Lab 6: DFT and FFT

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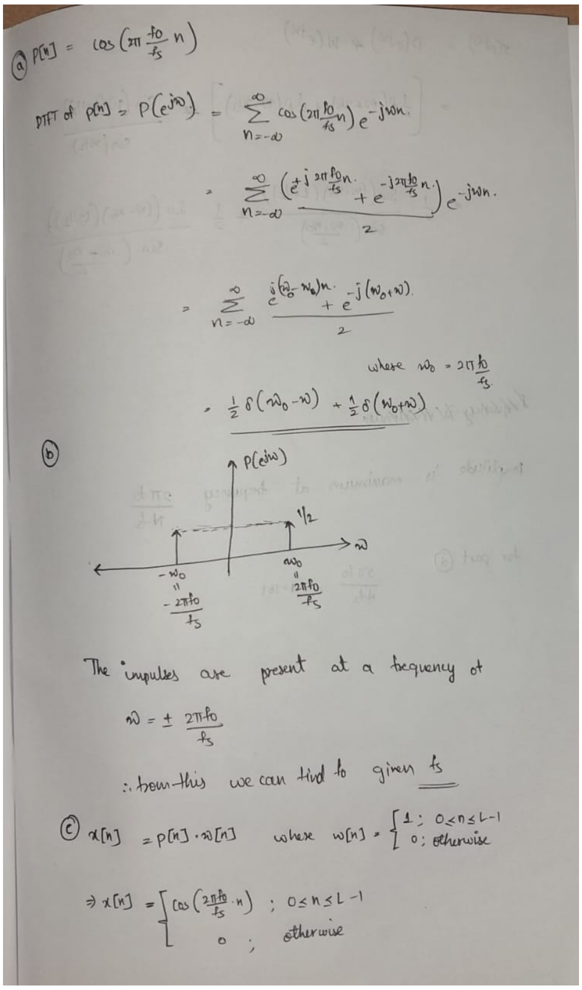
Team: Noicifiers

6.1 DFT for frequency analysis of CT signals:

a,b,c:



(a) $p[n] = \cos\left(2\pi \frac{f_0}{f_s} n\right)$

DTFT of $p[n] = P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \cos\left(2\pi \frac{f_0}{f_s} n\right) e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} \left(\frac{e^{j 2\pi \frac{f_0}{f_s} n} + e^{-j 2\pi \frac{f_0}{f_s} n}}{2} \right) e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} \frac{e^{j(\omega_0 - \omega)n} + e^{-j(\omega_0 + \omega)n}}{2}$

where $\omega_0 = 2\pi \frac{f_0}{f_s}$

$= \frac{1}{2}\delta(\omega_0 - \omega) + \frac{1}{2}\delta(\omega_0 + \omega)$

(b)

The impulses are present at a frequency of

$\omega = \pm \frac{2\pi f_0}{f_s}$

$\therefore$ from this we can find f_0 given f_s

(c) $x[n] = p[n] \cdot w[n]$ where $w[n] = \begin{cases} 1; & 0 \le n \le L-1 \\ 0; & \text{otherwise} \end{cases}$

$\Rightarrow x[n] = \begin{cases} \cos\left(\frac{2\pi f_0}{f_s} \cdot n\right); & 0 \le n \le L-1 \\ 0; & \text{otherwise} \end{cases}$

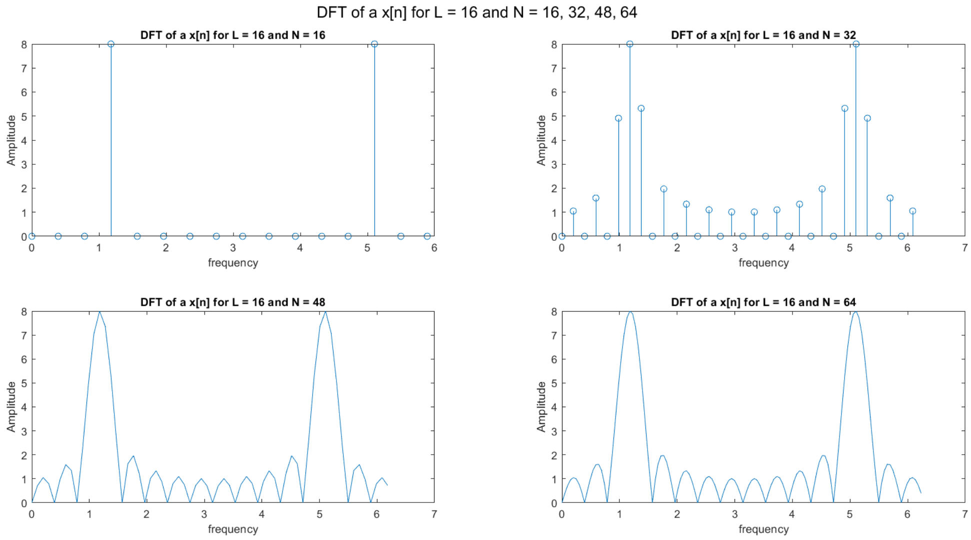
$$X(e^{j\omega}) = P(e^{j\omega}) * W(e^{j\omega})$$

$$= \left[\frac{1}{2}\delta(\omega+\omega_0) + \frac{1}{2}\delta(\omega-\omega_0)\right] * \frac{\sin\left(\omega\left(L+1/2\right)\right)}{\sin\left(\omega/2\right)}$$

$$= \frac{1}{2}\cdot\frac{\sin\left((\omega+\omega_0)\left(L+1/2\right)\right)}{\sin\left(\frac{\omega+\omega_0}{2}\right)} + \frac{1}{2}\,\frac{\sin\left((\omega-\omega_0)\left(L+1/2\right)\right)}{\sin\left(\frac{\omega+\omega_0}{2}\right)}$$

d:

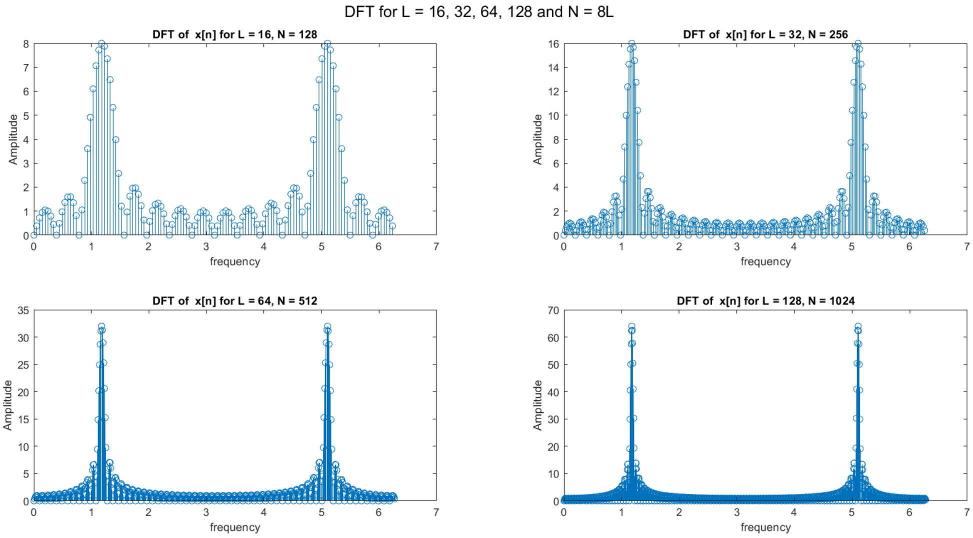
$$\mathit{Given},$$
$$f_0 = 12\ Hz \quad f_s = 64\ Hz$$
$$L = 16$$
$$m = \{1, 2, 3, 4\}$$
$$N = mL$$

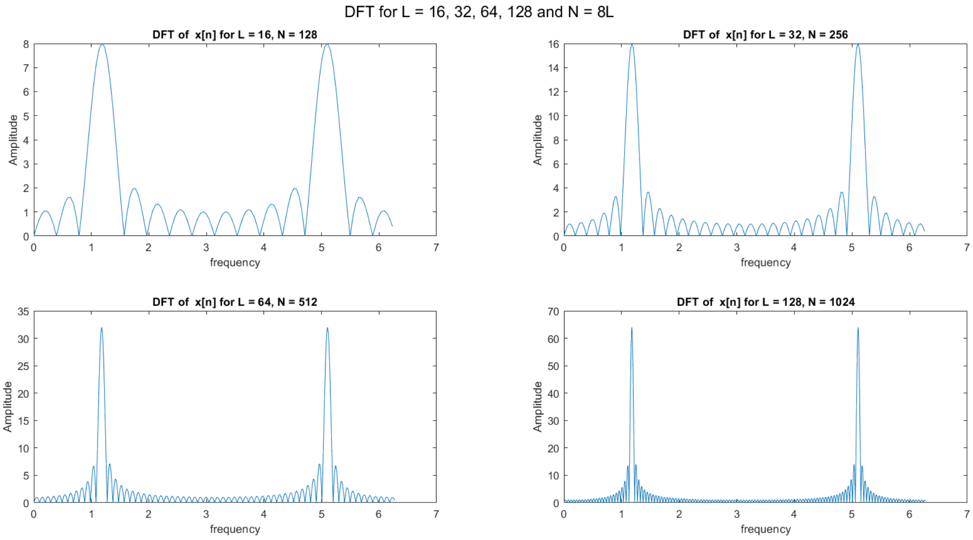


Yes, The plots were as expected by the equation obtained in part c of the question.

e:

$$\mathit{Given},$$
$$f_0 = 12\ Hz \quad f_s = 64\ Hz$$
$$L = \{16, 32, 64, 128\}$$
$$N = 8L$$

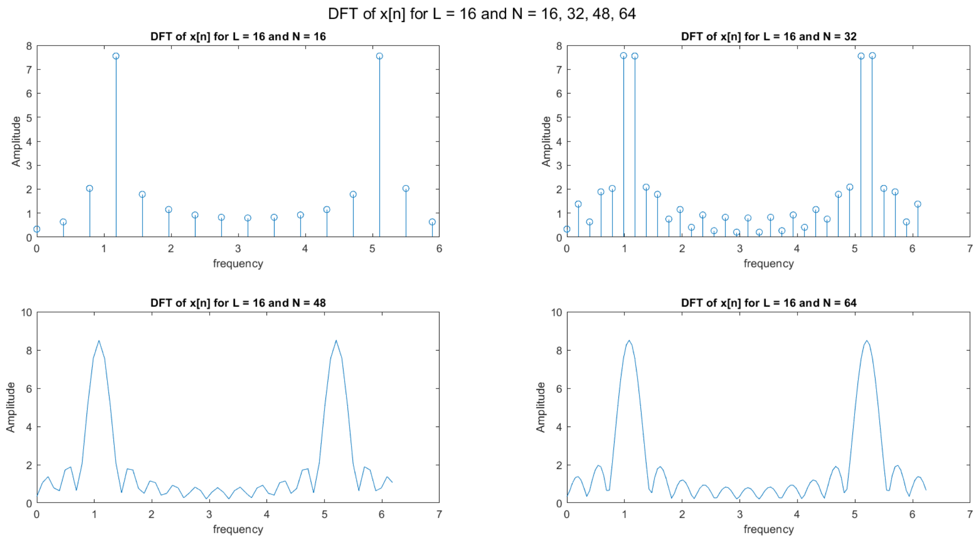




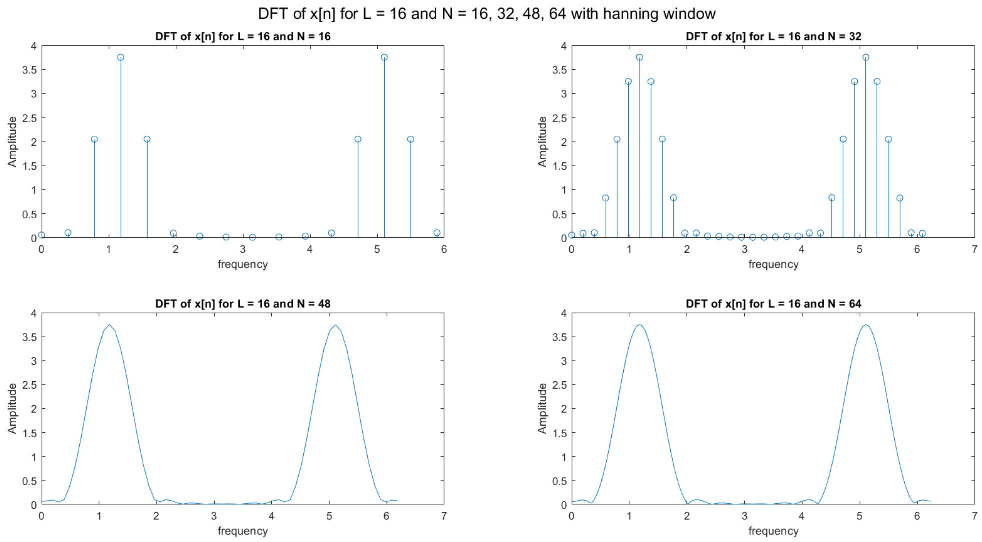
- As L increases no. of samples taken increase
- **Spectral leakage is decreased as L increases**
- The main lobe becomes narrower ⇒ DFT is **more sensitive to the frequency**
- **Frequency resolution increases as l increases**

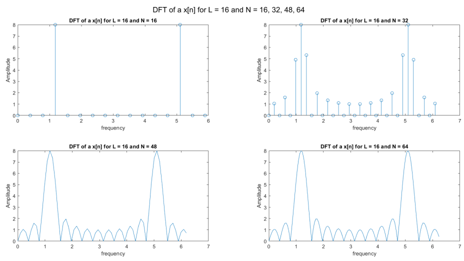
f:

$$Given,$$
$$f_0 = 11\ Hz \quad f_s = 64\ Hz$$
$$L = 16$$
$$m = \{1, 2, 3, 4\}$$
$$N = mL$$

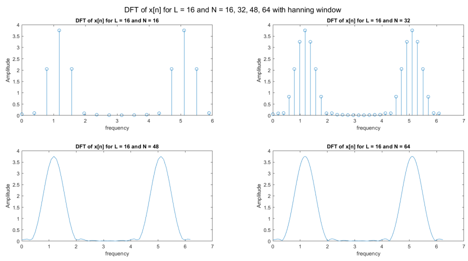


g:





the figure for part d (Rectangular window)



the figure for part g (Hanning window)

- The width of the main lobe increases.
- The Spectral Leakage is more in the Rectangular window when compared with the Hanning window.

h:

We get maximum Amplitude at ~~frequency~~ " $\frac{2\pi f_0}{f_s}$ "

part d

$\Rightarrow \frac{2\pi f_0}{f_s} = 1.1781 = \frac{3\pi}{8}$

$\Rightarrow f_0 = \frac{3 f_s}{16} = \frac{3}{16} \times 64 = 12$

$\Rightarrow \boxed{f_0 = 12 Hz}$

part f

$\frac{2\pi f_0}{f_s} = 1.07992 = \frac{11\pi}{32}$

$\frac{2\pi \times f_0}{64} = \frac{11\pi}{32} \Rightarrow \boxed{f_0 = 11 Hz}$

part g

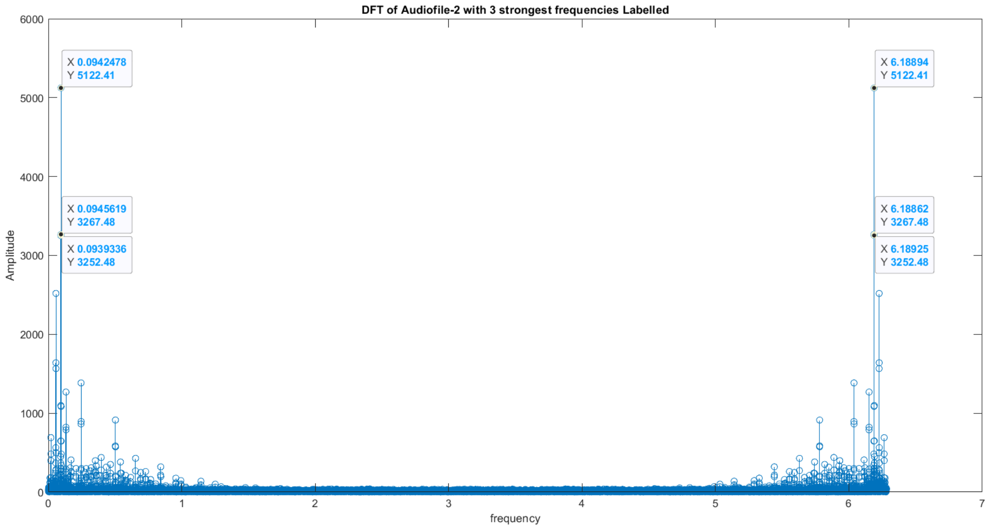
$\frac{2\pi f_0}{f_s} = 1.1781 = \frac{3\pi}{8}$

$\Rightarrow \boxed{f_0 = 12 Hz}$

* $N \geq L \Rightarrow N$ doesnot affect the answer

irrespective of N, the maximum amplitude ~~at~~ will be ~~frequency~~ at " $\frac{2\pi f_0}{f_s}$ "

i:



The three strongest frequencies are:

- 0.0942478 Hz or 6.18894 Hz
- 0.0945619Hz or 6.18862 Hz
- 0.0939336 Hz or 6.18925 Hz

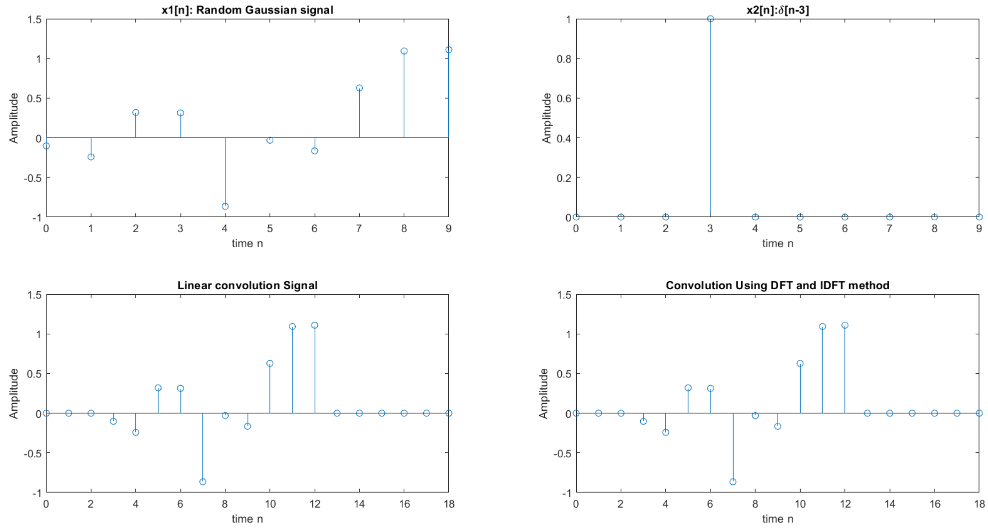
6.2 Direct and DFT-based convolutions:

a,b,c,d:

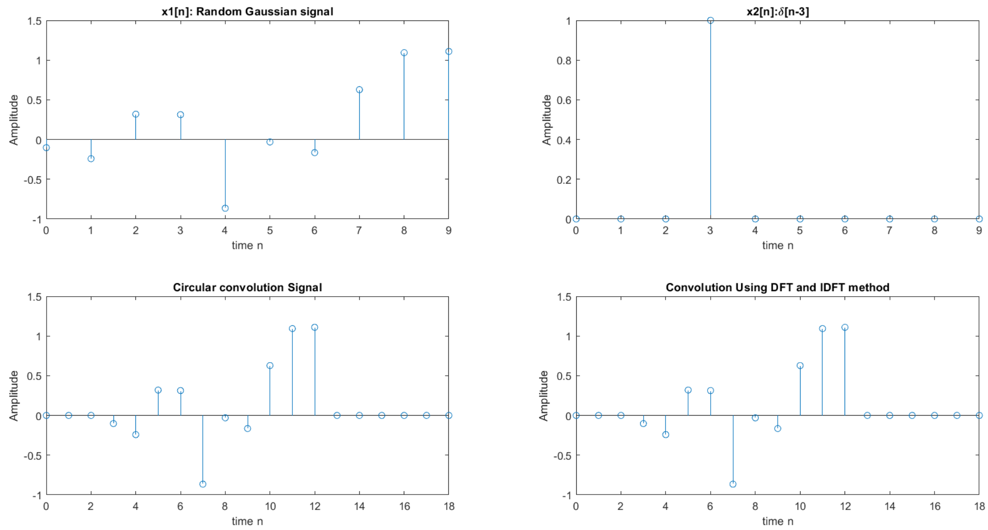
$x_1[n]$ is a random Gaussian sequence of length 10

$x_2[n]$ is the first 10 samples of the signal $\delta[n-3]$ starting from n = 0

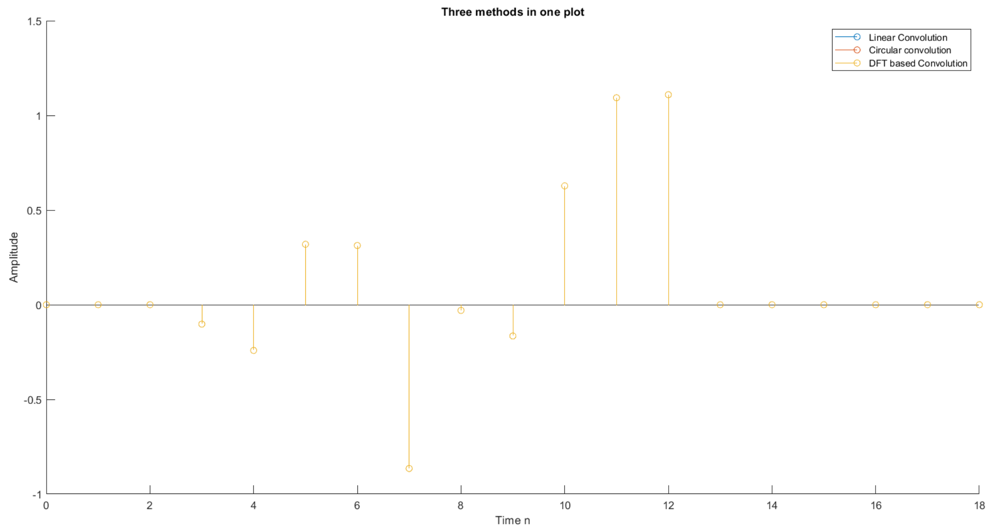
Linear Convolution v/s DFT-IDFT method:



Circular Convolution v/s DFT-IDFT method:

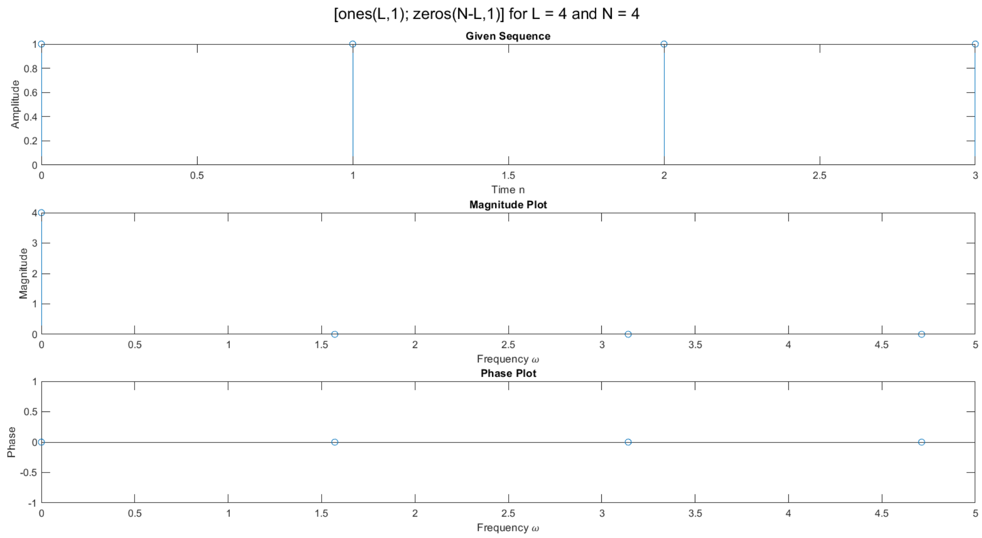


The three methods:

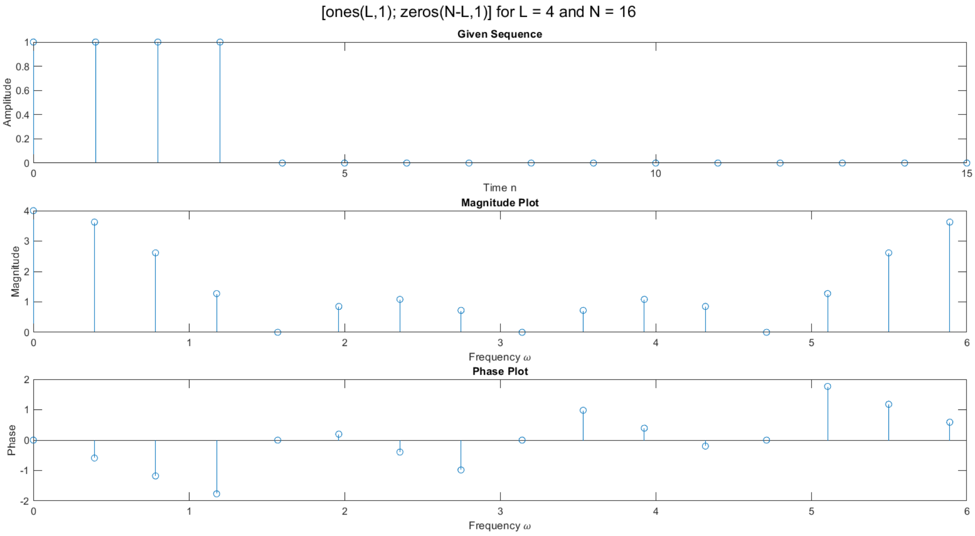


6.3 DFT of some signals:

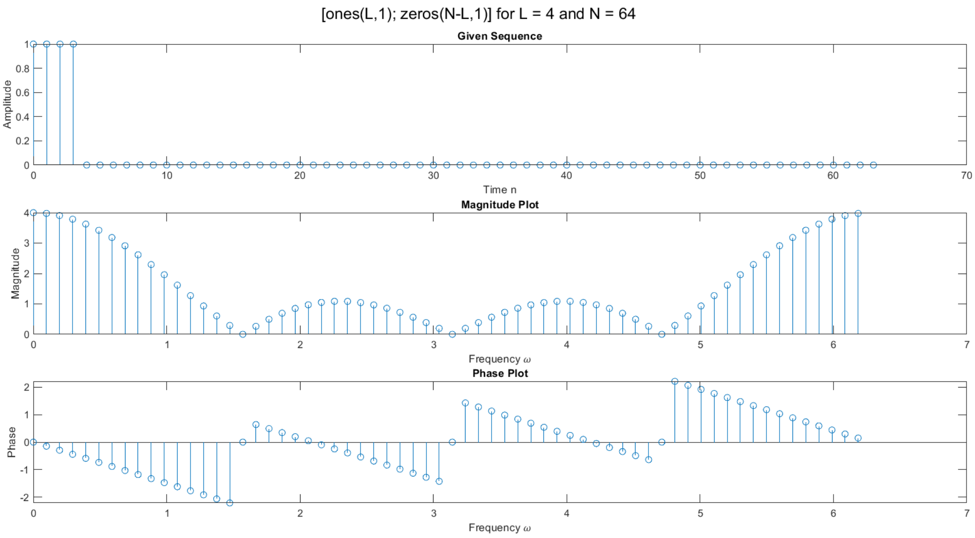
a: [ones(L,1); zeros(N-L,1)] l = 4 N = 4



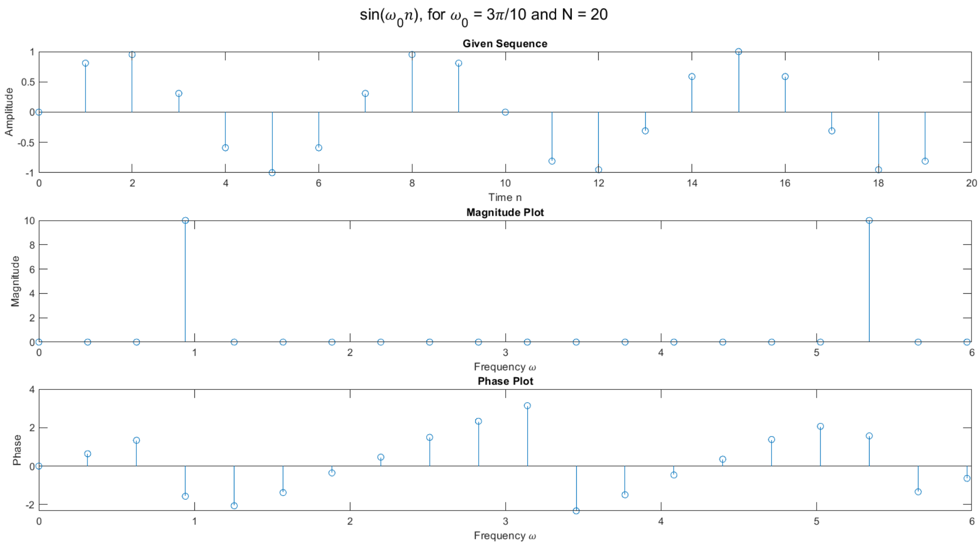
[ones(L,1); zeros(N-L,1)] l = 4 N = 16



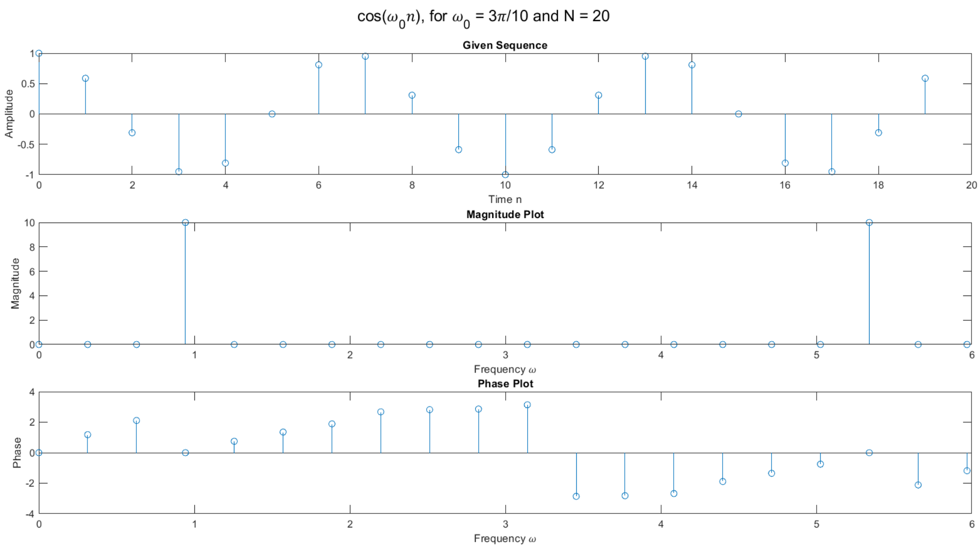
[ones(L,1); zeros(N-L,1)] l = 4 N = 64



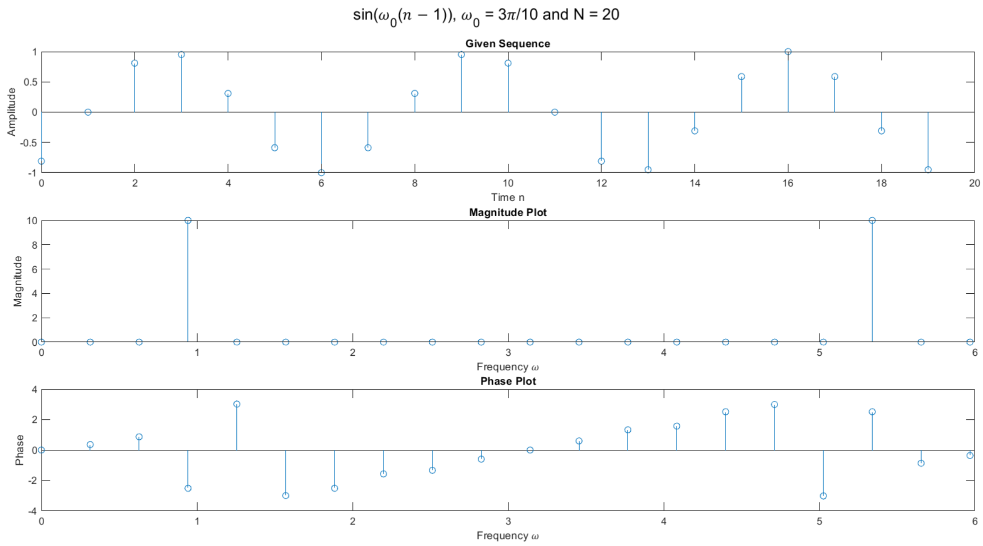
b: $sin(\omega_0 n)$, for ω_0 = 3π/10 and N = 20



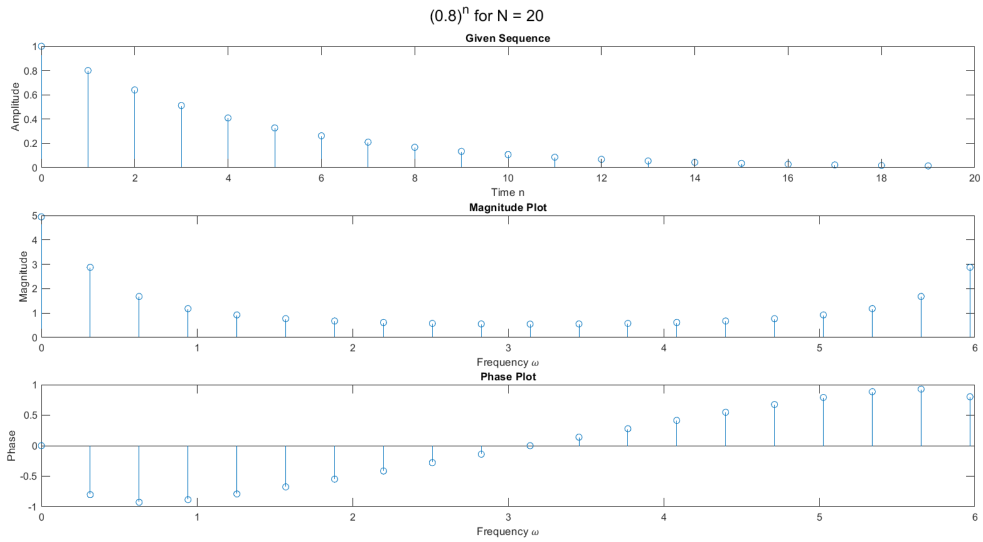
c: $cos(\omega_0 n)$, for ω_0 = 3π/10 and N = 20



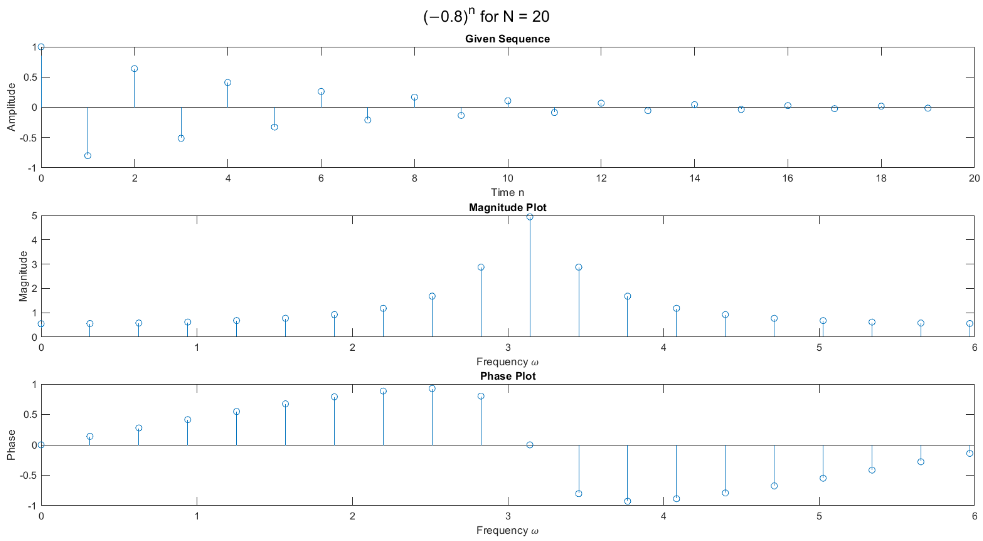
d: $sin(\omega_0(n-1))$, for ω_0 = 3π/10 and N = 20



e:$(0.8)^n$, for N = 20



f:$(-0.8)^n$, for N = 20



Yes, we can identify the low-frequency and high-frequency spectrums.