

## ECO423A: Financial Economics

### Assignment 1

Date: 25<sup>th</sup> May 2019

Last date of Submission: 14<sup>th</sup> June by midnight (positively)

Total Marks 100

#### Instructions:

1. All questions are compulsory
  2. It is an individual assignment and it is compulsory for everyone
  3. You can submit the assignment handwritten.
  4. Please go through the lecture notes and Bodie, Kane and Marcus (BKM) Chapters 5-10 to understand the problem statement.
  5. Spreadsheet work needs solver and for your convenience, I have attached the user document that explains the usage of Solver. An excel file is also attached for your reference. Please have a look.
  6. Considering the difficulty-adjusted return, all questions carry equal marks. Please try to maximize your difficulty based Sharpe ratio.
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1. Assume that you are considering selecting assets from among the following four candidates:

Asset 1		
Market condition	Return	Probability
Good	12	$\frac{1}{4}$
Average	10	$\frac{1}{2}$
Poor	8	$\frac{1}{4}$

Asset 2		
Market condition	Return	Probability
Good	3	$\frac{1}{4}$
Average	6	$\frac{1}{2}$
Poor	4	$\frac{1}{4}$

Asset 3		
Market condition	Return	Probability
Good	15	$\frac{1}{4}$
Average	11	$\frac{1}{2}$
Poor	4	$\frac{1}{4}$

Asset 4		
Rainfall	Return	Probability
Plentiful	12	$\frac{1}{3}$
Average	10	$\frac{1}{3}$
Light	5	$\frac{1}{3}$

Assume that there is no relationship between the amount of rainfall and the condition of the stock market.

- Solve for the expected return and the standard deviation of return for each separate investment.
- Solve for the correlation coefficient and the covariance between each pair of investments.
- Solve for the expected return and variance of each of the portfolios shown in the following

Portfolio	Asset 1	Asset 2	Asset 3	Asset 4
A	$\frac{1}{2}$	$\frac{1}{2}$		
B	$\frac{1}{2}$		$\frac{1}{2}$	
C	$\frac{1}{2}$			$\frac{1}{2}$
D		$\frac{1}{2}$	$\frac{1}{2}$	
E			$\frac{1}{2}$	$\frac{1}{2}$
F	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
G		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
H	$\frac{1}{3}$		$\frac{1}{3}$	$\frac{1}{3}$
I	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

- Plot the original assets and each of the portfolios from Part C in expected return and standard deviation space.
- A portfolio consists of 4 securities, 1, 2, 3, and 4. The proportions of these securities are:  $w_1=0.3$ ,  $w_2=0.2$ ,  $w_3=0.2$ , and  $w_4=0.3$ . The standard deviations of returns on these securities (in percentage terms) are:  $\sigma_1=5$ ,  $\sigma_2=6$ ,  $\sigma_3=12$ , and  $\sigma_4=8$ . The correlation coefficients among security returns are:  $\rho_{12}=0.2$ ,  $\rho_{13}=0.6$ ,  $\rho_{14}=0.3$ ,  $\rho_{23}=0.4$ ,  $\rho_{24}=0.6$ , and  $\rho_{34}=0.5$ . Assume equal-proportional investment.
    - What is the standard deviation of portfolio return?

- The returns of 4 assets, A, B, C, and D over a period of 5 years have been as follows:

Portfolio	Year 1	Year 2	Year 3	Year 4	Year 5
A	8%	10%	-6%	-1%	9%
B	10%	6%	-9%	4%	11%
C	9%	6%	3%	5%	8%
D	10%	8%	13%	7%	12%

Calculate the expected return on:

- portfolio of one stock at a time
  - portfolios of two stocks at a time
  - portfolios of three stocks at a time.
  - a portfolio of all the four stocks.
- The required return on the market portfolio is 16 percent. The beta of stock A is 1.6. The required return on the stock is 22 percent. The expected dividend growth on stock A is 12 percent. The price per share of stock A is Rs.260.
    - What is the expected dividend per share of stock A next year?

- b. What will be the combined effect of the following on the price per share of stock?
- The inflation premium increases by 5 percent.
  - The decrease in the degree of risk-aversion reduces the differential between the return on market portfolio and the risk-free return by one-half.
  - The expected growth rate of dividend on stock A decrease to 10 percent.
  - The beta of stock A falls to 1.1
5. You decide to invest your money in a stock portfolio consisting of 60% TATA motors and 40% in Flipkart. Using the data in the following table, you find that TATA has an annual standard deviation of 0.363 and Flipkart 0.34. The correlation coefficient between the returns of both stock is 0.34.
- Calculate the variance and standard deviation of this portfolio.
  - Calculate the relative contribution of each stock to this portfolio's variance
  - Calculate the  $\beta$  of each stock relative to this two-stock portfolio. Check your results
  - Now calculate portfolio risk (standard deviation) and return using some different values for the weights and plot the results in risk-return space. Using the given table.
  - The graph you plotted under (d) gives you a good idea what the minimum variance portfolio looks like, but can you calculate the properties exactly? What weights give the portfolio its minimum variance? What are this portfolio's standard deviation and return?
  - How would the graph under (d) look if TATA and Flipkart were perfectly positively correlated?

	Correlation Matrix							
	Factset	Tata	ABC	Flipkart	TCS	Return	Ann. St.dev	weight
Factset	1					0.08	0.287	0.1
TATA	0.43	1				0.075	0.363	0.1
ABC	0.34	0.28	1			0.06	0.462	0.6
Flipkart	0.55	0.34	0.35	1		0.125	0.340	0.1
TCS	0.62	0.43	0.39	0.58	1	0.10	0.250	0.1

6. Assume that you are assigned the task of evaluating the stock of Reliance Industries. To evaluate stock, you calculate its required return using the CAPM. The following information is available:
- Expected market risk premium      5%
- Risk-free rate                              2%
- Reliance industry' beta                1.2
- Using CAPM, calculate and interpret the expected return of reliance industries.

7. Suppose Godrej has the beta of 0.75 and an expected return of 13%. The risk free rate is 4%. Calculate the market risk premium and the expected return on the market portfolio.
8. Suppose you have two portfolios A & B. A has 300 stocks which are worth Rs. 10/stocks. Portfolio B has 50 stock which are worth Rs. 40/stock. You expect a return of 8% for stock A and a return of 13% for stock B.
- (a). What is the total value of the portfolio, what are the portfolio weights and what is the expected return?
- (b) Suppose stocks in portfolio A's price rise up to Rs 12 and portfolio B's stocks price fall to Rs. 36. What is the new value of the portfolio? What return did it earn? After the price change, what are the new portfolio weights?
9. Consider a portfolio of two stocks.

Stock	Expected return	Volatility
Stock X	15%	40%
Stock Y	7%	30%

Let  $w$  denote the weight on Stock A and  $1 - w$  denote the weight on Stock B. Correlation coefficient equals  $\rho_{X,Y}$ .

- (a). Write down a mathematical expression for the portfolio's mean return and volatility (standard deviation) as a function of  $w$ .
- (b). What is the portfolio's mean return and volatility when  $w = 0.4$  if  $\rho_{X,Y} = 0$ ?  $\rho_{X,Y} = +1$ ?  $\rho_{X,Y} = -1$ ?
- (c). Suppose  $\rho_{X,Y} = -1$ ? Are there portfolio weights that will result in a portfolio with no volatility? If so, what are the weights?
10. Suppose you are hired buy a Hedge Fund firm and the firm has asked to give the buy/sell recommendation using CAPM. You derive the following information for the broad market and for the stock of ICICI bank.

Expected market risk premium	8%
Risk free rate	5%
Historical beta for ICICI	1.50

You believe that the historical betas do not provide good forecasts of future beta, and therefore uses the following formula to forecast beta.

$$\text{Forecasted beta} = 0.80 + 0.20 * \text{historical beta}$$

After conducting a thorough examination of market trends and the ICICI financial statements, you predict that the ICICI return will equal 10%. You should now derive the following required return for ICICI along with the following valuation decision:

Valuation

CAPM Required Return

- i. Overvalued 8.3%
- ii. Overvalued 13.8%
- iii. Undervalued 8.3%
- iv. Undervalued 13.8%

11. Suppose we have portfolio of two stocks. We assume that the risk free rate is 3%.

Stock	Expected return	Volatility
Stock X	15%	40%
Stock Y	7%	30%

- a. What is the minimum variance portfolio when  $\rho_{X,Y} = 0$ ? What is its expected return and volatility?
- b. What is the minimum variance portfolio when  $\rho_{X,Y} = 0.4$ ? What is its expected return and volatility?
- c. What is the minimum variance portfolio when  $\rho_{X,Y} = -0.4$ ? What is its expected return and volatility?
- d. Determine the tangent portfolios and their respective mean returns and volatilities.

12. An investor had invested Rs.8 million each in DLF and Reliance Industries and Rs. 4 million in BATA, only a week before his untimely demise. As per his WILL this portfolio of stocks were to be inherited by his wife alone. As the partition among the family members had to wait for one year as per the terms of the will, the portfolio of shares had to be maintained as they were for the time being. The WILL had stipulated that the job of administering the estate for the benefit of the beneficiaries and partitioning it in due course was to be done by the reputed firm of Chartered Accountants, Menon Brothers. Meanwhile the widow of the deceased was very eager to know certain details of the securities and had asked the senior partner of Menon Brothers to brief her in this regard.

For this purpose, the senior partner has asked you to prepare a detailed note to him with calculations using CAPM, to answer the following possible doubts.

- a. What is the expected return and risk (standard deviation) of the portfolio?
- b. What is the scope for appreciation in market price of the three stocks-are they overvalued or undervalued?

You find that out the three stocks, your firm has already been tracking two viz. DLF (A) and Reliance Industries (B)-their betas being 1.7 and 0.8 respectively. Further, you have obtained the following historical data on the returns of Bata (BA):

Period	Market Return (%)	Return of Bata
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1	10	14
2	5	8
3	-2	-6
4	-1	4
5	5	10
6	8	11
7	10	15

On the future returns of the three stocks, you are able to obtain the following forecast from a reputed firm of portfolio managers.

State of the economy	Probability	Treasury bills (%)	DLF (%)	Reliance Industries (%)	BATA (%)	BSE-Sensex
Recession	0.3	7	5	15	-10	-2
Normal	0.4	7	18	8	16	7
Boom	0.3	7	30	12	24	26

Required: Prepare your detailed note to the senior partner.

13. Suppose you have two stocks,  $P$  and  $Q$ , such that  $\sigma_P = 0.30$ ,  $\sigma_Q = 0.80$ ,  $\bar{R}_P = 0.10$ ,  $\bar{R}_Q = 0.06$  and  $r_f = 0.02$ .

- What is the minimum variance portfolio when  $\rho_{P,Q} = 0$  and what is its volatility?
- What is the minimum variance portfolio when  $\rho_{P,Q} = 0.6$  and what is its volatility?
- What is the minimum variance portfolio when  $\rho_{P,Q} = -0.6$  and what is its volatility?
- Also determine the tangent portfolios and their respective mean returns and volatilities

14. Under the similar scenario of above question, suppose you have the investment exposure in three risky assets whose covariance matrix  $\Sigma$  is

$$\Sigma = \begin{pmatrix} 0.09 & 0.045 & 0.01 \\ 0.045 & 0.25 & 0.06 \\ 0.01 & 0.06 & 0.04 \end{pmatrix}$$

The expected returns are  $\bar{R}_{P1} = 0.11$ ,  $\bar{R}_{P2} = 0.09$ ,  $\bar{R}_{P3} = 0.05$ . The risk-free rate is  $r_f = 0.02$ .

Solve for the minimum variance portfolio using the first-order optimality conditions, i.e., without computing the inverse of the covariance matrix. What is the minimum variance?

15. We assume that the expected return on the tangent portfolio is 10% and its volatility is 40%. The risk-free rate is 2%.
- What is the equation of the Capital Market Line (CML)?
  - What is the standard deviation of an efficient portfolio whose expected return of 8%? How would you allocate Rs.1000 to achieve this position?

16. Take another example and assume that the expected return on the tangent portfolio is 12% and its volatility is 30%. The risk-free rate is 3%.

- a. What is the equation of the Capital Market Line (CML)?
  - b. What is the standard deviation of an efficient portfolio whose expected return of 16.5%? How would you allocate Rs. 3000 to achieve this position?
17. Suppose you are planning to invest in stock market with the assumption that the market premium will be around 9%, market volatility will be 30% and the risk-free rate is hovering around 3%.
  - a. Given your assumption, could you please write-down the equation of the SML?
  - b. Suppose a stock invested has a beta of 0.6. According to the CAPM, what is its expected return?
  - c. That same stock has a volatility of 60% and a correlation with the market portfolio of 25%. According to the CAPM, what is its expected return?
  - d. Under another scenario assume that stock invested has a volatility of 80% and a correlation with the market portfolio of -25%. According to the CAPM, what is its expected return?
18. Suppose the stock invested (call it X) in the above questions has a beta of 1.20 and you buy another stock Y which has a beta of 0.8. Suppose  $r_f = 2\%$  and  $\bar{R}_M = 12\%$ .
  - a. Following CAPM, what are the expected returns for each stock?
  - b. What is the expected return of an equally weighted portfolio of these two stocks?
  - c. What is the beta of an equally weighted portfolio of these two stocks?
  - d. How can you use your answer to part (c) to answer part (b)?
19. Suppose you are considering two risky assets for investment, X and Y, and a risk-free asset. The two risky assets are in equal supply in the market, i.e., the market portfolio  $M = 0.5X + 0.5Y$ . It is known that  $\bar{R}_M = 11\%$ ,  $\sigma_X = 20\%$ ,  $\sigma_Y = 40\%$  and  $\sigma_{X,Y} = 0.75$ . The risk-free rate is 2%. Assume CAPM holds.
  - a. What is the beta for each stock?
  - b. What are the values for  $\bar{R}_X$  and  $\bar{R}_Y$ ?
20. Suppose we have two assets, X and Y, and a risk-free asset. Stock X has 200 shares outstanding, a price per share of Rs. 3.00, an expected return of 16% and a volatility of 30%. Stock Y has 300 shares outstanding, a price per share of Rs. 4.00, an expected return of 10% and a volatility of 15%. The correlation coefficient  $\rho_{X,Y} = 0.4$ . Assume CAPM holds.
  - a. What is expected return of the market portfolio?
  - b. What is volatility of the market portfolio?
  - c. What is the beta of each stock?
  - d. What is the risk-free rate?
21. Now assume two mutually exclusive portfolios of growth or value stocks (please read about growth and value stocks). Suppose the growth stock portfolio and value stock portfolio have equal size in terms of total value. Furthermore, suppose that the expected return of the value stocks is 13% with a volatility of 12%, whereas the expected return of the growth stocks is 17% with a volatility of 25%. The correlation of the returns of these two portfolios is 0.50. The risk-free rate is 2%.
  - a. What is the expected return and volatility of the market portfolio (which is a 50-50 combination of the two portfolios)?

- b. Does CAPM hold in this economy?
22. Suppose you are working as an Investment Analyst in an investment firm and you have been assigned one client. Your client has decided to invest in exactly one of two risky funds,  $P_1$  and  $P_2$ . He comes to you for an investment advice. Whichever fund you recommend he will combine it with the risk-free asset. Expected returns are  $\bar{R}_{P_1} = 13\%$  and  $\bar{R}_{P_2} = 18\%$ . Assume the risk-free rate is  $4\%$ . Volatilities are  $\sigma_{P_1} = 20\%$  and  $\sigma_{P_2} = 30\%$ . Without knowing your client's tolerance for risk, which fund would you recommend?
23. Suppose you are now investor and you have hired a wealth manager who recommends you to invest in Mutual Fund M. It has an expected return of  $14\%$  with a volatility of  $20\%$ . The risk-free rate is  $3.8\%$ . Your Wealth Manager suggests you to add Stock B to your portfolio with a positive weight. Stock B has an expected return of  $20\%$ , a volatility of  $60\%$  and a correlation of  $0$  with Fund M.
- Is your wealth manager right?
  - You follow your broker's advice and make a substantial investment in Stock B so that now  $60\%$  is in Fund M and  $40\%$  is in Stock B. You tell your friend about your investment and he says you made a mistake and should reduce your investment in Stock B. Is your friend right?
  - You decide to follow your friend's advice and reduce your exposure to Stock B. Now Stock B represents only  $15\%$  of your risky portfolio with the rest invested in Fund M. Is the correct amount to hold of Stock B?
24. Suppose now you are a trained investor and have standalone exposure in mutual funds which has strong exposure to infrastructure (Say DSP Blackrock) and have investment in risk free asset as well. The mutual funds (NAVs) have an expected return of  $12\%$  and a volatility of  $25\%$ . The risk-free rate is  $4\%$ . Your old wealth manager is still in touch with you and suggests you to add another mutual funds to your current portfolio (say SBI Magnum). The Magnum has an expected return of  $20\%$ , a volatility of  $80\%$  and a correlation of  $0.2$  with the DSP mutual funds.
- Is your Wealth Manager (WM) right?
  - Suppose you follow your WM's advice and put  $50\%$  of your money in the SBI fund. (You sell  $50\%$  of your value of the DSP). What is the Sharpe ratio of your new portfolio?
  - What is the optimal fraction of your wealth to invest in the SBI fund?
25. Download the monthly data (at-least five years) of five stocks of your choice from Yahoo Finance and calculate the following:
- Calculate the minimum variance portfolio with optimal weights by considering only two stocks. Also, draw the efficient frontier of the same.
  - Find the optimal weights and maxim Sharpe ratio by considering all five stocks portfolio and draw the efficient frontier.

\*\*\*\*\*THE END\*\*\*\*\*