

Submission Deadline: August 14, 2024, 4:30 PM

1. Implement the acceptance rejection method to generate samples from a distribution with PDF

$$f(x) = 20x(1-x)^3 \quad \text{for } 0 < x < 1.$$

Please use the smallest value of c such that $f(x) \leq cg(x)$ for your choice of g .

- (a) What is the average of number of iterations needed to generate a random number from above PDF and why?
- (b) Generate 10000 random numbers from the distribution. Compute the sample mean and compare it with (theoretical) expectation of the PDF f .
- (c) What is the approximate value of $P(0.25 \leq X \leq 0.75)$ based on the generated sample in the part (b)? What is the exact value of the probability? Compare them.
- (d) Keep a count of number of iterations needed to generate each of the random numbers in part (b). Compute the average of all these values and compare it with the value obtained in part (a).
- (e) Draw the histogram of the sample obtained in part (b). Also, draw the PDF f on the same plot. Compare them.
- (f) Repeat parts (a)–(d) above with two values of c higher than the smallest value that you had chosen earlier. What are your observations ?

2. Implement acceptance rejection method to generate random number from the PDF

$$f(x) \propto x^{\alpha-1}e^{-x} \quad \text{for } 0 < x < 1.$$

- (a) Generate 10000 random numbers form the above PDF with $\alpha = 0.7$. Please mention the rejection constant and dominating PDF in the report.
 - (b) Generate 10000 random numbers form the above PDF with $\alpha = 3$.
 - (c) Generate 10000 random numbers form the above PDF with $\alpha = 3.7$.
 - (d) Calculate the sample mean and variance in each of the above cases and report.
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