Example: find a CFG for all binary strings with

L {even number of 0's        $\Sigma = \{0, 1\}$

$L = \{ \epsilon, 00, 010, 001,^{100} - - etc \}$

Solut:-

Cases

1) 1st symbol start with 1, followed by even number of 0's

2) 1st symbol start with 0, then followed by even 0's.

Example: find a CFG for all binary strings with $L$ {even number of 0's}

$\Sigma = \{0,1\}$

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Soluti:-

Cases

1) 1st symbol start with 1, followed by even number of 0's

and having

2) 1st symbol start with 0, ~~then followed by~~ even 0's.

$G = (V, T, P, S)$

$S \rightarrow 1S \mid 0A0S \mid \epsilon$

$A \rightarrow 1A \mid \epsilon$

$L = \{ w \in (0,1)^* / w \text{ has even number of } 0\text{'s} \}$

$G_1 = (\{S,A\}, \{0,1\}, P_1, S\}$

$P_1 : S \rightarrow 1S / 0A0S / \epsilon$

$A \rightarrow 1A / \epsilon$

$$L = \{ w \in (0,1)^* \mid w \text{ has even number of } 0's \}$$

$$G_1 = ( \{S, A\}, \{0,1\}, P_1, S )$$

$P_1 :$   $S \rightarrow 1S \mid 0A0S \mid \epsilon$

$A \rightarrow 1A \mid \epsilon$

$$G_2 = \{ \{S, T\}, \{0,1\}, P_2, S \}$$

$P_2 :$   $S \rightarrow 1S \mid 0T \mid \epsilon$

$T \rightarrow 1T \mid 0S$

$$L(G_1) = L = L(G_2)$$

Two Equivalent CFG's.

# Content Free Grammar

$$G = (V, T, P, S)$$

$$P: \{A \rightarrow \alpha\}$$

$$A \in V, \quad \alpha \in (V \cup T)^*$$

$$A \in (V \cup T)^*$$

Example:- $L(G) = \{0^n 1^n 2^n \mid n \geq 1\}$

$G = (\{S, A\}, \{0, 1, 2\}, P, S)$

$P:$

$S \rightarrow 0SA2$

$S \rightarrow 012$

$2A \rightarrow A2$

$1A \rightarrow 11$

Example:- $L(G) = \{0^n 1^n 2^n \mid n \geq 1\}$

$G = (\{S, A.\}, \{0, 1, 2\}, P, S)$

P:  $S \rightarrow 0SA2$

$S \rightarrow 012$

$2A \rightarrow A2$

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$S \rightarrow 012 \in L(G)$

$S \rightarrow 0\textcircled{S}A2$

$S \rightarrow 0\ 012\ \textcircled{2\ A}2$

$S \rightarrow 00\ \textcircled{1\ A}22$

$S \rightarrow 00\ 11\ 22$

# Derivation tree / Parse tree

$G = (V, T, P, S)$, a Parse tree for $G$ ~~for~~ if :-

(i) Every vertex has label which is symbol of $V \cup T \cup \{\epsilon\}$

(ii) root is $S$.

(iii) If a vertex is interior and has label by $A \in V$.
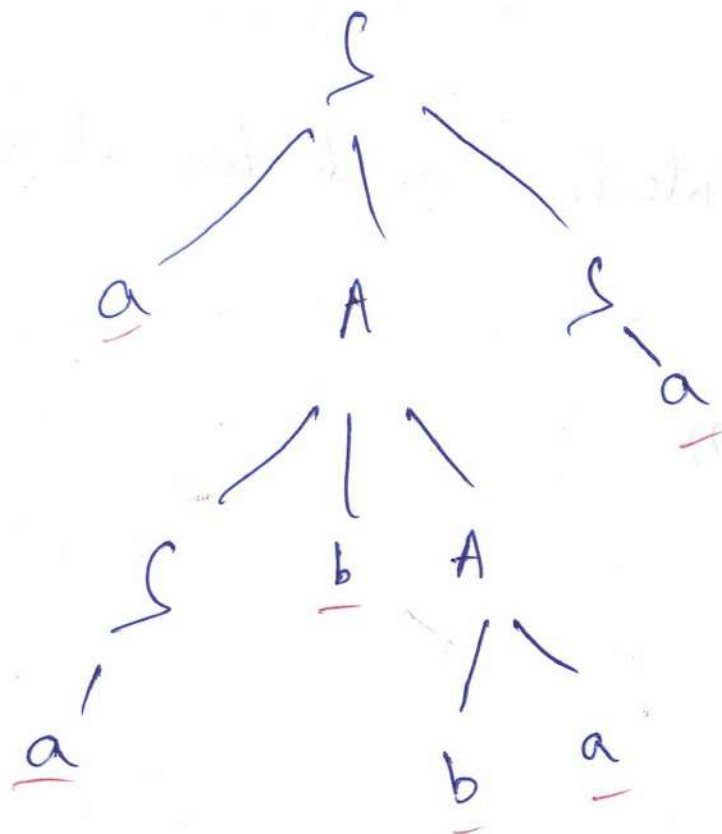
(iv) $A \rightarrow X_1 X_2 \cdots X_n$.

$\mathcal{R}$

Example    $G = (\{S, A\}, \{a, b\}, P, S)$    ⑰

$$P: \quad S \longrightarrow aAS \mid a$$

$$A \longrightarrow SbA \mid SS \mid ba$$

Parse tree



yields

aabbaa

Parse tree of this string

aabbaa

yield $\in (V \cup T)^*$

yield string

$$\alpha \in (V \cup T)^*$$

$$S \xrightarrow[G]{*} \alpha \quad \text{iff we have a derivation tree}$$
$$\text{that yield } \alpha.$$

---

Example: P:  $S \longrightarrow aAS \,/\, a$

$A \longrightarrow SbA \,/\, SS \,/\, ba$

---

$S \Longrightarrow aAS \Longrightarrow aSSS \Longrightarrow aSbaS \Longrightarrow aSbAbaS$

$$\alpha = a\underline{S}b\underline{A}baS \in (V \cup T)^*$$

Example $\qquad$ $G = (\{S, A\}, \{a, b\}, P, S)$

$p:\quad S \to aAS \,/\, a$ $\qquad\qquad$ $\alpha = \underline{aabbaa}$

$\qquad\quad A \to SbA \,/\, SS \,/\, ba$

Leftmost derivation ??

Example $\qquad$ $G = ( \{S, A\}, \{a, b\}, P, S)$

$p: \quad S \longrightarrow aAS \mid a$ $\qquad$ $\alpha = aa\,\underline{bb}\,aa$

$\qquad A \rightarrow SbA \mid SS \mid ba$

Leftmost derivation ??

$$S \Rightarrow a\,\underline{A}\,S \Rightarrow a\,\underline{S}\,bAS \Rightarrow aab\underline{A}S \Rightarrow aabba\,\underline{S}$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\boxed{aabbaa}$ ↙

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\alpha$.

$$S \overset{*}{\underset{G}{\Rightarrow}} aabbaa$$

Example :-  $\quad G = (\{S, A\}, \{a, b\}, P, S)$  (21)

$$P: \quad S \rightarrow aAS \mid a$$

$$A \rightarrow SbA \mid SS \mid ba \qquad \alpha = aabbaa$$

Right most derivation ??

Example:- $G = (\{S, A\}, \{a, b\}, P, S)$

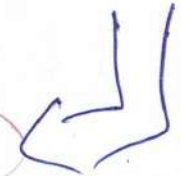$$P: S \rightarrow aAS | a$$
$$A \rightarrow SbA | SS | ba$$

$\alpha = aabbaa$

Right most derivation ??

$$S \Rightarrow aA\underline{\underline{S}} \Rightarrow aA\underline{\underline{a}} \Rightarrow aSb\underline{\underline{A}}a \Rightarrow a\underline{\underline{S}}bbaa$$

$$aabbaa$$

$\alpha$.

$$S \overset{*}{\underset{G.}{\Rightarrow}} aabbaa$$