01/07/24

Module – 03

Principle of Counting – I

Sum and product rule :-

① Sum Rule :-

Suppose two tasks T_1 and T_2 are to be performed. If the task T_1 can be performed in M different ways and the task T_2 can be performed in N different ways and if these two tasks cannot be performed simultaneously, then one of the two task (T_1 or T_2) can be performed in M+N ways.

Generally,

$T_1, T_2, ----- T_K$ are 'k' task such that no two of ~~this~~ these task can be performed at the same time and if the task T_i can be performed in n_i ways. Then one of the k task (~~like~~ a T_1, or T_2 or ---- or T_K) can be performed in $n_1+n_2+ ----- +n_k$ ways.

Example :-

Suppose there are 10 boys and 12 girls in a class we wish to select one of the students (either boy or girl) as class Representative.

Soln:- The number of ways ~~of one~~ selecting boy is 10 and the number of ways of selecting girl is 12.

∴ The number of ways of selecting a student is $10+12=22$

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① Suppose Hostel library has 12 books on physics, 10 books in chemistry, 16 books on Computer Science and 11 books on Electronics. Suppose a student wishes to choose one of these books for study. Then how many ways he can select a books.

Soln:- The number of ways of selecting physics books $=12$

——— " ——— Chemistry $=10$

——— " ——— Computer Science $=16$

——— " ——— Electronics $=11$

$=12+10+16+11$

$=49$

② Suppose T_1 is the task selecting a prime number less than 10 and T_2 is the task selecting an even number less than 10. T_1 and T_2 can be performed in how many ways.

Soln:-

prime number, $T_1 \rightarrow$ 2, 3, 5, 7

Even number, $T_2 \rightarrow$ 2, 4, 6, 8

T_1 or T_2 = 4 + 3

= 7

② Product rules:-

① Suppose two tasks T_1 and T_2 are to be performed one after the other. If T_1 can be performed in n_1 different ways and T_2 can be performed in N_2 different ways. Then both the tasks can be performed in $n_1 \times n_2$ ways.

Eg:-

① Suppose a person as 8 shirts and 5 ties. In how many ways we can choose.

Soln:-

$T_1 \longrightarrow$ 8 shirts

$T_2 \longrightarrow$ 5 ties

$8 \times 5 = 40$

shirts	Ties
1	1
2	2
3	3
4	4
5	5
6	
7	
8	

② There are 20 married couples in a party. Find number of ways of choosing one women and one man from the party. Such that two are not married to each other.

Soln:-

Men $\longrightarrow$ 20 (1, 2, - - - - 20)

Women $\longrightarrow$ 19 (19, 19, 19 - - - 19)

20×19

$= 380$

③ Cars of a particular manufacturer come in 4 models, 12 colours, 3 engines sizes and 2 transmission types.

(a) How many distinct cars can be manufactured.

(b) of these how many have the same colour.

Soln:- T_1 = models → 4

T_2 = Colors → 12

T_3 = Engine size → 3

T_4 = Transmission type → 2

(a) $4 \times 12 \times 3 \times 2 = \underline{288}$ ∴ 288 distinct cars can be manufactured.

(b) $4 \times 3 \times 2 = \underline{24}$ ∴ 24 cars have the same color.

(4) A bit is either 0 or 1. A byte is a sequence of 8 bits. Find

(i) Number of bytes

(ii) The Number of bytes starting and ending with 11.

(iii) The Number of bytes that begin with 11 and do not end with 11.

(iv) The Number of bytes that begin with 11 or end with 11.

Soln:- (i) □ □ □ □ □ □ □ □

2 chances

byte is a sequence of 8 bits

$\Rightarrow 2^8$

$= \underline{256}$

(ii) $\boxed{1}$ $\boxed{1}$ □ □ □ □ $\boxed{1}$ $\boxed{1}$

2 chances 2 2 2

$\Rightarrow 2^4 = \underline{16}$

(iii)

$\boxed{1}$ $\boxed{1}$ □ □ □ □ □ □

$\Rightarrow 2^6 - 16$

$= 64 - 16$

$= \underline{48}$

(iv) $(2^6 - 16) + (2^6 - 16) - 16$

$\Rightarrow (64 - 16) + (64 - 16) - 16$

$= 48 + 48 - 16$

$= \underline{80}$

(5) Find number of 2 digit even number.

$\Rightarrow$ □ □ $9 \times 5 = \underline{45}$

1–9 0, 2, 4, 6, 8

41 — without repetition

4 — with repetition

∴ 45 even numbers.

⑥ In how many ways three different points (coins) be placed in 2 different purses.

Soln:-

□	□
A	BC
B	AC
C	AB
BC	A
AC	B
AB	C
—	ABC
ABC	—

2 purses , 3 different points (A, B, C)

$2^3 = \underline{8 \text{ ways}}$.

Permutation and Combination :-

Permutation → [arrangement of objects]

Combination → [selection of objects]

* Repetation is not allowed.

* $\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$

* Type - I :-

Suppose it is required to find number of permutation that can be formed from a collection of n objects of which ~~of~~ n_1 are of first type and n_2 are of second type ----- n_k are of k^{th} type with $n_1 + n_2 + ---- + n_k = n$

Then the number of permutations of n objects (Taken all of them at a time) is $\frac{n!}{n_1!\, n_2! ---- n_k!}$

Example :-

① How many 9 letter words can be formed using the letters of the word ~~different~~ "DIFFICULT".

Soln:-

DIFFICULT

D → 1
I → 2
F → 2
C → 1
U → 1
L → 1
T → 1

$$= \frac{n!}{n_1!\, n_2! \cdots n_k!}$$

$\boxed{n=9}$

$$= \frac{9!}{1! \times 2! \times 2! \times 1! \times 1! \times 1!}$$

$$= 90720$$

② How many 7 letter words can be formed using the letters of the word "SUCCESS".

Soln :- SUCCESS

$S \to 3$
$U \to 1$
$C \to 2$
$E \to 1$

$\boxed{n=7}$

$$= \frac{n!}{n_1!\, n_2! \cdots n_k!}$$

$$= \frac{7!}{3! \times 1! \times 2! \times 1!}$$

$$= 420$$

③ ~~MASSA~~

(i) How many 10 letter words can be formed using the letters of the word "MASSASAUGA".

Soln :- "MASSASAUGA"

$\boxed{n=10}$

$M \to 1$
$A \to 4$
$S \to 3$
$U \to 1$
$G \to 1$

$$= \frac{n!}{n_1!\, n_2! \cdots n_k!}$$

$$= \frac{10!}{1! \times 4! \times 3! \times 1! \times 1!} = \frac{907200}{1 \times 24 \times 6 \times 1 \times 1}$$

$$= 6300$$

(ii) How many of them begin with S.

Soln :-

S S S M A U G A A A

9 boxes

$$= \frac{9!}{2! \times 4! \times 1! \times 1! \times 1!}$$

(Except 1 S)
$S \to 2$
$M \to 1$
$A \to 4$
$U \to 1$
$G \to 1$

$$= \frac{9!}{2 \times 24 \times 1 \times 1 \times 1}$$

$$= \frac{362880}{48}$$

$$= 7560$$

④ Find the value of 'n' so that $2p(n,2) + 50 = p(2n,2)$.

Soln :-

$$\boxed{p(n,r) = {}^nP_r = \frac{n!}{(n-r)!}}$$

$$p(n,2) = {}^nP_2 = \frac{n!}{(n-2)!}$$

$$\Rightarrow 2 \cdot p(n,2) + 50 = p(2n,2)$$

$$\Rightarrow 2 * \frac{n!}{(n-2)!} + 50 = \frac{2n!}{(2n-2)!}$$

$n! = n(n-1)*(n-2)!$

$$\Rightarrow \frac{2 * n(n-1)*\cancel{(n-2)!}}{\cancel{(n-2)!}} + 50 = \frac{2n(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$$

$$\Rightarrow 2n(n-1) + 50 = 2n(2n-1)$$

$$\Rightarrow 2n^2 - 2n + 50 = 4n^2 - 2n$$

$$50 = 4n^2 - 2n^2$$

$$2n^2 = 50$$

$$n^2 = 25$$

$$n = \pm 5$$

$$\boxed{n = 5}$$

⑤ Find the value of 'n' so that $p(n,2) = 90$.

Soln :-

$$P(n,2) = 90$$

$$\Rightarrow \frac{n!}{(n-2)!} = 90$$

$$\Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$$

$$\Rightarrow n(n-1) = 90$$

$$n^2 - n = 90$$

$$n^2 - n - 90 = 0$$

$-90n^2$
$-10n + 9n$

$$n^2 - 10n + 9n - 90 = 0$$

$$n(n-10) + 9(n-10) = 0$$

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$(n-10)(n+9)=0$

$n-10=0$, $n+9=0$

$\boxed{n=10}$ $n=-9$

$\therefore \boxed{n=10}$

⑥ ~~How~~ How many arguments are there for all the letters in the word "SOCIOLOGICAL".

In how many of the arrangements

(i) A and G are adjacent

(ii) All the vowels are adjacent.

Soln :-

SOCIOLOGICAL $\Rightarrow$

$S \to 1$
$O \to 3$
$C \to 2$
$I \to 2$
$L \to 2$
$A \to 1$
$G \to 1$

Total number of arrangements $= \dfrac{12!}{1! \times 3! \times 2! \times 2! \times 2! \times 1! \times 1!}$

$= 9979200$

(i) A and G are adjacent :-

11 $\boxed{\boxed{AG}\ \boxed{S}\boxed{O}\boxed{O}\boxed{O}\boxed{C}\boxed{C}\boxed{I}\boxed{I}\boxed{L}\boxed{L}}$

$= \dfrac{11!}{3! \times 2! \times 2! \times 2! \times 1! \times 1!}$

$= 831600$

A and G can be arranged in $2! = 2$

$\left[AG = \dfrac{2!}{1! \times 1!} = 2 \right]$

$\Rightarrow 831600 \times 2 = 1663200$

(ii) All the vowels are adjacent.

7 $\boxed{\boxed{AIO}\ \boxed{S}\ \boxed{C}\ \boxed{C}\ \boxed{L}\ \boxed{L}\ \boxed{G}}$

$= \dfrac{7!}{1! \times 1! \times 2! \times 2! \times 1!}$

$= 1260$

AIIOOO $= \frac{6!}{1! \times 2! \times 3!}$

$= 60$

$\Rightarrow 1260 \times 60 = 75600.$

(7) How many integers 'n' we can form using digits 3, 4, 4, 5, 5, 6, 7, if we want 'n' to exceed 5000000.

Soln:-

'n' must be of the form

$n = x_1 x_2 x_3 x_4 x_5 x_6 x_7$

n can be 5 or 6 or 7

(i) if it starts from 5

$n = \boxed{5}\ \square\square\square\square\square\square$ 6

it is fixed

$= \frac{6!}{1! \times 1! \times 2! \times 1! \times 1!}$

$= 360$

(ii) if it starts from 6

$n = \boxed{6}\ \square\square\square\square\square\square$

$= \frac{6!}{1! \times 2! \times 2! \times 1!}$

$= 180$

(iii) if it starts from 7

$n = \boxed{7}\ \square\square\square\square\square\square$

$= \frac{6!}{1! \times 2! \times 2! \times 1!}$

$= 180$

$= 360 + 180 + 180 = 720$ //

(3) Consider the permutation of the letters A, C, F, G, I, T, W, X.

(i) How many of these start with T.

(ii) How many start with T & end with C.

Soln:- a, c, f, g, i, t, w, x.

(i) Start with 't'

$\boxed{t}\ \square\square\square\square\square\square\square$

$= \frac{7!}{1! \times 1! \times 1! \times 1! \times 1! \times 1! \times 1!} = 5040$ //

(ii) Start with 't' and end with 'c'.

$\boxed{t}\ \square\ \square\ \square\ \square\ \square\ \square\ \boxed{c}$

$$= \frac{6!}{1! \times 1! \times 1! \times 1! \times 1! \times 1! \times 1! \times 1!}$$

$$= 720$$

Combinations :- (selection of objects)

$$c(n,r) = {}^nC_r = \frac{n!}{(n-r)!\, r!}$$

$${}^nC_r = \frac{{}^nP_r}{r!} \quad \left({}^nP_r = \frac{n!}{(n-r)!} \right)$$

① A certain question paper contains 2 parts A and B and each containing 4 questions. How many different way a student can answer five (5) questions by selecting atleast 2 questions from each part.

Soln :-

A(4) B(4)

5 questions A(2) B(3) $\Rightarrow c(4,2) \times c(4,3) = 24$

5 questions A(3) B(2) $\Rightarrow c(4,3) \times c(4,2) = 24$

$\Rightarrow 24 + 24 = 48$

$c(n,r) = {}^nC_r = \frac{n!}{n-r}$

$$c(4,2) = \frac{4!}{(4-2)! * 2!} = \frac{4!}{2! * 2!}$$

$$c(4,3) = \frac{4!}{(4-3)! * 3!} = \frac{4!}{1! * 3!}$$

$$\Rightarrow \frac{4!}{(4-2)! * 2!} * \frac{4!}{(4-3)! * 3!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{2! \times 2!} * \frac{4 \times 3 \times 2 \times 1}{1! \times 3!}$$

$$= 6 \times 4 = 24$$

(2) A women has 11. close relatives and she has wish to invite 5 of them to dinner. In how manys she can invite them in the following situation.

(i) There is no restriction on choice.

(ii) Two perticular person will not attend separately.

(iii) Two perticular person will not attend together.

Ans:- 11 → close relatives
5 → wish to invite

(i) $^{11}C_5 = \frac{11!}{(11-5)! \times 5!} = 462$

(ii) both attend → $^9C_3 = 84 = \frac{9!}{(9-3)! \times 3!}$

both not attend → $^9C_5 = 126$

$84 + 126 = 210$ ways

(iii) P_1 or P_2

(i) P_1 attend, P_2 will not attend → $^9C_4 = 126$

(ii) P_1 will not attend, P_2 will attend → $^9C_4 = 126$

(iii) both P_1 & P_2 will not attend → $^9C_5 = 126$

$126 + 126 + 126 = 378$ ways.

(3) Find number of arrangements (permutation) of all letters in the word TALLAHASSEE. How many of these arrangements have no-adjacent A's?

Soln:- TALLAHASSEE

T → 1
A → 3
L → 2
H → 1
S → 2
E → 2

Total = 11

$$= \frac{11!}{1! \times 3! \times 2! \times 1! \times 2! \times 2!}$$

$$= 831600$$

Have no adjacent A's (Not considering A) $= \dfrac{8!}{1! \times 2! \times 1! \times 2! \times 2!} = 5040$

T → 1
L → 2
H → 1
S → 2
E → 2

(Considering A) = _T_L_L_H_S_S_E_E_

$= {}^{9}C_{3}$ (9 → Number of ways, 3 → Number of A)

$= \dfrac{9!}{(9-3)! \times 3!} = 84$

$= 5040 \times 84$

$= 423360$ ways //

(4) How many arrangements of the letters in MISSISSIPPI have not consecutive S?

Soln:-
M → 1
I → 4
S → 4
P → 2

(considering S) _M_I_I_I_P_P_I_

$= {}^{8}C_{4} = \dfrac{8!}{(8-4)! \times 4!}$ (8 → No of ways, 4 → No of S)

$= 70$ ways //

(Not considering S) $= \dfrac{7!}{1! \times 4! \times 2!}$

$= 105$ ways

$= 105 \times 70$

$= 7350$ ways //

(5) From 7 consonents and 5 vowels how many sets consisting of 4 different consonents and 3 different vowels can be formed.

Soln-
C (7,4)
C (5,3)

$\boxed{C(7,4)}\ \underbrace{\Box\ \Box\ \Box}_{7-4=3}\ \boxed{C(5,3)}\ \underbrace{\Box\ \Box}_{5-3=2}$

$\Rightarrow C(7,4) \times C(5,3) \times 7!$

$C(7,4) = \dfrac{7!}{(7-4)! \times 4!} = 35$

$C(5,3) = \dfrac{5!}{(5-3)! \times 3!} = 10$

$\Rightarrow C(7,4) \times C(5,3) \times 7!$

$= 35 \times 10 \times 5040$

$= 1764000$ //

(6) Find How many distinct 4 digit integers one can make from a digits 1, 3, 3, 7, 7, 8.

Soln:- if there is exactly one 7 & two 3's, the other digit must be 1 or 8. We have two ways of choosing this number. We have $\binom{4}{2}$ ways of choosing the positions of the two 3's, which leaves two ways to choose the position of the 7, and one way to place the remaining digit.

$2 \cdot \binom{4}{2} \cdot 2 \cdot 1 = 2 \cdot \dfrac{4!}{2!\,2!} \cdot 2! = 2 \cdot \dfrac{4!}{2!}$

The factor of 2! in the denominator represents the number of ways we could permute the two 3's within a given arrangement without producing an arrangement distinguishable from that arrangement.

There is 3 cases:-

1) All digits are distinct $= 4! = 24$.

2) Exactly two digits are the same $= 2 \times 3 \times \dfrac{4!}{2!} = 72$.

for choosing 3 or 7 to coming twice; for choosing 2 digits from others; the set for ordering these 4 digits. Division by 2 is for repeated counting.

3) Numbers consist of 3, 7 $= \dfrac{4!}{2! \times 2!} = 6$.

[2! appear because of double counting]

Total $= 24 + 72 + 6 = 102$ ways //

Binomial and Multinomial Theorems

① $(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r} = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$

② $(x_1+x_2+\dots+x_k)^n = \sum_{n_i} \binom{n}{n_1, n_2, \dots, n_k} x_1^{n_1}, x_2^{n_2} \dots x_k^{n_k}$

* **Note :-**

$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!\, n_2! \dots n_k!}$

① Find coefficient of

(i) x^9y^3 in the expansion of $(2x-3y)^{12}$

Sol^n:- $(2x-3y)^{12} = \sum_{r=0}^{12} \binom{12}{r} (2x)^r (-3y)^{12-r}$

$\boxed{x=2x \quad y=-3y \quad n=12}$

$= \sum_{r=0}^{12} \binom{12}{r} 2^r x^r (-3)^{12-r} (y)^{12-r}$

$x^9 \qquad y^{12-9} \qquad y^3$

If $r=9$

$\binom{12}{9} 2^9 (-3)^3$

$\boxed{r=9} = \frac{12!}{(12-9)!\times 9!} \times 2^9 \times (-3)^3$

(ii) x^0 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$.

Sol^n:- $\left(3x^2-\frac{2}{x}\right)^{15} = \sum_{r=0}^{15} \binom{15}{r} (3x^2)^r \left(\frac{-2}{x}\right)^{15-r}$ $\quad (x^m)^n = x^{mn}$

$x = 3x^2$

$y = -\frac{2}{x}$

$n = 15$

$= \sum_{r=0}^{15} \binom{15}{r} 3^r x^{2r} (-2)^{15-r} \frac{1}{x^{15-r}}$

$= \sum_{r=0}^{15} \binom{15}{r} 3^r (-2)^{15-r} x^{2r-15+r} \quad 2r-(15-r)$

$= \sum_{r=0}^{15} \binom{15}{r} 3^r (-2)^{15-r} x^{3r-15}$

$x^{3(5)-15}$

$\boxed{x^0}$

$\boxed{r=5} = \binom{15}{5} 3^5 (-2)^{10}$

② Find coefficient of

(i) x^{12} in the expansion of $x^3(1-2x)^{10}$

Sol^n:- $x^3(1-2x)^{10}$

$= x^3 \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2x)^{10-r}$

$= x^3 \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2)^{10-r} x^{10-r}$

$= \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2)^{10-r} x^{10-r+3}$

$= \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2)^{10-r} x^{13-r}$

$x^{13-1} \quad x^{12} \quad \boxed{r=1}$

$= \binom{10}{1} (1)^1 (-2)^{10-1}$

$= \frac{10!}{(10-1)!\times 1!} \times 1 \times (-2)^9$

$= \frac{10\times 9!}{9!} \times (-2)^9$

$= 10 \times (-2)^9$

$= -5120$

(ii) x^k in the expansion of $(1+x+x^2)(1+x)^n$.

Sol^n:- $(1+x+x^2)(1+x)^n$

$= (1+x+x^2) \sum_{r=0}^{n} \binom{n}{r} (1)^r x^{n-r}$

$= (1+x+x^2) \sum_{r=0}^{n} \binom{n}{r} x^{n-r}$

$= \sum_{r=0}^{n} \binom{n}{r} x^{n-r} + \sum_{r=0}^{n} \binom{n}{r} x^{n+1-r} + \sum_{r=0}^{n} \binom{n}{r} x^{n+2-r}$

$= \binom{n}{n-k} + \binom{n}{n+1-k} + \binom{n}{n+2-k}$

$r = n-k \qquad r = n+1-k \qquad r = n+2-k$

$x^{n-r} \qquad x^{n+1-r} \qquad x^{n+2-r}$

$x^{n-(n-k)} \qquad x^{n+1-(n+1-k)} \qquad x^{n+2-(n+2-k)}$

x^{n-n+k}

$\boxed{x^k} \qquad \boxed{x^k} \qquad \boxed{x^k}$

Multinomial :-

① Determine coefficient of

(i) xyz^2 in the expansion of $(2x-y-z)^4$

Soln :-

$$\binom{n}{n_1\ n_2\ n_3} x_1^{n_1} x_2^{n_2} x_3^{n_3}$$

$n=4$ $\quad x_1 = 2x$

$n_1 = 1$ $\quad x_2 = -y$

$n_2 = 1$ $\quad x_3 = -z$

$n_3 = 2$

$$\Rightarrow \binom{4}{1\ 1\ 2} (2x)^1 (-y)^1 (-z)^2$$

$$= \binom{4}{1\ 1\ 2} 2^1 x^1 (-1)^1 y^1 (-1)^2 z^2$$

$$= \binom{4}{1\ 1\ 2} 2 \times -1 \times 1 \quad xyz^2$$

$$= \frac{4!}{1!\,1!\,2!} * -2 \qquad \binom{n}{n_1, n_2, \ldots n_k} = \frac{n!}{n_1!\,n_2!\ldots n_k!}$$

$$= \frac{4 \times 3 \times \cancel{2!}}{\cancel{2!}} * (-2)$$

$$= -24$$

(ii) $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$

Soln :-

$$\binom{n}{n_1\ n_2\ n_3} x_1^{n_1} x_2^{n_2} x_3^{n_3}$$

$n=7$ $\quad x_1 = 3x$

$n_1 = 2$ $\quad x_2 = -2y$

$n_2 = 2$ $\quad x_3 = -4z$

$n_3 = 3$

$$\Rightarrow \binom{7}{2\ 2\ 3} = (3x)^2 (-2y)^2 (-4z)^3$$

$$= \binom{7}{2\ 2\ 3} 3^2 x^2 (-2)^2 y^2 (-4)^3 z^3$$

$$= \binom{7}{2\ 2\ 3} 3^2 * (-2)^2 * (-4)^3 x^2 y^2 z^3$$

$$= \frac{7!}{2!\,2!\,3!} * 9 * (4) * (-64)$$

$$= 483840$$

② Find coefficient of v^2w^4xz in the expansion of $(3v+2w+x+y+z)^8$.

Soln :-

$$\binom{n}{n_1\ n_2\ n_3\ n_4\ n_5} x_1^{n_1} x_2^{n_2} x_3^{n_3} x_4^{n_4} x_5^{n_5}$$

$n=8$ $\quad x_1 = 3v$

$n_1 = 2$ $\quad x_2 = 2w$

$n_2 = 4$ $\quad x_3 = x$

$n_3 = 1$ $\quad x_4 = y$

$n_4 = 0$ $\quad x_5 = z$

$n_5 = 1$

$$\Rightarrow \binom{8}{2\ 4\ 1\ 0\ 1} (3v)^2 (2w)^4 (x)^1 (y)^0 (z)^1$$

$$= \binom{8}{2\ 4\ 1\ 0\ 1} 3^2 * v^2\, 2^4\, w^4\, x^1\, y^0\, z^1$$

$$= \binom{8}{2\ 4\ 1\ 0\ 1} 3^2 * 2^4 * v^2 w^4 x^1 y^0 z^1$$

$$= \binom{8}{2\ 4\ 1\ 0\ 1} 9 * 16$$

$$= \frac{8!}{2!\,4!\,1!\,0!\,1!} * 9 * 16$$

$$= 840 \times 9 \times 16$$

$$= 120960$$

Combination with Repitation:-

$C(n+r-1, r)$ — n = no of distinct objects, r = no of identical objects.

① In how many we can distribute 10 identical marbles among 6 distinct containers.

Soln:-

$r = 10$

$n = 6$

$C(n+r-1, r)$

$C(6+10-1, 10)$

$C(15, 10) = \frac{15!}{(15-10)! \times 10!}$

$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times \cancel{10!}}{5! \times \cancel{10!}}$

$= 3003$ //.

② In how many ways 10 identical pencils can be distributed among 5 children in the following cases

(i) There are no restriction

(ii) Each child gets atleast 1 pencil.

(iii) A youngest child gets atleast 2 pencils.

Soln:- $r = 10$, $n = 5$

(i) $C(n+r-1, r)$

$= C(5+10-1, 10)$

$= C(14, 10)$

$= {}^{14}C_{10}$

$= \frac{14!}{(14-10)! \times 10!}$

$= \frac{14!}{4! \times 10!}$

$= \frac{14 \times 13 \times 12 \times 11 \times \cancel{10!}}{4! \times \cancel{10!}}$

$= 1001$ //

(ii) if each child get one pencil

$\boxed{\boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}} = 5$

$\boxed{r=5}$ $\boxed{n=5}$

$= C(n+r-1, r)$

$= C(5+5-1, 5)$

$= C(9, 5)$

$= {}^{9}C_{5}$

$= \frac{9!}{(9-5)! \times 5!}$

$= \frac{9!}{4! \times 5!}$

$= \frac{9 \times 8 \times 7 \times 6 \times \cancel{5!}}{4! \times \cancel{5!}}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$

$= \cancel{756}\ 126$ //.

(iii) A youngest child gets atleast 2 pencils.

$\boxed{r=8}$ $\boxed{n=5}$

$= C(n+r-1, r)$

$= C(5+8-1, 8)$

$= C(12, 8)$

$= {}^{12}C_{8}$

$= \frac{12!}{(12-8)! \times 8!}$

$= \frac{12!}{4! \times 8!}$

$= \frac{\cancel{12} \times 11 \times 10 \times 9 \times \cancel{8!}}{\cancel{4} \times \cancel{3} \times 2 \times 1 \times \cancel{8!}}$

$= 495$ //

③ In how many ways can one distribute 8 identical balls into 4 distinct containers so ~~that~~

(i) No container is left empty.

(ii) The fourth container gets an odd number of balls.

Soln:-

$r = 8$, $n = 4$

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(i) $\boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}$ containers

$r=4,\ n=4$

$= C(n+r-1, r)$

$= C(4+4-1, 4)$

$= C(7,4)$

$= {}^{7}C_{4}$

$= \frac{7!}{(7-4)! \times 4!}$

$= \frac{7!}{3! \times 4!}$

$= \frac{7 \times 6 \times 5 \times \cancel{4!}}{3! \times \cancel{4!}}$

$= \frac{7 \times \cancel{6} \times 5}{\cancel{3} \times \cancel{2} \times 1}$

$= 35$

(ii) $\square\ \square\ \square\ \square$

odd balls

$\boxed{n=3}$

1 $\Rightarrow$ then $\boxed{r=7}$

$3 = \boxed{r=5}$

$5 = \boxed{r=3}$

$7 = \boxed{r=1}$

* if $n=3,\ r=7$

$= C(n+r-1, r)$

$= C(3+7-1, 7)$

$= C(9, 7)$

$= {}^{9}C_{7}$

$= \frac{9!}{(9-7)! \times 7!}$

$= \frac{9 \times 8 \times \cancel{7!}}{2! \times \cancel{7!}}$

$= \frac{9 \times \cancel{8}^{4}}{\cancel{2} \times 1}$

$= 36$

* if $n=3,\ r=5$

$= C(n+r-1, r)$

$= C(3+5-1, 5)$

$= C(7,5)$

$= {}^{7}C_{5}$

$= \frac{7!}{(7-5)! \times 5!}$

$= \frac{7 \times 6 \times \cancel{5!}}{2! \times \cancel{5!}}$

$= \frac{7 \times \cancel{6}^{3}}{\cancel{2} \times 1}$

$= 21$

* if $n=3,\ r=3$

$= C(n+r-1, r)$

$= C(3+3-1, 3)$

$= C(5,3)$

$= {}^{5}C_{3}$

$= \frac{5!}{(5-3)! \times 3!}$

$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2} \times 1 \times \cancel{3!}}$

$= 10$

* if $n=3,\ r=1$

$= C(n+r-1, r)$

$= C(3+1-1, 1)$

$= C(3,1)$

$= {}^{3}C_{1}$

$= \frac{3!}{(3-1)! \times 1!}$

$= \frac{3!}{2! \times 1!}$

$= \frac{3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times 1 \times \cancel{1}}$

$= 3$

$= 36 + 21 + 10 + 3$

$= 70$ ways

(4) In how many ways can we distribute 7 apples and 6 oranges among 4 children. So that each child gets atleast one apple.

Soln:- (i) if $n=4$, $r=7$

For Apples

$\cancel{= C(n+r-1, r)}$

if each child gets atleast one apple.

$n=4$, $r=3$

$= C(n+r-1, r)$

$= C(4+3-1, 3)$

$= C(6, 3)$

$= {}^{6}C_{3}$

$= \dfrac{6!}{(6-3)! \times 3!}$

$= \dfrac{\cancel{6} \times 5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{3!}}$

$= 20$

(ii)

For Oranges

if $n=4$, $r=6$

No restrictions

$= C(n+r-1, r)$

$= C(4+6-1, 6)$

$= C(9, 6)$

$= {}^{9}C_{6}$

$= \dfrac{9!}{(9-6)! \times 6!}$

$= \dfrac{{}_{3}\cancel{9} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}}$

$= 84$

$= 20 \times 84$

$= 1680$ ways

(5) Find the number of ways of giving 10 identical gift boxes to 6 persons A, B, C, D, E, F in a such a way that the total number of boxes given to A and B together does not exceed 4.

Soln:- $r=10$, $n=6$

(i) A and B.

$0 \le r \le 4$

$n=2$, $r=r$

$= C(n+r-1, r)$

$= C(2+r-1, r)$

$= C(1+r, r)$

$= {}^{1+r}C_{r}$

$= \dfrac{(1+r)!}{(1+r-r)! \times r!}$

$= \dfrac{(1+r)!}{1! \times r!}$

$= (r+1)$ if, $r=2$ $= \dfrac{3!}{2!} = \dfrac{3 \times \cancel{2!}}{\cancel{2!}}$

(ii) C, D, E, F

$\boxed{n=4}$ $\boxed{r=10-r}$

$= C(n+r-1, r)$

$= C(4+10-r-1, 10-r)$

$= C(13-r, 10-r)$

$= {}^{13-r}C_{10-r}$

$= \dfrac{(13-r)!}{((13-\cancel{r})-(10-\cancel{r}))! \times (10-r)!}$

$= \dfrac{(13-r)!}{3!\,(10-r)!}$

$$\boxed{\sum_{r=0}^{4} (r+1) * \frac{(13-r)!}{3!\,(10-r)!}}$$