

Minimization of FA

(86)

$(Q, \Sigma, \delta, q_0, F)$ DFA

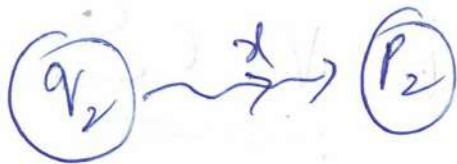
q_1, q_2

$$q_1 \equiv q_2$$

if $\hat{\delta}^1(q_1, a)$ and $\hat{\delta}^1(q_2, a)$ are both $\in F$
or both $\notin F$



either $p_1, p_2 \in F$ (Final)



or $p_1, p_2 \notin F$ (Final)

Φ

(1) $P \equiv P$ Equivalence (reflexive, symmetric, transitive) 87

(2) $P \equiv Q \Leftrightarrow Q \equiv P$

(3) $P \equiv Q$ and $Q \equiv R \Rightarrow P \equiv R$

Recursive Construction of $(k+1)^{\text{th}}$ equivalence.

(i) q_1, q_2 $(k+1)^{\text{th}}$ equivalent \Rightarrow q_1, q_2 must be k -equivalent

(ii) q_1, q_2 $(k+1)^{\text{th}}$ equivalent if

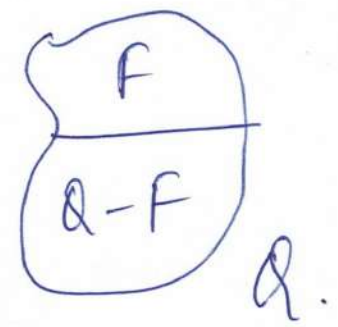
$\mathcal{L}(q_1, a), \mathcal{L}(q_2, a)$ are k -equivalent $\forall a \in \Sigma$

Q

Construction of Minimum DFA

1. $\pi_0 \rightarrow$ 0th equivalent class

$$\pi_0 = (Q_1^0, Q_2^0) \text{ i.e. } Q_1^0 = F, Q_2^0 = Q - F$$

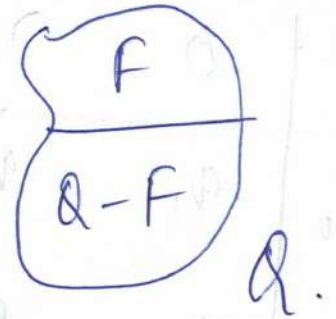


Construction of Minimum DFA

(88-89)

1. $\pi_0 \rightarrow 0^{\text{th}}$ equivalent class

$$\pi_0 = (Q_1^0, Q_2^0) \text{ i.e. } Q_1^0 = F, Q_2^0 = Q - F$$



2. Construction of π_{k+1} from π_k

$$q_1, q_2 \in Q_i^k \text{ and}$$

$$q_1, q_2 \in Q_j^{k+1} \text{ if } \delta(q_1, a), \delta(q_2, a) \text{ are } k\text{-equivalent} \\ \forall a \in \Sigma$$

Repeat until

$$\pi_n = \pi_{n+1} \checkmark$$

Q

Example:-

| δ | 0 | 1 |
|-------------------|-------|-------|
| $\rightarrow q_0$ | q_1 | q_5 |
| q_1 | q_6 | q_2 |
| $* q_2$ | q_0 | q_2 |
| q_3 | q_2 | q_6 |
| q_4 | q_7 | q_5 |
| q_5 | q_2 | q_6 |
| q_6 | q_6 | q_4 |
| q_7 | q_6 | q_2 |

$$\Pi_0 = \{ \{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\} \} \quad (q_0)$$

$$\Pi_1 = ??$$

$$q_0 \equiv q_1 ??$$

$$\delta(q_0, 0) = q_1 \quad \delta(q_0, 1) = q_1 \notin F \quad \therefore q_0 \equiv q_1$$

$$\delta(q_1, 0) = q_6 \quad \delta(q_1, 1) = q_2 \in F$$

$$q_0 \equiv q_4 ???$$

$$\delta(q_0, 0) = q_1 \quad \delta(q_4, 0) = q_7$$

$$\delta(q_0, 1) = q_5 \quad \delta(q_4, 1) = q_5$$

Example:-

| δ | 0 | 1 |
|-------------------|-------|-------|
| $\rightarrow q_0$ | q_1 | q_5 |
| q_1 | q_6 | q_2 |
| $*q_2$ | q_0 | q_2 |
| q_3 | q_2 | q_6 |
| q_4 | q_7 | q_5 |
| q_5 | q_2 | q_6 |
| q_6 | q_6 | q_4 |
| q_7 | q_6 | q_2 |

$$\pi_0 = \{ \{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\} \} \quad (90)-(91)$$

$$\pi_1 = ??$$

$$q_0 \equiv q_1 ??$$

$$\delta(q_0, 0) = q_1 \quad \delta(q_0, 1) = q_1 \notin F \quad \therefore q_0 \equiv q_1$$

$$\delta(q_1, 0) = q_6 \quad \delta(q_1, 1) = q_2 \in F$$

$$q_0 \equiv q_4 ???$$

$$\delta(q_0, 0) = q_1 \quad \delta(q_4, 0) = q_7$$

$$\delta(q_0, 1) = q_5 \quad \delta(q_4, 1) = q_5$$

$$\pi_1 = \{ \{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\} \}$$

$$\pi_2 = ??$$

$$\left. \begin{array}{l} \delta(q_4, 0) = q_7 \\ \delta(q_6, 0) = q_6 \end{array} \right\} \text{ belongs to diff. classes in } \pi_1$$

Example:-

| δ | 0 | 1 |
|-------------------|-------|-------|
| $\rightarrow a_0$ | a_1 | a_5 |
| a_1 | a_6 | a_2 |
| $* a_2$ | a_0 | a_2 |
| a_3 | a_2 | a_6 |
| a_4 | a_7 | a_5 |
| a_5 | a_2 | a_6 |
| a_6 | a_6 | a_4 |
| a_7 | a_6 | a_2 |

$$\pi_0 = \{ \{a_2\}, \{a_0, a_1, a_3, a_4, a_5, a_6, a_7\} \}$$

$$\pi_1 = ??$$

$$a_0 \equiv a_1 ??$$

$$\delta(a_0, 0) = a_1 \quad \delta(a_0, 1) = a_1 \notin F \quad \therefore a_0 \equiv a_1$$

$$\delta(a_1, 0) = a_6 \quad \delta(a_1, 1) = a_2 \in F$$

$$a_0 \equiv a_4 ???$$

$$\delta(a_0, 0) = a_1 \quad \delta(a_4, 0) = a_7$$

$$\delta(a_0, 1) = a_5 \quad \delta(a_4, 1) = a_5$$

$$\pi_1 = \{ \{a_2\}, \{a_0, a_4, a_6\}, \{a_1, a_7\}, \{a_3, a_5\} \}$$

$$\pi_2 = ??$$

$\delta(a_4, 0) = a_7$ belongs to diff. classes
 $\delta(a_6, 0) = a_6$ in π_1

$$\pi_2 = \{ \{a_2\}, \{a_6\}, \{a_0, a_4\}, \{a_1, a_7\}, \{a_3, a_5\} \}$$

$$\pi_3 = \pi_2$$



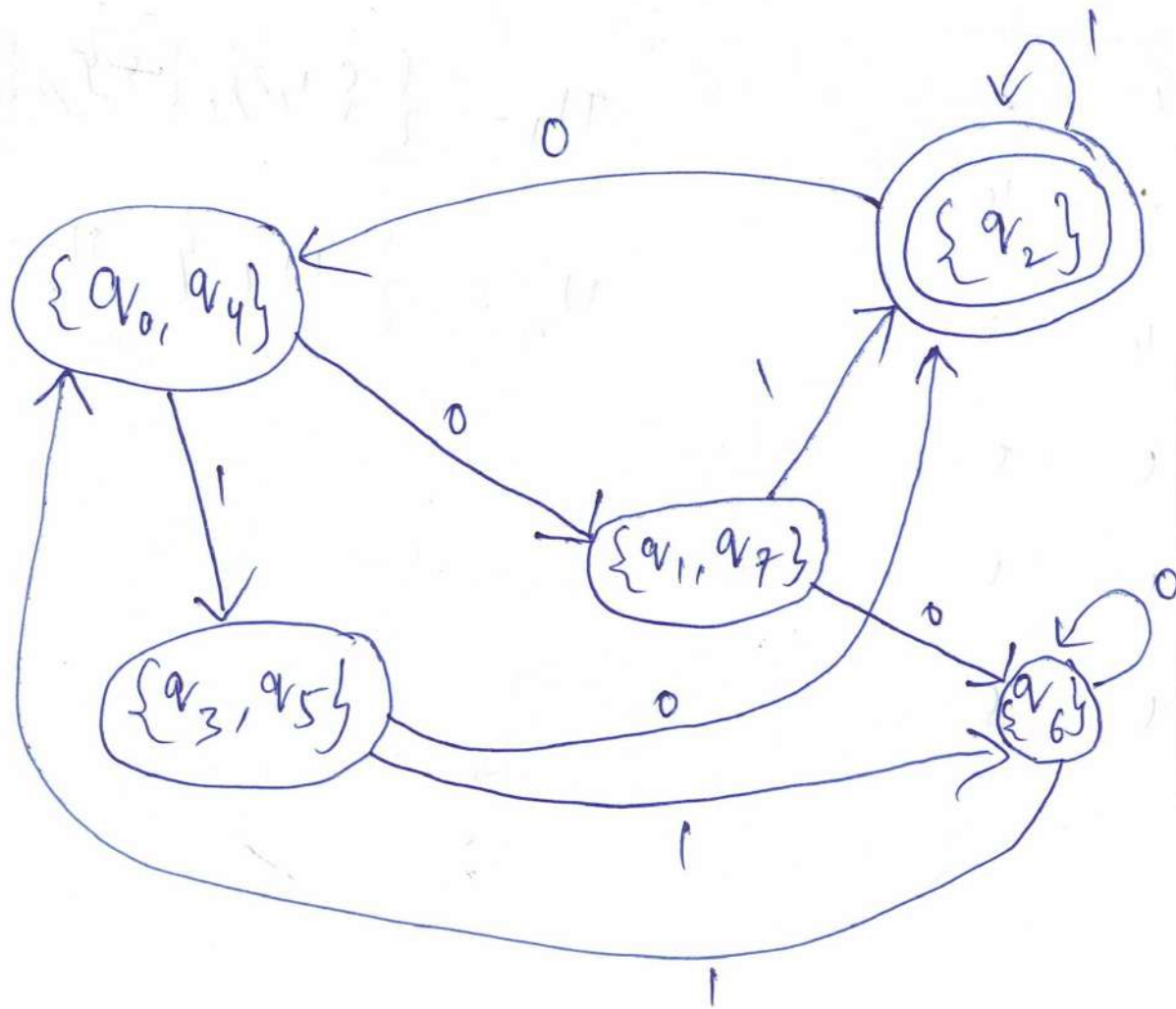
| δ | 0 | 1 |
|-------------------|-------|-------|
| $\rightarrow q_0$ | q_1 | q_5 |
| q_1 | q_6 | q_2 |
| $* q_2$ | q_0 | q_2 |
| q_3 | q_2 | q_6 |
| q_4 | q_7 | q_5 |
| q_5 | q_2 | q_6 |
| q_6 | q_6 | q_4 |
| q_7 | q_6 | q_2 |

$$\pi_3 = \pi_2 = \{\{q_2\}, \{q_6\}, \{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\} \quad (93)$$

P

| δ | 0 | 1 |
|----------|-------|-------|
| q_0 | q_1 | q_5 |
| q_1 | q_6 | q_2 |
| * q_2 | q_0 | q_2 |
| q_3 | q_2 | q_6 |
| q_4 | q_7 | q_5 |
| q_5 | q_2 | q_6 |
| q_6 | q_6 | q_4 |
| q_7 | q_6 | q_2 |

$$\pi_3 = \pi_2 = \{\{q_2\}, \{q_6\}, \{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\} \quad (q_3, q_4)$$



Example

$$\Sigma = \{a, b\}$$

95

| δ | a | b |
|-----------------|---|---|
| $\rightarrow 0$ | 1 | 2 |
| $*1$ | 3 | 4 |
| $*2$ | 4 | 3 |
| 3 | 5 | 5 |
| 4 | 5 | 5 |
| $*5$ | 5 | 5 |

$$\pi_0 = \{ \{1, 2, 5\}, \{0, 3, 4\} \}$$

$$\pi_1 = \{ \{1, 2\}, \{5\}, \{0\}, \{3, 4\} \}$$

$$\pi_2 = \{ \pi_1 \} \quad \pi_2 = \pi_1$$

Q

Example

$$\Sigma = \{a, b\}$$

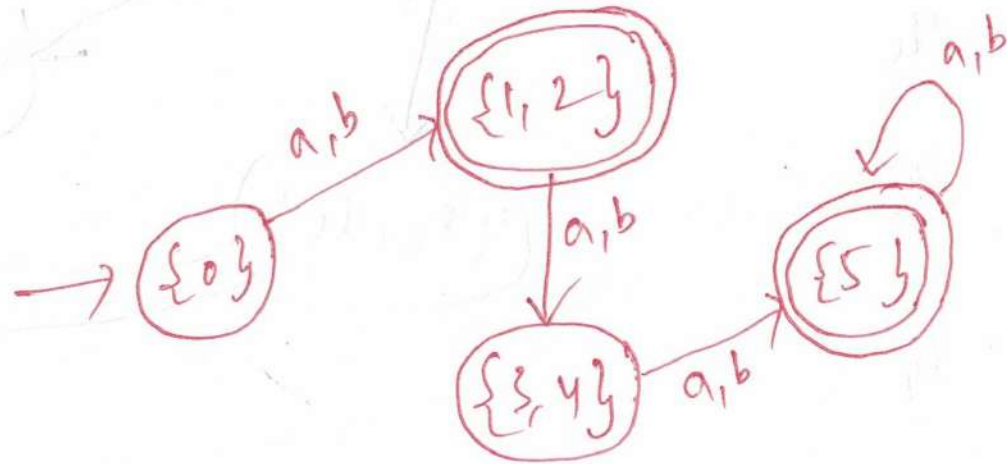
95-96

$$\pi_0 = \{ \{1, 2, 5\}, \{0, 3, 4\} \}$$

$$\pi_1 = \{ \{1, 2\}, \{5\}, \{0\}, \{3, 4\} \}$$

$$\pi_2 = \{ \pi_1 \} \quad \pi_2 = \pi_1$$

| δ | a | b |
|-----------------|---|---|
| $\rightarrow 0$ | 1 | 2 |
| *1 | 3 | 4 |
| *2 | 4 | 3 |
| 3 | 5 | 5 |
| 4 | 5 | 5 |
| *5 | 5 | 5 |



$$1 \approx 2, 3 \approx 4$$

Q.