HOMEWORK 6

Sriram Ashokkumar
908 216 3750
https://github.com/srirama02/CS760/tree/main/HW6

Instructions: Use this latex file as a template to develop your homework. Submit your homework on time as a single pdf file. Please wrap your code and upload to a public GitHub repo, then attach the link below the instructions so that we can access it. Answers to the questions that are not within the pdf are not accepted. This includes external links or answers attached to the code implementation. Late submissions may not be accepted. You can choose any programming language (i.e. python, R, or MATLAB). Please check Piazza for updates about the homework. It is ok to share the results of the experiments and compare them with each other.

1 Implementation: GAN (50 pts)

In this part, you are expected to implement GAN with MNIST dataset. We have provided a base jupyter notebook (gan-base.ipynb) for you to start with, which provides a model setup and training configurations to train GAN with MNIST dataset.

(a) Implement training loop and report learning curves and generated images in epoch 1, 50, 100. Note that drawing learning curves and visualization of images are already implemented in provided jupyter notebook. (20 pts)

Procedure 1 Training GAN, modified from Goodfellow et al. (2014)

Input: m: real data batch size, n_z: fake data batch size
Output: Discriminator D, Generator G
for number of training iterations **do**
$\#$ Training discriminator
Sample minibatch of n_z noise samples $\{z^{(1)}, z^{(2)}, \cdots, z^{(n_z)}\}$ from noise prior $p_g(z)$
Sample minibatch of $\{x^{(1)}, x^{(2)}, \cdots, x^{(m)}\}$
Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d}\big(\frac{1}{m}\sum_{i=1}^{m}\log D(x^{(i)}) + \frac{1}{n_z}\sum_{i=1}^{n_z}\log(1 - D(G(z^{(i)})))\big)$$

$\#$ Training generator
Sample minibatch of n_z noise samples $\{z^{(1)}, z^{(2)}, \cdots, z^{(n_z)}\}$ from noise prior $p_g(z)$
Update the generator by ascending its stochastic gradient:

$$\nabla_{\theta_g}\frac{1}{n_z}\sum_{i=1}^{n_z}\log D(G(z^{(i)}))\big)$$

end for
$\#$ The gradient-based updates can use any standard gradient-based learning rule. In the base code, we are using Adam optimizer (Kingma and Ba, 2014)

Expected results are as follows.

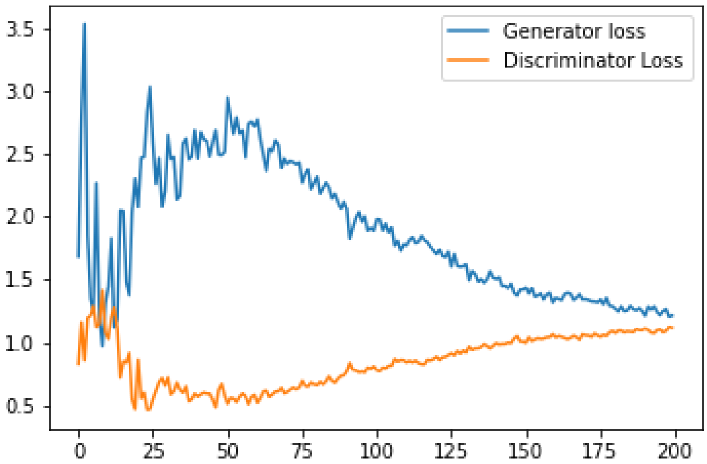
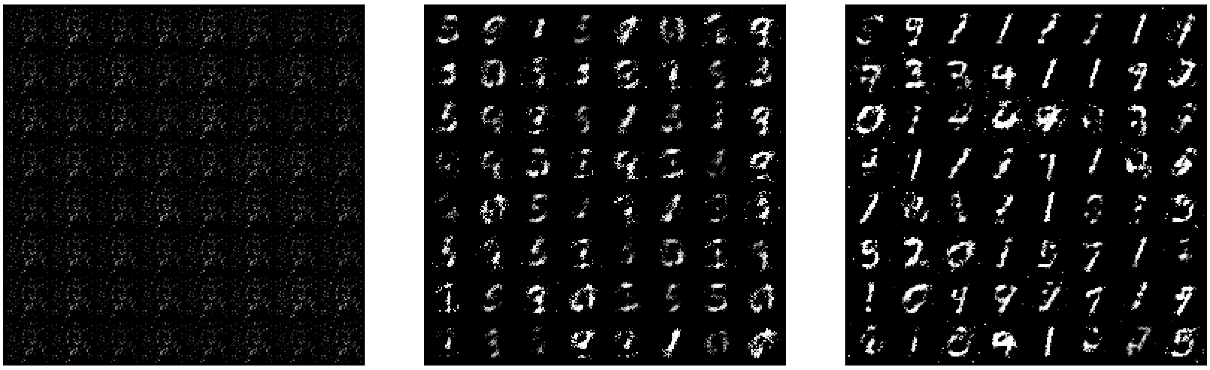


Figure 1: Learning curve



(a) epoch 1 (b) epoch 50 (c) epoch 100

Figure 2: Generated images by G

Solution goes here. Attach your learning curve and images.

(b) Replace the generator update rule as the original one in the slide,
"Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{n_z} \sum_{i=1}^{n_z} \log(1 - D(G(z^{(i)})))$$

" , and report learning curves and generated images in epoch 1, 50, 100. Compare the result with (a). Note that it may not work. If training does not work, explain why it doesn't work.
You may find this helpful: https://jonathan-hui.medium.com/gan-what-is-wrong-with-the-gan-cost-function-6f594162ce01 (10 pts)

Solution goes here. Attach your learning curve and images.

(c) Except the method that we used in (a), how can we improve training for GAN? Implement that and report your setup, learning curves, and generated images in epoch 1, 50, 100. This question is an open-ended question and you can choose whichever method you want. (20 pts)

Solution goes here. Attach your learning curve and images, and a short description of the method.

2 Directed Graphical Model [25 points]

Consider the directed graphical model (aka Bayesian network) in Figure 3.

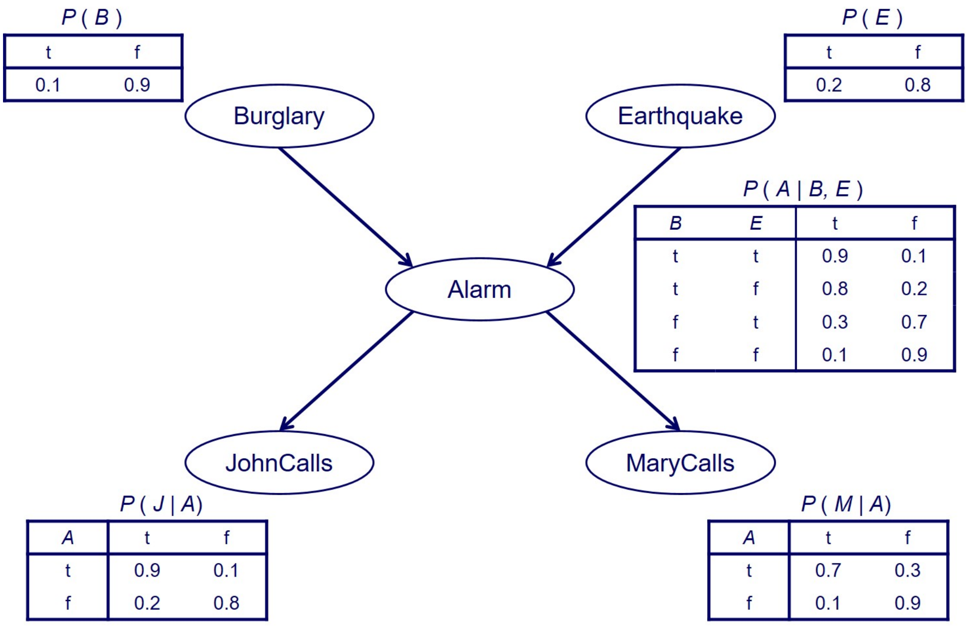


Figure 3: A Bayesian Network example.

Compute $P(B = t \mid E = f, J = t, M = t)$ and $P(B = t \mid E = t, J = t, M = t)$. (10 points for each) These are the conditional probabilities of a burglar in your house (yikes!) when both of your neighbors John and Mary call you and say they hear an alarm in your house, but without or with an earthquake also going on in that area (what a busy day), respectively.

The conditional probabilities are computed using the provided Conditional Probability Tables (CPTs) from the Bayesian network:

$$P(B = t \mid E = f, J = t, M = t) = \frac{P(B = t) \cdot P(E = f) \cdot P(A = t \mid B = t, E = f) \cdot P(J = t \mid A = t) \cdot P(M = t \mid A = t)}{P(E = f, J = t, M = t)}$$

$$P(B = t \mid E = f, J = t, M = t) \approx 0.471$$

Similarly,

$$P(B = t \mid E = t, J = t, M = t) = \frac{P(B = t) \cdot P(E = t) \cdot P(A = t \mid B = t, E = t) \cdot P(J = t \mid A = t) \cdot P(M = t \mid A = t)}{P(E = t, J = t, M = t)}$$

$$P(B = t \mid E = t, J = t, M = t) \approx 0.250$$

These conditional probabilities represent the likelihood of a burglary occurring given the absence or presence of an earthquake while both neighbors John and Mary call to report hearing the alarm.

3 Chow-Liu Algorithm [25 pts]

Suppose we wish to construct a directed graphical model for 3 features X, Y, and Z using the Chow-Liu algorithm. We are given data from 100 independent experiments where each feature is binary and takes value T or F. Below is a table summarizing the observations of the experiment:

X	Y	Z	Count
T	T	T	36
T	T	F	4
T	F	T	2
T	F	F	8
F	T	T	9
F	T	F	1
F	F	T	8
F	F	F	32

1. Compute the mutual information $I(X, Y)$ based on the frequencies observed in the data. (5 pts) Given probabilities:

   - $P(X = T) = \frac{\text{Count of } (X=T)}{\text{Total Count}} = \frac{36+4+2+8}{100} = 0.50$
   - $P(X = F) = \frac{\text{Count of } (X=F)}{\text{Total Count}} = \frac{9+1+8+32}{100} = 0.50$
   - $P(Y = T) = \frac{\text{Count of } (Y=T)}{\text{Total Count}} = \frac{36+4+9+1}{100} = 0.50$
   - $P(Y = F) = \frac{\text{Count of } (Y=F)}{\text{Total Count}} = \frac{2+8+8+32}{100} = 0.50$
   - $P(X = T, Y = T) = \frac{\text{Count of } (X=T,Y=T)}{\text{Total Count}} = \frac{36+4}{100} = 0.40$
   - $P(X = T, Y = F) = \frac{\text{Count of } (X=T,Y=F)}{\text{Total Count}} = \frac{2+8}{100} = 0.10$
   - $P(X = F, Y = T) = \frac{\text{Count of } (X=F,Y=T)}{\text{Total Count}} = \frac{9+1}{100} = 0.10$
   - $P(X = F, Y = F) = \frac{\text{Count of } (X=F,Y=F)}{\text{Total Count}} = \frac{8+32}{100} = 0.40$

   The mutual information $I(X, Y)$ is calculated as:

$$I(X, Y) = \sum_{x \in \{T,F\}} \sum_{y \in \{T,F\}} p(x, y) \log \left(\frac{p(x, y)}{p(x)p(y)} \right)$$

$$= P(X = T, Y = T) \log \left(\frac{P(X = T, Y = T)}{P(X = T)P(Y = T)} \right) + P(X = T, Y = F) \log \left(\frac{P(X = T, Y = F)}{P(X = T)P(Y = F)} \right)$$

$$+P(X = F, Y = T) \log \left(\frac{P(X = F, Y = T)}{P(X = F)P(Y = T)} \right) + P(X = F, Y = F) \log \left(\frac{P(X = F, Y = F)}{P(X = F)P(Y = F)} \right)$$

$$= 0.40 \cdot \log_2 \left(\frac{0.40}{0.50 \cdot 0.50} \right) + 0.10 \cdot \log_2 \left(\frac{0.10}{0.50 \cdot 0.50} \right)$$

$$+0.10 \cdot \log_2 \left(\frac{0.10}{0.50 \cdot 0.50} \right) + 0.40 \cdot \log_2 \left(\frac{0.40}{0.50 \cdot 0.50} \right)$$

$$= 0.278 \text{ bits}$$

   This calculation results in the mutual information $I(X, Y) \approx 0.278$ bits.

2. Compute the mutual information $I(X, Z)$ based on the frequencies observed in the data. (5 pts) Given the calculated probabilities:

   - $P(X = T) = 0.50$
   - $P(X = F) = 0.50$
   - $P(Z = T) = 0.57$
   - $P(Z = F) = 0.43$
   - $P(X = T, Z = T) = 0.38$
   - $P(X = T, Z = F) = 0.12$
   - $P(X = F, Z = T) = 0.17$

- $P(X = F, Z = F) = 0.33$

The mutual information $I(X, Z)$ is calculated as:

$$I(X, Z) = \sum_{x\in\{T,F\}} \sum_{z\in\{T,F\}} p(x, z) \log \left(\frac{p(x, z)}{p(x)p(z)}\right)$$

$$= 0.38 \cdot \log_2 \left(\frac{0.38}{0.50 \cdot 0.57}\right) + 0.12 \cdot \log_2 \left(\frac{0.12}{0.50 \cdot 0.43}\right)$$
$$+0.17 \cdot \log_2 \left(\frac{0.17}{0.50 \cdot 0.57}\right) + 0.33 \cdot \log_2 \left(\frac{0.33}{0.50 \cdot 0.43}\right)$$

$$= 0.134 \text{ bits}$$

This calculation results in the mutual information $I(X, Z) \approx 0.134$ bits.

3. Compute the mutual information $I(Z, Y)$ based on the frequencies observed in the data. (5 pts) Given the calculated probabilities:

   - $P(Y = T) = 0.50$
   - $P(Y = F) = 0.50$
   - $P(Z = T) = 0.57$
   - $P(Z = F) = 0.43$
   - $P(Z = T, Y = T) = 0.45$
   - $P(Z = T, Y = F) = 0.10$
   - $P(Z = F, Y = T) = 0.05$
   - $P(Z = F, Y = F) = 0.40$

   The mutual information $I(Z, Y)$ is calculated as:

$$I(Z, Y) = \sum_{z\in\{T,F\}} \sum_{y\in\{T,F\}} p(z, y) \log \left(\frac{p(z, y)}{p(z)p(y)}\right)$$

$$= 0.45 \cdot \log_2 \left(\frac{0.45}{0.57 \cdot 0.50}\right) + 0.10 \cdot \log_2 \left(\frac{0.10}{0.57 \cdot 0.50}\right)$$
$$+0.05 \cdot \log_2 \left(\frac{0.05}{0.43 \cdot 0.50}\right) + 0.40 \cdot \log_2 \left(\frac{0.40}{0.43 \cdot 0.50}\right)$$

$$= 0.398 \text{ bits}$$

   This calculation results in the mutual information $I(Z, Y) \approx 0.398$ bits.

4. Which undirected edges will be selected by the Chow-Liu algorithm as the maximum spanning tree? (5 pts) The mutual information values calculated earlier are:

   - $I(X, Y) \approx 0.278$ bits
   - $I(X, Z) \approx 0.134$ bits
   - $I(Z, Y) \approx 0.398$ bits

   The Chow-Liu algorithm constructs a maximum spanning tree (MST) by selecting edges based on the mutual information between pairs of variables. The edges with the highest mutual information values are included in the MST.

   Given the calculated mutual information values, the selected edges for the MST will be:

   - Edge between Z and Y, since $I(Z, Y)$ has the highest value.

- Edge between X and Y, since $I(X, Y)$ is higher than $I(X, Z)$.

Therefore, the maximum spanning tree will include the edges (Z, Y) and (X, Y).

5. Root your tree at node X, assign directions to the selected edges. (5 pts) Once the maximum spanning tree is constructed with the edges (Z, Y) and (X, Y), we root the tree at node X and assign directions to the edges as follows:

   - Since the tree is rooted at X, the edge between X and Y is directed from X to Y, resulting in $X \rightarrow Y$.
   - As Y is connected to the root X, the edge between Y and Z is directed from Y to Z, resulting in $Y \rightarrow Z$.

   Therefore, the directed edges in the tree, rooted at X, are $X \rightarrow Y$ and $Y \rightarrow Z$.

References

Goodfellow, I., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A., and Bengio, Y. (2014). Generative adversarial nets. *Advances in neural information processing systems*, 27.

Kingma, D. P. and Ba, J. (2014). Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*.