A simple application of Shannon's entropy

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1 Introduction

This document will proceed to explain a basic application of shannon's entropy on a web application such as 'Netflix'.
We will see how a user's watch time i.e., how much of the given content (A show or a movie) has been consumed. For example,
A person can watch half a movie and leave it giving a 50 percent progress.

2 Representation

A user's watch time of each movie will be stored in an array as such:
The rows representing the various movies / shows and the columns being the various users.

$$\begin{pmatrix} 7 & 0 & 1 \\ 9 & 1 & 3 \\ 8 & 6 & 4 \end{pmatrix}$$

For simplicity, we will assume that the percentage completetion of content is a discrete number between 1 to 10.
$\forall n \in \mathbb{N}$

3 Entropy

Let the probability of a user completing a given show a hundered percent i.e., in our scale, getting a 10 be 'p'.
$P(n = 10) = p$

Hence:

$P(n = 10) = (1 - p)$

We know, from shannon's Entropy:

$H(P) = \sum_x P(x) \cdot \log_2(1/P(x))$

plugging in the probabilities, we should get:

$p \cdot \log_2(\frac{1}{p}) + (1 - p) \cdot \log_2(\frac{1}{1-p})$

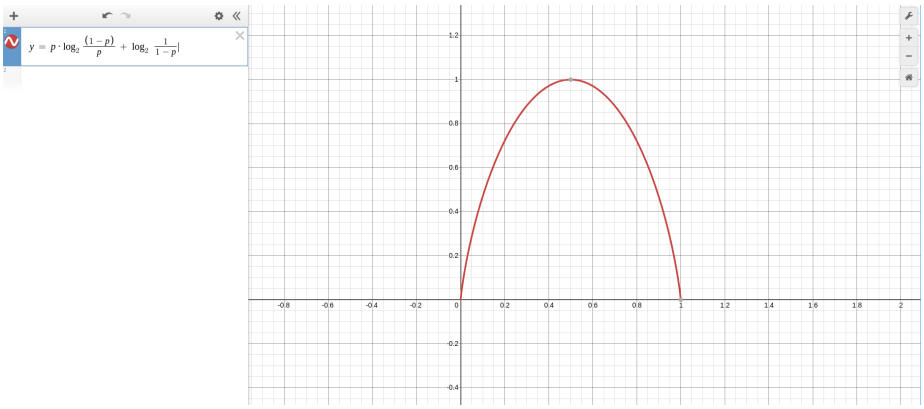
on simplifying it, we get:

$p \cdot \log_2(\frac{1-p}{p}) + \log_2(\frac{1}{1-p})$ bits.

Now, if we assume the number of total users to be u, then the total entropy would be $u \cdot H(P)$

4 Visualization

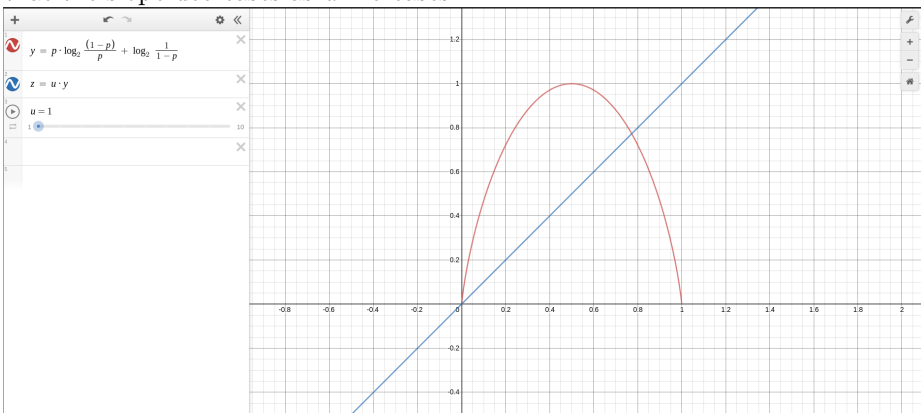
If we plot the above Entropy function on a graph, this is what we get:



Now the derivative of $H(P)$ would be:

$\frac{p^2+(1-p)^2}{1-p} + \log_2 \frac{1-p}{p}$

Plotting a line with the hypothetical number of users, u, we can see visually that the slope decreases as u increases



And when ‘u’ = 10: