Lab report - 5: Discrete-time FT and LTI systems

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QUESTION 1

a

$$The\ DTFT\ of\ any\ discrete-time\ signal\ x[n]\ is\ given\ as:$$
$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

Function For calculating the DTFT of a given signal:

```
function X = DT_Fourier(x,N0,w)
    X = zeros(1,length(w));
    for k = 1:length(x)
        X = X + (x(k)*exp(-1j*w*(k-N0)));
    end
end
```

Inputs:

- $x,$ a discrete-time signal of finite duration, the signal is zero elsewhere.
- $N_0,$ location of the sample x[0] in the given input signal x, note that $N0$ is a positive integer between 1 and length(x).
- $\omega,$ a vector of frequencies at which to compute the DTFT (though frequency is a continuous variable in DTFT we can evaluate it at only a finite set of points)

Output:

$X,$ a complex vector corresponding to the DTFT computed at the frequencies in ω.

b

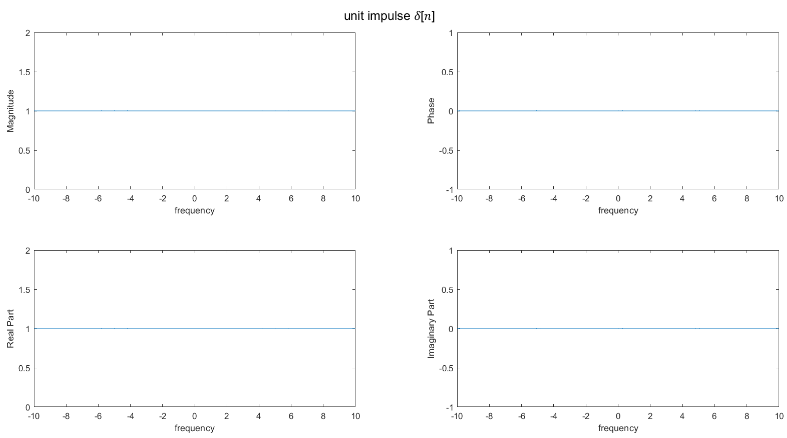


fig 1

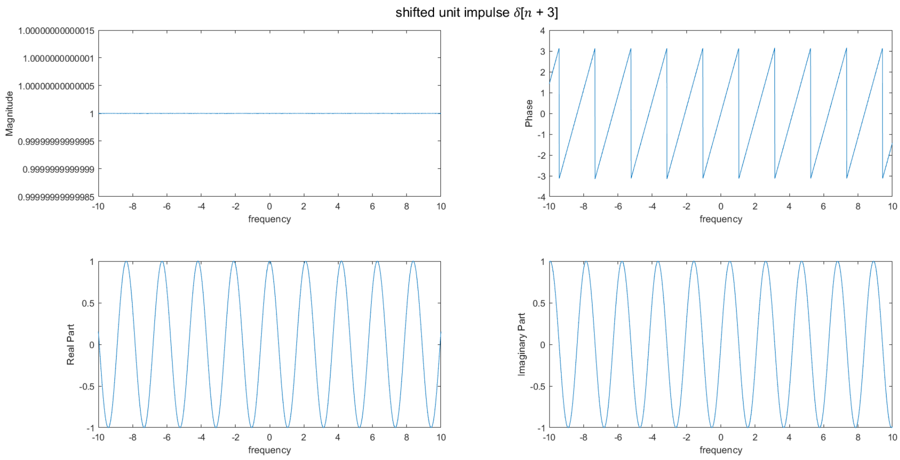


fig 2

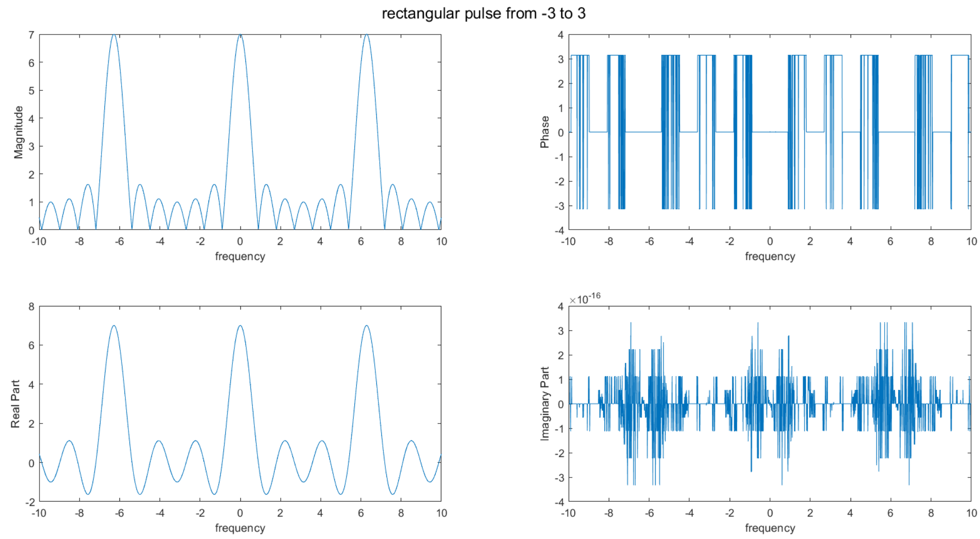


fig 3

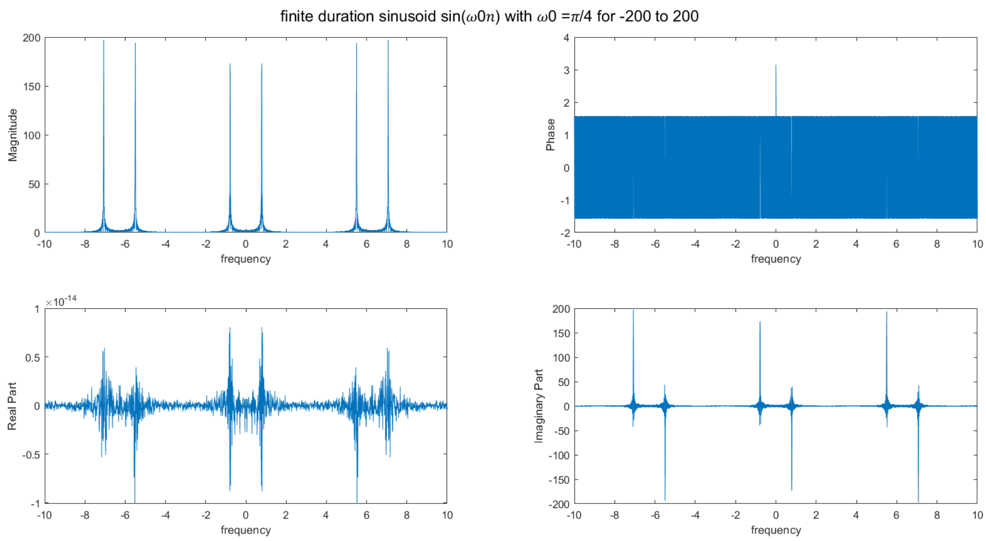


fig 4

c

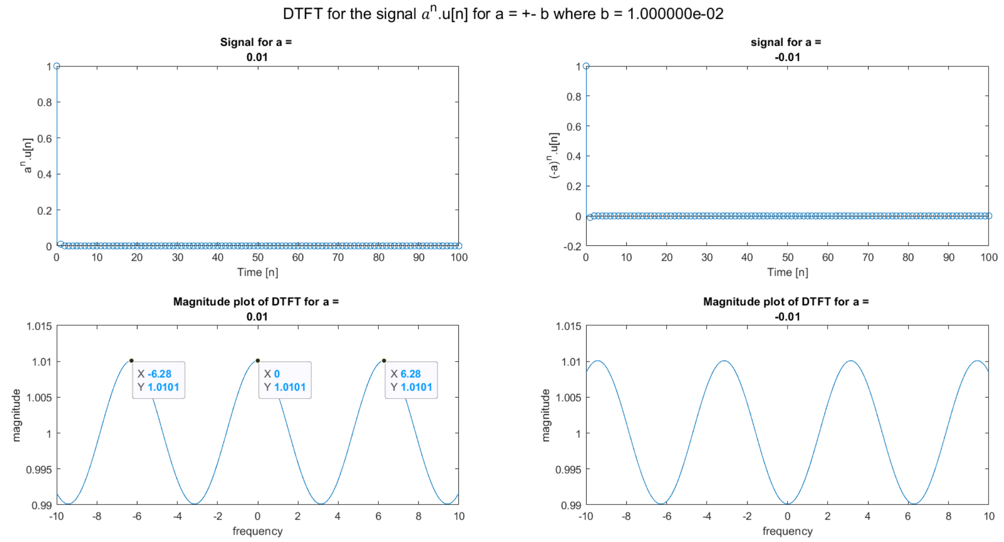

DTFT for the signal a^n.u[n] for a = +- b where b = 1.000000e-02
Signal for a = 0.01
signal for a = -0.01
Magnitude plot of DTFT for a = 0.01
Magnitude plot of DTFT for a = -0.01
Time [n]
frequency
magnitude
X -6.28 Y 1.0101
X 0 Y 1.0101
X 6.28 Y 1.0101

fig 1

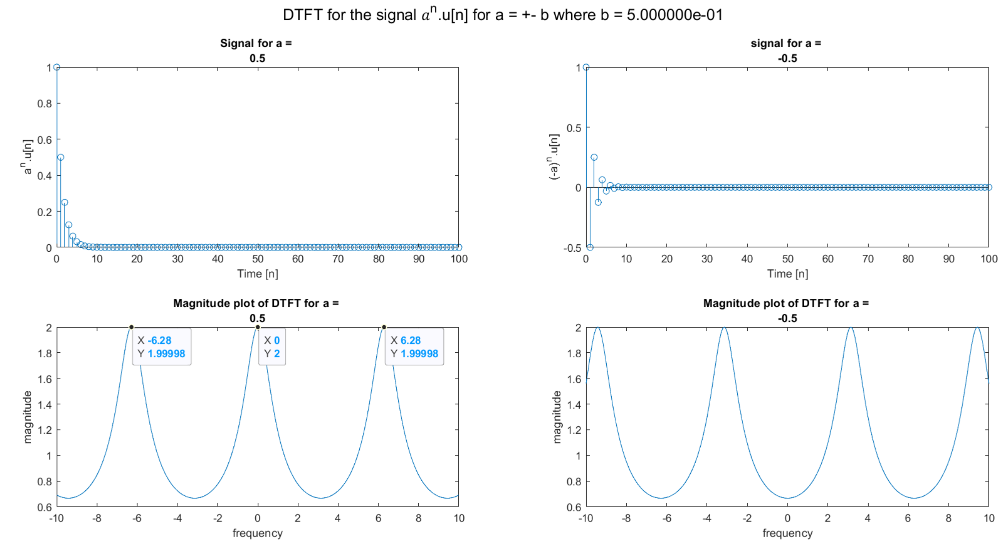

DTFT for the signal a^n.u[n] for a = +- b where b = 5.000000e-01
Signal for a = 0.5
signal for a = -0.5
Magnitude plot of DTFT for a = 0.5
Magnitude plot of DTFT for a = -0.5
Time [n]
frequency
magnitude
X -6.28 Y 1.99998
X 0 Y 2
X 6.28 Y 1.99998

fig 2

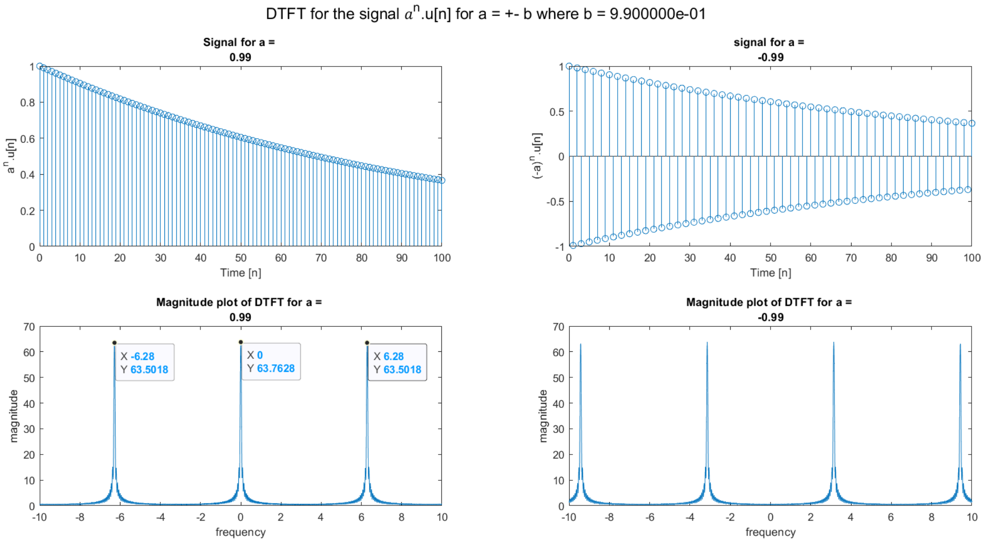
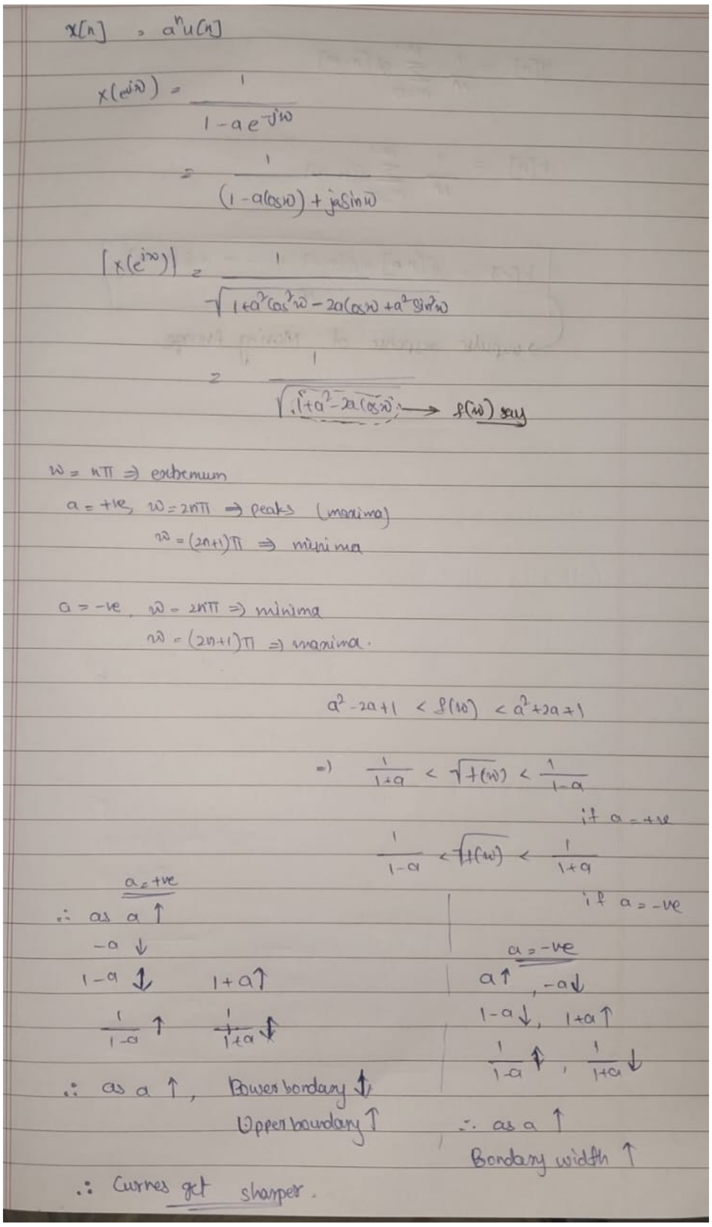

DTFT for the signal a^n.u[n] for a = +- b where b = 9.900000e-01
Signal for a = 0.99
signal for a = -0.99
Magnitude plot of DTFT for a = 0.99
Magnitude plot of DTFT for a = -0.99
a^n.u[n]
$(-a)^n$.u[n]
Time [n]
magnitude
frequency
X -6.28
Y 63.5018
X 0
Y 63.7628
X 6.28
Y 63.5018

fig 3



$x[n] = a^n u[n]$

$X(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}$

$= \frac{1}{(1 - a\cos\omega) + ja\sin\omega}$

$|X(e^{j\omega})| = \frac{1}{\sqrt{1 + a^2\cos^2\omega - 2a\cos\omega + a^2\sin^2\omega}}$

$= \frac{1}{\sqrt{1 + a^2 - 2a\cos\omega}} \rightarrow f(\omega)$ say

$\omega = n\pi \Rightarrow$ extremum

$a = +ve$, $\omega = 2n\pi \Rightarrow$ peaks (maxima)

$\omega = (2n+1)\pi \Rightarrow$ minima

$a = -ve$, $\omega = 2n\pi \Rightarrow$ minima

$\omega = (2n+1)\pi \Rightarrow$ maxima.

$a^2 - 2a + 1 < f(\omega) < a^2 + 2a + 1$

$\Rightarrow \frac{1}{1+a} < \sqrt{f(\omega)} < \frac{1}{1-a}$ if $a = +ve$

$\frac{1}{1-a} < \sqrt{f(\omega)} < \frac{1}{1+a}$ if $a = -ve$

$a = +ve$

$\therefore$ as $a \uparrow$

$-a \downarrow$

$1-a \downarrow$ $\quad 1+a \uparrow$

$\frac{1}{1-a} \uparrow$ $\quad \frac{1}{1+a} \downarrow$

$\therefore$ as $a \uparrow$, Lower bondary $\downarrow$

Upper boundary $\uparrow$

$a = -ve$

$a \uparrow$, $-a \downarrow$

$1-a \downarrow$, $1+a \uparrow$

$\frac{1}{1-a} \uparrow$, $\frac{1}{1+a} \downarrow$

$\therefore$ as $a \uparrow$

Bondary width $\uparrow$

$\therefore$ Curves get sharper.

Explanation for **varying a**

QUESTION 2

a

An order − M moving average filter is a discrete − time LTI system with input $x[n]$ *and output* $y[n]$ *rela*

$$y[n] = 1/M \sum_{m=0}^{M-1} x[n-m]$$

$$y[n] = \frac{1}{M} \sum_{m=0}^{M-1} x[n-m]$$

$$h[n] = \frac{1}{M} \sum_{m=0}^{M-1} \delta[n-m]$$

$$\boxed{h[n] = \frac{\delta[n] + \delta[n-1] + \dots + \delta[n-M]}{M}}$$

impulse response of Moving Average

c

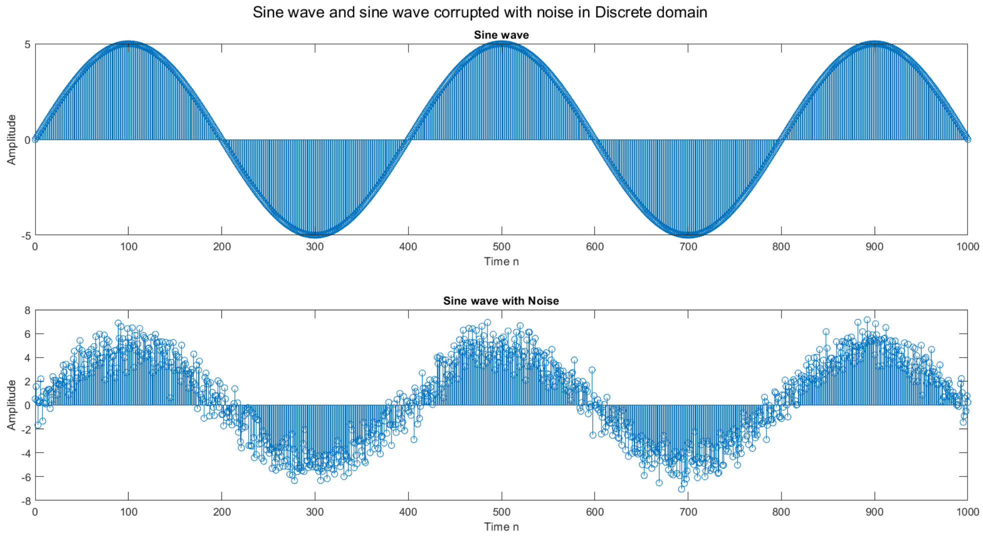


fig 1

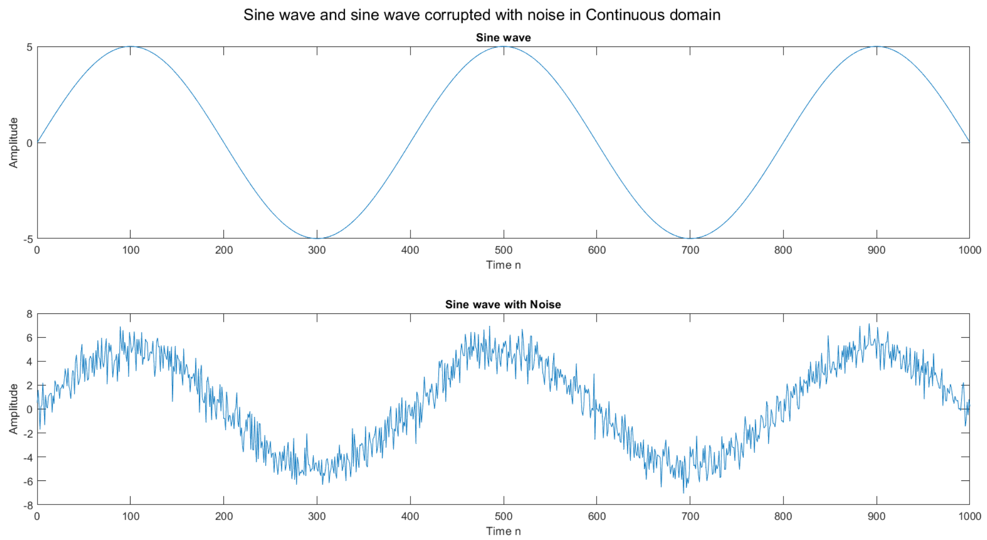


fig 2

d

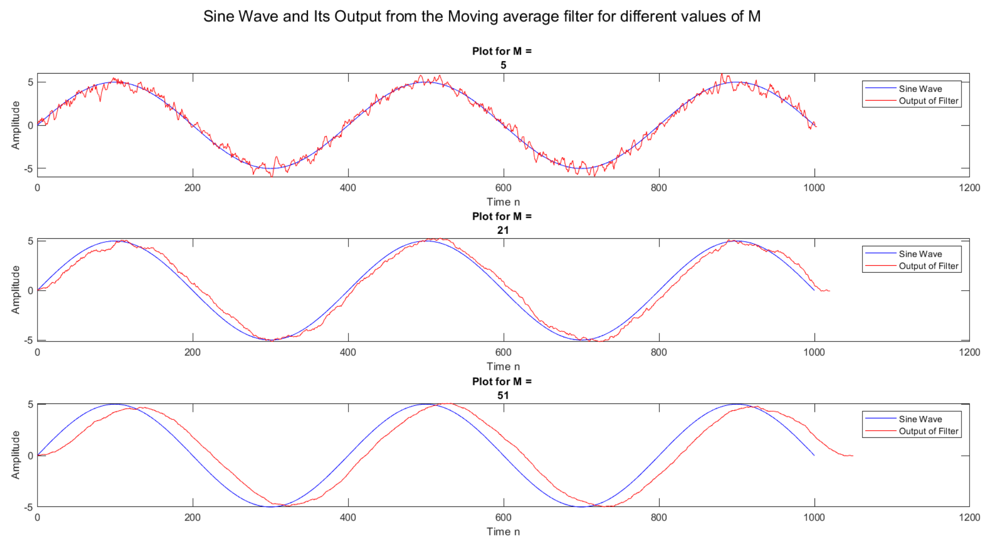
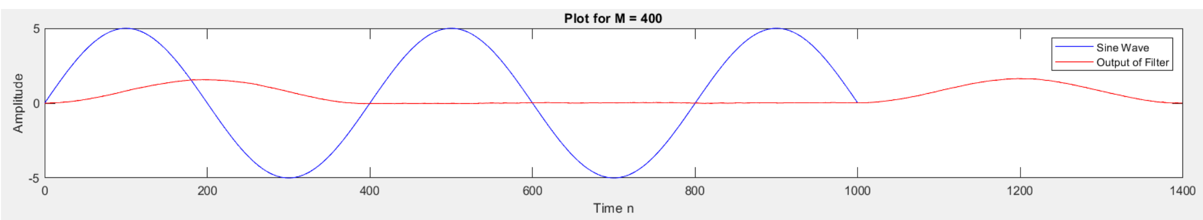


fig 1

e

- As M is increased, Noise is filtered out efficiently.
- This is because as the no of samples increases, as noise has a very high frequency, it gets canceled out easily averages out to a particular value causing a shift.



- Here time period of the sine wave is 400. So, average of the sine wave is zero, the output would be average noise causing a shift in the signal.
- However, with increasing M, the edges become less sharp.

f

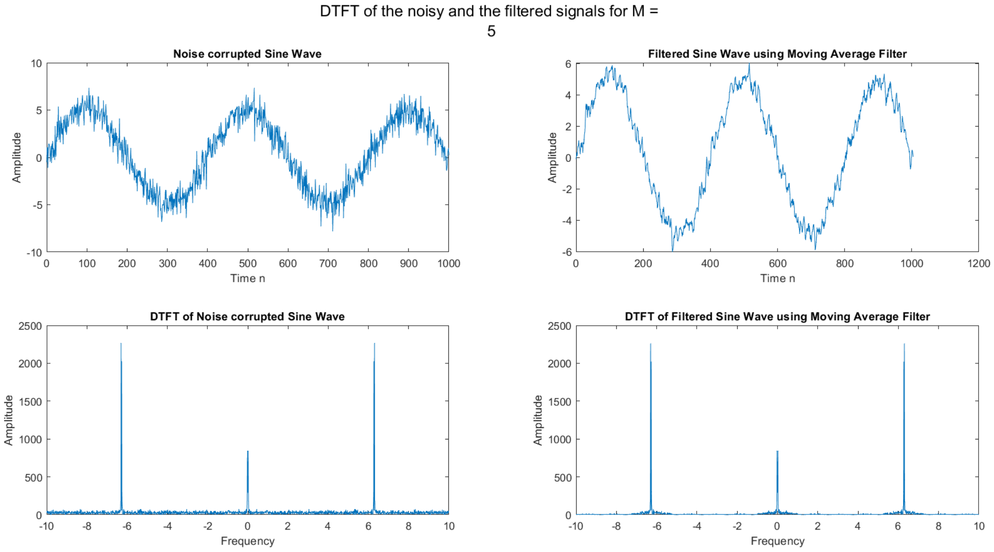


fig 1

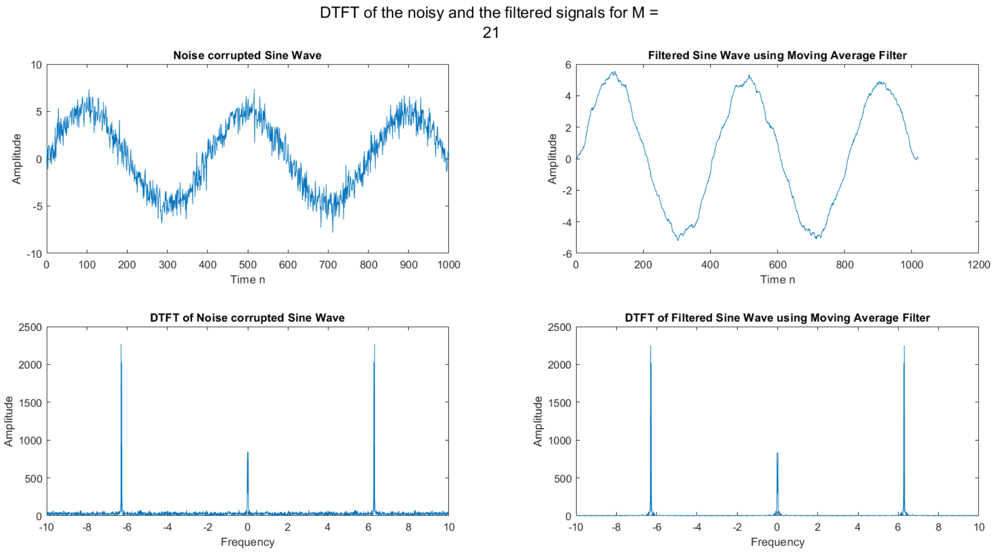


fig 2

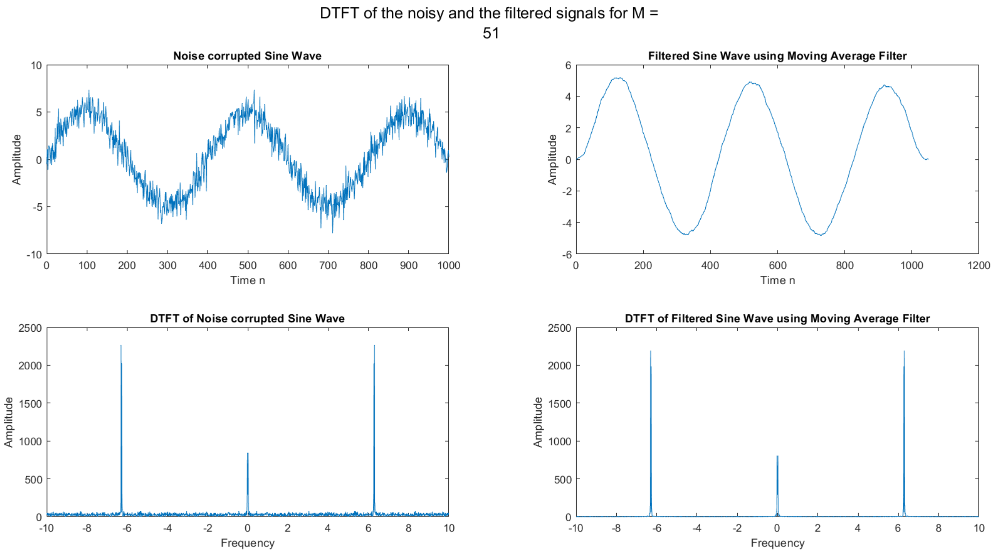


fig 3

- In Unfiltered Signal
  - Large peaks correspond to the DTFT of the sine wave.
  - There are some small peaks corresponding to noise.
- In filtered signals
  - as M increases, the number of small peaks decreases indicating that the noise is being efficiently filtered out with increasing M.

g

part a

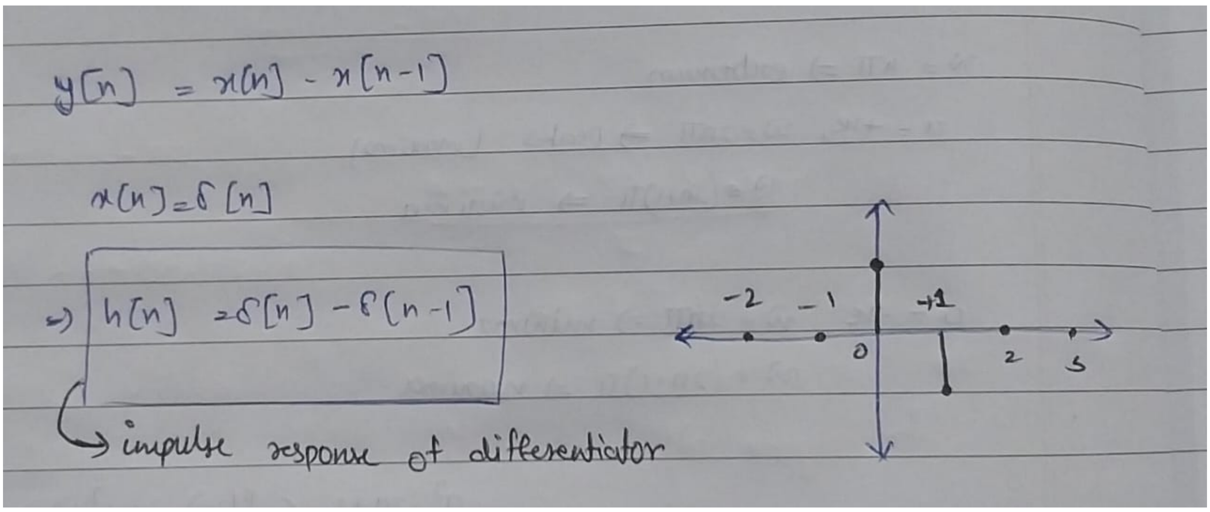


fig 1 Impulse Response of Differentiator Filter.

part c

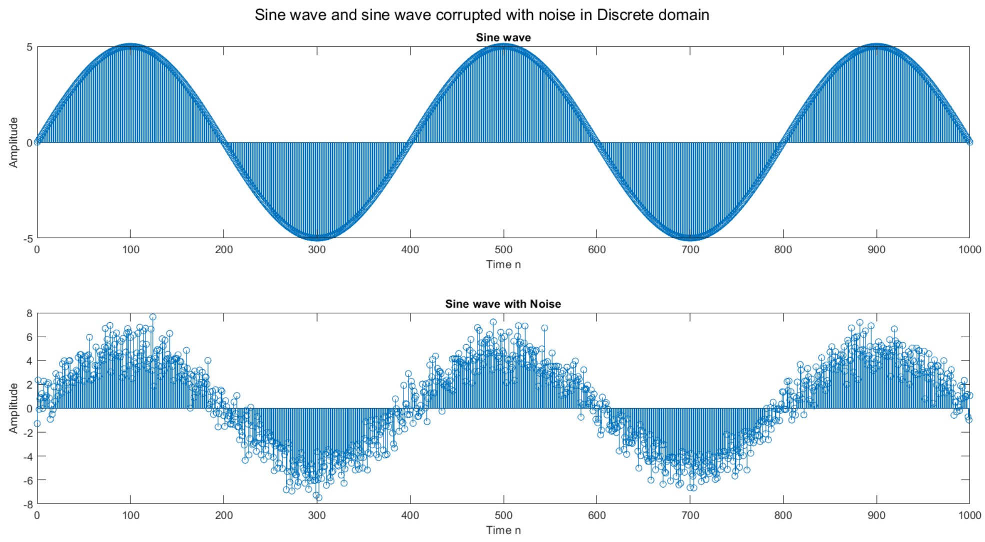


fig 2

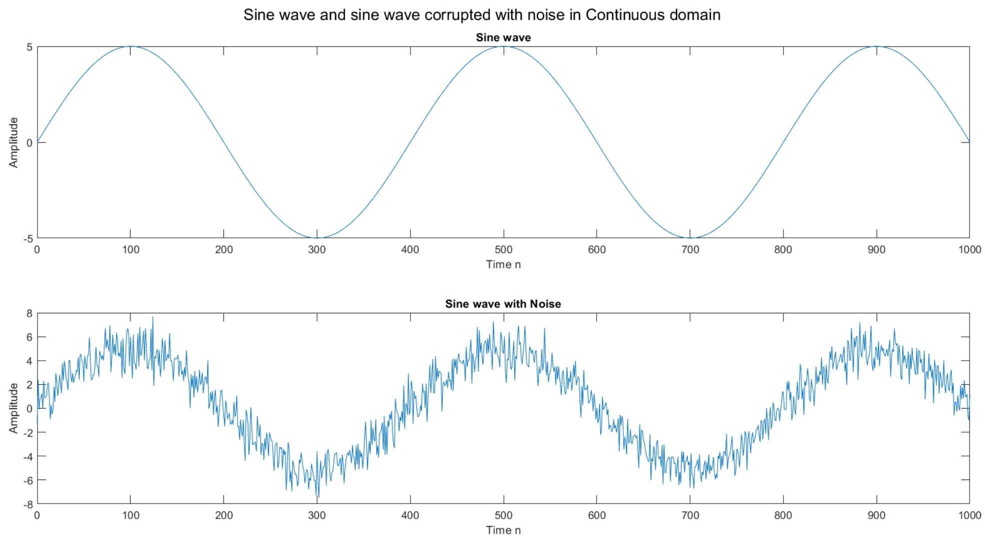


fig 3

part d

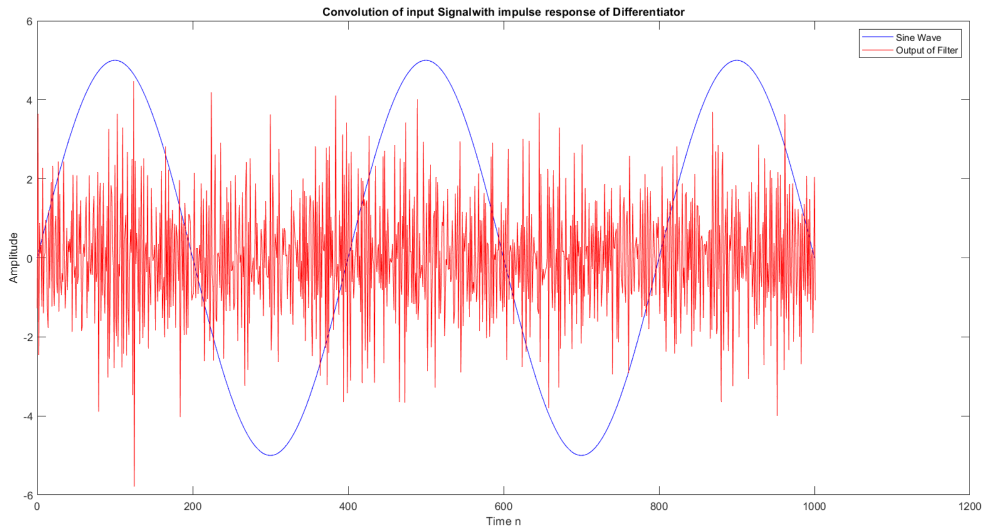


fig 4

part f

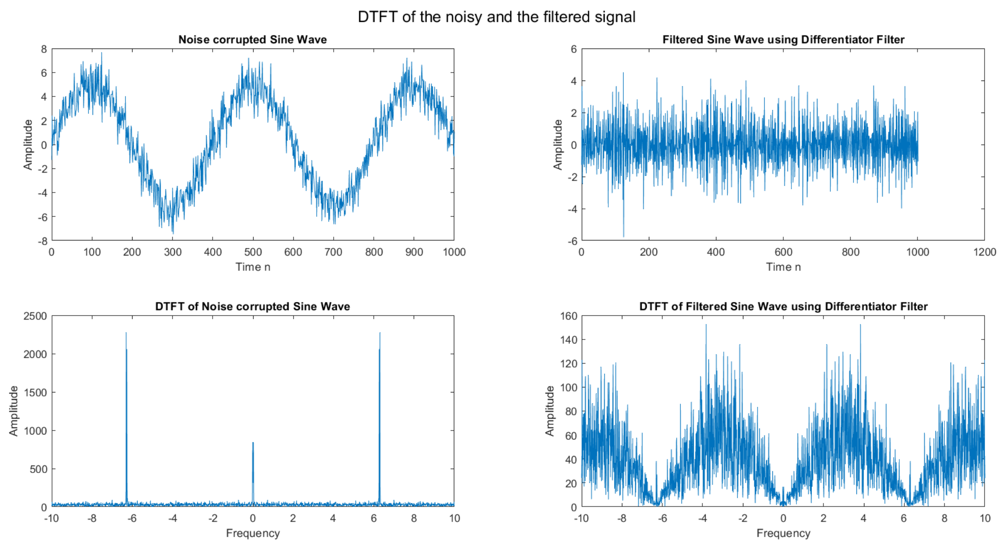


fig 5

- in the Noise-corrupted signal,
  - Large peaks correspond to the DTFT of the sine wave.
  - There are some small peaks corresponding to noise.
- In the filtered signal, only noise is present which is amplified 5 times due to the coefficient of sine wave. So the plot corresponds to the DTFT of the Noise signal.
- The sine wave has a frequency of 400 units. so here, $sin(n) - sin(n-1) \approx 0$. Therefore only noise remains.

h

- **MOVING AVERAGE FILTER:**
  - It serves as a **low-pass filter**, which smoothens out signals by eliminating short-term swings and keeping longer-term trends.

- Averaging eliminates high frequencies, making it equal to low-pass filtering.
- The moving average filter is a straightforward Low Pass **FIR filter** that is frequently used to smooth out a variety of collected data or signals.

- **DIFFERENTIATOR:**
  - it acts as a **high-pass** filter.
  - As we can see only noise that has a high frequency is passing out of the filter leaving behind the Sine wave which has a very low frequency when compared with the noise.
  - It is also an **FIR filter**.

QUESTION 3

$$The\ inverse\ DTFT\ is\ given\ by\ the\ expression:$$
$$x[n] = 1/2\pi \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

a

Function to calculate the Inverse DTFT of the Signal:

```
function x = Inv_DTFT(X,n,w)
    x = zeros(1,length(n));
    for k = 1:1:length(n)
        x = (1/2*pi)*int(X*exp(1j*w*n),w,-pi,pi);
    end
end
```

x[n] is expected to be a **complex-valued signal**. Hence, the Real part, Imaginary part, and magnitude are plotted with respect to time.

$$The\ frequency\ domain\ rectangular\ wave\ which\ in\ the\ interval\ [-\pi,\pi]\ is\ given\ by:$$
$$X(e^{j\omega}) = 1, if\ |\omega| \le \omega_c$$
$$0, if\ \omega_c < |\omega| < \pi$$

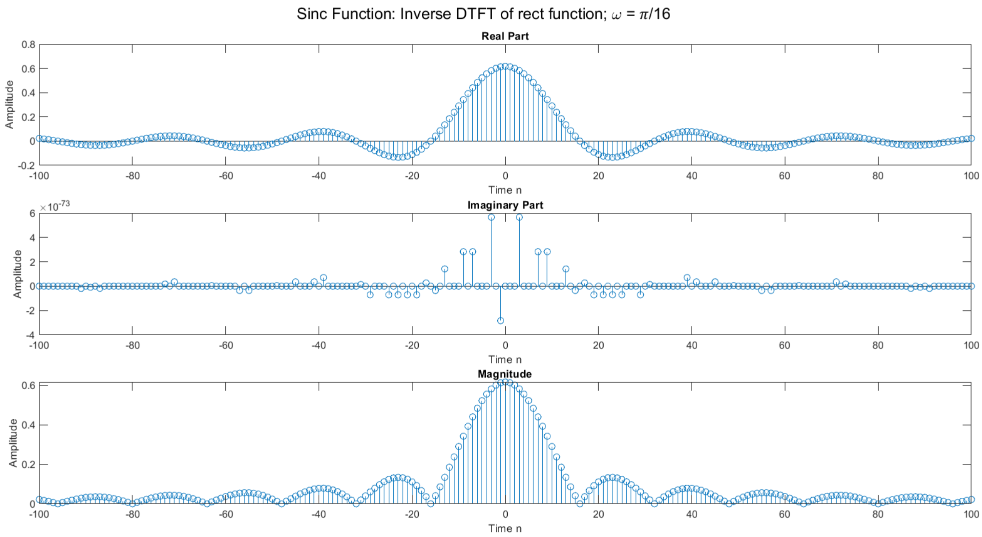


fig 1

b

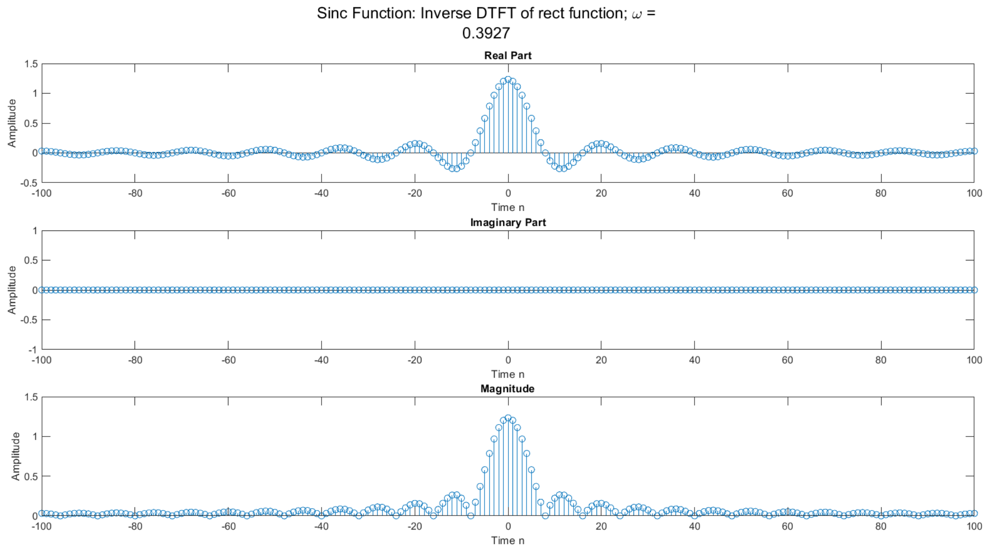

Sinc Function: Inverse DTFT of rect function; ω =
0.3927
Real Part
Imaginary Part
Magnitude
Amplitude
Time n

fig 1

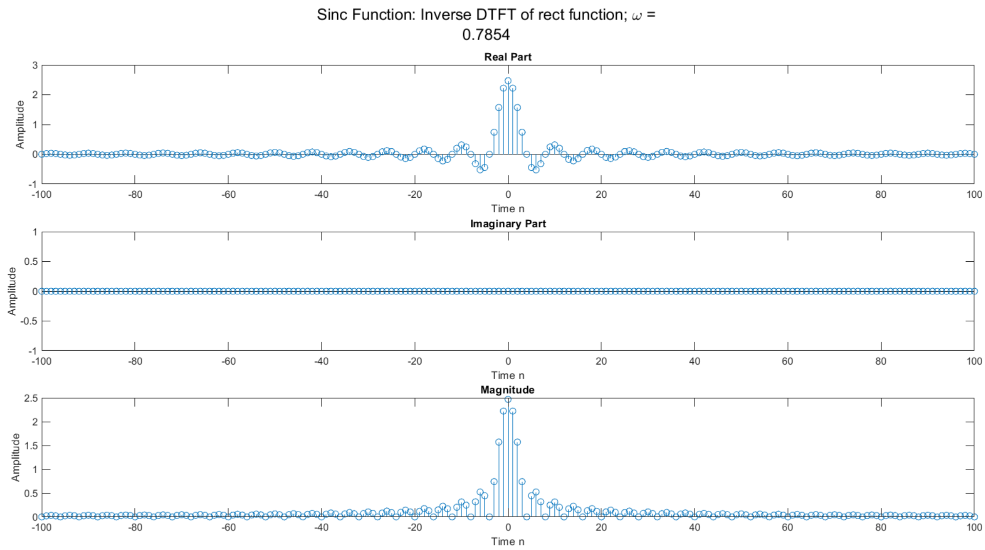

Sinc Function: Inverse DTFT of rect function; ω =
0.7854
Real Part
Imaginary Part
Magnitude
Amplitude
Time n

fig 2

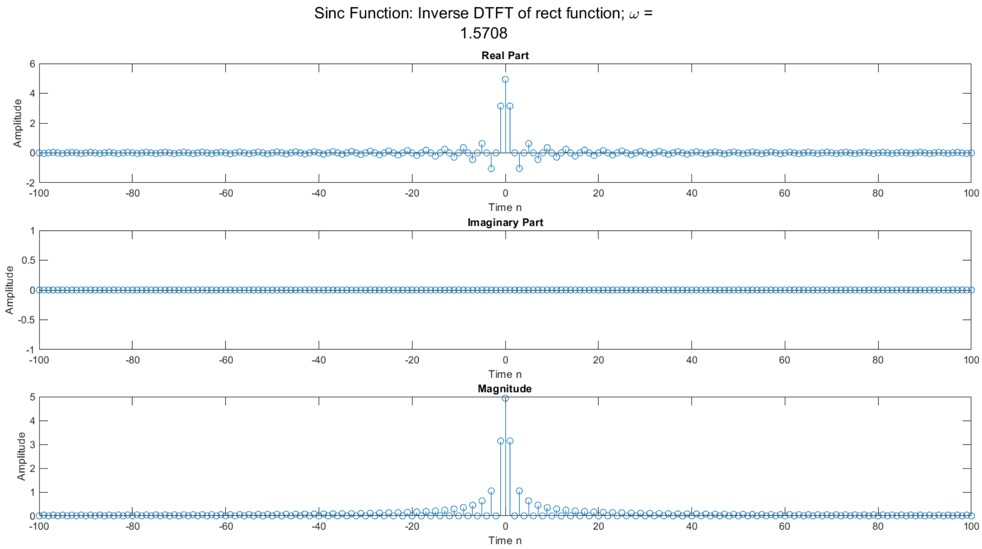

Sinc Function: Inverse DTFT of rect function; ω = 1.5708
Real Part
Imaginary Part
Magnitude
Amplitude
Time n

fig 3

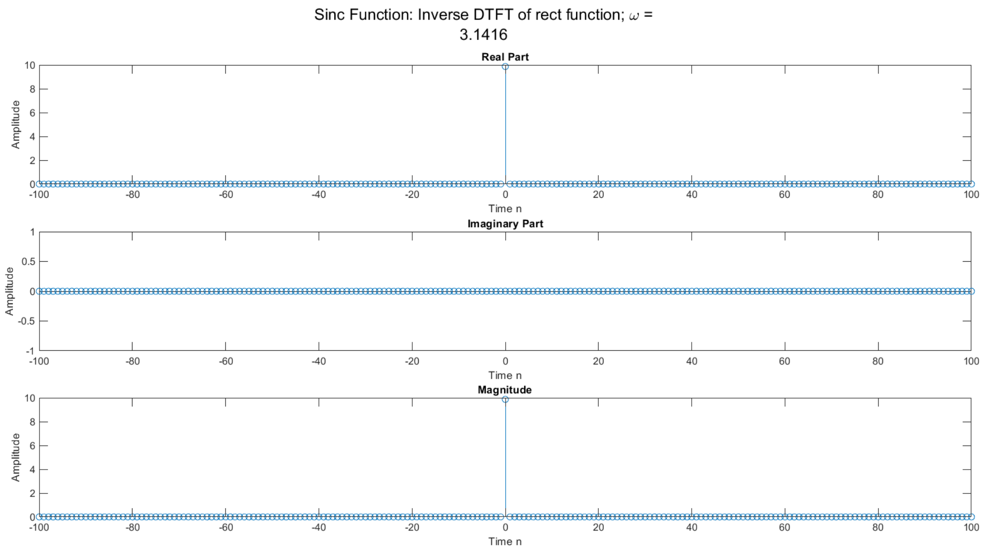

Sinc Function: Inverse DTFT of rect function; ω = 3.1416
Real Part
Imaginary Part
Magnitude
Amplitude
Time n

fig 4

$$X(e^{j\omega}) = \begin{cases} 1; & |\omega| \le \omega_c \\ 0; & \omega_c < |\omega| < \pi \end{cases}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(e^{j\omega}) d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} 1\, e^{+j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left(\frac{e^{j\omega n}}{jn} \right)_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi} \frac{e^{j\omega_c n} - e^{-j\omega_c n}}{2jn}$$

$$= \frac{1}{\pi} \frac{\sin(\omega_c n)}{n}$$

$$\boxed{x[n] = \frac{\omega_c}{\pi} \times \frac{\sin(\omega_c n)}{\omega_c n}}$$

$$\omega_c = \frac{\pi}{16} \Rightarrow x[n] = \frac{1}{16} \times \frac{\sin(\pi n/16)}{\pi n/16}$$

$$\omega_c = \pi/8 \Rightarrow x[n] = \frac{1}{8} \frac{\sin(\pi n/8)}{\pi n/8}$$

$$\omega_c = \pi/4 \Rightarrow x[n] = \frac{1}{4} \frac{\sin(\pi n/4)}{\pi n/4}$$

$$\omega_c = \pi/2 \Rightarrow x[n] = \frac{1}{2} \frac{\sin(n\pi/2)}{n\pi/2}$$

$\omega_c = \pi \Rightarrow x[n] = \frac{\sin(n\pi)}{n\pi} = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$

$\because \frac{\sin x}{x} \to 1$ as $x \to 0$

Theoritically:-

$X(e^{j\omega}) = 1 \; ; \; |\omega| \leq 1$

$\Rightarrow X(e^{j\omega}) = 1 \; \forall \omega \in \mathbb{R}$

this would be a dirac delta function

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} k \cdot \delta[n] \, e^{-j\omega n}$$

$$= k \cdot 1 \cdot e^{-j\omega(0)}$$

$$= k \quad \forall \omega \in \mathbb{R}$$

$X(e^{j\omega}) = K \Longleftrightarrow x[n] = K\delta[n]$

c

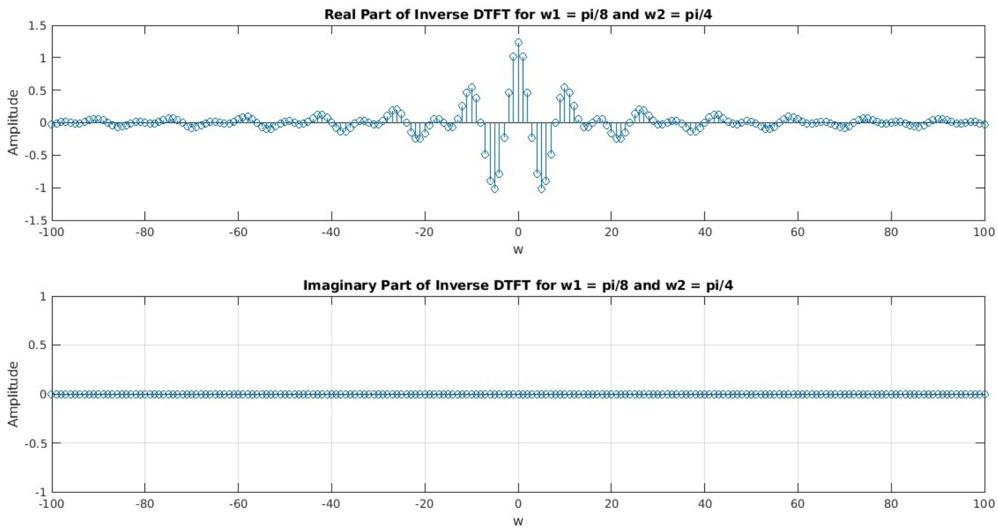


fig 1

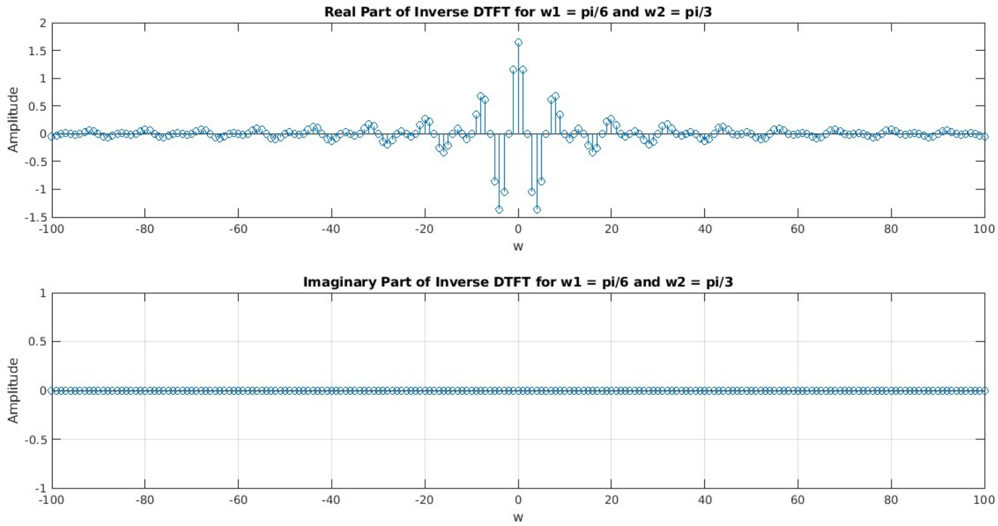


fig 2

$$x[n] = \frac{1}{2\pi}\int_{-w_2}^{-w_1} 1\cdot e^{+jwn}\,dw + \frac{1}{2\pi}\int_{w_1}^{w_2} 1\cdot e^{+jwn}\,dw \qquad w_1 < w_2,\ -w_2 < -w_1$$

$$= \frac{1}{2\pi}\left[\left(\frac{e^{+jwn}}{jn}\right)_{-w_2}^{-w_1} + \left(\frac{e^{+jwn}}{jn}\right)_{w_1}^{w_2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{-jw_1 n} - e^{-jw_2 n}}{jn} + \frac{e^{jw_2 n} - e^{+jw_1 n}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\left(e^{jw_2 n} - e^{-jw_2 n}\right) - \left(e^{jw_1 n} - e^{-jw_1 n}\right)}{jn}\right]$$

$$= \frac{1}{\pi}\left[\frac{\sin(w_2 n)}{n} - \frac{\sin(w_1 n)}{n}\right]$$

$$\boxed{x[n] = \frac{w_2}{\pi} \times \frac{\sin(w_2 n)}{w_2 n} - \frac{w_1}{\pi}\,\frac{\sin(w_1 n)}{w_1 n}}$$

$n = 0 \Rightarrow$ $\boxed{x[0] = \frac{w_2 - w_1}{\pi}}$ → max.

respectively, every $x[n]$ would come.

fig 3: explanation