Roulette Simulation

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Contents

Introduction

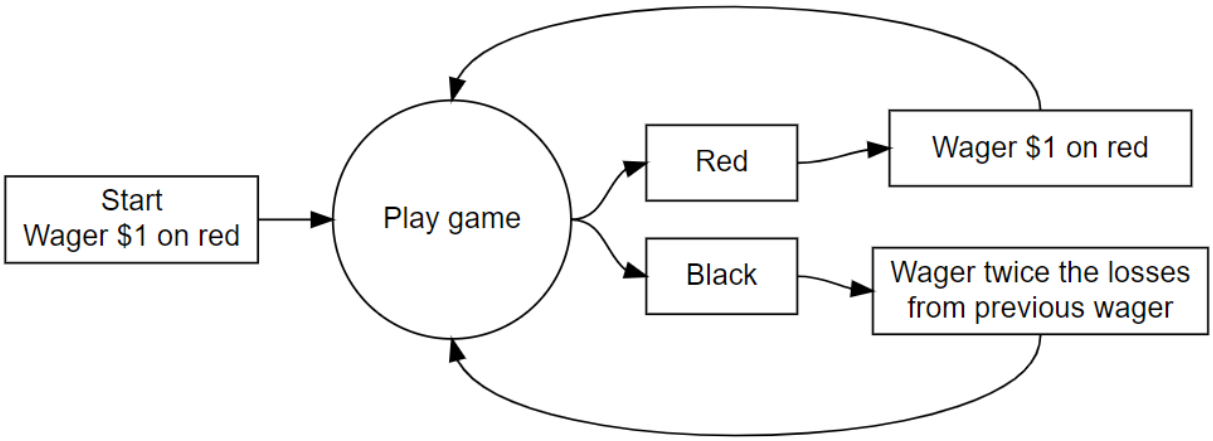
In this blog post, we explore Martingale's strategy and its viability through simulation and based on certain constraints applicable in a real world scenario(albeit with different parameter values depending on the setting).

Background

A brief blurb about Roulette in General :

A roulette table composed of 38 evenly sized pockets on a wheel. The pockets are colored red, black, or green. The pockets are also numbered. Roulette is a game of chance in which a pocket is randomly selected. Gamblers may wager on several aspects of the outcome. For example, one may place a wager that the randomly selected pocket will be red or odd numbered or will be a specific number. In our scenario, all one needs to know is that there are 38 pockets of which 2 are green, 18 are red, and 18 are black. The payout for a bet on black (or red) is $1 for each $1 wagered. This means that if a gambler bets $1 on black and the randomly selected pocket is black, then the gambler will get the original $1 wager and an additional $1 as winnings.

Here's a pictorial representation of Martingale's strategy. The Martingale strategy appears to always end in positive earnings, regardless of how unlucky a string of spins may be.



Constraints for the Simulation :

Stopping rule

A player will use the above strategy and play until

- The player has W(Winning Threshold for Stopping) dollars
- The player goes bankrupt
- The player completes L wagers (or plays)

In this scenario W is 300$ (Starting Budget + Winnings) abd L is 1000 plays.

Budget

The player starts with B(Starting Budget B - 200$) dollars. The player cannot wager more money than he/she has.

Maximum wager

Some casinos have a maximum bet. Call this parameter M. If the strategy directs the player to wager more than M dollars, then the player will only wager M dollars(100$ in this scenario).

Methods

The below chunks of code show the modular processes involved in simulation of the roulette game based on the above constraints. one_series() function is a culmination of the entire roulette spin which utilizes all the modular functions.

This chunk of code simulates a single spin of the roulette

```
#Simulates Single Spin
single_spin <- function(){
  possible_outcomes <- c(rep("red",18), rep("black",18), rep("green",2))
  sample(possible_outcomes, 1)
}
```

This chunk of code calculates the wager for the upcoming turn based on the previous wager as well as the previous outcome.

```r
#Simulates martingale wager
martingale_wager <- function(
  previous_wager
  , previous_outcome
  , max_wager
  , current_budget
){
  if(previous_outcome == "red") return(1)
  min(2*previous_wager, max_wager, current_budget)
}
```

This chunk of code simulates one turn of the roulette outputting specific parameters to be used as part of a series of spins

```r
#Simulates a single play/turn in a roulette game.
one_play <- function(previous_ledger_entry, max_wager){
  # Create a copy of the input object that will become the output object
  out <- previous_ledger_entry
  out[1, "game_index"] <- previous_ledger_entry[1, "game_index"] + 1
  out[1, "starting_budget"] <- previous_ledger_entry[1, "ending_budget"]
  out[1, "wager"] <- martingale_wager(
    previous_wager = previous_ledger_entry[1, "wager"]
    , previous_outcome = previous_ledger_entry[1, "outcome"]
    , max_wager = max_wager
    , current_budget = out[1, "starting_budget"]
  )
  out[1, "outcome"] <- single_spin()
  out[1, "ending_budget"] <- out[1, "starting_budget"] +
    ifelse(out[1, "outcome"] == "red", +1, -1)*out[1, "wager"]
  return(out)
}
```

This chunk of code establishes the stopping condition based on factors discussed above

```r
#Stopping Condition
stopping_rule <- function(
  ledger_entry
  , winning_threshold
){
  ending_budget <- ledger_entry[1, "ending_budget"]
  if(ending_budget <= 0) return(TRUE)
  if(ending_budget >= winning_threshold) return(TRUE)
  FALSE
}
```

This chunk of code simulates the entire series of roulette spins until a certain stopping condition is achieved

```r
#Simulation of the entire roulette game.
one_series <- function(
  max_games, starting_budget, winning_threshold, max_wager
){
  # Initialize ledger
  ledger <- data.frame(
```

```
      game_index = 0:max_games
    , starting_budget = NA_integer_
    , wager = NA_integer_
    , outcome = NA_character_
    , ending_budget = NA_integer_
  )
  ledger[1, "wager"] <- 1
  ledger[1, "outcome"] <- "red"
  ledger[1, "ending_budget"] <- starting_budget
  for(i in 2:nrow(ledger)){
    ledger[i,] <- one_play(ledger[i-1,], max_wager)
    if(stopping_rule(ledger[i,], winning_threshold)) break
  }
  # Return non-empty portion of ledger
  ledger[2:i, ]
}
```

Results

The below code chunk gives the profit over a full series of roulette spins which is a property of interest

```
#Profit function given the entire series of plays..
profit <- function(ledger){
  n <- nrow(ledger)
  profit <- ledger[n, "ending_budget"] - ledger[1, "starting_budget"]
  return(profit)
}
```

```
#All of the below function inputs are based on constraints described above
a <- one_series(1000,200,300,100)
a
```

```
##     game_index starting_budget wager outcome ending_budget
## 2            1             200     1   black           199
## 3            2             199     2     red           201
## 4            3             201     1   black           200
## 5            4             200     2     red           202
## 6            5             202     1     red           203
## 7            6             203     1   black           202
## 8            7             202     2   black           200
## 9            8             200     4   black           196
## 10           9             196     8     red           204
## 11          10             204     1   black           203
## 12          11             203     2     red           205
## 13          12             205     1   green           204
## 14          13             204     2   black           202
## 15          14             202     4     red           206
## 16          15             206     1   black           205
## 17          16             205     2     red           207
## 18          17             207     1     red           208
## 19          18             208     1     red           209
```

```
## 20          19          209     1     red          210
## 21          20          210     1     red          211
## 22          21          211     1   black          210
## 23          22          210     2   black          208
## 24          23          208     4   black          204
## 25          24          204     8   black          196
## 26          25          196    16     red          212
## 27          26          212     1     red          213
## 28          27          213     1     red          214
## 29          28          214     1   black          213
## 30          29          213     2     red          215
## 31          30          215     1   black          214
## 32          31          214     2   black          212
## 33          32          212     4   black          208
## 34          33          208     8   black          200
## 35          34          200    16     red          216
## 36          35          216     1   green          215
## 37          36          215     2     red          217
## 38          37          217     1   black          216
## 39          38          216     2   black          214
## 40          39          214     4   black          210
## 41          40          210     8   black          202
## 42          41          202    16     red          218
## 43          42          218     1   black          217
## 44          43          217     2   black          215
## 45          44          215     4     red          219
## 46          45          219     1   black          218
## 47          46          218     2   black          216
## 48          47          216     4   black          212
## 49          48          212     8   black          204
## 50          49          204    16     red          220
## 51          50          220     1     red          221
## 52          51          221     1   black          220
## 53          52          220     2   black          218
## 54          53          218     4   black          214
## 55          54          214     8     red          222
## 56          55          222     1     red          223
## 57          56          223     1     red          224
## 58          57          224     1   green          223
## 59          58          223     2     red          225
## 60          59          225     1     red          226
## 61          60          226     1     red          227
## 62          61          227     1   green          226
## 63          62          226     2   black          224
## 64          63          224     4   black          220
## 65          64          220     8     red          228
## 66          65          228     1   black          227
## 67          66          227     2   black          225
## 68          67          225     4   black          221
## 69          68          221     8   black          213
## 70          69          213    16   black          197
## 71          70          197    32     red          229
## 72          71          229     1   black          228
## 73          72          228     2     red          230
```

```
## 74          73          230     1   black          229
## 75          74          229     2     red          231
## 76          75          231     1   black          230
## 77          76          230     2   black          228
## 78          77          228     4     red          232
## 79          78          232     1   black          231
## 80          79          231     2   black          229
## 81          80          229     4   green          225
## 82          81          225     8     red          233
## 83          82          233     1     red          234
## 84          83          234     1     red          235
## 85          84          235     1     red          236
## 86          85          236     1     red          237
## 87          86          237     1     red          238
## 88          87          238     1   black          237
## 89          88          237     2     red          239
## 90          89          239     1     red          240
## 91          90          240     1   green          239
## 92          91          239     2   black          237
## 93          92          237     4   black          233
## 94          93          233     8   black          225
## 95          94          225    16     red          241
## 96          95          241     1   black          240
## 97          96          240     2     red          242
## 98          97          242     1     red          243
## 99          98          243     1     red          244
## 100         99          244     1     red          245
## 101        100          245     1   black          244
## 102        101          244     2   black          242
## 103        102          242     4     red          246
## 104        103          246     1     red          247
## 105        104          247     1   black          246
## 106        105          246     2     red          248
## 107        106          248     1     red          249
## 108        107          249     1     red          250
## 109        108          250     1   black          249
## 110        109          249     2   green          247
## 111        110          247     4     red          251
## 112        111          251     1   green          250
## 113        112          250     2     red          252
## 114        113          252     1   black          251
## 115        114          251     2   black          249
## 116        115          249     4     red          253
## 117        116          253     1   black          252
## 118        117          252     2   black          250
## 119        118          250     4   black          246
## 120        119          246     8     red          254
## 121        120          254     1     red          255
## 122        121          255     1   black          254
## 123        122          254     2   black          252
## 124        123          252     4   black          248
## 125        124          248     8   black          240
## 126        125          240    16   green          224
## 127        126          224    32   black          192
```

```
## 128        127             192    64     red           256
## 129        128             256     1     red           257
## 130        129             257     1     red           258
## 131        130             258     1     red           259
## 132        131             259     1   black           258
## 133        132             258     2     red           260
## 134        133             260     1     red           261
## 135        134             261     1     red           262
## 136        135             262     1     red           263
## 137        136             263     1     red           264
## 138        137             264     1     red           265
## 139        138             265     1   black           264
## 140        139             264     2     red           266
## 141        140             266     1   black           265
## 142        141             265     2     red           267
## 143        142             267     1   black           266
## 144        143             266     2   black           264
## 145        144             264     4   black           260
## 146        145             260     8   green           252
## 147        146             252    16     red           268
## 148        147             268     1   black           267
## 149        148             267     2     red           269
## 150        149             269     1     red           270
## 151        150             270     1     red           271
## 152        151             271     1     red           272
## 153        152             272     1   black           271
## 154        153             271     2   black           269
## 155        154             269     4   black           265
## 156        155             265     8   black           257
## 157        156             257    16     red           273
## 158        157             273     1   green           272
## 159        158             272     2   green           270
## 160        159             270     4   black           266
## 161        160             266     8     red           274
## 162        161             274     1   black           273
## 163        162             273     2   black           271
## 164        163             271     4     red           275
## 165        164             275     1   black           274
## 166        165             274     2     red           276
## 167        166             276     1   green           275
## 168        167             275     2     red           277
## 169        168             277     1   black           276
## 170        169             276     2     red           278
## 171        170             278     1   black           277
## 172        171             277     2     red           279
## 173        172             279     1   black           278
## 174        173             278     2   black           276
## 175        174             276     4     red           280
## 176        175             280     1   black           279
## 177        176             279     2     red           281
## 178        177             281     1   black           280
## 179        178             280     2   black           278
## 180        179             278     4     red           282
## 181        180             282     1   black           281
```

```
## 182        181             281     2     red           283
## 183        182             283     1     red           284
## 184        183             284     1     red           285
## 185        184             285     1   black           284
## 186        185             284     2     red           286
## 187        186             286     1   black           285
## 188        187             285     2     red           287
## 189        188             287     1   green           286
## 190        189             286     2     red           288
## 191        190             288     1     red           289
## 192        191             289     1   black           288
## 193        192             288     2     red           290
## 194        193             290     1   green           289
## 195        194             289     2     red           291
## 196        195             291     1     red           292
## 197        196             292     1     red           293
## 198        197             293     1     red           294
## 199        198             294     1     red           295
## 200        199             295     1     red           296
## 201        200             296     1     red           297
## 202        201             297     1     red           298
## 203        202             298     1     red           299
## 204        203             299     1   black           298
## 205        204             298     2     red           300
```

```
profit(a)
```

```
## [1] 100
```

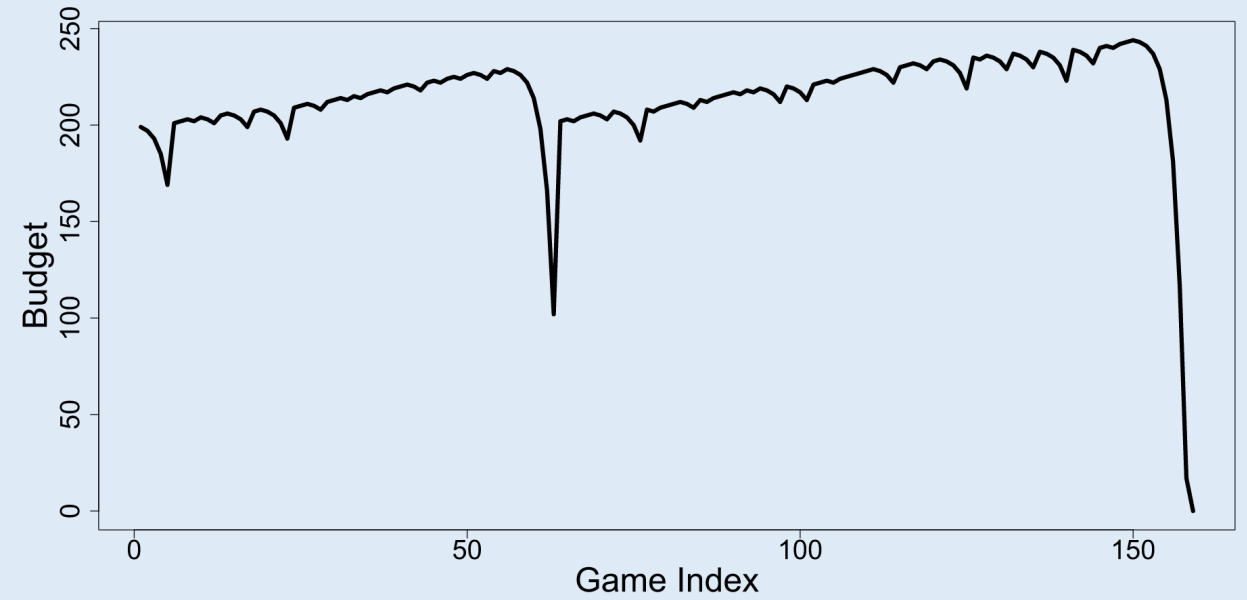
The results are not always positive in the sense that the chances of winning and losing seem to be random and the Martingale strategy **under these constraints is not** a guaranteed success. A specific observation is that a vast majority of profit results tend to be either -200 or 100. This is due to the either one of the stopping conditions being achieved - 1000 turns (extremely low chance of happening due to the small difference between starting amount and the winning threshold and the starting amount itself being low enough that 8-9 continuous losses would cause the player to lose the entire initial capital) or achieving the winning threshold or losing the initial capital/budget.

```
library(magrittr)

svg(filename = "pattern.svg", width=16, height =8.25)
par(cex.axis=2.0, cex.lab = 2.5, mar = c(8,8,2,2), bg = rgb(222, 235, 247, max = 255))
#set.seed(1)
#Constraints chosen based on above stipulations.
ledger <- one_series(1000,200,300,100)
plot(ledger[,c(1,5)], type = "l", lwd = 5, xlab = "Game Index", ylab = "Budget")
dev.off()
```

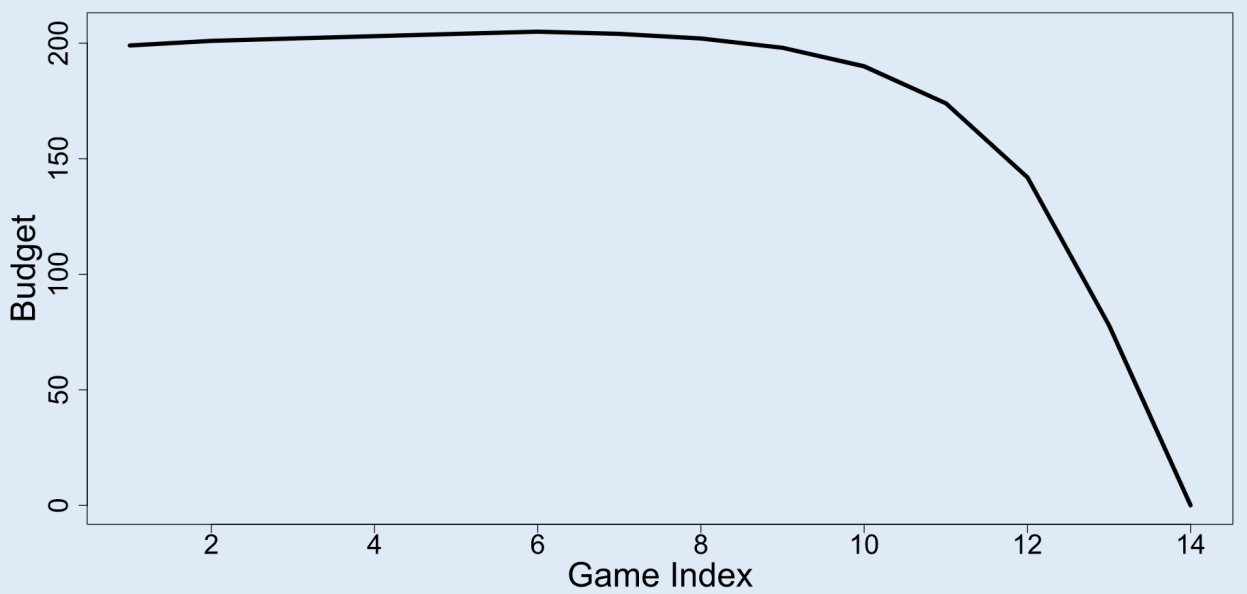
```
## pdf
##   2
```

The below figures show both gamblers winning and losing over the series of wagers based on above constraints.

Lost Wager



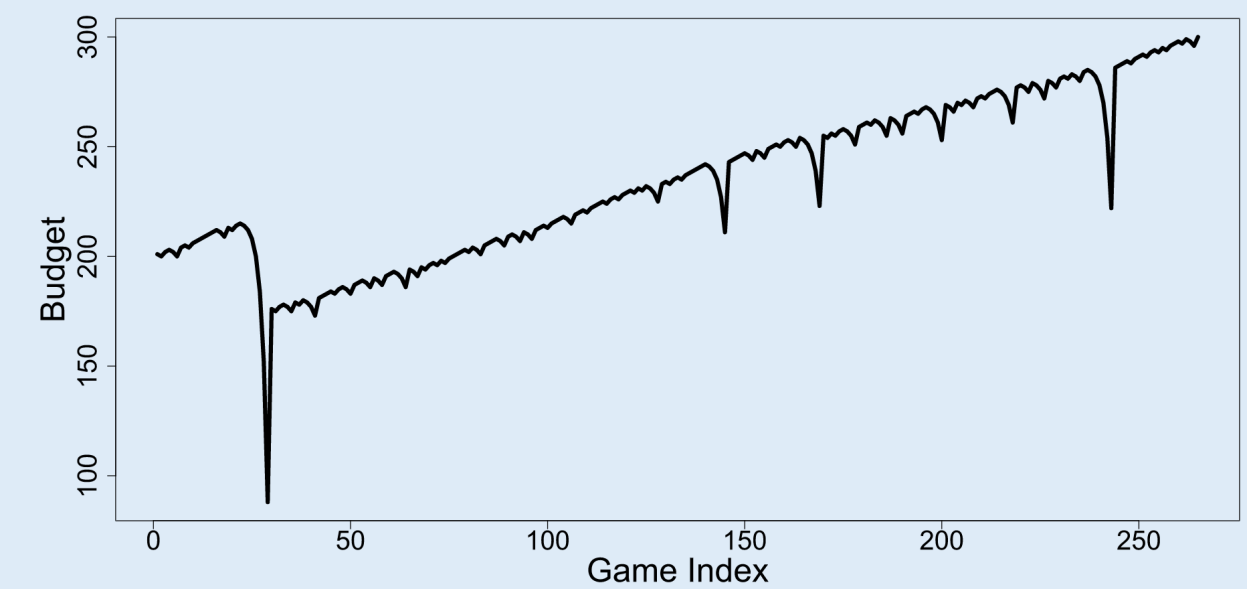
Here we see an example of a player get half way to his win condition but drops off sharply at his 250th turn.



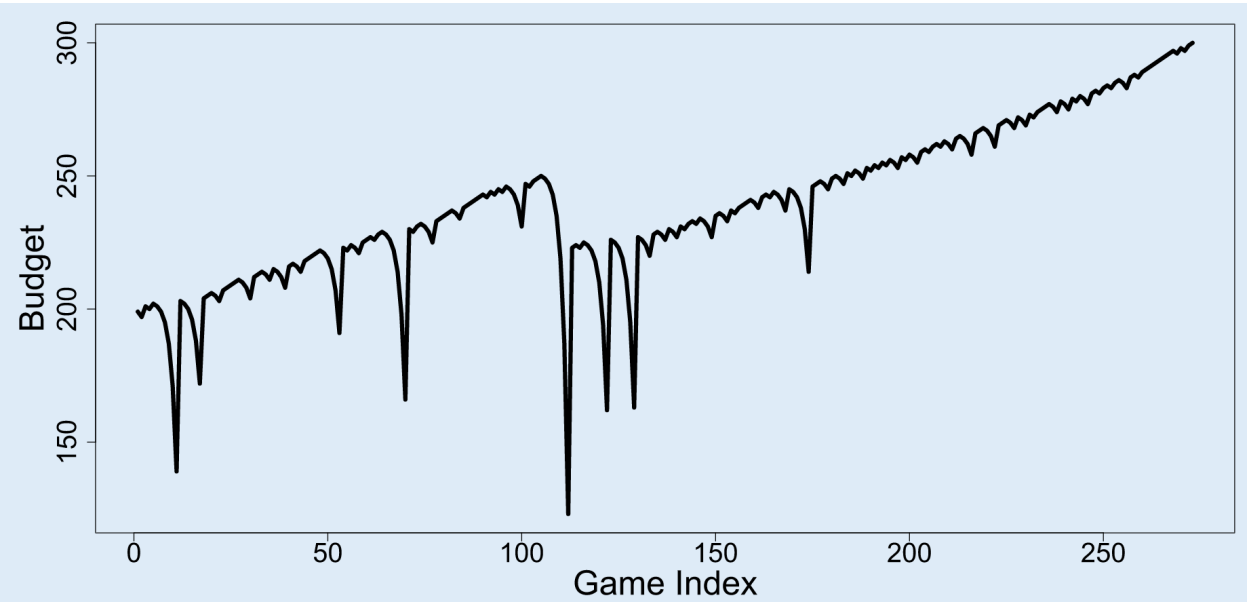
Here we see an example of a very swift loss right at turn 14.

Here we see a

Won Wager



Here is an example of a player almost losing at around turn 30 but managing to use the strategy and reach his win condition at turn 270 or so.



Here is another example of a player almost losing at many points in the game but managing to eek out the win eventually.

Conclusion and Discussion

A massive limitation with this simulation is the chosen constraint value. It can be concluded that the Martingale's strategy cannot guarantee a win with the above constraints. But, modification of said constraints can increase the probability of positive results. It can also be effectively concluded that the Martingale's strategy **guarantees** positive net earnings when there are no constraints and that the gambler has full control as to how many turns he gets to play, has an infinite (or a very large capital to spend) and there isn't a maximum bet restriction. Modifying some of these constraints can shift the odds in the gambler's favor though.

For instance,

- The budget of the gambler is directly proportional to the chance of a positive outcome.
- The number of chances to play is also directly proportional to the chance of a positive outcome. (In our scenario this played a very minor role due to how the stopping conditions were set up.)
- The maximum bet amount is inversely (although not in any linear fashion) proportional to the chance of a positive outcome.

All of the above listed points are highly situational and only applicable in a specific scenario to successfully execute the Martingale strategy as will be discussed in the conclusion.

The function below returns the total profits/losses of many replicates of roulette games using different constraints.

```
PResult <- function(r, max_games, starting_budget, winning_threshold, max_wager)
{
rep <- c(replicate(r, profit(one_series(max_games, starting_budget, winning_threshold, max_wager))))
return (rep)
}
```

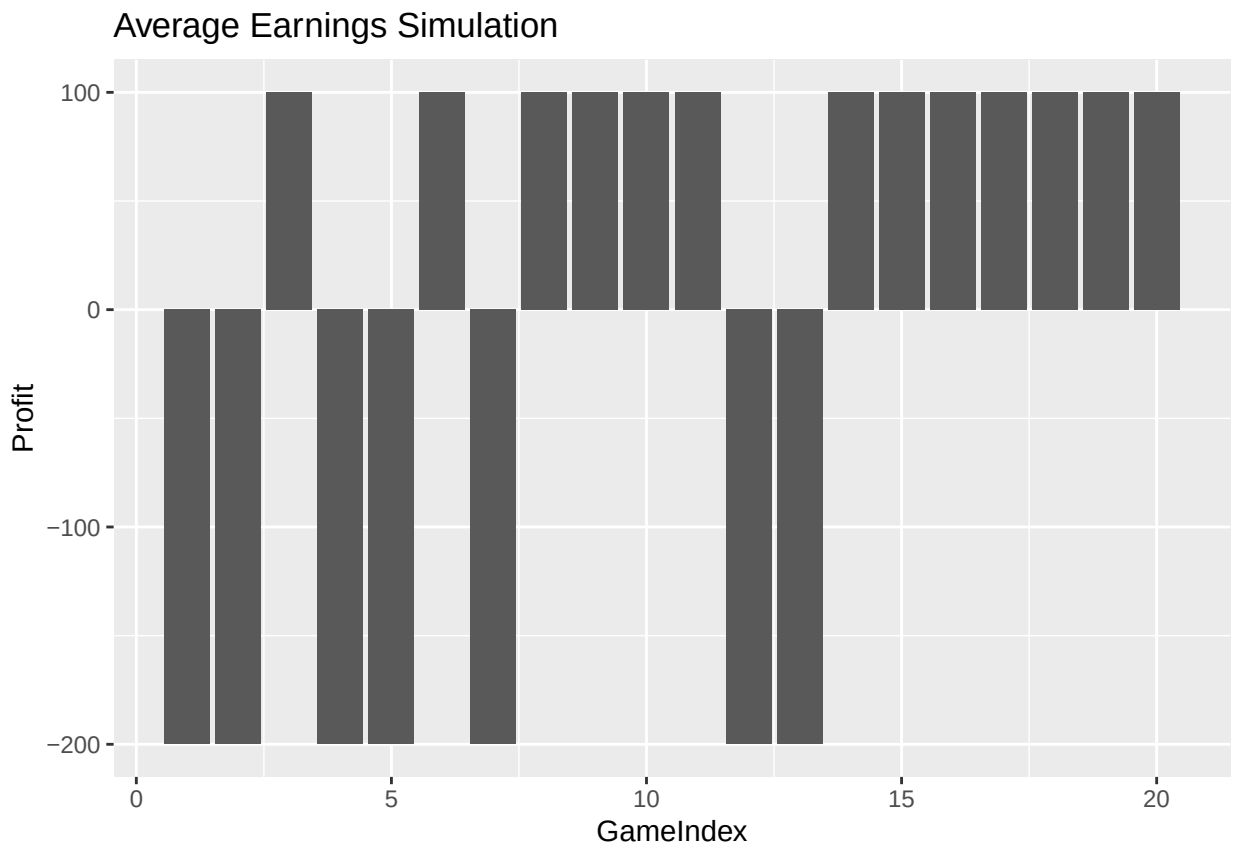
```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.4     v dplyr   1.0.7
## v tidyr   1.1.3     v stringr 1.4.0
## v readr   2.0.1     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x tidyr::extract()   masks magrittr::extract()
## x dplyr::filter()    masks stats::filter()
## x dplyr::lag()       masks stats::lag()
## x purrr::set_names() masks magrittr::set_names()
```

```
library(ggplot2)
```

This chunk of code shows how changing parameters based on the discussion above affects the outcome of the series of wagers i.e, gives us the distribution of earnings over games as well as average earnings over a series of games.

```
#.
set.seed(1)
a <- PResult(20, 1000, 200, 300, 100)
a1 <- as.data.frame(a)
a1 <- mutate(a1, GameIndex = 1:length(a))
colnames(a1) <- c("Profit", "GameIndex")
ggplot(data = a1, aes(x = GameIndex, y = Profit)) + geom_bar(stat = "identity") + labs(title = "Average
```



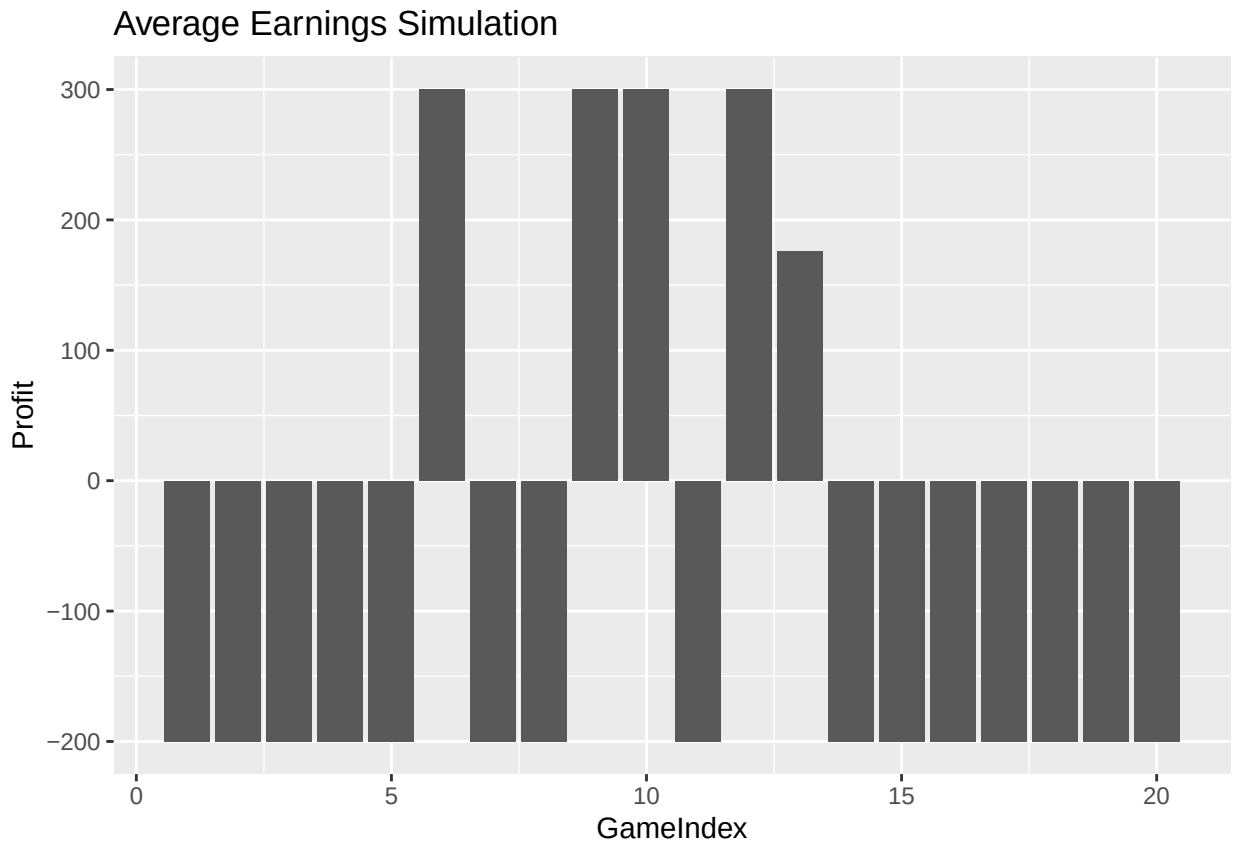

```
mean(a1$Profit)
```

```
## [1] -5
```

```
# get_last <- function(x) x[length(x)]
#
#
# # Simulation
# walk_out_money <- rep(NA, 50)
# for(j in seq_along(walk_out_money)){
#   walk_out_money[j] <- one_series(starting_budget = 200, winning_threshold = 300, max_games = 1000, m
# }
#
# lapply(walk_out_money, function(x) x[length(x)] - 200)
#
# # Walk out money distribution
# #hist(as.numeric(walk_out_money), breaks = 100)
#
# d1 <- as.data.frame(lapply(walk_out_money, function(x) x[length(x)] - 200))
# index(d1, col = 1:length(d1))
# dim(d1)
```

With these set of parameters, we have a mean earnings of -5.

```
set.seed(1)
b<- PResult(20, 1000, 200, 500, 100)
b1 <- as.data.frame(b)
b1 <- mutate(b1, GameIndex = 1:length(b))
colnames(b1) <- c("Profit", "GameIndex")
ggplot(data = b1, aes(x = GameIndex, y = Profit)) + geom_bar(stat = "identity") + labs(title = "Average
```

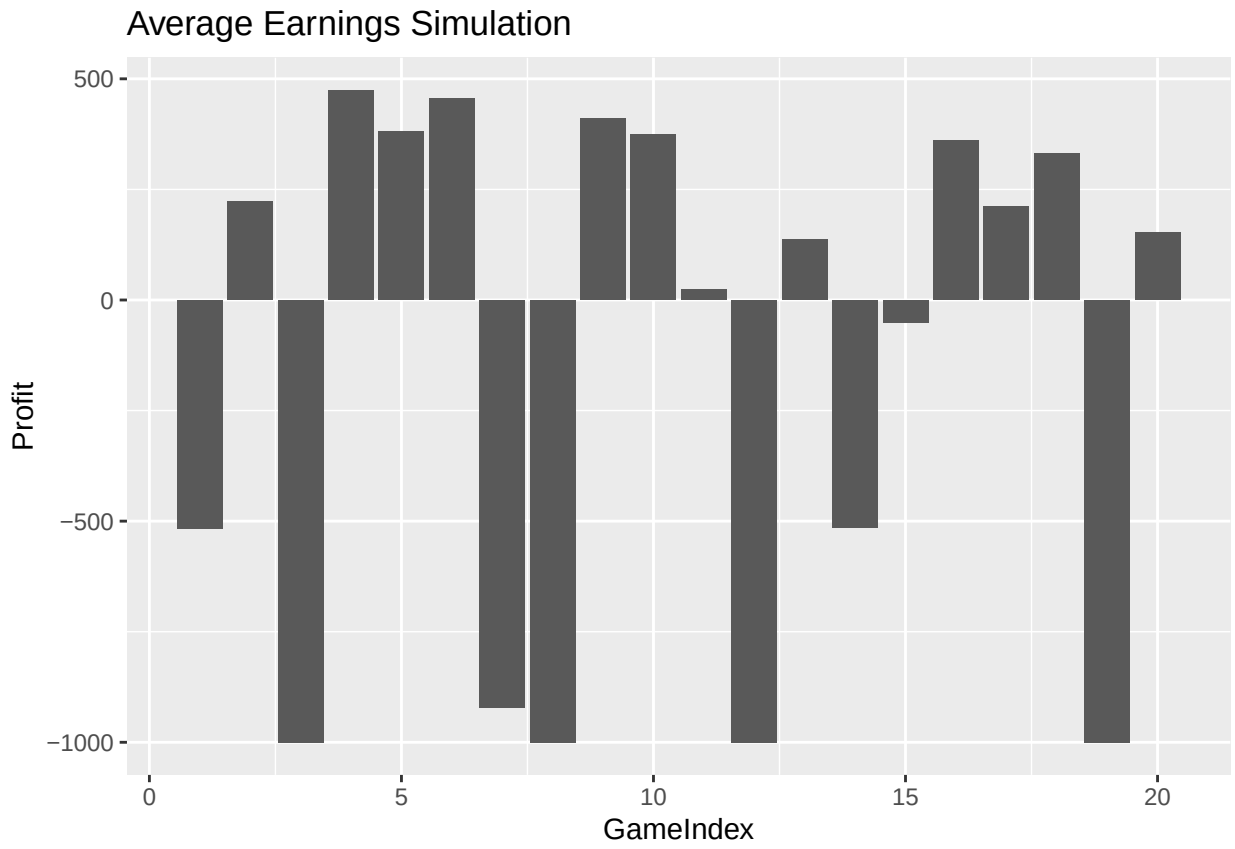



```
mean(b1$Profit)
```

```
## [1] -81.2
```

With these set of parameters, the mean earnings is -81.2. We had increased the winning threshold which proves to be problematic for the most part as the player would have to risk more to reach the higher winning threshold(to attain stopping condition) while having only 200$ to play around with.

```
set.seed(1)
c <- PResult(20, 1000, 1000, 2000, 200)
c1 <- as.data.frame(c)
c1 <- mutate(c1, GameIndex = 1:length(c))
colnames(c1) <- c("Profit", "GameIndex")
ggplot(data = c1, aes(x = GameIndex, y = Profit)) + geom_bar(stat = "identity") + labs(title = "Average
```

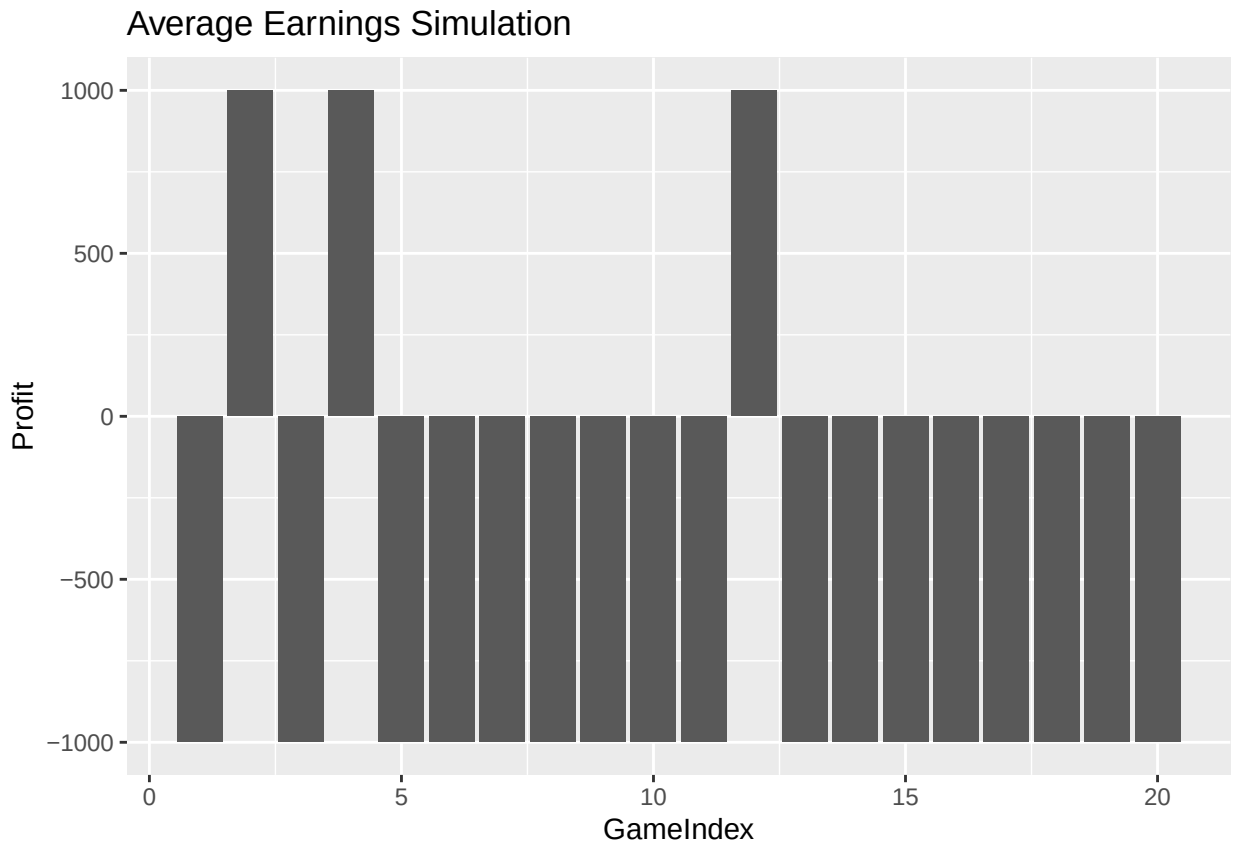



```
mean(c1$Profit)
```

```
## [1] -123.35
```

With these set of parameters, the mean earnings is -123.35 (We seem to have some sorely unlucky series of games so far!). We had increased the starting threshold and adjusted the winning threshold as well and the results don't seem too promising in this case either. An interesting observation is that the winning threshold is never reached (it would be reached with much higher repetitions but clearly, the probability is low. And the 1000 turn limit was actually used up in case of positive profits.). Perhaps more turns could deliver a full 1000$ profit? Let's explore that below.

```
set.seed(1)
d <- PResult(20, 10000, 1000, 2000, 200)
d1 <- as.data.frame(d)
d1 <- mutate(d1, GameIndex = 1:length(d))
colnames(d1) <- c("Profit", "GameIndex")
ggplot(data = d1, aes(x = GameIndex, y = Profit)) + geom_bar(stat = "identity") + labs(title = "Average
```




```
mean(d1$Profit)
```

```
## [1] -700
```

As expected, with a higher number of chances, we are able to reach the full 1000$ profit(1000$ being the difference between the winning_threshold and the starting_budget) but we still end up with a poor mean earnings of -700.

#Conclusion

Using the above observations, we can clearly see that Martingale's strategy while not a slam dunk in every scenario. This can be extended to state that based on a set of static conditions, Martingale's strategy is not a definite win strategy. It can however be considered a definite win strategy if there are no constraints(i.e, no budget limitations, no betting limitations etc) or if the constraints the dynamic (i.e, the starting budget can be completely controlled by the player as the game progresses - this alone could greatly increase the winning chances if the number of plays or the betting limit is high enough.). The more dynamic constraints, the faster/less number of turns it'll take to gain a positive outcome. Dynamically changing the winning threshold based on previous turns by the player also would guarantee a success provided the dynamic starting budget and reasonably high max number of turns.