1) Since its following poisson distribution. The probability of k cases being effected is given by

$$P(x=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

here $\lambda = E(x) = x \cdot P(x) = 10000 \times 1/1000 = 10$

$$P(x=0) = \frac{(10)^0 e^{-10}}{0!}$$

$$= 0.0000454$$

$$= \underline{4.54 \times 10^{-5}}$$

$$P(x=1) = \frac{(10)^1 e^{-10}}{1!}$$

$$= 0.000454$$

$$= \underline{4.54 \times 10^{-4}}$$

$$P(x=2) = \frac{(10)^2 e^{-10}}{2!}$$

$$= 0.00226$$

$$= \underline{2.26 \times 10^{-3}}$$

2) ∴ This is a poisson process with $\lambda = 2$
the probability that K cases occurs in next three months is given by

$$P(X=K) = \frac{(\lambda t)^K}{K!} e^{-\lambda t}$$

where $t = 3$

i) $$P(X=4) = \frac{(2\times3)^4}{4!} e^{-2\times3}$$

$$= \underline{0.1334}$$

ii) $$P(X=6) = \frac{(2\times3)^6}{6!} \times e^{-(2\times3)}$$

$$= \underline{0.1606}$$

3) Since it follows Poissons process the equation is as follows.

$$P(X=K) = \frac{(\lambda t)^K e^{-\lambda t}}{K!}$$

i) the probability that they will finish the arrangement is given by = probability that no customer arrives in first 30 mins. and is given by

$$P(X=0) = \frac{(10 \times 0.5)^0}{0!} \times e^{-(10 \times 0.5)}$$

$$= \underline{0.00673}$$

ii) Expected arrival time of one customer $= 1/\lambda$

$$= 1/10 = 0.1 \text{ hr}$$

The expected arrival time of next customer = 9.13 + 6 mins

$$= \underline{9:19 \text{ am}}$$

iii) The probability that next customer will arrive in next 10 mins = 1 − no customer arrives in next 10 mins

$$= 1 - P(X=0)$$

$$= 1 - \left(\frac{(10 \times 1/6)^0}{0!} \times e^{-(10 \times 1/6)} \right)$$

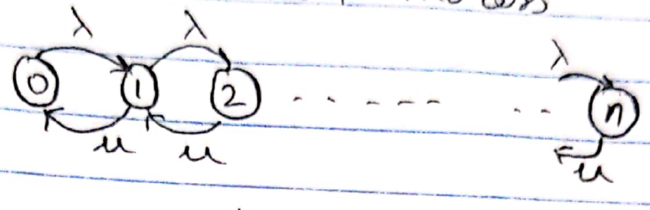
$$= 1 - 0.188875$$

$$= \underline{0.8111}$$

4) given $\lambda = 8$ customers/hr

$\mu = 60/5 = 12$ customers/hr

It's a birth death Process



This is an M/M/1 system, $\rho = \lambda/\mu = 8/12 = 2/3$

i) probability that teller is idle $= \pi_0 = 1-\rho$

$$= 1-\rho$$
$$= 1-2/3$$
$$= \underline{1/3}$$

ii) average no customers waiting for service $= E(L^q)$

$$E(L^q) = \frac{\rho^2}{1-\rho}$$
$$= \frac{(2/3)^2}{1/3}$$
$$= \underline{4/3}$$

iii) on average customer spend $= E(W)$

$$E(W) = \frac{\rho}{\lambda(1-\rho)}$$
$$= \frac{2/3}{8(1-2/3)}$$
$$= \frac{2/3}{8/3} = \underline{1/4}$$

Scanned with CamScanner

iv) Probability that there are more than 5 customers in bank $= \sum_{n=6}^{\infty} \pi_n$ ~~where~~

$= 1 -$ Probability that there are less than 6 customers in bank

$$= 1 - [\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5]$$

$$= 1 - [(1-\rho) + (1-\rho)\rho + (1-\rho)\rho^2 + \dots]$$

$$= 1 - (1-\rho)[1 + \rho + \rho^2 + \rho^3 + \rho^4 + \rho^5]$$

$$= 1 - \cancel{(1-\rho)}\frac{(1-\rho^6)}{\cancel{(1-\rho)}}$$

$$= 1 - 1 + \rho^6$$

$$= \rho^6$$

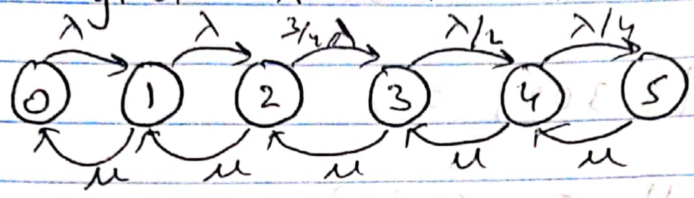
$$= (2/3)^6$$

$$= \underline{0.0877}$$

Scanned with CamScanner

5) given $\lambda = 1/7$, $\mu = 1/6$

a)



b) detailed balanced equation are given as

$$\lambda \pi_0 = \mu \pi_1 \Rightarrow \pi_1 = \lambda/\mu \, \pi_0$$
$$\lambda \pi_1 = \mu \pi_2 \Rightarrow \pi_2 = (\lambda/\mu)^2 \pi_0$$
$$3/4 \lambda \pi_2 = \mu \pi_3 \Rightarrow \pi_3 = 3/4 (\lambda/\mu)^3 \pi_0$$
$$\lambda/2 \, \pi_3 = \mu \pi_4 \Rightarrow \pi_4 = 3/8 (\lambda/\mu)^4 \pi_0$$
$$\lambda/4 \, \pi_4 = \mu \pi_5 \Rightarrow \pi_5 = 3/32 (\lambda/\mu)^5 \pi_0$$

c) average no of jobs = $E[n]$

$$E[n] = \sum_k k P_k$$

$$E[n] = \pi_1 + 2\pi_2 + 3\pi_3 + 4\pi_4 + 5\pi_5$$

$$= \pi_0 \left[(\lambda/\mu) + 2(\lambda/\mu)^2 + \frac{9}{4}(\lambda/\mu)^3 + \frac{3}{2}\left(\frac{\lambda}{\mu}\right)^4 + \frac{15}{32}\left(\frac{\lambda}{\mu}\right)^5 \right]$$

$$\sum_i \pi_i = 1$$

$$\pi_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \frac{3}{4}\left(\frac{\lambda}{\mu}\right)^3 + \frac{3}{8}\left(\frac{\lambda}{\mu}\right)^4 + \frac{3}{32}\left(\frac{\lambda}{\mu}\right)^5 \right] = 1$$

$\rho = \lambda/\mu = 0.857$

$\pi_0 [3.304] = 1$

$\rightarrow \pi_0 = \underline{0.302}$

$E[n] = \pi_0 \times 4.767$
$= 0.302 \times 4.76$

$\rightarrow E[n] = \underline{1.439}$

→ average no of Jobs = $\underline{1.439}$

d) no of states entering = no of states leaving

no of states leaving = $\mu \times$ (Probability of server busy)
$= \mu \times (1 - \pi_0)$

no of states leaving in 100 secs $= 100 \times \mu \times (1 - \pi_0)$
$= 100 \times 1/6 \times 0.698$

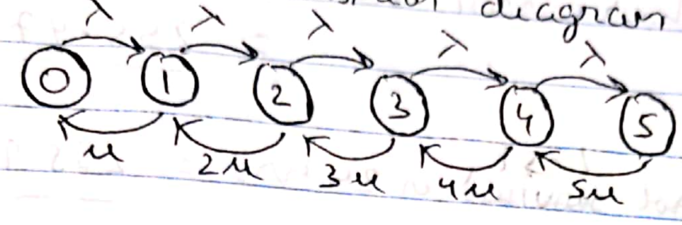
$= 11.63$

→ no of states entering in 100 secs = $11.63 \simeq \underline{12}$

Scanned with CamScanner

6) $\lambda = 50$ cars/hr , $\mu = 60/5 = 12$ cars/hr

a) The state transition diagram is as follows.



b) detailed balanced equations are as follows

$\lambda \Pi_0 = \mu \Pi_1 \Rightarrow \Pi_1 = \lambda/\mu \, \Pi_0$

$\lambda \Pi_1 = 2\mu \Pi_2 \Rightarrow \Pi_2 = \lambda/2\mu \, \Pi_1$

$\lambda \Pi_2 = 3\mu \Pi_3 \Rightarrow \Pi_3 = \frac{\lambda}{3\mu} \Pi_2$

$\lambda \Pi_3 = 4\mu \Pi_4 \Rightarrow \Pi_4 = \lambda/4\mu \, \Pi_3$

$\lambda \Pi_4 = 5\mu \Pi_5 \Rightarrow \Pi_5 = \lambda/5\mu \, \Pi_4$

$$\sum \Pi_i = 1$$

$$1 = \Pi_0 \left[1 + \lambda/\mu + \frac{1}{2}(\lambda/\mu)^2 + \frac{1}{6}(\lambda/\mu)^3 + \frac{1}{24}(\lambda/\mu)^4 + \frac{1}{120}(\lambda/\mu)^5\right]$$

$$1 = \Pi_0 \, [48.67]$$

$$\rightarrow \Pi_0 = \underline{0.0205}$$

c) probability that an arriving car will NOT enter the gas station $= \Pi_5$

$$\Pi_5 = 1/120 \, (\lambda/\mu)^5 \, \Pi_0$$

$$\rightarrow \underline{\Pi_5 = 0.213}$$

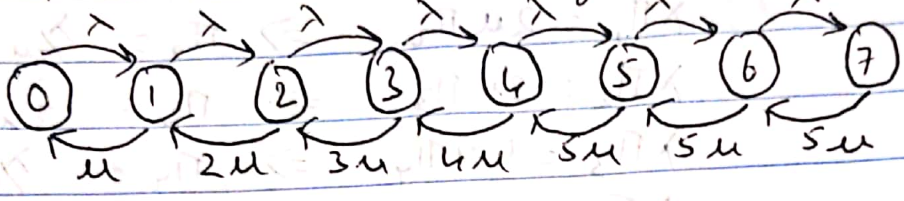
No of Cars which have to leave in 24 hrs $= 24 \times \lambda \times \Pi_5$

$\rightarrow$ No of cars which cannot enter $= 24 \times 50 \times 0.213$

$= \underline{255.97}$

$\rightarrow$ No of Cars not ~~serviced~~ Lost in 24 hrs $= \underline{255.97} \simeq \underline{256 \text{ Cars}}$

d) The state transition diagram for the scenario is as follows



detailed balance equations are

$$\Pi_0 \lambda = \mu \Pi_1 \Rightarrow \Pi_1 = \lambda/\mu \, \Pi_0$$
$$\Pi_2 2\mu = \Pi_1 \lambda \Rightarrow \Pi_2 = \tfrac{1}{2}(\lambda/\mu)^2 \Pi_0$$
$$\Pi_3 3\mu = \Pi_2 \lambda \Rightarrow \Pi_3 = \tfrac{1}{6}(\lambda/\mu)^3 \Pi_0$$
$$\Pi_4 4\mu = \Pi_3 \lambda \Rightarrow \Pi_4 = \tfrac{1}{24}(\lambda/\mu)^4 \Pi_0$$
$$\Pi_5 5\mu = \Pi_4 \lambda \Rightarrow \Pi_5 = \tfrac{1}{120}(\lambda/\mu)^5 \Pi_0$$
$$\Pi_6 5\mu = \Pi_5 \lambda \Rightarrow \Pi_6 = \tfrac{1}{600}(\lambda/\mu)^6 \Pi_0$$
$$\Pi_7 5\mu = \Pi_6 \lambda \Rightarrow \Pi_7 = \tfrac{1}{3000}(\lambda/\mu)^7 \Pi_0$$

$$\sum_{i=0}^{7} \Pi_i = 1$$

$$\Pi_0 \left[1 + \tfrac{1}{2}(\lambda/\mu)^2 + \tfrac{1}{6}(\lambda/\mu)^3 + \tfrac{1}{24}(\lambda/\mu)^4 + \tfrac{1}{120}(\lambda/\mu)^5 + \tfrac{1}{600}(\lambda/\mu)^6 + \tfrac{1}{3000}(\lambda/\mu)^7 \right] = 1$$

$$\Pi_0 (64.49) = 1$$

$\rightarrow \underline{\Pi_0 = 0.0155}$

$$\pi_7 = \frac{1}{3000}\left(\frac{\lambda}{\mu}\right)^7 \pi_0$$

$\rightarrow$ $\underline{\pi_7 = 0.111}$

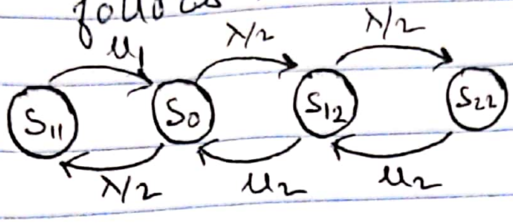
No of cars lost in 24 hours = $24 \times \lambda \times \pi_7$

$= 24 \times 50 \times 0.111$

$= 133.72$.

$\rightarrow$ Business Lost in 24 hours = $\underline{133.72} \approx \underline{134 \text{ cars}}$

7) a) The state transition diagram for the system is as follows:



Where S_{11} = 1 packet of Type 1

S_0 = 0 packets in system

S_{12} = 1 Packet of Type 2

S_{22} = 2 Packets of Type 2

The detailed balance equations are given as follows

$$\lambda/2\, \Pi_0 = \mu_1 \Pi_{11} \Rightarrow \Pi_{11} = \lambda/2\mu_1\, \Pi_0$$

$$\lambda/2\, \Pi_0 = \mu_2 \Pi_{12} \Rightarrow \Pi_{12} = \lambda/2\mu_2\, \Pi_0$$

$$\lambda/2\, \Pi_{12} = \mu_2 \Pi_{22} \Rightarrow \Pi_{22} = (\lambda/2\mu_2)^2\, \Pi_0$$

$$\sum_i \Pi_i = 1$$

$$\Pi_0 \left[1 + \lambda/2\mu_1 + \lambda/2\mu_2 + (\lambda/2\mu_2)^2\right] = 1$$

$\lambda = 10$ messages/second , $\mu_1 = 16$, $\mu_2 = 32$

$$\Pi_0\,(1.4931) = 1$$

$$\rightarrow \Pi_0 = \underline{0.669}$$

$\Pi_{11} = 0.209$, $\Pi_{12} = 0.104$, $\Pi_{22} = 0.016$

b) An accepted message of Type 1 can only arrive at state 0, otherwise its rejected. So its average service time will be $E[T_1]$

$\rightarrow E[T_1] = 300/4800 = \underline{1/16 \text{ secs}}$

An accepted message of type 2 can arrive from S_0 and S_{12}. The average service time will be

$$E_2 = \cancel{\frac{E[T_2]\Pi_0 + 2E[T_2]\Pi_{12}}{\Pi_0 + \Pi_{12}}} \quad \frac{E[T_2]\Pi_0 + 2E[T_2]\Pi_{12}}{\Pi_0 + \Pi_{12}}$$

$E[T_2] = 150/4800 = 1/32$ secs

~~$\rightarrow E_2 = 0.027/0.774 = 0.035$~~

$\rightarrow E_2 = \underline{0.0244 \text{ sec}}$

c) The loss probability are equal to probability of system being in blocking state for two packets.

loss probability of type 1 $= 1 - \Pi_0$
$= \underline{0.331}$

loss probability of type 2 $= 1 - (\Pi_0 + \Pi_{12})$
$= \underline{0.226}$
$=$