Lab 9 – Digital filter design

9.1 Filtering of periodic signals with LTI systems

a) LPF Code:

```
function  B = myLPF(A,w0_FS,wc)
    N = (length(A)-1)/2;
    l = 0;
    for k = 1:1:N
        if(k*w0_FS <= wc)
            l = k;
        end
    end
    B = A((N+1-l):(N+1+l));
end
```

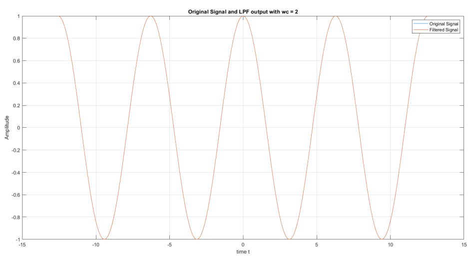
Inputs:

- input signal FS coefficients A
- , frequency of the input periodic signal w0_FS
- cut-off frequency wc

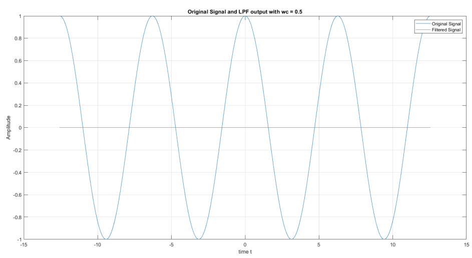
Outputs:

output signal FS coefficients in the vector B

b)Plots for the filtered signals



Plot Corresponding to wc= 2



Plot Corresponding to wc= 0.5

When wc = 0.5, the cutoff frequency is less than the Nyquist rate so the signal is lost

c)

Function:

```
function  B = myHPF(A,w0_FS,wc)
    N = (length(A)-1)/2;
    l = 0;
    for k = 1:1:N
        if(k*w0_FS < wc)
            l = k;
        end
    end
    B = [A(1:(N-l)) zeros(1,(2*l)+1) A((N+2+l):end)];
end
```

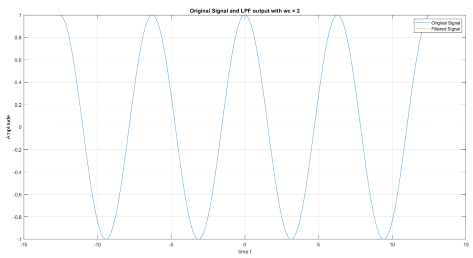
Inputs:

- input signal FS coefficients A
- , frequency of the input periodic signal w0_FS
- cut-off frequency wc

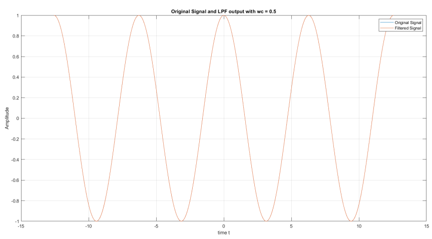
Outputs:

output signal FS coefficients in the vector B

Plots for Filtered Signals:



Plot Corresponding to wc= 2



Plot Corresponding to wc= 0.5

d)

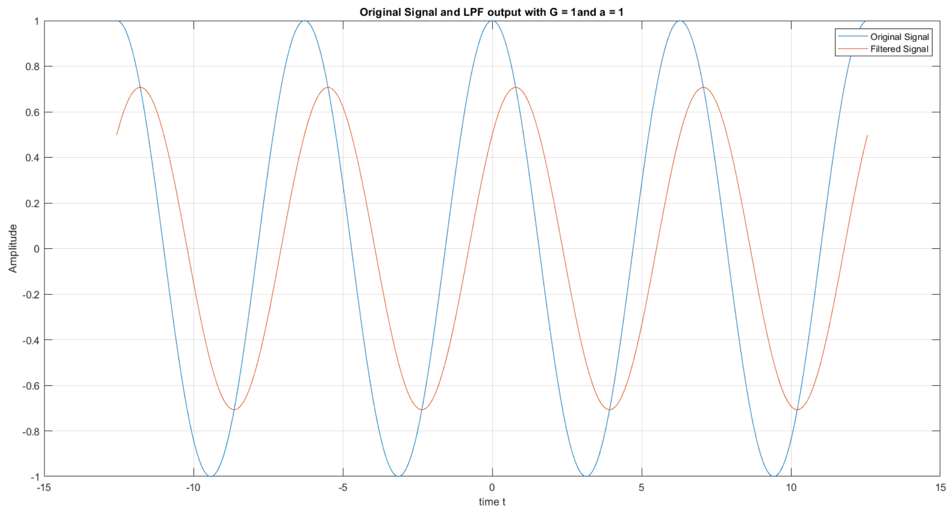
```
function  B = NonIdeal(A,w0_FS,G,a)
    N = (length(A)-1)/2;
    H = zeros(1,length(A));
    for k = -N:1:N
        H(k+N+1) = G/(a+1j*(k*w0_FS));
    end
    B = A.*H;
end
```

Inputs:

- input signal FS coefficients A
- frequency of the input periodic signal w0_FS
- G and a correspond to the frequency response of the window

Outputs:

Output signal FS coefficients in the vector B

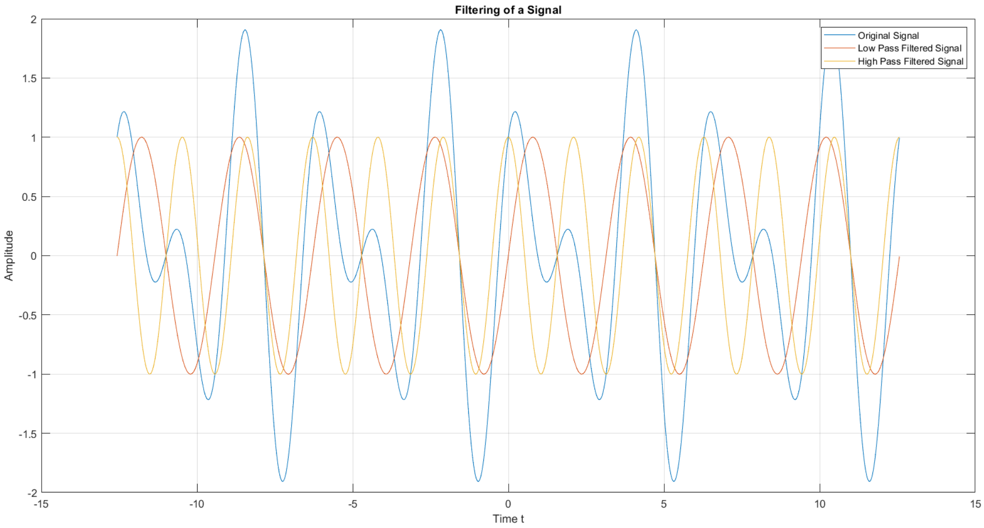


We give Real Signal as input.

We made the frequency response of the LTI system such that the OUTPUT Signal would also be real valued.

So, here we were required to make a Complex valued

e)



Time Period of Signal = LCM of periods of both = $LCM(\pi, \frac{2\pi}{3}) = 2\pi$

$\omega_0 = 1 = HCF(2, 3)$

9.2 Low-pass FIR filter design using windows

Impulse Response $h_{LPF}[n]$:

$$h_{LPF}[n] = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} 1\cdot e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{e^{j\omega n}}{jn}\Bigg|_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{\cancel{2}\pi}\times\frac{\cancel{2j}\sin\left(\frac{\pi n}{6}\right)}{\cancel{j}n}$$

$$= \frac{1}{6}\cdot\frac{\sin(\pi n/6)}{\pi n/6}$$

$$\boxed{h_{LPF}[n] = \frac{1}{6}\cdot \mathrm{sinc}(n/6)}$$

Impulse Response $h_d[n]$:

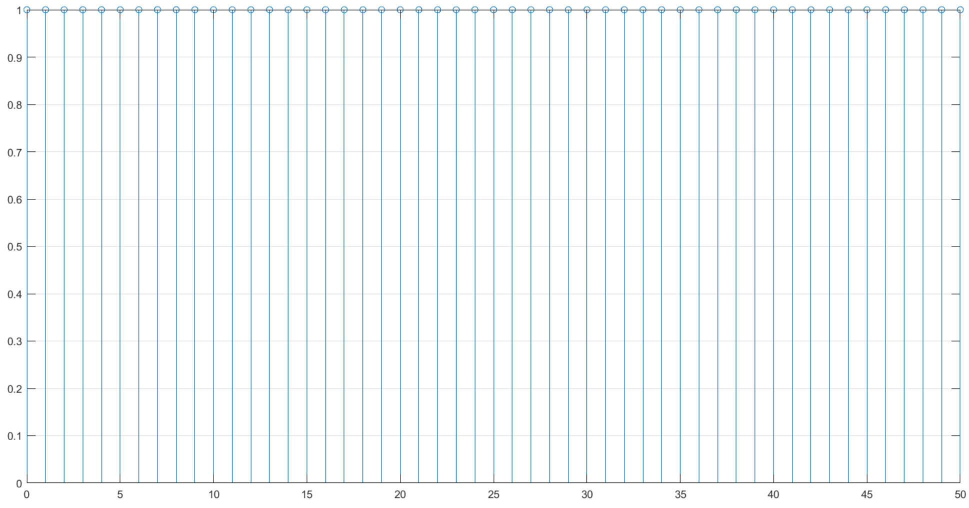
$$h_d[n] = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{-j\omega n_c}\cdot e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{j\omega(n-n_c)}\,d\omega$$

$$= \frac{1}{6}\,\frac{\sin\frac{\pi}{6}(n-n_c)}{\frac{\pi}{6}(n-n_c)}$$

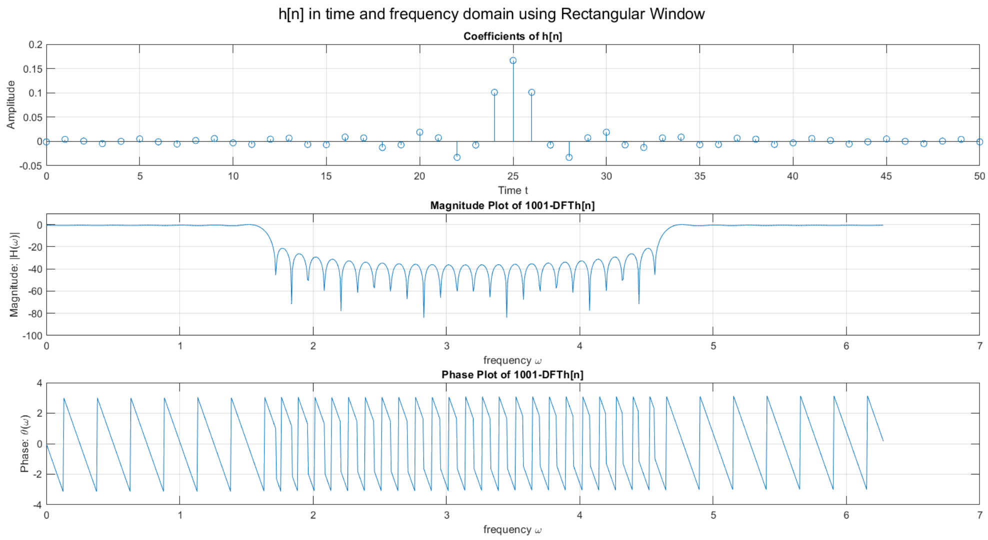
$$= \frac{1}{6}\,\mathrm{sinc}\left(\frac{n-n_c}{6}\right)$$

$$h_d[n] = \frac{1}{6}\,\mathrm{sinc}\left(\frac{n-n_c}{6}\right)$$

a,b)



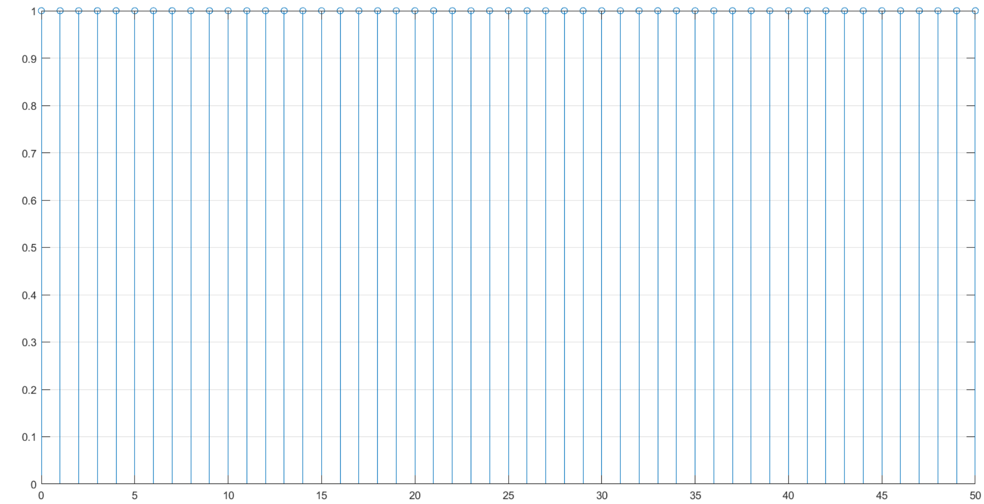
Rectangular Window



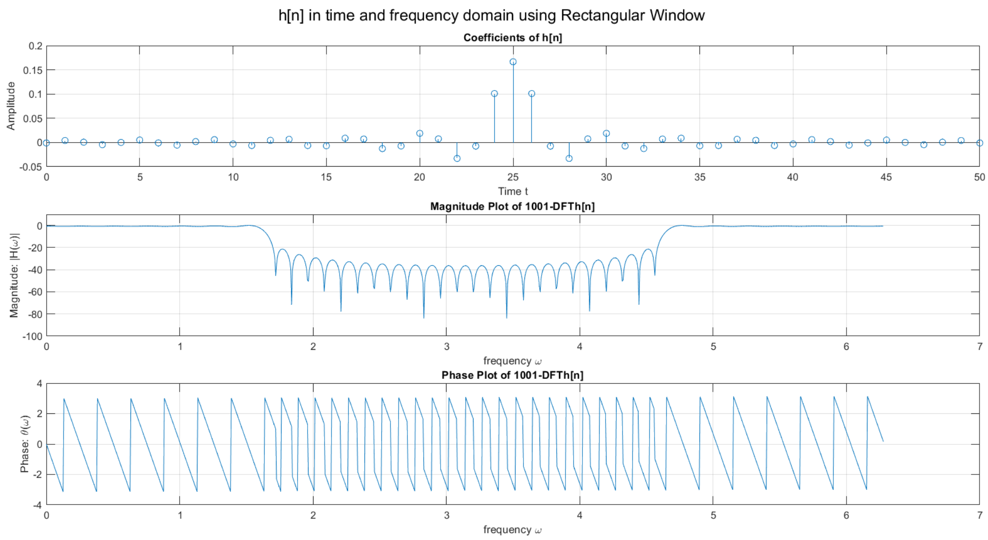
Yes, the Phase is linear.

the impulse Response is also symmetric n = 25.

c)



Blackman window

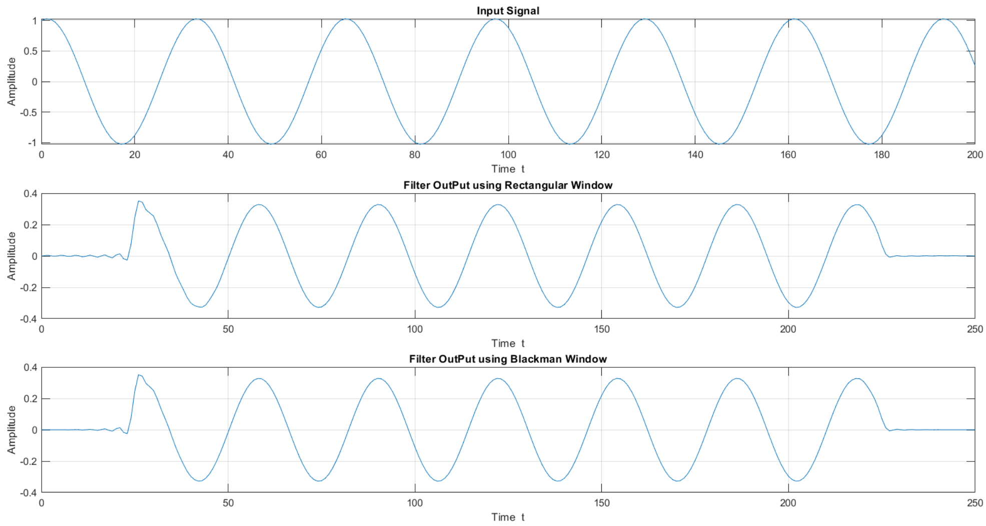


d)

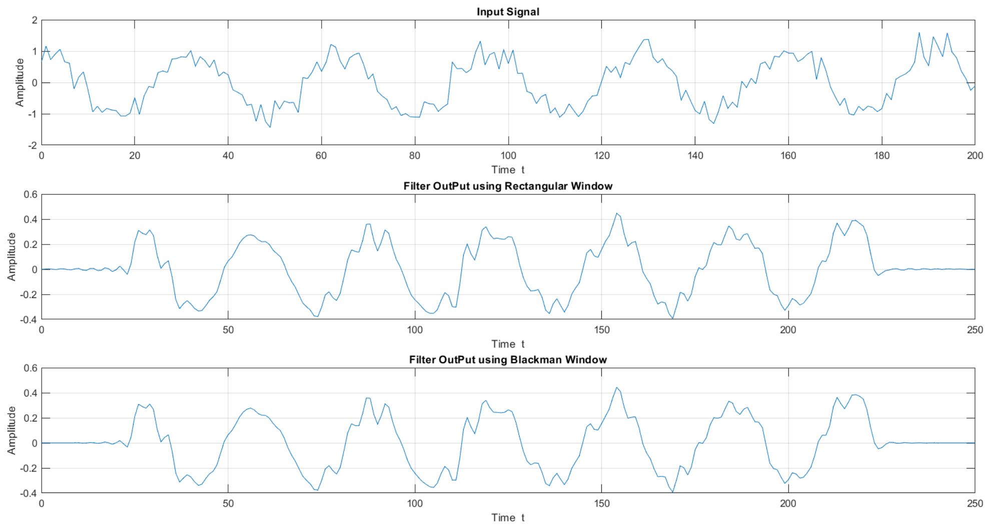
We can observe that ripple magnitude will decrease as n increases in the Blackman windowing filter and the amplitude of ripples in the stopband also decreases. But in a rectangular windowing filter, ripple magnitude and amplitude will be approximately the same. Blackman is desirable compared to rectangular windowing for its lower stopband magnitude but undesirable for its larger transition band.

e)

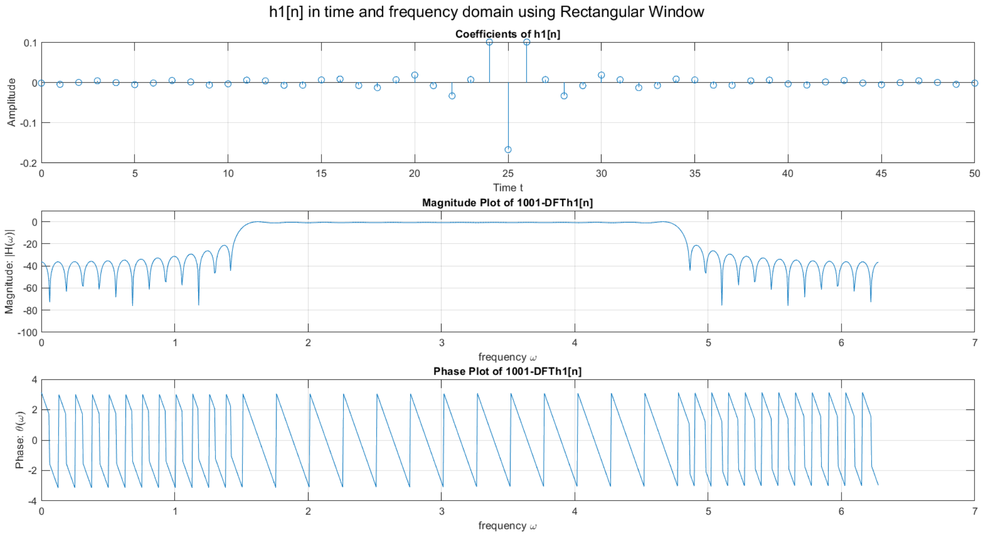
for $x[n] = cos(\frac{\pi n}{16}) + 0.25sin(\frac{\pi n}{16})$



for $x[n] = cos(\frac{\pi n}{16}) + 0.25randn(1,201)$



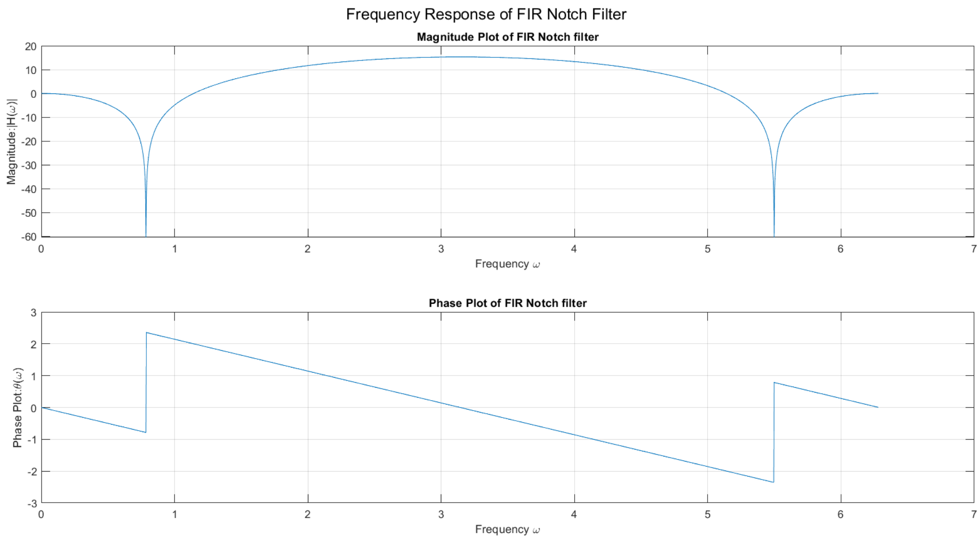
f)



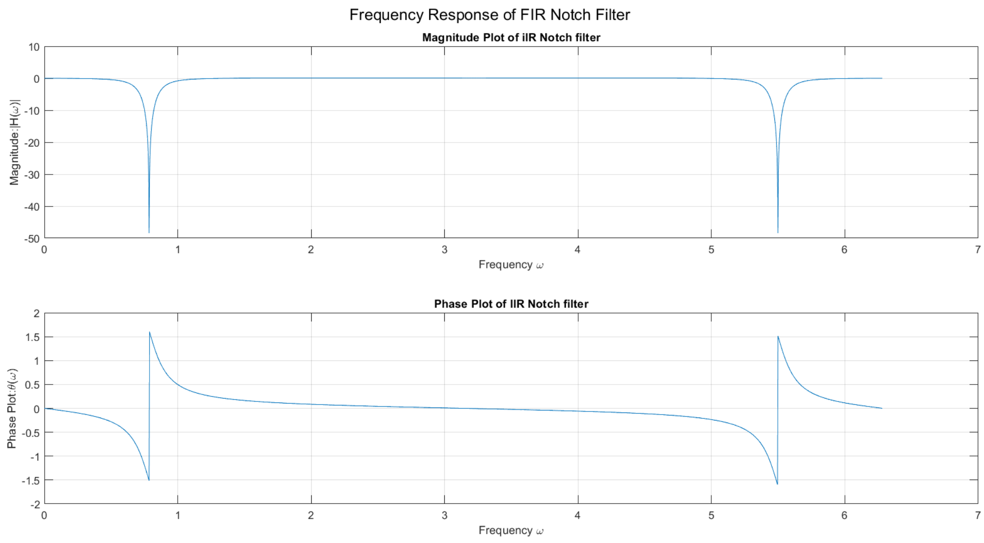
We can see that h1[n] is a high pass filter and h[n] is a low pass filter.

9.3 Notch filter

a)



b)

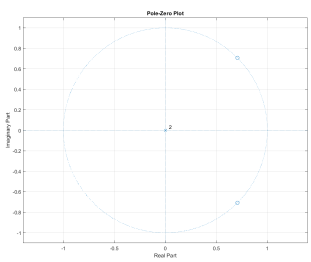


c)

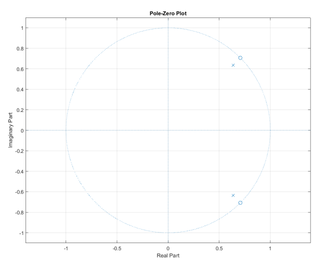
All the poles of both filters lie inside the unit circle. So, it is stable

Impulse Response of both the filters is Causal

it is evident from the Plots attached below



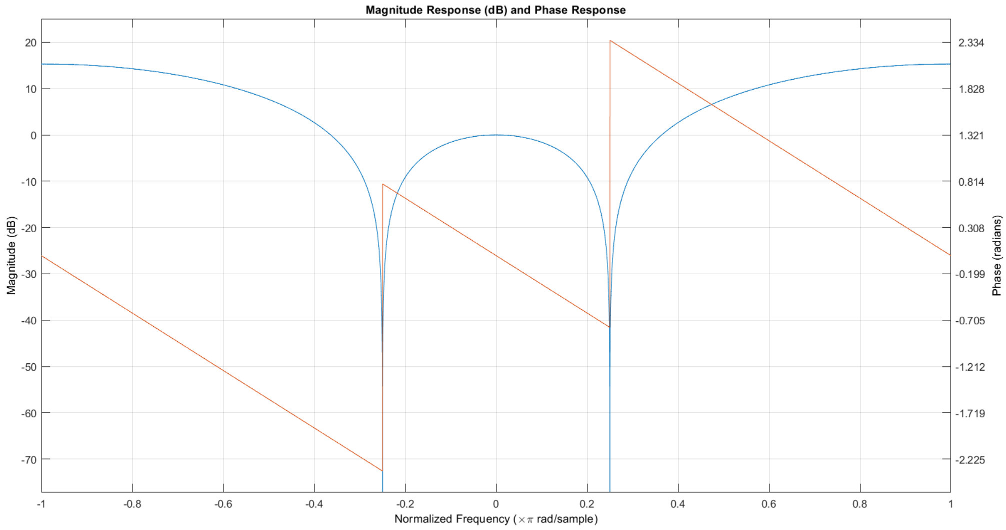
for FIR Notch Filter

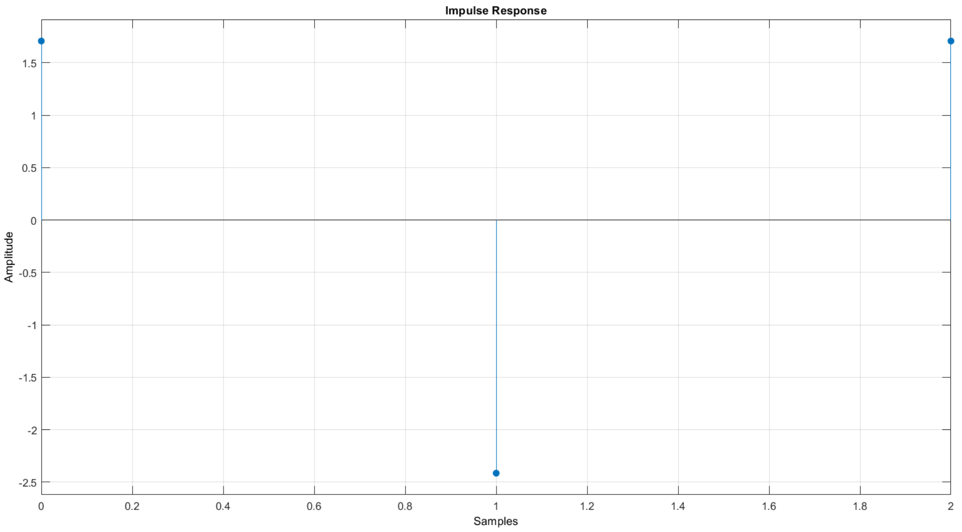


for IIR Notch Filter

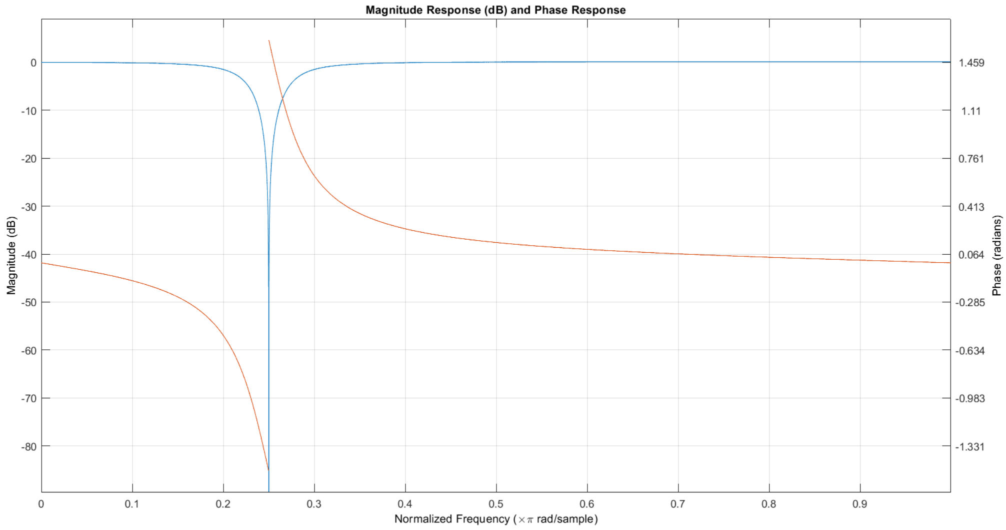
d)

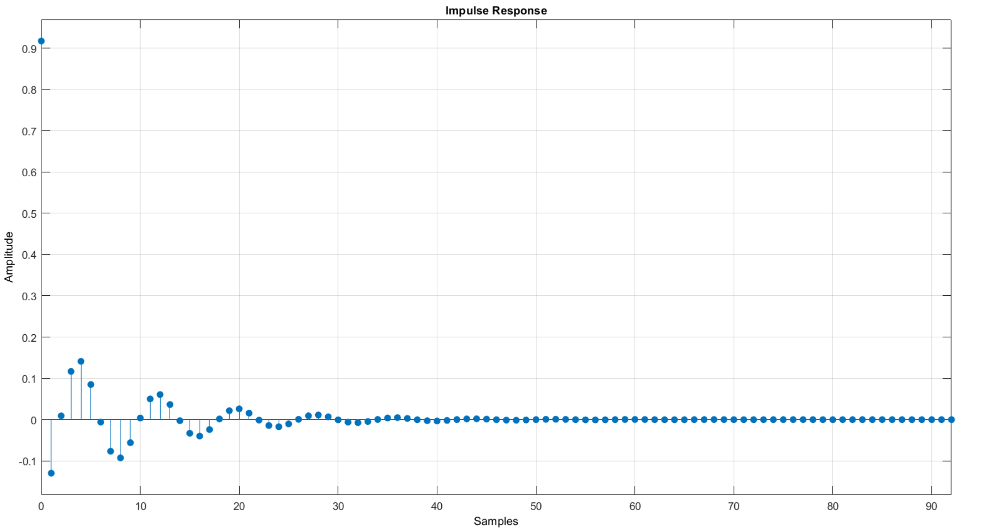
FIR NOTCH FILTER:



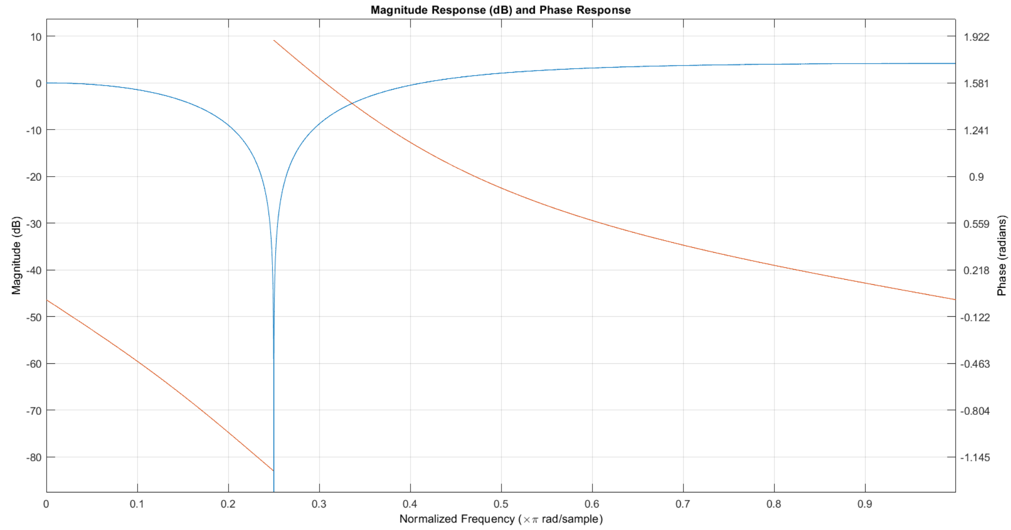


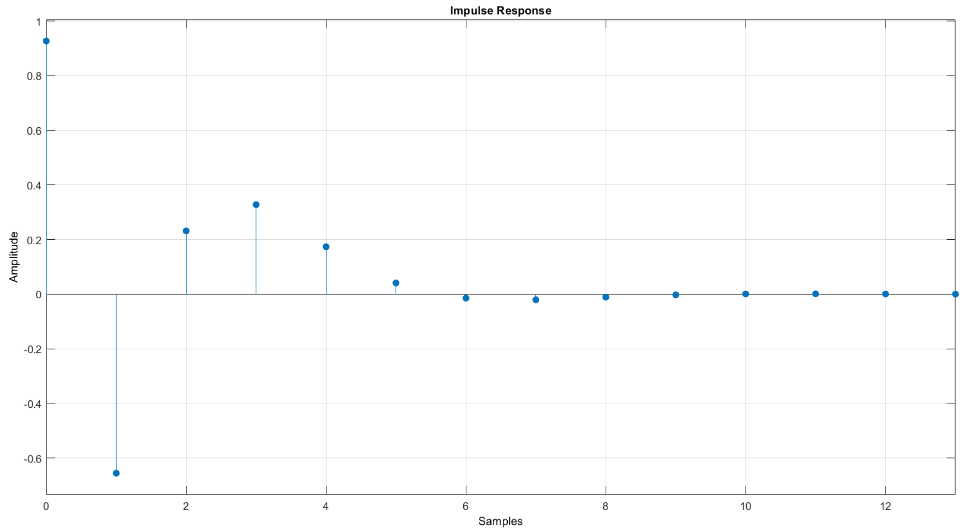
IIR NOTCH FILTER FOR r0 = 0.9:

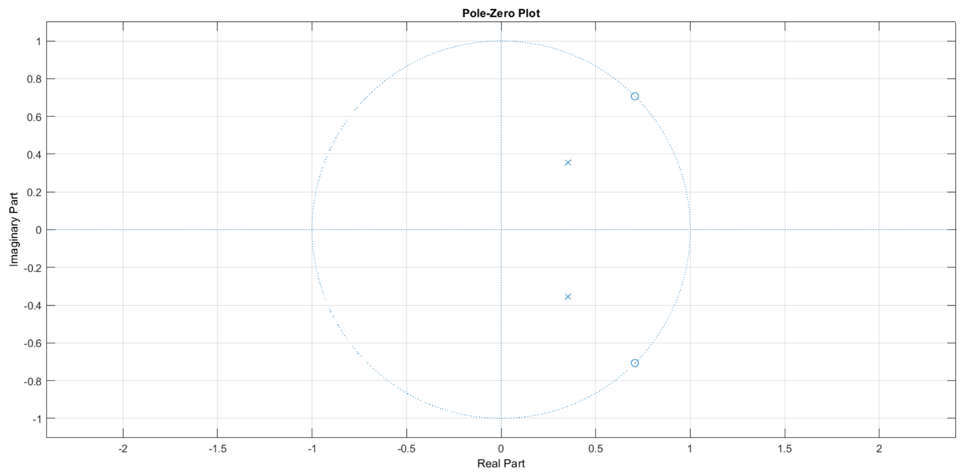


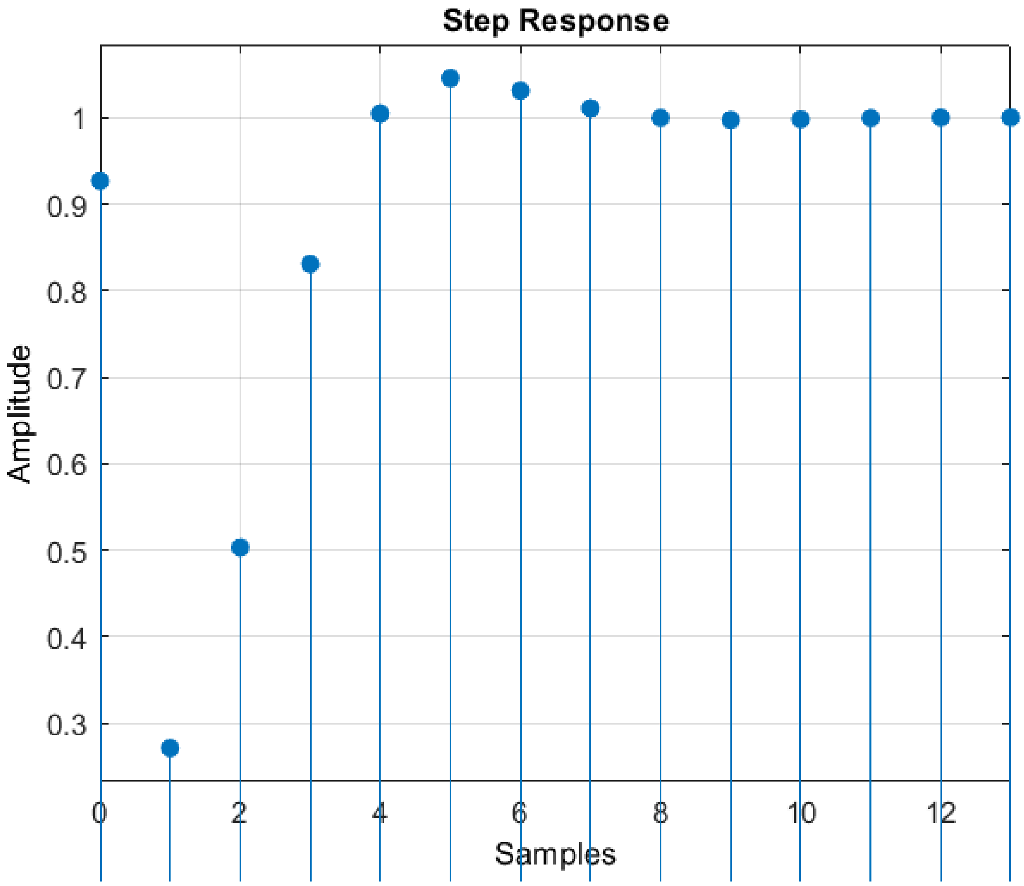


IIR NOTCH FILTER FOR r0 = 0.5:

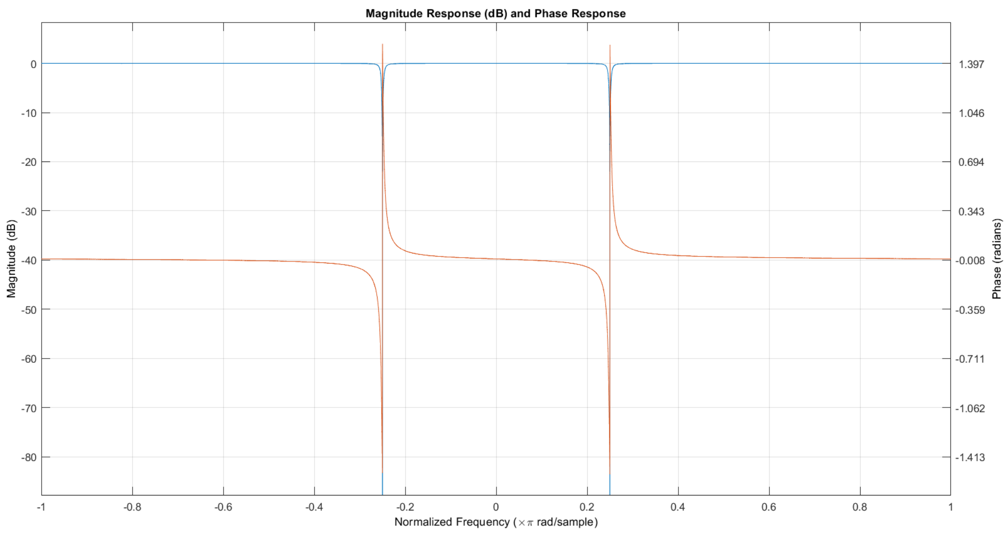

Magnitude Response (dB) and Phase Response
Magnitude (dB)
Phase (radians)
Normalized Frequency (×π rad/sample)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
1.922
1.581
1.241
0.9
0.559
0.218
-0.122
-0.463
-0.804
-1.145
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

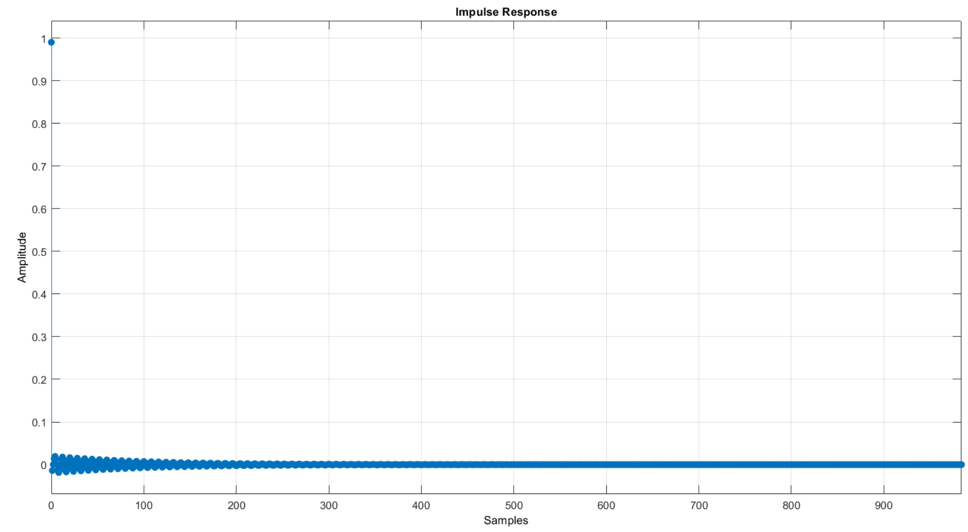

Impulse Response
Amplitude
Samples
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
2
4
6
8
10
12

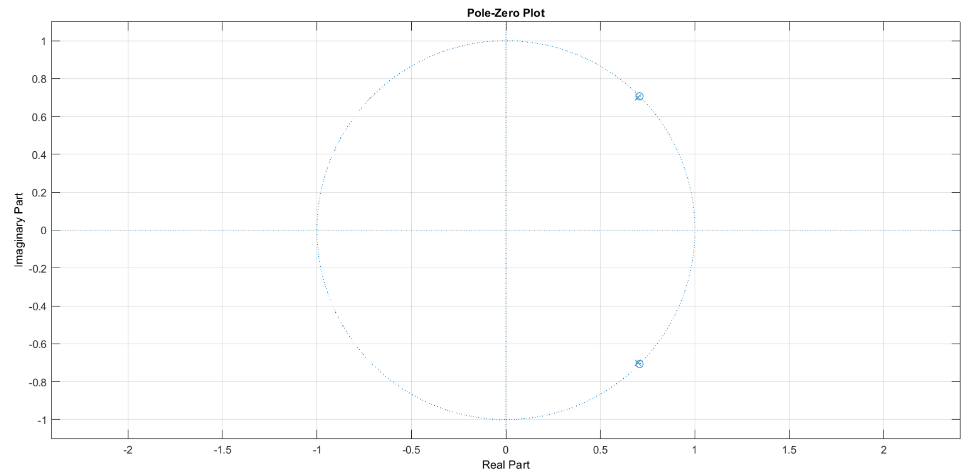

Pole-Zero Plot
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

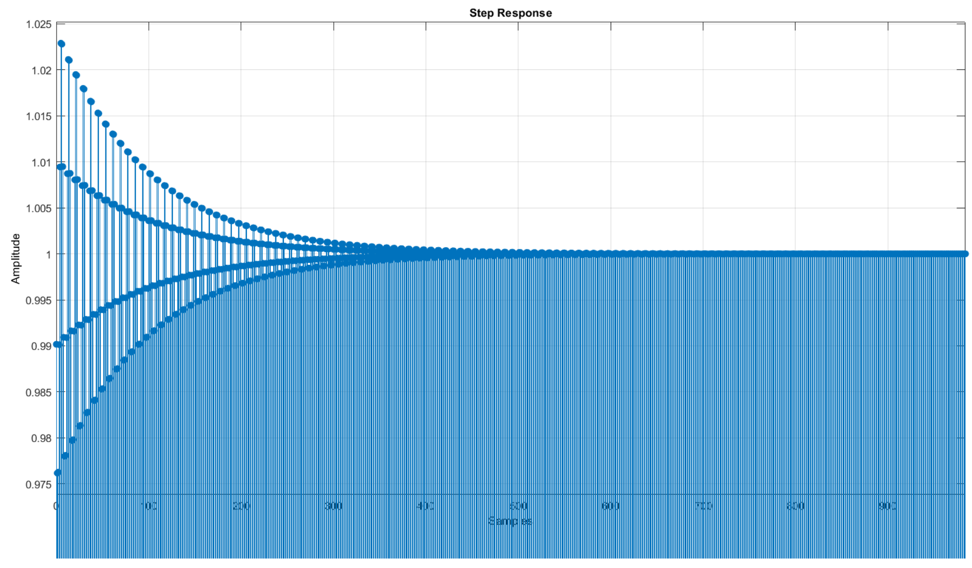


IIR NOTCH FILTER FOR r0 = 0.99:



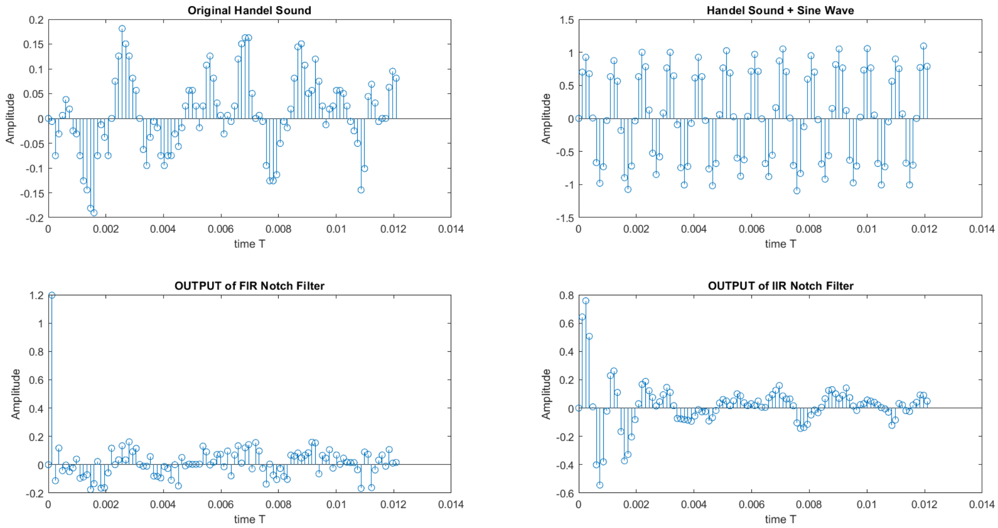

Impulse Response
Amplitude
Samples


Pole-Zero Plot
Imaginary Part
Real Part


Step Response
Amplitude
Samples

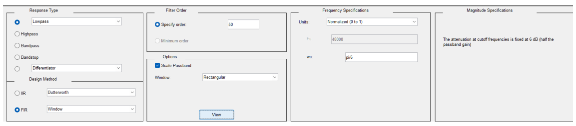
e,f)

We observe that noise is filtered out in both filters but in the IIR filter the quality of output audio was much better.

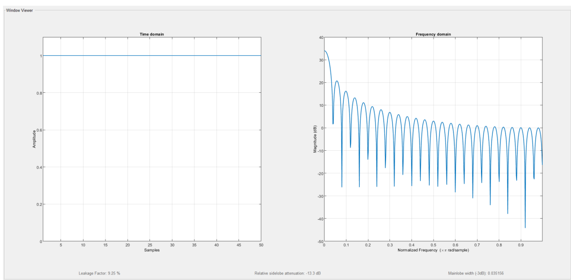


9.4 Filter design using filterDesigner

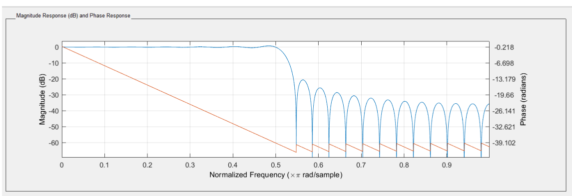
a,b)

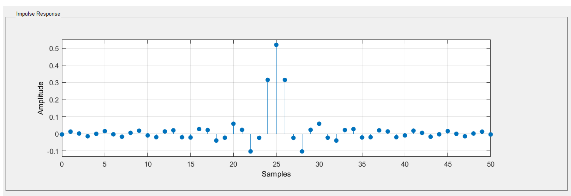


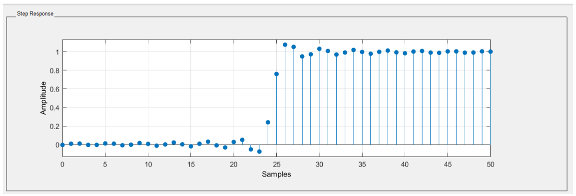
Specifications

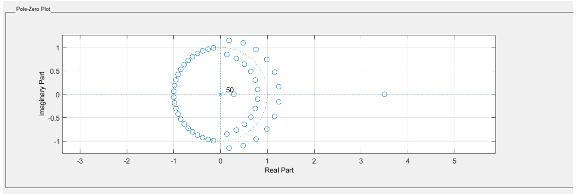


Window

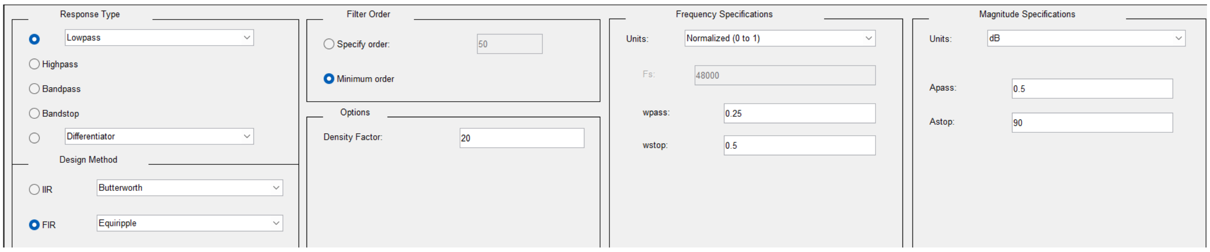








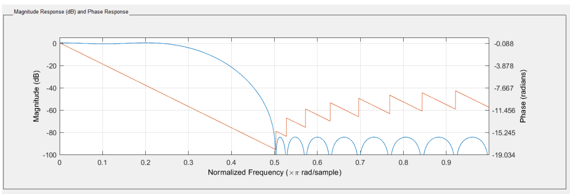
c)

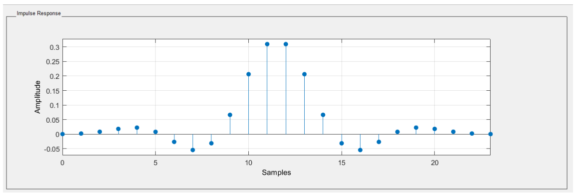


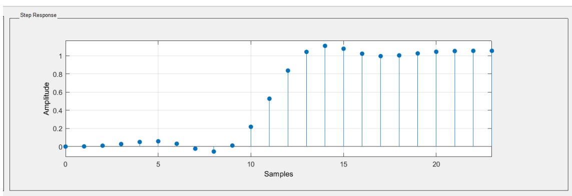
Specifications

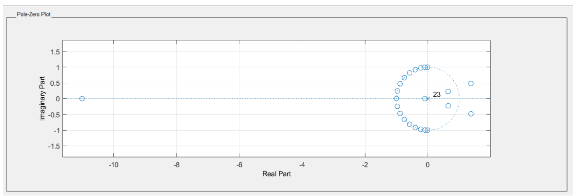
a low-pass FIR Equiripple filter

The passband attenuation of 0.5 dB, Stopband attenuation of 90 dB, normalized passband frequency of 0.25 normalized stopband frequency of 0.5

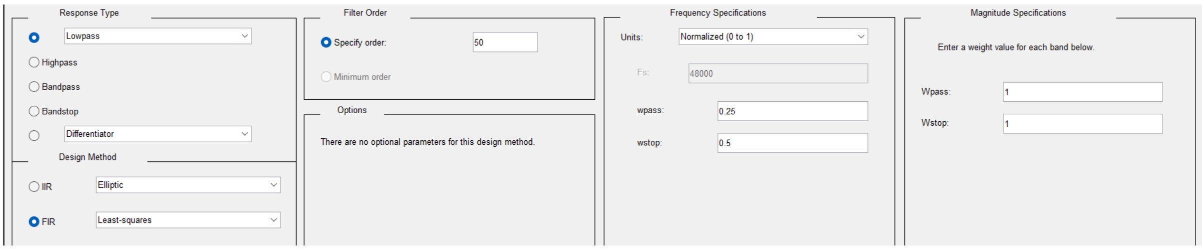








d)



Specifications