Sample

$S \rightarrow 0B / 1A$, $A \rightarrow 0 / 0S / 1AA$, $B \rightarrow 1 / 1S / 0BB$

$\alpha = 00110101$

$G = (\{S, A, B\}, \{0, 1\}, P, S)$

derivation tree for $\alpha = 00110101$

Left most derivation. ??

Example (23)—(24)

$$S \to 0B/1A, \quad A \to 0/0S/1AA, \quad B \to 1/1S/0BB$$

$$\alpha = 00110101$$

$$G = (\{S, A, B\}, \{0, 1\}, P, S)$$

derivation tree for $\alpha = 00110101$

Left most derivation. !!

$$S \Rightarrow 0\underline{B} \Rightarrow 00\underline{B}B \Rightarrow 001\underline{S}B \Rightarrow 0011\underline{A}B \Rightarrow 00110\underline{S}B$$
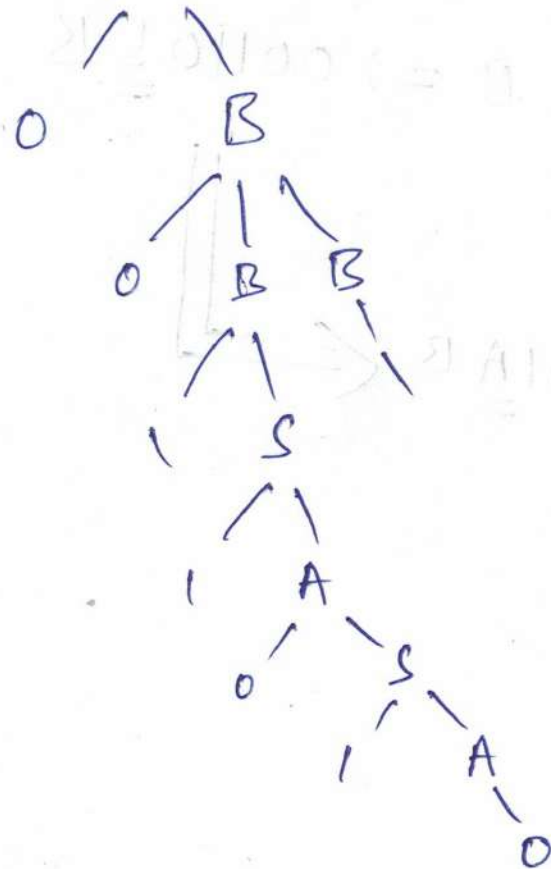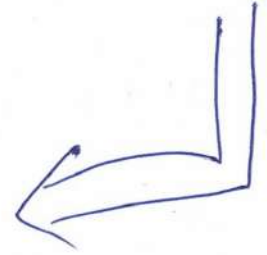
$$\boxed{00110101} \Leftarrow 0011010\underline{B} \Leftarrow 00110\underline{1A}B \Leftarrow$$
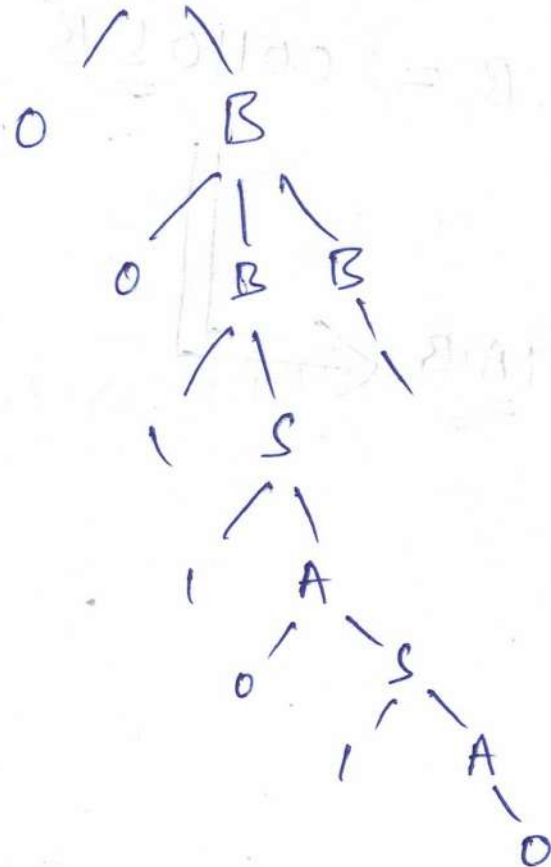
$$\alpha.$$

P.

$$S \rightarrow 0B/1A, \quad A \rightarrow 0/0S/1AA, \quad B \rightarrow 1/1S/0BB$$

$$\alpha = 00110101$$

derivation tree for $\alpha = 00110101$

Left most

S

$0 \quad B$

$0 \quad B \quad B$

$1 \quad S$

$1 \quad A$

$0 \quad S$

$1 \quad A$

0

S → 0B/1A,   A → 0/0S/1AA,   B → 1/1S/0BB

$$\alpha = 0011\,0101$$

Right most derivation??

S ⇒ 0B ⇒ 00BB ⇒ 00B1 ⇒ 001S1 ⇒ 0011A1

⇐ 001100S1

0011101A1 ⇐ 001100S1

(0011 0101) ⇐ 0011101A1

α

$S \to 0B / 1A, \quad A \to 0 / 0S / 1AA, \quad B \to 1 / 1S / 0BB$
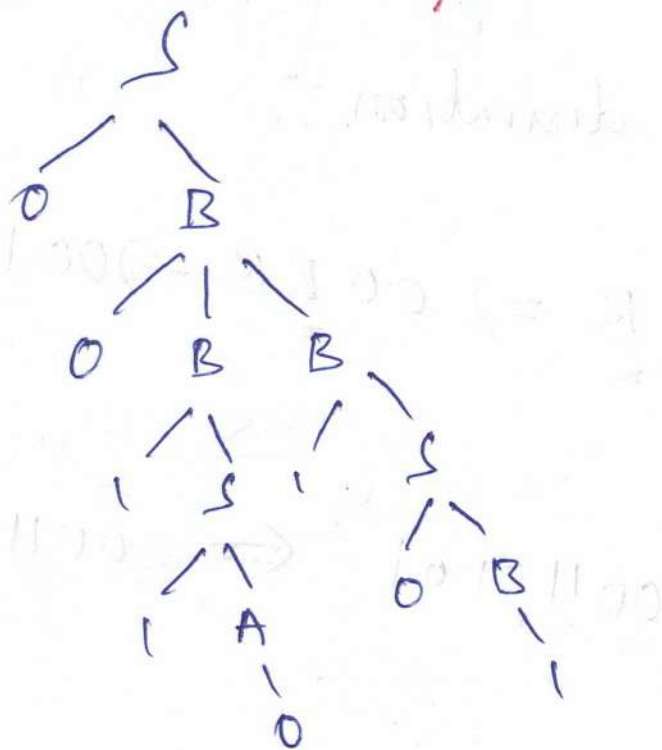
$\alpha = 0011010l$

derivation tree for $\alpha = 0011010l$

Left most

Right most   Another Left most derivation tree

$S \rightarrow OB / IA, \quad A \rightarrow O/OS/IAA, \quad B \rightarrow 1/IS/OBB$ (28)

$\alpha = 0011010 1$

Another left most derivation. for $\alpha = 0011 010 1$??

$S \Rightarrow O\underline{B} \Rightarrow OO\underline{BB} \Rightarrow OO1\underline{S}B \Rightarrow OO11AB$

$0011010B \Leftarrow 001101\underline{S} \Leftarrow 0011O\underline{B}$

$\boxed{0011010 1} \Leftarrow 0011010\underline{B}$

α.

Same way Another Rightmost derivation also Exists. for
Same derivation tree.

Ambiguity in CFG.

Example:- "In books selected information is given."

selected ← books / Information.

Definition:- A terminal string $w \in L(G)$ is ambiguous if

$\exists$ two or more derivation tree for w.

($\exists$ two or more left most derivation of w)

— A CFG G is ambiguous if $\exists$ some $w \in L(G)$ which is ambiguous.

— A CFL for which every CFG is ambiguous then CFL is called inherently ambiguous.

Example!- $G = (\{S\}, \{a, b, +, *\}, P, S)$ ③⓪

$P: S \longrightarrow S+S \mid S*S \mid a \mid b$ $\qquad \omega = a + a*b.$
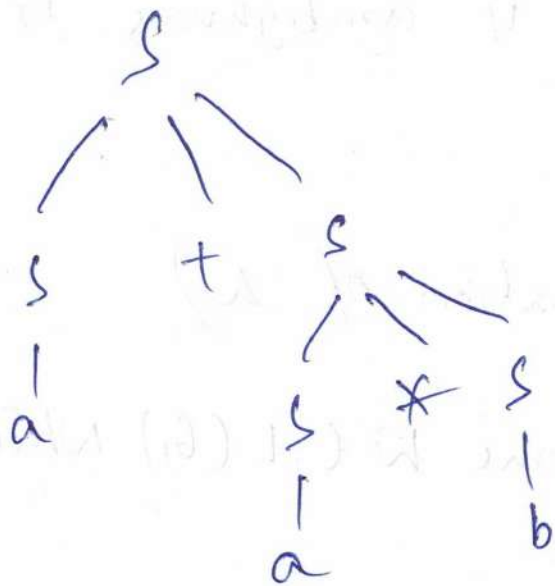
one derivation tree.

Example !:- $G = (\{S\}, \{a, b, +, *\}, P, S)$

P: $S \longrightarrow S + S \mid S * S \mid a \mid b$ $\omega = a + a * b$.
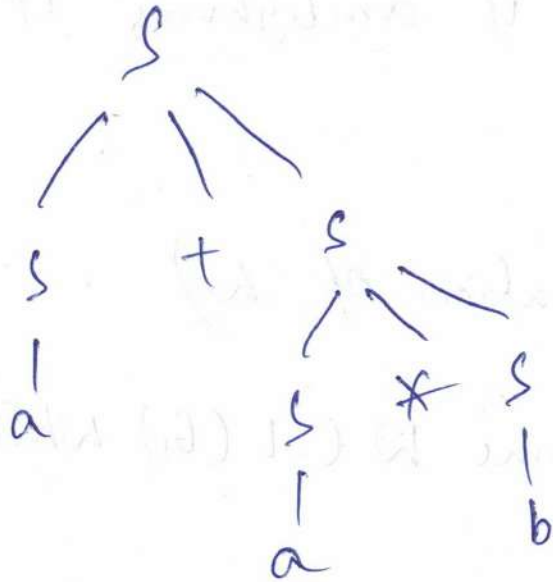
One derivation tree:

Example :- $G = (\{s\}, \{a, b, +, *\}, P, S)$
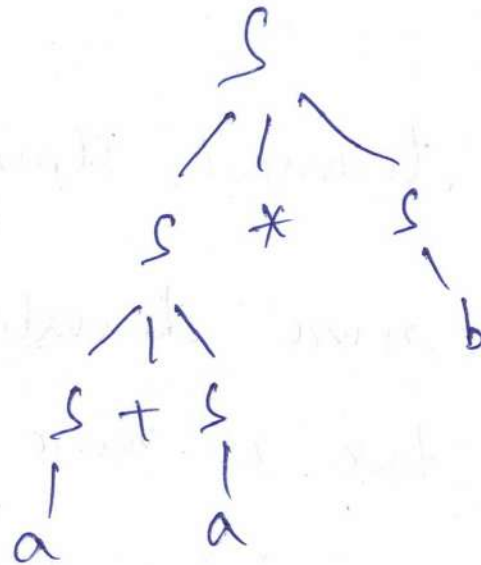
P: $S \longrightarrow S+S \mid S*S \mid a \mid b$ $W = a + a * b.$

One derivation tree.

Another derivation tree

Two derivation trees exist so $W = a + a*b$ is ambiguous

and the G is also Ambiguous.
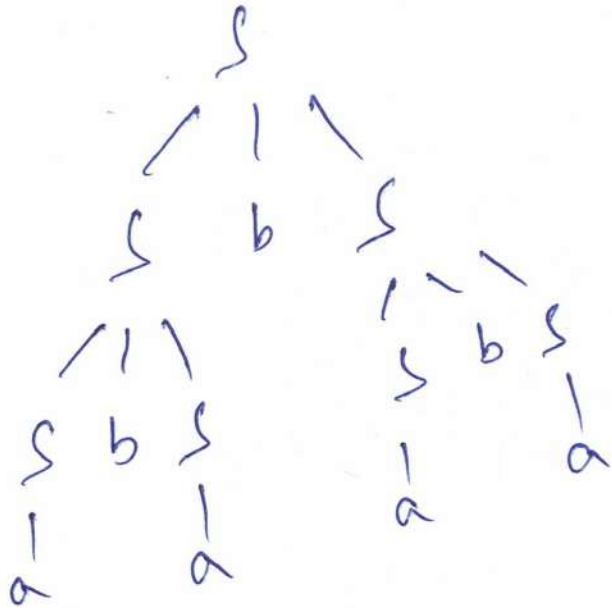
Another Example:-

$$G = (\{s\}, \{a, b\}, P, S)$$

P: $S \rightarrow SbS \mid a$
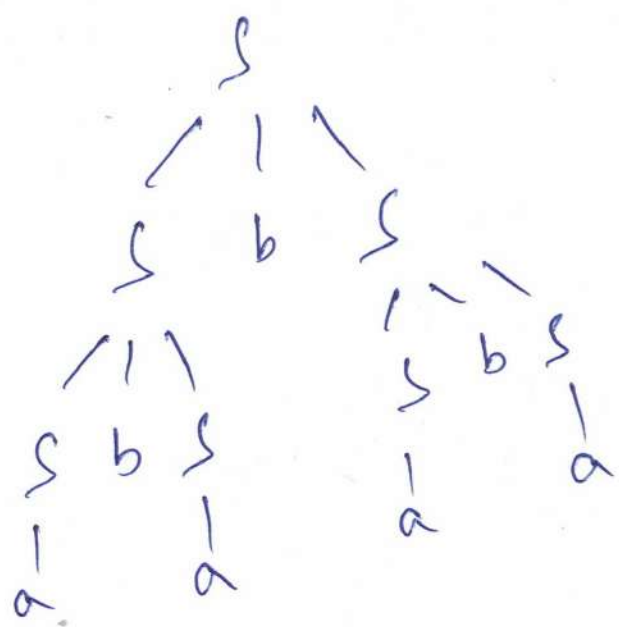
$W = abababa$

One derivation tree

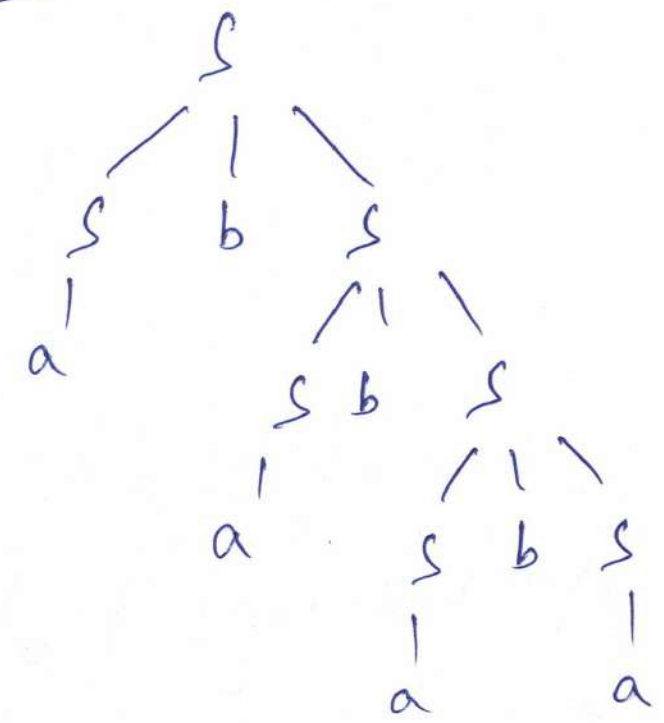**Another Example:-**

$$G = (\{S\}, \{a, b\}, P, S)$$

$$P: \quad S \rightarrow SbS / a \qquad\qquad W = ababab\,a$$

One derivation tree

Another derivation tree

∴ W = abababa is Ambiguous ⟹ G is ambiguous

Simplification of CFG

$$G = (V, T, P, S) \quad \text{starting variable } S \in V$$

Variables   terminals   Productions

$$L(G) = \{ \, w \in T^* \mid S \underset{G}{\overset{*}{\Rightarrow}} w \, \}$$

↳ May be not all symbols $V \cup T$ are used

There is a possibility for Simplification.

Example:

$$G_1 = \left(\{S, A, B, D, E\}, \{a, b, c\}, P, S \right)$$

$$P = \{ S \to AB, \quad A \to a, \quad B \to b, \quad B \to D, \quad E \to c/\epsilon \}$$

$$L(G_1) = \{ \omega \in T^* \mid S \underset{G}{\overset{*}{\Rightarrow}} \omega \} = \{ab\}$$

Example:-

$$G = (\{S, A, B, D, E\}, \{a, b, c\}, P, S)$$

$$P = \{S \rightarrow AB, \quad A \rightarrow a, \quad B \rightarrow b, \quad \boxed{B \rightarrow D}, \quad \boxed{E \rightarrow c / \epsilon}\}$$

$$L(G) = \{w \in T^* / S \underset{G}{\overset{*}{\Rightarrow}} w\} = \{ab\}$$

derivation

$$S \Rightarrow AB \Rightarrow aB \Rightarrow ab$$

$$G' = (V', T', P', S)$$

Remove Revamp V, T, P which are not useful.

$$V' = \{S, A, B\}, \quad T' = \{a, b\}$$

$$P' = \text{remove red colored from } P$$

$$= \{S \rightarrow AB, \quad A \rightarrow a, \quad B \rightarrow b\}.$$