Function Sequences

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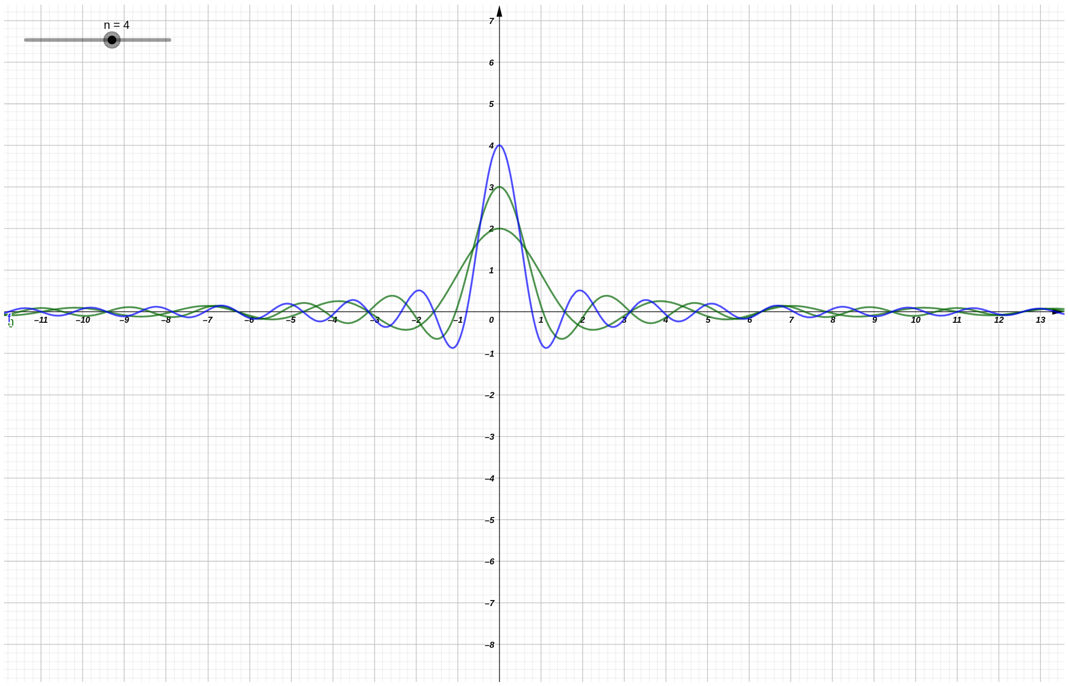
1 Definition

We call a sequence of functions from I $\rightarrow$ R defined by $(f_n)_{n \in N} \in (R^I)^N$
Below we can see an example of what a sequence of functions looks like.
The function defined is $f_n(x) = \frac{sin(n \cdot x)}{x}$
We can see the sine wave vary from each n value.



2 Convergence

2.1 Point Wise Convergence

We say that a sequence (f_n) is pointwise convergent if:
$\lim\limits_{n \to +\infty} f_n(x) = f(x)$
This goes to say that the sequence has to converge to a function f from the same input set I.
Example:

$$f_n : \begin{cases} [0,1] \longmapsto R \\ x \longmapsto x^n \end{cases}$$

setting $x = 0$ or $x = \frac{1}{2}$ we get $\lim\limits_{n \to +\infty} f_n(x) = 0$
But if x is 1, then the limit is 1 and it converges to that function.
Basically, for any given value of x we check the limit when n reaches infinity,

and if it's zero, it's pointwise convergent.

2.2 Uniform Convergence

Definition let f_n and $f \in R^I$ be the function sequence and the limit respectively. We say that f_n is uniformly convergent, if for any given value of n, the supremum $\mid f_n(x) - f(x) \mid$ exists and the supremum (which is a numerical sequence) has to tend to zero around positive infinity.
Example