GAUS AG, SS 2025

TALK 0 : OVERVIEW

AIM OF THE SEMINAR : To study the preprint "$\mathcal{D}$-elliptic sheaves and Hasse principle" by Arai-Hattori-Kondo-Papikian, on violation of Hasse principle for the Drinfeld-Stuhler variety X^D for a division algebra D over F.

DATES : Each session of the seminar will be held on Fridays, starting from May 2nd till July 18th (except for a three week pause in June).

TIMINGS : We meet from 9:15 CET (15:15 TAIWAN TIME) to 10:45 CET (16:45 TAIWAN TIME).

§0. NOTATION

p a prime, $q = p^n$. $C = \mathbb{P}^1_{\mathbb{F}_q}$ and $\infty \in C(\mathbb{F}_q)$ is the place $\left(\frac{1}{t}\right)$.

$A = \mathbb{F}_q[t]$	$\mathbb{Z}$	(classical analogues)
$F = \mathbb{F}_q(t)$	$\mathbb{Q}$	
$F_\infty = \mathbb{F}_q((\frac{1}{t}))$	$\mathbb{R}$	
$\mathbb{C}_\infty = \widehat{\overline{F_\infty}}$	$\mathbb{C}$	

§1. BRIEF HISTORY OF $\mathcal{D}$-ELLIPIC SHEAVES

(For more details, refer to [LRS93] and [BS97])

If k is a field of characteristic 0 (eg. $\mathbb{C}$), Krichever found the following algebro-geometric intepretation of certain commutative subrings of $k[[T]]\left[\frac{d}{dT}\right]$

$$\left\{\begin{array}{l}\text{(a) } X \text{ a proper curve}/k\\ \text{(b) } P\in X(k) \text{ smooth}\\ \quad \& \ T_{X,P}\simeq k \text{ an isom.}\\ \text{(c) } \mathcal{F} \text{ a torsion}\\ \quad \text{free sheaf of}\\ \quad \text{rank } \underline{1} \text{ on } X\\ \quad \text{s.t. } h^0(\mathcal{F})=h^1(\mathcal{F})=0\end{array}\right\} \underset{1-1}{\longleftrightarrow} \left\{\begin{array}{l}\text{commutative}\\ \text{subrings}\\ R\subset k[[T]]\left[\frac{d}{dT}\right],\\ k\subset R \text{ s.t. } \exists\\ A,B\in R \text{ with}\\ A=\left(\frac{d}{dT}\right)^{\alpha}+\text{lower order terms}\\ B=\left(\frac{d}{dT}\right)^{\beta}+\text{lower order term}\\ \& \ (\alpha,\beta)=\underline{1}\end{array}\right\}\Big/\sim$$

where $R_1\sim R_2$, if $\exists\, u(T)\in k[[T]]^{\times}$ s.t.

$$R_1 = u(T)\,R_2\,u(T)^{-1}.$$

Also multiplication in $k[[T]]\left[\frac{d}{dT}\right]$ is defined s.t.

$$\frac{d}{dT}\cdot f = f\cdot\frac{d}{dT} + f'$$

For a field L over $\mathbb{F}_q$, Drinfeld realized that the analogous non-commutative ring is $L[\tau]$ with

$$\tau\cdot a = a^q\tau \quad ; \ \forall\, a\in L$$

With this analogy, Drinfeld obtained the following generalization to the above dictionary

(i) $X_0/\mathbb{F}_q$ a reduced, geom. irreducible proper curve

(ii) $P_0 \in X_0(\mathbb{F}_q)$ regular and $P := P_0 \times_{\mathbb{F}_q} L$

(iii) A torsion free sheaf $\mathcal{F}$ on $X := X_0 \times_{\mathbb{F}_q} L$ s.t. $\chi(\mathcal{F}) = 0$

(iv) A ladder of sheaves

$$\begin{tikzcd}
\cdots \mathcal{F}_0 = \mathcal{F} \arrow[r, hook, "j_0"] & \mathcal{F}_1 \arrow[r, hook, "j_1"] & \cdots \arrow[r, hook, "j_{d-1}"] & \mathcal{F}_d = \mathcal{F}(P) \arrow[r, hook] & \cdots \\
{}^{\tau}\mathcal{F}_{-1} \arrow[u, "t_{-1}"] \arrow[r, hook, "{}^{\tau}j_0"] & {}^{\tau}\mathcal{F}_0 \arrow[u, "t_0"] & \cdots & {}^{\tau}\mathcal{F}_{d-1} \arrow[u, "t_{d-1}"]
\end{tikzcd}$$

s.t. $\mathrm{len}(\mathrm{coker}\, j_i) = 1$ & $\mathrm{len}(\mathrm{coker}(t_i)) = 1$

(Elliptic sheaves over L)

$\xleftrightarrow{1-1}$

Commutative subrings $R \subset L[\tau]$ s.t. $R \supsetneq \mathbb{F}_q$, $R \cap L = \mathbb{F}_q$ / $\sim$

$R_1 \sim R_2 \iff \exists\, u \in L^\times$ s.t. $R_1 = u R_2 u^{-1}$

(Drinfeld modules) previously known as Elliptic modules

If we denote by $\mathcal{E}ll(L)$ and $Dr(L)$ the two data described above, the bijection is given by

$$\mathcal{E}ll(L) \longleftrightarrow Dr(L)$$

$$(X_0, P_0, \{\mathcal{F}_i\}, \{t_i\}) \longmapsto \left(\begin{array}{l} R := \Gamma(X_0 \setminus \{P_0\}, \mathcal{O}_{X_0}) \\ \text{\& } R \subset L[\tau] \\ \text{determined by} \\ \text{action of } R \text{ on} \\ \Gamma(X \setminus \{P\}, \mathcal{F}) \end{array} \right)$$

$$\left(\mathrm{Proj}\left(\bigoplus_{n\geq 0}^{\infty} R_n\right), V(e), \{\widetilde{\mathfrak{m}[i-1]}\}, \{t_i\}\right) \longleftarrow\!\!\dashv (R \subset L[\tau])$$

where $R_n = \{x \in R \mid \deg(x) \leq n\}$ $e = 1 \in R_1$, $M_n = \{x \in L\{\tau\} \mid \deg x \leq n\}$ & $\mathfrak{m} = \bigoplus_{n=0}^{\infty} M_n \cdot \{t_i\}$ are canonically determined.

More generally, one also has the above dictionary for L replaced by an $\mathbb{F}_q$-scheme S

$$\mathcal{E}\ell\ell(S) \longleftrightarrow Dr(S)$$

If we restrict to $(X_0, P_0) = (\mathbb{P}^1_{\mathbb{F}_q}, \infty)$ and $\mathrm{rank}(\mathcal{F}) = d$, the corresponding objects on the right are "Drinfeld A-modules of rank d"

$$\mathcal{E}\ell\ell^d_A(S) \longleftrightarrow Dr^d_A(S)$$

For example, $Dr^d_A(\mathbb{C}_\infty)$ has objects isomorphic to $\mathbb{C}_\infty/\Lambda$ for an A-lattice $\Lambda \subset \mathbb{C}_\infty$ of rank d. So both objects above are function field analogues of elliptic curves. As for the latter, we can also introduce level I-structures on Drinfeld A-modules and elliptic A-sheaves to get

$$\mathcal{E}\ell\ell^d_{A,I}(S) \longleftrightarrow Dr^d_{A,I}(S)$$

It turns out that $Dr^d_{A,I}$ is a DM stack over $\mathbb{F}_q$ which is representable over $\mathbb{P}^1_{\mathbb{F}_q} \setminus (\{\infty\} \cup V(I))$ if $I \neq \phi$. Relevant details about Stacks will be discussed in Talks 1 & 3. The corresponding moduli space is denoted by M^d_I. Drinfeld constructed explicitly a compactification of M^2_I and proved

$$\varinjlim_{\substack{H \\ \text{open cpt} \\ \text{in } GL_2(\hat{A})}} H^1_{\text{ét},c}(M^2_H, \overline{\mathbb{Q}}_\ell) \underset{\substack{\text{as } GL_2(\mathbb{A}^f_F) \\ \times \\ \text{Gal}(F^s|F)\text{-repn's}}}{=} \bigoplus_{\pi \in \Pi_\infty} \pi \otimes \sigma(\pi)$$

where Π_∞ are irreducible, admissible $GL_2(\mathbb{A}^f_F)$-repn's s.t.

$$\pi_\infty = St_2(F_\infty) \quad \text{and}$$

$$\sigma(\pi)|_{\text{Gal}(F^s_\infty|F_\infty)} = Sp_{\text{Gal}}$$

For more details, see [DH87]

Deligne used the correspondence

$$\pi \leftrightarrow \sigma(\pi)$$

to prove local Langlands correspondence for GL_{2,F_∞}. This fits well with how local class field theory was first proved using the global class field theory.

It is natural to expect the above credo to work also for proving local Langlands for GL_{n,F_∞}, once a weak global reciprocity law as above is estabilished.

But compactifications of M_I^d are complicated and non-unique for $d>2$. Also this has been achieved only in the past decade or so, by the work of Pink, Fukaya-Kato-Sharifi, Hartl-Yu and others.

Following the credo that Drinfeld modules/elliptic sheaves generalize elliptic curves, we have

$$M^d_I \underset{F}{\times} F_\infty = GL_2(F) \backslash \left(\Omega \times GL_2(\mathbb{A}^f_F) / K(I) \right)$$

where $\Omega = \mathbb{C}_\infty \setminus F_\infty$

modular curve of level N

$$Y(N) \underset{\mathbb{Q}}{\times} \mathbb{C} = GL_2(\mathbb{Q}) \backslash \left(\mathbb{H}^\pm \times GL_2(\mathbb{A}^f_\mathbb{Q}) / K(N) \right)$$

where $\mathbb{H}^\pm = \mathbb{C} \setminus \mathbb{R}$

In the classical case, a compact analogue of R.H.S. above is also known. Namely, if $B/\mathbb{Q}$ is division algebra and $G/\mathbb{Q}$ its associated group scheme, then

$$V_{B,K} \underset{\mathbb{Q}}{\times} \mathbb{C} = G(\mathbb{Q}) \backslash \left(\mathbb{H}^\pm \times G(\mathbb{A}^f_\mathbb{Q}) / K \right)$$

is a compact Riemann surface and parametrizes certain Abelian surfaces over $\mathbb{C}$ having multiplication by a maximal order of B, called QM-abelian surfaces. In an attempt to generalize such an idea for elliptic sheaves, in [LRS93] $\mathcal{D}$-elliptic sheaves were introduced.

§2. $\mathcal{D}$-elliptic sheaves (Talk 2)

Let $d \geq 2$ be an integer and D be a central division algebra over F of dimension d^2 s.t. $D \otimes F_\infty \cong M_d(F_\infty)$. R denote the set of places of F where D ramifies (i.e. doesn't split) and we assume

$$\mathrm{inv}_x(D) = \frac{1}{d} \quad \forall\ x \in$$

$\mathcal{D}$ a locally free $\mathcal{O}_C$-algebra s.t. $\mathcal{D}|_{\mathrm{Spec}(F)} = D$ and $\widehat{\mathcal{D}_x} := \mathcal{D} \otimes_{\mathcal{O}_{C,x}} \widehat{\mathcal{O}_{C,x}}$ is a maximal order of $\widehat{D}_x \doteq D \otimes_F \widehat{F}_x$. Then $\mathcal{O}_D = H^0(C \setminus \{\infty\}, \mathcal{D})$ is a maximal A-order of D.

Let S be an $\mathbb{F}_q$-scheme.

DEF'N ÷ A sequence of triples $(\mathcal{E}_i, j_i, t_i)$ is said to be a $\mathcal{D}$-elliptic sheaf over S if $\mathcal{E}_i$ are locally free sheaf of rank d^2 over $C \times S$ equipped with an $\mathcal{O}_X$-linear action of $\mathcal{D}$ and injective $\mathcal{O}_{C\times S}$-linear $j_i : \mathcal{E}_i \to \mathcal{E}_{i+1}$, $t_i : {}^{\tau}\mathcal{E}_i \to \mathcal{E}_{i+1}$ which are $\mathcal{D}$-linear s.t.

(i)
$$\begin{tikzcd} \mathcal{E}_i \arrow[r, "j_i"] & \mathcal{E}_{i+1} \\ {}^{\tau}\mathcal{E}_{i-1} \arrow[u, "t_{i-1}"] \arrow[r, "{}^{\tau}j_{i-1}"] \arrow[ur, phantom, "\square"] & {}^{\tau}\mathcal{E}_i \arrow[u, "t_i"'] \end{tikzcd}$$
commutes $\forall\, i \in \mathbb{Z}$

(ii) $\mathcal{E}_{i+d} = \mathcal{E}_i \otimes (\mathcal{O}_C(\infty) \boxtimes \mathcal{O}_S)$ &

$\mathcal{E}_i \to \mathcal{E}_{i+1} \to \cdots \to \mathcal{E}_{i+d}$ is the natural inclusion induced by $\mathcal{O}_C \hookrightarrow \mathcal{O}_C(\infty)$

(iii) For the projection, $pr_S : C \times S \to S$, $(pr_S)_*(\mathrm{coker}(j_i))$ is a rk d sheaf.

(iv) $\mathrm{Coker}(t_i)$ are supported by the graph of $i_0 : S \to C$ (ind. of i) & it is the pushforward of a rank d sheaf via $S \xrightarrow[(i_0, id)]{} C \times S$

$Z(\underline{\mathcal{E}}) := i_0(S)$ is called the "zero of $\underline{\mathcal{E}}$".

(v) $\chi(\mathcal{E}_0|_{C \times \{s\}}) \in [0, d^2) \;\; \forall \; \underset{\text{geom. pt.}}{\{s\} \to S}$

(t-motive of $\underline{\mathcal{E}}$ over L) Since the $\mathcal{E}_i$'s are modification at ∞, we can form the well defined $A \otimes L$-module $P = H^0(C \setminus \{\infty\} \times L, \mathcal{E}_i)$ of rank d^2. This is called the t-motive of $\underline{\mathcal{E}}$. If L is perfect and $\infty \notin Z(\underline{\mathcal{E}})$, then P is a $L[\tau]$-module of rank d.

§3. Main result and Strategy of a proof / Structure of talks

As alluded to before, $\mathcal{D}$-elliptic sheaves are function field analogue of QM-abelian surfaces, so we can expect results about the latter to reflect on the former as well. It is with this credo in mind that the authors in [AHKP24] generalize a result in [Jor80] regarding sufficient conditions to estabilish an obstruction to Hasse principle for V_B, the coarse moduli space parametrizing QM abelian surfaces w.r.t. B. We also have the coarse moduli space parametrizing $\mathcal{D}$-elliptic sheaves, denoted $X^{\mathcal{D}}$ (to be defined in Talk 4), one can expect to get similar sufficient conditions for non-existence of $X^{\mathcal{D}}(K)$ for $K|\mathbb{F}_q(t)$ field ext'n.

We state the main result and explain the strategy of proof of both V_B and X^D, side by side to see the parallelism in the skeleton of proof's of both.

Below $B/\mathbb{Q}$ resp. D/F are division algebras of dimensions 4 resp. d^2. Moreover, B is indefinite.

<table>
<tr><th>QM-abelian surfaces</th><th>$\mathcal{D}$-elliptic sheaves</th></tr>
<tr><td>(i) $V_B/\mathbb{Q}$ coarse moduli space of QM-abelian surface with mult. by a maximal order $\mathcal{N}$ of B. If an extension $M/\mathbb{Q}$ splits B, then $V_B(M) \neq 0$ gives rise to a QM-abelian surface defined over M.</td><td>X_D/F coarse moduli space of $\mathcal{D}$-elliptic sheaves. If an extension $K\mid F$ splits D, then $X_D(K) \neq \phi$, implies the existence of $\mathcal{D}$-elliptic sheaf $\underline{\mathcal{E}}$ over K. (Talk 4)</td></tr>
<tr><td>(ii) ([Thm. 6.3, Jor 80]) For a prime χ, let</td><td>(Main result, Talk 9)
([Thm. 9.1, AHKP24]) Let $K\mid F$ be a field extension of degree d.</td></tr>
</table>

$\mathcal{B}(x) := \{ B/\mathbb{Q}$ quaternions s.t. $\mathbb{Q}(\sqrt{-x})$ doesn't split B if $x \neq 2$; $\mathbb{Q}(\sqrt{-2})$ & $\mathbb{Q}(\sqrt{-1})$ doesn't split B if $x = 2 \}$

Then there is a finite subset $\mathcal{P}(x) \subset \mathcal{B}(x)$ s.t. if M is im. quad ext'n of $\mathbb{Q}$ in which x ramifies, $B \in \mathcal{B}(x) \setminus \mathcal{P}(x)$ & M splits B, then $V_B(M) = \phi$.

Ex: $V_{B(39)}(\mathbb{Q}(\sqrt{-13})) = \phi$.

s.t.

- $D \otimes_F K \simeq M_d(K)$;
- $\exists$ place $\eta \neq \infty$ of F s.t. it totally ramifies in K and D splits at η;
- $P(\eta)$ is a finite computable set of places of F and D ramifies at some $p \notin P(\eta)$;
- $D \otimes_F F(\sqrt[d]{\mu\eta}) \not\simeq M_d(F(\sqrt[d]{\mu\eta}))$ $\forall \mu \in \mathbb{F}_q^\times$

Then $X_D(K) = \phi$.

Structure of proof

(ii) Suppose $V_B(M) \neq \phi$. Gives rise to a QM-abelian surface A over M.

Suppose $X_D(K) \neq \phi$. Gives rise to a $\mathcal{D}$-elliptic sheaf $\underline{E}$ over K.

(iv) For any finite place v of M, $\exists$ totally ramified extension $M'_{\tilde v} | M_v$ s.t. $A \underset{M}{\times} M'_{\tilde v}$ has good reduction. Denote by $\tilde A$ the reduction mod $\tilde v$.

(v) For a prime $s \neq \chi$, consider the Tate module

$$T_s(A) \simeq \mathcal{N} \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_s =: \mathcal{N}_s$$

Define

$$R_s : G(M^s | M) \to \underset{\mathcal{M}}{\mathrm{Aut}}(T_s(A))$$

$$\mathcal{N}_s^\times \subset B_s^\times$$

Then,

$$P_{n,v}(t) = \mathrm{Nrd}_{B_s/\mathbb{Q}_s}(1 - R_s(Fr_v^n)t)$$

is in $\mathbb{Z}[t]$ and ind. of s for $v \nmid s$. In fact,

$$P_{n,v}(t) = t^2 f^\circ_{\tilde A}\left(\frac{1}{t}\right)$$

where $f^\circ_{\tilde A}(-)$ is the reduced char. polynomial of Frobenius of $\tilde A / \mathbb{F}_v$.

(Talk 5) $\exists$ totally ramified extension $K'_{\tilde\eta} | K_\eta$, for $\eta \neq \infty$, s.t. $\underline{\mathcal{E}} \underset{K}{\times} K'_{\tilde\eta}$ has good reduction. Denote by $\bar{\underline{\mathcal{E}}}$, the red'n mod $\tilde\eta$.

(Talk 6) Using the t-motive of $\bar{\underline{\mathcal{E}}}$ and G_r construction of Drinfeld, one forms

$$T_p(\bar{\underline{\mathcal{E}}}) = \varprojlim_n \bar{\underline{\mathcal{E}}}[p^n](\bar{\mathbb{F}}_\eta)$$

and $p \neq \eta$ and $\infty \notin \mathcal{Z}(\bar{\underline{\mathcal{E}}})$,

$$\mathcal{O}_D / p^j \mathcal{O}_D \simeq \bar{\underline{\mathcal{E}}}[p^j](\bar{\mathbb{F}}_\eta)$$

$\forall j \geq 1$. This gives rise to

$$i_p : G(\bar{\mathbb{F}}_\eta | \mathbb{F}_\eta) \to \mathrm{Aut}_{D_p}(F_p \underset{\mathcal{O}_p}{\otimes} T_p(\bar{\underline{\mathcal{E}}}))$$

$$\simeq (D_p^{op})^\times$$

Define

$$P_{\bar{\underline{\mathcal{E}}}, Fr_\eta^j}(X) = \mathrm{Nrd}_{D_p^{op}/F_p}(X - i_p(Fr_\eta^j))$$

(vi) If $x \mid \mathrm{disc}(B)$, then $A[x] \simeq N/xN$ has exactly one subgroup of order x^2, called canonical torsion subgroup C_x. Fixing an isom. $C_x \simeq \mathbb{F}_2$, gives the canonical isogeny character

$$\begin{tikzcd} \rho : G(M^s/M) \arrow[r] \arrow[d] & \mathrm{Aut}(C_x) \arrow[d, phantom, "\wr"] \\ G(M^{ab}/M) \arrow[r, dashed] & \mathbb{F}_2^\times \end{tikzcd}$$

$$\begin{tikzcd} r_x(v) : \mathcal{O}_v^\times \arrow[r] & G(M^{ab}/M) \arrow[d, "\rho"] \\ & \mathbb{F}^\times \end{tikzcd}$$

be the induced map from Local CFT. Then $r_x(v)^{12} = 1$ for $v \nmid x$, and

$$r_x(v)\left(\frac{1}{s}\right) \equiv \pm s \pmod{x}$$

if $v \mid x$ is the unique prime in M.

(Talk 8) One has

$$\mathcal{O}_D/p\mathcal{O}_D \simeq \bigoplus_{j=0}^{d-1} \mathbb{F}\Pi^j$$

where $[\mathbb{F}:\mathbb{F}_p] = d$, $\Pi \in \mathcal{O}_D$ a prime elt s.t. $\Pi^d = 0$. For $L/\mathbb{F}_q$ a field ext^n and $\underline{\mathcal{E}}'$ a $\mathcal{D}$-ell. sh over L

$\Rightarrow \underline{\mathcal{E}}'[P](L^{sep})[\Pi] \simeq \mathbb{F}$ & gives rise to a canonical isogeny character

$$\rho_{\underline{\mathcal{E}}',P} : G(L^s|L) \longrightarrow \mathbb{F}^\times$$

Via Local CFT form

$$\begin{tikzcd} \widetilde{r}_{\underline{\mathcal{E}},P}(m) : K_m^\times \arrow[r] & G(K^{ab}/K) \arrow[d, "\rho_{\underline{\mathcal{E}},P}"] \\ r_{\underline{\mathcal{E}},P}^{(m)} = \widetilde{r}_{\underline{\mathcal{E}},P}^{(m)} \big| \mathcal{O}_m^\times & \mathbb{F}^\times \end{tikzcd}$$

Then

$$(r_{\underline{\mathcal{E}},P}(m))^{q^d-1} = 1; \; m \nmid P\infty$$

$$\widetilde{r}_{\underline{\mathcal{E}},P}(m)^{\ell_q(d)^2} = 1; \; m \mid \infty$$

$$r_{\underline{\mathcal{E}},P}(m)(q^{-1})^n \equiv q^{\frac{n}{d}[K_m:\mathbb{F}_p]} \pmod{P}$$

if $m \mid P$

(vii) One has

$$P_{n,v}(t) \bmod x = (1-\rho_p(Fr_v^n)t)(1-\rho_p(Fr_v^n)^p t)$$

The fact on $r_x(v)$ above, together with the Honda-Tate classification of isogeny classes of QM abelian surfaces over finite fields, $\widetilde{A}$ via the reduced characteristic polynomial $f^0_{\widetilde{A}}(t)$, one obtains contradiction to hypothesis on $B(x)$.

The three results above are proved by relating $\rho_{\underline{\mathcal{E}},p}$ to determinant char. of the t-motive of $\underline{\mathcal{E}}$ as well as the Carlitz character. (Will be discussed in Talk 7)

(Talk 9) One has

$$P_{\underline{\mathcal{E}},Fr_\eta^{(dn)}}(X) \bmod p = \prod_{j=0}^{d-1}\left(X-\rho_{\underline{\mathcal{E}},p}(Fr_{\mathbb{F}_\eta^{dn}})^{|p|^j}\right)$$

& $\rho_{\underline{\mathcal{E}},p}(Fr_{\mathbb{F}_\eta}^{dn}) = \rho_{\underline{\mathcal{E}},p}(Fr_\eta^{dn})$.

Using the facts on $\tilde{r}_{\underline{\mathcal{E}},p}$ and using results from Talks 5 & 6, we get the contradiction that $F(\sqrt[d]{\mu\eta})$ splits D.

In the last talk, we will also briefly discuss the work of Papikian on K_v-points of X_D. The authors also have a PARI-GP code giving rise to infinitely many quadratic extensions satisfying the conditions of main theorem as well as for Papikian's results. This aspect will also be touched upon.

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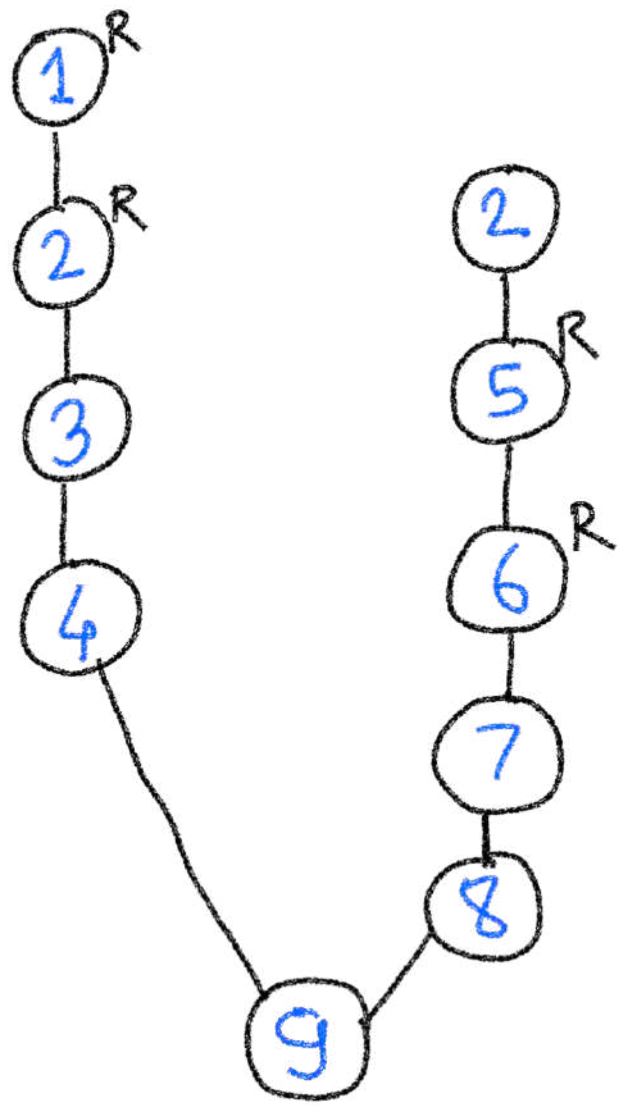

R = Reserved

i = Talk i

$1 \le i \le 9$

REFERENCES

[AHKP24] "D-elliptic sheaves & the Hasse principle" by Arai-Hattori-Kondo-Papikian

[LRS93] "D-elliptic sheaves & the Langlands correspondence" by Laumon-Rapoport-Stuhler.

[Jor80] "Points on Shimura curves over number field" by B.W. Jordan

[DH87] "Survey of Drinfeld modules" by Deligne-Husemöller