Topic 9: Homography

Dr. V Masilamani
masila@iiitdm.ac.in

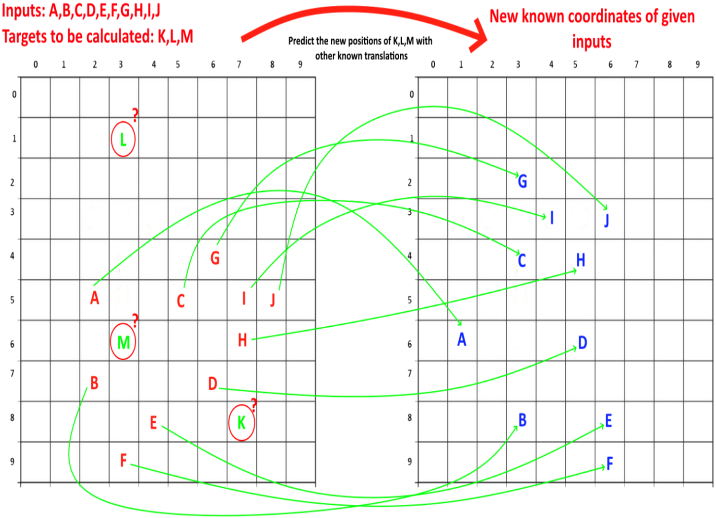
Department of Computer Science and Engineering
IIITDM Kancheepuram
Chennai-127

1. What is homography
2. Does homography exist?
3. Application of Homography -Image Mosaic
4. Image Rectification using homography
5. How to find homography

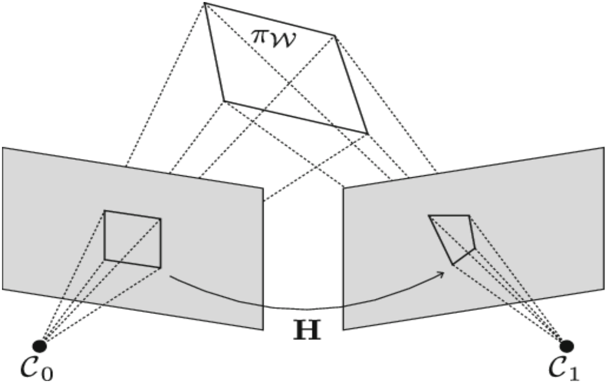
What is Homography

- 2D-projective transformation from 2D point x to 2D point x' in cartesian CS:
  - Let X and X' be the homogeneous coordinate representations of x and x' respectively
  - If $X' = AX$ and A is invertible, then A called as 2D-projective transformation
- 2D-homography:
  - Given a set of 2D-points x_i and their corresponding points x_i', compute the 2D-projective transformation that maps x_i to x_i'

What is Homography (cont.)



Does homography exist?



Yes, any two images of the same planar surface in space can be related by a homography.

Does homography exist? (cont.)

We know that the camera matrix transforms a point on object in WCS to its corresponding point in PCS as follows $\begin{bmatrix} x \\ y \\ w \end{bmatrix} =$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$u = x/w$
$v = y/w$

If the scene is planar, Assuming Z=0 in WCS, the above equations become

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{14} \\ c_{21} & c_{22} & c_{24} \\ c_{31} & c_{32} & c_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

$u = x/w$
$v = y/w$

Does homography exist? (cont.)

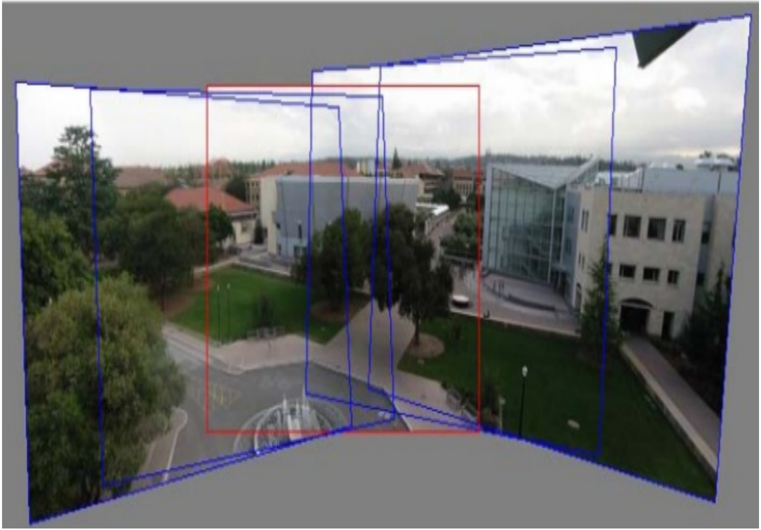
- The camera matrix becomes 3×3 matrix, and also invertible
- Let C_1 and C_2 be camera matrices for left and right camera respectively, and they capture the images of a planar scene
- Let x and x' be corresponding points in left and right images of point p on the scene
- $X = C_1 p$; $X' = C_2 p$
- $C_1^{-1} X = p$
- $X' = C_2 C_1^{-1} X$
- $X' = HX$, where $H = C_2 C_1^{-1}$
- H is he homography Matrix

Does homography exist? (cont.)

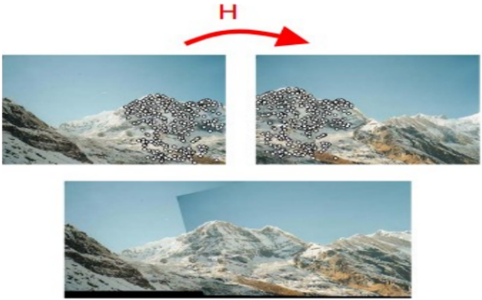
:

- If imaging surface is planar, homography exists.
- The converse of the above need not be true
- If the imaging surface is not planar, how to find the point correspondences
  - Use fundamental matrix

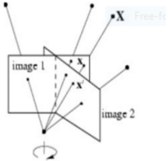
Application of Homography - Mosaic



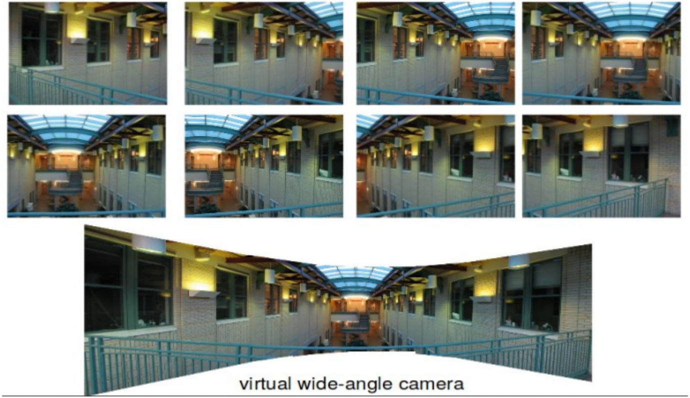
Application of Homography - Mosaic (cont.)



Rotating camera, arbitrary world

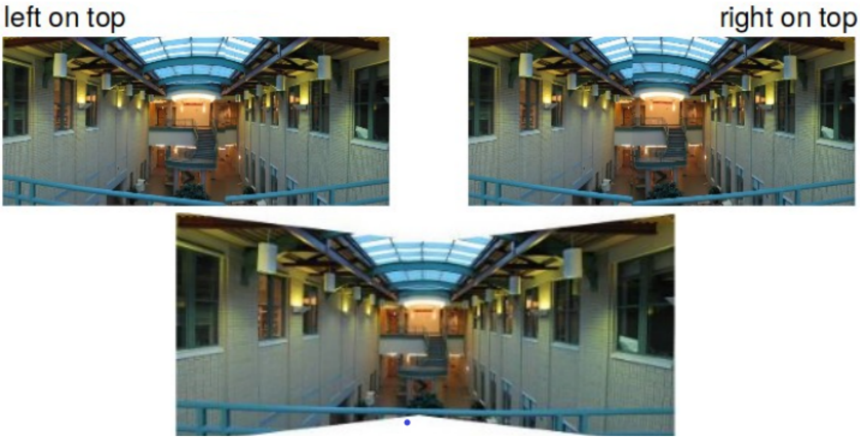


Application of Homography - Mosaic (cont.)



How to align the images

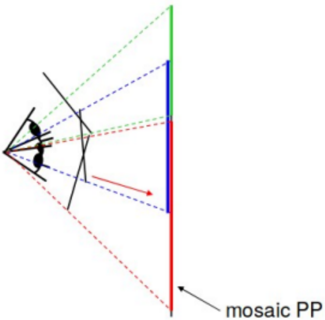
Is the translation sufficient?



How to do Image Mosaic?

- Take a sequence of images from the **same position**
  - Rotate the camera about its optical center
- **Compute Homography** between second image and first
- Transform the second image to **overlap** with the first
- Blend the two images together to create a mosaic
- If there are more images, repeat

Image Reprojection

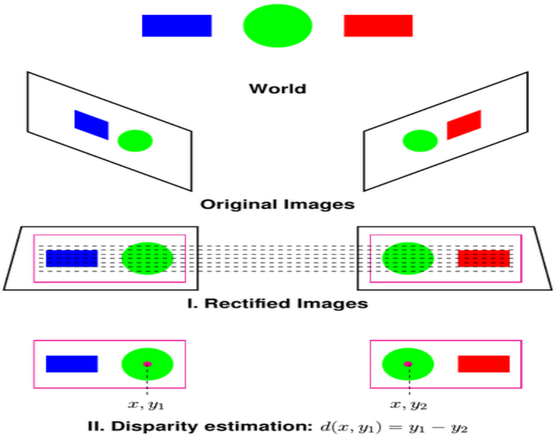


- The mosaic has a natural interpretation of scene
- The images are reprojected onto a common plane
- The mosaic is formed on this plane

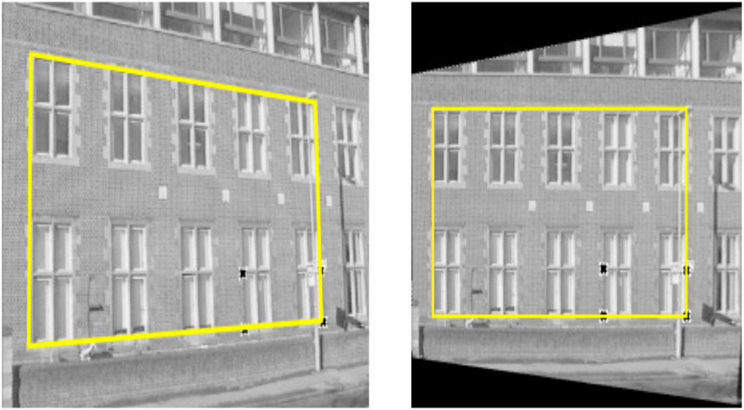
Image Rectification using Homography

- **Image Rectification:** The process of transforming images into images on a same plane
- **Stereo Rectification:** Image Rectification applied to stereo images
- **Perspective Rectification:** Image Rectification applied to image so that the perspective effect(distortion) can be removed

Image Rectification using Homography (cont.)



Perspective Rectification



from Hartley & Zisserman

Image Rectification using Homography (cont.)

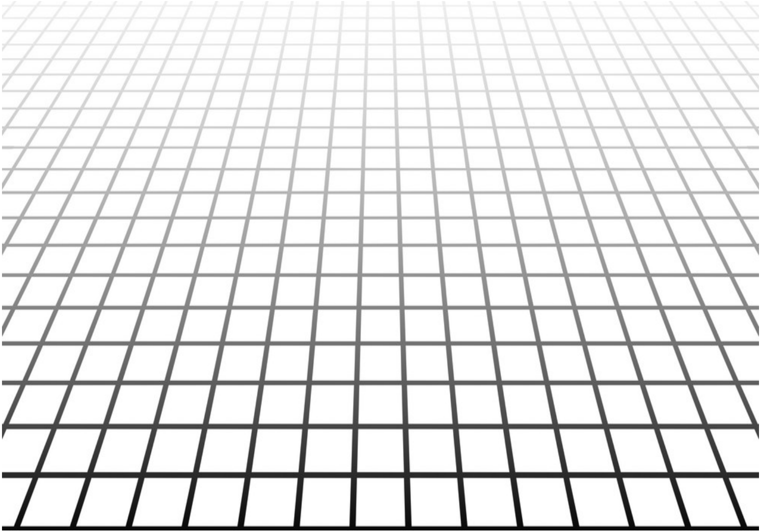


Image Rectification using Homography (cont.)

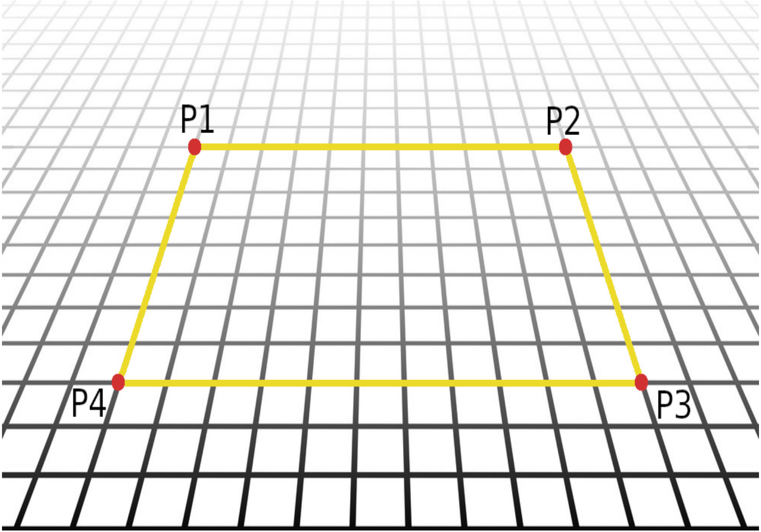


Image Rectification using Homography (cont.)

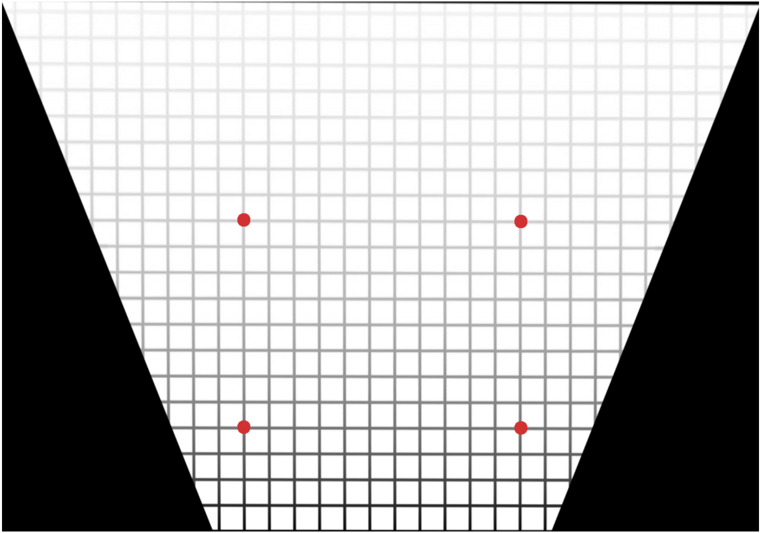


Image Rectification using Homography (cont.)

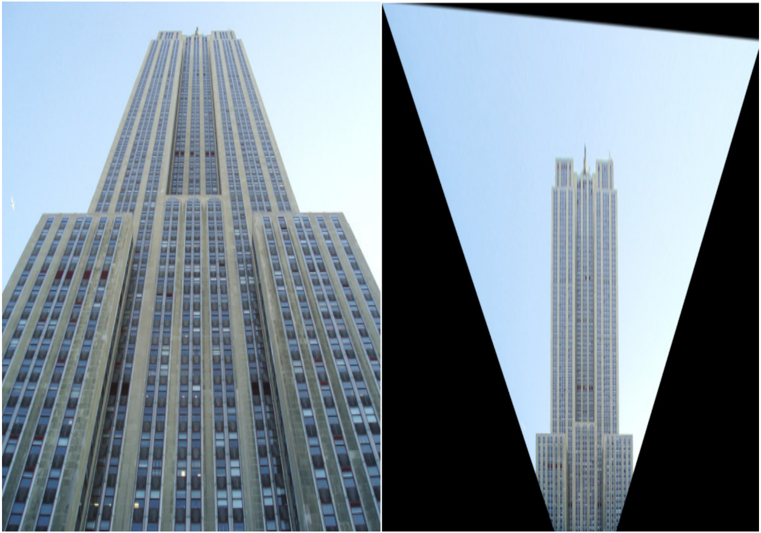


Image Rectification using Homography (cont.)

Image Rectification using Homography (cont.)

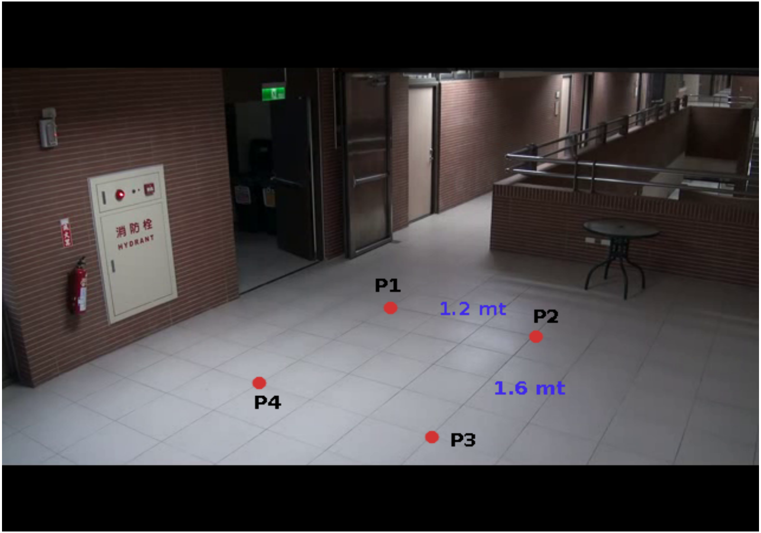
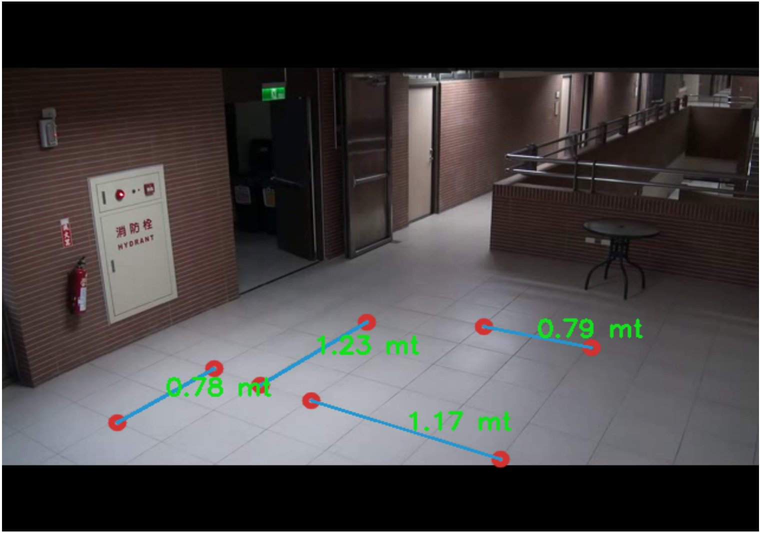


Image Rectification using Homography (cont.)

Image Rectification using Homography (cont.)



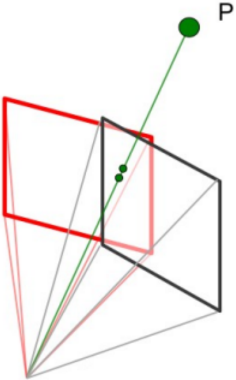
An Exampe of Homography

Two images are related by a homography when

- Both images are taken by the same camera but from a different angle
  - Camera is rotated about its center of projection without any translation

How to find homography?

Find homography, when the same camera is used to take image of the planar surface, after rotating the camera



- If world plane coordinate is P, then
- $x = A\ P$ and $x' = A'\ P$. (ie A'=AR)
- $x' = A'\ A^{-1}\ x.(sub : P = A^{-1}x)$
- $x' = ARA^{-1}x$
- $x' = H\ x$, where $H = ARA^{-1}$

Find homography when CCS1 is WCS and CCS2 is rota from CCS1

- If there is no translation $x' = R\,x$
- projection equation for x and x'

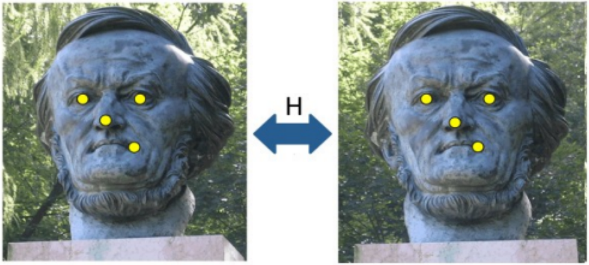
$$x = K\begin{bmatrix} I \mid 0 \end{bmatrix}\begin{bmatrix} X \\ 1 \end{bmatrix} = K\,X$$

$$\mathrm{x'} = \mathrm{K}\begin{bmatrix} R \mid 0 \end{bmatrix}\begin{bmatrix} X \\ 1 \end{bmatrix} = \mathrm{K\;R\;X}$$

- So $x' = K\;R\;K^{-1}\;x$ where K is intrinsic matrix
- Where $K\;R\;K^{-1}$ is a 3 by 3 matrix H called a homography

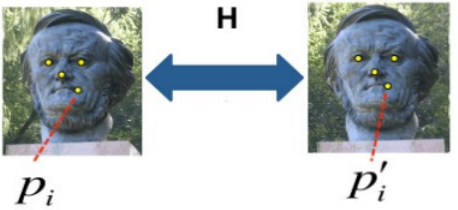
Computing homography when no information about camera is known

Estimate the homographic tranformation between two images



Assumption: Given a set of corresponding points.

Homography H?



$$p'_i = H \, p_i$$

Computing Homography

$$\begin{bmatrix} x'' \\ y'' \\ s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x'' / s$$
$$y' = y'' / s$$

$$\begin{bmatrix} x" \\ y" \\ s \end{bmatrix} = \text{k} \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

will also give the same solution as

$x' = x" / s$

$y' = y" / s$

put $k = 1/h_{33}$

$$\begin{bmatrix} x" \\ y" \\ s \end{bmatrix} = \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where $h'_{ij} = h_{ij}/h_{33}$

will also give the same solution as

$x' = x" / s$

$y' = y" / s$

The equations can be rewritten as

- $sx' - x'' = 0$
- $sy' - y'' = 0$

Sub value of s

- $(xh'_{31} + yh'_{32} + 1)x' - xh'_{11} - yh'_{12} - h'_{13} = 0$
- $(xh'_{31} + yh'_{32} + 1)y' - xh'_{21} - yh'_{22} - h'_{23} = 0$

which can be rewritten as

- $(xx'h'_{31} + yx'h'_{32} + x') - xh'_{11} - yh'_{12} - h'_{13} = 0$
- $(xy'h'_{31} + yy'h'_{32} + y') - xh'_{21} - yh'_{22} - h'_{23} = 0$

which can be rewritten as

- $-xx'h'_{31} - yx'h'_{32} + xh'_{11} + yh'_{12} + h'_{13} = x'$
- $-xy'h'_{31} - yy'h'_{32} + xh'_{21} + yh'_{22} + h'_{23} = y'$

For the n corresponding points (p_i, p_i') we get

$$
\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1 x_1' & -y_1 x_1' \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1 y_1' & -y_1 y_1' \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2 x_2' & -y_2 x_2' \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2 y_2' & -y_2 y_2' \\
x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3 x_3' & -y_3 x_3' \\
0 & 0 & 0 & x_3 & y_3 & 1 & -x_3 y_3' & -y_3 y_3' \\
x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4 x_4' & -y_4 x_4' \\
0 & 0 & 0 & x_4 & y_4 & 1 & -x_4 y_4' & -y_4 y_4' \\
 & & & & . & & & \\
 & & & & . & & & \\
 & & & & . & & & \\
 & & & & . & & &
\end{bmatrix}
\begin{bmatrix}
h'_{11} \\ h'_{12} \\ h'_{13} \\ h'_{21} \\ h'_{22} \\ h'_{23} \\ h'_{31} \\ h'_{32}
\end{bmatrix}
=
\begin{bmatrix}
x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \\ x_4' \\ y_4' \\ . \\ . \\ . \\ .
\end{bmatrix}
$$

Computing Homography (cont.)

Linear equation $A\ h\ =\ b$

Solve:

$$A^T\ A\ h\ =\ A^T\ b$$

$$(\mathrm{A}^T\ A)\ h\ =\ (A^T\ b)$$

$$\mathrm{h}\ =\ (\mathrm{A}^T\ A)^{-1}\ (A^T\ b)$$