

# Question - 1

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$$I = \{1, 2, 3, 4, 5, 6\}$$

Index of Products.

Data

Product	Octane	Sulfur	Cost \$/gall
Fuel 1	92	5	\$2.98
Fuel 2	93	6	\$3.17
Fuel 3	94	7	\$3.23
Additive 1	99	0	\$20.50
Additive 2	101	-1	\$23.75
Additive 3	103	-2	\$27.43

$\uparrow$   
 $P_i$

$\uparrow$   
 $O_i$

$\uparrow$   
 $S_i$

$\uparrow$   
 $C_i$

$$\text{MIN cost} = \sum_{i \in I} P_i C_i$$

ST

$$\sum_{i \in I} P_i \geq 250 ; \text{Constrain on gallons.}$$

$$10\% \cdot \sum_{i=1}^3 P_i - \sum_{i=4}^6 P_i \geq 0 ; \text{Constrain on additive.}$$

$$\frac{\sum_{i \in I} P_i O_i}{\sum_{i \in I} P_i} \geq 95 ; \text{constrain on Octane.}$$

$$3 \leq \frac{\sum_{i \in I} S_i P_i}{\sum_{i \in I} P_i} \leq 6 ; \text{Sulfur constrain.}$$



## Question - 2

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$I \in \{1, 2, 3, 4, 5, 6, 7\}$  ; Ingot indices.

$J \in \{1, 2, 3, 4\}$  ; Properties of material ore indices.

$K \in \{1, 2, 3, 4, 5\}$  ; Ore indices.

### Data

Ingot Specification matrix

	$j$
Ingot 1 $\rightarrow$	9
Ingot 2 $\rightarrow$	6
Ingot 3 $\rightarrow$	11
Ingot 4 $\rightarrow$	4
Ingot 5 $\rightarrow$	7
Ingot 6 $\rightarrow$	8
Ingot 7 $\rightarrow$	9
	$i = S$
	Tensile strength
	Compression strength
	Flex
	Rust proof

$S_{ij}$  is the  $[i^{th}, j^{th}]$  element in 'S' matrix.

### Demand row matrix.

$$D = [10,000 \quad 12,000 \quad 8,000 \quad 4,000 \quad 10,000 \quad 11,500 \quad 13,750]$$

$i$

$D_i$  is the  $i^{th}$  element in 'D' matrix.

### Cost row matrix.

$$C = [1,000 \quad 1,300 \quad 4,250 \quad 2,150 \quad 9,750]$$

$K$

$C_K$  is the  $K^{th}$  element in 'C' matrix.



Tungsten	4	2	3	5
Manganese	6	3	5	7
chromite	8	5	7	9
Magnetite	10	6	9	10
Cobaltite	14	8	11	13
	Test strength	compression strength	flex strength	rust proof

$K = A$

$A_{jk}$  is the  $[j^{\text{th}}, k^{\text{th}}]$  element in 'A' matrix

Decision Variable matrix. Amount of material ore per Ingot.

$O =$

	Tungsten	Manganese	chromite	Magnetite	Cobaltite
Ingot 1	$Tu_1$	$Mn_1$	$ch_1$	$Ma_1$	$Co_1$
Ingot 2	$Tu_2$	$Mn_2$	$ch_2$	$Ma_2$	$Co_2$
Ingot 3	$Tu_3$	$Mn_3$	$ch_3$	$Ma_3$	$Co_3$
Ingot 4	$Tu_4$	$Mn_4$	$ch_4$	$Ma_4$	$Co_4$
Ingot 5	$Tu_5$	$Mn_5$	$ch_5$	$Ma_5$	$Co_5$
Ingot 6	$Tu_6$	$Mn_6$	$ch_6$	$Ma_6$	$Co_6$
Ingot 7	$Tu_7$	$Mn_7$	$ch_7$	$Ma_7$	$Co_7$
	$K$				

$i$

$O_{ik}$  is the  $[i^{\text{th}}, k^{\text{th}}]$  element in 'O' matrix.



MIN

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$$\text{Cost} = \sum_{k \in K} \sum_{i \in I} O_{ik} \cdot C_k$$

ST

Demand constraint

$$\frac{\sum_{j \in J} \sum_{k \in K} O_{ik} \cdot A_{kj}}{\sum_{k \in K} A_{kj}} \geq D_i ; i \in I$$

Specification constraint

$$\frac{\sum_{k \in K} O_{ik} \cdot A_{kj}}{\sum_{k \in K} O_{ik}} = S_{ij} ; i \in I, j \in J$$

Constraint on material count.

$$O_{ik} \geq \begin{cases} \text{matrix of zeros of order } i \times k; \\ i \in I, k \in K. \end{cases}$$



## Question - 3

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indices

$i \in \{1, 2, 3, 4, 5, 6, 7\} \rightarrow$  Slot machine types.

$j \in \{1, 2, 3, 4\} \rightarrow$  floors.

Data

$H$  = on-hand machine count.

$H_i$  is  $i^{\text{th}}$  element in  $H$ .

$A$  = Area per square feet for machine type.

$A_i$  is  $i^{\text{th}}$  element in  $A$ .

$E$  = Expected revenue column ~~for~~ per machine type.

$E_i$  is  $i^{\text{th}}$  element in  $E$ .

$O$  = Operation cost per machine type.

$O_i$  is  $i^{\text{th}}$  element in  $O$ .

$M$  = Maintenance effor per machine type.

$M_i$  is  $i^{\text{th}}$  element in  $M$ .

$F$  = Floor area set.

$F_j$  is  $j^{\text{th}}$  element in  $F$ .

Total Area available ~~in~~ = 3 3000 sq. feet.

Total hours available for maintenance =  $5 \times 167$  hours.



Decision Variable matrix.

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$$x_{ij} = \begin{matrix} & \xrightarrow{\text{Floors}} j & \\ \begin{matrix} \downarrow \text{Type of machine} \\ i \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \\ x_{51} & x_{52} & x_{53} & x_{45} \\ x_{61} & x_{62} & x_{63} & x_{46} \\ x_{71} & x_{72} & x_{73} & x_{47} \end{bmatrix} \end{matrix}$$

$$\text{MAX} \sum_{i \in I} \sum_{j \in J} (E_i - O_i) x_{ij}$$

ST

constrain on machine count.

$$0 \leq \sum_{j \in J} x_{ij} \leq H_i ; i \in I$$

constrain on worker effort.

$$0 \leq \sum_{j \in J} \sum_{i \in I} M_i \cdot x_{ij} \leq 5 \times 167$$

constrain on square feet.

$$0 \leq \sum_{i \in I} A_i \cdot x_{ij} \leq F_j ; j \in J$$