Roulette Simulation

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Contents

Introduction

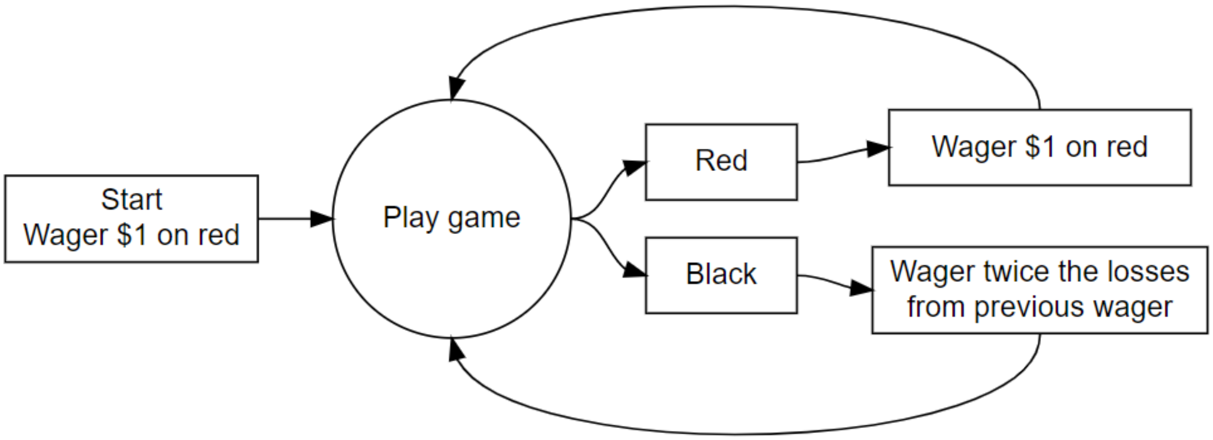
In this blog post, we explore Martingale's strategy and its viability through simulation and based on certain constraints applicable in a real world scenario(albeit with different parameter values depending on the setting).

Background

A brief blurb about Roulette in General :

A roulette table composed of 38 evenly sized pockets on a wheel. The pockets are colored red, black, or green. The pockets are also numbered. Roulette is a game of chance in which a pocket is randomly selected. Gamblers may wager on several aspects of the outcome. For example, one may place a wager that the randomly selected pocket will be red or odd numbered or will be a specific number. In our scenario, all one needs to know is that there are 38 pockets of which 2 are green, 18 are red, and 18 are black. The payout for a bet on black (or red) is $1 for each $1 wagered. This means that if a gambler bets $1 on black and the randomly selected pocket is black, then the gambler will get the original $1 wager and an additional $1 as winnings.

Here's a pictorial representation of Martingale's strategy. The Martingale strategy appears to always end in positive earnings, regardless of how unlucky a string of spins may be.



Constraints for the Simulation :

Stopping rule

A player will use the above strategy and play until

- The player has W(Winning Threshold for Stopping) dollars
- The player goes bankrupt
- The player completes L wagers (or plays)

In this scenario W is 300$ (Starting Budget + Winnings) abd L is 1000 plays.

Budget

The player starts with B(Starting Budget B - 200$) dollars. The player cannot wager more money than he/she has.

Maximum wager

Some casinos have a maximum bet. Call this parameter M. If the strategy directs the player to wager more than M dollars, then the player will only wager M dollars(100$ in this scenario).

Methods

The below chunks of code show the modular processes involved in simulation of the roulette game based on the above constraints. one_series() function is a culmination of the entire roulette spin which utilizes all the modular functions.

This chunk of code simulates a single spin of the roulette

```
#Simulates Single Spin
single_spin <- function(){
  possible_outcomes <- c(rep("red",18), rep("black",18), rep("green",2))
  sample(possible_outcomes, 1)
}
```

This chunk of code calculates the wager for the upcoming turn based on the previous wager as well as the previous outcome.

```
#Simulates martingale wager
martingale_wager <- function(
  previous_wager
  , previous_outcome
  , max_wager
  , current_budget
){
  if(previous_outcome == "red") return(1)
  min(2*previous_wager, max_wager, current_budget)
}
```

This chunk of code simulates one turn of the roulette outputting specific parameters to be used as part of a series of spins

```
#Simulates a single play/turn in a roulette game.
one_play <- function(previous_ledger_entry, max_wager){
  # Create a copy of the input object that will become the output object
  out <- previous_ledger_entry
  out[1, "game_index"] <- previous_ledger_entry[1, "game_index"] + 1
  out[1, "starting_budget"] <- previous_ledger_entry[1, "ending_budget"]
  out[1, "wager"] <- martingale_wager(
    previous_wager = previous_ledger_entry[1, "wager"]
    , previous_outcome = previous_ledger_entry[1, "outcome"]
    , max_wager = max_wager
    , current_budget = out[1, "starting_budget"]
  )
  out[1, "outcome"] <- single_spin()
  out[1, "ending_budget"] <- out[1, "starting_budget"] +
    ifelse(out[1, "outcome"] == "red", +1, -1)*out[1, "wager"]
  return(out)
}
```

This chunk of code establishes the stopping condition based on factors discussed above

```
#Stopping Condition
stopping_rule <- function(
  ledger_entry
  , winning_threshold
){
  ending_budget <- ledger_entry[1, "ending_budget"]
  if(ending_budget <= 0) return(TRUE)
  if(ending_budget >= winning_threshold) return(TRUE)
  FALSE
}
```

This chunk of code simulates the entire series of roulette spins until a certain stopping condition is achieved

```
#Simulation of the entire roulette game.
one_series <- function(
  max_games, starting_budget, winning_threshold, max_wager
){
  # Initialize ledger
  ledger <- data.frame(
```

```
      game_index = 0:max_games
    , starting_budget = NA_integer_
    , wager = NA_integer_
    , outcome = NA_character_
    , ending_budget = NA_integer_
  )
  ledger[1, "wager"] <- 1
  ledger[1, "outcome"] <- "red"
  ledger[1, "ending_budget"] <- starting_budget
  for(i in 2:nrow(ledger)){
    ledger[i,] <- one_play(ledger[i-1,], max_wager)
    if(stopping_rule(ledger[i,], winning_threshold)) break
  }
  # Return non-empty portion of ledger
  ledger[2:i, ]
}
```

Results

The below code chunk gives the profit over a full series of roulette spins which is a property of interest

```
#Profit function given the entire series of plays..
profit <- function(ledger){
  n <- nrow(ledger)
  profit <- ledger[n, "ending_budget"] - ledger[1, "starting_budget"]
  return(profit)
}
```

```
#All of the below function inputs are based on constraints described above
a <- one_series(1000,200,300,100)
a
```

```
##      game_index starting_budget wager outcome ending_budget
## 2             1             200     1     red           201
## 3             2             201     1   black           200
## 4             3             200     2   black           198
## 5             4             198     4   black           194
## 6             5             194     8     red           202
## 7             6             202     1     red           203
## 8             7             203     1     red           204
## 9             8             204     1     red           205
## 10            9             205     1     red           206
## 11           10             206     1     red           207
## 12           11             207     1   black           206
## 13           12             206     2     red           208
## 14           13             208     1     red           209
## 15           14             209     1     red           210
## 16           15             210     1   black           209
## 17           16             209     2   black           207
## 18           17             207     4     red           211
## 19           18             211     1   black           210
```

```
## 20      19      210  2    red      212
## 21      20      212  1    red      213
## 22      21      213  1  black      212
## 23      22      212  2  black      210
## 24      23      210  4    red      214
## 25      24      214  1  black      213
## 26      25      213  2    red      215
## 27      26      215  1  green      214
## 28      27      214  2    red      216
## 29      28      216  1    red      217
## 30      29      217  1    red      218
## 31      30      218  1  black      217
## 32      31      217  2    red      219
## 33      32      219  1    red      220
## 34      33      220  1  green      219
## 35      34      219  2  black      217
## 36      35      217  4  black      213
## 37      36      213  8    red      221
## 38      37      221  1  black      220
## 39      38      220  2    red      222
## 40      39      222  1    red      223
## 41      40      223  1  black      222
## 42      41      222  2    red      224
## 43      42      224  1    red      225
## 44      43      225  1  black      224
## 45      44      224  2    red      226
## 46      45      226  1    red      227
## 47      46      227  1    red      228
## 48      47      228  1    red      229
## 49      48      229  1    red      230
## 50      49      230  1    red      231
## 51      50      231  1    red      232
## 52      51      232  1    red      233
## 53      52      233  1    red      234
## 54      53      234  1    red      235
## 55      54      235  1  black      234
## 56      55      234  2    red      236
## 57      56      236  1    red      237
## 58      57      237  1  black      236
## 59      58      236  2    red      238
## 60      59      238  1    red      239
## 61      60      239  1  black      238
## 62      61      238  2    red      240
## 63      62      240  1    red      241
## 64      63      241  1  black      240
## 65      64      240  2    red      242
## 66      65      242  1  black      241
## 67      66      241  2    red      243
## 68      67      243  1  black      242
## 69      68      242  2    red      244
## 70      69      244  1  black      243
## 71      70      243  2  black      241
## 72      71      241  4    red      245
## 73      72      245  1    red      246
```

```
## 74          73             246     1   black           245
## 75          74             245     2     red           247
## 76          75             247     1   black           246
## 77          76             246     2   black           244
## 78          77             244     4   black           240
## 79          78             240     8   black           232
## 80          79             232    16     red           248
## 81          80             248     1   black           247
## 82          81             247     2   black           245
## 83          82             245     4   black           241
## 84          83             241     8     red           249
## 85          84             249     1   black           248
## 86          85             248     2     red           250
## 87          86             250     1   black           249
## 88          87             249     2     red           251
## 89          88             251     1   green           250
## 90          89             250     2   black           248
## 91          90             248     4   black           244
## 92          91             244     8     red           252
## 93          92             252     1   black           251
## 94          93             251     2     red           253
## 95          94             253     1     red           254
## 96          95             254     1     red           255
## 97          96             255     1   black           254
## 98          97             254     2   black           252
## 99          98             252     4   black           248
## 100         99             248     8     red           256
## 101        100             256     1     red           257
## 102        101             257     1   black           256
## 103        102             256     2     red           258
## 104        103             258     1     red           259
## 105        104             259     1   black           258
## 106        105             258     2   black           256
## 107        106             256     4     red           260
## 108        107             260     1   black           259
## 109        108             259     2   black           257
## 110        109             257     4   black           253
## 111        110             253     8   black           245
## 112        111             245    16   black           229
## 113        112             229    32     red           261
## 114        113             261     1   black           260
## 115        114             260     2     red           262
## 116        115             262     1   black           261
## 117        116             261     2     red           263
## 118        117             263     1   black           262
## 119        118             262     2   black           260
## 120        119             260     4   black           256
## 121        120             256     8     red           264
## 122        121             264     1     red           265
## 123        122             265     1     red           266
## 124        123             266     1     red           267
## 125        124             267     1     red           268
## 126        125             268     1   black           267
## 127        126             267     2   black           265
```

```
## 128         127              265     4     red              269
## 129         128              269     1   black              268
## 130         129              268     2   black              266
## 131         130              266     4     red              270
## 132         131              270     1     red              271
## 133         132              271     1     red              272
## 134         133              272     1   black              271
## 135         134              271     2   black              269
## 136         135              269     4     red              273
## 137         136              273     1   black              272
## 138         137              272     2     red              274
## 139         138              274     1     red              275
## 140         139              275     1   black              274
## 141         140              274     2     red              276
## 142         141              276     1   black              275
## 143         142              275     2   black              273
## 144         143              273     4     red              277
## 145         144              277     1     red              278
## 146         145              278     1     red              279
## 147         146              279     1   black              278
## 148         147              278     2     red              280
## 149         148              280     1     red              281
## 150         149              281     1     red              282
## 151         150              282     1     red              283
## 152         151              283     1   black              282
## 153         152              282     2     red              284
## 154         153              284     1   black              283
## 155         154              283     2     red              285
## 156         155              285     1   black              284
## 157         156              284     2     red              286
## 158         157              286     1   black              285
## 159         158              285     2   black              283
## 160         159              283     4   black              279
## 161         160              279     8     red              287
## 162         161              287     1   green              286
## 163         162              286     2   black              284
## 164         163              284     4     red              288
## 165         164              288     1   green              287
## 166         165              287     2   black              285
## 167         166              285     4     red              289
## 168         167              289     1   black              288
## 169         168              288     2   black              286
## 170         169              286     4     red              290
## 171         170              290     1     red              291
## 172         171              291     1   green              290
## 173         172              290     2   black              288
## 174         173              288     4     red              292
## 175         174              292     1   black              291
## 176         175              291     2     red              293
## 177         176              293     1   black              292
## 178         177              292     2     red              294
## 179         178              294     1   black              293
## 180         179              293     2     red              295
## 181         180              295     1   black              294
```

```
## 182        181              294     2   black          292
## 183        182              292     4     red          296
## 184        183              296     1     red          297
## 185        184              297     1     red          298
## 186        185              298     1     red          299
## 187        186              299     1     red          300
```

```
profit(a)
```

```
## [1] 100
```

The results are not always positive in the sense that the chances of winning and losing seem to be random and the Martingale strategy **under these constraints is not** a guaranteed success. A specific observation is that a vast majority of profit results tend to be either -200 or 100. This is due to the either one of the stopping conditions being achieved - 1000 turns (extremely low chance of happening due to the small difference between starting amount and the winning threshold and the starting amount itself being low enough that 8-9 continuous losses would cause the player to lose the entire initial capital) or achieving the winning threshold or losing the initial capital/budget.

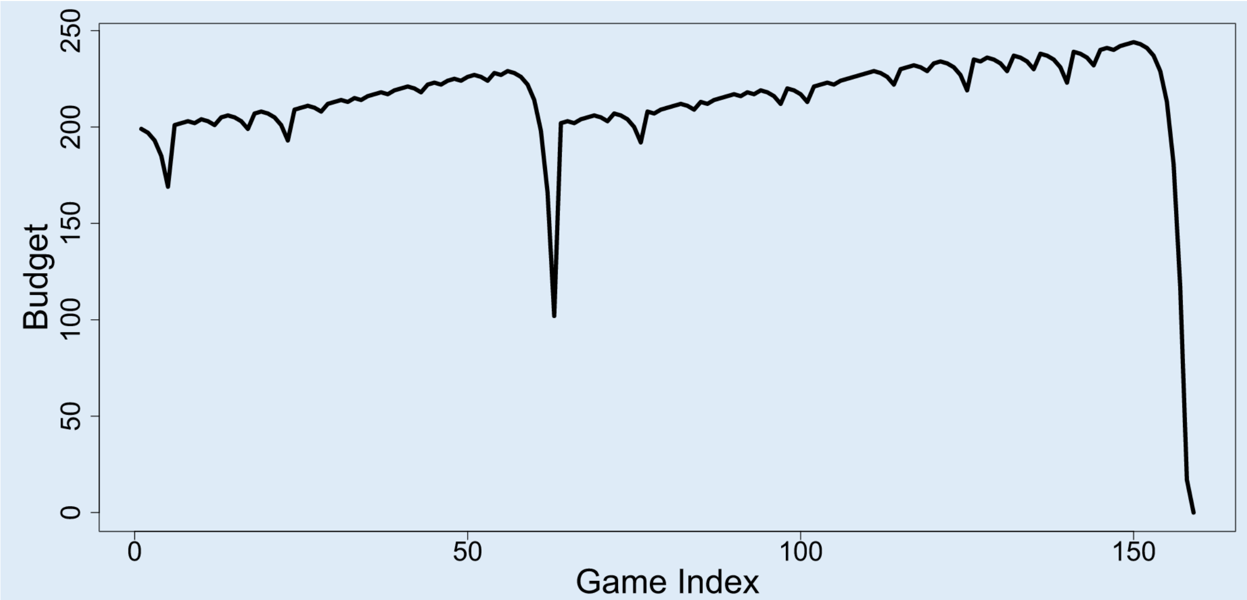
```
library(magrittr)

svg(filename = "pattern.svg", width=16, height =8.25)
par(cex.axis=2.0, cex.lab = 2.5, mar = c(8,8,2,2), bg = rgb(222, 235, 247, max = 255))
#set.seed(1)
#Constraints chosen based on above stipulations.
ledger <- one_series(1000,200,300,100)
plot(ledger[,c(1,5)], type = "l", lwd = 5, xlab = "Game Index", ylab = "Budget")
dev.off()
```

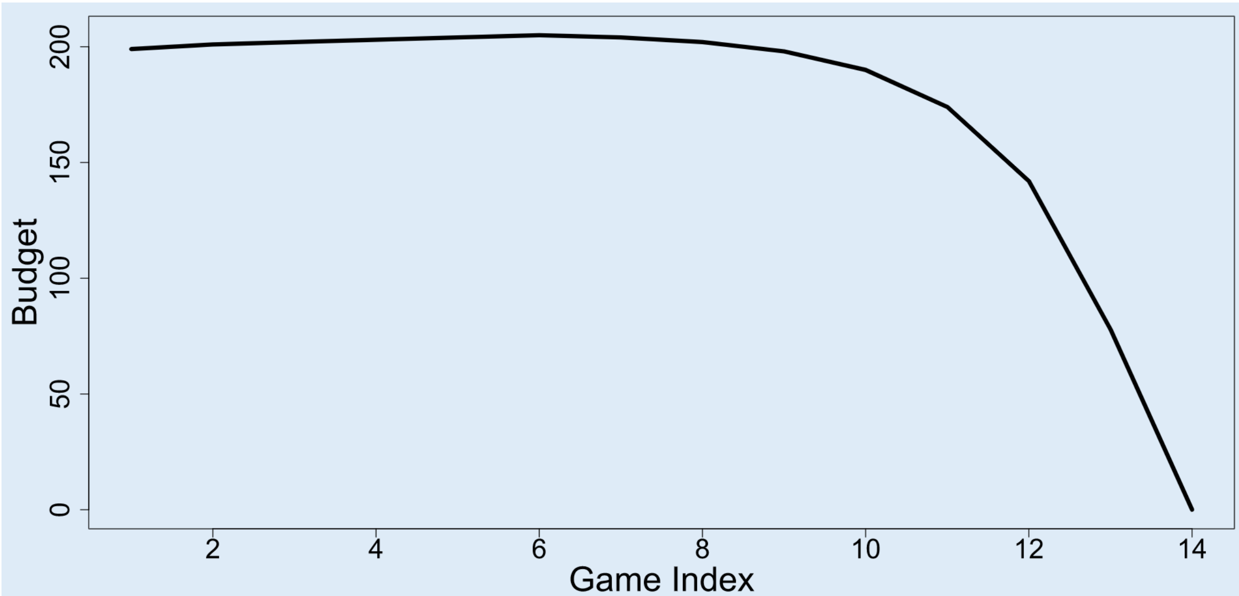
```
## pdf
##   2
```

The below figures show both gamblers winning and losing over the series of wagers based on above constraints.

Lost Wager



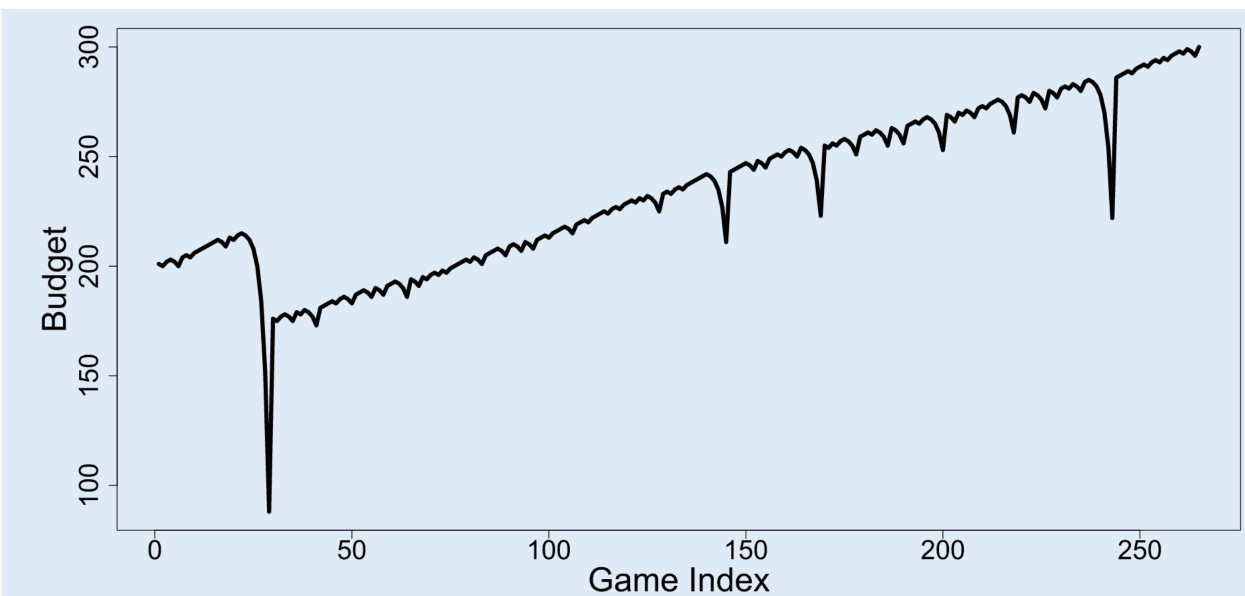
Here we see an example of a player get half way to his win condition but drops off sharply at his 250th turn.



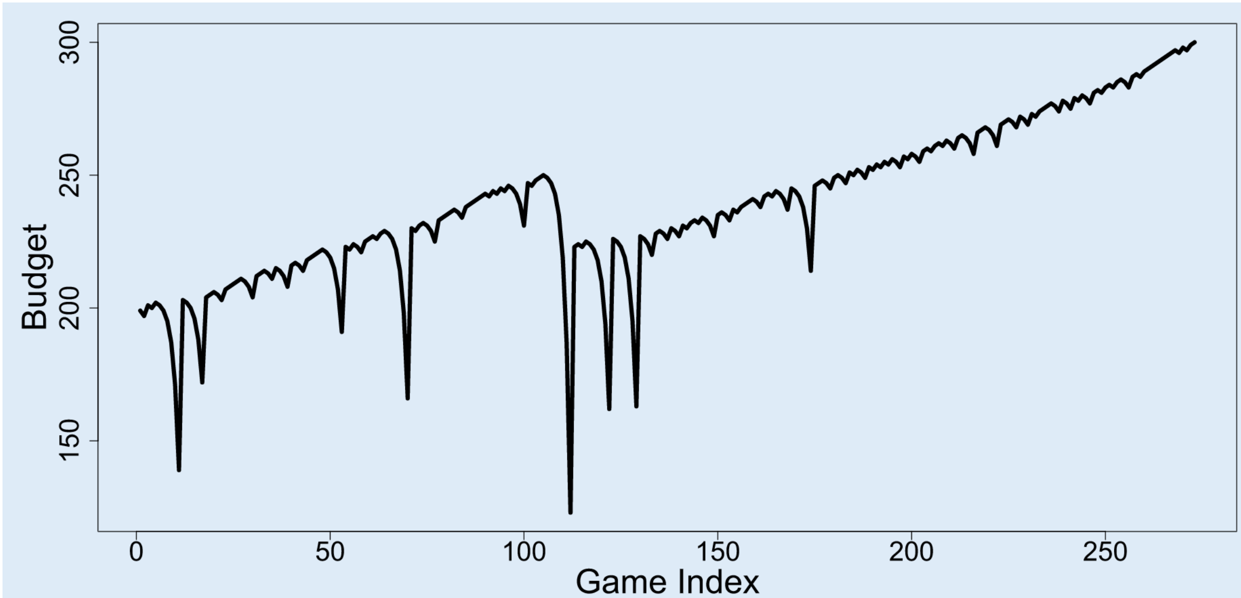
Here we see an example of a very swift loss right at turn 14.

Here we see a

Won Wager



Here is an example of a player almost losing at around turn 30 but managing to use the strategy and reach his win condition at turn 270 or so.



Here is another example of a player almost losing at many points in the game but managing to eek out the win eventually.

Conclusion and Discussion

A massive limitation with this simulation is the chosen constraint value. It can be concluded that the Martingale's strategy cannot guarantee a win with the above constraints. But, modification of said constraints can increase the probability of positive results. It can also be effectively concluded that the Martingale's strategy **guarantees** positive net earnings when there are no constraints and that the gambler has full control as to how many turns he gets to play, has an infinite (or a very large capital to spend) and there isn't a maximum bet restriction. Modifying some of these constraints can shift the odds in the gambler's favor though.

For instance,

- The budget of the gambler is directly proportional to the chance of a positive outcome.
- The number of chances to play is also directly proportional to the chance of a positive outcome. (In our scenario this played a very minor role due to how the stopping conditions were set up.)
- The maximum bet amount is inversely (although not in any linear fashion) proportional to the chance of a positive outcome.

All of the above listed points are highly situational and only applicable in a specific scenario to successfully execute the Martingale strategy as will be discussed in the conclusion.

The function below returns the total profits/losses of many replicates of roulette games using different constraints.

```
PResult <- function(r, max_games, starting_budget, winning_threshold, max_wager)
{
rep <- c(replicate(r, profit(one_series(max_games, starting_budget, winning_threshold, max_wager))))
return (rep)
}
```

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.4     v dplyr   1.0.7
## v tidyr   1.1.3     v stringr 1.4.0
## v readr   2.0.1     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x tidyr::extract()   masks magrittr::extract()
## x dplyr::filter()    masks stats::filter()
## x dplyr::lag()       masks stats::lag()
## x purrr::set_names() masks magrittr::set_names()
```

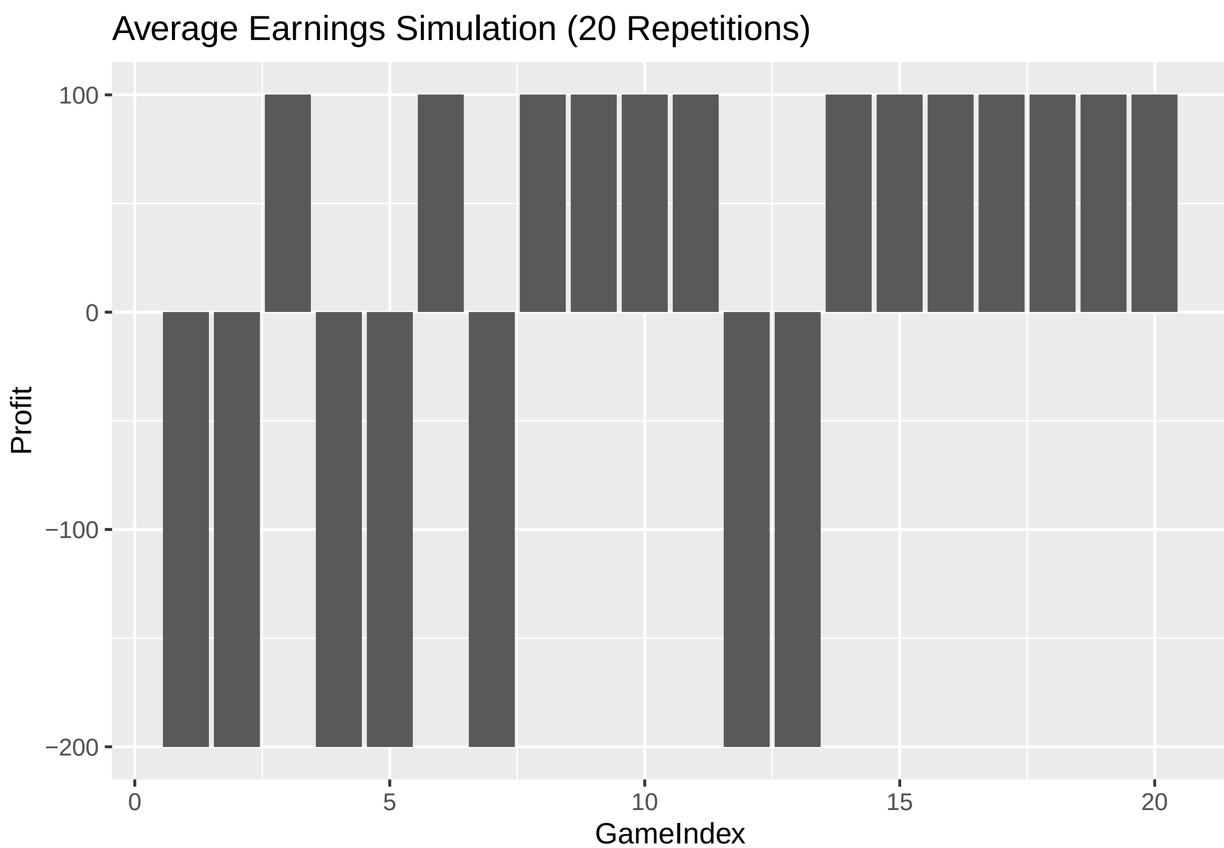
```
library(ggplot2)
```

This chunk of code shows how changing parameters based on the discussion above affects the outcome of the series of wagers i.e, gives us the distribution of earnings over games as well as average earnings over a series of games.

In order to make the histogram viewable, it's shown for 20 repetitions but the distribution can be extended upto any number of required repetitions. The means/average earnings shown will be for 1000 repetitions.

```
#.
set.seed(1)
a <- PResult(20, 1000, 200, 300, 100)
a1 <- as.data.frame(a)
a1 <- mutate(a1, GameIndex = 1:length(a))
colnames(a1) <- c("Profit", "GameIndex")
ggplot(data = a1, aes(x = GameIndex, y = Profit)) + geom_histogram(stat = "identity") + labs(title = "A
```

```
## Warning: Ignoring unknown parameters: binwidth, bins, pad
```




```r
a2 <- PResult(1000, 1000, 200, 300, 100)
a2 <- as.data.frame(a2)
a2 <- mutate(a2, GameIndex = 1:length(a2))
colnames(a2) <- c("Profit", "GameIndex")
mean(a2$Profit)
```

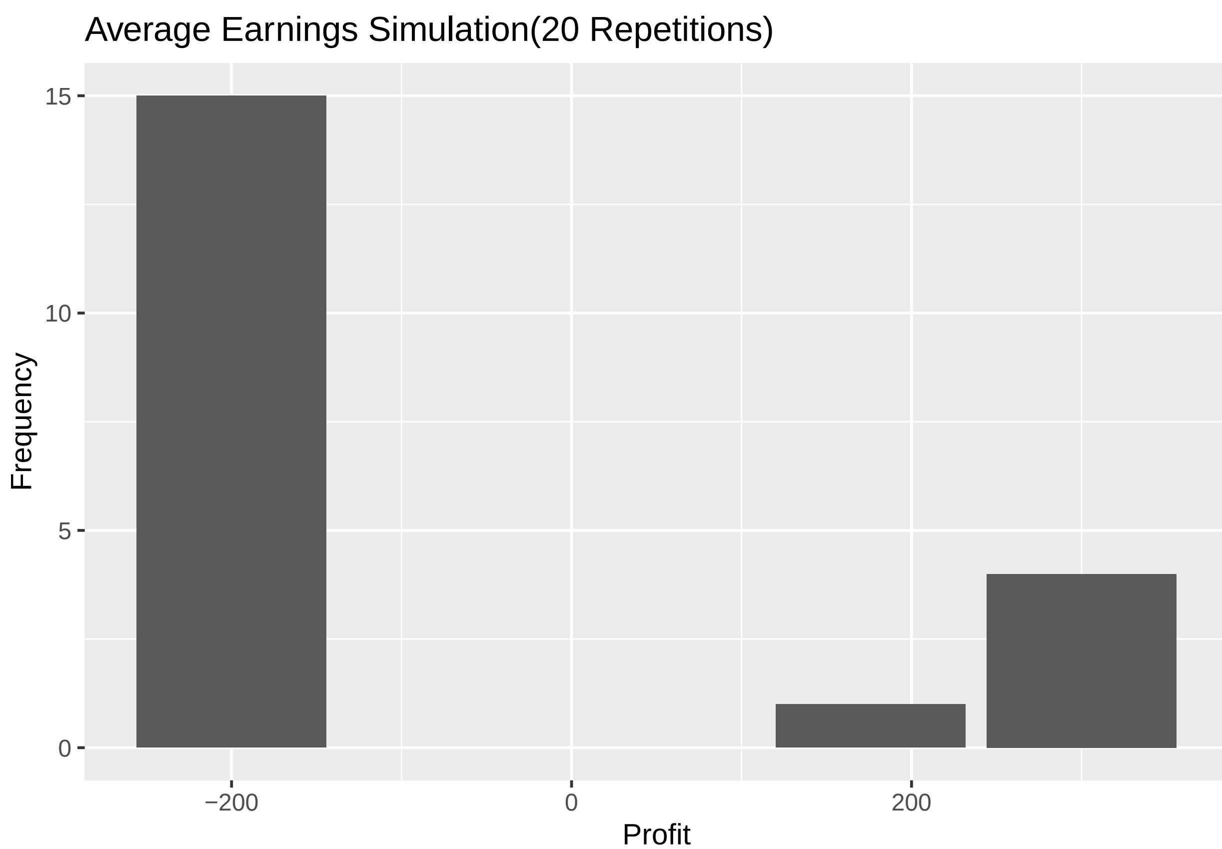
```
## [1] -48.316
```

```r
# get_last <- function(x) x[length(x)]
#
#
# # Simulation
# walk_out_money <- rep(NA, 50)
# for(j in seq_along(walk_out_money)){
#   walk_out_money[j] <- one_series(starting_budget = 200, winning_threshold = 300, max_games = 1000, m
# }
#
# lapply(walk_out_money, function(x) x[length(x)] - 200)
#
# # Walk out money distribution
# #hist(as.numeric(walk_out_money), breaks = 100)
#
# d1 <- as.data.frame(lapply(walk_out_money, function(x) x[length(x)] - 200))
# index(d1, col = 1:length(d1))
# dim(d1)
```

With these set of parameters, we have a mean earnings of -48.316.

```
set.seed(1)
b<- PResult(20, 1000, 200, 500, 100)
b1 <- as.data.frame(b)
b1 <- mutate(b1, GameIndex = 1:length(b))
colnames(b1) <- c("Profit", "GameIndex")
b2 <- b1 %>% group_by(Profit) %>% count()
colnames(b2) <- c("Profit", "Frequency")
ggplot(data = b2, aes(x = Profit, y = Frequency)) + geom_col(stat = "identity") + labs(title = "Average
```

```
## Warning: Ignoring unknown parameters: stat
```



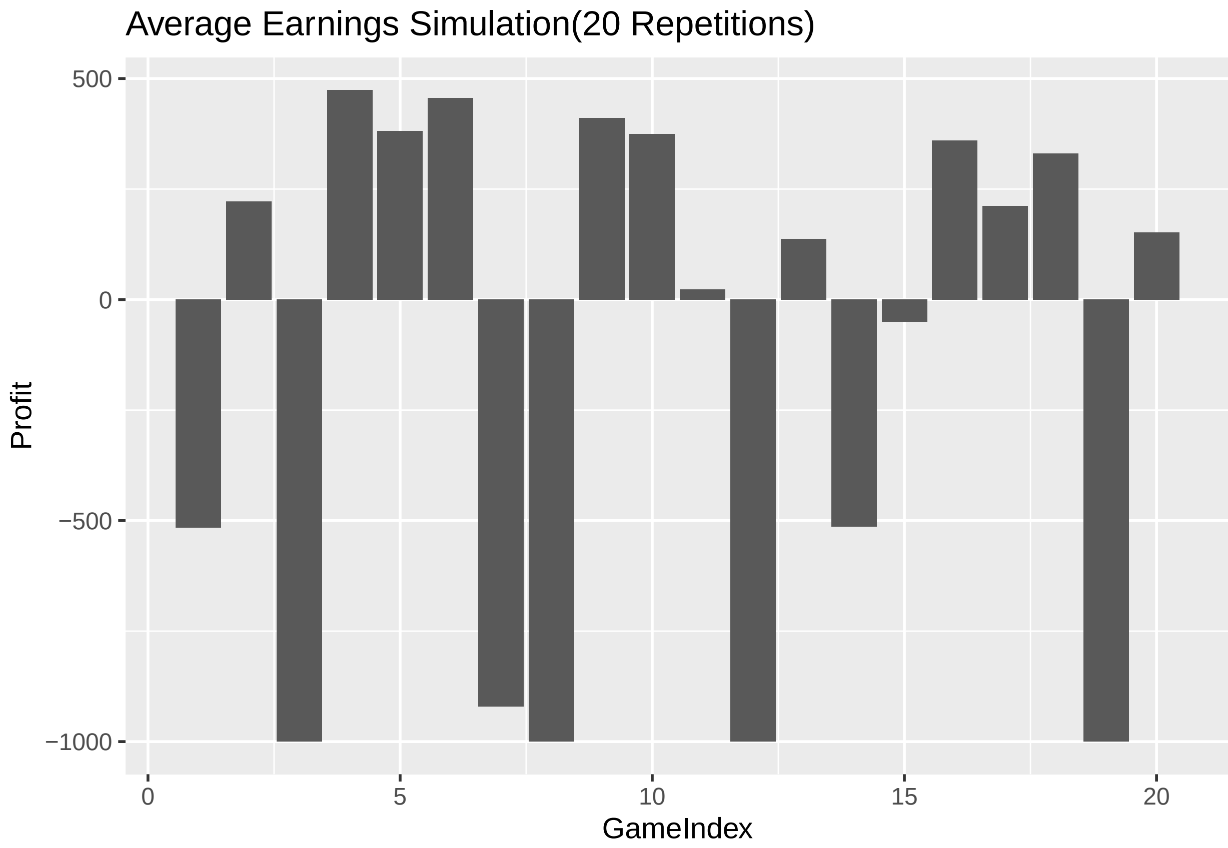

```
b2<- PResult(1000, 1000, 200, 500, 100)
b2 <- as.data.frame(b2)
b2 <- mutate(b2, GameIndex = 1:length(b2))
colnames(b2) <- c("Profit", "GameIndex")
mean(b2$Profit)
```

```
## [1] -96.324
```

With these set of parameters, the mean earnings is -96.324. We had increased the winning threshold which proves to be problematic for the most part as the player would have to risk more to reach the higher winning threshold(to attain stopping condition) while having only 200$ to play around with.

```
set.seed(1)
c <- PResult(20, 1000, 1000, 2000, 200)
c1 <- as.data.frame(c)
c1 <- mutate(c1, GameIndex = 1:length(c))
colnames(c1) <- c("Profit", "GameIndex")
ggplot(data = c1, aes(x = GameIndex, y = Profit)) + geom_histogram(stat = "identity") + labs(title = "A
```

```
## Warning: Ignoring unknown parameters: binwidth, bins, pad
```



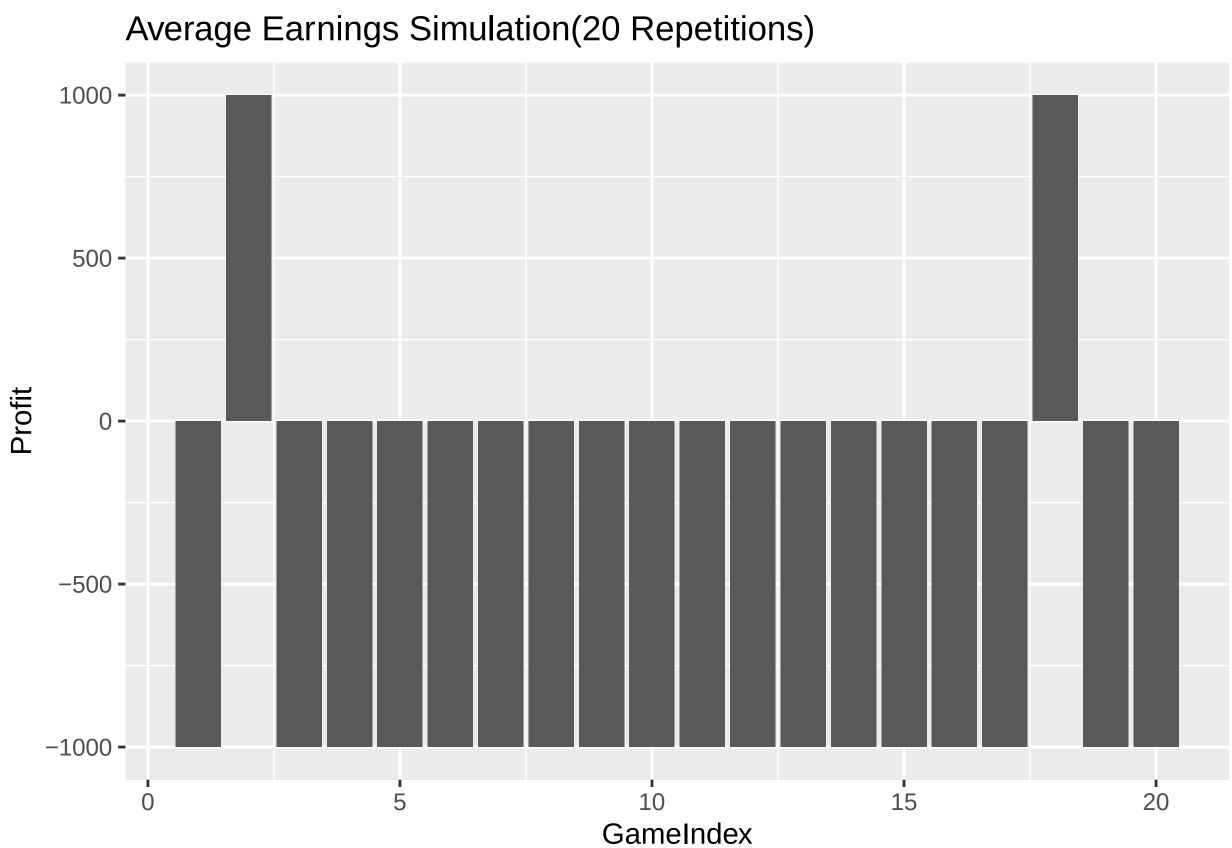

```
c2<- PResult(1000, 1000, 200, 500, 100)
c2 <- as.data.frame(c2)
c2 <- mutate(c2, GameIndex = 1:length(c2))
colnames(c2) <- c("Profit", "GameIndex")
mean(c2$Profit)
```

```
## [1] -97.798
```

With these set of parameters, the mean earnings is -97.798 (We seem to have some sorely unlucky series of games so far!). We had increased the starting threshold and adjusted the winning threshold as well and the results don't seem too promising in this case either. An interesting observation is that the winning threshold is never reached (it would be reached with much higher repetitions but clearly, the probability is low. And the 1000 turn limit was actually used up in case of positive profits.). Perhaps more turns could deliver a full 1000$ profit? Let's explore that below.

```
set.seed(2)
d <- PResult(20, 10000, 1000, 2000, 200)
d1 <- as.data.frame(d)
d1 <- mutate(d1, GameIndex = 1:length(d))
colnames(d1) <- c("Profit", "GameIndex")
ggplot(data = d1, aes(x = GameIndex, y = Profit)) + geom_histogram(stat = "identity") + labs(title = "A
```

```
## Warning: Ignoring unknown parameters: binwidth, bins, pad
```




```
d2<- PResult(1000, 1000, 200, 500, 100)
d2 <- as.data.frame(d2)
d2 <- mutate(d2, GameIndex = 1:length(d2))
colnames(d2) <- c("Profit", "GameIndex")
mean(d2$Profit)
```

```
## [1] -100.776
```

As expected, with a higher number of chances, we are able to reach the full 1000$ profit(1000$ being the difference between the winning_threshold and the starting_budget) but we still end up with negative mean earnings of -100.76.

The below chunk of code calculates the average number of plays before a stopping condition is achieved.

```r
NumberofPlays <- function(ledger){
  n <- nrow(ledger)
  return(n)
}

AverageNumberofPlays <- function(r, max_games, starting_budget, winning_threshold, max_wager)
{
rep <- c(replicate(r, NumberofPlays(one_series(max_games, starting_budget, winning_threshold, max_wager
return(mean(rep))
}
```

```r
AverageNumberofPlays(1000, 1000, 200, 300, 100)
```

```
## [1] 202.771
```

Here we find that the average number of plays using the initial given parameters is 208.807

#Conclusion

Using the above observations, we can clearly see that Martingale's strategy while not a slam dunk in every scenario. This can be extended to state that based on a set of static conditions, Martingale's strategy is not a definite win strategy. It can however be considered a definite win strategy if there are no constraints(i.e, no budget limitations, no betting limitations etc) or if the constraints the dynamic (i.e, the starting budget can be completely controlled by the player as the game progresses - this alone could greatly increase the winning chances if the number of plays or the betting limit is high enough.). The more dynamic constraints, the faster/less number of turns it'll take to gain a positive outcome. Dynamically changing the winning threshold based on previous turns by the player also would guarantee a success provided the dynamic starting budget and reasonably high max number of turns.

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