1.1 $y \cdot z = 11$

1.2 $X Y = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

1.3 $\det(X) = ad - bc = 6 - 4 = 2$

since $\det(X) \neq 0$, its invertible

$$X^{-1} = \frac{1}{\det(X)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$X^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

1.4 since $\det(X) \neq 0$ the rank of X is $\underline{2}$

2.1 $$\frac{dy}{dx} = \frac{d}{dx}(x^3 + x - 5)$$

$$= 3x^2 + 1$$

2.2 $f(x_1, x_2) = x_1 \sin(x_2) e^{-x_1}$

$$\nabla f(x) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix}$$

$$\frac{\partial f}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1 \sin(x_2) e^{-x_1})$$

$$= \sin(x_2)\left[e^{-x_1} + -x_1 e^{-x_1}\right]$$

$$\frac{\partial f}{\partial x_2} = \frac{\partial}{\partial x_2}(x_1 \sin x_2 e^{-x_1})$$

$$= x_1 e^{-x_1} \cos x_2$$

$$\nabla f(x) = \begin{pmatrix} \sin x_2 (e^{-x_1} - x_1 e^{-x_1}) \\ x_1 e^{-x_1} \cos x_2 \end{pmatrix}$$

3.1 Mean $= 3/5 = 0.6$

3.2 Variance $= 0.24$

3.3 $\frac{1}{32}$

3.4 To find the maximum we have

$P(x = head) + P(x = tail) = 1$

$P(outcome) = x^3(1-x)^2$

The maximum of this function occurs at $x = 0.6$

3.5 $P(z = T \text{ and } y = b) = 0.1$

$P(z = T \mid y = b) = 0.1/0.25$

$= 0.4$

3.6 False

True

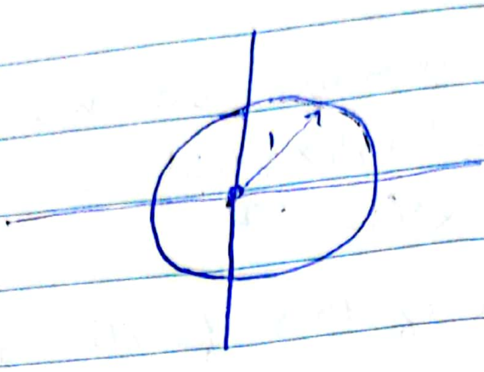
False

False

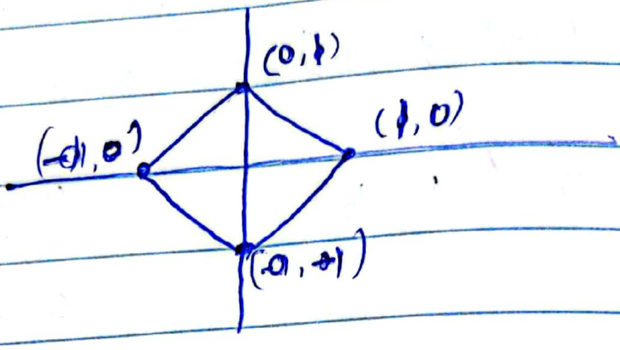
True

3.7 ~~Ds~~ D, A, B, C order is

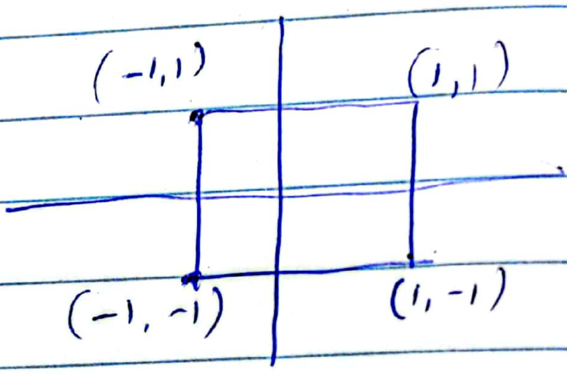
4.1



4.2



4.3



Geometry

4.1 $w^T(x_1 - x_2) = 0$, since dot product is constant they are orthogonal

2

Since ~~w is orthogonal~~ to ~~the line. Let x_d~~ be a point ~~on the line which is~~ closest to

4.2 Since w is orthogonal to the line. Let x_d be the point on the line $w^T x + b = 0$, which is also present on the line perpendicular to $w^T x + b = 0$ and passing through origin. The distance between origin to the line $w^T x + b = 0$ is the projection of the point x_d along the vector $\vec{w}$

$$\text{Project}_{\vec{w}} \vec{x}_d = \|\vec{x}_d\| \cos(x_d, \vec{w}) = \frac{x_d \cdot w}{\|w\|}$$

as x_d is present on $w^T x + b = 0$, $w \cdot x_d = -b$

$$\|\text{Projection}_w \vec{x}_d\| = \frac{b}{\|w\|}$$

5.1) Scatter plot code

5.2 a) 12382

b) 6186

c) 2913

d) 46.26

e) 46.57