Assignment No 7

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1 Abstract

Digital Signal Processing is very important as we deal with signals almost every single day. For example we answer many phone calls and browse the internet very frequently. The processing of all these signals comes under Digital Signal Processing or DSP in short.

2 Introduction

In this assignment we are going to find out DFT of various signals and plot the magnitude and phase plots.

3 Basics Of Signal Processing

When we have a signal, observing it in time domain alone may not give us enough information. We also need the signal's response in frequency domain.For that we have various types of transforms and series for different signals.

- Fourier Series of a signal is found out if the signal is continuous and periodic.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnt}$$

$$c_n = \frac{1}{2\pi}\int_{t_o}^{t_o+2\pi} f(t)e^{-jnt}dt$$

- Fourier Transform of a signal is found out if signal is continuous and non periodic.

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$f(t) = \int_{-\infty}^{\infty} F(j\omega t)e^{j\omega t}d\omega$$

- If we take samples of an non periodic signal f(t) then we define Discrete time fourier transform or DTFT. F(jθ) is periodic with 2π.

$$F(e^{j\theta}) = \sum_{n=-\infty}^{\infty} f[n]e^{-jn\theta}$$

- If f[n] is now periodic we then calculate what is known as Discrete Fourier Series or DFS of f[n] or DFT in the case N samples of a non periodic f[n] are taken and its DFS is calculated.Let period of f[n] be N then the DFT or DFS of a signal is also discrete and periodic with period N.

$$F[k] = \sum_{n=0}^{N-1} f[n]exp(-2\pi\frac{nk}{N}j) = \sum_{n=0}^{N-1} f[n]W^{nk}$$

$$f[n] = \sum_{n=0}^{N-1} F[k]W^{-nk}$$

Since DFT is both periodic and discrete we can store its values in a computer and process the signal.

4 Computing DFT in Python

The commands used for computing DFT and inverse DFT of signal are in a module called numpy or pylab(which contains numpy). The commands are

```
numpy.fft.fft()
numpy.fft.ifft()
```

We also have another command called **fftshift()** which essentially places zero frequency in the frequency spectrum in the center of the graph.

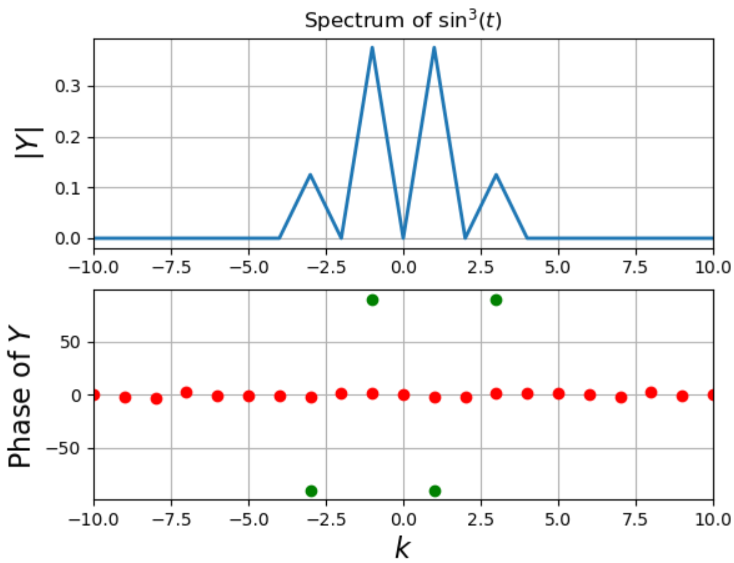
5 The Assignment

5.1 Problem 1

In the first problem we have to find the frequency spectrum of sin^3t and cos^3t.
We first define the number of samples that we need for the signal. The more samples we gather, the better the spectrum but takes more time. For sin^3t and cos^3t2^7 samples are enough to identify the spectrum. Then we find DFT of the sampled signal and plot its magnitude and phase.

```
n=2**7
x=linspace(0,2*pi,n+1);x=x[:-1]
y=sin(x)**3
Y=(fftshift(fft(y))/n)
w=linspace(-64,63,n)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin^3(t)$")
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii])*180/pi,'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
```

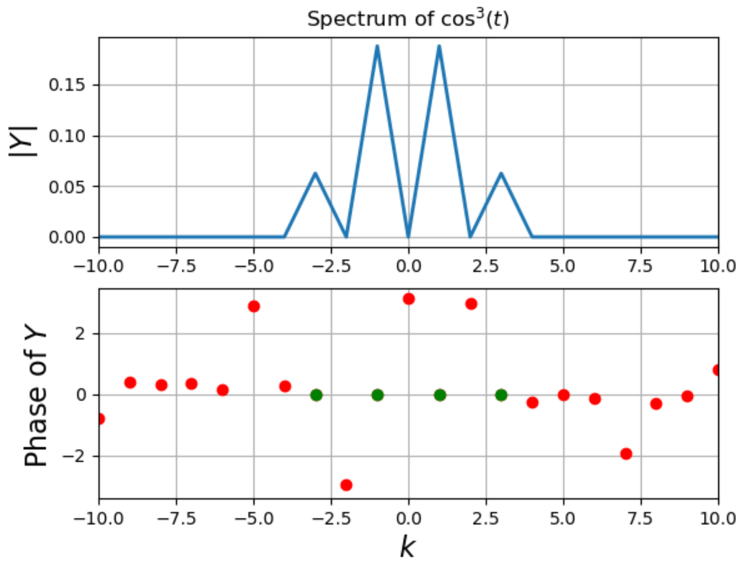
We use **linspace** command to create samples of x at which $sin^3(x)$ is computed.
To avoid repletion of 0 and 2π both we divide x into n+1 samples and delete the last sample.
After plotting we limit the x axis using **xlim** command to observe the signal better.



Similar process is done for cos^3x

```
y=cos(x)**3
Y=(fftshift(fft(y))/n)
w=linspace(-64,63,n)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\cos^3(t)$")
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii])*180/pi,'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
```

If we plot phase of DFT at all points we get a messed up graph. So we compute phase for points at which magnitude is greater than 0.001.



5.2 Problem 2

In this problem we have to find DFT of cos(20 + 5cos(x)).Compared to $sin^3(x)$ this signal has a wider frequency band and peaks in magnitude closer to each other. So we need slightly higher samples for computation.We use 2^10 samples.

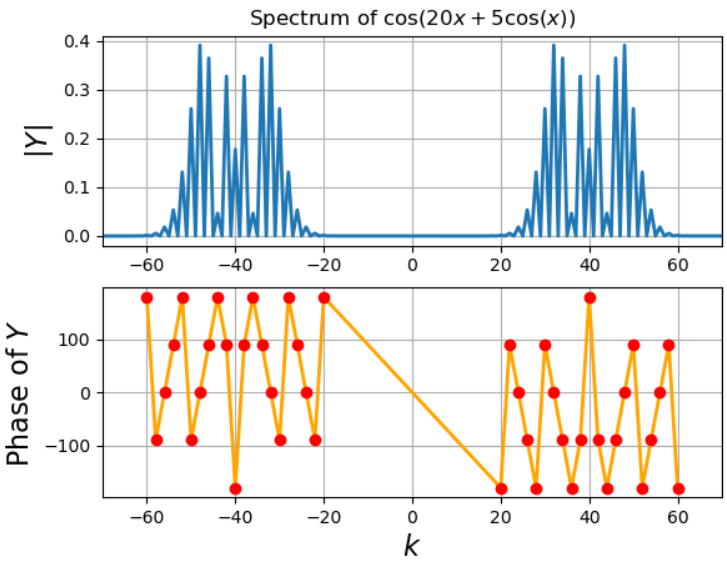
```
n=2**10
```

```
x=linspace(-10*pi,10*pi,n+1);x=x[:-1]
y=cos(20*x+5*cos(x))
Y=(fft(y)/n)*2
```

In this case we divided x from -10π to 10π instead of 0 to 2π because we need a wider spectrum to observe the peaks easily and also higher samples to distinguish between the peaks.

```
w=linspace(-64,64,n+1);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-60,60])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\cos(20x+5\cos(x))$")
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii])*180/pi,color='orange',lw=2)
xlim([-60,60])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
```

Rest of the process is similar to the previous problem. We define **w** and plot the magnitude and phase spectrum.



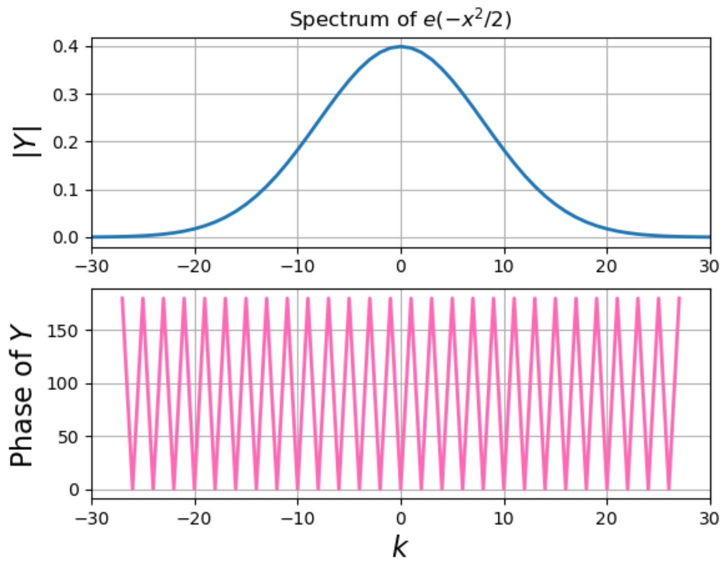
There are many peaks in the graph centered around -40 and 40. Phase at these points are either $+/-\pi$ or $+/-\frac{\pi}{2}$.

5.3 Problem 3

In the final problem we have to find the spectrum of $exp(-\frac{x^2}{2})$.This type of function is classified as Gaussian function and is not band-limited in frequency.
To get highly accurate(to 6 digits) magnitude of DFT of signal we have to take a large amount of samples.So we take 2^18 samples from -8π to 8π.

```
n=2**18
x=linspace(-8*pi,8*pi,n+1);x=x[:-1]
y=exp(-x**2/2)
Y=(fftshift(fft(y))/n)*8
w=linspace(-n/2,n/2,n+1);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-30,30])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $e(-x^2/2)$")
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii])*180/pi,color='hotpink',lw=2)
xlim([-30,30])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
```

We take frequency axis from -2^{19} to 2^{19} and limit the axis to length 30 on either side of 0.



In this plot we can observe that the points are represented up to 6 significant digits.

6 Observation

Computing DFT of signals is very easy using these commands. However we have to be careful with the sampling and defining the frequency axis as things can get messy and the plots obtained may not be accurate. We also have to worry about under-sampling as we may not be able to observe distinct peaks in the frequency spectrum. The phase spectrum of the signals may not be accurate and so we should consider phase only near the peaks.