# Minimization of FA

$(Q, \Sigma, \delta, q_0, F)$ DFA

$q_1 . q_2$

$q_1 \equiv q_2$

if $\hat{\delta}(q_1, x)$ and $\hat{\delta}(q_2, x)$ are both $\in F$

or both $\notin F$

$q_1 \xrightarrow{x} P_1$ either $P_1, P_2 \in F$ (Final)

$q_2 \xrightarrow{x} P_2$ or $P_1, P_2 \notin F$ (Final)

(1) $P \equiv P$ Equivalence ( reflexive , symmetric, transitive)

(2) $P \equiv q \Leftrightarrow q \equiv P$

(3) $P \equiv q$ and $q \equiv r \Rightarrow P \equiv r$

Recursive Construction of $(k+1)^{th}$ equivalence.

(i) $q_1, q_2$ $(k+1)$ th equivalent $\Rightarrow$ $q_1, q_2$ must be $k$ - equivalent

(ii) $q_1, q_2$ $(k+1)^{th}$ equivalent if

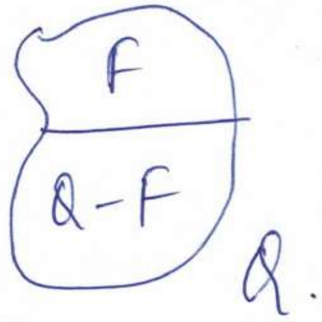
$\delta(q_1, a)$ , $\delta(q_2, a)$ are $k$ - equivalent $\underline{\forall a \in \Sigma}$

P

## Construction of Minimum DFA

1. $\Pi_0 \rightarrow$ 0th equivalent class

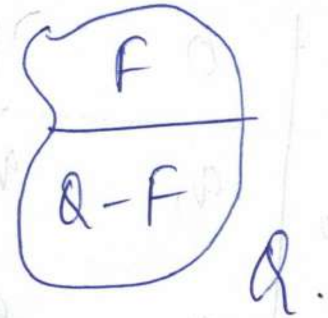
$\Pi_0 = (Q_1^0, Q_2)$ i.e. $Q_1^0 = F$, $Q_2^0 = Q - F$



# Construction of Minimum DFA

1. $\Pi_0 \rightarrow$ 0th equivalent class

$$\Pi_0 = (Q_1^0, Q_2^0) \quad \text{i.e. } Q_1^0 = F, \; Q_2^0 = Q - F$$




2. Construction of $\Pi_{k+1}$ from $\Pi_k$

$q_1, q_2 \in Q_i^k$ and

$q_1, q_2 \in Q_J^{k+1}$ if $\delta(q_1, a), \delta(q_2, a)$ are k-equivalent $\forall a \in \Sigma$

Repeat until

$\Pi_n = \Pi_{n+1}$ ✓

Example:-

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $* q_2$ | $q_0$ | $q_2$ |
| $q_3$ | $q_2$ | $q_6$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

$\Pi_0 = \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

$\Pi_1 = ??$

$q_0 \equiv q_1 ??$

$\delta(q_0, 0) = q_1 \quad \delta(q_0, 1) = q_1 \notin F \quad \therefore q_0 \equiv q_1$

$\delta(q_1, 0) = q_6 \quad \delta(q_1, 1) = q_2 \in F$

$q_0 \equiv q_4$ ???

$\delta(q_0, 0) = q_1 \quad \delta(q_4, 0) = q_7$

$\delta(q_0, 1) = q_5 \quad \delta(q_4, 1) = q_5$

Example:-

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $* q_2$ | $q_0$ | $q_2$ |
| $q_3$ | $q_2$ | $q_6$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

$\pi_0 = \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

$\pi_1 = ??$

$q_0 \equiv q_1 ??$

$\delta(q_0, 0) = q_1 \quad \delta(q_0, 1) = q_1 \notin F \quad \therefore q_0 \equiv q_1$

$\delta(q_1, 0) = q_6 \quad \delta(q_1, 1) = q_2 \in F$

$q_0 \equiv q_4 ???$

$\delta(q_0, 0) = q_1 \quad \delta(q_4, 0) = q_7$

$\delta(q_0, 1) = q_5 \quad \delta(q_4, 1) = q_5$

$\pi_1 = \{\{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

$\pi_2 = ??$

$\left.\begin{array}{l}\delta(q_4, 0) = q_7 \\ \delta(q_6, 0) = q_6\end{array}\right\}$ belongs to diff. classes in $\pi_1$

Example:-

90 — 91 — 92

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $* q_2$ | $q_0$ | $q_2$ |
| $q_3$ | $q_2$ | $q_6$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

$\Pi_0 = \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

$\Pi_1 = ??$

$q_0 \equiv q_1$ ??

$\delta(q_0, 0) = q_1 \quad \delta(q_0, 1) = q_1 \notin F \quad \therefore q_0 \not\equiv q_1$

$\delta(q_1, 0) = q_6 \quad \delta(q_1, 1) = q_2 \in F$

$q_0 \equiv q_4$ ???

$\delta(q_0, 0) = q_1 \quad \delta(q_4, 0) = q_7$

$\delta(q_0, 1) = q_5 \quad \delta(q_4, 1) = q_5$

$\Pi_1 = \{\{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

$\Pi_2 = ??$

$\left.\begin{array}{l}\delta(q_4, 0) = q_7 \\ \delta(q_6, 0) = q_6\end{array}\right\}$ belongs to diff. classes in $\Pi_1$

$\Pi_2 = \{\{q_2\}, \{q_6\}, \{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

$\Pi_3 = \Pi_2$

P

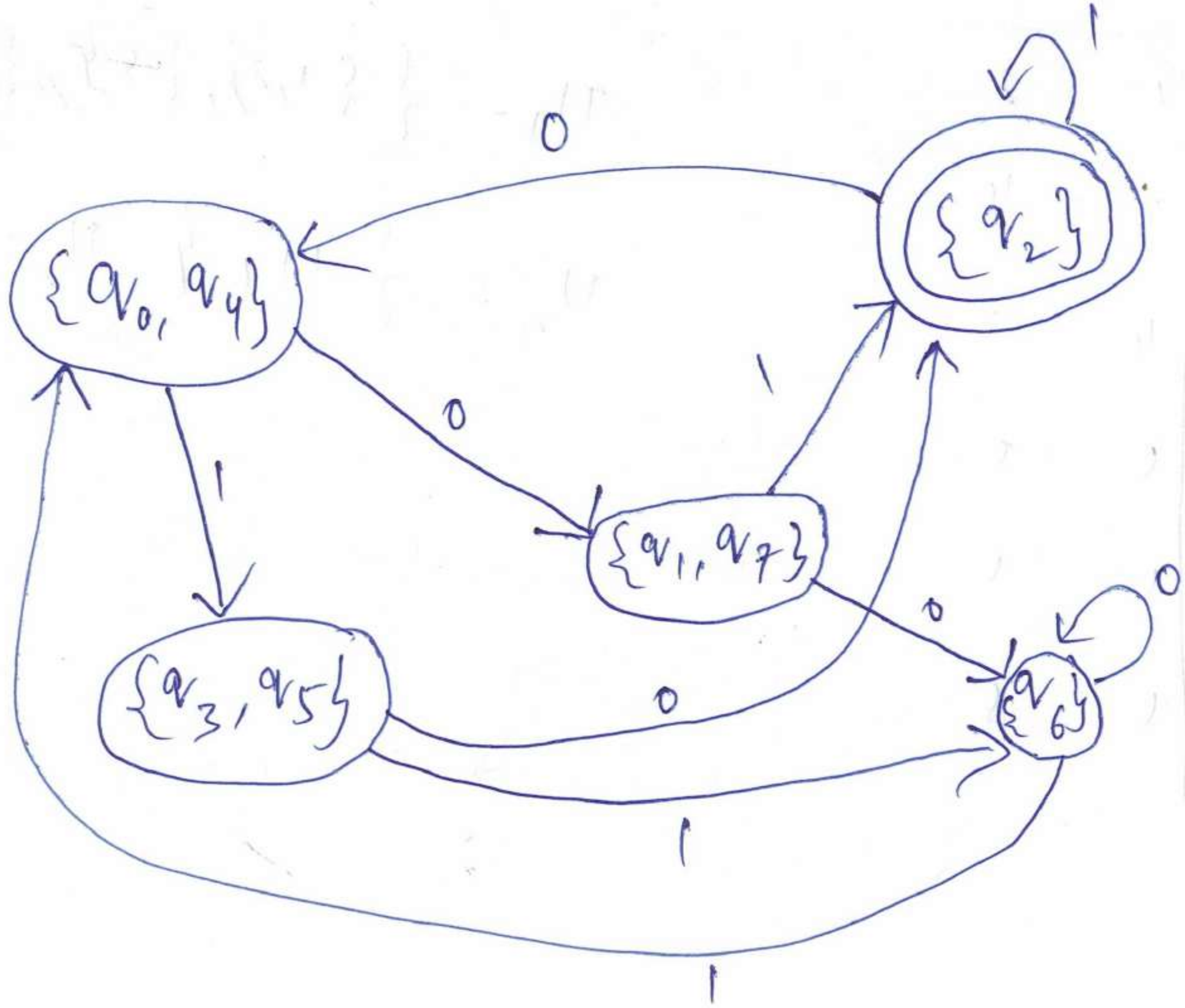
| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $* q_2$ | $q_0$ | $q_2$ |
| $q_3$ | $q_2$ | $q_6$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

$\Pi_3 = \Pi_2 = \{\{q_2\}, \{q_6\}, \{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

93

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $* q_2$ | $q_0$ | $q_2$ |
| $q_3$ | $q_2$ | $q_6$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

$\pi_3 = \pi_2 = \{\{q_2\}, \{q_6\}, \{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\}$ q3 q4



<u>Example</u> $\Sigma = \{a, b\}$

| $\delta$ | $a$ | $b$ |
|---|---|---|
| $\rightarrow 0$ | 1 | 2 |
| $*1$ | 3 | 4 |
| $*2$ | 4 | 3 |
| 3 | 5 | 5 |
| 4 | 5 | 5 |
| $*5$ | 5 | 5 |

$\Pi_0 = \{\{1, 2, 5\}, \{0, 3, 4\}\}$

$\Pi_1 = \{\{1, 2\}, \{5\}, \{0\}, \{3, 4\}\}$

$\cancel{\Pi_2 = \{\Pi_1\}}$ $\Pi_2 = \Pi_1$

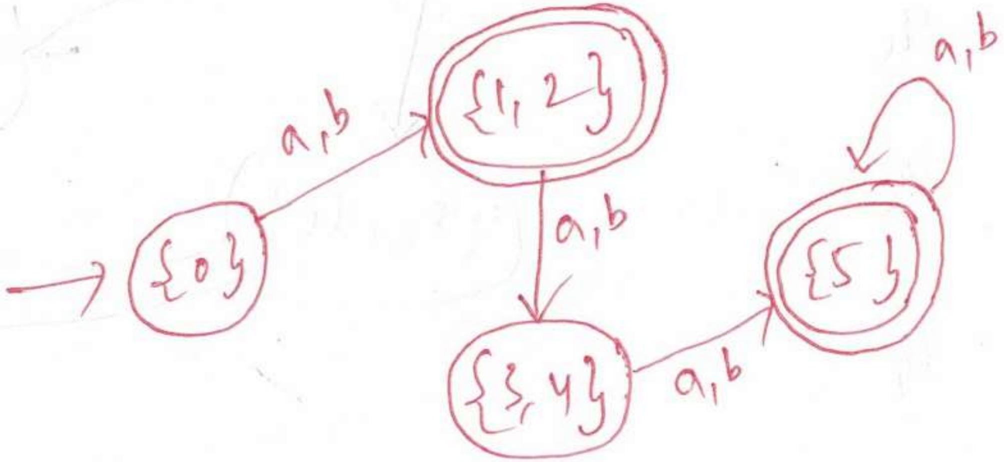
Example $\Sigma = \{a, b\}$

| $\delta$ | a | b |
|---|---|---|
| $\rightarrow 0$ | 1 | 2 |
| $*1$ | 3 | 4 |
| $*2$ | 4 | 3 |
| 3 | 5 | 5 |
| 4 | 5 | 5 |
| $*5$ | 5 | 5 |

$$\Pi_0 = \{\{1, 2, 5\}, \{0, 3, 4\}\}$$

$$\Pi_1 = \{\{1, 2\}, \{5\}, \{0\}, \{3, 4\}\}$$

$$\cancel{\Pi_2 = \{\Pi_1\}} \quad \Pi_2 = \Pi_1$$




$1 \approx 2, \; 3 \approx 4$