Assignment No 4

M Srinivas Reddy

February 16, 2020

1 Abstract

Fourier Series enables us to split a function into cosines and sines of different frequencies. We have formulas for determining the exact coefficients but we can also try and estimate them using **lstsq** function inbuilt in python. So we are going try to estimate them and observe the deviation.

2 Introduction

In this assignment we are going to first find the exact Fourier coefficients of two functions, **exp(x) and cos(cos(x))**.
We are then going to estimate the coefficients using **lstsq** function and plot various graphs.

3 Functions and their Fourier Coefficients

3.1 Plotting Functions

We have to plot two functions **exp(x) and cos(cos(x))**.
We use a semi-log plot (Log scaling on y-axis) to plot **exp(x)** because it's value blows up very quickly. We plot the functions from, $x = [-2\pi, 4\pi)$.

```
figure(1)
x1 = arange(-2*p,4*p,p/100,dtype=float)
semilogy(x1,exp(x1),color='r',label='exp(x)')
figure(2)
plot(x1,cos(cos(x1)),color='r',label='cos(cos(x))')
```

The figures are at the end of the report along with the estimated function.

3.2 Calculating Fourier Coefficients

To calculate the Fourier Coefficients we need to integrate the function. We have an integration function called **'quad'**. We first multiply the corresponding functions with cos(kx) and sin(kx) and store them as a function. we the integrate them from $(0, 2\pi)$

```
    def u1(x,k):
        return exp(x)*cos(k*x)
    def v1(x,k):
        return exp(x)*sin(k*x)
for i in range(0,26):
 if i==0:
  vect1[0]=(1/(2*p))*(integrate.quad(u1,0,2*p,args=(i))[0])
 else:
  vect1[2*i-1]=(1/p)*(integrate.quad(u1,0,2*p,args=(i))[0])
  vect1[2*i]=(1/p)*(integrate.quad(v1,0,2*p,args=(i))[0])
```

Similar technique is followed for **cos(cos(x)**.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)cos(kx)dx$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)sin(kx)dx$$

We calculate 51 coefficients in total. Note that we store the coefficients in an array, which will be useful while estimating the Coefficients. We can then plot them.

```
figure(3)
semilogy(n,vect1,'ro',markersize=3,label='Fourier Coeff')
figure(4)
loglog(n,vect1,'ro',markersize=3,label='Fourier Coeff')
```

4 Estimating Coefficients

We first build a matrix **A** consisting of the cosines and sines for 400 values of x between (0,2π) and **b** is the column matrix containing the estimated coefficients.

$$\begin{bmatrix} 1 & cos(x_1) & sin(x_1) & ... & cos(25x_1) & sin(25x_1) \\ 1 & cos(x_2) & sin(x_2) & ... & cos(25x_2) & sin(25x_2) \\ ... & ... & ... & ... & ... & ... \\ 1 & cos(x_{400}) & sin(x_{400}) & ... & cos(25x_{400}) & sin(25x_{400}) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ ... \\ a_{25} \\ a_{25} \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \\ ... \\ f(x_{399}) \\ f(x_{400}) \end{bmatrix}$$

$$a_0 + \sum_{n=0}^{25} a_n cosnx_i + \sum_{n=0}^{25} b_n sinnx_i \simeq f(x_i)$$

We now try to find **b** matrix using **lstsq** function and store the in c1 and c2 respectively for both the functions.

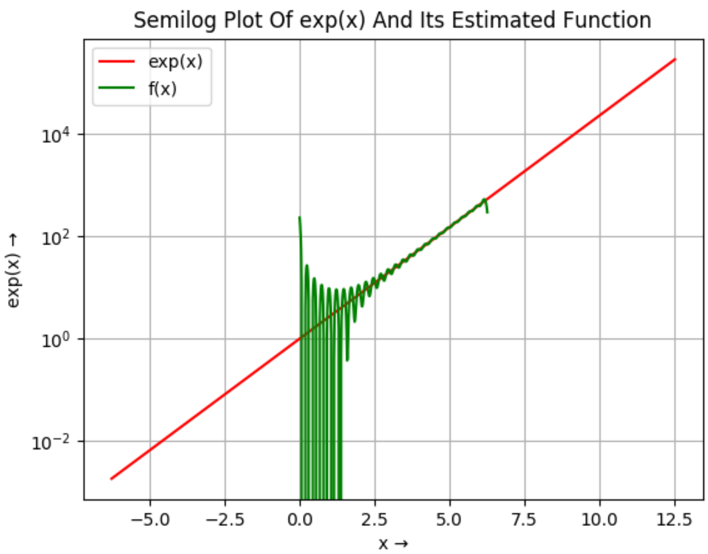
```
c1=lstsq(A,exp(x),rcond=-1)[0]
c2=lstsq(A,cos(cos(x)),rcond=-1)[0]
```

5 Plots

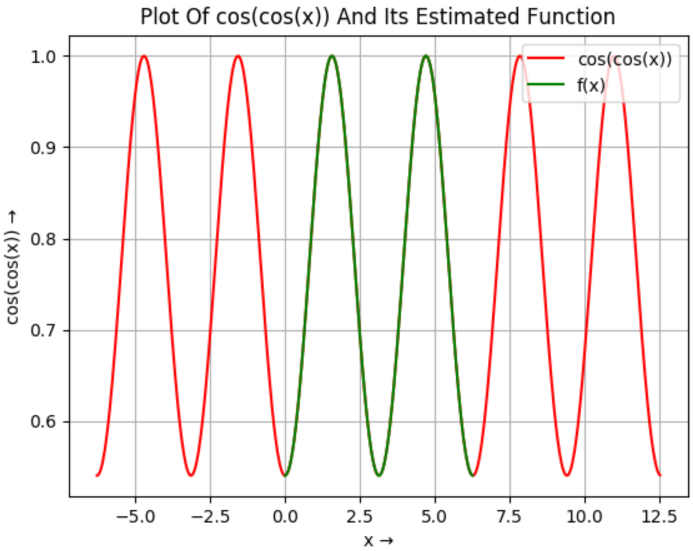
5.1 Function Plots

We plot the estimated function along with the actual function. We find out the estimated function by matrix multiplication of **A** and **b**.

```
matmul1=matmul(A,c1)
figure(1)
semilogy(x,matmul1,color='g',label='f(x)')
```



Similarly for cos(cos(x)).

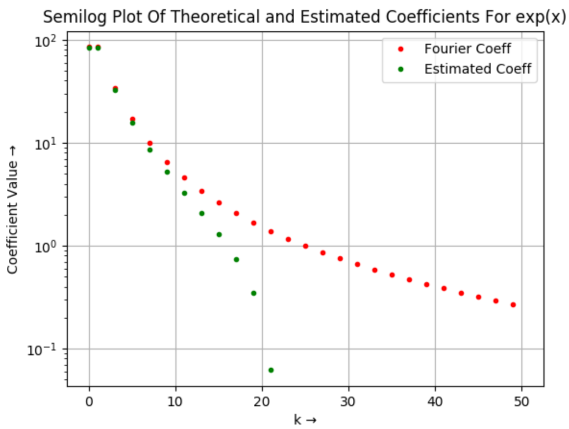


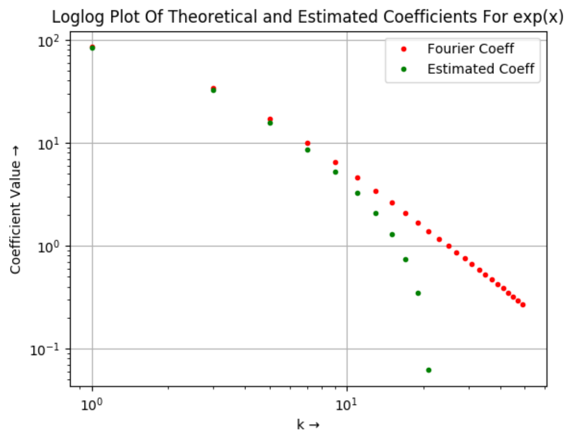
5.2 Coefficient Plots

Similar to the previous section we plot **semilog** and **loglog** plots of both theoretical and estimated coefficients for both the functions

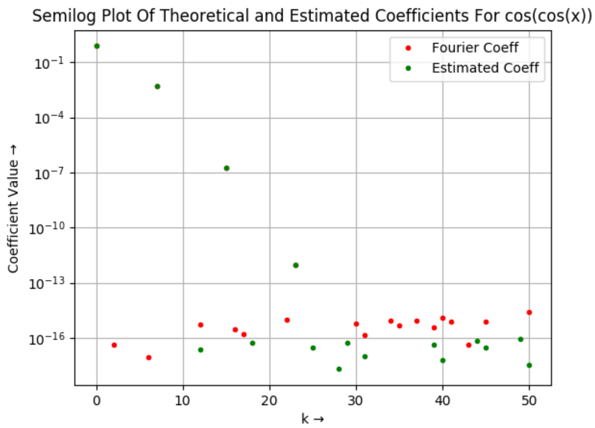
```
figure(3)
semilogy(n,c1,'go',markersize=3,label='Estimated Coeff')
figure(4)
loglog(n,c1,'go',markersize=3,label='Estimated Coeff')
```

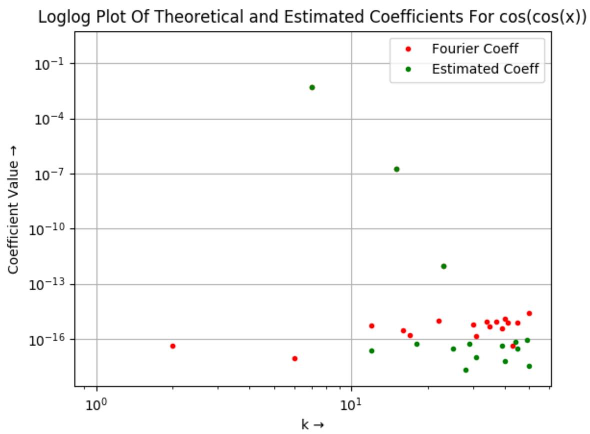
For **exp(x)**:





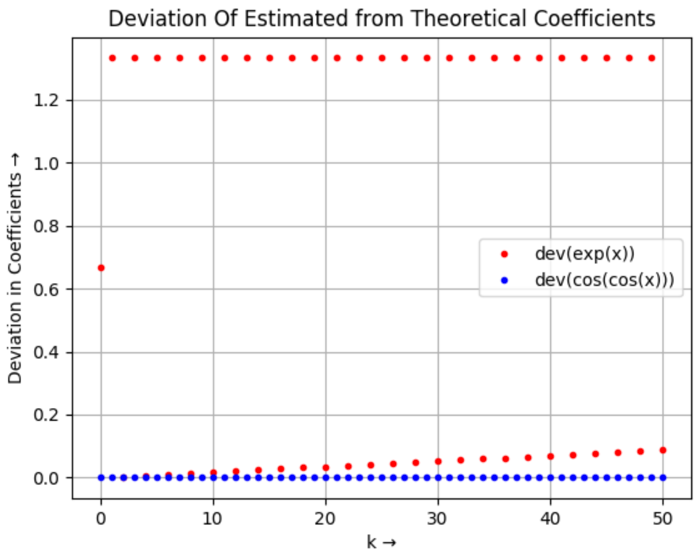
For $\mathbf{cos(cos(x))}$:





Lastly we plot a graph comparing the deviation between Estimated and Theoretical coefficients of **exp(x)** and **cos(cos(x))**.

```
figure(7)
plot(n,abs(c1-vect1),color='red',marker='o')
plot(n,abs(c2-vect2),color='blue',marker='o')
```



6 Observation

In this assignment we can observe that Fourier Coefficients are better estimated for periodic functions like cos(cos(x)) than for exponential functions.