CHAPTER ٤

LINEAR EQUATIONS IN TWO VARIABLES

The principal use of the Analytic Art is to bring Mathematical Problems to Equations and to exhibit those Equations in the most simple terms that can be.

—Edmund Halley

٤.١ Introduction

In earlier classes، you have studied linear equations in one variable. Can you write down a linear equation in one variable؟ You may say that x + ١ = ٠، x + √ ٢ = ٠ and √٢٣y+ = ٠ are examples of linear equations in one variable. You also know that such equations have a unique (i.e.، one and only one) solution. You may also remem how to represent the solution on a number line. In this chapter، the knowledge of lir equations in one variable shall be recalled and extended to that of two variables. You will be considering questions like: Does a linear equation in two variables have a solution؟ If yes، is it unique؟ What does the solution look like on the Cartesian plane؟ You shall also use the concepts you studied in Chapter ٣ to answer these questions.

٤.٢ Linear Equations

Let us first recall what you have studied so far. Consider the following equation:

$$٢x + ٥ = ٠$$

Its solution، i.e.، the root of the equation، is $-\frac{٥}{٢}$. This can be represented on the number line as shown below:

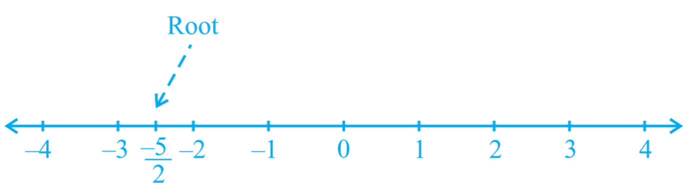

Fig. ٤.١

While solving an equation، you must always keep the following points in mind:

The solution of a linear equation is not affected when:

(i) the same number is added to (or subtracted from) both the sides of the equation

(ii) you multiply or divide both the sides of the equation by the same non-zero number.

Let us now consider the following situation:

In a One-day International Cricket match between India and Sri Lanka played in Nagpur، two Indian batsmen together scored ١٧٦ runs. Express this information in t form of an equation.

Here، you can see that the score of neither of them is known، i.e.، there are two unknown quantities. Let us use x and y to denote them. So، the number of runs scor by one of the batsmen is x، and the number of runs scored by the other is y. We know that

$$x + y = ١٧٦،$$

which is the required equation.

This is an example of a linear equation in two variables. It is customary to denote the variables in such equations by x and y، but other letters may also be used. Some examples of linear equations in two variables are:

$$١.٢s + ٣t = ٥،\ p + ٤q = ٧،\ \pi u + ٥v = ٩ \text{ and } ٣ = \sqrt{٢}x - ٧y.$$

Note that you can put these equations in the form $١.٢s + ٣t - ٥ = ٠$، $p + ٤q - ٧ = ٠$، $\pi u + ٥v - ٩ = ٠$ and $\sqrt{٢}x - ٧y - ٣ = ٠$، respectively.

So، any equation which can be put in the form $ax + by + c = ٠$، where a، b and c are real numbers، and a and b are not both zero، is called a linear equation in two variables. This means that you can think of many many such equations.

Example ١ : Write each of the following equations in the form $ax + by + c = ٠$ and indicate the values of a، b and c in each case:

(i) $٢x + ٣y = ٤.٣٧$ (ii) $x - ٤ = \sqrt{٣}y$ (iii) $٤ = ٥x - ٣y$ (iv) $٢x = y$

Solution : (i) $٢x + ٣y = ٤.٣٧$ can be written as $٢x + ٣y - ٤.٣٧ = ٠$. Here $a = ٢$، $b = ٣$ and $c = -٤.٣٧$.

(ii) The equation $x - ٤ = \sqrt{٣}y$ can be written as $x - \sqrt{٣}y - ٤ = ٠$. Here $a = ١$، $b = -\sqrt{٣}$ and $c = -٤$.

(iii) The equation $٤ = ٥x - ٣y$ can be written as $٥x - ٣y - ٤ = ٠$. Here $a = ٥$، $b = -٣$ and $c = -٤$. Do you agree that it can also be written as $-٥x + ٣y + ٤ = ٠$؟ In this case $a = -٥$، $b = ٣$ and $c = ٤$.

(iv) The equation $٢x = y$ can be written as $٢x - y + ٠ = ٠$. Here $a = ٢$، $b = -١$ and $c = ٠$.

Equations of the type $ax + b = ٠$ are also examples of linear equations in two variables because they can be expressed as

$$ax + ٠.y + b = ٠$$

For example، $٤ - ٣x = ٠$ can be written as $-٣x + ٠.y + ٤ = ٠$.

Example ٢ : Write each of the following as an equation in two variables:

(i) $x = -٥$ (ii) $y = ٢$ (iii) $٢x = ٣$ (iv) $٥y = ٢$

Solution : (i) $x = -٥$ can be written as $١.x + ٠.y = -٥$، or $١.x + ٠.y + ٥ = ٠$.

(ii) $y = ٢$ can be written as $٠.x + ١.y = ٢$، or $٠.x + ١.y - ٢ = ٠$.

(iii) $٢x = ٣$ can be written as $٢x + ٠.y - ٣ = ٠$.

(iv) $٥y = ٢$ can be written as $٠.x + ٥y - ٢ = ٠$.

EXERCISE ٤.١

١. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be ₹x and that of a pen to be ₹y).

٢. Express the following linear equations in the form $ax + by + c = ٠$ and indicate the values of a، b and c in each case:

(i) $٢x + ٣y = ٩.٣\overline{٥}$ (ii) $x - \frac{y}{٥} - ١٠ = ٠$ (iii) $-٢x + ٣y = ٦$ (iv) $x = ٣y$

(v) $٢x = -٥y$ (vi) $٣x + ٢ = ٠$ (vii) $y - ٢ = ٠$ (viii) $٥ = ٢x$

٤.٣ Solution of a Linear Equation

You have seen that every linear equation in one variable has a unique solution. What can you say about the solution of a linear equation involving two variables؟ As there are two variables in the equation، a solution means a pair of values، one for x and one for y which satisfy the given equation. Let us consider the equation $٢x + ٣y = ١٢$. Here، $x = ٣$ and $y = ٢$ is a solution because when you substitute $x = ٣$ and $y = ٢$ in the equation above، you find that

$$٢x + ٣y = (٢ \times ٣) + (٣ \times ٢) = ١٢$$

This solution is written as an ordered pair (٣، ٢)، first writing the value for x and then the value for y. Similarly، (٠، ٤) is also a solution for the equation above.

On the other hand، (١، ٤) is not a solution of ٢x + ٣y = ١٢، because on putting x = ١ and y = ٤ we get ٢x + ٣y = ١٤، which is not ١٢. Note that (٠، ٤) is a solution but not (٤، ٠).

You have seen at least two solutions for ٢x + ٣y = ١٢، i.e.، (٣، ٢) and (٠، ٤). Can you find any other solution؟ Do you agree that (٦، ٠) is another solution؟ Verify the same. In fact، we can get many many solutions in the following way. Pick a value of your choice for x (say x = ٢) in ٢x + ٣y = ١٢. Then the equation reduces to ٤ + ٣y = ١٢، which is a linear equation in one variable. On solving this، you get $y = \frac{٨}{3}$. So $\left(٢، \frac{8}{3}\right)$ is another solution of ٢x + ٣y = ١٢. Similarly، choosing x = – ٥، you find that the equation becomes –١٠ + ٣y = ١٢. This gives $y = \frac{٢٢}{3}$. So، $\left(-5، \frac{٢٢}{3}\right)$ is another solution of ٢x + ٣y = ١٢. So there is no end to different solutions of a linear equation in two variables. That is، a linear equation in two variables has infinitely many solutions.

Example ٣ : Find four different solutions of the equation x + ٢y = ٦.

Solution : By inspection، x = ٢، y = ٢ is a solution because for x = ٢، y = ٢

x + ٢y = ٢ + ٤ = ٦

Now، let us choose x = ٠. With this value of x، the given equation reduces to ٢y = ٦ which has the unique solution y = ٣. So x = ٠، y = ٣ is also a solution of x + ٢y = ٦. Similarly، taking y = ٠، the given equation reduces to x = ٦. So، x = ٦، y = ٠ is a solution of x + ٢y = ٦ as well. Finally، let us take y = ١. The given equation now reduces to x + ٢ = ٦، whose solution is given by x = ٤. Therefore، (٤، ١) is also a solution of the given equation. So four of the infinitely many solutions of the given equation are:

(٢، ٢)، (٠، ٣)، (٦، ٠) and (٤، ١).

Remark : Note that an easy way of getting a solution is to take x = ٠ and get the corresponding value of y. Similarly، we can put y = ٠ and obtain the corresponding value of x.

Example ٤ : Find two solutions for each of the following equations:

(i) ٤x + ٣y = ١٢

(ii) ٢x + ٥y = ٠

(iii) ٣y + ٤ = ٠

Solution : (i) Taking x = ٠، we get ٣y = ١٢، i.e.، y = ٤. So، (٠، ٤) is a solution of the given equation. Similarly، by taking y = ٠، we get x = ٣. Thus، (٣، ٠) is also a solution.

(ii) Taking $x = ٠$, we get $٥y = ٠$, i.e., $y = ٠$. So $(٠, ٠)$ is a solution of the given equation. Now, if you take $y = ٠$, you again get $(٠, ٠)$ as a solution, which is the same as the earlier one. To get another solution, take $x = ١$, say. Then you can check that the corresponding value of y is $-\frac{٢}{٥}$. So $\left(١, -\frac{٢}{٥}\right)$ is another solution of $٢x + ٥y = ٠$.

(iii) Writing the equation $٣y + ٤ = ٠$ as $٠.x + ٣y + ٤ = ٠$, you will find that $y = -\frac{٤}{٣}$ for any value of x. Thus, two solutions can be given as $\left(٠, -\frac{٤}{٣}\right)$ and $\left(١, -\frac{٤}{٣}\right)$.

EXERCISE ٤.٢

١. Which one of the following options is true, and why?

$y = ٣x + ٥$ has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions

٢. Write four solutions for each of the following equations:

(i) $٢x + y = ٧$ (ii) $\pi x + y = ٩$ (iii) $x = ٤y$

٣. Check which of the following are solutions of the equation $x - ٢y = ٤$ and which are not:

(i) $(٠, ٢)$ (ii) $(٢, ٠)$ (iii) $(٤, ٠)$

(iv) $(\sqrt{٢}, ٤\sqrt{٢})$ (v) $(١, ١)$

٤. Find the value of k, if $x = ٢$, $y = ١$ is a solution of the equation $٢x + ٣y = k$.

٤.٤ Summary

In this chapter, you have studied the following points:

١. An equation of the form $ax + by + c = ٠$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.

٢. A linear equation in two variables has infinitely many solutions.

٣. Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph of the linear equation.