Passivity based Energy Shaping Control for Quadruped Robots

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***Abstract*—Real world mechanical systems are composed of several subsystems with complex interconnections between each system. Designing controllers for each subsystem in local position and velocity coordinates to achieve a global objective poses a huge challenge. Further, this approach does not account for the interactions of the system with the environment. Modelling systems through energy exchange with the actuators and environment provides a intuitive way to analyse the underlying structure of the system. Since most mechanical systems dissipate energy through heat or friction, they are passive in nature. Hence, modelling the system using a port Hamiltonian approach and designing controllers using a passivity based control provides an unified framework to design stable controllers to achieve a desired energy level. In this report, a passivity based energy shaping controller using the port Hamiltonian setting is proposed to achieve active damping properties for a quarter car model and a wheeled quadruped robot.**

***Index Terms*—port Hamiltonian, passivity, energy shaping, quarter car, wheeled quadruped**

I. Introduction

Most suspension systems are passive in nature. The system energy dies down to zero eventually in the presence of zero control input. Such systems can be modelled using the energy coordinates through the port Hamiltonian formulation.[1] In passive suspension systems (where the damping is constant) a lot of offline tuning is involved to reduce the bumpiness of the ride. On the other hand, active suspension systems consists of a force controller that is used to inject energy as well as to remove energy (leading a variable damping). From vehicle dynamics perspective, the goal of the suspension system is to minimize the vertical oscillations of the body with respect to the wheel disturbances.

In this report an active suspension system is proposed for on-road and off-road vehicles where a force controller is added to the system to achieve the desired equilibrium positions through energy shaping method exploiting the passivity proprieties of the system[2−4]. Through this approach one can easily shape the Hamiltonian energy of the system to reach a desired equilibrium. We show that the closed loop system is globally asymptotically stable when the input disturbances at the wheel are bounded. Compared to other control methods such an LQR and PID controllers which involve tuning of the control parameters which little to no physical meaning, the passivity based energy shaping based control method (PB-ESC) provides an intuitive way to design controllers.[5] Section II introduces the port Hamiltonian formulation for a spring-mass system with a simple energy shaping control. In section III, the port Hamiltonian formulation is used to model a quarter car and a wheeled quadruped for off-road navigation applications. A PB-ESC based controller design is proposed to maintain a desired equilibrium for both cases and the controller implementation in MATLAB is discussed in section IV.

II. Preliminaries

A. Port Hamiltonian Formulation

The port-Hamiltonian formulation is a powerful tool to combine the frameworks of system modelling, geometric analysis and control through an unified approach. In the section, we review a spring mass system as shown in Figure 1a in the port Hamiltonian setting. In the case of no control, the equation of motion of the spring is given by

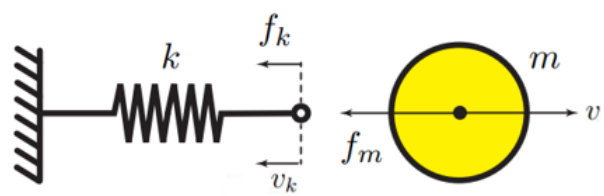

Fig. 1. PHS of spring mass system

$$m\ddot{q} = -kq \tag{1}$$

where m is the mass of the spring, k is the stiffness, q is the displacement from the rest length. Let the Hamiltonian energy level of the system, $\mathcal{H}$ given by the sum of potential $\mathcal{T}$ and kinetic $\mathcal{V}$ energies

$$\mathcal{H} = \mathcal{T} + \mathcal{V} = \frac{p^2}{2m} + \frac{kq^2}{2} \tag{2}$$

The momentum p and generalized position coordinates q with velocity v is given by

$$\frac{dp}{dt} = -\frac{\partial \mathcal{H}}{\partial q}(p,q) = -kq \tag{3}$$

$$\frac{dq}{dt} = \frac{\partial \mathcal{H}}{\partial p}(p,q) = v \tag{4}$$

The spring system contributes to the potential energy part in the Hamiltonian and the displacement q

$$spring : \begin{cases} \dot{q} = -v_k \\ F_k = \dfrac{\partial \mathcal{H}}{\partial q}(p, q) \end{cases} \tag{5}$$

where the force exerted on the spring is given by F_k and the velocity of the endpoint of the spring (where the mass is attached) is $-v_k$. The mass system contributed to the kinetic energy part in the Hamiltonian and the momentum p

$$mass : \begin{cases} \dot{p} = F_m \\ v = \dfrac{\partial \mathcal{H}}{\partial p}(p, q) \end{cases} \tag{6}$$

From Newton's third law, we can define interconnection g as follows

$$interconnection : \begin{cases} -v_k = v \\ F_m = -F_k \end{cases} \tag{7}$$

where F_m is the force exerted on the mass and v is its velocity. By defining the state $x = [p; q]$ one can write the set of equations defined in (3) in state space form as

$$\dot{x} = \Big[J(x) - R(x)\Big] \frac{\partial \mathcal{H}}{\partial x}(x) + g(x)u \tag{8}$$

$$y = g^T(x) \frac{\partial \mathcal{H}}{\partial x}(x) \tag{9}$$

Here, $J(x)$ is a skew symmetric internal interconnection matrix and $R(x)$ is a symmetric resistive structure, $g(x)$ represents the interconnection i.e, the effect of port variables on the state and vice versa and u is the control input. Note that $J(x)$ is power continuous and $R(x)$ models pure resistive losses of the system. For the spring mass system from the example, the state space formulation of the port-Hamiltonian system (PHS) becomes

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathcal{H}}{\partial q}(p, q) \\ \dfrac{\partial \mathcal{H}}{\partial p}(p, q) \end{bmatrix} \tag{10}$$

$$v = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathcal{H}}{\partial q}(p, q) \\ \dfrac{\partial \mathcal{H}}{\partial p}(p, q) \end{bmatrix} \tag{11}$$

B. Passivity Based Energy Shaping Control (PB-ESC)

Adding $F_{control}$ to the system (10-11), the new port Hamiltonian formulation is given as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathcal{H}}{\partial q}(p, q) \\ \dfrac{\partial \mathcal{H}}{\partial p}(p, q) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_{control} \tag{12}$$

$$v = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathcal{H}}{\partial q}(p, q) \\ \dfrac{\partial \mathcal{H}}{\partial p}(p, q) \end{bmatrix} \tag{13}$$

Here the controller $F_{control}$ can be used to inject the desired energy into the system in order to shape the Hamiltonian to the desired energy level. A simple energy shaping control law that results in a desired oscillation amplitude x_s can be designed in terms of the energy error $\tilde{E}$ and the system Hamiltonian $\mathcal{H}(x)$ as follows[3]

$$F_{control} = -\frac{\lambda}{2}\tilde{E} \tag{14}$$

$$\tilde{E} = \mathcal{H}(x) - \frac{k}{2}x_s^2 \tag{15}$$

Comparing the results with an LQR controller as shown in Figure 1d, we can see that the energy injected by the PB-ESC controller takes a much more natural shape compared to LQR. Further, the LQR controller also leads to dissipative energy as seen from the negative area of its control input.

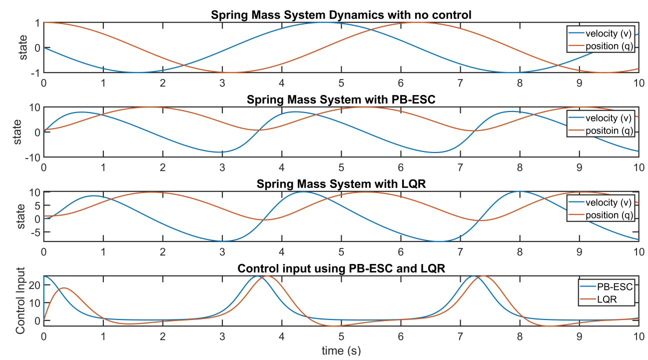

Fig. 2. Comparison of PB-ESC with LQR control for a spring mass system.

III. Main Contributions

A. PB-ESC for a quarter car model

A lumped parameter quarter car model is shown in Figure 3. The body mass m_b is linked to the wheel mass m_w through a spring and damper system with stiffness k_s, and damping ratio b_S. The wheel mass is supported by the tire stiffness and damping k_t, b_t.

System coordinates x_r, x_w, x_b represent the road profile input, wheel center displacement, and vehicle body displacement respectively. F_s is the active suspension control parameter, active as a damping injection element.
The generalized coordinates of the PHS for this model are defined as $q_1 = x_w - x_t$ and $q_2 = x_b - x_w$. And p_1, p_2 are the system momenta (see appendix 1).// System Hamiltonian is defined by $\mathcal{H}(\mathbf{p}, \mathbf{q}) = \mathbf{T}(\mathbf{p}) + \mathbf{V}(\mathbf{q})$, where $V(q)$ is the potential energy and the kinetic energy is $\mathbf{T}(\mathbf{p}) = \frac{1}{2}\mathbf{p}^\mathbf{T}\mathbf{M}^{-}\mathbf{1}\mathbf{p}$ where M is the mass matrix.
State space system is can then be represented as the following

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -D & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{H}}{\partial p} \\ \frac{\partial \mathcal{H}}{\partial q} \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} F_s + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \dot{x}_r(t) \tag{16}$$

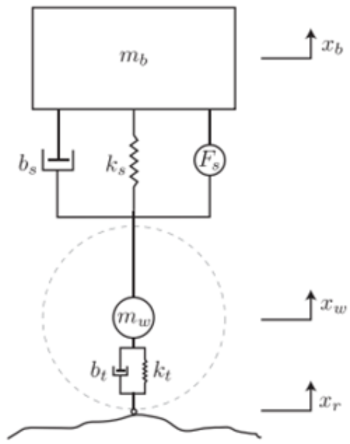

Fig. 3. Quarter car suspension model

Where

$$D = \begin{bmatrix} bt & 0 \\ 0 & bs \end{bmatrix},\ G = [0,1]^T,\ G_1 = [b_t, 0]^T,\ G_2 = [-1, 0]^T$$

The state space system (12) can be further simplified into the following form: (see appendix for detailed equations)

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -D & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{E}}{\partial \zeta} \\ \frac{\partial \mathcal{E}}{\partial \eta} \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} F_s \tag{17}$$

$$\mathcal{E}(\zeta, \eta, t) = \mathcal{W}(\zeta, t) + \mathcal{U}(\eta, t) \tag{18}$$

B. PB-ESC for a wheeled quadruped

A basic sketch of the wheeled quadruped leg replacement is shown in figure 4. Hip joint motors act as the suspension element of the system, motor torque contributes the the dampening of the wheeled link's angle replacing the conventional spring and damper seen in a vehicle suspension.

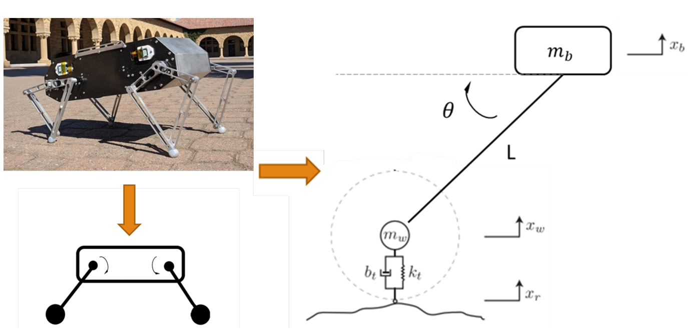

Fig. 4. Proposed design

Using the model described in (A), the model can be represented as shown in Fig 4b. The generalized coordinates here are $q_1 = x_w - x_r$ and $q_2 = \theta$.

The suspension of this system in produced through a motor at the hip joint, and the motor torque τ acts as an injection damping control input. The state space model of the system is written as:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -D & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{H}}{\partial p} \\ \frac{\partial \mathcal{H}}{\partial q} \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} /L + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \dot{x}_r(t) \tag{19}$$

Where

$$D = \begin{bmatrix} bt & 0 \\ 0 & bs \end{bmatrix},\ G = [0,1]^T,\ G_1 = [b_t, 0]^T,\ G_2 = [-1, 0]^T$$

And the system can be further simplified into:

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -D & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{E}}{\partial \zeta} \\ \frac{\partial \mathcal{E}}{\partial \eta} \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} \tau/L \tag{20}$$

$$\mathcal{E}(\zeta, \eta, t) = \mathcal{W}(\zeta, t) + \mathcal{U}(\eta, t) \tag{21}$$

C. Controller Design

We write the closed loop control system in the form [6−7]

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} J_a - D_d & -M_d M^{-1} 1 \\ M_1 M_d & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{H}_d}{\partial p} \\ \frac{\partial \mathcal{H}_d}{\partial q} \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \dot{x}_r \tag{22}$$

$$\mathcal{H}_d(p, q) = T_d(p) + V_d(q) \tag{23}$$

Where J_a in an interconnection term, D_d is the desired damping matrix, and M_d is the desired inertia matrix. The desired potential energy V_d aims to assign the desired equilibrium.

$$M_d = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix},\ D_d = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix},\ J_a = \begin{bmatrix} 0 & -j_a \\ j_a & 0 \end{bmatrix} \tag{24}$$

IV. Simulation Results

To test the validity of the control model, we simulated the PHS for both quarter car and quadruped model with and without a control input. Results show that the controller successfully dampens the main body oscillation (x_b) when subjected to road profile disturbance.

It can be seen from Figure 5 that the control strategy has successfully dampened body oscillation and reduced the

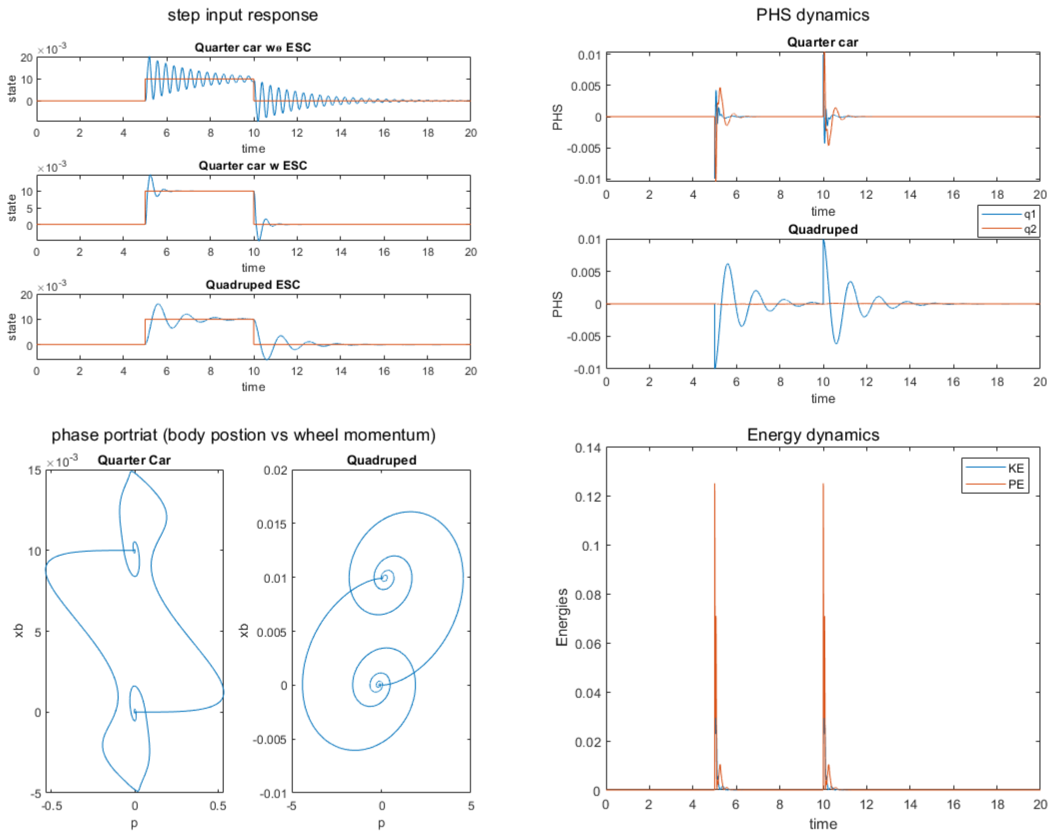

Fig. 5. stability and convergence of PB-ESC based controller with a step input for quarter car model and a wheeled quadruped

displacement with time by adding damping to the system based on the desired energy as shown in the energy dynamics figure.

A sine wave response of the quadruped model is shown in Figure 6. Results show the body response is damped throughout the road profile input, provided a smooth floating body dynamic behavior with minimum vibration.

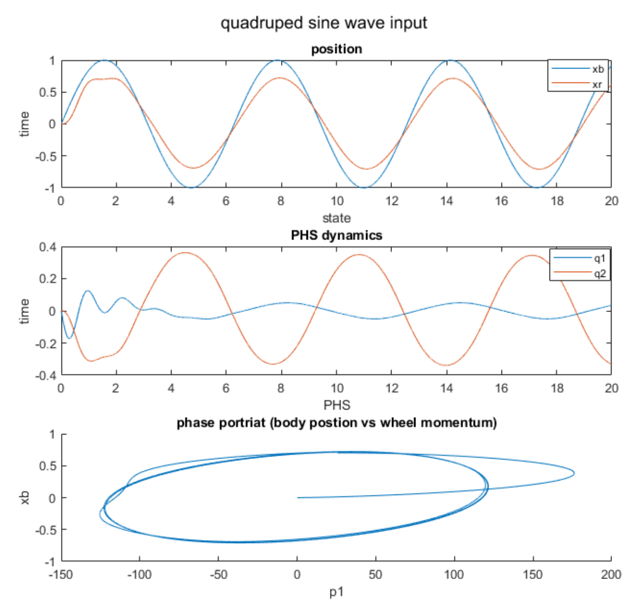

Fig. 6. Wheeled quadruped dynamics with a sinusoidal disturbance applied at its wheel

V. Conclusions

This report considers the design of an active suspension system for a quarter car model and a wheeled quadruped robot. The system is modelled using the port Hamiltonian setting and passivity based energy shaping control is used as the force controller in order to drive the system to the desired equilibrium energy level. Designing the controller in energy coordinates provides an intuitive method for control design. This leads to a more natural control input with minimum loss compared to LQR based control.

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VI. APPENDICES

A. Appendix 1

Quarter car model energies are derives as follows

$$T(q) = \frac{1}{2}m_w(\dot{q}_1 + \dot{x}_r)^2 + \frac{1}{2}m_b(q_1 + \dot{q}_2 + \dot{x}_r)^2 \tag{25}$$

$$V(q) = \frac{1}{2}k_t q_1^2 + \frac{1}{2}k_s q_2^2 \tag{26}$$

$$\mathcal{D}(q) = \frac{1}{2}b_t \dot{q}_1^2 + \frac{1}{2}b_s \dot{q}_2^2 \tag{27}$$

Conjugate momentum is found by taking the first partial derivative of the Lagrangian with respect to q.

$$p_1 = \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m_w(\dot{q}_1 + \dot{x}_r) + m_b(q_1 + \dot{q}_2 + \dot{x}_r) \tag{28}$$

$$p_2 = \frac{\partial \mathcal{L}}{\partial \dot{q}_2} = m_b(q_1 + \dot{q}_2 + \dot{x}_r) \tag{29}$$

$$T(p,q) = \frac{1}{2}pM^-1p^T \tag{30}$$

$$M(car) = \begin{bmatrix} m_w + m_b & m_b \\ m_b & m_b \end{bmatrix}$$

$$M(quadruped) = \begin{bmatrix} m_w + m_b & m_b \\ m_b & m_b + m_l \end{bmatrix}$$

from these equations, we find the derivative of the momentum and generalized coordinate as (Note that the road disturbance affects q_1, and force/torque control input contributes to p_2

$$\dot{q}_1 = \frac{\partial \mathcal{H}}{\partial p_1} - \dot{x}_r \tag{31}$$

$$\dot{q}_2 = \frac{\partial \mathcal{H}}{\partial p_2} \tag{32}$$

$$\dot{p}_1 = -\frac{\partial \mathcal{H}}{\partial q_1} - \frac{\partial \mathcal{D}}{\partial \dot{q}_1} \tag{33}$$

$$\dot{p}_2 = -\frac{\partial \mathcal{H}}{\partial q_2} - \frac{\partial \mathcal{D}}{\partial \dot{q}_2} + (F_s, T_\theta/L) \tag{34}$$