Lab Report - 4

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Team: Noicifiers

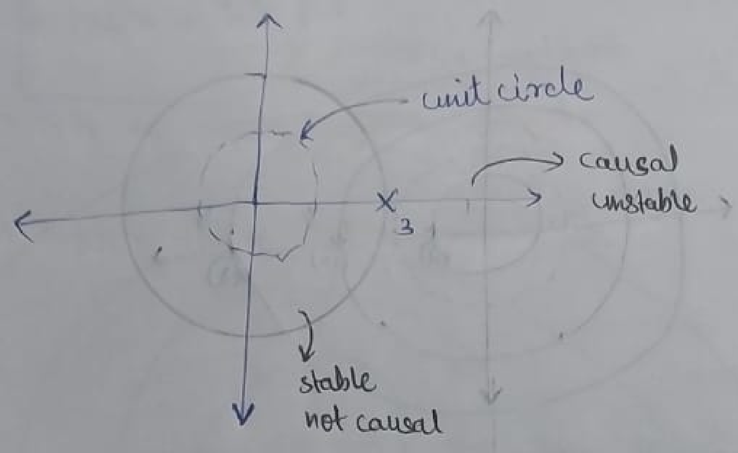
QUESTION 1a

```
function [N,ROC,C,S] = roc_cs(p)
    p = abs(p);
    p = unique(p,"sorted");
    if p(1) == 0
        N = length(p);
    else
        N = length(p)+1;
    end
    if p(1) == 0 && N == 1
        ROC(1,1) = 0;
        ROC(1,2) = Inf;
        C = 1;
        S = 1;
    else
         ROC = zeros(N,2);
        C = zeros(N,1);
        S = zeros(N,1);
        for k = 1:1:N
            if k == N
                if p(1) ~= 0
                    ROC(k,1) = p(k-1);
                    ROC(k,2) = Inf;
                elseif p(1) == 0
                    ROC(k,1) = p(k);
                    ROC(k,2) = Inf;
                end
                if ROC(k,1) < 1
                    S(k) = 1;
                end
            elseif  k==1
                if p(1) ~= 0
                    ROC(k,2) = p(k);
                elseif p(1) == 0
                    ROC(k,2) = p(k+1);
                end
                if ROC(k,2) > 1
                    S(k) = 1;
                end
            else
                if p(1) ~= 0
                    ROC(k,1) = p(k-1);
                    ROC(k,2) = p(k);
```

```
                elseif p(1) == 0
                    ROC(k,1) = p(k);
                    ROC(k,2) = p(k+1);
                end
                    if (ROC(k,1)<1) && (1<ROC(k,2))
                        S(k) = 1;
                    end
            end
            C(N) = 1;
        end
    end
end
```

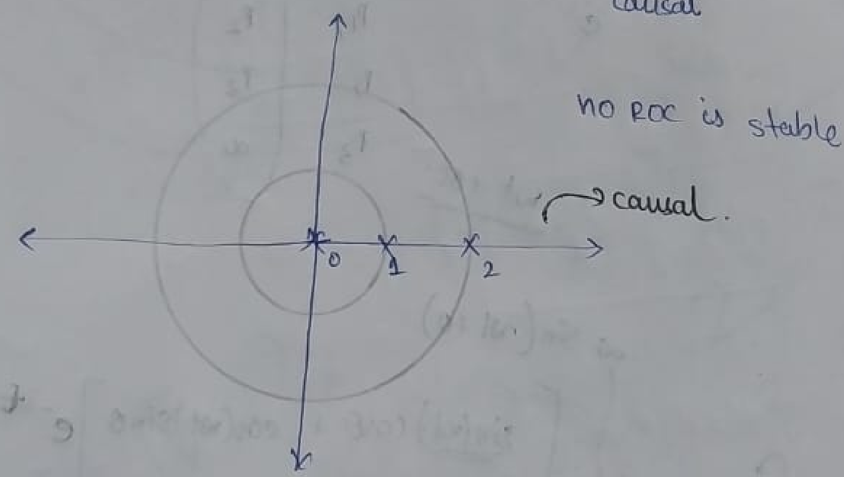
QUESTION 1b

$P > 3$ ⇒ ROC : $(0,3)$, $(3,\infty)$

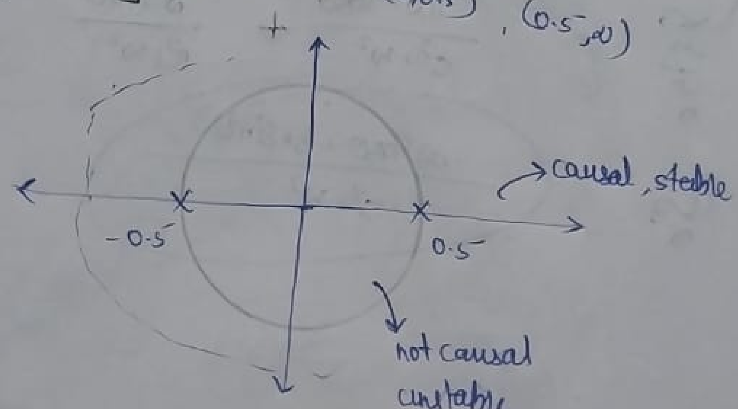


$P = [0,1,2]$ ⇒ ROC : $(0,1)$ $(1,2)$ $(2,\infty)$

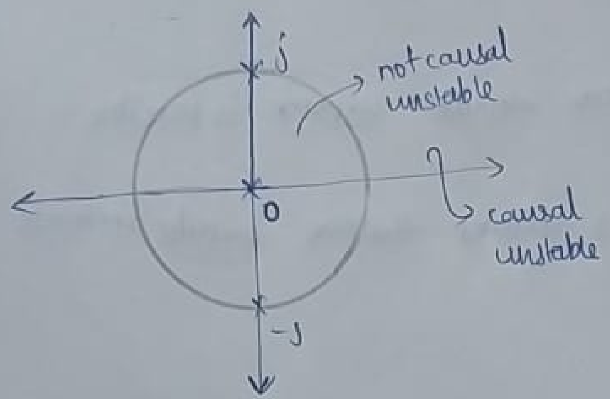
causal



$P = [0.5; -0.5]$ ⇒ ROC : $(0,0.5)$, $(0.5,\infty)$



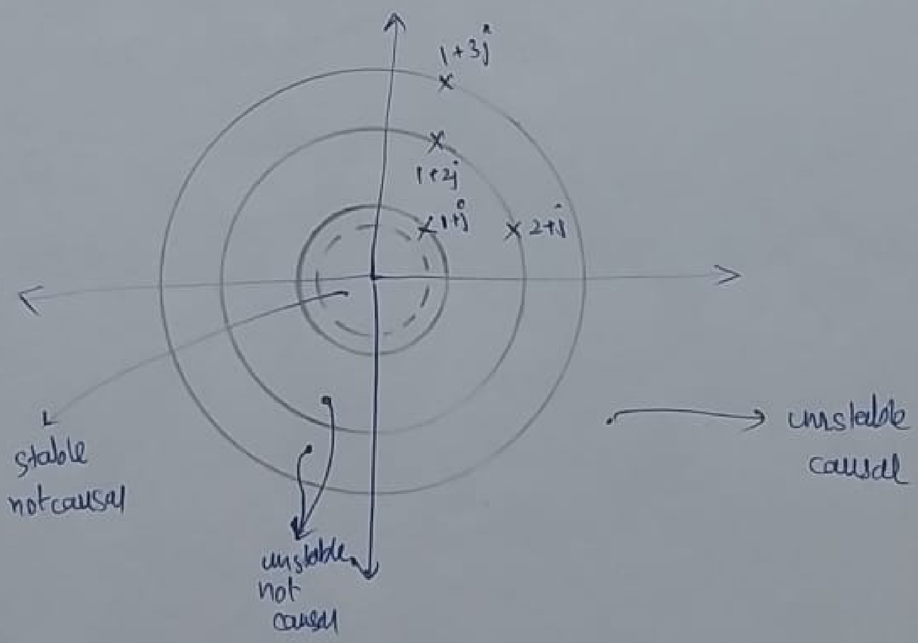
$P = [0, j, -j] \Rightarrow$ ROC: $(0,1)$, $(1,\infty)$



$\rightarrow\sqrt{2} \quad \rightarrow\sqrt{5} \quad \rightarrow\sqrt{10} \quad \rightarrow\sqrt{5}$

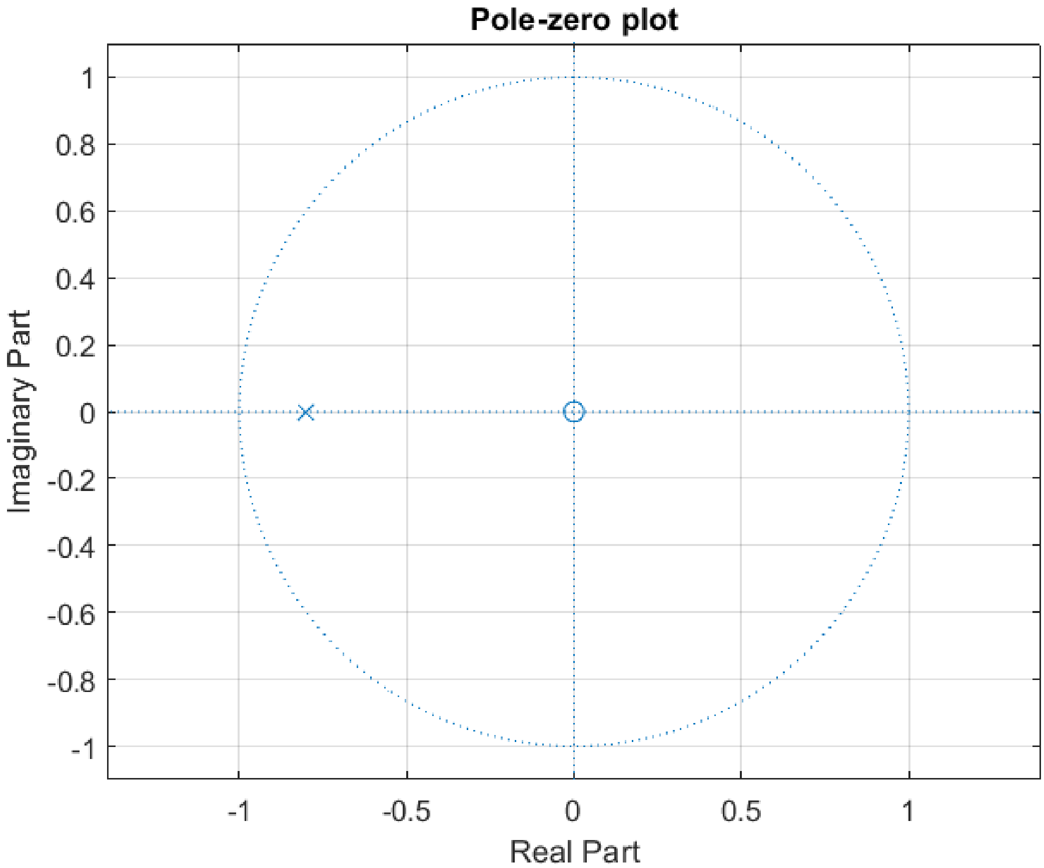
$P = [1+j, 2+j, 1+3j, 2+j]$

$\Rightarrow$ ROC: $(0, \sqrt{2})$, $(\sqrt{2}, \sqrt{5})$, $(\sqrt{5}, \sqrt{10})$, $(\sqrt{10}, \infty)$

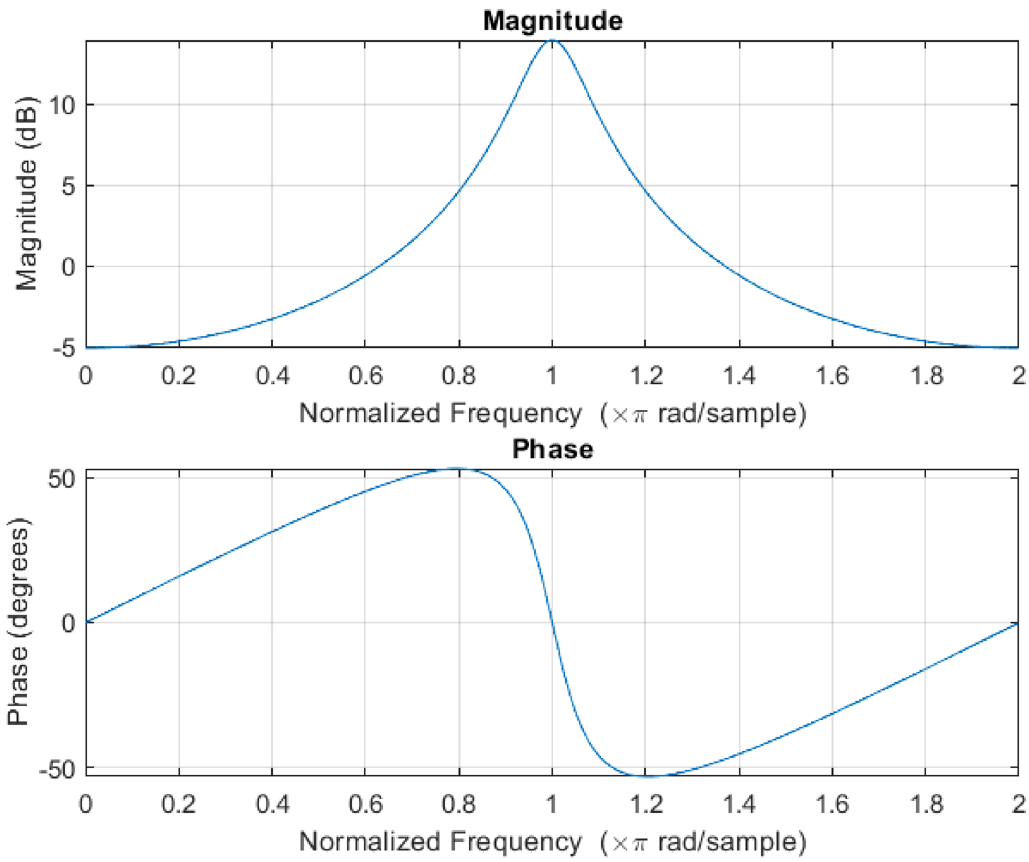


QUESTION 2a

$$\begin{gathered} Given, \\ H(z) = \frac{z}{z+p} = \frac{1}{1+pz^{-1}} \\ p \in (-1, 1) \\ p = 0.8 \end{gathered}$$

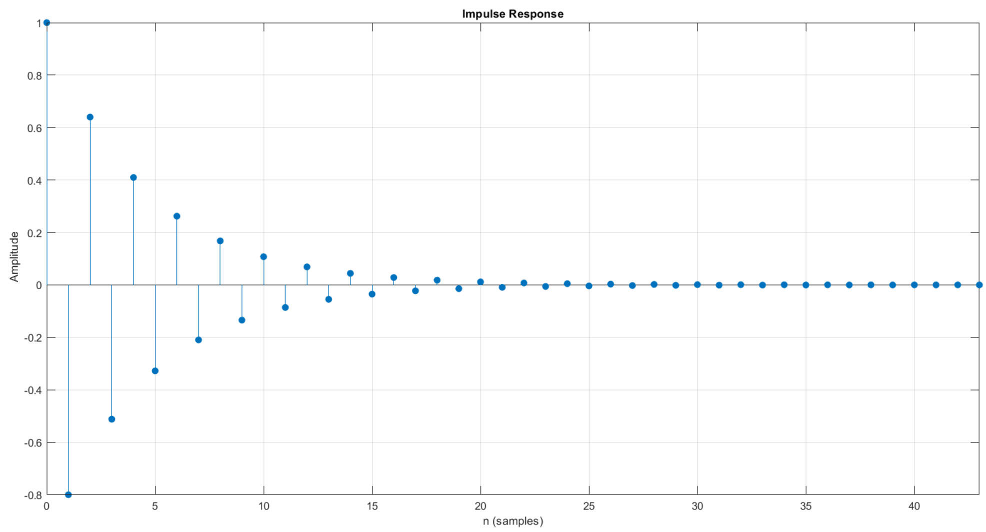


QUESTION 2b



Frequency response plot

QUESTION 2c



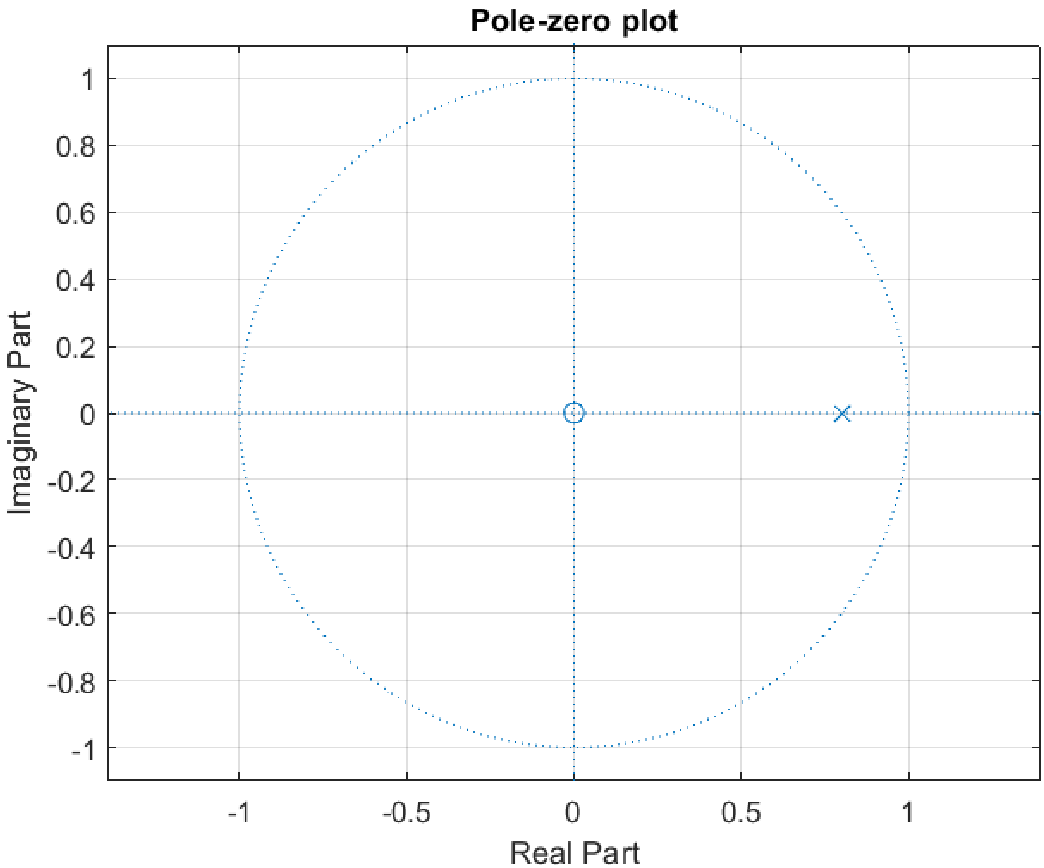
For linear systems, there will be only one input-output pair.

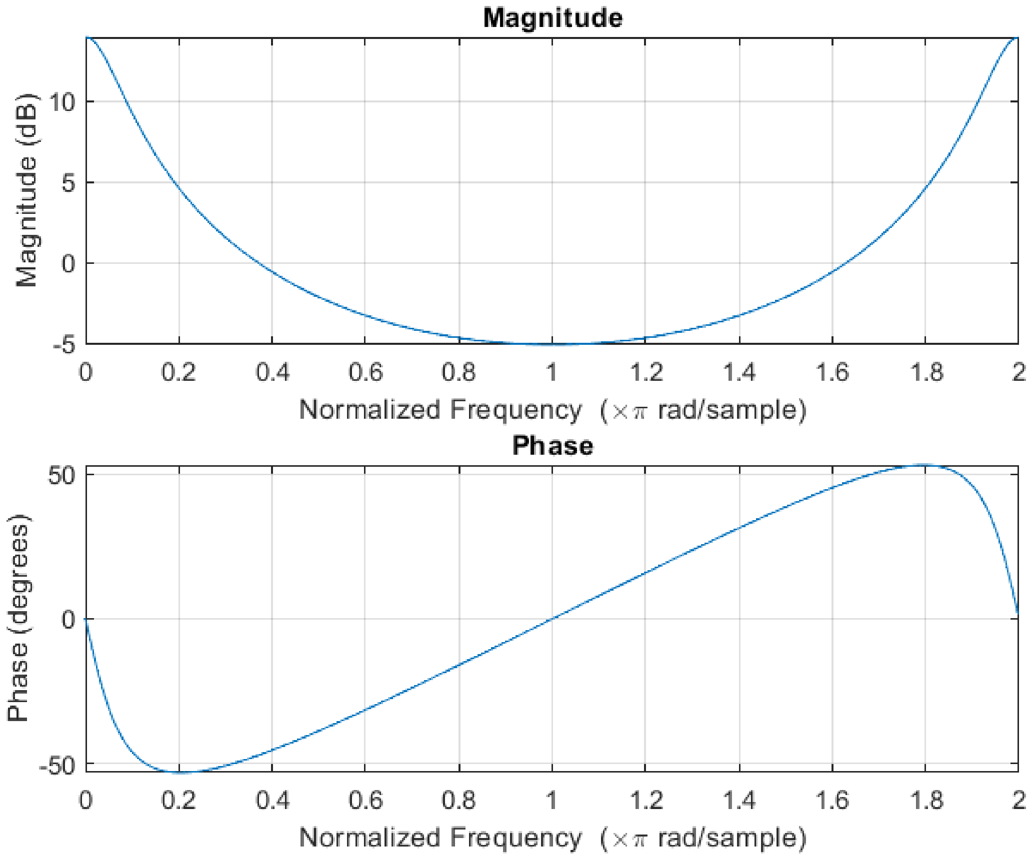
⇒ Only one Impulse response is possible.

⇒ That single Impulse response is returned by the impz() function.

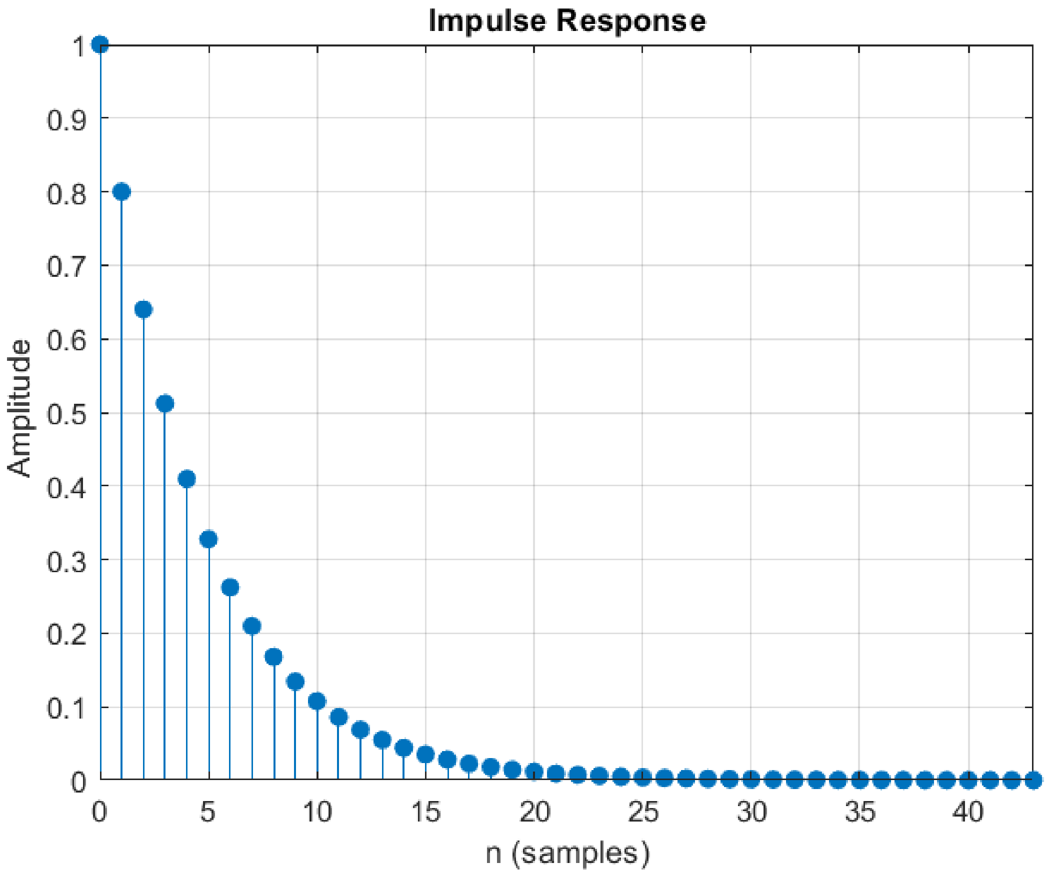
QUESTION 2d

FOR p = -0.8:

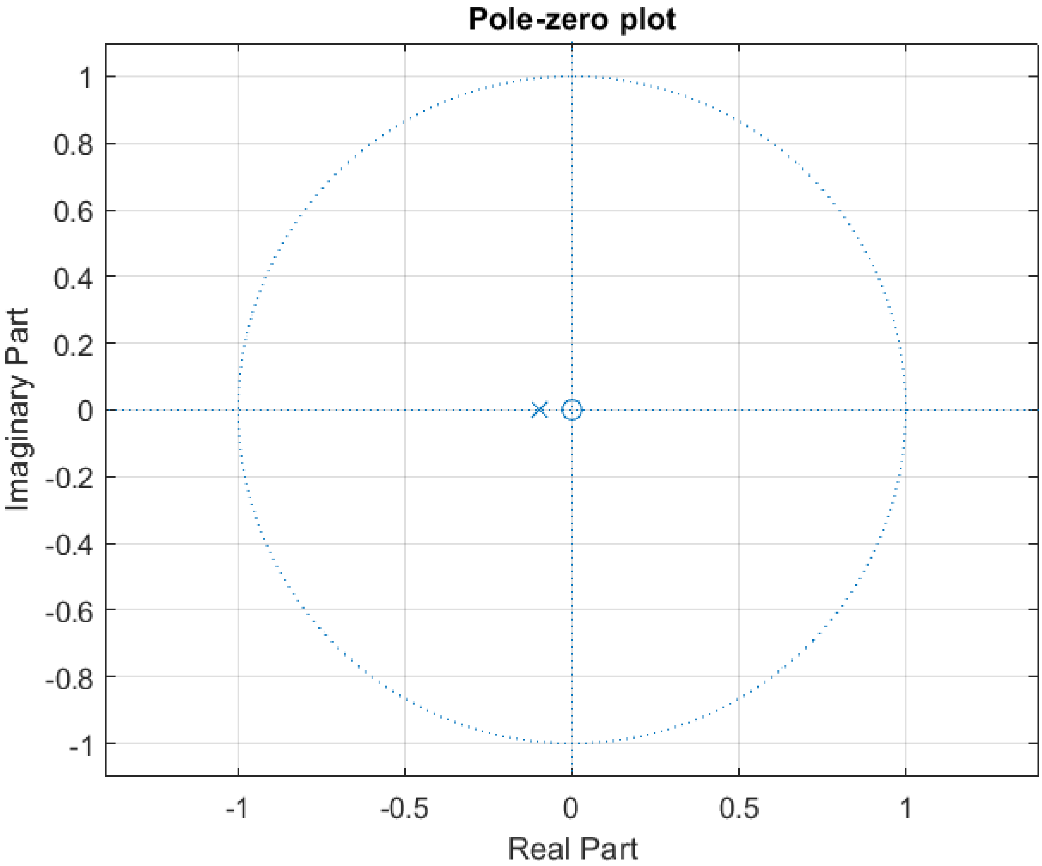


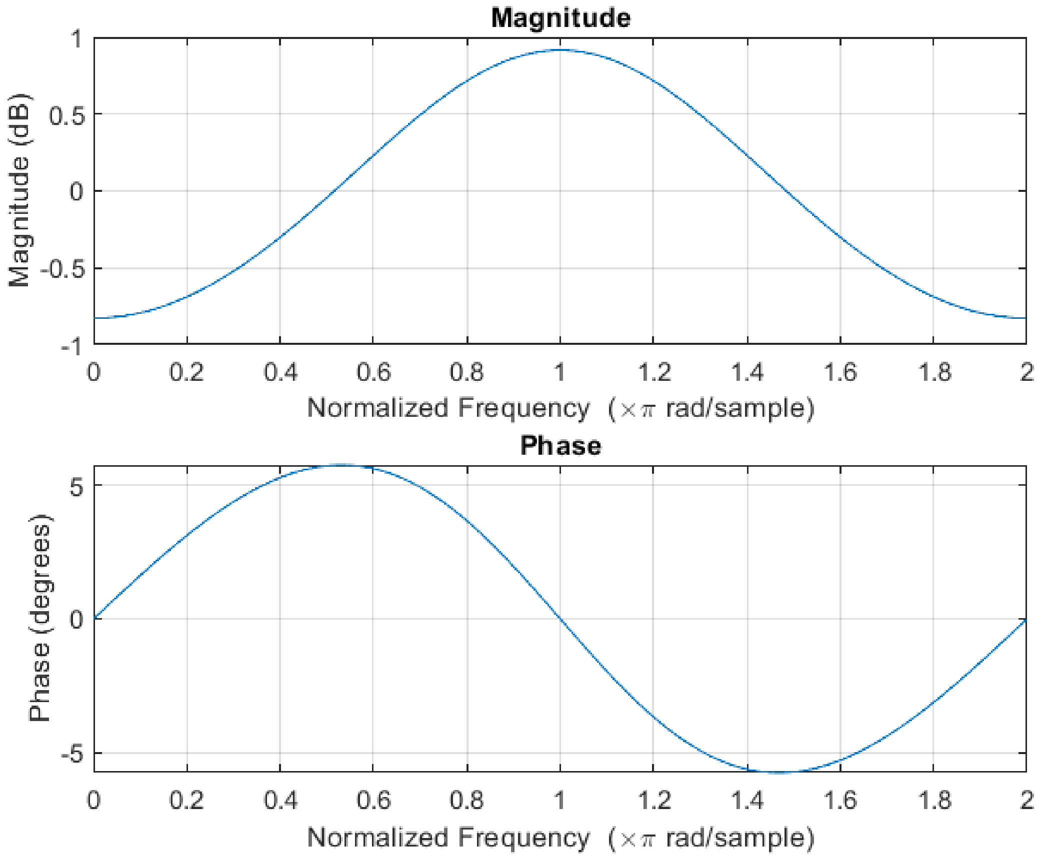


Frequency Response Plot

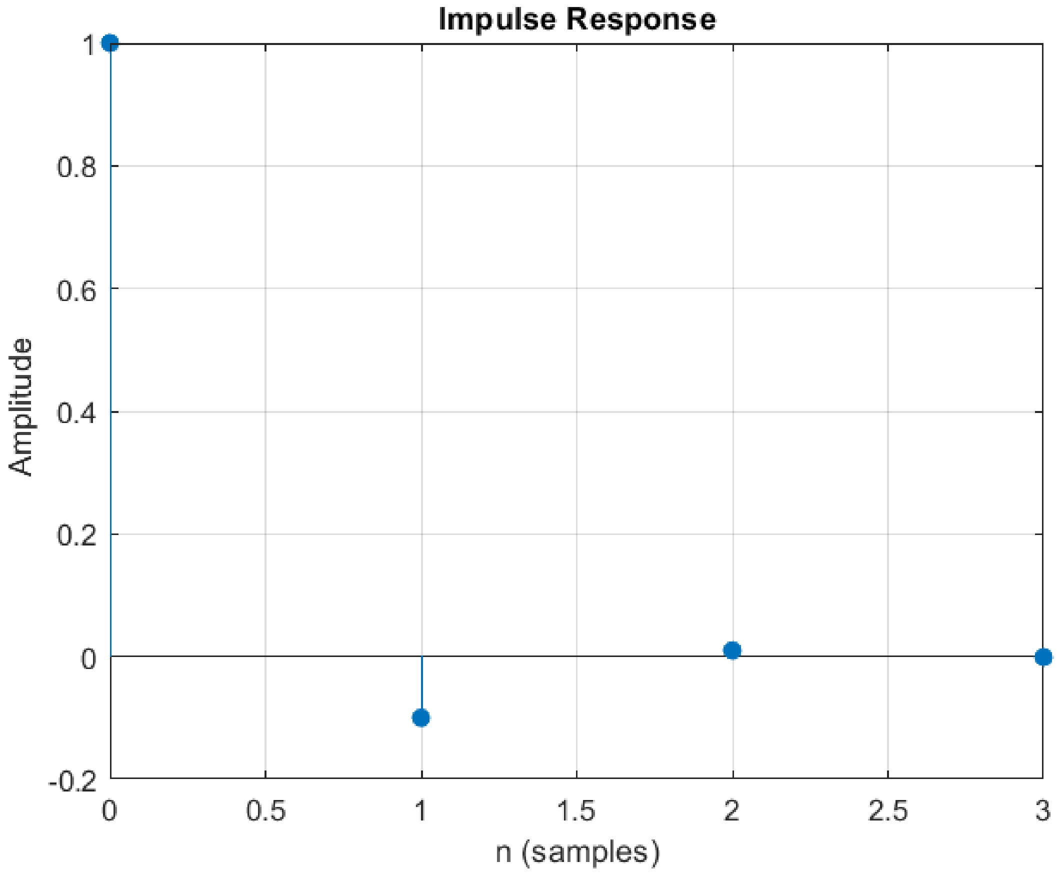

Impulse Response
Amplitude
n (samples)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40

FOR p = 0.1:


Pole-zero plot
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

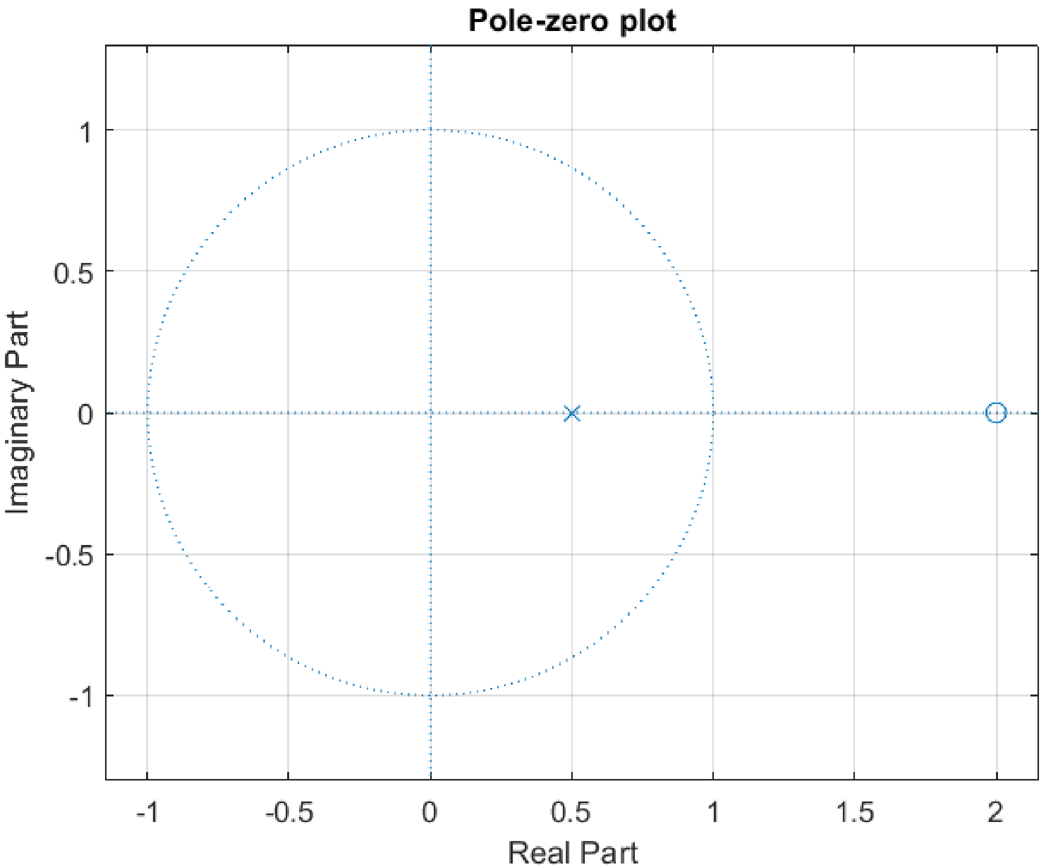


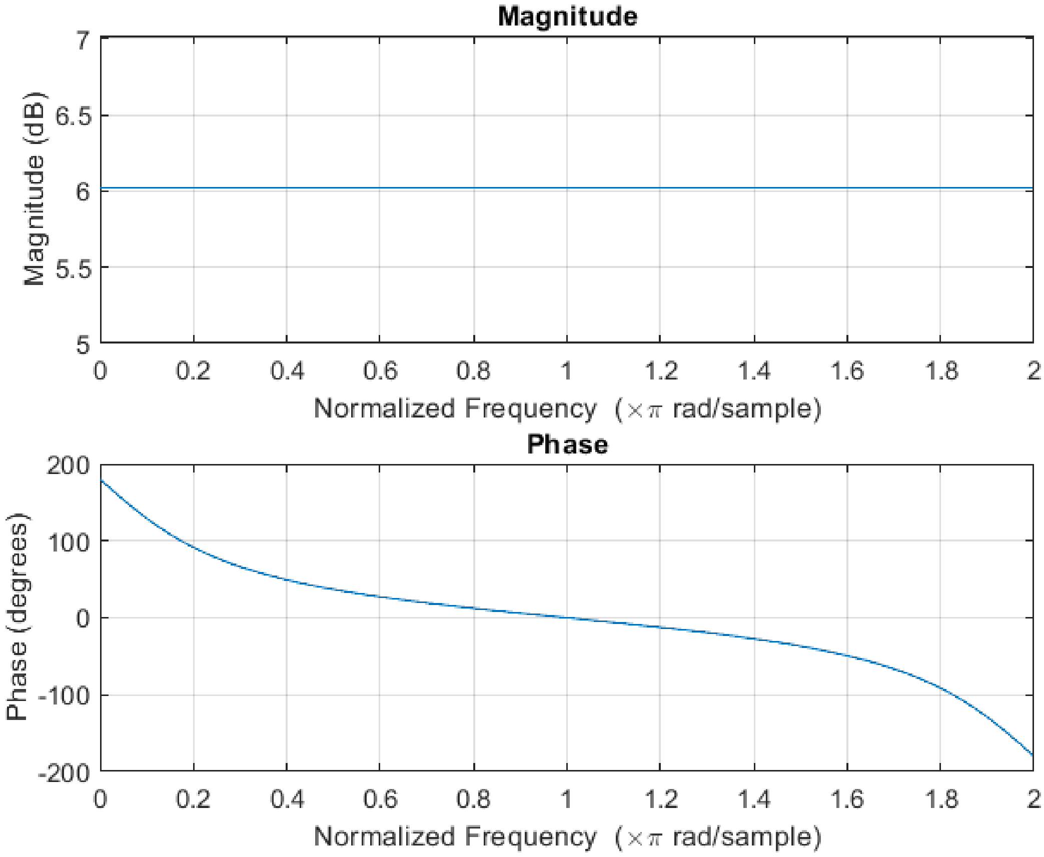
Frequency Response Plot



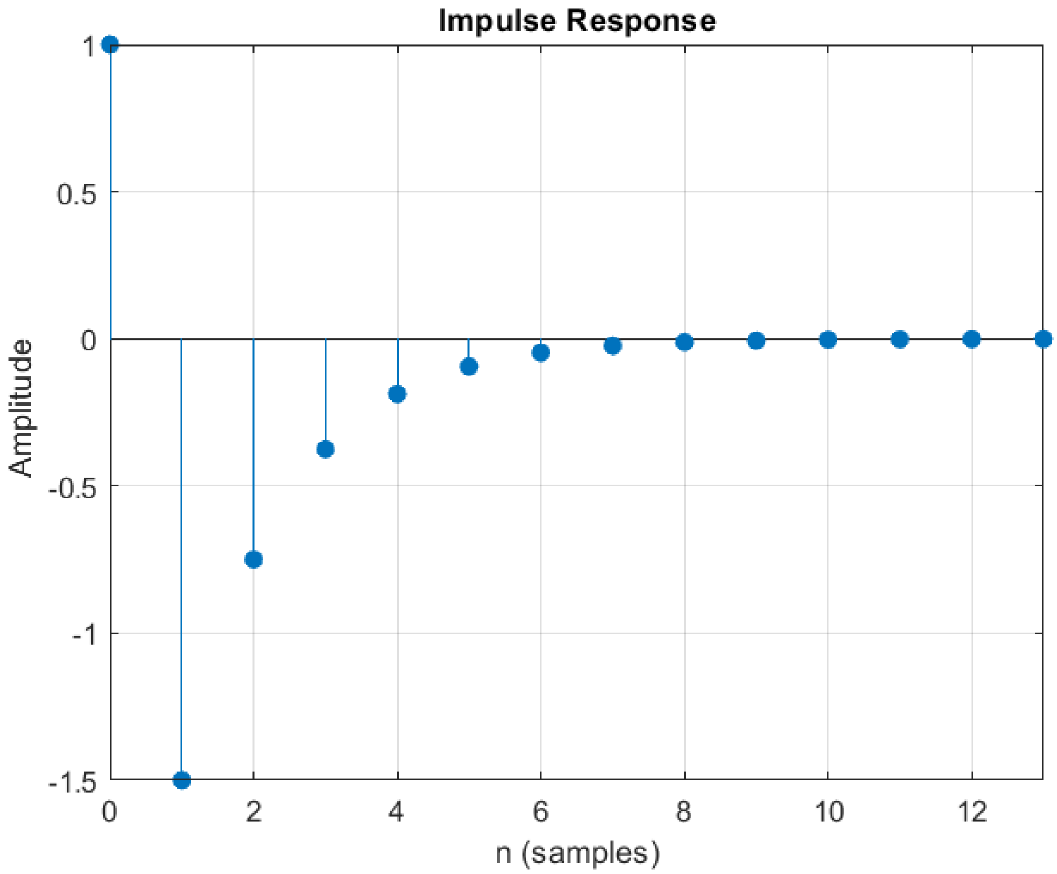
QUESTION 2e

$$Given,$$
$$H(z) = \frac{z - p^{-1}}{z - p}, p \in (0, 1)$$
$$p = 0.5$$


Pole-zero plot
Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
1.5
2



Frequency response plot



Dependence of Impulse response on poles

- The magnitude of poles < 1 and ROC is outside then the Impulse response Converges
- The magnitude of poles > 1 and ROC is outside then the Impulse response Diverges
- The magnitude of poles > 1 and ROC is inside then the Impulse response Converges
- The magnitude of poles < 1 and ROC is inside then the Impulse response Diverges

Dependence of Frequency Response on Poles:

- Peaks(Maxima) in the Frequency response correspond to the poles of the system
- Poles closer to the unit circle have sharp peaks
- Minimas in the frequency response correspond to the zeros of the system

QUESTION 3a

$$H(z) = \frac{z^2 - 2cos(\theta)z + 1}{z^2 - 2rcos(\theta)z + r^2};$$
$$r \in (0,1); \theta \in [0,\pi]$$

Finding poles and zeros of the function:

$$H(z) = \frac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta)z + r^2}$$

zeros:-

$$z^2 - (2\cos\theta)z + 1 = 0$$

$$z = \frac{\cancel{2}\cos\theta \pm \sqrt{\cancel{4}\cos^2\theta - \cancel{4}}}{\cancel{2}}$$

$$= \cos\theta \pm i\sin\theta$$

$$\Rightarrow \boxed{z = e^{\pm i\theta}}$$

poles:-

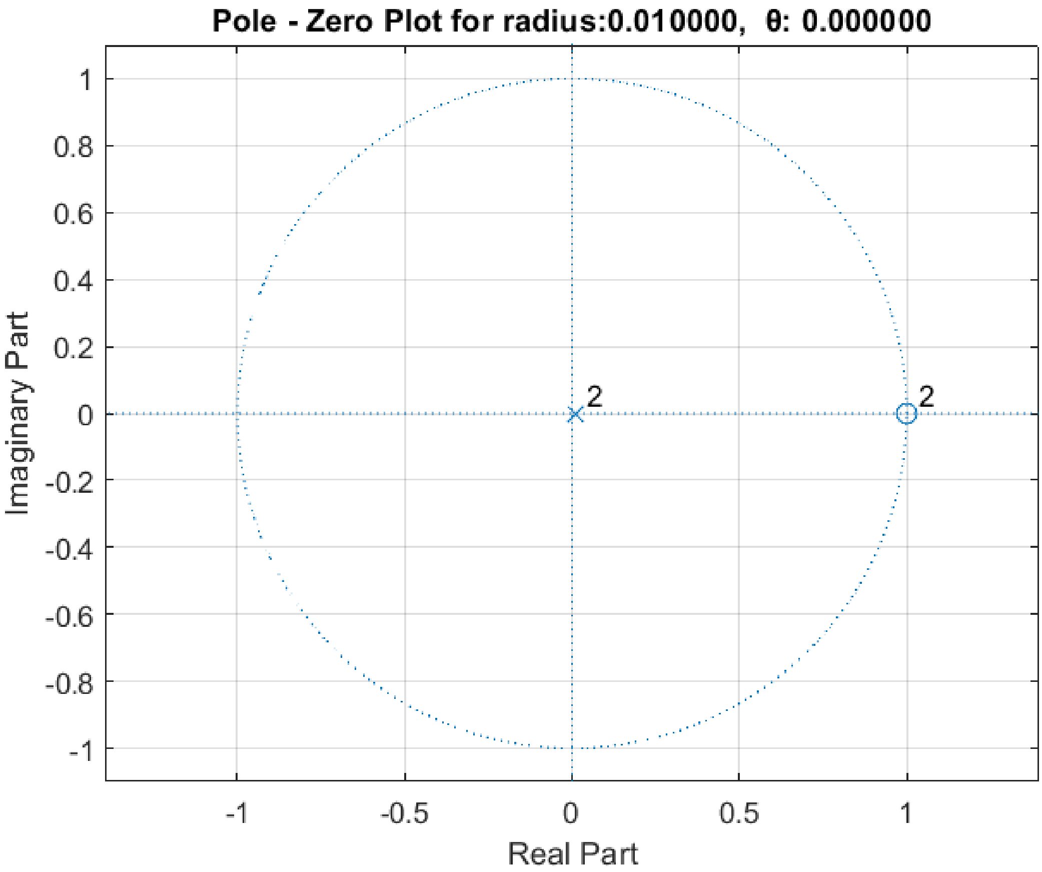
$$z^2 - (2r\cos\theta)z + r^2 = 0$$

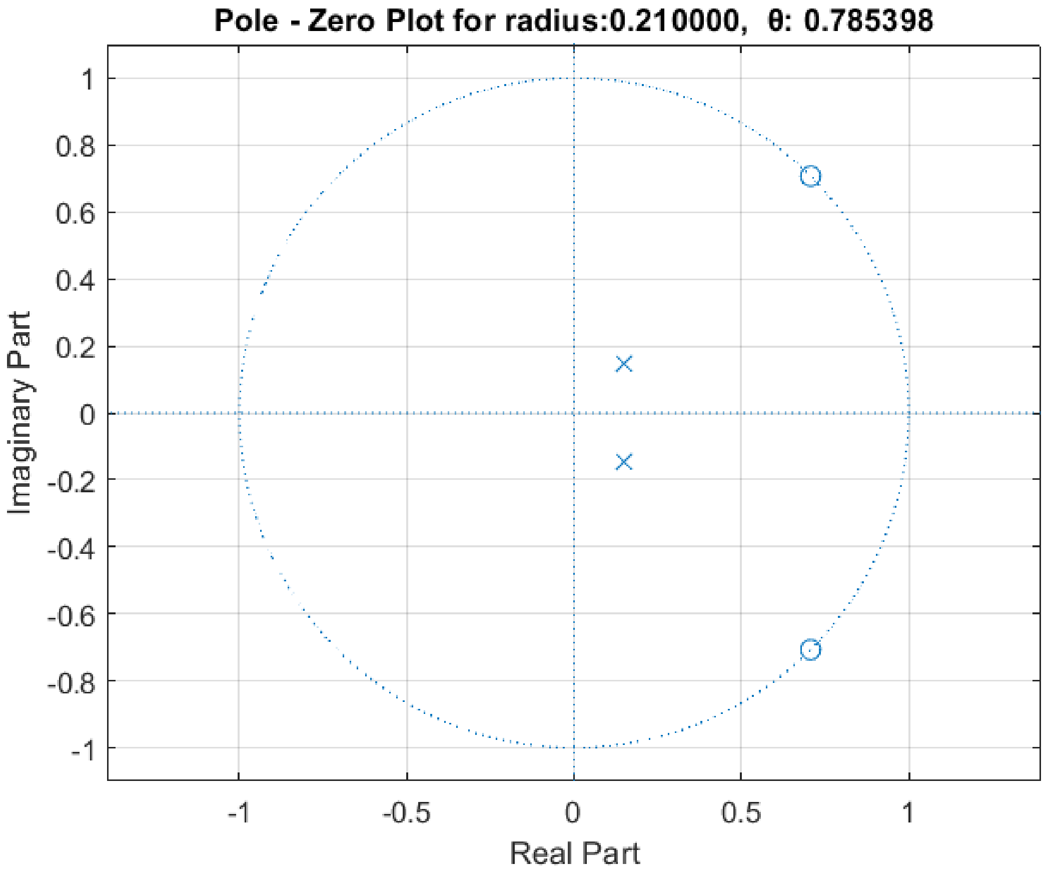
$$\Rightarrow z = \frac{\cancel{2}r\cos\theta \pm \sqrt{\cancel{4}r^2\cos^2\theta - \cancel{4}r^2}}{\cancel{2}}$$

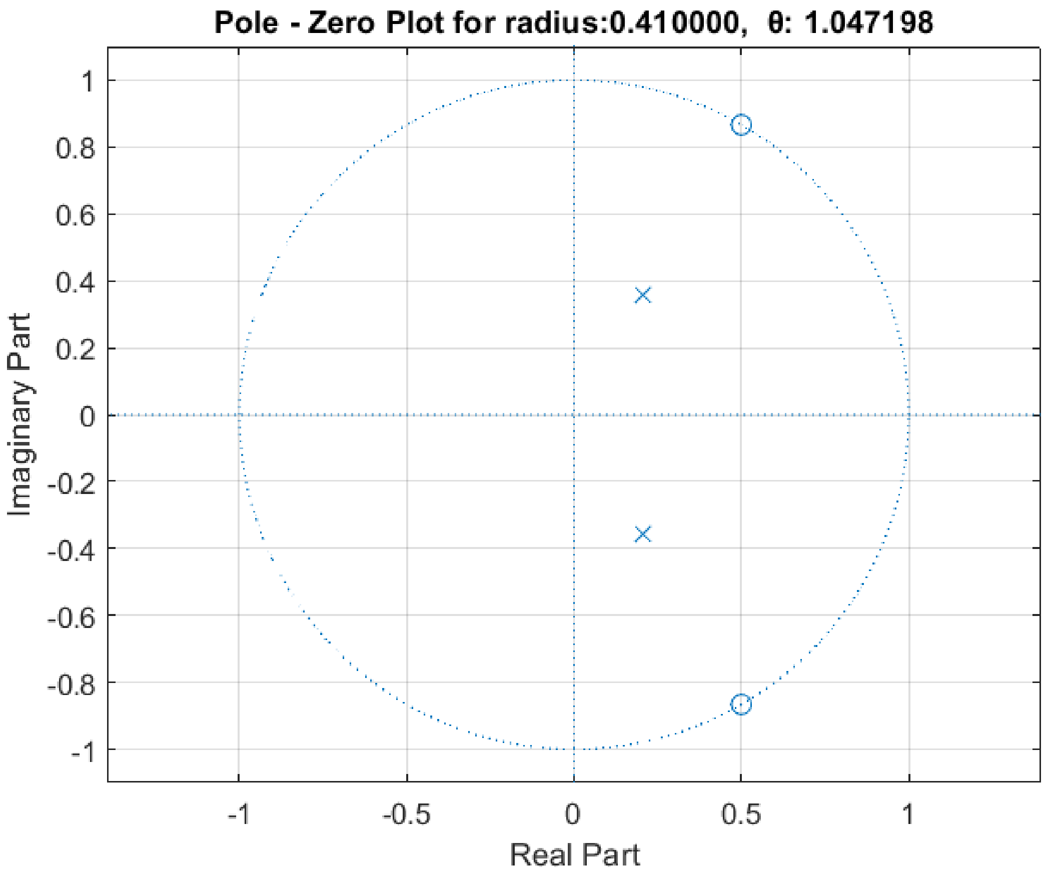
$$= r\cos\theta \pm i r\sin\theta$$

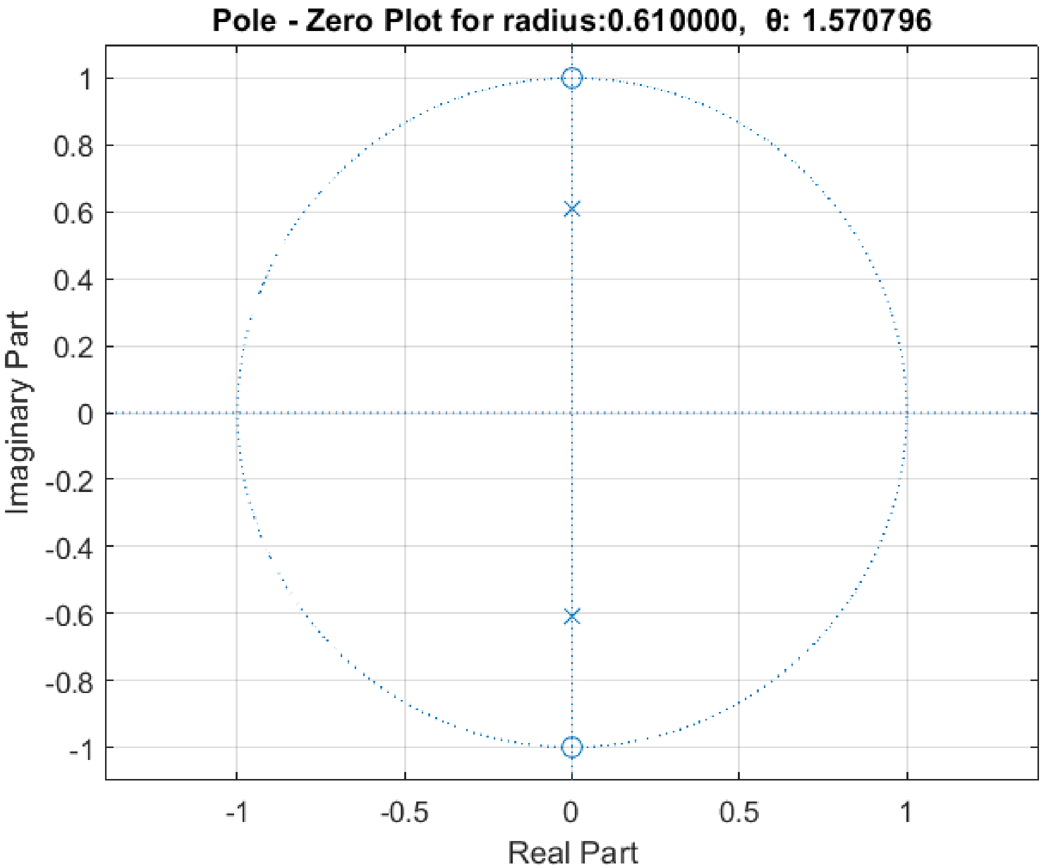
$$\boxed{z = re^{\pm i\theta}}$$

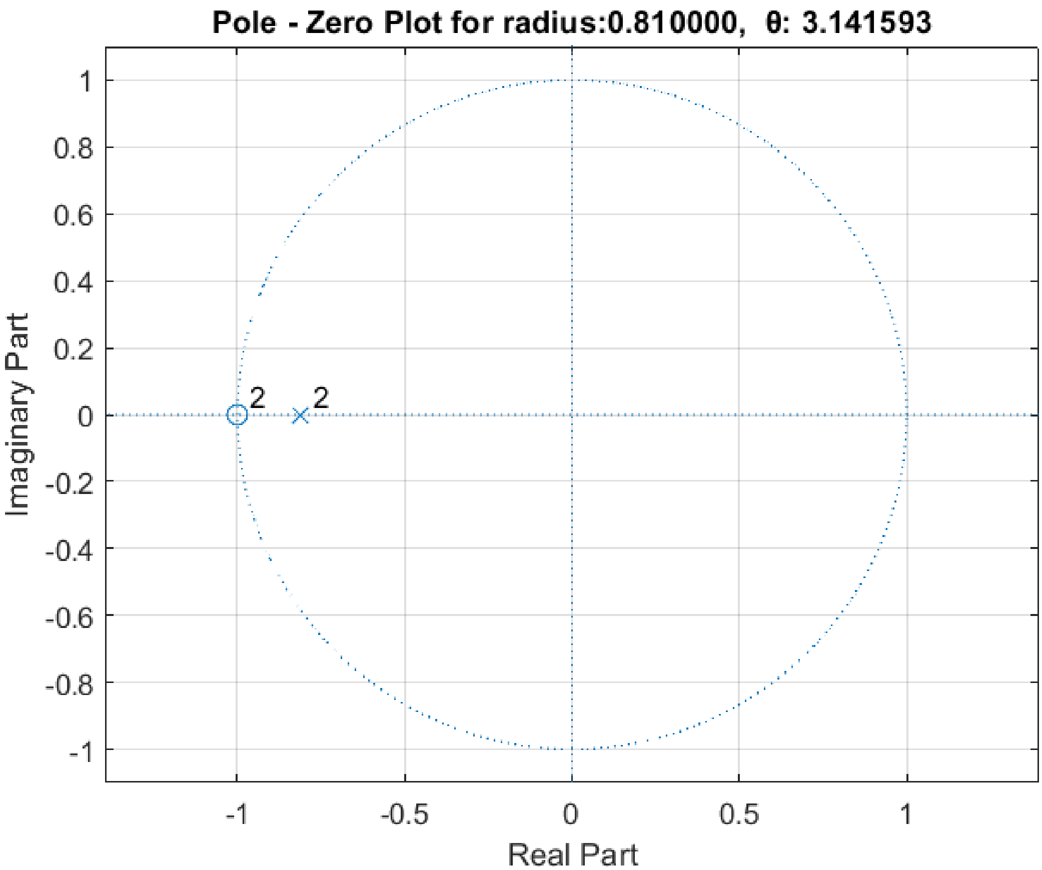
Pole -Zero Plots for Varying radius and phase:




Pole - Zero Plot for radius:0.210000, θ: 0.785398
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


Pole - Zero Plot for radius:0.410000, θ: 1.047198
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


Pole - Zero Plot for radius:0.610000, θ: 1.570796
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


Pole - Zero Plot for radius:0.810000, θ: 3.141593
Imaginary Part
Real Part
2
2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

QUESTION 3b

$$\frac{(z - e^{i\theta})(z - e^{-i\theta})}{(z - re^{i\theta})(z - re^{-i\theta})}$$

$$\frac{(1 - z^{-1}e^{i\theta})(1 - z^{-1}e^{-i\theta})}{(1 - rz^{-1}e^{i\theta})(1 - rz^{-1}e^{-i\theta})}$$

$$|rz^{-1}e^{i\theta}| < 1 \qquad |rz^{-1}e^{-i\theta}| < 1$$

$$r|z^{-1}| < 1 \qquad r|z^{-1}| < 1$$

$$\boxed{|z| > r} \qquad \boxed{|z| > r}$$

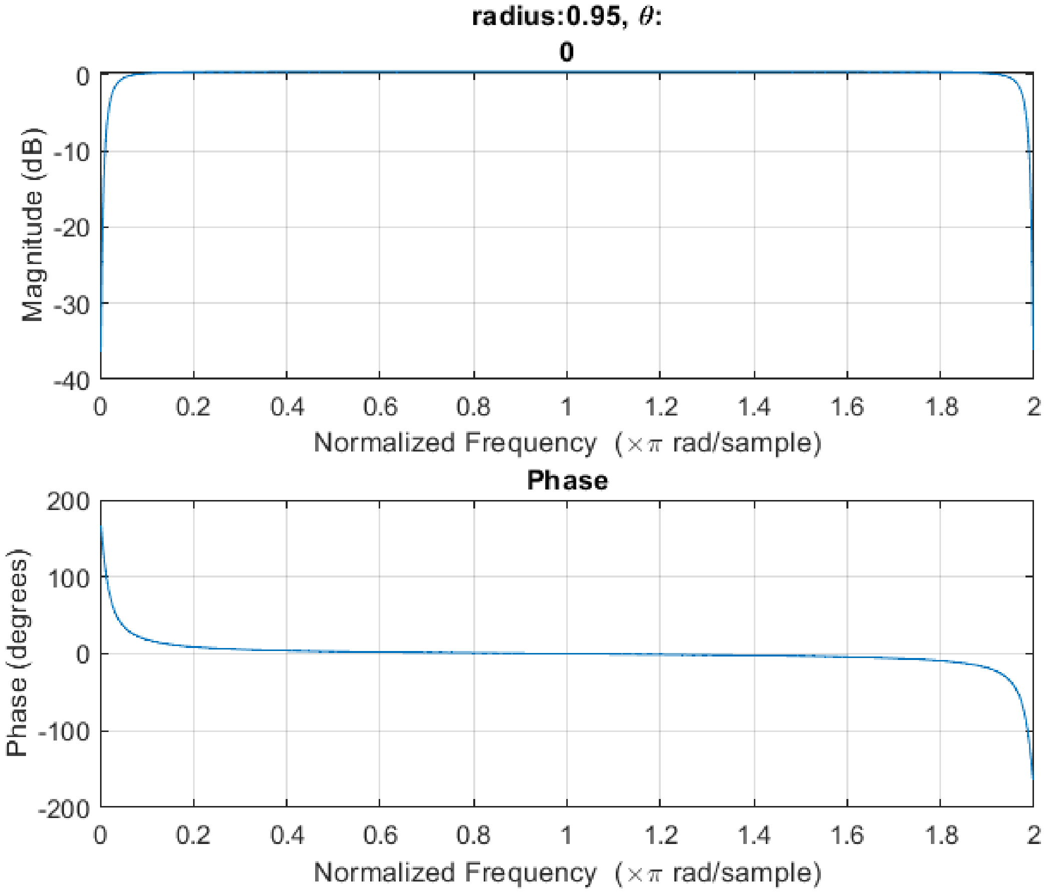
$$\boxed{\text{ROC}: |z| > r}$$

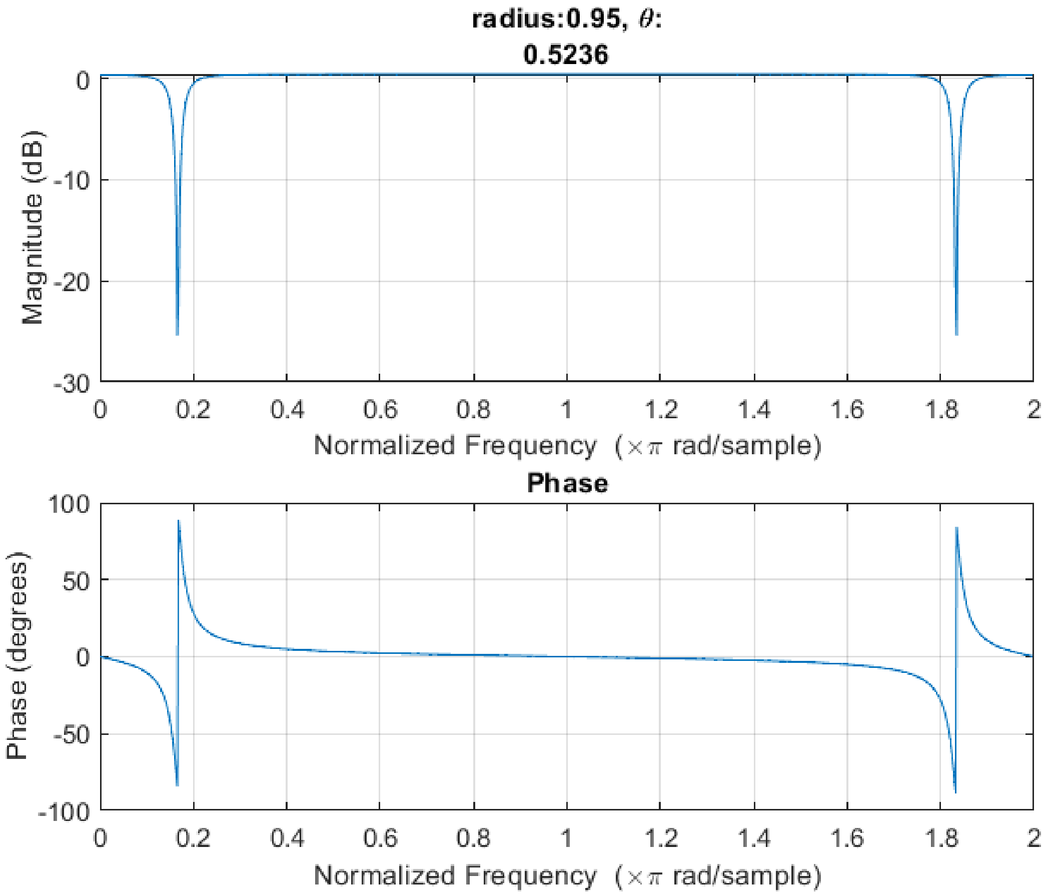
$\therefore$ for the system to be both stable and causal,

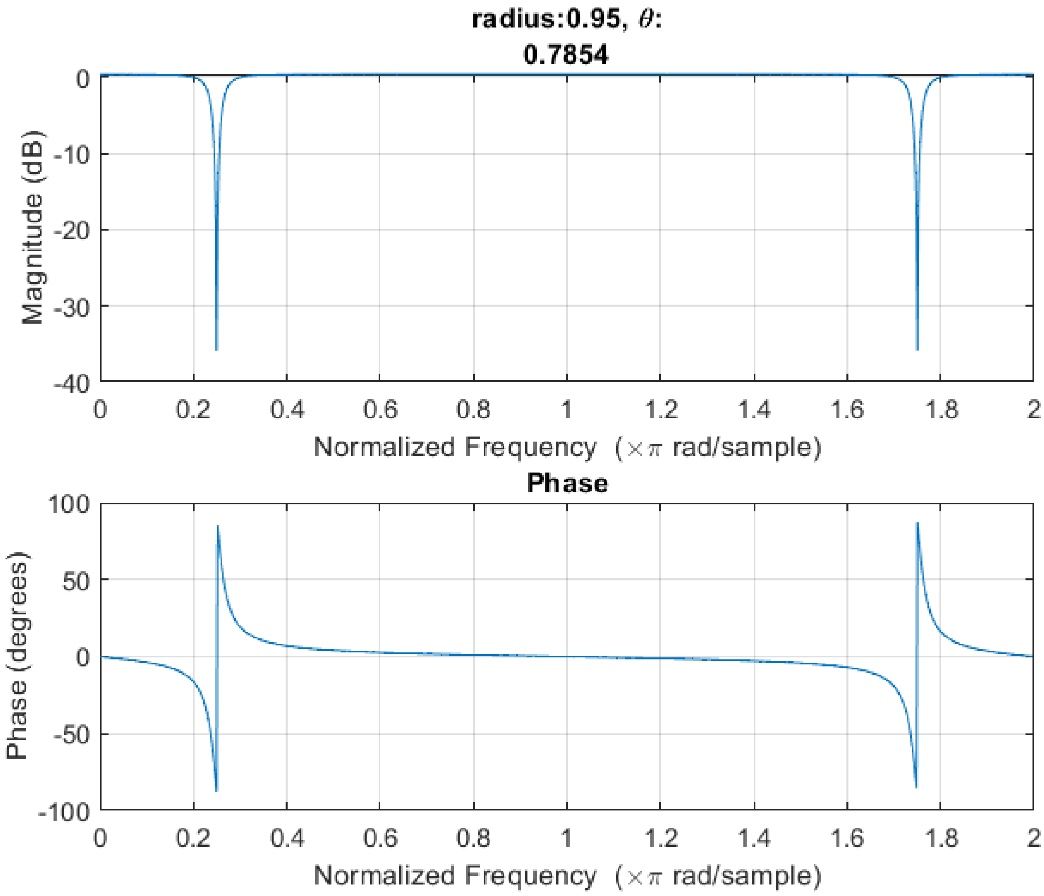
$$\boxed{0 < r < 1}$$

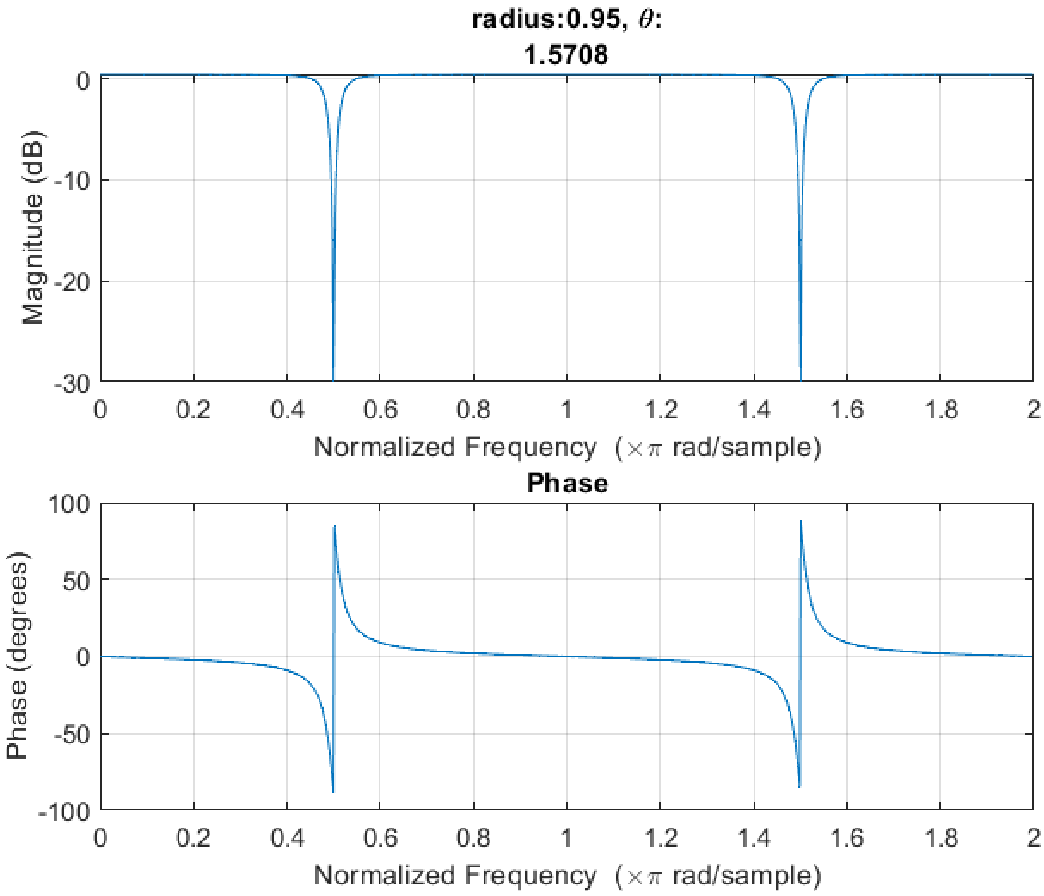
QUESTION 3c

Frequency response plots for varying θ:

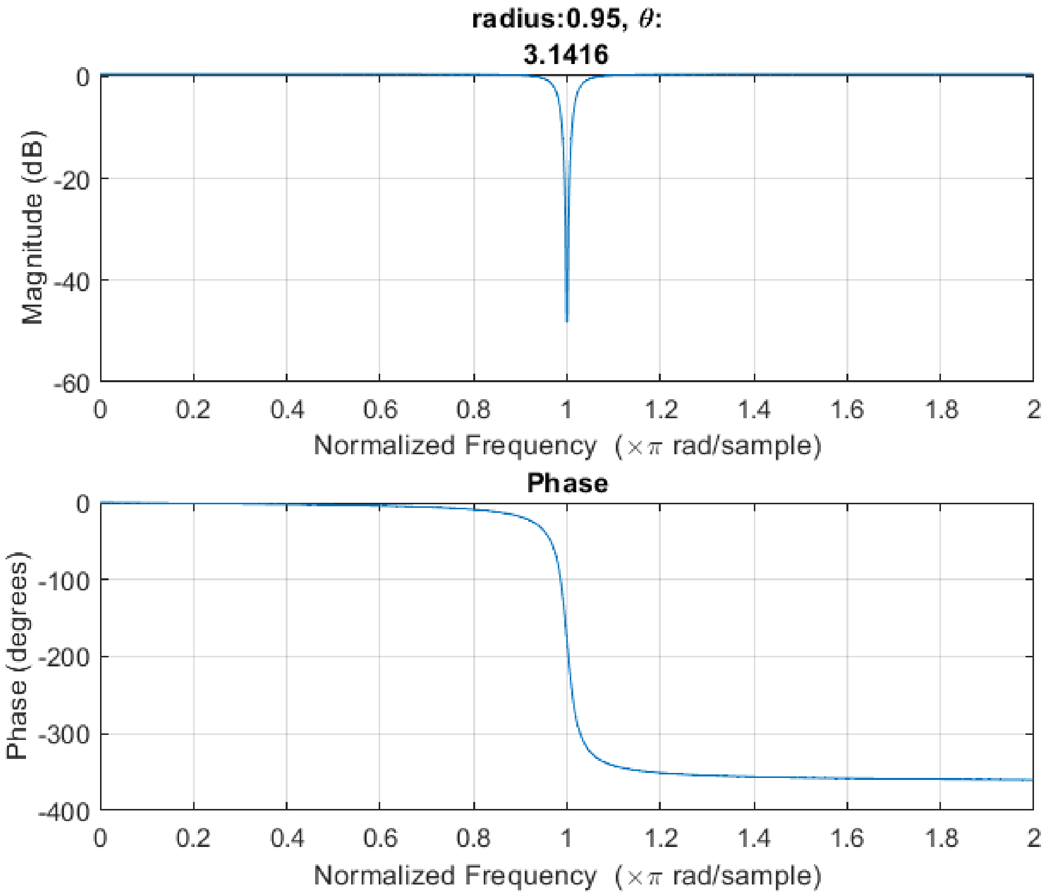



radius:0.95, θ:
0.5236
Magnitude (dB)
Normalized Frequency ($\times\pi$ rad/sample)
Phase
Phase (degrees)
Normalized Frequency ($\times\pi$ rad/sample)

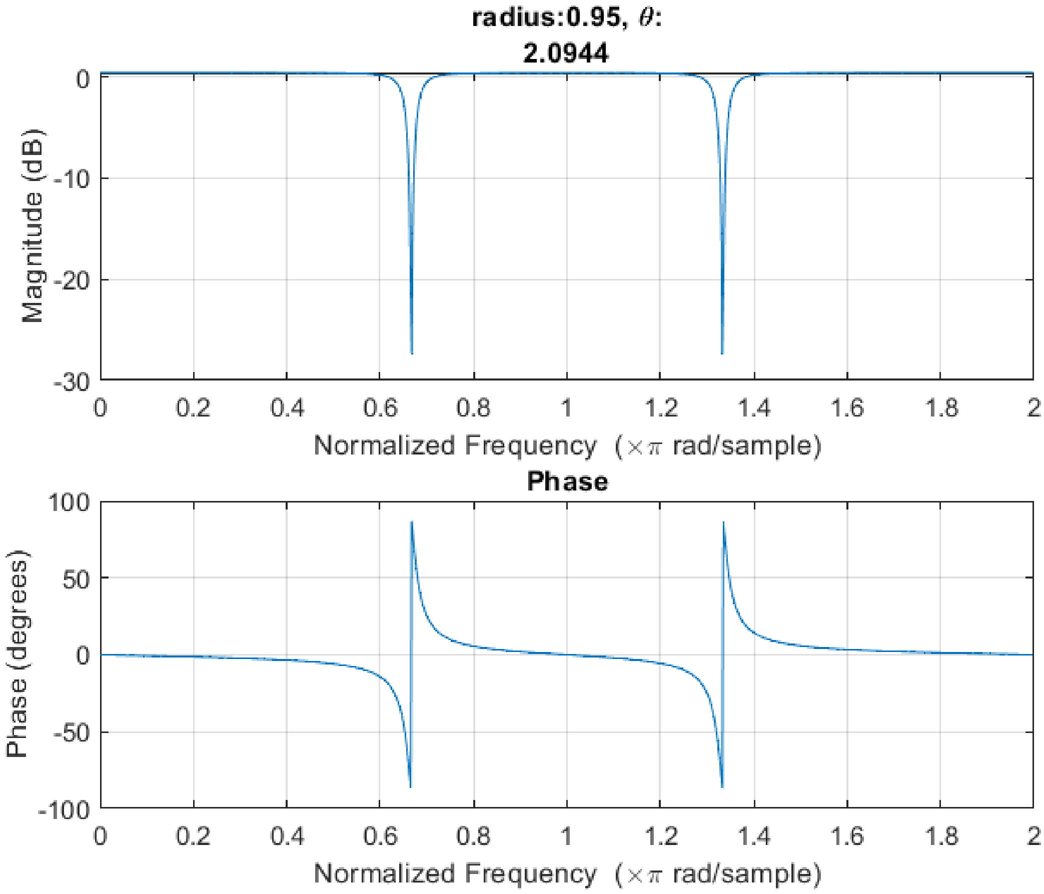

radius:0.95, θ:
0.7854
Magnitude (dB)
Normalized Frequency ($\times\pi$ rad/sample)
Phase
Phase (degrees)
Normalized Frequency ($\times\pi$ rad/sample)



radius:0.95, θ: 1.5708
Magnitude (dB)
0
-10
-20
-30
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)



radius:0.95, θ: 3.1416
Magnitude (dB)
0
-20
-40
-60
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
0
-100
-200
-300
-400
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)



For a fixed radius and varying **θ,**

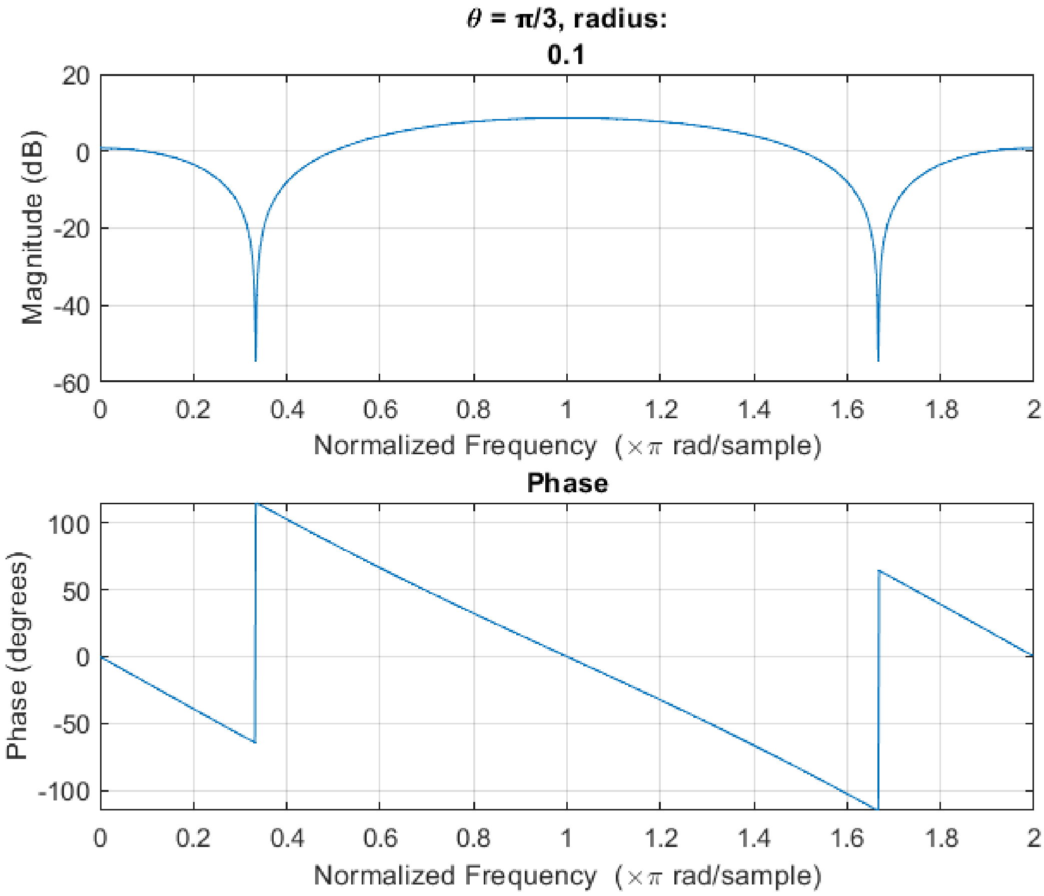
Zeros always line on the circle of radius 1 but phase changes with the given θ, i.e., +θ for one zero and -θ for another zero.

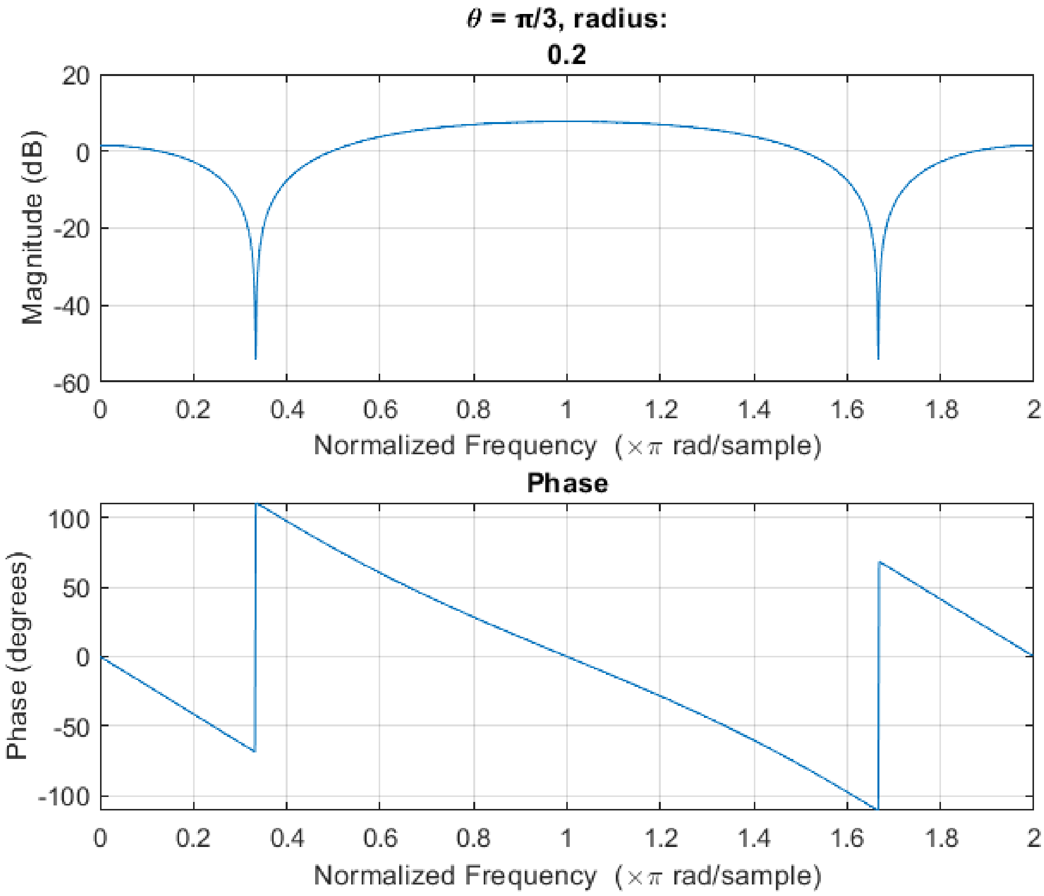
Poles always line on the circle of radius 0.95 (since the given r value is 0.95) but phase changes with the given θ, i.e., +θ for one pole and -θ for another pole.

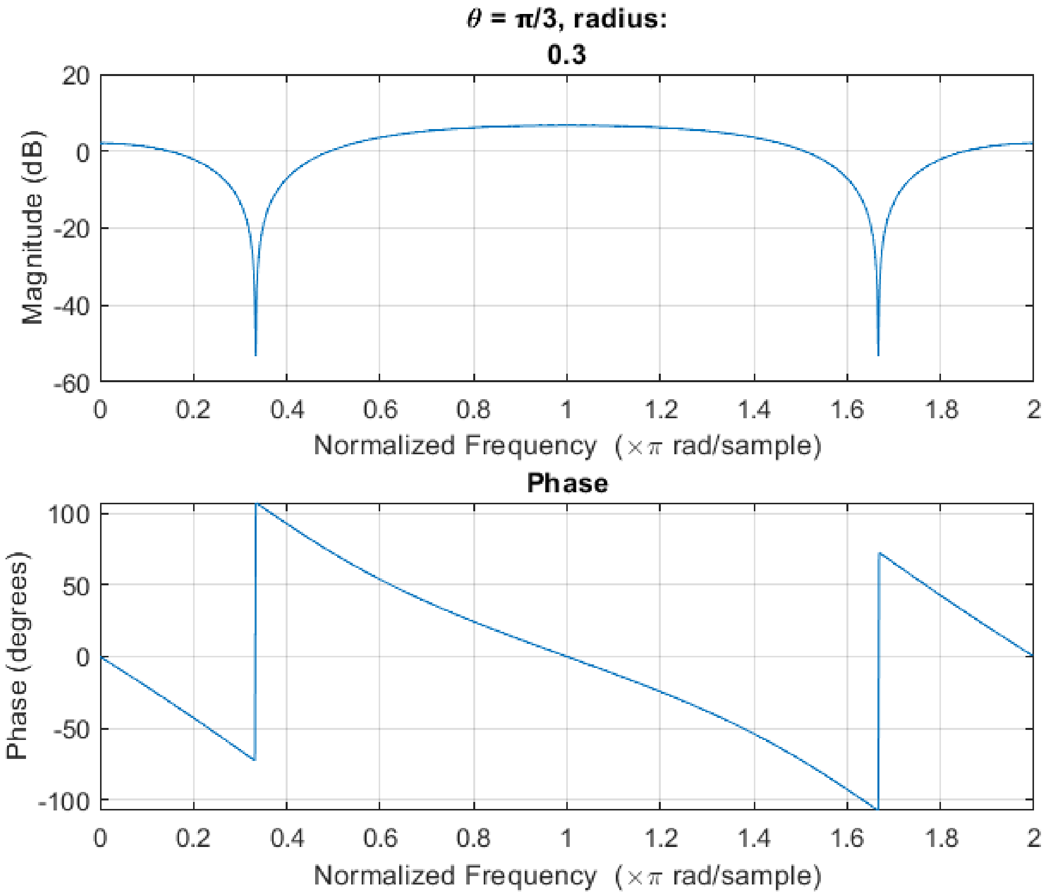
In the frequency response plot, there will be a discontinuity at the given theta.

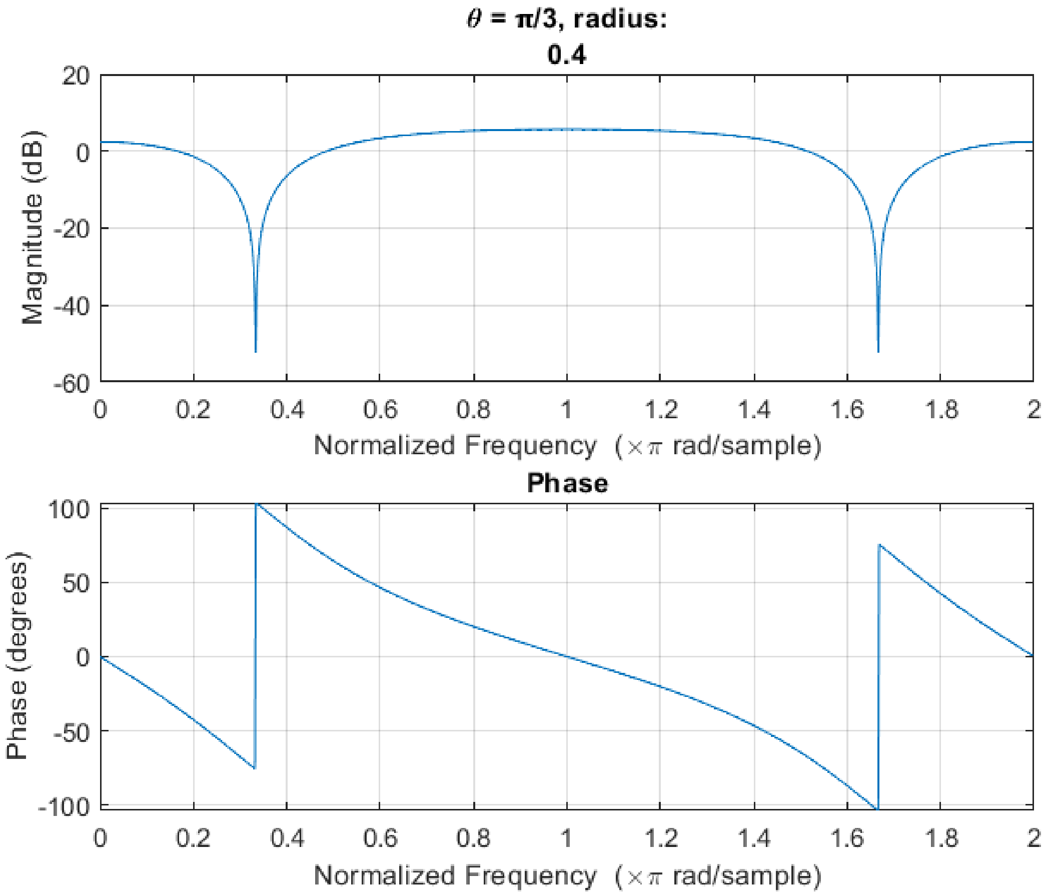
QUESTION 3d

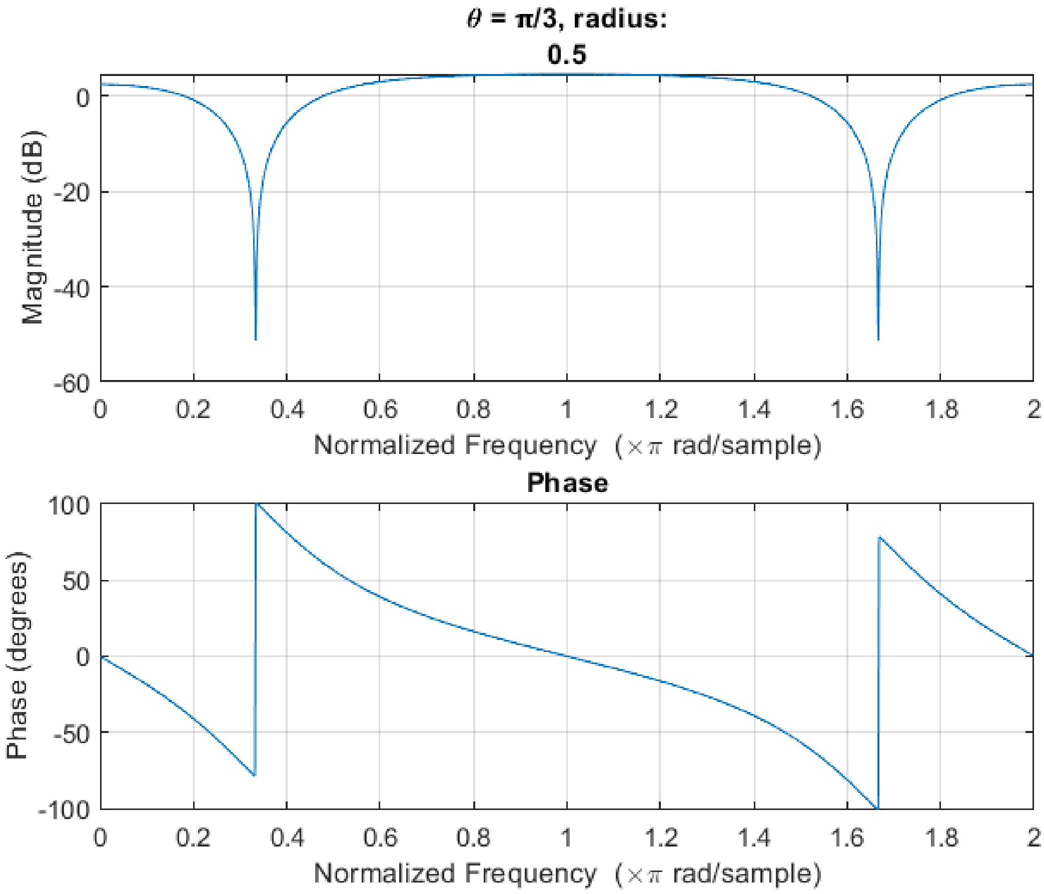
Frequency Response plots for varying radius(r):

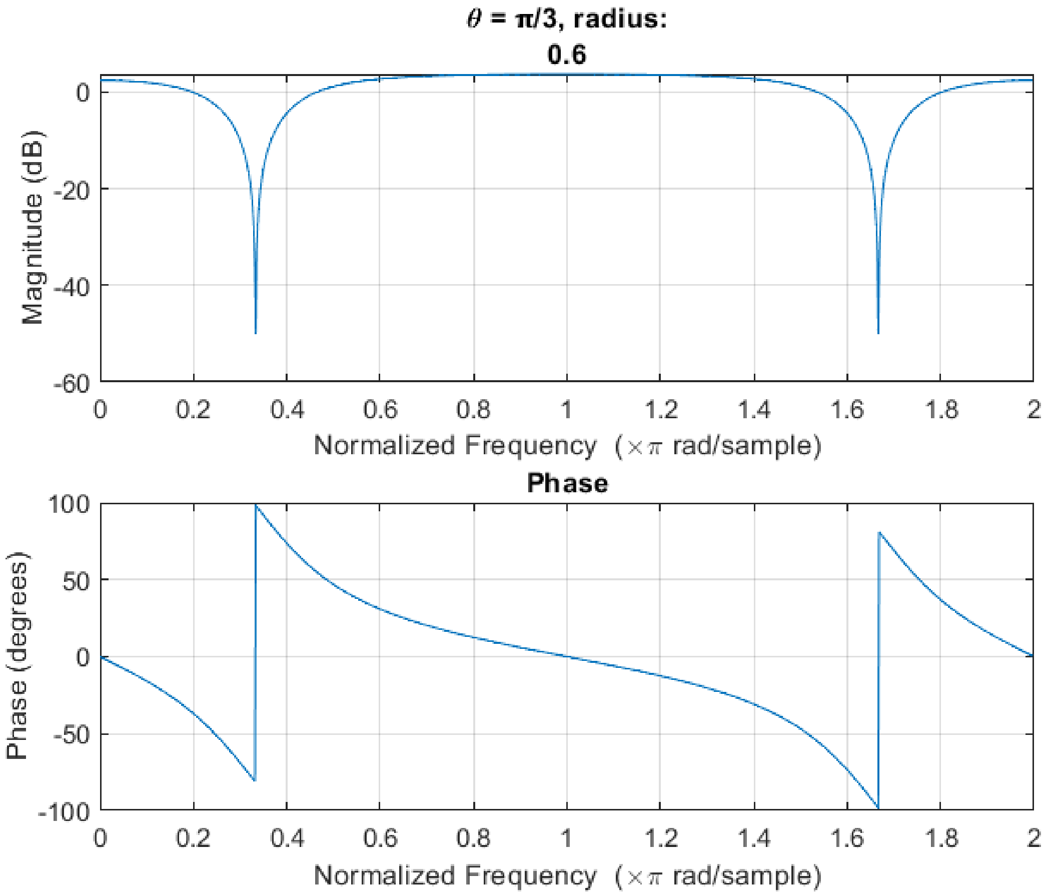

θ = π/3, radius:
0.1
Magnitude (dB)
20
0
-20
-40
-60
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Normalized Frequency ($\times\pi$ rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
Normalized Frequency ($\times\pi$ rad/sample)

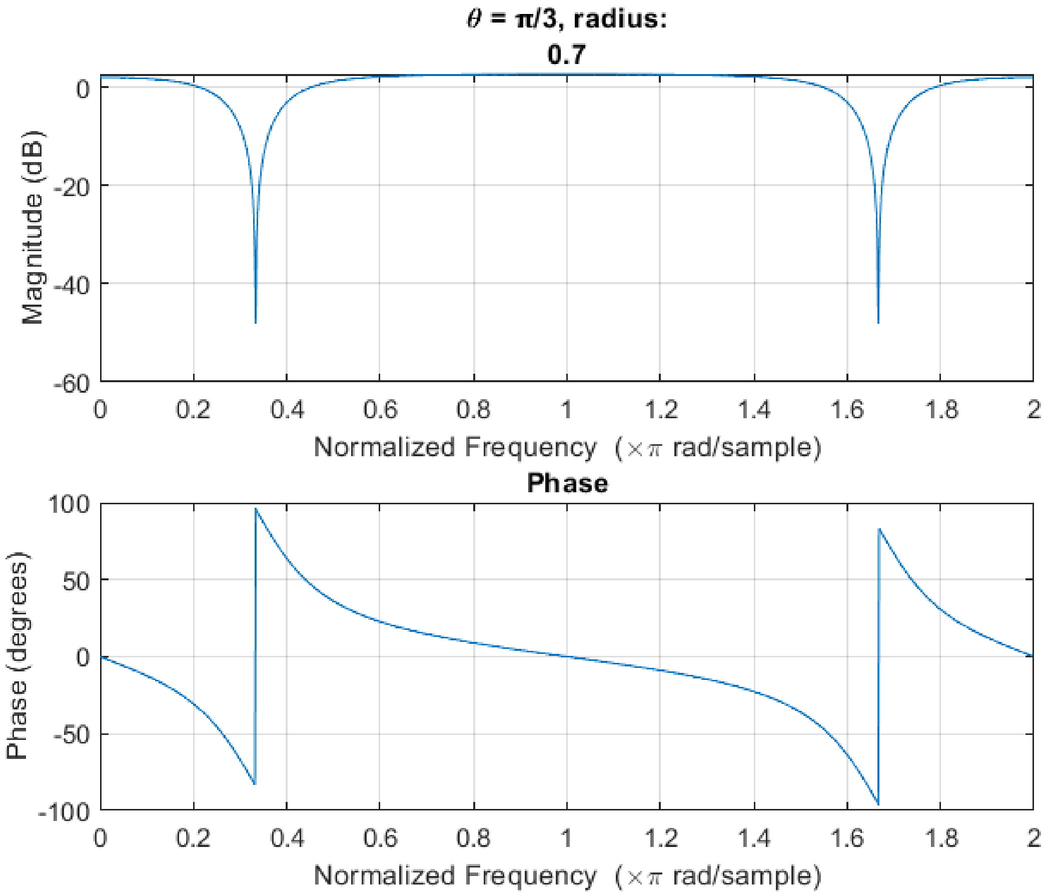

θ = π/3, radius:
0.2
Magnitude (dB)
20
0
-20
-40
-60
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Normalized Frequency ($\times\pi$ rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
Normalized Frequency ($\times\pi$ rad/sample)

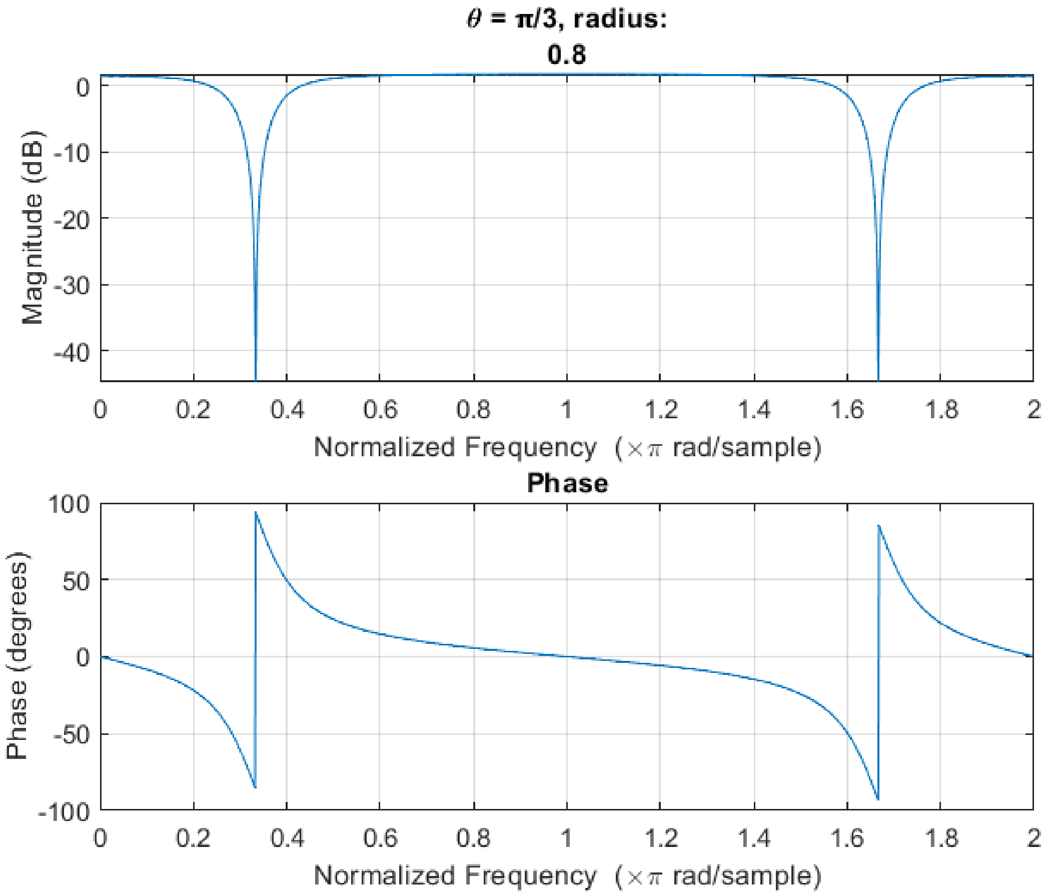

θ = π/3, radius:
0.3
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
Normalized Frequency (×π rad/sample)

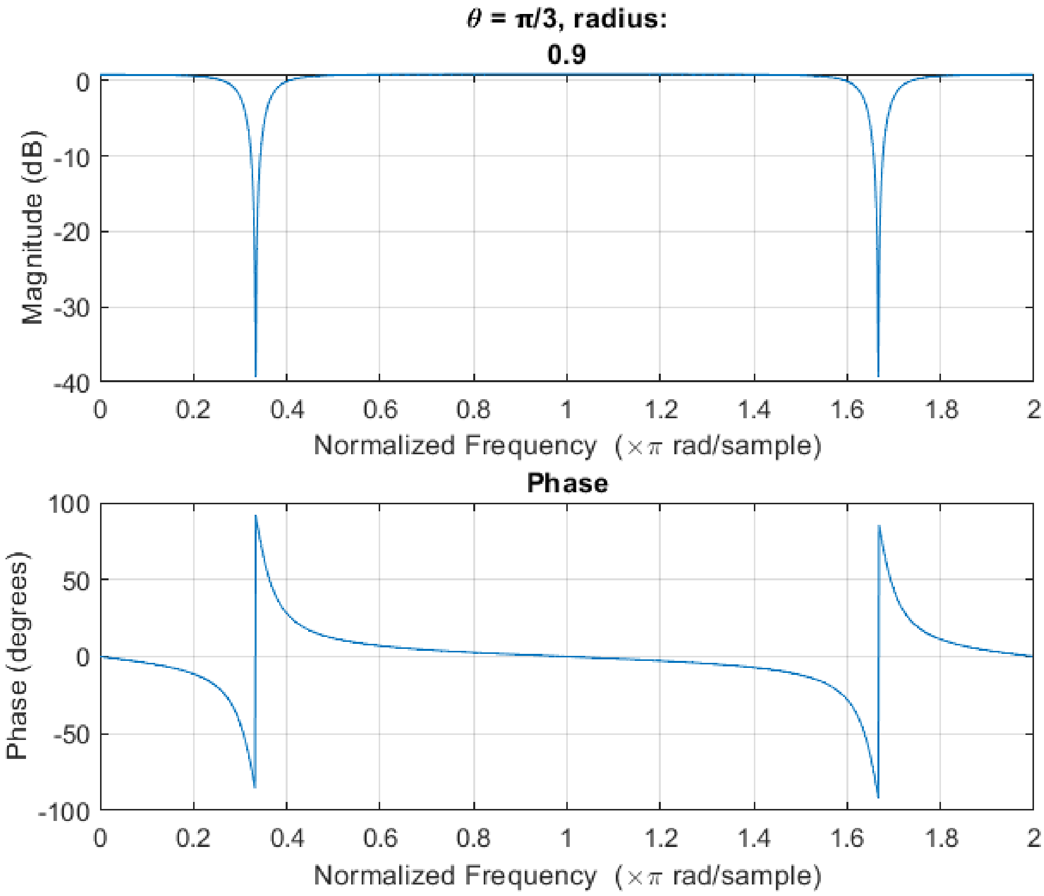

θ = π/3, radius:
0.4
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
Normalized Frequency (×π rad/sample)


θ = π/3, radius:
0.5
Magnitude (dB)
0
-20
-40
-60
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)


θ = π/3, radius:
0.6
Magnitude (dB)
0
-20
-40
-60
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Normalized Frequency (×π rad/sample)


θ = π/3, radius:
0.7
Magnitude (dB)
0
-20
-40
-60
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
Normalized Frequency (×π rad/sample)


θ = π/3, radius:
0.8
Magnitude (dB)
0
-10
-20
-30
-40
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
100
50
0
-50
-100
Normalized Frequency (×π rad/sample)



For a fixed **θ** and varying radius**,**

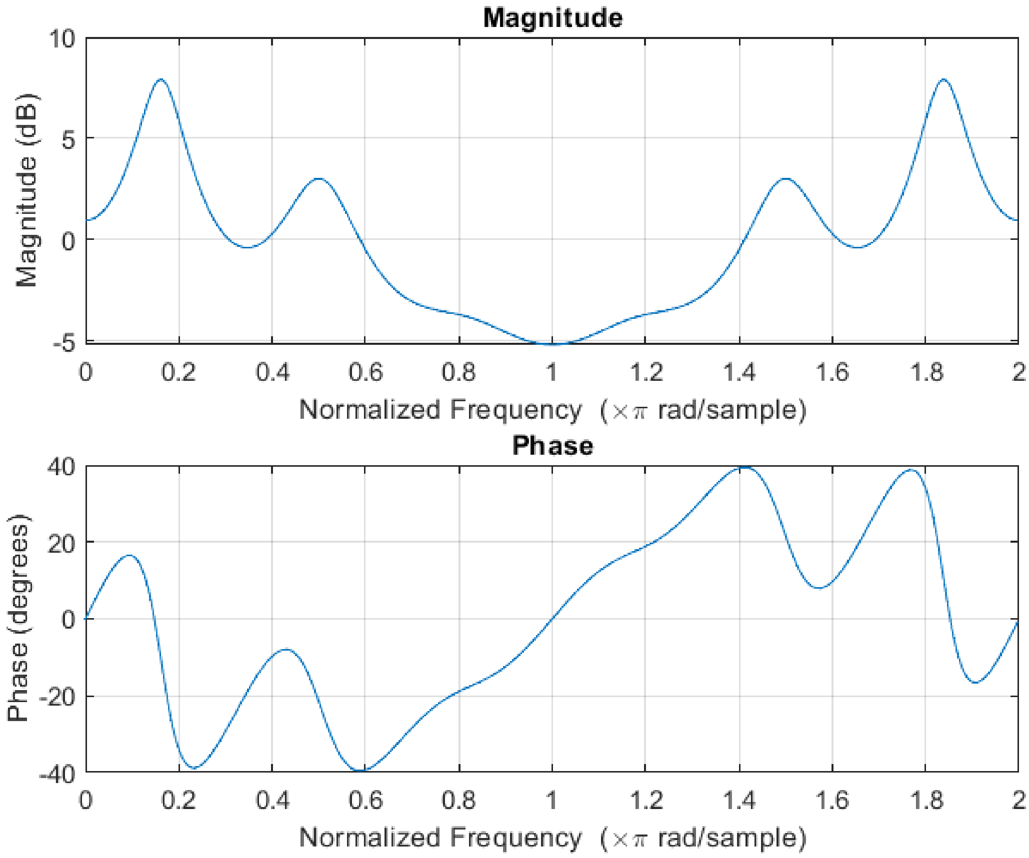
Zeros always lie on the unit circle at the same position irrespective of radius r.

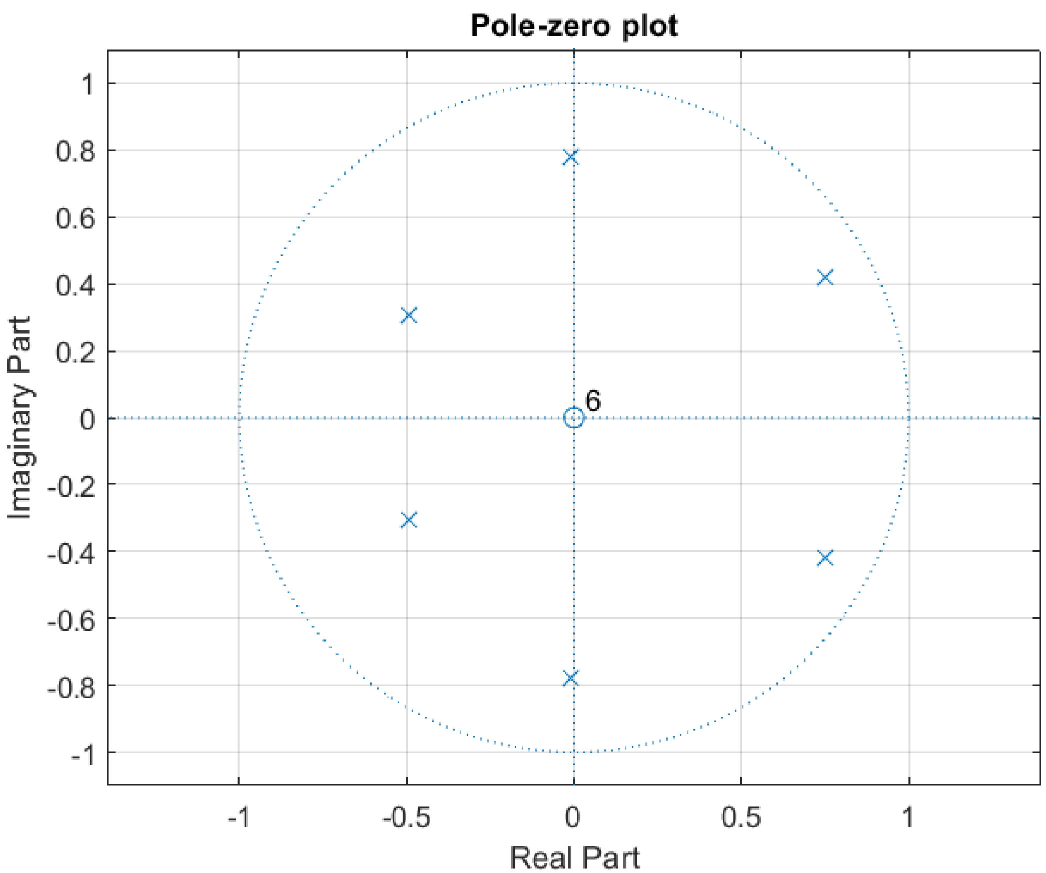
Poles have the same phase but the magnitude of the poles changes with radius r.

As the radius increases, the rate at which the magnitude is changing is very high at the phase of the pole.

QUESTION 4

$$H(z) = \frac{1}{1 - 0.5z^{-1} + 0.2z^{-2} - 0.1z^{-3} + 0.007z^{-4} + 0.14z^{-5} + 0.15z^{-6}}$$


Magnitude
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
Normalized Frequency (×π rad/sample)


Pole-zero plot
Imaginary Part
Real Part
6

$$H(z) = \frac{1}{1 - 0.5z^{-1} + 0.2z^{-2} - 0.1z^{-3} + 0.007z^{-4} + 0.14z^{-5} + 0.15z^{-6}}$$

$$H(z) = \frac{z^6}{z^6 - 0.5z^5 + 0.2z^4 - 0.1z^3 + 0.007z^2 + 0.14z^1 + 0.15}$$

- The system has 6 zeros, each zero has a value of 0.
- The poles of the equation are:
  - -0.009+j0.78
  - -0.009-j0.78
  - 0.75+j0.42
  - 0.75-j0.42
  - -0.491+j0.306
  - -0.491-j0.306
- Let a pole of the system have magnitude **p** and phase **θ** then, The Frequency response plot will have a maxima at the point **(log(|p|),θ)**.
- Let a zero of the system have magnitude **p** and phase **θ** then, The Frequency response plot will have a minima at the point **(log(|p|),θ)**.