isometric projection and isometric (view) drawing appear alike. However, the students are advised to note what he is required to draw, isometric projection or isometric (view) drawing.



**Fig.16.6**



**Fig.16.7**

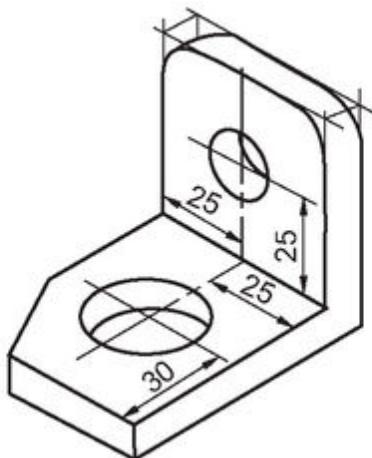
#### **16.3.4 Positioning of Isometric Axes**

Isometric axes may be placed in any position demanded by the problem, to reveal certain faces of the object to a better advantage. This should cause no problem, since the angle between the axes and the procedure followed in constructing the projection or drawing are the same for any position. [Figure 16.7](#) shows two orientations of the cube in isometric form, revealing different faces.

#### **16.3.5 Centre Lines on an Isometric Projection**

If an isometric projection or drawing is to be dimensioned and if it has holes, which must be located, centre lines must be drawn. The centre lines are placed on a plane in which the hole is shown and dimensions are placed, parallel to the

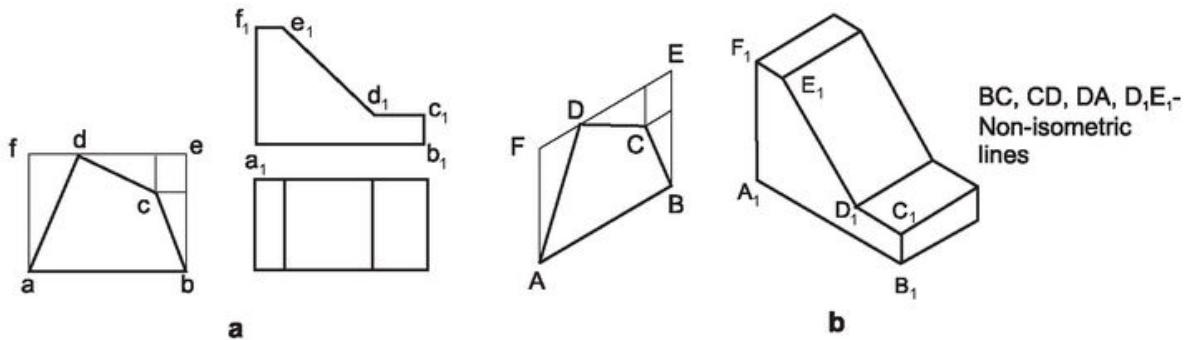
planes. The centre lines must be drawn parallel to the isometric axes, as shown in Fig.16.8.



**Fig.16.8 Use of centre lines for dimensioning**

### 16.3.6 Non-isometric Lines

When an object contains lines (edges), which are not parallel to the isometric axes, these are known as non-isometric lines. Non-isometric lines cannot be represented directly on isometric projection (or drawing), as these are not parallel to the isometric axes and also do not follow the rules of isometric lines. These lines are drawn either by the box method or the off-set method. In the first method, the object is imagined to be enclosed in a square/rectangular box. This is done, by enclosing the views in squares/rectangles. The box is first drawn in isometric and the object is located in it by marking its points of contact with the box.



**Fig.16.9 Isometric views with non-isometric lines**

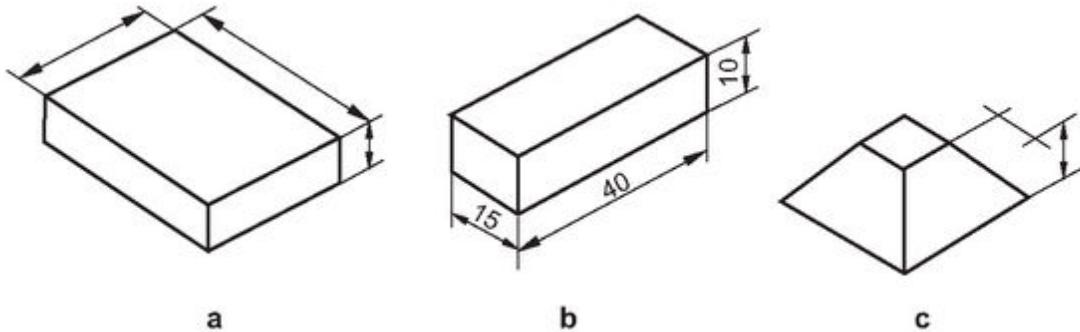
The off-set method is better suited to objects made-up of a number of planes at different angles. The ends of the edges are located by dropping perpendiculars from each point to an isometric reference plane. The perpendiculars may be located on the drawing, as these are isometric lines and the respective magnitudes are transferred from the orthographic views. [Figure 16.9a](#) shows the orthographic views of a plane figure and solid, having non-isometric lines. [Figure 16.9b](#) shows the corresponding isometric views.

### 16.3.7 Hidden Lines

It is the usual practice to omit the hidden lines, unless they are needed to make the projection or drawing clear.

### 16.3.8 Isometric Dimensioning

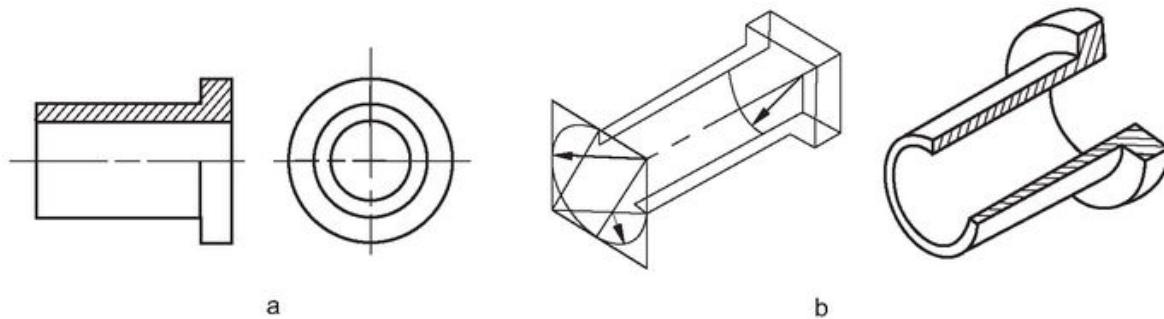
If an isometric projection or drawing is to be used as a working drawing, it must be completely dimensioned. Extension lines, dimension lines and dimensions for the isometric projection or drawing must be placed in isometric planes of the faces, as shown in [Fig.16.10](#).



**Fig.16.10 Dimension lines and numerals on isometric projection**

### 16.3.9 Isometric Sections

The interior details of an object may be shown by using isometric sectional views. The cutting planes should always be isometric planes. [Figure 16.11a](#) shows the orthographic views of an object and [Fig.16.11b](#) shows the isometric sectional view.



**Fig.16.11 Isometric view in section**

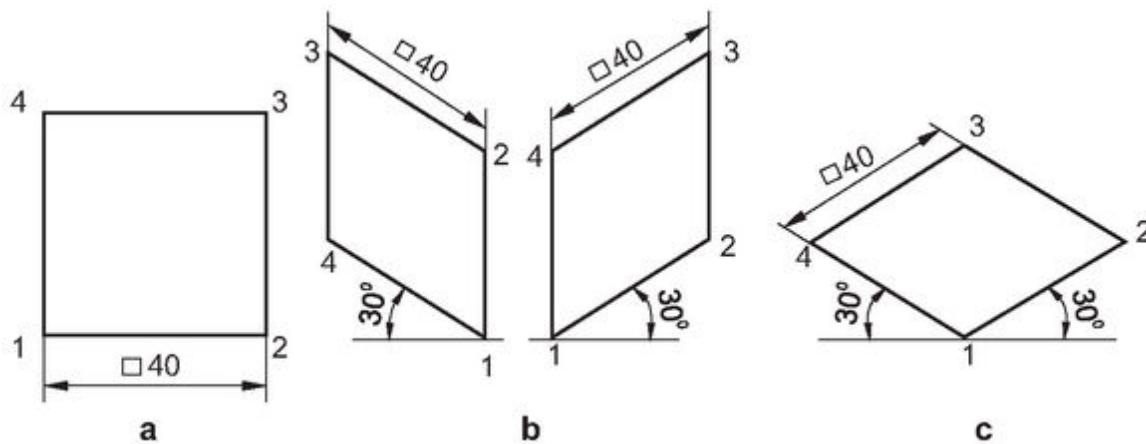
### 16.3.10 Isometric Projection of Plane Figures

**Problem 1** Draw an isometric projection of a square plane of side 40.

**HINT** It may be noted from Figs.16.5b and c, that a horizontal line is inclined at  $30^\circ$  in the isometric projection, whereas a vertical line remains vertical always.

**Case I Vertical plane**

**Construction (Fig.16.12b)**



**Fig.16.12**

1. Draw a line at  $30^\circ$  to the horizontal and mark the isometric length on it.
2. Draw verticals at the ends of the line and mark the isometric length on these parallel lines.
3. Join the ends by a straight line, which is also inclined at  $30^\circ$  to the horizontal.

As shown in Fig.16.12b, there are two possible positions for the plane.



- (i) The shape of the isometric projection or drawing of a square is a rhombus.

- (ii) While dimensioning an isometric projection or isometric drawing, true dimensional values only must be used.

### **Case II Horizontal plane**

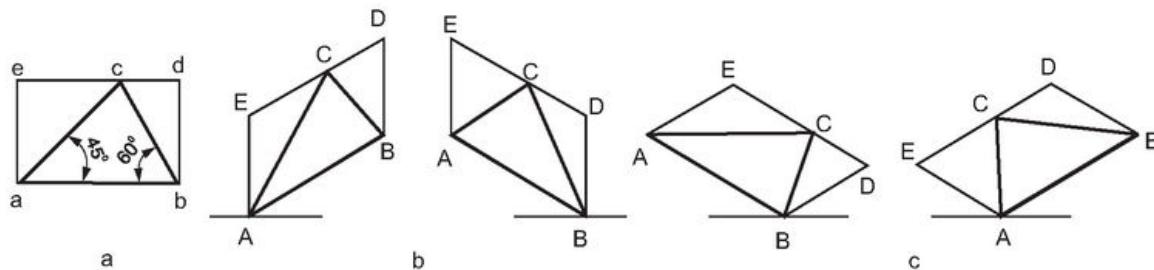
#### **Construction ([Fig.16.12c](#))**

1. Draw two lines at  $30^\circ$  to the horizontal and mark the isometric lengths, along the same.
2. Complete the figure, by drawing  $30^\circ$  inclined lines at the ends, till the lines intersect.

**Problem 2** *Figure 16.13a shows the front view of a triangular plane, having its surface parallel to V.P. Draw its isometric drawing (view).*

**HINT** The surface of the triangle is vertical and the base AB is horizontal. The two sides of the triangle AC and BC are inclined and not parallel to any isometric axis. In an isometric projection or drawing, angles do not change in any fixed proportion. These inclined lines are non-isometric lines and hence, these are drawn after locating their ends, on isometric lines.

#### **Construction ([Fig.16.13](#))**



**Fig.16.13**

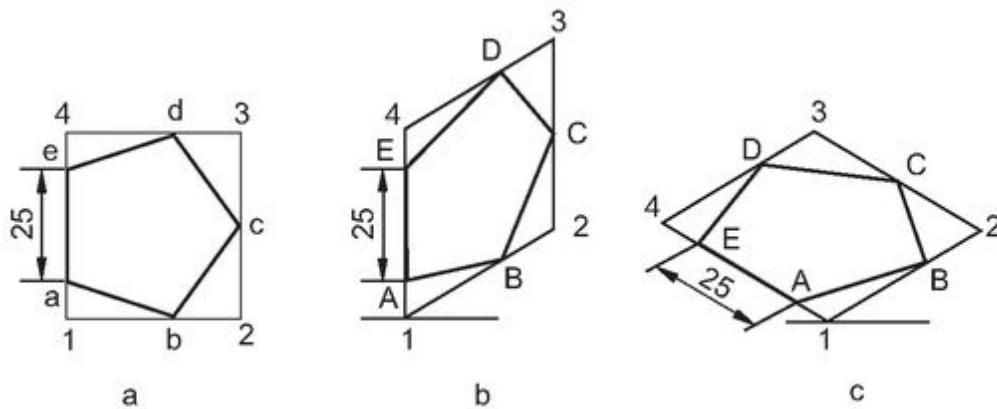
1. Enclose the front view of the triangle in the rectangle abde ([Fig.16.13a](#)).

2. Make the isometric drawing ABDE, of the rectangle, by using true lengths.
3. Locate the point C on DE such that,  $DC = dc$ .
4. Join B, C and A, C and obtain the isometric drawing of the plane.

[Figure 16.13b](#) shows the two possible orientations. [Figure 16.13c](#) shows the two possible orientations, when the plane is parallel to H.P.

**Problem 3** [Figure 16.14a](#) shows the projection of a pentagonal plane. Draw the isometric drawing of the plane, assuming the surface of the plane to be (i) parallel to V.P and (ii) parallel to H.P.

Following the principle of Construction: [Figs.16.13b](#) and [c](#); [Figs.16.14b](#) and [c](#) show the construction for obtaining the isometric drawings for the cases (i) and (ii) respectively.



**Fig.16.14**

### 16.3.10.1 Circles, Arcs and Curves in Isometric Projection or Drawing

Circular features of an object will not appear as circles in the isometric projection or drawing, but appear elliptical in

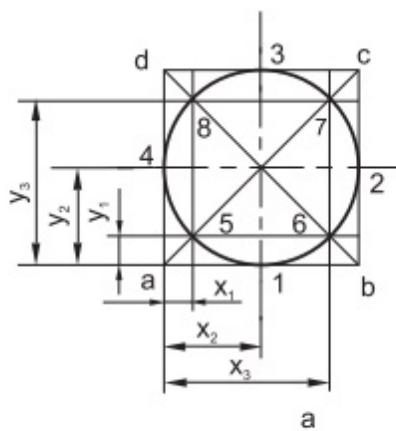
shape. These may be constructed by means of either an off-set (co-ordinate) method or four-centre method.

**Problem 4** *Figure 16.15a shows the projection of a circle. Draw the isometric drawing of the circle.*

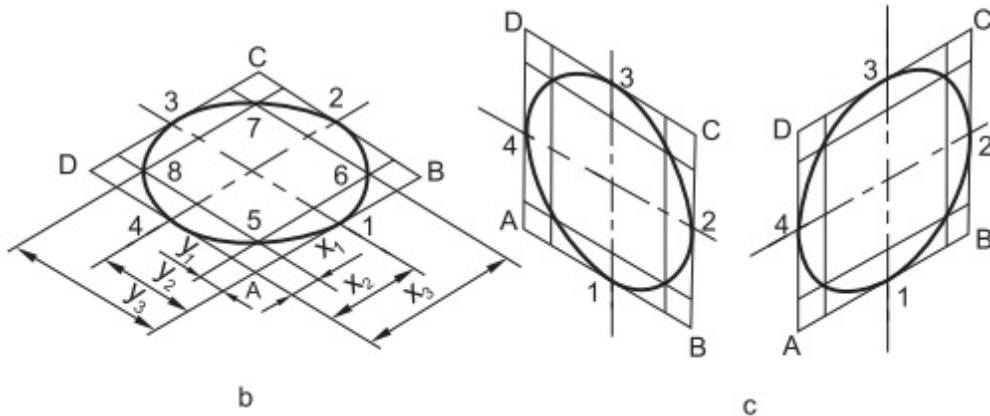
**Method I Off-set or co-ordinate method**

**Construction (Fig.16.15b)**

1. Enclose the circle in a square, touching at points 1, 2, 3 and 4.



a



c

**Fig.16.15**

2. Draw diagonals to the square, intersecting the circle at points 5, 6, 7 and 8.

3. Determine the distances (off-sets) of the division points ( $x_1, x_2, x_3$  and  $y_1, y_2, y_3$ , in the present case) from the edges of the square.
4. Draw the isometric drawing of the square and mark the off-sets, corresponding to the division points of the circle.
5. Join these points by a smooth curve.

Figure 16.15b shows the isometric drawing of the square, by treating it as a horizontal plane. Figure 16.15c shows the two possible orientations of the plane, by treating it as a vertical one.

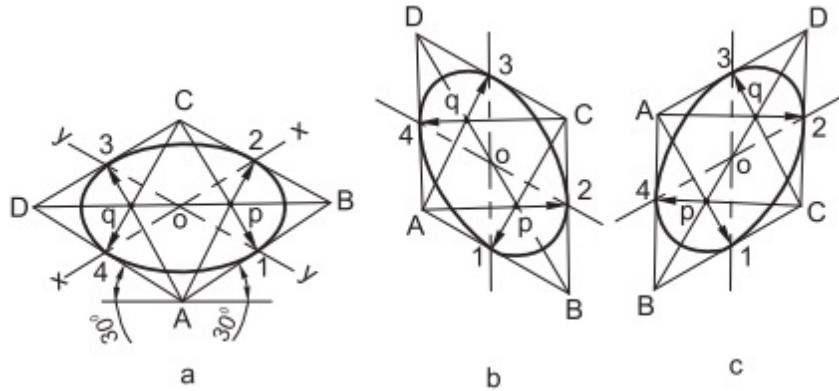


In an isometric drawing of a circle, which is an ellipse, the major axis of the ellipse is longer than the diameter of the circle. However, in an isometric projection, the length of the major axis of the ellipse is equal to the true diameter of the circle.

### **Method II Four-centre method**

#### **Construction (Fig.16.16 a)**

1. Enclose the circle in a square (Fig.16.15 a).
2. Construct the isometric drawing of the square, which is a rhombus and locate the mid-points 1, 2, 3 and 4 of the sides.
3. Join these mid-points to the nearest corners A and C of the rhombus, intersecting at p and q.
4. With centres p and q and radius p1 (=q3), draw two arcs.
5. With A and C as centres and radius A2 (= C4), draw two arcs meeting the above arcs tangentially.



**Fig.16.16 Isometric drawing of a circle- Four-centre method**



- (i) The above procedure may be adopted, whatever may be the orientation of the rhombus (Figs.16.16 b and c).
- (ii) Depending upon the orientation, a part of the above construction may be used to draw the isometric drawing of a semi-circle.
- (iii) The ellipse obtained by the four-centre method is an approximate one and for all practical purposes, it is generally acceptable.

**Problem 5** Draw the isometric view of a circle of 50 diameter, with the given point o as centre.

**Case I** Horizontal plane

**Construction (Fig.16.16a)**

1. Through the given centre o, draw centre lines x-x and y-y, parallel to the isometric axes and making  $30^\circ$  with the horizontal.
2. On these lines, mark points 1, 2, 3 and 4 at a distance equal to 25 from o.

3. Through these points, draw lines parallel to the centre lines and obtain the rhombus ABCD of sides equal to 50.
4. Draw the ellipse in the rhombus, by four -centre method.

**Case II Vertical plane**

Follow the principle of Construction: [Fig.16.16 a](#) suitably and obtain the ellipses in the two possible orientations, as shown in [Figs.16.16 b and c](#).

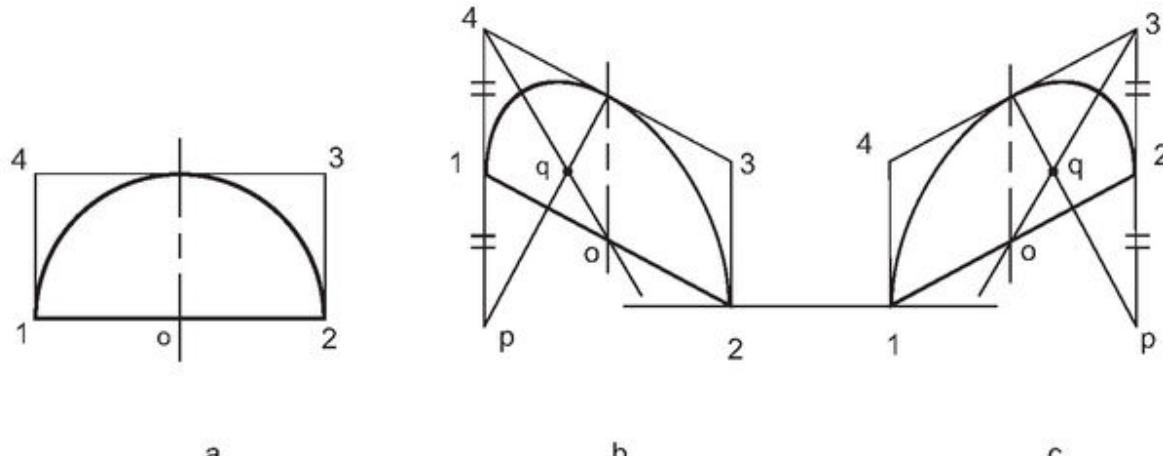


The above construction is very useful while drawing isometric projections or drawings of solids with circular holes.

**Problem 6** [Figure 16.17 a](#) shows the front view of a semi-circle, whose surface is parallel to V.P. Draw its isometric view.

**Construction ([Fig.16.17 b](#))**

1. Enclose the semi-circle in a rectangle.

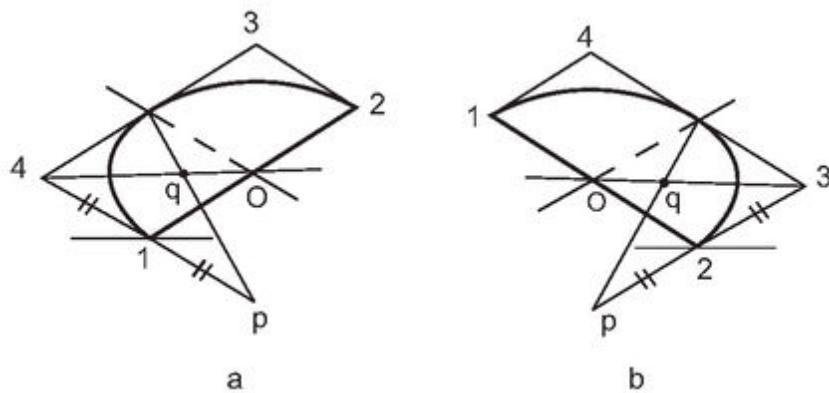


**Fig.16.17**

2. Draw the isometric view of the rectangle.

3. Using the four-centre method, draw the half ellipse in it, which is the required view. The centres for the larger arc, p and smaller arc, q, may be obtained as shown or by completing the rhombus.

[Figure 16.17 c](#) shows the construction for another possible orientation of the plane. [Figures 16.18a and b](#) show the construction for the two possible orientations of the plane, when it is horizontal.

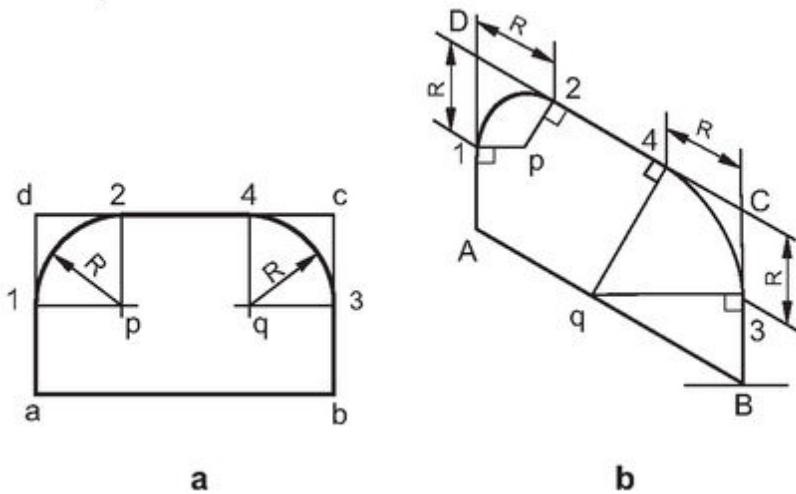


**Fig.16.18**

**Problem 7** [Figure 16.19a](#) shows the front view of a plane, parallel to VP. The upper two corners of the plane are rounded with quarter circles. Draw its isometric drawing.

**HINT** Referring [Fig.16.16a](#), the lines 2A, 3A, 1C and 4C are respectively, the perpendicular bisectors of BC, CD, AB and DA.

**Construction ([Fig.16.19b](#))**



**Fig.16.19**

1. Enclose the given view in a rectangle abcd.
2. Draw the isometric view of the rectangle, which is a parallelogram ABCD.
3. From upper two corners of the parallelogram C and D, mark points on the sides, at a distance equal to R, the radius of the arcs.
4. At these points, erect perpendiculars to the respective sides, intersecting at points p and q, as shown.
5. With centres p and q and radii p1 and q3 respectively, draw arcs and obtain the required view.

It may be noted that although, the arcs are of the same radius in the orthographic projection; in isometric projection or view, the arcs are drawn with different radii.

### 16.3.11 Isometric Projections of Prisms and Pyramids

**Problem 8** *Figure 16.20a shows the orthographic views of an irregular pyramid. Draw its isometric view.*

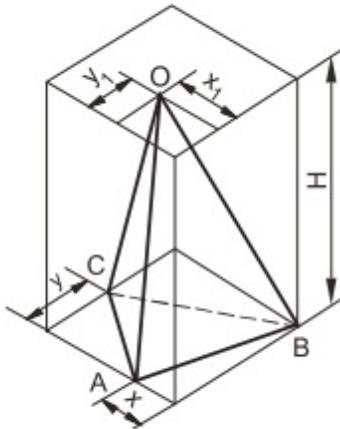
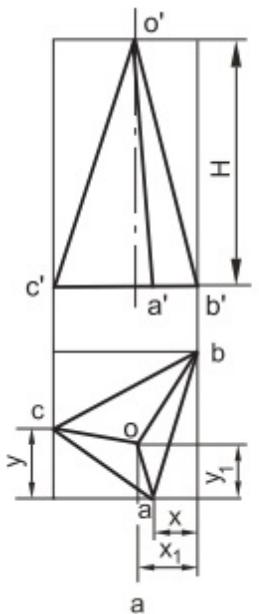
### **Construction (Fig.16.20)**

1. Enclose the orthographic views of the solid in rectangles and locate the corners of the base  $a$ ,  $b$  and  $c$  by off-sets  $x$  and  $y$ . Similarly, locate the vertex by using off-sets  $x_1$  and  $y_1$  (Fig.16.20a).

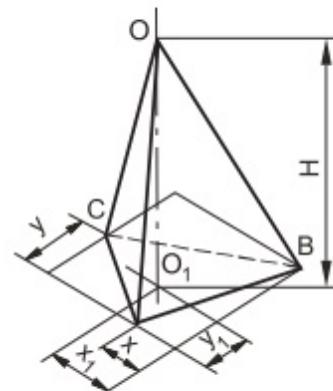
### **Box method (Fig.16.20b)**

2. Construct the isometric drawing of the box enclosing the solid and locate the corners of the solid  $A$ ,  $B$  and  $C$ .
3. Determine the position of the vertex on the top surface of the box, by using off-sets  $x_1$  and  $y_1$ .
4. Complete the isometric drawing, by joining the vertex to the corners.

### **Off-set method (Fig.16.20c)**



b-Box method



c-Off-set method

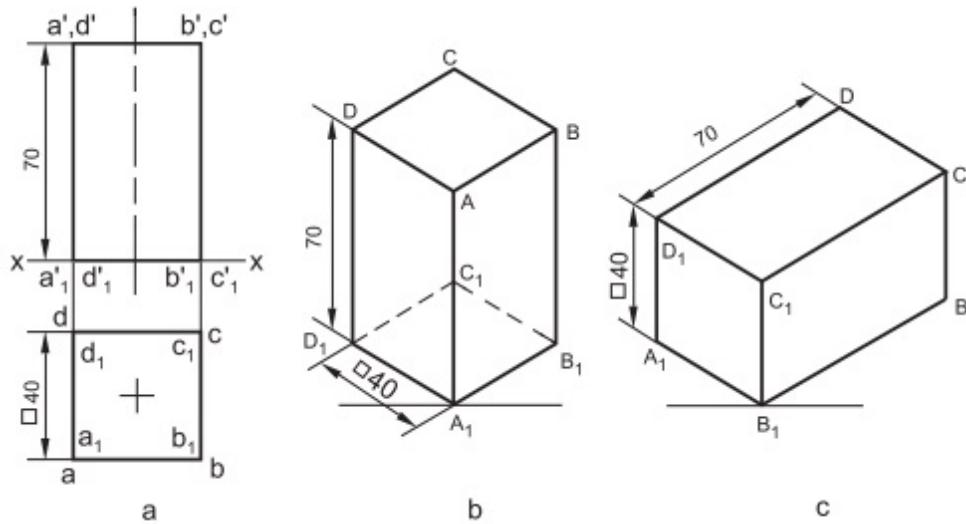
**Fig.16.20**

1. Draw the isometric view of the base and locate the corners A, B and C, by using off-sets x and y.
2. Locate the projection of the vertex O<sub>1</sub> on the base by using off-sets x<sub>1</sub> and y<sub>1</sub>.
3. Draw a vertical centre line O<sub>1</sub>O, equal to the height of the solid H.
4. Complete the isometric drawing, by joining the vertex to the corners of the base.

**Problem 9** Draw the isometric view of a square prism, with side of base 40 and length of axis 70, when its axis is (i) vertical and (ii) horizontal.

**Case I Axis vertical**

**Construction (Fig.16.21b)**



**Fig.16.21**

1. Draw the isometric view A<sub>1</sub> B<sub>1</sub> C<sub>1</sub> D<sub>1</sub> of the bottom base of the prism, which is a square of 40 side.
2. Through the points A<sub>1</sub>, B<sub>1</sub>, C<sub>1</sub> and D<sub>1</sub>, draw vertical lines A<sub>1</sub>A, B<sub>1</sub>B, C<sub>1</sub>C and D<sub>1</sub>D, of length equal to 70.

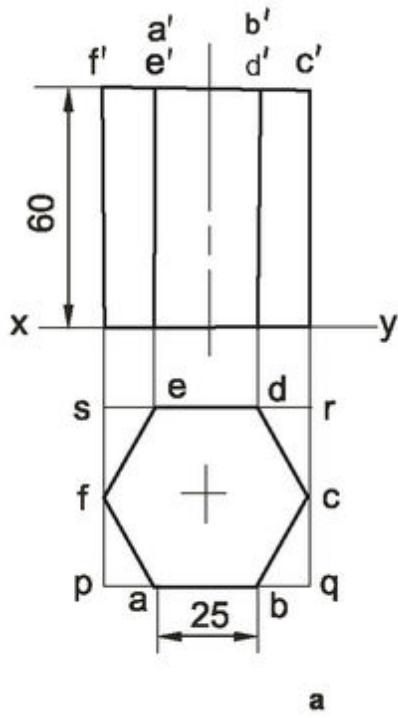
3. Join A, B; B, C; C, D and D, A.
4. Darken the visible edges and obtain the isometric view of the prism.

Figure 16.21c shows one of the two possible orientations of the prism, when its axis is horizontal.

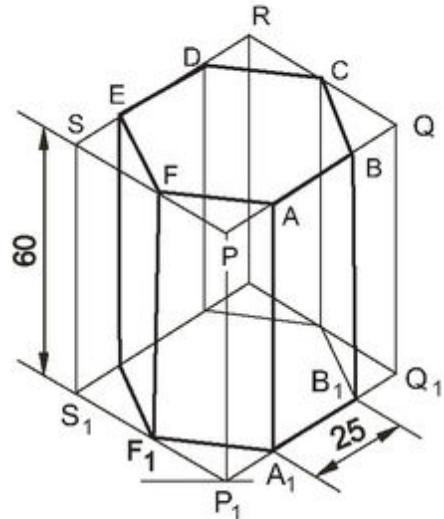
**Problem 10** *Draw the isometric view of a hexagonal prism, with side of base 25 and axis 60 long. The prism is resting on its base on H.P, with an edge of the base parallel to V. P. Use the box method.*

### **Construction (Fig.16.22)**

1. Draw the projections of the prism, satisfying the given conditions.
2. Enclose the views in rectangles.
3. Draw the isometric view of the rectangular box.
4. Determine the distances (off-sets) of the corners of the bases, from the edges of the box.
5. Using the off-sets, locate the corners of the bases at the top and bottom bases of the prism, in the isometric view.
6. Join A, A<sub>1</sub>; B, B<sub>1</sub>, etc., and darken the visible edges and obtain the required isometric view.



a



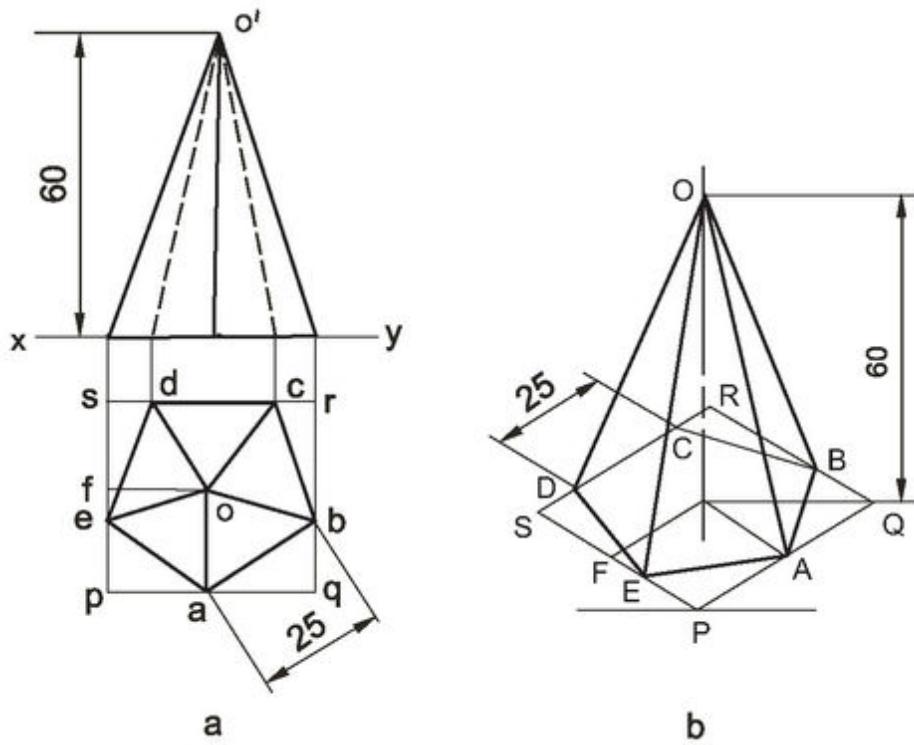
b

**Fig.16.22**

**Problem 11** Draw the isometric view of a pentagonal pyramid, with side of base 25 and axis 60 long. The pyramid is resting on its base on H.P with an edge of the base (away from the observer) parallel to V.P. Use the off-set method.

**Construction (Fig.16.23)**

1. Draw the projections of the pyramid.

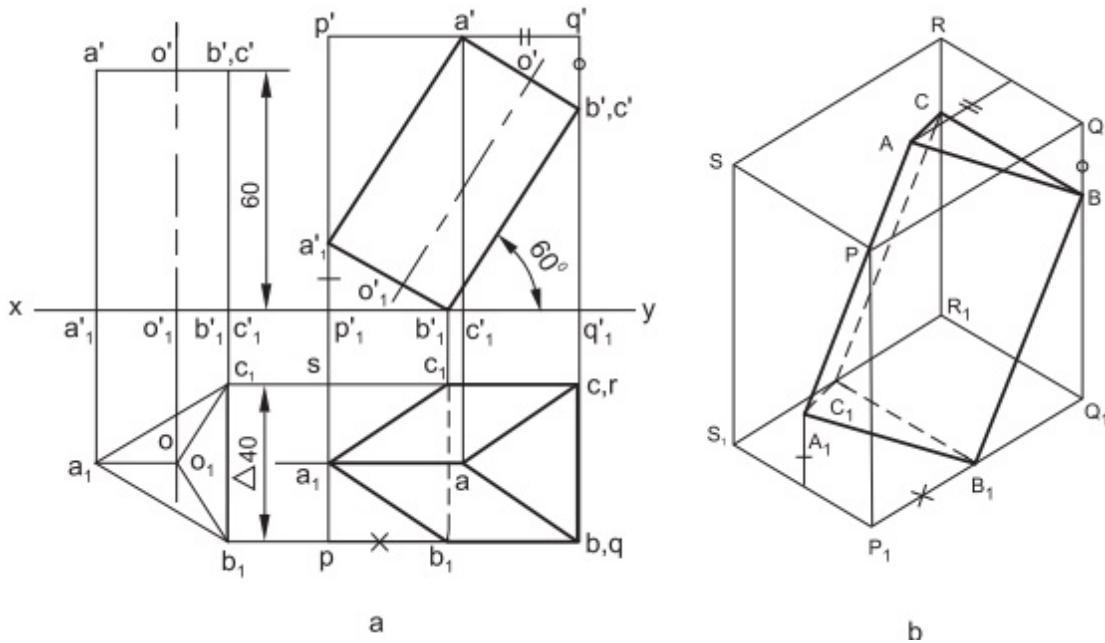


**Fig.16.23**

2. Enclose the projections in rectangles and locate the corners of the base and vertex by off-sets in the top view.
3. Draw the isometric view of the rectangular box.
4. Using the off-sets, locate the corners of the base A, B, C, etc., and the vertex O.
5. Join O, A; O, B; O, C; etc., and darken the visible edges and obtain the required isometric view.

**Problem 12** An equilateral triangular prism, with edge of base 40 and axis 60 long, lies on one of its base edges on H.P and the rectangular face containing that edge is inclined at  $60^\circ$  to H.P. Draw the isometric view of the solid.

**Construction (Fig.16.24)**

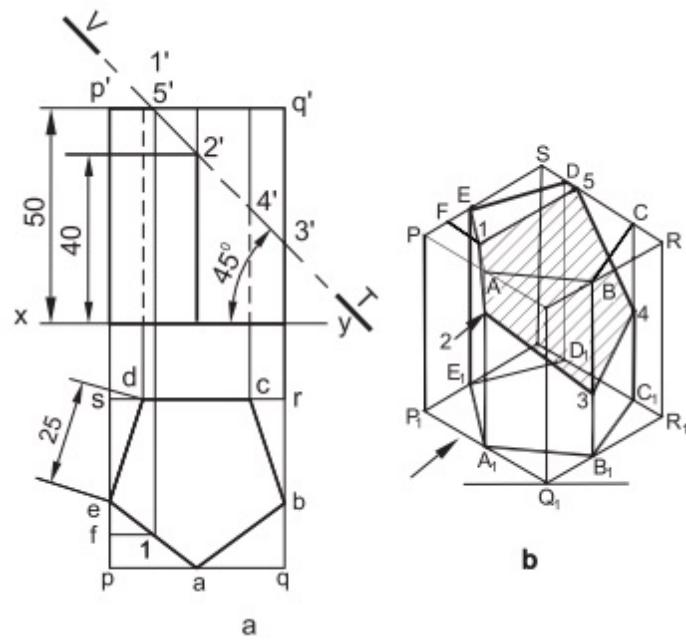


**Fig.16.24**

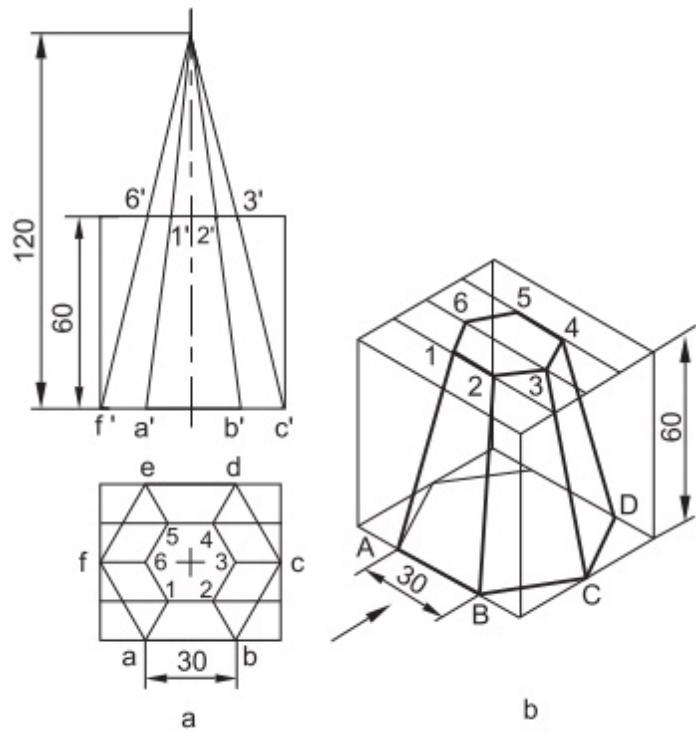
1. Draw the projections of the prism, satisfying the given conditions.
2. Enclose the final projections in rectangles and draw the isometric view of the rectangular box.
3. Determine the off-sets of the points of contact, between the corners of the bases and the box.
4. Using the off-sets, locate the above points.
5. Join the points in the order and obtain the isometric view.

**Problem 13** A pentagonal prism of side of base 25 and axis 50 long, is resting on its base on H.P with an edge of its base (away from the observer) parallel to V.P. A section plane, perpendicular to V.P and inclined at  $45^\circ$  to H.P, passes through a point on the axis at 40 above the base. Draw the isometric projection of the truncated portion of the prism.

**Construction (Fig.16.25)**



**Fig.16.25**



**Fig.16.26**

1. Draw the projections of the truncated prism.

2. Enclose the projections in rectangles and draw the isometric projection of the box.
3. Locate the corners  $A_1, B_1$ , etc., of the bottom base and  $A, B$ , etc., of the top base and complete the isometric projection of the complete prism.
4. Determine the off-sets of the points of intersection  $1', 2'$ , etc., between the V.T of section plane and the edges of the prism.
5. Using the isometric lengths of the off-sets, locate the points  $1, 2$ , etc., corresponding to the above points.
6. Join the points  $1, 2, 3$ , etc., by straight lines and obtain the sectioned portion.
7. Draw the vertical edges of the truncated prism.
8. Darken the visible edges of the prism and cross-hatch the sectioned portion.

**Problem 14** A hexagonal pyramid with side of base 30 and axis 120 long, is resting on its base on H.P. An edge of the base is parallel to V.P. A horizontal section plane passes through a point on the axis, at a distance of 60 from the base. Draw the isometric view of the frustum of the pyramid.

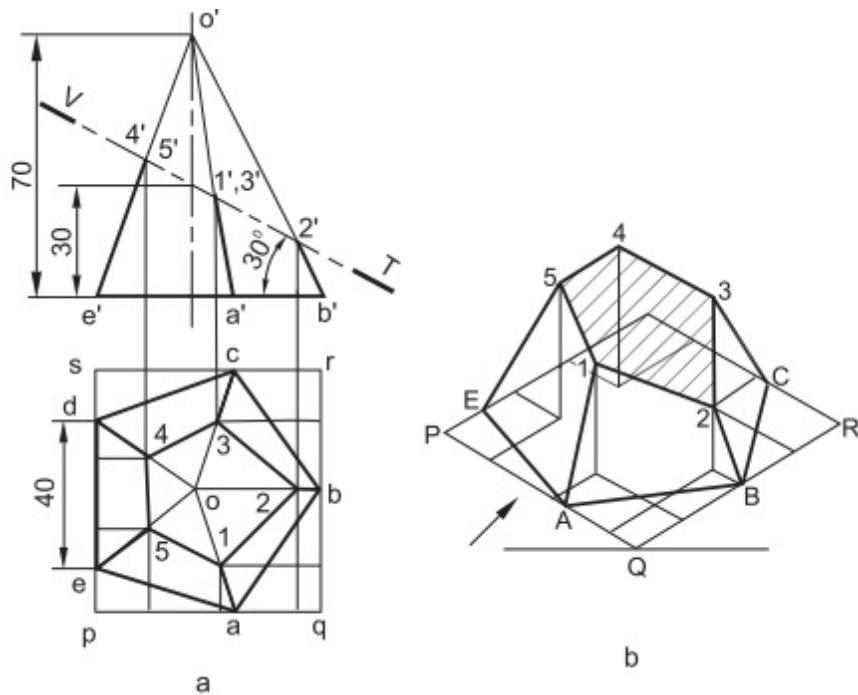
### **Construction (Fig.16.26)**

1. Draw the projections of the frustum of the pyramid.
2. Enclose the projections in rectangles and draw the isometric view of the rectangular box.
3. Determine the off-sets of the corners of the bottom base and top end of the pyramid.
4. Using the off-sets, locate the above corners  $A, B$ , etc., and  $1, 2$ , etc.

5. Join 1, A; 2, B; etc.
6. Darken the visible edges of the frustum of the pyramid.

**Problem 15** A pentagonal pyramid with edge of base 40 and axis 70 long, is resting on its base on H.P. One of the base edges of the pyramid is perpendicular to V.P. A section plane, perpendicular to V.P and inclined to H.P at  $30^\circ$ , passes through the axis, at a height of 30 from the base. Draw the isometric projection of the truncated pyramid.

Following the principle of Construction: Fig.16.25 suitably, obtain the isometric projection of the truncated pyramid, as shown in Fig.16.27.

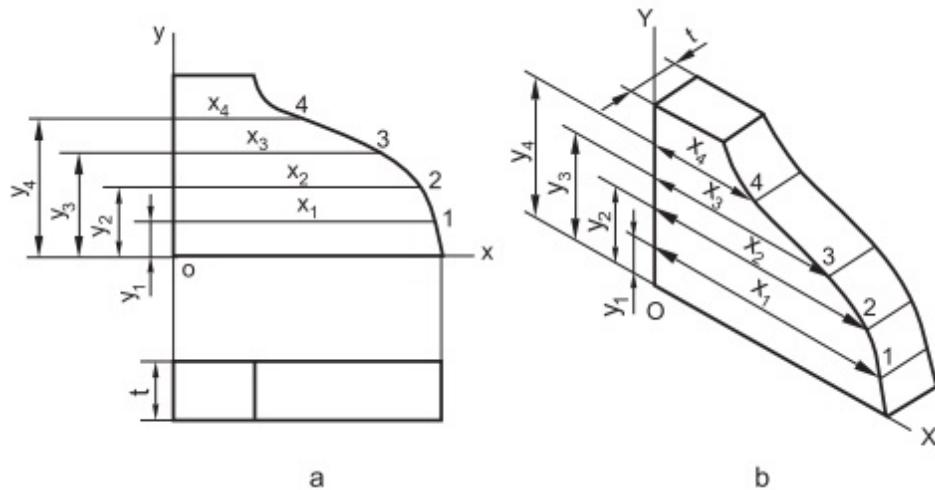


**Fig.16.27**

**Problem 16** Figure 16.28a shows the orthographic views of an object, having an irregular curved surface. Draw the isometric drawing of it.

**Construction (Fig.16.28)**

1. Select a number of points 1, 2, 3, etc., on the given curve (front view), to get a good representation of it in the isometric drawing.
2. Determine the off-sets (co-ordinates) of the points, parallel to the two principal axes  $ox$  and  $oy$ .
3. Draw the isometric axes  $OX$  and  $OY$ , through any point  $O$ .
4. Locate the points 1, 2, 3, etc., by means of co-ordinates, drawn parallel to the two axes.
5. Draw a smooth curve passing through the points.
6. To draw the curve on the other parallel face, project each point, parallel to the  $Z$ -axis through a distance equal to the thickness of the object and join by a smooth curve.

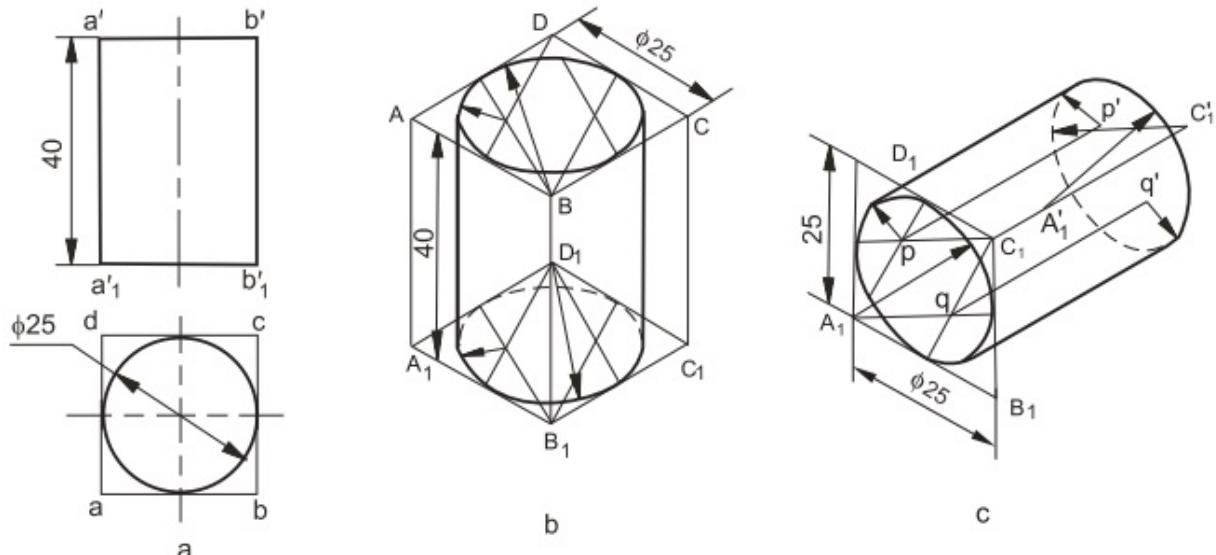


**Fig.16.28 Construction of an irregular curve in isometric**

### 16.3.12 Isometric Projections of Cylinders and Cones

**Problem 17** Make the isometric drawing of a cylinder of base diameter 25 and axis 40 long.

**Construction (Fig.16.29b)**



**Fig.16.29**

1. Enclose the cylinder in a box and draw its isometric drawing.
2. Draw ellipses, corresponding to the bottom and top bases, by using the four-centre method.
3. Join the two bases by two common tangents.



**Figure 16.29c** shows the isometric drawing of a cylinder, using the off-set method and assuming horizontal orientation. Following are the stages of construction:

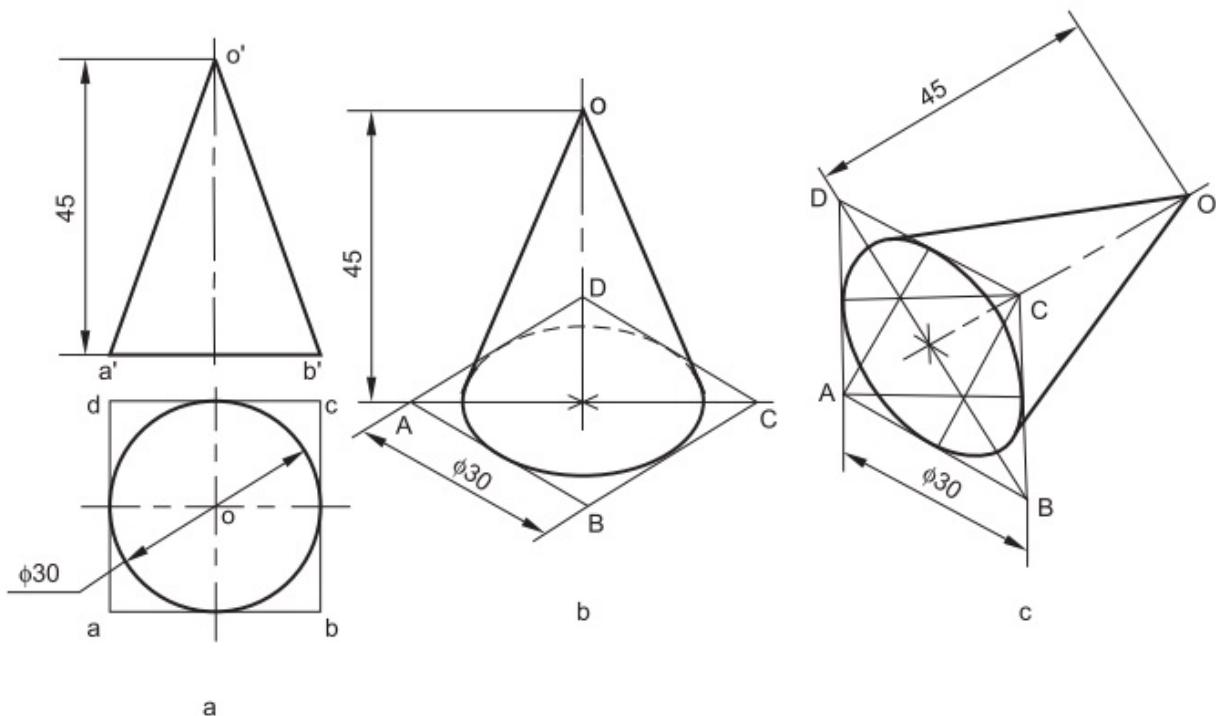
- (i) Construct the isometric drawing, corresponding to the bottom base of the cylinder, by the four-centre method.
- ii) Project the four centres through a distance equal to the length of the axis and draw the ellipse,

corresponding to the top base of the cylinder.

- Join the two ellipses by two common tangents and complete the drawing.

**Problem 18** Draw the isometric drawing of a cone of base diameter 30 and axis 45 long. Use the off-set method.

**Construction (Fig.16.30b)**



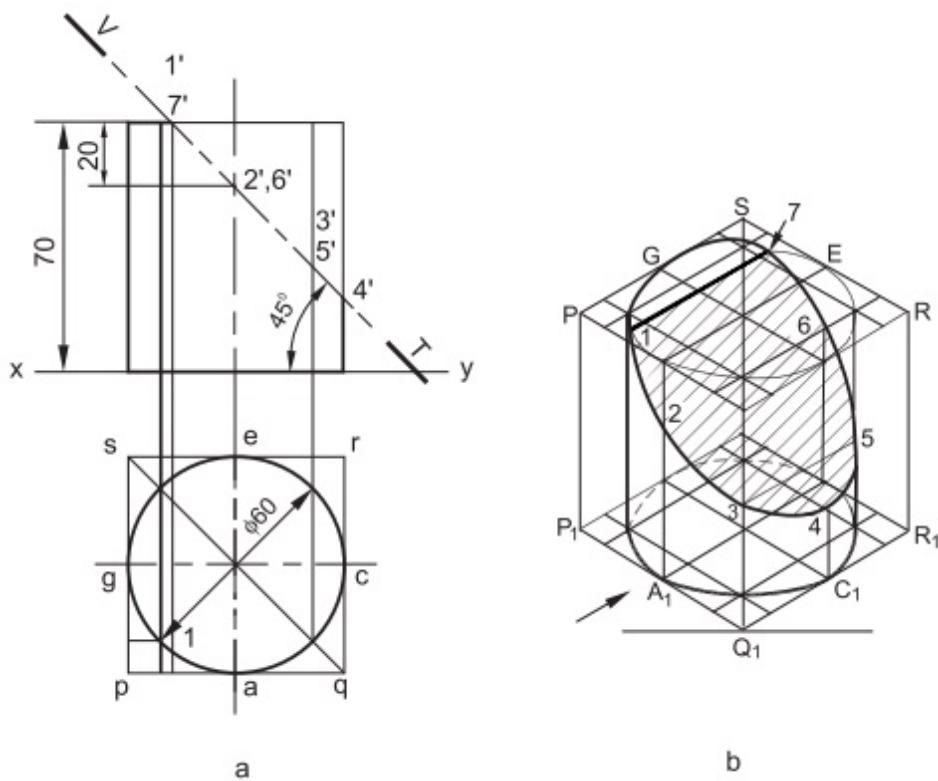
**Fig.16.30**

- Enclose the base of the cone in a square (Fig.16.30a).
- Draw the ellipse, corresponding to the circular base of the cone.
- From the centre of the ellipse, draw a vertical centre line and locate the apex, at a height of 45.
- Draw the two outer-most generators from the apex to the ellipse and complete the drawing.

Figure 16.30c shows the construction, assuming horizontal orientation.

**Problem 19** A cylinder diameter of base 60 and axis 70 long, is resting on its base on H.P. A section plane, perpendicular to V.P and inclined at  $45^\circ$  to H.P, passes through the axis at a distance of 20 from its top end. Draw the isometric projection of the truncated cylinder.

**Construction (Fig.16.31)**



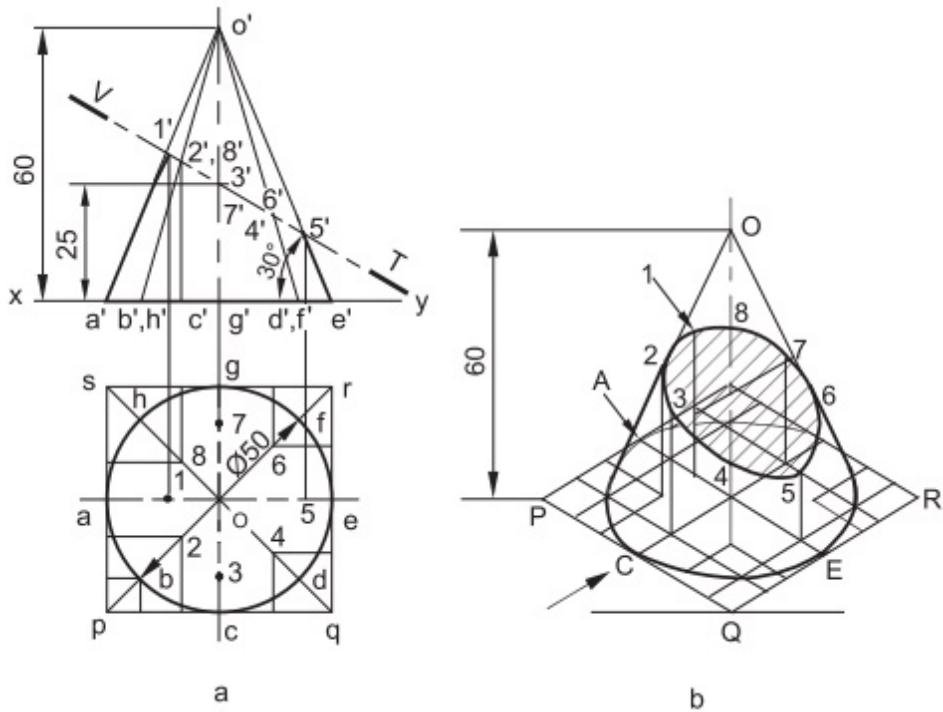
**Fig.16.31**

1. Draw the projections of the truncated cylinder.
2. Enclose the truncated cylinder, in a square box.
3. Divide the circle (top view) into some equal parts, say 8 and draw the corresponding generators in the front view.

4. Draw the isometric projection of the square box.
5. Locate the points of intersection 1', 2', etc., between the V.T of section plane and the generators and top base of the cylinder.
6. Determine the off-sets of the above points as well as those of the division points of the bottom base.
7. Using the off-sets, draw the isometric projection of the bottom base.
8. Using the off-sets, locate the points 1, 2, etc.
9. Join the points in the order suitably and obtain the isometric projection of the truncated cylinder.

**Problem 20** A cone of base diameter 50 and axis 60, rests with its base on H.P. A section plane perpendicular to V.P and inclined at  $30^\circ$  to H.P passes through the axis, at a distance of 25 above the base. Draw the isometric projection of the truncated cone. Use off-set method.

Following the principle of Construction: [Fig.16.31](#) suitably, draw the isometric projection of the truncated cone, as shown in [Fig.16.32](#).

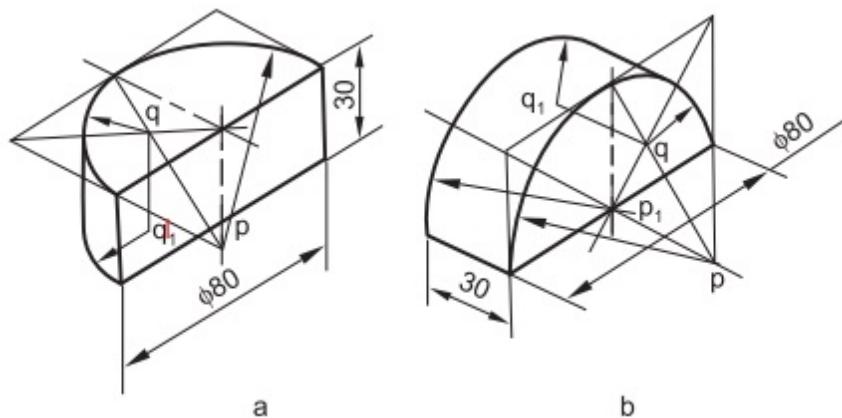


**Fig.16.32**

**Problem 21** Draw the isometric view of a semi-cylindrical disc of 40 radius and 30 thick, keeping its axis (i) vertical and (ii) horizontal.

**Case I Axis vertical - Four-centre method**

**Construction (Fig.16.33a)**



**Fig.16.33**

1. Using the four-centre method, locate the centres p and q and draw the isometric view of the top face of the disc.
2. Through q, draw a vertical projector and locate the centre  $q_1$  ( $qq_1$  = thickness of the disc).
3. With centre  $q_1$ , draw the smaller arc and complete the view.

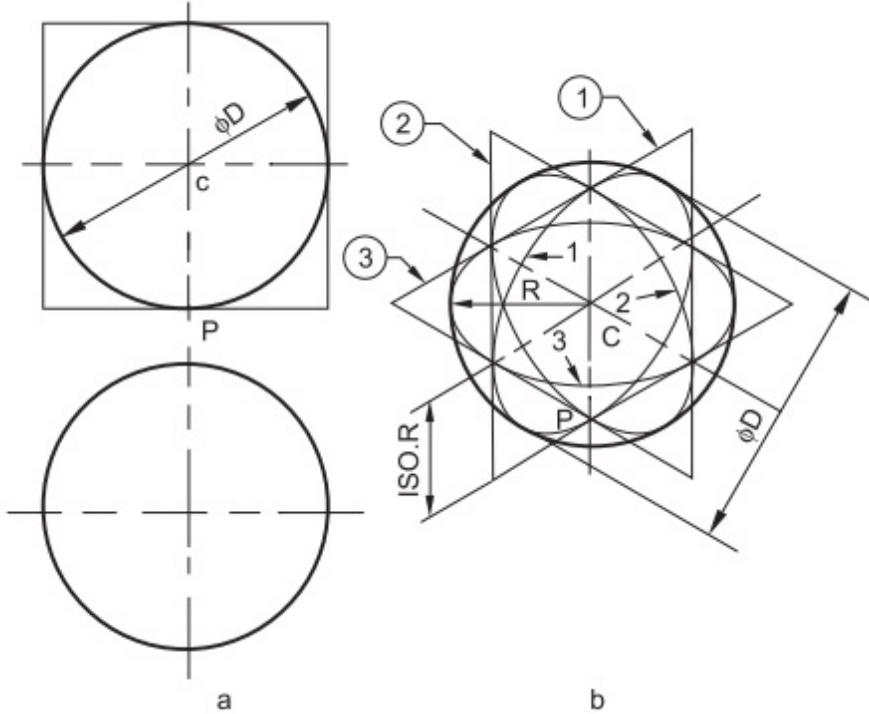
***Case II Axis horizontal-Four-centre method***

Figure 16.33b shows the construction of the isometric view of the disc, by keeping the axis horizontal.

### 16.3.13 Isometric Projection of a Sphere

A sphere appears as a circle of diameter equal to the diameter of sphere when seen from any direction. Hence, the isometric projection of a sphere is also a circle of diameter equal to the true diameter of the sphere, as explained below.

The projections of a sphere, resting on H.P are shown in Fig.16.34a. It is to be noted from the figure that C is its centre, D is the diameter and P is the point of contact with H. P.



**Fig.16.34**

Consider vertical sections 1 and 2 through the centre of the sphere. The sections produced are circles of diameter  $D$ . These circles are known as greater circles. The isometric projections of these circles are shown in Fig.16.34b, by ellipses 1 and 2, drawn around the centre  $C$ . The length of the major axis in each case is  $D$ . However, the distance of the point  $P$  from the centre  $C$  is equal to the isometric radius of the sphere.

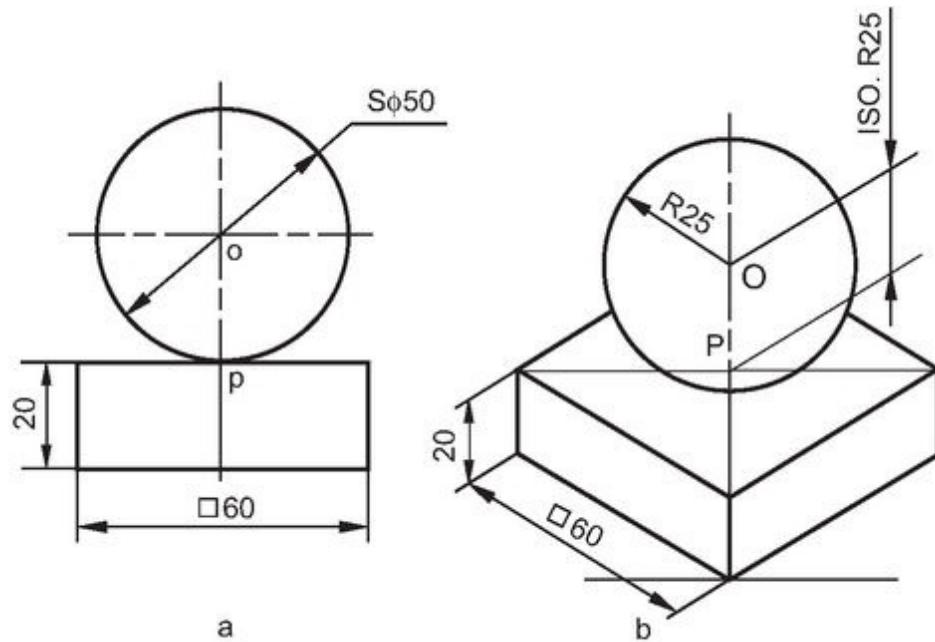
Consider a horizontal section plane 3, through the centre of the sphere. The isometric projection of the circle produced as a section is also an ellipse 3. In all the above cases, the distance of the outer-most points on the ellipses from the centre is equal to  $D/2$ . Thus, it can be noted that in an isometric projection, the distances of all the points on the surface of a sphere from its centre, are equal to the true radius of the sphere.

Hence, the isometric projection of a sphere is a circle, whose diameter is equal to the true diameter of the sphere. However, the distance of the centre of the sphere, from its point of contact with H.P is equal to the isometric radius of the sphere.

Therefore, it is to be noted that isometric scale must be used while constructing isometrics of spheres or spherical parts.

**Problem 22** *Figure 16.35a shows the front view of a sphere, resting centrally on the top of a square block. Draw the isometric projection of the arrangement.*

**Construction (Fig.16.35b)**



**Fig.16.35**

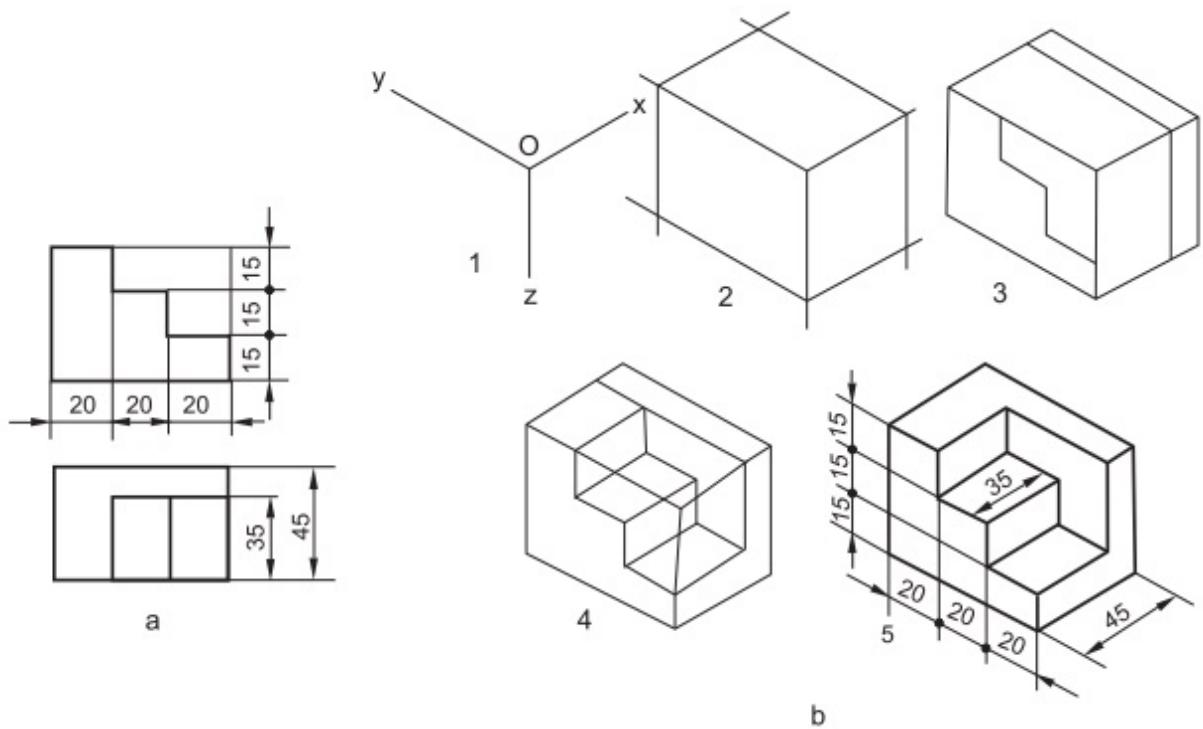
1. Draw the isometric projection of the square block, using the isometric scale.
2. Locate the centre P of the top surface of the block, the point of contact of the sphere.

3. Draw a vertical line PO, equal to the isometric radius of the sphere.
4. With centre O and radius equal to the true radius of the sphere, draw a circle, completing the isometric projection of the combined solids.

## 16.4 EXAMPLES

For all the examples that follow, Figure a represents the orthographic projections and Figure b, the corresponding isometric projection/drawing. The students are advised to refer the following steps in making an isometric of an object, from the given orthographic views. As an example, refer [Fig.16.36](#) for the application of the steps.

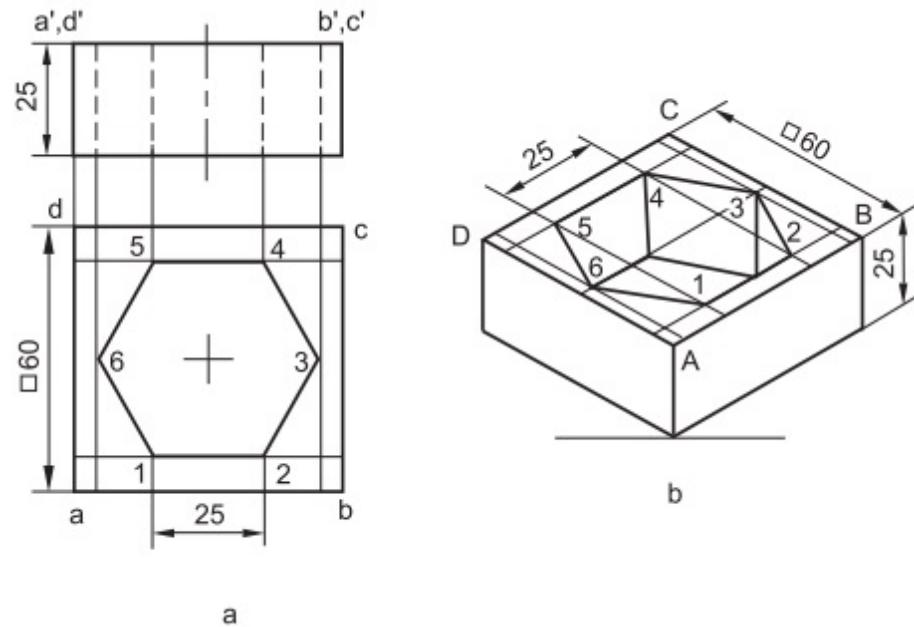
1. Study the given views and note the principal dimensions and other features of the object.
2. Draw the isometric axes.
3. Mark the principal dimensions along the isometric axes.
4. Complete the housing block, by drawing lines parallel to the isometric axes and passing through the above markings.
5. Locate the principal corners of all the features of the object, on the three faces of the housing block.
6. Draw lines parallel to the axes and passing through the above points and obtain the isometric projection/drawing of the object, by darkening the visible edges.



**Fig.16.36**

**Problem 23** A square slab of  $60 \times 60 \times 25$ , has a hexagonal hole through it. The side of the hexagon is 25 and one side of it is parallel to a face of the slab. Draw its isometric projection.

**Construction (Fig.16.37)**

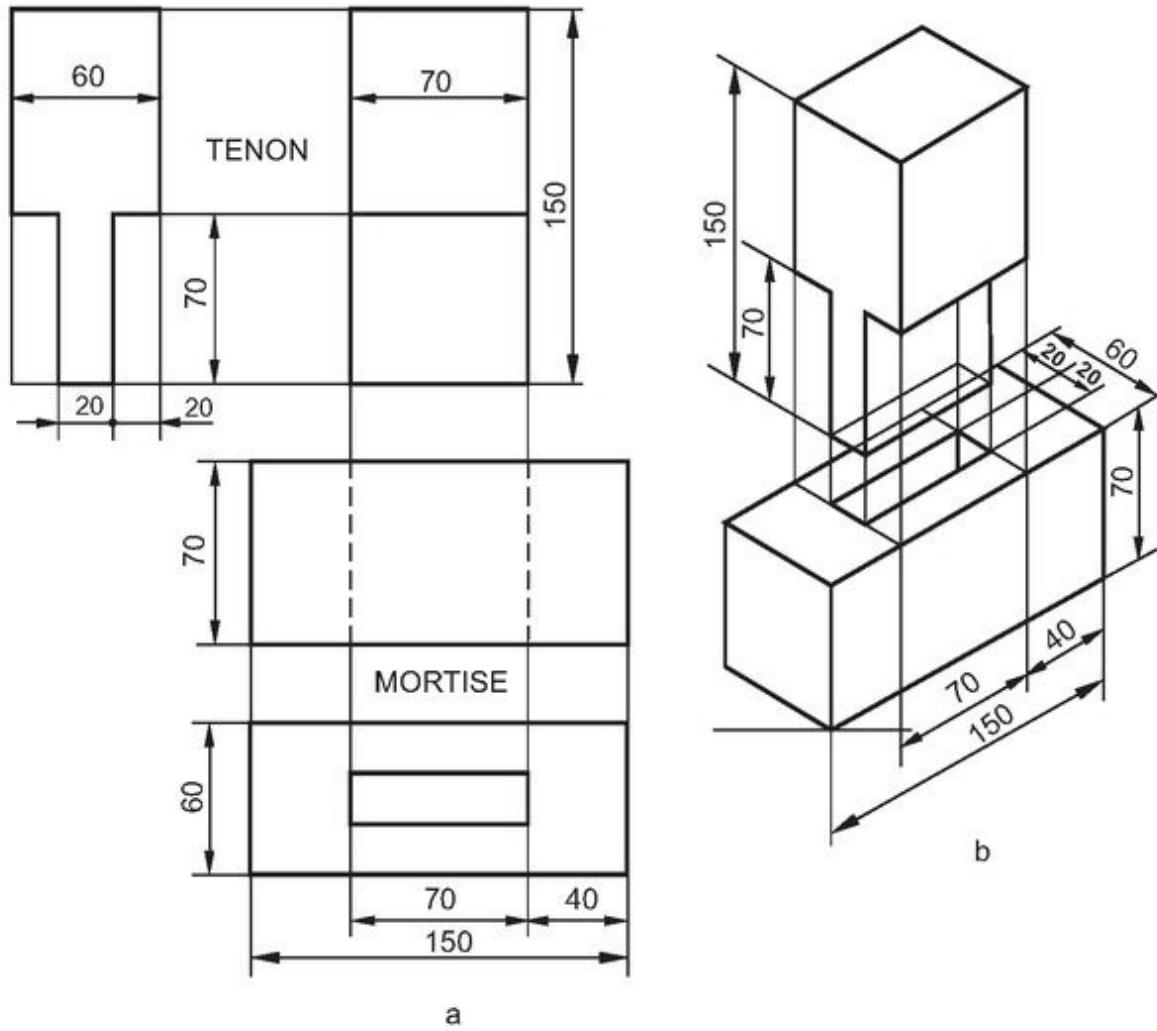


**Fig.16.37**

1. Draw the projections of the slab, with the hexagonal hole through it.
2. Using isometric scale, draw the isometric projection of the complete slab.
3. Using the off-ssets, locate the corners of the hexagonal hole, in the top base of the slab.
4. Join 1,2; 2,3; etc., thus completing the isometric projection of the hexagon in the top base of the slab.
5. Show the internal visible edges of the hole by dark lines.

**Problem 24** Draw the isometric projections of a mortise and tenon joint, from two *pieces of size 150×60×70*. *The length of the tenon is 70 and thickness is 20. The two pieces must be shown separately, in a position ready for fitting.*

**Construction (Fig.16.38)**



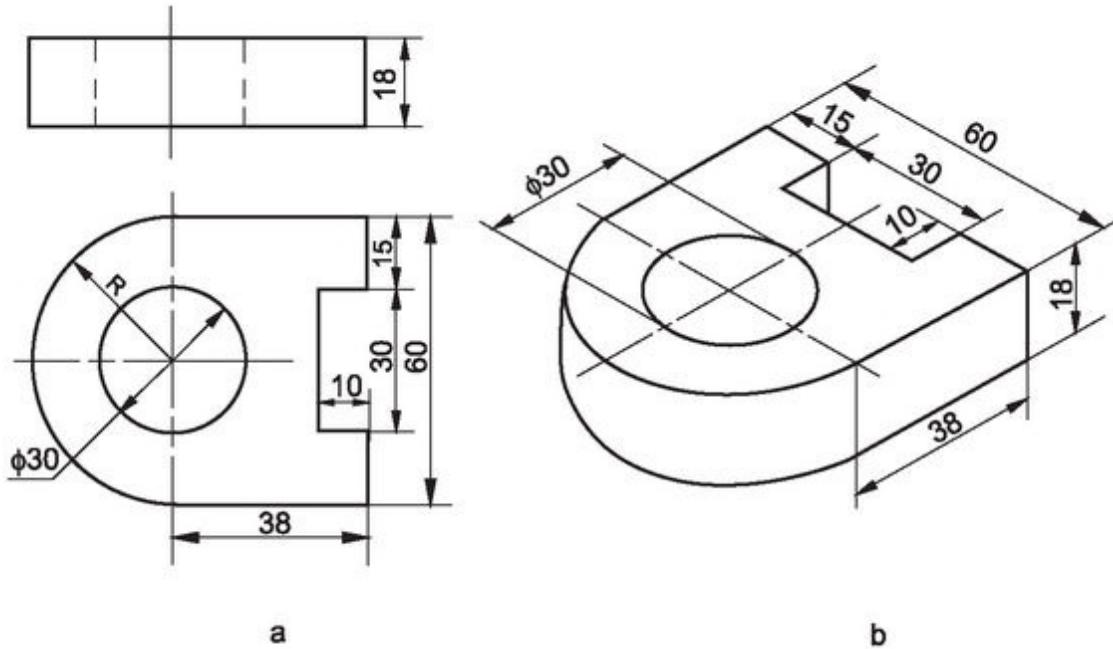
**Fig.16.38**

1. Draw the projections of the mortise and tenon pieces, in their proper orientation ([Fig.16.38a](#)).
2. Using the isometric scale and following the box method, draw the isometric projections of the mortise and tenon as shown in [Fig.16.38b](#), which are in a position, ready for fitting.

**Problem 25** *Draw the isometric view of the block, two views of which are shown in [Fig.16.39a](#).*

**Construction ([Fig.16.39b](#))**

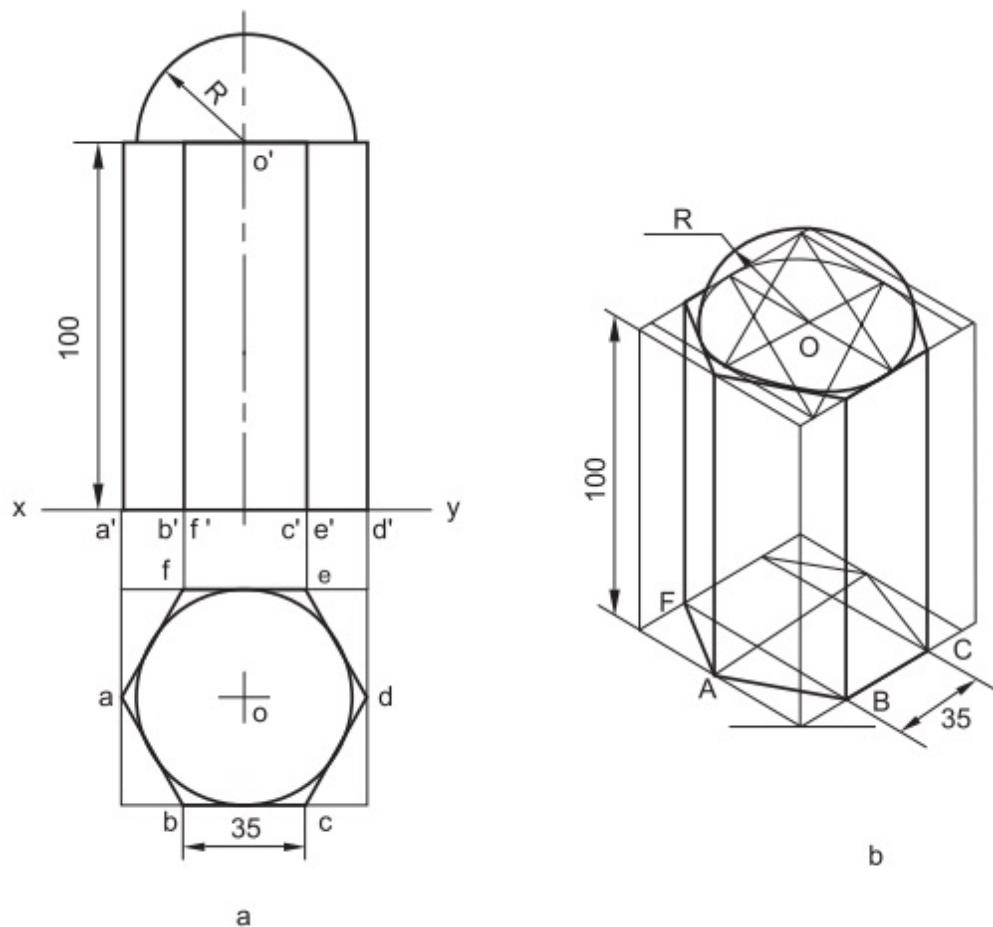
Figure 16.39b shows the isometric view of the block, using the box method for the main body of the block and four-centre method for the ellipse.



**Fig.16.39**

**Problem 26** A hemi-sphere is resting on the top of a hexagonal prism of 35 side and axis 100 long. Draw the isometric projection of the set-up, when the hemi-sphere is touching all the edges of the top base.

**Construction (Fig.16.40)**

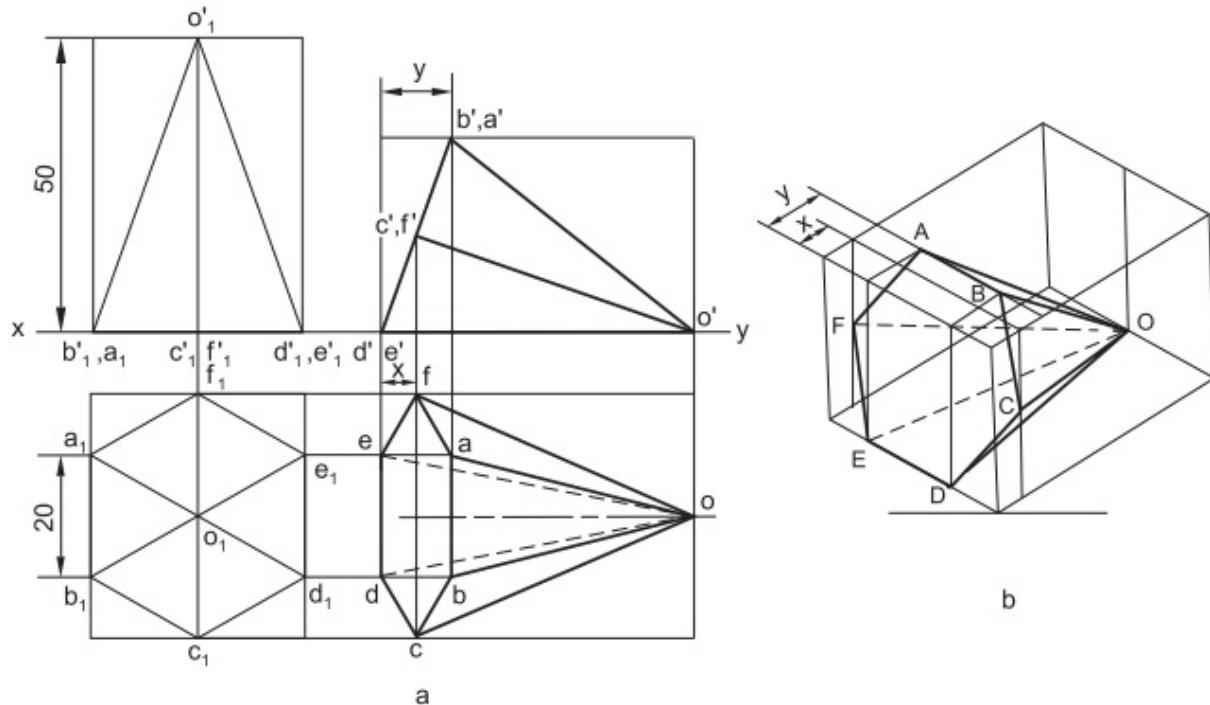


**Fig.16.40**

1. Draw the projections of the combination of the solids, satisfying the given conditions.
2. Using the isometric scale and following the box method, draw the isometric projection of the prism.
3. Locate the centre O of the top base of the prism and draw the isometric projection (ellipse) of the circular base of the hemi-sphere, by following the four-centre method.  
The ellipse touches all the top base edges of the prism.
4. With O as centre and radius equal to the true radius of the hemi-sphere, draw a circular arc touching the above ellipse tangentially.

**Problem 27** Draw the isometric projection of a regular hexagonal pyramid, with side of base 20 and height 50, when lying with one of its triangular faces on H.P and its base at right angle to V.P.

**Construction (Fig.16.41)**



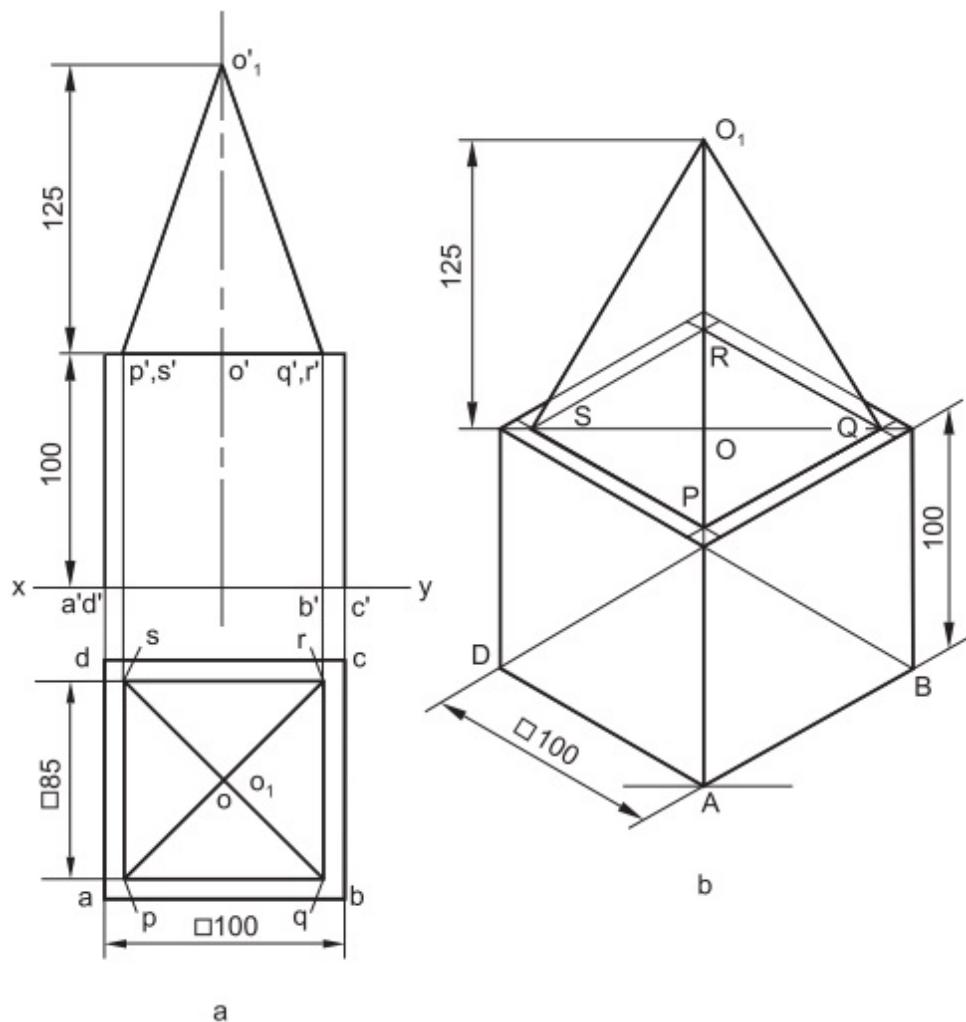
**Fig.16.41**

1. Draw the projections of the pyramid, satisfying the given conditions.
2. Enclose the final projections in rectangles and using the isometric scale, draw the isometric projection of the rectangular box.
3. Determine the off-sets of the points of contact, between the corners of the base, apex and the box, in both the projections.
4. Using the off-sets, locate the above points on the surfaces of the box.

5. Join the above points in the order and obtain the isometric projection of the pyramid.

**Problem 28** A square pyramid with base edge 85 and height 125, is resting on a cube of side 100. The axes of the solids coincide along a line. The two sides of the base of the pyramid are parallel to the edges of the cube. Draw the isometric view of the combination of the solids.

**Construction (Fig.16.42)**



**Fig.16.42**

1. Draw the projections of the combination of the solids, satisfying the given conditions.

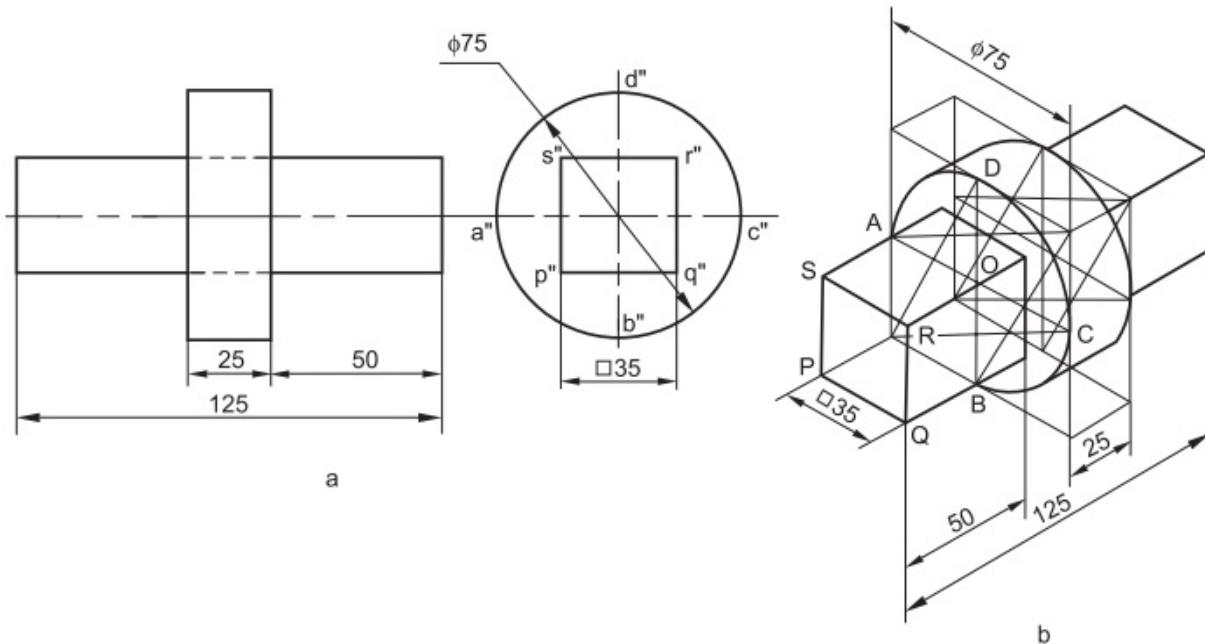
2. Using the true lengths, draw the isometric view of the cube.
3. Locate the centre O of the top face of the cube and using the off-sets, locate the corners P, Q, R and S of the pyramid base.
4. Through O, draw the axis and mark  $O_1$  on it such that,  $OO_1 = 125$ .
5. Join  $O_1, P; O_1, Q; O_1, R$  and  $O_1, S$ .
6. Darken the visible edges and obtain the required isometric view.

**Problem 29** A circular block of 75 diameter and 25 thick, is pierced centrally through its flat faces, by a square prism of base 35 side and 125 long, which comes out equally on both sides of the block. Draw the isometric projection of the combination when the combined axis is horizontal.

### **Construction (Fig.16.43)**

1. Draw the projections (front and side views) of the circular block, with the square prism pierced through it.
2. Enclosing the projections of the cylindrical block in a box using isometric scale, draw the isometric projection of the box first and then that of the cylindrical block (use four-centre method for obtaining the ellipses, corresponding to the two faces of the cylindrical block).
3. Locate the centre O of the visible base of the circular block and draw the isometric projection of the square prism such that, the axis of the prism coincides with the axis of the circular block. Ensure that the prism

comes out equally from both the bases of the cylindrical block.



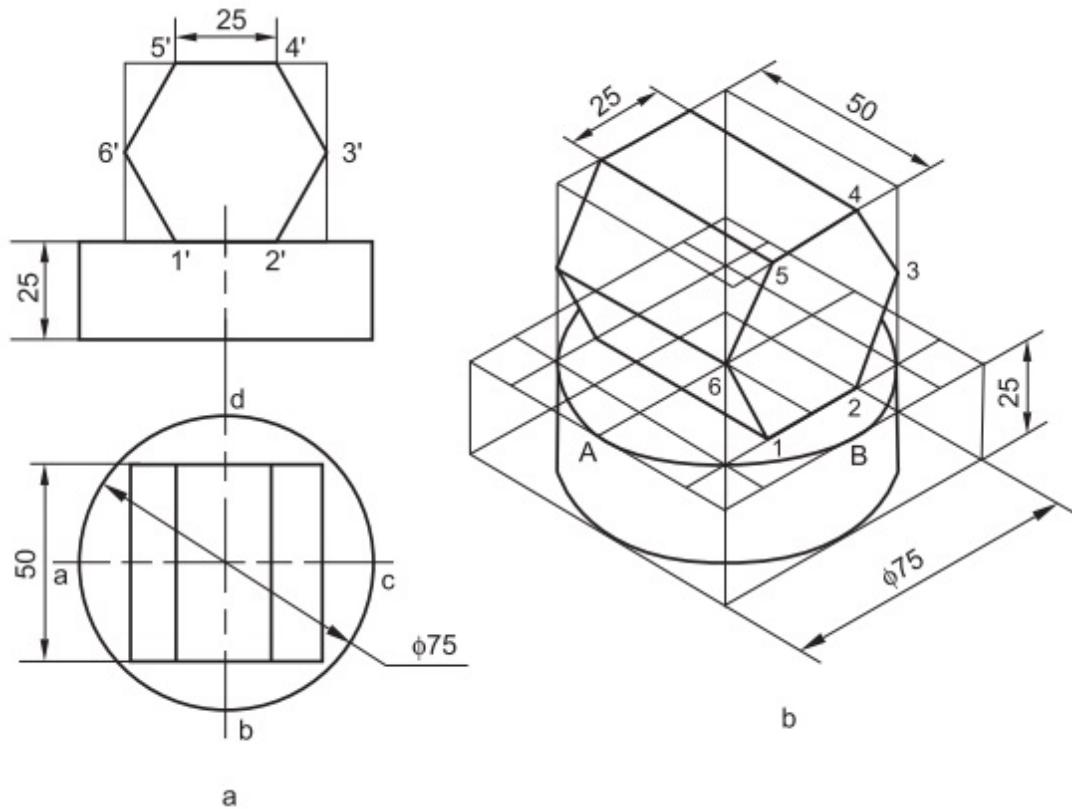
**Fig.16.43**

**Problem 30** A hexagonal prism with side of base 25 and axis 50 long, rests centrally on one of its rectangular faces, on a cylindrical block of 75 diameter and 25 thick. The cylindrical block rests on its base on H.P and the axis of the prism is perpendicular to V.P. Draw the isometric view of the combination of the two solids.

**Construction (Fig.16.44)**

1. Draw the projections of the combination of the solids, satisfying the given conditions.
2. Following the box method and using the true lengths, draw the isometric view of the cylindrical block. Use four-centre method to draw the ellipses, corresponding to the bases of the block.
3. Following the box method, draw the isometric view of the prism in its proper position.

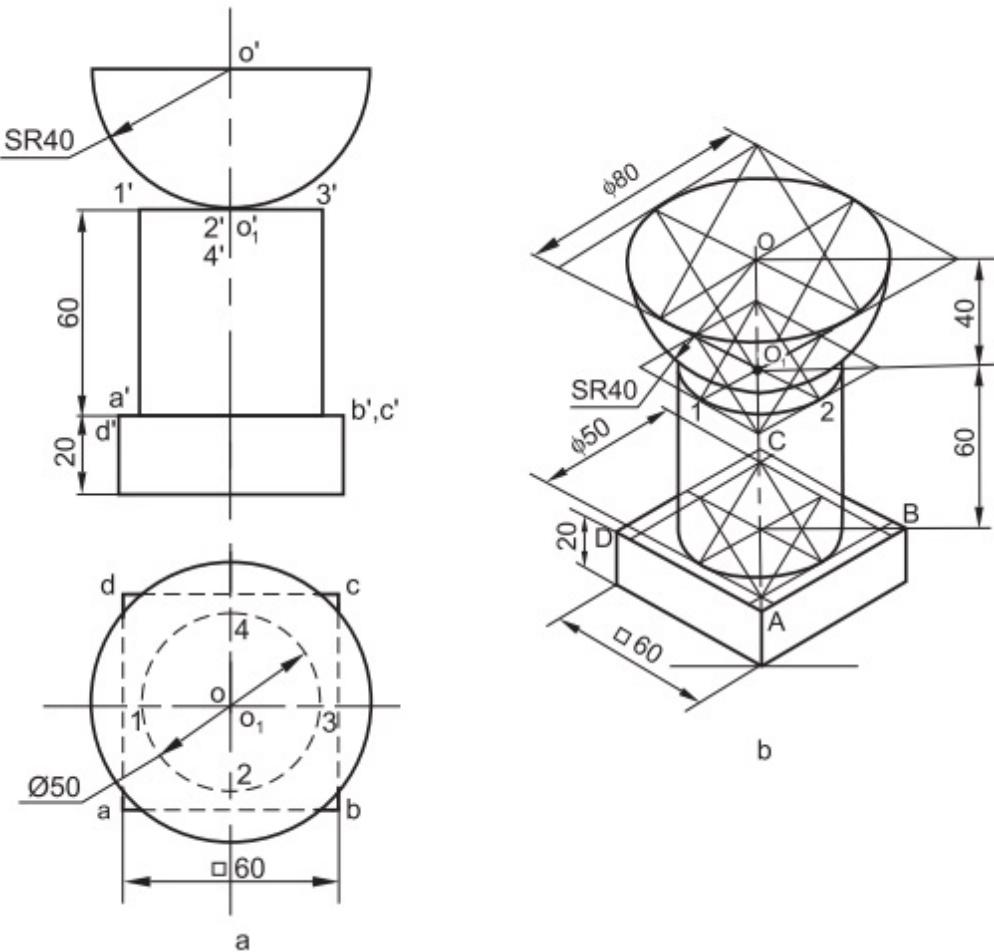
**Problem 31** A hemi-sphere of 80 diameter is placed centrally over a cylinder of 50 diameter and 60 long such that, the hemi-sphere rests on a point on the cylinder. The cylinder is placed centrally over a square block, with side of base 60 and 20 thick. Draw the isometric projection of the combination of the solids.



**Fig.16.44**

**Construction ([Fig.16.45](#))**

1. Draw the isometric projection of the square block and cylinder, using isometric scale.

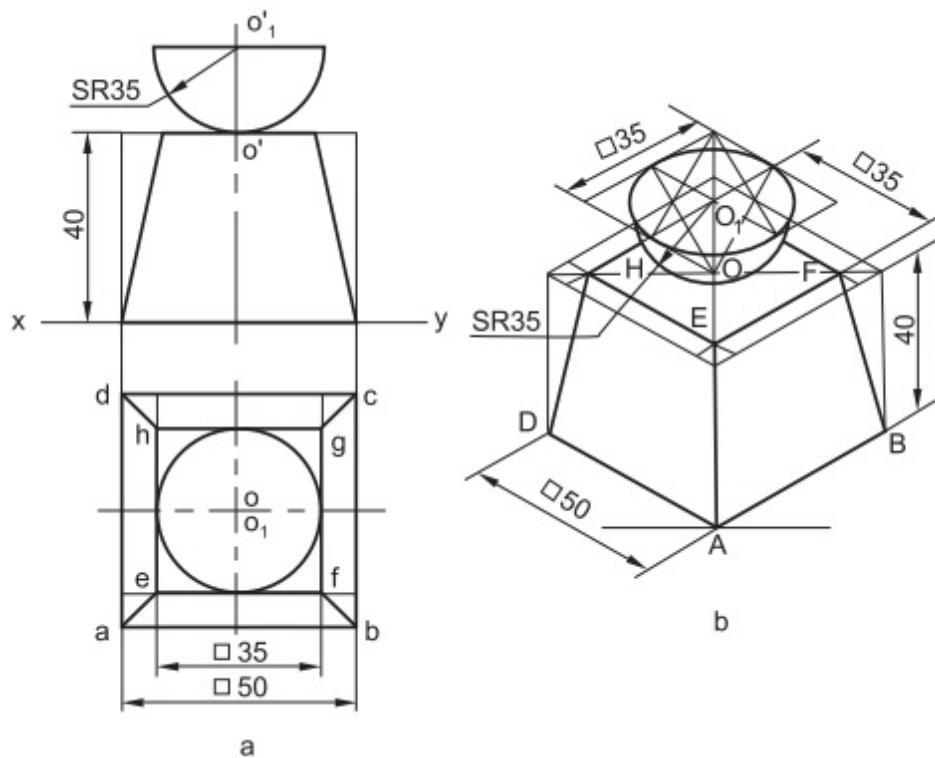


**Fig.16.45**

2. Locate the centre,  $O_1$  of the top base of the cylinder.
3. Through  $O_1$ , draw a vertical line and locate  $O$  on it such that,  $O_1O$  is equal to the isometric length of the radius of the hemi-sphere.
4. With  $O$  as centre and using the four-centre method, draw the ellipse, corresponding to the top face of the hemi-sphere.
5. With  $O$  as centre and true radius of the hemi-sphere as radius, draw an arc meeting the above ellipse tangentially.
6. Darken the visible boundaries of the solids.

**Problem 32** A frustum of a square pyramid (Base, 50mm square, top surface, 35mm square and axis 40mm) rests on its base on H.P. A hemi-sphere of 70mm diameter is centrally placed over the top surface of the pyramid and the hemi-sphere rests on a point on the surface of the pyramid. Draw the isometric projection of the combination of the solids.

**Construction (Fig.16.46)**



**Fig.16.46**

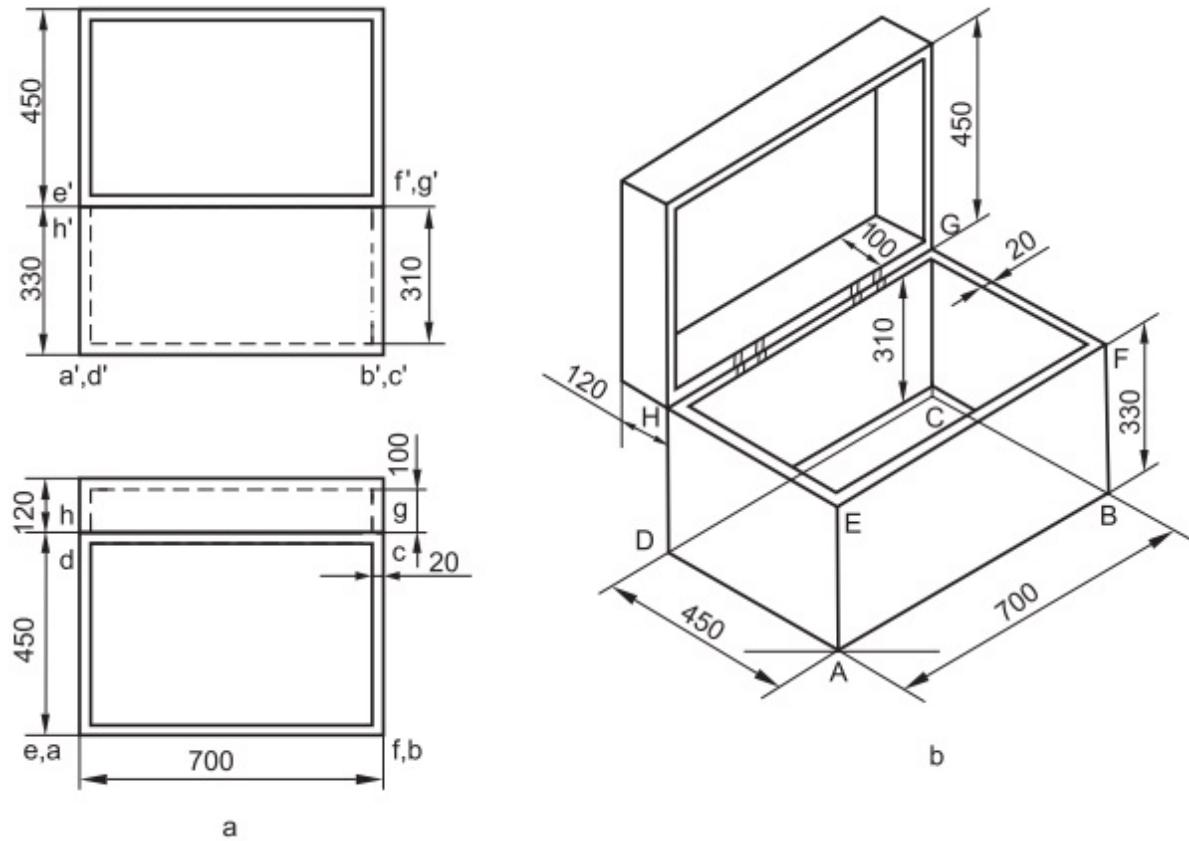
1. Draw the projections of the combination of the solids.
2. Enclose the frustum of the pyramid in a box and draw its projections (front view).
3. Using isometric scale, draw the isometric projection of the box.

4. Using the off-sets, locate the corners (A, B, C, D, E, F, G and H) of the pyramid in the box.
5. Join the above points properly and complete the isometric projection of the pyramid.
6. Locate O, the centre of the top base of the pyramid.
7. Locate  $O_1$  on the vertical line through O such that,  $OO_1$  is equal to the isometric radius of the hemisphere.
8. With  $O_1$  as centre, complete the rhombus, representing the isometric projection of the square.
9. Using the four-centre method, draw the ellipse, within the above rhombus, representing the isometric projection of the top face of the hemi-sphere.
10. With  $O_1$  as centre and true radius of the hemi-sphere as radius, draw an arc meeting the above ellipse tangentially.
11. Darken the visible part of the combination of the solids.

**Problem 33** *The outside dimensions of a box, made of 20mm thick wooden planks are  $700 \times 450 \times 450$  mm. The height of the lid on the outside is 120mm. Draw the isometric view of the box, with the lid open at right angle.*

### **Construction ([Fig.16.47](#))**

1. Draw the projections of the box, keeping the lid in open position, at right angle to the top open surface of the box ([Fig.16.47a](#)). Note that the lid appears as a rectangle of size  $700 \times 450$  mm in the front view and a rectangle of size  $700 \times 120$  mm in the top view.

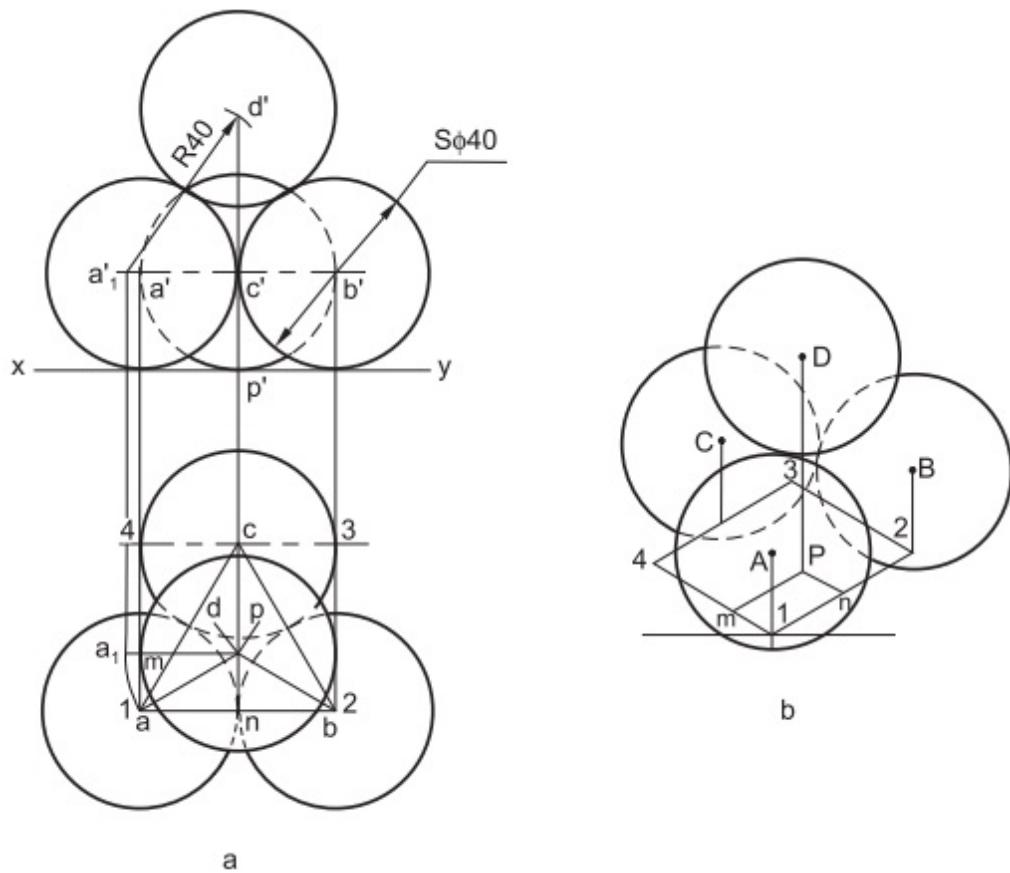


**Fig.16.47**

- Using the true lengths, draw the isometric view of the main body of the box in the horizontal position.
- Draw the isometric view of the lid such that, its front bottom edge coincides with the top rear edge of the main body of the box.
- Darken the visible edges of the box and complete the isometric view of the box.

**Problem 34** Draw the isometric projection of four equal spheres of 40 diameter, which are arranged in a tetrahedral form such that, each sphere touches the other three.

**Construction (Fig.16.48)**



**Fig.16.48**

To draw the projections of the spheres ([Fig.16.48a](#))



For keeping the four spheres of equal size, in tetrahedral form, three spheres must be kept on H.P such that, each touches the other two. The fourth sphere must be kept centrally on the top of the three spheres such that, it touches all the three spheres.

1. In the top view, locate the centres of the bottom three spheres a, b and c, at the corners of an equilateral triangle abc of 40 side, with one side, say ab parallel to xy.
2. In the front view, locate the centres a', b' and c' by projection and on a line parallel to and 20 above xy.

3. With centres  $a$ ,  $b$ ,  $c$ ,  $a'$ ,  $b'$  and  $c'$  and radius 20, draw circles forming the projections of the bottom spheres.

When the fourth sphere is placed on top of the three spheres, its centre  $d$  will lie above the centre of the triangle  $abc$ , while  $a$ ,  $b$ ,  $c$  and  $d$  form the corners of a triangular pyramid (tetrahedron).

**To locate  $d'$**

- (i) Rotate  $da$ , parallel to  $xy$  to  $da_1$ .
- (ii) Through  $a_1$ , draw a projector meeting the line of centres in the front view at  $a'_1$ .
- (iii) With  $a'_1$  as centre and radius 40, draw an arc intersecting the projector through  $d$  at  $d'$ .
4. With  $d'$  and  $d$  as centres and radius 20, draw circles.
5. Following the rules of visibility, complete the projections.

*To draw isometric projection of spheres ([Fig.16.48b](#)):*

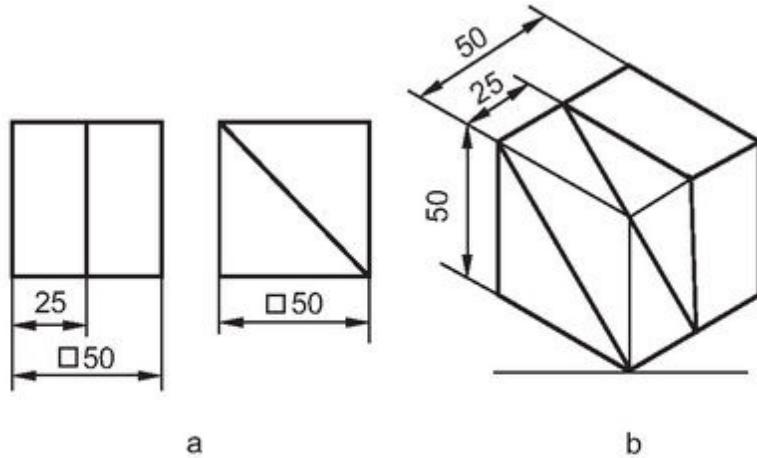
1. Draw a rectangle 1-2-3-4, enclosing the centres of the spheres and using isometric lengths.
2. Locate the centres  $A$ ,  $B$  and  $C$  of the bottom spheres, at a height equal to the isometric radius of the spheres.

*To locate the centre  $D$  of the top sphere:*

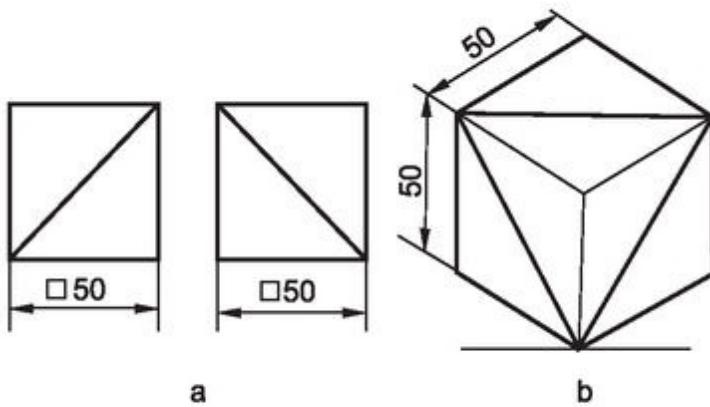
- (i) Using the off-sets,  $am$  ( $=l-m$ ) and  $an$  ( $=l-n$ ), locate the centre  $P$  of the triangle  $abc$ .
- (ii) Through  $P$ , draw a vertical line.
- (iii) Locate  $d$  on the above line such that,  $PD =$  isometric length of the distance  $p'd'$ .

3. With D as centre and true radius of the sphere as radius, draw a circle.
4. Following the rules of visibility, complete the isometric projection.

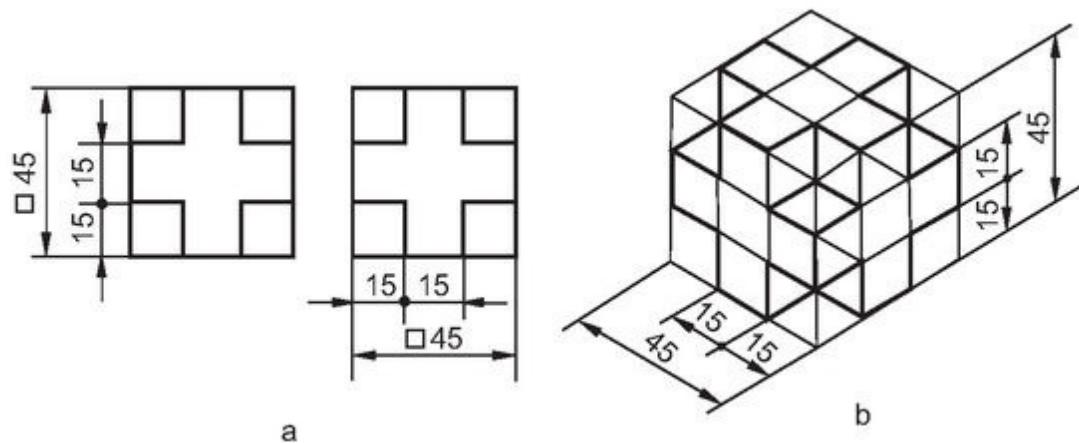
**Problems 35 to 53** Figures 16.49a to 16.70a show the orthographic projections of certain objects and Figs.16.49b to 16.70b represent the respective isometrics; the constructions of which are self explanatory.



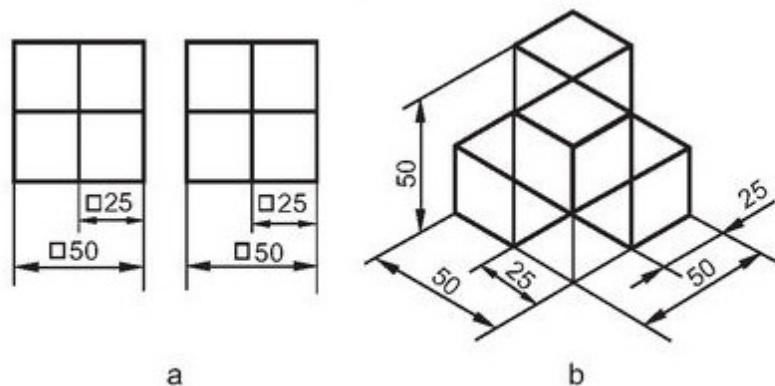
**Fig.16.49**



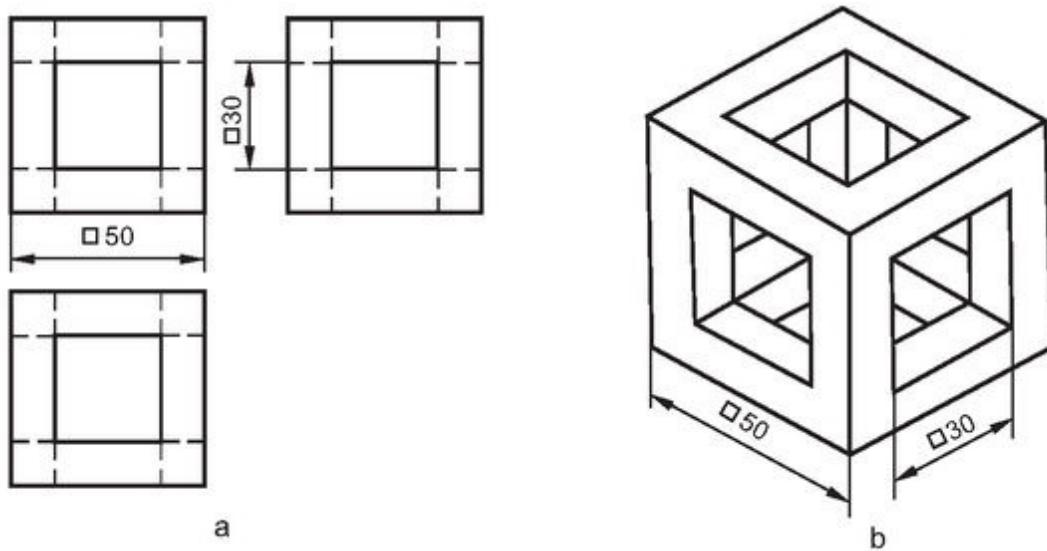
**Fig.16.50**



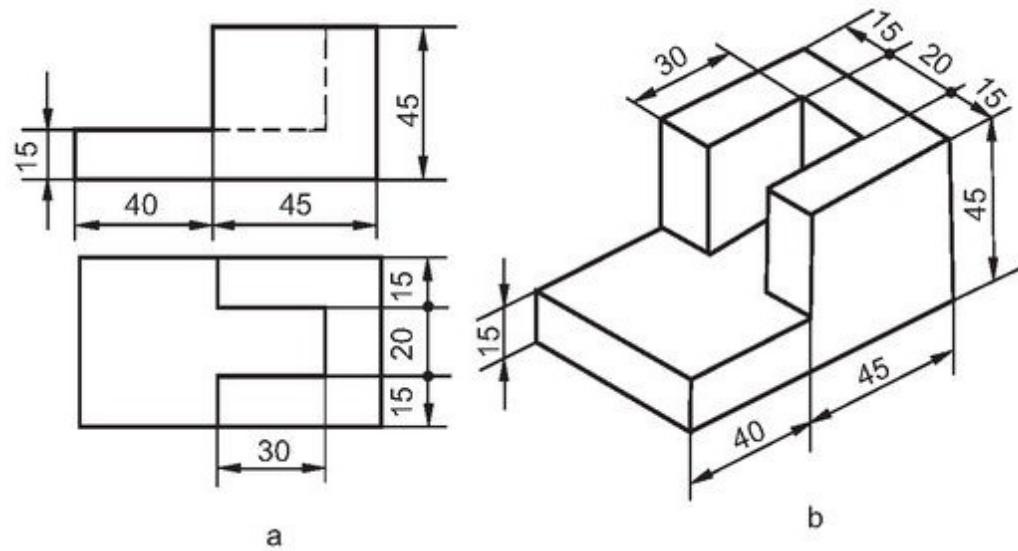
**Fig.16.51**



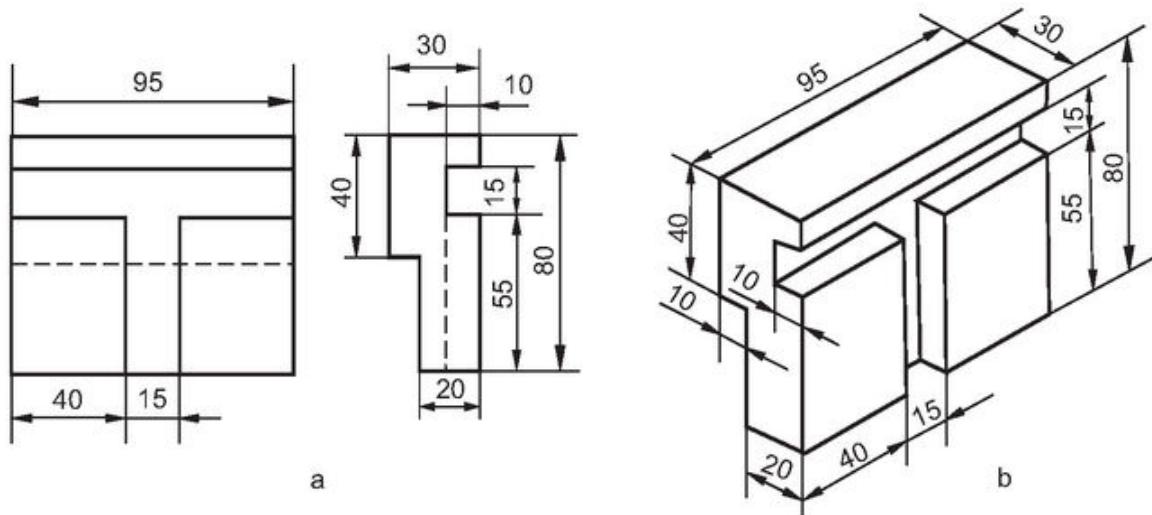
**Fig.16.52**



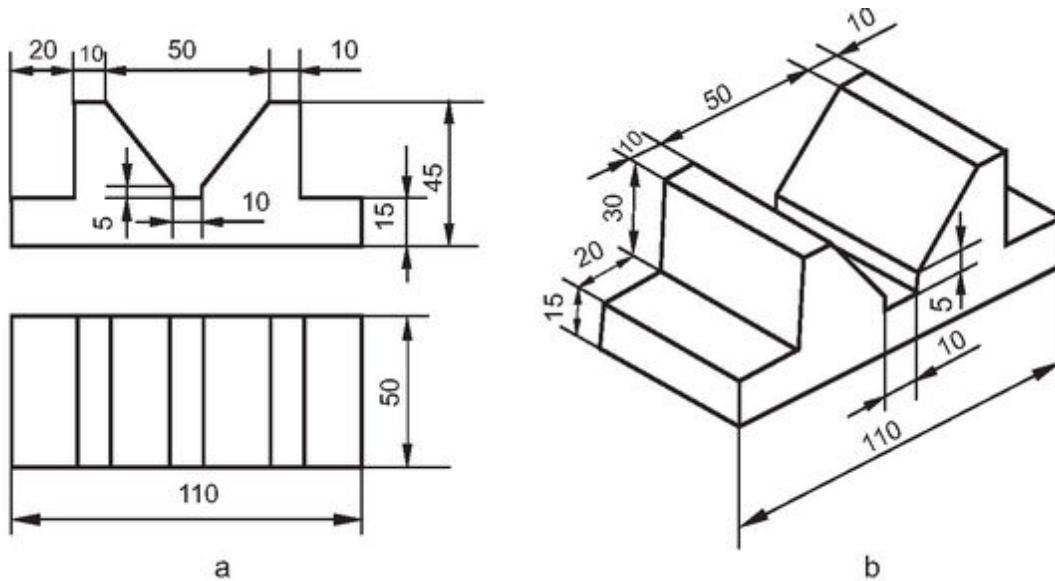
**Fig.16.53**



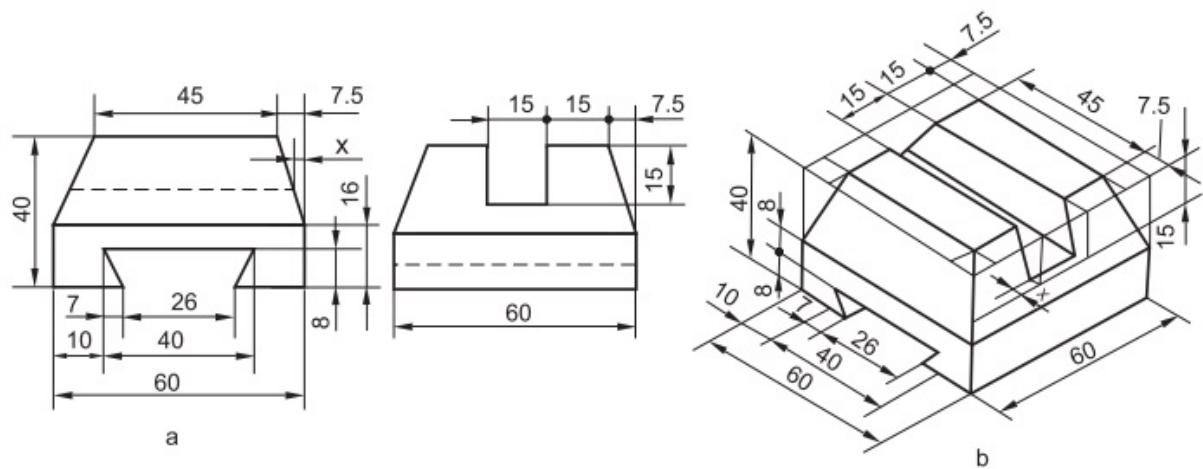
**Fig.16.54**



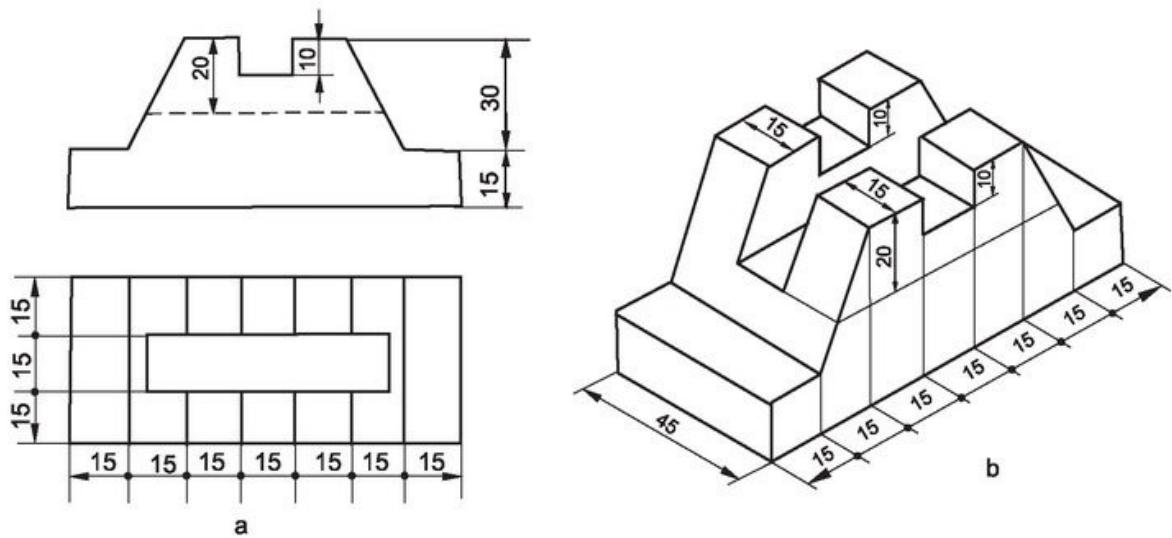
**Fig.16.55**



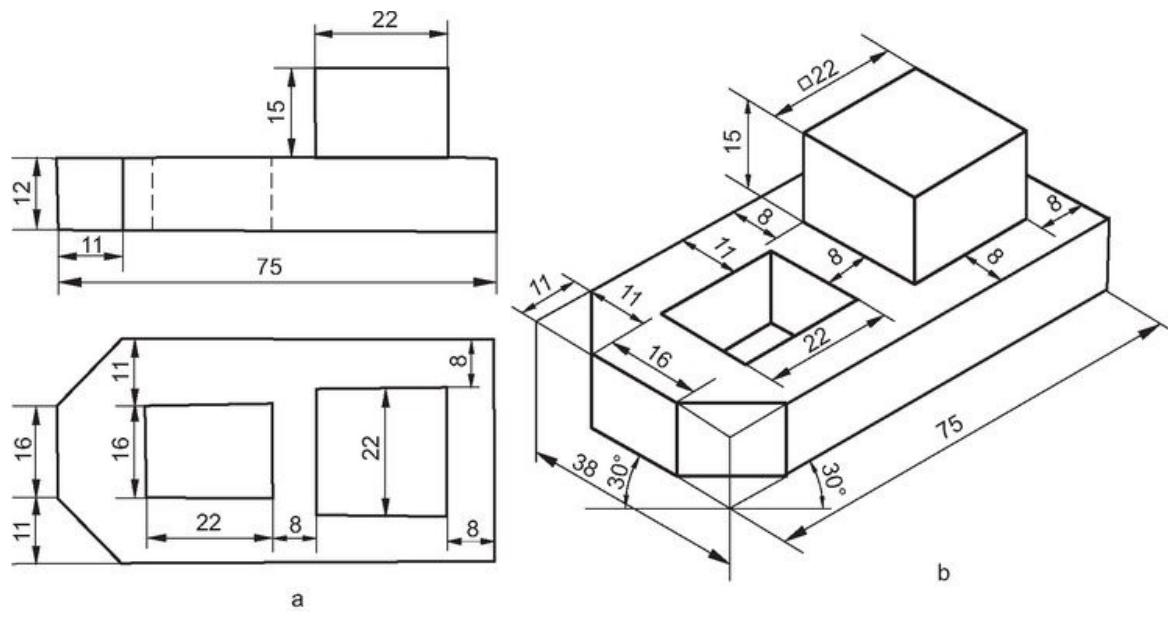
**Fig.16.56**



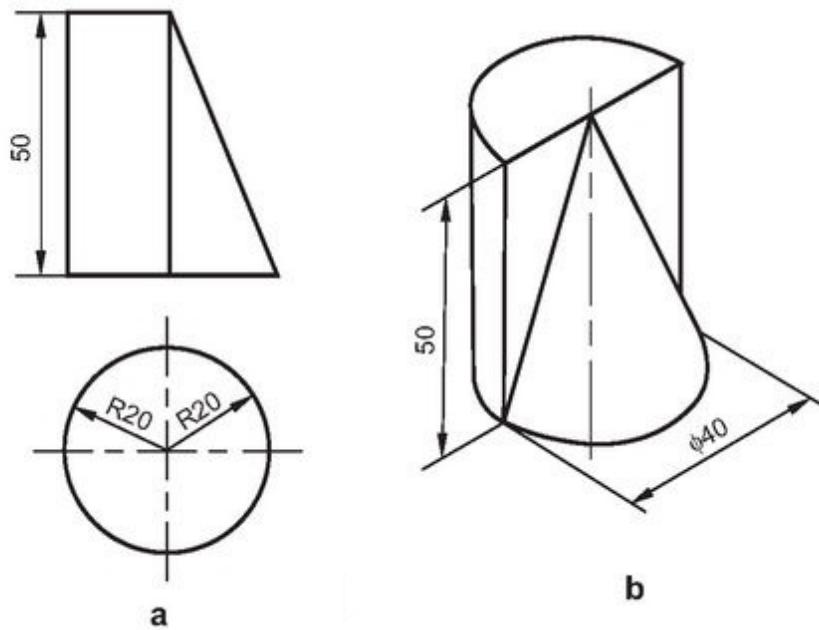
**Fig.16.57**



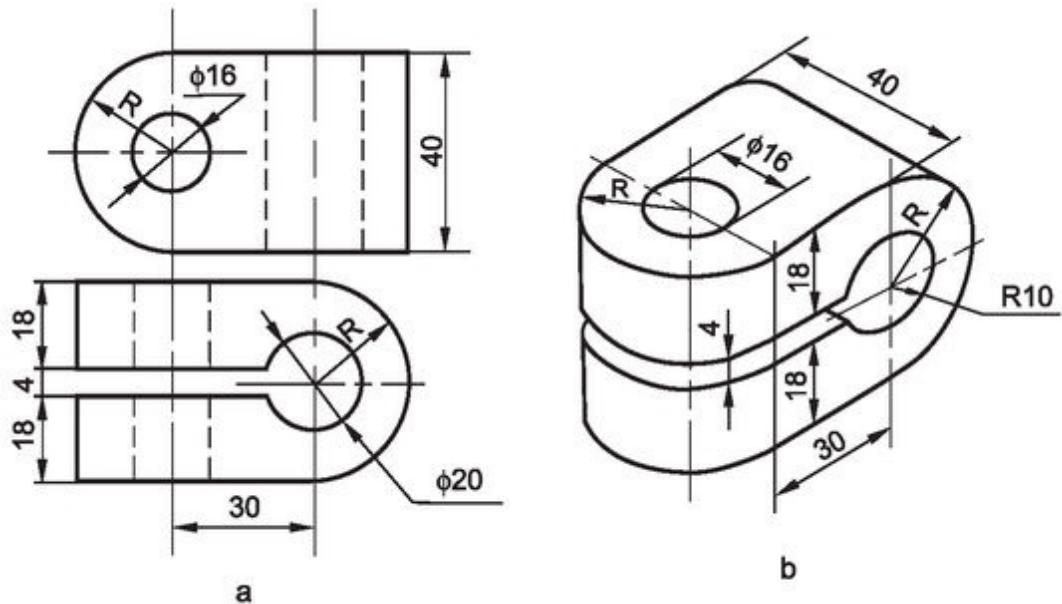
**Fig.16.58**



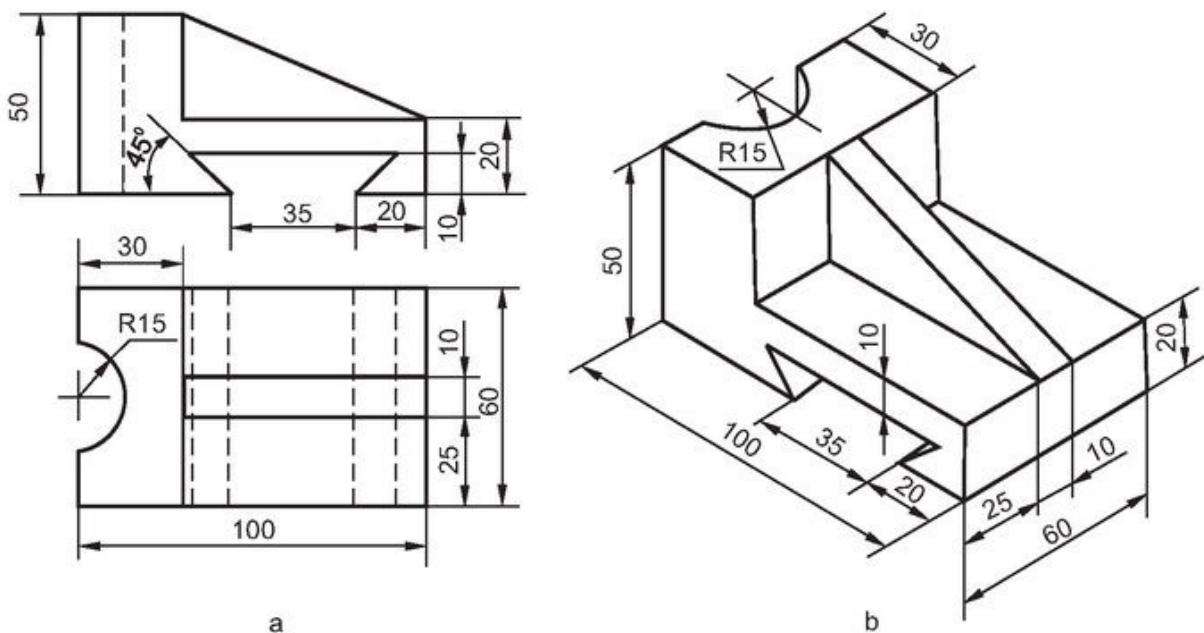
**Fig.16.59**



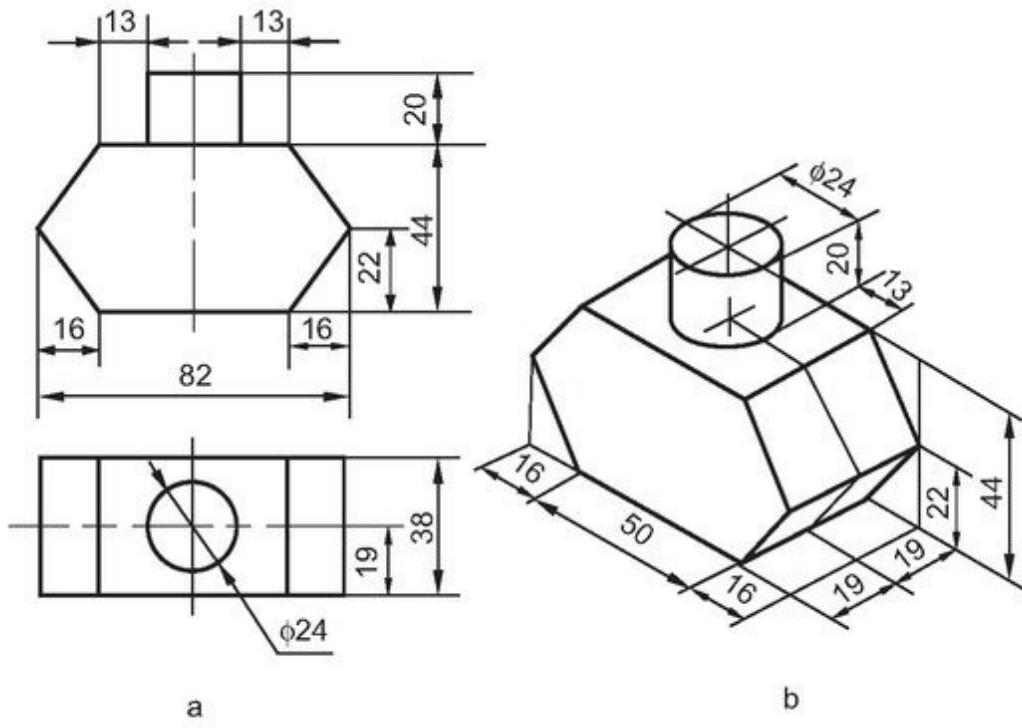
**Fig.16.60**



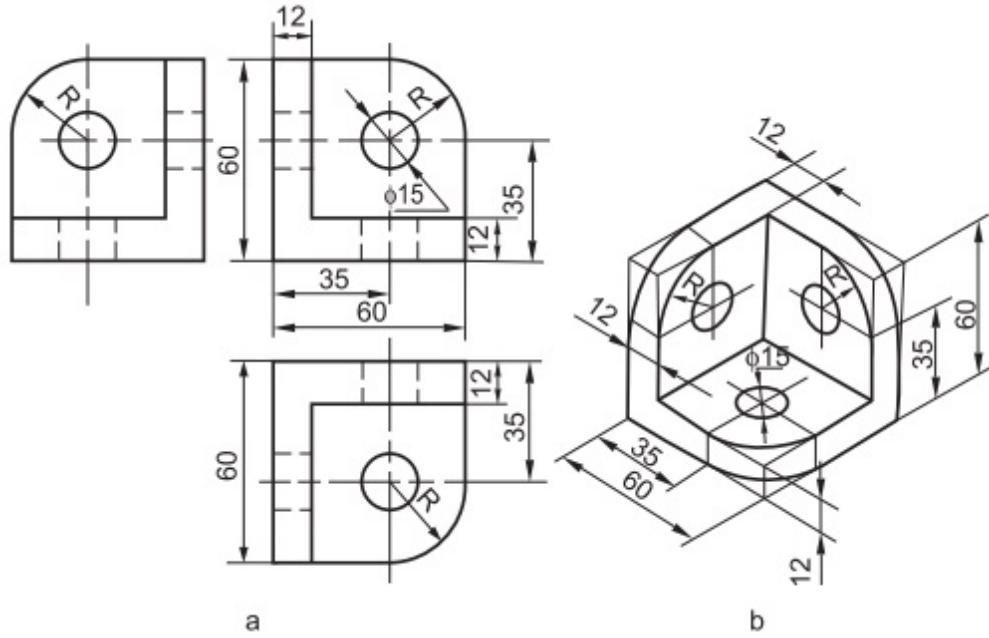
**Fig.16.61**



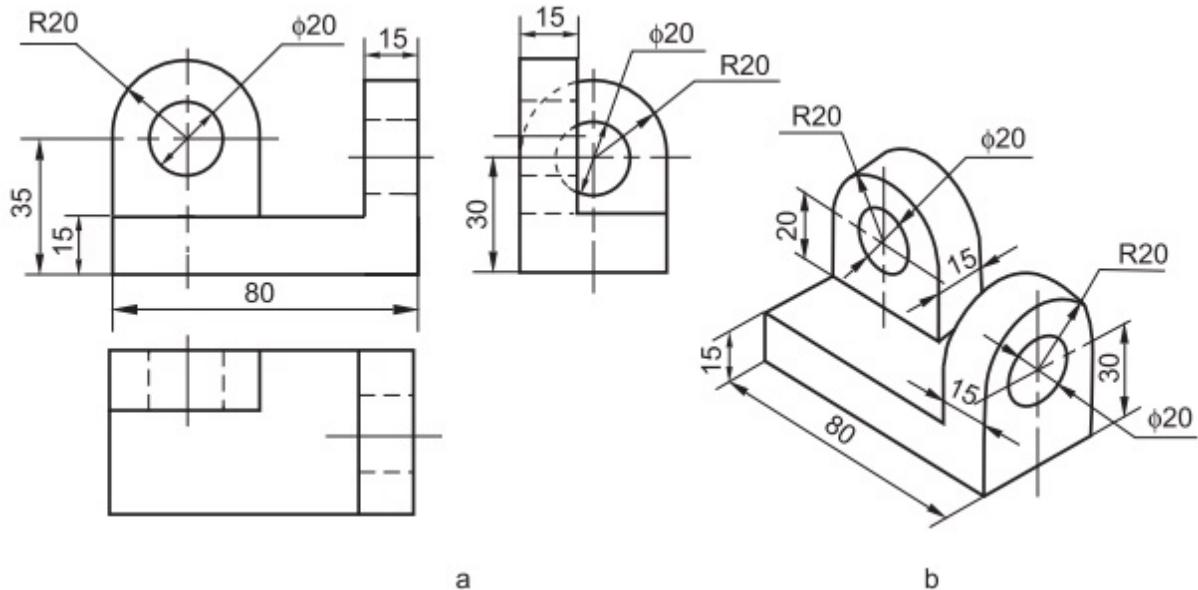
**Fig.16.62**



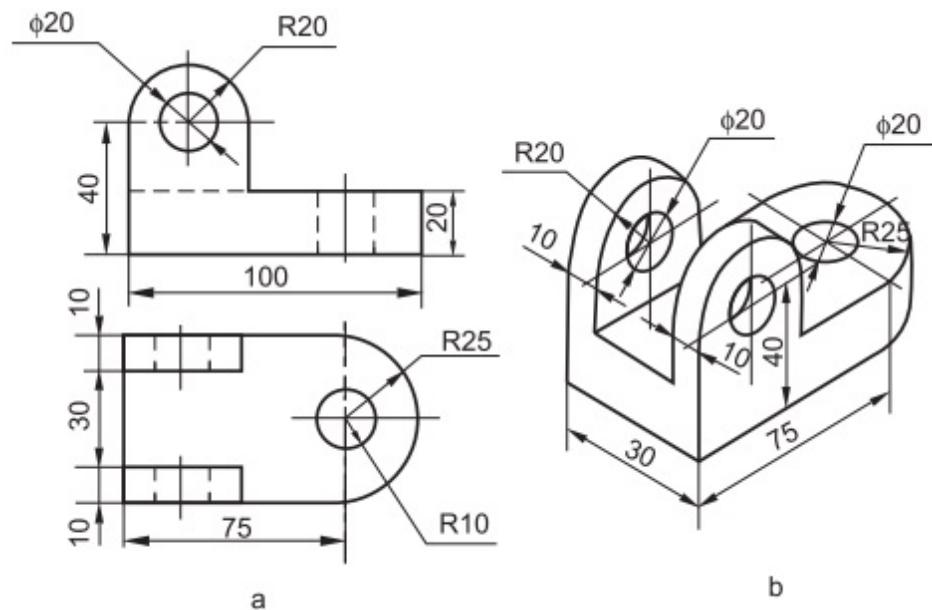
**Fig.16.63**



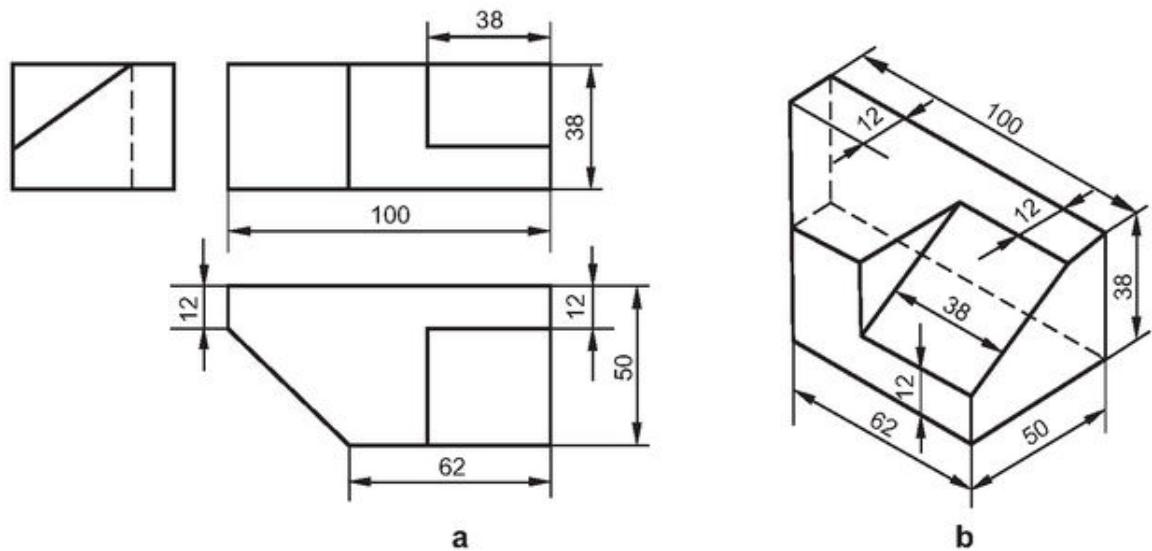
**Fig.16.64**



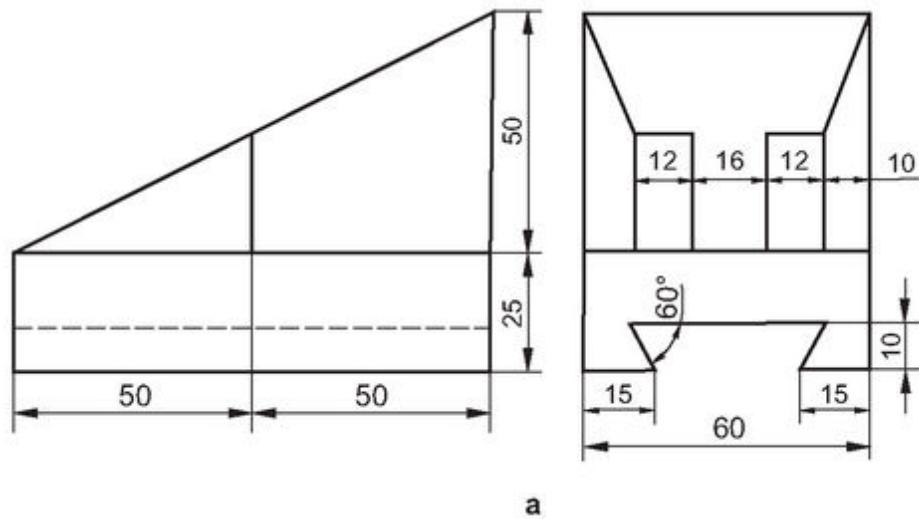
**Fig.16.65**



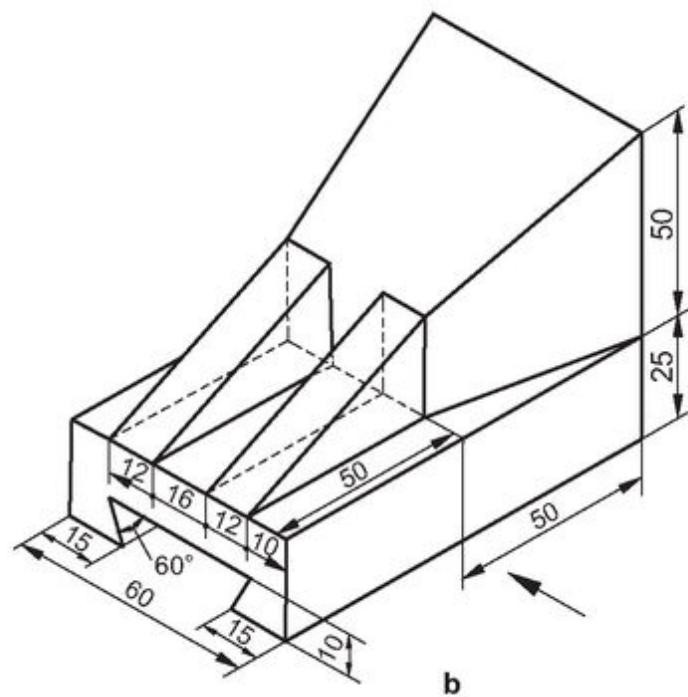
**Fig.16.66**



**Fig.16.67**

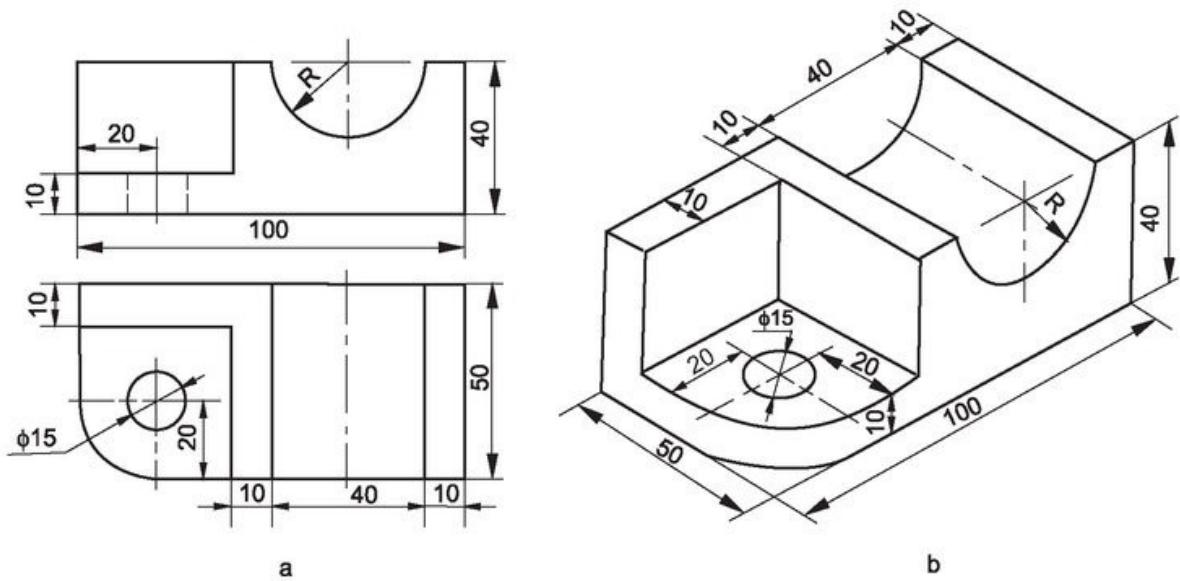


a

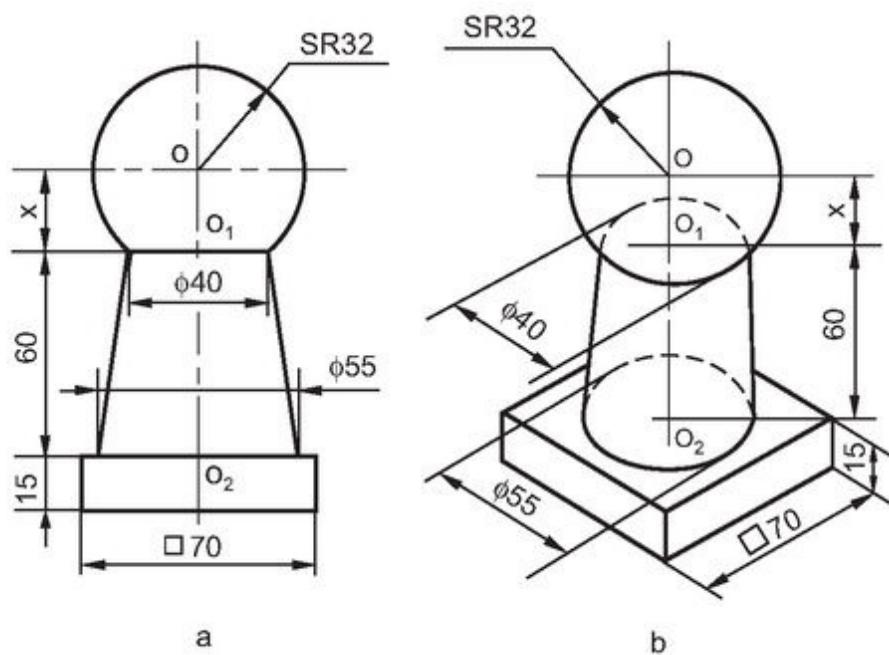


b

**Fig.16.68**



**Fig.16.69**



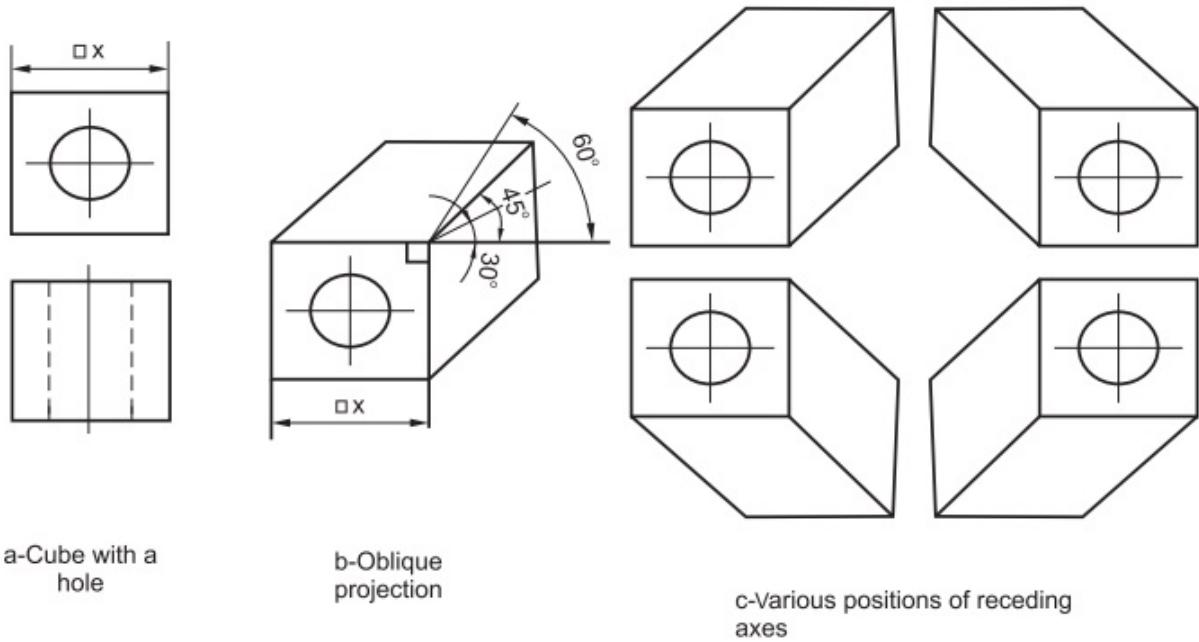
**Fig.16.70**

## 16.5 OBLIQUE PROJECTIONS

## 16.5.1 Introduction

Oblique projection is a form of pictorial projection, similar to the isometric projection, but differing from it in the direction of the projectors. In isometric projection, the projectors are perpendicular to the plane of projection and the principal faces of the object are oblique to it. In oblique projection, the projectors are oblique to the plane of projection but parallel to each other, while one principal face of the object is parallel to the plane of projection.

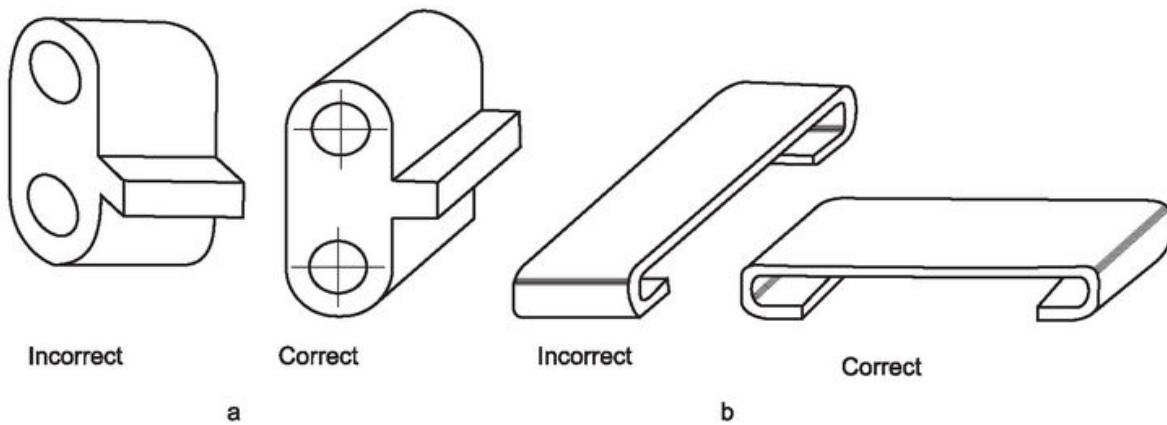
Oblique projection differs from the isometric projection in that, two axes are always perpendicular to each other while the third, known as the receding axis is at some convenient angle, say  $30^\circ$ ,  $45^\circ$  or  $60^\circ$  with the horizontal, as shown in [Fig.16.71b](#). Oblique projection is more flexible and also has advantages, like (i) irregular outlines on the front face appear in their true shape, (ii) distortion can be reduced by foreshortening the receding axis and (iii) choice permits the selection of the axes, to obtain the realistic appearance ([Fig.16.71 c](#)).



**Fig.16.71 Oblique Projections of a cube**

## 16.5.2 Rules for Positioning an Object

The following are the rules recommended while considering the position of the object to obtain the oblique projection of it, for a better realistic appearance:



**Fig.16.72 Orientations of objects in oblique projection**

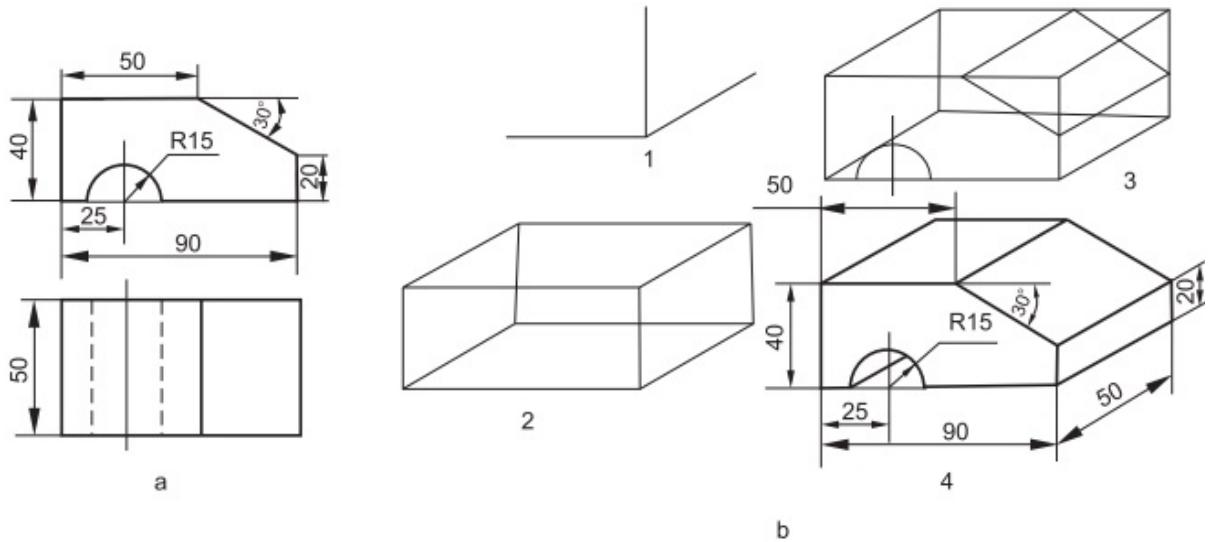
1. Place the most irregular face or face containing more number of curved outlines, parallel to the principal plane, to minimize distortion and simplify the construction.
2. Place the longest face parallel to the plane of projection.

The advantages of following these rules are evident from the [Fig.16.72](#).

### 16.5.3 Method of Drawing Oblique Projection

1. Study the given views carefully and select the face that is either the most irregular one or the one with circular features, if any. Let this face be made parallel to the plane of projection.
2. Draw this face to its true size and shape.
3. Draw the receding lines, through all the visible corners of the front face.
4. Mark the length of the object, along the receding lines and join these in the order.
5. Add other features, if any, on the top and side faces.

[Figure 16.73](#) shows the application of the above steps in developing the oblique projection of an object.

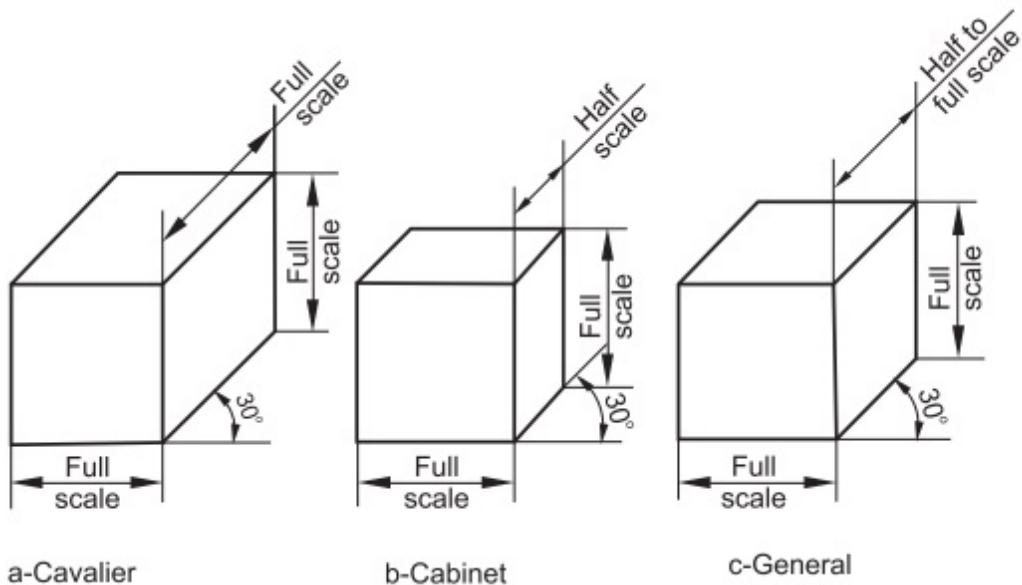


**Fig.16.73 Stages in obtaining an oblique projection**

## 16.5.4 Lengths of the Receding Lines

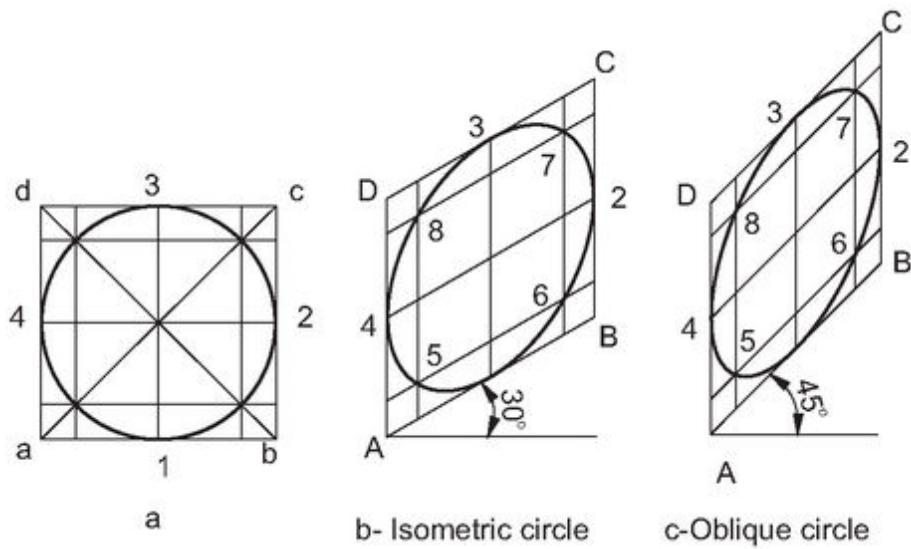
Depending upon the scale of measurement followed along the receding lines, oblique projections are classified as cavalier, cabinet and general.

The fact that the eye is accustomed to seeing the objects with the receding lines appearing to converge, the oblique projection shown in Fig.16.74a presents a distorted appearance. To reduce the amount of distortion, and to have a more realistic appearance, the lengths of the receding lines are reduced as shown, either in Fig.16.74b or in Fig.16.74c. If the receding lines are measured to the true size, the projection is known as cavalier projection. If these are reduced to one half of the true length, the projection is called cabinet projection. In general oblique, the measurements along the receding lines vary from half to full size.

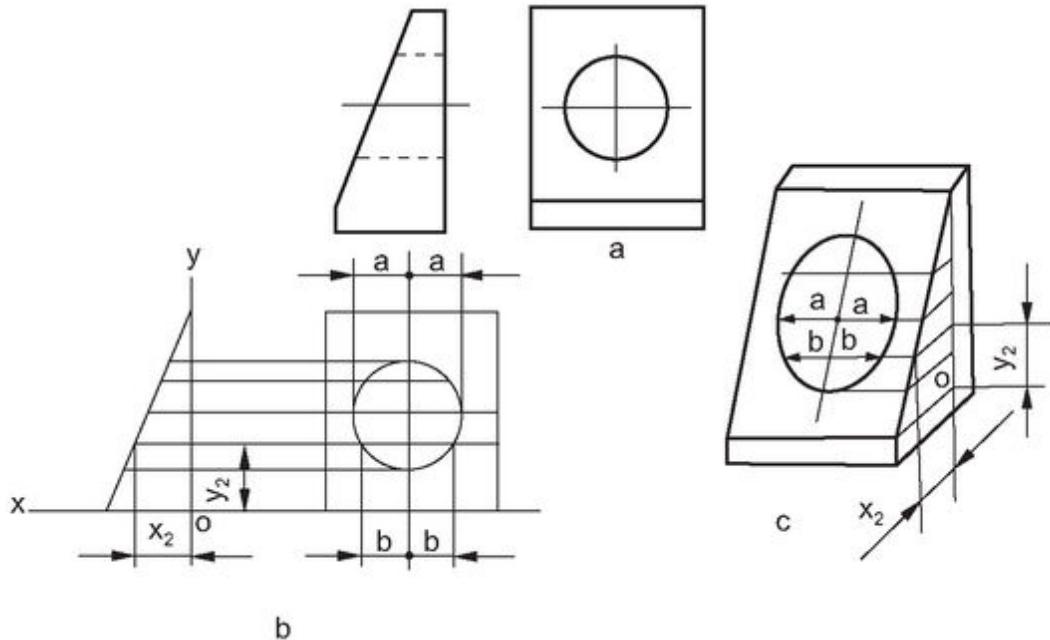


### **Fig.16.74 Types of obliques**

## **16.5.5 Angles, Circles and Curves in Oblique Projection**



**Fig.16.75 Comparison of isometric and oblique circles**



**Fig.16.76 Construction of circular features on inclined surface**

As already mentioned, angles, circles and irregular curves on the surfaces, parallel to the picture plane, appear in true size and shape. However, when these are located on receding faces, the construction methods, similar to isometric drawing may be followed.

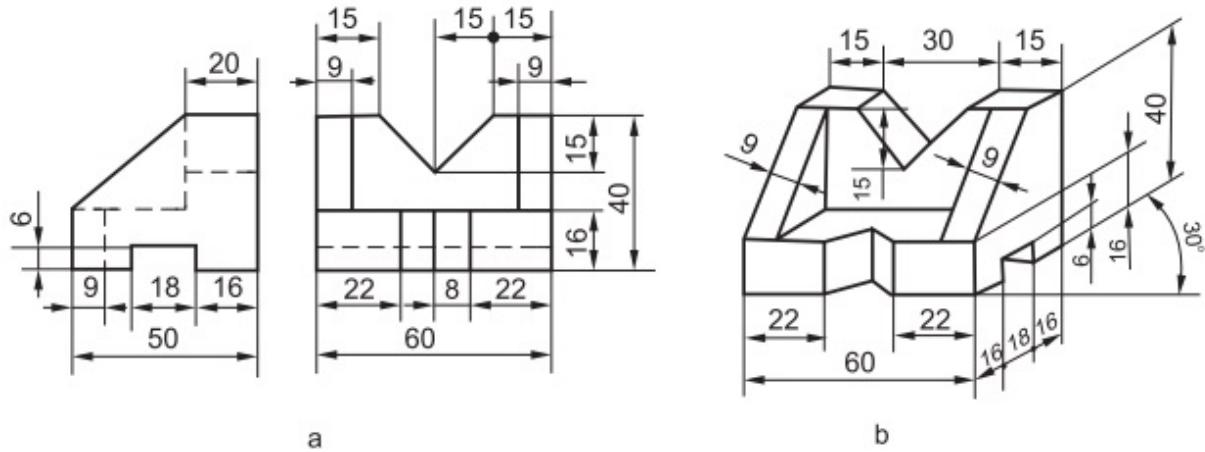
For example, the method of representing a circle on an oblique face may be carried out by the off-set method where as the four-centre method cannot be used. In case of cavalier oblique, the method and the result is the same as that of the isometric drawing, since the angle and the length of the receding axis can be the same as in the isometric projection. [Figure 16.75](#) shows the projections of circles of same size in both isometric and oblique presentation, however, the receding axis of  $45^\circ$  is used in the latter case.

Curved features of all sorts may be plotted by the off-set or co-ordinate method, as shown in Fig.16.76.

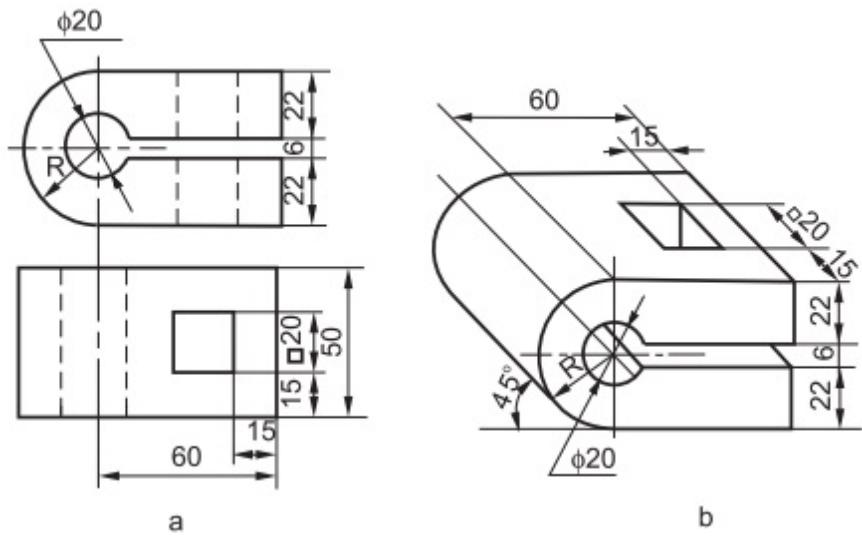
## 16.6 EXAMPLES

The examples that follow, Figure a represents the orthographic projection and Figure b, the corresponding oblique projection. The solutions given in the following examples are self-explanatory.

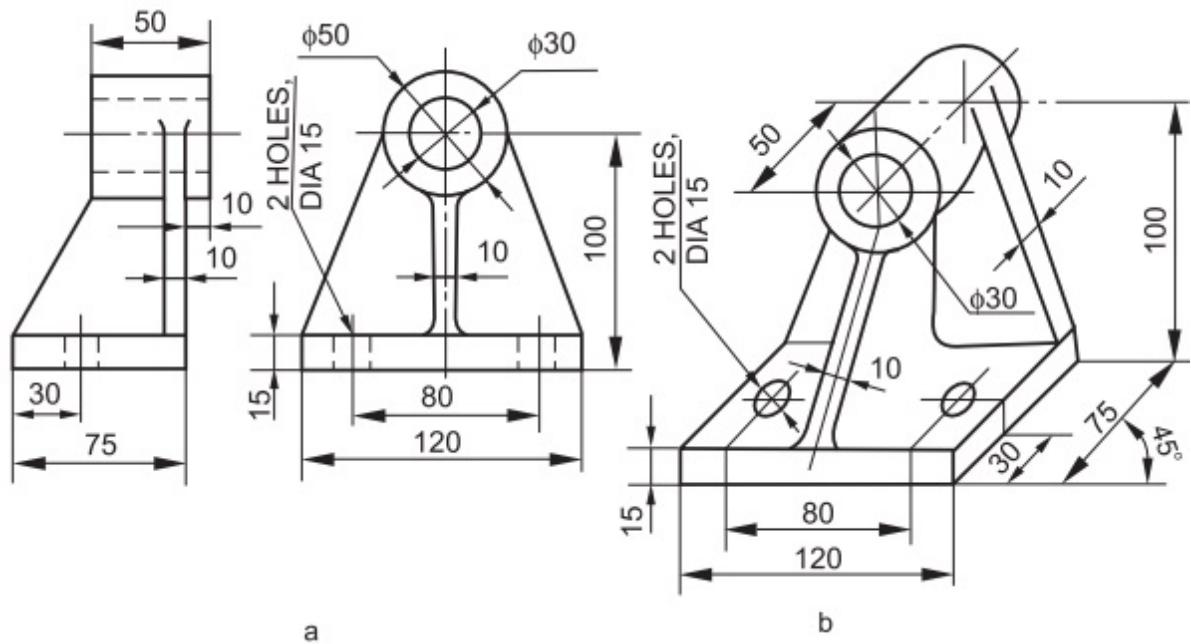
**Problem 54 to 59** Figures 16.77a to 16.82a show the orthographic views of certain objects and Figs.16.77b to 16.82b represent the respective oblique projections.



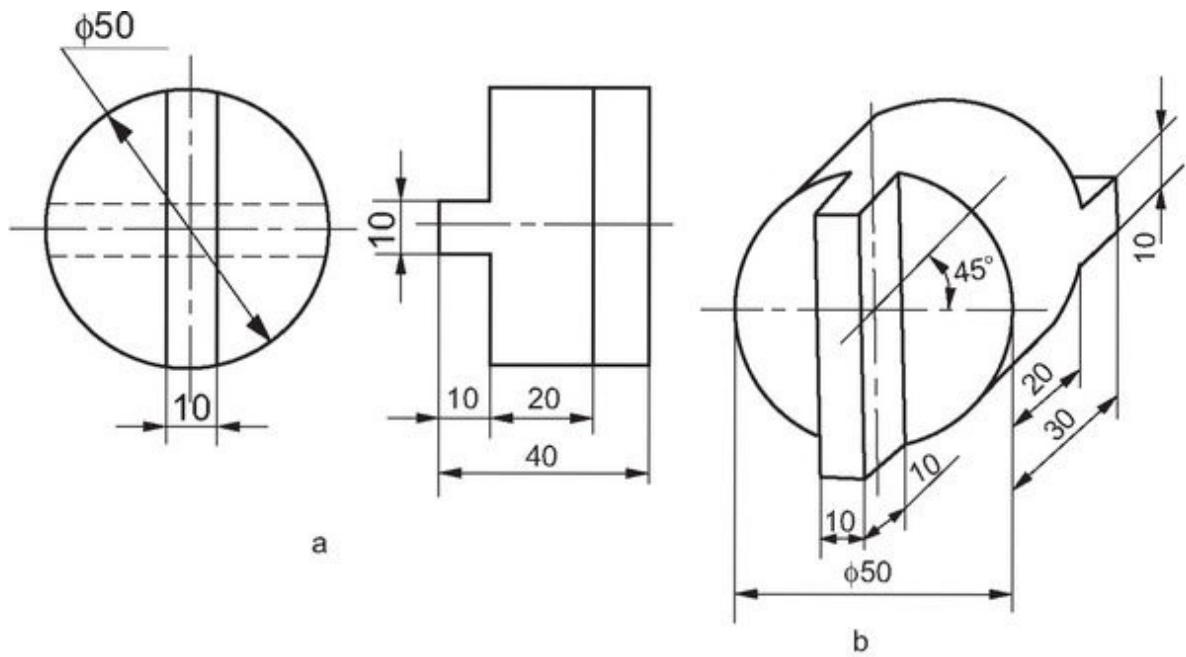
**Fig.16.77**



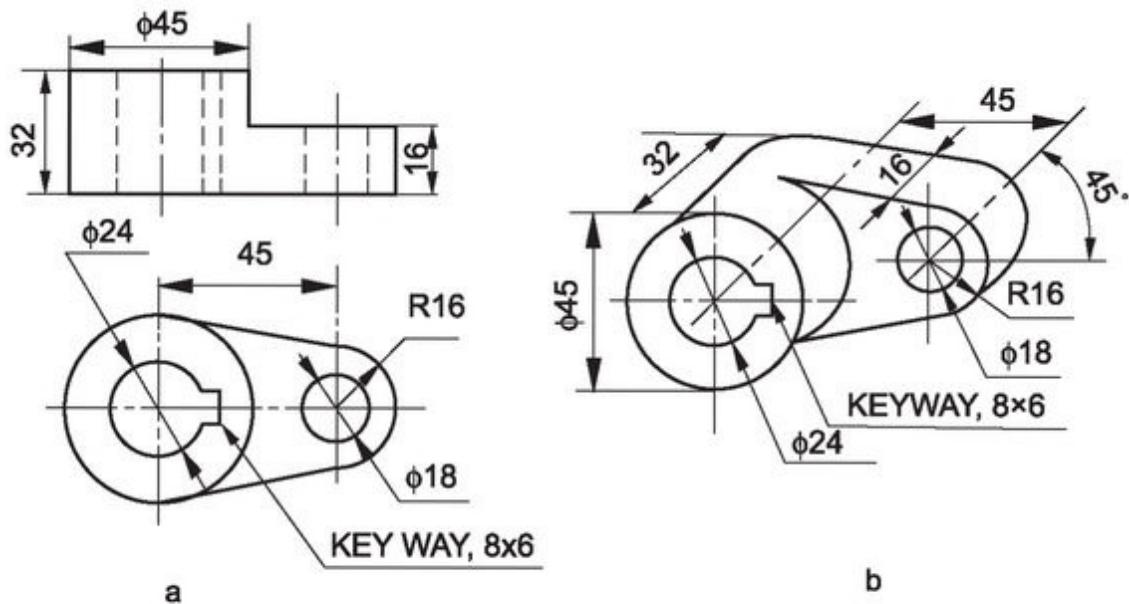
**Fig.16.78**



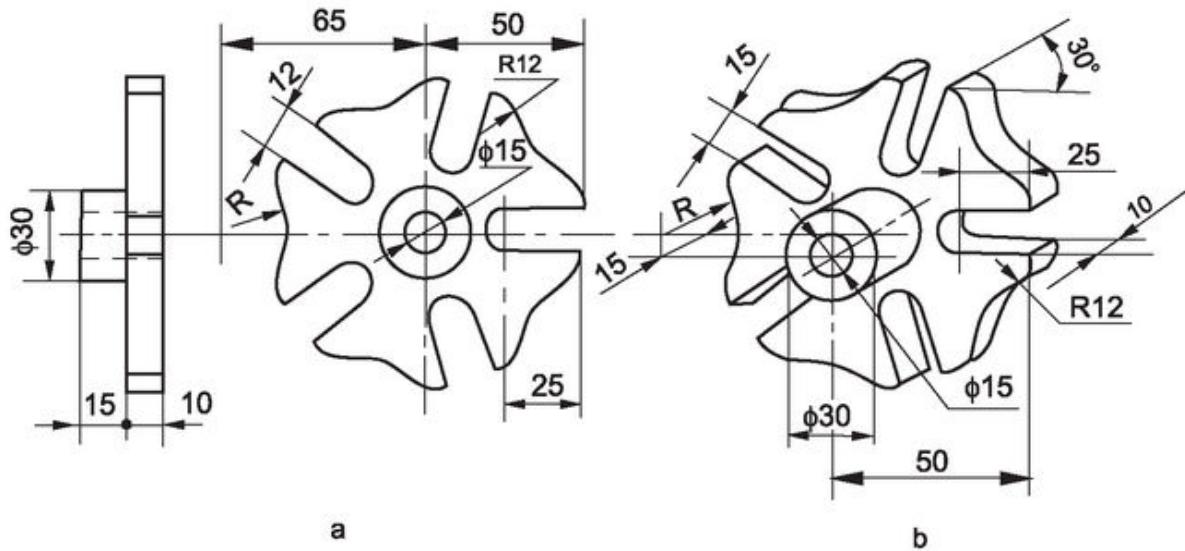
**Fig.16.79**



**Fig.16.80**



**Fig.16.81**

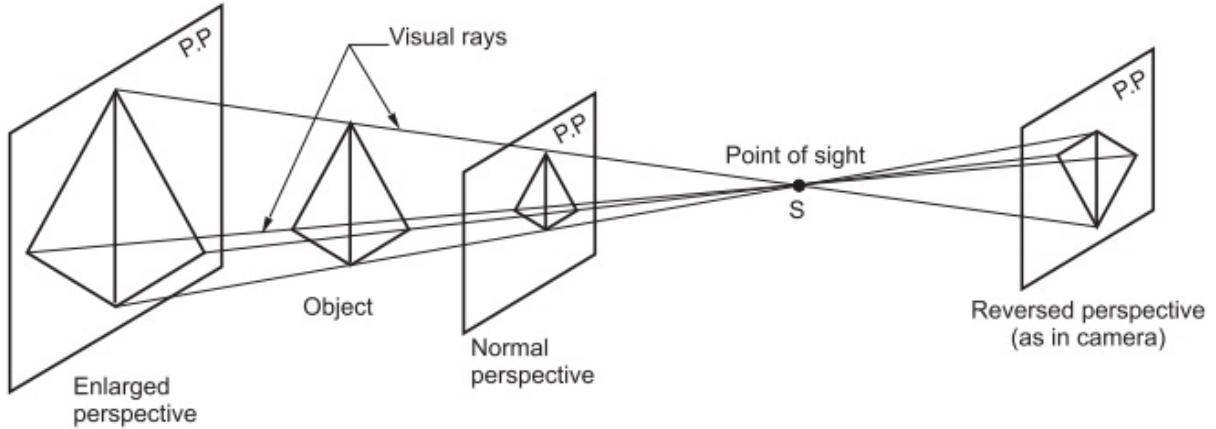


**Fig.16.82**

## 16.7 PERSPECTIVE PROJECTIONS

### 16.7.1 Introduction

Like isometric and oblique projections, perspective projection is another method of pictorial projection. But, it differs from each other in depiction. It is the view of an object on a plane surface, as it appears to an observer, stationed at a particular position, relative to the object. The perspective is different in appearance, from its actual form and it is due to optical illusion. A picture taken by a camera is a real perspective. The four elements involved in the photography; namely the object, light rays, camera lens and film are represented in a perspective by the object, visual rays, point of sight and picture plane (P.P) respectively.



**Fig.16.83 Relation between P.P, object and points of sight**

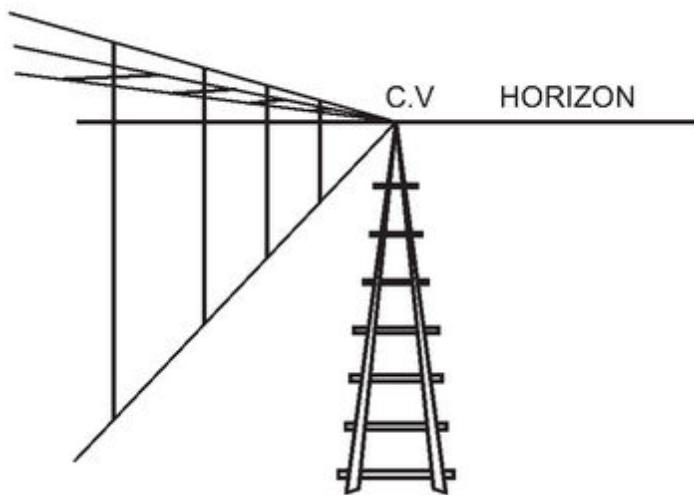
If the position of the object and the point of sight are assumed to be fixed, the following relationship exists:

1. When the object is in-between the plane of projection and the point of sight, the perspective obtained is larger than the object.
2. When P.P. is in-between the object and the point of sight, one gets a normal perspective. When P.P. is farther away from the object, the perspective becomes smaller.
3. If the point of sight is in-between the object and P.P., the perspective is reversed, as in the case of a camera ([Fig.16.83](#)).

Perspective projection/view of an object has considerable advantage over the other types of pictorials, since it represents the object as seen by the observer. A perspective view of a building along with trees, etc., is shown frequently by an architect. It is often shaded and tastefully colored sometimes, to get more realistic appearance. These drawings are used in advertising and commercial art.

## 16.7.2 Perspective Projections

The object is assumed to be located in IIIrd quadrant and hence the top view is above the front view (third angle projection). The objects appear to be different from their actual form, due to illusion. The following examples may be quoted for illusion ([Fig.16. 84](#)):



**Fig.16.84 Perspective projection**

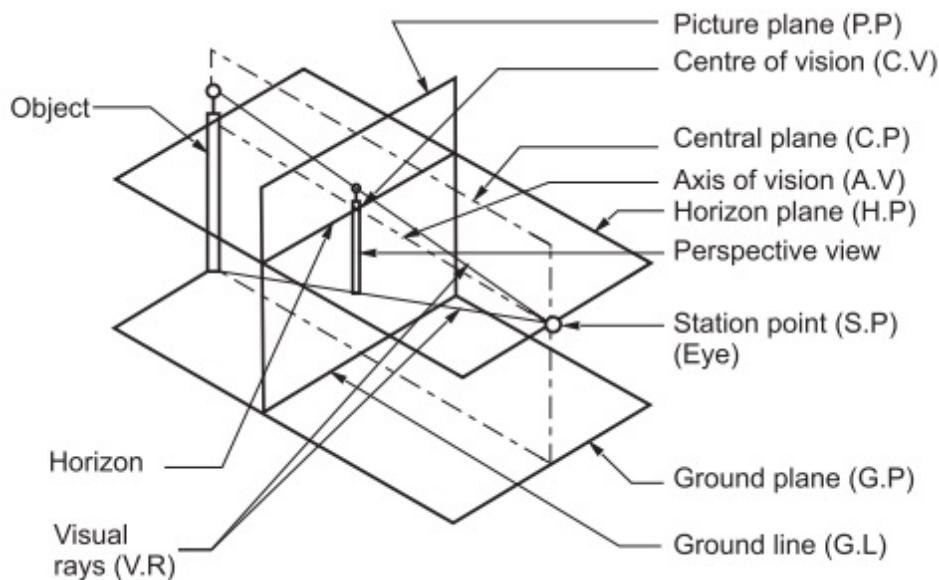
1. The heights of series of lamp posts / telephone poles, farther away from the observer appear to be shorter.
2. The horizontal parallel edges/rail track along the direction of sight of the observer, appear to converge to a point. The perspective view is represented as seen from one point.

The theory of perspective considers the artist's canvas (P.P) as a transparent screen, through which he looks from a fixed vantage point at the scene he is painting. Light rays coming from each point of the scene are imagined to enter his eye and the totality of these lines is called a projection. The point where any line in the projection pierces the

viewing screen is the image of the corresponding point on the scene. If all such piercing points are joined in the order, the perspective of the object is obtained.

### 16.7.2.1 *Elements of a Perspective Projection*

In Fig.16. 85, the observer's eye is shown viewing the object. This is called the station point (S.P). When the station point is selected, the position of the horizon plane is determined. The horizon plane (H.P) passes through the station point and is parallel to the ground plane (G.P). The picture plane (the plane on which the perspective appears) is located in-between the station point and the object. The picture plane is perpendicular to the horizon plane and ground plane. The perspective projection is obtained on the picture plane, by joining the points of intersection of the visual rays with P.P, in the order. The line of intersection between the P.P and H.P is called the horizon.



**Fig.16.85 Nomenclature of perspective projection**

Ground plane is the plane on which the object is assumed to be present. The ground line (G.L) is the line of intersection between the P.P and ground plane. The axis of vision is the line passing through the station point and is perpendicular to the picture plane. Centre of vision (C.V) is the point of intersection between the P.P and axis of vision, and it lies on the horizon. The central plane is a plane passing through the station point and perpendicular to both G.P and P.P as shown in [Fig.16.85](#).

### 16.7.3 Selection of Station Point

Care must be exercised in selecting the location of the station point, since poor choice of the position may distort the perspective and it may be displeasing to the eye. The rules to be followed while selecting the station point are:

1. The position of the station point should be so selected that the centre of vision must be nearer to the centre of interest for the viewer.
2. It is desirable that the station point be at a distance from the picture plane, equal to at least twice the maximum dimension of the object, facilitating the object to be viewed naturally, without turning the head.
3. A wide angle view should be avoided. It has been observed that the best results are obtained when visual rays are kept within a cone of not more than  $30^\circ$  between diametrically opposite elements.
4. The point of sight may also be placed above the object, without violating the rule of distance. In such case, the perspective reveals the features on the top of the object.

In locating an object in relation to the picture plane, it is advisable to place it such that, both the side faces do not make the same angle of inclination with P.P.

## 16.7.4 Positioning the Object

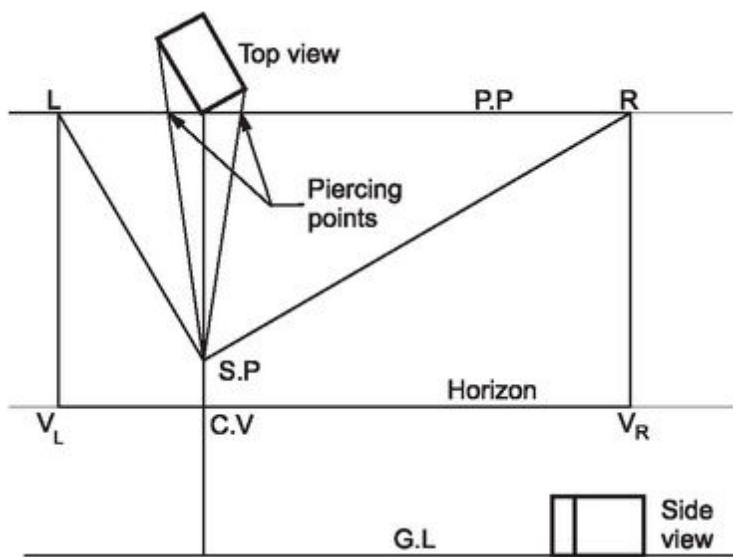
Depending upon the details available on the top or bottom faces of the object, the location of the object with respect to the horizon plane is to be determined. Locating the object above the horizon reveals the details of the bottom face of the object (ground view). Positioning the object below the horizon reveals the details of the top face (aerial view); whereas, both the bases are not seen in the perspective, if the object is located on the horizon (general view).

The size of the perspective or the magnitude of a line of the object decreases as the distance of it from the picture plane increases. This continues until the view becomes a point at the horizon. The following facts are worth remembering about the lines:

1. A set of perpendicular horizontal lines vanish at a single vanishing point (C.V).
2. A system of horizontal lines has one V.P on the horizon.
3. Vertical lines appear vertical in the perspective.
4. A line in the picture plane will have its true length in its perspective.

## 16.7.5 Vanishing Points and its Location

The real or imaginary parallel lines appear to come together at the vanishing point. This point is located on the horizon (front view). In the rail track, the rails appear to come closer at C.V, as shown in Fig.16.84. This is called the perspective effect. Referring to Fig. 16.86, the following steps may be used to obtain the vanishing point(s):



**Fig.16.86 Location of vanishing points**

1. Draw two lines parallel to the sides of the top view of the object, from the station point, meeting P.P at R and L (top view).
2. Project the points R and L to the horizon line and mark  $V_R$  (Vanishing right point) and  $V_L$  (Vanishing left point-front view).

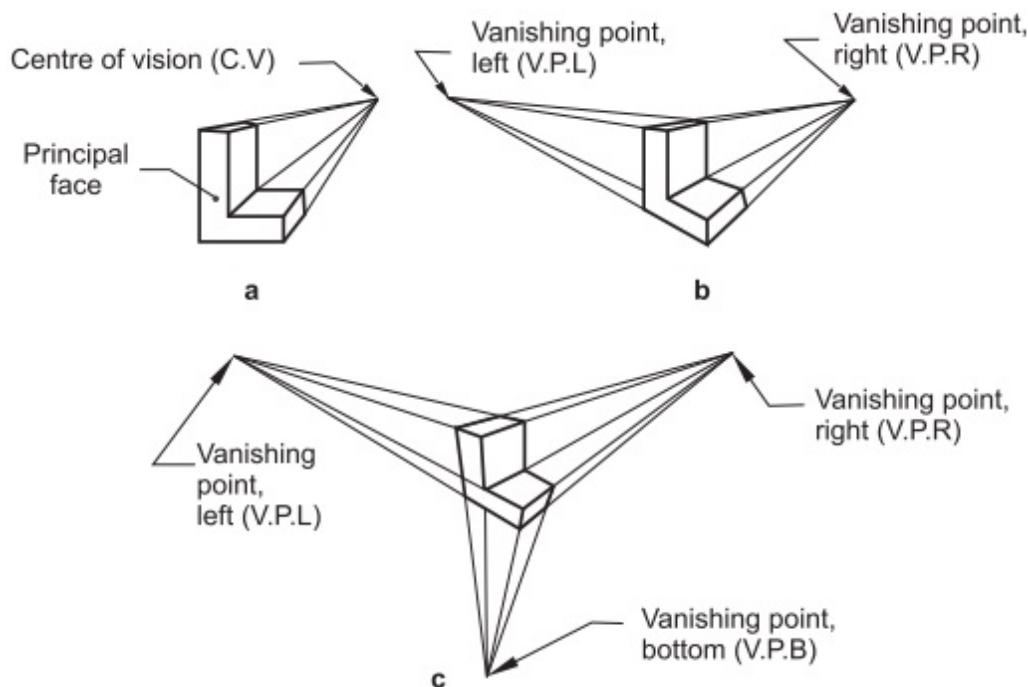
All the parallel edges of the object inclined to the right of P.P, have their vanishing point  $V_R$  on the horizon and the edges inclined to the left, have the vanishing point  $V_L$ . The vanishing point is that point at which all the parallel edges appear to coincide on the horizon.

[Figure 16.86](#) represents the projections of the perspective elements. The front and top views of the object are shown in the figure (third angle projection is always used). The picture plane is seen in its edge view (PP similar to xy in orthographic projections). The object is above P.P, while the station point (S.P) is below it as the object and station point are on either side of the P.P. The ground and horizon planes are seen in their edge views (horizon and G.L) in the front view. The centre of vision (C.V) is on the horizon and is just below S.P. Any convenient distance between S.P and C.V may be chosen without affecting the perspective.

## 16.7.6 Classification of Perspectives

Perspective projections of the objects such as lines, planes and solids may be broadly classified into three categories:

1. Single point (parallel) perspective
2. Two point (angular) perspective
3. Three point (oblique) perspective



**Fig.16.87 Classification of perspective projections**

The perspective projections are classified, based on their relative positions of the object with respect to the picture plane. When a solid is situated with one of its vertical surfaces parallel to P.P, and for all the horizontal edges perpendicular to P.P, there exists a single vanishing point (C.V.). The perspective drawn using single vanishing point is known as single point perspective. When a vertical surface of the object is inclined to P.P, the two sets of horizontal edges have two vanishing points and the perspective obtained with two vanishing points is called two-point perspective. When all the three surfaces of the object are inclined to P.P, the preparation of its perspective needs three vanishing points and is known as three-point perspective or oblique perspective ([Fig.16.87](#)).

## 16.7.7 Methods of Drawing Perspective Views

Perspective views of an object may be drawn by any one of the following methods:

### I Visual ray method

The perspectives developed using this method, are known as single point perspectives. Two views of the object (top and side/front views) are required for developing a perspective view. The top and front views of the perspective set-up (picture plane, station point, horizon and ground line along with the two views of the object) are represented in third angle method of projection.

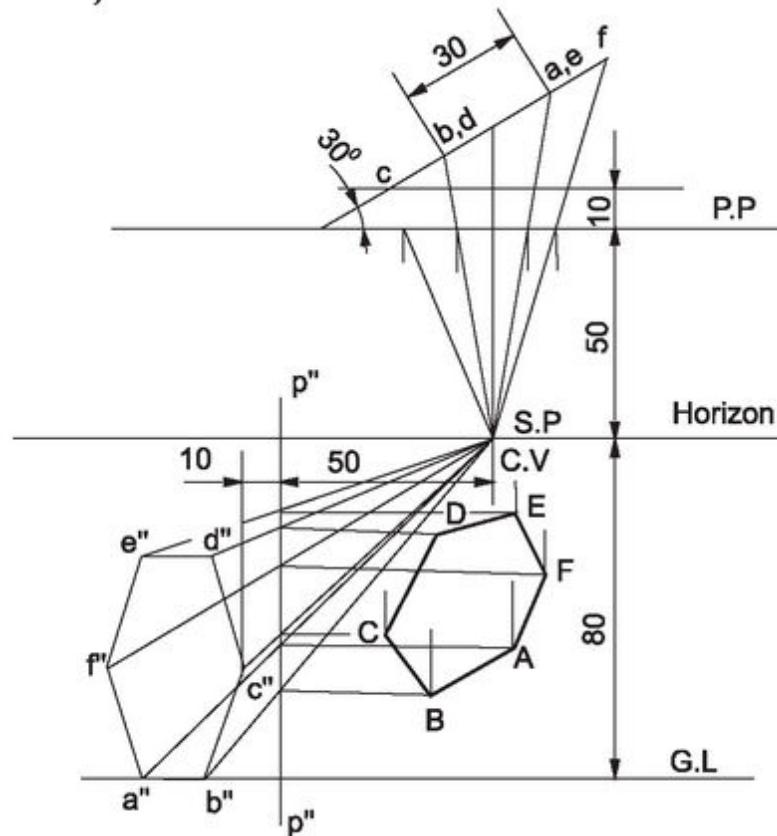
*Procedure to be followed when top and side views are used:*

1. Locate the perspective elements (P.P, horizon, G.L and S.P) and draw the top view of the object.
2. Join all the points in the top view of the object to S.P representing the visual rays and locate the piercing points of the visual rays with the P.P (x co-ordinates of the various points).
3. Draw side view of the object, P.P and S.P (C.V).
4. Join all the points of the side view of the object to C.V and locate the piercing points with  $p''-p''$  (y co-ordinates of various points).
5. Using x and y co-ordinates, locate the points on P.P (front view) and obtain the perspective projection.

**Problem 60** A hexagonal plane of side 30 is resting on an edge on the ground with its surface inclined at  $30^\circ$  to P.P. The nearest corner of the plane is 10 away from P.P. The station point is 50 in front of P.P and 80 above G.L and in the central plane of the object. Draw its perspective.

**Construction (Fig.16.88)**

1. Locate the perspective elements P.P, horizon and ground line (G.L).
2. Draw the top view of the hexagonal plane, satisfying the given conditions and locate the station point (S.P) and C.V in the central plane, passing through the centre of the plane.
3. Draw the side view of the picture plane ( $p''-p''$ ) at 50 from C.V and locate the side view of the hexagonal plane.
4. Join the corners of the top view of the object to S.P and locate the piercing points with P.P.
5. Join the corners of the side view of the object to C.V and locate the piercing points with  $p''-p''$ .
6. Draw the projectors from the piercing points of picture plane and complete the perspective of the object.

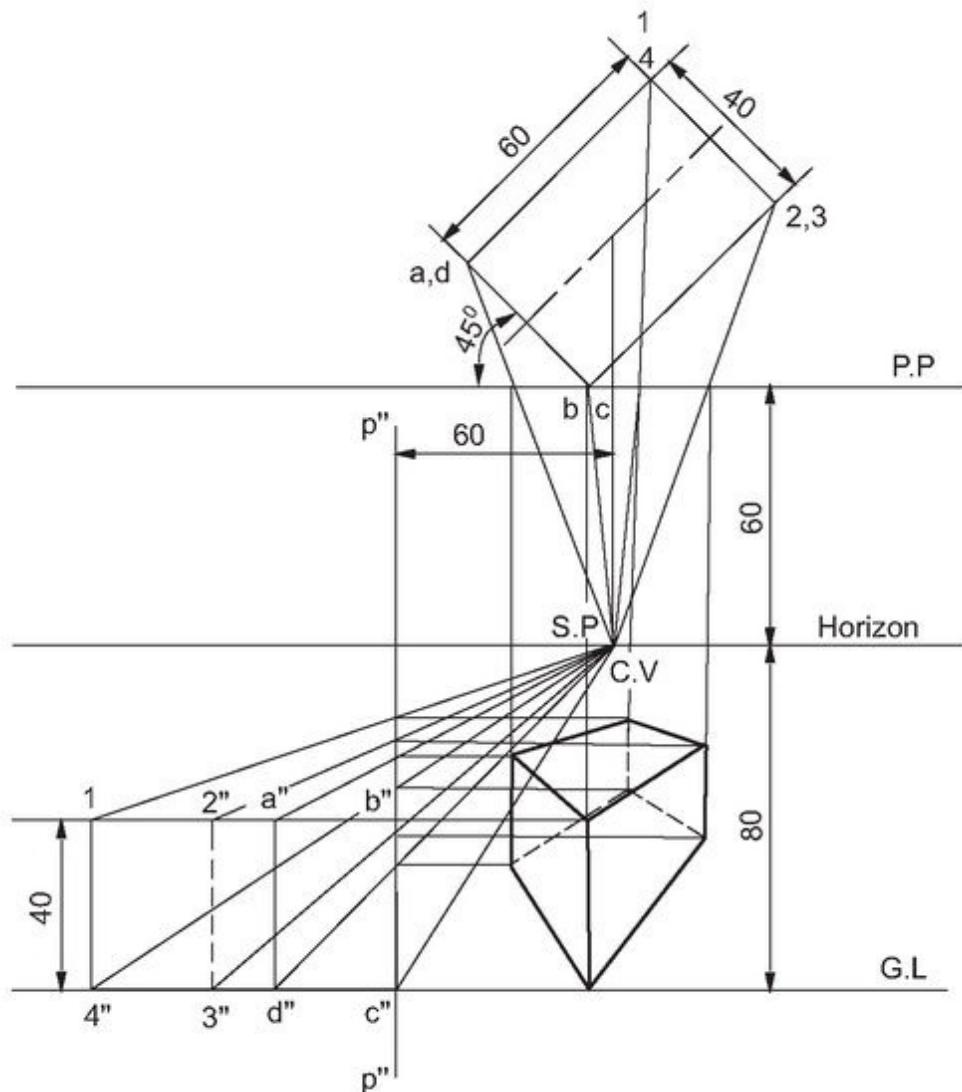


### **Fig.16.88**

**Problem 61** Draw the perspective of a square prism of edge of base 40 and of length 60, lying on a rectangular face on the ground, with a corner on P.P and the bases are inclined at  $45^\circ$  to P.P. The station point is 60 in front of P.P and 80 above G.L and lies in a central plane, which is passing through the centre of the prism.

#### **Construction (Fig.16.89)**

1. Draw the top view of the square prism and locate the perspective elements P.P, station points (S.P, C.V), horizon and G.L.
2. Draw the side view of the picture plane  $p''-p''$  at 60 from C.V and also draw the side view of the prism.
3. Join S.P to the corners of the solid in top view and locate the piercing points with P.P.
4. Join C.V to the corners of the solid in the side view and locate the piercing points with  $p''-p''$ .
5. Project the piercing points on P.P and  $p''-p''$  and complete the perspective view of the solid, following the rules of visibility.



**Fig.16.89**

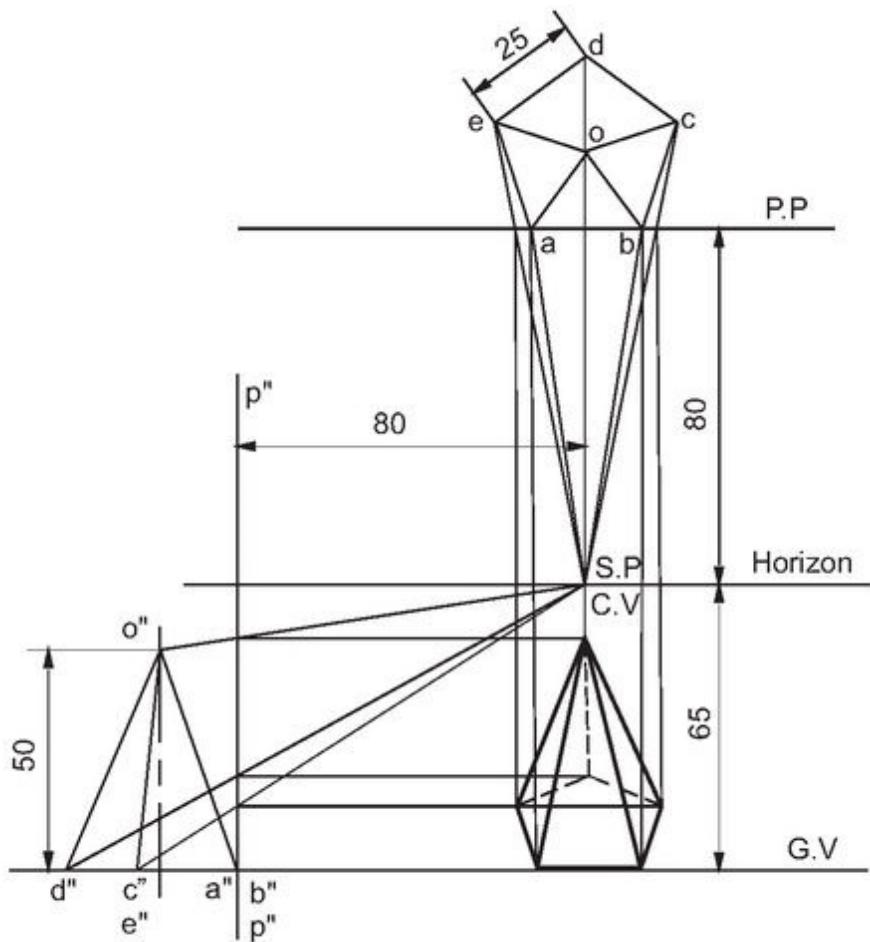


Perspective is the image of the solid formed on the P.P, as the observer views it through P.P.

**Problem 62** A pentagonal pyramid of side of base 25 and height 50, rests with an edge of the base, touching the P.P. The station point is on the central line passing through the apex and 80 from P.P and 65 above the ground. Draw the perspective view of the solid.

### ***Construction (Fig.16.90)***

1. Draw the top view of the pentagonal pyramid and locate the perspective elements, satisfying the given conditions.
2. Locate p"-p" (side view of P.P) at 80 from C.V and draw the side view of the solid.
3. Join the corners of the base and apex (top view) to S.P and locate the piercing points with P.P.
4. Join the corners of the solid and its apex (side view) to C.V and locate the piercing points with p"-p".
5. Project the corresponding piercing points and complete the perspective view of the solid, following the rules of visibility.



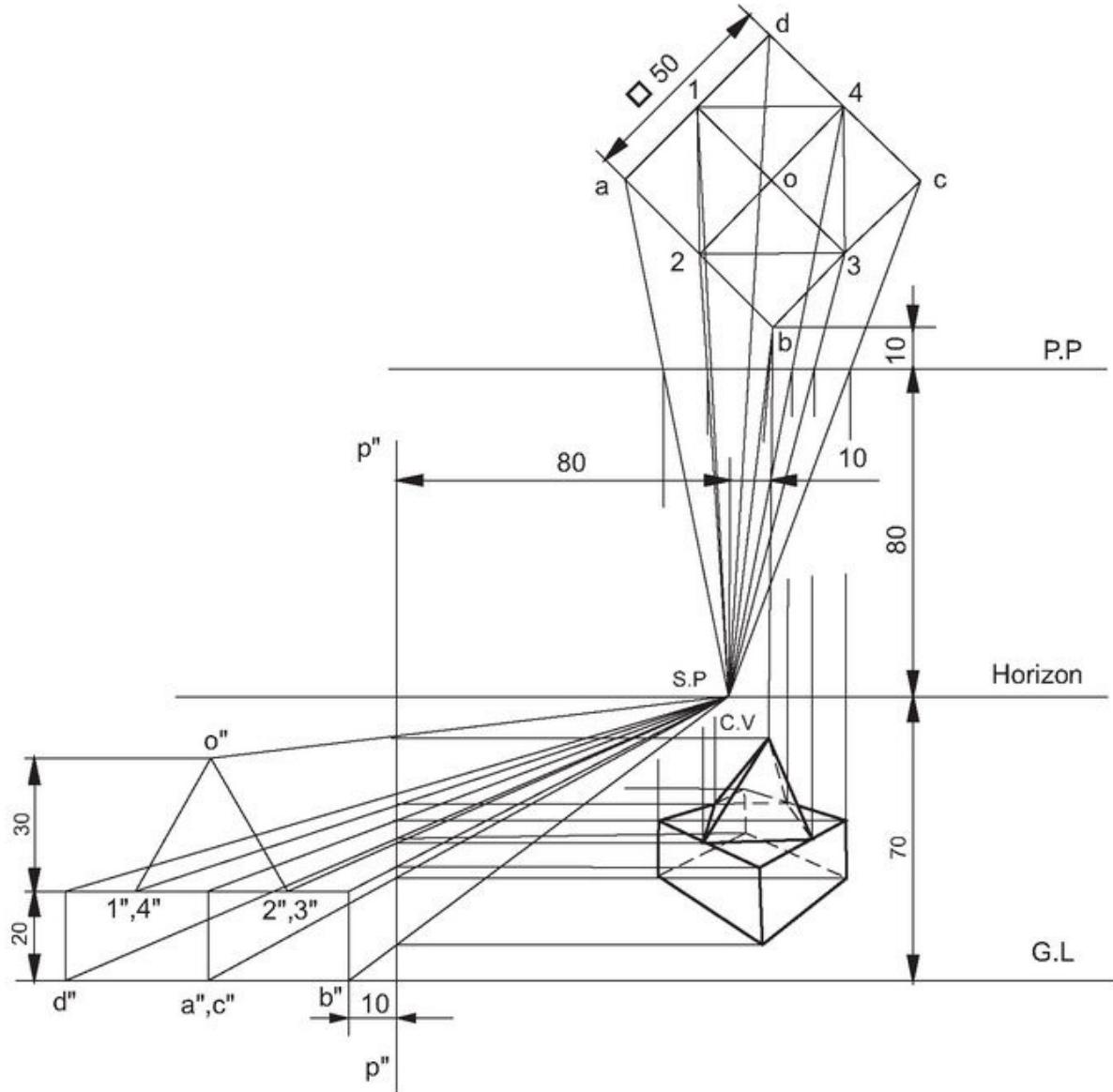
**Fig.16.90**

**Problem 63** A square prism of side of base 50 and height 20, supports a square pyramid of 30 height, with the corners of the base located at the mid-points of the edges of the prism. A vertical edge of the prism is 10 away from P.P and the vertical surfaces are equally inclined to P.P. The station point is 70 above the ground and 80 in front of P.P. It is 10 to the left of the apex of the solid. Draw the perspective of the combination of the solids.

**Construction (Fig.16.91)**

1. Draw the top view of the combination of the solids and locate the perspective elements.

2. Locate the side view of P.P and draw the side view of the combination of the solid, satisfying the given conditions.
3. Join all the corners of the combination of the solids with S.P and locate the piercing points with P.P.
4. Join all the corners of the solids (side view) with C.V and locate the corresponding piercing points with  $p''-p'$ .
5. Project the corresponding piercing points (with P. P and  $p''-p''$ ) and complete the perspective view, following the rules of visibility.



**Fig.16.91**

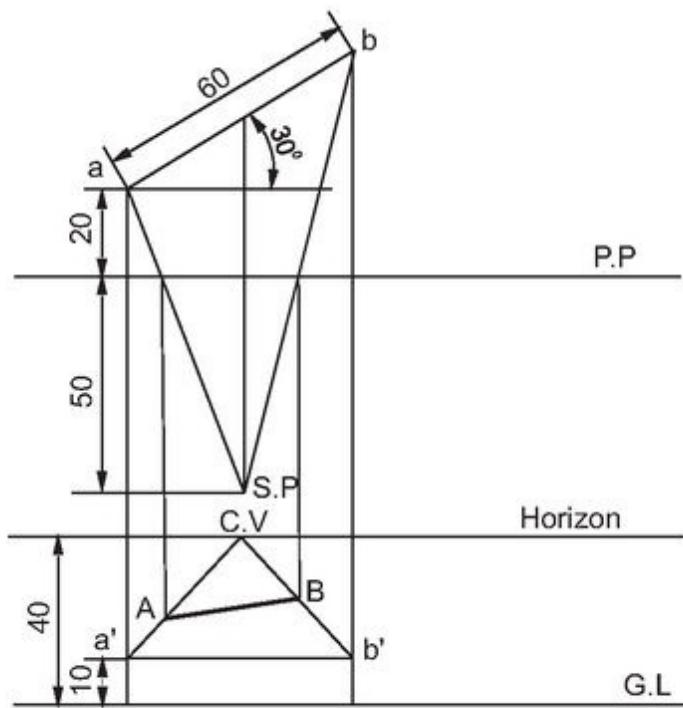
*Procedure to be used when top and front views are used:*

1. Locate the perspective elements (P.P, horizon, G.L and S.P) and draw the top view of the object.
2. Join all the points in the top view of the object to S.P, representing the visual rays and locate the piercing points of the visual rays with the P.P.

3. Draw the front view of the object and join all the points to C.V representing the visual rays.
4. Draw the projectors from the piercing points (on P.P) to the above respective visual rays.
5. Join all these intersecting points and obtain the perspective.

**Problem 64** A straight line AB of 60 long, is parallel to and 10 above G.P and inclined at  $30^\circ$  to P.P and one end A is 20 from P.P. The station point is 50 in front of P.P, 40 above G.L and on the central plane passing through the mid-point of the line. Draw the perspective of the line.

**Construction (Fig.16.92)**



**Fig.16.92**

1. Draw the top view of the line ab and locate the perspective elements P.P, station point (S.P), horizon,

centre of vision (C.V), ground line (G.L) and the front view of the line  $a'b'$ , satisfying the given conditions.

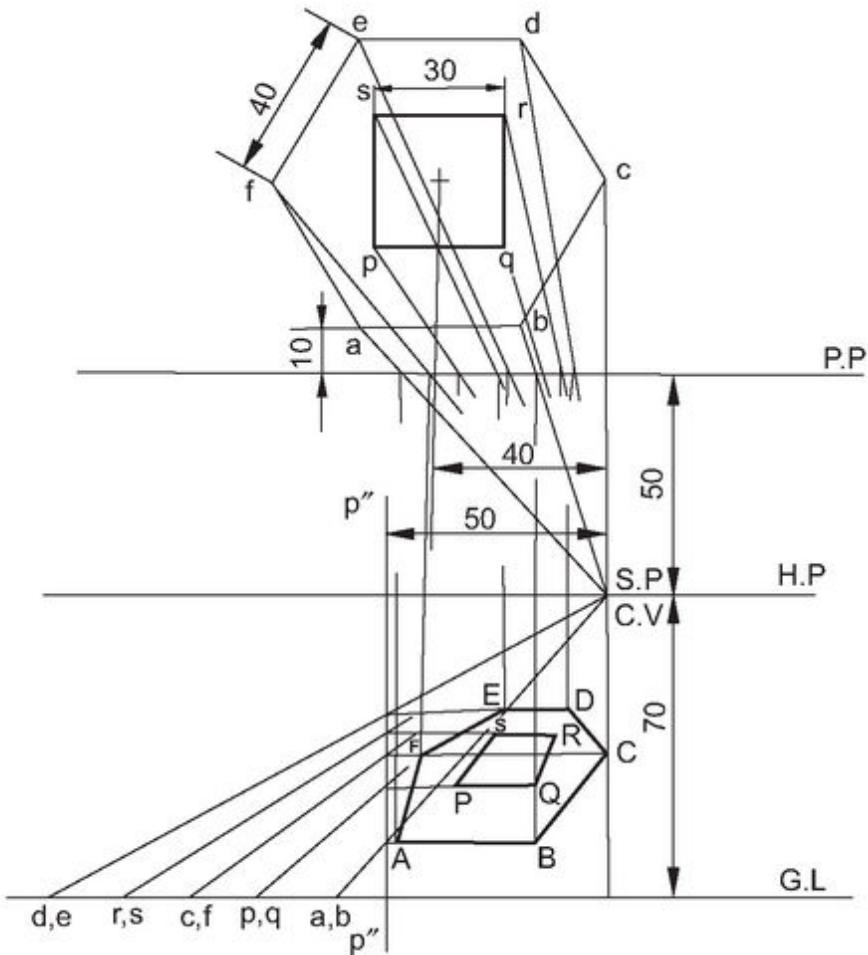
2. Join  $a', b'$  to C.V.
3. Join  $a, b$  to S.P and locate the points of intersection with P.P.
4. Project these intersection points to  $a'-C.V$  and  $b'-C.V$  and mark A and B.

AB represents the perspective view of the given line.

**Problem 65** A hexagonal plane with 40 side, has a centrally cut square hole with 30 side such that, a side of the hole and a side of the hexagon are parallel to P.P. It lies on the G.P with the nearer edge of the hexagon 10 behind the P.P. The station point is 50 in front of P.P, 70 above G.P and lies in a central plane which is at a distance of 40 towards right of the centre of the object. Draw the perspective view.

### **Construction (Fig.16.93)**

1. Draw the top view of the hexagonal plane and locate the perspective elements satisfying the given conditions.
2. Locate  $p'' - p''$  (side view of P.P) and draw the side view of the plane.
3. Join the corners of the plane in the top view to S.P and the corners in the side view to C.V and locate the piercing points with P.P and  $p'' - p''$  respectively.
4. Project the corresponding points and complete the perspective view of the plane.



**Fig.16.93**

**Problem 66** A composite plate is made of a rectangle with 60 and 40 sides and a semi-circle on its longer edge. Draw its perspective view when it is lying on G.P. The longer edge is perpendicular to P.P and shorter edge is 10 behind it. The station point is 50 in front of P.P, 60 above G.P and lies in the central plane which is 50 to the right of the centre of the semi-circle.

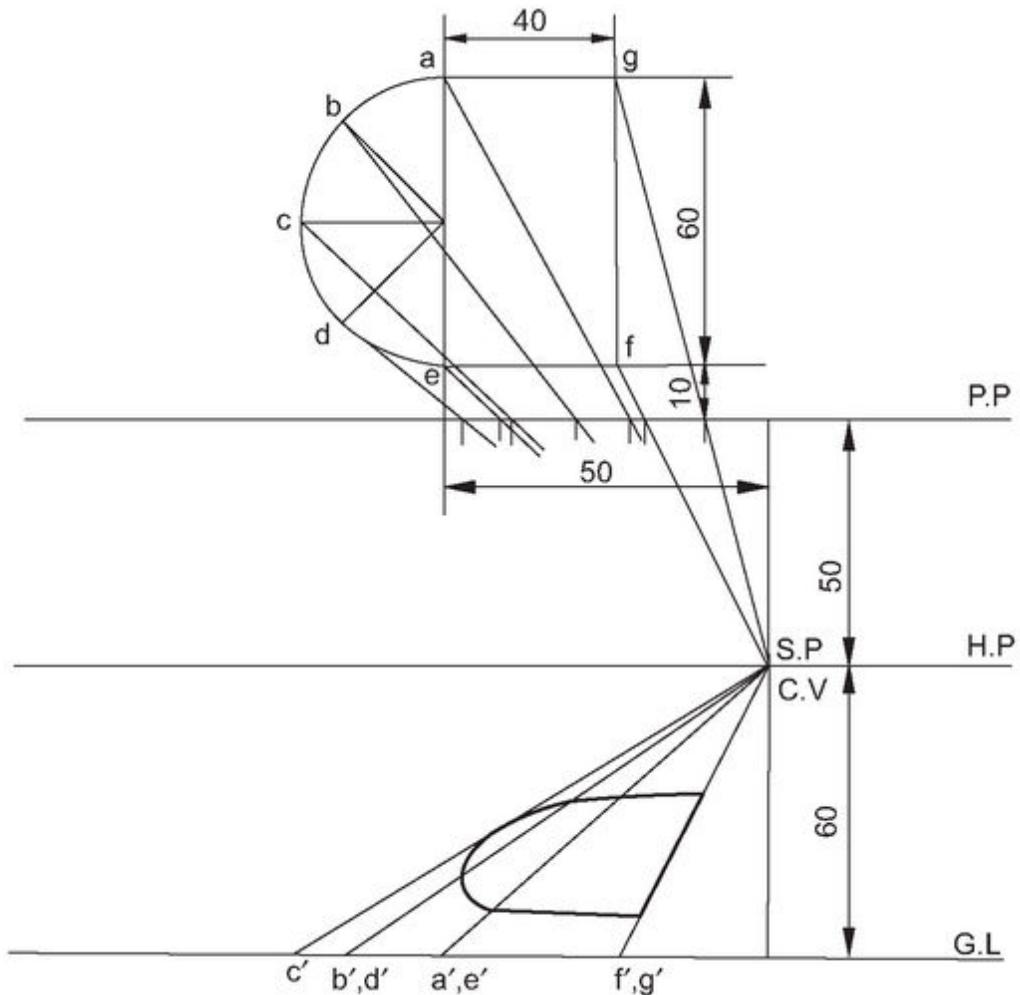
**Construction (Fig.16.94)**

1. Draw the top view of the rectangular plane and locate the perspective elements, satisfying the given conditions. Also locate a number of points on the semi-

circle to facilitate the location of these points in the perspective.

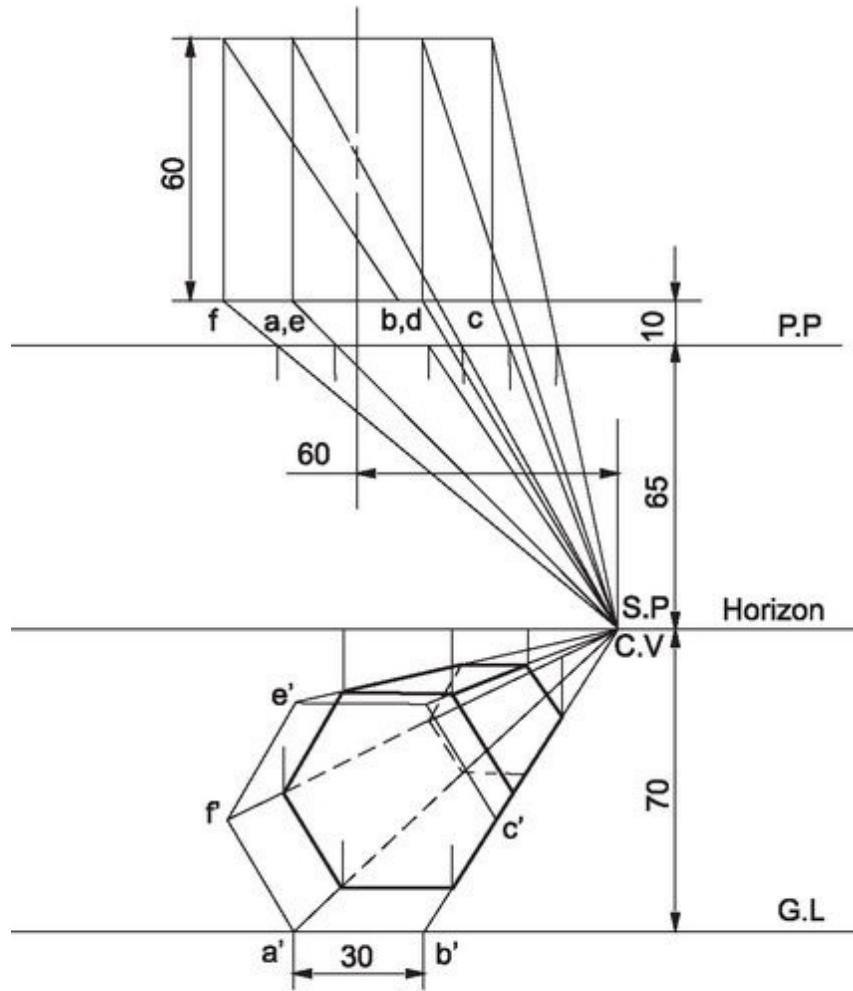
2. Draw the front view of the plate and locate the corners and other points.
3. Join the points on the plane in the top view to S.P and locate the piercing points with P.P and join the points in the front view to C.V.
4. Project and obtain the perspective view as shown.

**Problem 67** A hexagonal prism of side of base 30 and length of axis 60, rests with one of its rectangular faces on the ground such that, the axis is perpendicular to P.P. The nearest base is 10 behind P.P. The station point is 60 to the right of the centre of the prism, 65 in front of P.P and 70 above the ground. Draw the perspective view of the solid.



**Fig.16.94**

**Construction (Fig.16.95)**



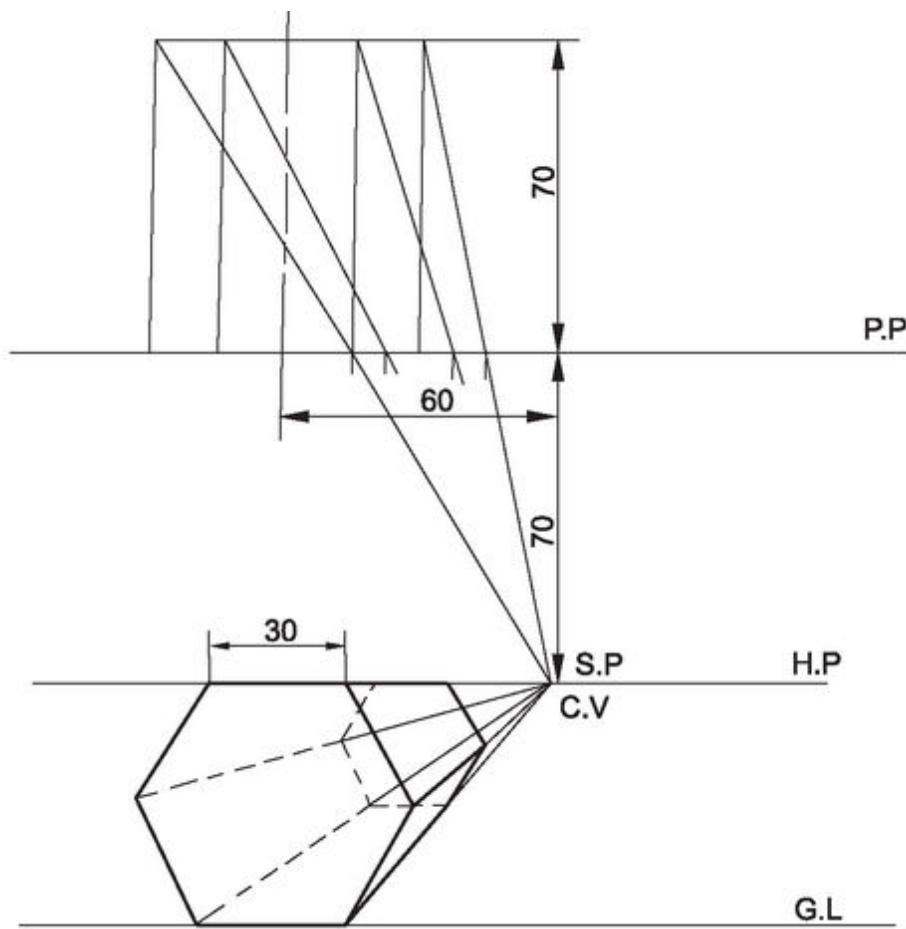
**Fig.16.95**

1. Draw the top view of the hexagonal prism and locate the perspective elements P.P, horizon, S.P, C.V and G.L.  
A clipboard icon with a checkmark. All the lateral edges of the prism are perpendicular to P.P and hence C.V will be the vanishing point to all these parallel edges.
2. Locate C.V and draw the front view of the prism.
3. Join all the corners of the solid to S.P and locate the intersection points with P.P.

4. Join all the corners of the front view to C.V and project the corresponding intersection points from P.P.
5. Complete the perspective view, following the rules of visibility.

**Problem 68** A hexagonal prism of base edge 30 and 70 long, is resting on one of its rectangular faces on the ground with the hexagonal face touching the P.P. The station point is 60 to the right of the axis of the object; 70 away from P.P. The top-most rectangular face of the object touches the horizon plane. Draw the perspective view of the object.

**Construction (Fig.16.96)**

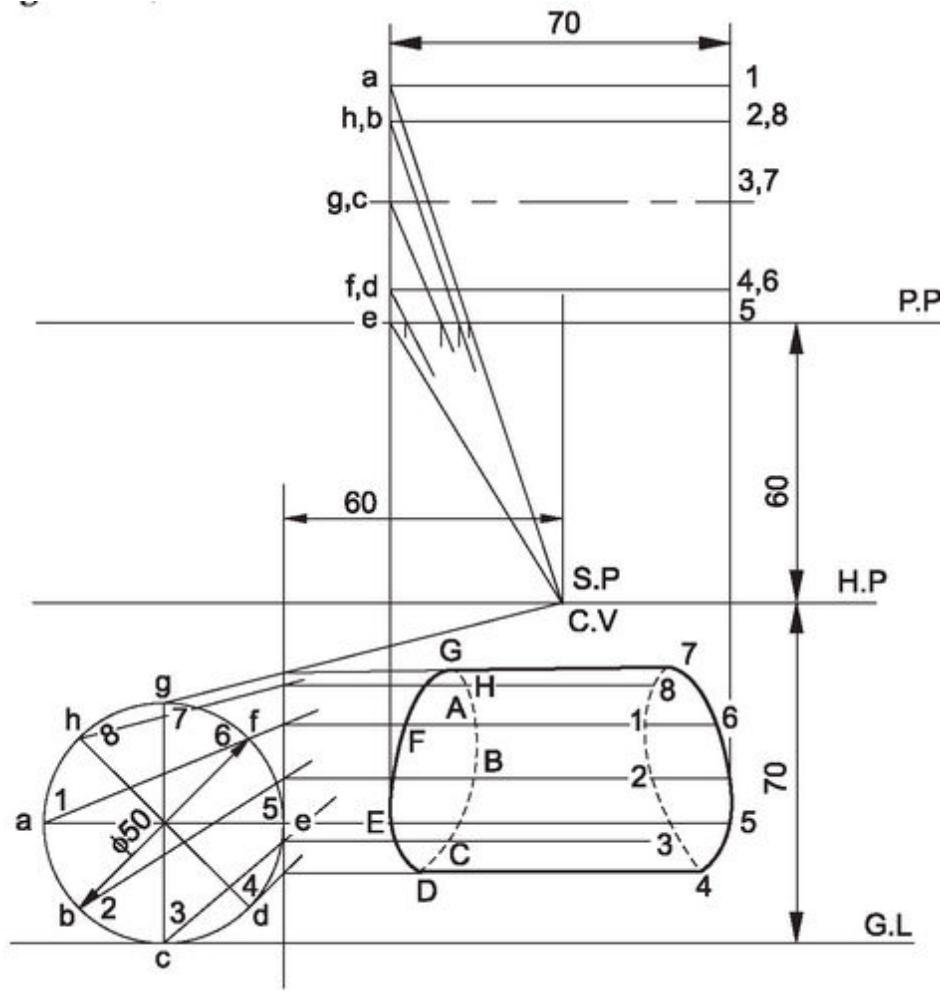


**Fig.16.96**

1. Draw the top view of the hexagonal prism and locate the perspective elements, satisfying the given conditions.
2. Draw the front view of the prism and locate the C.V.
3. Join all the corners of the solid (top view) to the S.P and locate the piercing points with the P.P.
4. Join all the corners of the front view to C.V and project the corresponding piercing points from P.P.
5. Complete the perspective view, following the rules of visibility.

**Problem 69** A cylinder of base diameter 50 and axis 70 long, is resting on one of its generators on the ground such that, it is parallel to P.P. One of the generators is touching the P.P. The station point is along the central line of the object, 60 away from the P.P and 70 above the ground. Draw the perspective view of the object.

**Construction (Fig.16.97)**



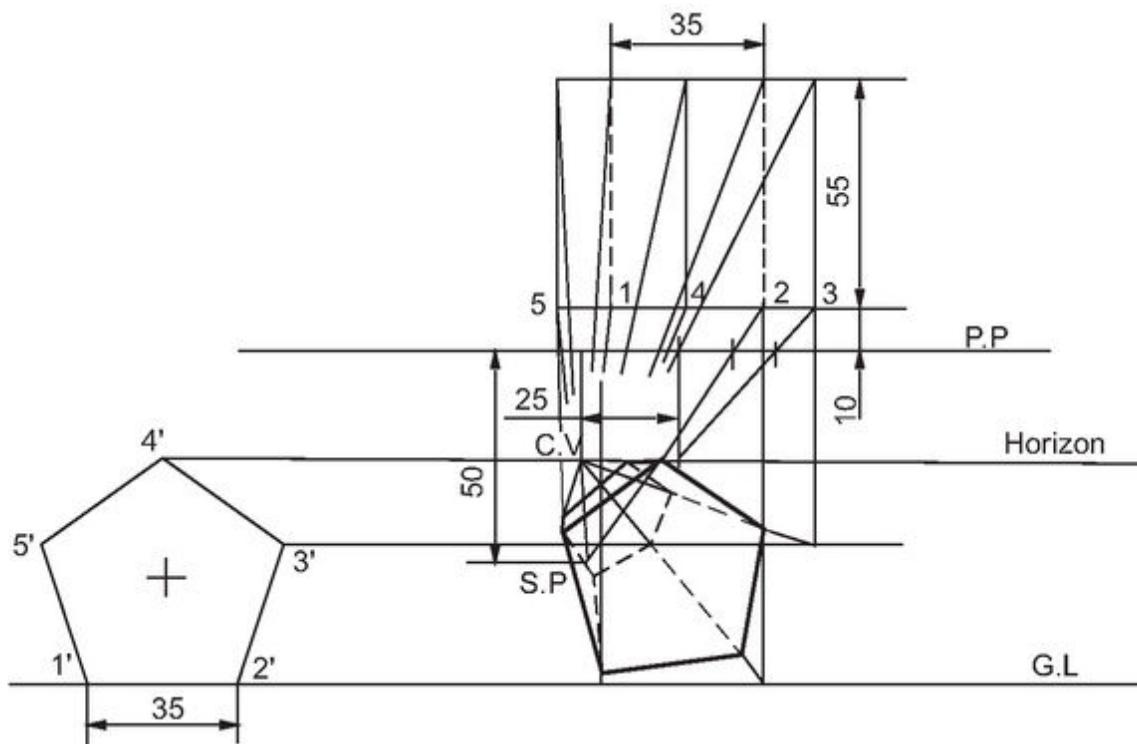
**Fig.16.97**

1. Draw the top view of the cylinder and locate the perspective elements, satisfying the given conditions.
2. Locate  $p'' - p''$  (side view of P.P) at 60 from S.P and draw the side view of the cylinder.
3. Divide the base of the cylinder (side view) into 8 equal parts and locate the corresponding division points on both ends of the cylinder in the top view.
4. Join these points (on the left side) to S.P and locate the piercing points with P.P.

5. Join the division points in the side view with C.V and locate the piercing points with  $p'' - p''$ .
6. Project and locate the points in the perspective view on the left side of the cylinder.
7. Obtain by symmetry, the points on the right side and obtain the perspective view of the cylinder by joining these points.

**Problem 70** A pentagonal prism (base edge 35, axis 55), rests on one of its rectangular faces on the ground, with its axis perpendicular to P.P. The nearest pentagonal face is 10 behind P.P. The station point is 50 in front of P.P and 25 to the left of the axis. The horizon plane contains the top-most edge of the solid. Draw the perspective projection of the solid.

**Construction (Fig.16.98)**

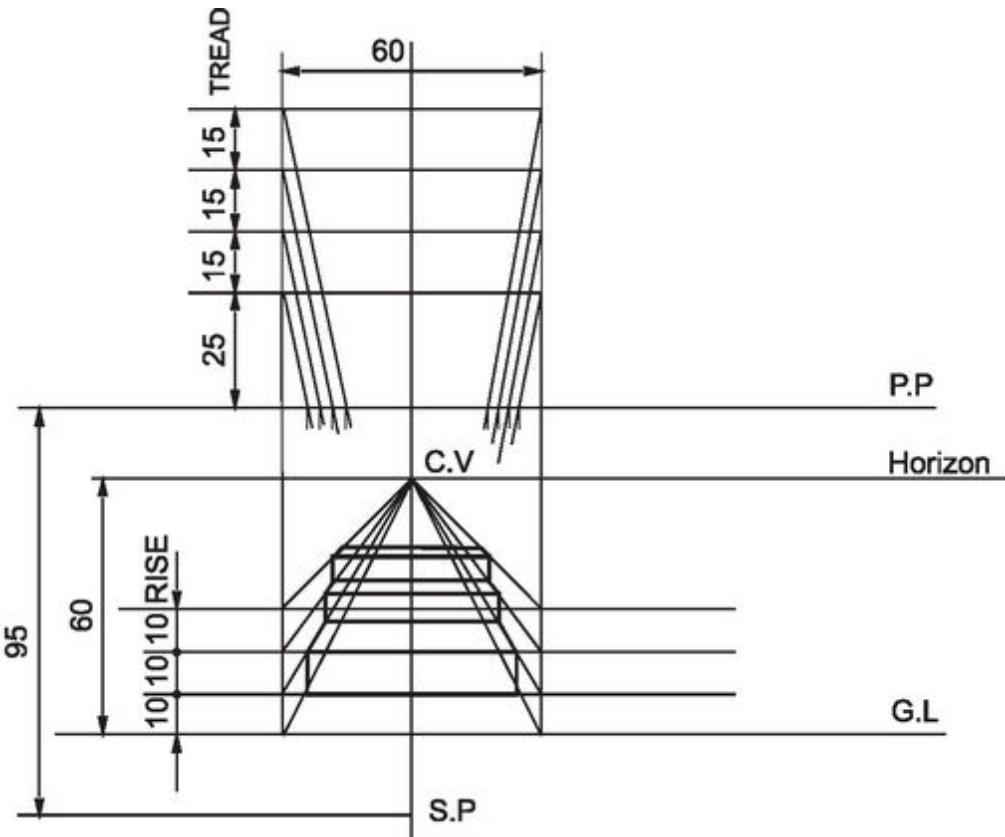


**Fig.16.98**

1. Draw the top view of the solid, satisfying the given conditions and locate the perspective elements.
2. Draw the front view of the solid on G.L to help locate the elevations of various corners above G.L.
3. Draw parallel lines to G.L, passing through the corners of 3', 4' and 5' of the front view.
4. Join S.P to various corners of the solid in the top view and locate the piercing points with P.P.
5. Draw projectors from the corners of the solid (1, 2, etc.) from top view to G.L; meeting the respective horizontal lines from the front view.
6. Join these points to C.V and transfer the piercing points with P.P, to the corresponding lines.
7. Join these points in the order, keeping in mind the visible and invisible edges and obtain the required perspective.

**Problem 71** A model of steps has three steps of 15 tread and 10 rise. Steps measure 60 width-wise. Draw the perspective of the model, when placed with its first step, 25 within the perspective plane and the longer edge being parallel to it. The station point is 95 from P.P and 60 above the ground and lies on the central line.

**Construction (Fig.16.99)**

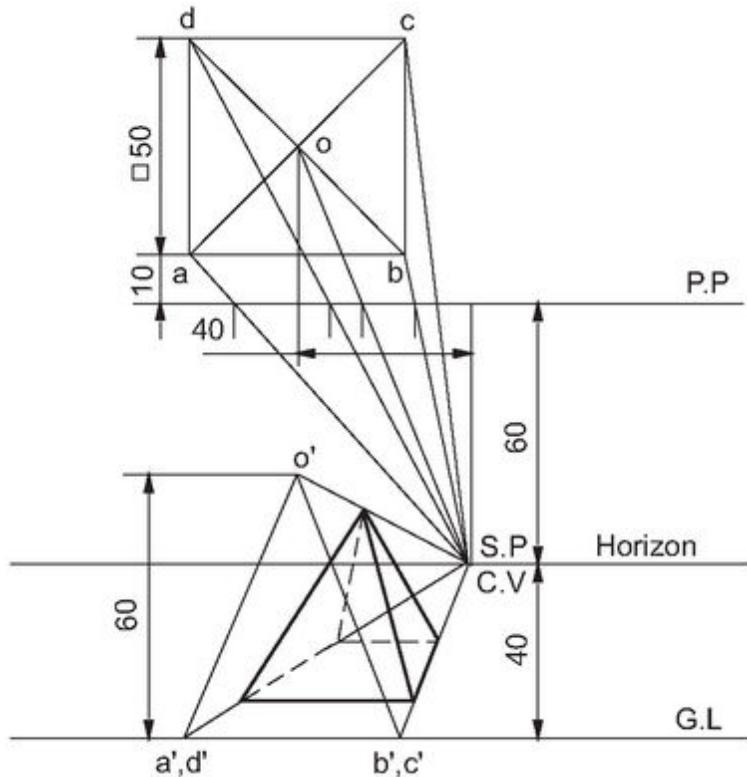


**Fig.16.99**

1. Draw the top view of the model of steps and locate the perspective elements, satisfying the given conditions.
2. Draw three parallel lines above H.P and at different elevations, corresponding to the rise of steps.
3. Join various corners in the top view of the model to S.P and locate the piercing points with P.P.
4. Extend the perpendicular edges (to P.P) of model of steps to G.L; intersecting the parallel lines drawn above G.L.
5. Join these intersection points to C.V (C.V is the vanishing point to edges perpendicular to P.P).
6. Transfer the piercing points on P.P, to the above lines and complete the perspective of the model of steps.

**Problem 72** Draw the perspective view of a square pyramid of base 50 side and 60 height. An edge of the base is parallel to and 10 behind P.P. The station point is situated at a distance of 60 in front of P.P, 40 above the ground and 40 to the right of the apex.

**Construction (Fig.16.100)**



**Fig.16.100**

1. Draw the top and front views of the square pyramid, after locating the perspective elements.
  - i) The edges ab and cd which are parallel to P.P w'll not have a vanishing point.
  - ii) The edges perpendicular to P.P (ad and bc) have C.V as their vanishing point.



2. Join the corners in the top view to S.P and locate the intersection points with P.P.
3. Join the corners and apex in the front view to C.V.
4. Project the points of intersection on P.P, on to the above lines.

Following the rules of visibility, complete the perspective view of the solid.

## **II Vanishing point method**

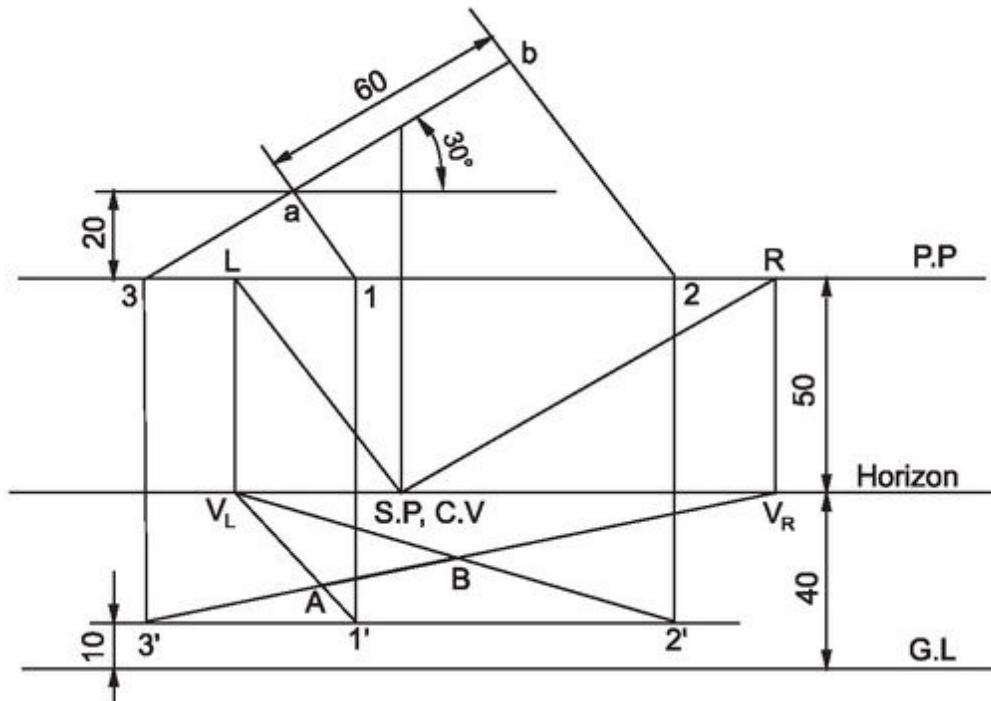
Determination of the vanishing point is based on the optical illusion that all parallel edges of an object converge to a single point, known as vanishing point.

When the object is so placed that, its two surfaces are inclined to P.P; the horizontal edges of the object will converge to two vanishing points. The perspective obtained using two vanishing points is known as two point/angular perspective. However, only one orthographic view (top view) is sufficient to obtain the perspective of the solid by this method. For the purpose of locating the elevations of various points/lines from horizon, a measuring line is made use of.

When an object is so placed, that all the three surfaces are inclined to P.P, the edges will converge to three vanishing points. The perspective of the object drawn by using three vanishing points is known as three point perspective. However, developing a three point perspective is outside the scope of this book.

**Problem 73** *A straight line AB of 60 long, is parallel to and 10 above G.P and inclined at  $30^\circ$  to P.P and one end A is 20 from P.P. The station point is 50 in front of P.P, 40 above G.L and on the central plane passing through the mid-point of the line. Draw the perspective of the line.*

### **Construction (Fig.16.101)**



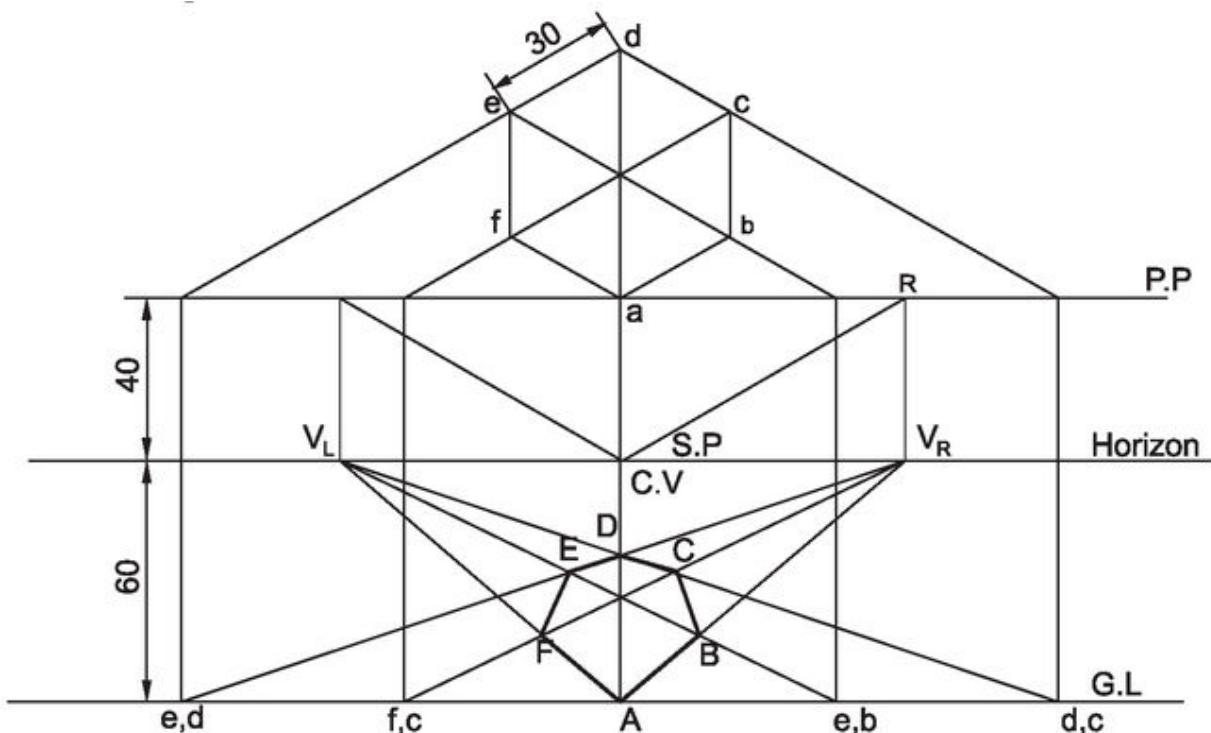
**Fig.16.101**

1. Draw the top view of the line and locate the perspective elements.
2. Assume two imaginary parallel lines (a<sub>1</sub> and b<sub>2</sub>) through a and b.
3. Locate V<sub>L</sub> and V<sub>R</sub>, the vanishing points for the imaginary lines and the line ab respectively.
4. Extend ba to meet P.P at 3.  
Points 1, 2 and 3 are on P.P.
5. Locate 1', 2' and 3' on a line parallel to and 10 above G.L, corresponding to 1, 2 and 3.
6. Join points 1' and 2' to V<sub>L</sub> on which the two ends of the line AB lie.
7. Join 3' to V<sub>R</sub>, on which the line AB lies.

Locate AB, the perspective view of the given line AB, as shown.

**Problem 74** A hexagonal plane of side 30 is resting on H.P with a corner in P.P and the two sides equally inclined to P.P. The station point is 40 in front of P.P on the central line of the plane. The station point is 60 above the ground. Obtain the perspective projection of the plane.

**Construction (Fig.16.102)**



**Fig.16.102**

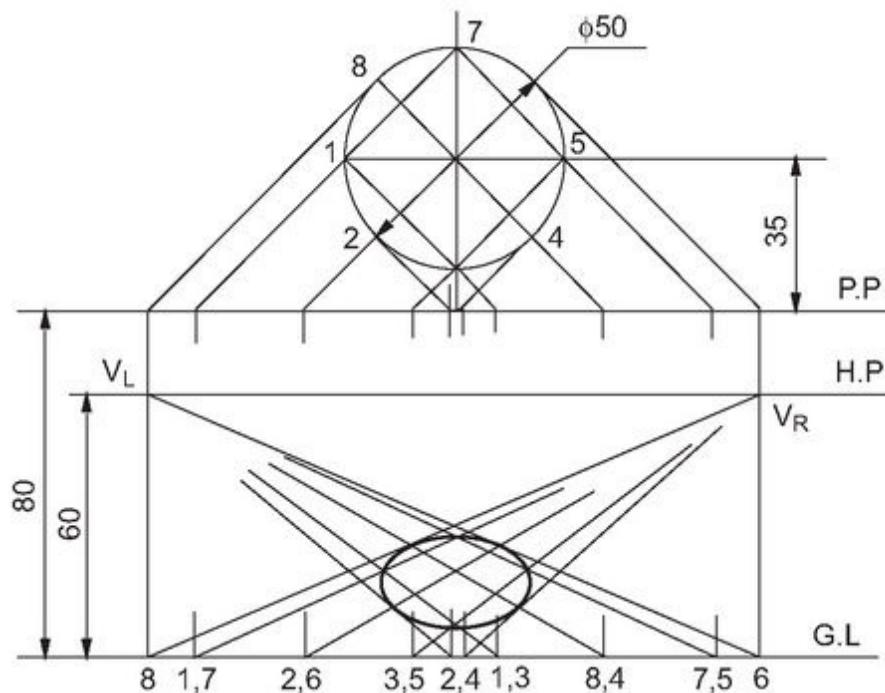
1. Draw the top view of the plane and locate the perspective elements, satisfying the given conditions.
2. Locate  $V_L$  and  $V_R$  on the horizon plane, corresponding to the inclined edges of the plane.
3. Extend the imaginary lines passing through the corners of the plane to meet P.P and locate the

corresponding points on G.L.

4. Join these points to  $V_L$  and  $V_R$  and locate the points of intersection and name the corners as shown.
5. Join these points in the order and obtain the perspective projection of the plane.

**Problem 75** Draw the perspective of a horizontal circular lamina of 50 diameter resting on the ground. The centre of the plane is 35 behind P.P. The station point is in the central plane, passing through the centre of the circular plane and 80 in front of P.P and 60 above the ground.

**Construction (Fig.16.103)**



**Fig.16.103**

1. Draw the top view of the circular lamina and divide it into, say 8 equal parts. Locate the perspective elements.

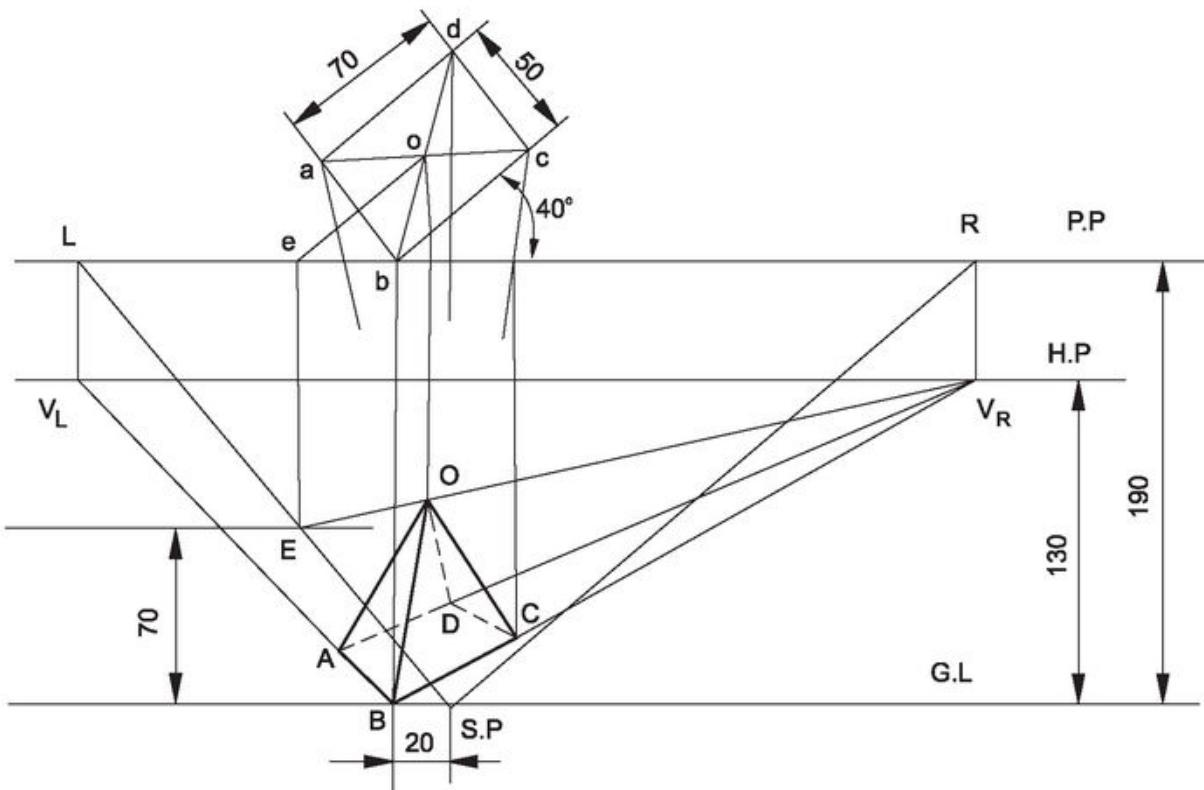
2. Locate  $V_L$  and  $V_R$  on H.P, the vanishing points, corresponding to the imaginary parallel lines, drawn through the division points of the circle, as shown.
3. Extend the imaginary lines to meet P.P and locate the corresponding points on G.L.
4. Join these points to  $V_L$  and  $V_R$  and locate the points of intersection on the perspective.
5. Join these points by a smooth curve and obtain the perspective view of the lamina.

**Problem 76** A rectangular pyramid of base  $70 \times 50$  and altitude 70, rests with its base on the ground. One corner of the base is 20 to the left of the eye and in P.P. The 70 long side of the base recedes to the right at  $40^\circ$ . The eye is 190 from P.P and 130 above G.L. Draw the perspective of the pyramid.

### **Construction ([Fig.16.104](#))**

1. Draw the top view of the pyramid and locate the perspective elements.
2. Locate  $V_R$  and  $V_L$ , corresponding to the edges bc, ad and ab, dc.
3. Locate B on G.L, as b is in contact with P.P.
4. Join a, d, c and apex to S.P and locate the intersection points with P.P.
5. Join B to  $V_R$  and  $V_L$ .
6. Locate A and C on these lines, by projection.
7. Join A to  $V_R$  and locate D on it, by projection.
8. Draw a line through o parallel to bc, meeting P.P at e.

- Draw a projector through e and mark E at 70 from G.L , equal to the height of the pyramid.
- Join E to  $V_R$  and locate o on it.
- Following the rules of visibility, complete the perspective of the pyramid.



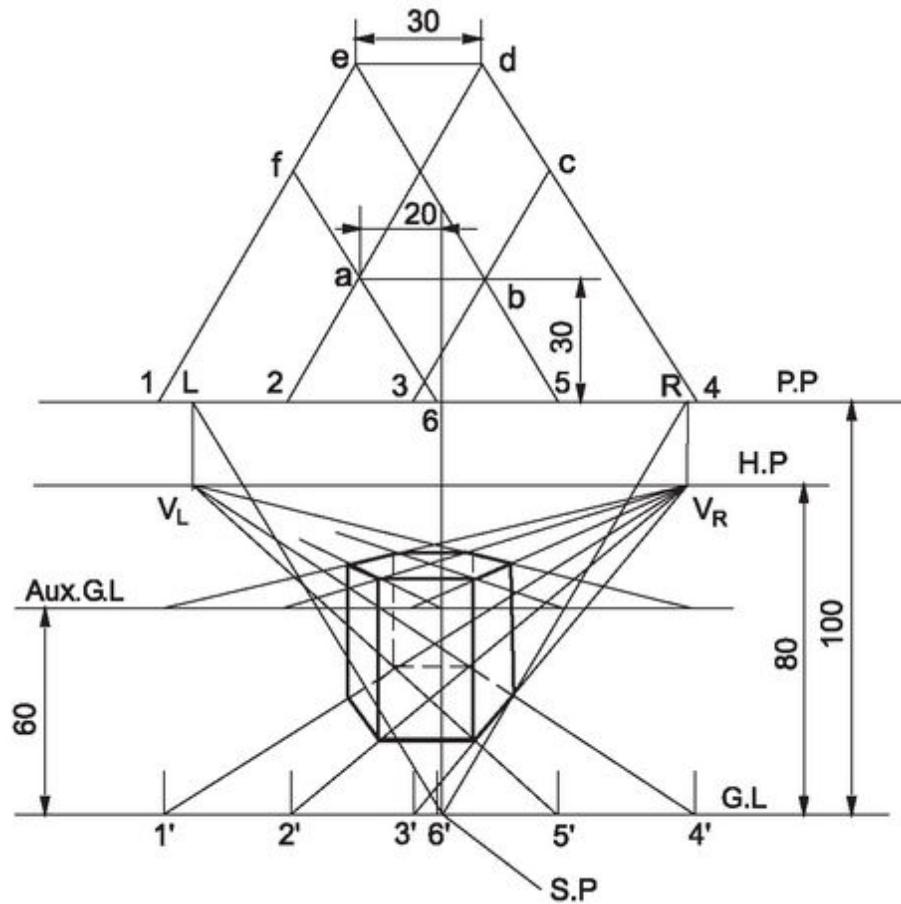
**Fig.16.104**

**Problem 77** A hexagonal prism of side of base 30 and height 60 stands with its base on the ground. One rectangular face is parallel to P.P and 30 away from it and the nearest vertical edge of this face is 20 to the left of the eye (S.P). The eye is 100 from P.P and 80 above G.L. Draw the perspective of the prism.

### **Construction ([Fig.16.105](#))**

- Draw the top view of the prism and locate the perspective elements.

2. Extend the edges cb, da and ef to meet P.P at 3, 2 and 1 respectively.
3. Extend dc, eb and fa to meet P.P at 4, 5 and 6 respectively.
4. Locate 1', 2', 3' etc., on G.L, corresponding to 1,2, 3, etc.
5. Locate  $V_L$  and  $V_R$  on the horizon, the vanishing points for the above lines.
6. Join 1', 2' and 3' to VR and 4', 5' and 6' to VL.
7. Locate the corners of the bottom base of the solid at the intersection of these lines, as shown.
8. Draw an auxiliary ground line at 60 from G.L, equal to the height of the prism.
9. Repeat the steps 4 to 7 and locate the top base of the prism and complete the perspective of the solid.



**Fig.16.105**

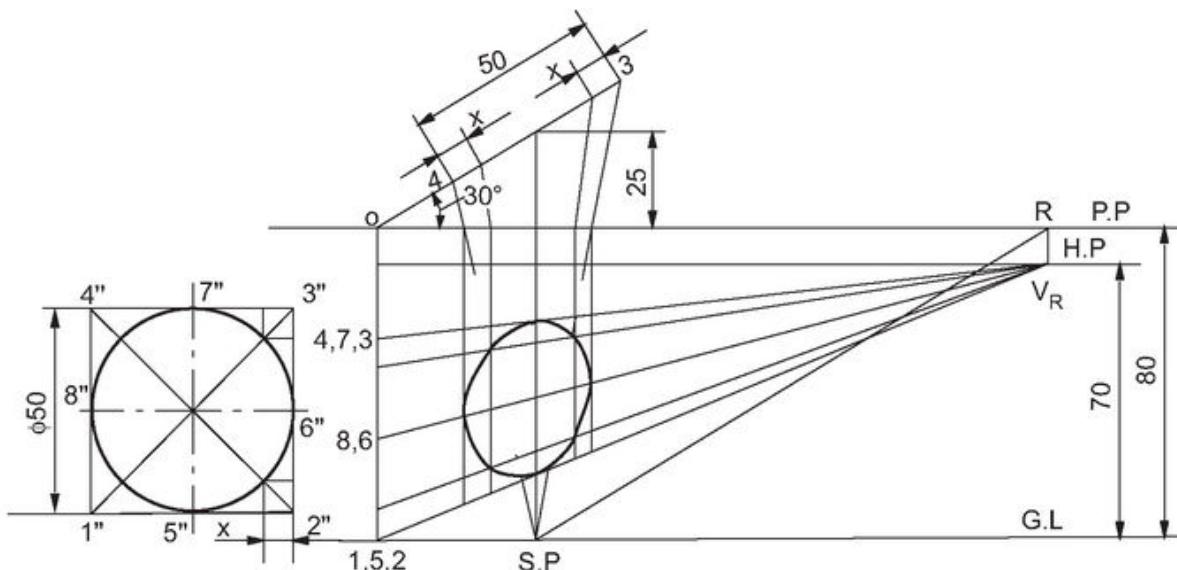
### III Combination method

Both visual ray method and vanishing point method may be combined for drawing the perspective view of an object with ease. By combining both the methods, it is possible to reduce the time required in drawing the perspective of an object. For example, when visual ray method alone is used, two views of the object are necessary to draw the perspective. Also, when vanishing point method is used, more sets of lines are to be drawn, connecting the corners and the vanishing points at various elevations of the object. These intersecting lines sometimes cause confusion to the beginner where as, the combination method makes use of the advantages of both the methods. In this method, the

piercing points of visual rays with P.P in the top view are used to locate the corners and edges of the object in perspective.

**Problem 78** Draw the perspective of a vertical circular plane of 50 diameter inclined at  $30^\circ$  to P.P. The centre of the plane is 25 behind P.P. It is resting on a point on the circumference on the ground. The station point is located in the central plane, passing through the centre of the circular plane and 80 in front of P.P and 70 above the ground.

**Construction (Fig.16.106)**



**Fig.16.106**

1. Draw the top view of the vertical circular plane, satisfying the given conditions and locate the perspective elements.
2. Extend the top view to meet P.P at o.
3. Project o to the ground line and draw the front view of the plane, in contact with the projector. Enclose the circle in a square.



The elevations of various points on the circle are obtained from the front view. Hence, this view serves as a measuring line.

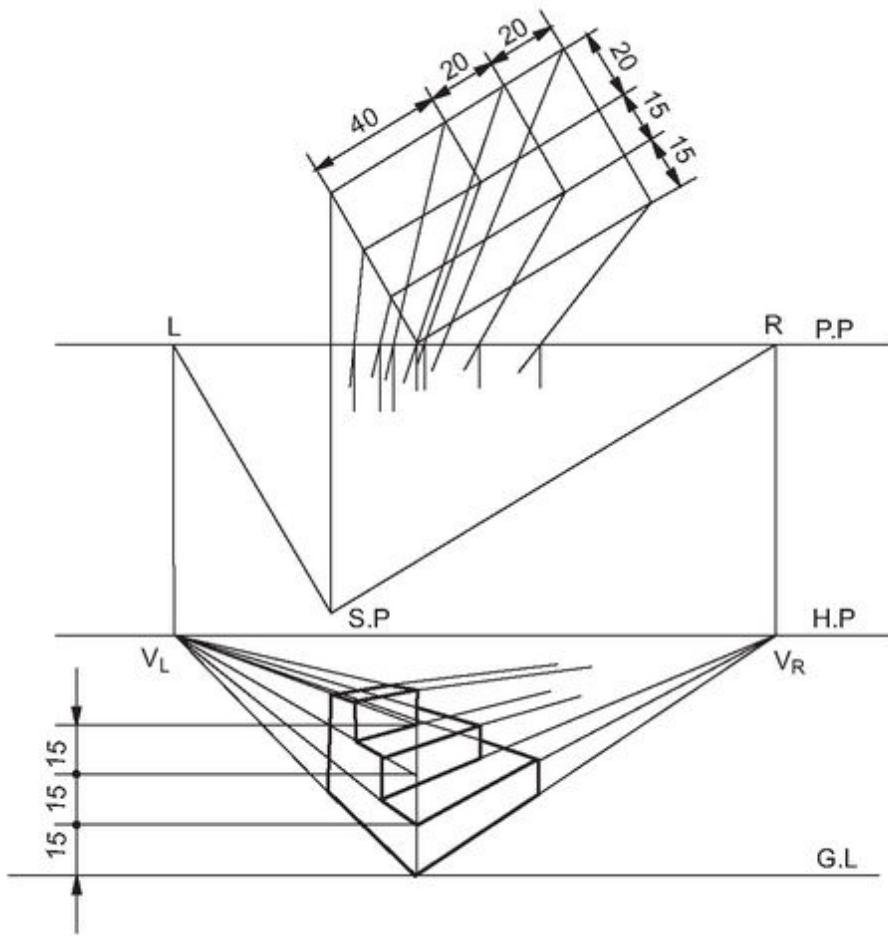
4. Draw a line from S.P, parallel to the top view of the plane and locate R on P.P. Locate  $V_R$  (vanishing point on the right side) on the horizon.
5. Join S.P to the various points in the top view and locate the piercing points on P.P.
6. Join the points on the measuring line to  $V_R$  and project the piercing points on P.P to the corresponding lines and obtain the points in the perspective.
7. Obtain the perspective of the circle, by joining the above points by a smooth curve.

**Problem 79** *For the model of steps shown in Fig.16.107, draw the perspective view by suitably selecting the station point.*

**HINT** A vertical edge of the model is taken, coinciding with P.P. The height of each step is chosen as 15.

Using the combination of visual ray and vanishing point methods, the perspective of the model is obtained, as shown in the above figure.

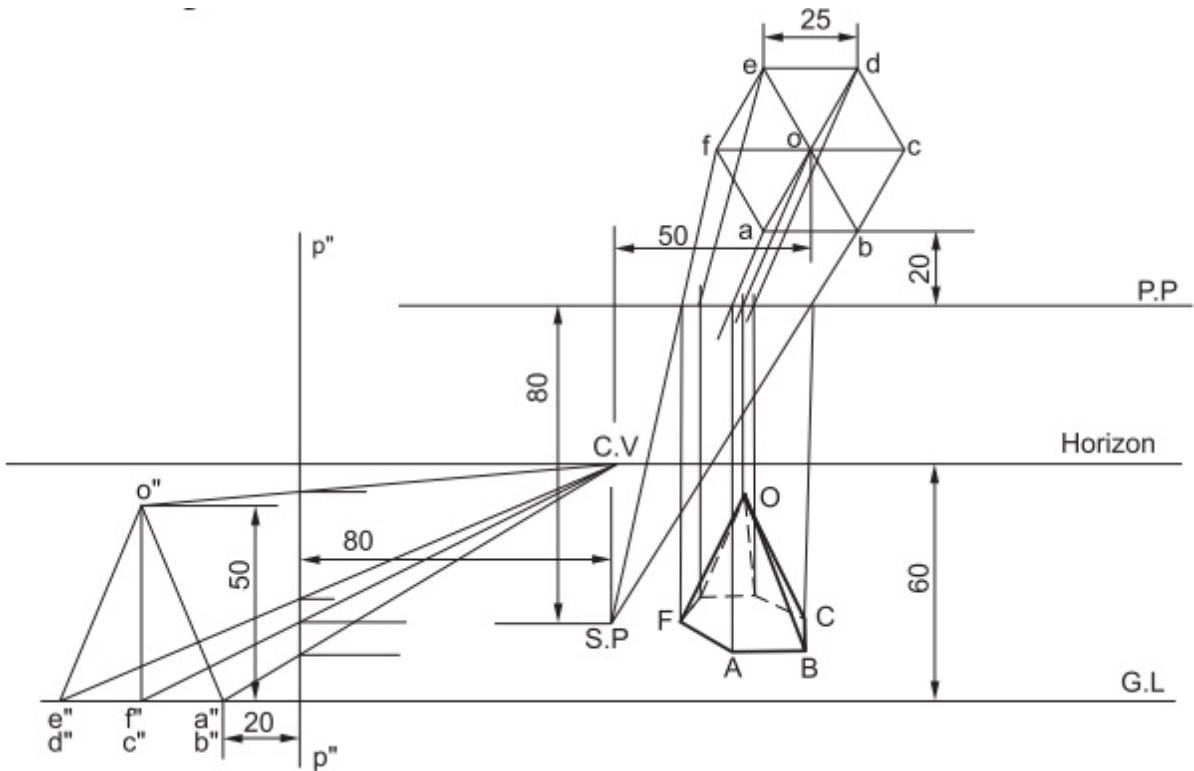
**Problem 80** *A hexagonal pyramid of base 25 side and 50 height, is resting on the ground on its base with one side of the base parallel to and 20 behind P.P. The station point is 60 above H.P and 80 in front of P.P and lies 50 to the left of the central plane, passing through the axis of the solid. Draw the perspective view of the pyramid.*



**Fig.16.107**

1-a *Visual ray method, making use of the side view of the object*

**Construction ([Fig16.108 a](#))**

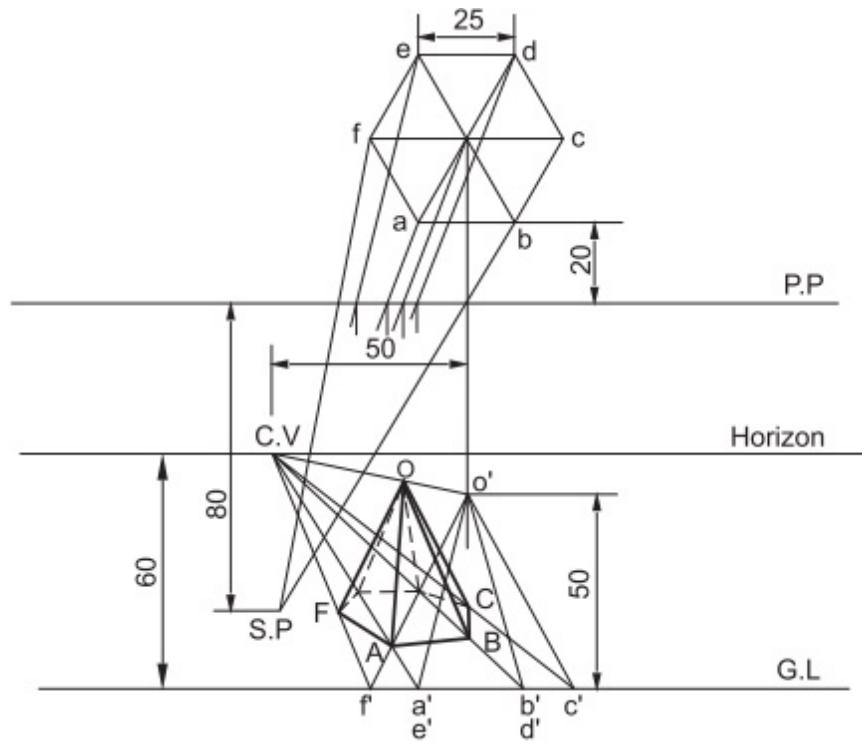


**Fig.16.108a**

1. Draw the top view of the pyramid and locate the perspective elements, satisfying the given conditions.
2. Locate the side view of P.P ( $p''-p''$ ) and draw the side view of the pyramid.
3. Join the corners of the solid (top view) with S.P and locate the piercing points of these visual rays with P.P.
4. Join all the corners and apex (side view) with C.V and locate the piercing points of these visual rays with  $p''-p''$ .
5. Project the corresponding piercing points (with P.P and  $p''-p''$ ) and complete the perspective view, following the rules of visibility.

*1-b Visual ray method, using the front view of the solid*

**Construction (Fig.16.108b)**

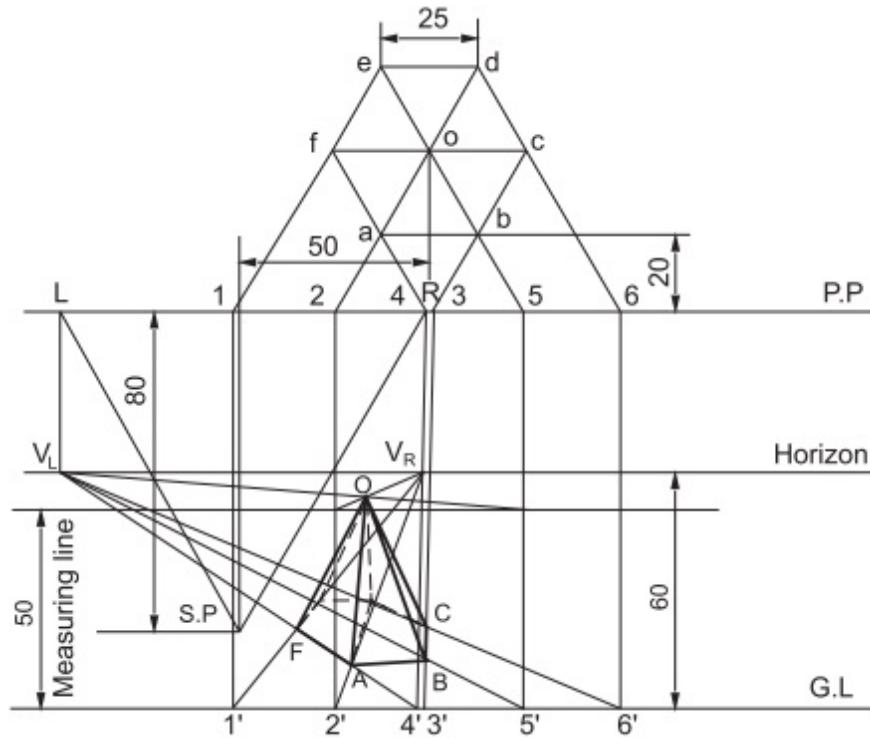


**Fig.16.108b**

1. Draw the top and front views of the pyramid, after locating the perspective elements, satisfying the given conditions.
2. Join all the corners in the top view to S.P and locate the piercing points of the visual rays with P.P.
3. Join the corners and apex in the front view to C.V.
4. Project the piercing points on P.P to the corresponding (above) lines.
5. Following the rules of visibility, complete the perspective view of solid.

## II Vanishing point method

**Construction (Fig.16.108c)**



**Fig.16.108c**

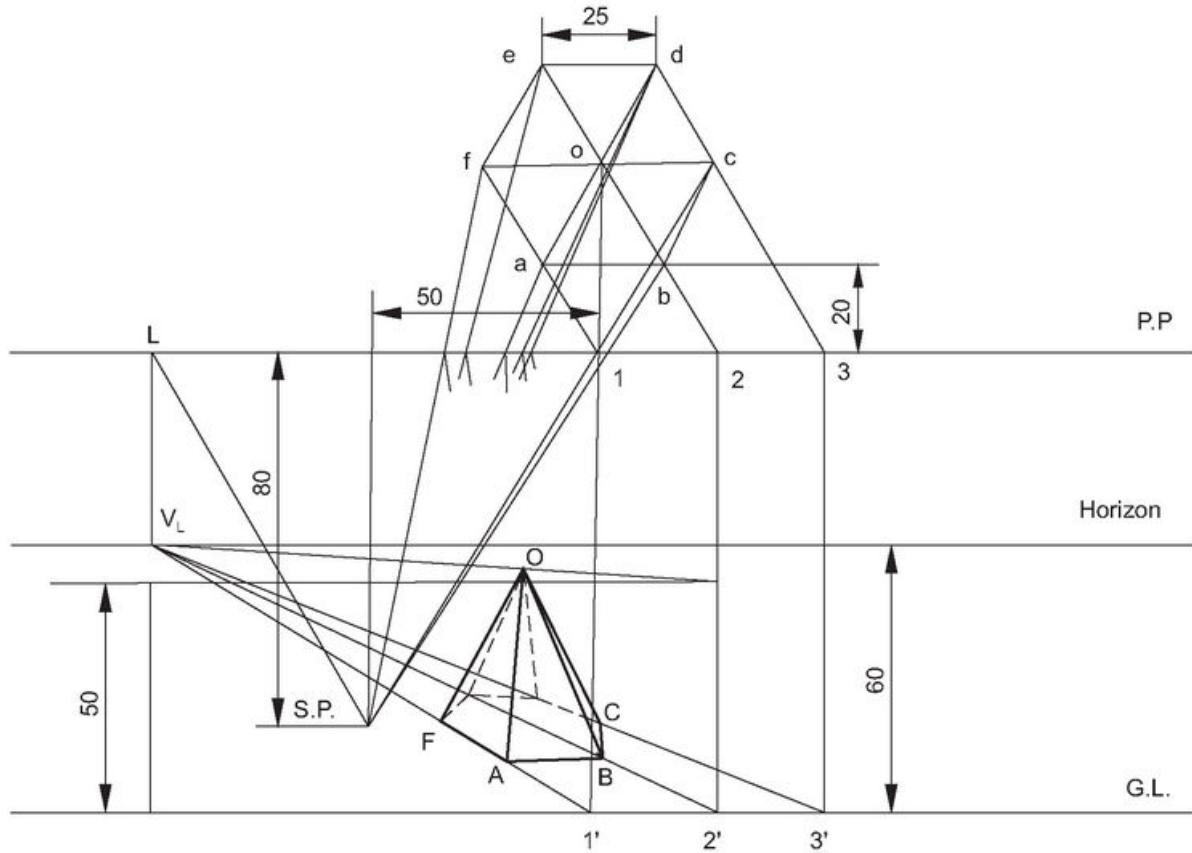
1. Draw the top view of the solid and locate the perspective elements, satisfying the given conditions.  
📎 Edges parallel to P.P will not have any vanishing point.
2. Locate  $V_L$  and  $V_R$  on horizon; the vanishing points corresponding to the imaginary parallel lines, passing through the corners of the base of the solid in the top view.
3. Extend the two sets of imaginary parallel lines to meet P.P at 1, 2, 3, etc., and locate the corresponding points on G. L, by projection.
4. Join these points to  $V_L$  and  $V_R$  and locate the points of intersection corresponding to the corners of the base.

5. Locate a measuring line and obtain the apex of the solid in the perspective.
6. Join the corners and corners to the apex and following the rules of visibility, obtain the perspective of the solid.

### **III Combination method**

#### ***Construction ([Fig.16.108d](#))***

1. Draw the top view of the pyramid and locate the perspective elements, satisfying the given conditions.
2. Join the corners of the top view to S.P and locate the piercing points with P.P.
3. Locate  $V_L$ , the vanishing point, corresponding to the imaginary parallel lines passing through the corners of the top view of the solid.
4. Extend the imaginary lines to meet P.P at 1, 2 and 3 and locate the corresponding points on G.L.
5. Join these points with  $V_L$  and project the piercing points on P.P to the corresponding lines.
6. Draw a line parallel to and 50 above G.L, to locate the apex of the solid in the perspective.
7. Following the rules of visibility, complete the perspective of the solid.



**Fig.16.108d**

## 16.7.8 Properties of Perspectives

From the subject content presented so far, the following may be noted:

- | Position of line                                     | Appearance in perspective                          |
|--|--|
| 1. Parallel to P.P and G.P                           | 1. Parallel to G.L                                 |
| 2. Parallel to P.P and inclined at $\theta$ to G. P. | 2. Inclined at $\theta$ to G.L                     |
| 3. Inclined to P.P (set of parallel lines)           | 3. Converge and vanish at a single vanishing point |
| 4. Parallel to P.P and perpendicular to G.P          | 4. Vertical to G.L                                 |

- 5. Set of lines perpendicular to P.P
- 5. Converge to C.V
- 6. In P.P
- 6. True length in perspective

## EXERCISES

### Isometric projections

- 16.1 Draw the isometric projection of (i) a rectangle of 80 and 50 sides; its plane being horizontal and (ii) a regular pentagon of 25 side; its plane being vertical and one of its sides, horizontal.
- 16.2 Draw the isometric projection of a circular lamina of 80 diameter when its surface is (i) vertical and (ii) horizontal.
- 16.3 A hexagonal prism with side of base 25 and axis 50 long, rests on its base in H.P and axis parallel to V.P. Draw orthographic projections and provide the isometric projection of the solid. Show the isometric scale.
- 16.4 Draw the isometric projections of a square prism of base 30 side and axis 60 long, when its axis is (i) vertical and (ii) horizontal.
- 16.5 A hexagonal prism with side of base 25 and axis 50 long, rests on its base in H.P. Its axis is parallel to V.P. Draw orthographic projections and provide the isometric projection of the solid. Show the isometric scale.
- 16.6 A hexagonal prism of side of base 30 and 70 long, has a concentric square hole of side 20. One face of

the square hole is parallel to a face of the prism.  
Draw the isometric view of the solid.

16.7 A triangular prism with edge of base 40 and 80 long, lies on one of its base edges on H.P and the rectangular face containing that edge is inclined at  $60^\circ$  to H.P. Taking the resting edge normal to V.P, draw the isometric projection of the solid.

16.8 Draw the isometric projection of a hexagonal prism of side of base 35 and altitude 50, surmounting a tetrahedron of side 45 such that, the axes of the solids are collinear and one of the edges of the two solids is parallel.

16.9 A hexagonal prism of side of base 30 and 70 long, has a square hole of side 20 at the centre. The axes of the hole and the solid coincide and one of the faces of the hole is parallel to the faces of the hexagon. Draw the isometric projection of the prism with the hole.

16.10A triangular prism of base edge 30 and height 60, stands on one of its corners on the ground, with the axis inclined at  $30^\circ$  to H.P and  $45^\circ$  to the V.P. The base of the object is nearer to V.P compared to the top. Draw the isometric view of the object.

16.11A hexagonal prism having the side of the base 30 and height 70, is resting on one of the corners of the base and its axis is inclined at  $30^\circ$  to H.P. Draw its projections and also prepare the isometric view of the prism in the above stated condition.

16.12A hexagonal prism with side of base 25 and height 50mm, rests on H.P and one of the edges of its base is parallel to V.P. A section plane perpendicular to V.P and inclined at  $50^\circ$  to H.P bisects the axis of the

prism. Draw the isometric projection of the truncated prism.

16.13 Draw the isometric projection of a vertical pentagonal hollow prism of 30 side of base at the outside, height 70 and 6 thick, when resting with two of its rectangular faces equally inclined to V.P.

16.14 A pentagonal pyramid of 40 side of base and height 70, rests with its base on H.P. One edge of the base is perpendicular to V.P. A section plane perpendicular to V.P and inclined at  $30^\circ$  to H.P cuts the axis of the pyramid at a point 30 above the base. Draw the isometric projection of the solid.

16.15 The frustum of a hexagonal pyramid, sides of top and bottom bases, 25 and 40 respectively with axis 50, rests on its base in H.P. Its axis is parallel to V.P. A sphere of diameter 40 is placed centrally on top of the prism. Draw the orthographic projections and provide the isometric projection of the solid.

16.16 A hexagonal pyramid with side of base 30 and axis 120 long, is resting on H.P, an edge of the base is parallel to V.P, a horizontal section plane passes through a point on the axis, at a distance 60 from the base. Draw an isometric view of the frustum of the pyramid.

16.17 A square pyramid of 20 side and height 40, is placed on the top of a cylinder of 40 diameter and height 60. Draw the isometric projection of the compound solids.

16.18 Draw an isometric projection of frustum of a hexagonal pyramid, with its base and top surfaces as hexagons of sides 80 and 40 respectively and height 70.

16.19 Draw the isometric view of door-steps having three steps of 220 tread and 150 rise. The steps measure 750 width-wise.

16.20 A pedestal in the form of a square prism, with side of base 80 and height 100, is surmounted by a cylindrical slab of 50 diameter and height 40. Draw the isometric projection of the pedestal, when the axes of the prism and cylinder coincide.

16.21 A box made of 20 thick wooden planks, has the overall dimensions of  $200 \times 100 \times 150$ , including the lid. The lid is of outside dimensions  $200 \times 100 \times 40$ . Draw the isometric projection, using the isometric scale, when the lid is open through  $120^\circ$ .

16.22 A stool with a circular top of 400 diameter and 30 thick, has 3 vertical cylindrical legs of 40 diameter each and 500 long. The legs are symmetrically positioned on a pitch circle diameter of 350, supporting the top. Draw the isometric projection of the stool.

16.23 A cylinder of diameter of base 60 and height 70, rests with its base on H.P. A section plane perpendicular to V.P and inclined at  $45^\circ$  to H.P, cuts the cylinder such that, it passes through a point on the axis at 50 above its base. Draw the isometric projection of the truncated cylinder.

16.24 Draw the isometric projection of a cylinder of 75 diameter and 100 long, with a concentric 20 square hole. The cylinder is lying on H.P with its axis parallel to V.P and the side of the square hole making an angle of  $45^\circ$  with H.P.

16.25 A horizontal cylinder of 20 diameter and 40 long, is resting centrally on the top of a vertical hexagonal

prism of 40 height and side of base 30. One edge of the prism is perpendicular to V.P and the axis of the cylinder is parallel to V.P. Draw the isometric projection of the two solids in position.

16.26A hollow cylinder of 50 and 30 outside and inside diameters and height 65, stands vertically on a square prism of 60 side and 35 height. Draw the isometric view.

16.27A cylinder 50 diameter and 60 height, stands on H.P. A section plane perpendicular to V.P, inclined at  $55^\circ$  to H.P, cuts the cylinder and passes through a point on the axis at a height of 45 above the base. Draw the isometric projection of the truncated portion of the cylinder, when the cut surface is clearly visible to the observer.

16.28A cylinder of 50mm diameter and 60 height, stands on H.P.A section plane perpendicular to V.P, inclined at  $55^\circ$  to HP, cuts the cylinder and passes through a point on the axis at a height of 45mm above the base. Draw the isometric projection of the truncated portion of the cylinder, when the cut surface is clearly visible to the observer.

16.29The frustum of a cone with base diameter 50, diameter at the top 30 and 40 height, rests on a cube of side 70 such that, their axes are collinear. A hemisphere of 30 diameter is placed centrally, with its curved surface in contact with the top surface of the frustum. Draw the isometric projection of the arrangement.

16.30A cone of base 20 diameter and 30 high, rests on the frustum of a hexagonal pyramid of base 25 side, 15 side at the top and 25 height such that, their axes

coincide. Draw the isometric projection of the arrangement.

16.31A cone of diameter of base 60 and height 65, rests with its base on H.P. A cutting plane perpendicular to V.P and inclined at  $30^\circ$  to H.P, cuts the cone such that, it passes through a point on the axis at a distance of 30 above the base. Draw the isometric projection of the cone.

16.32A cone of base diameter 30 and height 40, is resting over a frustum of a hexagonal pyramid of base side 40, top base side 25 and height 60. Draw the isometric projection of the solids.

16.33A frustum of a cone having 25 as top diameter, 50 as bottom diameter and axis length 50, is placed vertically on a cylindrical block of 75 diameter and is 25 thick such that, both the solids have the common axis. Draw the isometric projection of the combination of these solids.

16.34A paper weight consists of three portions. Bottom-most portion is a hexagonal prism of side of base 60 and height 15. Middle portion is the frustum of a hexagonal pyramid of base 60 side, and side at the top 50; height being 25. Top portion is a hemisphere, touching all the sides of the hexagon. Draw the isometric projection of the solid.

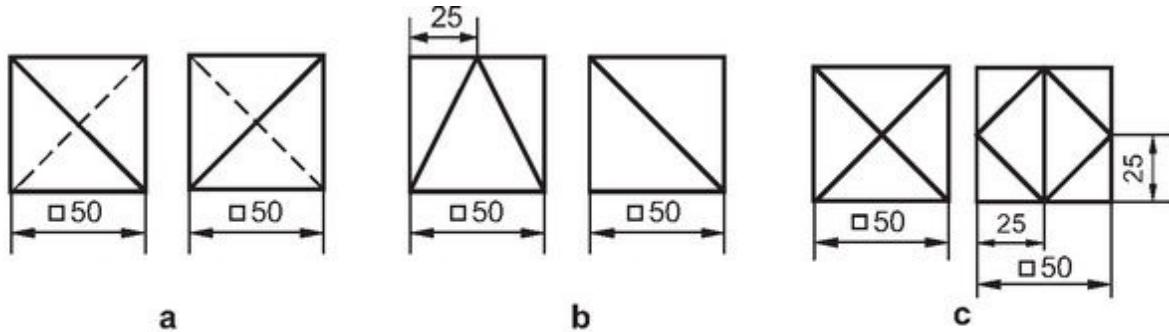
16.35A paper weight consists of three portions. The bottom-most portion is a cylinder of 60 diameter and 20 height. The middle portion is a frustum of cone of height 20 and bottom is 60 diameter and top is 30 diameter. The top-most part is a hemi-sphere of 15 radius. Draw the isometric projection of the paper weight.

16.36 A hemi-sphere of 40 diameter is nailed on the top surface of a frustum of a square pyramid. The sides of the top and bottom faces of the frustum are 20 and 40 respectively and its height is 50. The axes of both the solids coincide. Draw the isometric projection of the assembly.

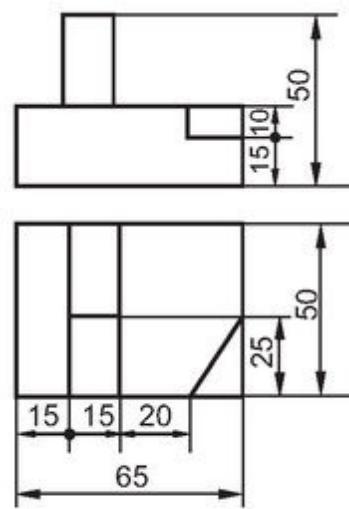
16.37 A hemi-sphere of 60 diameter is resting on a point on the top of an octagonal slab of side of base 30 and 30 height. Another hemi-sphere of 40 diameter, is placed on the flat surface of the above hemi-sphere with its flat surface at the top. Draw the isometric projection of the combination of solids.

16.38 Draw the isometric projection of a sphere of 60 diameter, resting centrally on the top of a square prism with side of base 60 and 20 height.

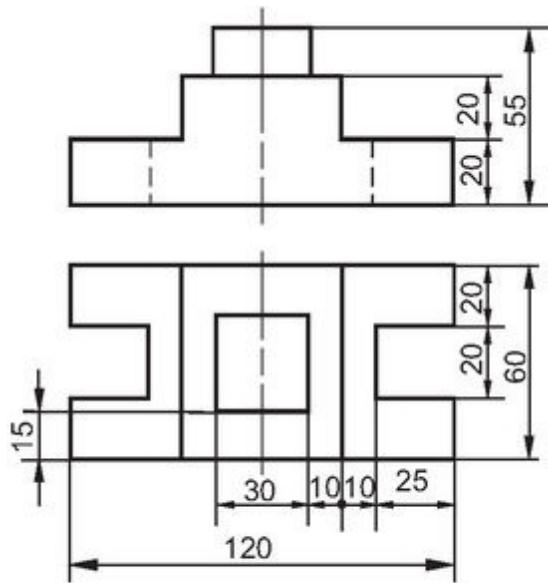
16.39 Figures 16.109 to 16.123 show the orthographic projections of certain objects. Draw the isometric projections of each.



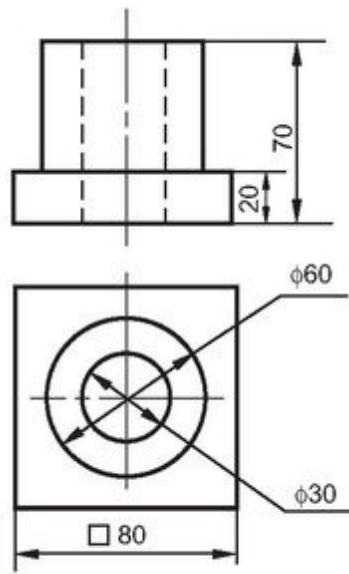
**Fig.16.109**



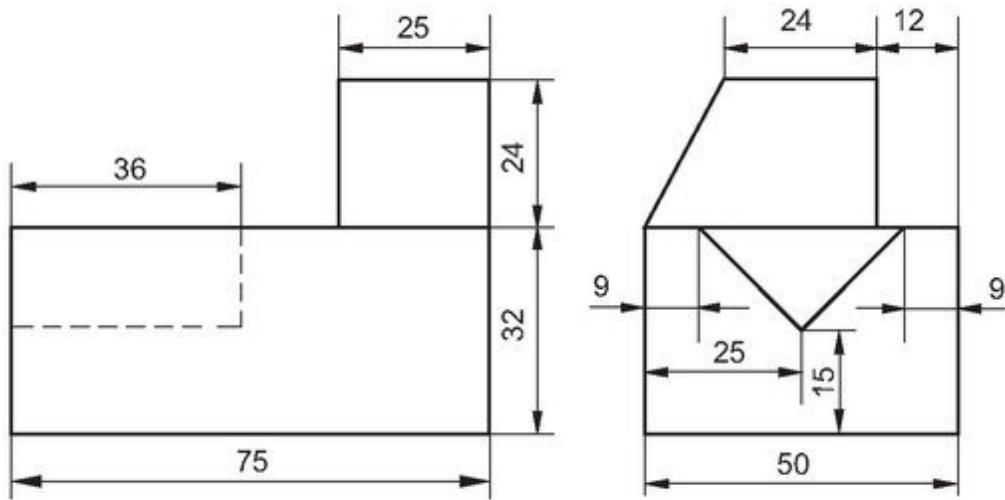
**Fig.16.110**



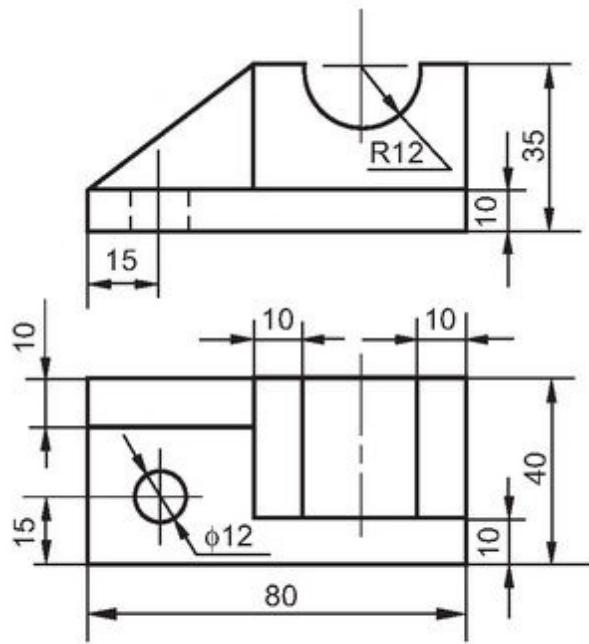
**Fig.16.111**



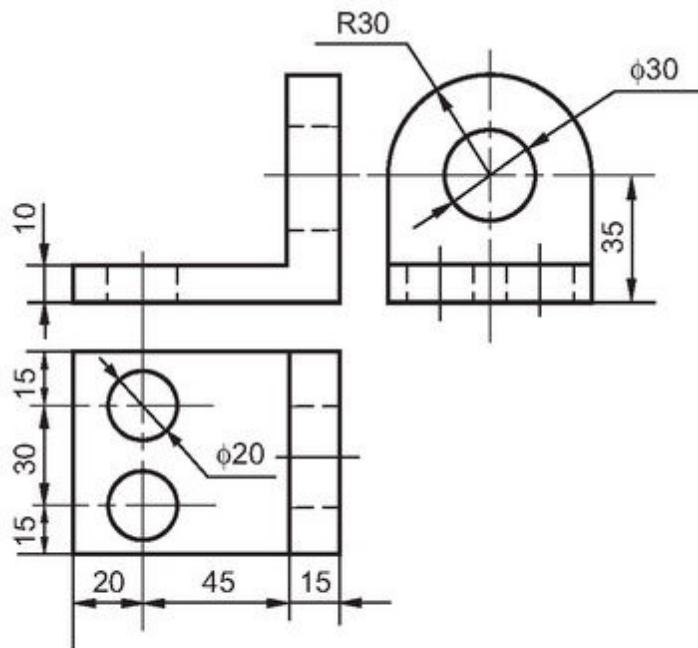
**Fig.16.112**



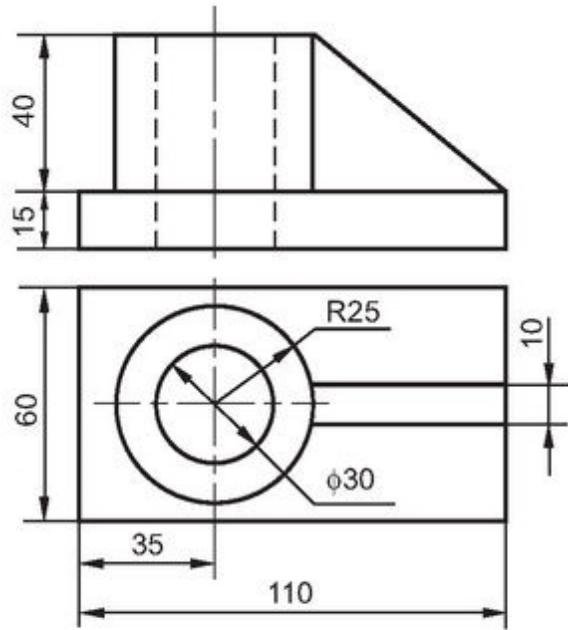
**Fig.16.113**



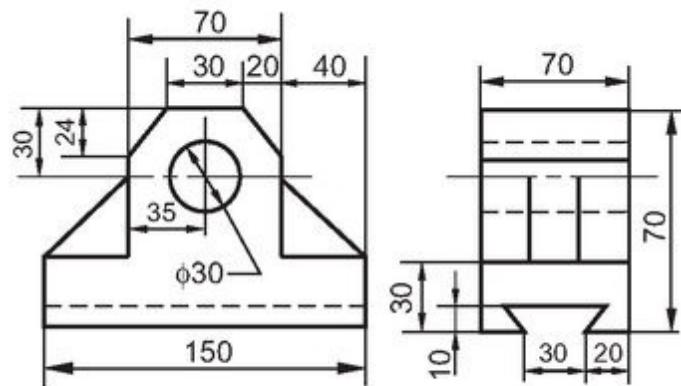
**Fig.16.114**



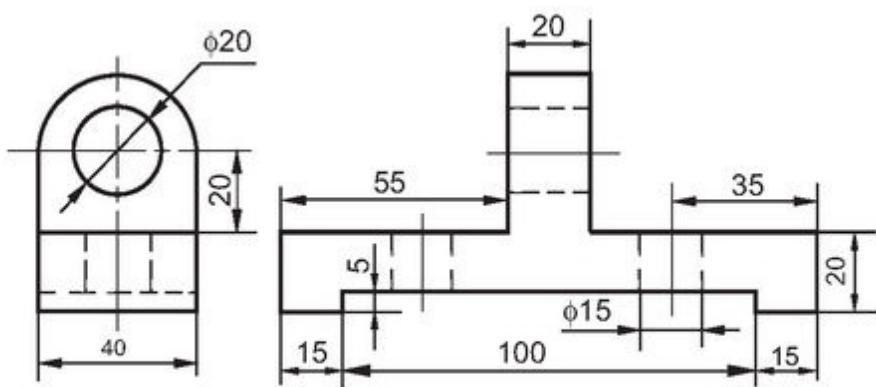
**Fig.16.115**



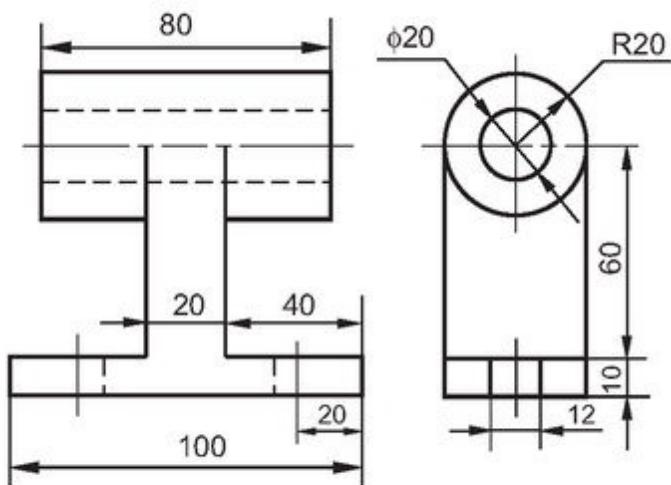
**Fig.16.116**



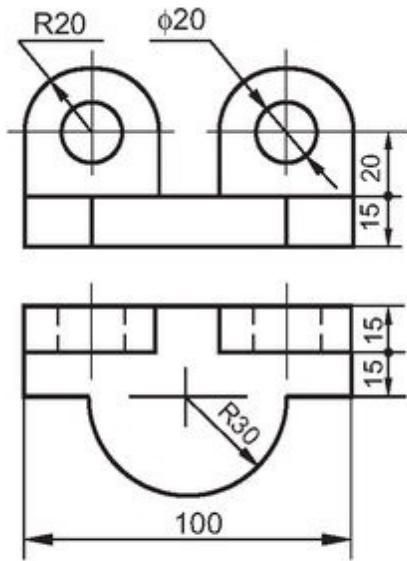
**Fig.16.117**



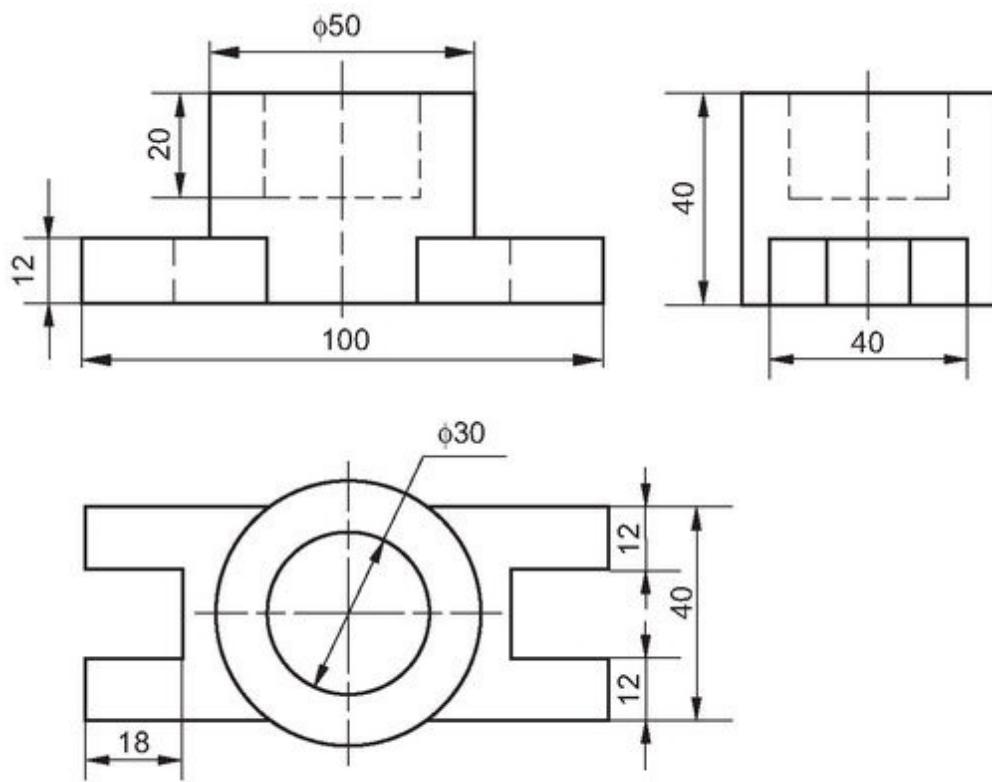
**Fig.16.118**



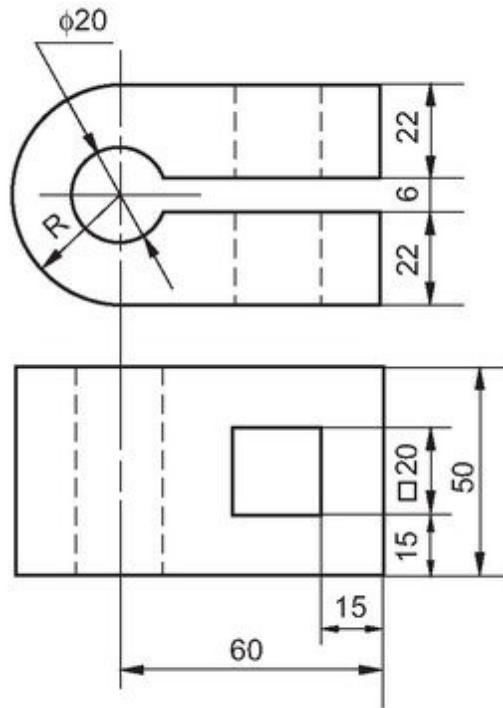
**Fig.16.119**



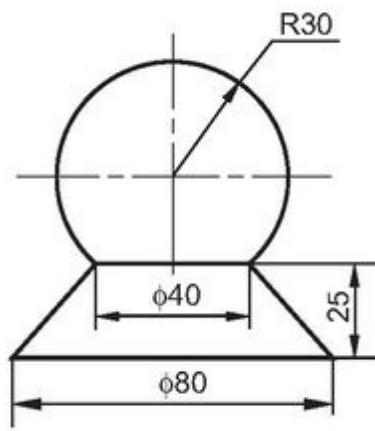
**Fig.16.120**



**Fig.16.121**



**Fig.16.122**



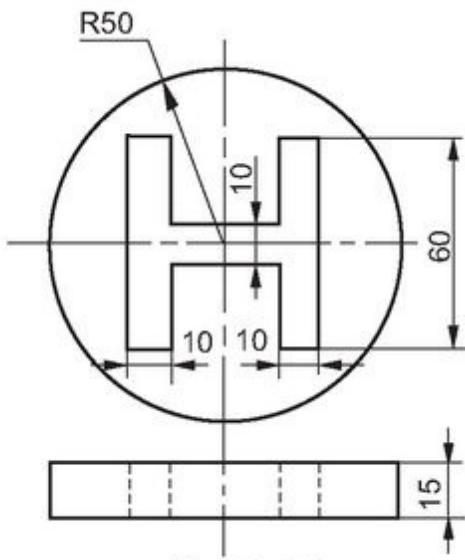
**Fig.16.123**

**Oblique projections**

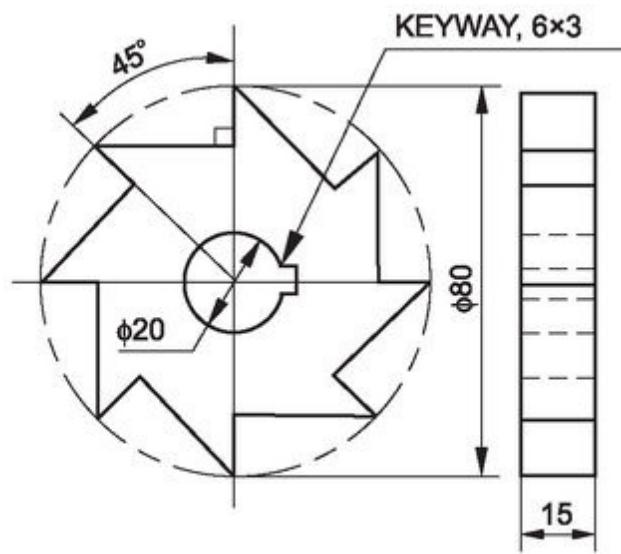
16.40 Figures 16.124 to 16.133 show the orthographic projections of certain objects. Draw the oblique projections of each.

### Perspective projections

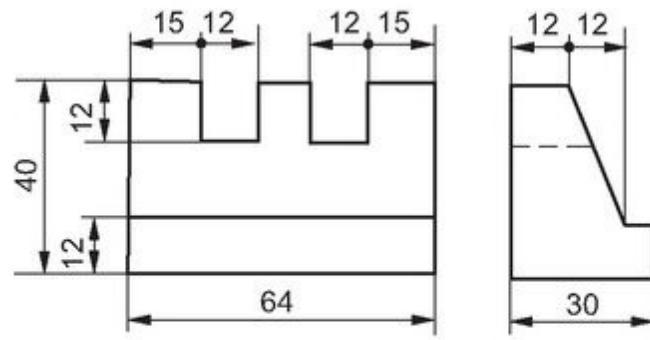
16.41 Draw the perspective view of a point A situated 20 behind the picture plane and 15 above the ground plane. The station point is 30 in front of the picture plane, 40 above the ground plane and lies in a central plane which is 35 to the left of the given point.



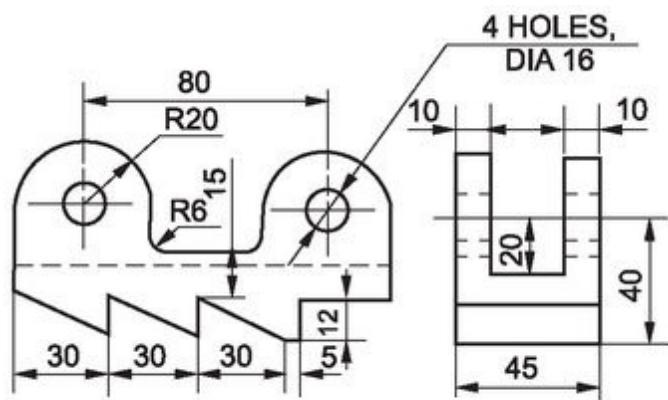
**Fig.16.124**



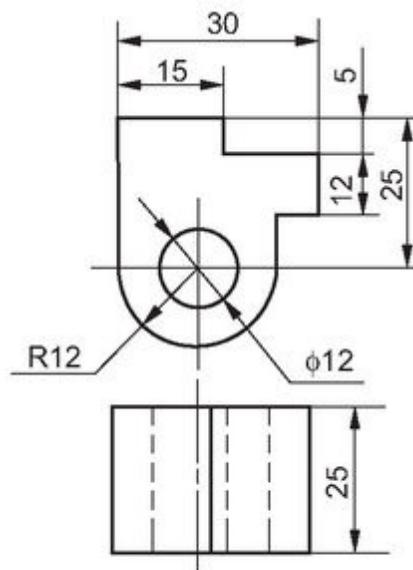
**Fig.16.125**



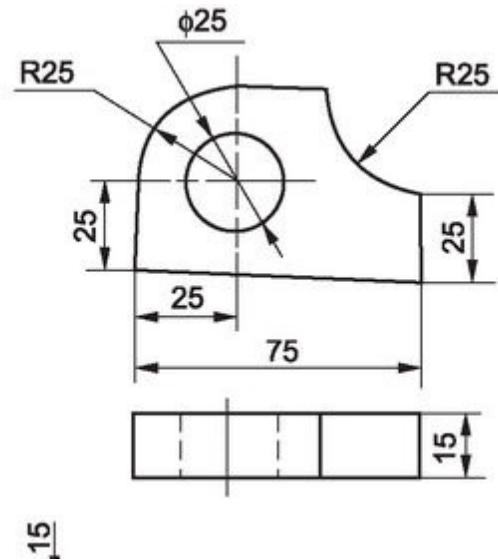
**Fig.16.126**



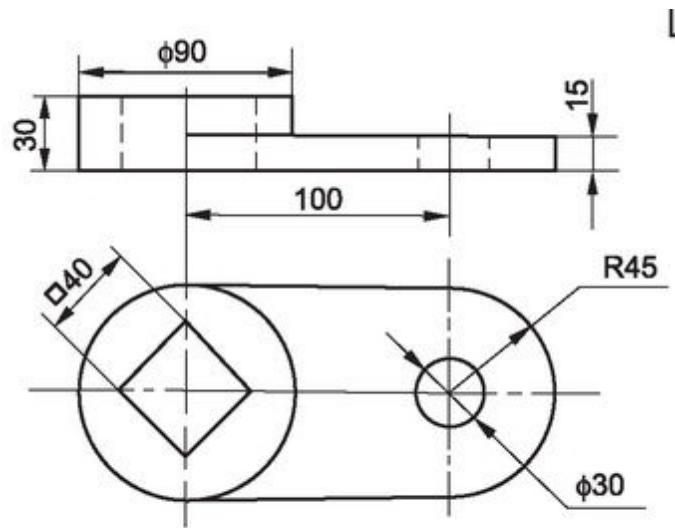
**Fig.16.127**



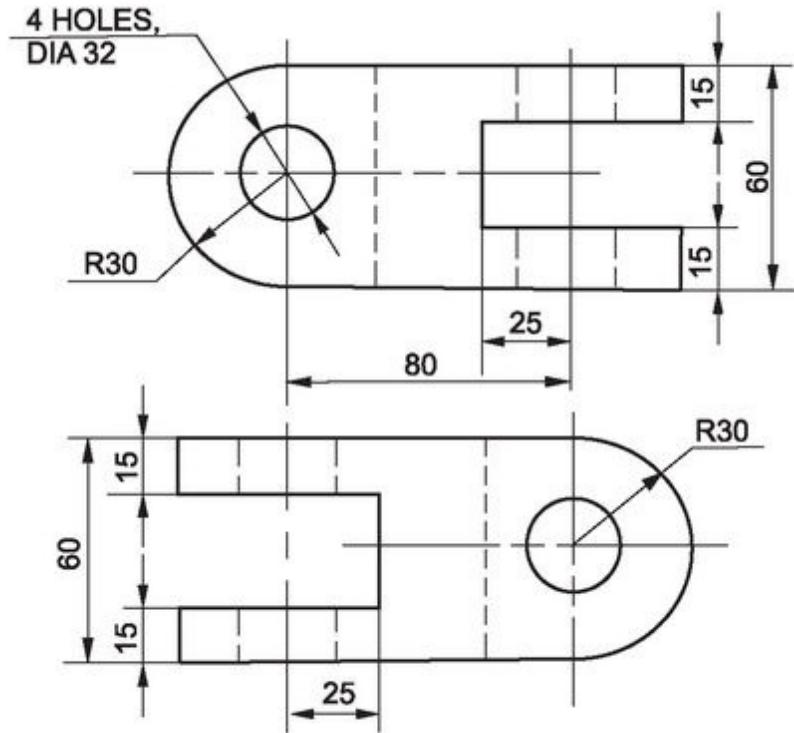
**Fig.16.128**



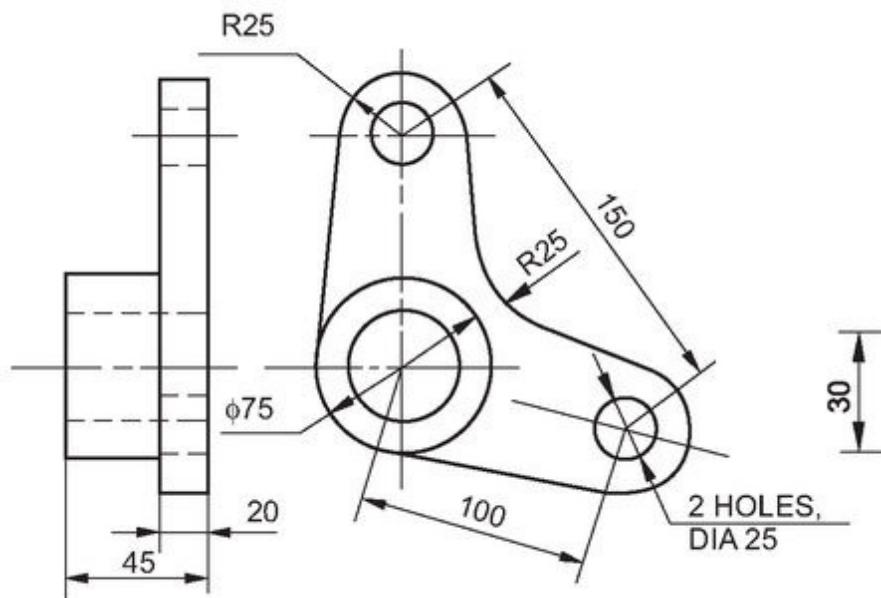
**Fig.16.129**



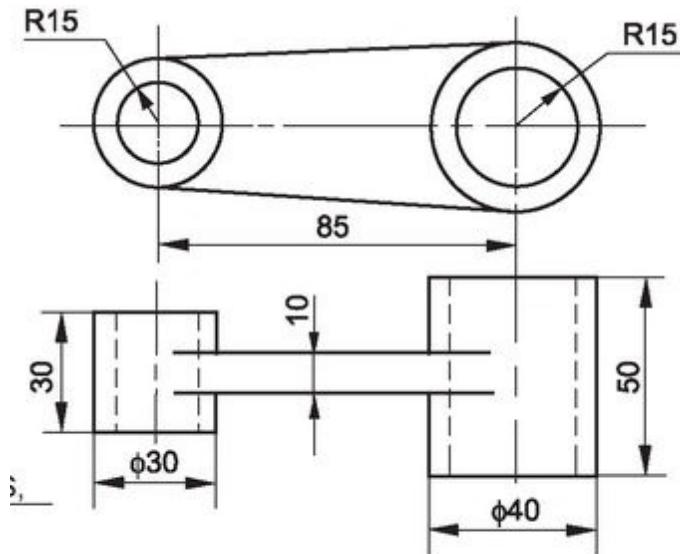
**Fig.16.130**



**Fig.16.131**



**Fig.16.132**



**Fig.16.133**

16.42A straight line AB of 35 long, is parallel to both P.P and G.P. The line is 10 above the G.P and 20 behind the P.P. Central plane is 10 left of A. The station point is 30 in front of picture plane, 25 above the ground plane and lies in the central plane. Draw the perspective view.

16.43Draw the perspective view of a straight line AB, 30 long, which is in the G.P and inclined at  $30^\circ$  to the P.P. End A is in P.P. Central plane is 10 right of A. The station point is 30 in front of P.P, 30 above the G.P and lies in the central plane.

16.44A straight line AB of 40 length, is parallel to G.P and 20 above it. It is inclined at  $45^\circ$  to the P.P. End A is 10 behind the P.P. Central plane is 10 right of A. The station point is 25 in front of P.P, 35 above G.P and lies in the central plane. Draw the perspective view.

16.45Draw the perspective of a straight line AB of 30 long, parallel to G.P and 5 above it. It is inclined at

$30^\circ$  to P.P; the end A being 10 behind P.P. The station point is 40 in front of P.P, 30 above G.P and 10 to the right of A. Solve the problem by (i) visual ray method and (ii) vanishing point method.

16.46 Draw the perspective view of a rectangular plane of  $40 \times 30$ , which lies on the ground plane. One of the corners is touching the picture plane and an edge is inclined at  $55^\circ$  to picture plane. The station point is 30 in front of P.P, 65 above the ground and lies in central plane, which is at a distance of 30 to the right of the corner, touching the P.P.

16.47 A hexagonal plane of side 30, is resting on horizon plane with a corner in P.P and the two sides are equally inclined to P.P. The station point is 40 in front of P.P and on the central line of the plane. The station point is 60 above ground. Draw the perspective projection of the plane.

16.48 A rectangle of size  $30 \times 20$ , has its surface parallel to and 10 above G.P. The shorter edge is inclined at  $60^\circ$  to P.P and the nearest corner is 10 in front of P.P. The station point is 60 in front of P.P and 40 above G.P and in the central plane, passing through the nearest corner to P.P. Draw the perspective of the rectangle.

16.49 A square plane with 60 side, lies on G.P with the edge nearer to he observer lying in the P.P. The station point is 50 in front of P.P, 60 above G.P which is 50 towards the right centre of the object. Draw its perspective view.

16.50 A hexagonal vertical plane of side 25, is resting on an edge on the ground and inclined at  $45^\circ$  to P.P. The nearest corner of the plane is 10 in front of P.P. The station point is in the central plane, passing through

the centre of the lamina and 45 behind P.P and 50 above the G.P. Draw the perspective of the lamina.

16.51 Draw the perspectives of the following solids:

- (i) A hexagonal prism with side of base 25 and axis 80 long, rests on a longer edge on the ground, with one base, 10 behind and parallel to P.P.
- (ii) A hexagonal pyramid of side of base 25 and axis 45 long, rests on its base, parallel to and 5 behind P.P.
- (iii) A hollow cylinder is of 60 external diameter and 80 long, with a wall thickness of 10. It is resting on a generator on the ground, with its axis inclined at  $60^\circ$  to and touching the P.P.
- (iv) A tetrahedron of 50 side, resting on a face on the ground, with an edge parallel to and 10 behind P.P, when, the station points are located,
  - (a) on a central line of the solid, 20 in front of P.P and 75 above the ground,
  - (b) 60 in front of P.P, 50 to the right of the axis of the solid and 55 above the ground, 35 to the left of the centre of the nearest base, 60 in front of P.P and 80 above the ground, and
  - (d) 60 in front of P.P, 80 above the ground and 40 to the left of the axis of the solid.

16.52 A pentagonal prism with side of base 25 and axis 60 long, rests with one of its rectangular faces on the ground such that, a base is touching the P.P. The station point is 20 in front of P.P, 55 above the ground and lies in a central plane which is at 80 to the right of the centre of the prism. Draw the perspective of the solid.

16.53A hexagonal prism of side of base 25 and 50 long, is resting on a rectangular face on the ground with a base in P.P. The station point is 65 above the ground, 60 from P.P and 50 to the right of the axis of the prism. Draw the perspective of the solid.

16.54A hexagonal prism with side of base 40 and height 95, rests on one of its rectangular faces on the ground. The axis is receding away from P.P towards the right and at an angle of  $50^\circ$ . The nearest corner of the rectangular face on the ground is 45 to the left of the eye and 50 inside P.P. The eye is 120 from P.P and 100 above the ground. Draw the perspective of the solid.

16.55A rectangular prism of base  $50 \times 40$  and axis 90 long, has a concentric circular hole of 30 diameter. The solid rests on a base on the ground, with a vertical edge in P.P. The longer edges of the base are inclined at  $30^\circ$  to P.P and towards left. The station point is 30 to the left of the vertical edge (in P.P), 50 in front of P.P and 100 above the ground. Draw the perspective of the prism.

16.56Draw the perspective of a model of steps, having 3 steps of 10 rise and 10 tread, the length being 50. The model is so placed that the vertical edge of the first step is in P.P while the longer edge is inclined at  $30^\circ$  to P.P. The station point is 60 in front of P.P, 50 above the ground and 30 from the vertical edge in P.P.

16.57A square prism with side of base 40 and 20 height, rests with its base on G.P and with a rectangular face in P.P. A square pyramid of 30 height is centrally placed on the prism and the four corners of the pyramid coincide with the midpoints of the edges of

the prism. The station point is 70 in front of P.P, 60 above the ground and 50 to the right of the axis of the pyramid. Draw the perspective of the combination of the solids.

16.58A pentagonal prism with side of base 20 and 40 height, rests with its base on the ground, with one side parallel to P.P. The nearest vertical edge is 5 behind P.P. The station point is 40 in front of P.P and 60 above the ground and lies on the axis of the prism. Draw the perspective of the solid.

16.59Draw the perspective of the following combination of solids: A 25 thick octagonal slab rests with its base on the ground, supporting a square pyramid of base 40 side and 50 height. The corners of the pyramid coincide with the alternate corners of the slab at the top. A rectangular face of the slab is parallel to and 30 behind P.P. The station point is 120 in front of P.P and 40 above the ground and 50 to the right of apex of the pyramid.

16.60A cylinder of base 50 diameter and 75 long, has a coaxial square hole of 25 side. The cylinder is resting on the ground, with its base parallel to P.P and 10 behind it. The faces of the hole are equally inclined to G.P. The station point is 50 to the left of the axis of the solid, 45 in front of P.P and 70 above G.P. Draw the perspective of the solid.

16.61Draw the perspective projection of a cube of 50 side, resting on one of its bases on the ground, with a vertical edge, 20 behind P.P. A vertical face containing the nearest vertical edge is inclined at  $60^\circ$  to P.P. The station point is 10 to the right of vertical edge, 75 above G.P and 20 in front of P.P.

16.62 A square prism of edge of base 40 and 60 long, has a concentric circular hole of 30 diameter. It is resting on its base on G.P, with a vertical face inclined at  $30^\circ$  to P.P. The nearest vertical edge is in P.P. The station point is 50 above G.P, 80 in front of P.P and in the central plane passing through the axis of the solid. Draw the perspective of the prism.

16.63 Draw the perspective of a pentagonal prism of side of base 20 and 40 long, resting on one of the rectangular faces on G.P, with its axis inclined at  $60^\circ$  to P.P, with a corner of the base in P.P. The station point is 60 in front of P.P and in the central plane, bisecting the axis. The horizon is at the level of the top edge of the prism.

16.64 A hexagonal prism of side of base 25 and 50 long, has a concentric circular hole of 40 diameter. It is resting on one of its longer edges on G.P such that, two rectangular faces are vertical and inclined at  $45^\circ$  to P.P. The nearest vertical edge of the base is 10 behind P.P. Draw perspective of the prism, when the station point is 50 in front of P.P, 60 above G.P and 20 to the left of the nearest edge.

16.65 A circular slab with diameter of base 120 and thickness 50, rests with its base on the ground. The axis of the slab is 80 behind P.P and 80 to the left of the eye. The eye is 160 from P.P and 120 above the ground. Draw the perspective.

16.66 Draw the perspective of a cylinder of 40 diameter and 50 height, lying on the ground, with its axis perpendicular to P.P and one of its ends touching P.P. The eye is 35 to the left of the axis of the cylinder, 50 in front of P.P and 60 above the ground.

16.67 A square pyramid of side of base 30 and 50 long, rests centrally on the top of a square prism, with side of base 50 and 20 long. An edge of the base of both the solids is inclined at  $30^\circ$  to P.P and the nearest corner is 20 from it. The station point is on the central plane passing through the apex of the pyramid and is located 80 in front of P.P and 60 above G.P. Draw the perspective of the solids.

16.68 Draw the perspective of a rectangular pyramid of base  $30 \times 20$  and axis 35 long, when resting on the ground on its base, with the longer edge parallel to and 30 behind P.P. The central plane is 30 to the left of the apex and the station point is 50 in front of P.P and 30 above G.P.

16.69 A rectangular pyramid of base  $30 \times 20$  and 45 height, rests with its base on the ground. One longer edge of the base is parallel to P.P and 30 behind it. The station point is 50 in front of P.P and 25 to the left of the axis of the solid and 50 above the ground. Draw the perspective projection.

16.70 A pentagonal pyramid of side of base 30 and 60 height, rests on its base on the ground, with a corner in P.P. The edge of the base opposite to the corner, is parallel to P.P. Draw the perspective of the solid, if the station point is on the central line, 90 in front of P.P and 75 above the ground.

16.71 A triangular pyramid of base edges 40 long and axis 70, is resting on one of the base edges on the ground with the base being parallel to the P.P. The apex is nearer to the P.P and 20 behind it. The station point is 50 to the right of the axis and 60 from P.P. The horizon is 70 from the ground. Draw the perspective view of the object.

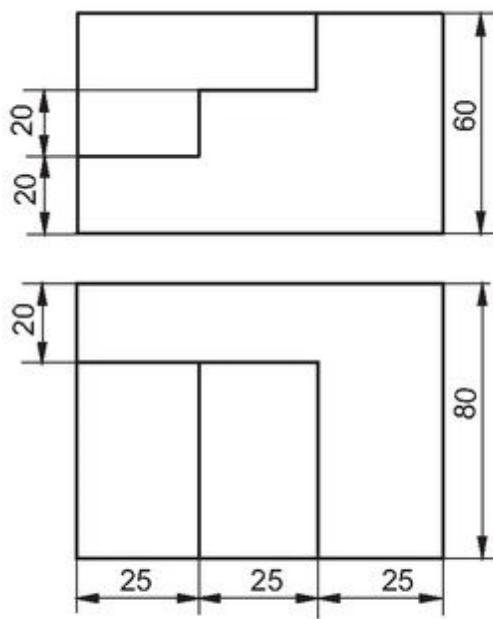
16.72 A hollow cylinder of base diameter 50 and axis 80 long, is resting on one of its generators on the ground with the axis making an angle  $30^\circ$  to the P.P ad leaning towards right. The thickness of the cylinder is 10. A point on the circumference of the outer circle of the base nearest to the P.P is 20 behind it. The station point which is on the central line of the axis is 60 from P.P and 70 above the ground. Draw the perspective projection of the object.

16.73 Draw the perspective projection of a rectangular block of  $3m \times 2m \times 1.5m$ , resting on a horizontal plane with one of its sides of the rectangular plane making an angle of  $45^\circ$  with V.P. The observer is at a distance of 6m from the P.P. Assume eye level as 1.5m.

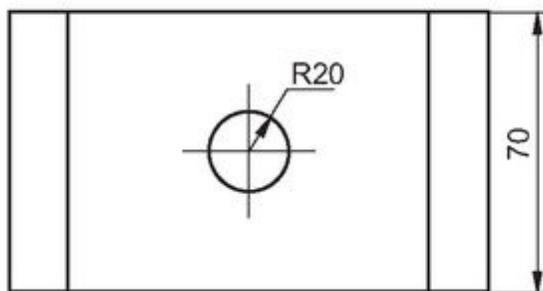
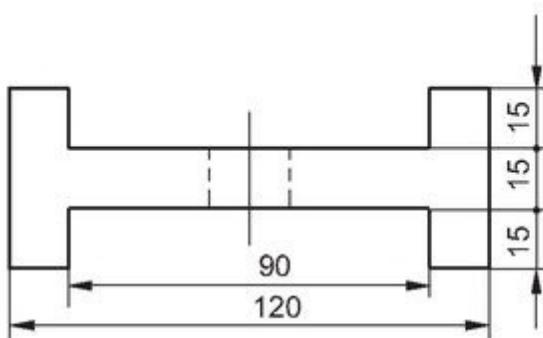
16.74 A composite solid is in the form of a square prism of side of base 50, upto a height of 85 and tapers into a frustum of a square pyramid, whose top surface is a square of 25 side. The total height of the solid is 125. Draw the solid in perspective, when it is resting on its base on the ground, with a side inclined at  $30^\circ$  to P.P and a corner containing the side is in P.P and 50 to the right of the station point. The eye is 175 in front of P.P and 150 above the ground.

16.75 Figures 16.134 to 16.138 represent the orthographic projections of certain objects. Draw the perspective of each.

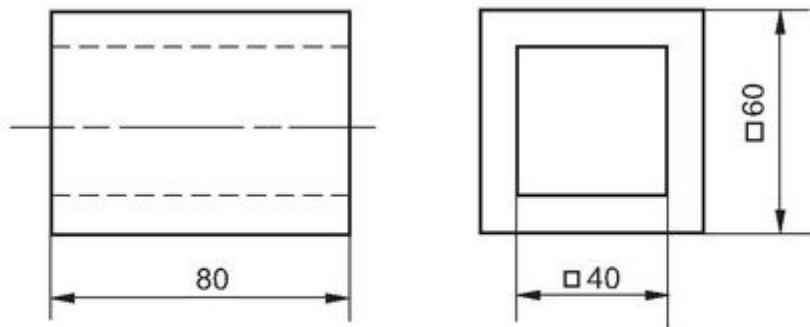
**HINT** Draw the top view such that, one edge coincides with P.P. Select the station point that will represent the shape of the object in the best possible manner.



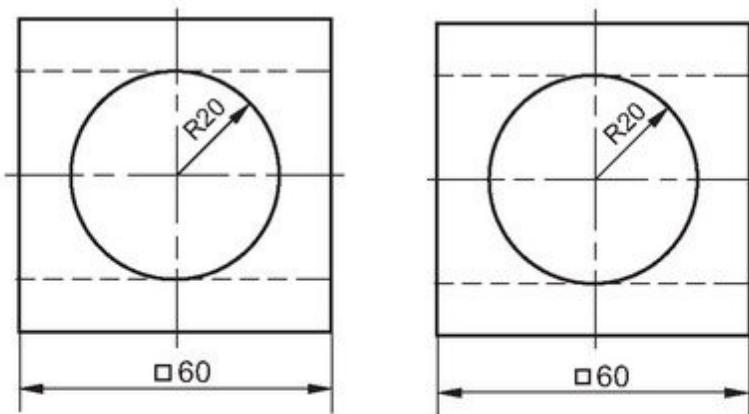
**Fig.16.134**



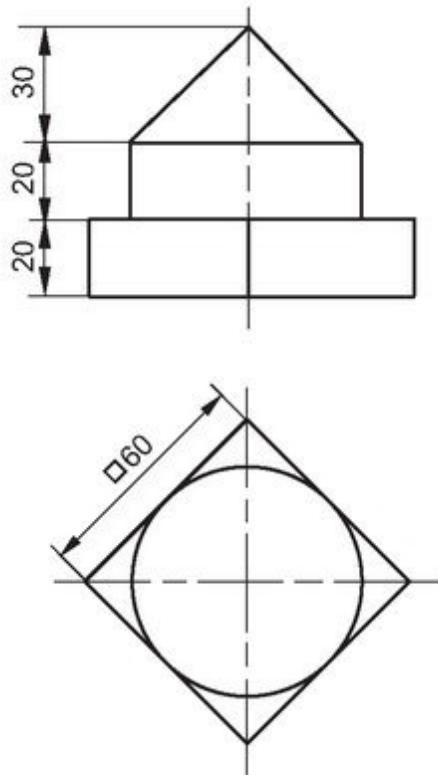
**Fig.16.135**



**Fig.16.136**



**Fig.16.137**



**Fig.16.138**

16.76A 25 thick octagonal slab rests with its base on the ground and supports a square pyramid of 50 high and edge of base 40 on it such that, each corner of the base of the pyramid rests on a top corner of the slab. Draw the perspective projection of the arrangement with the axis of the pyramid 75 behind the P.P and 60 to the left of the eye. One of the rectangular faces of the octagonal slab is parallel to P.P. The eye is 125 in front of the P.P and 100 above the ground.

16.77Draw the perspective projection of a shed with one corner of the longer side of the roof touching the P.P at a point. The eye is 5m in front of the point, touching the P.P and 2m above the G.P. The roof of

the shed is supported on four pillars of  $50 \times 50 \times 6$ m high. The roof comprises of two rectangular surfaces of  $15\text{m} \times 5\text{ m}$  inclined mutually at  $120^\circ$ . Assume that the outer surfaces of the pillars are in flush with the sides of the roof at the corners.

16.78A man stands at a distance of 5m from a flight of four stone steps having a width of 2m, tread 0.3m and rise 0.2m. The flight makes an angle of  $30^\circ$  with the PP and touches the same at a distance of 2m to the right of the C.V. Take horizon level to be 1.5m above the ground level. Draw the perspective projection of the flight.

## REVIEW QUESTIONS

- 16.1 What is the purpose of pictorial drawings?
- 16.2 Mention some of the applications of pictorial drawings.
- 16.3 Name and explain the different forms of pictorial projections.
- 16.4 Differentiate between isometric, non-isometric (i) lines and (ii) planes.
- 16.5 What is the difference between isometric projection and isometric drawing?
- 16.6 Describe the four-centre method of drawing isometric projection of a circle.
- 16.7 What is the principle followed in the box method of obtaining the isometric projection of an object?
- 16.8 Differentiate between isometric and oblique projections.

- 16.9 List out the rules to be followed while drawing oblique projections of an object.
- 16.10 When will the size of the perspective be larger than the object?
- 16.11 What is a normal perspective?
- 16.12 What is the main difference between a perspective and other types of pictorial projections?
- 16.13 Define the terms (i) station point, (ii) horizon, (iii) ground line, (iv) axis of vision, (v) centre of vision and (vi) vanishing point.
- 16.14 What are the rules to be followed while selecting a station point?
- 16.15 What is a parallel perspective?
- 16.16 When does one require measuring lines?

## OBJECTIVE QUESTIONS

- 16.1 In isometric projection, the three principal axes of the object will be equally/ unequally fore-shortened.
- 16.2 Isometric projection is preferred for \_\_\_\_ size objects and perspective is for \_\_\_\_ size objects.
- 16.3 The ratio between the isometric and true length is  
(a)  $2/\sqrt{3}$ , (b)  $\sqrt{2}/3$ , (c)  $\sqrt{2}/\sqrt{3}$ .  
( )
- 16.4 The angle between isometric axes is (a)  $90^\circ$  (b)  $120^\circ$  (c)  $60^\circ$ . ( )
- 16.5 In isometric projection, dimension lines are drawn parallel to \_\_\_\_ \_\_\_\_.

- 16.6 A circle in isometric projection appears as \_\_\_\_\_.  
16.7 Isometric projection of a sphere is a circle having a diameter equal to \_\_\_\_\_ of sphere.  
16.8 Isometric scale must be used while drawing isometric views of spherical objects.  
(True/False)  
16.9 A picture taken with a camera is a real isometric projection. (True/False)  
16.10 When the object is above the horizon, the perspective projection reveals the details of the top face of the object.  
(True/False)  
16.11 The vanishing point is located (a) above horizon, (b) below horizon, (c) on the horizon.  
( )  
16.12 In oblique perspective, the number of vanishing points are: (a) one, (b) two, (c) three.  
( )  
16.13 The perspective of a circle is a circle/ line, when its plane is inclined /parallel to P.P.  
16.14 Concentric circles appear as concentric ellipses in perspective projection.  
(True/ False)

## ANSWERS

- 16.1 equally  
16.2 small; large

- 16.3 c
- 16.4 b
- 16.5 isometric axes
- 16.6 ellipse
- 16.7 true diameter
- 16.8 True
- 16.9 False
- 16.10 False
- 16.11 c
- 16.12 c
- 16.13 circle; parallel
- 16.14 False

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## 17.1 INTRODUCTION

Free-hand sketching is one of the effective ways employed to communicate ideas and designs. This is the primary language of the scientists and engineers, who are incharge of design and development of various projects. This is also defined as thinking with a pencil.

It is usually assumed that, skill in sketching may be acquired more easily than proficiency in instrumental drawings. This is not so because, both in sketching and instrumental drawings, the same basic principles of drawing must be employed. A lot of effort and practice are required to sketch say, two parallel lines, a circle, etc., than to draw the same. A student is therefore advised to acquire the sketching skills through constant effort and practice because later in his career, he must be in a better position to express his ideas through, on the spot sketches, not only to his superiors but also to his sub-ordinates.

## 17.2 USE OF SKETCHES

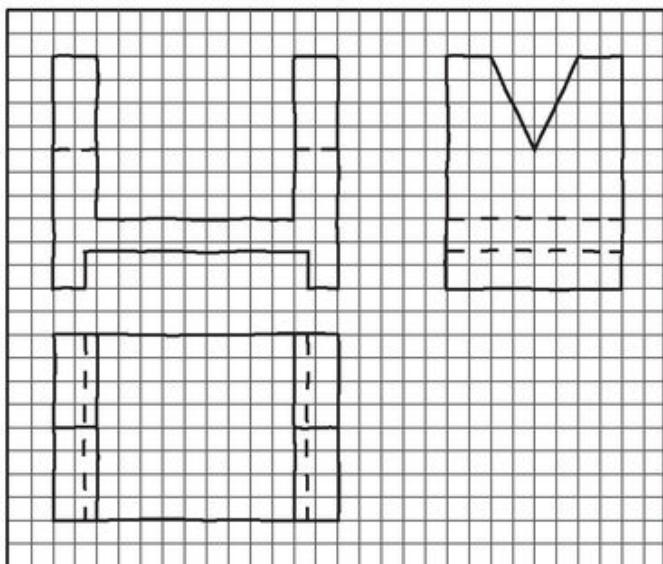
Free-hand sketches of technical nature are employed for a variety of purposes, the following being the important ones:

1. To convey information regarding a repair or modification needed in an existing structure or machine
2. To help the designer in developing new ideas
3. To convey the ideas of the designer to the draughtsman
4. To present the ideas of the designer to the management
5. To serve as a basis for discussion between engineers and workmen
6. To serve as teaching aid during discussion in the classroom

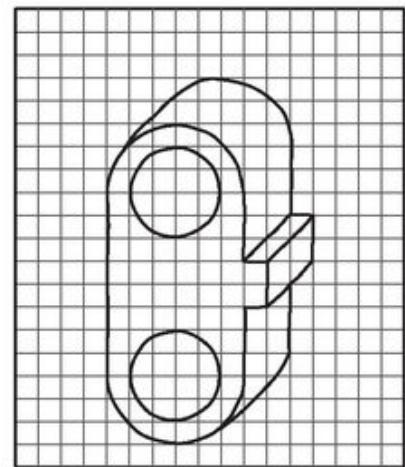
### **17.3 MATERIALS FOR SKETCHING**

A fairly soft pencil of the grade HB is preferred for sketching works. The pencil used for sketching work should have a conical point. A good quality eraser that will not spoil the paper is also necessary. A sketch pad of plain unruled paper is sufficient for sketching purposes, because this helps the student in achieving accuracy of observation, a good sense of proportion and, sureness in handling the pencil.

Rectangular co-ordinate papers, isometric ruled papers and perspective grids are also available, either to produce dimensioned free-hand sketches or to provide guidelines for sketching (refer [Figs.17.1 to 17.3](#)).

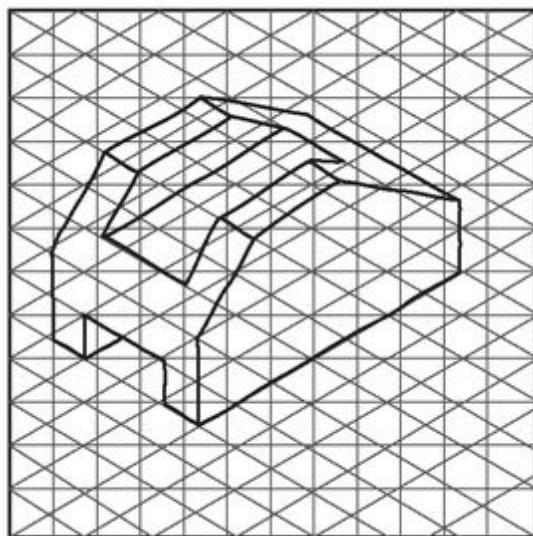


a

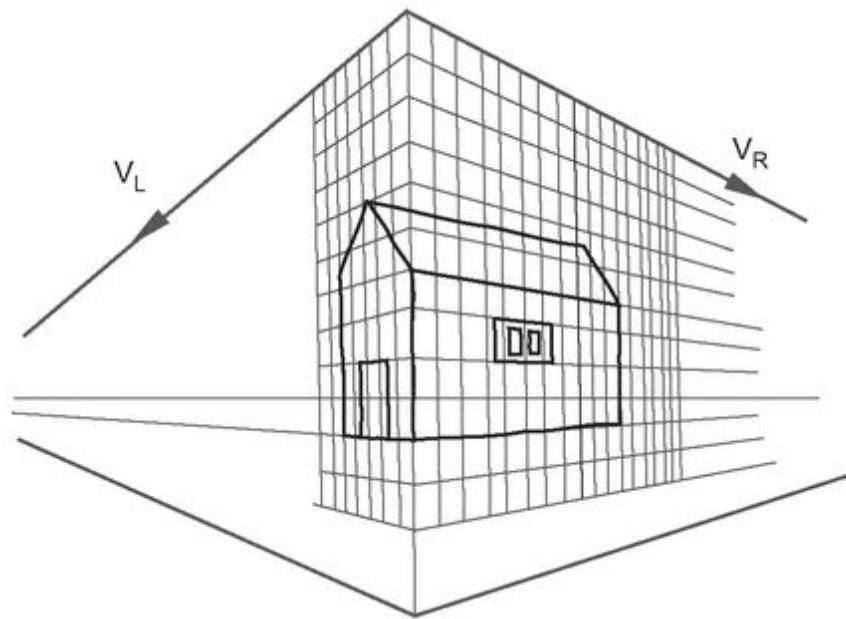


b

**Fig.17.1 Rectangular co-ordinate paper used for sketching**



**Fig.17.2 Isometric paper used for sketching**



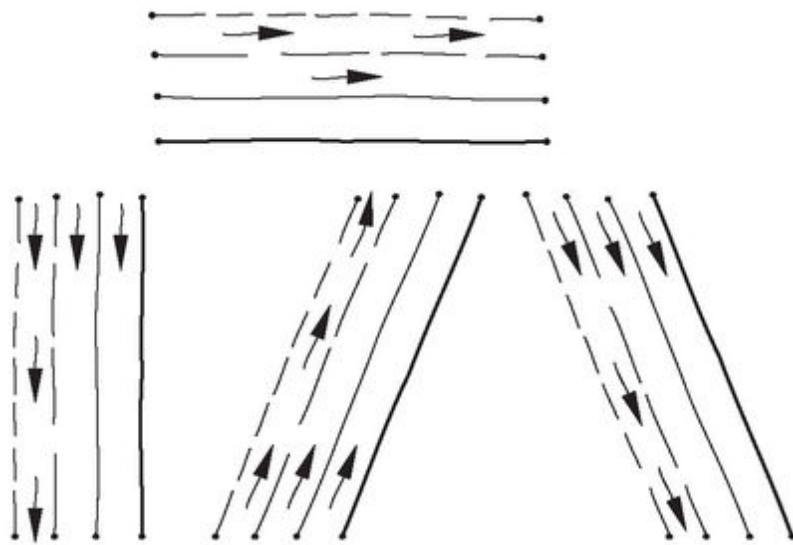
**Fig.17.3 Perspective grid used for sketching**

## 17.4 SKETCHING STRAIGHT LINES

The shapes of objects are made-up of flat and curved surfaces, which are represented by straight and curved lines respectively. The student must be able to sketch these elements rapidly and accurately.

The lines may be either horizontal, vertical or inclined. [Figure 17.4](#) shows the directions in which these lines must be sketched to attain the required straightness.

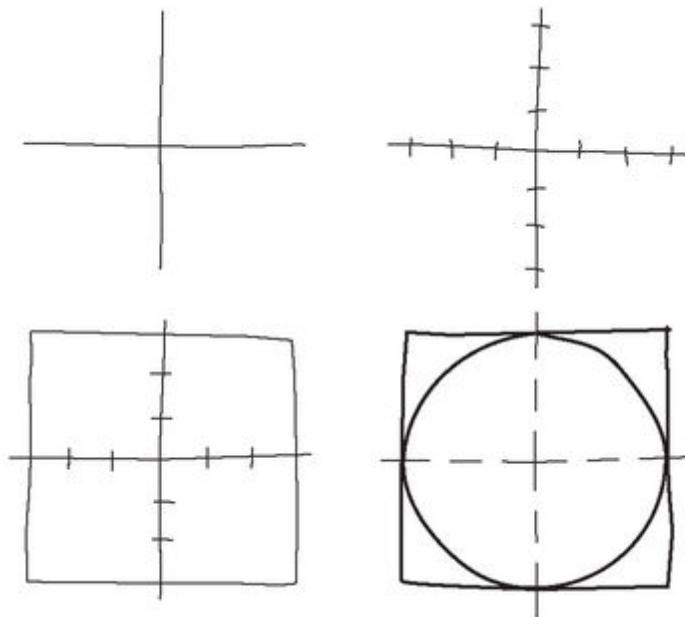
The first step in sketching any straight line is to determine and mark the end points of the line. A light line with one or series of strokes may then be tried between the end points. Any waviness or roughness if present, may then be corrected and finally darkened to the required extent.



**Fig.17.4 Sketching of straight lines**

## 17.5 SKETCHING A SQUARE

This also forms an exercise on straight lines. This is also useful in sketching circles or arcs of circles. [Figure 17.5](#) shows the steps involved in sketching a square.



**Fig.17.5 Sketching of a square / circle - Method I**

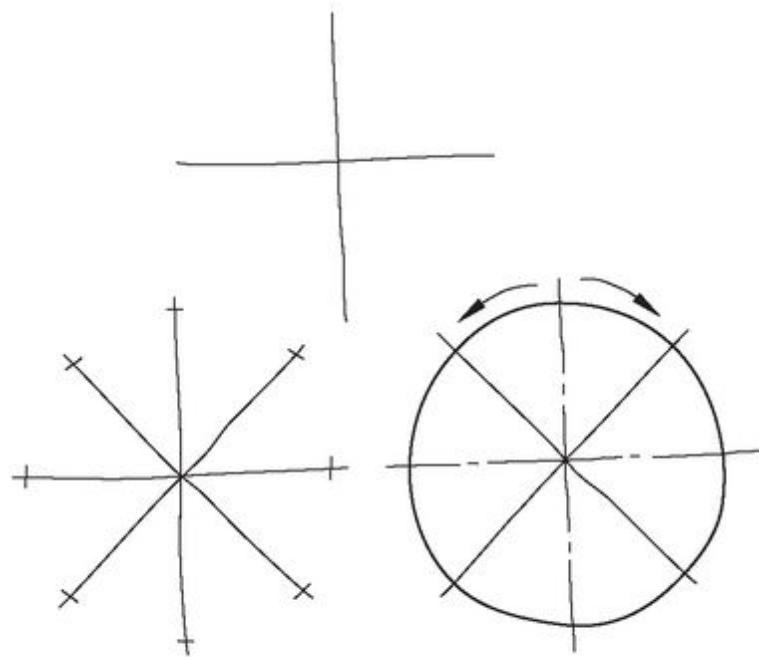
## 17.6 SKETCHING CIRCLES

Small circles and arcs may be sketched in either one or two strokes, without the help of any guiding blocks. However, it is better to follow some systematic method for sketching larger circles and arcs.

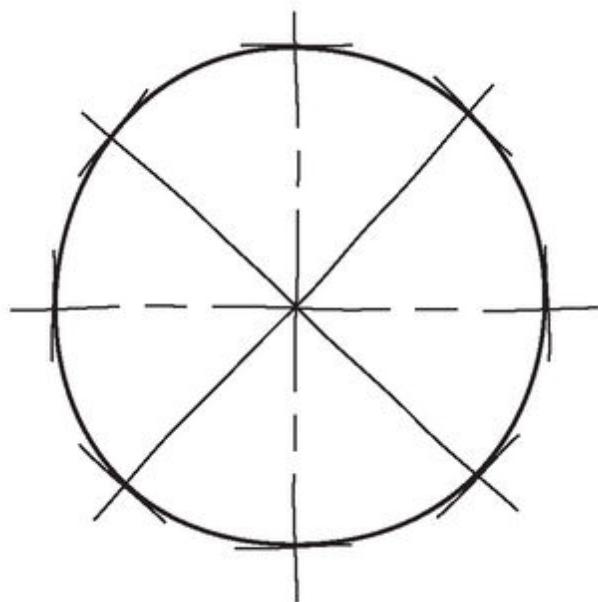
**Method I** The radius of the circles is estimated and marked-off on horizontal and vertical centre lines and a square is completed, with its side being equal to the diameter of the circle. The sketching of the circle is then completed with the help of the arcs drawn, tangent to the sides of the square, at the mid-points (refer [Fig.17.5](#)).

**Method II** Radial lines are sketched, preferably eight in number, with the help of four straight lines, as shown in [Fig.17.6](#). Along each radial line, the radius of the circle is estimated and marked-off. A smooth curve passing through these points, results in the required circle.

For sketching very big circles, the method shown in [Fig.17.7](#) may be followed.



**Fig.17.6 Sketching of a circle-Method II**



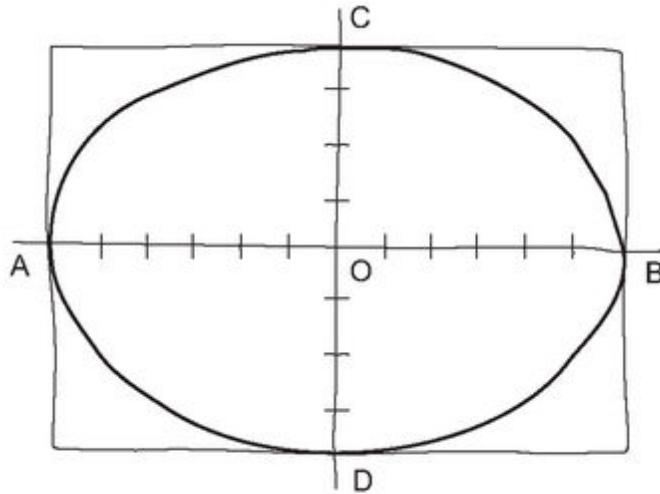
**Fig.17.7 Sketching of a large circle**

## 17.7 SKETCHING AN ELLIPSE

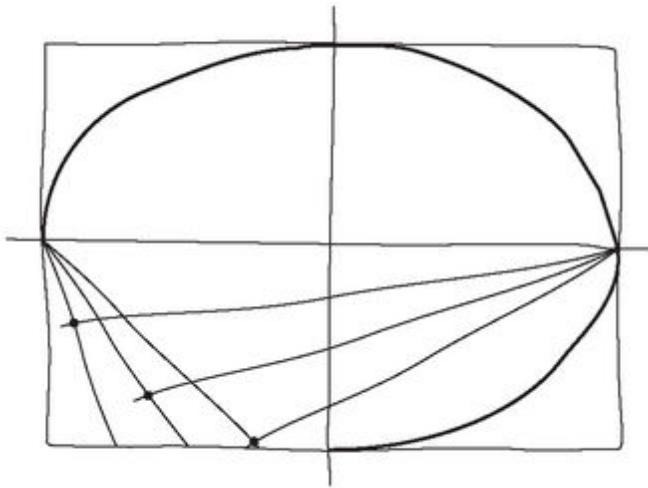
Ellipses may form part of the draughting work, either in the orthographic views or in the pictorial representations. Following are the steps for sketching an ellipse, as shown in [Fig.17.8](#):

1. Draw two lines at right angle to each other and mark by estimation, the semimajor OA (=OB) and semiminor OC (=OD) axes.
2. Complete the rectangle with the major and minor axes as sides.
3. Sketch the arcs tangentially at the mid points of the sides, resulting in the required ellipse.

For a more accurate work, additional points on the curve may be obtained, as shown in [Fig.17.9](#).

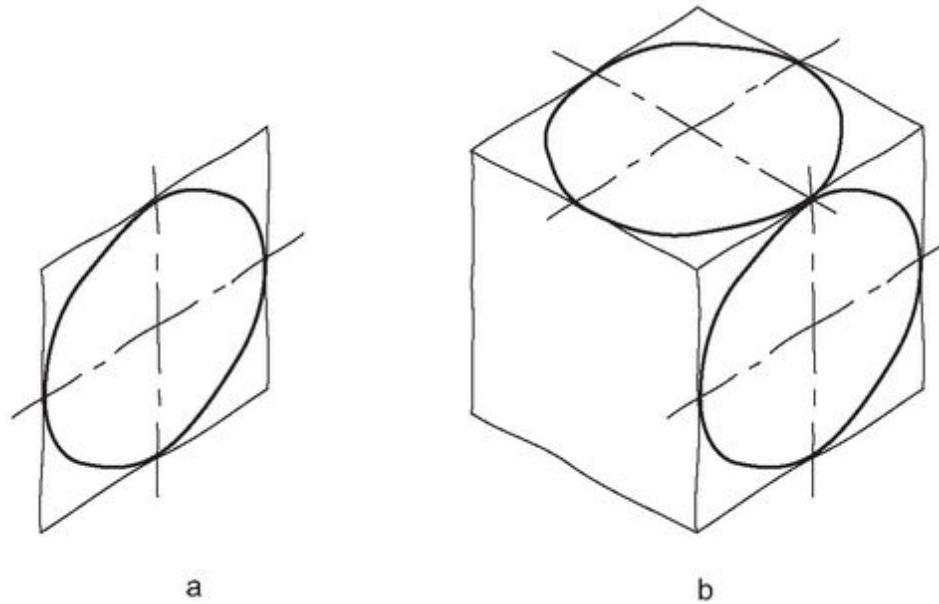


**Fig.17.8 Sketching of an ellipse**



**Fig.17.9 Method of obtaining additional points on an ellipse**

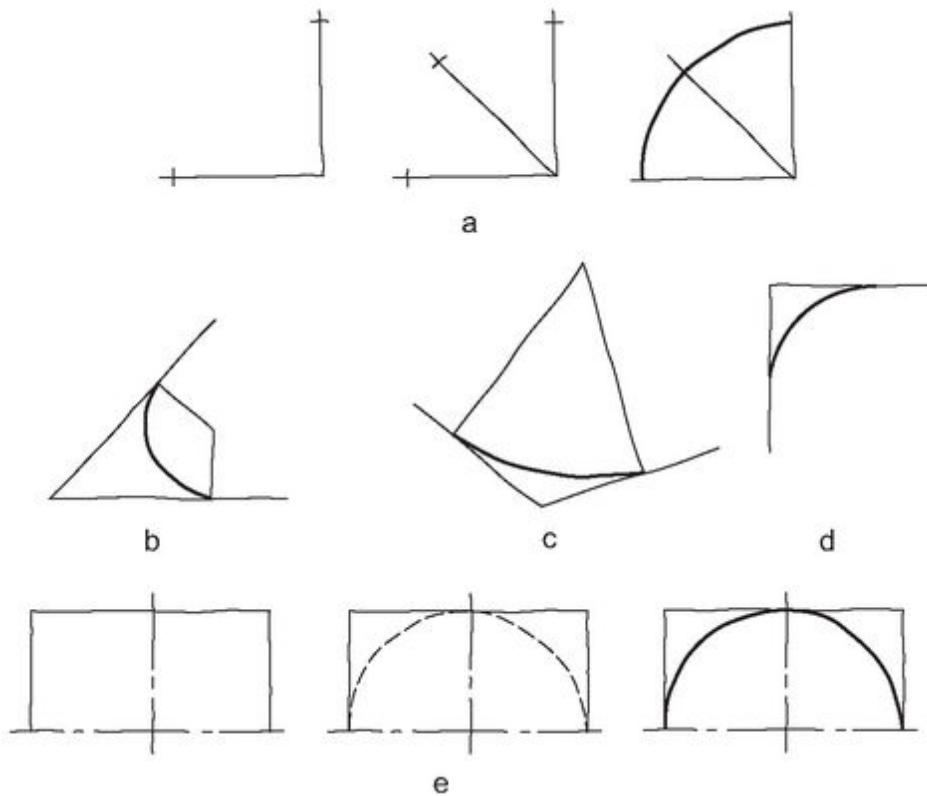
Circular features in all the types of pictorial projections, except those contained by the front face of an object in oblique projection, appear as elliptical features. Ellipses in a pictorial projection may be sketched, by enclosing it in a parallelogram, representing the square, super-scribing the original circle. Here also, as shown in [Fig.17.10](#), the ellipse consisting of arcs is tangential to the sides of the parallelogram at their mid-points.



**Fig.17.10 Sketching of an ellipse in pictorial projection**

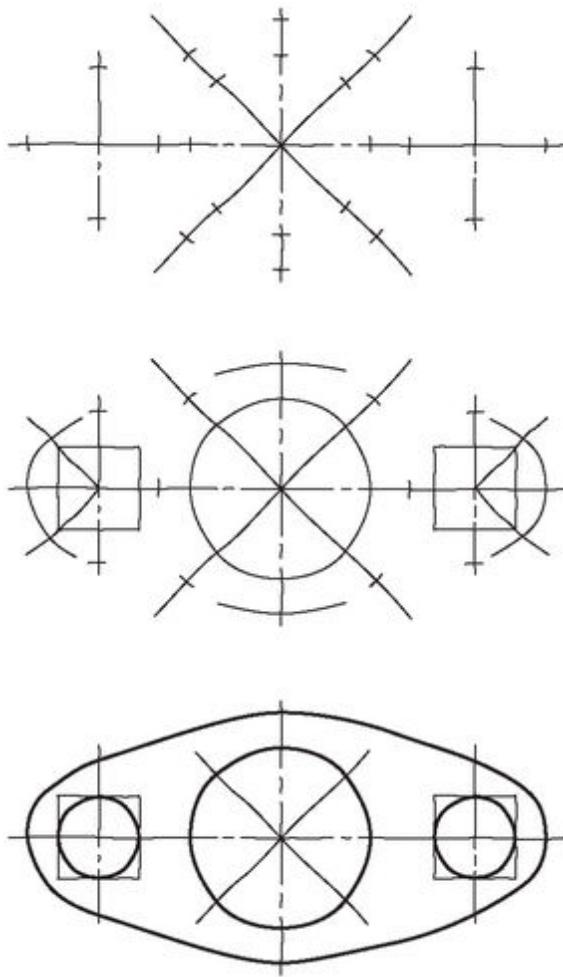
## 17.8 CIRCULAR ARCS

Circular arcs are sketched in a manner, similar to that adopted for sketching circles. [Figure 17.11](#) shows the constructions to obtain certain circular arcs. While sketching tangent arcs, one should have in mind, the actual geometrical constructions involved, so as to get a better result.



**Fig.17.11 Sketching of arcs**

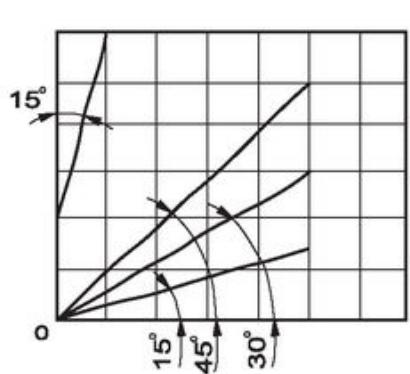
Figure 17.12 shows the stages involved in sketching a figure having circles and arcs.



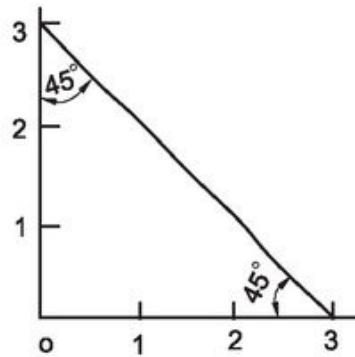
**Fig.17.12 Steps in sketching an orthographic view**

## 17.9 ANGLES

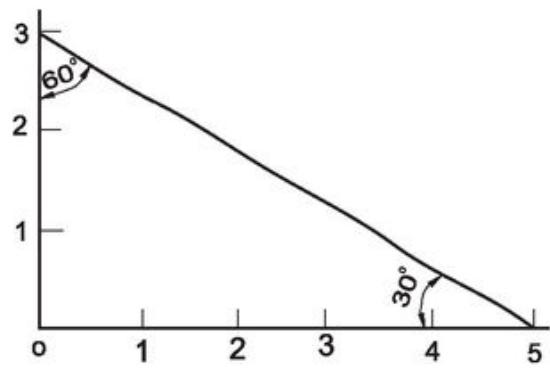
Figure 17.13 b, c and d show the methods of sketching inclined lines, making some commonly used angles. Figure 17.13a shows the way of using a co-ordinate paper to obtain angles of 15, 30, 45, 60 and 75 degrees, with sufficient accuracy.



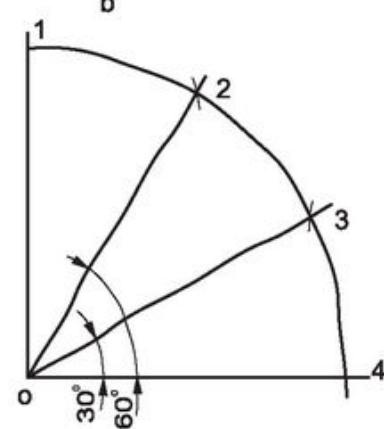
a



b



c



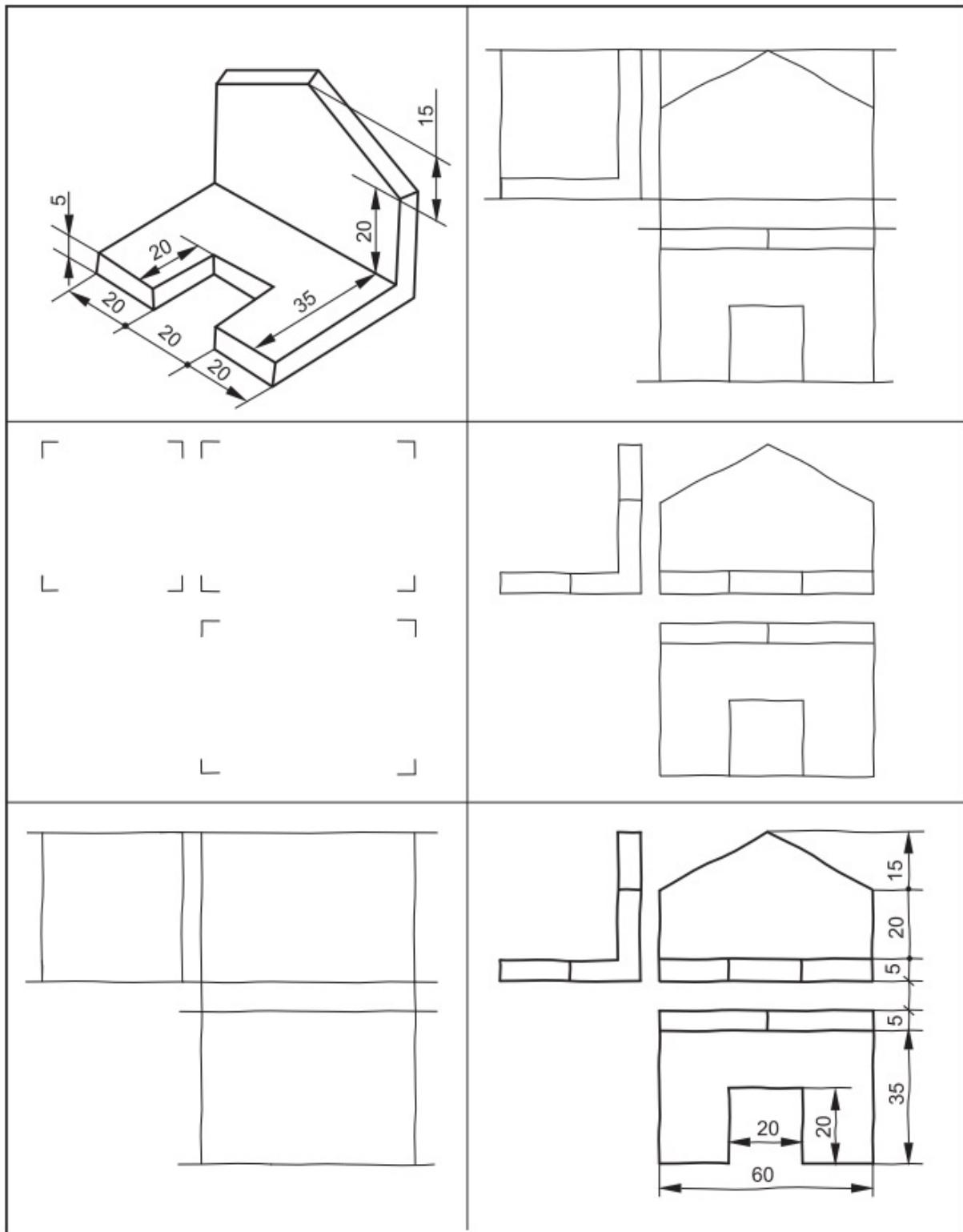
d

**Fig.17.13 Sketching of angles**

## 17.10 SKETCHING ORTHOGRAPHIC VIEWS OF AN OBJECT

Very often, when repairs are to be made or changes are proposed in an existing machine, the production shop is asked to produce some parts. This is done, by supplying shop drawings in the form of orthographic views. The following are the steps in sketching the orthographic views of an object (refer Fig.17.14):

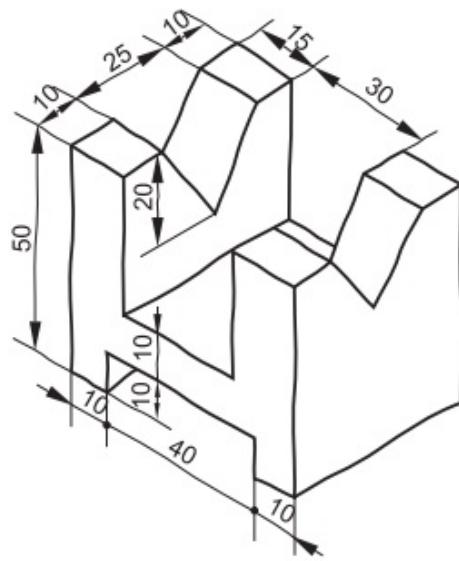
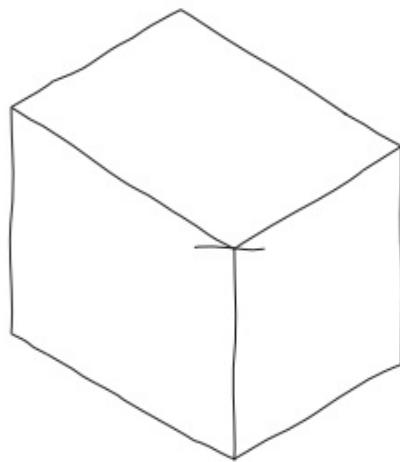
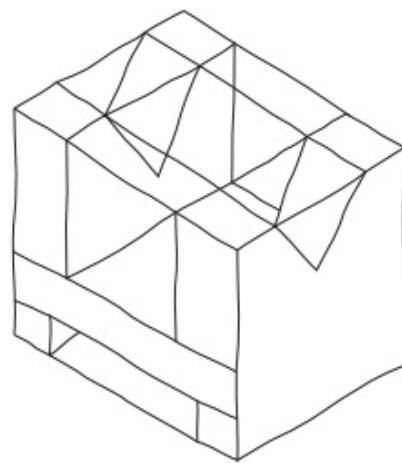
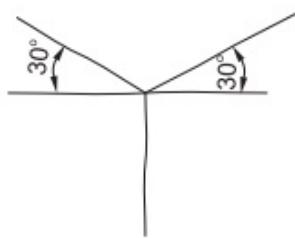
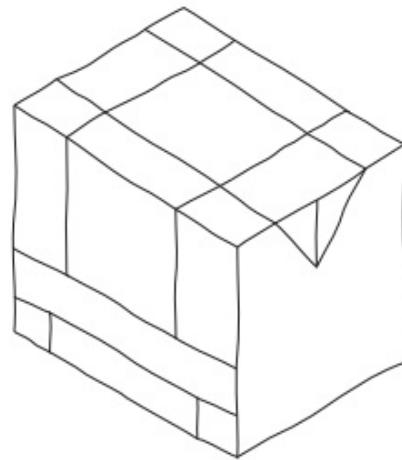
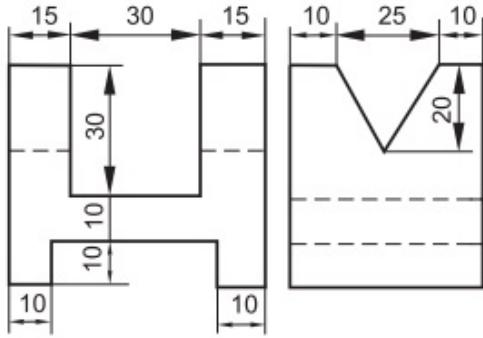
1. Observe the object critically (in case changes being proposed, have the change in mind clearly).
2. Decide the necessary views.
3. Estimate the proportions carefully.
4. Sketch lightly, the overall size of each view as a rectangular or square block, keeping sufficient distance between the views.



**Fig.17.14 Sketching orthographic views from an isometric view**

5. Add lines to represent the details in each view.
6. Complete each view, by darkening all the lines, forming the view.
7. Dimension the views and add the necessary notes.

## **17.11 SKETCHING ISOMETRIC VIEWS**



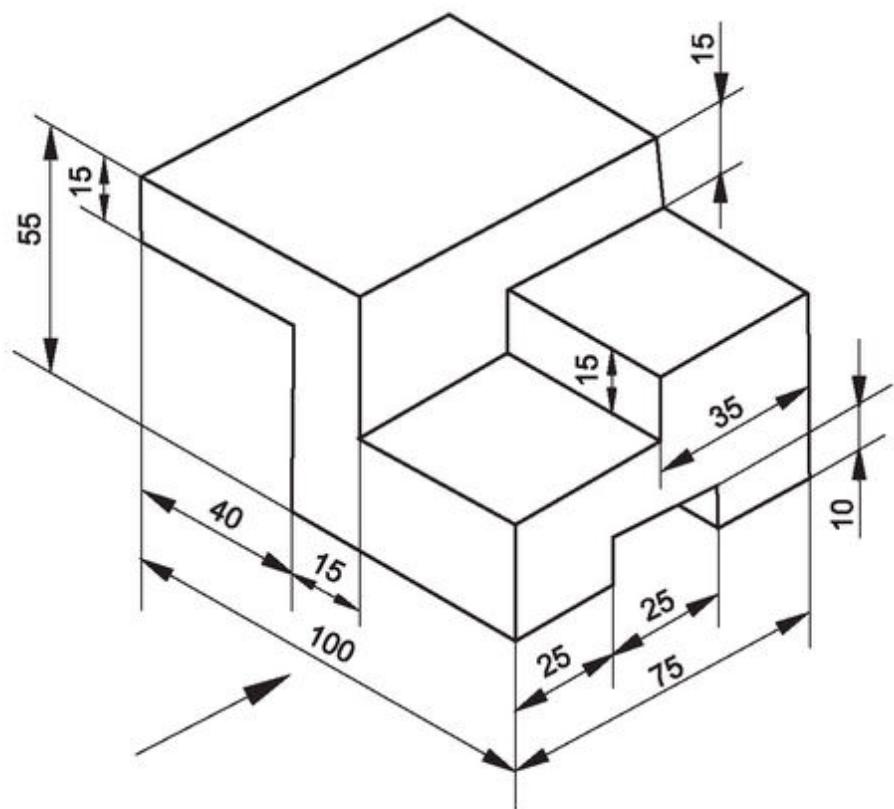
### **Fig.17.15 Sketching isometric view from orthographic view**

For a designer, while developing new ideas, isometric sketching plays an important role in arriving at a final acceptable solution. The designer will be in a better position to incorporate his ideas, when he has his preliminary isometric sketch in front of him. He goes on modifying the figure, till he is satisfied about the arrangement and function of the parts involved in the unit. The following are the steps involved in sketching an isometric, from the orthographic views (refer Fig.17.15):

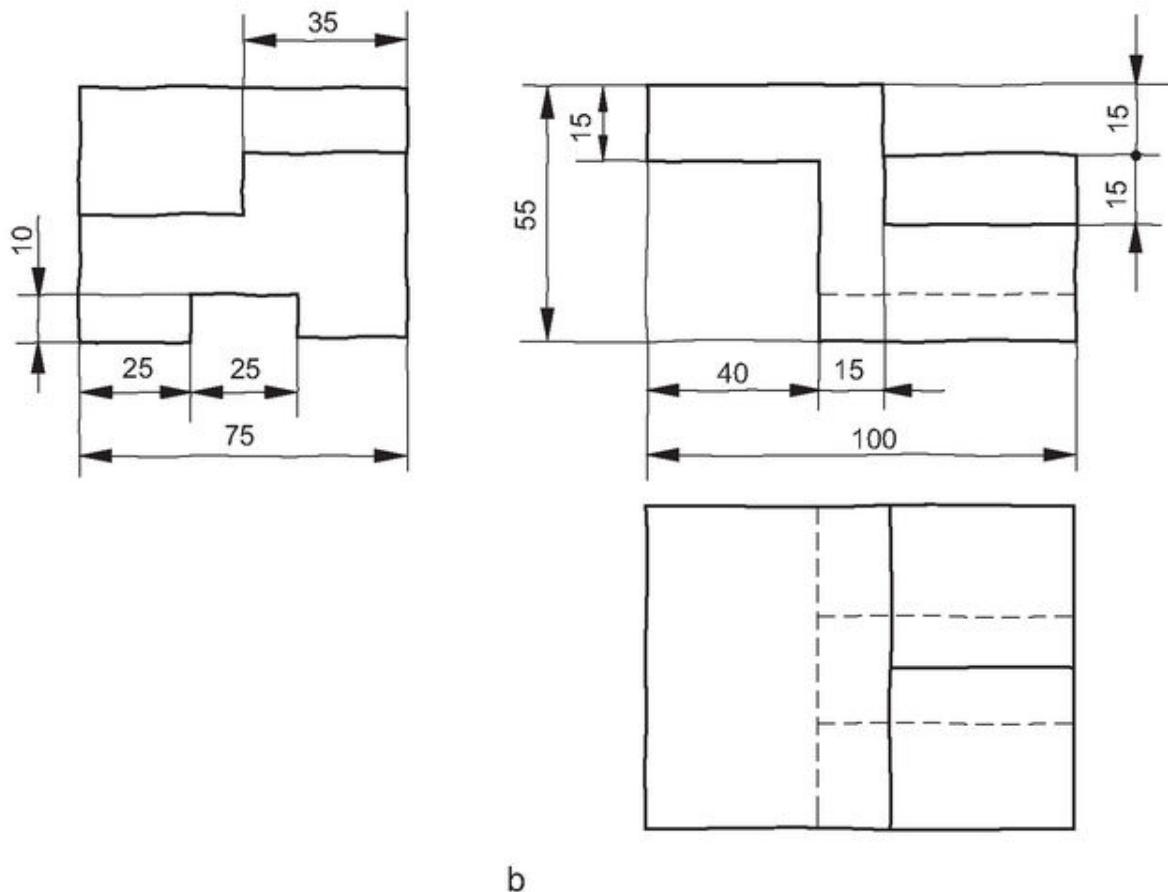
1. Study the views critically.
2. Sketch the isometric axes.
3. Estimate and mark-off the principal dimensions along the isometric axes.
4. Sketch the box lightly, enclosing the principal dimensions.
5. Lay-off the distances in each face, to locate all the features.
6. Sketch the lines through these points and parallel to the isometric axes.
7. Complete the view, by darkening the required lines.
8. Dimension the view and add the necessary notes.

## **17.12 EXAMPLES**

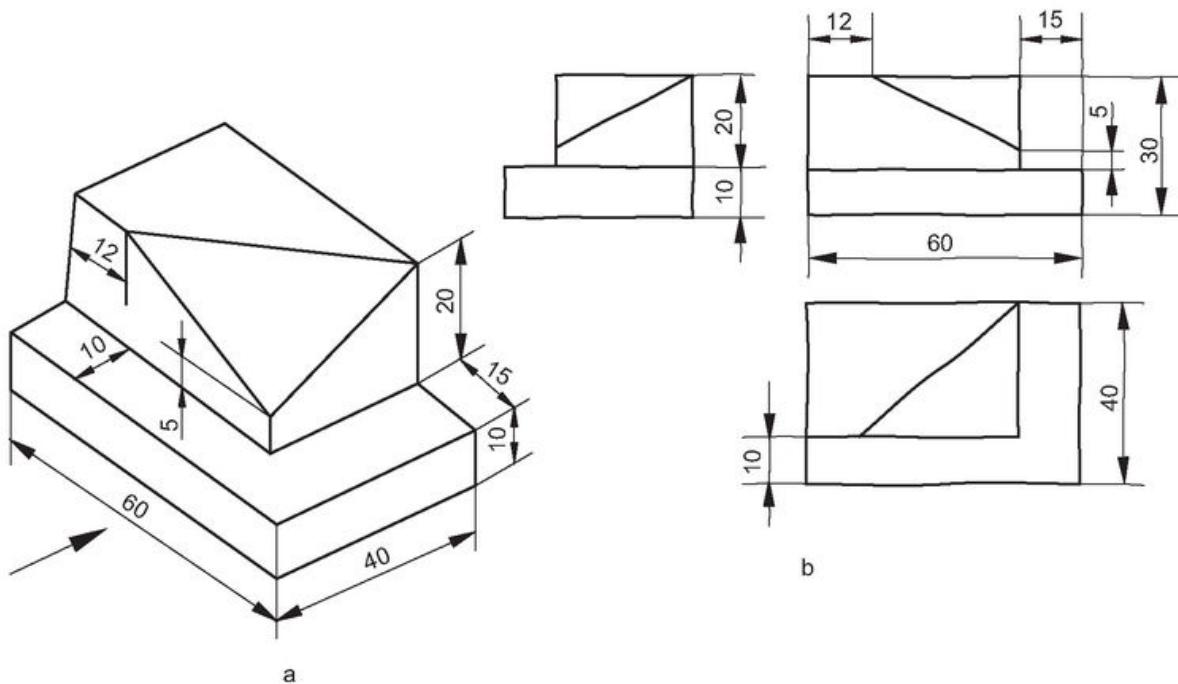
1. Figures 17.16a to 17.20a show isometric views of certain objects. Figures 17.16b to 17.20b show the free-hand sketches of the orthographic views.



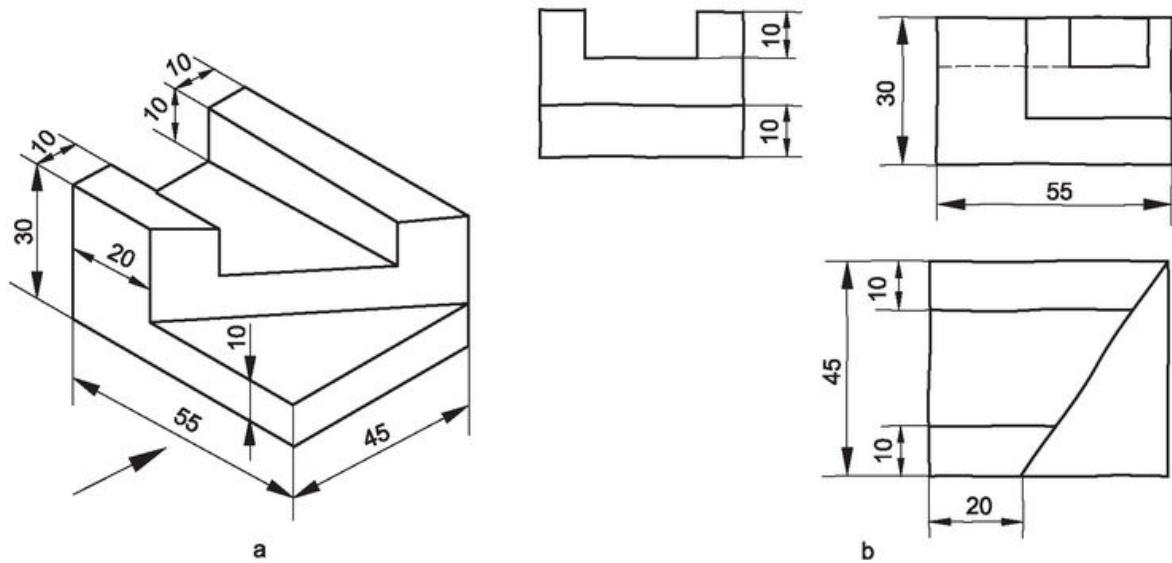
a



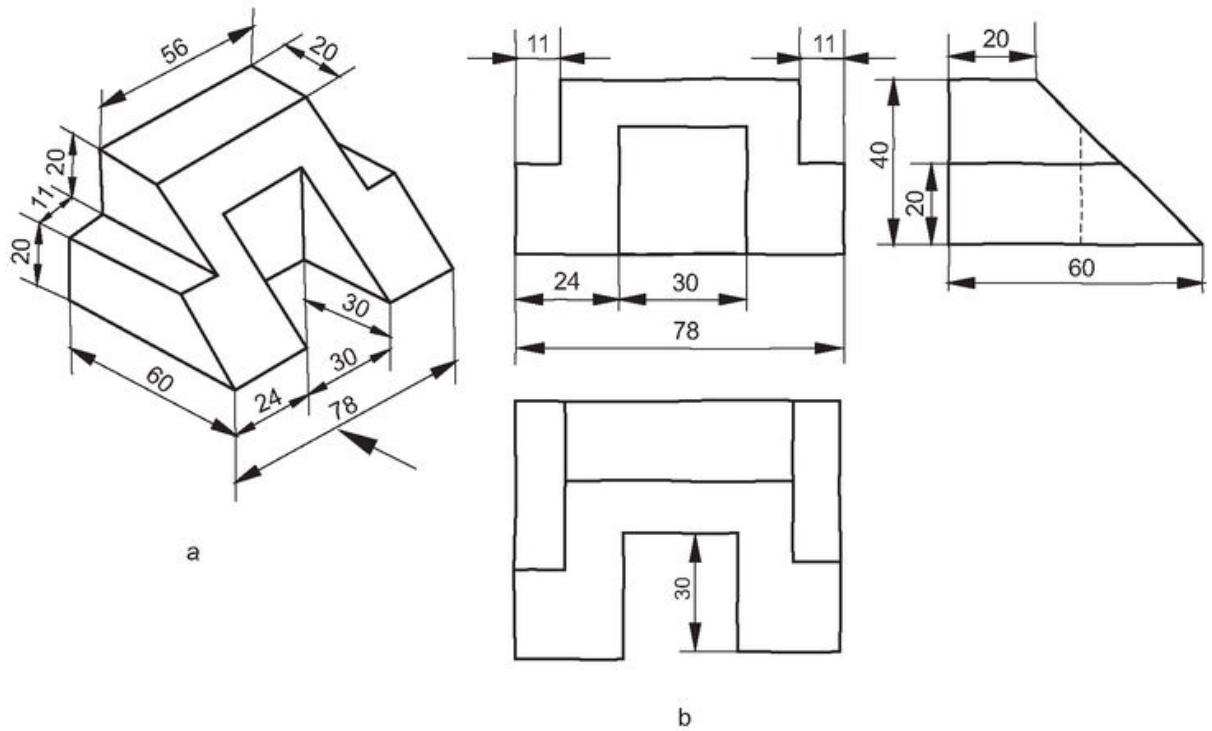
**Fig. 17.16**



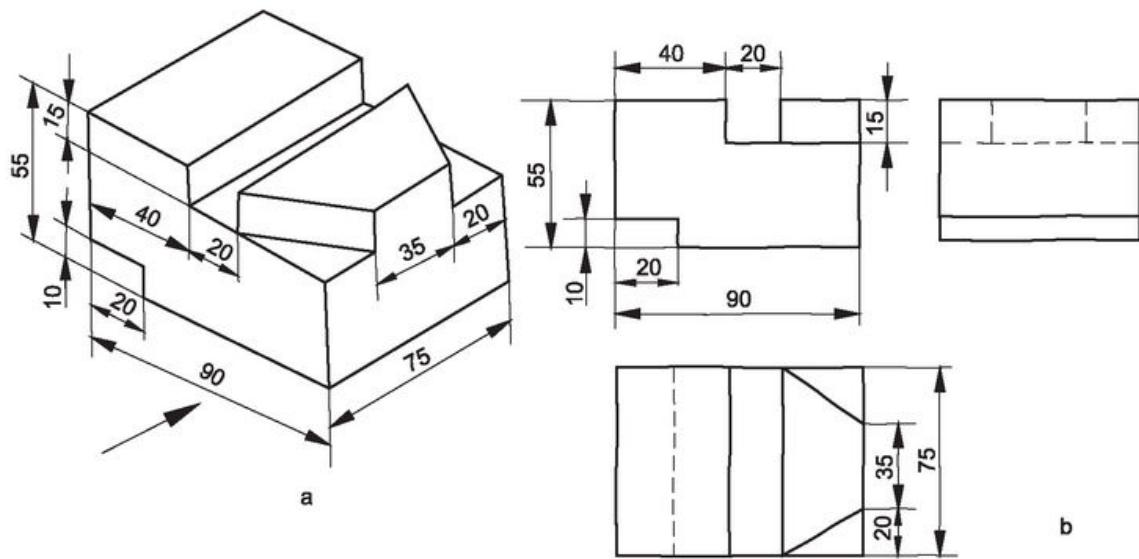
**Fig. 17.17**



**Fig. 17.18**



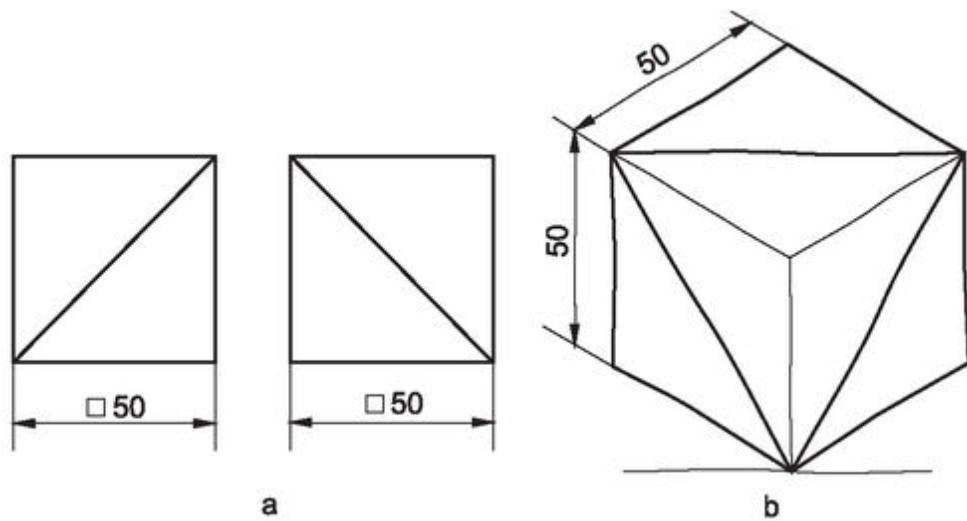
**Fig.17.19**



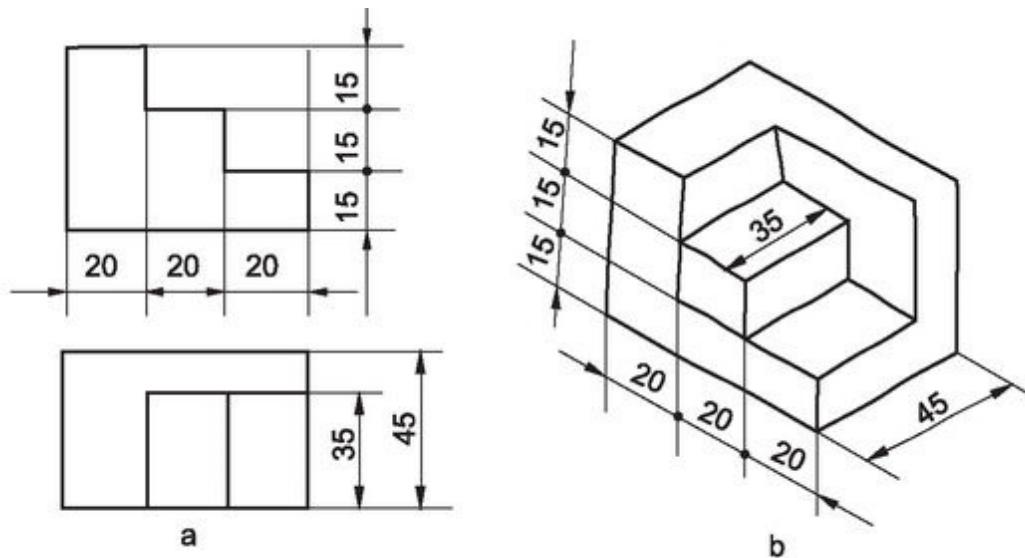
**Fig.17.20**

2. Figures 17.21a to 17.24a show the orthographic views of certain objects. Figures 17.21b to 17.24b show the

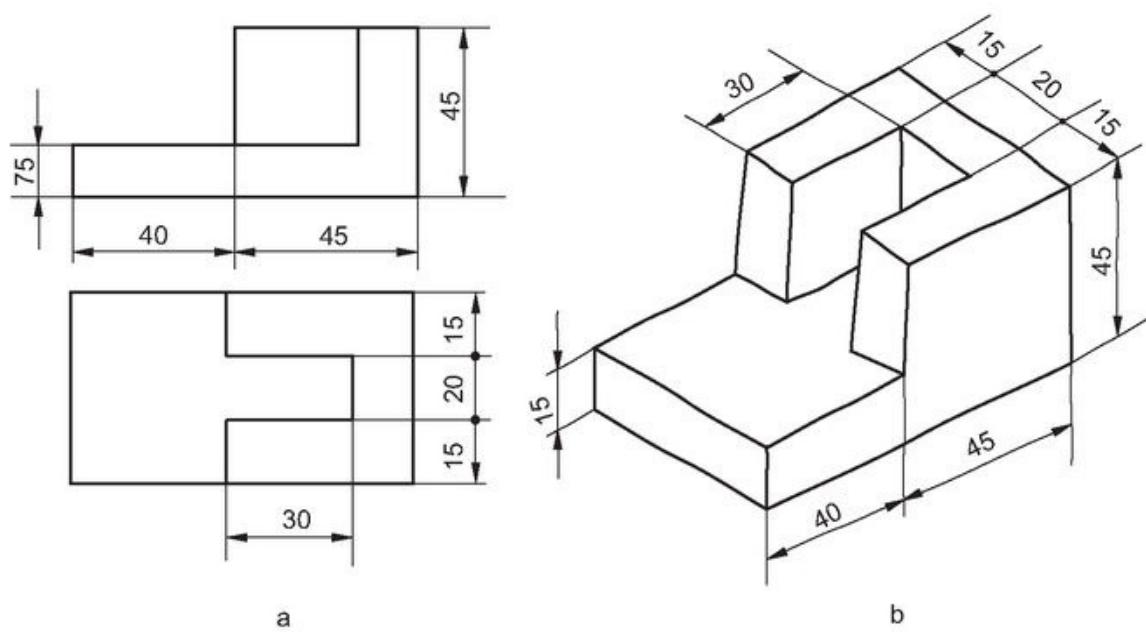
free-hand sketches of the isometric projections.



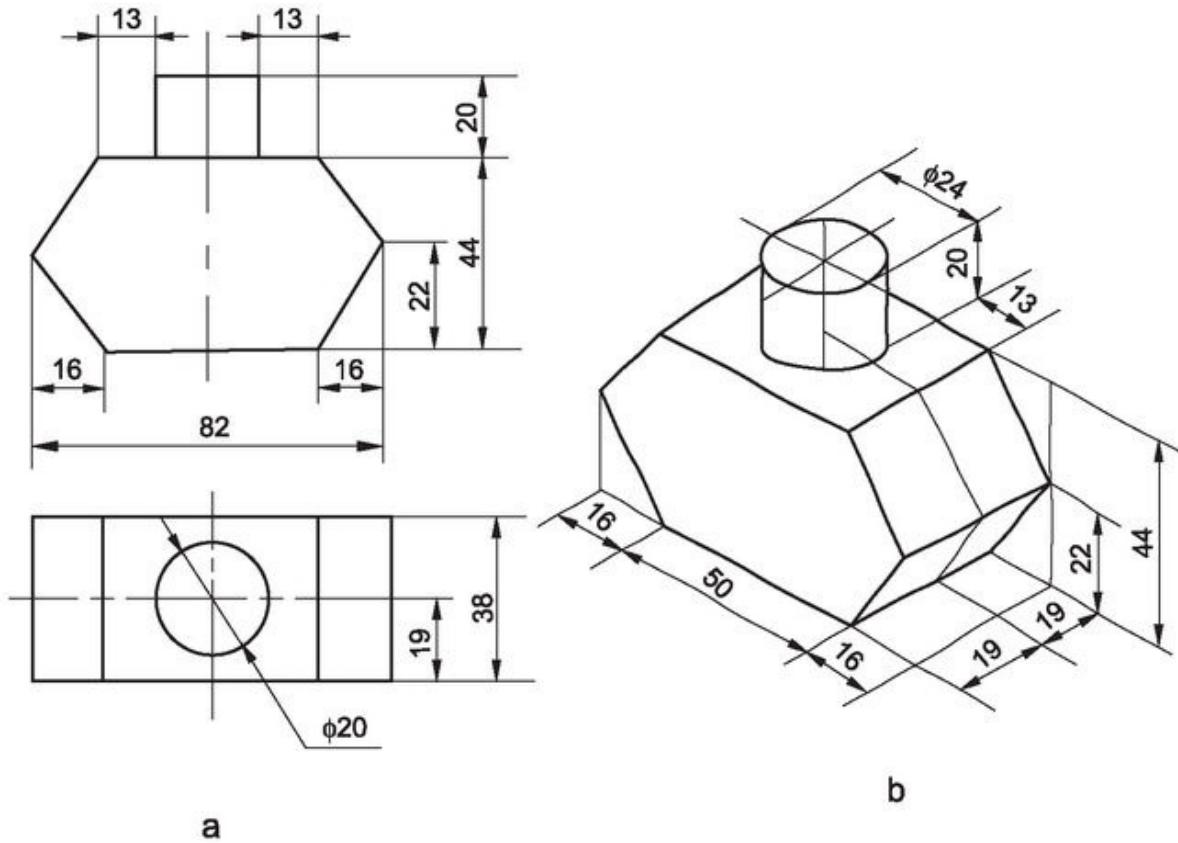
**Fig.17.21**



**Fig.17.22**



**Fig.17.23**

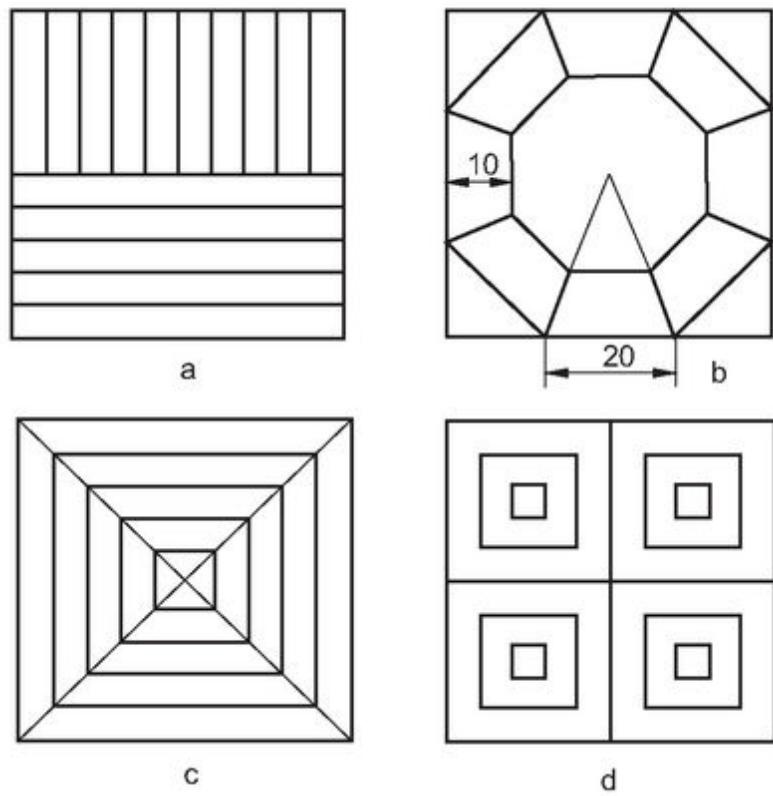


**Fig.17.24**

## EXERCISES

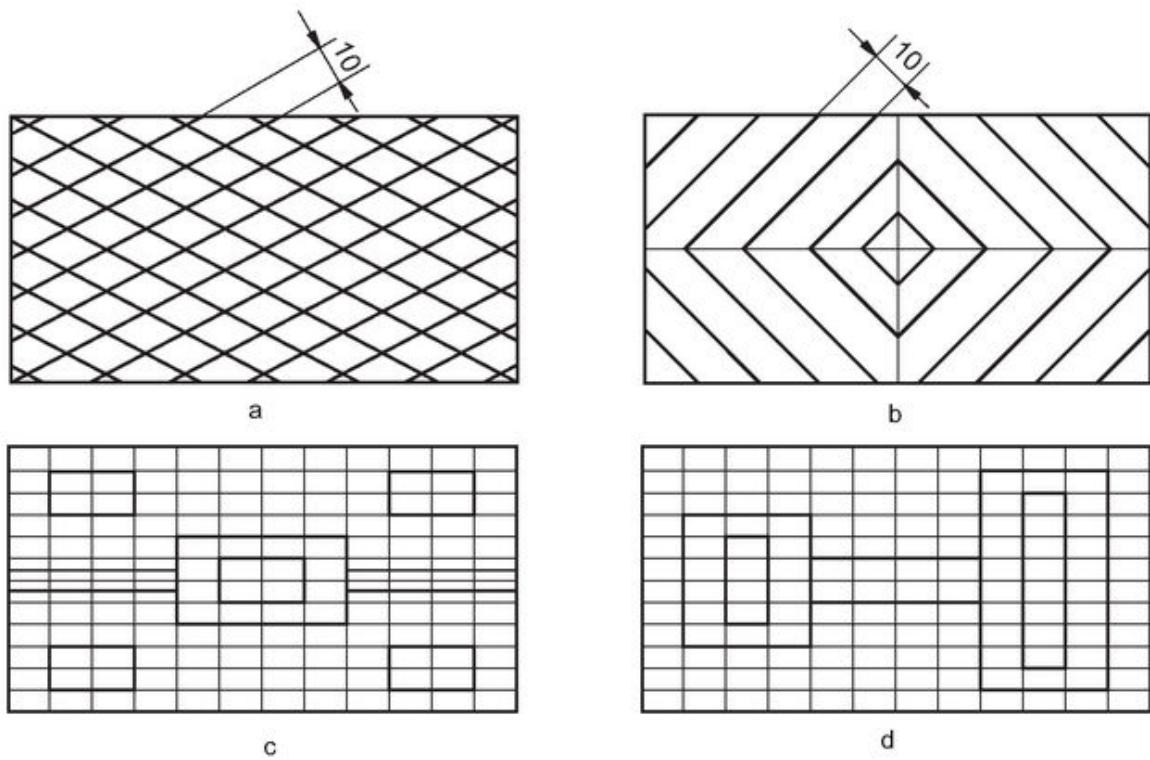
Sketch the following:

- 17.1 (i) [Figures 17.25 a, b, c and d](#) in square blocks of 50 side.



**Fig.17.25**

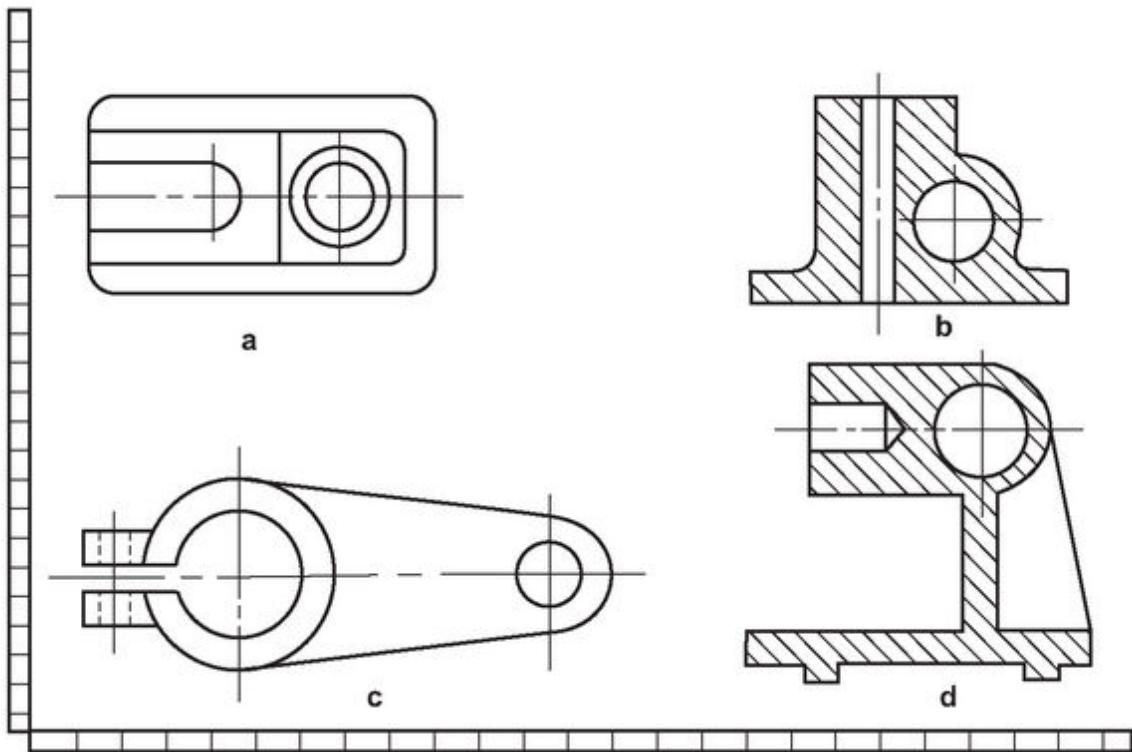
(ii) Figures 17.26 a, b, c and d in rectangular blocks of  $120 \times 60$ .



**Fig.17.26**

(iii) Circles of diameters 50, 100 and 150.

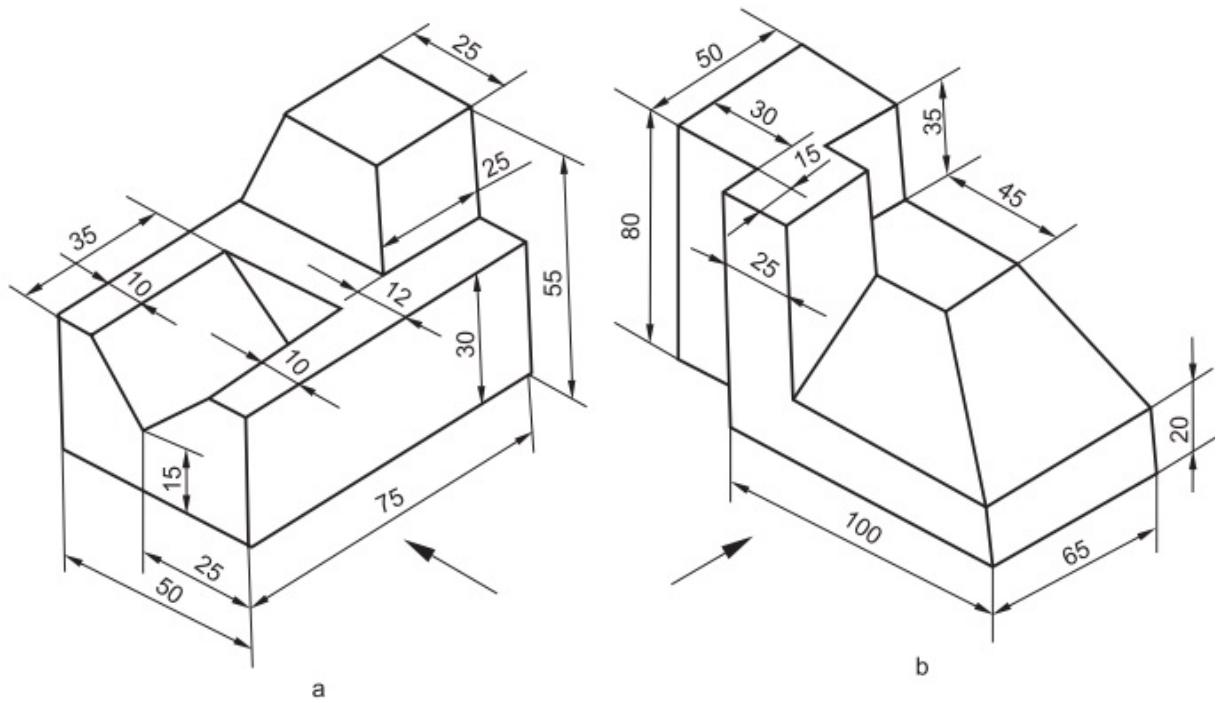
(iv) [Figures 17.27 a, b, c and d.](#)



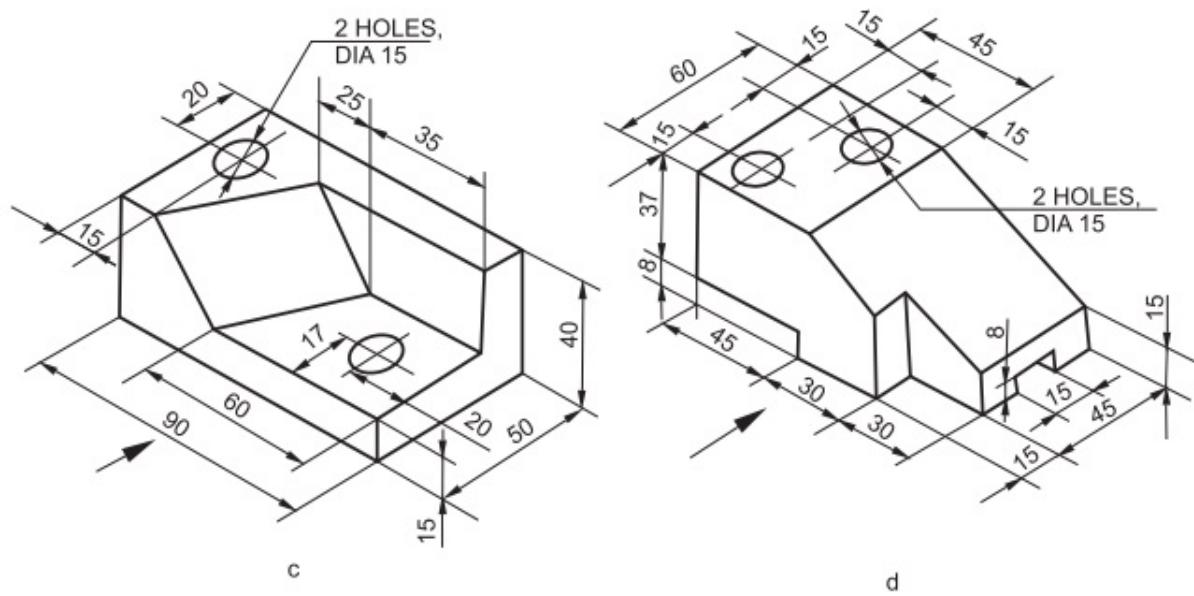
**Fig.17.27**

(v) An ellipse of major axis 120 and minor axis 70.

17.2 [Figure 17.28](#) shows the isometric projections of certain objects. Sketch the orthographic views of each.

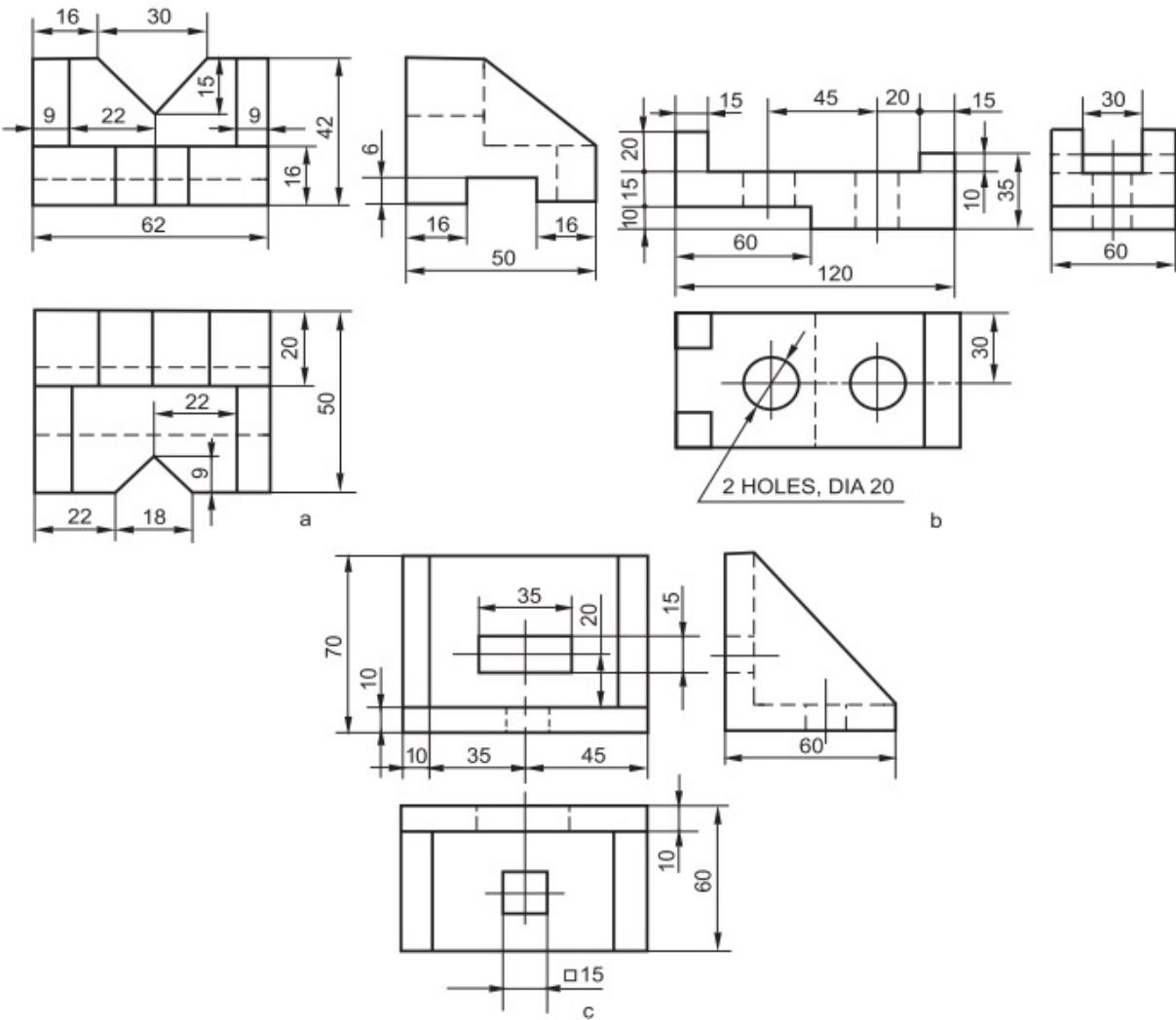


**Fig.17.28 - a,b**



**Fig.17.28 - c,d**

17.3 Figure 17.29 shows the orthographic views of certain objects. Sketch the isometric view of each.



**Fig.17.29**

## REVIEW QUESTIONS

- 17.1 What are the applications of free-hand sketching?
- 17.2 List the materials for free-hand sketching.
- 17.3 What are the steps to be followed in sketching (i) a straight line, (ii) a square, (iii) a circle and (iv) an ellipse?

- 17.2 Mention the steps to be followed in sketching orthographic views of an object.
- 17.3 What are the steps to be followed in sketching isometric projection of an object, given its orthographic views?

## OBJECTIVE QUESTIONS

17.1 Free-hand sketches are employed to present the ideas of the designer to the management.

(True/False)

17.2 A pencil of the grade \_\_\_\_\_ is preferred for sketching works.

17.3 Sketching a square is also useful in sketching circles.

(True/False)

117.4 Circular feature in isometric projection, appear as \_\_\_\_\_ feature.

17.5 Shop drawings are supplied in the form of (a) orthographic views, (b) isometric projection, (c) perspective projection.

( )

## ANSWERS

17.1 True

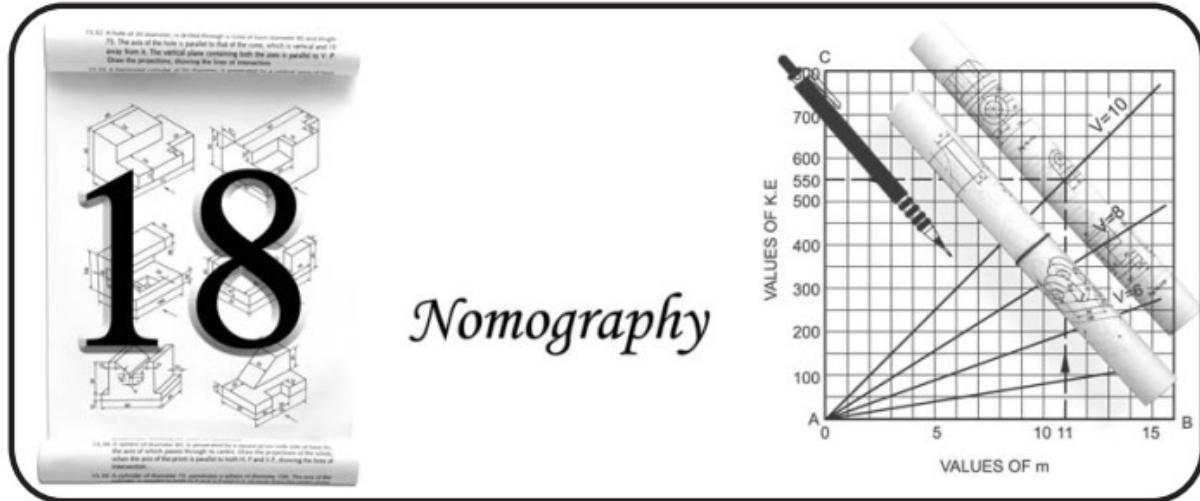
17.2 HB

17.3 True

17.4 elliptical

17.5 a

*OceanofPDF.com*



## 18.1 INTRODUCTION

Nomograms are drawings made in order to replace cumbersome and repeated calculations and thus save time. Engineering students find a potential use of the nomograms in their fields of study and professional practice. Only certain types of nomograms, i.e., concurrency charts, alignment charts and Z-charts are presented in this chapter.

## 18.2 CONCURRENCY CHARTS

These charts look like rectangular cartesian co-ordinate charts. These charts or cartesian co-ordinate systems are made use of very frequently, for the graphical representations and solutions of some equations.

### 18.2.1 Construct an Addition Chart, for $2a + b = c$ .

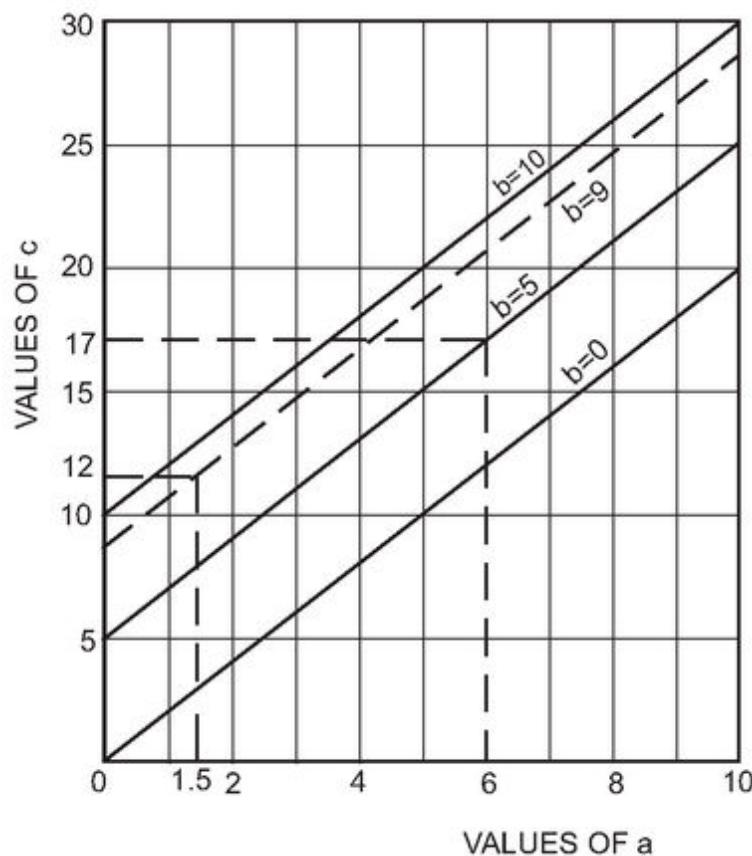
## Construction (Fig.18.1)

1. Assign different values for  $a$  and mark along  $x$ -axis.
2. Obtain the limiting values of  $c$  based on the values of  $a$  and  $b$  and mark along the  $y$ -axis.
3. For a particular value of  $b$ , obtain a number of points by using the given equation and draw a line through the points.
4. Obtain a set of parallel lines for different values of  $b$ , in a similar way.

For a value of  $a = 6$  and  $b = 5$ , the value of  $c$  is 17, as shown.



The same chart may be used for subtraction, in which case the equation takes the form,  $b = c - 2a$ .



### **Fig.18.1**

For a value of  $c = 12$  and  $a = 1.5$ , the value of  $b$  is 9, as marked.

## **18.2.2 Solve the Following Equation involving Addition and Subtraction.**

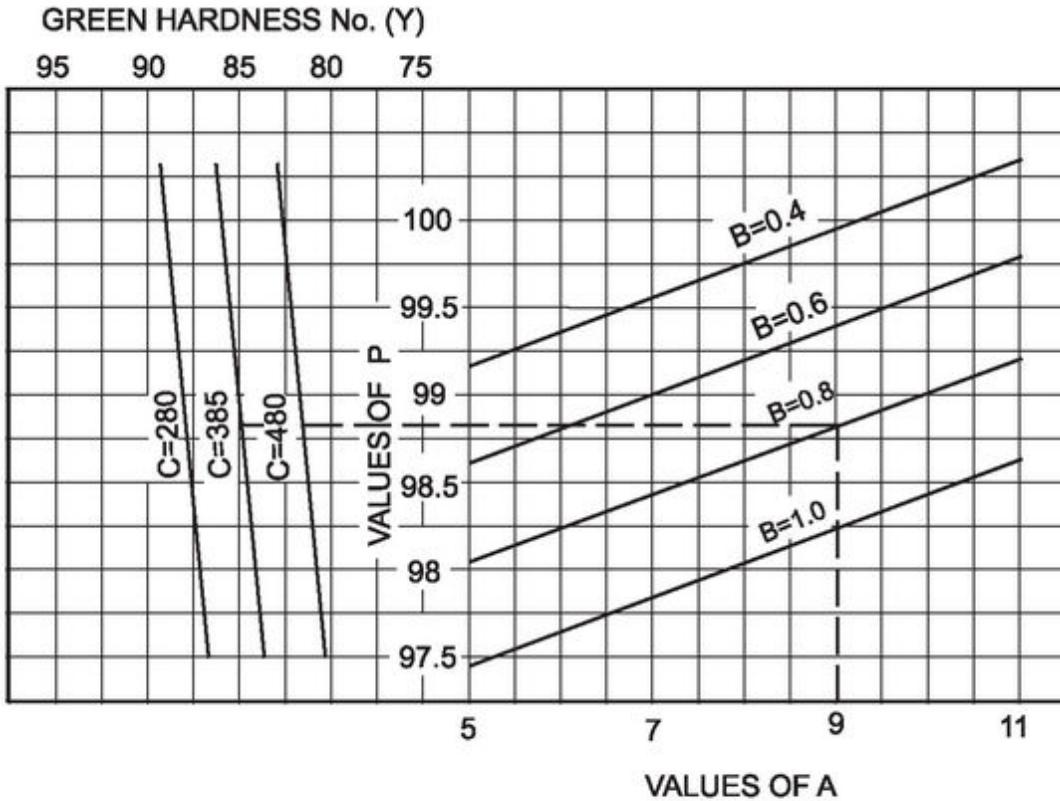
*Green hardness number =  $85.6 + 1.6 \text{ clay, percent} - 2.3 \text{ water/clay, ratio} - 0.034 \text{ grain size in microns}$ , given the limits of variation for each variable as:*

1. Clay, percent      5, 7, 9 and 11
2. Water / clay, ratio    0.4, 0.6, 0.8, 1.0
3. Grain size, microns 280, 385 and 480

### ***Construction (Fig.18.2)***

1. Consider the part equation,  $P = 85.6 + 1.6 A - 2.3 B$ .
2. Follow the steps similar to Construction: [Fig.18.1](#) and obtain a set of lines for different values of  $B$ .
3. Write the given equation as  $Y = P - 0.034 C$ .
4. Follow the steps similar to Construction: [Fig.18.1](#) and obtain a set of lines for different values of  $C$ .

For values of  $A = 9$ ,  $B = 0.8$  and  $C = 385$ , the value of  $Y$  is 85 as marked.

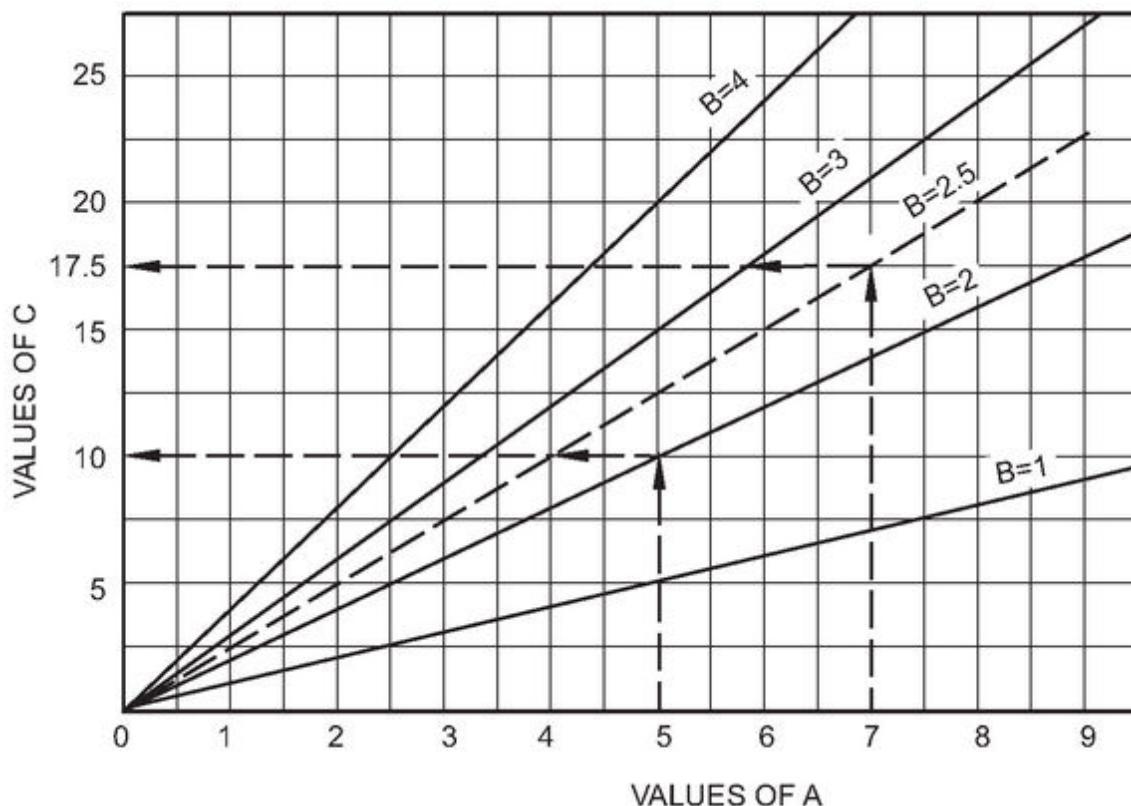


**Fig.18.2**

### 18.2.3 Construct a Concurrency Chart for the Equation, AB = C.

***Construction (Fig.18.3)***

1. Assign a number of values to A in the given equation and mark along the x-axis.



**Fig.18.3**

2. Obtain the limiting values of C, based on the values of A and B and mark along the y-axis.
3. For a particular value of B, obtain a number of points using the given equation and draw a line through the points. It may be noted that the line passes through the origin.
4. Obtain a set of lines, passing through the origin for different values of B.

For a value of  $A = 5$ ,  $B = 2$ , the value of  $C$  is 10, as marked.



The same chart may also serve as a division chart, i.e.,  $C/B = A$ .

For a value of  $C = 17.5$  and  $B = 2.5$ , the value of  $A$  is 7, as shown.

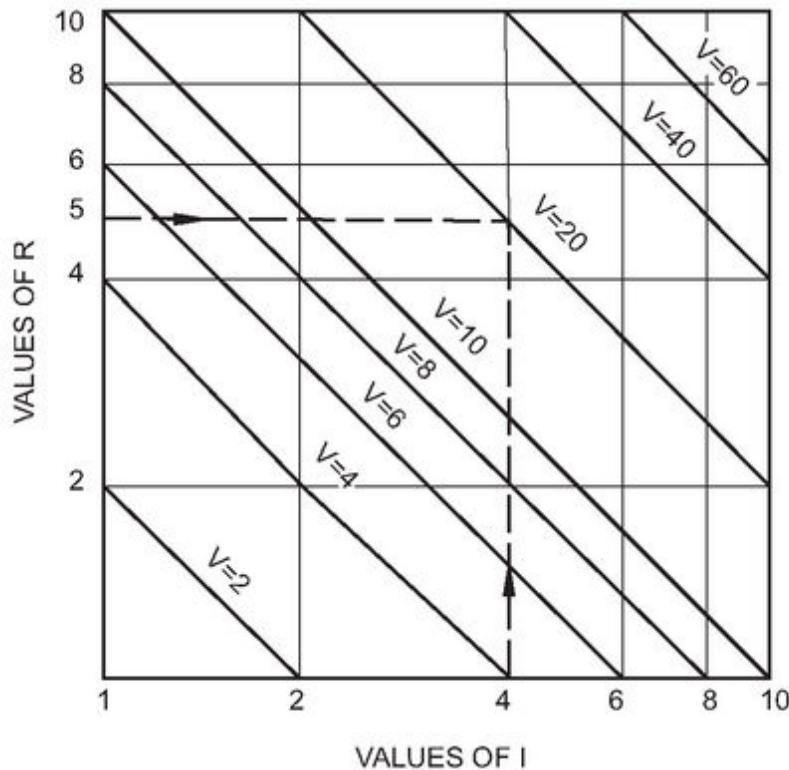
#### **18.2.4 Draw a Concurrency Chart for the Equation, $V = I R$ , Where $V$ =Voltage, $I$ = Current and $R$ = Resistance, Given $I$ Varies from 0 to 10 Amperes and $R$ from 0 to 10 Ohms.**

**HINT** Solve the equation by converting into an addition equation.

##### ***Construction (Fig.18.4)***

1. Take logarithms on both sides of the given equation, obtaining,  $\log V = \log I + \log R$ .
2. Mark the values of  $I$  and  $R$  on a log-log graph.
3. Join the corresponding values of  $I$  and  $R$  and mark the values of  $V$  on the lines.

For a value of  $I = 4$  amps and  $R = 5$  ohms,  $V$  is 20 volts, as shown.

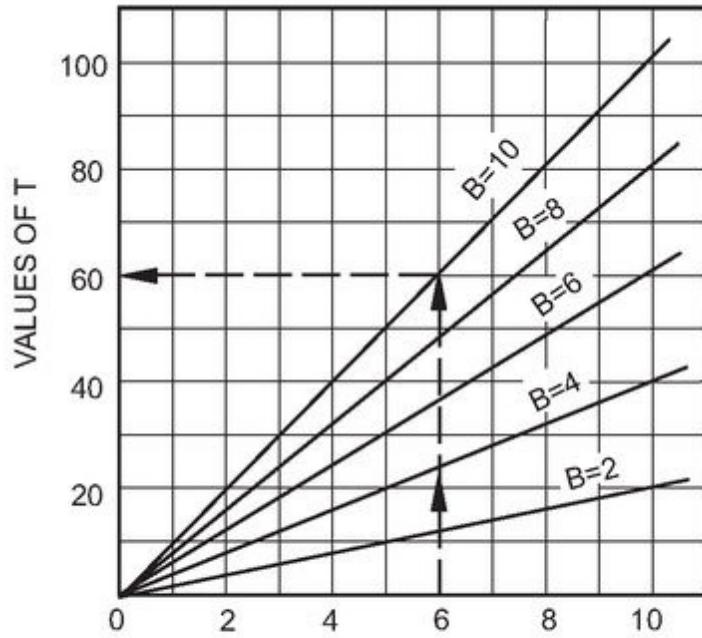


**Fig.18.4**

### 18.2.5 Construct a Multiplication Chart for an Equation Involving 3 Variables, Such as ABC = D.

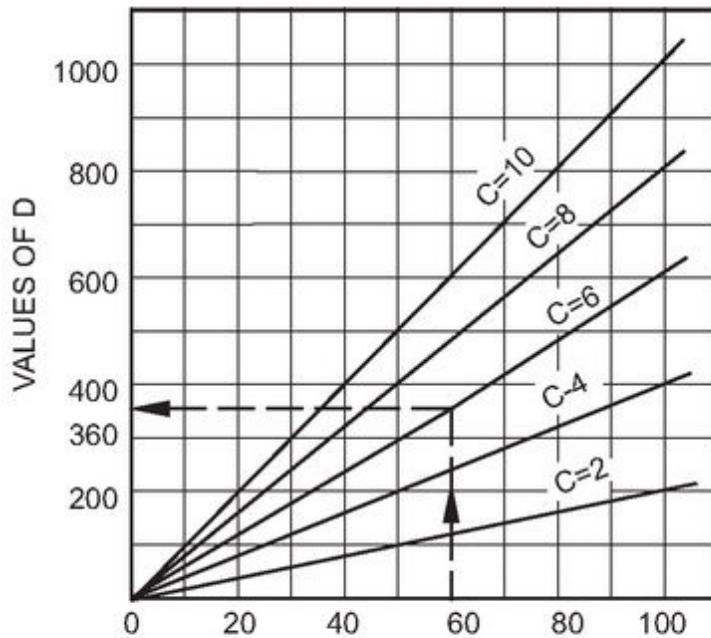
***Construction (Figs.18.5 and 18.6)***

1. Treat  $AB = T$ , reducing the equation to  $TC = D$ .
2. Construct the chart (Fig.18.5) for  $AB = T$ , by following Construction: Fig.18.3.
3. Solve the equation  $TC = D$ , on similar lines (Fig.18.6).



**VALUES OF A**

**Fig.18.5**



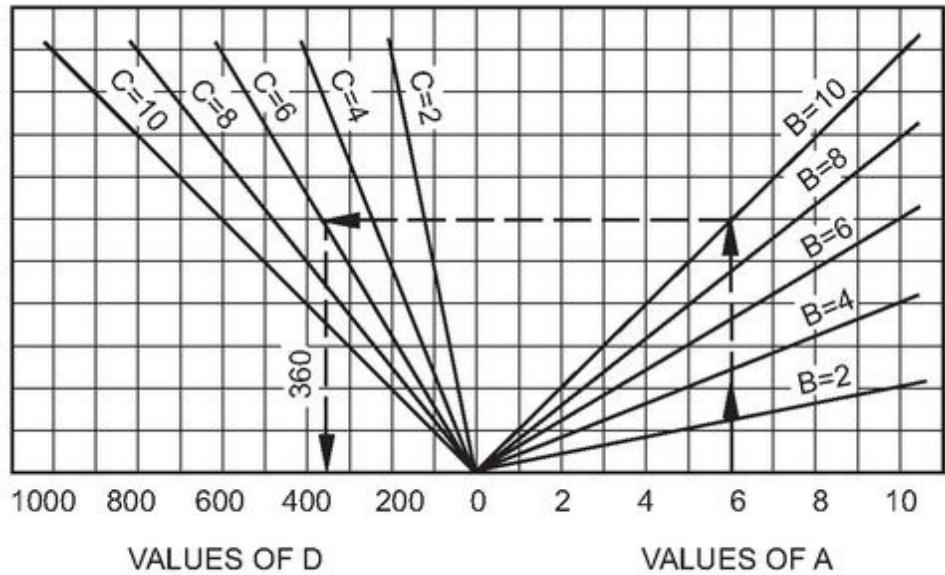
## VALUES OF T

**Fig.18.6**



The above two charts may also be drawn into a single chart, as shown in [Fig.18.7](#). However, values of T need not be marked on the graph.

For a value of  $A = 6$ ,  $B = 10$  and  $C = 6$ , the value of  $D$  is 360, as shown in [Figs. 18.5, 18.6](#) and [Fig.18.7](#).



**Fig.18.7**

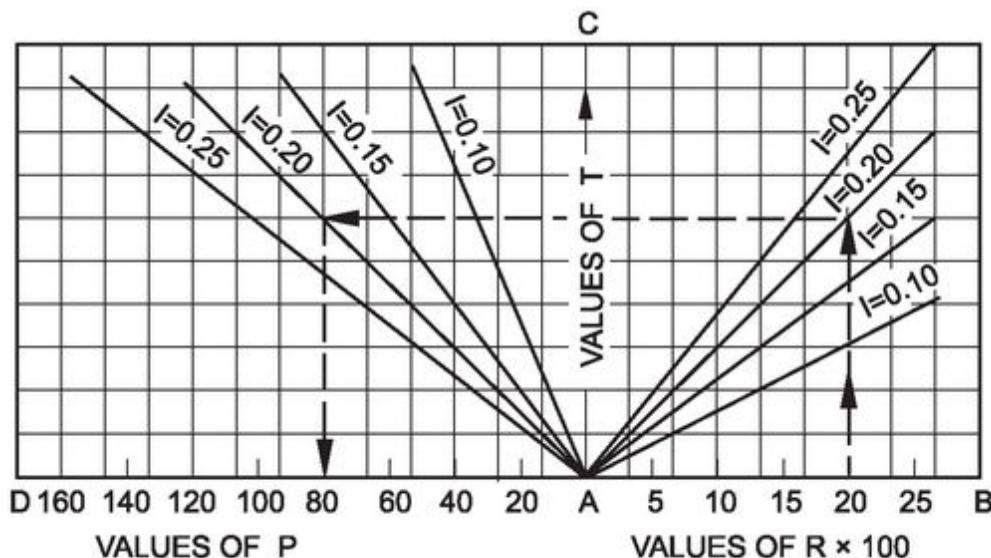
**18.2.6 Draw a Nomogram for the Equation,  $P = I^2R$ , Where  $P$  = Power in Watts,  $I$  = Current in Amperes (0.1 to 0.25) and  $R$  = Resistance in Ohms (1000 to 2500).**

***Construction (Fig.18.8)***

1. Mark values of  $R$  along the line AB.
2. Divide the above equation  $P = I^2R$  into two equations  $IR = T$  and  $IT = P$  and the values of  $T$  are taken along AC.
3. Mark the values of  $P$  along the line AD; the maximum value being 160.

4. Draw two sets of lines, passing through the origin, satisfying the equations  $IR = T$  and  $TI = P$ , for the given value of  $I$ .

For values of  $R = 2000$  ohms and  $I = 0.2$  amperes, the value of  $P$  is 80 watts, as shown.



**Fig.18.8**

**18.2.7 Design a Nomogram for the Equation,  $K.E = \frac{1}{2} mv^2$ , Where K.E represents Kinetic Energy, m represents Mass and v represents Velocity, given m varies from 0 to 15 and v from 0 to 10.**

***Construction (Fig.18.9)***

1. Mark the values of  $m$  along  $AB$ .
2. Mark the values of  $K.E$  along  $AC$  ( $K.E_{max} = 750$ ).

3. Draw lines corresponding to  $v = 4$  to  $v = 10$ , satisfying the equation and passing through the origin.

For values of  $m = 11$  and  $v = 10$ , the value of K.E is 550 as shown.

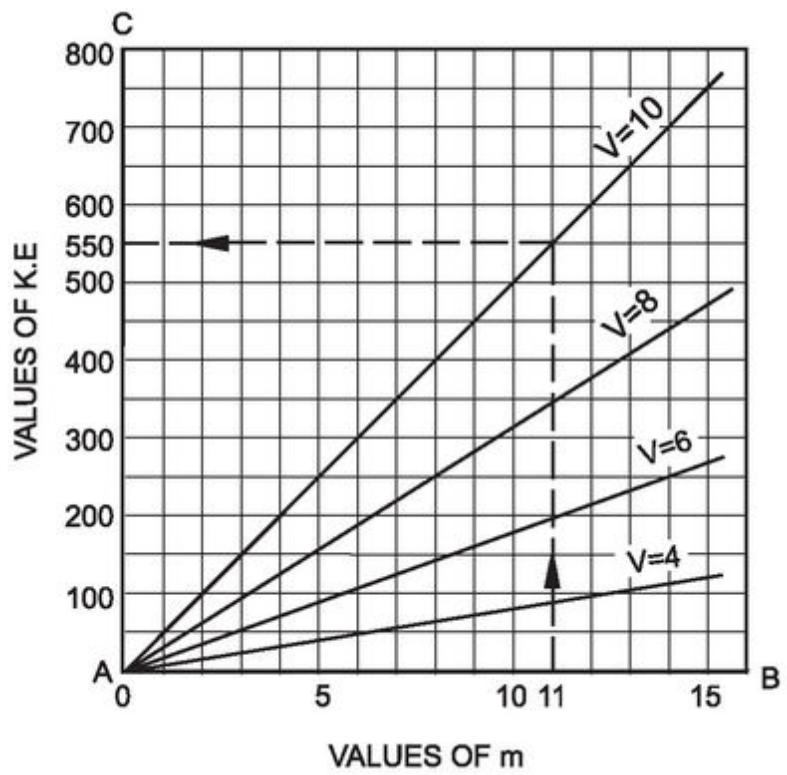
## 18.3 ALIGNMENT CHARTS

These charts are designed to solve equations of the form,  $f_1(u) + f_2(v) = f_3(w)$ . They consist of three parallel scales, so graduated that, if a line is drawn connecting the points on two of the scales, it will intersect the third scale at a point, satisfying the condition.

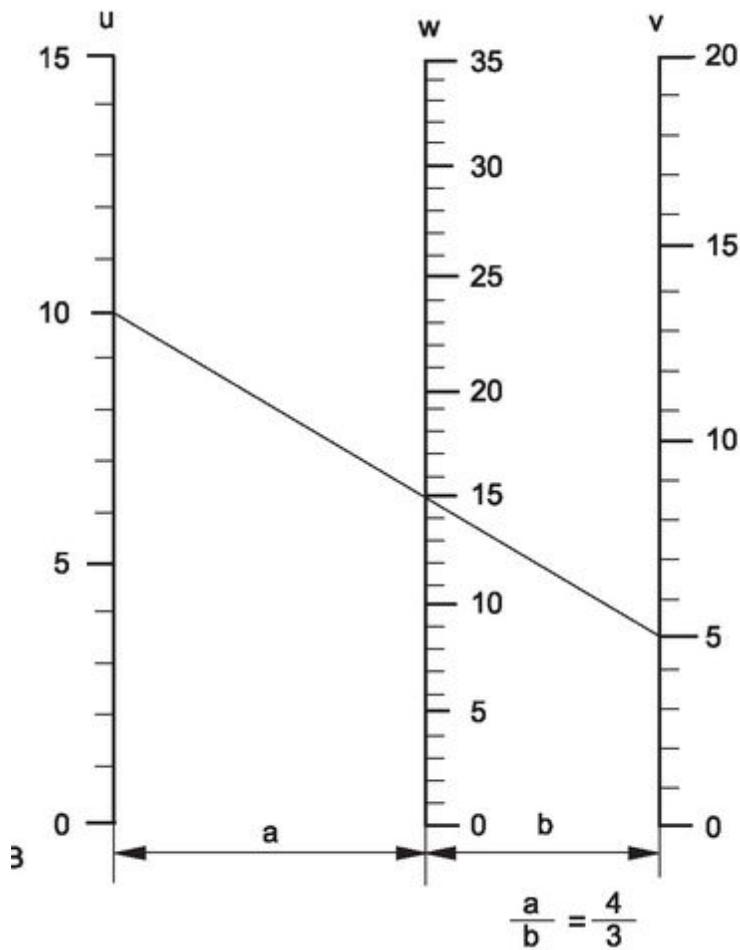
### 18.3.1 Construct a Chart of the Type, $u + v = w$ , given $u$ varies from 0 to 15 and $v$ from 0 to 20.

#### ***Construction (Fig.18.10)***

1. Construct  $u$  scale, say of length 15 cm such that, each division is 1 cm long ( $m_u$ , the multiplying factor of  $u$  scale is 1.0).
2. Construct  $v$  scale parallel to  $u$  scale, each division length being 0.75 cm ( $m_v$ , the multiplying factor of  $v$  scale is 0.75).



**Fig.18.9**



**Fig.18.10**

3. Obtain the length of one division on w scale ( $m_w$ ) from the equation:

$$m_w = \frac{\text{maximum length of line}}{(u_{\max} + v_{\max}) - (u_{\min} + v_{\min})}$$

$$= 15 / 35 \cong 0.43$$

4. Locate w scale in-between u and v scales such that,

$$\frac{a}{b} = \frac{m_u}{m_v} = \frac{4}{3}$$

5. Obtain the limiting values of w and construct the scale.

For a value of  $u = 10$  and  $v = 5$ , the value of  $w$  is 15, as shown.

A chart of the form  $u - v = w$  may be obtained by reversing the  $v$  scale and changing the markings on the  $w$  scale to suit the maximum and minimum values.



To solve an equation consisting of three and more variables, it may be suitably split into a number of equations, each consisting of only two variables.

For example, the equation  $u + v + w = s$ , can be split into (i)  $u + v = p$  and (ii)  $p + w = s$ .

Following the above construction, the chart may be constructed, consisting of five scales, representing  $u$ ,  $v$ ,  $p$ ,  $w$  and  $s$ .

### 18.3.2 Construct an Alignment Chart for Moment of Inertia, I of a Rectangular Beam, given by the Equation, $I = 1/12 BD^3$ , Where B is the Base of the Rectangle (1 to 10 cm) and D is the Height of the Rectangle (1 to 10 cm).

#### ***Construction (Fig. 18. 11)***

1. Take logarithms on both sides of the equation, obtaining,

$$\log I + \log 12 = \log B + 3 \log D.$$

2. Draw two parallel lines at convenient distance and plot the scales of  $B$  and  $D$ , using logarithmic scales.

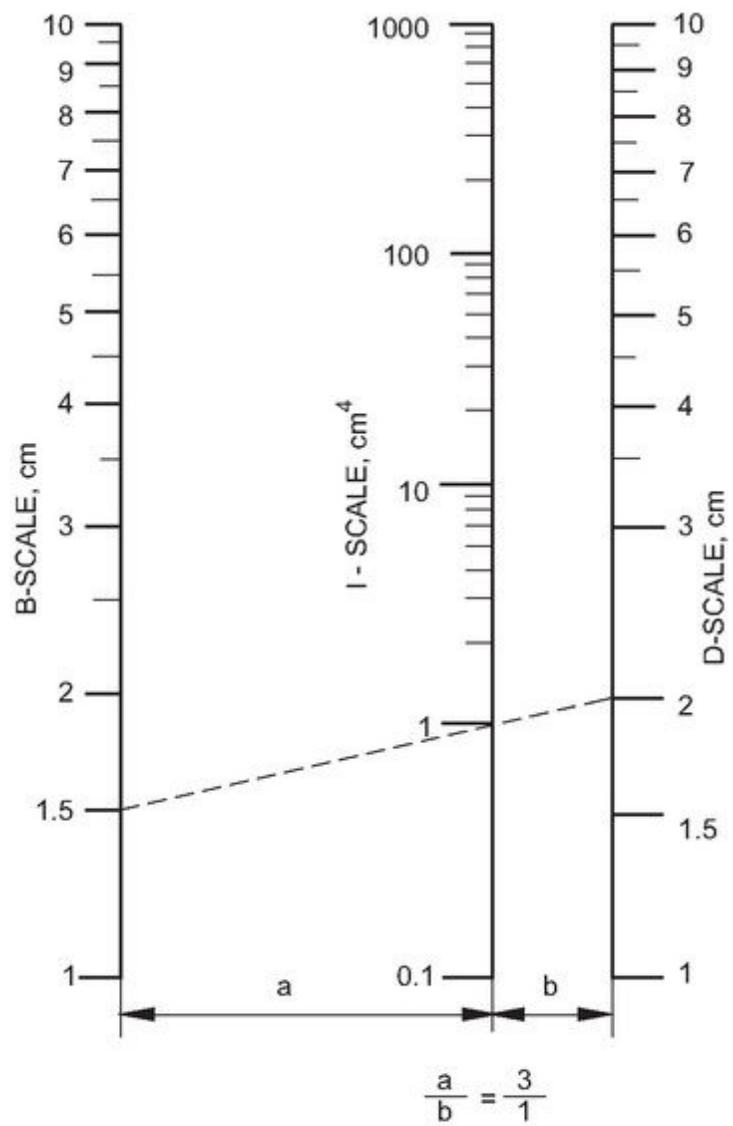
3. Draw a parallel line between D and B scales such that,  $a : b = 3 : 1$  (right hand side of the above equation shows a ratio of 1: 3) and construct the I scale.

To locate a point on the I scale, say at  $D = 2$  for a value of  $I = 1$ , B should take a value of 1.5. Hence, the line joining the values  $D = 2$  and  $B = 1.5$ , intersect the I scale at 1. Similarly, other graduations on the I scale may be determined.

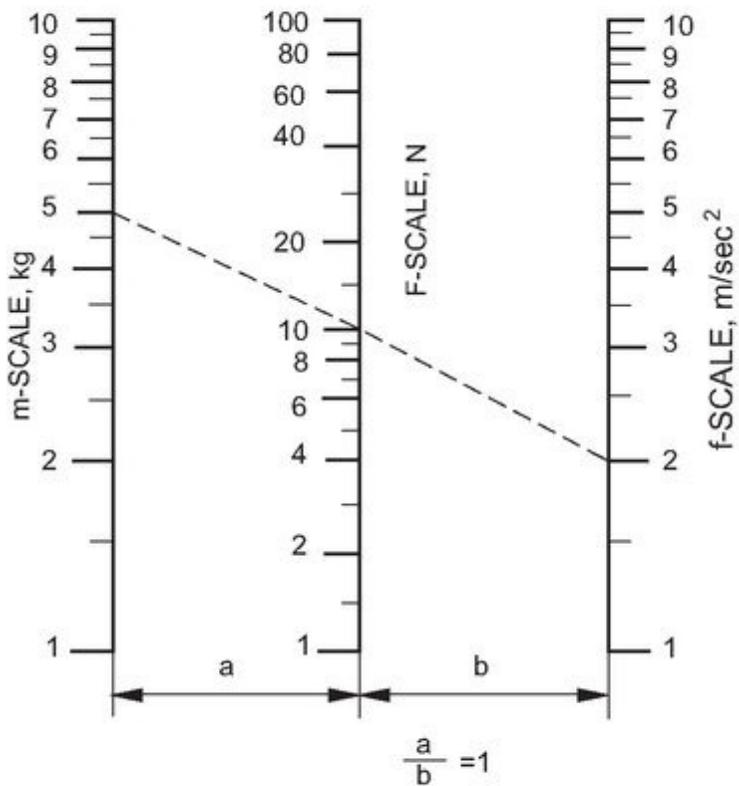
### **18.3.3 Construct an Alignment Chart, representing Newton's Second Law of Motion, $F = mf$ , where, F = Applied Force, N; m = Mass of Body (1 to 10 Kg) and f = Acceleration ( 1 to 10 m/sec<sup>2</sup>).**

#### ***Construction (Fig.18.12)***

1. Follow steps 1 and 2 of the above construction and plot the scales of m and f.



**Fig.18.11**



**Fig.18.12**

2. Draw F scale parallel to and mid-way between the f and m scales.
3. Locate the division points of F scale, by following the procedure explained in the previous construction.

For a value of  $m = 5$  and  $f = 2$ , the value of  $F$  is 10, as shown.

#### 18.3.4 Construct an Alignment Chart, representing $K.E = \frac{1}{2} I \omega^2$ , Where $I = \text{Mass Moment of Inertia}$ (10)

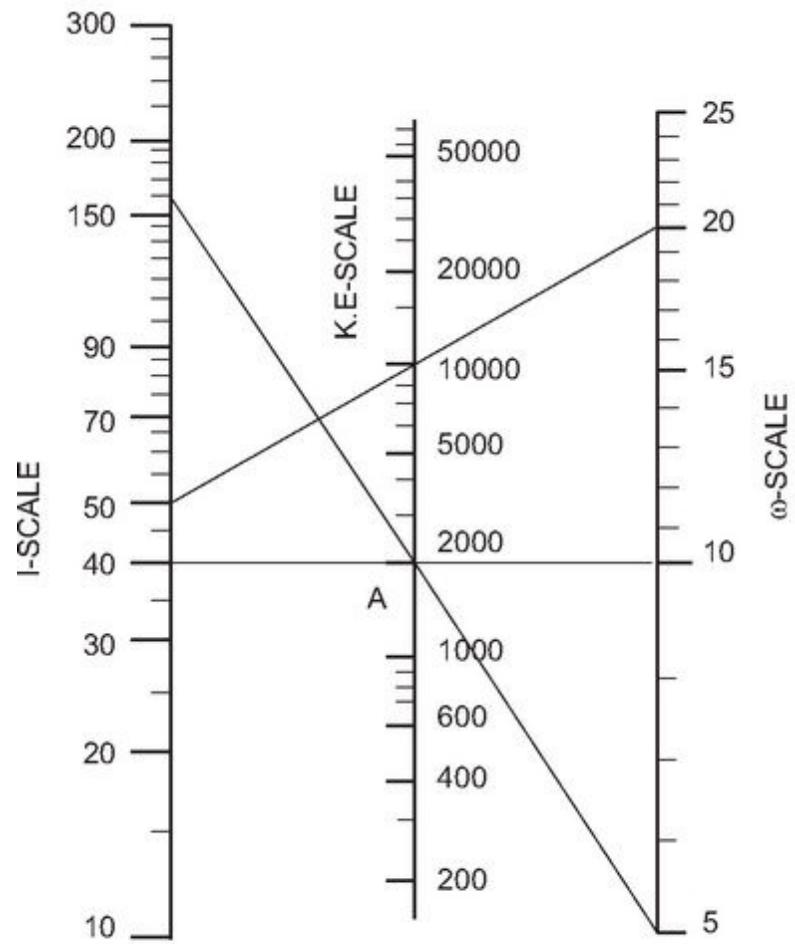
**to 300 Kg-ms<sup>2</sup>) and  $\omega$  = Angular Velocity (5 to 25 rad/sec).**

***Construction (Fig. 18.13)***

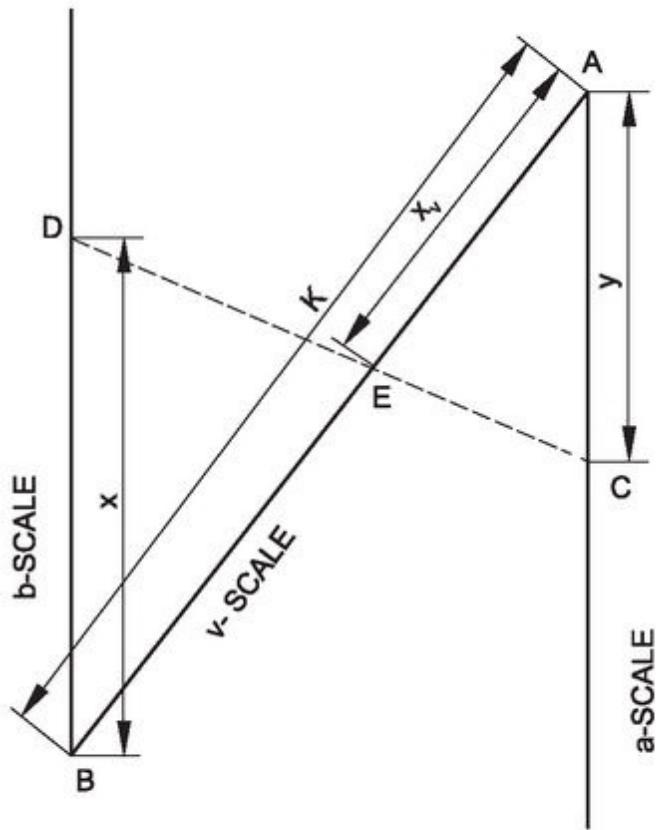
1. Follow the steps 1 and 2 of Construction: [Fig. 18. 11](#) and draw the I and w scales.
2. To locate the K.E scale: Join, say  $\omega = 10$  and  $I = 40$  and  $w = 5$  and  $I = 160$  and obtain the intersection point A. This point corresponds to K.E = 2000. The K.E scale should pass through A and parallel to the other two scales.
3. Obtain a value of 10,000 on the K.E scale at, say  $\omega = 20$  and  $I = 50$ .
4. Construct the K.E scale by projecting from a log scale.

## **18.4 Z-CHARTS**

Z - charts are used for solving equations of the form,  $f_1$  (a) =  $f_2$  (b).  $f_3$  (v). A Z-chart consists of two parallel scales and one diagonal scale connecting the minimum values of both the scales. [Figure 18.14](#) shows the construction of a Z-chart. Following are the steps involved in the construction:



**Fig.18.13**



**Fig.18.14**

1. Draw two scales  $a$  and  $b$  at a convenient distance apart.
2. On the vertical scale  $a$ , locate the fixed point  $C$  at distance  $y$  from  $A$ .
3. On the right side of  $b$  scale, mark a temporary  $v$  scale of length, say  $x$ , satisfying the scale equation given below:

From the similar triangles,  $BED$  and  $AEC$ ,

$$\begin{aligned}
 \frac{x}{y} &= \frac{k - x_v}{x_v} \\
 &= \frac{m_b}{m_a} f_3(v) \\
 x &= y \frac{m_b}{m_a} f_3(v), \text{ the scale equation}
 \end{aligned}$$

where,

$m_a$  = scale modulus of a scale (multiplier), and

$m_b$  = scale modulus of b scale

4. Lines joining C with the graduations on temporary v scale, will intersect the diagonal (v-scale) at points having the same value of v.

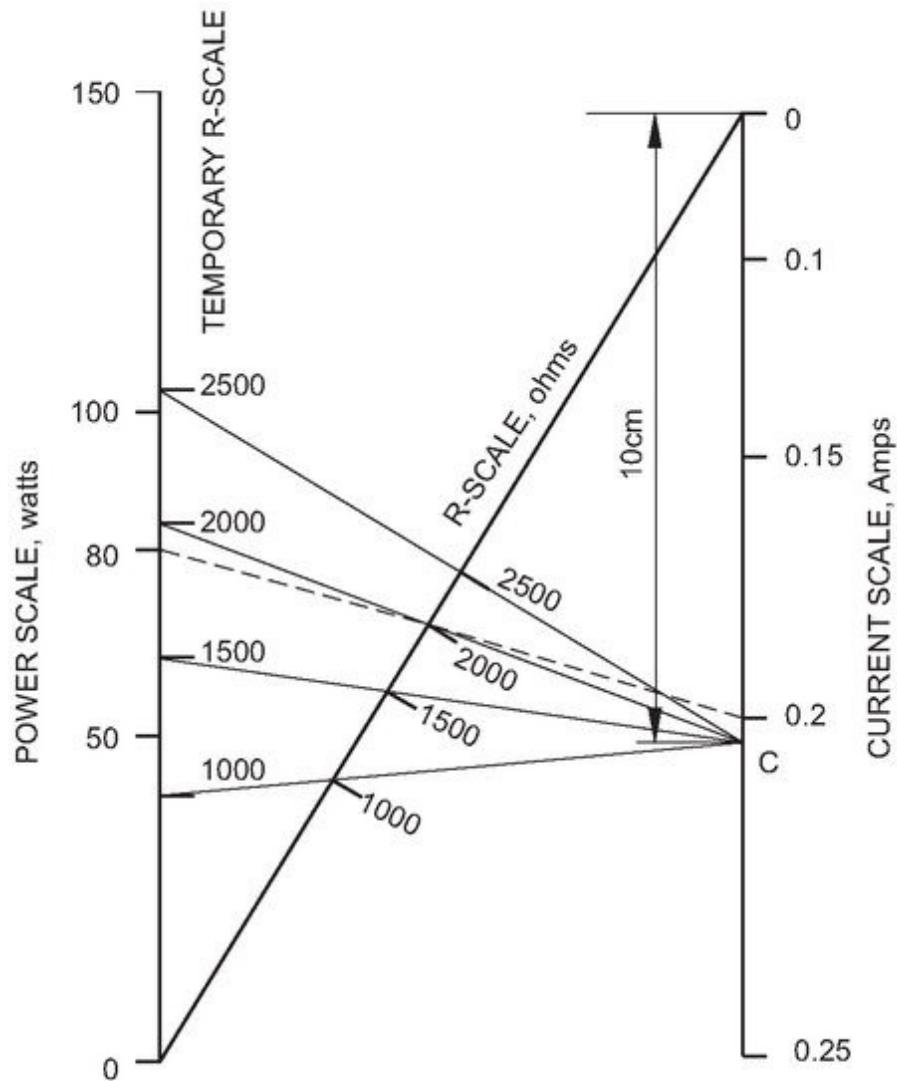
 The modulus of a scale is the length of the scale to the value it represents.

#### 18.4.1 Design a Z-type Alignment

**Chart for the Equation,  $P = I^2R$ ,  
Where P= Power in Watts, I =  
Current in Amperes (0.10 to 0.25)  
and R = Resistance in Ohms  
(1000 to 2500).**

#### ***Construction (Fig.18.15)***

1. Construct P and I scales of any convenient length, say 15 cm, after obtaining the modulii of the scales as:



**Fig.18.15**

$$m_p = \frac{15}{P_{\max}} = \frac{15}{(0.25)^2 (2500)} = 0.102, \text{ and}$$

$$m_i = \frac{15}{I_{\max}^2} = \frac{15}{(0.25)^2} = 240$$

2. Mark a point C along the I-scale at a distance, y (10 cm) from 0.
3. On the right side of the power scale, mark a temporary R scale of any length, say x, satisfying the scale

equation,

$$x = y \frac{m_p}{m_i} R$$

The equation, after substituting the above values, reduces to  $x = 0.00425 R$ .

4. Join the point C to the values of R on the temporary scale and transfer the division points on to the diagonal scale.

For a value of  $I = 0.2$  amps and  $R = 2000$  ohms, the value of  $P$  is 80 watts, as marked.

## EXERCISES

18.1 Construct an addition chart for the equation,  $2x + y = 8$ , when  $x$  varies from 0 to 10 and  $y$  from -7 to 8.

18.2 Construct a subtraction chart for the equation,  $3u - 2v = 5$ , when  $u$  varies from 5 to 15 and  $v$  from 0 to 10.

18.3 Construct a multiplication chart for the following equation by any two methods:

$A = st$ , where,  $A$  = shear area of punched hole in a plate,  $\text{cm}^2$

$s$  = perimeter of the hole, cm (2 to 10)

$t$  = thickness of the plate, cm (0.5 to 2.5)

18.4 Solve the following equations by concurrency charts:

(a)  $P = \frac{2\pi NFR}{60}$ , where,  $P$  = Power, watts

$$N = \text{RPM} \quad (200 \text{ to } 1000)$$

F = force, N (500 to 10000)

R = radius, m (0.02 to 0.08)

(b)  $T = \frac{\pi d^3 \tau}{16}$ , where, T = torque, N-mm

16                   d = diameter of the shaft,  
                        mm (20 to 100)

τ = shear stress, N/mm<sup>2</sup>  
(45 to 95)

18.5 Construct an alignment chart for the equation,  $V = c \sqrt{2gH}$

where,

V = velocity of water in m/s, through an orifice

g = 9.81 m/s<sup>2</sup>

c = coefficient of orifice, depending upon the shape (0.6 to 1.0)

H = head of water (0.5 to 4m)

18.6 Construct a chart for equation, for the maximum surge pressure due to the instantaneous valve shut-off,  $H = a V_o / g$

where,

H = maximum surge pressure rise in metres of water

a = pressure wave velocity (600 to 1200 m/s)

$V_o$  = initial velocity of water flow in pipe line (0.25 to 3 m/s)

g = 9.81 m/s<sup>2</sup>

18.7 Vieker's formula for hardness number,

$$V = \frac{P}{0.5383d^2}$$

where,

V = Vicker's hardness number

P = applied load (5 to 120 kg)

d = the diagonal of indentation (0.2 to 2mm)

Construct an alignment chart for the equation.

18.8 Specific speed of a centrifugal pump,  $N_s = NQ / H^{1/4}$

where,

$N_s$  = specific speed of a centrifugal pump

Q = flow rate (5 to 50 m<sup>3</sup>/min)

N = speed (100 to 4000 rpm)

H = head (5 to 50m)

Construct an alignment chart for the equation.

18.9 Bending stress in the outer fibre of a section of a rectangular beam,  $\sigma = 6M / bh^2$

where,

M = bending moment on the section ( $1 \times 10^6$  to  $30 \times 10^6$  N-mm)

b = breadth of the section (50 to 400 mm)

h = depth of the section (80 to 500 mm)

$\sigma$  = bending stress, N/mm<sup>2</sup>

Construct an alignment chart for the equation.

18.10 Discharge from a steam nozzle,  $M = 0.3155 A_t \sqrt{P_1/V_1}$

where,

$A_t$  = exit area of the nozzle,  $\text{cm}^2$ (0 to 35)

$P_1$  = pressure of steam,  $\text{kg}/\text{cm}^2$  (1 to 20)

$V_1$  = specific volume,  $\text{m}^3/\text{kg}$  (0 to 1.5)  $M$  = discharge,  $\text{kg}/\text{s}$

Construct an alignment chart for the equation.

18.11 Shaper machining time is given by,  $T = w/fN$

where,

$T$  = time, min

$w$  = width of the workpiece, cm ( 5 to 45)

$f$  = feed, mm/stroke (0.125 to 0.75)

$N$  = number of strokes/min (10 to 100)

Construct an alignment chart for the equation.

18.12 Resistance of a wire in ohms,  $R = rL/A$

where,

$r$  = specific resistance, ohm cm (0 to 5)

$L$  = length of wire, m (0 to 300)

$A$  = cross-sectional area of wire,  $\text{cm}^2$  (0 to 0.0025)

Construct an alignment chart for the equation.

18.13 Current in amperes for a three phase, three wire, a.c circuit,  $I = 580P/EF_p$

where,

$P$  = load (0 to 15 Kw)

$E$  = circuit voltage (100 to 500V)

$F_p$  = power factor (0.5 to 1.0)

Construct an alignment chart for the equation.

18.14 Simple interest law,  $I = PRT$

where,

$I$  = interest in rupees

$P$  = principal amount, Rs (100 to 10, 000)

$R$  = rate of interest per year ( 8 to 15%)

$T$  = time, years (0 to 5)

Construct an alignment chart for the equation.

18.15 Brake horse power of an engine,  $B = (d^2n/2.5)$

where,

$d$  = diameter of engine cylinder, cm ( 0 to 12.5)

$n$  = number of cylinders ( 2, 4, 6, 8 10 and 12)

Construct a Z-chart.

18.16 H.P of a refrigeration compressor =  $4.71 T / COP$

where,

$T$  = tons of refrigeration required (10 to 100)

$COP$  = coefficient of performance (1 to 6)

Construct a Z-chart.

18.17 Starting torque in foot-pounds of a motor,  $T = 7.05P/N$

where,

$P$  = rotor input (5000 to 25000 watts)

$N$  = rotor synchronous speed (600 to 800 rpm)

Construct a Z-chart.

## **REVIEW QUESTIONS**

- 18.1 What is a nomogram?
- 18.2 What is a concurrency chart?
- 18.3 What is the slope of a line represented by the equation,  $4a + b = c$ ?
- 18.4 What is an alignment chart?
- 18.5 What is Z- chart? Why is it given that name?

## **OBJECTIVE QUESTIONS**

- 18.1 Multiplication equation can be solved by converting it, into an addition/subtraction equation.
- 18.2 A straight line, represented by,  $y = ax$ , passes through the origin, when plotted in a cartesian coordinate system.  
(True/False)
- 18.3 The equation of the type,  $v = wr$  may be represented by either the concurrency chart or alignment chart.  
(True/False)
- 18.4 Z- chart is similar to an alignment chart.  
(True/False)

## **ANSWERS**

18.1 addition

18.2 True

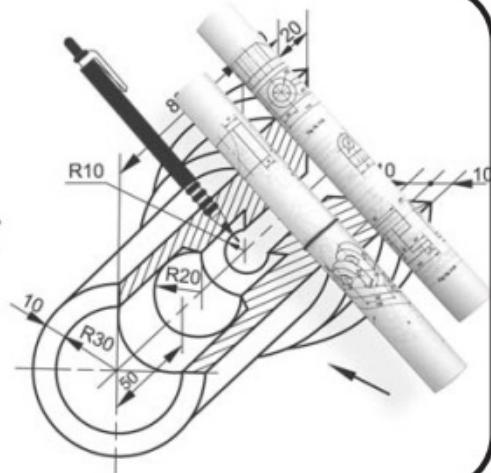
18.3 True

18.4 True

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# 19

## Miscellaneous Topics



### 19.1 INTRODUCTION

The topics, which are essential and useful to the students during the study of certain courses later, are presented under this chapter. In this chapter, loci of points, visibility, sectional views and graphical representation of data are covered.

### 19.2 LOCI OF POINTS

The term locus of a point is defined as the path of a point, moving in space. The locus of any point on a link in a mechanism represents the motion characteristic of that particular point. In other words, the motion characteristic of a link in a mechanism may be checked, by determining the loci of a number of points on it.

Link mechanisms only are considered here as the curves such as cycloid, parabola, trochoid, etc., have already been dealt in Chapter 5.

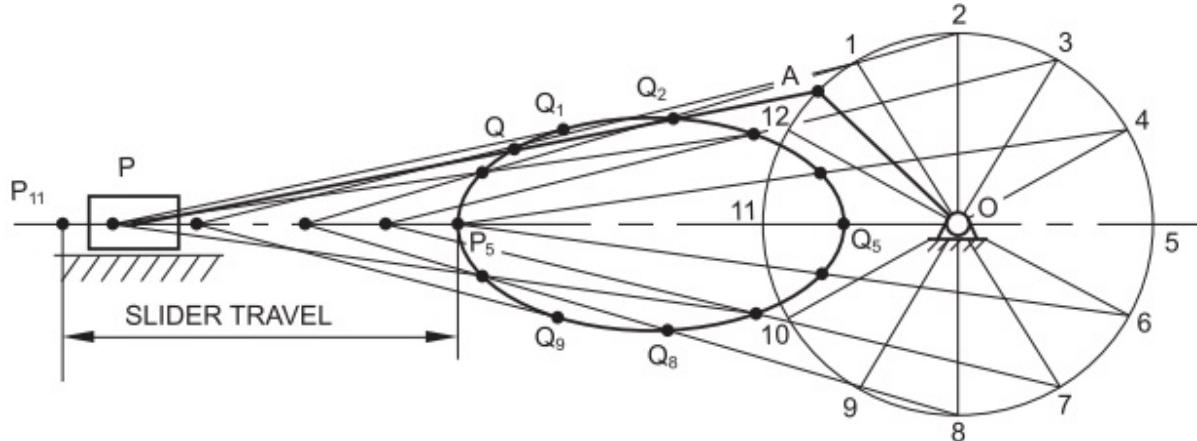
The problems may be solved by applying common sense rather than by applying the knowledge of geometrical principles. The solutions may be found either by (i) drawing different positions for the links or component parts of the mechanism, locating the tracing point for each position and joining the points thus obtained or (ii) using a paper trammel. Here, the solutions based on the first method only are presented.

## 19.2.1 Slider Crank Mechanism

It is one of the simplest mechanisms, consisting of four links, with one sliding pair and three turning pairs. In Fig.19.1, AP is the connecting rod and OA is the crank. The end of the connecting rod P is connected to a slider. The locus of A, the crank pin, is a circle and the locus of P is a straight line.

**Problem 1** Determine the locus of a point Q on the connecting rod, given the link lengths and the position of Q on AP.

**Construction (Fig.19.1)**



**Fig.19.1**

1. Assume different positions of the crank and determine the corresponding positions for the connecting rod.
2. Locate the position of the point Q for each position of the crank.
3. Join all the points by a smooth curve, which is the locus of the point Q.



When the angle between the crank and connecting rod is  $90^\circ$ , the connecting rod occupies its extreme position in its oscillation. There are two such positions. The position of the point Q corresponding to the above two positions of the connecting rod gives the turning points on the curve.

## 19.2.2 Four-bar Mechanism

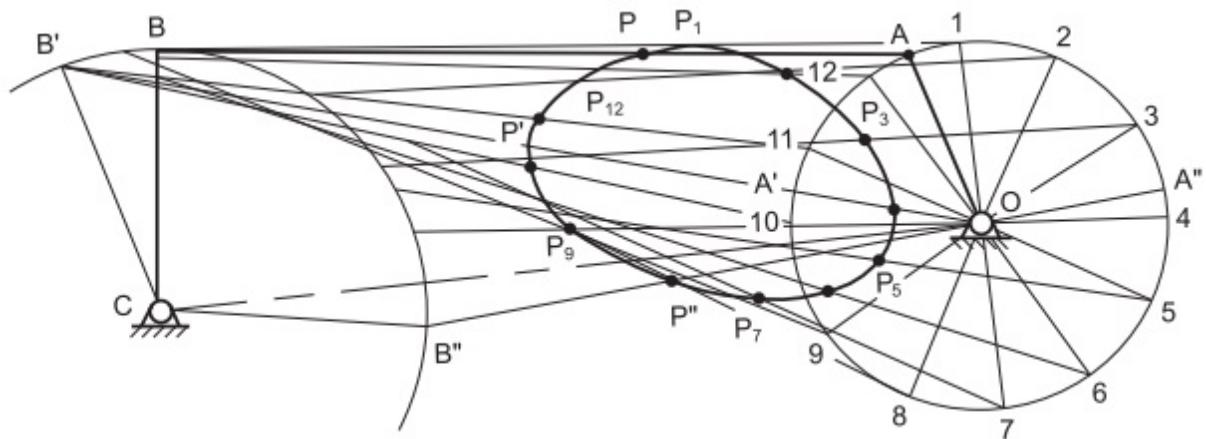
This is also one of the simplest mechanisms, consisting of four links, with four turning pairs. Referring [Fig.19.2](#), the link OA may be treated as the input link, rotating about the fixed point O, while the link CB may either rotate completely or oscillate about the fixed point C. The link AB is called a coupler.

**Problem 2** *Trace the path of a point, P on the coupler in the four-bar mechanism, shown in [Fig.19.2](#).*

### **Construction ([Fig.19.2](#))**

1. With centre O and radius OA, draw a circle representing the locus of A and with centre C and radius CB, draw an arc, representing the locus of B.
2. Locate the limiting positions of the point B, i.e., B' and B'', when the input link and the coupler lie in a line ( $OA + AB = OB'$  and  $AB - OA = OB''$ ).

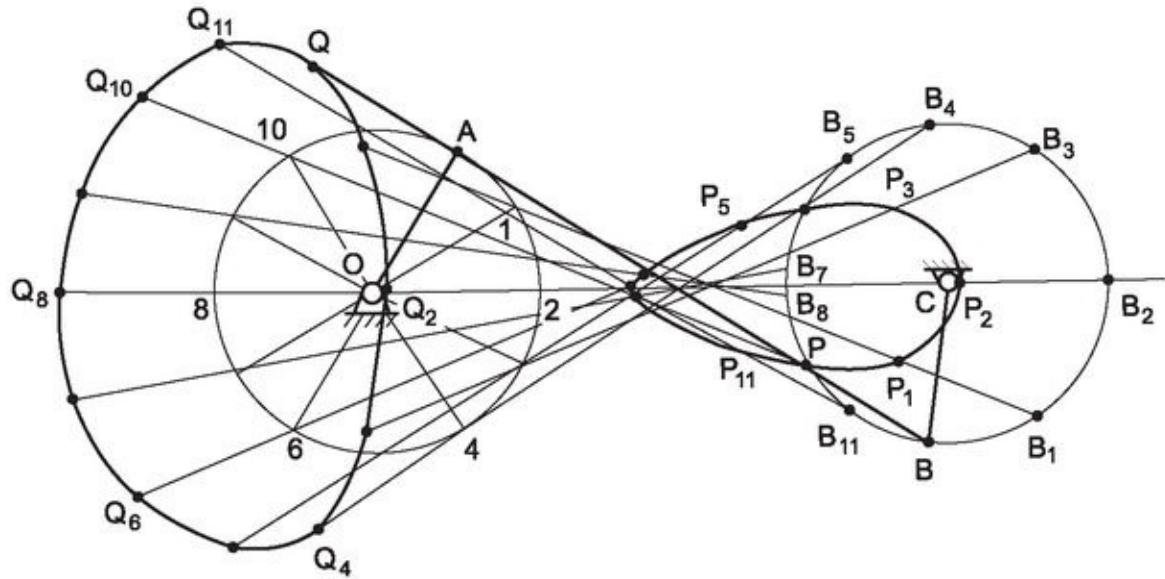
3. Consider different positions of A and locate the corresponding positions of the coupler AB.
4. Locate the point P for each position of AB.
5. Locate the turning points  $P'$  and  $P''$  on the curve, corresponding to the output link positions  $CB'$  and  $CB''$ .
6. Join all the points by a smooth curve, which is the required locus.



**Fig.19.2**

**Problem 3** Draw the loci of points P and Q on a four-bar mechanism, shown in Fig.19. 3. Links OA and CB are of equal length and rotate about O and C respectively, through  $360^\circ$ .

**Construction (Fig.19.3)**

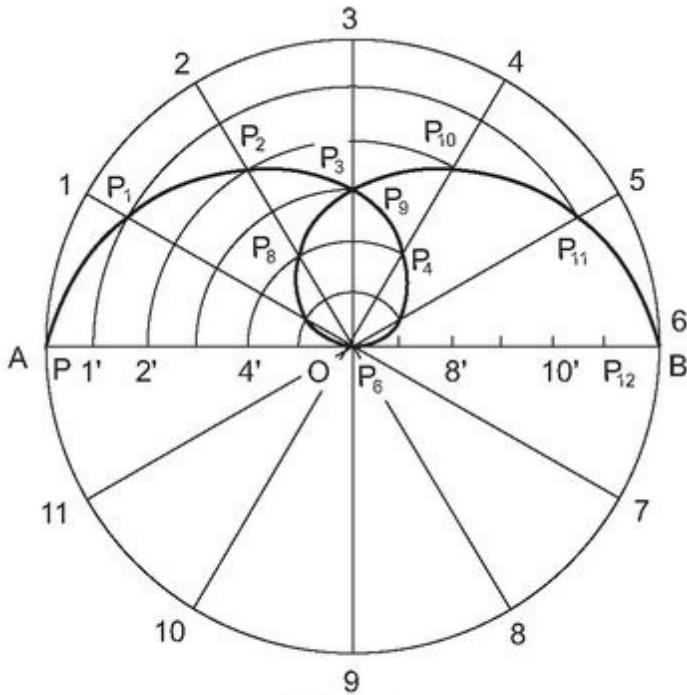


**Fig.19.3**

1. Draw two circles, representing the paths of A and B.
2. Divide the path of A into some number of equal parts and locate the position of link AB for each position of A.
3. Locate the points P and Q for each position of AB and join by a smooth curve, obtaining the required loci of P and Q.

**Problem 4** A rod AB rotates about its mid-point and a point P moves from A to B in one complete rotation of AB. Trace the path of the point P.

**Construction (Fig.19.4)**

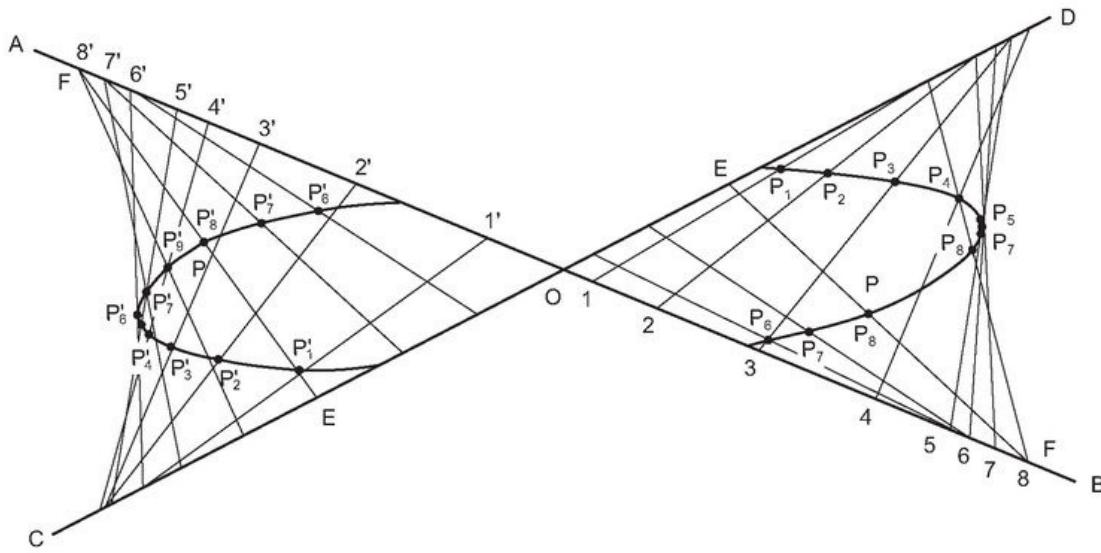


**Fig.19.4**

1. Divide the path of the rod, viz., the circle and the length of the rod AB into the same number of equal parts, say 12.
2. When the rod occupies the position 1-7, draw an arc with centre O and radius O1' to intersect at  $P_1$ , the position of P along the rod.
3. Repeat the procedure for different positions of the rod and locate the points  $P_2, P_3$ , etc.
4. Join the points  $P_1$ , to  $P_{12}$  by a smooth curve, which is the required locus.

**Problem 5** *AB and CD are two fixed bars intersecting at O. A rod EF of constant length slides between the bars so that, its ends E and F are always on the bars CD and AB respectively. Trace the locus of P lying on EF.*

### **Construction (Fig.19.5)**



**Fig.19.5**

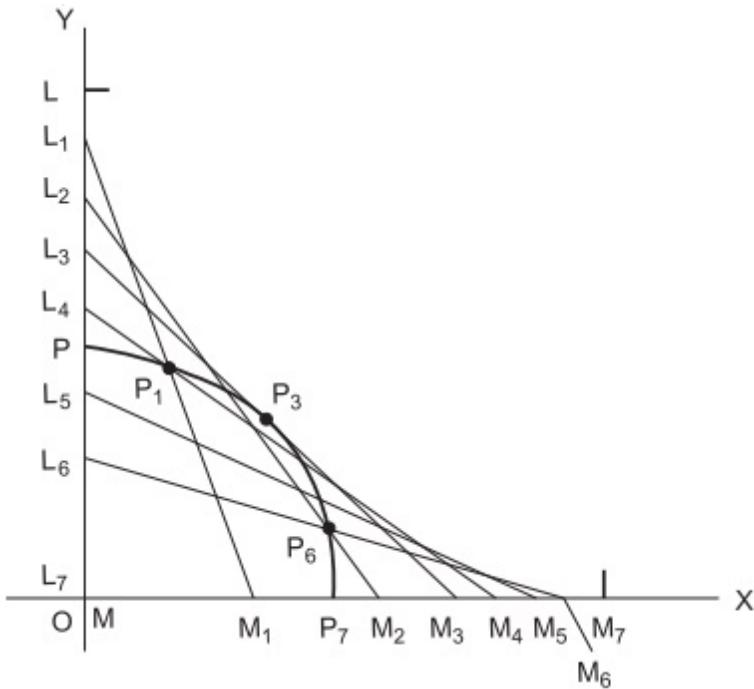
1. Mark a number of points 1, 2, 3, etc., and 1', 2', 3', etc., along OB and OA respectively.
2. Locate the positions of the rod EF, corresponding to the above positions for F.
3. Mark the position of P, for each of the above positions.
4. Draw a smooth curve joining these points, which forms the locus of P.



At the points 6, 7, 8 and 6', 7', 8' considered, the rod EF may occupy two possible positions and thus two positions for the point P, are marked.

**Problem 6** One end of a rod slides along a vertical surface, while the other end slides along a horizontal surface. Draw the locus of the mid-point on the rod for a complete movement of the rod.

### **Construction (Fig.19.6)**



**Fig.19.6**

1. Draw the lines OX and OY, perpendicular to each other; representing the horizontal and vertical surfaces respectively.

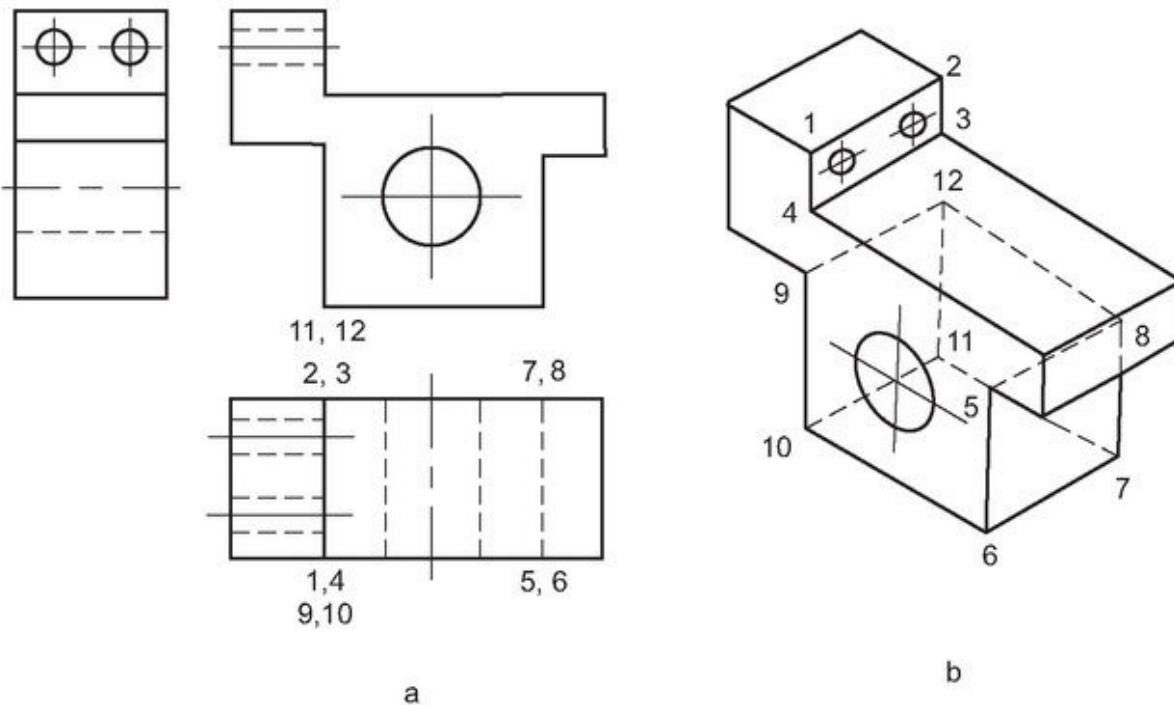
Let LM represents the position of the rod, when it coincides with the vertical surface and P be its mid-point. It may be noted that the length of the rod may be chosen arbitrarily.

2. Locate number of points  $L_1, L_2, \dots, L_7$  on the vertical surface, which need not be equi-distant.
3. Using the chosen length LM of the rod, locate the corresponding end points  $M_1, M_2, \dots, M_7$  on the horizontal surface.
4. Locate the points  $P_1, P_2, \dots, P_7$  at the middle of  $L_1M_1, L_2M_2, \dots, L_7M_7$  respectively.

5. Join the points  $P, P_1, P_2, \dots, P_7$  by a smooth curve, forming the required locus.

## 19.3 VISIBILITY

Problems dealing with solids or with opaque surfaces require the use of dotted lines to represent invisible features. [Figure 19.7](#) shows the pictorial representation of an object along with its orthographic projections. The rules of visibility may be understood from the following observations:



**Fig.19.7**

*In the top view,*

1. The face 1-2-3-4 appears edge-wise and lines 4-3, 9-12 and 10-11 coincide with 1-2. As these edges are just

behind 1-2, they are not shown dotted.

2. The edges of the holes and the face 5-6-7-8 are shown dotted.

*In the side view,*

1. The top edge of the hole is just behind the visible bottom edge of the projection.
2. The lower edge of the hole is shown dotted because it is invisible.

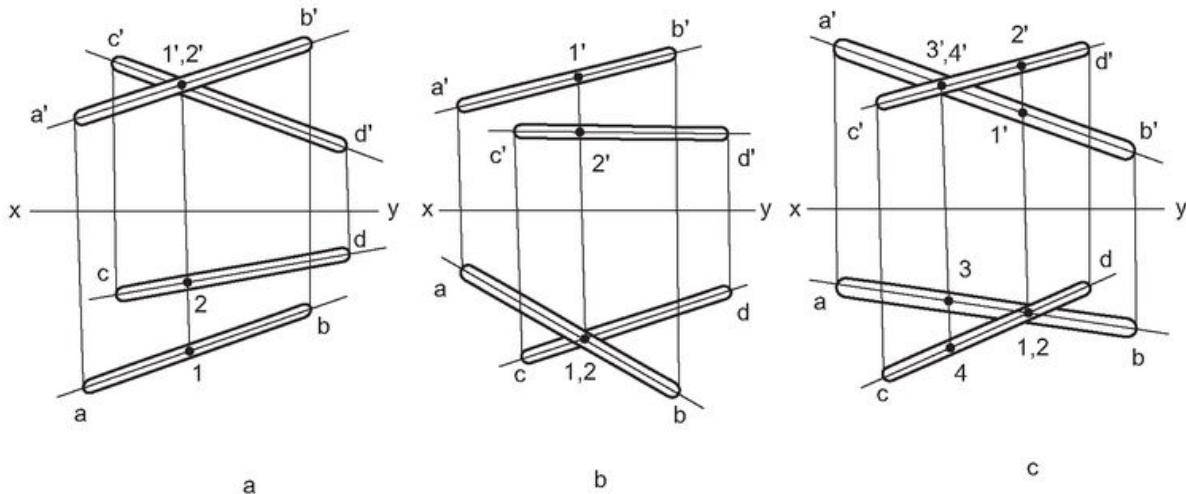
*In general, the rules governing visibility in any view are:*

1. In the front view, the parts nearer to the observer will be visible. These are determined from the top view.
2. In the top view, the highest parts are visible. These are determined from the front view.
3. In the right side view, the parts on the right are visible. These are determined from the front and top views.

Some times, there is an advantage in choosing a number of views to reduce the number of invisible lines to be represented. This simplifies both the making and reading of the drawing.

### 19.3.1 Determination of Visibility

**Problem 7** Figures 19.8a, b and c show the projections of two lines in three possible positions. Study the views and determine the visibility of one line relative to the other.



**Fig.19.8**

1. In Fig.19.8a, it may be observed that the point 1 on AB is in front of the point 2 on CD in the top view. Hence, the rod AB is visible and shown in front of CD, in the front view.
2. In Fig.19.8b, the front view shows that the point 1 on AB is nearer to the observer than 2 on CD. Hence, at the crossing point shown in the top view, AB is completely visible and shown over CD.
3. In Fig.19.8c, the point 2 on CD is nearer to the observer than 1 on AB in the front view. Hence, in the top view, CD is shown over AB. In the top view, the point 4 on CD is in front of the point 3 on AB. Hence, in the front view CD is shown in front of AB.

**Problem 8** *Figure 19.9 shows the projections of a line, intersecting an oblique plane. Determine the visibility of the line with respect to the plane.*

*I. Edge view method*

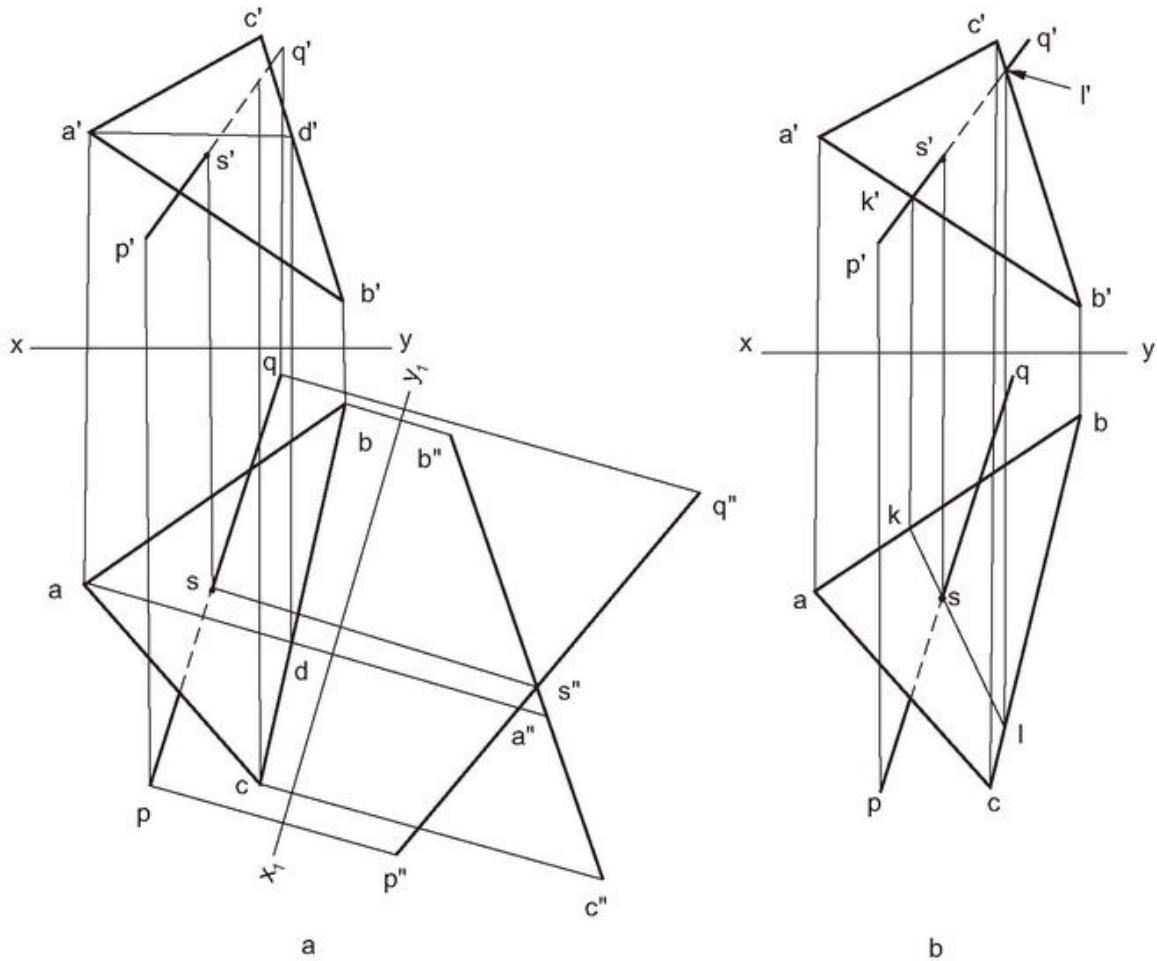
**Construction (Fig.19.9a)**

1. Obtain the edge view of the plane and determine the intersection point  $s''$ , between the plane ABC and the line PQ.
2. In the edge view,  $q''$  is nearer to the observer and hence,  $q''s$  is visible in the top view. Similarly, in the top view,  $p$  is nearer to the observer and hence,  $p's'$  is visible in the front view.

*II. Cutting plane method*

**Construction ([Fig.19.9b](#))**

1. Imagine a vertical cutting plane KL containing the line  $p'q'$  and obtain its projections  $kl$  and  $k'l'$ .
2. Locate the intersection point  $s$  between  $kl$  and  $pq$ .
3. Project and obtain the point  $s'$  in the front view.



**Fig.19.9**

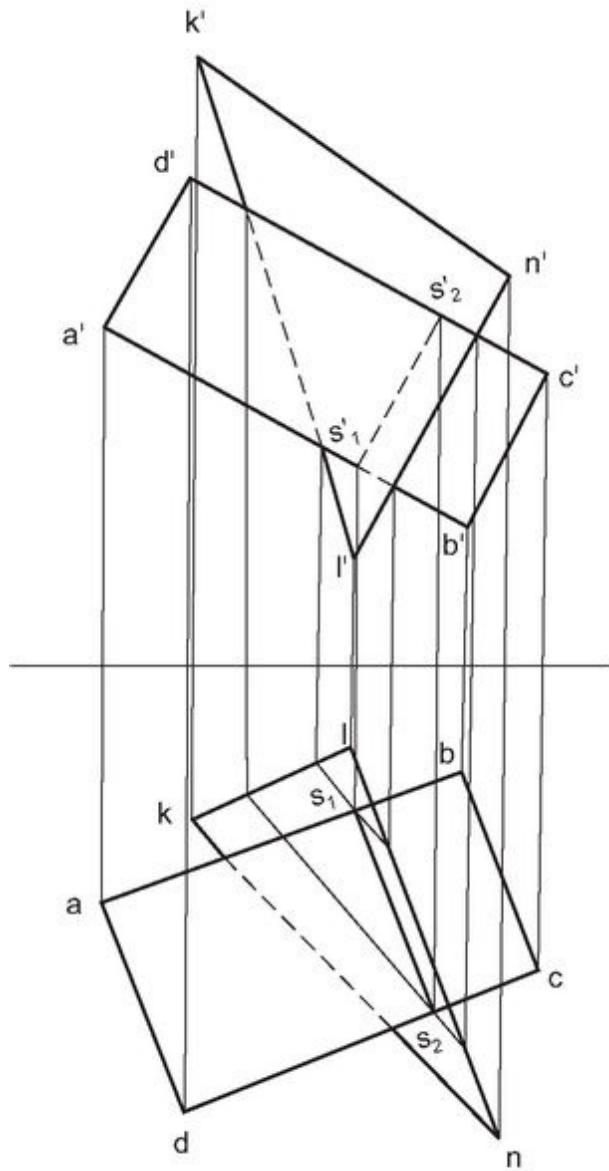
In the top view,  $kl$  is the line of intersection between the vertical plane considered and the given plane  $ABC$ . The point of intersection between the vertical plane and the line  $PQ$ , viz.,  $s$  also lies on  $kl$ . Hence,  $s$  lies on the plane  $ABC$  and thus it is the piercing point.

Since  $q'$  is nearer to the observer, the line  $qs$  is visible. Also, since  $p$  is nearer to the observer,  $p's'$  is visible.

**Problem 9** *Figure 19.10 shows the projections of two intersecting oblique planes. Determine the visibility of the planes.*

1. Consider the lines AB and CD of the given plane ABCD and determine the projections of the piercing points  $S_1$  and  $S_2$  between the lines and the plane KLN, following the method explained in the preceding section (cutting plane method).
2. Join  $s_1^{\text{f}}$ ,  $s_2^{\text{f}}$  and  $s_1$ ,  $s_2$  in the two views, forming the projections of the line of intersection between the planes.

The edge AD is nearer to the observer in both the views. Hence, the portion of the plane ABCD, viz.,  $S_1S_2DA$  is visible. The portion of the plane covered by the plane KLN is not visible.



**Fig.19.10**

**Problem 10** *Figure 19.11 shows the front, top and auxiliary views of an irregular triangular pyramid. Determine the visibility of the edges of the solid.*

1. Consider the intersection points 3, 4 between oc and ab in the top view. Let the point 3 lie on OC and 4 on AB.

2. Locate the points  $3'$  and  $4'$  in the front view, by projection.

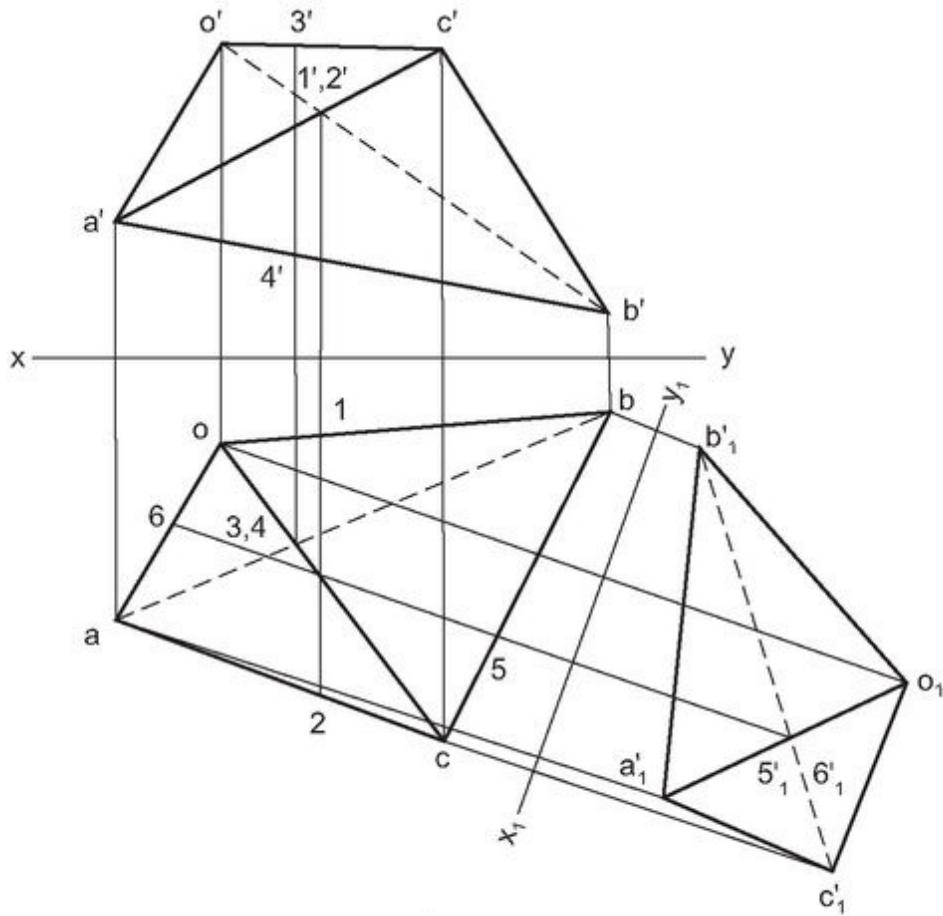
From the front view, it is clear that the point  $3$  is above  $4$ . Hence, the edge  $OC$  is visible in the top view.

3. Consider the intersection points  $1, 2$  between  $a'c'$  and  $o'b'$  in the front view. Let the point  $1$  lie on  $OB$  and  $2$  on  $AC$ .
4. Locate the points  $1$  and  $2$  in the top view.

From the top view, it can be observed that the point  $2$  is in front of the point  $1$ ; therefore, in the front view, the line  $a'c'$  is visible.

5. Consider the intersection points  $5, 6$  between  $o_1'a_1'$  and  $b_1'c_1'$  in the auxiliary view. Let the point  $5$  lie on  $BC$  and  $6$  on  $OA$ .

From the top view, it is clear that the point  $6$  is nearer to the observer. Hence, in the auxiliary view,  $b_1'c_1'$  is invisible



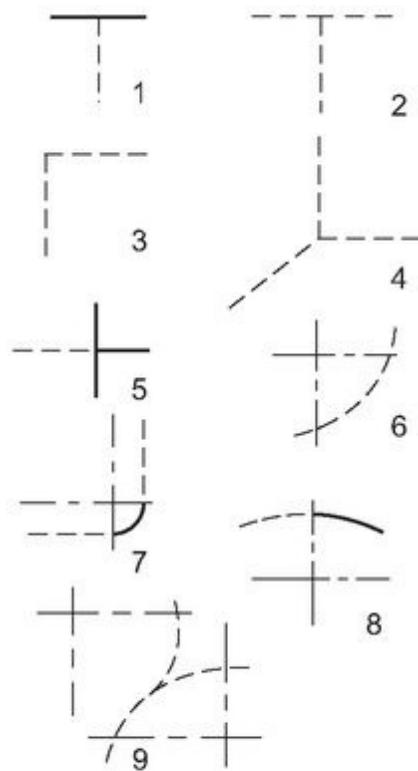
**Fig.19.11**

### 19.3.2 Techniques of Drawing Invisible Lines

Various conditions arising in the representation of the invisible features are standardized in the method of showing them. These are illustrated in Fig.19.12.

1. An invisible line intersects a visible line with a dash in contact.
2. An invisible line intersects another invisible line at the crossing point of two dashes in contact.

3. Invisible lines meeting at a corner have two dashes meeting at the corner.
4. Three invisible lines meeting at a corner, have three dashes intersecting at the corner.
5. When an invisible line shown as a continuation of a visible line, the invisible line begins with a space.
6. Invisible arcs begin with a dash.



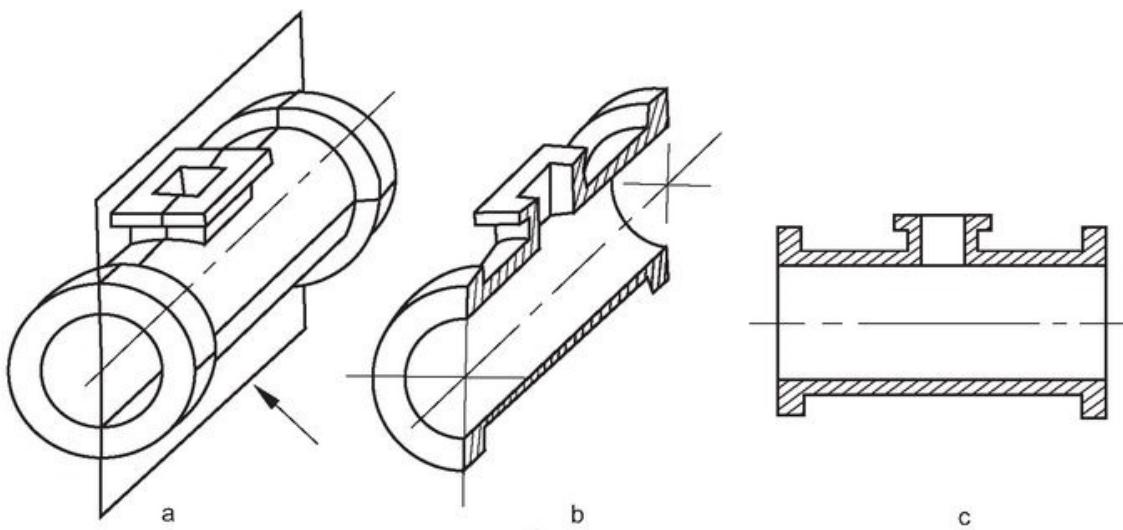
**Fig.19.12**

7. When arcs are too small, they may be made continuous.
8. When an invisible arc is a continuation of a visible arc, the invisible arc begins with a space.

- When two invisible arcs meet, the intersection at the point of tangency is located with a dash on each arc.

## 19.4 SECTIONAL VIEWS

Conventional orthographic views, when carefully selected, can reveal only the external features of an object. The number and types of views will depend upon the complexity of the object. It is the usual practice to represent the hidden details of the object by dotted lines. But, when the object contains complicated hidden details, the too many dotted lines required, will only cause confusion. In such cases, one or more number of views are drawn assuming as if, a portion of the object is cut and removed, to reveal the interior details. Such views are called sectional views. To explain the principle of obtaining a sectional view, only a relatively simple object is selected.



**Fig.19.13**

Referring Fig.19.13a, to obtain a sectional view, a section plane is assumed to be passing through the object.

[Figure 19.13b](#) shows the portion of the object lying behind the section plane. To draw the sectional view, it is the rule to assume as if that portion of the object lying between the observer and the section plane is removed. [Figure 19.13c](#) shows the sectional (orthographic) front view, drawn with the actual sectioned zone cross-hatched by lines inclined at  $45^\circ$  to the principal edges of the view.

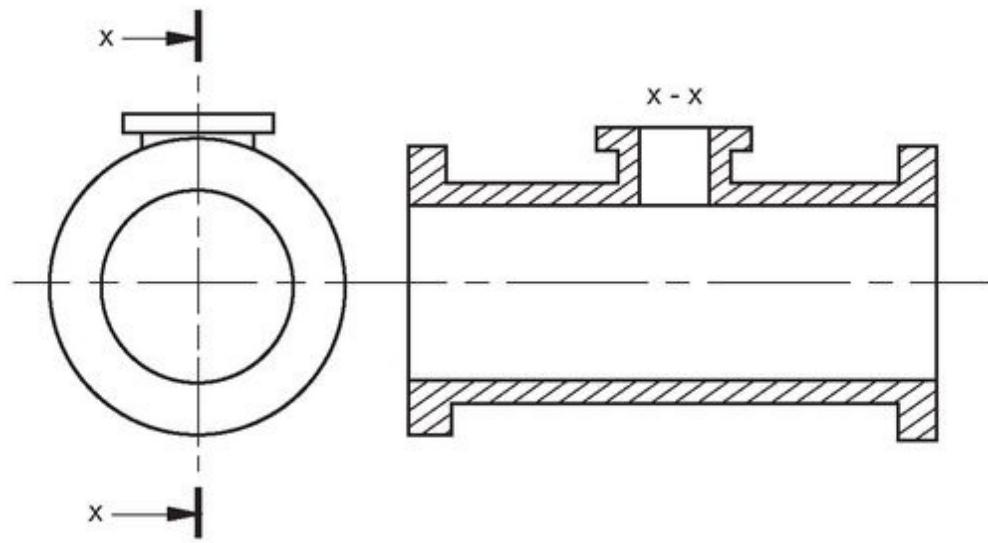


1. The section plane is usually assumed to be passing through the axis of symmetry of the object. In case of un-symmetric objects, the position of the section plane must be specified.
2. In a sectional view, the cross-hatched areas are those portions that have been actually cut by the section plane and the visible parts behind the section plane must be shown without hatching.

#### 19.4.1 Full Section

A sectional view obtained by assuming that the object is completely cut by a plane is called a full sectional view. That is, to draw a full section, one half of the object only, is imagined to be removed.

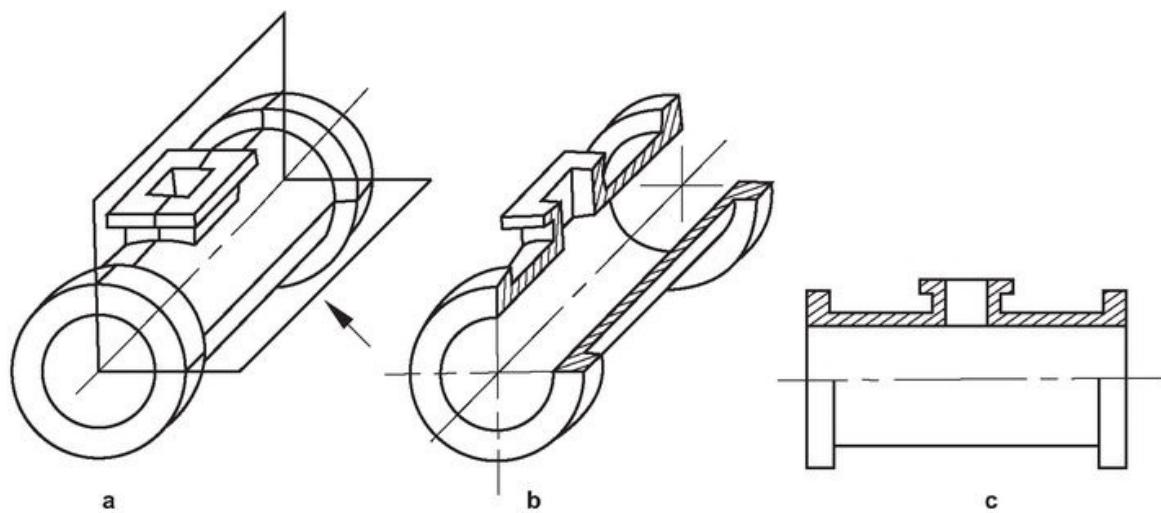
[Figure 19.14](#) shows the sectional front view and right side view, for the object shown in [Fig. 19.13a](#). It may be noted that the object is shown as if cut only in the sectional view and the sectioning effect should not be shown in the other views, which are plain.



**Fig.19.14**

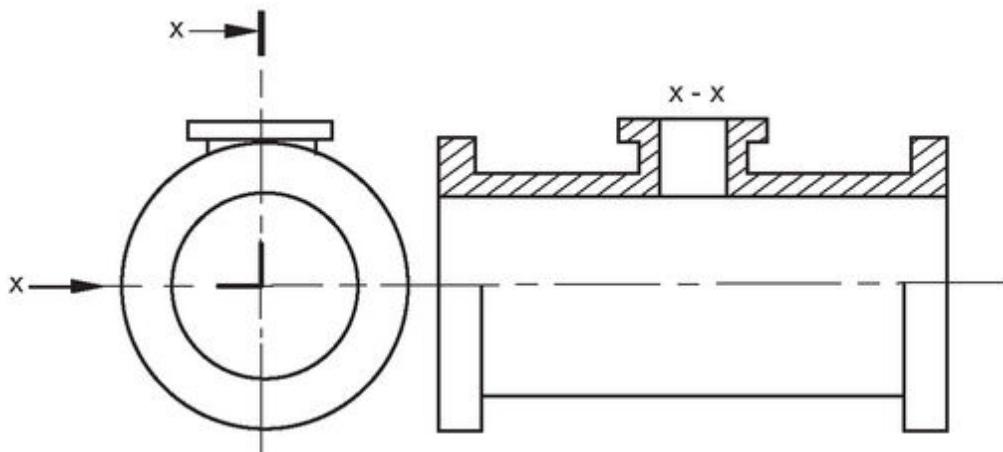
Note that the section plane appears as a line in the side view with the arrow heads indicating the direction of sight for obtaining the sectional view.

#### 19.4.2 Half Section



**Fig.19.15**

For a symmetrical object, the purpose of sectioning may be achieved by assuming a section plane, removing only one quarter of the object. In such a case, the view obtained is called the half-sectional view. [Figure 19.15a](#) shows the position of the section plane, for obtaining the half-sectional view. It may be noted that a half-sectional view reveals both the exterior and interior details of a symmetrical object. [Figure 19.15c](#) shows the halfsectional front view. [Figure 19.16](#) shows the half-sectional front view and right side view.



**Fig.19.16**

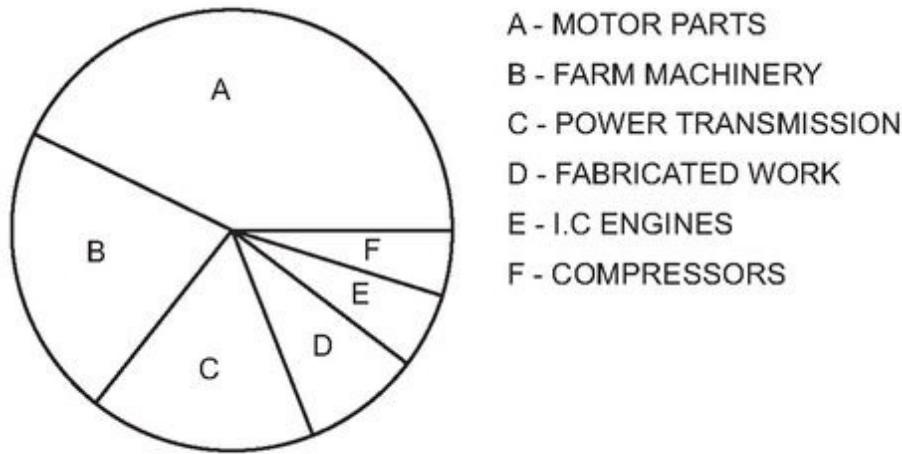


It is the usual practice to omit dotted lines in the sectioned portion of the sectional view.

## 19.5 GRAPHICAL REPRESENTATION

An effective method of conveying correlated data is by way of graphical representation. Graphical methods are used in tabulating data for analysis, solving problems and presenting facts. Properly constructed charts, graphs and diagrams form a powerful tool for a statistician for the future prediction. Also, industries prepare popular types of

graphs in order to strengthen their relationship with the public. The charts require mental effort to comprehend the facts (refer Fig.19.17).



**Fig.19.17 Uses of ductile iron**

### 19.5.1 Classification of Charts

According to use, the two types are (i) those used for technical purpose and (ii) those used for popular appeal. In general, the classification can be as follows:

1. Rectilinear charts
2. Semi-logarithmic charts
3. Logarithmic charts
4. Bar and area charts
5. Percentage bar charts
6. Polar charts
7. Trilinear charts
8. Pictorial charts

## 9. Nomograms

Out of the above, nomograms are dealt in Chapter 18.

### 19.5.1.1 **Rectilinear Charts**

A rectilinear chart is made on a sheet, ruled with equi-spaced horizontal lines, crossing equi-spaced vertical lines. The charts are available in centimetre and inch units, with optional spacing between the lines. It is assumed that the student is familiar with the terms such as abscissa, ordinate, axes, etc. The chart, when prepared purely for reading the values, it is then called a quantitative and when it is prepared for comparison, it is then called a qualitative chart.

A title has to be provided for each curve and when more than one curve is shown on a sheet, the different curves should be drawn so as to be easily distinguishable by varying the character of the lines, with a tabular key for identification.

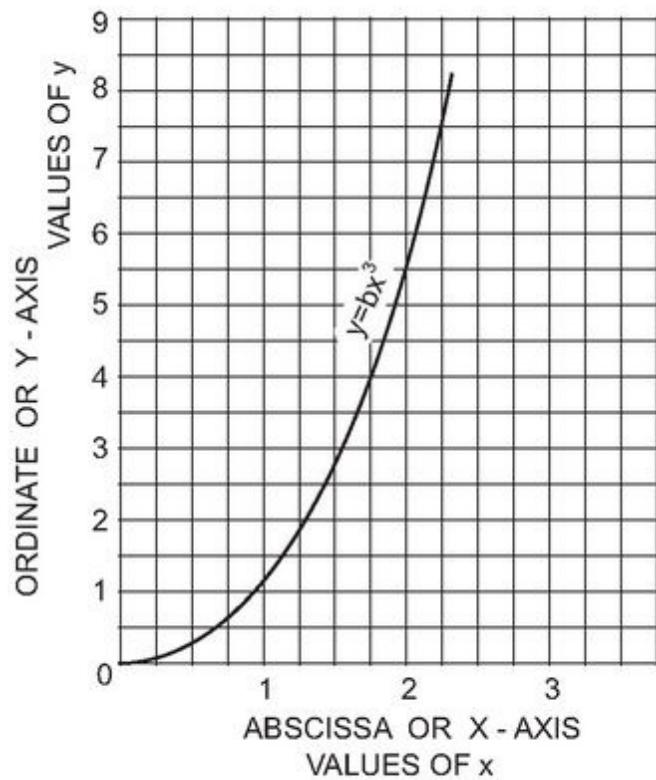
The zero line should be included in the chart, if visual comparison of the plotted magnitudes is desired. The independent variable values should usually increase from left to right and the vertical scale from bottom to top.

To draw a rectilinear chart (graph) for a relation,  $y = bx^3$

#### **Construction (Fig. 19.18)**

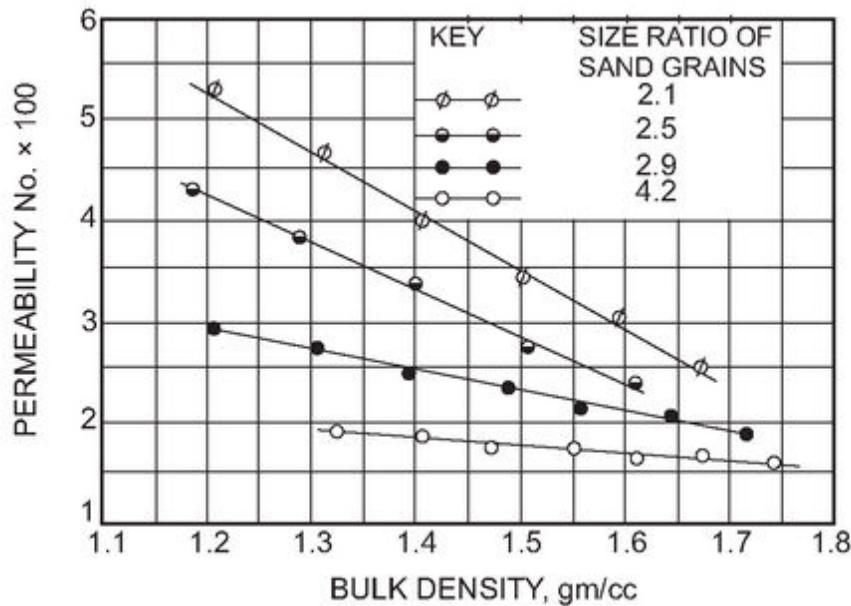
1. Mark the independent variable, viz.,  $x$  along the abscissa to a suitable scale, to (i) create the right impression of the relationship to be shown and (ii) ensure effective and efficient use of co-ordinate area.
2. Mark the dependent variable, viz.,  $y$  along the ordinate to a suitable scale.

3. By assuming suitable values to  $x$ , obtain the corresponding values for  $y$ .



**Fig.19.18**

4. Mark the points corresponding to the above values and draw a smooth curve through the points.



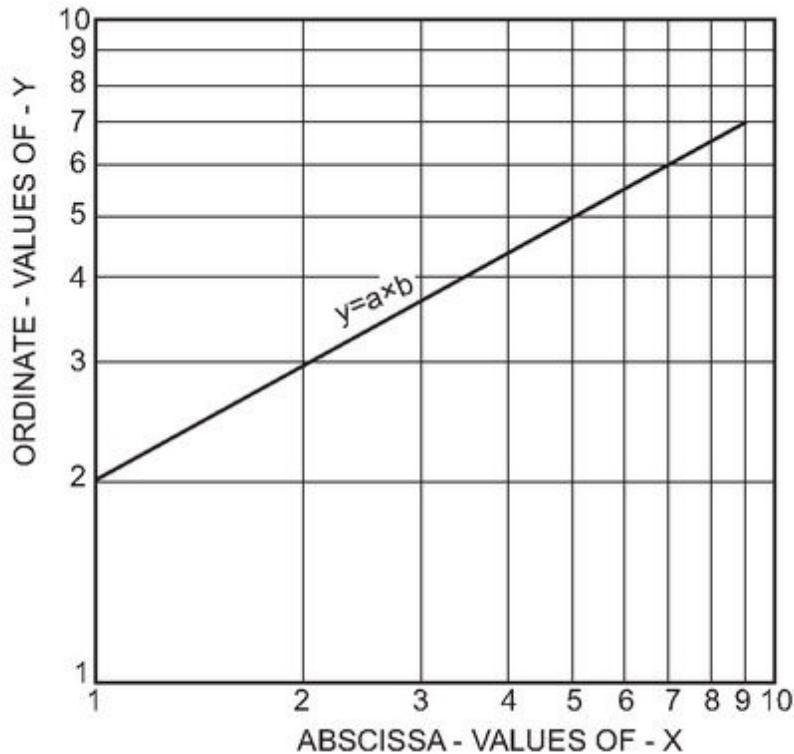
**Fig.19.19**



If the chart is drawn for representing an experimental observation, the plotted points should be marked by small circles. Open circles, filled-in circles and partially filled-in circles are preferred for identification symbols. Mathematical curves are drawn without distinguishing marks at the computed positions.

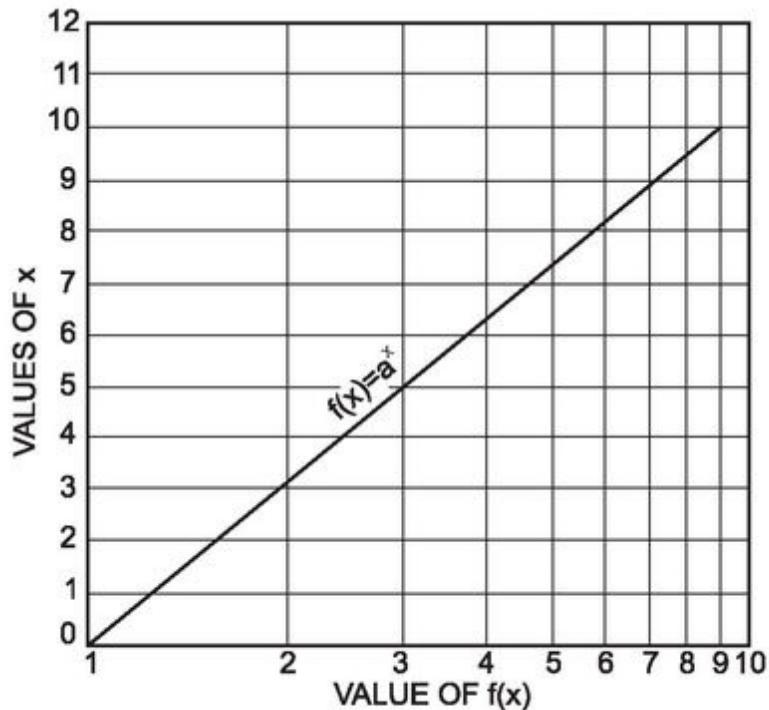
Figure 19.19 shows a chart, representing experimental observations. The construction particulars for the Figs.19.19 to 19.27 are not given, as they are self explanatory.

### 19.5.1.2 *Logarithmic Charts*



**Fig.19.20**

In these charts, horizontal and vertical rulings are spaced proportional to the logarithms of numbers. Main advantages of these graphs over the co-ordinate graphs are: (i) The error in plotting or reading the values is a constant percentage and (ii) an algebraic equation of the form,  $y = ax^b$ , appears as a straight line, if  $x$  has a value other than zero, as shown in Fig.19.20. Hence, the exponent of the equation may be determined by measuring the slope of the line.



**Fig.19.21**

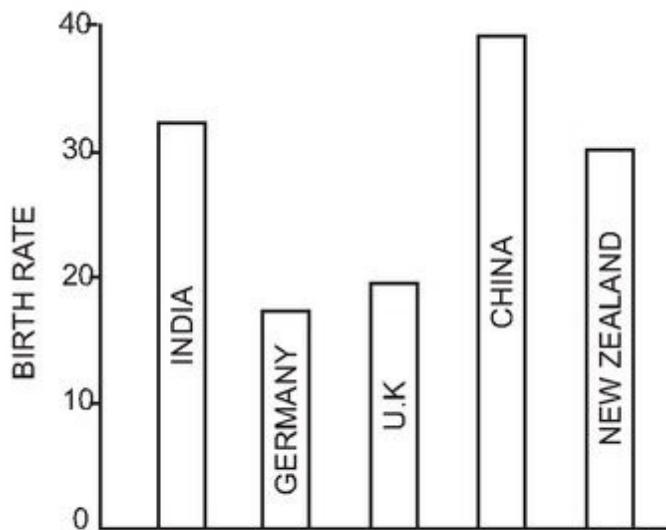
If the paper has ruled lines that are spaced to a uniform scale in one direction and to a logarithmic scale in the other direction, the chart is called a semi-logarithmic chart (Fig. 19.21). The slope of the curve on this chart indicates the rate of change rather than the amount of change. This paper should be used whenever percentage changes, rather than quantity change is to be shown, i.e., when the value of one variable increases in geometric progression and the other in an arithmetic progression, this form is more useful.

### 19.5.1.3 ***Bar Charts and Area Charts***

Bar charts are used principally to cover economic and industrial surveys representing the summary of statistical data, as in Fig.19.22. They can be easily understood by any average person. The bars may be drawn either horizontally

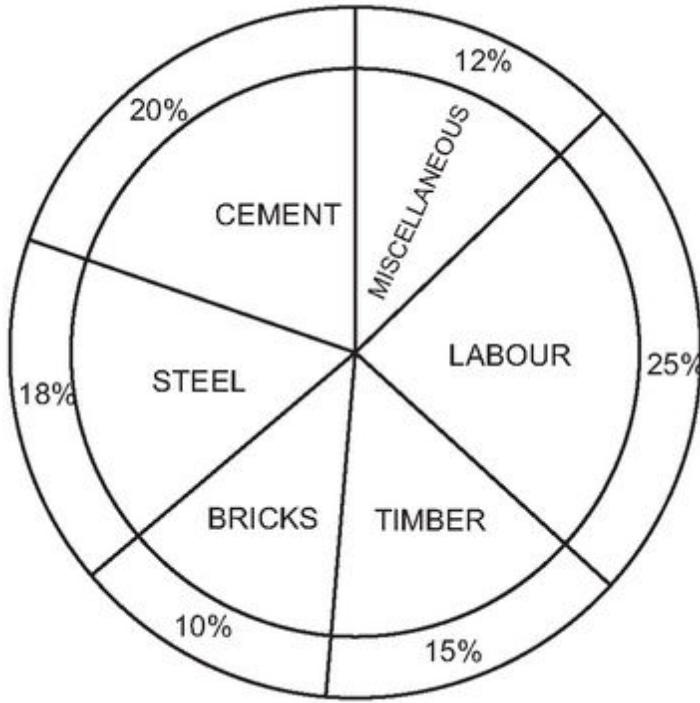
or vertically, but all should start at the same zero line. The bars should be proportional to the amounts they represent.

An area diagram, also known as pie chart can be used for pictorial representation, but it is much inferior to the bar chart. It may be regarded as a 100



**Fig.19.22 Birth rate per thousand**

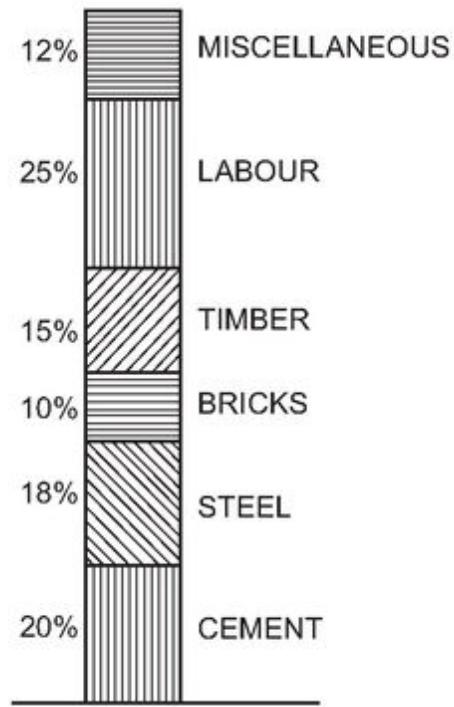
percent bar, bent into circular form. The circumference of the circle is divided into 100 parts and sectors are used to represent percentage, as shown in [Fig.19.23](#).



**Fig.19.23**

#### **19.5.1.4 Percentage Bar Chart**

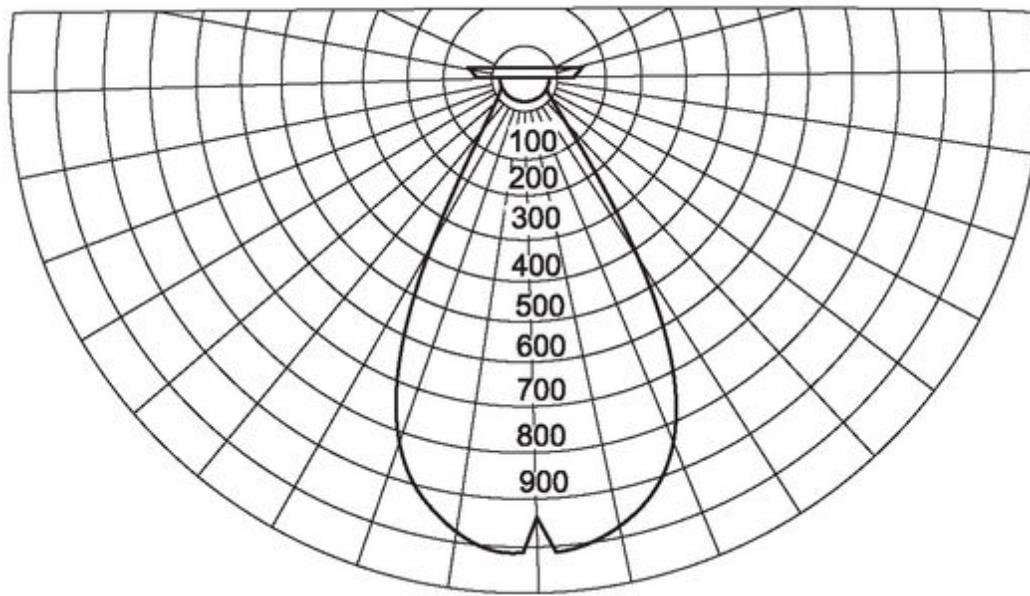
This chart fulfills the same purpose as the pie chart, i.e., unbending of the area chart. The descriptions may be placed on either side of the bar, as shown in Fig.19.24.



**Fig.19.24**

#### **19.5.1.5 *Polar Chart***

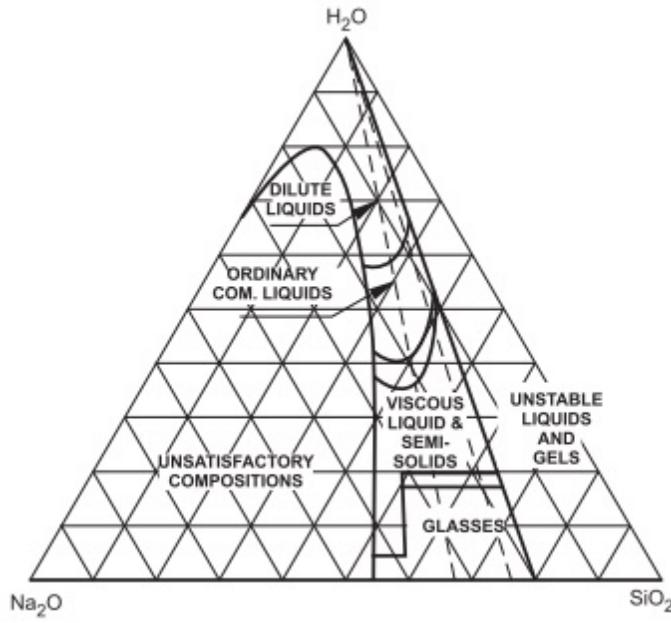
The chart may be constructed by self recording instruments to represent certain types of technical data. Polar curves are used to represent intensity of a light bulb ([Fig.19.25](#)) or heat, etc.



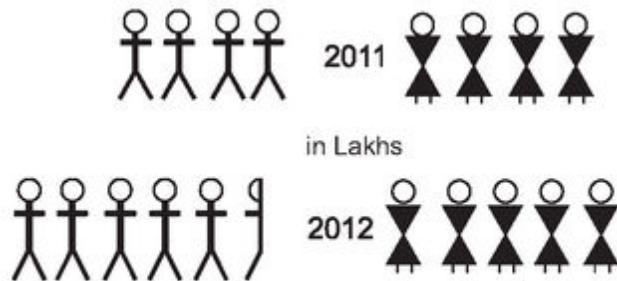
**Fig.19.25 Intensity of light bulb**

#### **19.5.1.6 Trilinear Chart**

They are used to study the properties of chemical compounds consisting of three elements, compounds or three variables, e.g., ternary equilibrium diagrams in metallurgy, the properties of sodium silicate ([Fig.19.26](#)) used in foundries, etc.



**Fig.19.26 Physical characteristics of sodium silicate**



**Fig.19.27 Voters in a particular area**

**Fig.19.27 Voters in a particular area**

The chart has the form of an equilateral triangle, the altitude of which represents 100 percent of each of the three constituents. Its use depends upon the geometrical principle, that the sum of the three perpendiculars from any point is equal to the altitude.

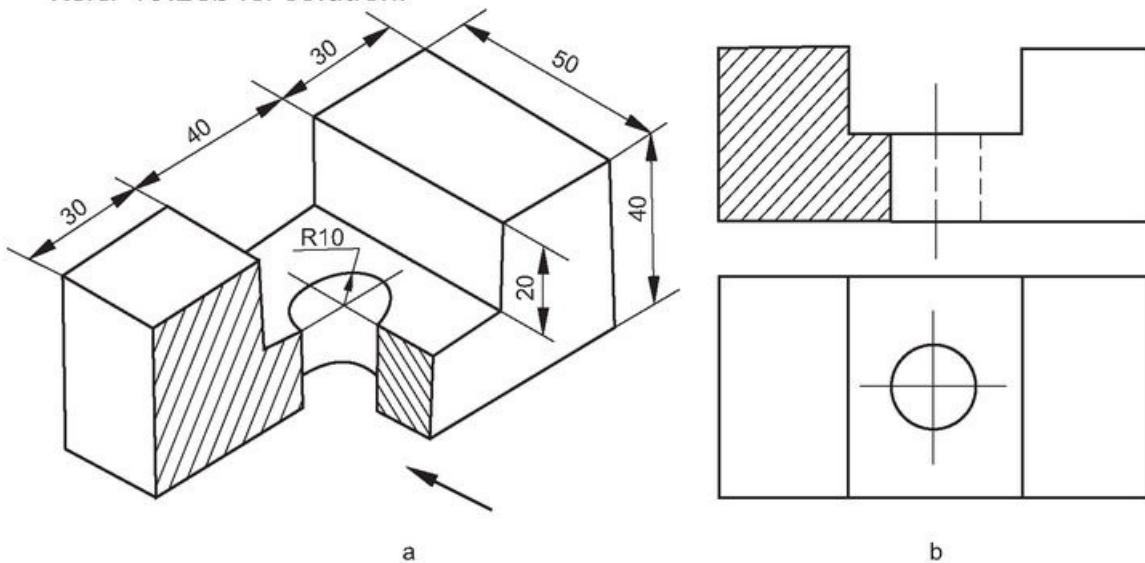
#### 19.5.1.7 *Pictorial Charts*

These charts are used to convey the information to non-technical persons. They are generally used to present the information such as the growth of population, illiterate and literate personnel, per capita income, etc. [Figure 19.27](#) shows voters in a particular area.

## 19.6 EXAMPLES

**19.6.1** [Figure 19.28a](#) shows a machine block. Draw the half sectional front view and top view.

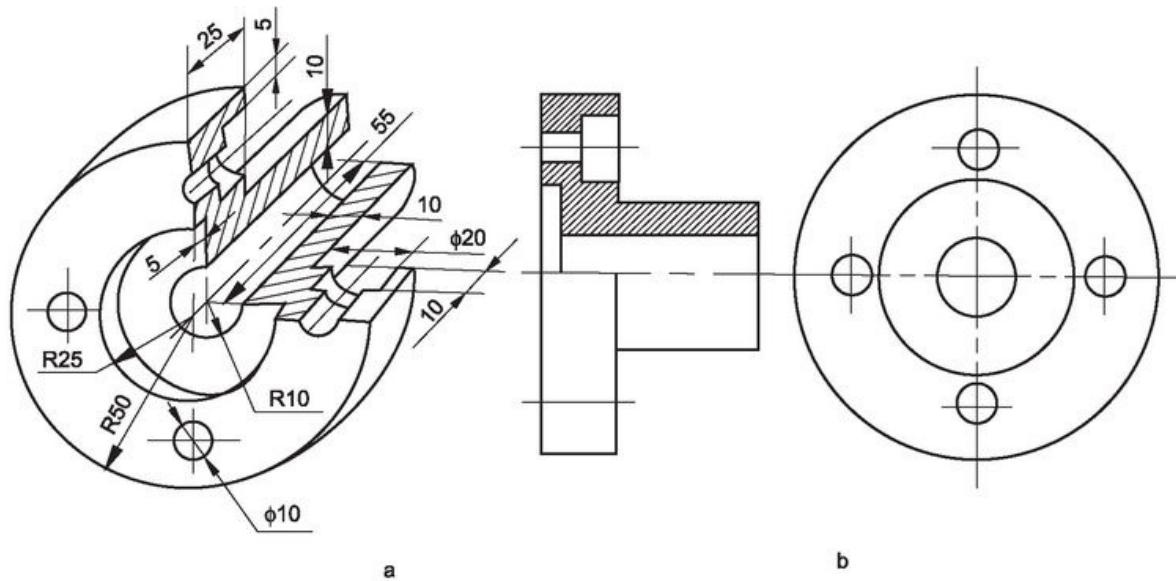
Refer [19.28b](#) for solution.



**Fig.19.28**

**19.6.2** For the flange shown in [Fig.19.29a](#), draw the half sectional front view and side view.

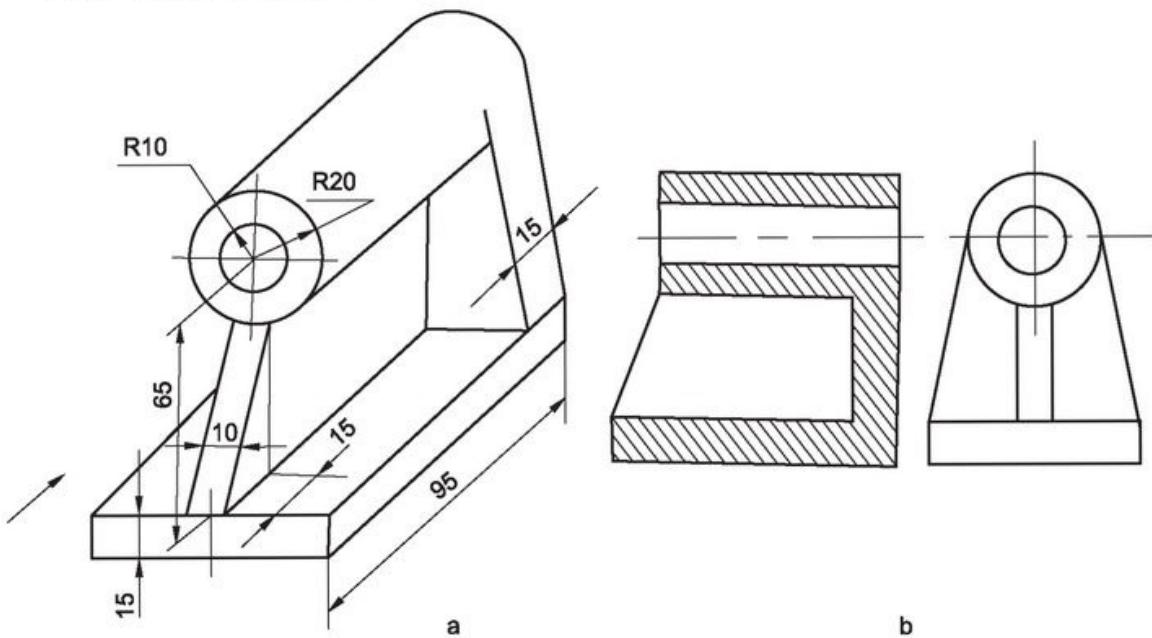
Refer 19.29b for solution.



**Fig.19.29**

**19.6.3** Figure 19. 30a shows a shaft support. Draw the front view and sectional right side view.

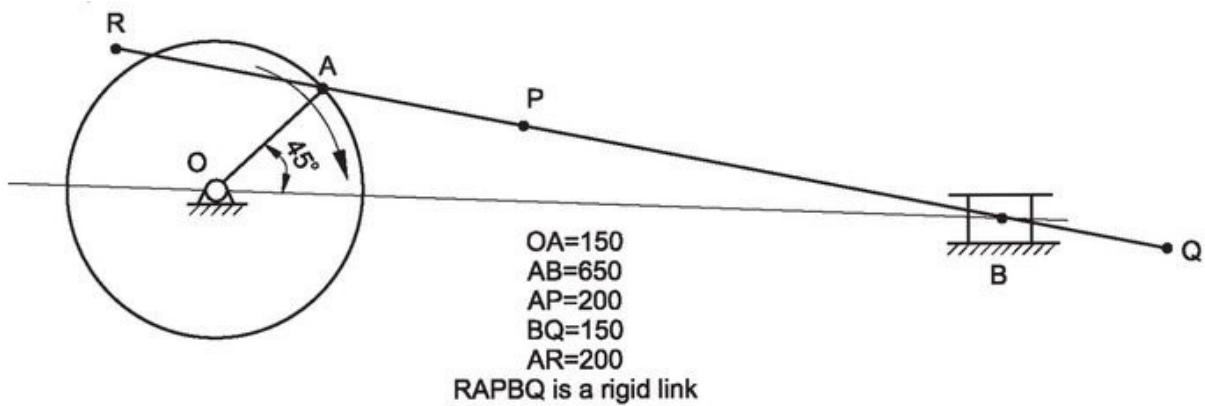
Refer Fig.19.30b for solution.



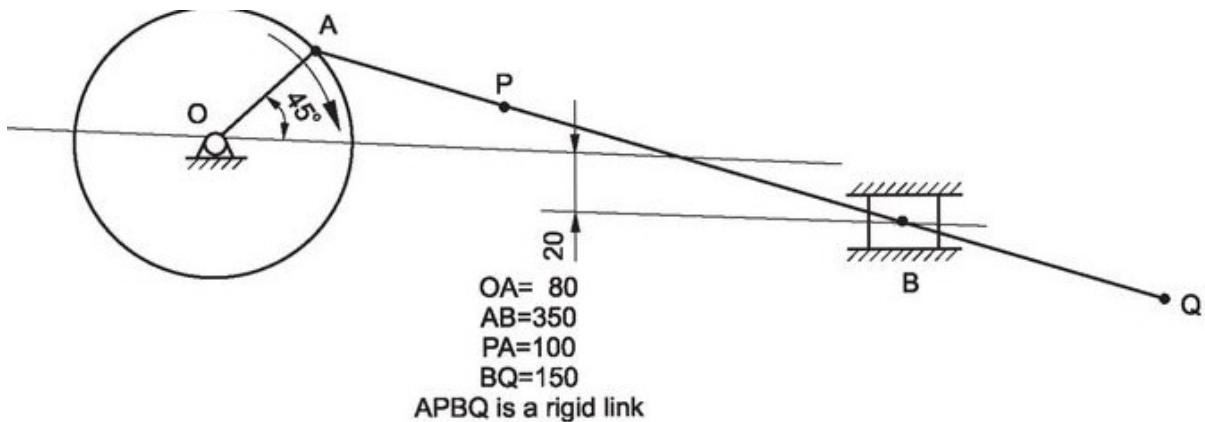
**Fig.19.30**

## EXERCISES

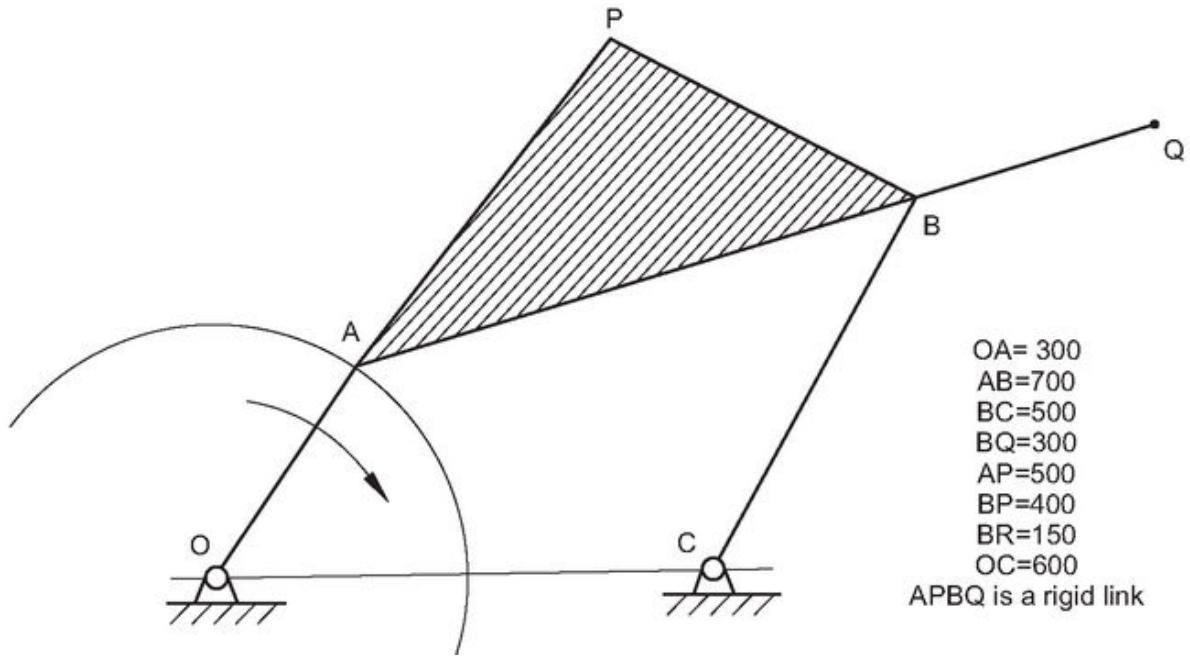
19.1 Figures 19.31 to 19.35 show certain mechanisms.  
Draw the loci of the points P, Q and R wherever marked (Hint: OA is the input link).



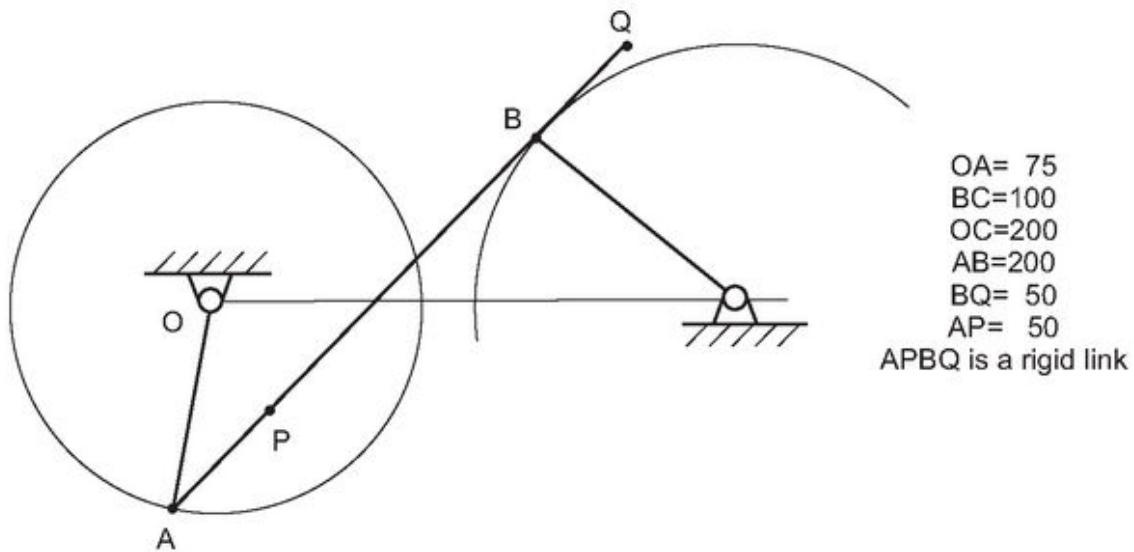
**Fig.19.31**



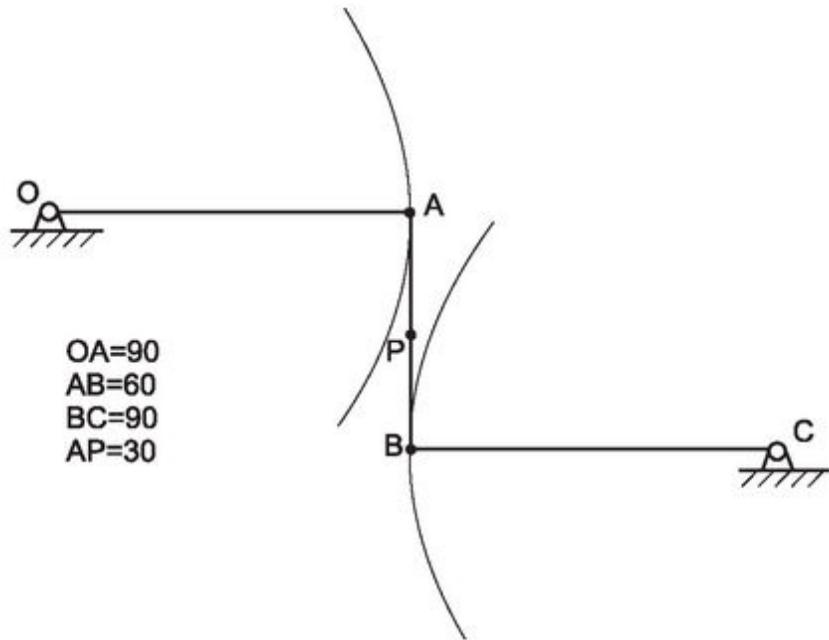
**Fig.19.32**



**Fig.19.33**

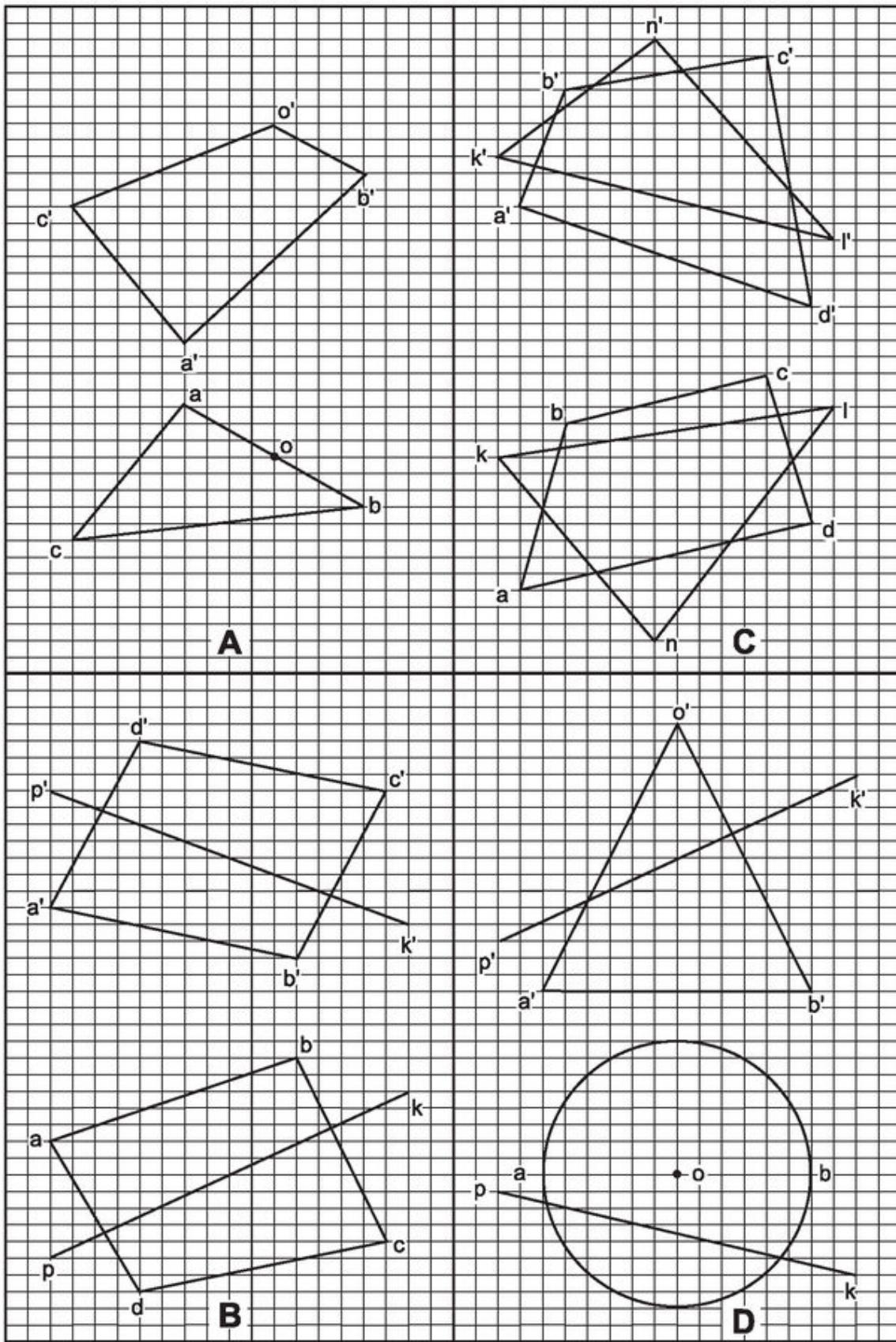


**Fig.19.34**



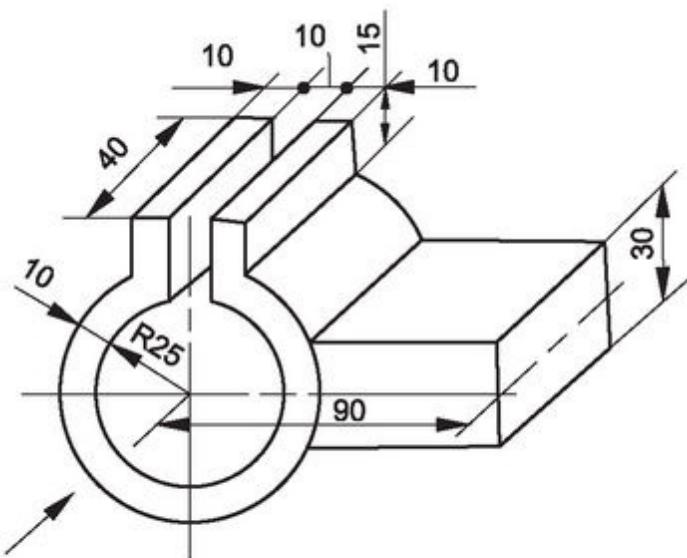
**Fig.19.35**

- 19.2 [Figure 19.36A](#) shows the projections of a tetrahedron. Determine the visibility of the lines.
- 19.3 [Figure 19.36B](#) shows the projections of a plane and a line. Determine the piercing point between the plane and line.

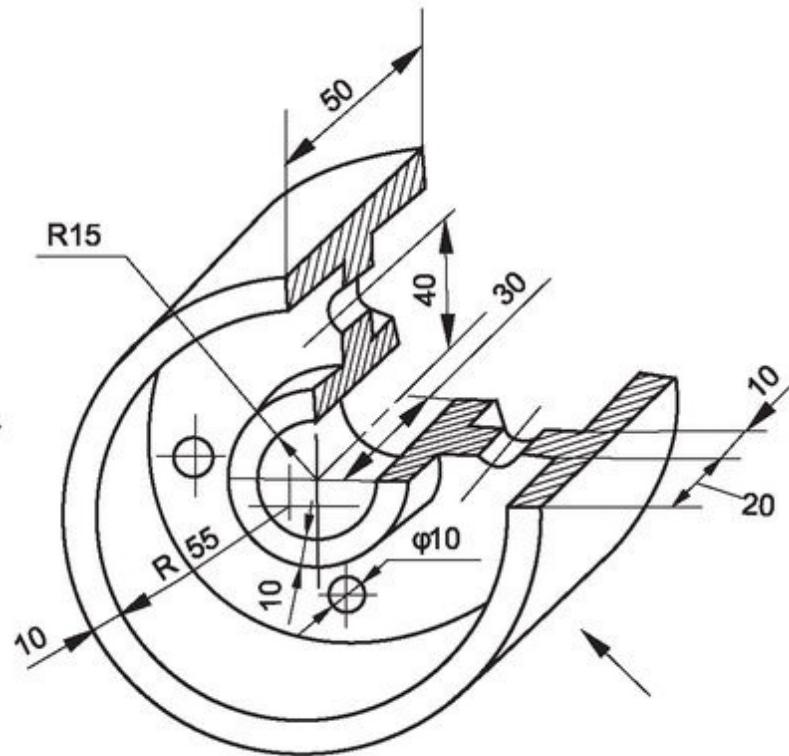


**Fig.19.36**

- 19.4 Figure 19.36C shows the projections of two planes. Determine the intersection line between the planes.
- 19.5 Determine the piercing points between a cone and a line, the projections of which are shown in Fig.19.36D.
- 19.6 Figure 19.37 shows a clamp. Draw the front view and sectional top view.
- 19.7 Figure 19.38 shows a pulley. Draw the half sectional front view and side view.



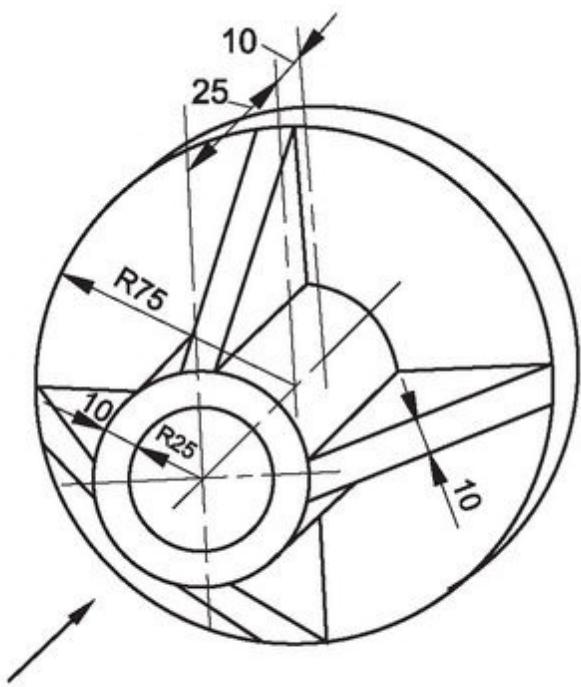
**Fig.19.37**



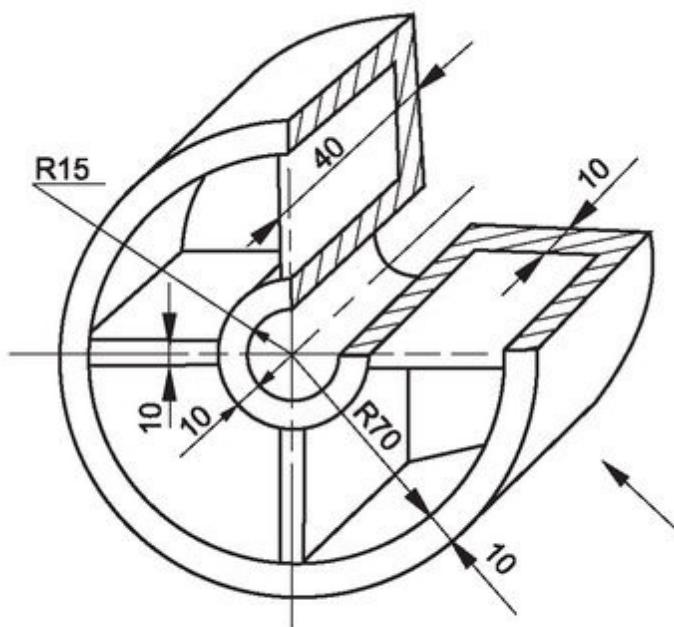
**Fig.19.38**

19.8 [Figure 19.39](#) shows a shaft support. Draw the front view and sectional top view.

19.9 [Figure 19.40](#) shows a pulley. Draw the half sectional front view and top view.

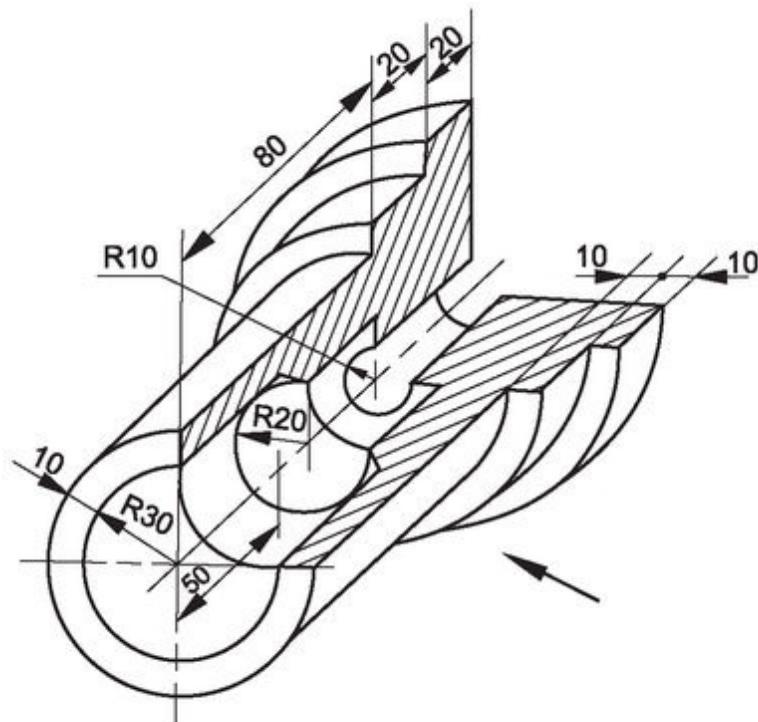


**Fig.19.39**



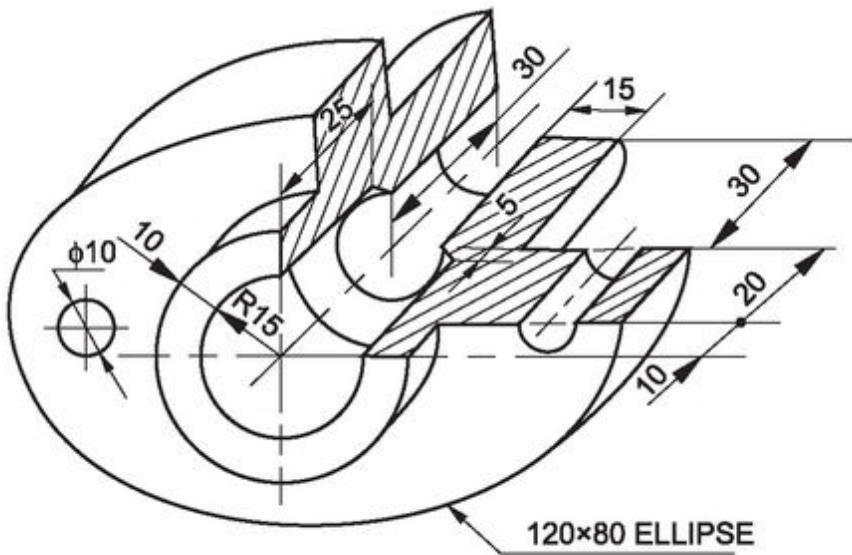
**Fig.19.40**

19.10 Figure 19.41 shows a bush. Draw the half sectional front view and left side view.



**Fig.19.41**

19.11 Figure 19.42 shows a gland. Draw the front view, half sectional top view and half-sectional left side view.



**Fig.19.42**

## REVIEW QUESTIONS

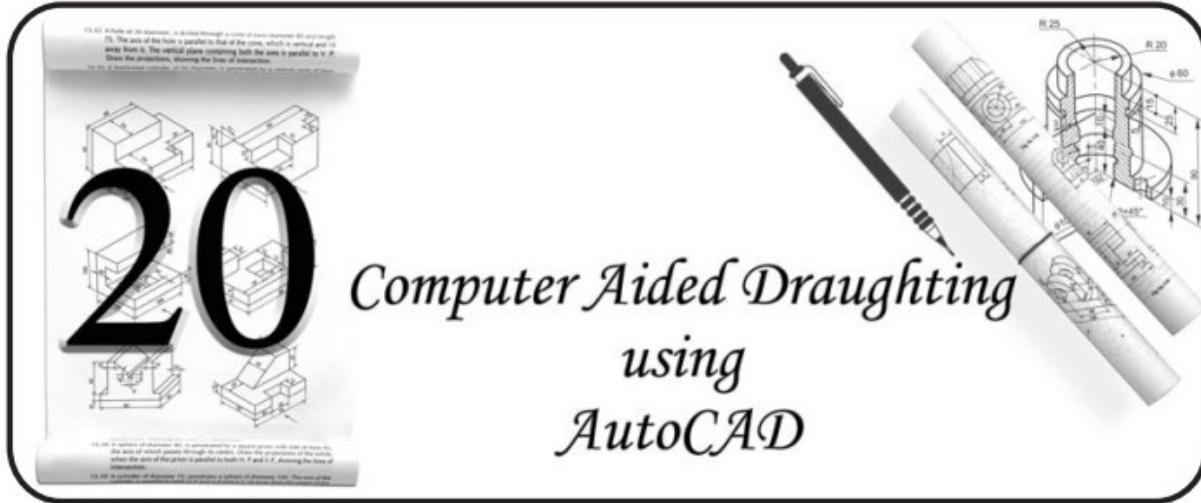
- 19.1 What is the locus of a point on a link in a mechanism?
- 19.2 When are sectional views preferred in the place of plain views?
- 19.3 What is the principle of obtaining a sectional view?
- 19.4 Differentiate between a (full) sectional view and a half sectional view.
- 19.5 When are the following views preferred: (i) Full sectional view and (ii) half sectional view.
- 19.6 For any object considered, sketch sectional front view and top view; indicating the section plane in the top view.

## OBJECTIVE QUESTIONS

- 19.1 The locus of a crank pin in a slider crank mechanism is a — — — — .
- 19.2 — — — lines are used to represent the invisible features.
- 19.3 An invisible line intersects a —— line with a —— in contact.
- 19.4 An invisible line intersects another ——line at the crossing point of two dashes in contact.
- 19.5 Invisible lines at a corner should be represented by two dashes meeting at the corner. (True/False)
- 19.6 When an invisible arc is a continuation of a visible arc, the invisible arc begins with a space/dash.

## ANSWERS

- 19.1 circle
- 19.2 Dotted
- 19.3 visible; dash
- 19.4 invisible
- 19.5 True
- 19.6 space



## 20.1 INTRODUCTION

Most of the companies and students of Engineering Institutions are replacing their draughting boards with Computer Aided Draughting Systems which have a tremendous advantage over conventional draughting systems. CAD system cannot do anything unless it is operated by a technical person who has a thorough knowledge of drawing fundamentals. In fact, a CAD system requires a technical person to have a more thorough knowledge of orthographic projections, conventional practices and standard codes of drawing than ever before. A person who wants to learn and use computer aided drawing must first learn engineering drawing on a drawing board as before. The computer would then come handy in doing the task in less time and more accurately than is possible by the hand.

Computer aided draughting involves conventional computer techniques with the help of processing systems to develop graphical solutions on the monitor. It deals with creation, storage, and manipulation of models through

computer. The engineer and designer find it most convenient to prepare the sketches through AutoCAD. There are many graphics software packages available in the market and AutoCAD occupies the first position, making it the world standard for generating drawings. It can be used on any micro-computer system. AutoCAD provides facilities that allow users to customize AutoCAD to make it more efficient and thus increase their productivity.

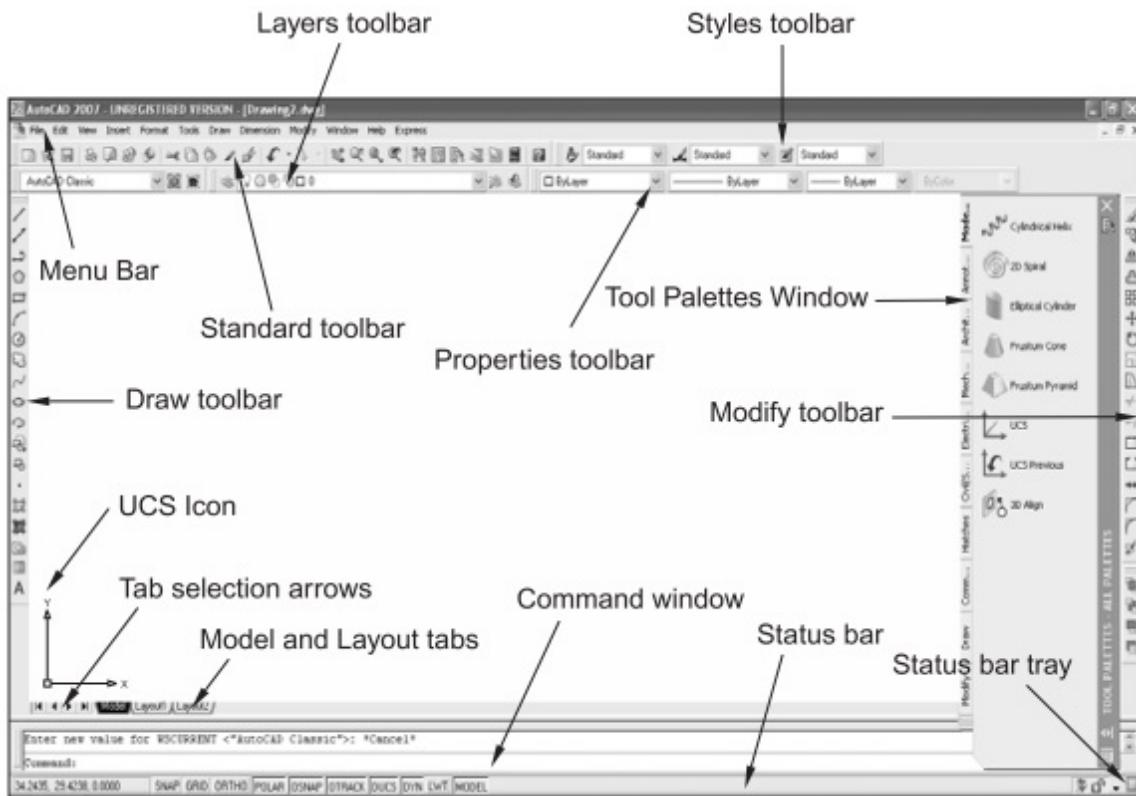
The advantages of CAD system include:

1. High productivity with reduced lead time,
2. Standardization of the drawing,
3. Possibility of easy and quick modifications,
4. Automatic creation of documentation,
5. Compact storage (CD or hard disk),
6. Excellent drawing quality,
7. Possibility of colored drawings,
8. Possibility of pre-storing of commonly used components in a library,
9. Possibility of seeing three dimensional drawings for better view, and
10. Better communication and presentation of the work.

## **20.2 STARTING AUTOCAD**

Once the computer is turned on, the operating system gets loaded and AutoCAD can be started by double clicking on the AutoCAD icon. The various components of the AutoCAD screen are: The drawing area, command window, menu bar, tool bars model and layout tabs, tool palettes, and status

bar (Fig.20.1). AutoCAD is designed to run on Windows 2000 and Windows XP.



**Fig.20.1**

- i) Menu bar consists of items such as file, edit, view, insert, format, tools, draw, dimension, modify, window, help, express, etc.
- ii) Tool bars consist of a set of small graphical icons assigned to do a specific job. Only the required icons out of many should be displayed on the screen. Tool bars can be chosen by first clicking VIEW on menu bar and resulting popup menu.

The menu bar located at the top of the screen displays the menu bar titles. As the cursor is moved over this, various titles are highlighted and by means of pick button, a desired item can be chosen. Once it is

selected, the corresponding menu is displayed directly under the title. A command can be invoked by picking from this menu.

The standard tool bar contains applications such as open a new file/existing file, save, print, cut, paste, etc.

- iii) Drawing area - major central area on which drawings can be created and a cross hair which can be moved by a mouse, is displayed in this area.
- iv) Command area - commands can be typed in this area and response to various prompts should be given here and press enter key.
- v) Status line - on the left hand corner, it displays x, y, and z co-ordinates indicating the position of the cross hair in the drawing area. Drawing aids such as SNAP, GRID, ORTHO, etc., are displayed here.

## 20.2.1 Invoking AutoCAD Commands

After starting the AutoCAD and when the cursor is in the drawing area to perform an operation, one has to invoke the AutoCAD commands. The following methods are provided to invoke the commands:

1. **Key board** Using the key board, command name can be typed at the command prompt and by pressing ENTER or SPACE BAR, the command can be invoked.
2. **Menu** The menu bar is at the top of the screen which displays the menu bar titles. As the cursor is moved over this, various titles are highlighted and by means of pick button, a desired item can be chosen. Once it is selected, the corresponding menu is displayed directly

under the title. A command can be invoked by picking from this menu ([Fig.20.2](#)).

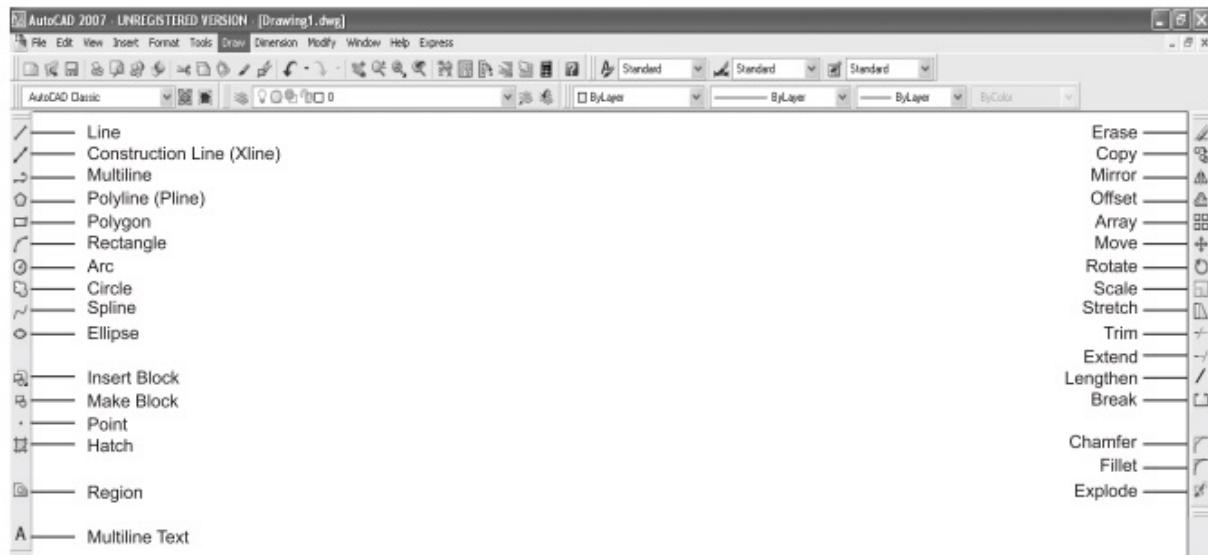
3. **Draw tool bar** This is an easy and convenient way to invoke a command. This is displayed on the left extreme of the initial AutoCAD screen ([Fig.20.3](#)) and is very easy to choose by picking.
4. **Tool palettes** These are shown on the right side of the monitor screen. It is an easy and convenient way of placing and sharing hatch pattern and blocks.
5. **Modify tool bar** This is docked on the right side of the drawing area, and contains commands to edit existing drawing ([Fig.20.3](#)).



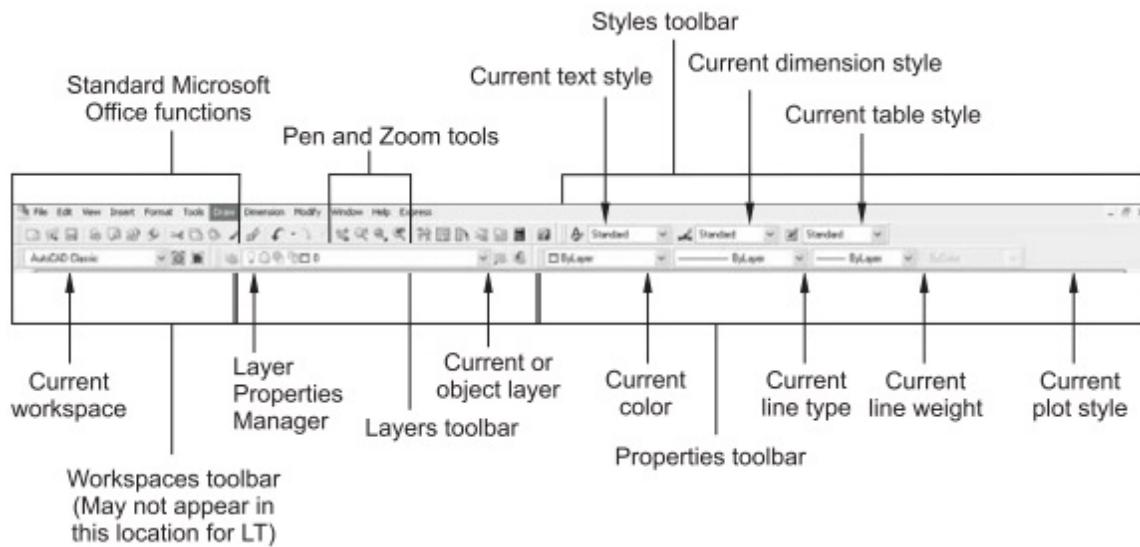
**Fig.20.2**

Most of the commands are contained in the Draw and Modify tool bars. When a command is activated by clicking on it, an active assistance dialogue box appears on the

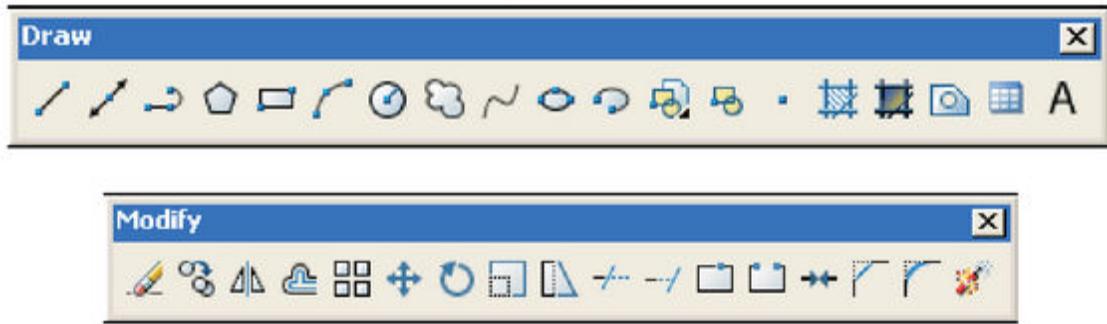
screen to help. The menu bar, the standard tool bar, and properties tool bar are shown in [Fig.20.4a](#). Draw and modify tool bars, as they appear floating, is represented in [Fig.20.4b](#). The command window and the status bar are shown separately in [Fig.20.4c](#).



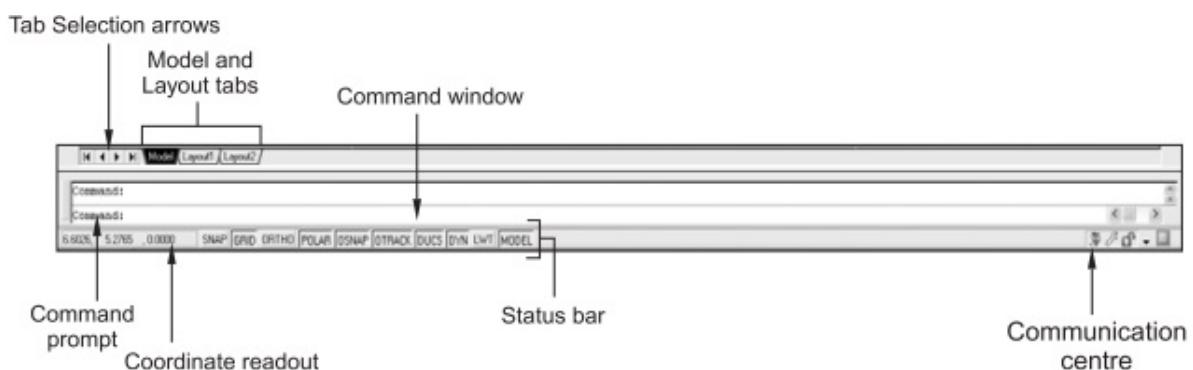
**Fig.20.3**



**Fig.20.4(a)**



**Fig.20.4(b)**



**Fig.20.4(c)**

## 20.2.2 Interactive Techniques

Developing a drawing by AutoCAD is done by interactive techniques, so that it is easy to follow and achieve the results. The popular interactive techniques are: Layers, drawing insertion, object, snap, zooming, panning, plan view and 3D views, view ports, resolution, editing the drawing and many more.

The layering concept is similar to the transparent overlays used in many draughting applications. This allows the user to view the plot-related aspects of a drawing separately or in any combination. In drawing insertion, a drawing can be stored in a drawing file, and this may be

inserted in subsequent drawings for any number of copies. To refer to the geometric features of existing objects when entering points, the object snap may be used. The visual image of the drawing on the screen may be magnified or shrunk by zooming, whereas, panning allows viewing different portions of the drawing without changing the magnification. In plan view (top view), the construction plane of the current user co-ordinate systems is parallel to the screen. The drawing may also be viewed from any point in space (even from inside an object). The graphics area of the screen can be divided into several view ports, each displaying a different view of the drawing. This resolution can be changed at any time. The editing facilities of AutoCAD make it easy to correct or revise a drawing. Multiple copies of an object arranged in rectangular or circular patterns are easy to create.

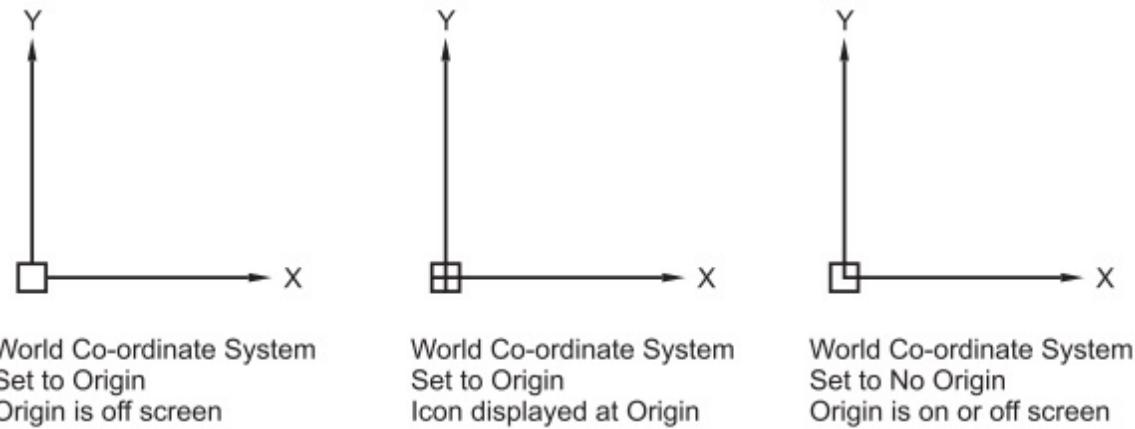
## **20.3 PLANNING FOR A DRAWING**

While planning a drawing in AutoCAD, one has to organize some of the information such as choosing the units, co-ordinates, etc.

### **20.3.1 World Co-ordinate System (WCS)**

In this system, the x, y and z co-ordinates of any point are measured from the origin, which is located at the lower left hand corner of the screen ([Fig.20.5](#)). This system is fixed and used in 2D drawings, wire frame models, and surface models.

However, the origin can be relocated/ the x,y and z axes can be re-oriented by the user co-ordinate system (UCS). The UCS is used in 3D drawings. It is also useful for relocating the origin or rotating z and y axes in 2D drawings. UCS command is also used to set a new co-ordinate system by shifting the working plane to the desired location. This makes dimensioning of the object easier. One can also fix the screen size equal to the standard sheet size chosen, by setting the x, y and z coordinates to any given scale.



**Fig.20.5**

## 20.3.2 User Co-ordinate System (UCS)

The UCS command is used to set a new co-ordinate system by shifting the xy plane to the desired location. This makes locating the features and dimensioning the objects easier. The changes in the UCS can be viewed by the change in position and orientation of the icon, which is by default placed at the lower left corner of the drawing window.

### 20.3.2.1 Units and Scales

As a general rule, everything drawn with AutoCAD will be drawn to full size. However, when it comes to printing, only then a scale has to be chosen so that the drawing can be accommodated to suit the paper.



**Fig.20.6**

One can use the units on the monitor, either metres or millimetres, depending upon the size of the drawing and convention followed. Decimal units are the most commonly used ones; however, AutoCAD can be configured to work with other types of drawing units as well. To change the units, one must use the drawing UNITS dialogue box.

When the command UNITS is used, the dialogue box appears on the screen (Fig.20.6). The upper two sections refer to length, i.e., linear or angular units. Setting/precision required in each case, can be set independently. For measuring angular units, the direction can be chosen. Drawing units also can be chosen from the dialogue box. There are 5 different linear units to choose from the dialogue box and decimal units are the default units.

**Direct distance entry** Direct distance entry is a feature of AutoCAD, which saves time. The distance at the command line may be entered and the cursor may be used to point the direction.

### **20.3.2.2 *Setting Limits of the Drawing***

The limits of the drawing area are usually determined by,

1. Actual size of the drawing,
2. Space needed for necessary details,
3. Space between various views, and
4. Space for border and title block, if any.

To change the limits from default value (12" × 9"), the **LIMITS** command can be used.

#### **Command Sequence**

Command: LIMITS

Reset model space limits:

Specify lower left hand corner or [ON/OFF]:0, 0

Specify upper right hand cover <current>: 24, 18

## **20.4 STARTING A NEW DRAWING**

A new drawing can be opened using **QNEW** command. This command displays select template dialogue box, which displays a list of default templates available. The desired template can be selected to open a new drawing.

#### ***Saving the work***

Before exiting, the work has to be saved and AutoCAD provides the following commands to save the work on hard disc of the computer:

### **QSAVE SAVE AS SAVE**

When the command **SAVE** is chosen, **QSAVE** command is invoked. When the drawing is saved first time, in the present session the **QSAVE** command prompts to enter the file name in the save drawing as dialog box. A name can be given and save button may be chosen.

### ***Opening an existing drawing***

Command **OPEN** can be used to open a drawing file. This command displays the select file dialogue box. Using this box, one can open the selected drawing for further processing.

### ***The HELP command***

This command displays Help dialogue box. One can access help on different topics and commands. It has 5 tabs: Contents, Index, Search, Favorites, and Ask Me. In the middle of a command, if help is needed, choose Help button to display information about that particular command in the dialogue box.

## **20.5 DIMENSIONING AND UTILITY COMMANDS**

The drawing must contain size descriptions to produce an object. The size description may include length, width, height and location features. With the help of dimensioning, this information is added to the drawing. This information is vital. The part should be drawn to actual size so that the

dimensioning reflects the actual size of the feature. However, the dimensions must be accurate.

## 20.5.1 Dimensioning in AutoCAD

This includes generation of arrows, lines-both dimension and extension lines and other things that form part of the dimensioning are automatically performed; thus saving the user's time. The default measurements can be overridden and settings can be changed.

The fundamental terms of dimensioning are:

Dimension line - this indicates that the distance/angle being measured.

Dimension text - this represents actual measurement between the selected points.

Arrow heads - this is the symbol used at the end of a dimension line. They are also called terminators of the dimension lines.

Extension lines -they are drawn from the object measured to the dimension line. Extension lines are used in linear and angular dimensioning. Generally, they are drawn perpendicular to the dimension line.

Leader - this is a line that stretches from the dimension text to the object being dimensioned. The text can be placed at the end of the leader line if it cannot be located near the object.

## 20.5.2 Selecting Dimensioning Commands

The dimension tool bar can be displayed by right clicking on any tool bar and dimension commands can be chosen from the tool bar / dimension menu.

By directly entering dimensioning command in the command line - DIMLINEAR

Command: DIMLINEAR ↵

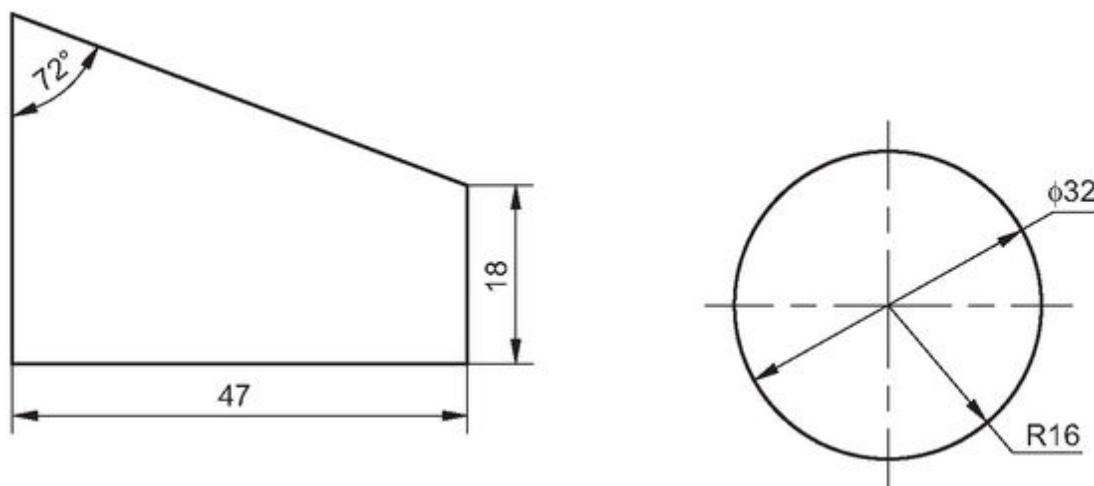
Specify first extension line origin or <select object>:  
Select a point

Specify second extension line origin: Select a point

Specify dimension line location (horizontal/ vertical, etc.): Select a point Command: ↵

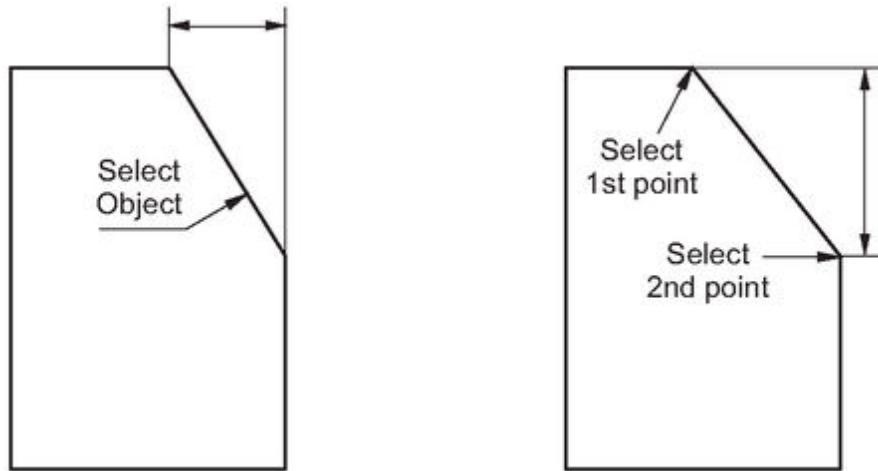
## 1.DIM and DIM1 Commands

The command **DIM** keeps always in the dimension mode, till one exits from the dimension mode. By entering **E**. **DIM1** command, allows to execute a single dimension only and automatically reverts back to normal command prompt. AutoCAD provides the following fundamental dimensioning types: Linear and angular dimensions, radius and diameter dimensioning ([Fig.20.7](#)).



**Fig.20.7 Linear, angular, diameter and radius dimensioning**

## 2. Creating linear dimensions



**Fig.20.8 Drawing linear dimensions**

Command **DIMLIN** or **DIMLINEAR** is used to measure the shortest distance between two points, for which two points/objects may be selected. If the selected object is aligned, the linear dimension will add **Horizontal** or **Vertical** dimension to the object.

The prompt sequence:

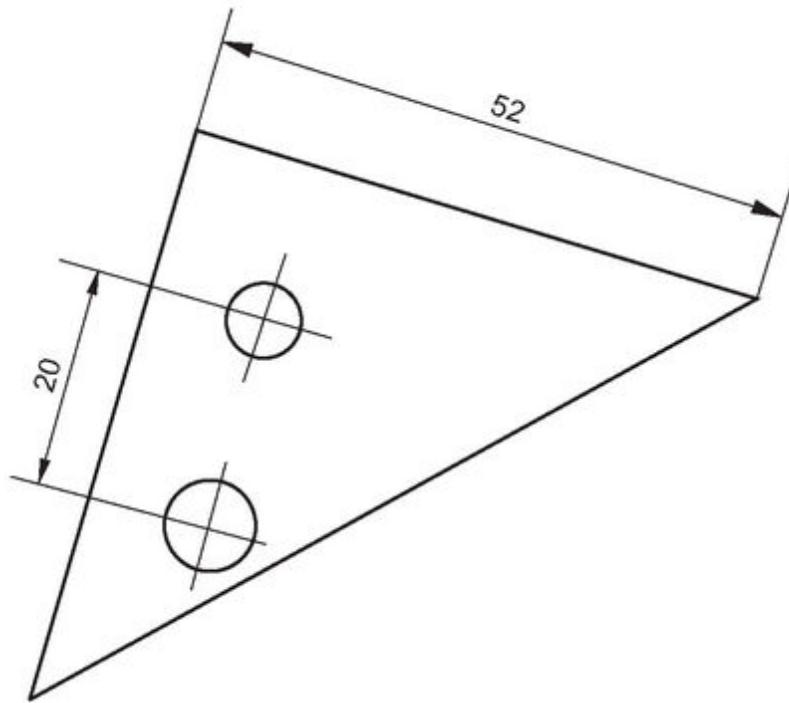
Specify first extension line origin or <select object>: ↵

Specify dimension line location or (MText...) : select a point

You can also select two end points of the line instead of the object ([Fig.20.8](#)).

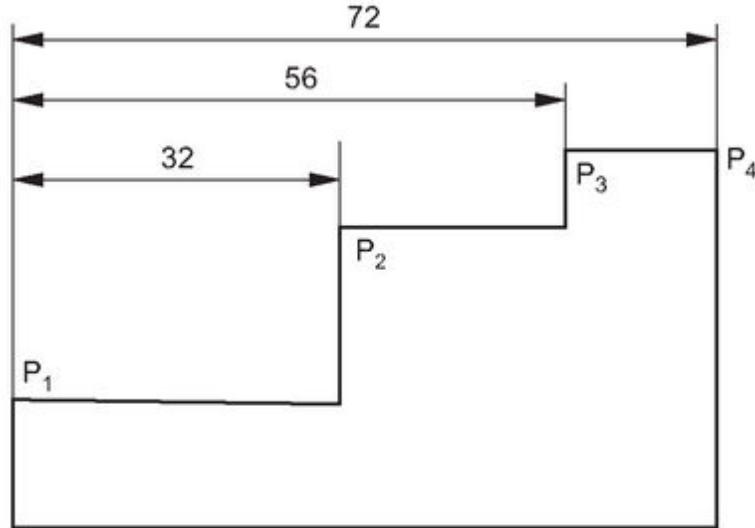
## 3. Creating aligned dimensions

If the object chosen is not parallel to either x/ y axis, then the command **DIMALIGNED** has to be used for dimensioning such objects. The dimension created with **ALIGNED** command is parallel to the object being dimensioned ([Fig.20.9](#)).



**Fig.20.9 Aligned dimensioning**

#### **4. Creating base line dimensions**



**Fig.20.10 Base line dimensioning**

If different features of a part with reference to a fixed point are to be located, then the command **DIMBASE** or **DIMBASE LINE** is used. When this option is chosen,

previous style of (angular, ordinate, linear) dimension that was created will be selected and used as the base line ([Fig.20.10](#)).

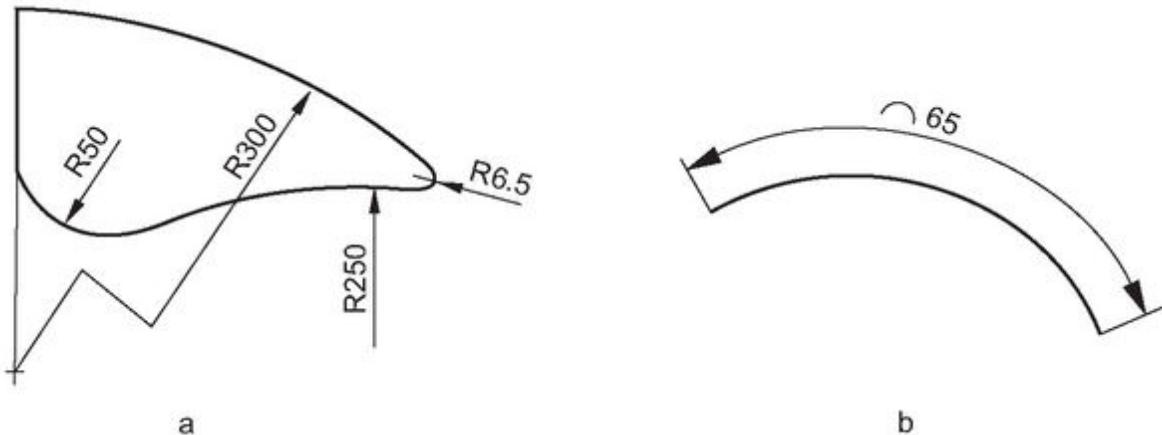
## 5. Dimensioning an arc

To dimension an arc whose centre is within the drawing area, one can use Radius dimension tool on the Dimension tool bar to get the radius dimension. However, if the centre of the arc is not within the drawing area, one can use jogged dimension.

### Sequence of operations

1. Click the Jogged Dimension tool from the Dimension tool bar, choose Dimension: Jogged, or enter Dimjogged.
2. At the select arc or circle: select the object
3. At the specify centre location override: select a point that indicates the general direction to the centre of the arc. A dimension line appears.
4. Position the dimension line and then click.
5. The jogged dimension line is placed in the drawing ([Fig.20.11 a](#)).

Arc length also can be given a dimension using the Arc Length tool. Choose the Arc Length tool from the Dimension tool bar.



**Fig.20.11**

### **Sequence of operations**

1. Dimension: Arc Length or enter Dimarc
2. Select arc: select arc one wants to dimension

Arc dimension appears and moves with the cursor (Fig.20.11 b).

## **20.6 DRAWING AIDS**

Drawing with AutoCAD is really like drawing on a drawing board. AutoCAD has several drawing aids, which can help to draw horizontal and vertical lines on a computer.

### **1. ORTHO Mode**

Ortho is the short form of orthogonal, which means either vertical or horizontal. It is not a command, but only a drawing mode. The option is available on the status bar. It does the very simple job of constraining the lines to be drawn either horizontally or vertically only.

### **2. Drawing GRID**

The GRID command is most often used when a new drawing is started. This is a regular pattern of dots displayed on the screen which acts as a visual aid. It has no effect on the drawing and does not force alignment of any type. It is purely a visual reference. The grid spacing can be controlled and it gives an idea about the size of the drawn objects.

Command Sequence:

Command: **GRID**

Specify grid spacing (x) or [ON/ OFF/ Snap/ Aspect] <0,000>: enter spacing

Grid can be displayed with different x and y spacing by using “Aspect” option.

Grid limits can be specified by the use of drawing limits. Thus, they can be used to define the extent of the drawing.

Command Sequence:

Command: **LIMITS**

Specify lower left corner or [ON/ OFF] <0.000, 0.000>: Enter co-ordinates or accept default value.

Specify upper right corner <420.000, 297.000>: Enter co-ordinates or accept the default value. Limits mode is useful to know the extent of the plotted drawing sheet. This will not allow to draw the objects outside the limits.



Using the Grid mode is like having a grid under drawing to help with layout. In AutoCAD, the grid mode allows to see the limits of the drawing. It also helps to visualize the distances within any given view. When this command is used, dots are displayed in a regular pattern in standard orthogonal fashion ([Fig.20.12a](#)). Only in the Snap

mode by Rotate option, the grid can be set to any angle ([Fig.20.12b](#)).

### 3. LAYERS

When a drawing is complex/ complicated and for sorting the objects, the technique of using Layers will help keep track of what is what. The LAYER command is used to draw objects having multiple planes.

Layers may be thought as overlays on which various types of information is kept ([Fig.20.12](#)). In a floor plan of a building, for example, plumbing fixtures, wiring, furniture, and ceiling, etc., are represented on a single sheet. It may be difficult to get the required details easily. However, once they are kept in the form of different layers, it becomes easy to understand and works may be allotted to different groups for execution without delay. With AutoCAD, one can turn off the layers not needed and plot a drawing containing only the required information. A carefully planned layering scheme helps to produce a document that combines the type of information needed in each case.

Use of layers also allows one to modify the drawings more easily. The various options of LAYER command are:

**? -** Displays information about one or more layers.

The list shows the layer name, color, and line type.

**Make -** Creates a new layer and assigns it as the current layer.

**Set -** Allows to specify the current layer. The set option is used to specify the current layer.

**New -** Creates one or more new layers. The NEW option does not make the new layer the current layer.

**ON/ OFF** - Controls whether or not a particular layer is displayed. The ON/OFF option does not affect the computer time required to redraw a drawing. It only controls what is displayed.

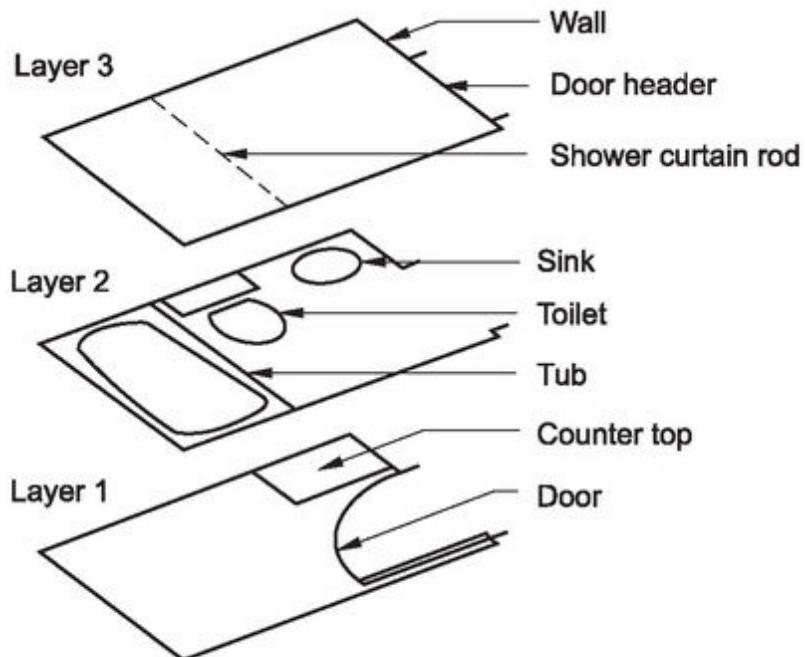
**Ltype** - Changes the line type used for specific layers of the drawing.

#### 4. SNAP Mode

This allows to pick points which lie on a regular grid. When the snap mode is turned on and the grid is displayed, the snap and grid spacing are the same and the cross hairs will jump from one grid point to another as we move across the screen.

Command Sequence:

Command: **SNAP**

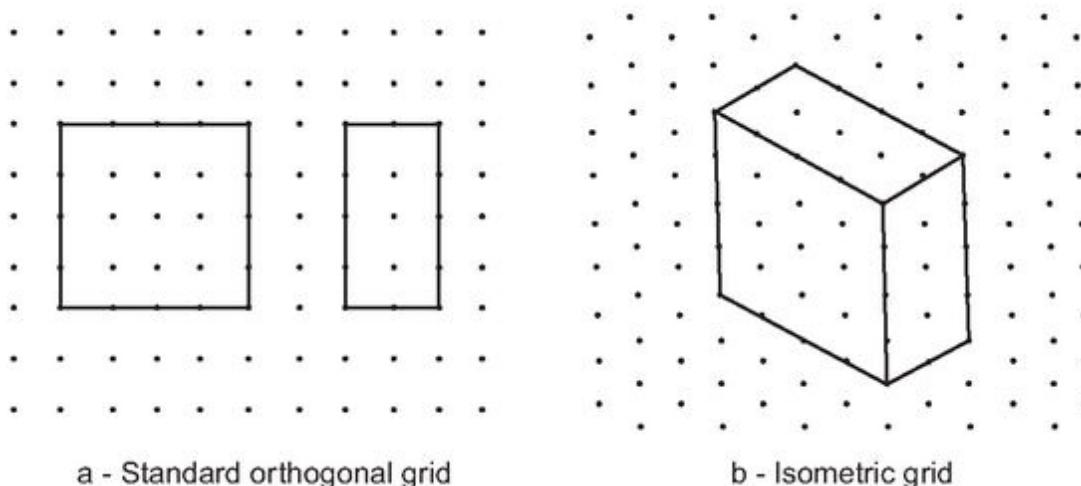


**Fig.20.12**

Specify snap spacing or [ON/OFF/ Aspect/ Rotate/ Style/ Type] <10.000>: enter the required spacing.

The “Aspect” option can be used to vary the horizontal and vertical snap spacing independently.

“Rotate” is used to set the snap grid to any angle. The snap style can be set to isometric/standard using the “Style” option ([Fig.20.13](#)).



**Fig.20.13**



The Snap mode forces the cursor to move in steps of a specific distance. This is useful when one wants to select points on the screen at a fixed interval. Both Polar and Grid Snap modes are available in AutoCAD. Command Snap may be selected from the status bar/ F9 key (Function Key).

Both Grid and Snap Modes can be used together and in such a case the spacing of grid and snap setting can be set equal to each other so that every snap point can be seen.

Polar Snap is similar to Grid Snap, forcing the cursor to move in exact increments. The main difference is that Polar

Snap snaps to distances from a previously selected point rather than to a fixed grid, i.e., a point may first be selected (not necessarily coinciding with grid point) and from this point, cursor moves in steps of a specific distance.

When using a pointing device, exact alignment of points can be tedious. To streamline the process, the SNAP command is used to force all points picked on the screen into alignment. If the grid is activated, SNAP produces a visible change on the screen. Without GRID, there is no change in the appearance of the screen.

## **5. Polar Tracking**

This allows to snap into whatever angles we choose to configure. One of the great benefits is that, when used in combination with direct distance entry, one can draw lines of a given length and at preset angle without using any construction lines and without the need for entering relative co-ordinates.

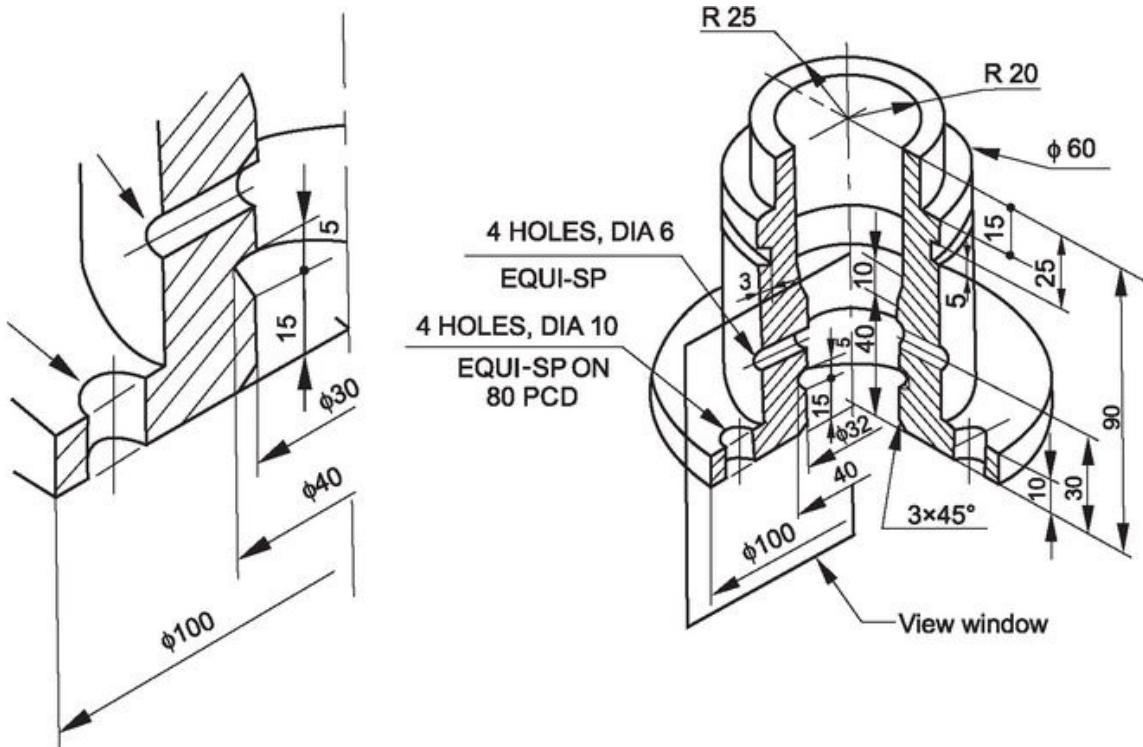
## **6. Plotting**

To get a hard copy output from AutoCAD, PLOT command is used. Prior to that, various views of the object are drawn using AutoCAD and saved. Next step is to select a particular view by the command VIEW for the purpose of plotting on a sheet. AutoCAD may be allowed to fit the drawing on to the sheet, by selecting Fit To Paper from the Plot Scale group commands. The plot displays exactly the same thing that one would see on the screen once the view is recalled.

## **7. Window**

The window option enables to use a window to indicate the area one wants to plot ([Fig.20.14](#)). Nothing outside the window prints. From the drop down list Window may be

selected. Indicate a window on the drawing area and AutoCAD may be allowed to fit this drawing area on to the sheet by using Fit To Paper option in the Plot Scale group and the plot displays exactly the same thing that is enclosed within the window.



**Fig.20.14**

## 20.7 THE FUNCTION KEYS

Many drawing modes can be controlled quickly using the key board function keys. In most of the cases, it is quicker.

F1 Key: brings in AutoCAD help and user Documentation dialogue box. One can search for help on any AutoCAD command or topic.

F2 Key: used to toggle the AutoCAD text window. This window contains the command history during the

drawing session. It is useful to view the results of the commands.

F3 Key: toggles running object snaps on and off

F4 Key: toggles tablet mode on and off

F5 Key: cycles through isoplanes. It also affects the orientation of isocircles drawn with the ellipse command

F6 Key: a three way toggle key which changes the coordinate reading in the status bar. By default, Cartesian co-ordinates are displayed in the status bar and F6 Key either turns off this/ changes to polar system.

F7 Key: is used to toggle grid mode on and off

F8 Key: is used to toggle ortho mode on and off

F9 Key: is used to toggle snap mode on and off

F10 Key: is used to switch polar tracking on and off

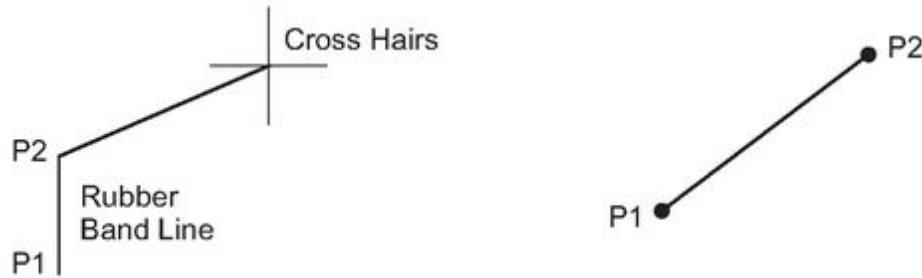
F11 Key: toggles object snap tracking on and off

## 20.7.1 Drawing Entities/Objects

The Draw commands can be invoked from the Draw tool bar located on the left side of the drawing area and are used to create new objects such as lines and circles. Most of the AutoCAD drawings are composed of these basic components. Efficient use of AutoCAD can be achieved by understanding the Draw commands.

### 1. LINE command

A line can be drawn between two points within the drawing area. The basic drawing methods are the same as creating drawings on a drawing board.



**Fig.20.15**

With the Line command, one can draw a simple line from one point to another. When the first point is picked and the cross hair is moved to the location of the second point, a rubber band line is seen indicating the line that will be drawn when the second point is picked ([Fig.20.15](#)). AutoCAD will draw a straight line between each picked point and the previous point. Lines can also be drawn by entering the coordinates of the points at the command prompt.

### **Drawing Entity-LINE**

Lines can be constrained to horizontal/ vertical by the ORTHO command. CLOSE option uses the starting point of the first line segment in the current LINE command as the next point.

1. Lines can be drawn using co-ordinate system (rectangular Cartesian co-ordinates). To draw a rectangle ([Fig.20.16a](#)):

Command: LINE

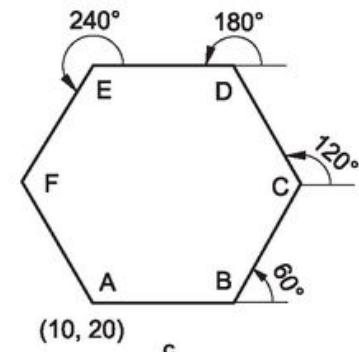
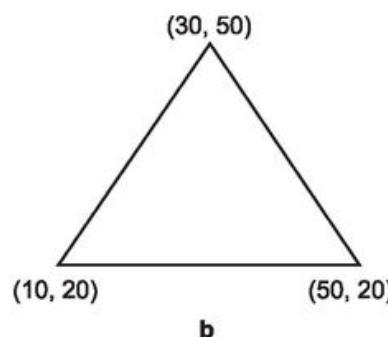
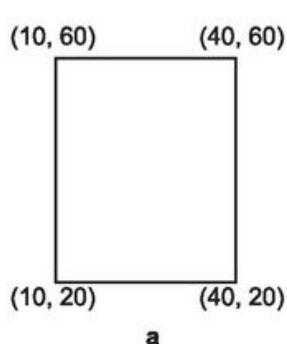
From point: 10, 20 ↵

To point: 40, 20 ↵

To point; 40, 60 ↵

To point: 10, 60 ↵

To point: ↵



**Fig.20.16**

2. It is also possible to specify the co-ordinates in the incremental format as the distances from the current cursor position in the drawing area. The distance is specified by using @ parameter before the actual value. To construct a triangle of given altitude (30) and base (40) ([Fig.20.16b](#)):

Command: LINE

From point: 10, 20 ↵

To point: @40, 0 ↵

To point;@ - 20, 30 ↵

To point: ↵

3. It is also possible to specify the point co-ordinate using the polar co-ordinate format. To construct a hexagon ([Fig.20.16c](#)) of side 30:

Command: LINE

From point: 10,20 ↵(A)

To point: @30 <0 ↵(B)

To point: @30 <60 ↲(C)

To point: @ 30 <120 ↲(D)

To point: @ 30 <180 ↲(E)

To point: @ 30 <240 ↲(F)

To point: close

## 2.RECTANGLE command

This is used to draw rectangle whose sides are vertical and horizontal. It is only a closed poly line and drawn by picking two diagonal corners ([Fig.20.17](#)).



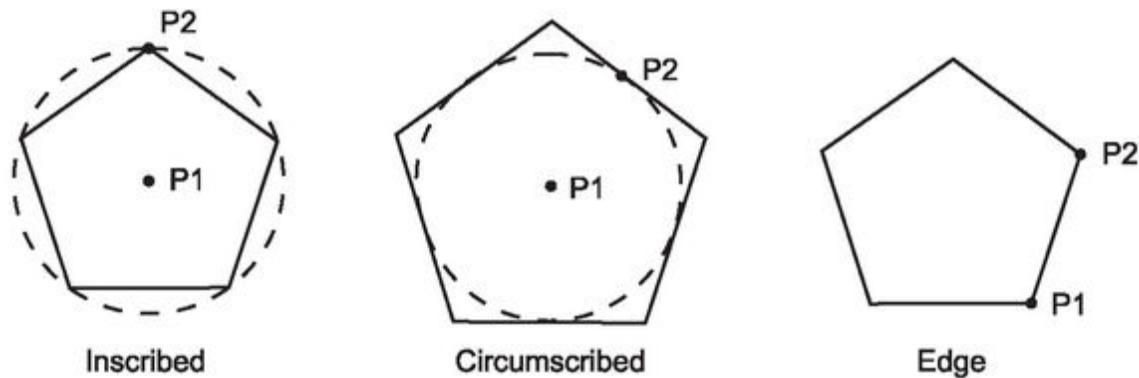
**Fig.20.17**

## 3. POLYGON command

This can be used to draw any regular polygon of sides upto 1024, which requires 4 inputs:

1. The number of sides,
2. Centre pick point,
3. A pick point to determine the radius (inscribed or circumscribed circle), and
4. Orientation of the polygon.

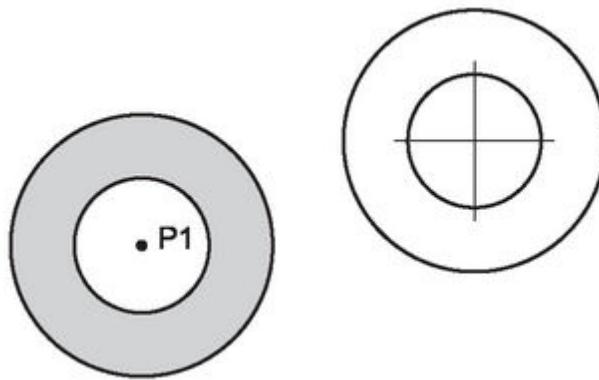
One may also specify the length of the side ([Fig.20.18](#)) using edge option.



**Fig.20.18**

#### 4. DONUT command

This command draws a solid donut shape. AutoCAD asks to define the hole and the outside diameter of the donut. By selecting the centre point, the donut is positioned (Fig.20.19).



**Fig.20.19**



**Fig.20.20**

## **5. CIRCLE Command**

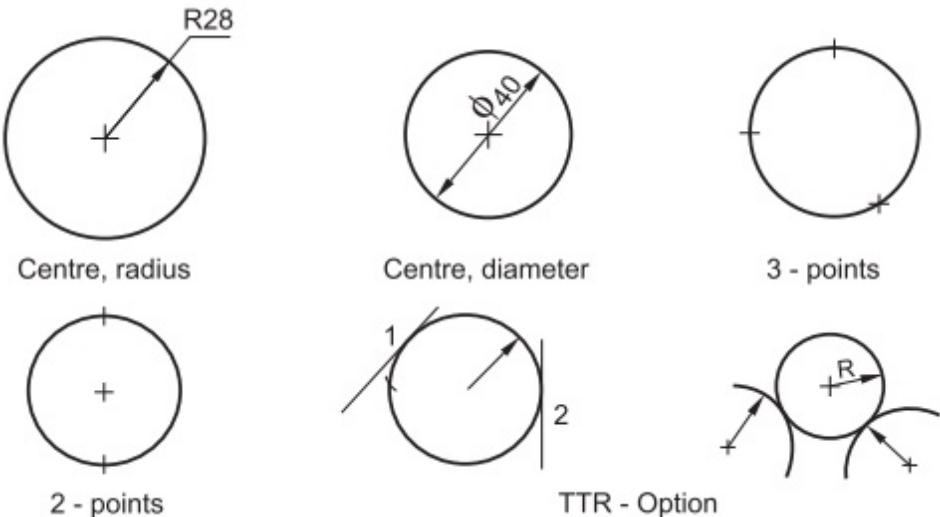
A circle is a single drawing entity in the same way that a line is a single drawing entity. CIRCLE is a frequently used command. It is used in the creation of mechanical, electrical, and architectural drawings. The ability to select diameter end points, the centre point and a radius, or three points on the circumference provides a great deal of flexibility. Circle command offers several ways to create a circle. The default method is to pick the centre point and then to pick a second point on the circumference or enter the radius at the keyboard.

### **Command sequence**

Command: CIRCLE

1. Specify centre point for circle or [3P/ 2P/ TTR] <centre point>: (pick P<sub>1</sub>)  
Specify radius of circle or [Diameter] <56.0>: (pick P<sub>2</sub> or enter the radius, [Fig.20.20](#)).
2. Diameter/ <Radius><current default>: select D or R
3. 3P (3 point) option: one is prompted for a first, second and third point. The circle will be drawn to pass through these points.
4. 2P (2 point) option: one is prompted for the selection of two points which form the opposite ends of the diameter.
5. TTR option: allows one to define a circle based on two tangent points and a radius. The tangent points can be on lines, arcs or circles.

The options appear within square brackets and default options appear in triangular brackets.



**Fig.20.20**

## 6. ARC command

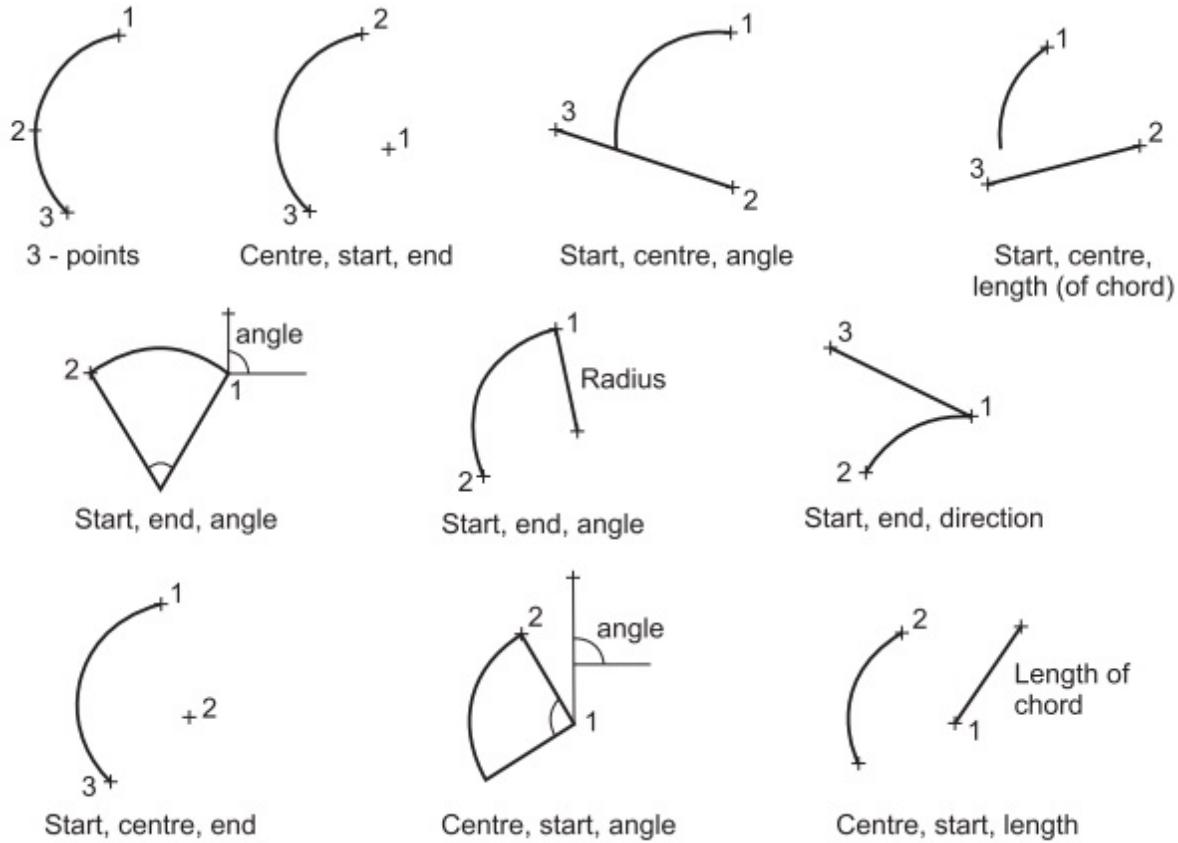
The ARC command permits to draw incomplete circles, using a variety of methods ([Fig.20.21](#)). Prompts are provided at each step of arc construction to indicate what options are available.

Command: ARC

1. Three points on arc
2. Start point, centre, end point
3. Start point, centre, included angle
4. Start point, centre, length of chord
5. Start point, end point, angle
6. Start point, end point, radius
7. Start point, end point, starting direction
8. Centre point, start point, end point
9. Centre point, start point, included angle
10. Centre point, start point, length of chord

## 11. Continuation of previous line or arc

The last option is available as a menu option only from the Text menu. It is not an option from the drop down menu system. It is quicker and easier to draw an arc than to draw a circle and break it.

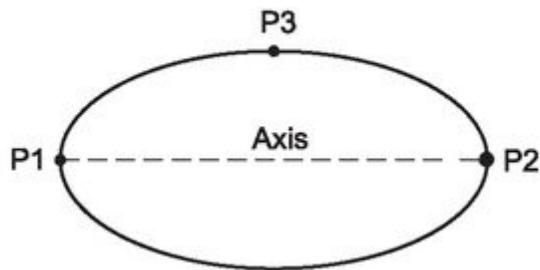


**Fig.20.21**

## 7. ELLIPSE command

This command gives a number of different creation options:

Pick two end points of an axis and then a third point to define the eccentricity of the ellipse - default option ([Fig.20.22](#)). Ellipse command can also be used to draw isometric circles.

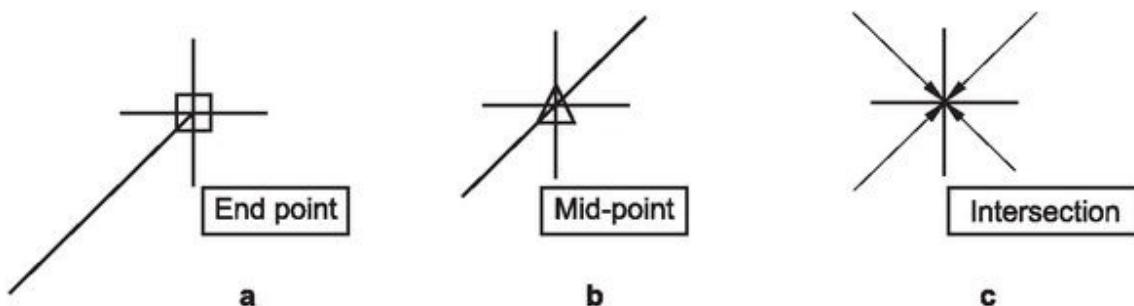


**Fig.20.22**

## 20.7.2 Object Snaps

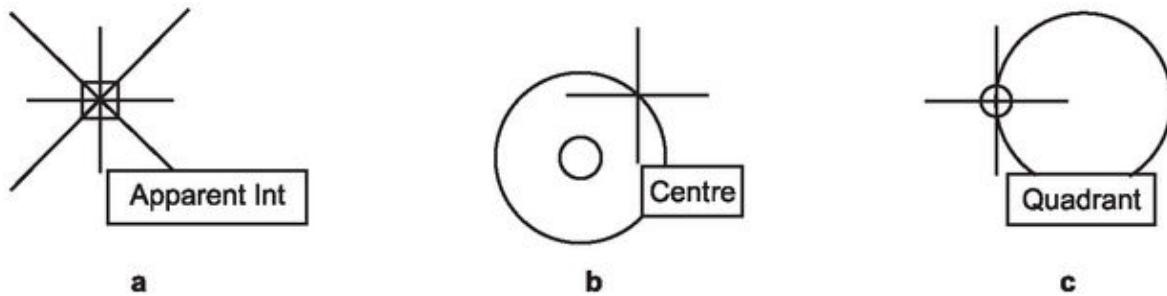
This is one of the most useful features of AutoCAD, which improves the accuracy of the drawing. This refers to the cursor's ability to snap exactly to a geometric point of an object. However, an object should exist before this command is used, as only with respect to the existing object, this command refers to.

Each time when **Osnaps** is used to pick a point, the aperture box appears at the centre of the cross hairs to indicate the area AutoCAD uses to search for object snaps, and the cursor jumps to the snap location. To locate the end point of all objects (line, arc, etc.), mid-point, intersection point, centre of a circle, quadrant, etc., one can use Quick find tool bar.



**Fig.20.23**

- End point** The end point Osnap snaps to the end points of lines, arcs, and poly line vertices (Fig.20.23a).
- Mid-point** The mid-point Osnap snaps to the mid-points of the above objects (Fig.20.23b).

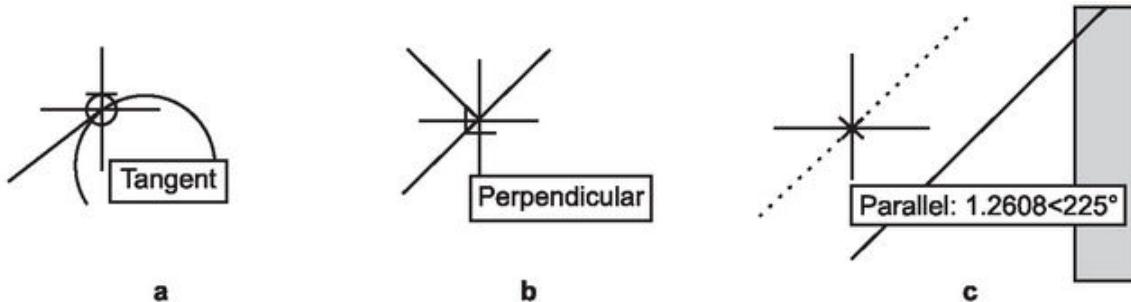


**Fig.20.24**

- Intersection** This snaps to the physical intersection of any two drawing objects.  
One must pick two points to indicate which two objects should be used (Fig.20.23c).
- Extension** This enables to snap to some point along the imaginary extension of a line, arc, etc. Once the end point of the existing object is found, a small cross appears at the end point and a dashed extension line is displayed from the end point to the cursor (Fig.20.24a).
- Centre** This snaps to the centre of a circle, arc, or poly line arc segment. The cursor must pass over the circumference of the circle, etc., so that the centre can be found (Fig.20.24b).
- Quadrant** This snaps to one of the four circle quadrants 0, 90, 270, and 180 degrees (Fig.20.24c).
- Tangent** This snaps to a tangent point on a circle. One can draw a line from point to the tangent point or draw

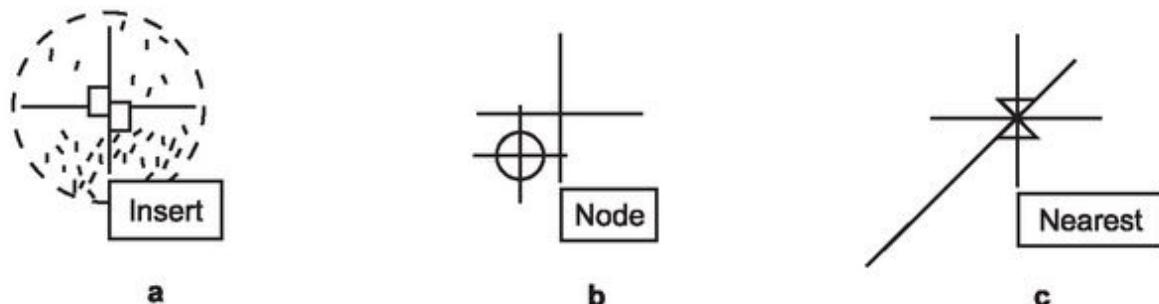
a line from a tangent point (Fig.20.25a).

8. **Perpendicular** This snaps to a point, which forms a perpendicular with the selected object. Perpendicular can be used to draw a line to a perpendicular point or from a perpendicular point (Fig.20.25b).



**Fig.20.25**

9. **Parallel** This is used to draw a line parallel to any other line. First start the line command, specify the first point when prompted and then start parallel Osnap. Now, move the cursor close to a parallel position and a dotted line will appear, indicating the parallel. Now, pick the second point of your line (Fig.20.25c).
10. **Insert** This snaps to insertion point of a block, text or an image (Fig.20.26a).



**Fig.20.26**

11. **Node** This snaps to the centre point of an object. One can insert a number of regularly spaced symbols or blocks along a line, if already a number of points were created with Divide command ([Fig.20.26b](#)).
12. **Nearest** This snaps to the nearest point on a drawing object. This is useful to make sure that a pick point lies on a drawing object ([Fig.20.26c](#)).

## 20.8 ENTITY SELECTION TYPES

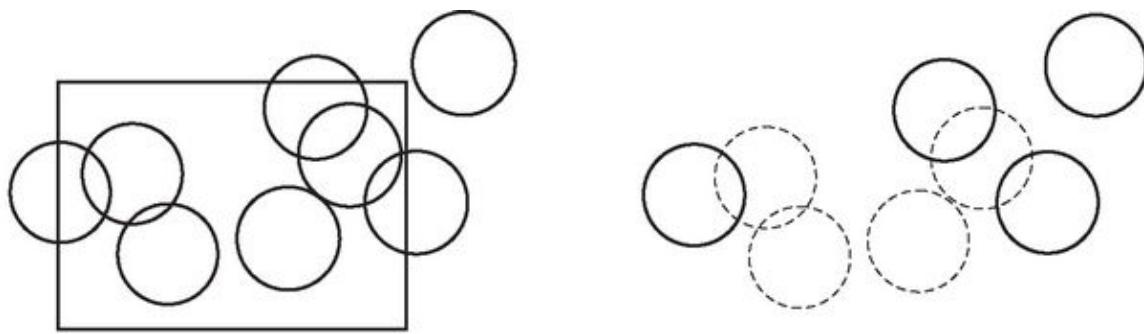
All the modify commands require that we make one or more object selections.

1. **Selection by picking** The most obvious method of selecting an object is by this method. When it is required to modify, for example, if the command **ERASE** is given, the cursor change from cross hair to pick box and the response “Select objects” appears at the command line. When an object is chosen by the mouse, it is highlighted in a dashed line ([Fig.20.27](#)). Selection sets are an important concept in AutoCAD as they can be used to a great effect. If more objects are to be selected, picking one at a time becomes tedious. More selection options are available in AutoCAD.



**Fig.20.27**

2. **Window Selection** This option is invoked by typing W in response to 'select objects' prompt. One can draw a rectangle and the objects present completely in the rectangle will be chosen (Fig.20.28). However, if 'C' is typed at the 'select object' prompt (crossing window selection), then the objects which lie entirely in the window and which cross the window border will also be selected (Fig.20.29).

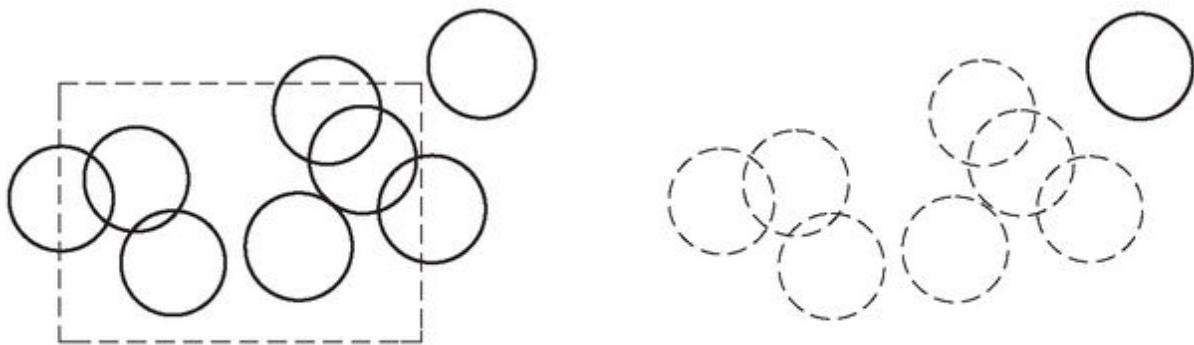


The window selection box is shown as a rectangle with a solid line

Only objects entirely within the window will be selected

**Fig.20.28**

3. **Implied windowing** Window and crossing window selection is facilitated without giving any command. If a point is picked in space on the graphic window that becomes the first point of window and if the cursor is moved to the right, window will be selected and if the cursor is moved to the left, crossing window option is invoked.

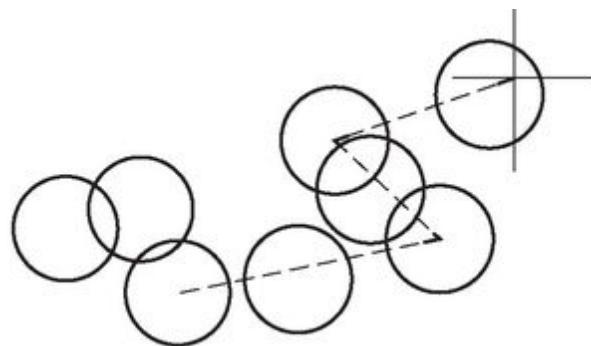


The window selection box is shown as a rectangle with a broken line

Objects within and crossing the window will be selected

**Fig.20.29**

4. **Undo option** AutoCAD allows to undo the last selection made during the compilation of a selection set. By entering U at the next 'Select objects' prompt the objects previously added are removed.
5. **Selecting ALL objects** By typing ALL as the 'select objects' prompt, all the objects are selected. No picking is required.
6. **Fence selection** This option allows a multi-line segment to be drawn and all objects crossing the fence will be selected ([Fig.20.30](#)). This option is invoked by typing F at the 'select objects' prompt.



**Fig.20.30**

## Command Sequence

Select objects: F

First fence point: (pick the point)

Specify end point of the line or [undo]: (pick second point)

Specify end point of the line or [undo]: (pick another point or ↵ to end fence selection)

select objects : ↵ (to complete the selection set )

7. **Window polygon selection** This command is invoked by typing WP at the 'select objects' prompt. This is same as window option. However, irregular shaped polygon can be drawn for selecting the objects.
8. **Crossing polygon selection** By typing CP, this command is invoked and it is similar to crossing window type. However, irregular polygon can be drawn for selecting the objects.
9. **Last object selection** The last object created can be selected by entering L at the 'select objects' prompt.

## 20.9 DISPLAY COMMANDS

In manual draughting, it is difficult to see and alter minute details. We can overcome this difficulty in AutoCAD, by viewing only a specific portion of the drawing. The commands **REDRAW**, **REGEN**, **PAN**, **ZOOM**, and **VIEW** are some of the display commands and they are transparent commands that can be used while another command is in progress.

1. **The REDRAW command** This command redraws the screen and is used to remove the small cross marks that appear when a pint is specified. This also redraws

the objects that do not display on the screen as a result of editing some other objects.

2. **The REGEN command** This command regenerates the entire drawing to update it. One advantage is that the drawing is redefined by smoothing out circles and arcs. This command can be aborted by ESC key.
3. **The ZOOM command** Getting close to or away from the drawing is the function of the ZOOM command. This command enlarges or reduces the view of the drawing on the screen, but it does not affect the actual size of the objects. A section of a drawing can be zoomed to see it in greater detail. To edit some minute details, a portion of the drawing may be enlarged by ZOOM command. Once the editing is over, one may want to return to previous view. This can be done using the PREVIOUS option of the zoom command.
4. **The PAN command** This command allows to bring into view the portion of the drawing that is outside the current display area. This is done without changing the magnification of the drawing. The command allows to slide the big drawing right, left, up or down to bring the part to view.
5. **The VIEW command** While working on a drawing, we may need to work on some portion of the drawing more often than others. Instead of wasting time by recalling, we can store the view under a name and restore it using the same name. We can also use the VIEW command to work with the views at the command prompt.
6. **The DSVIEWER command** As a navigational tool, AutoCAD provides the option of opening another drawing display window along with the graphics

screen window one is working on. The window (Aerial view window) can be used to view the entire drawing and select those portions one wants to quickly zoom/pan. The Aerial view window can be kept open as one works on the graphics screen or minimize it.

**Creating Text** In manual drawing, lettering is accomplished by hand using pen/ pencil. This is time consuming and tedious job. Computer aided drafting has made this process entirely simple.

7. **The TEXT command** This command displays a line in the drawing area where the start point has to be specified after entering the height and rotation angle. Irrespective of justify option chosen, the text is first left aligned at the selected point.

### **Command Sequence**

Command: TEXT

Specify the start point of the text or [Justify/ style]:  
(Specify start point).

Specify height <0.2000>: 0.15

Specify rotation angle of text <0>: 30°

Enter text (Enter first line of the text)

Enter text (Enter second line of the text)

Enter text: ↵ (to end)

8. **The MTEXT command** The text created by the MTEXT command is a single object regardless the number of lines it contains. This command can be used to write a multiline whose width can be specified by defining two corners of the text boundary.

9. **Concept of BLOCKS** The ability to store parts of a drawing, or the entire drawing such that they need not be drawn when required in the same drawing or another drawing, is a great benefit. A Block can be created using the BLOCK command. Objects in a drawing or an entire drawing as a drawing file can be saved using the WBLOCK command. The main difference between the two is that a WBLOCK can be inserted in any other drawing, but a block can be inserted only in the drawing file in which it was created. The blocks created in the current drawing, are inserted using the INSERT command.
10. **Concept of WBLOCK** The ability to store the drawing and save it to disk. In this respect, it is similar to SAVE command. A WBLOCK is also similar to a BLOCK. However, a BLOCK can only be inserted back into the drawing where it was created. While a WBLOCK can be inserted into any drawing. The DWG file extension is given to a WBLOCK file name. AutoCAD saves BLOCKS as entities within the drawing file, whereas, WBLOCKS saves as separate drawing files on the disk.

## 20.10 EDITING COMMANDS

Most of the time, the drawing made by lines and circles need modifications to create the image needed. Modify commands are provided by AutoCAD to help this situation. These commands can be accessed from the keyboard, pull-down menu and from the tool bar. Modify tool bar is located on the right side of the drawing area and it can be invoked from this tool bar.

1. **The ERASE command** This command can be invoked by typing E at the command prompt. This erases/removes the selected objects from the drawing. However, these objects can be got/made to appear by undo command.

### **Command sequence**

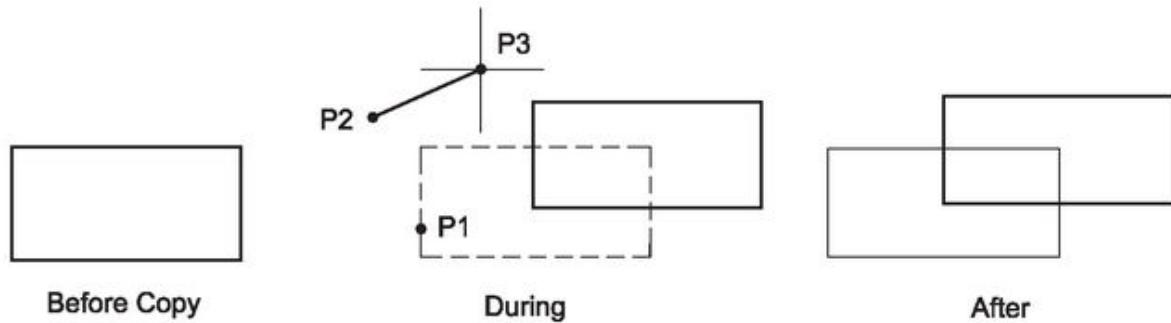
Command: ERASE

Select objects: (Pick an object)

Select objects: ↵ (to end selection process)

By typing L at the 'select objects' prompt, the last drawn object is selected.

2. **The COPY Command** This command can be used to create one or more replicas of any drawing object/objects. This is a very useful and time saving command and can be invoked by typing COPY at the command prompt ([Fig.20.31](#)).



**Fig.20.31**

### **Command Sequence**

Command: **COPY**

Select objects:(Pick object to copy,  $P_1$ )

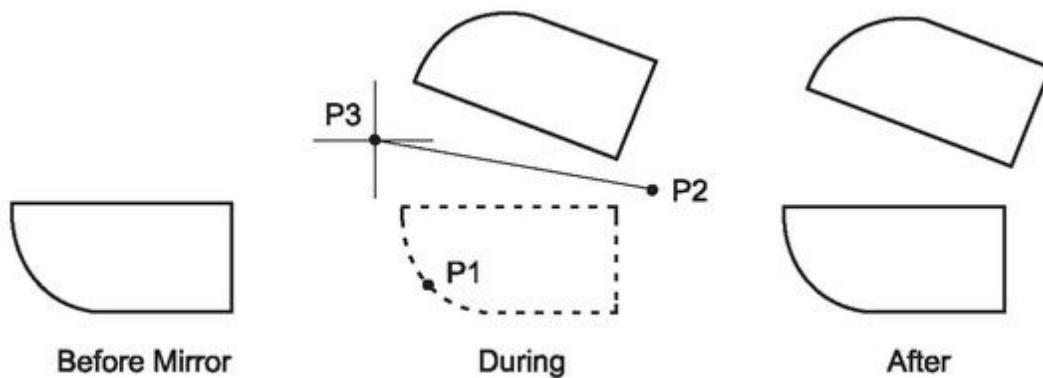
Select objects: ↵ (to end selection process)

Specify base point or displacement, or [Multiple]:  
PickP<sub>2</sub> or M for multiples)

Specify second point of displacement or <use first  
point as displacement>:pick P<sub>3</sub>

 Base point P<sub>2</sub> and second point P<sub>3</sub> are used to  
indicate the distance and direction of the copied  
object from the original object.

3. **The MIRROR command** This command allows to mirror selected objects in the drawing when the position of imaginary mirror line is defined by two points (Fig.20.32).



**Fig.20.32**

### **Command Sequence**

Command: **MIRROR**

Select objects:(Pick Object to copy, P<sub>1</sub>)

Select objects: ↵ (to end selection)

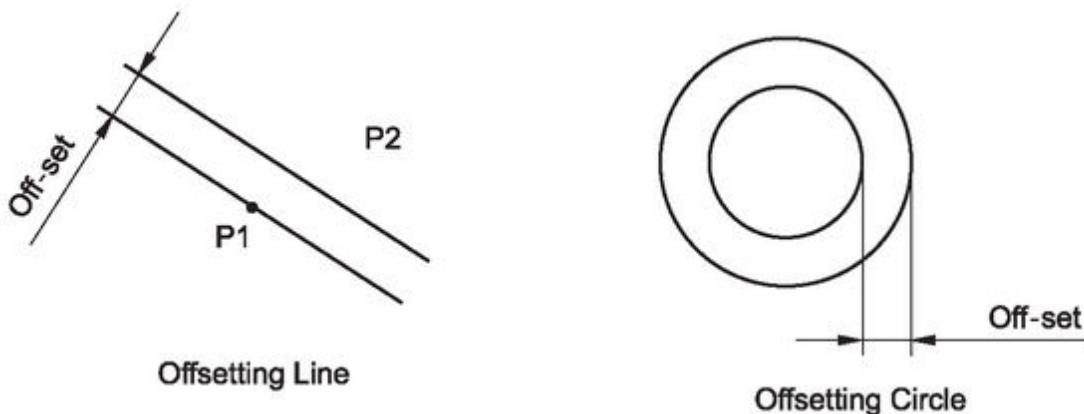
Specify first point of mirror line: (PickP<sub>2</sub>)

Specify second point of mirror line: (pick P<sub>3</sub>)

Delete source objects? (Yes/ No) <N>: ↵ (for No)

In order to create perfectly horizontal / vertical mirror lines, ORTHO should be used.

4. **THE OFFSET command** This command creates a new object parallel to or concentric with a selected object. The new object is drawn at the off-set distance desired and in the chosen direction ([Fig.20.33](#)).



**Fig.20.33**

### Command Sequence

Command: **OFFSET**

Specify offset distance or [Through]<1.00> (Specify distance)

Select objects offset or<exit>: (select, P<sub>1</sub>)

Specify point on side to offset: (pick direction, P<sub>2</sub>)

Select objects to offset or <exit >: ↵ (to end)

Circles can be off-set inside/ outside of themselves to create a new one, which is concentric with the original.

5. **The ARRAY command** This command makes multiple copies of selected objects in a rectangular matrix or circular pattern. The command minimizes the effort

required. When creating rectangular arrays, the original object is located in the left hand bottom position. The distance between rows DR and distance between columns DC is to be specified. The polar array works in a similar way to rectangular array. However, centre point and the total number of objects in the array are to be specified.

### ***To copy an object multiple times:***

- i. From the modify menu, choose Copy,
- ii. Select the objects to copy,
- iii. Enter m (multiple)
- iv. Specify the base point,
- v. Specify the second point of displacement, and
- vi. Specify the next point of displacement.

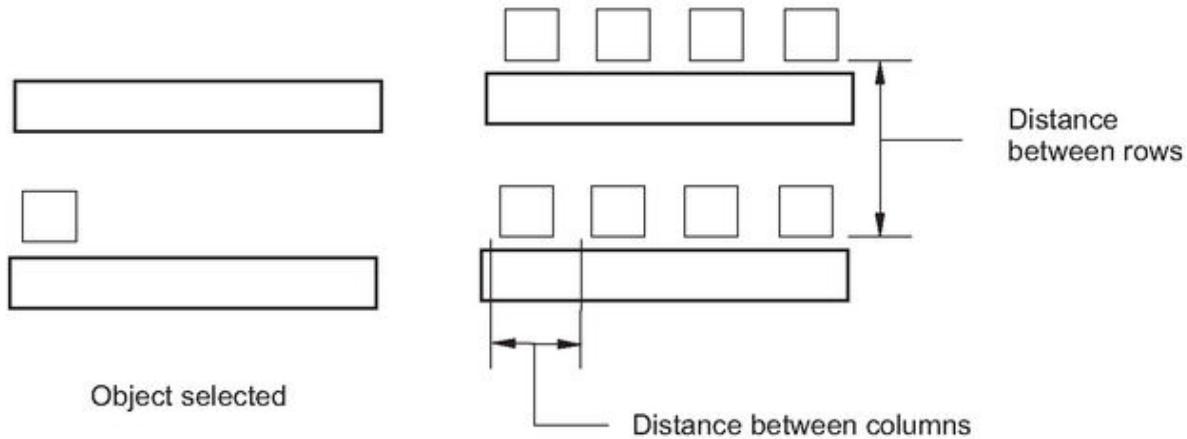
Continue inserting copies, or press Enter to end the command.

### ***Creating rectangular arrays***

AutoCAD builds a rectangular array along a base line defined by the current snap rotating angle. This angle is zero by default, so the rows and columns of a rectangular array are orthogonal with respect to the X and Y axes.

- i. From the modify menu, choose Array,
- ii. In the Array dialog box,choose Rectangular Array,
- iii. Choose select objects. The array dialog box closes and AutoCAD prompts for object selection,
- iv. Select the objects to be arrayed and press Enter,

- v. In the Rows and Columns boxes, enter the number of rows and columns in the array,
- vi. Specify the horizontal and vertical spacing between objects,
- vii. Choose OK to create the array ([Fig.20.34a](#)).



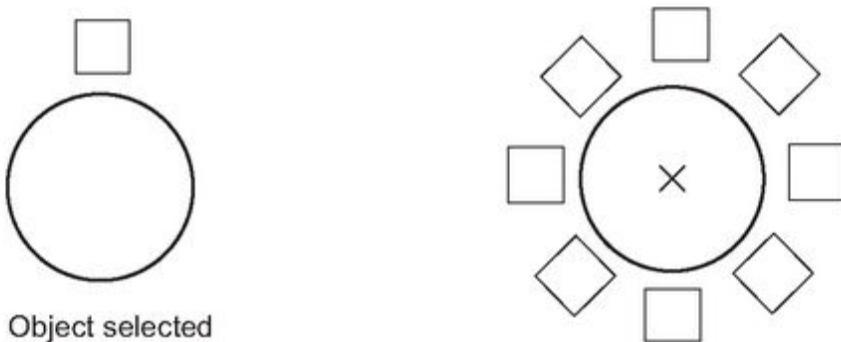
**Fig.20.34a Rectangular array**

### ***Creating Polar arrays***

When a polar array is created, the array is drawn clock-wise or otherwise, depending upon whether a positive or negative value has been entered for the angle that needs to be filled.

- i. From the modify menu, choose Array,
- ii. In the Array dialog box, choose Polar Array,
- iii. Next to Centre Point, enter co-ordinates,
- iv. Choose Select Objects, the Array dialog box closes and AutoCAD prompts for object selection,
- v. Select the object to be arrayed,
- vi. In the Methods box, select number of items, and angle to fill,

- vii. Enter the number of items
- viii. Enter the angle to fill and the angle between them,
- ix. Choose OK to create the array ([Fig.20.34b](#)).

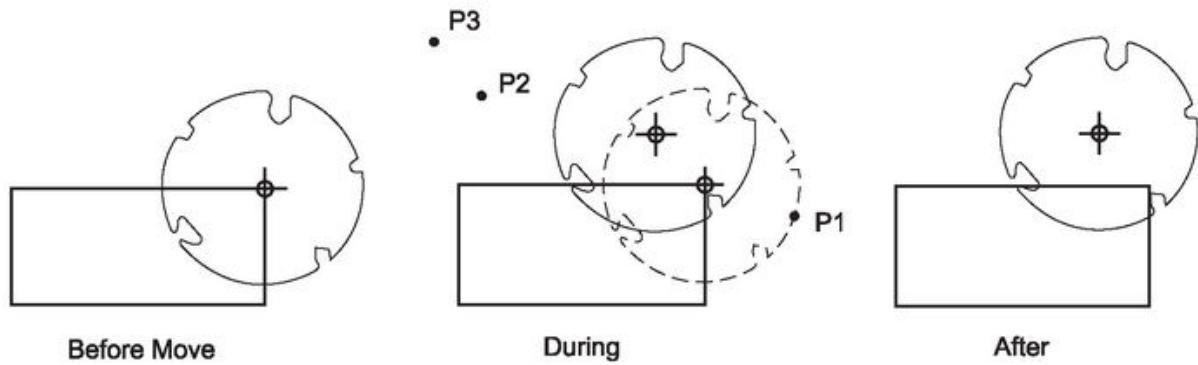


**Fig.20.34b Polar array**

In the mechanical design, draw a single tooth of a gear and use ARRAY to replicate the single tooth around the centre point of the gear. In landscape design, draw a single bush, then use a rectangular array to produce a row of plants along a drive.

In architectural design, draw one side of a geodesic dome, then use a circular array to complete the drawing of the periphery of the dome. When drawing a suspended ceiling, draw one panel and replicate it over the entire open space.

6. **The MOVE command** Move command works in a similar way to copy command, but no copy is made. The selected objects are simply moved from one location to another ([Fig.20.35](#)).



**Fig.20.35**

## Command Sequence

Command: MOVE

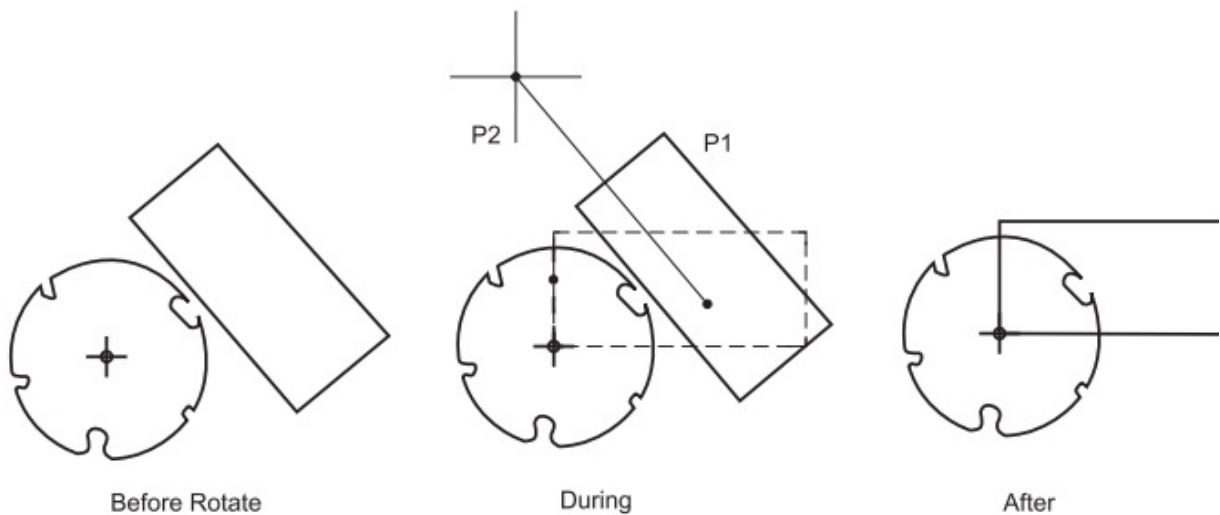
Select objects: (Pick object to move, P<sub>1</sub>)

Select objects: ↵ (to end)

Specify base point or displacement: (pick P<sub>2</sub>)

Specify second point of displacement or < use first point as displacement>: (pick P<sub>3</sub>)

 Points P<sub>2</sub> and P<sub>3</sub> are used to indicate the distance and direction of movement.



**Fig.20.36**

7. **The ROTATE command** This command allows the object to be rotated about a point selected. A second rotation point / angle can be specified ([Fig.20.36](#)).

### **Command Sequence**

Command: ROTATE

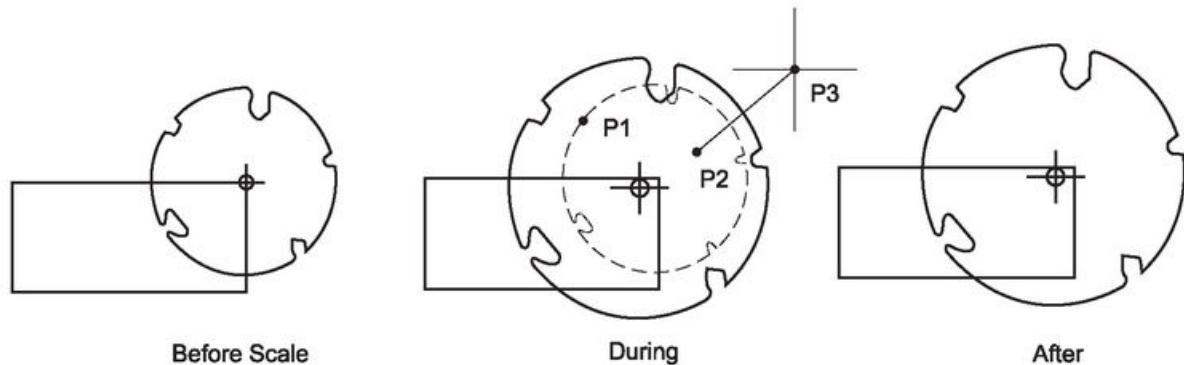
Select objects: (Pick object to rotate, P<sub>1</sub>)

Select objects: ↵ (to end)

Specify base point: (pick base point P<sub>2</sub>)

Specify rotation angle or [Reference]: (pick second point P<sub>3</sub> or enter angle).

8. **The SCALE command** This command can be used to change the size of an object or group of objects. A scale factor 2 given at the prompt, will double the size of the objects in the selection set ([Fig.20.37](#)).



**Fig.20.37**

### **Command Sequence**

Command: SCALE

Select objects: (Pick object to be scaled, P<sub>1</sub>)

Select objects: ↵ (to end)

Specify base point: (pick base point P<sub>2</sub>)

Specify scale factor or (Reference): (pick second point P<sub>3</sub> or enter scale factor)



The base point, P<sub>2</sub> is the centre of the object when it is scaled. IF P<sub>2</sub> is chosen at the previous centre of the object, the object would have remained in the same position.

9. **The STRETCH command** The stretch command can be used to move one or more vertices of an object while leaving the rest of the object unchanged ([Fig.20.38](#)).

### Command Sequence

Command: **STRETCH**

Select the objects to stretch by crossing window or crossing polygon

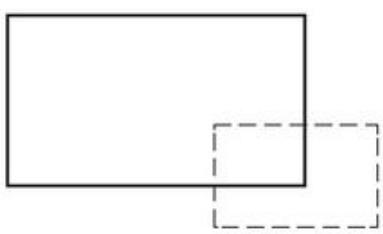
Select objects: (pick first point of crossing window)

Select opposite corner: (Pick second point of window)

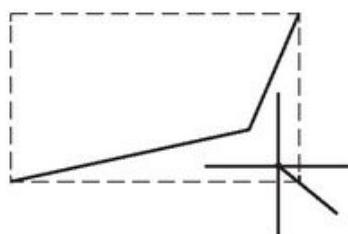
Select objects: ↵ (to end)

Specify base point of displacement: (pick base point)

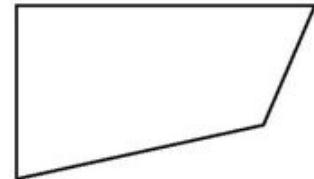
Specify second point of displacement: (pick second point)



Select Vertex



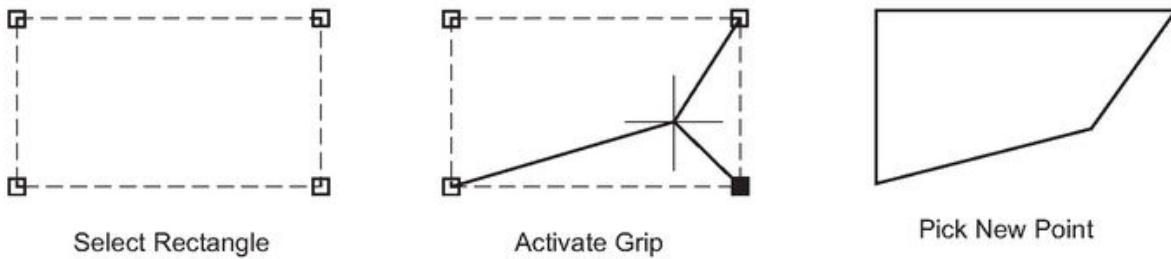
During Stretch



After Stretch

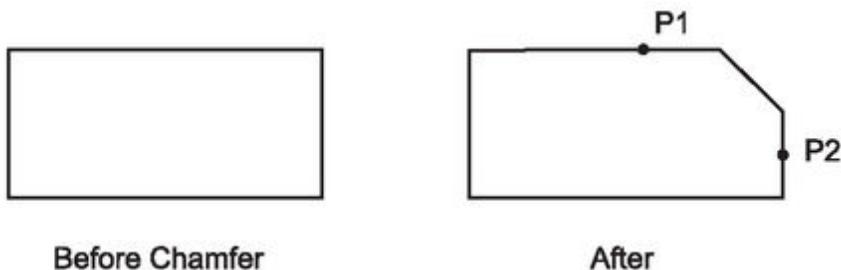
**Fig.20.38**

**10. Stretching with GRIPS** The object is simply selected, using grips by clicking on it to stretch an object. Small square grips appear at each vertex and the object will be highlighted (Fig.20.39). Click a grip to activate and click to reposition the same. Escape key is used to release the grips.



**Fig.20.39**

**11. The CHAMFER command** This command enables to create a chamfer between any two non-parallel lines as in the illustration (Fig.20.40). Chamfer distances are also set prior to this.



**Fig.20.40**

### **Command Sequence**

Command: **CHAMFER**

Specify first line or [Poly line / Distance / Angle/ Trim / Method]: D (to set distances)

Specify first Chamfer distance <10.00>: 20 (required distance)

Specify second chamfer distance < 20.00 >: ← (first distance value)

Select first line: (pick P<sub>1</sub>)

Select second line: (pick P<sub>2</sub>)

Command ends

The various options:

Poly line - Chamfer all vertices simultaneously

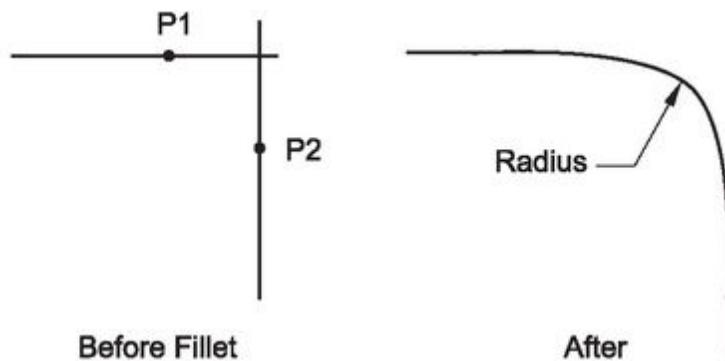
Distance - allows two chamfer distances

Angle - allows the angle between the first line and chamfer

Trim - whether original lines are trimmed / retained

Method - toggles between distance / angle mode

12. **The FILLET command** This command allows to draw an arc between two intersecting lines. Command is first used to set required radius and later two lines are selected ([Fig. 20.41](#)).



**Fig.20.41**

## Command Sequence

Command: **FILLET**

Current setting: Mode = Trim, Radius = 10.00

Select first object or [Poly line / Radius/ Trim] = R

Specify fillet radius <10.00>: 25

Select first object or [Poly line / Radius/ Trim] = (pick P<sub>1</sub>)

Select second object: (pick P<sub>2</sub>)

13. **The TRIM command** This is one of a few commands of AutoCAD that asks to select two objects: The first set defines the boundary, and the second is the set of the objects to be edited. The two sets of objects are not mutually exclusive.

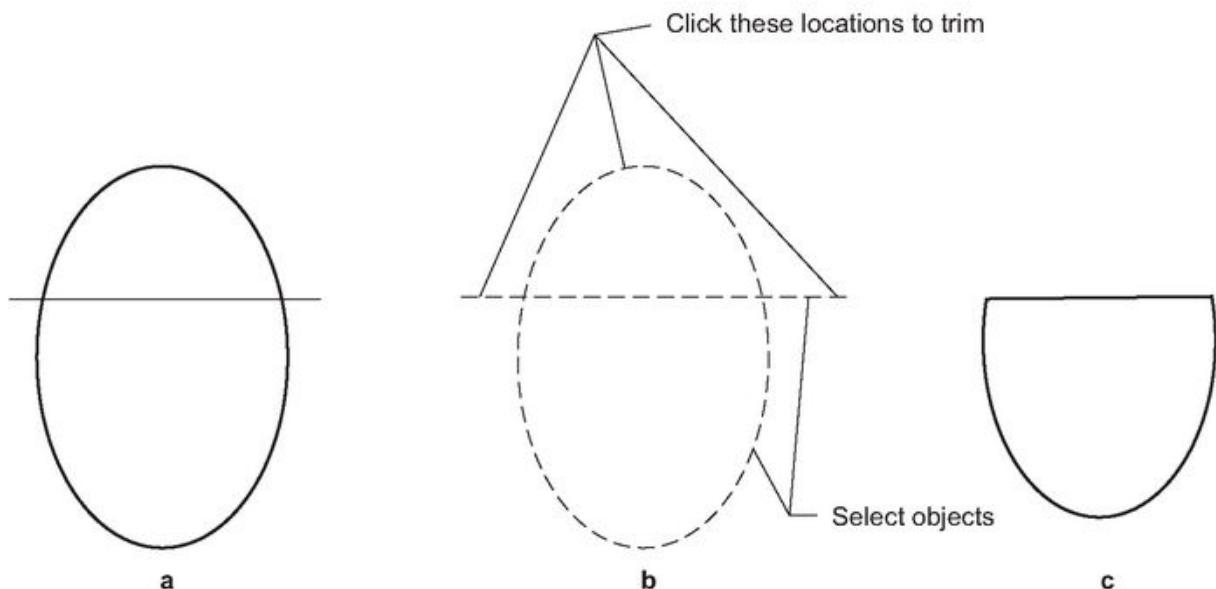
To draw the objects:

1. Draw an ellipse by one of the methods ([Fig.20.42a](#)).
2. Draw a line using LINE command over the ellipse where the ellipse has to be trimmed.

Sequence of operations:

- i) Select Trim tool from the modify tool bar
- ii) At the prompt “select objects or < select all >”: Click the ellipse and the line crossing it ([Fig.20.42b](#)).
- iii) Enter to finish the selection of the objects
- iv) At the prompt “select objects to trim or [Fence, etc.]: Click the portion of the ellipse for trimming.
- v) Click a point near the left end of the line and right end of the line. The line trims the ellipse ([Fig.20.42c](#)).

vi) Press “enter” to exit the Trim command.



**Fig.20.42**

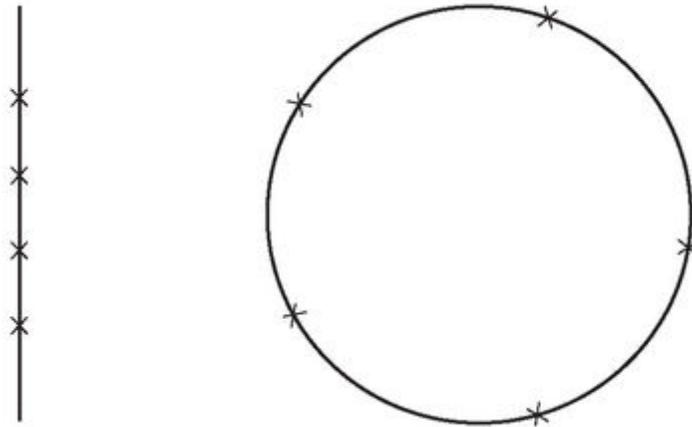
14. **THE EXPLODE command** This command is used to explode single object back to its constituent parts. In other words, the command is used to return blocks, poly lines, etc., (which may be composed of a number of component objects) back to their individual component parts. The change has no visible effect.
15. **The DIVIDE command** This command is used to divide the object into a specified number of equal length segments without actually breaking it ([Fig.20.43](#)). Markers are placed at equal intervals.

### **Command Sequence**

Command: **DIVIDE**

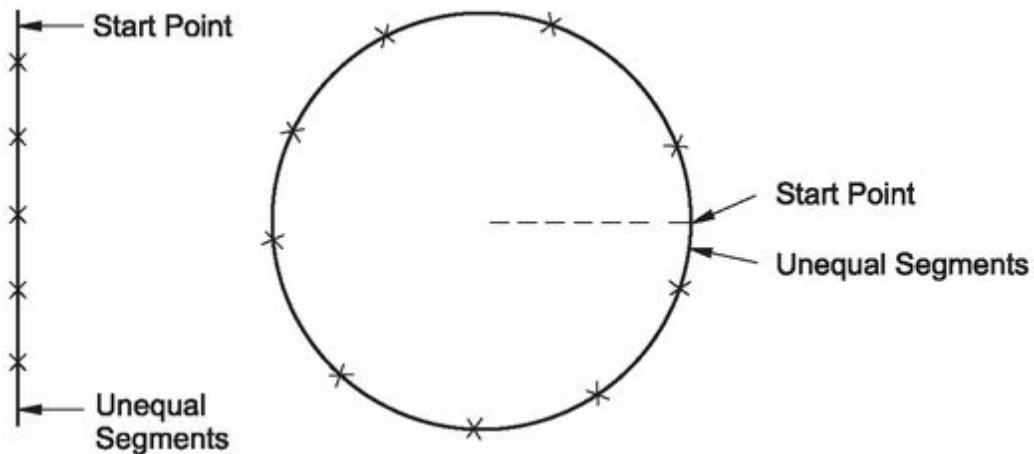
Select objects to divide: (select the object)

Enter number of segments or [Blocks]: (Specify the number)



**Fig.20.43 Dividing into 5 equal parts**

16. **The MEASURE command** This command segments an object at fixed distances without actually dividing it. This command places points or blocks on the given object at a specified distance. In this case, the last segment may or may not be the same as the other segments ([Fig.20.44](#)).



**Fig.20.44**

### **Command Sequence**

Command: **MEASURE**

Select objects to measure: (Select the object)

Specify the length of the segment or [Block]: (Specify)

17. **The PEDIT command** This command is used to edit any type of poly line

Command: **PEDIT**

Select poly line or [Multiple]: (Select)

Enter an option [Close / Join / Width / Edit vertex/  
Fit/Spline /Decurve /L type gen / Undo]: (enter option)

### **Options:**

1. Close: This option closes an open poly line, by creating a segment that connects the last segment to the first segment.
2. Join: This option can be used to join the ends, only if a point is open.
3. Width: This option changes the width of the poly line with a constant or varying width. The desired width has to be specified.
4. Edit vertex: Allows to select a vertex of a poly line and different editing operations can be performed.
5. Fit: This option generates a curve that passes through all the vertices of the poly line.
6. Spline: This option smoothens the corners of a straight segment poly line. However, it passes through only the first and last control points.
7. Decurve: This option straightens the curves generated after using the fit of spline option on a poly line.
8. L Type gen: This option is used to control the line type pattern generation.

9. Undo: This option negates the effect of the most recent PEDIT operations.
18. **The REDO command** This command brings back the process removed previously, using undo commands; but it must be entered immediately after the UNDO command.
19. **The OOPS command** This command restores the objects erased by the last Erase command. Oops command can be used after BLOCK or WBLOCK because these commands can erase the selected objects after creating a block.

## 20.11 USE OF THREE DIMENSIONAL DRAWINGS

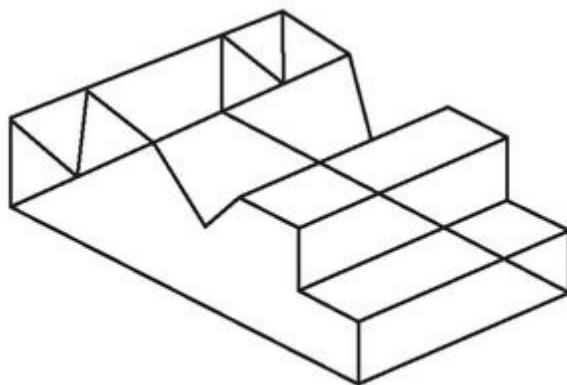
As the drawings are made on a 2D sheet, with x and y coordinates, making a 3D drawing is difficult. However, with the use of computers and required software, a 3D object can easily be created. Prototyping of an object is a time consuming and costly affair, whereas, creating a 3 D object on computer is easier as it permits edition also. From this 3D model, we can generate 2D drawing views, and realistic effects can be created to these models. Individual objects thus created can also be assembled and proper functioning of them can be checked and also the 2D assembled views can be generated. Animation of the assemblies and cut sections also can be created.

### Types of 3D Models

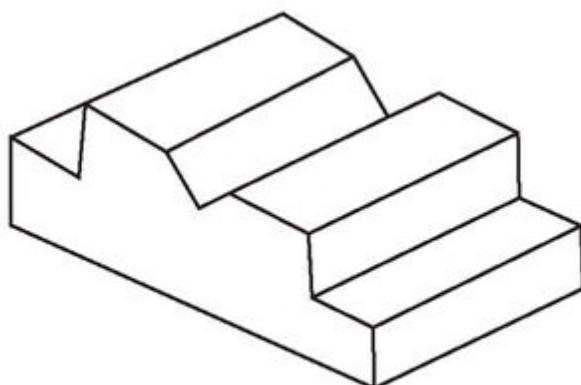
1. Wire frame models - by the use of simple entities of AutoCAD, these models can be created. Entities such as lines, rectangles, poly lines, etc., are used to create

the edges of these models. These models are used in the body buildings of vehicles ([Fig.20.45](#)).

2. Surface models - using one or more surfaces, these models are made. The wall thickness of these models is negligible, but one cannot see through these models. These are used in plastic moulding industry, utensils manufacturing, shoe manufacturing, etc. A surface model can be directly created and a wire frame model can be created and converted into a surface model ([Fig.20.46](#)).



**Fig.20.45**

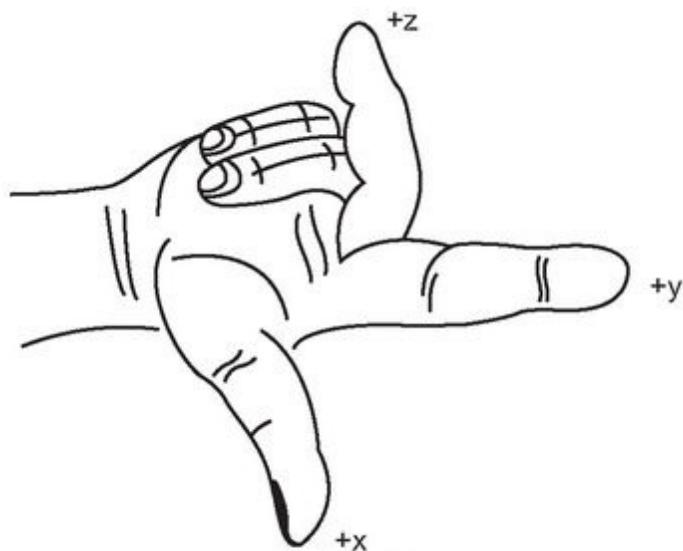


**Fig.20.46**

3. Solid model - these are solid filled models made of wood or metal. A hole/ recess can be cut through them and slices can be cut from these models.

### 20.11.1 Conventions in AutoCAD

For drawing 3D views, right hand rule is used to represent the x, y, and z co-ordinates while keeping the thumb, index finger, and the middle finger mutually perpendicular to each other ([Fig.20.47](#)). The Directions of these fingers represent positive directions of the axes.

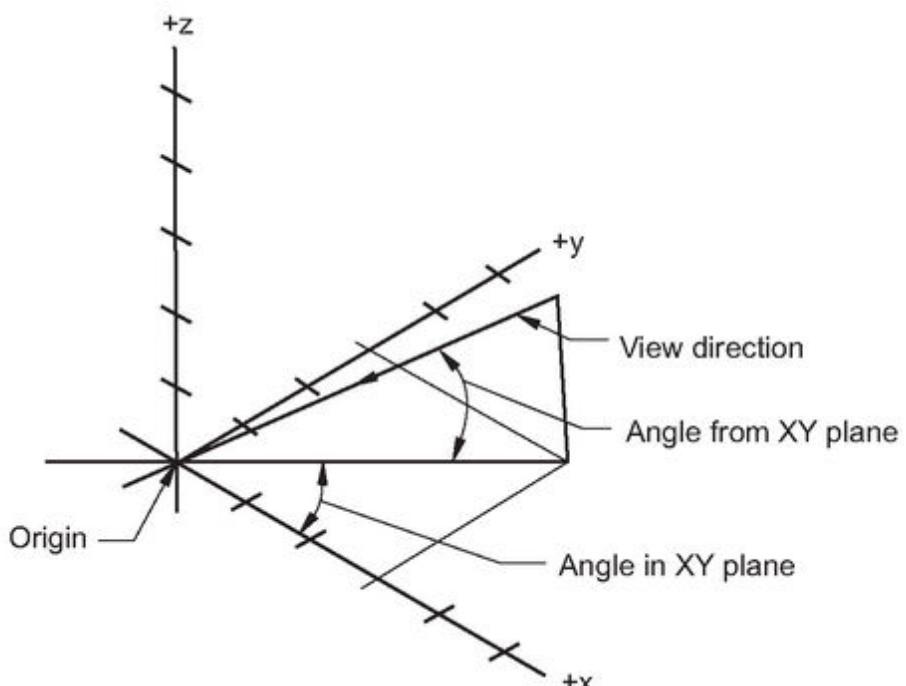


**Fig.20.47**

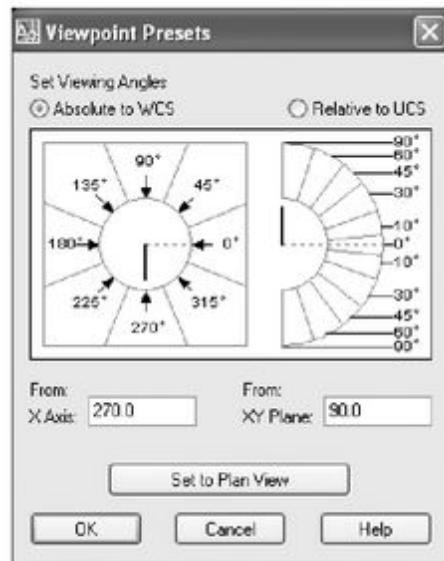
To view 3D models, the view point must be changed; otherwise the view will be a 2D entity. The view point can be changed by the **DDVPOINT** or **VPOINT** command or using the view tool bar. **Viewpoint presets** dialogue box can be invoked by **DDVPOINT** command. [Figure 20.48](#) shows various view direction parameters.

## 20.11.2 Viewpoint Presets

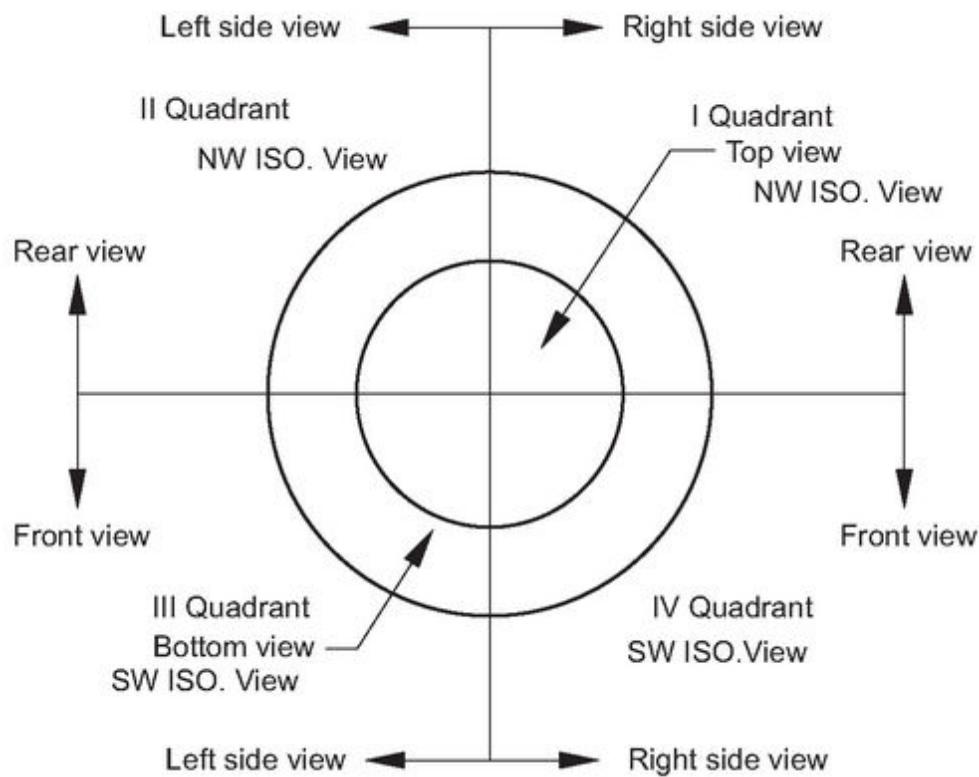
Viewpoint presets dialogue box provides ([Fig.20.49](#)) choice for any view by choosing angle from x axis and angle from xy plane. Consider a frustum of a cone and its top view representing two concentric circles and the centre point being located at the intersection point of x and y axes as shown in [Fig.20.50](#). Based on the selection of the viewpoint, the view appeared varies as referred in [Tables 20.1A](#) and [B](#) (z axis direction +ve if the view is within the smaller circle, -ve if the viewpoint is within the bigger circle).



**Fig.20.48**



**Fig.20.49**



**Fig.20.50**

### 20.11.2.1 Isometric Drawings

Viewing an object in three dimensions gives a sense of its true shape and form. It also helps to conceptualize the design resulting in better design decisions. In addition, three dimensional objects help communicate ideas well to non-technical persons also. A further advantage is one can derive 2D drawings from 3D models, which might otherwise take considerably more time with standard 2D drawing methods.

### ***Creating 3D Forms from 2D shapes***

1. By extruding a 2D object into a variety of shapes, 3D forms can be created
2. By setting thickness and elevation to a 2D object (20.11.4)
3. By converting the wire frame models to surface models (20.11.6)
4. Creating poly face meshes (20.11.7)
5. Creating revolved surfaces
6. By solid modeling
7. Composite solids can be obtained by various Boolean operations (20.11.10)

## **20.11.3 3D Co-ordinate Systems**

Two main types of co-ordinate systems are available as in the case of 2D system.

### **1. Absolute Co-ordinate System (ACS)**

The co-ordinates of a point are measured from the origin  $(0, 0, 0)$  as z co-ordinates are also included.

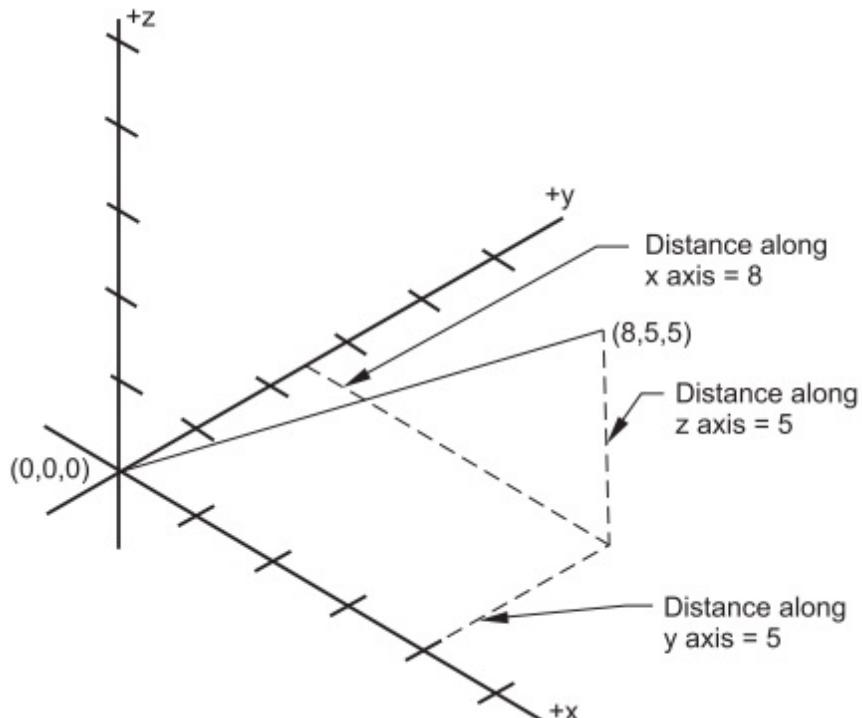
**Example 1** To draw a line from origin to a point  $(8, 5, 5)$

Command: LINE

Specify first point: 0, 0, 0

Specify next point or (undo): 8, 5, 5

Specify next point or (undo): ↵



**Fig.20.51**

Figure 20.51 shows the line drawn from origin using absolute co-ordinates.

**Table 1A - Details of the view obtained  
(orthographic)**

S.No	Direction of view	Name of the view	V point value
1	X axis +ve	Right side view	1, 0, 0
2	X axis -ve	Left side view	-1, 0, 0
3	Y axis +ve	Front view	0, 1, 0
4	Y axis -ve	Rear view	0, 0, -1
5	Z axis +ve	Top view	0, 0, 1
6	Z axis -ve	Bottom view	0, 0, -1

**Table 1B - Details of the views obtained (isometric)**

S.No.	Quadrant	View within	Views visible	V point value	Isometric view
1	Ist	Smaller circle	Right, rear, top	1, 1, 1	NE
2	Ist	Bigger circle	Right, rear, bottom	1, 1, -1	
3	IIInd	Smaller circle	Left, rear, top	-1, 1, 1	NW
4	IIInd	Bigger circle	Left, rear, bottom	-1, 1, -1	
5	IIIrd	Smaller circle	Left, front, top	-1, -1, 1	SW
6	IIIrd	Bigger circle	Left, front, bottom	-1, -1, -1	
7	IVth	Smaller circle	Right, front, top	1, -1, 1	SE
8	IVth	Bigger circle	Right, front, bottom	1, -1, -1	

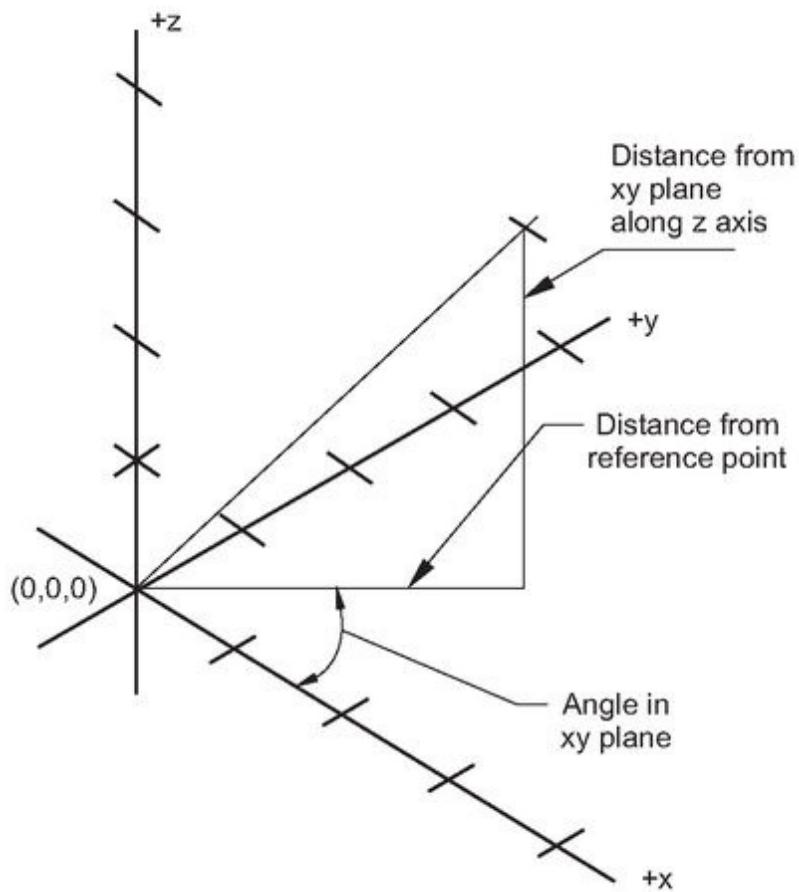
## 2. Relative co-ordinate system

1. Relative rectangular co-ordinate system - syntax for this is @ x co-ordinate, y co-ordinate, z co-ordinate.
2. Relative cylindrical co-ordinate system - syntax for this is @ distance from reference point in xy plane <angle in xy plane from x-axis. distance from xy plane along z axis ([Fig.20.52](#)),
3. Relative spherical co-ordinate system-syntax for this is @ length of the line from the reference point <angle in

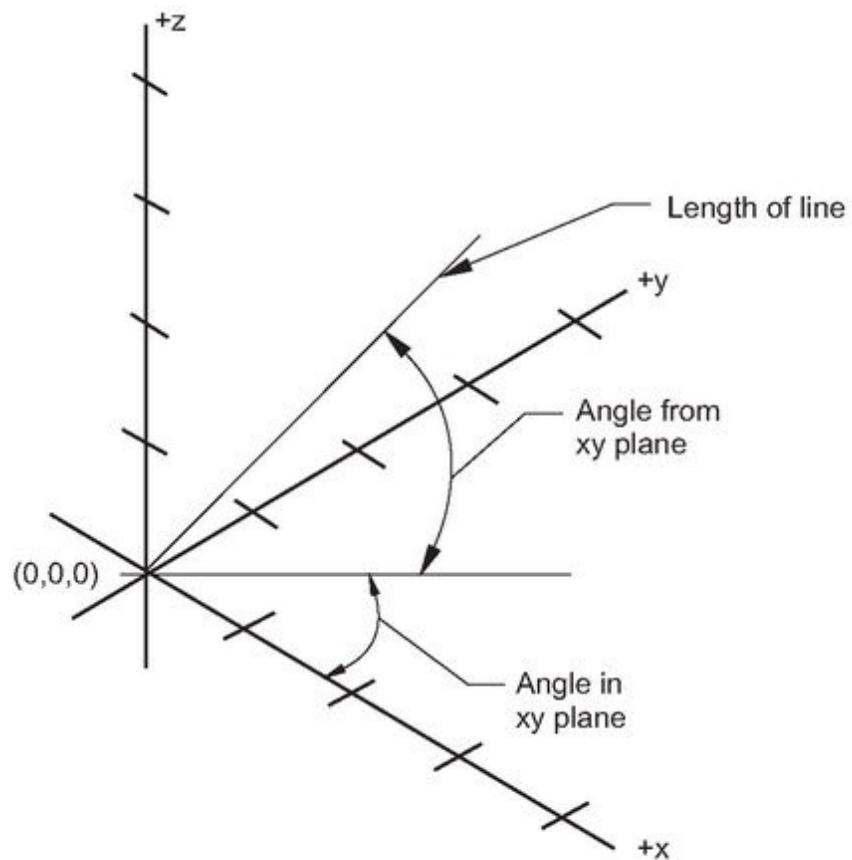
xy plane from x axis, <angle from xy plane (Fig.20.53).

**Example 2** Draw a wire frame model with ACS (Fig.20.54).

1. Choose 3D views> view point presets from the view menu and select the angle 225 from x axis edit box and angle 40 from xy plane and choose OK.
2. Command: LINE



**Fig.20.52**



**Fig.20.53**

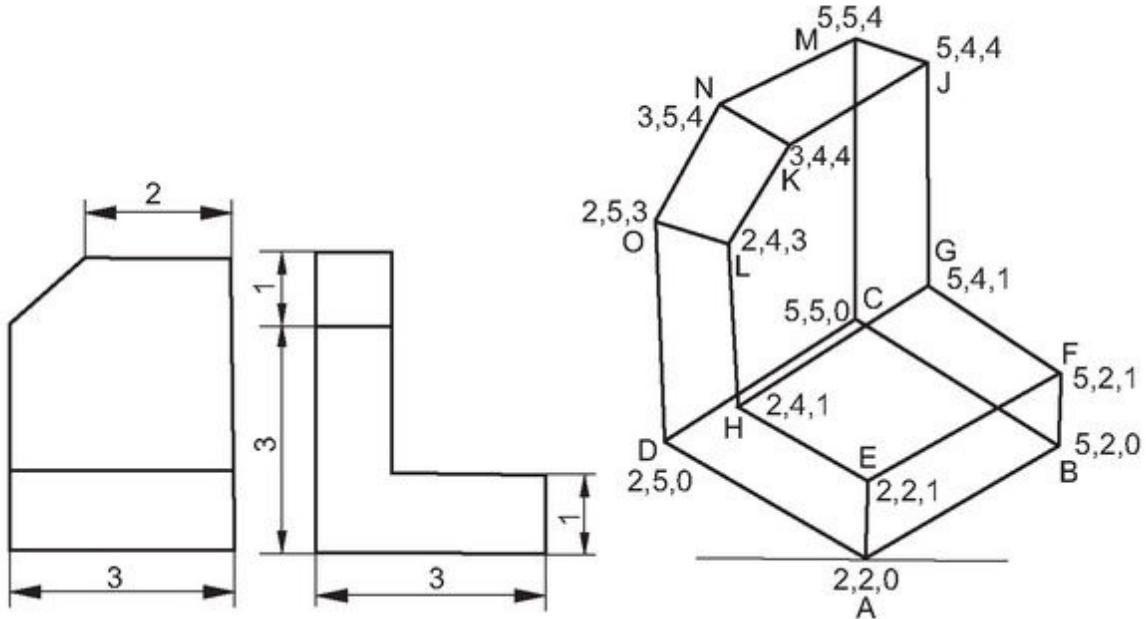
Specify first point: 2, 2, 0 A

Specify next point or (undo): 5,2,0 get **AB**

Specify next point or (undo): 5,5,0 get **BC**

Specify next point or (undo): 2, 5,0 get **CD**

Specify next point or (undo): C



**Fig.20.54**

3. Command: LINE

Specify first point: 2, 2, 0 A

Specify next point or (undo): 2,2, I get **AE**

Specify next point or (undo): 5, 2, I get **EJ**:

Specify next point or (undo): 5,4, I get **FG**

Specify next point or (undo): 2,4, I get **GH**

Specify next point or (undo): 2,2, I get **HE**

Specify next point or (undo): ↵

4. Command: LINE

Specify first point: 5,4, I G

Specify next point or (undo): 5, 4, 4 get **GJ**

Specify next point or (undo): 3,4,4 get **JK**

Specify next point or (undo): 2, 4, 3 get **KL**

Specify next point or (undo): 2,4, 1 get **LH**

Specify next point or (undo): ↵

5. Command: LINE

Specify first point: 5,4, 4 ↵

Specify next point or,(undo): 5,5,4 get **JM**

Specify next point or (undo): 3, 5,4 get **MN**

Specify next point or (undo): 2,5,3 get **NO**

Specify next point or (undo): 2, 5, 0 get **OD**

Specify next point or (undo): 5,5,0 get **DC**

Specify next point or undo): 5, 5,4 get **CM**

Specify next point or (undo):J

6. Command: LINE

Specify first point: 5,2, 0 B

Specify next point or (undo): 5,2, 1 get **BF**

Specify next point or (undo): ↵

7. Command: LINE Specify first point: 3,4,4 K

Specify next point or (undo): 3, 5,4 get **KN**

Specify next point or (undo): ↵

8. Command: LINE

Specify first point: 2, 4, 3 L

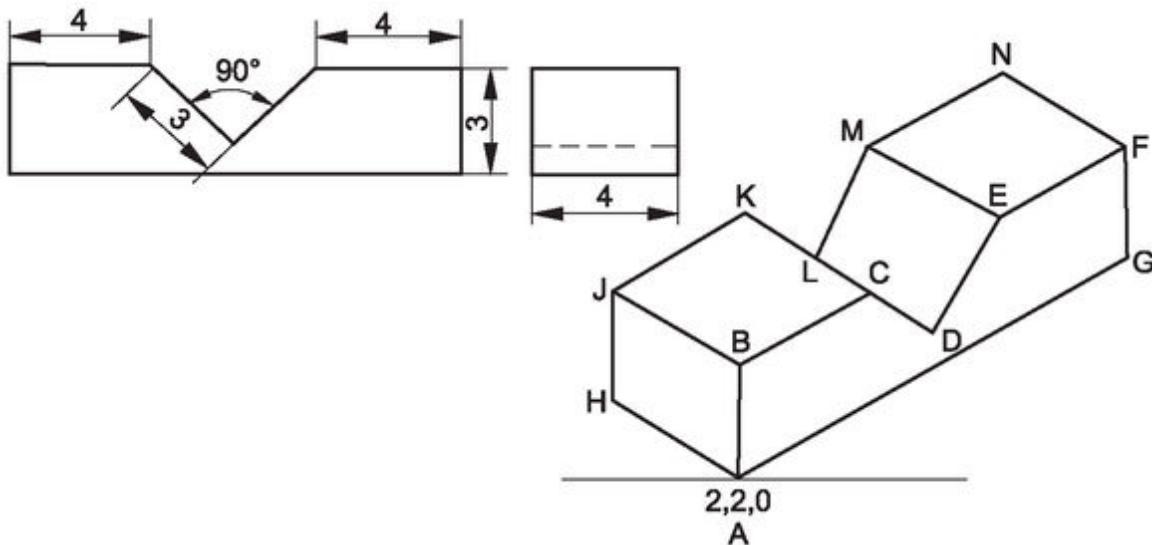
Specify next point or (undo): 3, 5,4 get **LO**

Specify next point or (undo): ↵

9. Command: VPOINT

Specify a view point: -1, -1, 1

**Example 3** Draw 3D wire frame model shown in Fig.20.55  
(use relative cylindrical co-ordinates/spherical co-ordinates)



**Fig.20.55**



From the view tool bar, 4 isometric views are seen when 3D views are chosen.

1. Choose SW isometric view from the view tool bar.
2. We can choose any point to start from, as the start point is not mentioned, Invoke line command  
Specify first point: 2, 2, 0 A  
Specify next point or (undo): @ 0, 0, 3 get **AB**  
Specify next point or (undo): @ 4, 0, 0 get **BC**  
Specify next point or (undo): @3<0<315 get **CD**  
Specify next point or (undo): @3<0<45 get **DE**  
Specify next point or (undo): @4, 0, 0 get **EF**  
Specify next point or (undo): @ 0, 0, -3 get **FG**  
Specify next point or (undo): ↵

3. Choose the line command

Specify first point: 2, 2, 0 A

Specify next point or (undo): @0, 4, 0 get **AH**

Specify next point or (undo): @0, 0, 3 get **HJ**

Specify next point or (undo): @4, 0, 0 get **JK**

Specify next point or (undo): @3<0<315 get **KL**

Specify next point or (undo): @3<0<45 get **LM**

Specify next point or (undo): @4, 0, 0 get **MN**

Specify next point or (undo): ↵

4. Complete the model by joining the remaining edges using LINE command.

## 20.11.4 Setting Thickness and Elevation

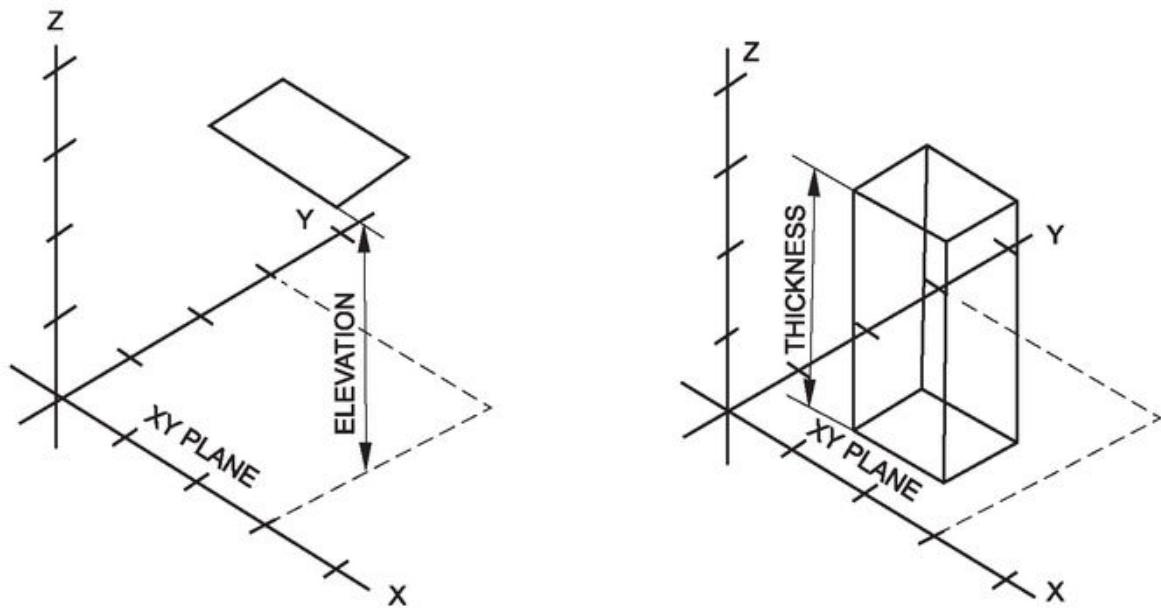
This command sets elevation and thickness for new objects and the existing objects are not modified.

Command: **ELEV**

Specify new default elevation <0.000>: Enter value

Specify new default thickness <0.000>: Enter value

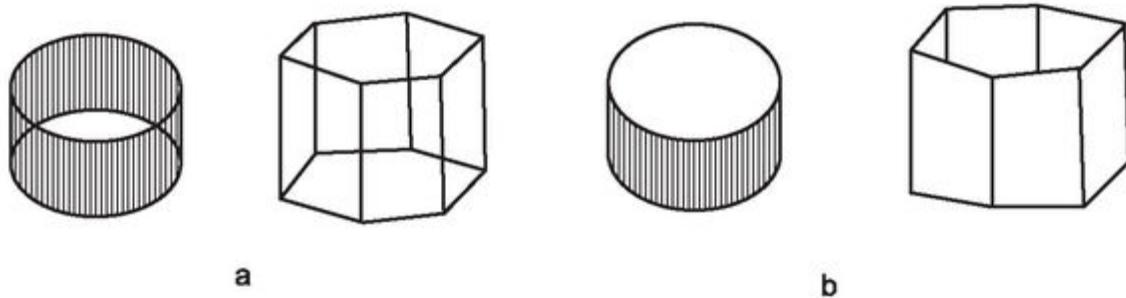
1. **Elevation** - By default, the working plane is on xy plane and this can be moved to any specified elevation by this command. This movement takes place along Z direction.
2. **Thickness** - This command creates extruded surface models, and the thickness is taken along the z- axis ([Fig.20.56](#)).



**Fig.20.56**

### 20.11.5 Suppressing the Hidden Edges

The HIDE command considers solids and 3D faces as opaque surfaces and the objects are non-transparent. When a solid model is created, the hidden edges are also seen as if the model is a wire frame model. However, when the command HIDE is used, the 3D models are displayed with hidden edges suppressed. [Figure 20.57](#) shows 3D models with all edges, however, the hidden edges are suppressed.



**Fig.20.57**

## **20.11.6 Wire Frame Models Converted to Surface Models**

PFACE/ 3D FACE command is used to create 3D faces. A prompt to specify first, second, third and fourth points of the 3D face is shown when this command is invoked. However, for the second time one has to specify third and fourth point only as the previous last 2 points are taken as first and second points. This process continues till one exits. Points must be specified either in clock-wise or anti-clockwise direction.

The unwanted edges can be avoided by using invisible option. While entering the fourth point to specify a 3D face in space, the third and fourth points corresponding to the invisible edges should be entered after entering I(invisible) option ([Fig.20.58](#)).

Command: **3D FACE**

Specify first point: P<sub>1</sub>

Specify next point: P<sub>2</sub>

Specify third point or (invisible) <exit>: I

Specify first point: P<sub>3</sub>

Specify second point: P<sub>4</sub>

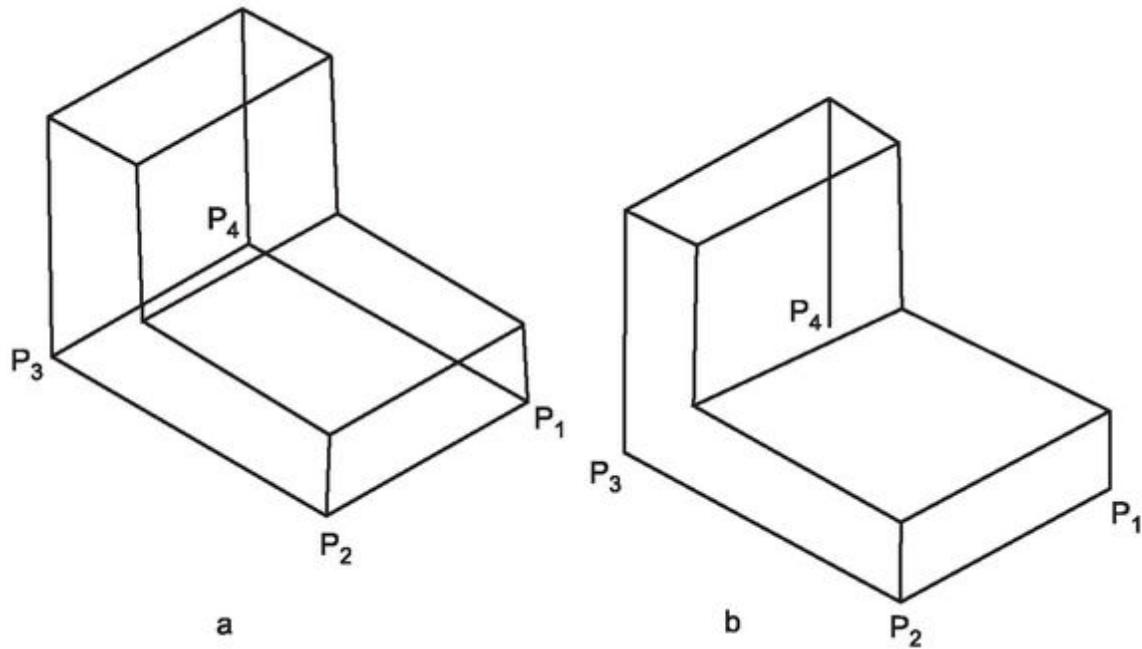
Specify third point or (invisible) <exit>: I

Specify first point: P<sub>4</sub>

Specify second point: P<sub>1</sub>

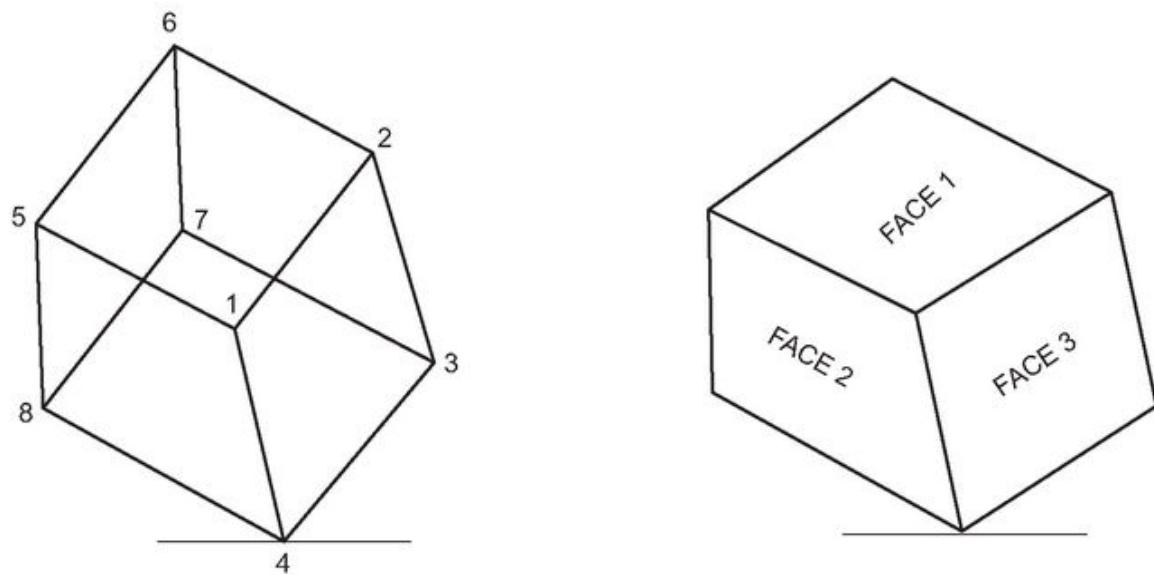
Specify third point or (invisible) <exit>: ↵

Thus, other edges can be eliminated which are not needed.



**Fig.20.58**

### 20.11.7 Creating Poly Face Meshes



## **Fig.20.59**

By choosing a command PFACE, a poly face mesh can be created by specifying the co-ordinates of the vertices. One needs to select the vertices if required and twice if they are coincident with another face. Thus, generation of unrelated 3D faces can be avoided and there is no restriction on the number of faces and vertices.

In the case of command **3D FACE**, the vertices of a face are not chosen twice even if they are coincident with another face.

### **Command: PFACE**

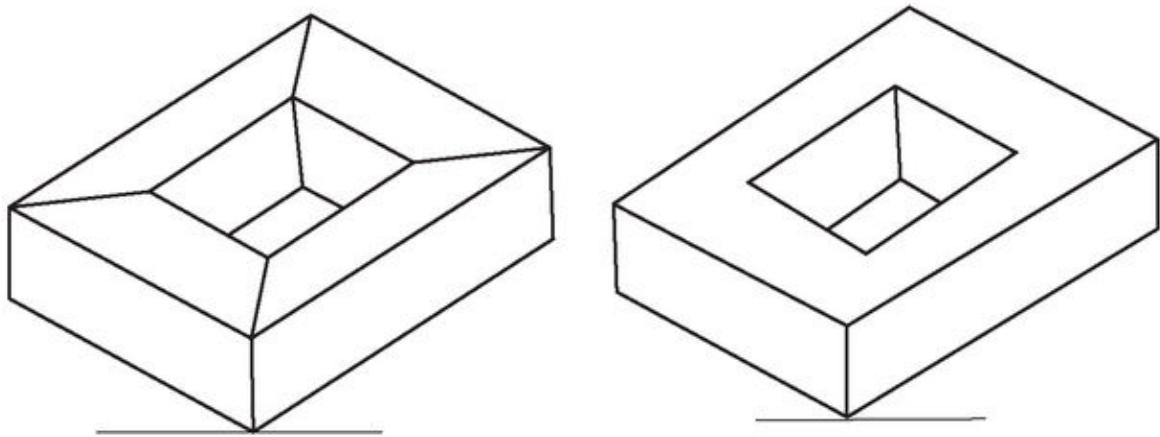
Specify location of vertex 1: First vertex ([Fig.20.59](#)) and thus complete the specification of all the 8 vertices. Later assign vertices to various faces.

**Example:** Specify 1 to 4 vertices for face 1 and enter.

Similarly for other faces also vertices may be specified to get the poly face mesh created. If an edge is to be made visible, a negative number may be provided for the first vertex of the edge.

## **20.11.8 Visibility of the 3D Face Edges**

An edge of the 3D face can be made visible/ invisible by this command - EDGE. When this command is invoked, one has to specify the edge when prompted, to toggle the visibility. If D is entered, the edge will be visible ([Fig.20.60](#)).

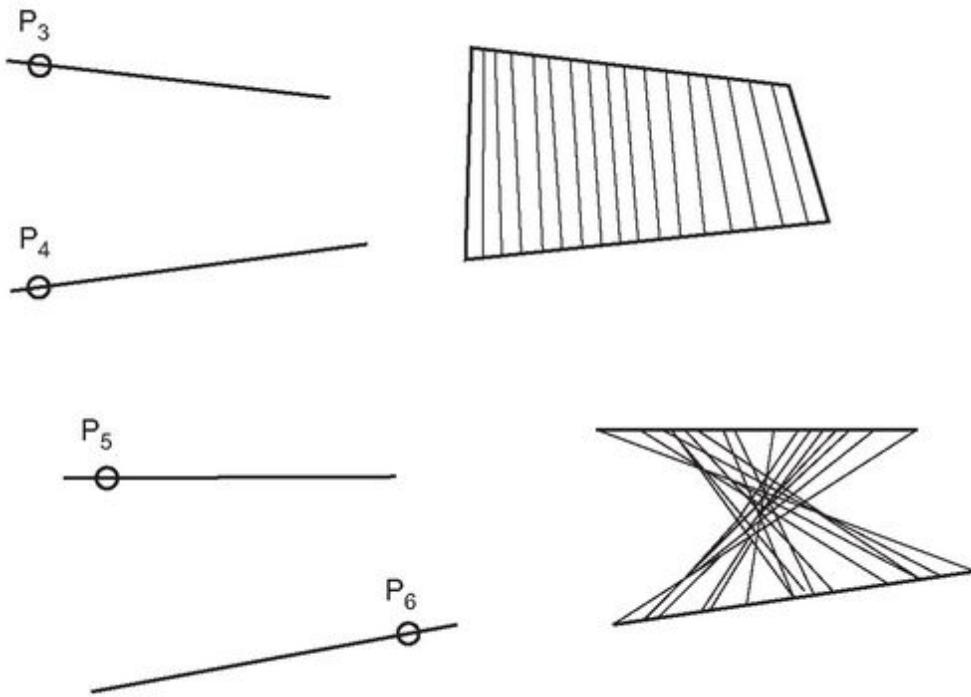


**Fig.20.60**

### 20.11.9 Directly Creating the Surface Models

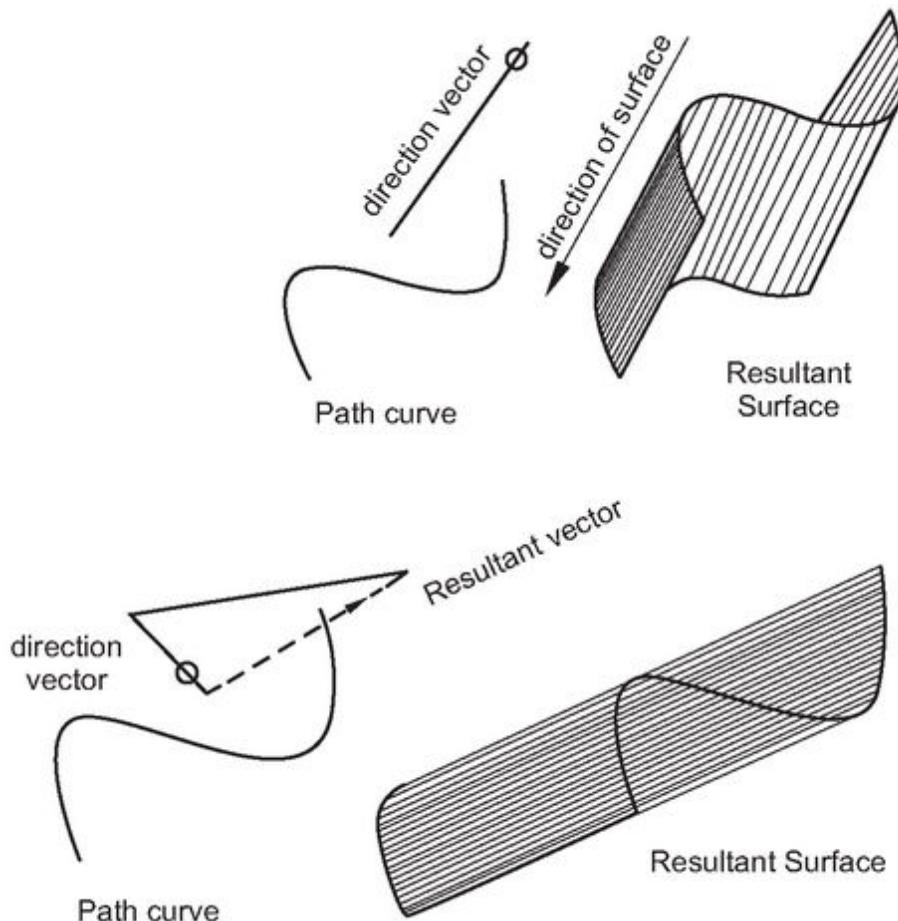
Surface models with curved surfaces cannot be created using 3D faces. However, one can directly create surface with straight edges or curved edges with the help of AutoCAD.

1. **Creating ruled surfaces** The command **RULE SURF** can create a ruled surface between two closed/open entities. Line, poly lines, splines, circles, axes, ellipses, or points can be chosen for this purpose. If two defining entities are open, the points selected will result in various polygon meshes as shown in [Fig.20.61](#). However, the smoothness of the ruled surface can be increased using the SURFTAB system variable.



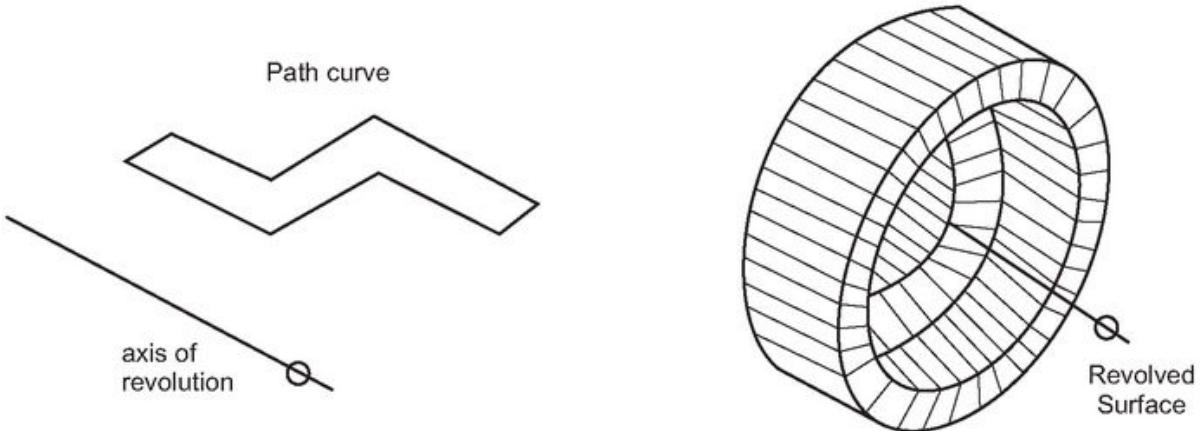
**Fig.20.61**

2. **Creating tabulated surface** The command **TABSURF** creates a surface along the direction defined by a direction vector using a path curve. Lines, curves, circles, ellipses, etc., can be used as base curves. The length of the tabulated surface will be equal to the length of the direction vector. The tabulated surface will be created along the imaginary vector in case the imaginary vector forms the resultant of all the direction vector segments ([Fig.20.62](#)).



**Fig.20.62**

3. **Creating revolved surface** The command **REVSURF** creates revolved surface by revolving the path curve about a specified axis of rotation. The path curve should be a single entity, but it can be an open/ closed entity. AutoCAD uses the right hand thumb rule to determine the direction of revolution ([Fig.20.63](#)). AutoCAD draws lines in the direction of the revolution and the number of tabulated lines is specified by the value of the **SURFTAB 1** system variable.



**Fig.20.63**

## 20.11.10 Solid Modeling

It is the process of building objects that have all the attributes of an actual solid object. Solid models also make it easy to visualize the objects because one always thinks of objects as solids. AutoCAD solid modeling is based on **AICS** solid modeler, which is a part of the core technology.

1. **Creating solid primitives** Solid primitives form the basic building blocks for a complex solid. Box, wedge, cone, cylinder, sphere, and torus are the six solid primitives, which are pre-defined by **AICS** for constructing a solid model. **ISOLINES**, called, tessellation lines determine the number of computations needed to generate a solid. The value assigned to **ISOLINES** variable should be realistic to generate a solid in a reasonable time. Just as line surface meshes, solids are also displayed as wire frame models and one has to hide/shade them. The smoothness of the objects (rounded) is controlled by the **FACETRES** system variable. The height of the

primitives is always along z axis (+ve). The **FACETRES** system variable can go upto 10.

## 2. Creating a solid box

Command: **BOX**

A rectangular box or a cube can be created by this command. A number of options are available:

- i. **Two corner option** Length of the box is taken along x axis and height along z axis. Two opposite corners on the xy plane are to be chosen and later on height is chosen.
- ii. **Centre length option** First, centre of the box (geometric centre) is chosen; later on length, width, and height are chosen to get the primitive.
- iii. **Corner-cube option** First, one specified corner is chosen and later on one edge is specified (magnitude of the edge).

## 3. Creating a solid cone

Command: **CONE**

One can create a solid cone with circular/elliptical base. We may choose to specify height of the cone/location of the apex. Once apex is chosen, the relation between xy plane and its base are automatically determined.

- i. **Circular cone** - to create a cone with circular base: Specify the centre point of the base, then either radius/end point of the diameter. Height of the cone or apex may be chosen later.
- ii. **Elliptical cone**- one has to enter E at the first prompt to specify the centre of the cone. Ellipse

can be created by any one method; later on height/apex may be chosen.

#### 4. Creating a solid cylinder

Command: **CYLINDER**

Both circular cylinder/elliptical cylinder can be created. Either height of the cylinder or the other end of the centre may be given to create inclined cylinder.

- i. ***Circular cylinder***- The sequence of prompts when command **CYLINDER** is chosen:

Current wire frame density: ISOLINES = 4

Specify centre point of the base of cylinder or (Ellipse) <0, 0, 0>:

Give specific radius of base of cylinder or (Diameter): Sp. Radius

Specify height of cylinder or (centre of the other end): Sp. ht. of cylinder Inclined cylinder may be created by entering C at the prompt

Specify height of the cylinder

- ii. ***Elliptical cylinder*** - Elliptical cylinder can be created by entering E at the first prompt - specify centre point for base of the cylinder or (elliptical) <0, 0, 0>: one can specify the height of the cylinder or the centre of the other end of the cylinder.

#### 5. Creating a solid sphere

Command: **SPHERE** creates sphere and the prompt sequence is given below:

Current wire frame density: **ISOLINES** = 4

Specify centre of the sphere <0, 0, 0>: give

Specify radius of sphere or <Diameter>: specify

## 6. Creating a solid torus

Command **TORUS** creates a torus. One half of the torus is above xy plane and the other half is below the plane. The radius of the torus is the distance from the centre to the centre line of the tube. This can have a -ve/+ve value and the radius of the tube can be more than the radius of the torus.

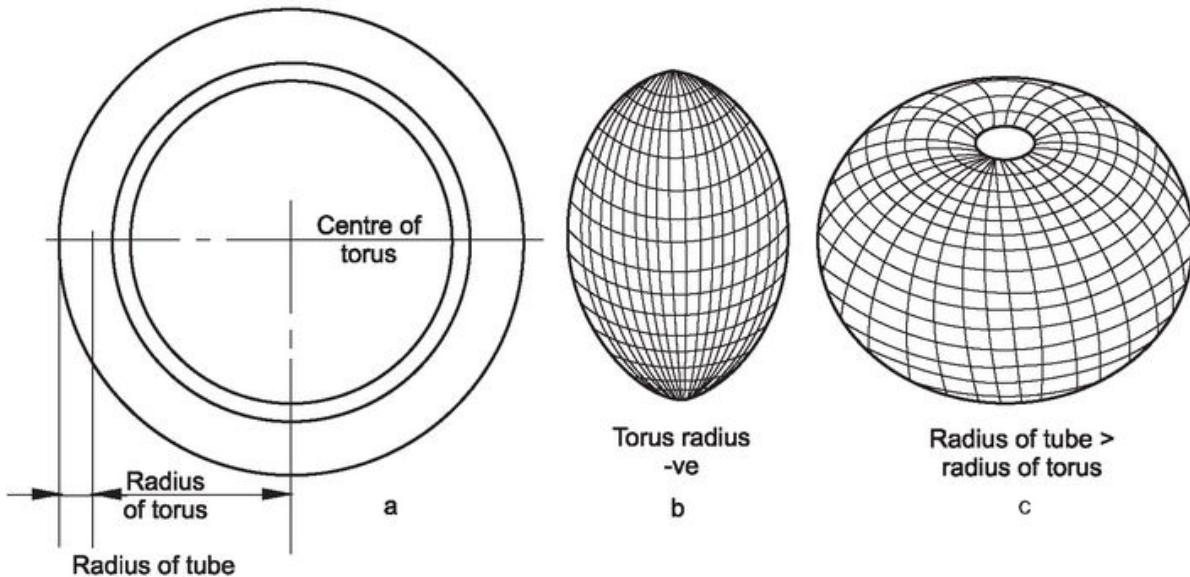
Command: **TORUS**

Current wire frame density; **ISOLINE** = 4

Specify centre of torus <0,0,0>: locate

Specify radius of torus or {Diameter}: Specify radius

Specify radius of the tube or {Diameter}: Specify radius ([Fig. 20.64a](#))



**Fig.20.64**

If the radius of the torus chosen is -ve, it results in a rugby ball shape.

If the radius of the tube is more than radius of torus, the solid looks like an apple ([Figs.20.64b, and c](#)).



Boolean operations are used to create complex solid models from the solid primitives.

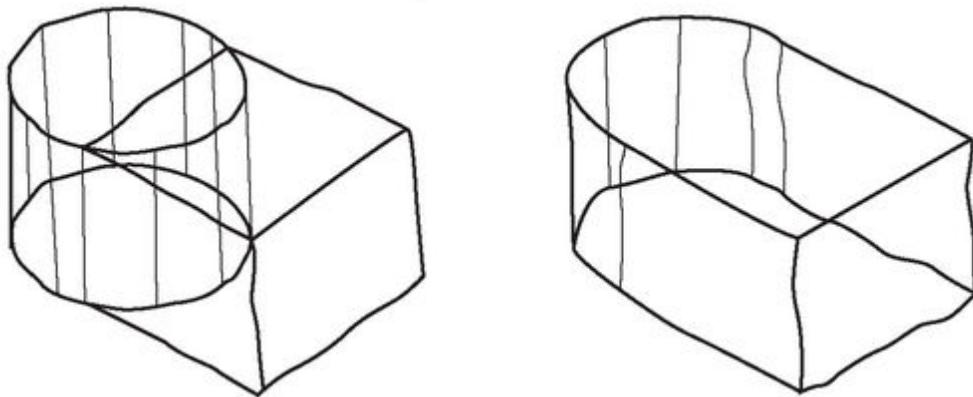
**The REGION** command is used to create regions from the selected closed entities. They are 2D entities with 3D properties of solids.

## 7. Creating complex solid models

The various Boolean operations that can be performed on solid primitives are: Union, subtract, intersect, and interfere.

## 8. Combining solid models

The command UNION combines solids/regions forming a complex solid model. Once this command is invoked, one is required to select the solids to be added. The resultant effect is shown in [Fig.20.65](#).

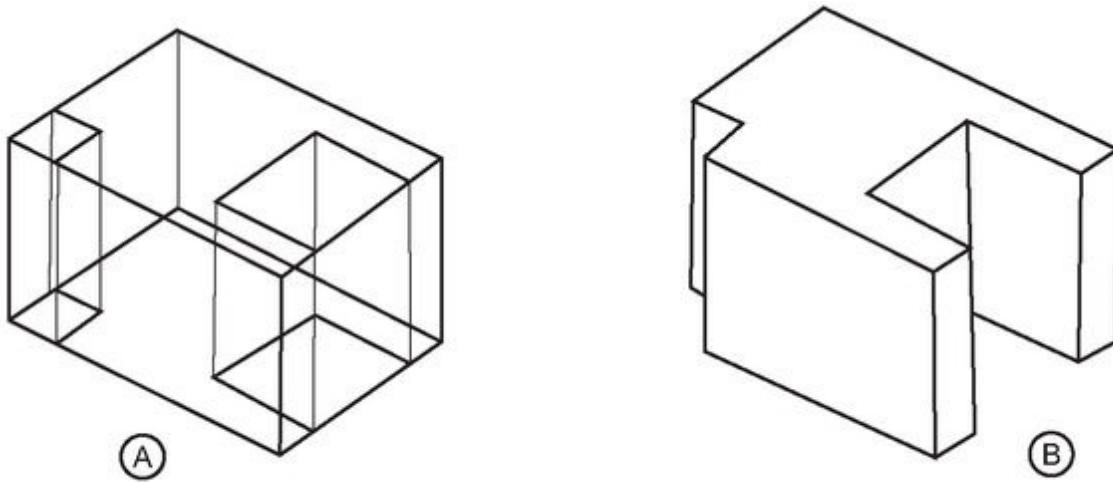


**Fig.20.65**

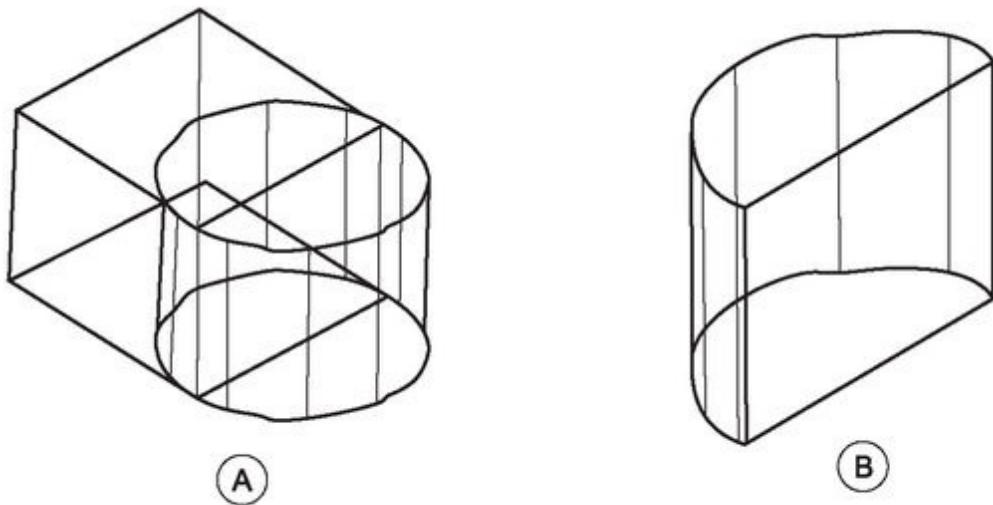
## 9. Subtracting one solid from the other

Command SUBTRACT creates a solid model by removing the material common to the set of regions. Once the set of solids is selected after the command is

invoked, the material common from the first selected set and second selected set is removed ([Fig.20.66](#)).



**Fig.20.66**



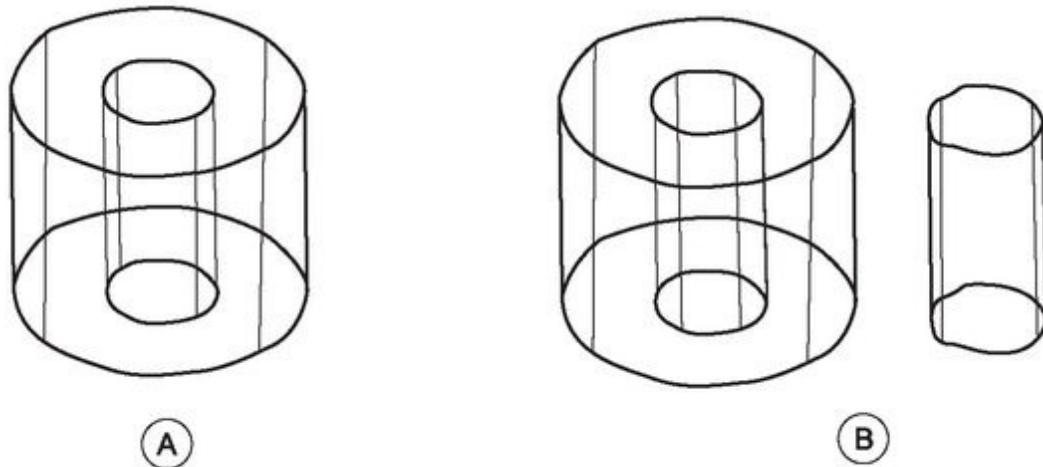
**Fig.20.67**

## 10. Intersecting solid models

**INTERSECT** command creates a composite solid by retaining the material common to selected regions ([Fig.20.67](#)).

## 11. Checking interference in solids

Command **INTERFERE** is used to create a composite solid model as in the case of command INTERSECT. However, the original model also is retained. In INTERFERENCE, a solid is created and moved out as shown ([Fig.20.68](#)).



**Fig.20.68**

## 20.12 BUILDING DRAWING

A building is a structure used for the purpose of providing sheltered accommodation. It may be a residential building, comprising of essential rooms like drawing room, bed room, dining hall, kitchen, toilet, etc., or a non-residential building like a shop, office, school, theatre, hostel, hospital, etc., whose components may vary with the nature of the building.

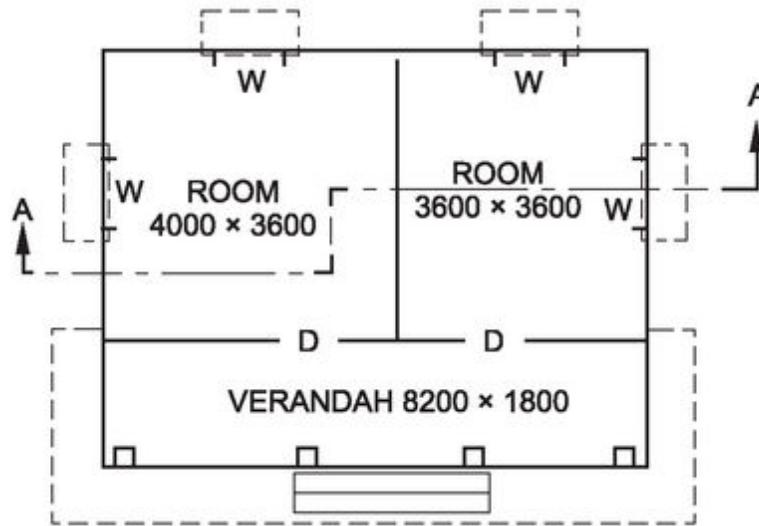
Any building essentially comprises of three parts; foundation, super-structure and roof. Foundation is the bottom-most part of a structure taken into ground to get good anchorage and to form a stable base for the structural

members to rest on. The structure above the ground level is known as super-structure. Roof is the upper-most part of a structure, shielding it from the weathering elements like sun, snow and rain.

The main aim of the building drawing is to give sufficient information by the design engineer to the construction engineer and the following views are generally presented:

- Top view (Plan)
- Front view (Elevation)
- Section at a particular plane or several planes

**Example 4** Using AutoCAD commands, draw the front and top views of a panchayat office, the line diagram of which is shown in Fig.20.69a. Also, draw a section on A -A. Make suitable assumptions wherever needed.



**Fig.20.69a**

### **Generating model:**

Command: limits

Reset Model space limits:

Specify lower left corner or [ON/OFF] <0.0000,0.0000>:

Specify upper right corner <420.0000,297.0000>:  
297,210

Command: z

ZOOM

Specify corner of window, enter a scale factor (nX or nXP), or

[All/Center/Dynamic/Extents/Previous/Scale/Window/Object] <real time>: a Regenerating model.

TO DRAW THE FRONT VIEW

Command: l

LINE Specify first point: 50,50

Specify next point or [Undo]:<Ortho on>150

Specify next point or [Undo]: 90

Specify next point or [Close/Undo]: 150

Specify next point or [Close/Undo]: 90

Specify next point or [Close/Undo]: Zero length line  
created at (50.0000,50.0000, 0.0000)

TRIM

Current settings: Projection=UCS, Edge=None

Select cutting edges ...

Select objects: all

9 found

Select objects:

Select object to trim or shift-select to extend or  
[Project/Edge/Undo]:

Command: o

OFFSET

Specify offset distance or [Through] <25.0000>: 15

Select object to offset or <exit>:

Specify point on side to offset:

Select object to offset or <exit>:

Command: o

OFFSET

Specify offset distance or [Through] <15.0000>: 10

Select object to offset or <exit>:

Specify point on side to offset:

Select object to offset or <exit>:

Command:

ARRAY

Select objects: Specify opposite corner: 2 found

Command: o

OFFSET

Specify offset distance or [Through] <3.0000>: 2

Command: tr

TRIM

Current settings: Projection=UCS, Edge=None

Select cutting edges ...

Select objects: Specify opposite corner: 11found

Command:

Command:\_erase 1 found]

Command: 1

LINE Specify first point: \*Cancel\*

Command: o

OFFSET

Specify offset distance or [Through] <2.0000>: 2

Command: tr

TRIM

Current settings: Projection=UCS, Edge=None

Select cutting edges ...

Select objects: Specify opposite corner: 14 found

Select objects: 1 found, 15 total

Select objects:

Select objects:

Command: Mirror

MIRROR

Specify first point of mirror line: Specify second point of mirror line:

Delete source objects? [Yes/No] <N>:

Command: 0

TO DRAW THE TOP VIEW

OFFSET

Specify offset distance or [Through] <5.0000>: 153

Command: 0

OFFSET

Specify offset distance or [Through] <153.0000>: 15

Command: tr

TRIM

Current settings: Projection=UCS, Edge=None Select cutting edges ...

Select objects: all 63 found

Command: \_offset

Specify offset distance or [Through] <1.0000>: 60

Command: L

LINE Specify first point:

Specify next point or [Undo]:<Ortho on> 100

Specify next point or [Undo]: 150

Specify next point or [Close/Undo]:

Command: tr

TRIM

Current settings: Projection=UCS, Edge=None

Select cutting edges ...

Select objects: all

58 found

Command: L

LINE Specify first point:

Specify next point or [Undo]: 45

Command: 0

OFFSET

Specify offset distance or [Through] <5.0000>: 2.5

Command: mirror

MIRROR

Select objects: Specify opposite corner: 2 found

Command: L

LINE Specify first point:

>>Specify first corner: >>Specify opposite corner:

Specify next point or [Undo]: 75

Specify next point or [Undo]: 150

Command: L

LINE Specify first point:

Specify next point or [Undo]: 10

Specify next point or [Close/Undo]:

Command: ar

ARRAY

Select objects: Specify opposite corner: 3 found

Command: L

LINE Specify first point:

Specify next point or [Undo]:<Ortho on> 10

Command: 0

OFFSET

Specify offset distance or [Through] <45.0000>: 10

Command: 0

OFFSET

Specify offset distance or [Through] <25.0000>: 25

Command: L

LINE Specify first point:

Command: 0

OFFSET

Specify offset distance or [Through] <25.0000>: 5

Command: 0

OFFSET

Specify offset distance or [Through] <5.0000>: 2

Command: L

LINE Specify first point:

Specify next point or [Undo]: 7

Specify next point or [Undo]: 45

Command: mirror

MIRROR

Select objects: Specify opposite corner: 7 found

Select objects:

Specify first point of mirror line: Specify second point of mirror line:

Delete source objects? [Yes/No] <N>:

Command: t

MTEXT Current text style: "Standard" Text height: 2.5

Specify first corner:

Specify opposite corner or [Height/Justify/Line spacing/Rotation/Style/Width]:

Command: co

COPY

Select objects: Specify opposite corner: 5 found

Select objects:

Specify base point or displacement: Specify second point of displacement or

<use first point as displacement>:

Specify second point of displacement:

Command: ro

ROTATE

Current positive angle in UCS:  
ANGDIR=counterclockwise ANGBASE=0

Select objects: Specify opposite corner: 5 found

Specify base point:

Specify rotation angle or [Reference]: 90

Command: m

MOVE

Select objects: Specify opposite corner: 5 found

Select objects:

Specify base point or displacement: Specify second point of displacement or

Command: mi

MIRROR

Select objects: Specify opposite corner: 5 found

TO DRAW THE SECTION A-A

Command: L

LINE Specify first point:397,490

Specify next point or [Undo]: 90

Specify next point or [Undo]: 150  
Specify next point or [Close/Undo]: '\_pan  
>>Press ESC or ENTER to exit, or right-click to display shortcut menu.

Resuming LINE command

Specify next point or [Close/Undo]: 90  
Specify next point or [Close/Undo]: c  
Command: 0

OFFSET

Specify offset distance or [Through] <1.0000>: 35  
Command: 0

OFFSET

Specify offset distance or [Through] <35.0000>: 10  
Command: 0

OFFSET

Specify offset distance or [Through] <10.0000>: 15  
Command: 0

OFFSET

Specify offset distance or [Through] <15.000> : 10  
Command: 0

OFFSET

Specify offset distance or [Through] <10.000> : 6  
Command: tr

TRIM

Current setting: Projection= UCS, Edge= None

Select cutting edges ...

Select objects: Specify opposite corner: 13 found

OFFSET

Specify offset distance or [Through] <6.0000>: 8

Select object to offset or <exit>:

Command: ar

ARRAY

Select objects: Specify opposite corner: 18 found

OFFESET

Specify offset distance or [Through] <8.0000>: 10

Select object to offset or <exit>:

OFFSET

Specify offset distance or [Through] <5.0000>: 15

Command: L

LINE Specify first point:

Command: o

OFFSET

Specify offset distance or [Through] <15.0000>: 25

Command: ar

ARRAY

Select objects: 1 found

Command: mi

MIRROR

Select Objects: Specify opposite corner: 8 found

Command: \_bhatch

Select internal point: \*Cancel\*

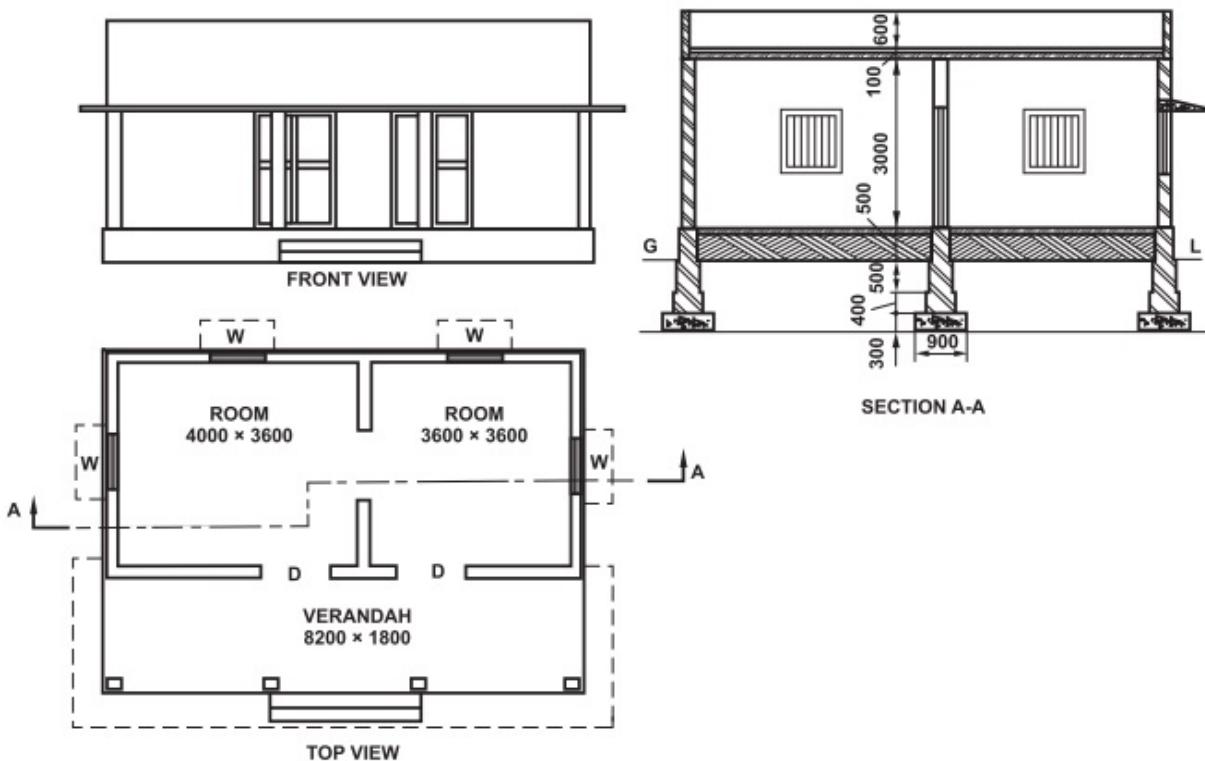
Select internal Point: Selecting everything..

Resuming DIMLINEAR command.

Specify first extension line origin or <select object>:

Specify second extension line origin:

Figure 20.69b shows the three views - front view, top view and section A-A for the panchayat office; the line diagram of which is shown in Fig.20.69a.



**Fig.20.69b**

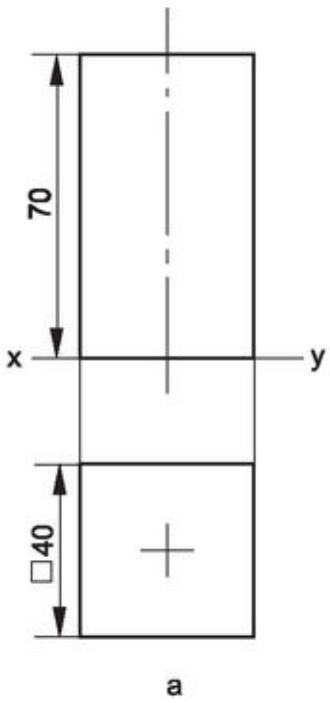
## EXERCISES

- 20.1 Draw the isometric views for the objects, the orthographic projections of which are shown in [Fig.20.70](#). Use absolute co-ordinates.

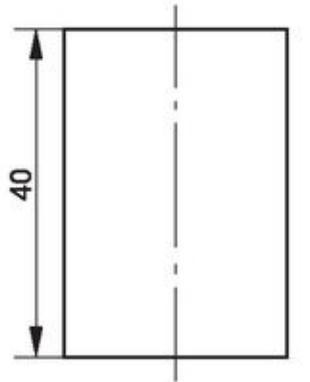
20.2 Draw the isometric views for the objects, the orthographic projections of which are shown in [Fig.20.71](#). Use relative co-ordinates.

20.3 Develop wire frame models for the following models:

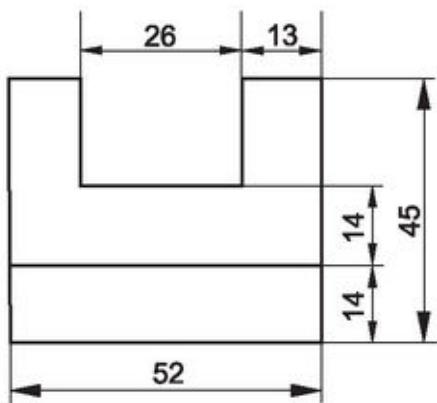
- (i) Rectangular prism of base  $40 \times 30$  and 60 high.
- (ii) Rectangular box of base  $40 \times 30$  and 20 high.
- (iii) Square prism of edge of base 25 and 50 high.
- (iv) Cube of edge 30.
- (v) Cylinder of base 30 diameter and 50 high.
- (vi) Cone of base 30 diameter and 50 high.
- (vii) Sphere of 40 diameter.
- (viii) Torus of tube diameter 20 and radius of 35.



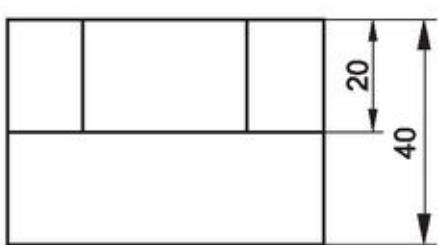
a



b



c



d

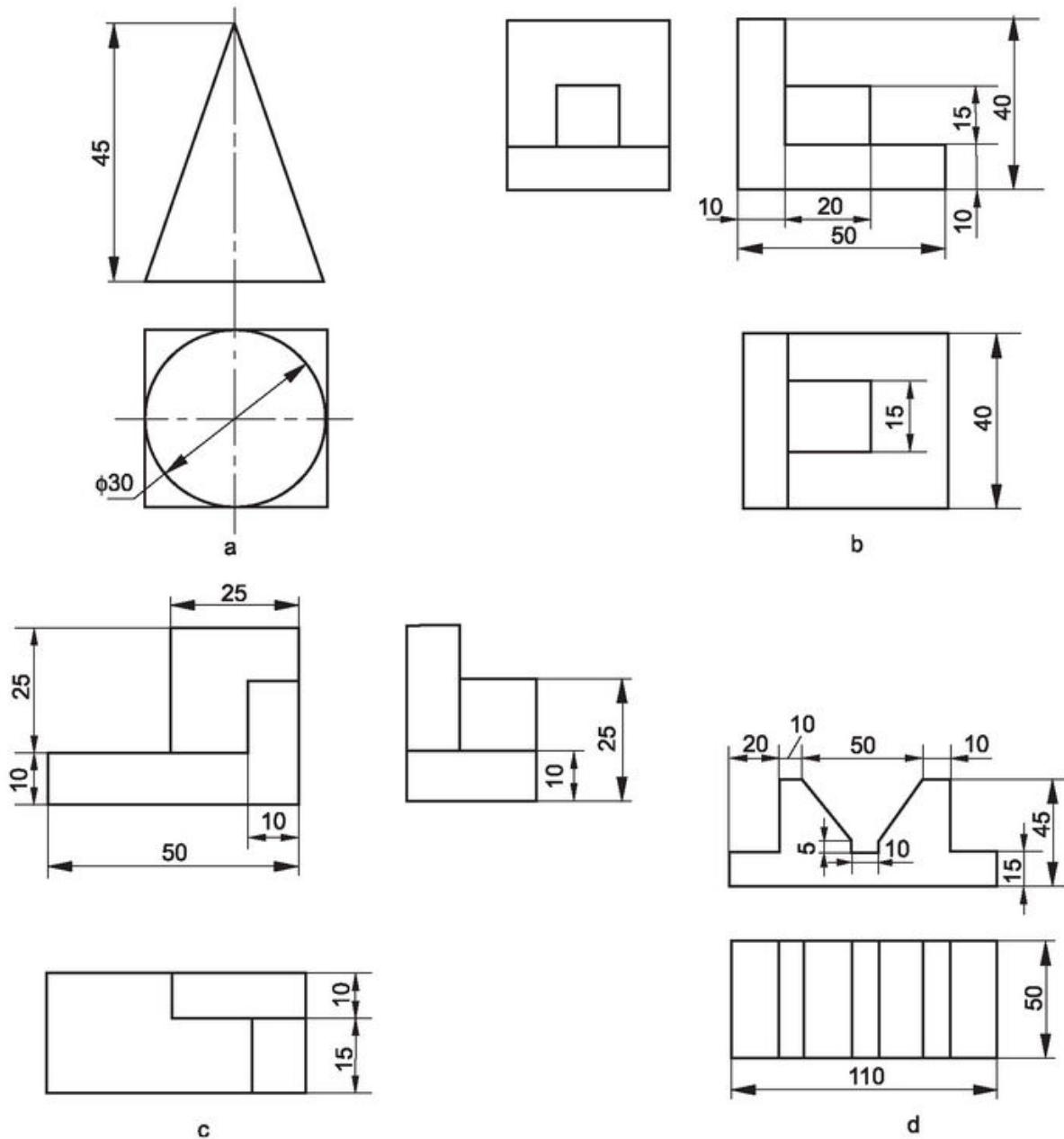
**Fig.20.70**

20.4 Develop surface models for the objects listed in Q.No. 20.3.

20.5 Develop solid models for the objects listed in Q.No.20.3.

20.6 Using various Boolean operations, develop the following combination of solids:

- (i) A cylindrical slab of 50 diameter containing a square hole of edge 20 centrally.
- (ii)A paper weight consisting of a frustum of a cone of height 25, bottom base 70 diameter and top surface of 40 diameter and a sphere of diameter 60 and resting on the frustum such that, the top surface of the frustum is in contact with the surface of the sphere.
- (iii)One half of the cylinder of base 40 diameter and 50 high, is in contact with half cone of diameter of base 40 and 50 high.
- (iv)A hemi-spherical bowl of 150 diameter and 20 thick.
- (v)A horizontal cylinder of diameter 60 and 30 long, having a central co-axial hexagonal hole with distance across corners 40.
- (vi)A vertical cone of base diameter 60 and 75 high, having a concentric circular hole of 30 diameter.



**Fig.20.71**

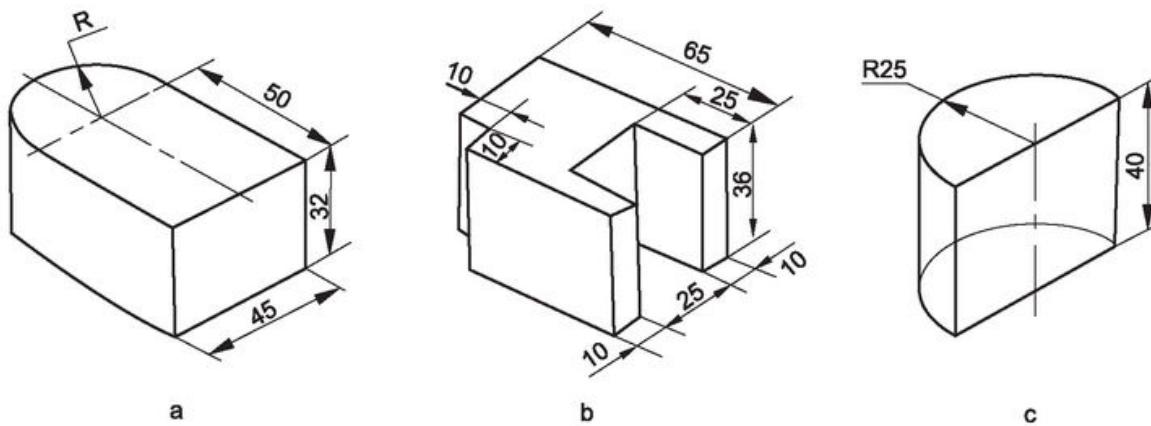
20.7 A square pyramid with side of base 60 and 75 high and square prism with the diagonal of the base 60 and 100 long, fully penetrates each other concentrically such that, the base diagonal of the prism is parallel to an edge of the base of pyramid.

Obtain the result of the Boolean operations such as UNION, SUBTRACT, and INTERSECT.

20.8 Two cylinders of base diameters 60 and 100 long, intersect with axes perpendicular to each other. Obtain the Boolean operation of UNION.

20.9 A cone of diameter of base 90 and axis 100 long, is penetrated by a cylinder of diameter of base 60 and 100 long, with their axes parallel to and 10 away from each other. Obtain the resulting Boolean operations such as UNION, and SUBTRACT.

20.10 Obtain the following composite solids shown in [Fig.20.72](#), by Boolean operations such as UNION, SUBTRACT and INTERSECT using solid primitives.



**Fig.20.72**

20.11 Set the monitor of the computer to the size of A4 sheet and draw the following, using AutoCAD commands:

- Four concentric squares of side 20, 30, 40 and 60.
- Three concentric circles of diameter 30, 45, and 65 using off-set command.
- A rectangle of 40 × 60mm side.

- d. A pentagon, hexagon and a heptagon of side 30.
  - e. A donut with a hole diameter 30mm and outside diameter 50.
- 20.12 Draw a circle of 60 diameter using 5 different options.
- 20.13 Draw a hexagon of side of base 15 and mirror the same to produce a honeycomb structure.
- 20.14 Make a rectangular array choosing the distance between rows 25 and distance between columns 30. The object chosen can be
- a. a pentagon of side 10,
  - b. a circle of diameter 15,
  - c. a square of side 10, and
  - d. a semi-circle of diameter 10.
- 20.15 Make a polar array of the objects described in Q.20.5 selecting 6, 8 and 12 objects at a time.
- 20.16 Using rotate command, reproduce the pentagon of side of base 20 such that, one edge makes an angle 25, 30, 45 and 60 degrees with vertical.
- 20.17 Draw a rectangle 80×100 and a hexagon of side of base 60:
- a. Chamfer the corners choosing chamfer distance 3,5, and 6, and
  - b. Fillet the corners with a fillet radius 12.
- 20.18 Divide the circumference of the following objects into 10, 15 and 20 equal parts:
- a. a circle of diameter 60, and

b. an ellipse having major diameter 60 and minor diameter 40.

20.19 Segment the objects listed in Q. 20.9 at fixed distances of 10 and 15.

20.20 Draw a rectangle of  $100 \times 50$ . Using an off-set of 10 each on both x and y directions, draw three rectangles.

20.21 Draw circle of diameter 15. Use array command and draw 5 rows and 5 columns. Set offset distance of 10 on both directions.

20.22 Draw 10 circles in a row of radius 10. Keep centre to centre distance as 20 and use mirror option to reflect on both sides of centre to centre distance 20.

20.23 Draw arcs, satisfying the following data:

i) Arc passing through the points A, B and C. The co-ordinates are (0, 30), (25,25) and (30,0), respectively..

ii) Arc passing through the points A and B ; the centre of the arc being O. The co-ordinates are (0,30), (30,0) and (0,0) respectively.

iii) Arc of radius 15 and passing through the points A and B. The co-ordinates are (15, 15) and (30,30)respectively.

iv) Radius of the arc 25 and length of chord 30 mm.

20.24 Draw two lines AB and AC, making an angle of  $45^\circ$  at A. Draw the angle bisector between the two lines.

20.25 An actual distance of 4km is shown by a line 8cm long on a map. Draw a plain scale reading in kilometres and hectometres. Mark on this scale, the

following distances: i) 5.7km, ii) 3km 3hm and iii) 220 decametres. What is the R.F of this scale?

20.26 Draw a fixed line MN (directrix) of any length. The distance of the focus from the directrix is 50. Trace the path of a point, P moving in such a way that the ratio of its distance from the fixed point, A to its distance from the fixed line MN is always unity. Name the curve. Plot at least 5 points.

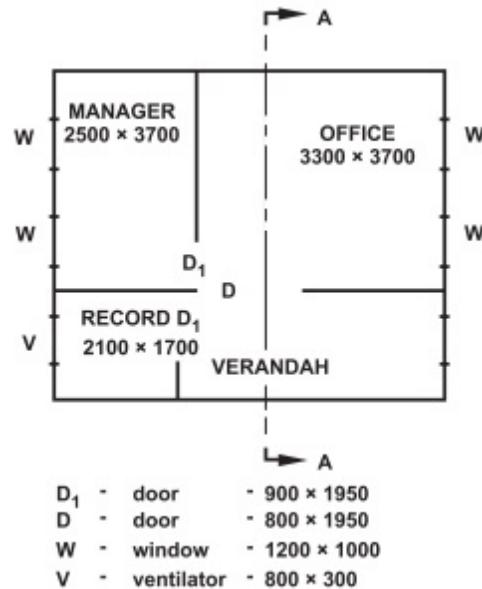
20.27 A point moves such that, the difference of its distances from two fixed points is constant and is equal to 100. The fixed points are located 200 apart. Draw the locus of the moving point. Name the curve.

20.28 In a wall clock, an ant centered inside, started crawling towards the periphery on the seconds hand at uniform speed. The ant reaches the tip of the seconds hand in one minute. The length of seconds hand is 100. Draw the locus of the ant.

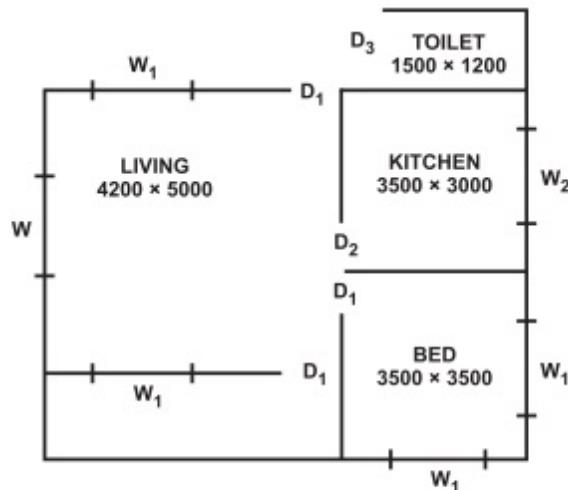
20.29 Draw a hexagon by using line command and convert all the lines into a single entity (PEDIT).

20.30 A hexagonal prism of side of base 25 and axis 50 long, is resting on H.P, with an edge of the base perpendicular to V.P. Draw the front and top views of the solid.

20.31 A pentagonal pyramid of side base 30 and axis is 60 long. Draw isometric view in 2D plane. Choose any orientation.



**Fig.20.73**



$W$ - window	- $1600 \times 1200$	$D_2$ - door	- $900 \times 1950$
$W_1$ - window	- $1200 \times 1200$	$D_3$ - door	- $800 \times 1950$
$W_2$ - window	- $1000 \times 1200$	Wall thickness	- 200
$D_1$ - door	- $1000 \times 1950$	Ht. of parapet wall	- 600
		Ceiling height	- 3000

**Fig.20.74**

20.32 Draw the 3D view of 4 equal spheres of 40 diameter which are arranged in tetrahedral form such that, each sphere touches the other three.

- 20.33 The plan of a three room office building is given in Fig.20.73. Draw to suitable scale, front view, top view, and a sectional view along A- A, using AutoCAD, making suitable assumptions.
- 20.34 Figure 20.74 represents line sketch of a single bed room house. Draw to a suitable scale, the front and sectional top view, using AutoCAD.

## OBJECTIVE/ REVIEW QUESTIONS

- 20.1 What is CAD?
- 20.2 Graphics can be converted into hard copy with a \_\_\_\_\_.
- 20.3 What is Graphics package?
- 20.4 Computer aided graphics systems have 3 major components. What are they?
- 20.5 What is a digitizer?
- 20.6 What are the applications of locators and selectors?
- 20.7 Two examples of single user operating systems are \_\_\_\_\_ and \_\_\_\_\_.
- 20.8 Two examples of multi-user systems are \_\_\_\_\_ and \_\_\_\_\_.
- 20.9 DOS is used on \_\_\_\_\_ computers.
- 20.10 Two types of DOS commands are \_\_\_\_\_ and \_\_\_\_\_.
- 20.11 What do you understand by drawing limits and extents?
- 20.12 What is layering concept?

20.13 How do you begin a new drawing?

20.14 How do you select an existing drawing for editing?

20.15 How to exit from AutoCAD?

20.16 Zooming shrinks the drawing for editing.

(True/False)

20.17 Panning Changes the magnification of the drawing.

(True/False)

20.18 What are the uses of editing facilities of AutoCAD?

20.19 AutoCAD editor Screen has \_\_\_\_\_ areas.  
What are they?

20.20 What is a status line?

20.21 In AutoCAD \_\_\_\_\_ helps to set-up a drawing.

20.22 \_\_\_\_\_ provides introduction to the methods for starting a new drawing.

20.23 \_\_\_\_\_ command constrains the lines drawn horizontal and vertical directions only.

20.24 \_\_\_\_\_ command sets the increments for cursor movements.

20.25 Grid command helps the user by \_\_\_\_\_.

20.26 Help can be obtained by \_\_\_\_\_ command.

20.27 \_\_\_\_\_ command saves the work.

20.28 List 4 important options of zoom command.

20.29 Object selection is achieved by \_\_\_\_\_ and \_\_\_\_\_.

20.30 List 5 important edit commands and mention their applications.

- 20.31 List the utility commands.
- 20.32 Differentiate between DIM and DIMI commands.
- 20.33 List the categories of dimensioning commands.
- 20.34 Distinguish between Normal, Outer-most, and Ignore styles of hatch command.
- 20.35 When do you use PLOT and PRPLOT commands?
- 20.36 Explain how a line can be drawn by (i) Cartesian co-ordinate methods, (ii) Incremental from and (iii) Polar co-ordinate form.
- 20.37 How to draw an ellipse?
- 20.38 Give the procedure for describing a polygon of 7 sides of side 30mm.
- 20.39 What are the various modeling techniques on ACAD?
- 20.40 What do you understand by VPOINT command?
- 20.41 What is VIEWPORTS?
- 20.42 What are the options available on VIEWPORTS command?
- 20.43 Which option is chosen to know the number and co-ordinates of current view port?
- 20.44 How do you draw a cylinder, cone and box?
- 20.45 What are the applications of primitives?
- 20.46 When do you choose the command PLINE?
- 20.47 What are the responses of a computer for the command OFFSET?
- 20.48 What are the features of the command ELEV and THICKNESS?

- 20.49 Explain the features of CHANGE PROP command.
- 20.50 What do you achieve by EXTRUSION command?
- 20.51 List the various options available to draw an arc?
- 20.52 What are the options available to draw a circle?
- 20.53 Distinguish between 2 point and 3 point option of drawing a circle.
- 20.54 List the various Boolean operations that may be performed on a computer.
- 20.55 \_\_\_\_\_ operations can be used to create composite solids.
- 20.56 Region command allows \_\_\_\_\_
- 20.57 \_\_\_\_\_ allows to combine the volume of two or more solids.
- 20.58 \_\_\_\_\_ allows to remove common area shared by two sets of solids.
- 20.59 Intersect allows to create a composite solid that contains only \_\_\_\_\_.
- 20.60 Mass prop provides \_\_\_\_\_ for solid.
- 20.61 Mass prop displays \_\_\_\_\_ for regions.
- 20.62 SOLVIEW command provides \_\_\_\_\_
- 20.63 Explain the functions of the commands ISOPLANE.

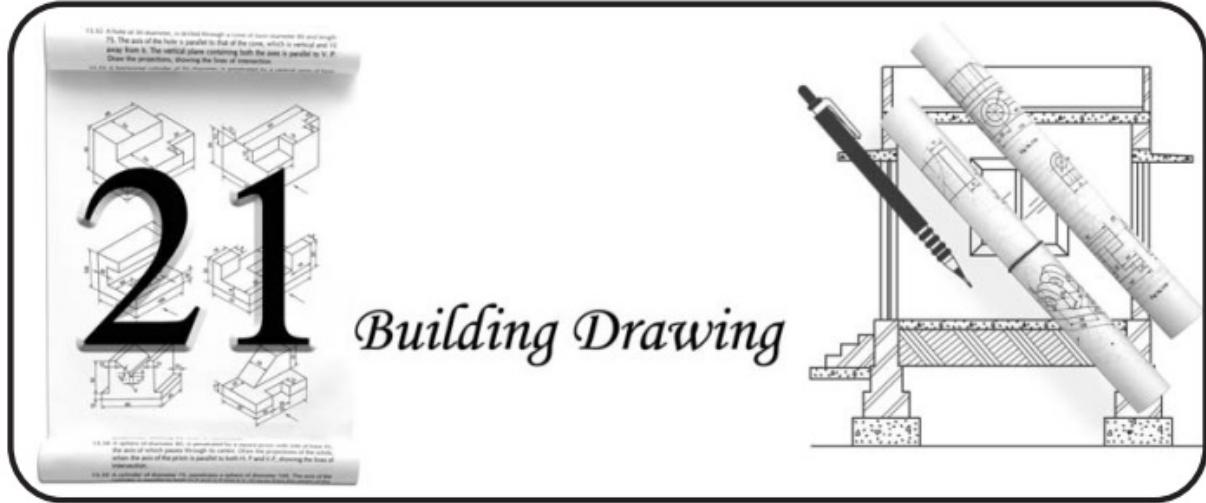
- 20.64 Explain the various dimensioning methods of AutoCAD.
- 20.65 Distinguish between base line and continue commands.
- 20.66 List the various options available for the command DIM.
- 20.67 Describe how to make centre lines for an arc or circle.
- 20.68 Solid modeling can be created by \_\_\_\_\_ a 2D object.
- 20.69 \_\_\_\_\_ models are true shape 3D objects.
- 20.70 Describe the responses of computer for command RECTANG.

## ANSWERS

- 20.2 plotter/printer
- 20.4 draughtsman, hardware, and software
- 20.7 Windows 2000, Windows XP
- 20.8 UNIX, LINUX
- 20.9. personal
- 20.10 internal, external
- 20.16 True
- 20.17 False
- 20.19 four
- 20.21 Use Wizard
- 20.22 Instruction

20.23ORTHO  
20.24SNAP  
20.25creating reference lines  
20.26HELP  
20.27SAVE  
20.29Pick box, Window option  
20.55Boolean  
20.56to create 2D enclosed areas  
20.57Union  
20.58Subtract  
20.59Common volume of two or more overlapping solids  
20.60volumetric information  
20.61area properties  
20.62multi- and sectional view drawings  
20.68extruding  
20.69Solid

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## 21.1 INTRODUCTION

A building is a structure used for the purpose of providing sheltered accommodation. It may be a residential building comprising of essential rooms like drawing room, bed room, dining hall, kitchen, toilet, etc., or a non-residential building like a shop, office, school, theatre, hostel, hospital, etc., whose components may vary with the nature of the building.

Any building essentially comprises of three parts: Foundation, super-structure and roof. Foundation is the bottom-most part of a structure taken into ground to get good anchorage and to form a stable base for the structural members to rest on. The structure above the ground level is known as super-structure. Roof is the upper-most part of a structure, shielding it from the weathering elements like sun, snow and rain.

The main aim of the building drawing is to give sufficient information by the design engineer to the construction engineer and the following views are generally presented:

- i. Top view (Plan)
- ii. Front view (Elevation)
- iii. Section at a particular plane or several planes

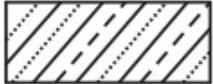
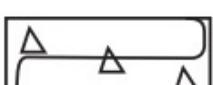
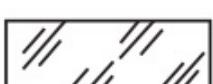
## **21.2 CONVENTIONAL SYMBOLS**

As per IS: 9621-1967, the code of practice for “Building Drawing - Preparation of various views of a building”, the following are the symbols of different building materials, as shown in [Table 21.1](#).

## **21.3 TERMINOLGY**

Terminology of both building materials and components is discussed below:

**Table 21.1 Conventional Symbols**

<i>Sl.No.</i>	<i>Name of the Material</i>	<i>Symbol</i>
1.	Brick	
2.	Stone	
3.	Sand	
4.	Earth	
5.	Plain cement concrete	
6.	Reinforced cement concrete	
7.	Glass	
8.	Wood across grain	
9.	Floor finish	
10.	Plywood	
11.	Ceramic tiles	
12.	Metal sections	

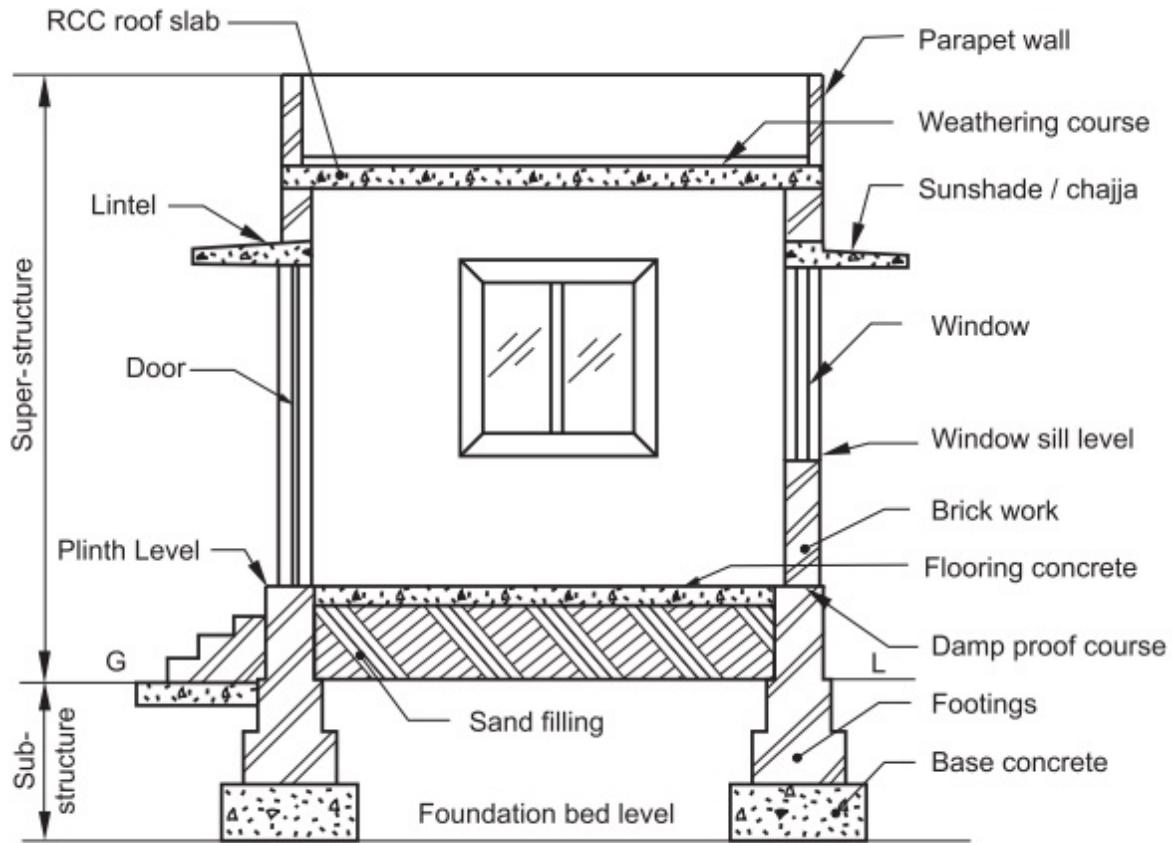
## **21.3.1 Terminology of Building Materials**

1. *Masonry* The term masonry refers to the construction using either bricks or stones with cement, lime or mud mortar; the cement mortar being the mostly used one now-a-days.
2. *Cement mortar* (CM) It is a substance produced from prescribed proportions of cement, sand and water which gradually sets hard after mixing. CM (1:4) means cement mortar prepared by using 1 part of cement and 4 parts of sand by weight and correct amount of water. Mortar is used as a binding material for stone or brick masonry and as a covering material to walls in the form of plaster to provide smooth and hard decorative surface.
3. *Plain cement concrete* It is a mixture of cement, sand, coarse aggregate and water, without any reinforcement, eg., Mix 1:2:4, consists of 1 part of cement, 2 parts of sand and 4 parts of aggregate with suitable quantity of water
4. *Reinforced cement concrete* It is the concrete, reinforced by the mild steel bars.

## **21.3.2 Terminology of Building Components**

Every building has two basic components, viz., Foundation (sub-structure) and Superstructure. The foundation is the

structure below the ground surface and super-structure is that above the ground level. The various building elements are as shown in Fig.21.1.



**Fig.21.1**

### 21.3.2.1 *Foundation*

Foundation is the portion of the building, which goes below the ground level. It transmits the load coming from the super-structure on to the sub-soil below it. Foundation may consist of concrete, stone and brick footings, above the base concrete.

*Footings* Footings are stepped courses in foundation. These are constructed in brick masonry or stone masonry

or concrete masonry under the walls or columns for distributing the load of the super-structure on to a larger area of the sub-soil.

*Base concrete* Base concrete is the very first course of the foundation of footings immediately above the levelling course, i.e., it is the bottom-most structural bed transferring the load of the structure to the soil below it. It provides stability and strength to the foundation.

Basement floor is the lower storey of a building below the ground level.

*Plinth* It is the portion of the structure above the ground and upto floor level. The level of the floor is usually known as the plinth level. The built-up covered area at the floor level is known as plinth area. The plinth height may be 300 to 600mm, but 450mm is more common.

*Flooring* The purpose of a floor is to provide a level surface for the occupants of a building, furniture, equipment, etc. Floor area is the utility area of a building at floor level, that can be occupied by men and material.

*Damp proof course* Dampness is the presence of hygroscopic or gravitational moisture. The dampness gives rise to un-hygienic conditions, apart from reduction in strength of structural components of the building. Damp proof course is a continuos layer of impervious materials as bitumen, slate or rich concrete provided at the plinth level beneath the walls to prevent the entry of moisture into the building.

### 21.3.2.2 *Super-structure*

It is the portion of the building above the ground level. Super-structure has the following components:

1. *Masonry walls and columns* - Masonry walls support vertical loads acting by gravitation, lateral forces and enclosed space in the building. Minimum thickness of a load bearing wall is 200mm and thickness of parapet wall and partition walls may be generally of 100mm.
2. *Doors, windows and ventilators* - A door is a movable barrier provided in the opening of a wall to provide access to various spaces of a building. A window is an opening in the wall for the purpose of providing daylight and ventilation. Ventilator is an opening made at roof level for the removal of exhaust air.
3. *Lintel* - This is a structural member to support the super-imposed load carried by the wall above the opening.
4. *Sunshades or chajjas* - It is a projected portion of building from the lintel of wall provided above the window, door or ventilators for the protection against the sun and rain.
5. *Roof* - Roof is the upper-most part of the building, which is constructed in the form of a frame work to protect the building from the natural elements such as rain, snow, wind, etc.
6. *Weathering course* - It is the layer of a brick, jelly, lime concrete with some water proofing compound or flat tiles in lime or cement mortar or two layers of bituminous felt provided at the top of the RCC roof to protect the slab from weathering, rain, snow, heat, etc., and render the roof water tight.
7. *Parapet wall* - It is a low height wall constructed along the edge of the roof. It prevents any body from falling, from the top of the roof and offers psychological protection even to those afraid of elevated places.

These provide an architectural means of improving the appearance.

## **21.4 PRINCIPLES OF BUILDING DRAWING**

Planning is the proper arrangement and allotment of the space contained by rooms and apartments. While planning, one has to remember that though each requirement is separate, but all considerations are inter-related and integrated. One more point to be kept in mind throughout the whole process of planning is the three dimensional aspect of the structure, ultimately the total effect depends upon many components.

The impression is imprinted on a person who enters the building while walking across the different rooms, passages, and the stairs, about the sizes of the rooms, light, ventilation and spacious arrangement.

The main aim of developing building drawing is to give sufficient information to the construction engineer. Hence, the following guidelines are kept in mind while planning residential or office buildings.

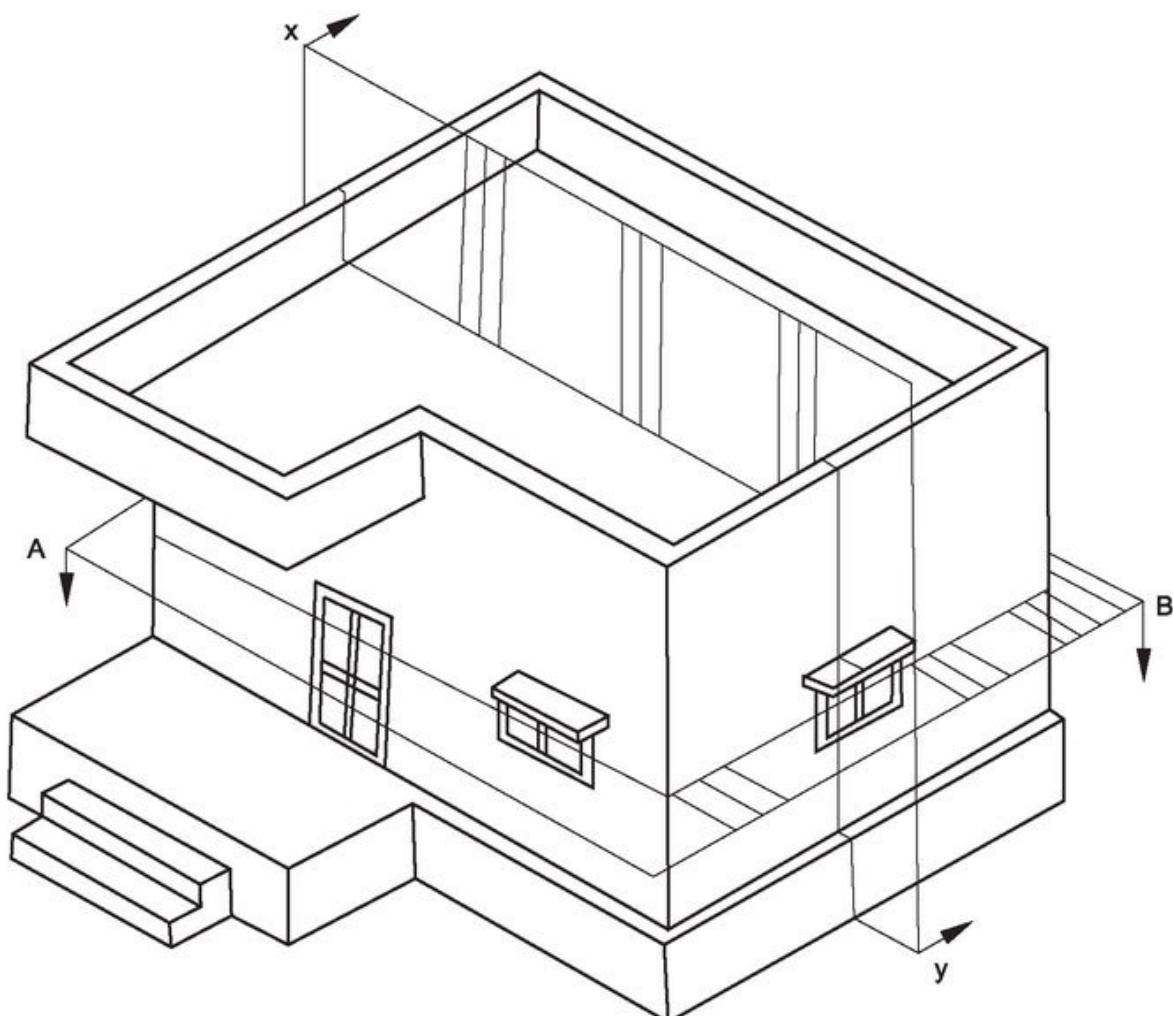
### **21.4.1 Planning**

1. Collect the information about the proposed building, its purpose and general scope of its components.
2. Place the plan elements with proper correlation, giving consideration to the orientation , convenience, comfort and attention to the other planning principles.

3. Adopt proper proportions and shape of units of building, from practical and aesthetic point of view.

## 21.4.2 Top View (Plan)

The building is imagined to be cut by a horizontal plane, slightly above the windowsill. Imagine the upper portion of the building above the horizontal plane that is removed. The building is seen from the top and its projection on a horizontal plane is known as top view.



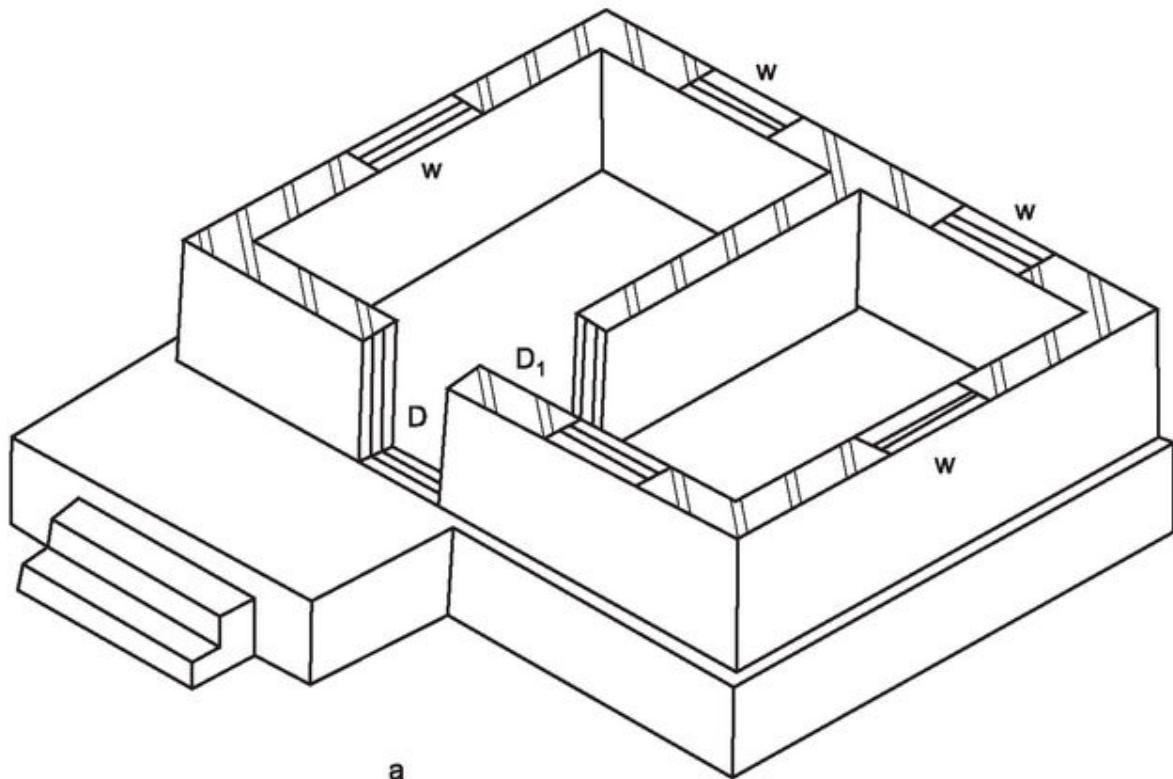
**Fig.21.2**

Consider a simple building shown in Fig.21.2. For obtaining the top view, the building is imagined to be cut by a horizontal plane, AB above window-sill. The remaining portion of the building is shown in Fig.21.3a, in isometric view. The conventional representation of the top view, in orthographic projection is shown in Fig.21.3b.

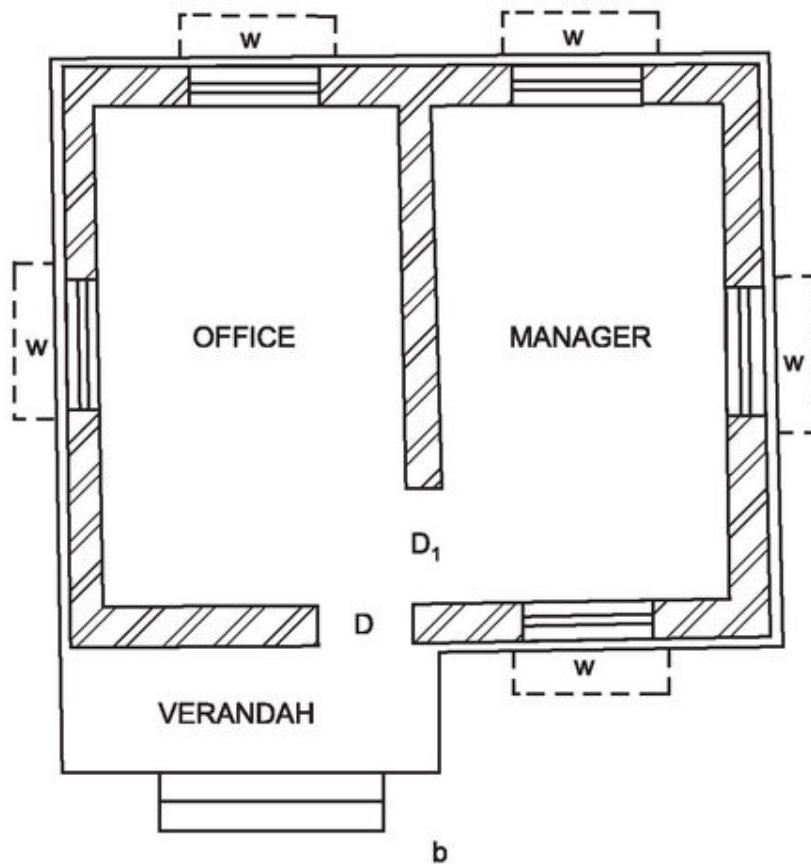
### 21.4.3 Section

The building is cut by a vertical plane xy, and the front portion is imagined to be removed and the remaining portion is shown in Fig.21.4a, in isometric view, as seen, in the direction of the arrows. Figure 21.4b shows the conventional representation of sectional view (orthographic projection). This view shows the constructional details from foundation bed level to roof. They provide the information about the thickness of walls, foundation for the walls, slab thickness, floor details, vertical heights above the floor, from window-sill, height of doors and windows, sunshade projections and thickness. In short, the section gives all information for an estimate and construction which are invisible in top view and front view.

The section plane should pass through doors, windows to provide the interior details.

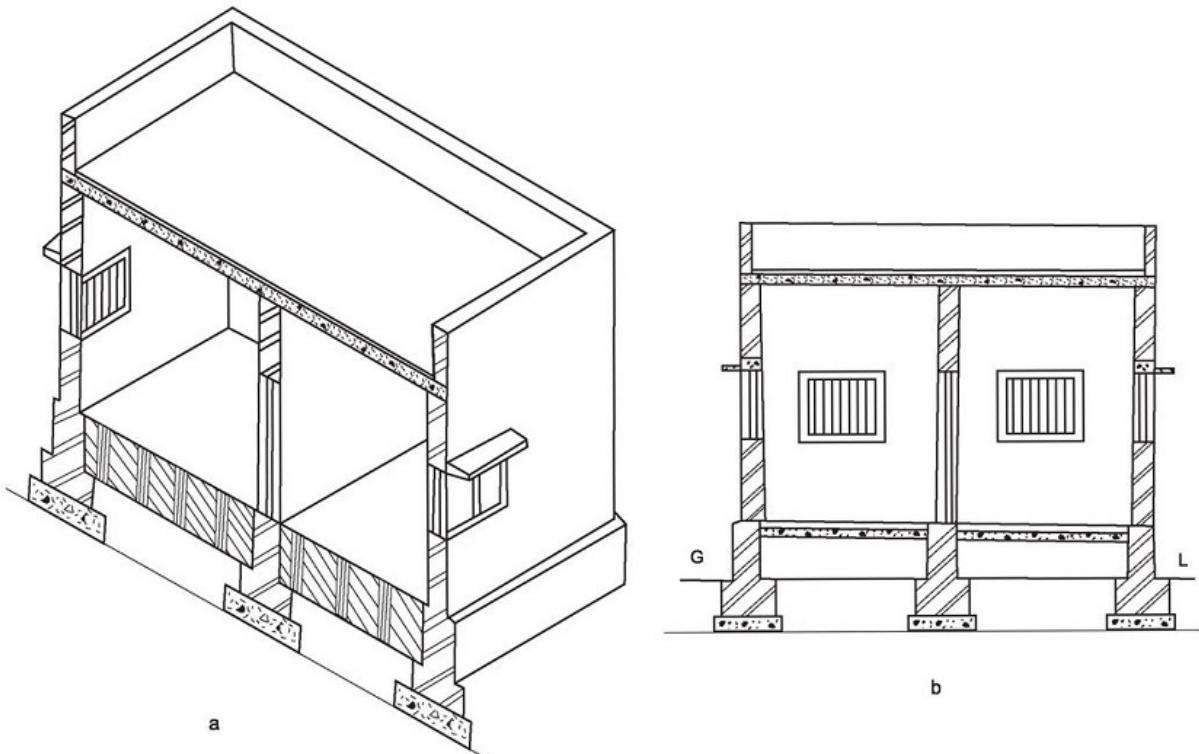


a



b

**Fig.21.3**



**Fig.21.4**

#### **21.4.4 Front View**

This is the view obtained by an observer, by standing in front of the building. This provides elevation of the building and overall view.

#### **21.5 METHODS OF CONSTRUCTION**

Buildings may be constructed for both office and residential purposes with either load bearing walls or framed

construction, based upon the number of storeys of the structures.

### **21.5.1 Load Bearing Walls**

A load bearing wall is one, which rests on the foundation, taken deep into the subsoil. It takes the super-imposed load, i.e., the load transmitted from slabs and beams. It transmits the load of the super-structure on to the sub-soil on which it rests. The entire wall should be taken deep into the ground where the enlarged footings provide enough stability for it. Also, the stress transmitted is considerably reduced because of the increase in width of the footings. A partition wall is an internal screen wall which rest above the floor level to create a room or enclosure. It may not be anchored deep into the ground. It may not also take any load of the super-structure. Most of the residential buildings that are small in size, and are one or two storeys, are generally constructed with load bearing walls.

### **21.5.2 Framed Construction**

It comprises of slabs resting on beams, which are supported by a network of columns. Only these columns should be taken below the ground level and provided with foundations and footings. All the walls either internal or external, are partition walls and none of them bear any load. They rest on the plinth beam and should not go below the floor level nor provide with any footing or foundation. Bigger multi-storeyed buildings or multiple residential apartments, public buildings and commercial complexes are generally constructed as framed structures.

### **21.5.3 Components of Residential Buildings**

A residential building is proposed to have the following components:

1. Drawing room
2. Bed room with attached toilet
3. Kitchen along with store
4. Dining room, guest room with attached toilet
5. Office room or study room
6. Verandah

### **21.5.4 Components of Office Buildings**

An office building is supposed to have the following components:

1. Office room
2. Waiting hall/ reception
3. Verandah
4. Separate water closet and bath room

Table 21.2 gives the standard dimensions for various building units such as drawing room, bed room, etc.

**Table 21.2 Standard dimensions for various building units (All dimensions are in millimetres)**

S.No.	Building unit	Standard dimensions
1.	Drawing room or living room	$4200 \times 4800$ to $5800 \times 7200$
2.	Bed room	$3000 \times 3600$ to $4200 \times 4800$
3.	Dining room	$3600 \times 4200$ to $4200 \times 4800$
4.	Kitchen	$2500 \times 3900$ to $3000 \times 3000$
5.	Bath, W.C combined	$1800 \times 1800$ to $1800 \times 2500$
6.	Bath (separate)	$1200 \times 1800$
7.	W.C (separate)	$1200 \times 1200$
8.	Plinth height	300 to 600
9.	Width of cement concrete bed of foundation	Width of brick footing + 200
10.	Ceiling height of 1. main rooms 2. W.C & bath	3000 to 3600 2000 to 2500
11.	Size of window	$750 \times 1000$ to $1200 \times 1200$

## 21.6 EXAMPLES

**Problem 1** The line diagram of a single room office building is shown in Fig.21.5a. Draw the front, top and sectional views to a suitable scale. Assume suitable data wherever necessary.

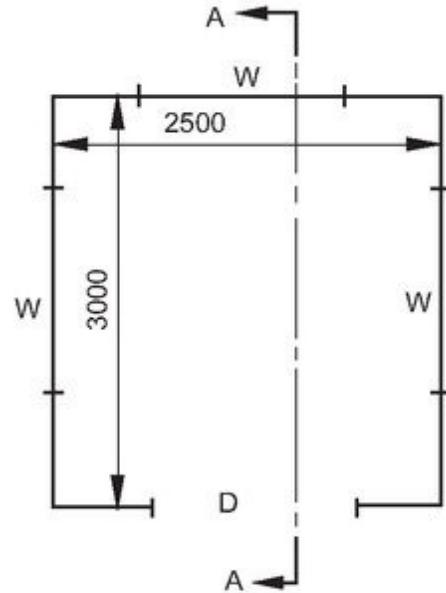
Specifications: Room measures  $2500 \times 3500$

Foundation: Depth below GL= 900; width of CC(1:4:8) is 900; thickness is 300. There are two footings of brick masonry in CM(1:5) of height 300 each; width of first footing 500 and that of second footing is 400.

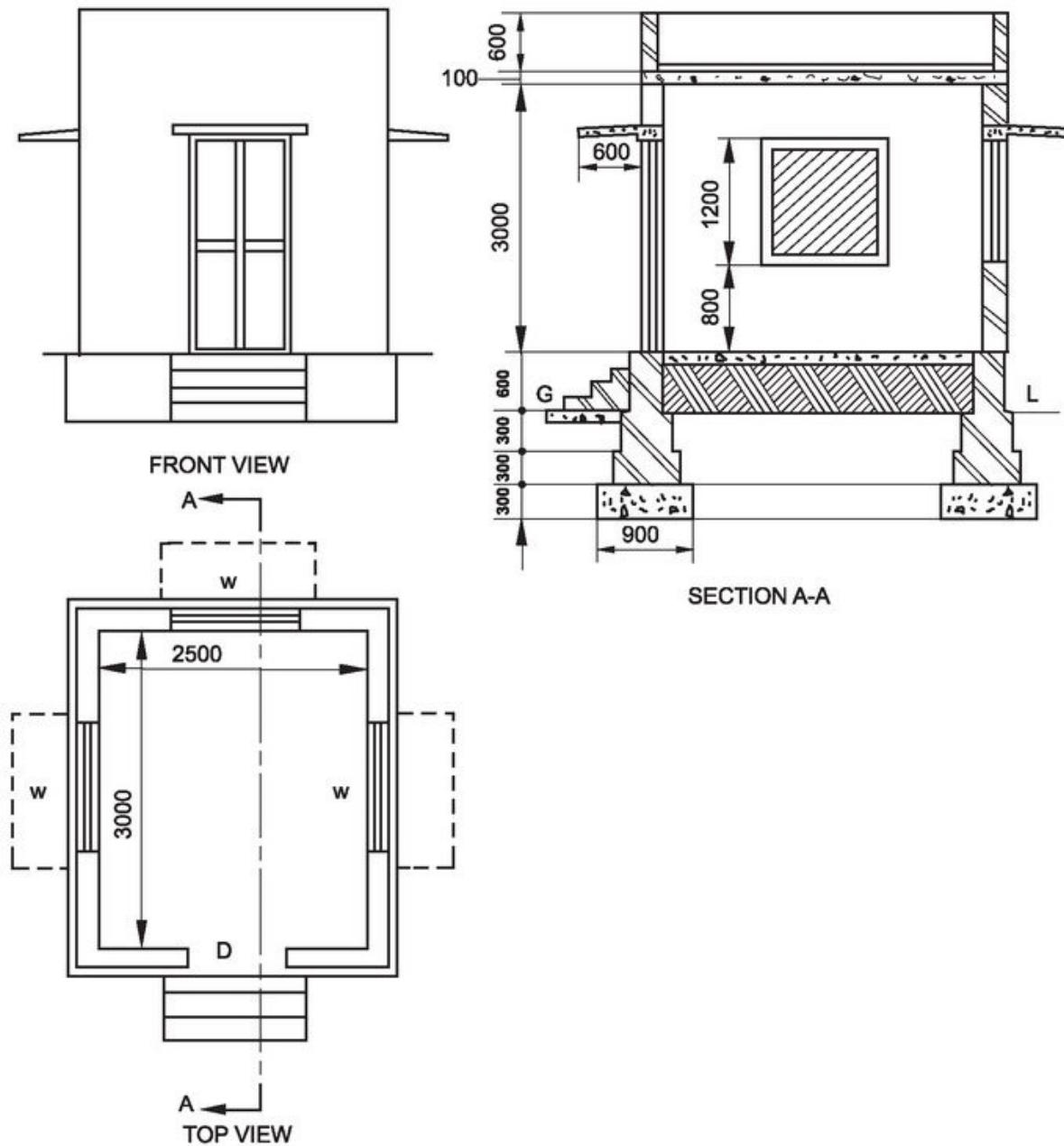
Super-structure: Brickwork CM(1:6) is 200 thick; height of wall is 3000; Roof-RCC slab of 100 thick of mix 1:2:4 (All

dimensions are in mm).

Figure 21.5b shows the necessary views of the single room office building as per specifications given.

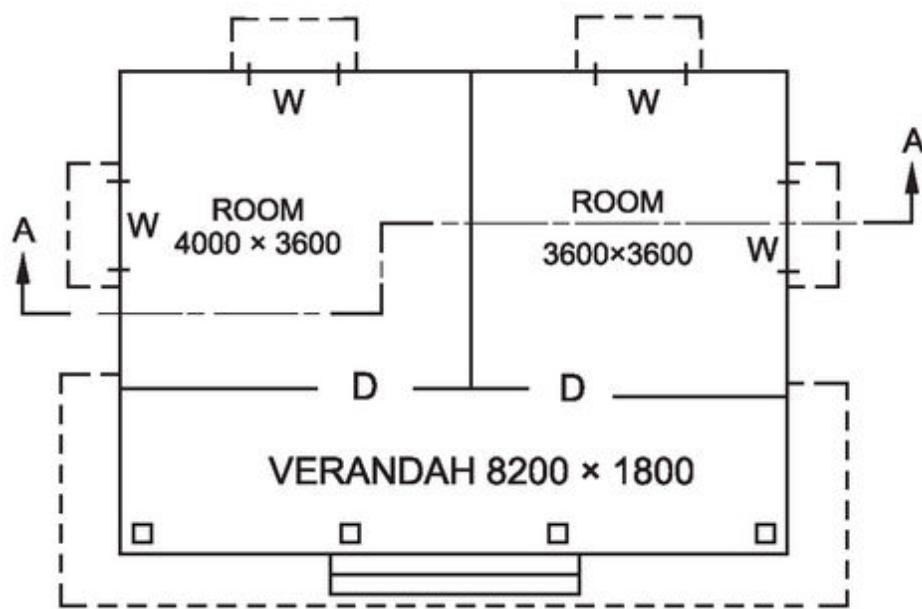


**Fig.21.5a**



**Fig.21.5b**

**Problem 2** Draw the front and top views of a panchayat office, the line diagram of which is shown in Fig.21.6a. Also, draw a section on AA. Make suitable assumptions wherever needed.



**Fig.21.6 a**

Figure 21.6b shows the front, top and sectional views of a panchayat office, to a suitable scale. The following assumptions are made:

Depth of the foundation below GL = 1200

Width of the cement concrete bed (1:4:8) = 900

Thickness of concrete footing = 300

First brick footing = 500 wide  $\times$  400 high

Second footing = 400 wide  $\times$  500 high

Other details are shown in sectional view.

**Problem 3** The line diagram ([Fig.21.7a](#)) represents a three room office building. Draw to a suitable scale the front view, top view and sectional view along AA. Assume suitable data.

[Figure.21.7b](#) represents the required views of the three room office building. The following standard dimensions for doors and windows are followed:

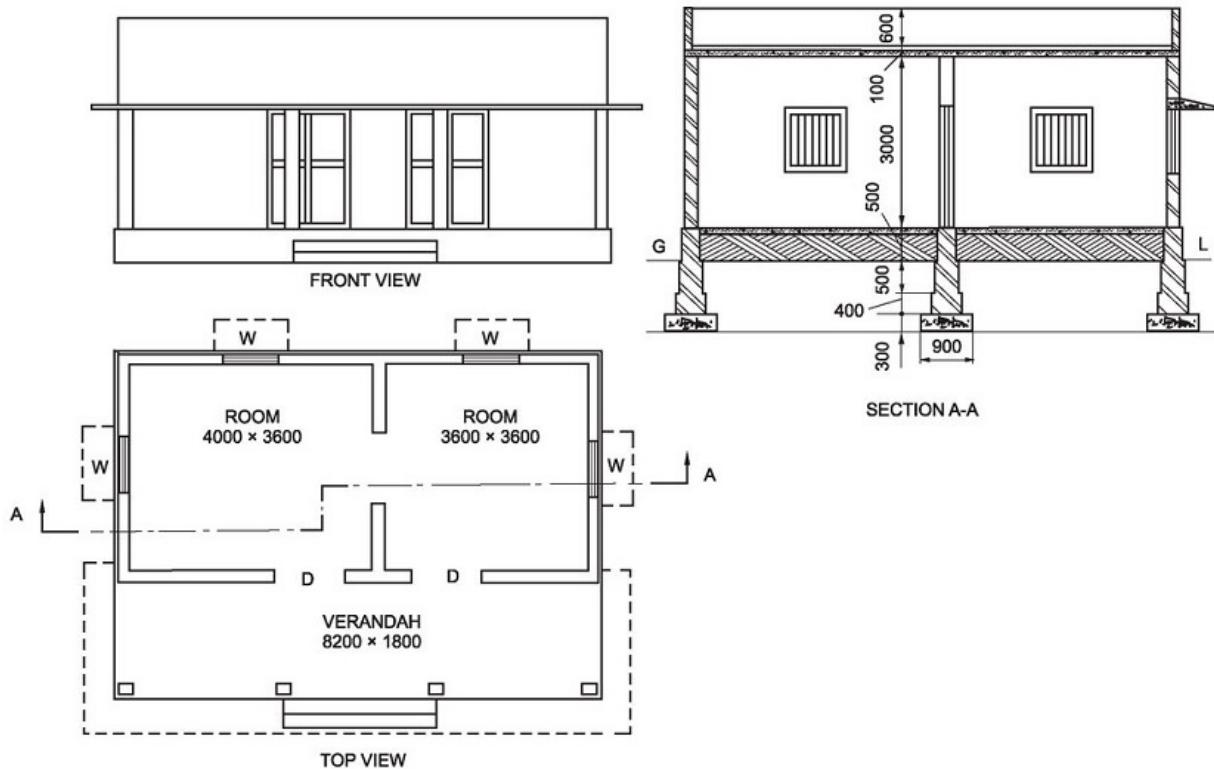
D - Panelled door -  $1000 \times 1200$

$D_1$  - Door -  $900 \times 2000$

W - Window -  $900 \times 1200$

$W_1$  - Window panelled -  $2000 \times 1000$

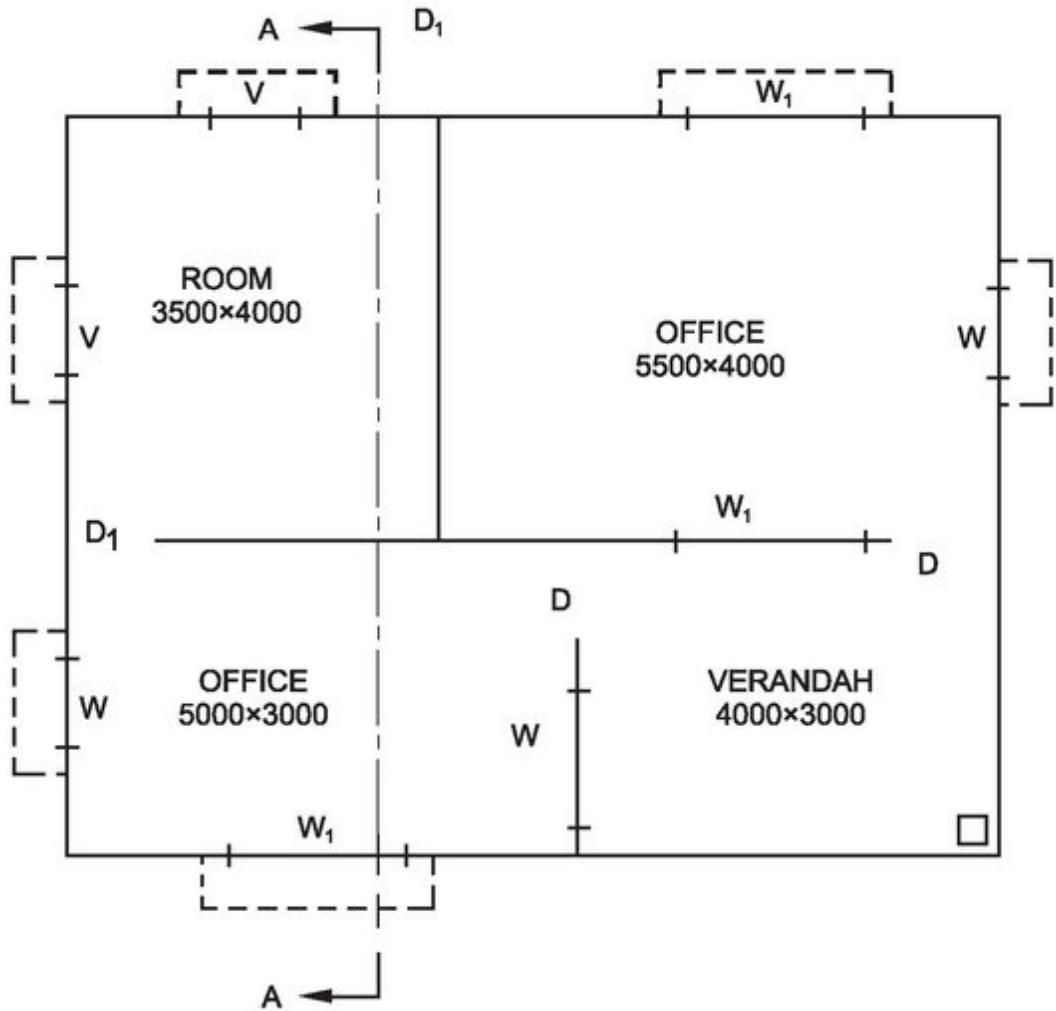
V - Ventilator -  $800 \times 300$



**Fig.21.6b**

**Problem 4** The line diagram shown in Fig.21.8a represents a two room residential building. Draw to a suitable scale, front view, top view and sectional view on AA. Make suitable assumptions wherever needed.

Figure 21.8b shows the required views of a two room apartment assuming suitable data. The following standard dimensions are taken for doors and windows:



**Fig.21.7a**

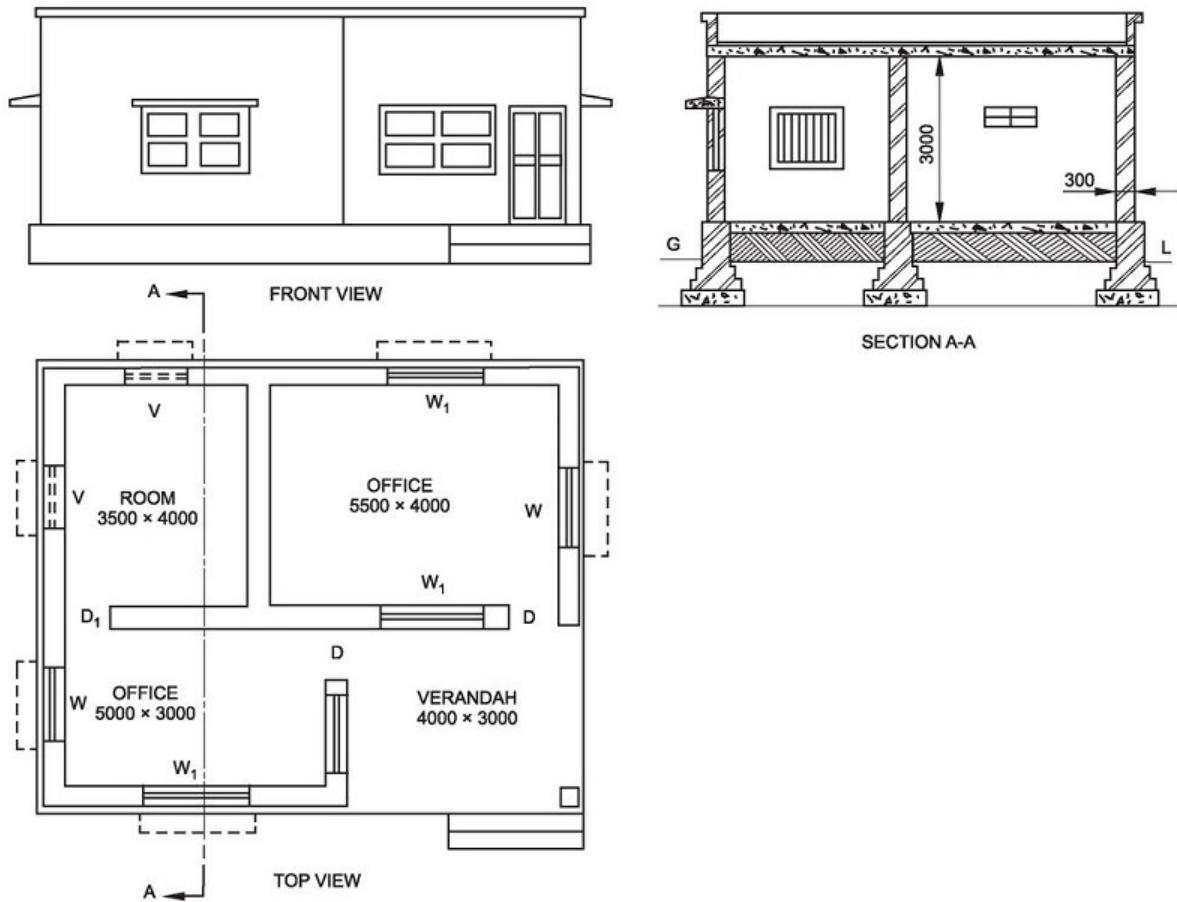
$D_1$  - Door -  $900 \times 1950$

$D$  - Door -  $800 \times 1950$

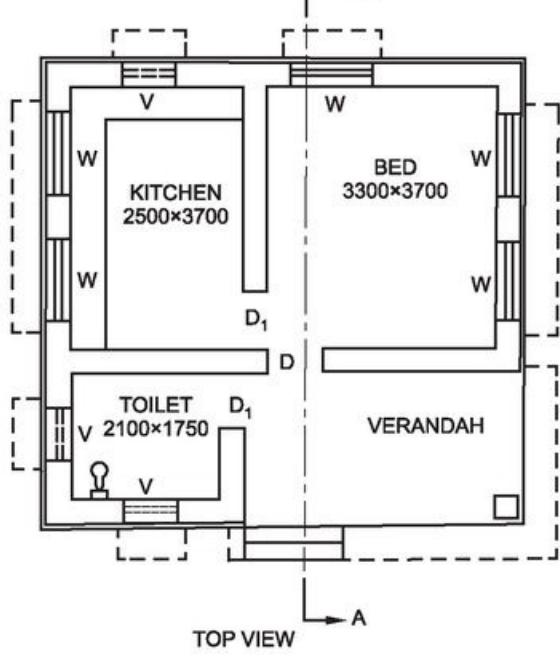
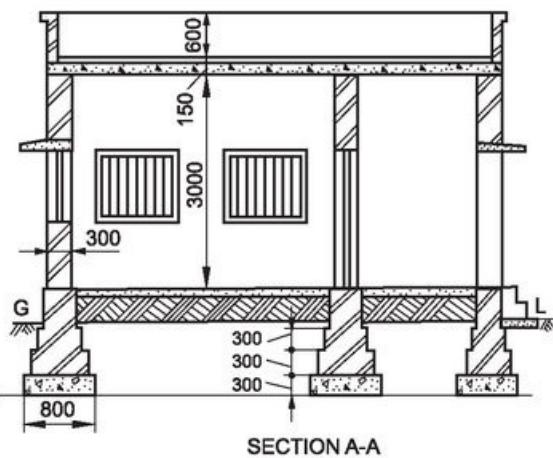
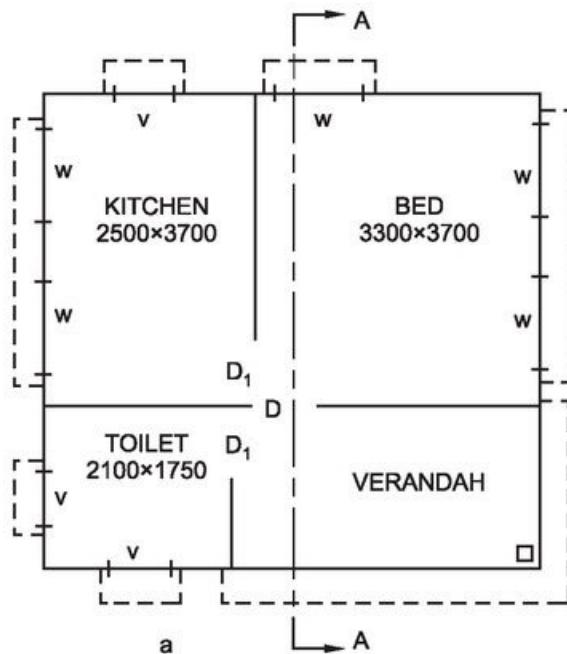
$W$  - Window panelled -  $1200 \times 1000$

$V$  - Ventilator -  $800 \times 300$

**Problem 5** The line diagram of an LIG house is shown in Fig.21.9a. Draw to a suitable scale, both the front view and sectional top view taking the section above window-sill level. Assume suitable data.



**Fig.21.7b**



b

## **Fig.21.8**

Figure 21.9b represents the front view and sectional top view of an LIG house, taking section above the window-sill level. The following assumptions and sizes for doors and windows are made:

Wall thickness - 200

Height of parapet - 600

Ceiling height above floor level - 300'

W - Window - 1600 × 1200

W 1 - Window - 1200 × 1200

W 2 - Window - 1000 × 1200

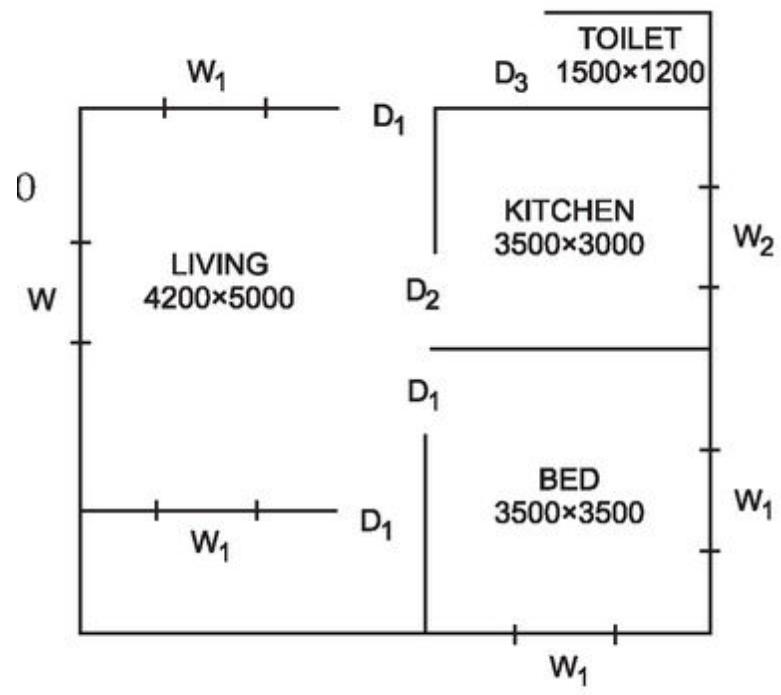
D 1 - Door - 1000 × 1950

D2 - Door - 900 × 1950

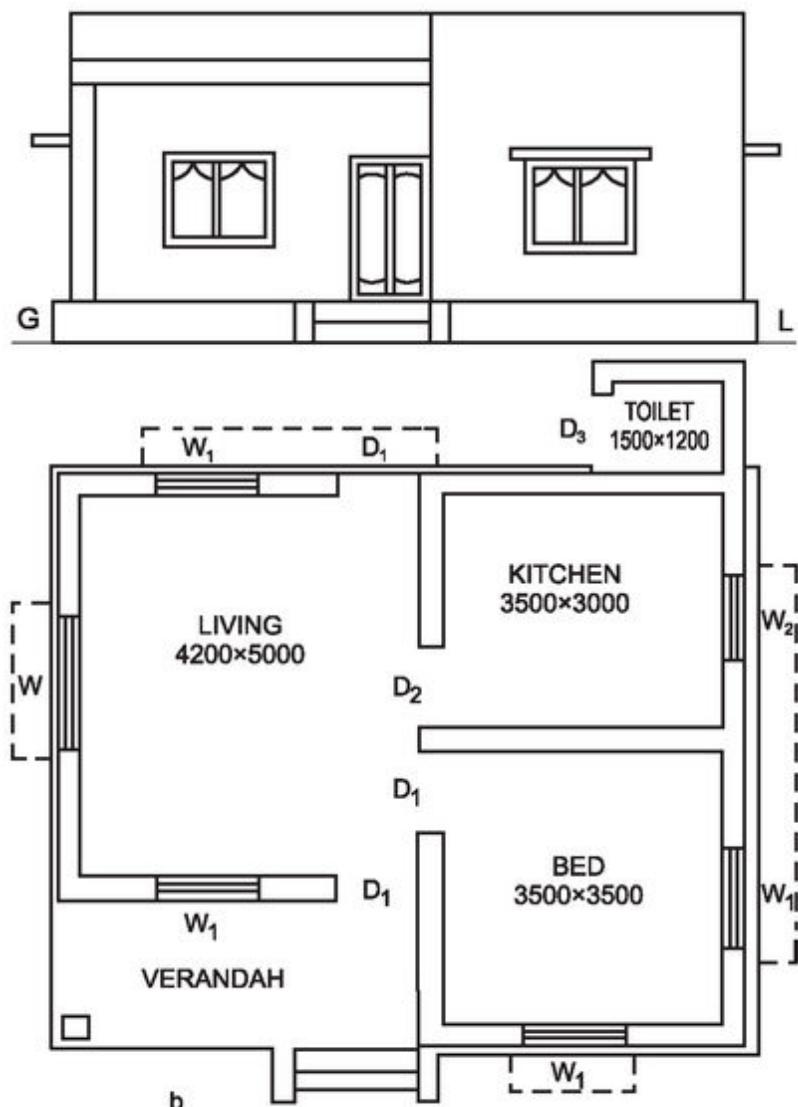
D3 - Door - 800 × 1950

V - Ventilator - 600 × 450

Width of sunshade - 600



a

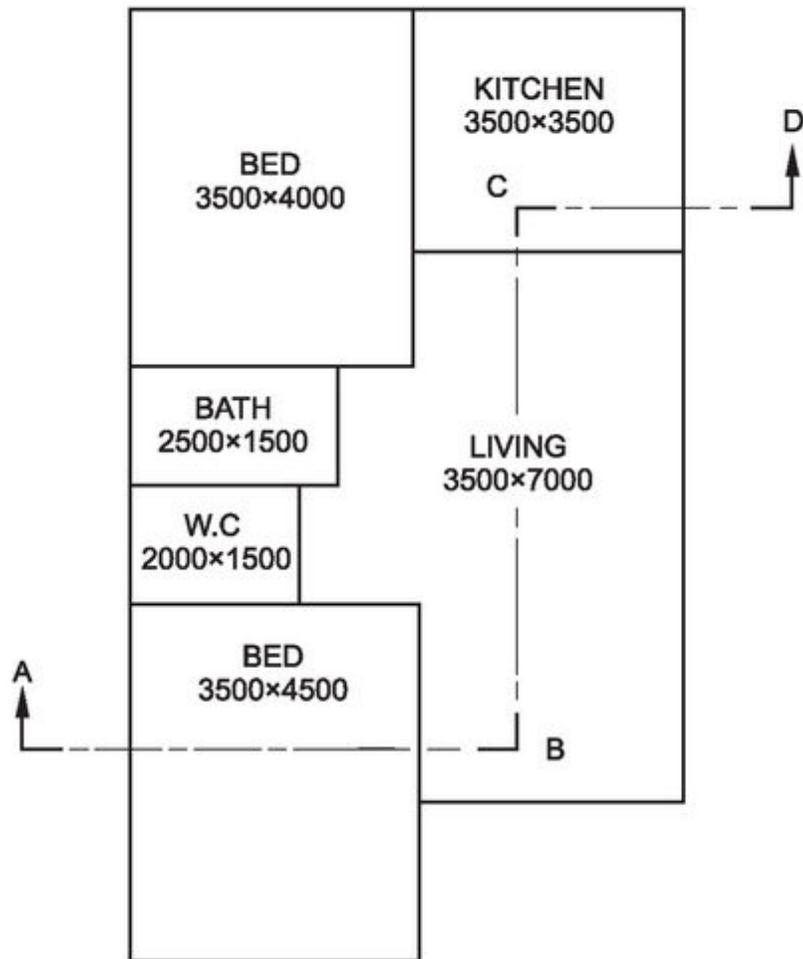


**Fig.21.9**

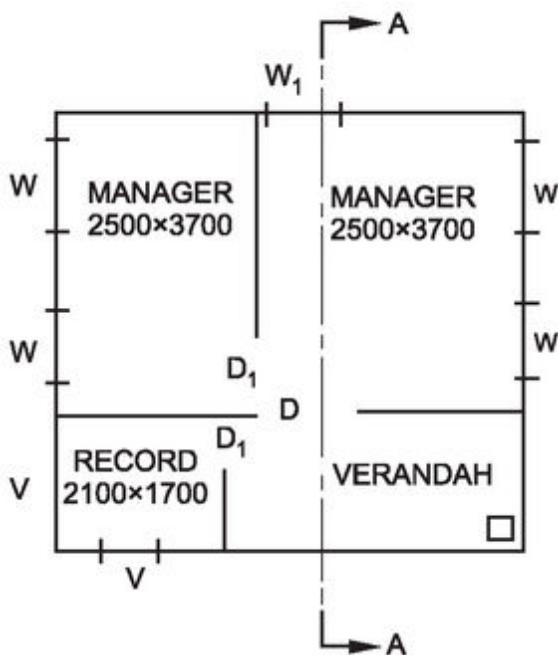
## EXERCISES

- 21.1 The line diagram of a residential building is shown in [Fig. 21.10](#). All the dimensions are clear internal dimensions. Draw (i) fully dimensioned top view with proper positioning of doors, windows and ventilators, (ii) a sectional view along ABCD and (iii) front view.

21.2 The line plan of a three room office building is shown in Fig.21.11. Draw to a suitable scale, front view, top view and sectional view along AB.



**Fig.21.10**



$D_1 - 900 \times 1950$   
 $D - 800 \times 1950$   
 $W - 1200 \times 1000$   
 $V - 800 \times 300$

**Fig.21.11**

21.3 The line diagram of a village library building is shown in Fig.21.12. Draw the top view of the building, taking section at window-sill level. Also, draw the front view of the building. Partition walls are of 100 thick and main walls are of 200 thick. Assume suitable data wherever necessary.

21.4 The line diagram of a two room office building is shown in Fig.21.13. Draw the dimensioned top view and front view of the building. Adopt the following:

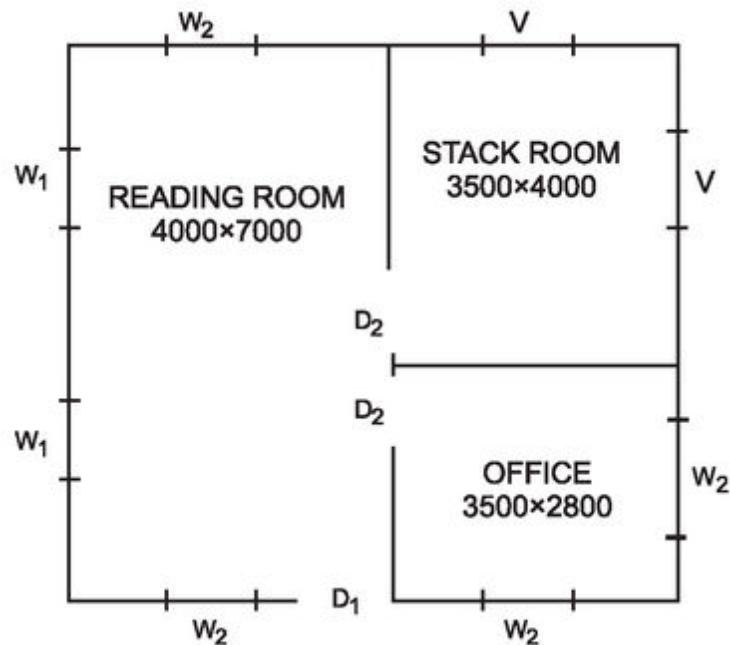
Sunshades to extend on either side of windows by 150.

W - Window-  $1500 \times 1200$ ,

$D_1$  - Door -  $1100 \times 2000$

$D_2$  - Door -  $900 \times 1950$

$V$  - Ventilator -  $600 \times 500$

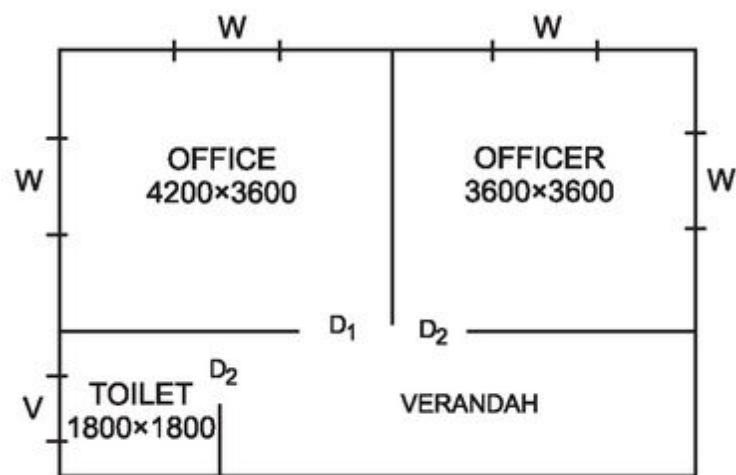


$D_1 - 1000 \times 1950 \quad w_1 - 1600 \times 1200$

$D_2 - 900 \times 1950 \quad w_2 - 1200 \times 1200$

$v - 600 \times 400$

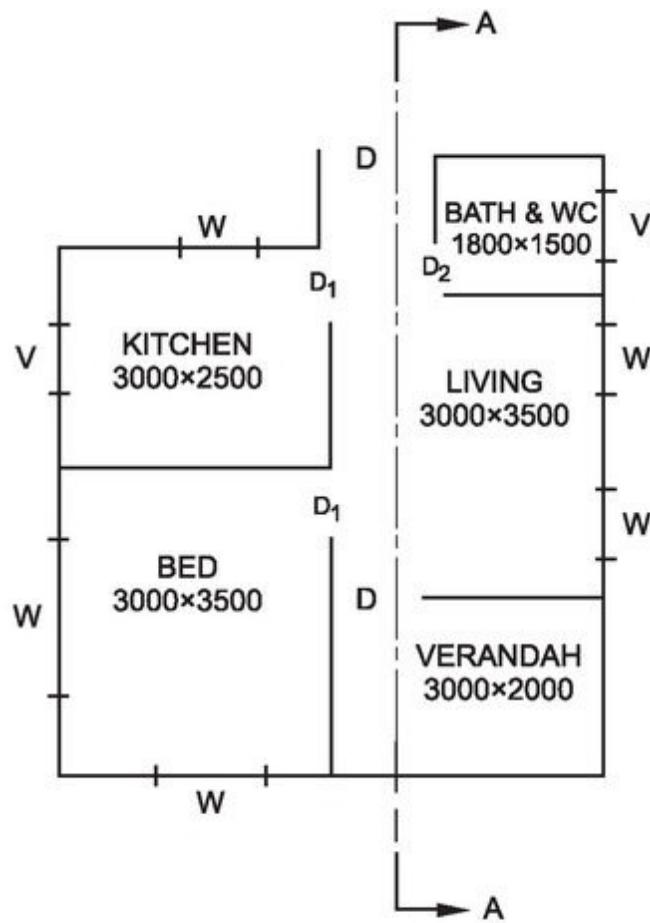
**Fig.21.12**



### **Fig.21.13**

21.5 The line diagram of a residential building is shown in [Fig.21.14](#). Draw to a suitable scale, front view, top view and section along AA. Make suitable assumptions. The following standard dimensions for doors and windows may be taken:

D - Panelled door	- $900 \times 1950$
D <sup>1</sup> - Door	- $800 \times 1950$
D <sub>2</sub> - Door	- $800 \times 1800$
W - Window	- $1200 \times 1000$
V - Ventilator	- $800 \times 600$



**Fig.21.14**

## OBJECTIVE QUESTIONS

21.1 Sub-soil influences \_\_\_\_\_. \_\_\_\_\_.

- a. foundation and drainage
- b. thickness of walls and numbers of storeys
- c. ground water quality
- d. type of building

21.2 Dampness causes \_\_\_\_\_. \_\_\_\_\_.

- a. unhygienic condition for inmates
- b. corrosion of metals
- c. determination of wood, stone, paints and electrical wires
- d. All the above

21.3 The store room should be attached with \_\_\_\_\_.

- a) bed, b) bath, c) kitchen, d) drawing room

21.4 The component of the building, which gives good anchorage to all structural members, is \_\_\_\_\_.

- (a)super-structure, (b) roof, (c) foundation, (d) all the above

21.5 The lower storey of the building below the ground level is \_\_\_\_\_.

- (a)basement floor, (b) foundation, (c) super-structure, (d) none

21.6 The portion of the structure above ground level upto the floor level is \_\_\_\_\_.

- (a)chajja, (b) lintel, (c) plinth, (d) all the above

21.7 The general mortar mix used for the plastering is \_\_\_\_\_.

- (a)1:6, (b) 1:4, (c) 1:3, (d) all the above

21.8 The general mortar mix used for brick work is \_\_\_\_\_.

- (a)1:4, (b) 1:6, (c) 1:5, (d) 1:3

21.9 The purpose of floor is to provide \_\_\_\_\_for the occupants of the building.

- a. uneven surface

- b. level surface
- c. hygienic surface
- d. none

21.10 A partition wall is a \_\_\_\_\_.

- a thin wall with no foundation
- b. internal wall to create room
- c. wall which divides a room into two
- d. wall that bears no load

21.11 Framed structure consists of only \_\_\_\_\_.

- a. partition walls
- b. walls with number of openings
- c. brick walls
- d. thin walls

21.12 Framed structures are preferred to load bearing wall structures when \_\_\_\_\_. \_\_\_\_\_.

- a. basement floor is to be provided
- b. residential houses are constructed
- c. more number of storeys are provided
- d. more number of openings are provided for walls

21.13 A load bearing wall is one which rests on \_\_\_\_\_  
(a) foundation, (b) plinth, (c) chajja, (d) none

21.14 The building is imagined to be cut by a horizontal plane slightly above the window-sill level. The upper portion, imagined to be removed and the view taken from above is \_\_\_\_\_

- (a) front view, (b) top view, (c) both, (d) none.

21.15 \_\_\_\_\_ gives all information for an estimate and construction details which are invisible in top and front views.

- (a) Sectional view, (b) Elevation, (c) both, (d) none

## FILL IN THE BLANKS

21.16 The essential parts of the building are \_\_\_\_\_.

21.17 The cement concrete without any reinforcement is termed as \_\_\_\_\_.

21.18 The cement concrete with reinforcement is known as \_\_\_\_\_.

21.19 In framed construction, the total load is borne by \_\_\_\_\_.

21.20 In load bearing wall construction, the total load is borne by \_\_\_\_\_.

21.21 The conventional symbol for RCC is \_\_\_\_\_.

21.22 The conventional symbol for glass is \_\_\_\_\_.

21.23 The conventional symbol for brick is \_\_\_\_\_.

21.24 The conventional symbol for stone is \_\_\_\_\_.

21.25 The conventional symbol for earth is \_\_\_\_\_.

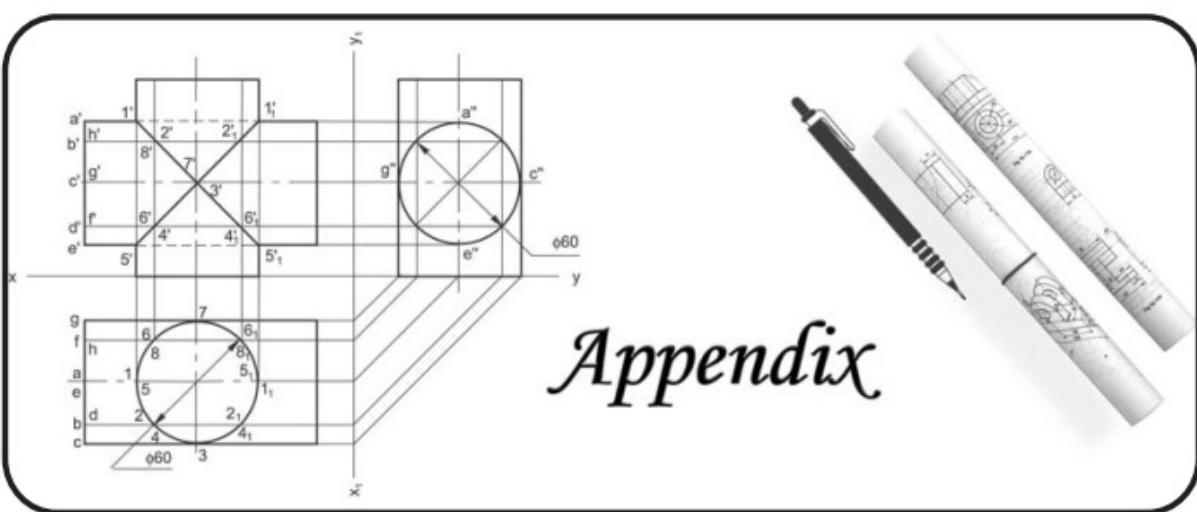
## ANSWERS

21.1.a

21.2.d

- 21.3.c
  - 21.4.c
  - 21.5.a
  - 21.6.c
  - 21.7.a
  - 21.8.a
  - 21.9.b
  - 21.10.c
  - 21.11.a
  - 21.12.c
  - 21.13.a
  - 21.14.b
  - 21.15.a
  - 21.16.substructure, super-structure and roof
  - 21.17.plain cement concrete
  - 21.18.reinforced cement concrete
  - 21.19.columns
  - 21.20.walls
- 21.21 to 21.25 Refer [Table 21.1](#)

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## 1. DRUGHTING TOOLS

- Engineering drawing is called the universal language of engineers. Explain.

Engineering drawing is the graphic language, from which any trained person can visualize the object. It conveys the same picture to every trained person. Drawings prepared in one country may be utilized in any other country, irrespective of the language spoken there. Hence, engineering drawing is called the universal language of engineers.

- What is the main purpose of the spring-bow compass?

Spring - bow compass is used for drawing very small circles.

- What are the uses of set-squares?

Set-squares, normally two in numbers, are the instruments used to draw lines inclined with the horizontal at  $30^\circ$ ,  $45^\circ$ , or  $60^\circ$ . Usually, these are used

in conjunction with T-square and these may also be used with the mini-draughtsman.

The two set-squares may be used in combination to draw a line parallel to any given line, not very far apart and also to draw a line perpendicular to any given line, either from a point on it or outside it.

4. Explain the method of using “French curves”?

French curves are used for drawing irregular curved lines, the radius of which is not constant. While using, first a series of points are marked along the desired path and then the most suitable curve is made to fit along it. A smooth curve is then drawn along the edge of the curve. This is repeated till a continuous smooth curve is obtained through all the points marked.

5. What are the different means of fastening the drawing sheet to the drawing board?

Thumb tacks, clips and adhesive tape.

## 2. PRINCIPLES OF GRAPHICS

6. State any two standard sizes of drawing sheets, according to BIS.

A0:  $1189 \times 841$ ; A1:  $841 \times 594$

7. Give the location of the title block in a drawing sheet.

At the bottom right hand corner.

8. List out the contents of the title block.

Title of the drawing, drawing number, scale, symbol denoting the method of projection, name of the firm

and initials of staff who have drawn, checked and approved.

9. What is the purpose of sectioning an object?

The purpose of sectioning an object is to reveal the interior details.

10. How hidden lines are represented?

Hidden lines are represented by thin (short) dashes.

11. What is a leader line?

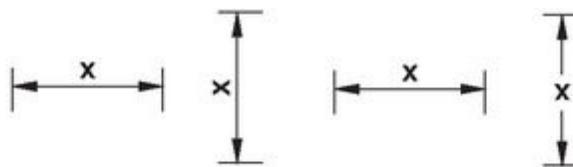
A leader line is a line referring to a feature (dimension, object, outline, etc.). It is drawn at an angle greater than  $30^\circ$ .

12. How are leader lines terminated?

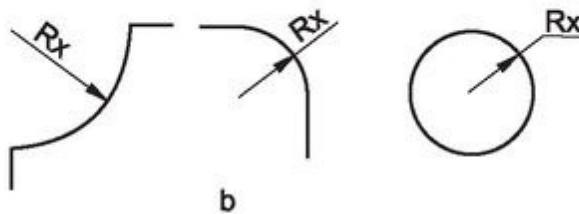
Leader lines should terminate (i) with a dot, if they end within the outline of an object, (ii) with an arrow head, if they end on the outline of an object, or (iii) without dot or arrow head, if they end on a dimension line.

13. Give the methods to represent dimensioning of

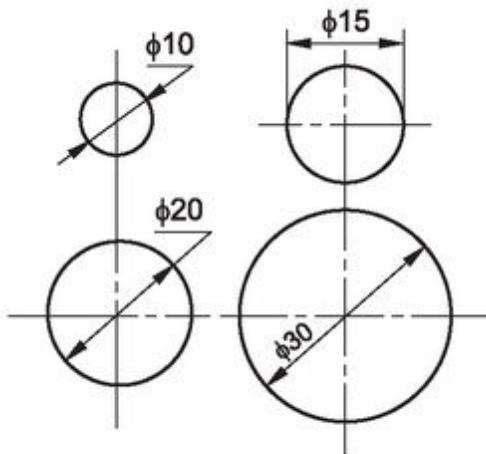
- (a) a straight line and (b) radius of a curve



a (i) Aligned system      a (ii) Uni-directional system



**Fig..13**



**Fig..14**

14. Show the different methods of indicating diameters.

### 3. GEOMETRICAL CONSTRUCTIONS

15. What is a regular polygon?

A polygon is a plane figure, bounded by straight edges. When all the edges are of equal length, the polygon is called a regular polygon.

16. What is meant by circum-circle of a triangle?

The circle passing through the vertices of a triangle is called the circum-circle of the triangle.

17. What is meant by in-circle of a triangle?

The circle drawn inside a triangle such that, all the sides of the triangle are tangential to the circle, is called the in-circle of the triangle.

18. Give the definition of tangency.

A line, meeting a curve at a point on it is called the tangent to that curve.

## 4. SCALES

19. Define the term “Scale” or “Scale Factor”.

The term, “Scale” or “Scale Factor” is defined as the ratio of the length of a line in the drawing to the actual length of the line on the object.

20. Name the different types of scales used in engineering draughting practice.

Plain scales, diagonal scales, comparative scales, isometric scales, vernier scales and scale of chords.

21. What is plain scale?

A line, suitably divided into equal parts (primary divisions) is called as plain scale. Generally the first part is sub-divided into smaller parts (secondary divisions). Thus, a plain scale is used to represent either two units or a unit and its fraction such as kilometres and hectometres.

22. What is the application of diagonal scale?

If a fractional portion of a secondary division is needed which is too small to be sub-divided, diagonal scales are used. Hence, diagonal scales are used to measure three consecutive units such as kilometres, hectometres and decametres.

23. Reducing scales are used in the preparation of building drawings.

(True / False)

True

24. Vernier scales are used to measure dimensions very accurately than plain scales.

(True / False)

True

25. What is least count?

Least count is the smallest distance that can be measured by vernier scale. It is given by the difference between one main scale division and one vernier scale division.

26. What are the applications of scale of chords?

A scale of chords may be used to measure an angle or to set the required angle.

## 5. CURVES USED IN ENGINEERING PRACTICE

27. Name the five conic sections.

Ellipse, parabola, hyperbola, circle and triangle.

28. Name the solids of revolution.

Cylinder, cone and sphere

29. Define the term, “eccentricity” as applied to conic sections.

Eccentricity of a conic section is the ratio of the distance from the focus to the distance of it from the directrix.

30. Define locus of a point on (a) ellipse and (b) parabola, with respect to fixed points.

Ellipse is a curve traced by a point moving such that, the sum of its distances from two fixed points known as foci, is constant and equal to the major axis.

Parabola is a curve traced by a point, moving such that, at any position, its distance from the fixed point (focus) is always equal to its distance from a fixed straight line (directrix).

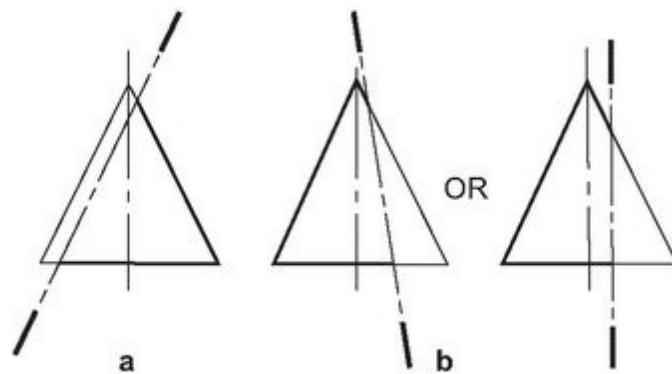
31. What are the asymptotes to a hyperbola?

Asymptotes are the lines, which pass through the centre of the major axis and tangential to the curve at infinity.

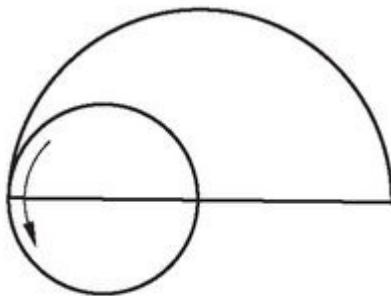
32. What is a rectangular hyperbola?

Rectangular hyperbola is a curve generated by a point which moves in such a way that the product of its distances from two fixed straight lines, the asymptotes, at right angle to each other is a constant.

33. A cone is cut by a cutting plane. Sketch the position of the cutting plane which will give in its true shape (a) parabola and (b) hyperbola.



**Fig..33**



**Fig..34**

34. When the diameter of the directing circle is twice the diameter of the rolling circle, show the path traced by the contacting point on the rolling circle, when the rolling circle rolls inside the directing circle.
35. What is an Archimedean spiral?

An Archimedean spiral is a curve traced by a point, moving with uniform velocity along a line, which is also rotating with uniform angular velocity.

36. What is a logarithmic spiral?

Logarithmic spiral is the curve traced by a point, which is moving along a rotating line such that, for equal angular displacements of the line, the ratio of the length of consecutive radius vectors is constant.

## 6. ORTHOGRAPHIC PROJECTIONS

37. What is projection?

A projection is defined as a representation of an object on a two dimensional plane.

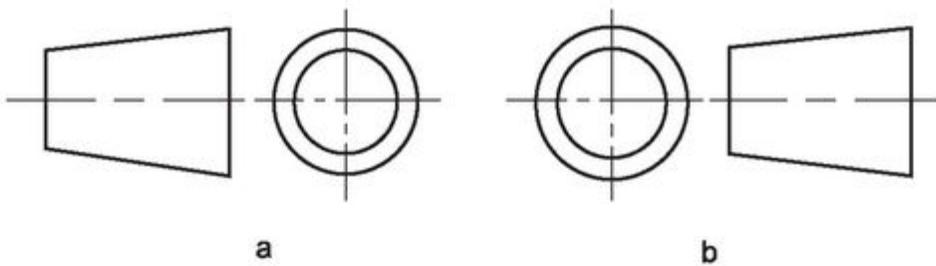
38. How the projection of an object is obtained?

A projection may be obtained by viewing the object from the point of sight and, tracing in correct sequence, the points of intersection between the rays of sight and the plane to which the object is projected.

39. What is meant by an orthographic projection?

A projection is called an orthographic projection, when the point of sight is imagined to be located at infinity so that the rays of sight are parallel to each other and intersect the plane of projection at right angle to it.

40. Sketch the symbols used to represent (a) first angle projection and (b) third angle projection.



**Fig.40**

41. H.P and V.P are always \_\_\_\_\_ to each other.

Perpendicular

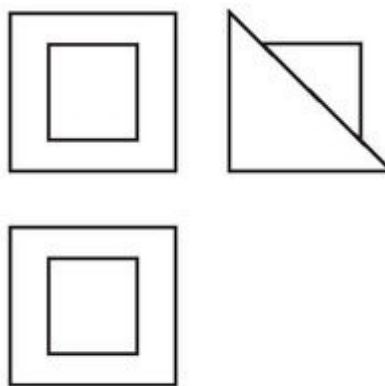
42. In first angle projection, to obtain right side view, the P.P is assumed to be on \_\_\_\_\_ side of the object.  
left

43. For what type of objects, one-view drawings are sufficient? How the shape and size of such features are indicated?

Objects with cylindrical, square or hexagonal features or, planes of any size with any number of features in it

may be represented by a single view. In such cases, the diameter of the cylinders, the side of the square, the side of the hexagon or the thickness of the plate may be expressed by a note or abbreviation.

44. Two concentric squares with corresponding edges parallel, are the views of a solid in both top and front views. How it looks like in end view?



**Fig.44**

## 7. PROJECTIONS OF POINTS

45. What is a projector line?

Projector line is the line joining the front and top views of any point. This is perpendicular to the reference line  $xy$ .

46. What is a projector?

A line drawn perpendicular to the reference line  $xy$  is called a projector.

47. Why second and fourth angles of projections are not followed in practice?

In second and fourth angles of projections; both the front and top views will lie on one side of the reference line, which may cause confusion in visualizing the object.

48. What is a reference line?

A reference line is the line of intersection between H.P and V.P.

49. State the location of the point, when its front view is on the reference line?

The point is on H.P.

50. State the location of the point, when its top view is on the reference line?

The point is on V.P.

51. The line joining the projections of a point intersects the reference line at  $90^\circ$ .

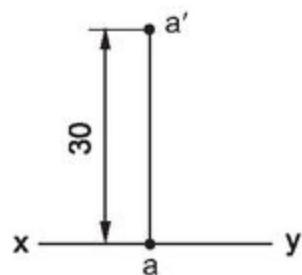
(True / False)

True

52. A point A is 30mm from H.P. Its shortest distance from the ground line XY is 50mm. Find its distance from V.P.

40mm

53. Show the projections of a point A which is common to I and II quadrants and 30mm from H.P.



### **Fig.53**

54. The point, whose projections coincide at 30mm above the reference line, lie on \_\_\_\_\_ quadrant.  
second
55. State the quadrant in which a point is situated, given that its top view is 40mm above xy and the front view is 10mm below the top view.  
Second quadrant
56. The top view of a point, a moves along a circular path. How does its front view, a' move with respect to the reference line?  
The front view, a' moves along a straight line and parallel to the reference line.

## **8. PROJECTIONS OF STRAIGHT LINES**

57. A straight line is generated as the \_\_\_\_\_ of a moving point.  
locus
58. When will be the front view of a straight line show the true length of the line?  
When the line is parallel to V.P; the front view reveals the true length of the line.
59. When will be the top view of a straight line show the true length of the line? When the line is parallel to H.P; the top view reveals the true length of the line.

60. When a straight line is parallel to both H.P and V.P; its side view is a \_\_\_\_\_.

point

61. When will be the apparent inclination and true inclination of a line, inclined to H.P will be equal?

When the line is parallel to V.P, the apparent inclination and true inclination of a line, inclined to H.P will be equal.

62. Name the two methods used to draw projections of a line, inclined to both H.P. and V.P.

Rotating line method and trapezoidal method.

63. The sum of inclinations of a straight line inclined to both H.P and V.P and whose projections are perpendicular to the reference line is \_\_\_\_\_.

90°

64. When the projections of a line lie along the same projector?

When the sum of inclinations of the line with H.P and V.P is equal to 90° (i.e.,  $\theta + \phi = 90^\circ$ ); the projections of a line lie along the same projector.

65. When the end projectors of a line coincide, give the relation between  $\theta$  and  $\phi$ .  $\theta = 90 - \phi$

66. There are two lines AB and CD. How to decide whether the lines intersect each other or not?

If the lines are intersecting, they appear to be intersecting in both the views and in addition, the intersecting points in both the views lie on a common projector.

67. Define the trace of a straight line.

The trace of a straight line is the point of intersection between the given line or its extension and the plane of projection.

68. A vertical line has \_\_\_\_\_ trace only.  
horizontal
69. A line AB has no V.T and another line CD has no H.T. State the positions of the lines AB and CD.  
The line AB is perpendicular to H.P and parallel to V.P.  
The line CD is perpendicular to V.P and parallel to H.P.
70. State whether a line inclined to V.P and parallel to H.P will have a vertical trace or not.  
The line will have a vertical trace.
71. When will be the H.T and V.T of a line lie on the same projector?  
When the sum of inclinations of the line with H.P and V.P is equal to  $90^\circ$  (i.e.,  $\theta + \phi = 90^\circ$ ).

## 9. PROJECTIONS OF PLANES

72. A lamina is seen in its true shape in end view. How it looks like in the front and top views?  
Straight lines, perpendicular to the reference line.
73. What is an oblique plane?  
The plane which is inclined to both the reference planes is called an oblique plane.
74. What are the traces of a plane?  
A plane extended if necessary, will meet the principal planes of projections along the lines, known as traces.

75. When are both the views (front and top) of a plane are straight lines?

When the surface of the plane is perpendicular to both H.P and V.P

76. When will be the traces (H.T and V.T) of a plane are perpendicular to the reference line?

When the surface of the plane is perpendicular to both H.P and V.P

77. What is an edge view of a plane?

An edge view of a plane is a straight line.

## 10. AUXILIARY PROJECTIONS

78. What is an auxiliary plane?

An auxiliary plane is a plane perpendicular to one of the principal planes of projection and inclined to the other.

79. When the auxiliary views preferred?

Auxiliary views are preferred when the object contains features on surfaces, which are inclined to the principal planes of projection.

80. Define A.V.P. what type of view is obtained on it?

A.V.P is a plane, perpendicular to H.P and inclined to V.P. Auxiliary front view is obtained on A.V.P.

81. Define A.I.P. What type of view is obtained on it?

A.I.P is a plane, inclined to H.P and perpendicular to V.P. Auxiliary top view is obtained on A.I.P.

82. What are the applications of the auxiliary projections with respect to (i) straight lines and (ii) planes?

Auxiliary projections of straight lines are used to obtain, (i) true lengths and true inclinations of lines, (ii) point or edge views of lines and (iii) conventional projections.

Auxiliary projections of planes are used to obtain, (i) edge views of planes, (ii) true shapes and true inclinations of planes and (iii) conventional projections.

## 11. PROJECTIONS OF SOLIDS

83. Define the term, "Polyhedron".

A polyhedron is a solid bounded by plane surfaces.

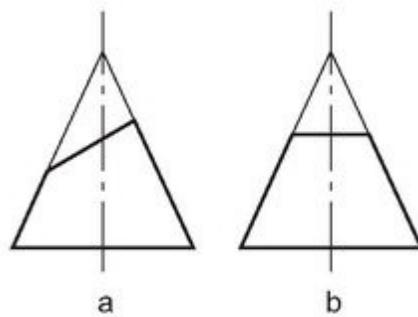
84. State the shape and number of faces of a "Dodecahedron".

Dodecahedron has twelve equal faces, each an equilateral triangle.

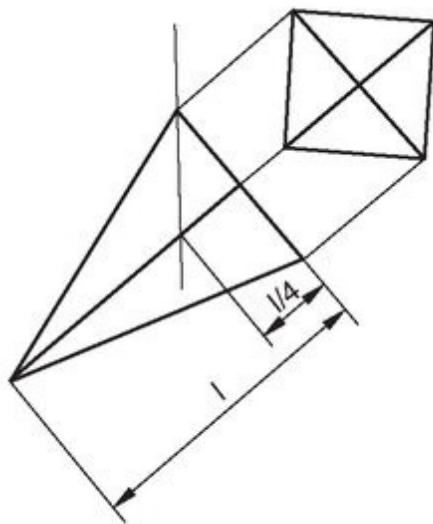
85. What is meant by frustum?

Frustum is a part of solid obtained, when it is cut by a section plane, parallel to the base.

86. Sketch (a) truncated cone and (b) frustum of a cone.



**Fig.86**



**Fig.87**

87. A pyramid is hung by a base corner, with its axis parallel to V.P. Sketch the location of its center of gravity, on the axis of the solid.
88. A cylinder has its axis parallel to profile plane. The axis is inclined to H.P at  $30^\circ$ . What is its inclination with V.P?

$60^\circ$

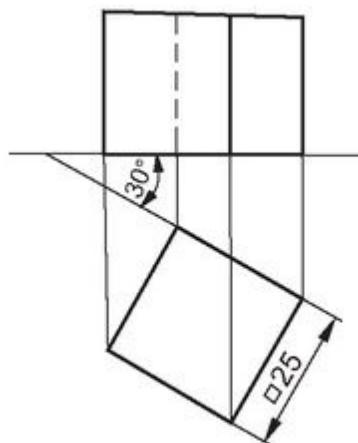
- For a particular position of a cube, the boundary of  
89. front view will be a true hexagon. State its position.

One of the solid diagonals is parallel to H.P and perpendicular to V.P.

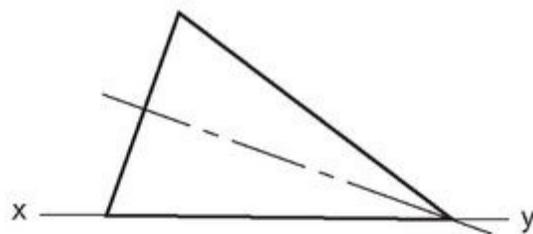
90. When a cube has its solid diagonal perpendicular to profile plane, what is the view of the solid in profile plane?

Isometric projection

91. Sketch the projections of a cube of edge 25mm, resting on one of its square faces on H.P and two parallel edges of the base make an angle of  $30^\circ$  with V.P.



**Fig.91**



**Fig.92**

92. When a cone lies on one generator on H.P. with its axis parallel to V.P, sketch the front view.

93. Differentiate between right regular and oblique solids.

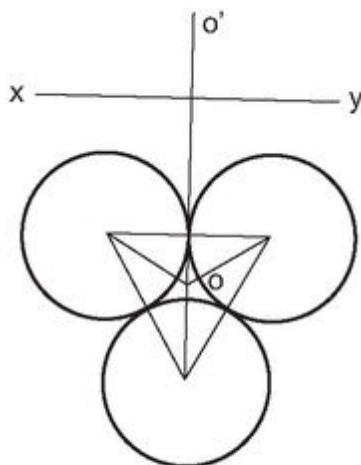
A right regular solid has its axis perpendicular to its base and all the faces are of the same size and shape. An oblique solid has its axis inclined to its base and the faces are not of the same size and shape.

94. Three spheres of equal radius lie on H.P, touching each other, with line joining centres of two spheres, parallel to H.P. Sketch the location of the centre of three spheres.

## 12. SECTIONS OF SOLIDS

95. How is the true shape of a section obtained?

The true shape of a section is obtained by projecting the section on a plane, parallel to the section plane.



**Fig.94**

96. When a cube is cut by a section plane, the true shape of the section produced can be a regular hexagon.

(True / False)

True

97. If a cylinder is cut by a section plane parallel to the axis; the true shape of the section produced is \_\_\_\_\_.

rectangle

98. If a hexagonal prism is cut by a section plane, parallel to the axis; the true shape of the section produced is \_\_\_\_\_.

rectangle

99. If a hexagonal prism is cut by a section plane, inclined to its axis; what will be the true shape of the section produced?

Six sided figure (irregular hexagon)

100. A solid is resting on its base on H.P. When the sectioned portion in the top view and the true shape of the section are identical?

When the section plane is parallel to H.P and perpendicular to V.P.

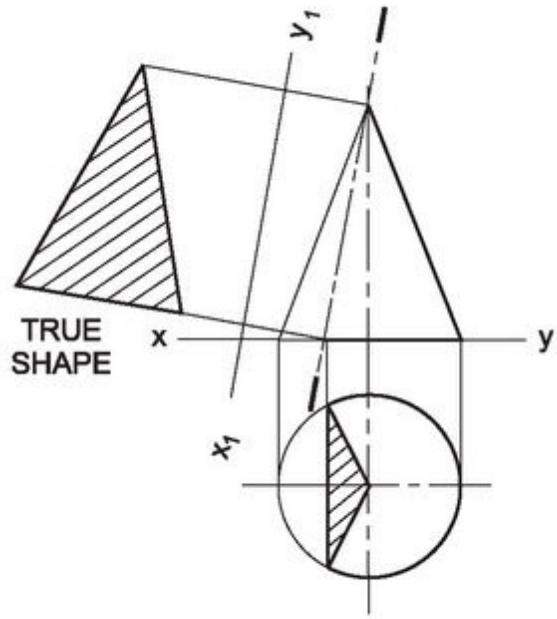
101. The true shape of the section formed by the intersection of a right circular cone, with a plane parallel to a generator of the cone is \_\_\_\_\_.

parabola

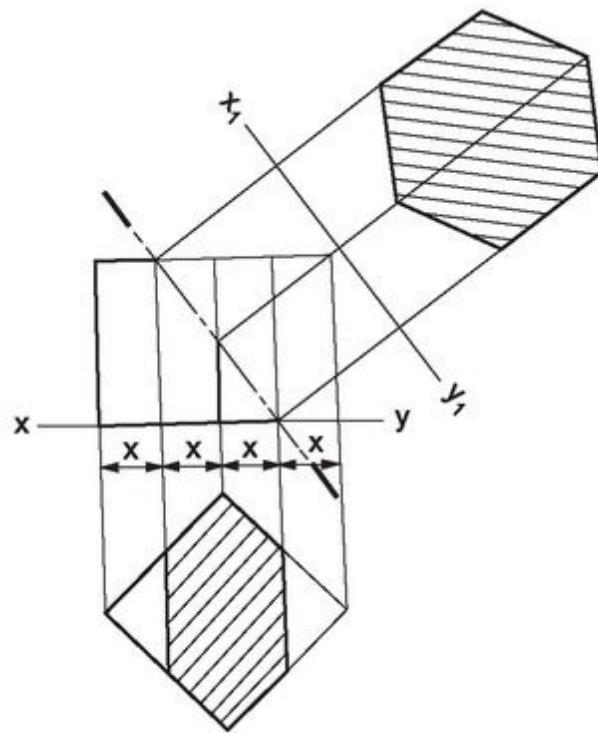
102. What is the shape of the curve generated, when a cone is cut by a plane, inclined to its base and cutting both the sides of the axis?

Ellipse

103.A cone rests on its base on H.P. An A.I.P inclined to H.P, cuts the cone but passes through the vertex. Sketch the cut section in the top view and the true shape.



**Fig.103**



**Fig.104**

104. Show a sketch of a cube's position and the cutting plane position to obtain a regular hexagon as true shape.

105. A sphere is resting on H.P. An A.I.P inclined to H.P. cuts the sphere. Name the curve in the true shape and in the top view.

True shape: circle

Top view: Ellipse

## 13. HELICAL SURFACES

106. What is a helix?

Helix is a curve, generated by a point moving on the surface of a cylinder or cone in a circular direction. The point moves at a constant angular velocity and with a simultaneous uniform rate of advance in axial direction; the ratio of the two movements bring constant.

107.What is meant by a convolution of a helix?

The helix corresponding to one revolution of the moving point is known as one convolution of the helix.

108.Define pitch of a thread.

In screw threads, pitch is the distance from a point on the thread to the corresponding point on the adjacent thread and parallel to the axis.

109.What is meant by lead of a thread?

Lead of a thread is equal to the axial advance of the thread per revolution.

110.How lead and pitch are related to single start and double start thread?

Lead is equal to the pitch in single start and twice the pitch in double start thread.

## 14. DEVELOPMENT OF SURFACES

111.What are the two principal methods of development?

Parallel line and radial line development methods.

112.Which method is used for drawing the developments of pyramids and cones?

Radial line method

In drawing the development of objects, isometric  
113.lengths are used.

(True / False)

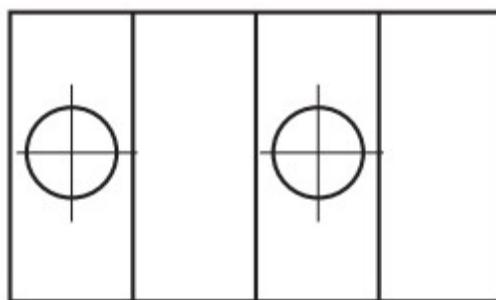
False

114.For what type of solids, may the triangulation method of development be followed?

Transition pieces and reducers

115.The development of a circular hole, through the flat face of a prism will appear as \_\_\_\_\_, in the development of the lateral surface of the prism.

circle.



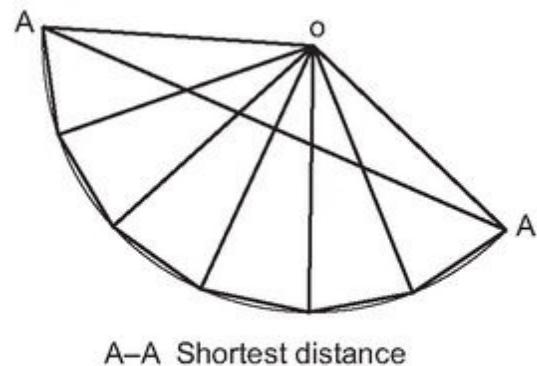
**Fig.116**

116.A through hole is drilled through the mid-point of a face of a square prism such that, the axis of the hole bisects the axis of the prism. Sketch the development of the prism with the hole.

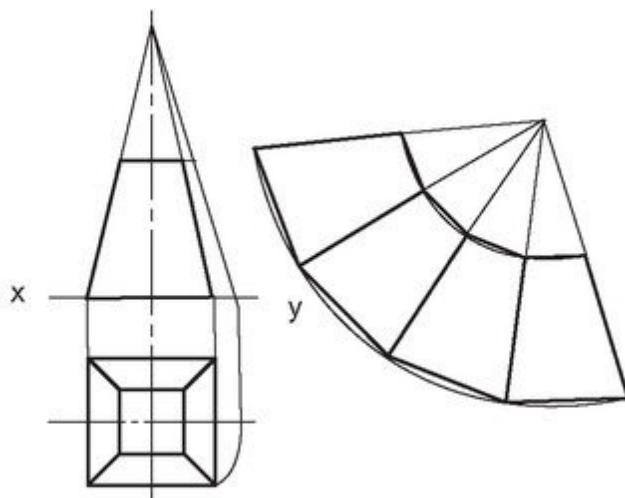
117.For which solid, the development will be four equilateral triangles?

Tetrahedron

118.Show the shortest distance to be traveled around the surface of a hexagonal pyramid, starting from one corner A and reaching the same corner.



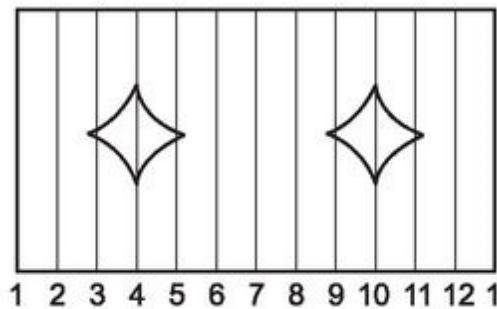
**Fig.118**



**Fig.119**

- 119.A square pyramid is cut by a plane parallel to its base.  
Sketch its development.
- 120.A cylinder has a square hole, the axes of hole and cylinder being intersecting each other. How the development of lateral surface will look like.
- 121.What is the shape of the development of the lateral surface of a cylinder?

## Rectangle



**Fig.120**

122.The development of a circular hole on a curved surface will appear as .

ellipse

123.How the angle of the sector is calculated in the development of a cone?

$$\theta = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

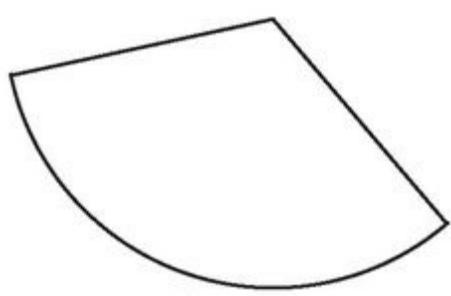
124.What is the shape of the development of the lateral surface of a cone?

Sector of a circle

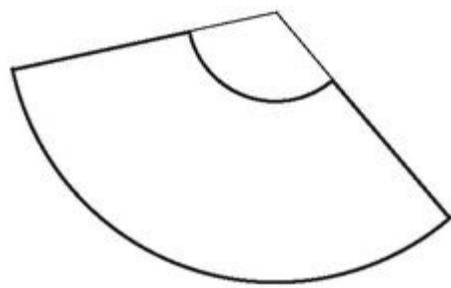
125.A cone is cut by a plane parallel to the base. How the development will look like?

126.The slant height of a cone is equal to its base circle diameter. What will be the shape of the development of the cone?

Semi-circle



**Fig.124**



**Fig.125**

## **15. INTERSECTION OF SURFACES**

127. Define the term “Line of intersection”.

When two surfaces intersect, the line of intersection is a line or curve, along which all the elements of one surface pierce the other. The line of intersection may be straight or curved, depending upon the nature of intersecting surfaces.

128. Name the methods of obtaining the line of intersection.

Cutting plane method and generator method.

129. Define key or critical points as applied to the intersection curves.

The key or critical points are the points at which the curve changes its direction and also change the visibility of the curve.

130. What are the rules of visibility, as applied to line of intersection?

The part of line of intersection, in front of solid axis is visible in the front view; other part being invisible.

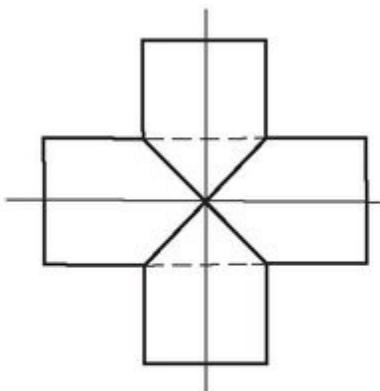
The part of line of intersection above the solid axis is visible in the top view; other part being invisible.

131. When the intersection lines will cross in the case of cylinders?

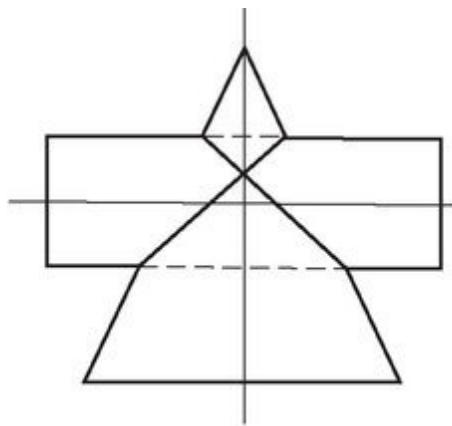
When two cylinders of the same size intersect each other such that, their axes intersect at right angle.

132. Sketch the interpenetration curve, when two cylinders of equal diameter join at right angle, with their axes

intersecting.



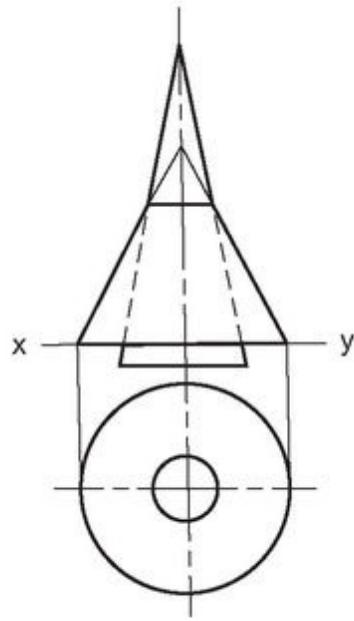
**Fig.132**



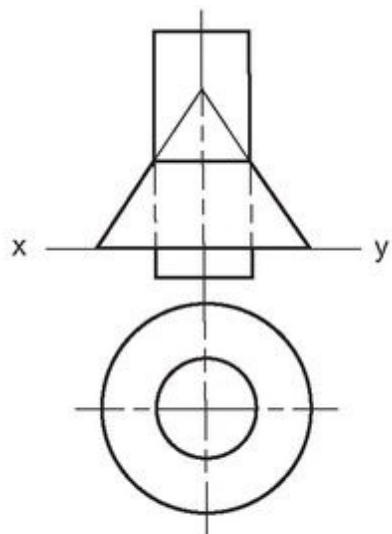
**Fig.133**

133.A cone, resting on its base on H.P, is penetrated by a horizontal cylinder such that, an imaginary common sphere is enveloped. Sketch the line of intersection in the front view.

134.A large vertical cone with base on H.P, is centrally penetrated by a small vertical cone. How the penetration curve will look like in the front and top views.



**Fig.134**



**Fig.135**

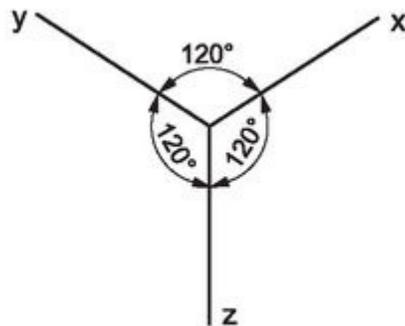
135.A large cone, standing on its base on H.P, is penetrated co-axially, by a small cylinder. Sketch the line of intersection in the front view.

136. When a vertical cone is penetrated by a vertical coaxial cylinder; the line of intersection in the top view appears as \_\_\_\_\_.  
circle

## 16. PICTORIAL PROJECTIONS

137. What is an isometric scale?

In isometric projection, the true lengths are to be converted into isometric lengths; by multiplying them with 0.82. The figure showing the conversion of true lengths into isometric lengths is called an isometric scale.



**Fig.139**

138. Give the ratio of the isometric length to true length.

0.82

139. How the isometric axes are positioned?

140. Give the difference between the isometric view and isometric projection.

In isometric view, actual measurements are used. In isometric projection, the actual measurements are reduced to 82%.

141. Isometric projection or isometric view of a square will be a \_\_\_\_\_.

rhombus

142. What are (a) isometric lines and non-isometric lines?

A line drawn parallel to any one of the isometric axes is called an isometric line. A line, not parallel to any one of the isometric axes is called non-isometric line.

143. The size of the isometric view is \_\_\_\_\_ larger than the isometric projection.

22%

144. What are the methods used for drawing an isometric projection of an object?

Co-ordinate or off-set method and box method

145. Isometric projection is preferred for \_\_\_\_\_ size objects and perspective projection is preferred for \_\_\_\_\_ size objects.

small, large

146. A sphere in isometric projection appears as \_\_\_\_\_.

circle

147. The diameter of the circle, representing the isometric projection of a sphere of 100mm diameter is \_\_\_\_\_.

100mm

148. Define horizon plane and central plane.

The horizon plane (H.P) is a plane that passes through the station point and parallel to the ground plane (G.P).

The central plane is a plane that passes through the station point and perpendicular to both the ground plane (G.P) and picture plane (P.P).

149. Define "Vanishing Point".

Vanishing point is the point at which the real or imaginary parallel lines appear to come together.

150. Define "Horizon line" and "Centre of vision", as applied to perspective projections.

Horizon line is the edge view of the horizon plane situated above H.P at the eye level of the observer.

Centre of vision (C.V) is the point of intersection between picture plane (P.P) and the axis of vision, and it lies on the horizon.

151. A line parallel to the picture plane has \_\_\_\_\_ vanishing point on the picture plane.

no

152. A straight edge of 50mm length will be seen in its true length in the perspective view, when it is located on \_\_\_\_\_.

picture plane

153. When a plane surface is seen as a straight line in perspective projection?

When it is perpendicular to the picture plane and along the central plane.

154. When will be the size of the perspective be larger than the object?

When the object is located between the observer and the picture plane.

155.What is normal perspective?

When the picture plane is located between the object and the observer, the perspective obtained is called normal perspective.

156.What is a parallel perspective?

When the front face of the object is parallel to the picture plane, the perspective obtained is called parallel perspective.

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# **Engineering Drawing**

## **Third Edition**

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- \* **Objective Type Questions - 325**

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