Chapter 10

Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Summary

For further details please refer the book

Introduction:

- Returns on a security come from two sources:
  - Common macro-economic factor
  - Firm specific events
- Possible common macro-economic factors
  - Gross Domestic Product growth
  - Interest rates

Single factor model:

$R_i = E(R_i) + \beta_i F + e_i$

R_i = Excess return on security

β_i= Factor sensitivity or factor loading or factor beta

F = Surprise in macro-economic factor

(F could be positive or negative but has expected value of zero)

e_i = Firm specific events (zero expected value)

- Use more than one factor:
  - Examples: Market Return, GDP, Expected Inflation, Interest Rates
  - Estimate a beta or factor loading for each factor using multiple regression

Multifactor Model Equation

$R_i = E\left(R_i\right) + \beta_{iGDP} GDP + \beta_{iIR} IR + e_i$

R_i = Excess return for security i

β_{GDP} = Factor sensitivity for GDP

β_{IR} = Factor sensitivity for Interest Rate

e_i = Firm specific events

Interpretation: The expected return on a security is the sum of:

1. The risk-free rate
2. The sensitivity to GDP times the GDP risk premium
3. The sensitivity to interest rate risk times the interest rate risk premium

Arbitrage Pricing Theory:

- Arbitrage occurs if there is a zero investment portfolio with a sure profit
    - No investment → investors create large positions to obtain large profits
    - All investors will want an infinite position in the risk-free arbitrage portfolio
    - In efficient markets, profitable arbitrage opportunities will quickly disappear
- The Law of One Price:
    - Enforced by arbitrageurs: If they observe a violation they will engage in *arbitrage activity*
    - This bids up (down) the price where it is low (high) until the arbitrage opportunity is eliminated
- **APT and Well-Diversified Portfolios**

$$R_P = E(R_P) + \beta_P F + e_P$$
$$where\ F = Systematic\ Risk$$
$$E(R_P) = \sum w_i E(R_i)$$
$$\beta_P = \sum w_i \beta_i$$
$$e_P = \sum w_i e_i$$

- For a well-diversified portfolio, e_P → 0 as the number of securities increases and their associated weights decrease

No-Arbitrage Equation of APT:

$$E(R_P) = \beta_P E(R_M)$$

Applies to well-diversified portfolios

Establishes that the SML of CAPM applies to well-diversified portfolios

Difference between APT and CAPM

APT	CAPM
• Assumes a well-diversified portfolio, but residual risk is still a factor • Does not assume investors are mean-variance optimizers • Uses an observable market index • Reveals arbitrage opportunities	• Model is based on an inherently unobservable "market" portfolio • Rests on mean-variance efficiency. The actions of many small investors restore CAPM equilibrium

Multifactor APT:

- Use of more than a single systematic factor
- Requires formation of factor portfolios
- Factor analysis (principle component analysis)
- What factors?
    - Factors that are important to performance of the general economy
    - What about firm characteristics? (Refer chapter)

Two-Factor Model:

- The multifactor APT is similar to the one-factor case

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + e_i$$

- Track with diversified factor portfolios:
    - β=1 for one of the factors and 0 for all other factors
    - The factor portfolios track a particular source of macroeconomic risk, but are uncorrelated with other sources of risk

Fama-French Three-Factor Model:

$$R_{it} = \alpha_i + \beta_{iM}R_{Mt} + \beta_{iSMB}SMB_t + \beta_{iHML}HML_t + e_{it}$$

- SMB = Small Minus Big (firm size)
- HML = High Minus Low (book-to-market ratio)
- Are these firm characteristics correlated with actual systematic risk factors? (Refer chapter)

Practice Questions

1. Consider the single factor APT. Portfolio A has a beta of 0.2 and an expected return of 13%. Portfolio B has a beta of 0.4 and an expected return of 15%. The risk-free rate of return is 10%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio __________ and a long position in portfolio __________.
   A. A, A
   B. A, B
   C. B, A
   D. B, B
   E. No arbitrage opportunity exists.

   Answer: A: 13% = 10% + 0.2F; F = 15%; B: 15% = 10% + 0.4F; F = 12.5%; therefore, short B and take a long position in A.

2. Consider a single factor APT. Portfolio A has a beta of 1.0 and an expected return of 16%. Portfolio B has a beta of 0.8 and an expected return of 12%. The risk-free rate of return is 6%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio ___________ and a long position in portfolio ________.
   A. A, A
   B. A, B
   C. B, A
   D. B, B
   E. A, the riskless asset

   Answer: A: 16% = 1.0F + 6%; F = 10%; B: 12% = 0.8F + 6%: F = 7.5%; thus, short B and take a long position in A.

3. Consider the single-factor APT. Stocks A and B have expected returns of 15% and 18%, respectively. The risk-free rate of return is 6%. Stock B has a beta of 1.0. If arbitrage opportunities are ruled out, stock A has a beta of:
   A. 0.67
   B. 1.00
   C. 1.30
   D. 1.69
   E. 0.75

   Answer: A: 18% = 6% + bF; B: 8% = 6% + 1.0F; F = 12%; thus, beta of A = 9/12 = 0.75.

4. Consider the one-factor APT. Assume that two portfolios, A and B, are well diversified. The betas of portfolios A and B are 1.0 and 1.5, respectively. The expected returns on portfolios A and B are 19% and 24%, respectively. Assuming no arbitrage opportunities exist, the risk-free rate of return must be:
   A. 4.0%
   B. 9.0%
   C. 14.0%
   D. 16.5%
   E. 8.2%

   Answer: A: 19% = r_f + 1(F); B:24% = r_f + 1.5(F); 5% = .5(F); F = 10%; 24% = r_f + 1.5(10); r_f = 9%.

5. Consider the multifactor APT. There are two independent economic factors, F_1 and F_2. The risk-free rate of return is 6%. The following information is available about two well-diversified portfolios:

Portfolio	β of F_1	β of F_2	Expected Return
A	1.0	2.0	19%
B	2.0	0.0	12%

   1. Assuming no arbitrage opportunities exist, the risk premium on the factor F_1 portfolio should be:
      A. 3%
      B. 4%
      C. 5%
      D. 6%
      E. 2%

   Answer: 2A: 38% = 12% + 2.0(RP1) + 4.0(RP2); B: 12% = 6% + 2.0(RP1) + 0.0(RP2); 26% = 6% + 4.0(RP2); RP2 = 5; A: 19% = 6% + RP1 + 2.0(5); RP1 = 3%.

   2. Assuming no arbitrage opportunities exist, the risk premium on the factor F_2 portfolio should be:
      A. 3%
      B. 4%
      C. 5%
      D. 6%
      E. 2%

   Answer: 2A: 38% = 12% + 2.0(RP1) + 4.0(RP2); B: 12% = 6% + 2.0(RP1) + 0.0(RP2); 26% = 6% + 4.0(RP2); RP2 = 5; A: 19% = 6% + RP1 + 2.0(5); RP1 = 3%.

6. Consider the single-factor APT. Stocks A and B have expected returns of 12% and 14%, respectively. The risk-free rate of return is 5%. Stock B has a beta of 1.2. If arbitrage opportunities are ruled out, stock A has a beta of:
   A. 0.67
   B. 0.93
   C. 1.30
   D. 1.69
   E. 1.27

   Answer: A: 12% = 5% + bF; B: 14% = 5% + 1.2F; F = 7.5%; Thus, beta of A = 7/7.5 = 0.93.

7. Consider a single factor APT. Portfolio A has a beta of 2.0 and an expected return of 22%. Portfolio B has a beta of 1.5 and an expected return of 17%. The risk-free rate of return is 4%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio and a long position in portfolio:
   A. A, A
   B. A, B
   C. B, A
   D. B, B
   E. A, the riskless asset

   Answer: A: 22% = 2.0F + 4%; F = 9%; B: 17% = 1.5F + 4%: F = 8.67%; thus, short B and take a long position in A.

8. Consider the single factor APT. Portfolio A has a beta of 0.5 and an expected return of 12%. Portfolio B has a beta of 0.4 and an expected return of 13%. The risk-free rate of return is 5%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio ____ and a long position in portfolio _________.
   A. A, A
   B. A, B
   C. B, A
   D. B, B
   E. No arbitrage opportunity exists.

   Answer: A: 12% = 5% + 0.5F; F = 14%; B: 13% = 5% + 0.4F; F = 20%; therefore, short A and take a long position in B.

9. Security A has a beta of 1.0 and an expected return of 12%. Security B has a beta of 0.75 and an expected return of 11%. The risk-free rate is 6%. Explain the arbitrage opportunity that exists; explain how an investor can take advantage of it. Give specific details about how to form the portfolio, what to buy and what to sell.

Answer: An arbitrage opportunity exists because it is possible to form a portfolio of security A and the risk-free asset that has a beta of 0.75 and a different expected return than security B. The investor can accomplish this by choosing .75 as the weight in A and .25 in the risk-free asset. This portfolio would have $E(r_p) = 0.75(12\%) + 0.25(6\%) = 10.5\%$, which is less than B's 11% expected return. The investor should buy B and finance the purchase by short selling A and borrowing at the risk-free asset.

10. In the APT model, what is the nonsystematic standard deviation of an equally-weighted portfolio that has an average value of $\sigma(e_i)$ equal to 25% and 50 securities?
    A. 12.5%
    B. 625%
    C. 0.5%
    <u>D.</u> 3.54%
    E. 14.59%

Answer: Find the solution