



Inspire...Educate...Transform.

Supervised models

Time Series Forecasting

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Thanks to Dr.Sridhar Pappu for the material



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"Prediction is very difficult, especially if it's about the future."

--Niels Bohr, Nobel laureate in Physics

What is Time Series data?

- A sequence of data points in successive order, indexed by time.

$$y_t, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}, \dots$$

- Eg: Population of the country listed year-wise, Temperature in the city listed by the hour, Number of iPhones sold listed for each quarter



Forecasting

- Factors needed to forecast the next month's stock price of Tata Motors (\hat{y}_{t+1})
 - Current price (y_t)
 - Current Sales, Revenue and profit data (x_1)
 - Sales trend (x_2)
 - Level debt carried by the company (x_3)
 - Competition (x_4)
 - Import/export rules (x_5)
 - Interest rate environment (x_6)
 - US/INR exchange rate (x_7)
 - Tax rates (x_8)
 - Crack down on black money? (x_9)
 - Cost of steel? (x_{10})
 - Number of smart phones sold? (x_{11})



Forecasting

$$\hat{y}_{t+1} = g(t, x_1, x_2, x_3 \dots, y_t, y_{t-1}, y_{t-2}, \dots)$$

g might be some complex linear or nonlinear function.

Time series forecasting attempts to do same forecast just using the past data of y , without relying on any other external predictors (x_i).



Typical Time Series

$$\hat{y}_{t+1} = f(t, y_t, y_{t-1}, y_{t-2} \dots)$$

f can be linear or nonlinear function

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Why Time Series forecasting?

- Causal independent variables are
 - Unknown to us
 - Not available
 - Might not fit the data well
 - Difficult to forecast

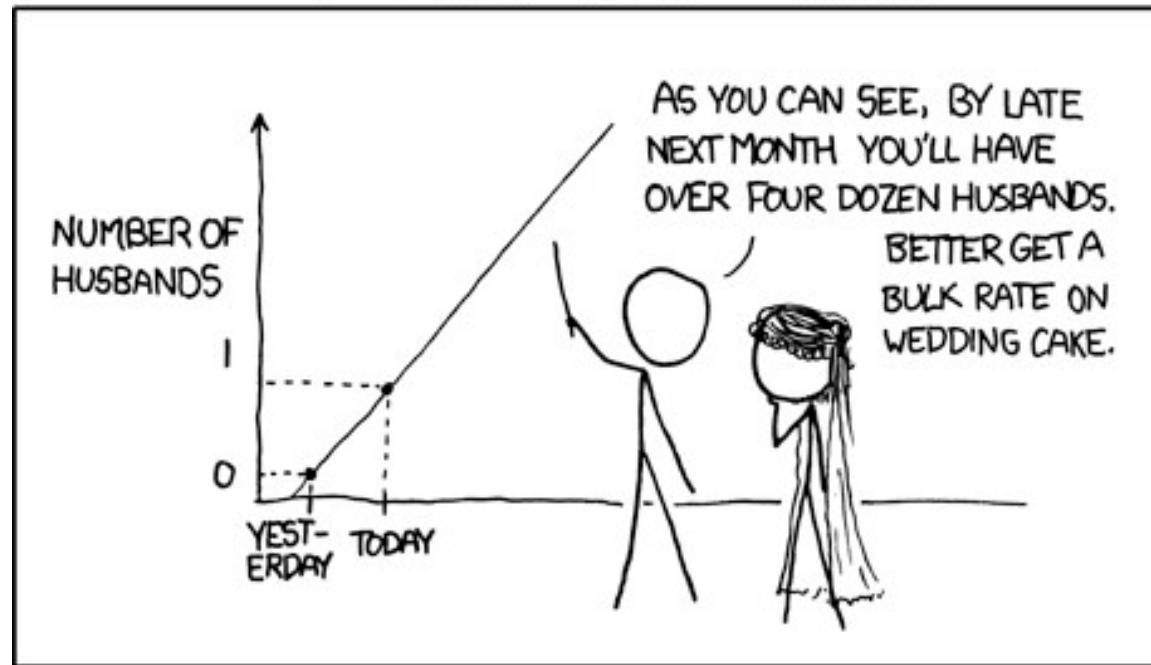


Q2C





MY HOBBY: EXTRAPOLATING



FORECASTING THROUGH TREND ANALYSIS

Regression with time

$$\hat{y}_{t+1} = f(t)$$

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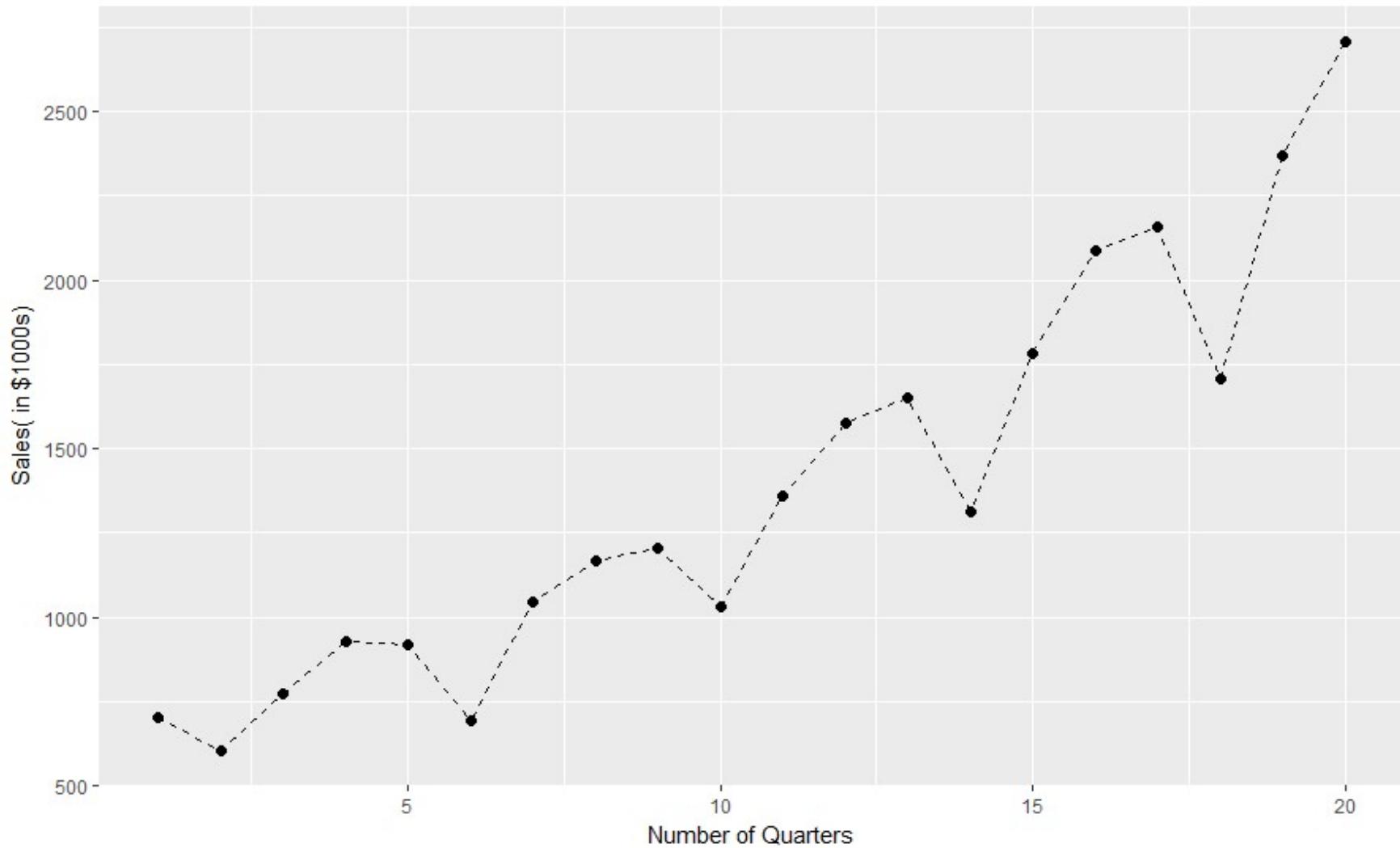
Regression on Time

- Use when trend is the most pronounced

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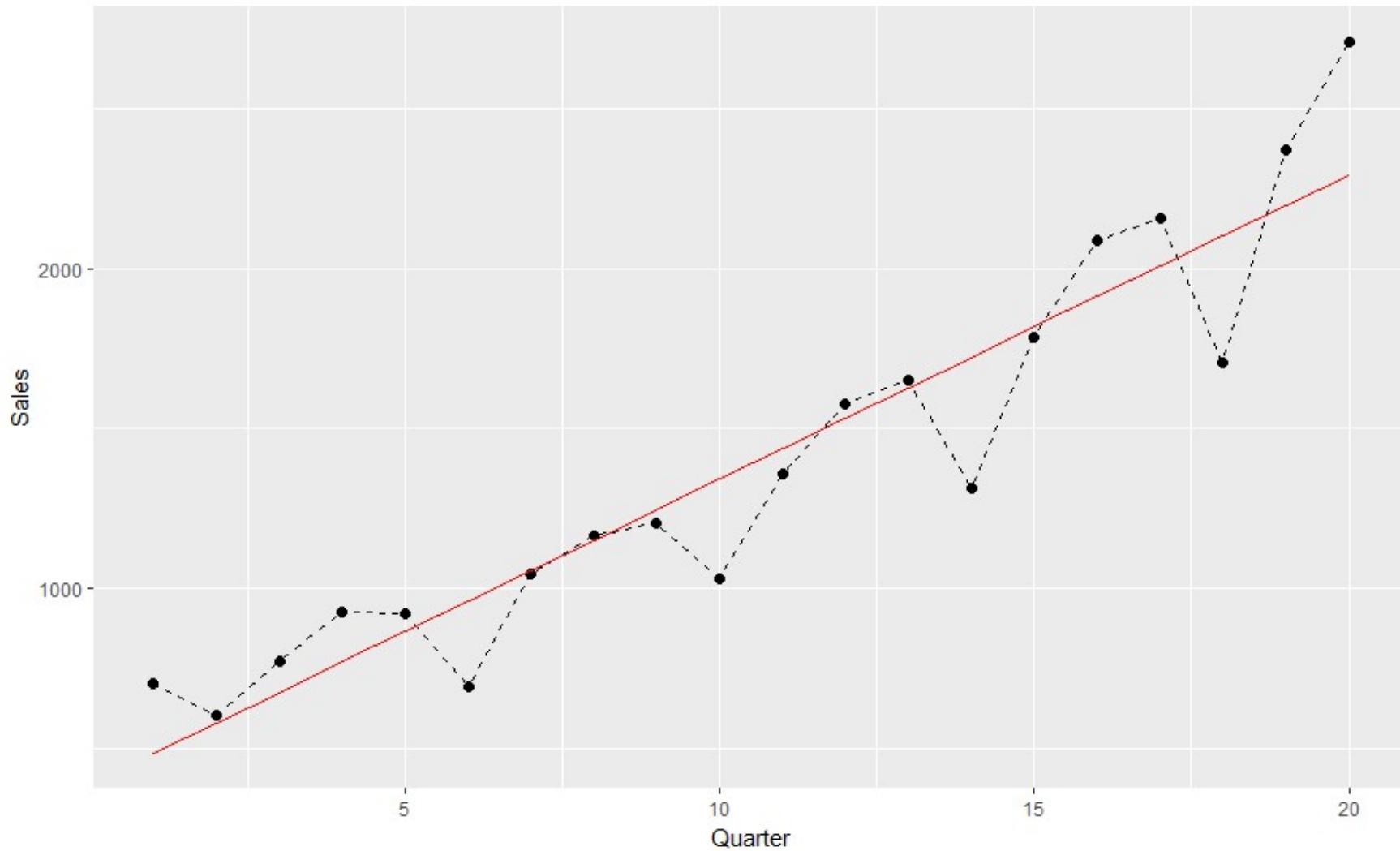
Seasonality



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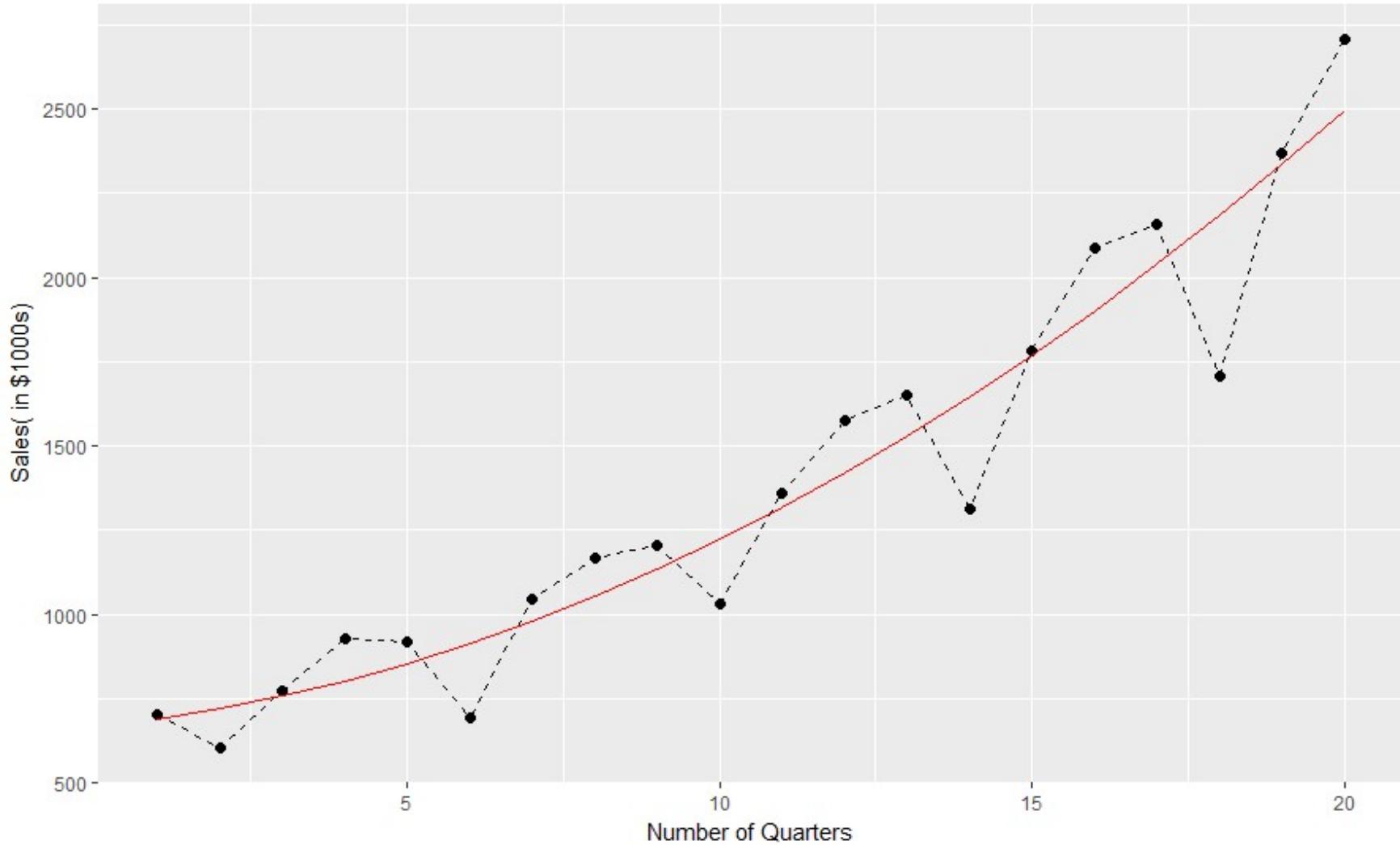
Regression Analysis – Linear fit



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Quadratic Trend



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Seasonal Regression Models

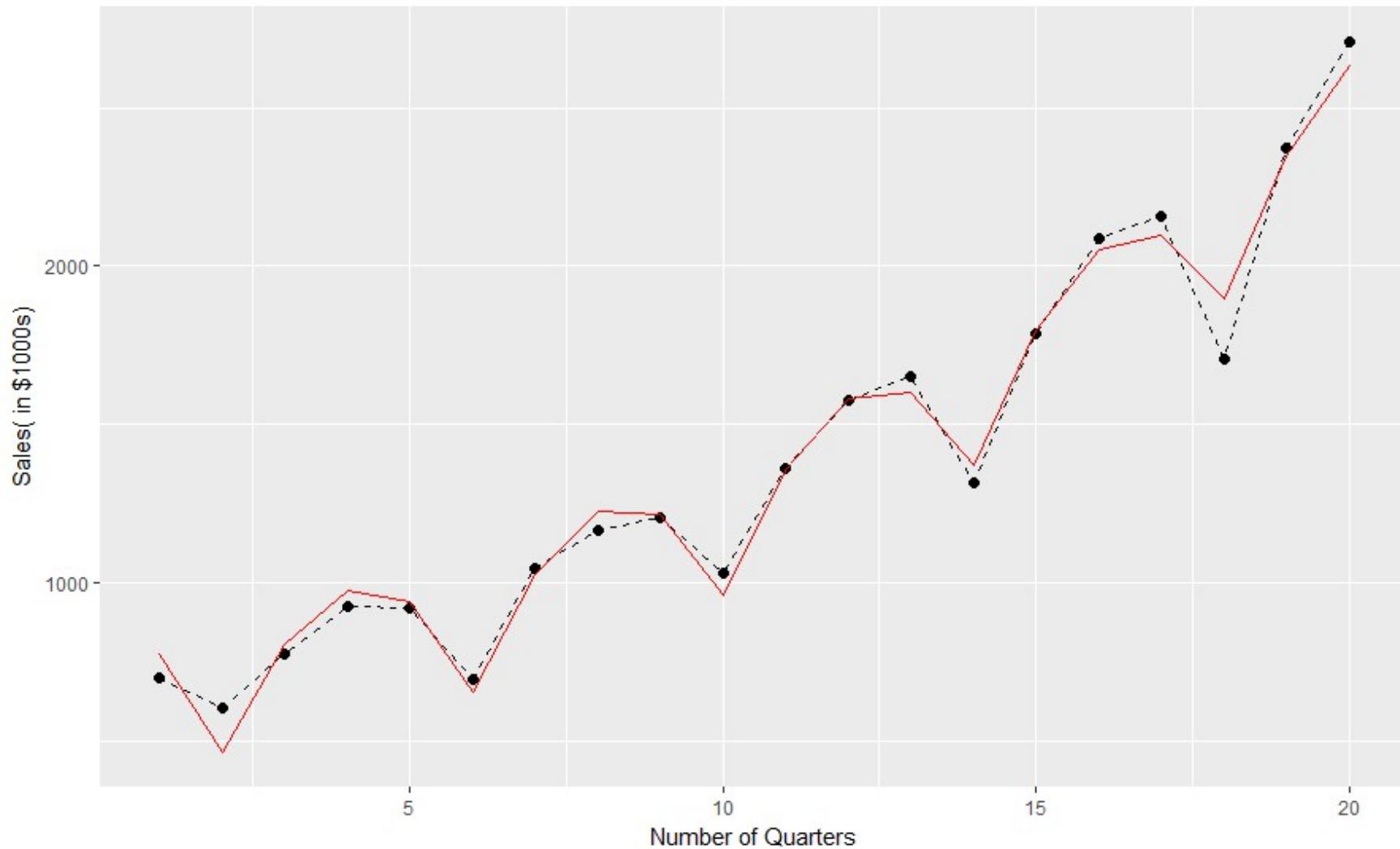
Quarter	Value of		
	X_{3t}	X_{4t}	X_{5t}
1	1	0	0
2	0	1	0
3	0	0	1
4	0	0	0

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \beta_5 X_{5t} + \epsilon_t$$

where, $X_{1t} = t$ and $X_{2t} = t^2$.



Seasonal Regression Models

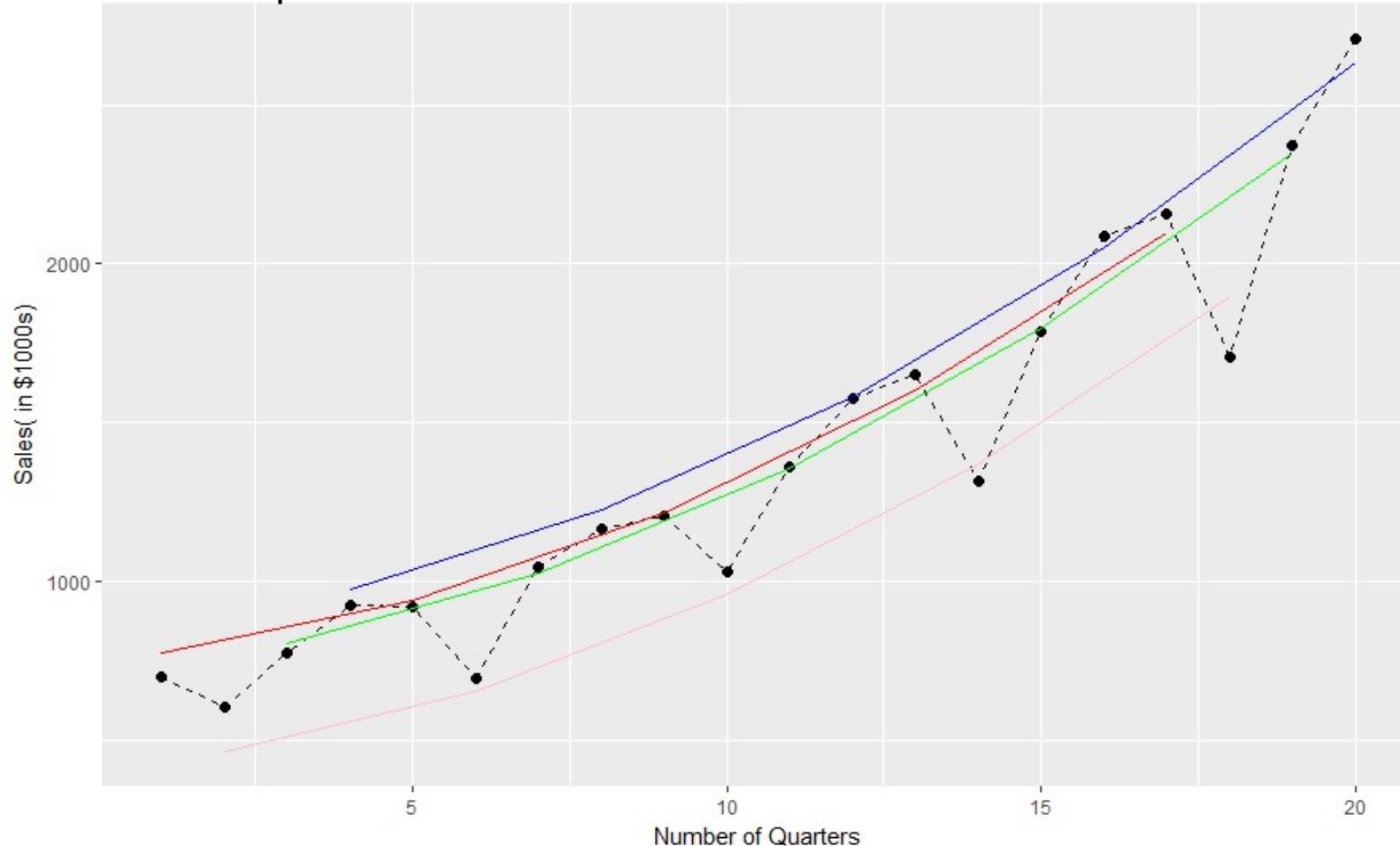


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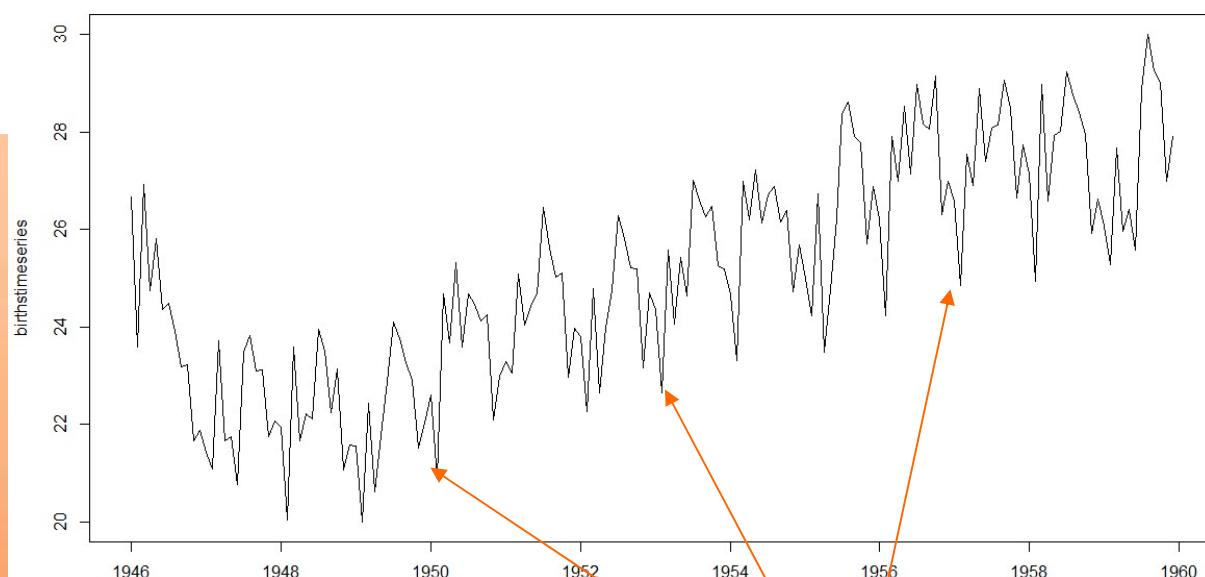


Quadratic fit with seasonality

Plotting the fitted data-points separately for each quarter, shows how R manages to do such a good fit. Its basically fitting a quadratic line for each quarter with a different intercept.



Births in NY



```
> birthstimeseries <- ts(births,
+                         frequency=12,
+                         start=c(1946,1))
> birthstimeseries
   Jan    Feb    Mar    Apr    May    Jun    Jul    Aug    Sep    Oct    Nov    Dec
1946 26.663 23.598 26.931 24.740 25.806 24.364 24.477 23.901 23.175 23.227 21.672 21.870
1947 21.439 21.089 23.709 21.669 21.752 20.761 23.479 23.824 23.105 23.110 21.759 22.073
1948 21.937 20.035 23.590 21.672 22.222 22.123 23.950 23.504 22.238 23.142 21.059 21.573
1949 21.548 20.000 22.424 20.615 21.761 22.874 24.104 23.748 23.262 22.907 21.519 22.025
1950 22.604 20.894 24.677 23.673 25.320 23.583 24.671 24.454 24.122 24.252 22.084 22.991
1951 23.287 23.049 25.076 24.037 24.430 24.667 26.451 25.618 25.014 25.110 22.964 23.981
1952 23.798 22.270 24.775 22.646 23.988 24.737 26.276 25.816 25.210 25.199 23.162 24.707
1953 24.364 22.644 25.565 24.062 25.431 24.635 27.009 26.606 26.268 26.462 25.246 25.180
1954 24.657 23.304 26.982 26.199 27.210 26.122 26.706 26.878 26.152 26.379 24.712 25.688
1955 24.990 24.239 26.721 23.475 24.767 26.219 28.361 28.599 27.914 27.784 25.693 26.881
1956 26.217 24.218 27.914 26.975 28.527 27.139 28.982 28.169 28.056 29.136 26.291 26.987
1957 26.589 24.848 27.543 26.896 28.878 27.390 28.065 28.141 29.048 28.484 26.634 27.735
1958 27.132 24.924 28.963 26.589 27.931 28.009 29.229 28.759 28.405 27.945 25.912 26.619
1959 26.076 25.286 27.660 25.951 26.398 25.565 28.865 30.000 29.261 29.012 26.992 27.897
> |
```

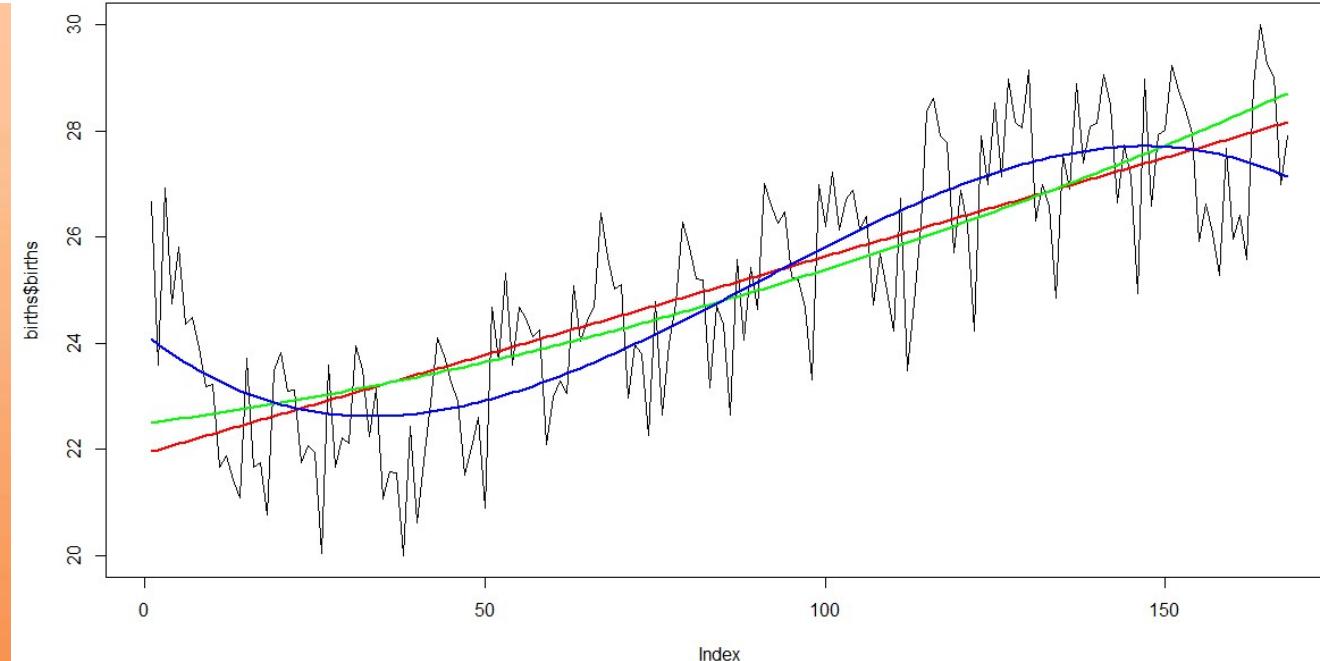
Seasonal Regression Models



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Seasonal Regression Models - Births

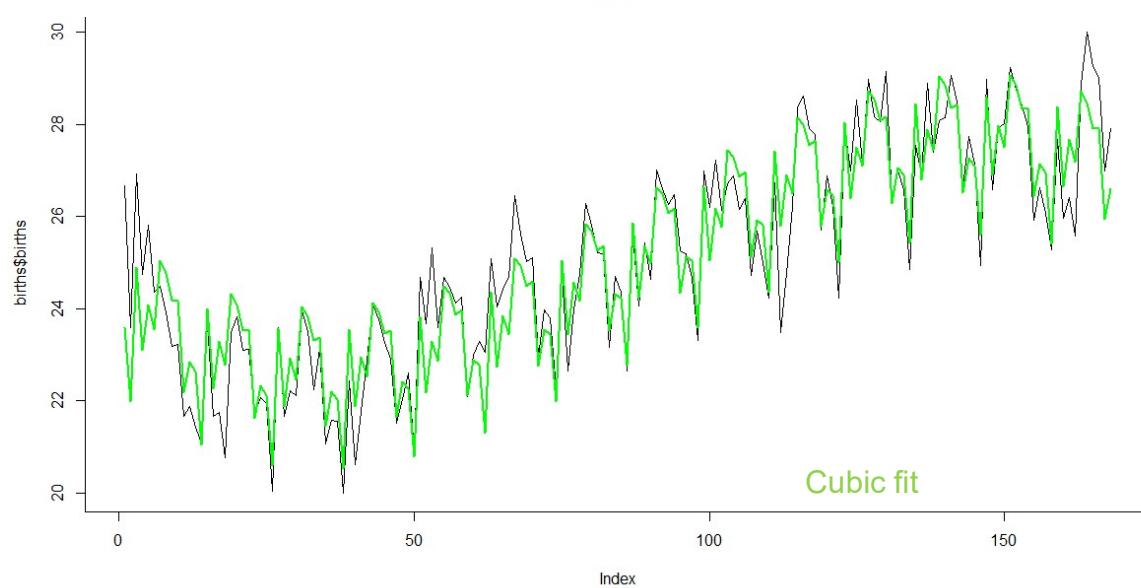
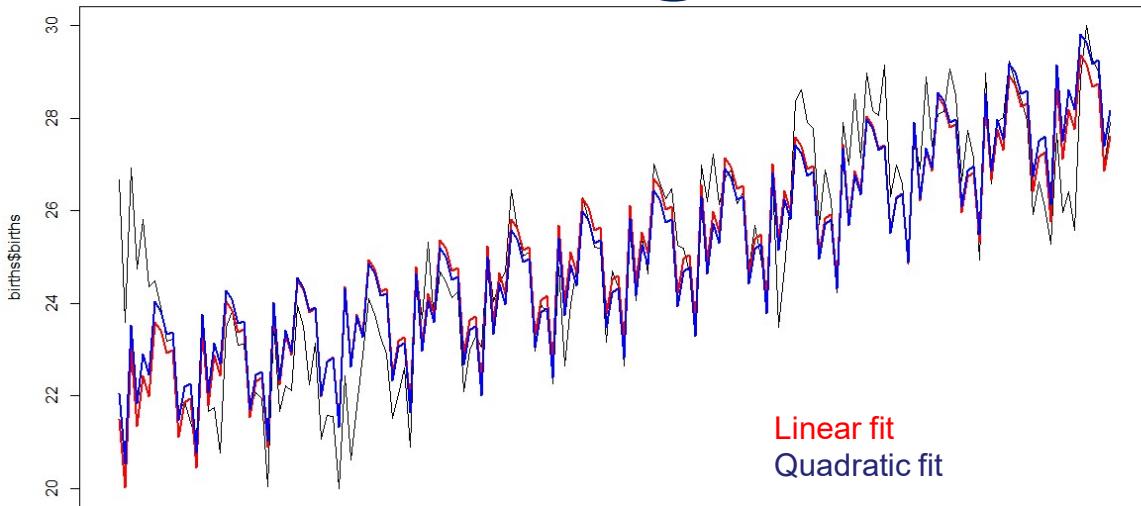


Data Editor						
	births	time	var3	var4	var5	var6
1	26.663	1				
2	23.598	2				
3	26.931	3				
4	24.74	4				
5	25.806	5				
6	24.364	6				
7	24.477	7				
8	23.901	8				
9	23.175	9				
10	23.227	10				
11	21.672	11				
12	21.87	12				
13	21.439	13				
14	21.089	14				
15	23.709	15				
16	21.669	16				
17	21.752	17				
18	20.761	18				
19	23.479	19				

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Seasonal Regression Models - Births



Data Editor

	births	time	seasonal	var4	var5
1	26.663	1	1		
2	23.598	2	2		
3	26.931	3	3		
4	24.74	4	4		
5	25.806	5	5		
6	24.364	6	6		
7	24.477	7	7		
8	23.901	8	8		
9	23.175	9	9		
10	23.227	10	10		
11	21.672	11	11		
12	21.87	12	12		
13	21.439	13	1		
14	21.089	14	2		
15	23.709	15	3		
16	21.669	16	4		
17	21.752	17	5		
18	20.761	18	6		
19	23.479	19	7		

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forecast package in R

- The *forecast* package in R contains several time-series forecasting methods.
- *tslm* function captures this break-up of linear model with seasonality

```
>  
> tslmfit <- tslm(birthstimeseries ~ poly(trend,3) + season)  
>
```

Another Simple Way of Incorporating Seasonality

- Take the trend prediction and actual prediction.
- Depending on additive or multiplicative model compute the deviation and map it as seasonality effect for each prediction.
- Take averages of the seasonality value. Use this to make future predictions.

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Case

Year	Quarter	Time variable (this is created)	Revenues (in \$M)
2008	I	1	10.2
	II	2	12.4
	III	3	14.8
	IV	4	15
2009	I	5	11.2
	II	6	14.3
	III	7	18.4
	IV	8	18



Call:
lm(formula = y ~ x)

Residuals:

Min	1Q	Median	3Q	Max
-3.5595	-0.9384	0.4405	1.3265	1.9286

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	10.0393	1.5531	6.464	0.00065 ***
x	0.9440	0.3076	3.069	0.02196 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.993 on 6 degrees of freedom

Multiple R-squared: 0.6109, Adjusted R-squared: 0.5461

F-statistic: 9.422 on 1 and 6 DF, p-value: 0.02196

What is the Regression equation?

$$y = 10.0393 + 0.9440x$$



Seasonality: Multiplicative

Time	Observed values TSI* (assuming no impact of cyclicality)	Predicted values (per the regression) T*	SI* = TSI/T
1	10.2	10.983	0.929
2	12.4	11.927	1.040
3	14.8	12.871	1.150
4	15.0	13.815	1.086
5	11.2	14.759	0.759
6	14.3	15.703	0.911
7	18.4	16.647	1.105
8	18.0	17.591	1.023

* T: Trend; S: Seasonal; I: Irregular



Quarterly Seasonality

Time	Average seasonality factor
Q1	$0.844 \left(= \frac{0.929+0.759}{2} \right)$
Q2	0.975
Q3	1.127
Q4	1.054

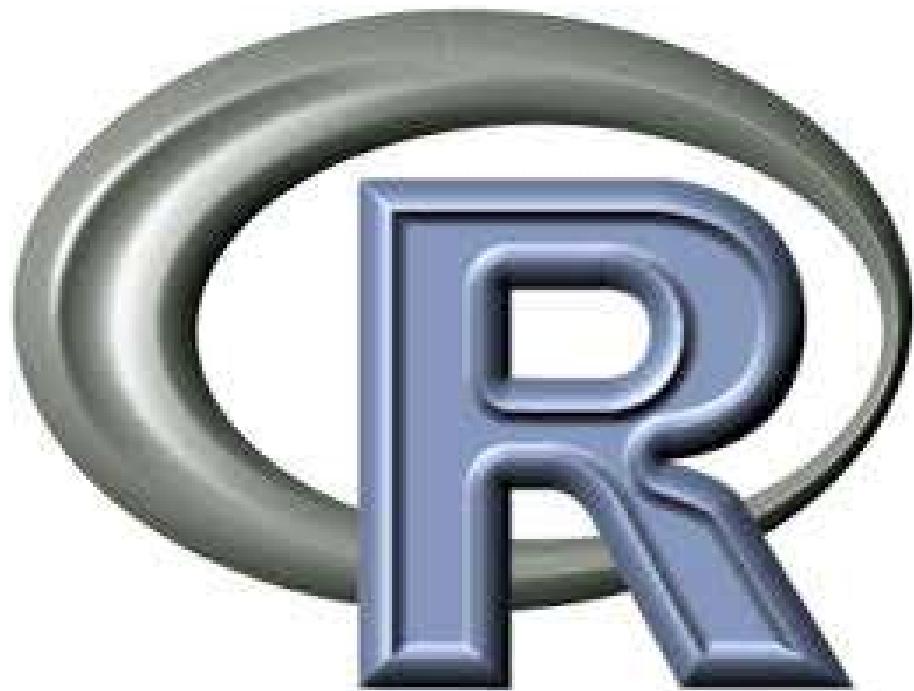
Time	Observed values TSI* (assuming no impact of cyclicalty)	Predicted values (per the regression) T*	SI* = TSI/T
1	10.2	10.983	0.929
2	12.4	11.927	1.040
3	14.8	12.871	1.150
4	15.0	13.815	1.086
5	11.2	14.759	0.759
6	14.3	15.703	0.911
7	18.4	16.647	1.105
8	18.0	17.591	1.023

Computations

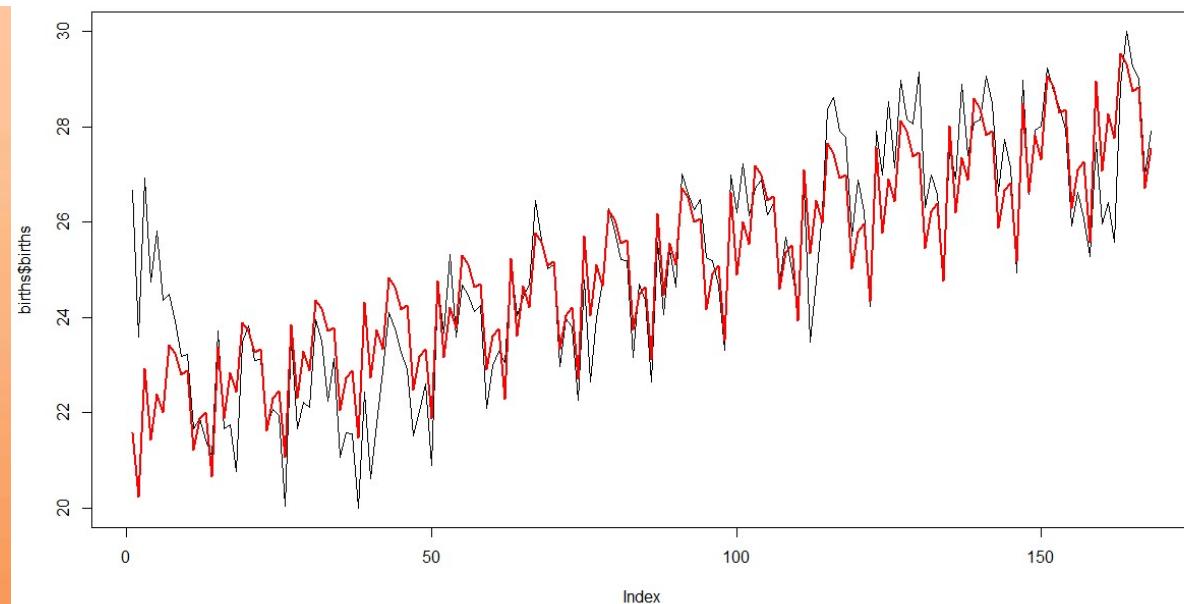
- Trend $Y_9 = 10.039 + 0.944(9) = 18.535$
- Corrected for seasonality and randomness: $18.535 * 0.844 = 15.643$

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Seasonality: Multiplicative

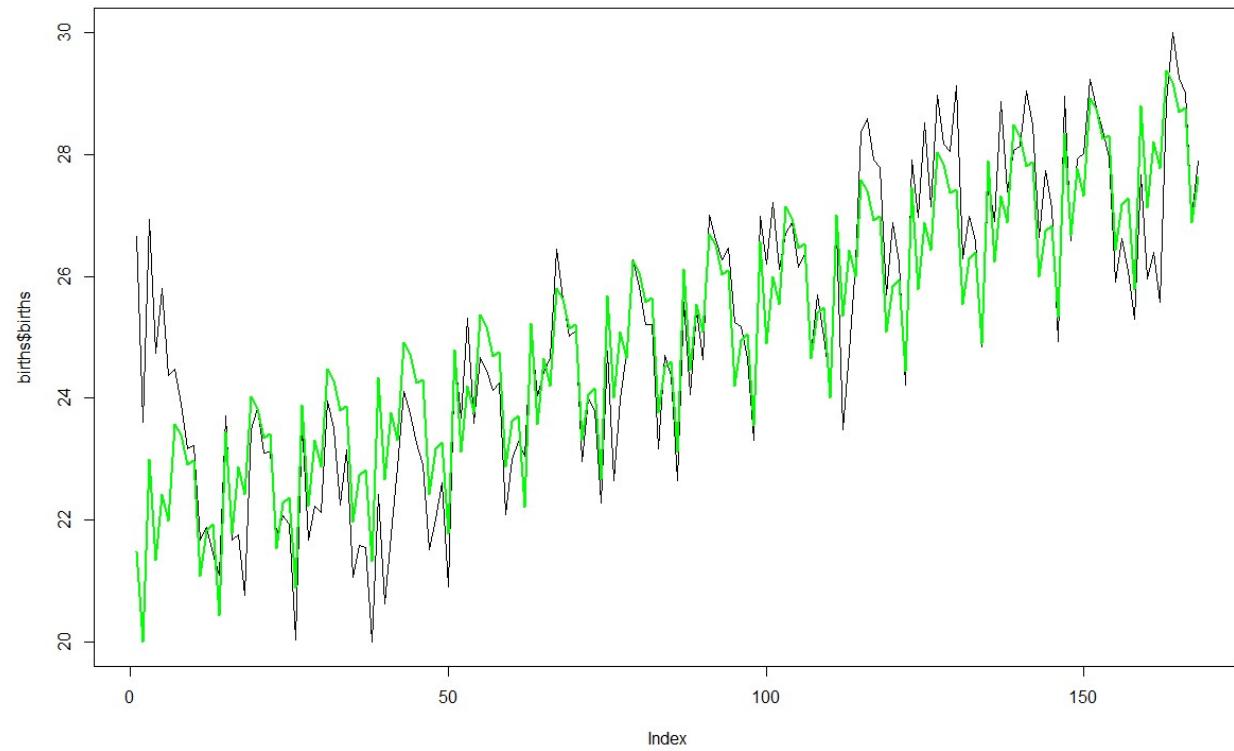


```
> births$SeasonalFactor <- births$births/predict(lm1)
> seasonalAdustFactor <- tapply(births$SeasonalFactor,
+                                     births$seasonal, mean)
> birthspr <- predict(lm1)*rep(seasonalAdustFactor,14)
> plot(births$births, type="l")
> points(births$time, birthspr, type="l", col="red", lwd=2)
```

births	time	seasonal	SeasonalFactor
26.663	1	1	1.2143042
23.598	2	2	1.0729008
26.931	3	3	1.2223736
24.740	4	4	1.1210359
25.806	5	5	1.1673742
24.364	6	6	1.1002941
24.477	7	7	1.1035459
23.901	8	8	1.0757752
23.175	9	9	1.0413570
23.227	10	10	1.0419543
21.672	11	11	0.9705802
21.870	12	12	0.9778208
21.439	13	1	0.9569611
21.089	14	2	0.9397800
23.709	15	3	1.0547879
21.669	16	4	0.9624398
21.752	17	5	0.9645349



Seasonality: Additive



```
> births$mae <- births$births-predict(lm1)
> seasonalAdd <- tapply(births$mae,
+                         births$seasonal, mean)
> birthspr <- predict(lm1)+rep(seasonalAdd,14)
> plot(births$births, type="l")
> points(births$time, birthspr, type="l", col="green", lwd=2)
>
```

Data Editor

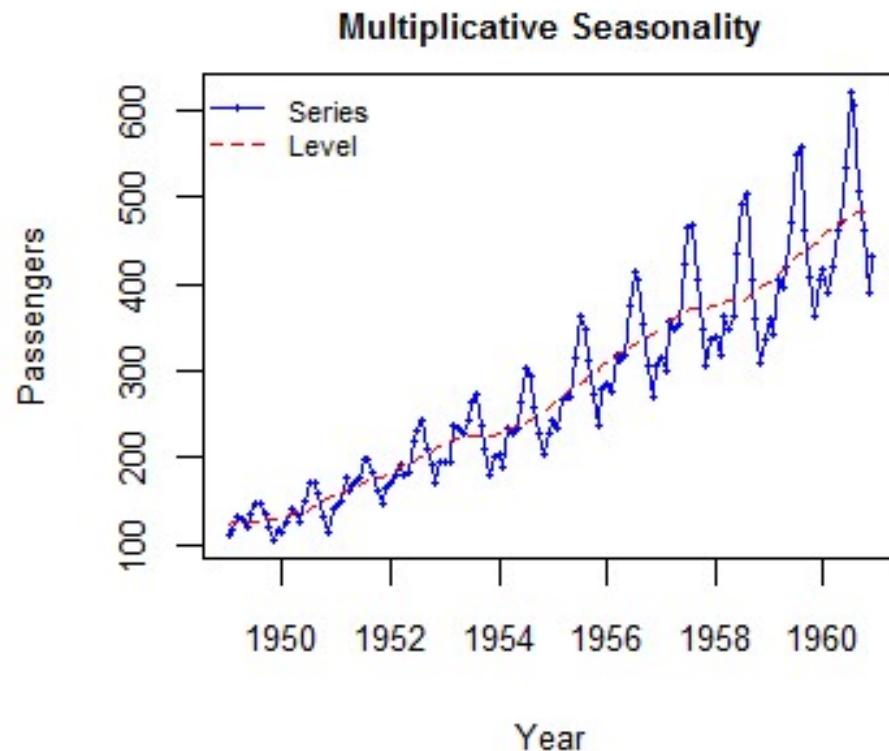
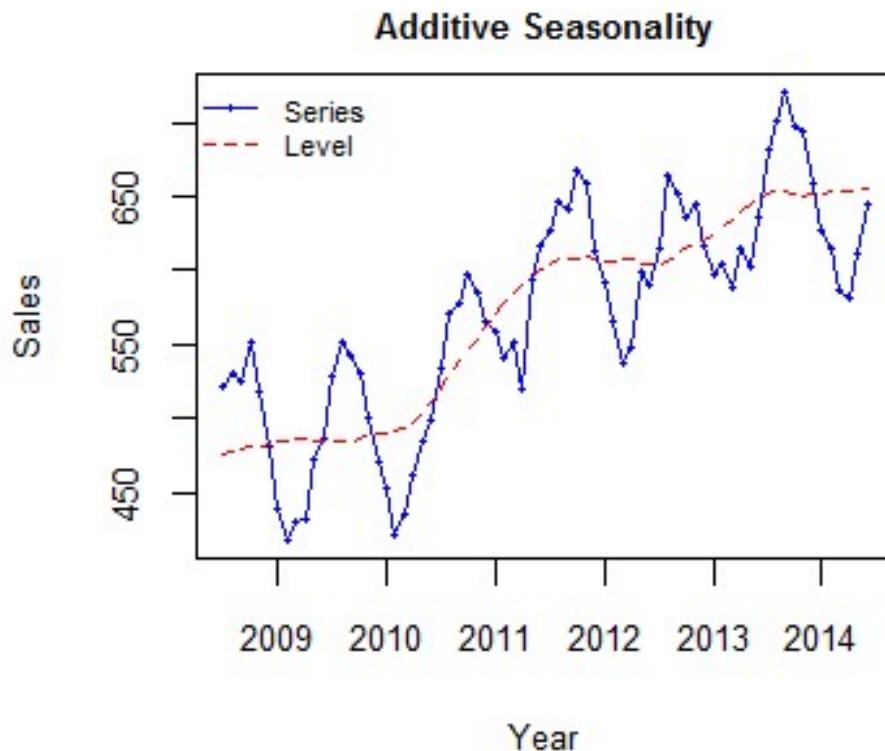
File Edit Help

	births	time	seasonal	mae
1	26.663	1	1	4.70557
2	23.598	2	2	1.603422
3	26.931	3	3	4.899274
4	24.74	4	4	2.671125
5	25.806	5	5	3.699977
6	24.364	6	6	2.220829
7	24.477	7	7	2.29668
8	23.901	8	8	1.683532
9	23.175	9	9	0.920384
10	23.227	10	10	0.9352357
11	21.672	11	11	-0.6569126
12	21.87	12	12	-0.4960608
13	21.439	13	1	-0.9642091
14	21.089	14	2	-1.351357
15	23.709	15	3	1.231494
16	21.669	16	4	-0.8456539
17	21.752	17	5	-0.7998021
18	20.761	18	6	-1.82795
19	23.479	19	7	0.8529014

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Additive or Multiplicative



Source: <http://www.forsoc.net/2014/11/11/can-you-identify-additive-and-multiplicative-seasonality/>

Goodness of Fit

- MSE (Mean square error)
- MAE (Mean absolute error)
- RMSE (Root mean square error)
- MAPE (Mean absolute percent error)
- NMSE (Normalized mean square error)
- NMAE (Normalized mean absolute error)
- NMAPE (Normalized mean absolute percent error)



Issues with Regressing on Time

- If there is no trend or if seasonality and fluctuations are more important than trend, then the coefficients behave weirdly

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TIME SERIES: AUTO REGRESSIVE METHODS

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Auto Regressive Methods

$$\hat{y}_{t+1} = f(y_t, y_{t-1}, y_{t-2} \dots)$$

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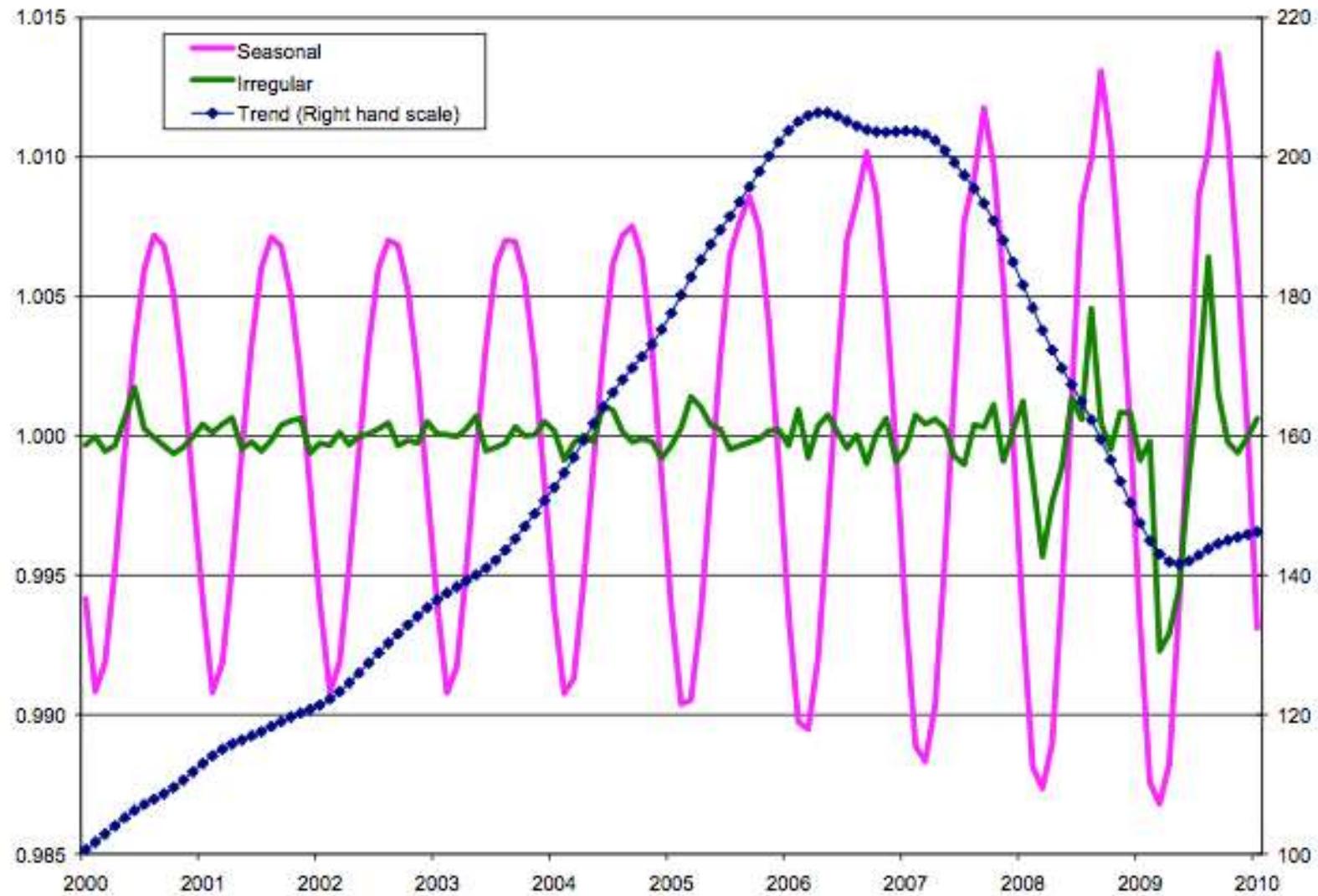


Components of time series

- We use different techniques for time series with different characteristics
 - Trend
 - Seasonal
 - Random stationary
- First we need to identify them



Trend, Seasonality and Randomness



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Time Series Descriptive Statistics

- In descriptive statistics covered earlier (central tendencies, measures of variability, skewness, kurtosis, distributions, correlations, etc.), the order of observations in the data was of no consequence.
- In time series descriptive statistics, order of observations is of primary importance and so autocorrelations, etc. play a vital role in identifying the models and their characteristics.
- Autocorrelation is a metric that allows us to understand the strength of order in the time-series

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AUTOCORRELATION AND PARTIAL AUTOCORRELATION

Autocorrelation (ACF) and Partial ACF (PACF)

- ACF: n^{th} lag of ACF is the correlation between a day and n days before that.
- PACF: The same as ACF with all intermediate correlations removed. It is the k^{th} coefficient of the ordinary least squares regression.

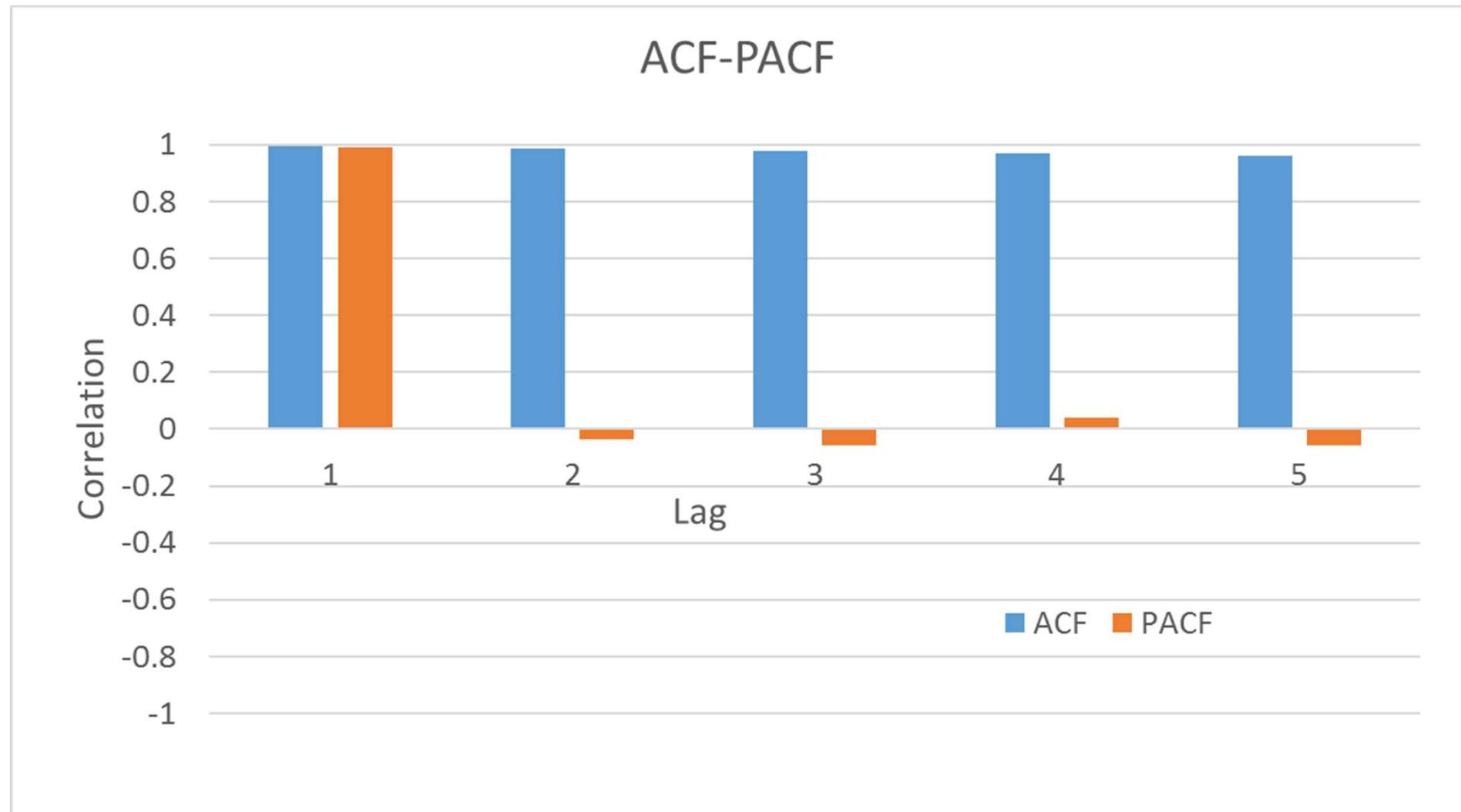
$$[y_t] = \beta_0 + \sum_{i=1}^k \beta_i [y_{t-i}] \text{ where}$$

$[y_t]$ is the input time series, k is the lag order and β_i is the i^{th} coefficient of the linear multiple regression.

EXCEL ACTIVITY



Autocorrelation (ACF) and Partial ACF (PACF)

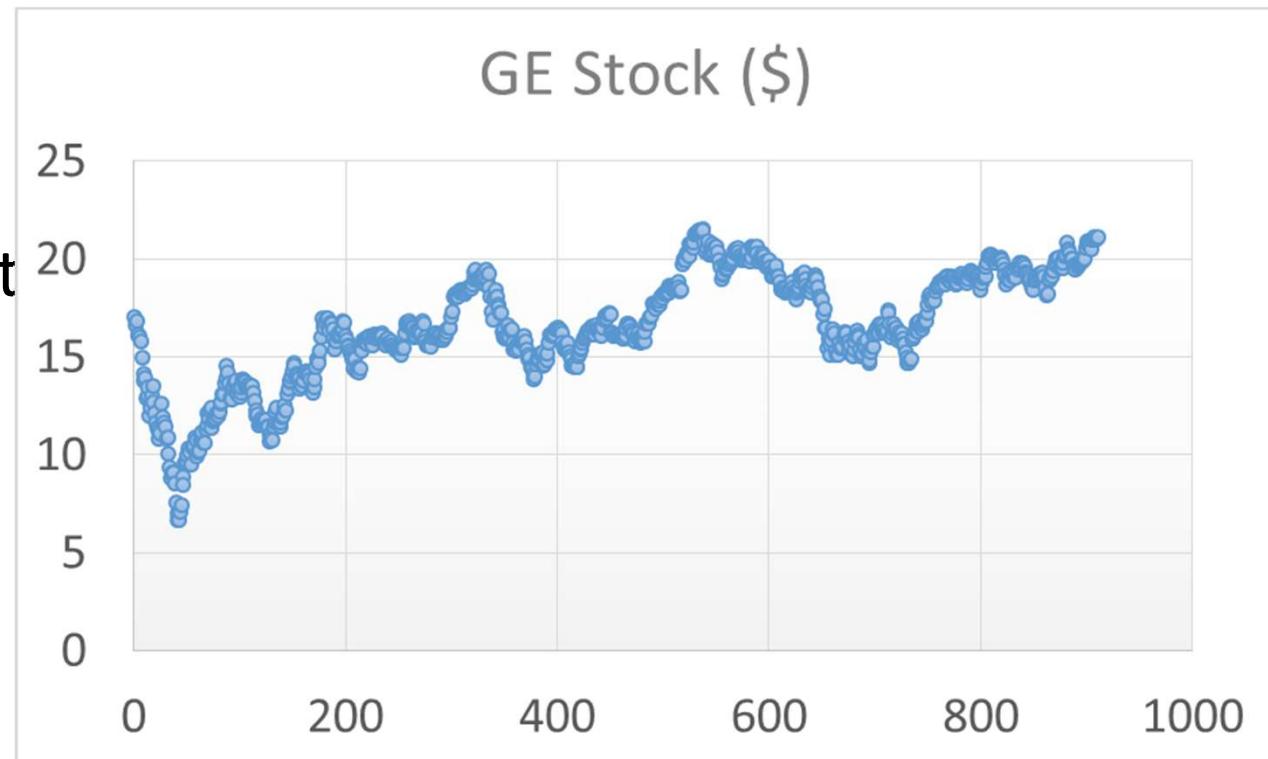


See the attached file 01Correlations.xlsx



Components of Time Series

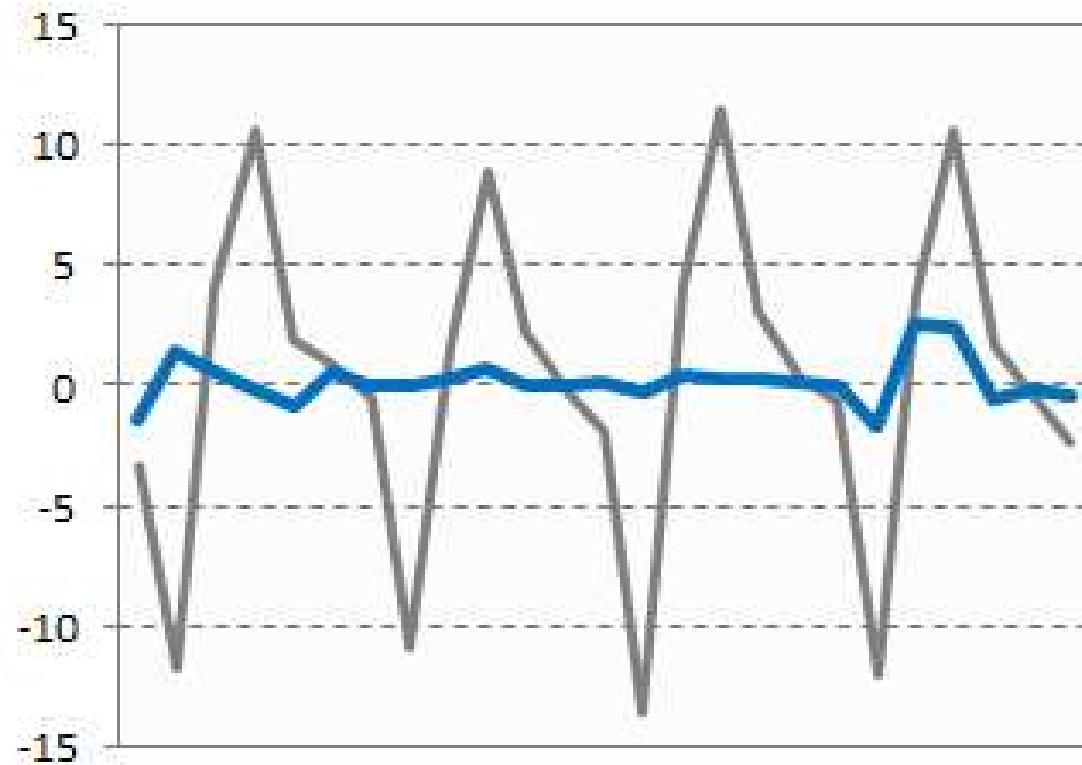
- Trend
- Seasonality
- Random component



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Seasonality



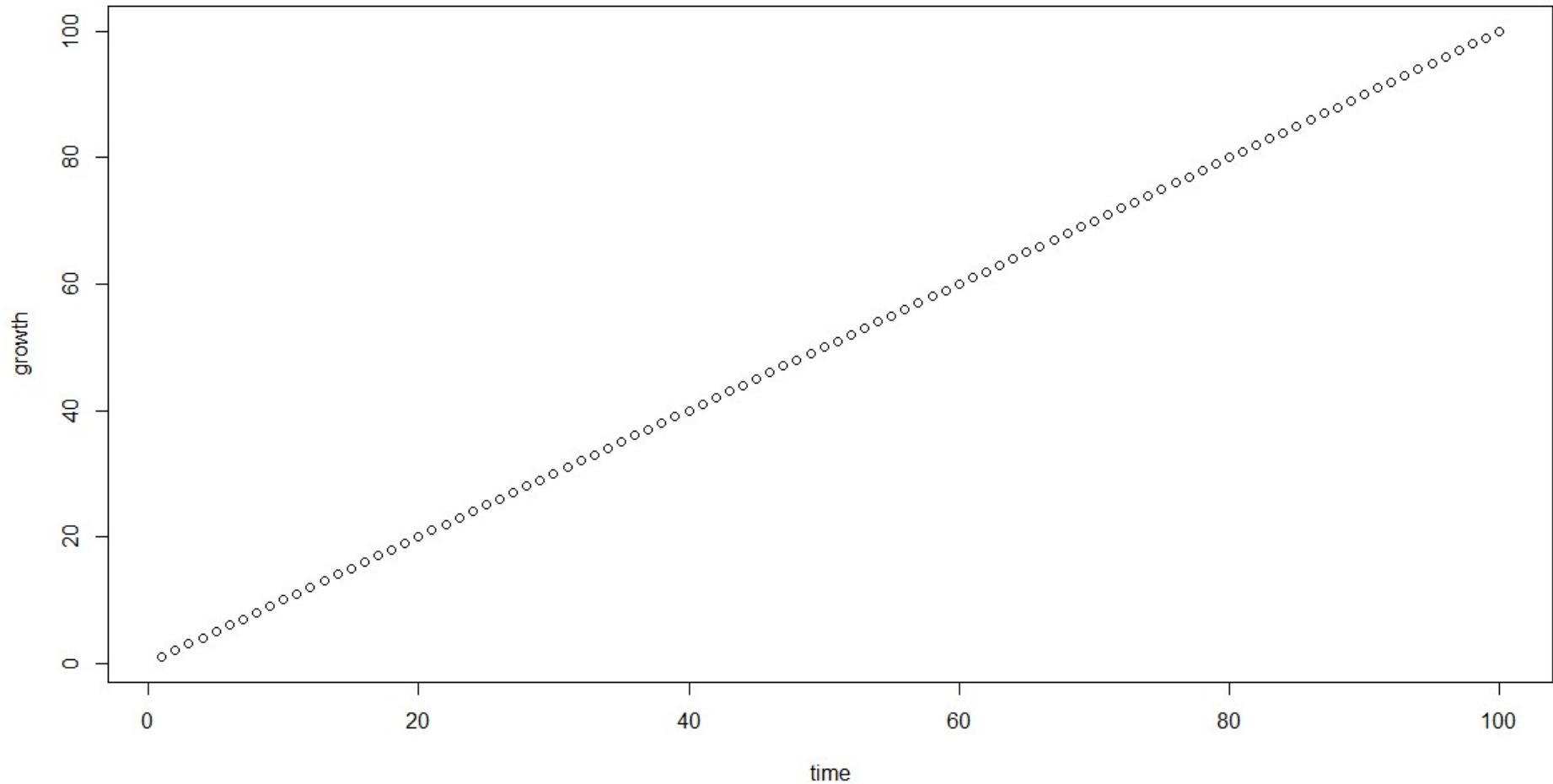
ACF and PACF – Idealized Trend, Seasonality and Randomness



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ACF and PACF – Idealized Trend

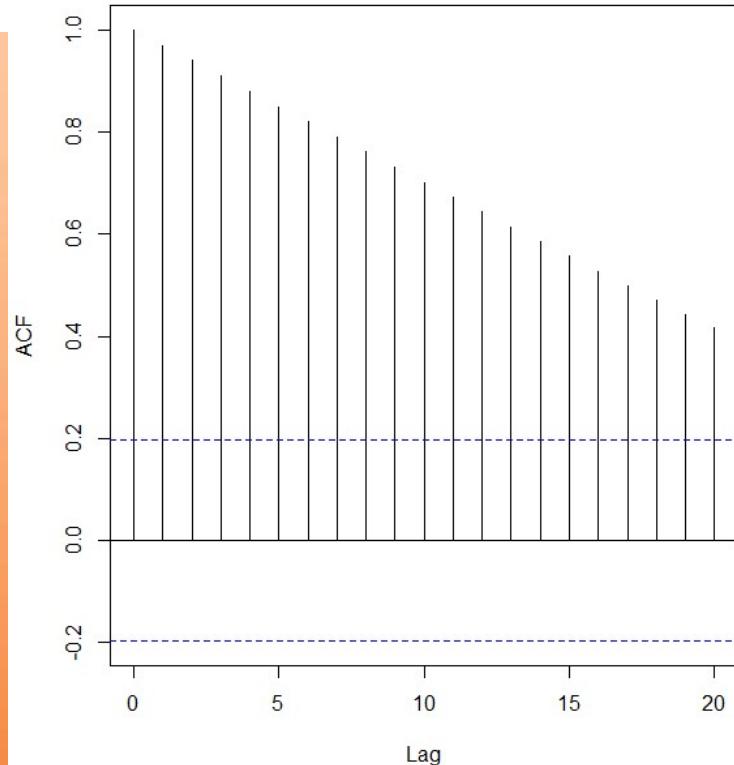


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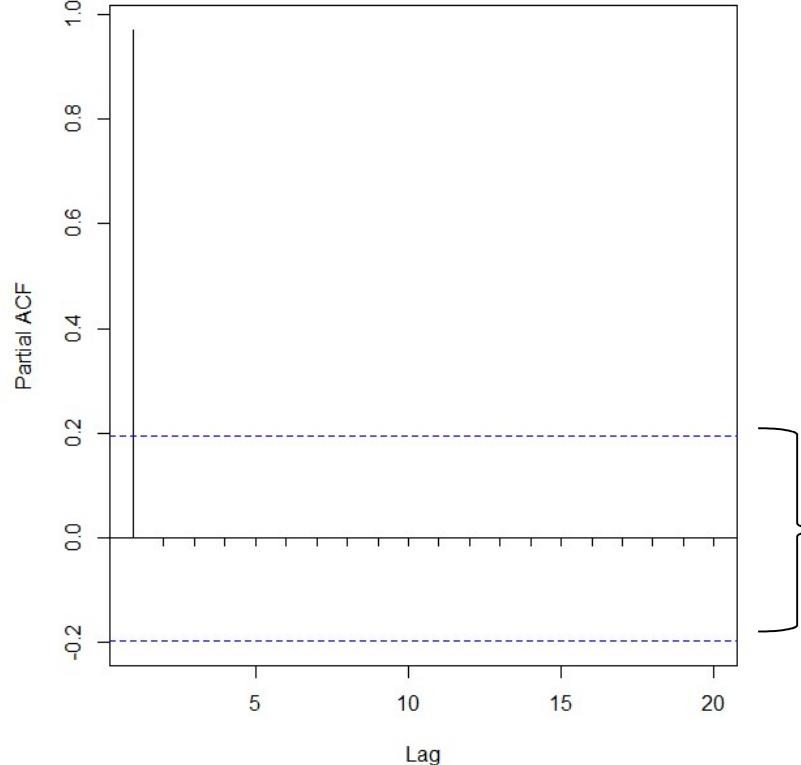


ACF and PACF – Idealized Trend

Series growth



Series growth



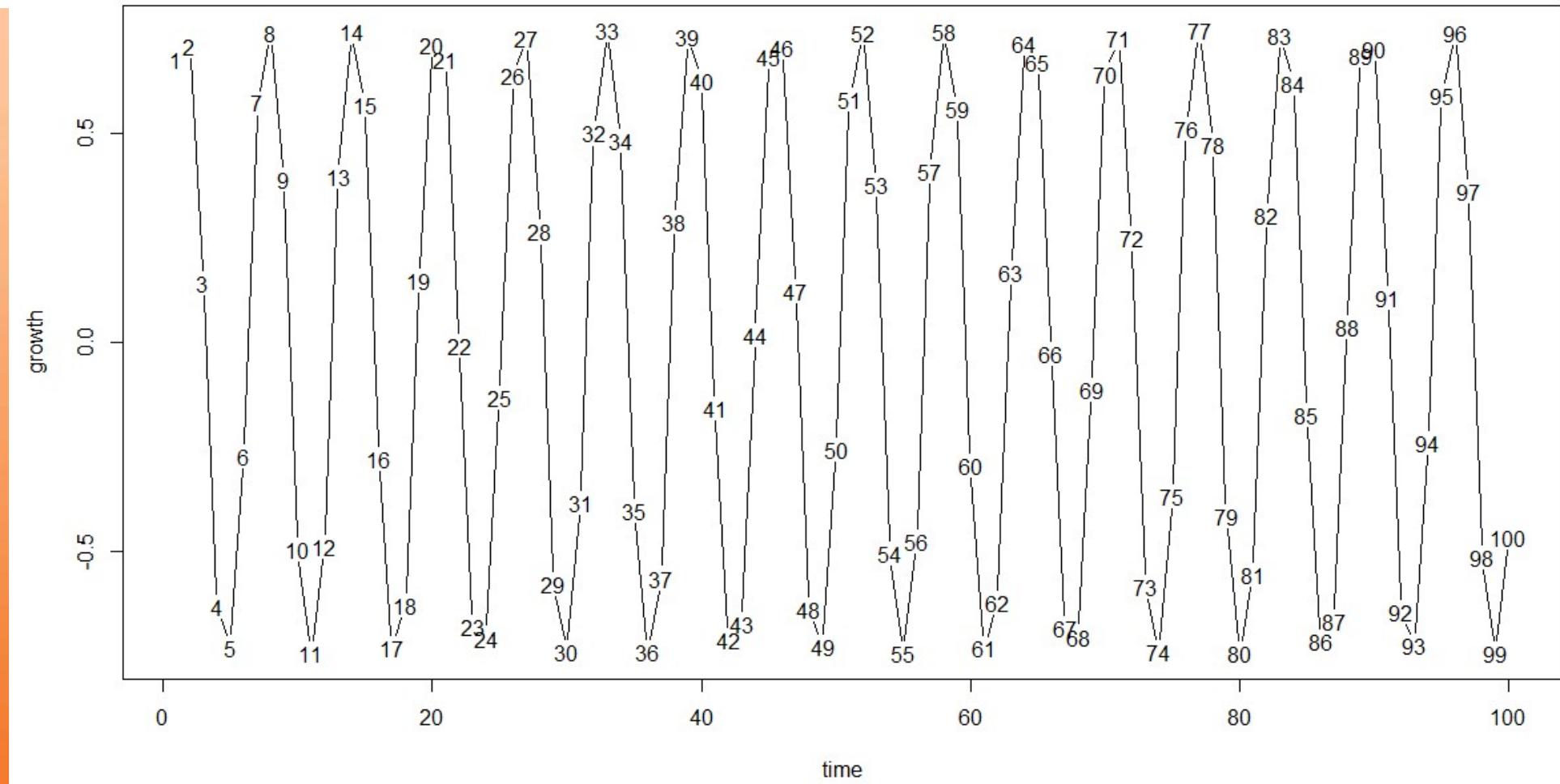
$$95\% \text{ CI: } 0 \pm \frac{1.96}{\sqrt{n}}$$

- ACF is a bar chart of correlation coefficients of the time series and its lags.
- PACF is a plot of the partial correlation coefficients of the time series and its lags.

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ACF and PACF – Idealized Seasonality

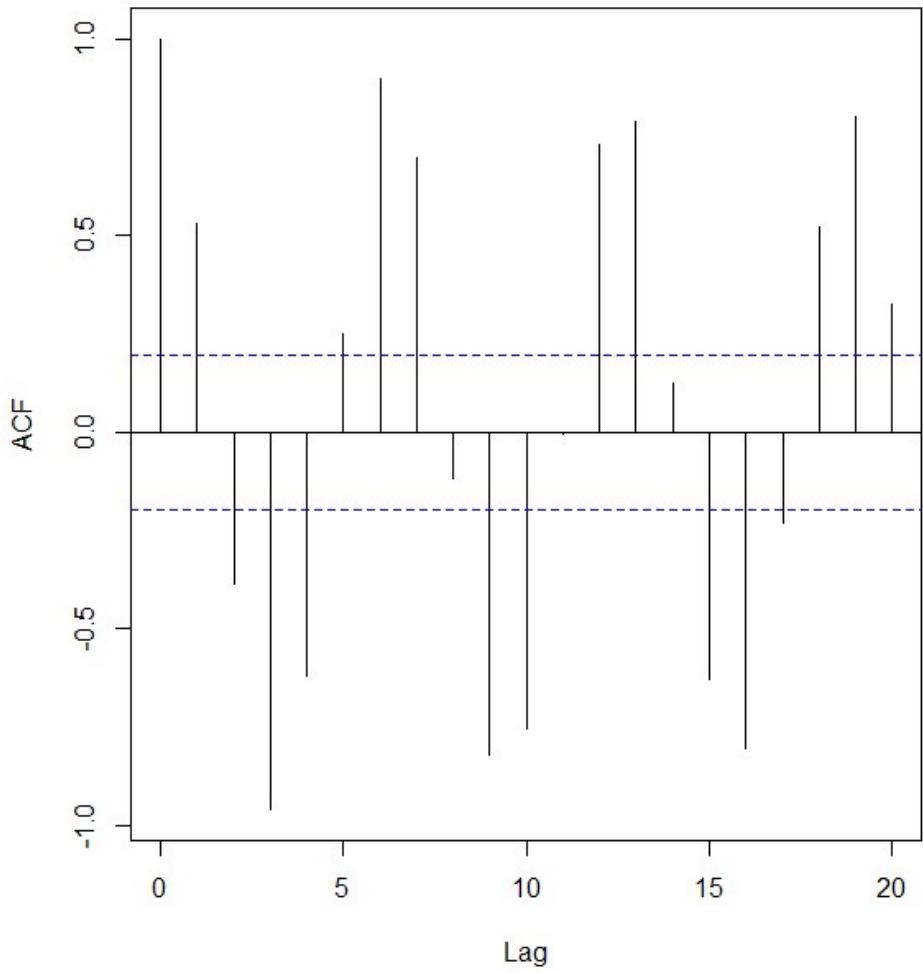


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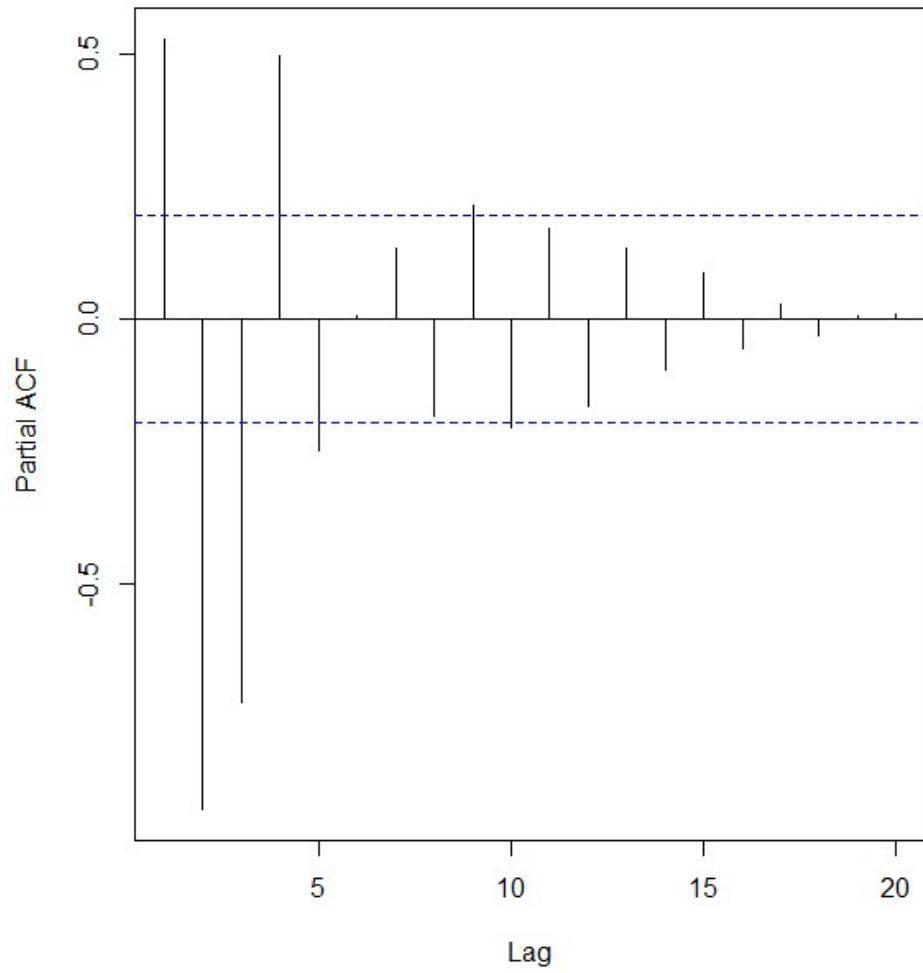


ACF and PACF – Idealized Seasonality

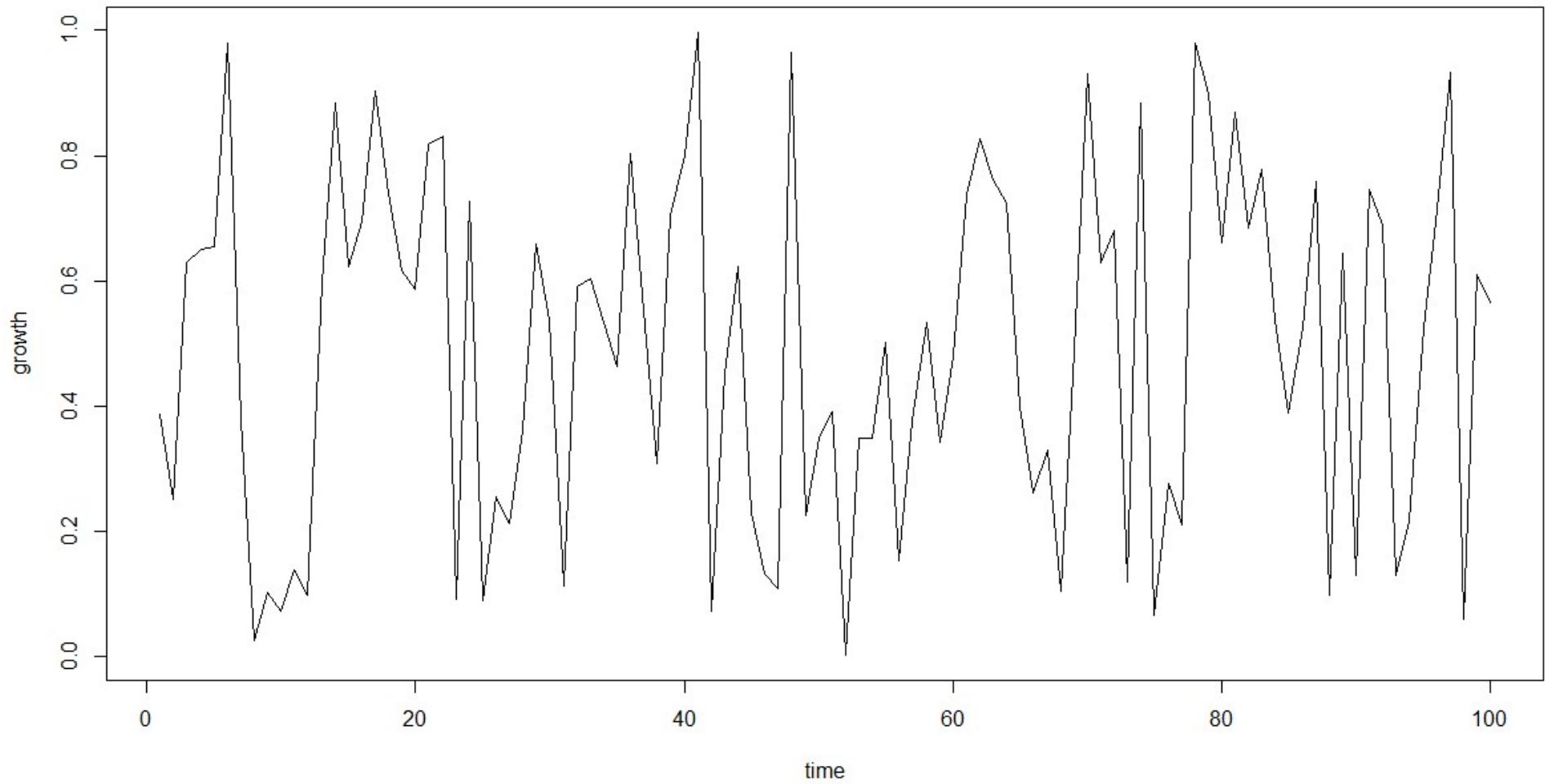
Series growth



Series growth



ACF and PACF – Idealized Randomness

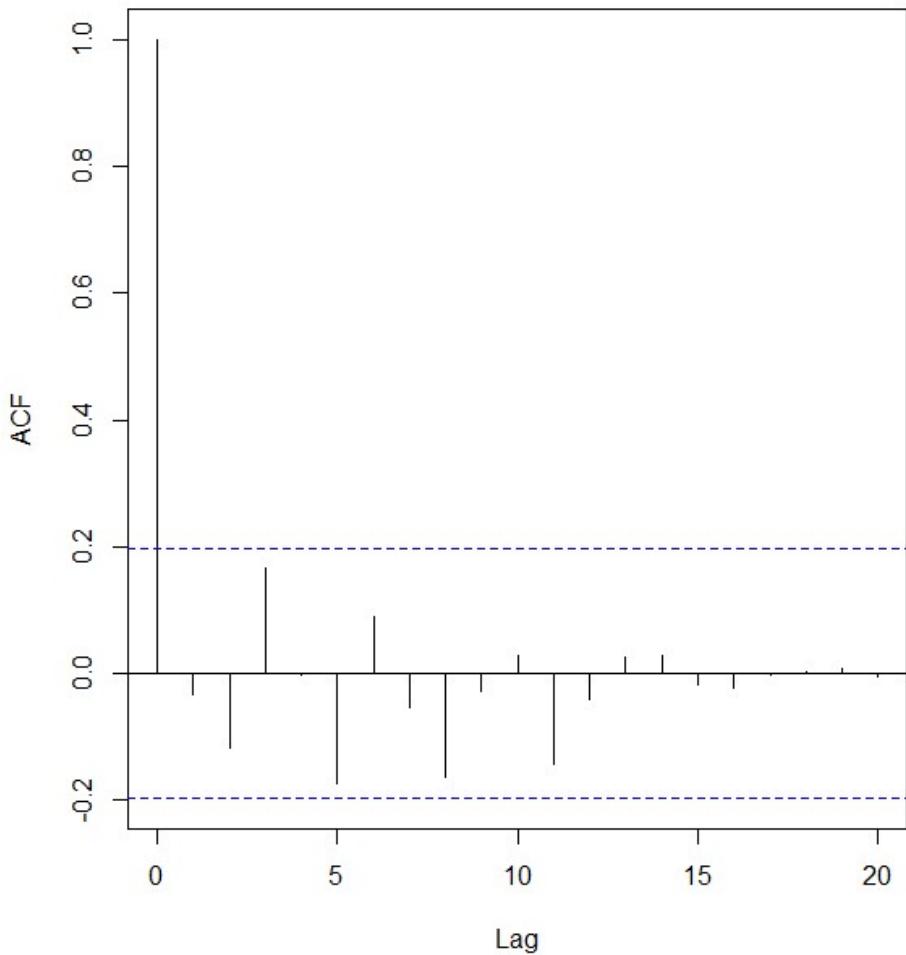


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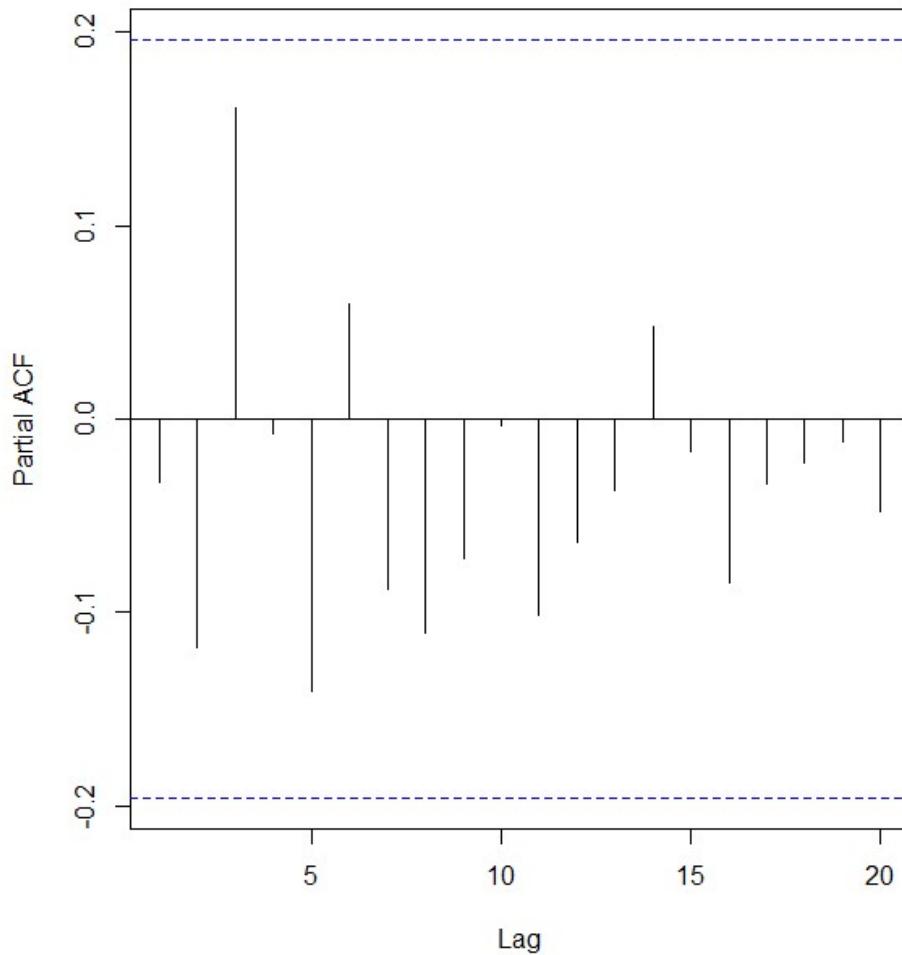


ACF and PACF – Idealized Randomness

Series growth



Series growth



ACF and PACF – Idealized Trend, Seasonality and Randomness

- Ideal Trend: Decreasing ACF and 1 or 2 lags of PACF
- Ideal Seasonality: Cyclical in ACF and a few lags of PACF with some positive and some negative
- Ideal Random: A spike may or may not be present; even if present, magnitude will be small

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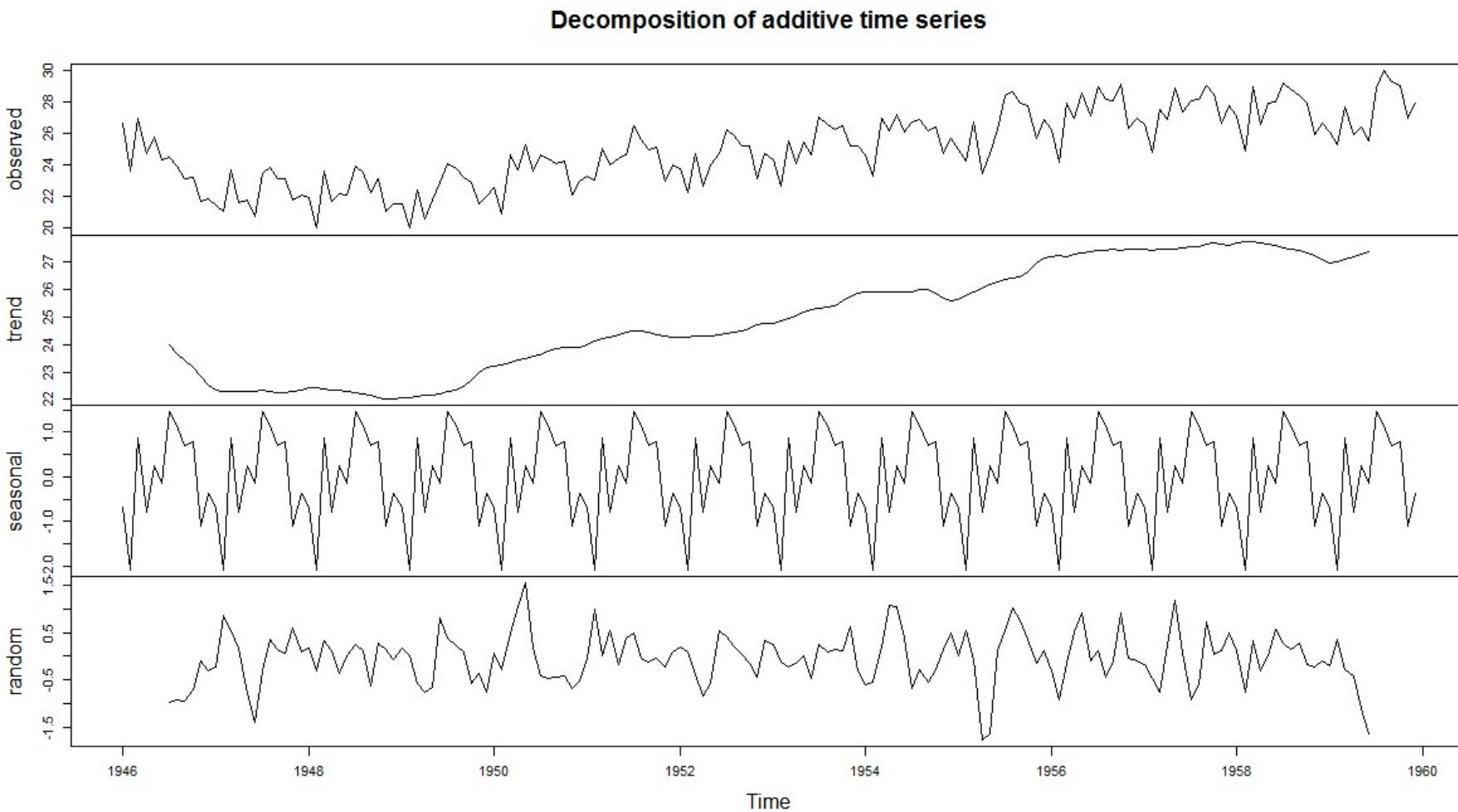
ACF and PACF (Real-world): Decomposing Time Series into the 3 Components



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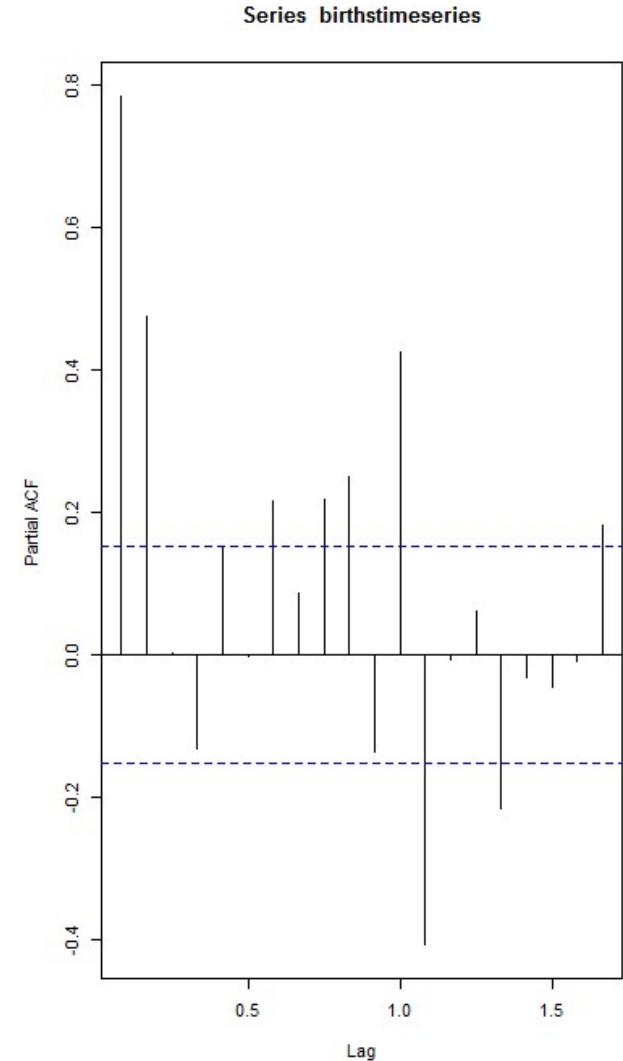
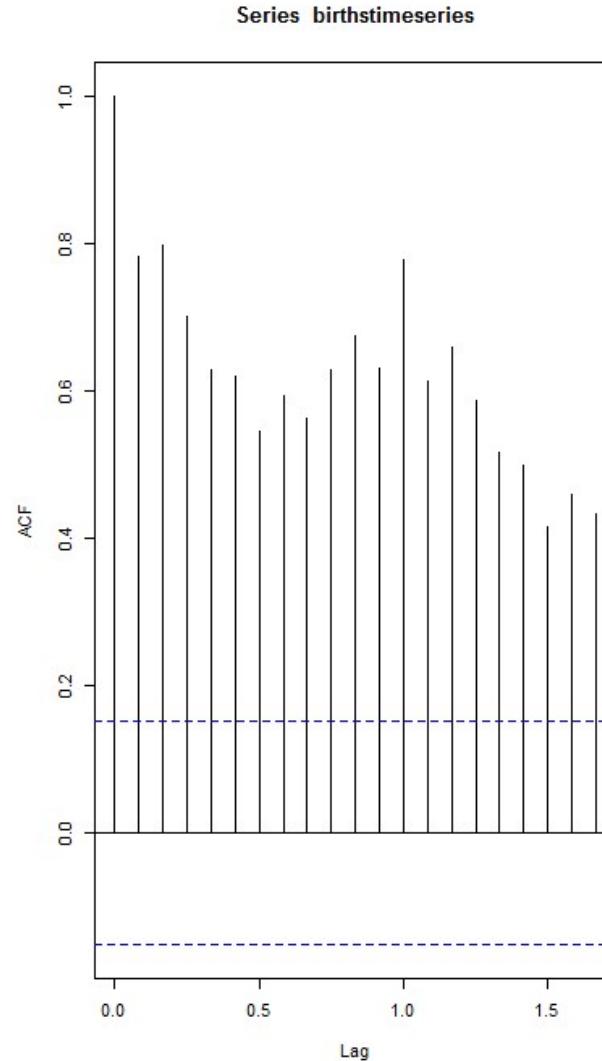
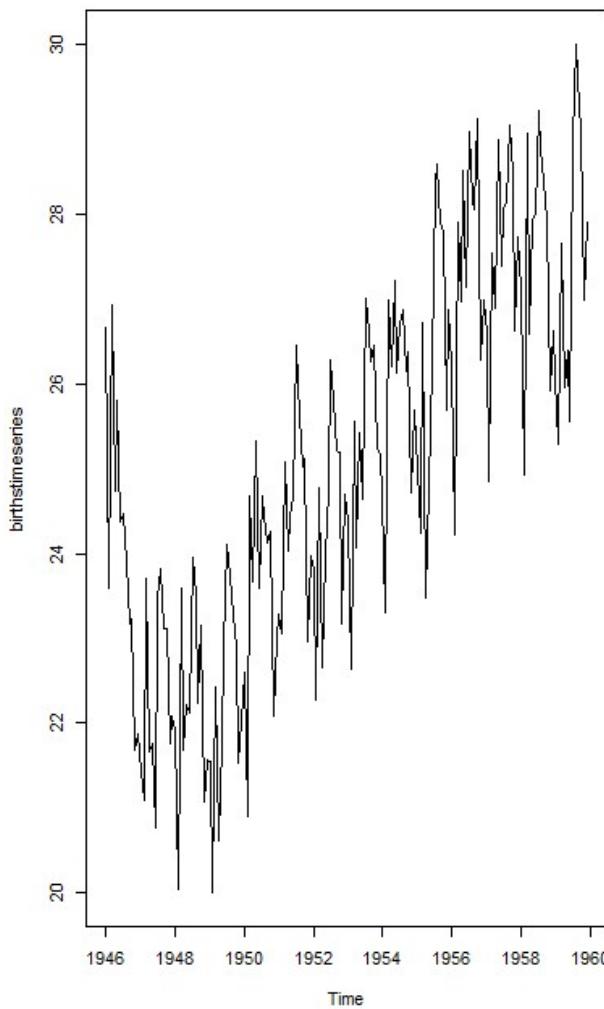
ACF and PACF (Real-world): Decomposing Time Series into the 3 Components – Births in NY



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ACF and PACF (Real-world): Decomposing Time Series into the 3 Components – Births in NY



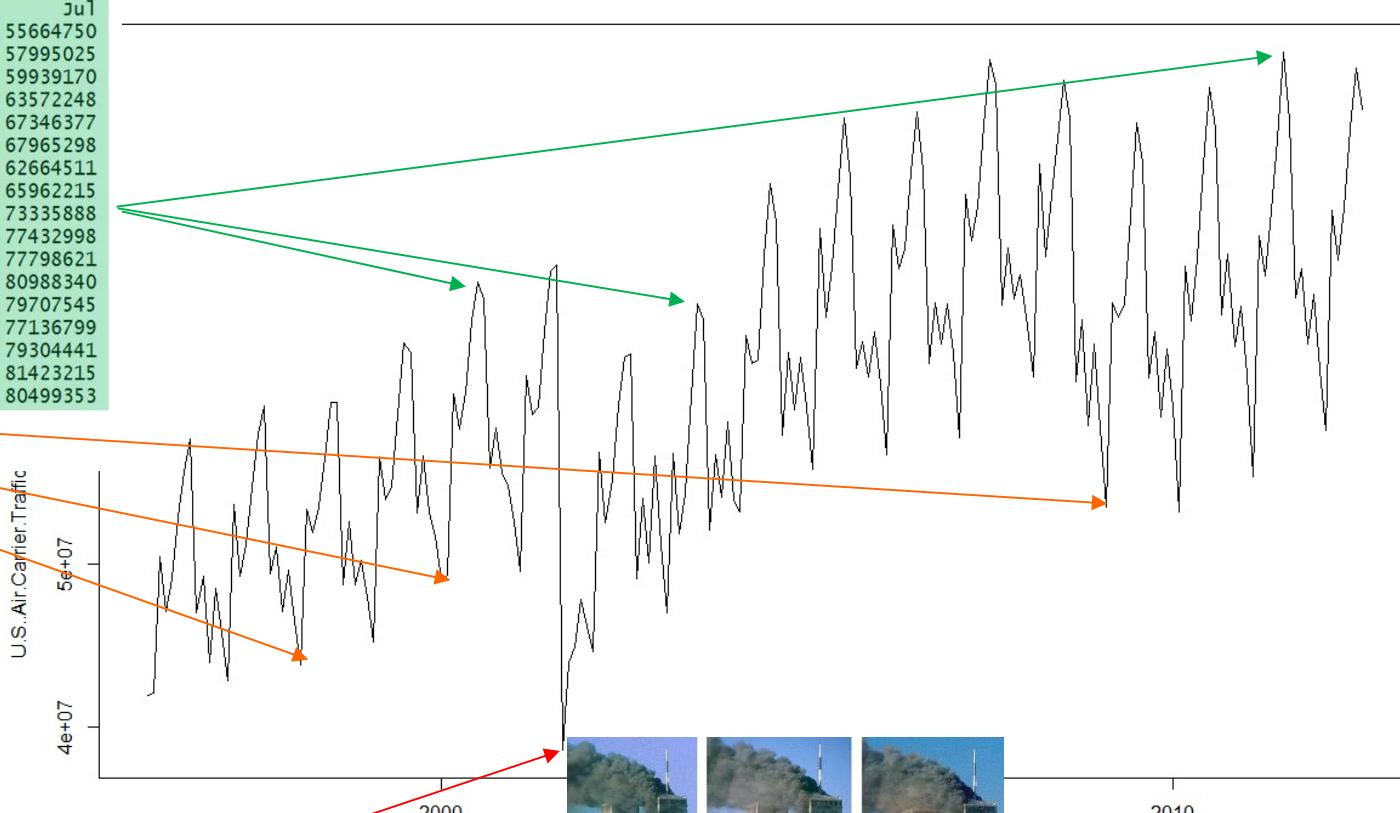
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US Air Carrier Traffic – Revenue Passenger Miles ('000)

RPM

	Jan	Feb	Mar	Apr	May	Jun	Jul
1996	41972194	42054796	50443045	47112397	49118248	52880510	55664750
1997	45850623	42838949	53620994	49282817	51191842	54707221	57995025
1998	46514139	43769273	53361926	51968480	53515798	56460422	59939170
1999	47988560	45241211	56555731	53920855	54674958	59213000	63572248
2000	49045412	49306303	60443541	58286680	60533783	64903295	67346377
2001	52634354	49532578	61575055	59151645	59662416	64353323	67965298
2002	46224031	44615129	56897729	52542164	55116060	59745343	62664511
2003	51197175	47040806	56766580	51857453	54335598	60272900	65962215
2004	53979786	53179693	64035864	62340117	62530704	68866398	73335888
2005	59629608	55795165	70595861	65145552	68268899	72952959	77432998
2006	61035027	56729212	70799794	68120559	69352606	74099239	77798621
2007	63016013	57793832	72700241	69836156	71933109	76926452	80988340
2008	64667106	61504426	74575531	68906882	72725750	76162105	79707545
2009	58373786	53506580	66027341	65166300	65868254	71350227	77136799
2010	59651061	53240066	68307090	64953250	68850904	74474550	79304441
2011	61630362	55391206	70158268	67683558	71711448	76057910	81423215
2012	61940180	58243763	71696039	68669228	71887523	76760759	80499353
	Aug	Sep	OCT	Nov	Dec		
1996	57723208	47035464	49263120	43937074	48539606		
1997	59715433	49418190	51058879	47056048	49654209		



5

Data sources:

http://www.bts.gov/xml/air_traffic/src/index.xml

and <https://datamarket.com/data/set/281x/us-air-carrier-traffic-statistics-revenue-passenger-miles>

Last accessed: 31-Mar-2016

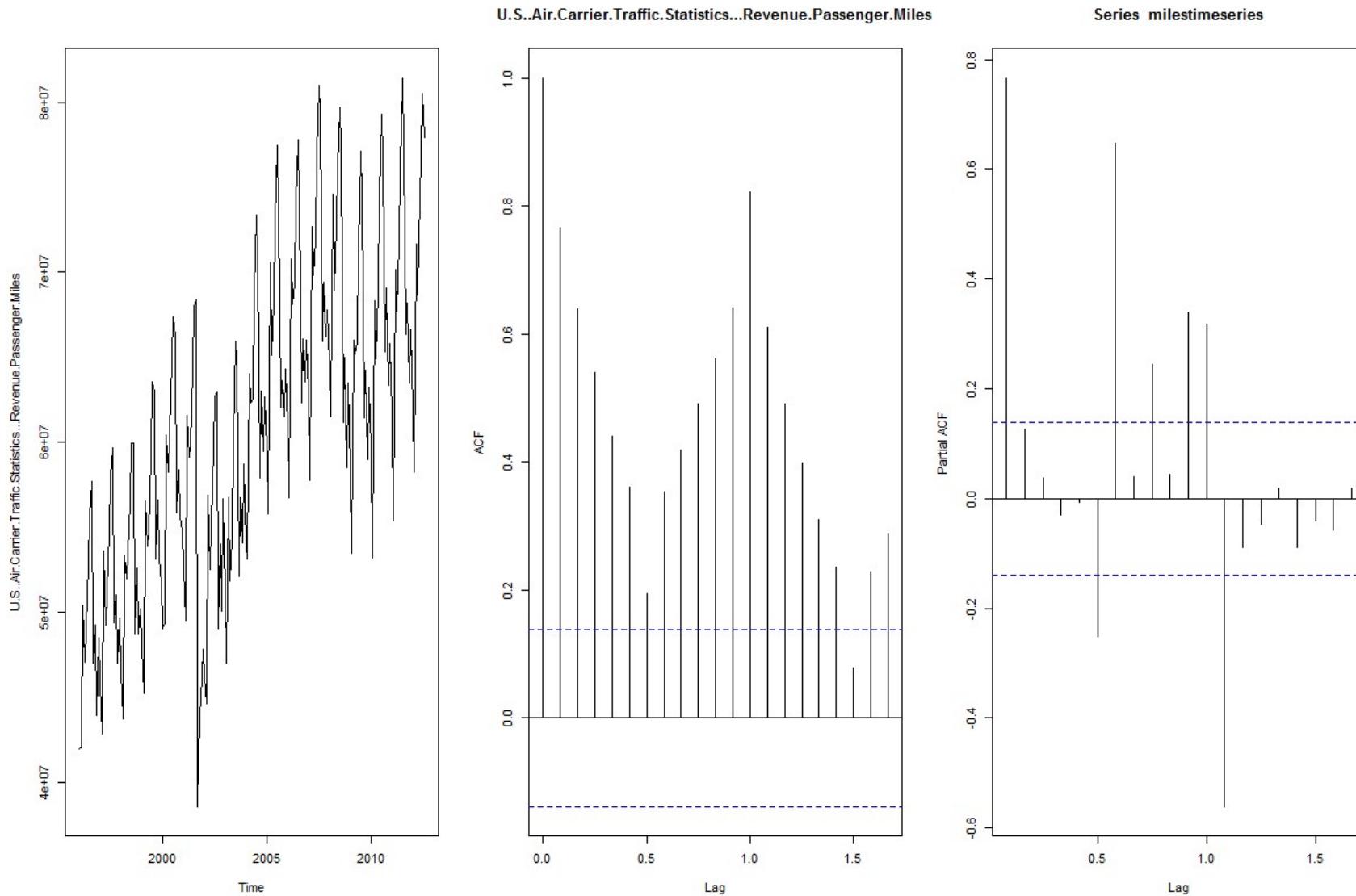
	Aug	Sep	Oct	Nov	Dec
1996	57723208	47035464	49263120	43937074	48539606
1997	59715433	49418190	51058879	47056048	49654209
1998	59927214	48751280	52578217	48734375	50208641
1999	63003663	53131972	56653901	53215500	51746821
2000	66256804	55900504	58373996	55590325	54822970
2001	68377080	38601868	43964788	44915764	47836501
2002	62944816	49096035	54019748	50106814	56656594
2003	64989766	52121480	56724551	54128776	58739845
2004	70961522	57881042	63021142	59453943	62680310



2000 2010



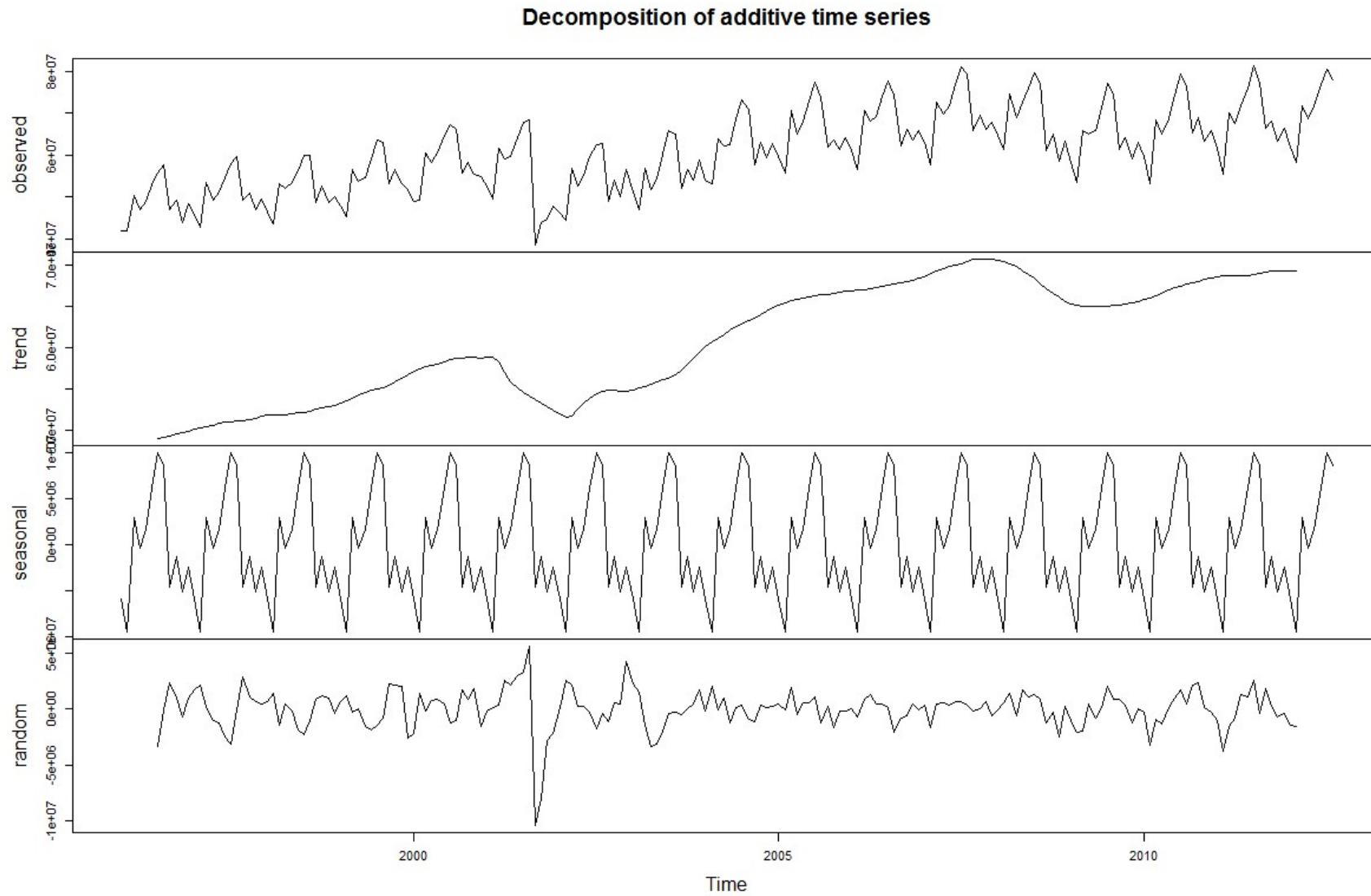
Revenue Passenger Miles: ACF and PACF



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ACF and PACF (Real-world): Decomposing Time Series into the 3 Components – Revenue Passenger Miles (RPM)



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Stationary and Non-Stationary

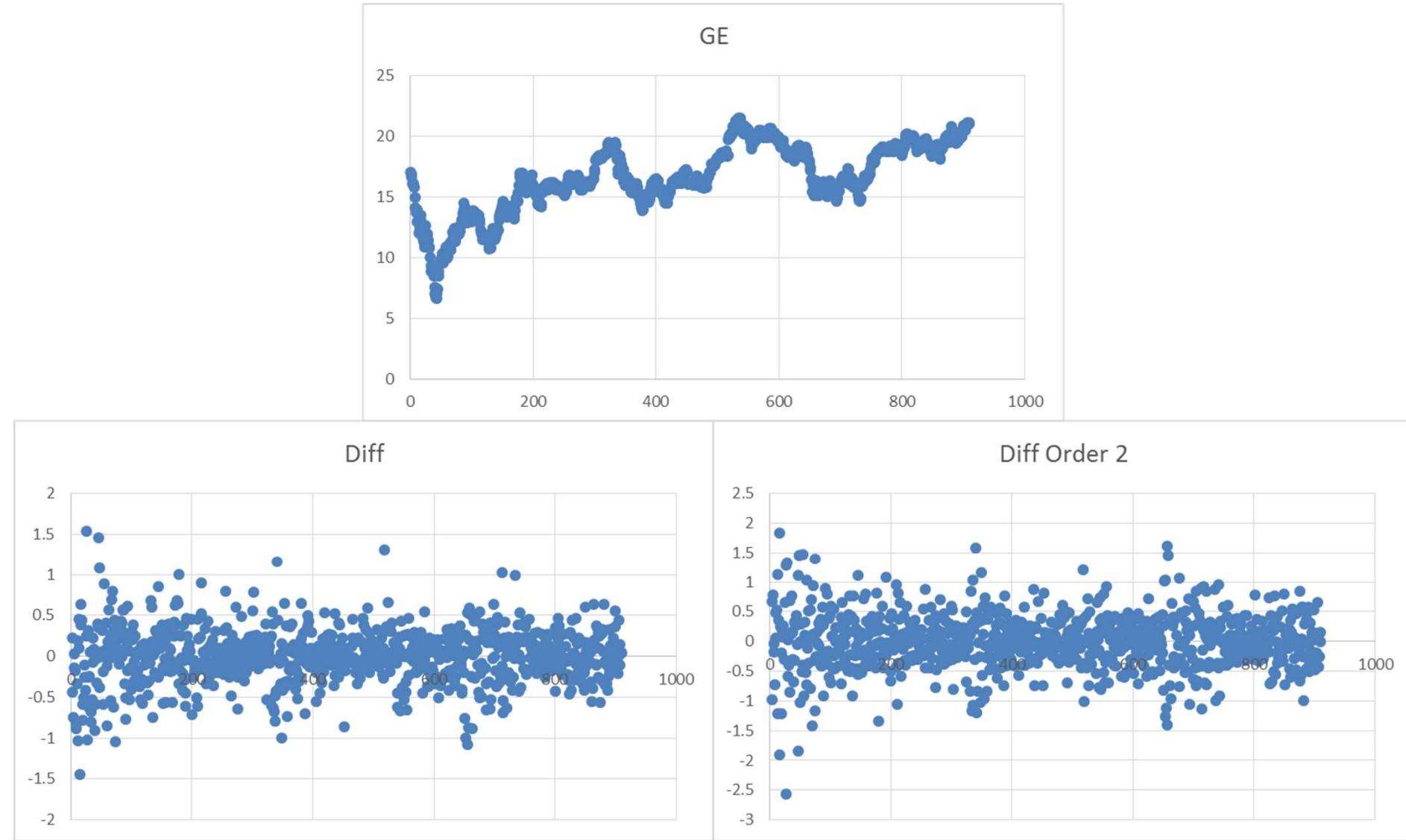
- Stationary data has constant statistical properties – mean, variance, autocorrelation, etc. – over time
- If the data is stationary, forecasting is easier!
- Differencing to convert non-stationary to stationary

EXCEL ACTIVITY

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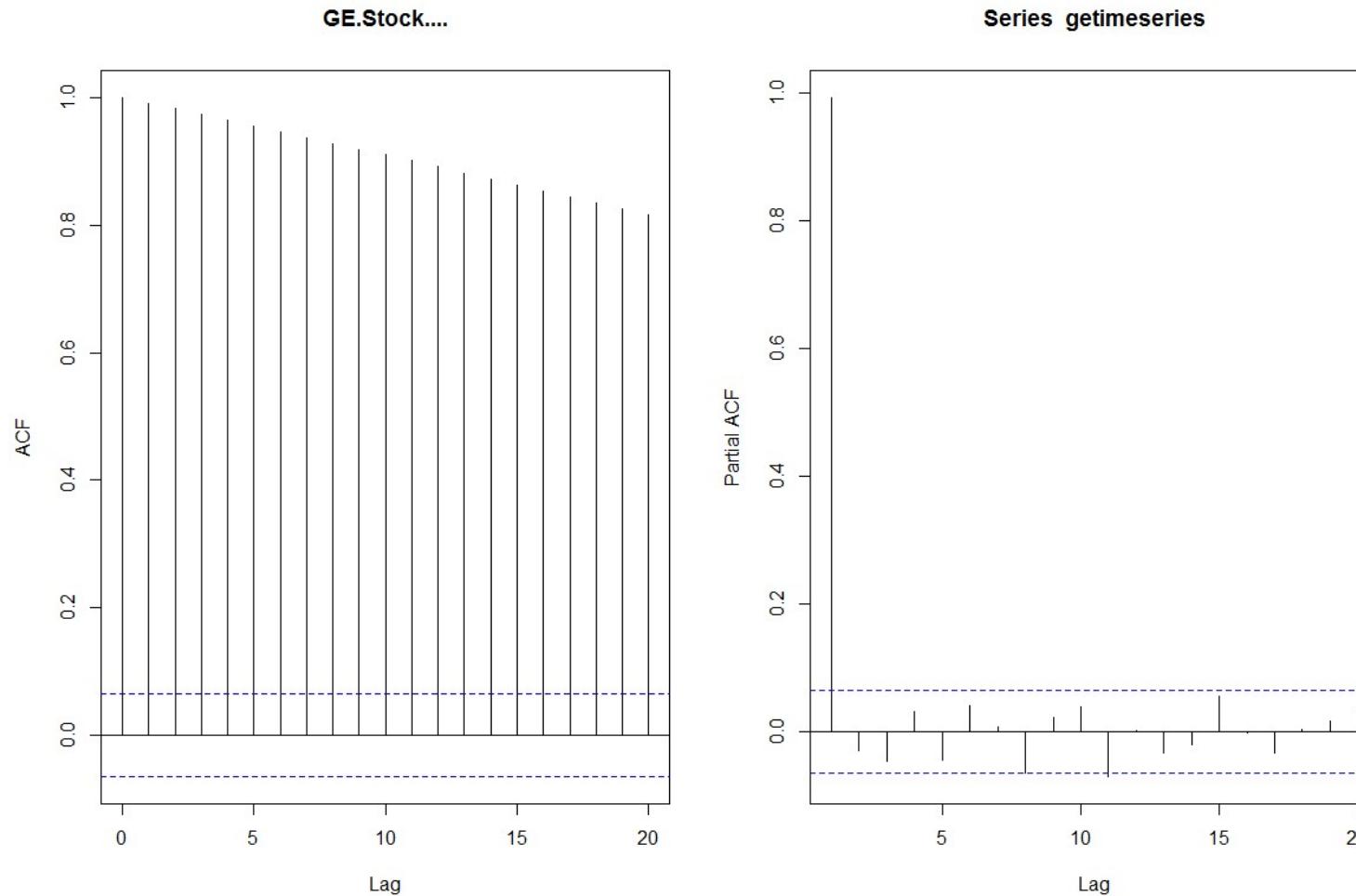
Removing Trend from Data



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ACF and PACF of Stationary and Non-Stationary

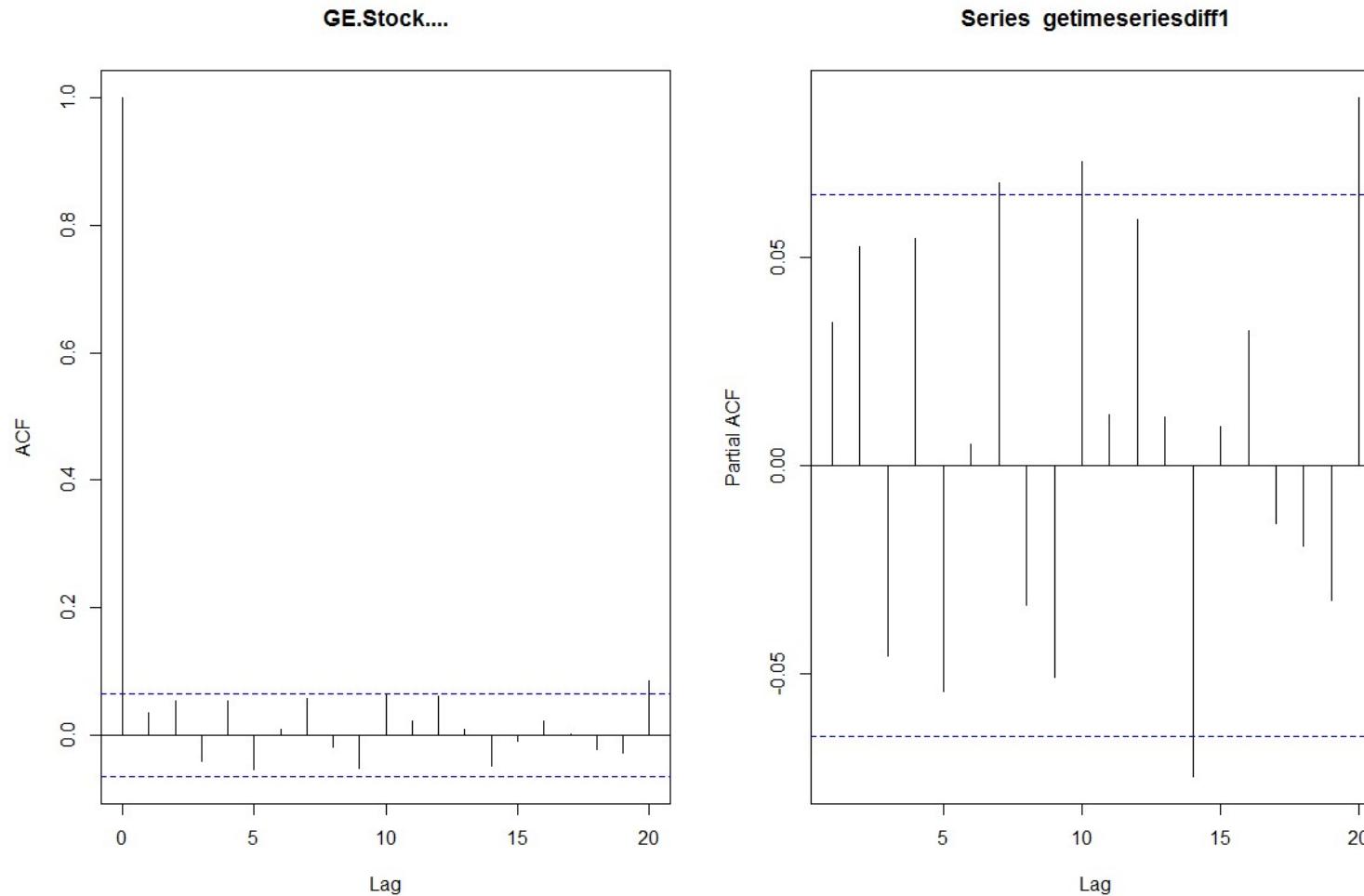


Price of GE stock is highly correlated with the previous day's value.

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ACF and PACF of Stationary and Non-Stationary



Daily changes in GE stock price are essentially random.

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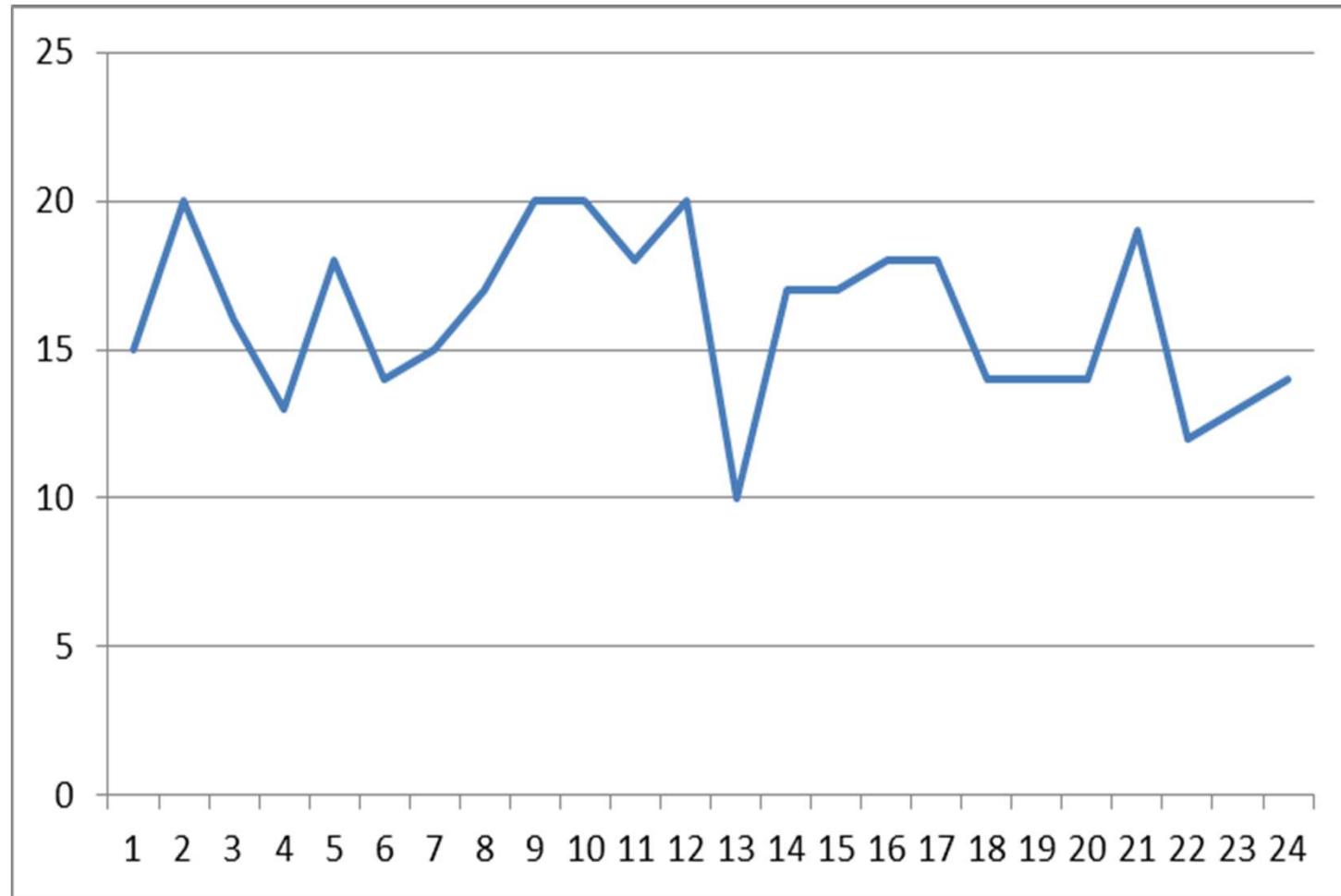
ACF and PACF of Stationary and Non-Stationary

- Non-stationary series have an ACF that remains significant for half a dozen or more lags, rather than quickly declining to zero.
- You must difference such a series until it is stationary before you can identify the process.

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Stationary Model: Moving Averages



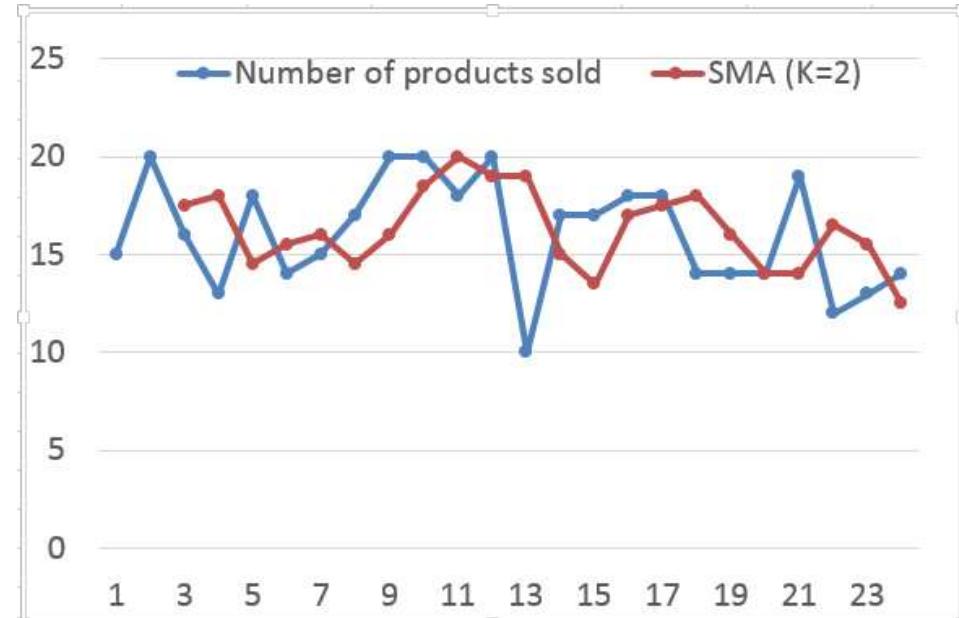
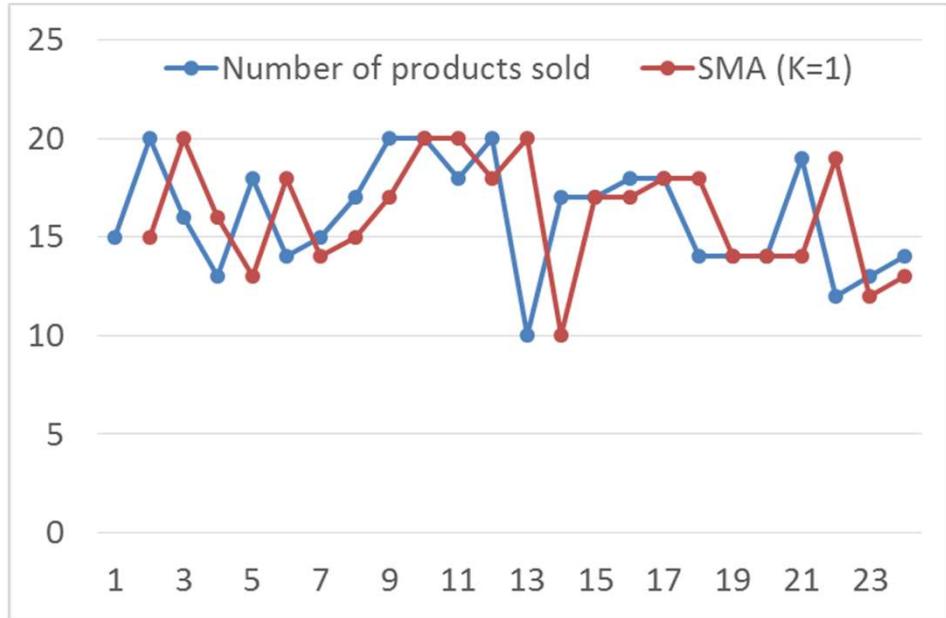
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Stationary Model: Case 1 – Simple Moving Averages

Number of products sold	SMA (K=1)	Error	SMA (K=2)	Error	SMA (K=3)	Error
15						
20	15	5				
16	20	4	17.5	1.5		
13	16	3	18	5	17	4
18	13	5	14.5	3.5	16.333333	1.66667
14	18	4	15.5	1.5	15.666667	1.66667
15	14	1	16	1	15	0
17	15	2	14.5	2.5	15.666667	1.33333
20	17	3	16	4	15.333333	4.66667
20	20	0	18.5	1.5	17.333333	2.66667
18	20	2	20	2	19	1
20	18	2	19	1	19.333333	0.66667
10	20	10	19	9	19.333333	9.33333
17	10	7	15	2	16	1
17	17	0	13.5	3.5	15.666667	1.33333
18	17	1	17	1	14.666667	3.33333
18	18	0	17.5	0.5	17.333333	0.66667
14	18	4	18	4	17.666667	3.66667
14	14	0	16	2	16.666667	2.66667
14	14	0	14	0	15.333333	1.33333
19	14	5	14	5	14	5
12	19	7	16.5	4.5	15.666667	3.66667
13	12	1	15.5	2.5	15	2
14	13	1	12.5	1.5	14.666667	0.66667
		2.91304		2.68182		2.49206





Only decision point is K

Stationary Model: Case 2 – Weighted Moving Averages

$$\hat{Y}_{t+1} = w_1 Y_t + w_2 Y_{t-1} + \cdots + w_k Y_{t-k+1}$$

- Typically we choose a time period of moving average and weights are chosen such that the error is minimized

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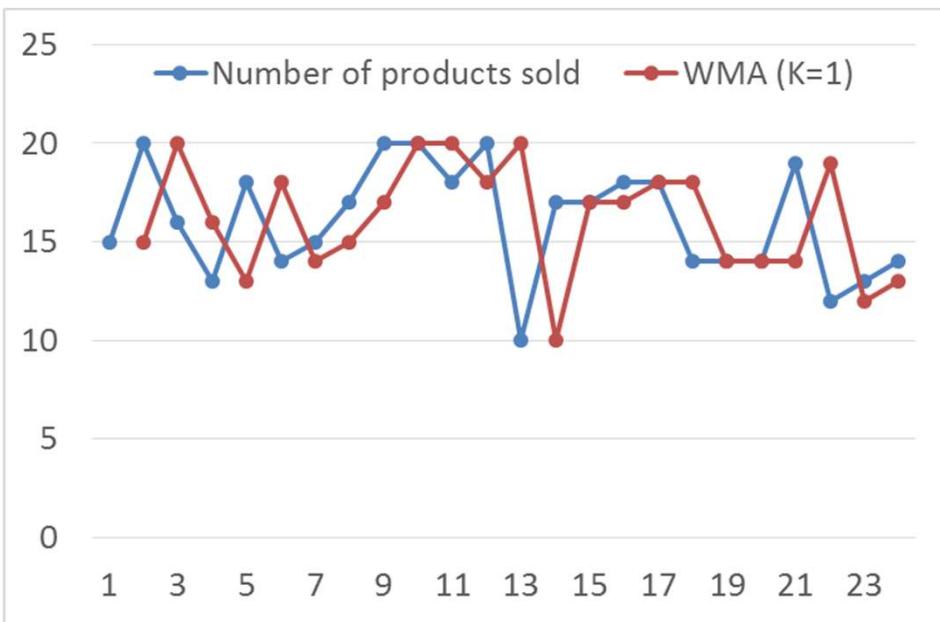


Stationary Model: Case 2 – Weighted Moving Averages

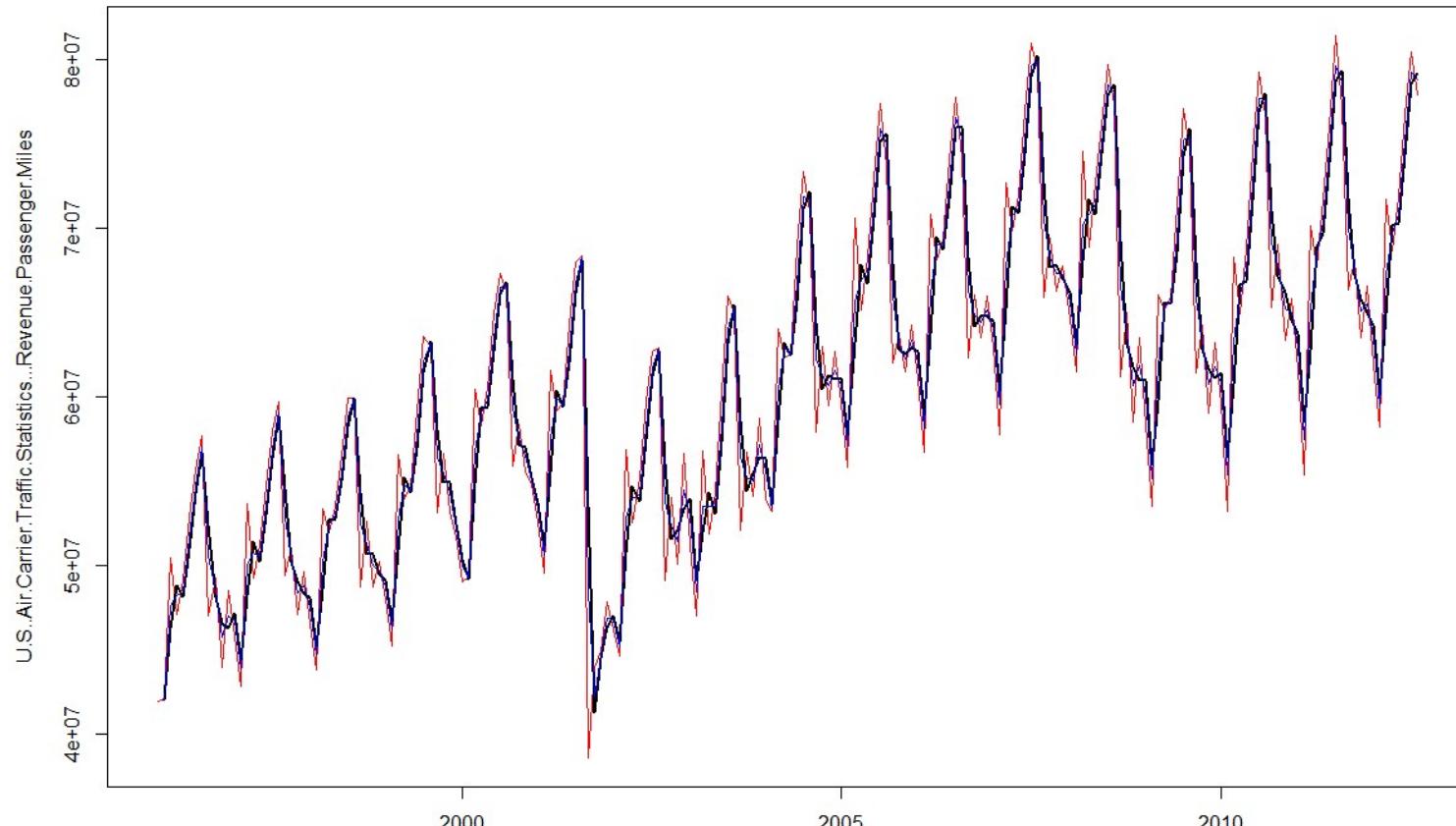
Number of products sold	WMA (K=1)	Error	WMA (K=2)	Error	WMA (K=3)	Error
15						
20	15	5				
16	20	4	18.3333333	2.3333333		
13	16	3	17.3333333	4.3333333	17.1666667	4.16666667
18	13	5	14	4	15.1666667	2.8333333
14	18	4	16.3333333	2.3333333	16	2
15	14	1	15.3333333	0.3333333	15.1666667	0.16666667
17	15	2	14.6666667	2.3333333	15.1666667	1.8333333
20	17	3	16.3333333	3.6666667	15.8333333	4.16666667
20	20	0	19	1	18.1666667	1.8333333
18	20	2	20	2	19.5	1.5
20	18	2	18.6666667	1.3333333	19	1
10	20	10	19.3333333	9.3333333	19.3333333	9.3333333
17	10	7	13.3333333	3.6666667	14.6666667	2.3333333
17	17	0	14.6666667	2.3333333	15.1666667	1.8333333
18	17	1	17	1	15.8333333	2.1666667
18	18	0	17.6666667	0.3333333	17.5	0.5
14	18	4	18	4	17.8333333	3.8333333
14	14	0	15.3333333	1.3333333	16	2
14	14	0	14	0	14.6666667	0.66666667
19	14	5	14	5	14	5
12	19	7	17.3333333	5.3333333	16.5	4.5
13	12	1	14.3333333	1.3333333	14.6666667	1.66666667
14	13	1	12.6666667	1.3333333	13.6666667	0.3333333
		2.91304348		2.66666667		2.55555556

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SMA and WMA – Revenue Passenger Miles



> MAPE-SMA 4.093731 > MAPE-WMA 2.729154

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~~Stationary Model: Case 3 – Exponential Weighted Moving Averages or Exponential Smoothing~~

Averaging over long periods dampens fluctuations, removing not only the noise but also trend and seasonality.

Moving averages over short recent periods maintains trend and seasonality but determining an optimum number for periods is tricky, even when using metrics like MAE. If averaged over too few periods, irregularities continue to remain and if averaged over long periods, dampening again becomes a problem.

Exponential smoothing **retains all older periods** while giving a greater weight to more recent periods (hence not a MOVING average).

Caution: It doesn't make any one method superior for all situations.



Stationary Model: Case 3 – Exponential ~~Weighted Moving Averages~~ or Exponential Smoothing

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$

Above equation indicates that the predicted value for time period $t+1$ (\hat{Y}_{t+1}) is equal to the predicted value for the previous period (\hat{Y}_t) plus an adjustment for the error made in predicting the previous period's value ($\alpha(Y_t - \hat{Y}_t)$).

The parameter α can assume any value between 0 and 1 ($0 \leq \alpha \leq 1$).

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Exponential Smoothing in Other Ways

$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$ can be rewritten variously as

$$\begin{aligned}\hat{Y}_{t+1} &= \alpha Y_t + (1 - \alpha) \hat{Y}_t \\ &= Y_t - (1 - \alpha)(Y_t - \hat{Y}_t) \\ &\Rightarrow \hat{Y}_{t+1} = \alpha Y_t + \alpha(1 - \alpha)Y_{t-1} + \alpha(1 - \alpha)^2 Y_{t-2} + \dots + \alpha(1 - \alpha)^n Y_{t-n} + \dots\end{aligned}$$



Exponential Smoothing

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t) \quad \alpha = \frac{2}{N+1}$$

- Forecasting at time t+1 requires both the forecasted value and the True Value at time t
- So if you want to forecast more than 1 time period into the future, the best you can do is to use the last available value
- All future predictions are same! This is in accordance with stationary assumption.



Exponential Smoothing

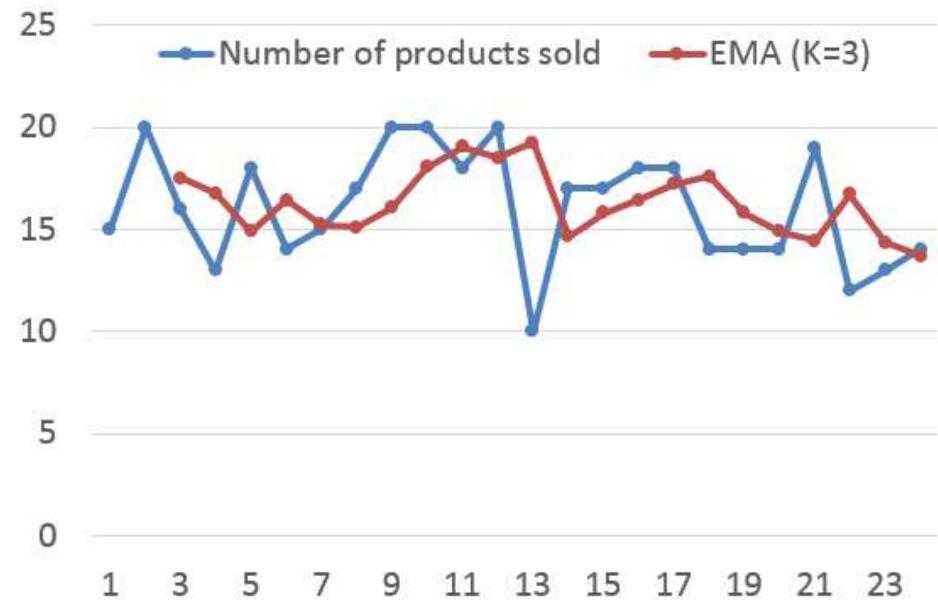
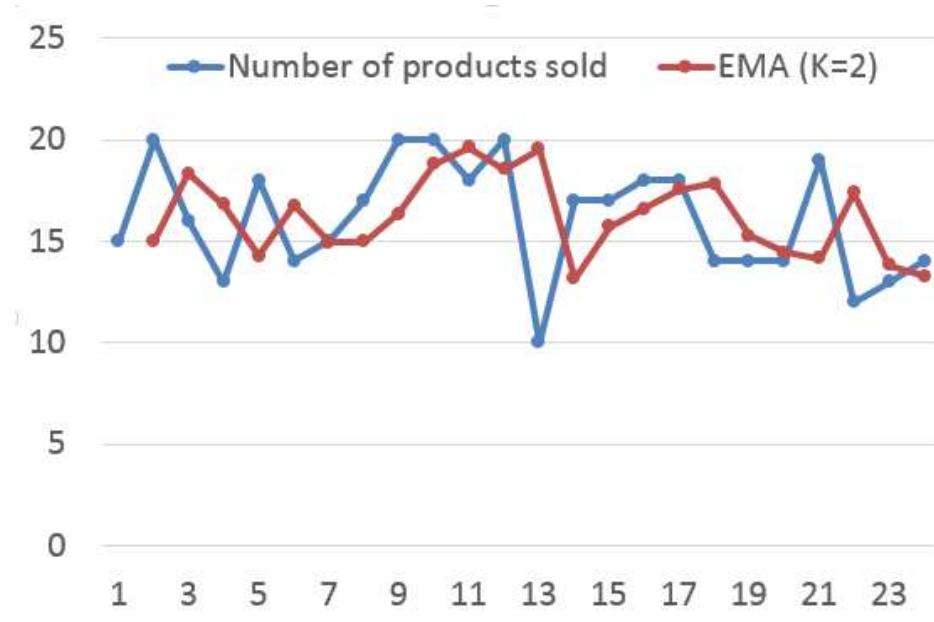
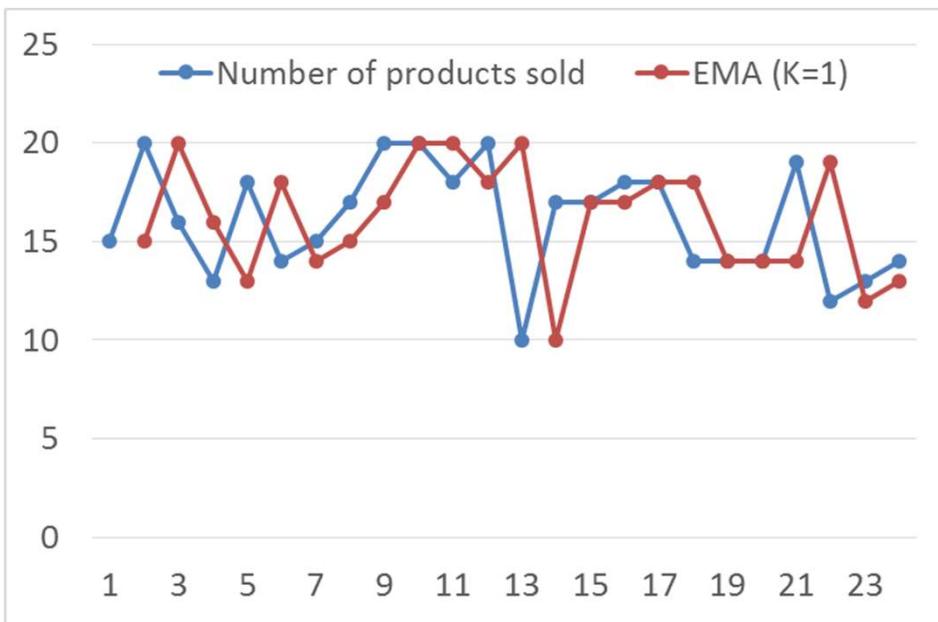
	A	B	C	D	E	F	G
1	Numbe	EMA (K=1)	Error	EMA (K=2)	Error	EMA (K=3)	Error
2	15						
3	20	=A2*1	=ABS(B3-A3)	=15			
4	16	=A3*\$K\$2+B3*\$L\$2	=ABS(B4-A4)	=A3*\$K\$3+D3*\$L\$3	=ABS(A4-D4)	=AVERAGE(A2:A3)	
5	13	=A4*\$K\$2+B4*\$L\$2	=ABS(B5-A5)	=A4*\$K\$3+D4*\$L\$3	=ABS(A5-D5)	=A4*\$K\$4+F4*\$L\$4	=ABS(A5-F5)
6	18	=A5*\$K\$2+B5*\$L\$2	=ABS(B6-A6)	=A5*\$K\$3+D5*\$L\$3	=ABS(A6-D6)	=A5*\$K\$4+F5*\$L\$4	=ABS(A6-F6)
7	14	=A6*\$K\$2+B6*\$L\$2	=ABS(B7-A7)	=A6*\$K\$3+D6*\$L\$3	=ABS(A7-D7)	=A6*\$K\$4+F6*\$L\$4	=ABS(A7-F7)
8	15	=A7*\$K\$2+B7*\$L\$2	=ABS(B8-A8)	=A7*\$K\$3+D7*\$L\$3	=ABS(A8-D8)	=A7*\$K\$4+F7*\$L\$4	=ABS(A8-F8)
9	17	=A8*\$K\$2+B8*\$L\$2	=ABS(B9-A9)	=A8*\$K\$3+D8*\$L\$3	=ABS(A9-D9)	=A8*\$K\$4+F8*\$L\$4	=ABS(A9-F9)
10	20	=A9*\$K\$2+B9*\$L\$2	=ABS(B10-A10)	=A9*\$K\$3+D9*\$L\$3	=ABS(A10-D10)	=A9*\$K\$4+F9*\$L\$4	=ABS(A10-F10)
11	20	=A10*\$K\$2+B10*\$L\$2	=ABS(B11-A11)	=A10*\$K\$3+D10*\$L\$3	=ABS(A11-D11)	=A10*\$K\$4+F10*\$L\$4	=ABS(A11-F11)
12	18	=A11*\$K\$2+B11*\$L\$2	=ABS(B12-A12)	=A11*\$K\$3+D11*\$L\$3	=ABS(A12-D12)	=A11*\$K\$4+F11*\$L\$4	=ABS(A12-F12)
13	20	=A12*\$K\$2+B12*\$L\$2	=ABS(B13-A13)	=A12*\$K\$3+D12*\$L\$3	=ABS(A13-D13)	=A12*\$K\$4+F12*\$L\$4	=ABS(A13-F13)
14	10	=A13*\$K\$2+B13*\$L\$2	=ABS(B14-A14)	=A13*\$K\$3+D13*\$L\$3	=ABS(A14-D14)	=A13*\$K\$4+F13*\$L\$4	=ABS(A14-F14)
15	17	=A14*\$K\$2+B14*\$L\$2	=ABS(B15-A15)	=A14*\$K\$3+D14*\$L\$3	=ABS(A15-D15)	=A14*\$K\$4+F14*\$L\$4	=ABS(A15-F15)
16	17	=A15*\$K\$2+B15*\$L\$2	=ABS(B16-A16)	=A15*\$K\$3+D15*\$L\$3	=ABS(A16-D16)	=A15*\$K\$4+F15*\$L\$4	=ABS(A16-F16)
17	18	=A16*\$K\$2+B16*\$L\$2	=ABS(B17-A17)	=A16*\$K\$3+D16*\$L\$3	=ABS(A17-D17)	=A16*\$K\$4+F16*\$L\$4	=ABS(A17-F17)
18	18	=A17*\$K\$2+B17*\$L\$2	=ABS(B18-A18)	=A17*\$K\$3+D17*\$L\$3	=ABS(A18-D18)	=A17*\$K\$4+F17*\$L\$4	=ABS(A18-F18)
19	14	=A18*\$K\$2+B18*\$L\$2	=ABS(B19-A19)	=A18*\$K\$3+D18*\$L\$3	=ABS(A19-D19)	=A18*\$K\$4+F18*\$L\$4	=ABS(A19-F19)
20	14	=A19*\$K\$2+B19*\$L\$2	=ABS(B20-A20)	=A19*\$K\$3+D19*\$L\$3	=ABS(A20-D20)	=A19*\$K\$4+F19*\$L\$4	=ABS(A20-F20)
21	14	=A20*\$K\$2+B20*\$L\$2	=ABS(B21-A21)	=A20*\$K\$3+D20*\$L\$3	=ABS(A21-D21)	=A20*\$K\$4+F20*\$L\$4	=ABS(A21-F21)
22	19	=A21*\$K\$2+B21*\$L\$2	=ABS(B22-A22)	=A21*\$K\$3+D21*\$L\$3	=ABS(A22-D22)	=A21*\$K\$4+F21*\$L\$4	=ABS(A22-F22)
23	12	=A22*\$K\$2+B22*\$L\$2	=ABS(B23-A23)	=A22*\$K\$3+D22*\$L\$3	=ABS(A23-D23)	=A22*\$K\$4+F22*\$L\$4	=ABS(A23-F23)
24	13	=A23*\$K\$2+B23*\$L\$2	=ABS(B24-A24)	=A23*\$K\$3+D23*\$L\$3	=ABS(A24-D24)	=A23*\$K\$4+F23*\$L\$4	=ABS(A24-F24)
25	14	=A24*\$K\$2+B24*\$L\$2	=ABS(B25-A25)	=A24*\$K\$3+D24*\$L\$3	=ABS(A25-D25)	=A24*\$K\$4+F24*\$L\$4	=ABS(A25-F25)
26			=AVERAGE(C3:C25)		=AVERAGE(E3:E25)		=AVERAGE(G3:G25)

J	K	L
K	2/(K+1)	1-[2/(K+1)]
1	1	=1-K2
2	=2/(J3+1)	=1-K3
3	=2/(J4+1)	=1-K4
4	=2/(J5+1)	=1-K5

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha) \hat{Y}_t$$

Exponential Smoothing

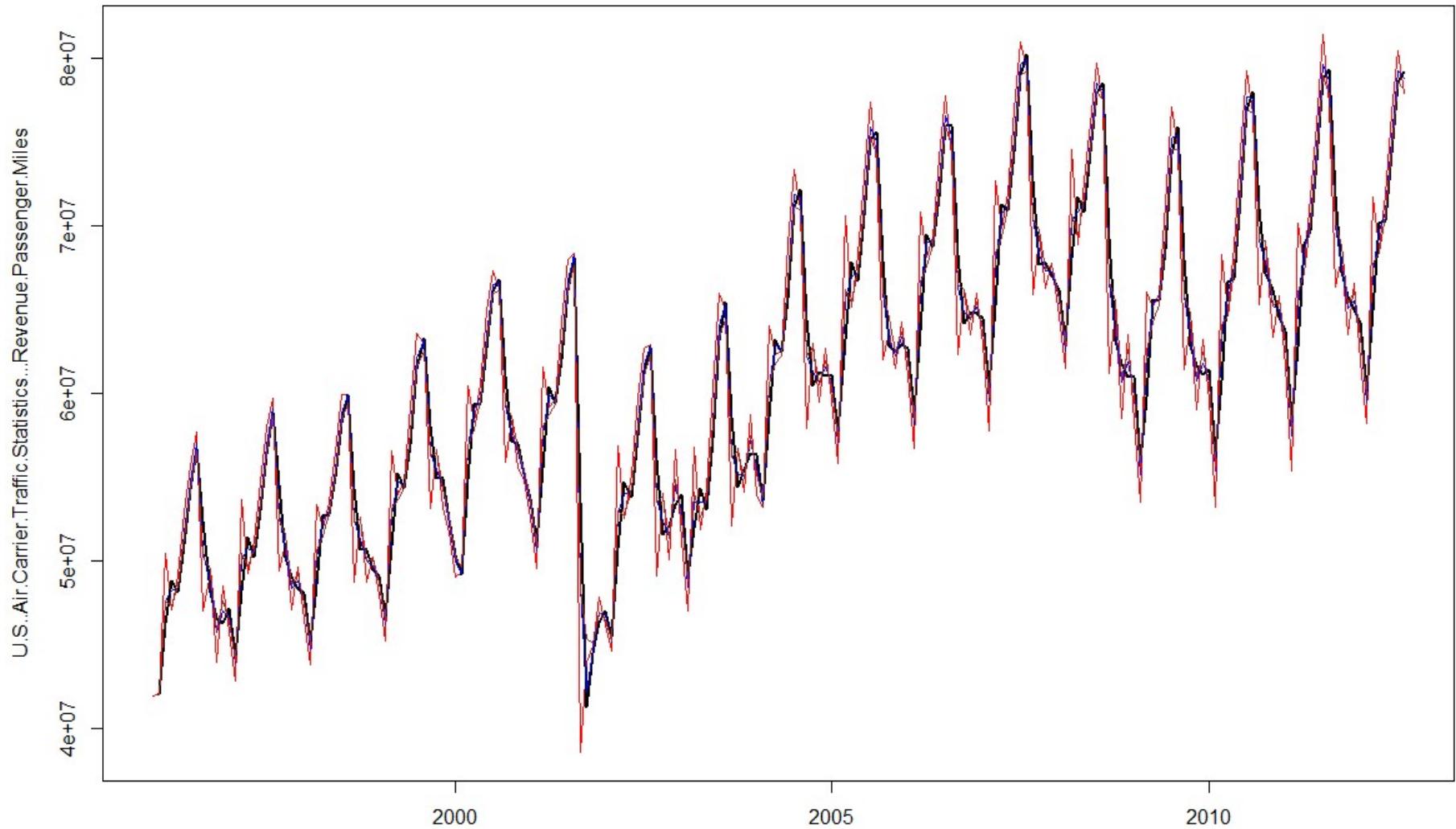
	A	B	C	D	E	F	G	H	I
1	Number of	EMA (K=1)	Error	EMA (K=2)	Error	EMA (K=3)	Error	EMA (K=4)	Error
2	15								
3	20	15	5	15					
4	16	20	4	18.3333333	2.33333	17.5			
5	13	16	3	16.7777778	3.77778	16.75	3.75	17	
6	18	13	5	14.2592593	3.74074	14.875	3.125	15.4	2.6
7	14	18	4	16.7530864	2.75309	16.4375	2.4375	16.44	2.44
8	15	14	1	14.9176955	0.0823	15.21875	0.21875	15.464	0.464
9	17	15	2	14.9725652	2.02743	15.109375	1.890625	15.2784	1.7216
10	20	17	3	16.3241884	3.67581	16.054688	3.945313	15.96704	4.03296
11	20	20	0	18.7747295	1.22527	18.027344	1.972656	17.580224	2.41978
12	18	20	2	19.5915765	1.59158	19.013672	1.013672	18.548134	0.54813
13	20	18	2	18.5305255	1.46947	18.506836	1.493164	18.328881	1.67112
14	10	20	10	19.5101752	9.51018	19.253418	9.253418	18.997328	8.99733
15	17	10	7	13.1700584	3.82994	14.626709	2.373291	15.398397	1.6016
16	17	17	0	15.7233528	1.27665	15.813354	1.186646	16.039038	0.96096
17	18	17	1	16.5744509	1.42555	16.406677	1.593323	16.423423	1.57658
18	18	18	0	17.524817	0.47518	17.203339	0.796661	17.054054	0.94595
19	14	18	4	17.8416057	3.84161	17.601669	3.601669	17.432432	3.43243
20	14	14	0	15.2805352	1.28054	15.800835	1.800835	16.059459	2.05946
21	14	14	0	14.4268451	0.42685	14.900417	0.900417	15.235676	1.23568
22	19	14	5	14.1422817	4.85772	14.450209	4.549791	14.741405	4.25859
23	12	19	7	17.3807606	5.38076	16.725104	4.725104	16.444843	4.44484
24	13	12	1	13.7935869	0.79359	14.362552	1.362552	14.666906	1.66691
25	14	13	1	13.264529	0.73547	13.681276	0.318724	14.000144	0.00014
26			2.913043		2.56867		2.49091		2.3539



$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$

$$\alpha = \frac{2}{K + 1}$$

SMA, WMA and Exponential Smoothing – RPM



> MAPESMA 4.093731 > MAPEWMA 2.729154 > MAPEEMA 2.541979

CSE 72026



ADDING TREND AND SEASONALITY TO MOVING AVERAGE PROCESSES

Holt-Winters Method

- This method separates the forecast into 3 components – Base Level + Trend + Seasonal
- It updates each of the level using a Exponential smoothing with a different smoothing factor.
- The algorithm computes the best possible smoothing factors based on the given data and does subsequent predictions using fitted smoothing constants.



Holt-Winters Method

- 3 components – Trend (T), Seasonality(S) and Base Level/Expectation(B).
- 3 weights – smoothing parameters – are used to update components at each period.
- Initial values for Base Level/Expectation and trend components are obtained using linear regression on time.
- Initial values for seasonal component are obtained from a dummy variable regression using de-trended data.
- In the Expectation equation, the series is seasonally adjusted by subtracting the seasonal component.



Holt-Winters Method

Additive Seasonality

$$\hat{Y}_t = B_{t-1} + T_{t-1} + S_{t-p}$$

$$\hat{Y}_{t+n} = B_t + nT_t + S_{t+n-p}$$

Multiplicative Seasonality

$$\hat{Y}_t = (B_{t-1} + T_{t-1})S_{t-p}$$

$$\hat{Y}_{t+n} = (B_t + nT_t)S_{t+n-p}$$

The 3 smoothing equations are:

$$B_t = \alpha(Y_t - S_{t-p}) + (1 - \alpha)(B_{t-1} + T_{t-1})$$

$$T_t = \beta(B_t - B_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma(Y_t - B_t) + (1 - \gamma)S_{t-p}$$

$$B_t = \alpha \frac{Y_t}{S_{t-p}} + (1 - \alpha)(B_{t-1} + T_{t-1})$$

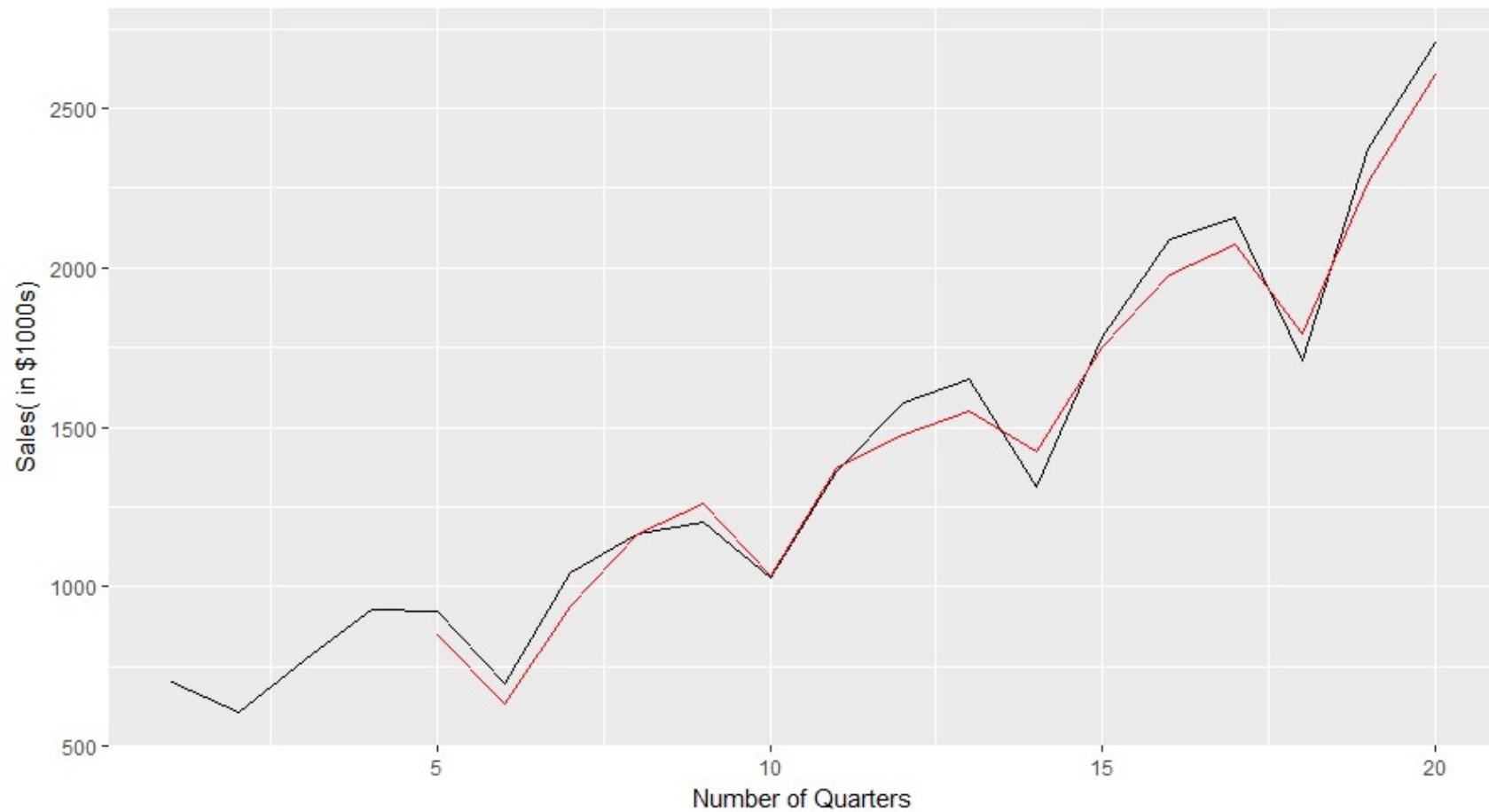
$$T_t = \beta(B_t - B_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma \frac{Y_t}{B_t} + (1 - \gamma)S_{t-p}$$

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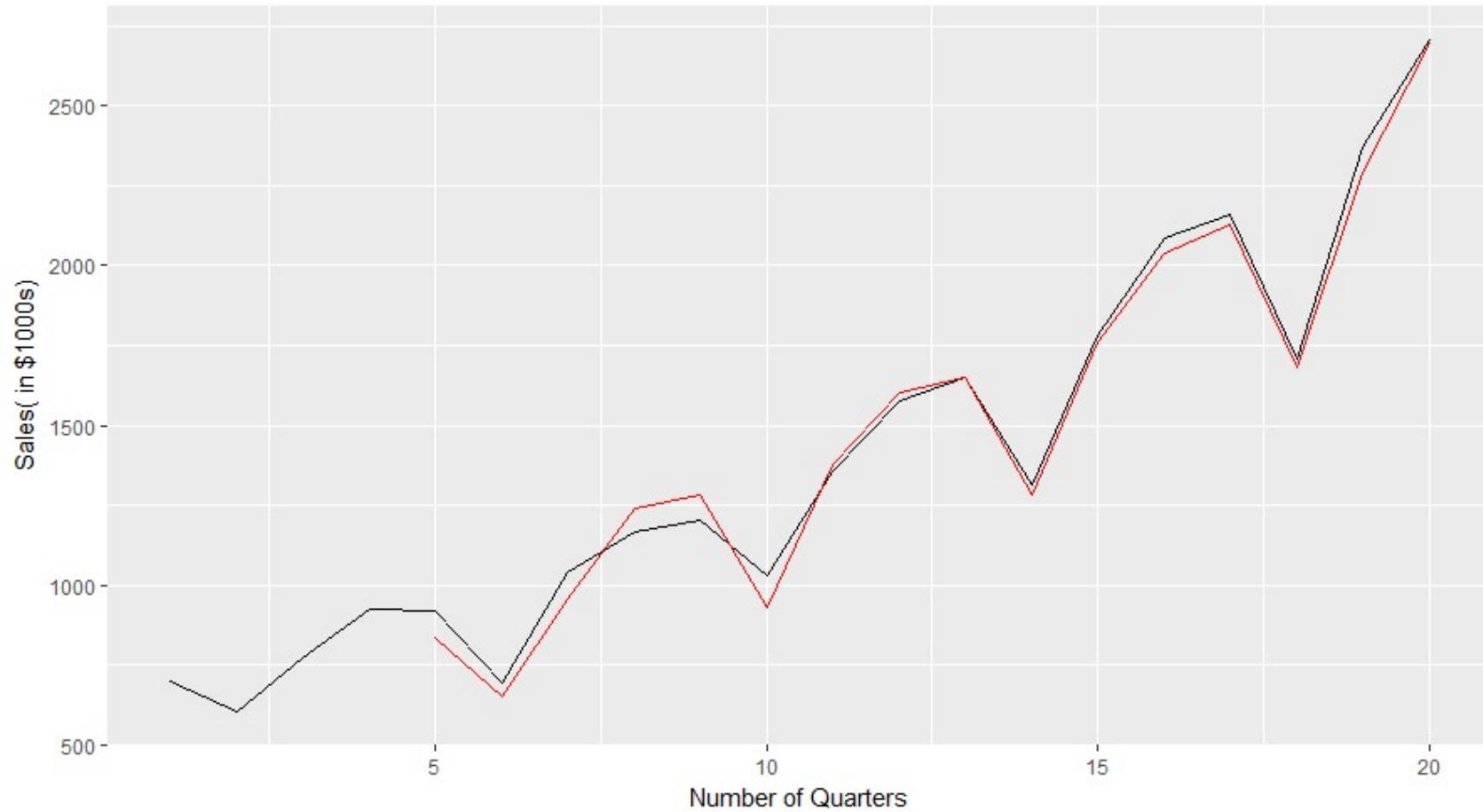


Holt-Winters Additive Seasonal Model

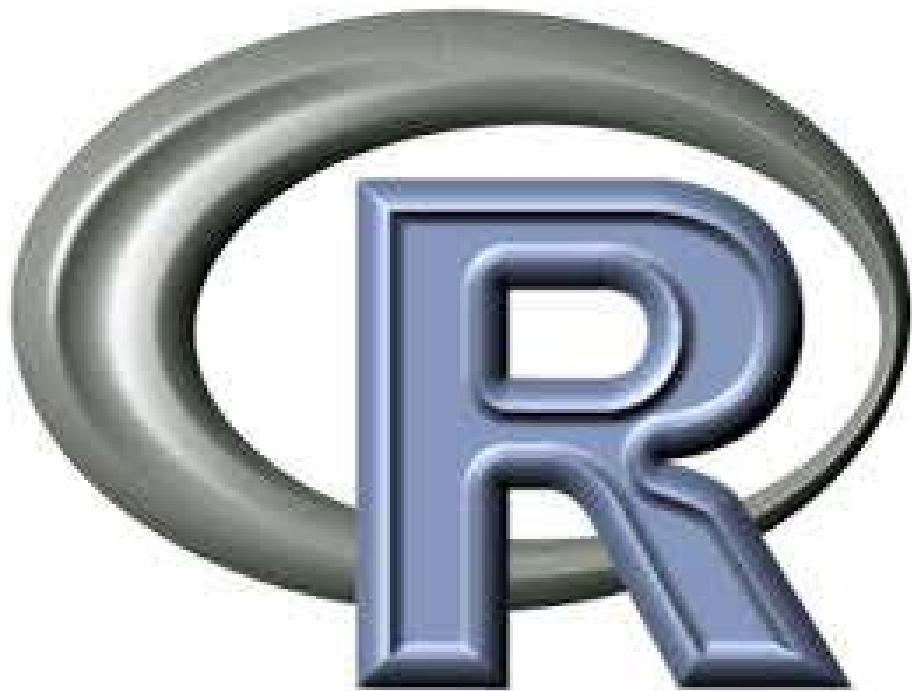


SSE = 103627

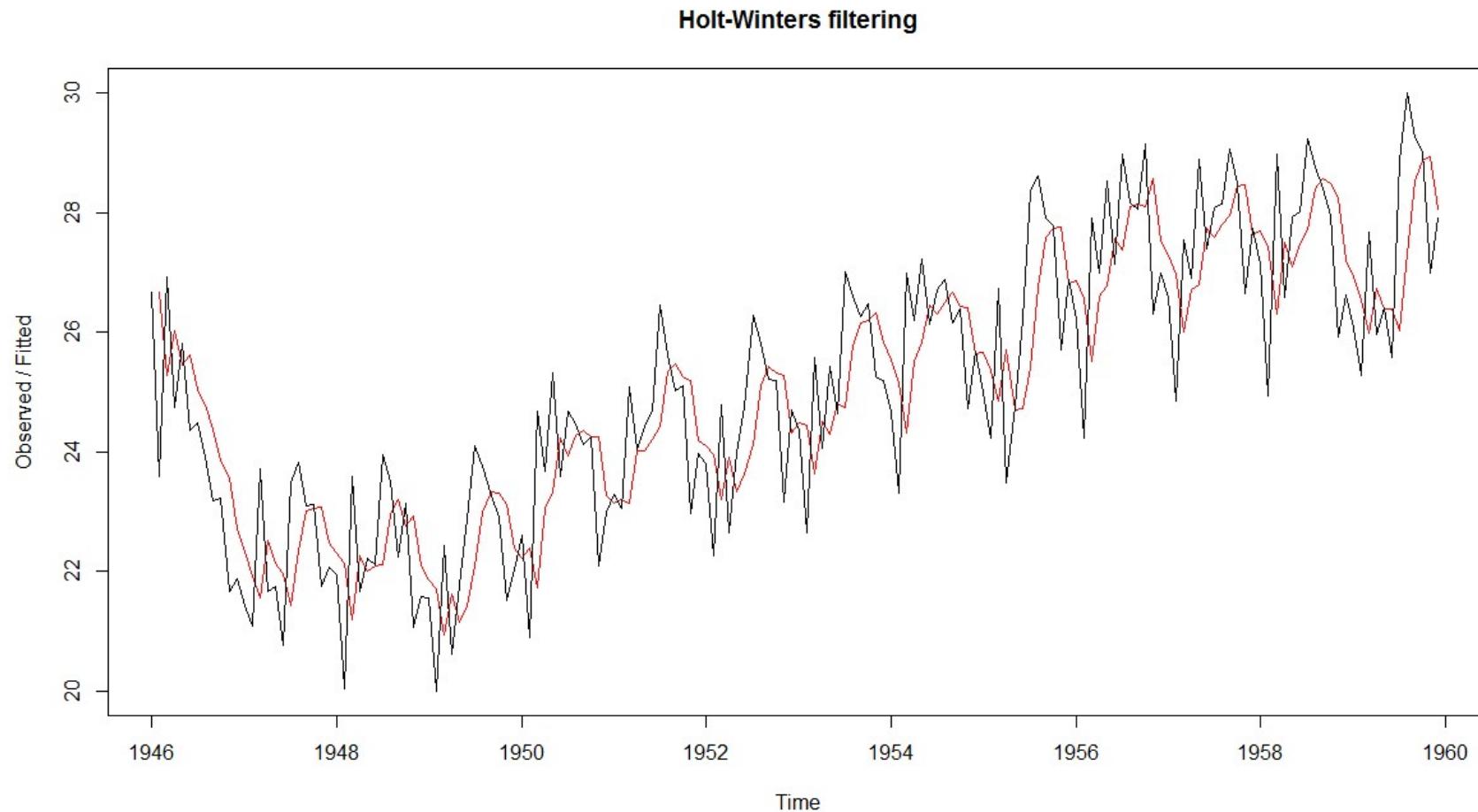
Holt-Winters Multiplicative Seasonal Model



$$\text{SSE} = 49816$$



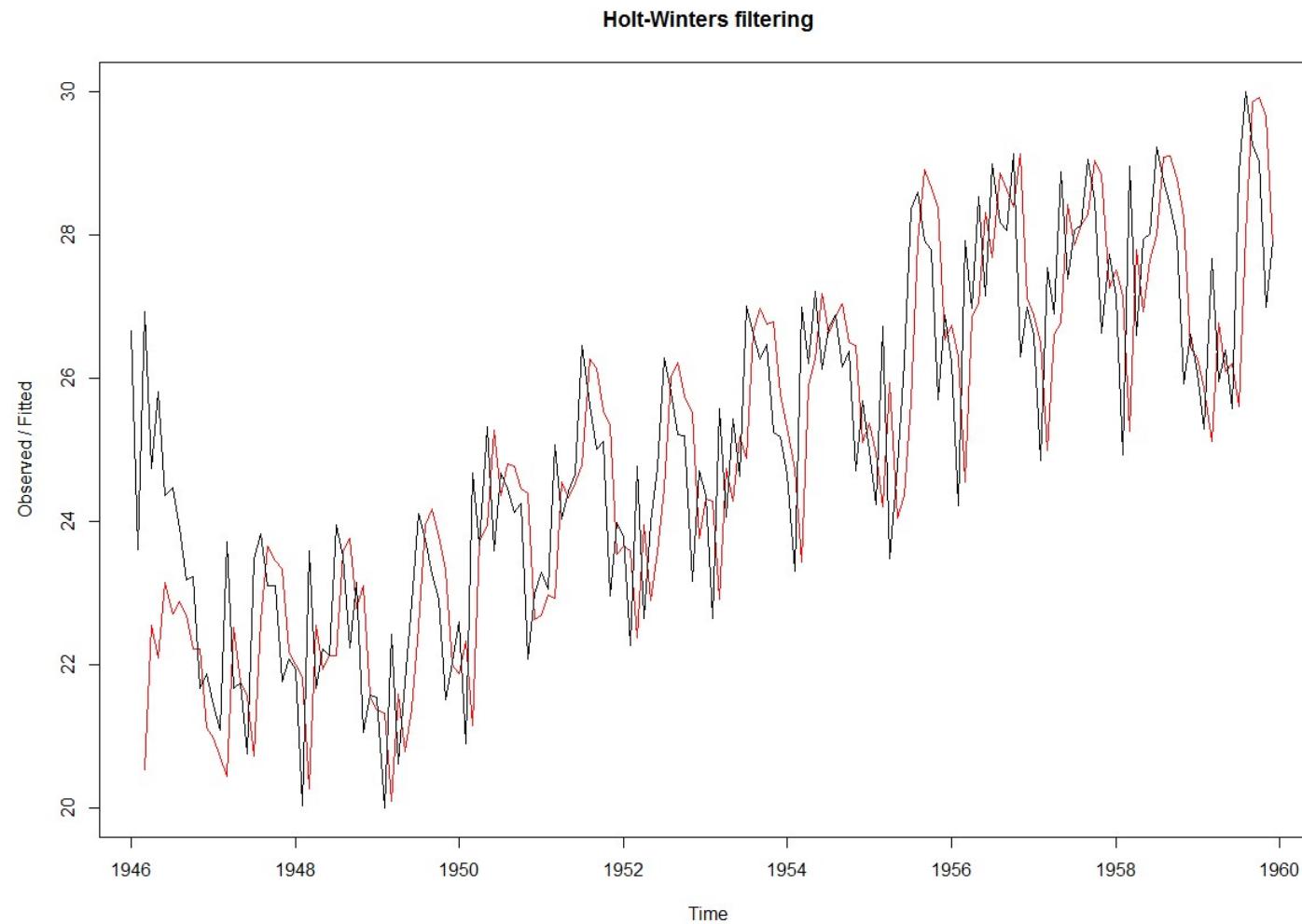
Holt-Winters Method: Only Level



```
> birthsforecast$sse  
[1] 281.8759
```



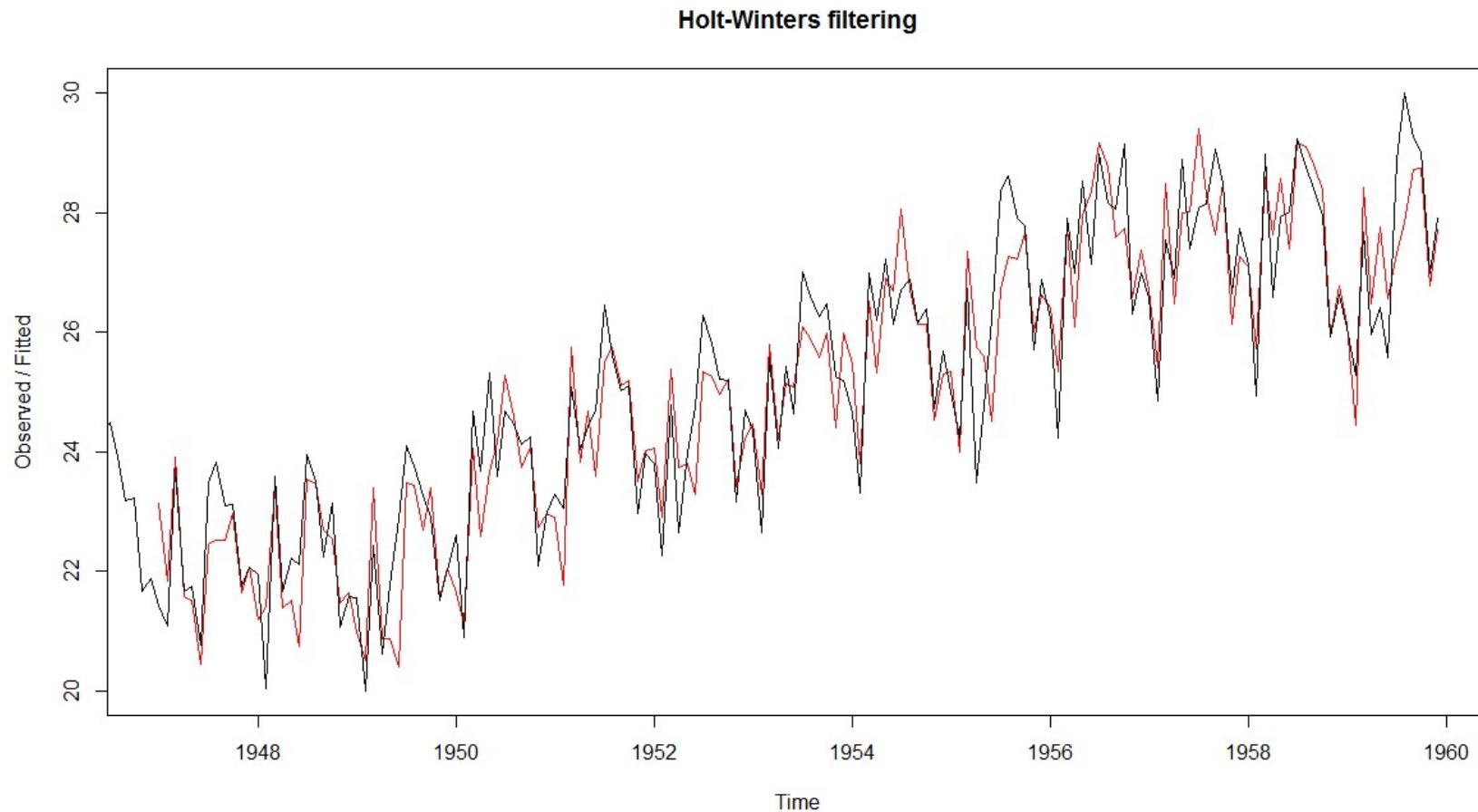
Holt-Winters Method: Level & Trend Only



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Holt-Winters Method: All Components



```
> birthsforecast$sse  
[1] 90.94058
```

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Holt-Winters Method: All Components

Holt-winters exponential smoothing with trend and additive seasonal component.

Call:

```
Holtwinters(x = birthstimeseries)
```

Smoothing parameters:

```
alpha: 0.4823655  
beta : 0.02988495  
gamma: 0.563186
```

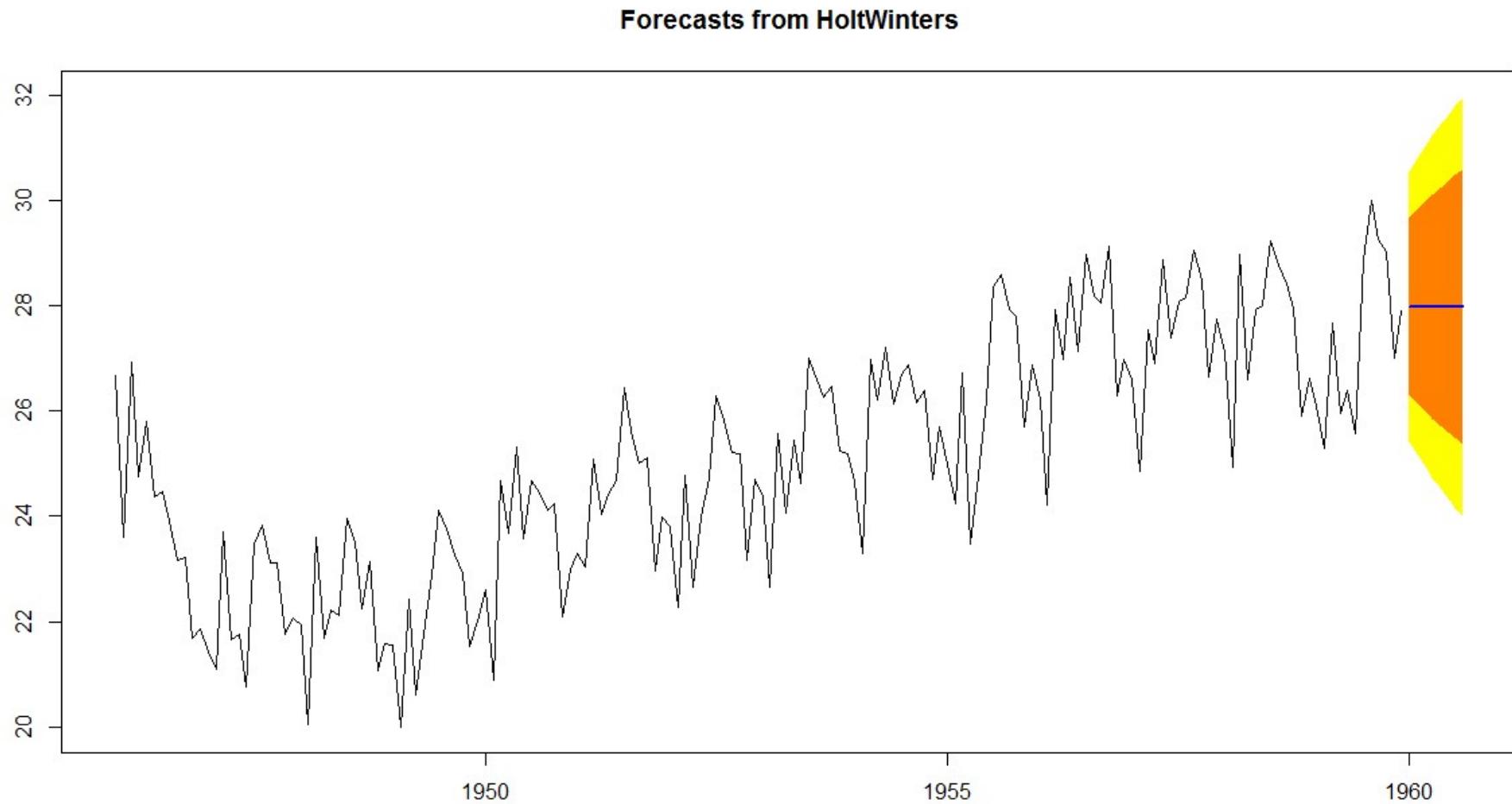
Coefficients:

```
[,1]  
a 28.04366357  
b 0.04199921  
s1 -0.78546221  
s2 -2.19944507  
s3 0.87813012  
s4 -0.65164728  
s5 0.63427267  
s6 0.21182821  
s7 2.23177191  
s8 2.17167733  
s9 1.52077678  
s10 1.16900861  
s11 -0.97500043  
s12 -0.18636055
```

```
> birthsforecast$fitted
```

	xhat	level	trend	season
Jan 1947	23.13579	23.81055	-0.1567618007	-0.51798958
Feb 1947	21.87080	22.87571	0.1817718860	0.87710702

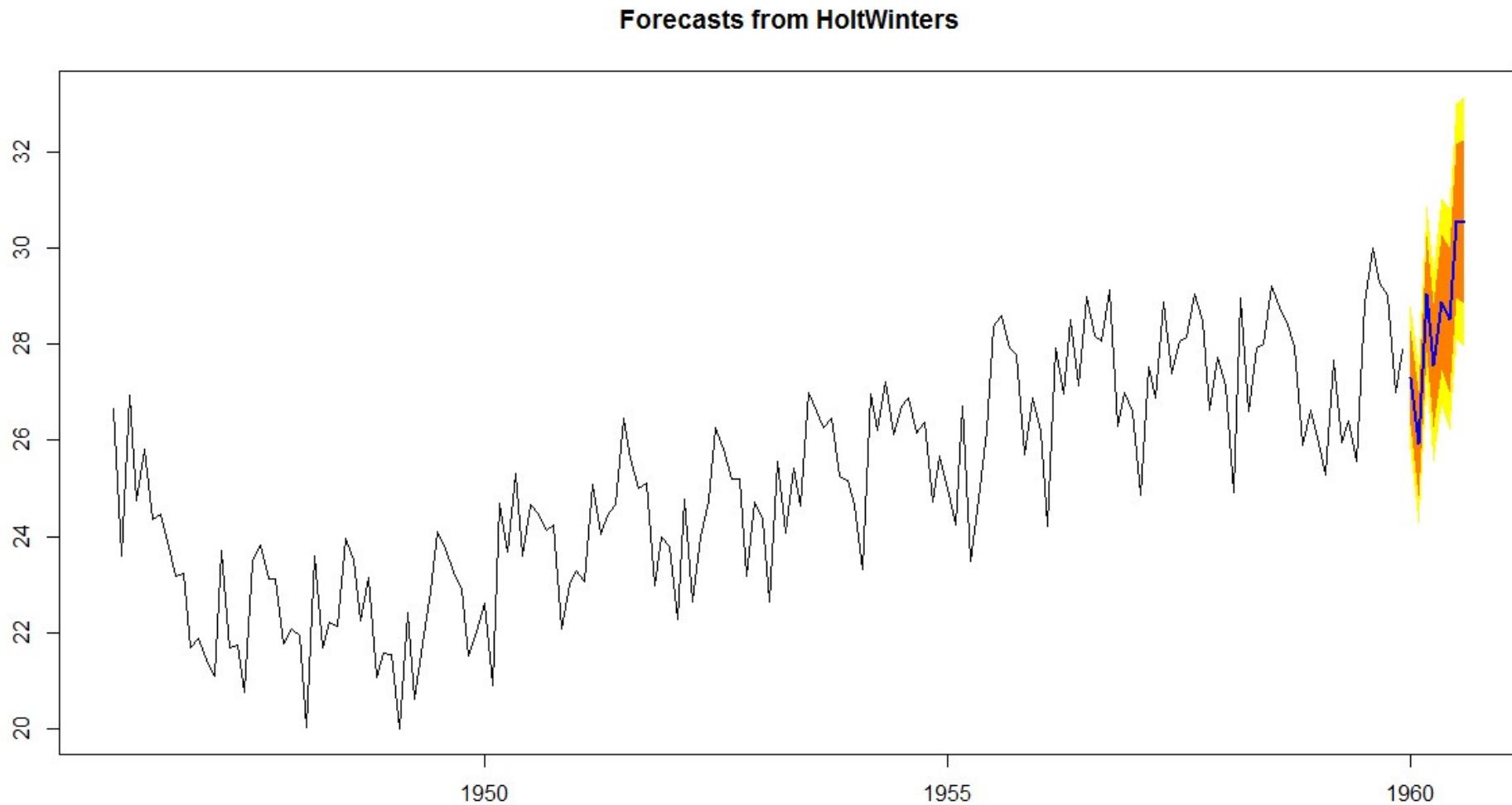
Holt-Winters Method: Forecasting without Trend or Seasonality



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Holt-Winters Method: Forecasting with all components

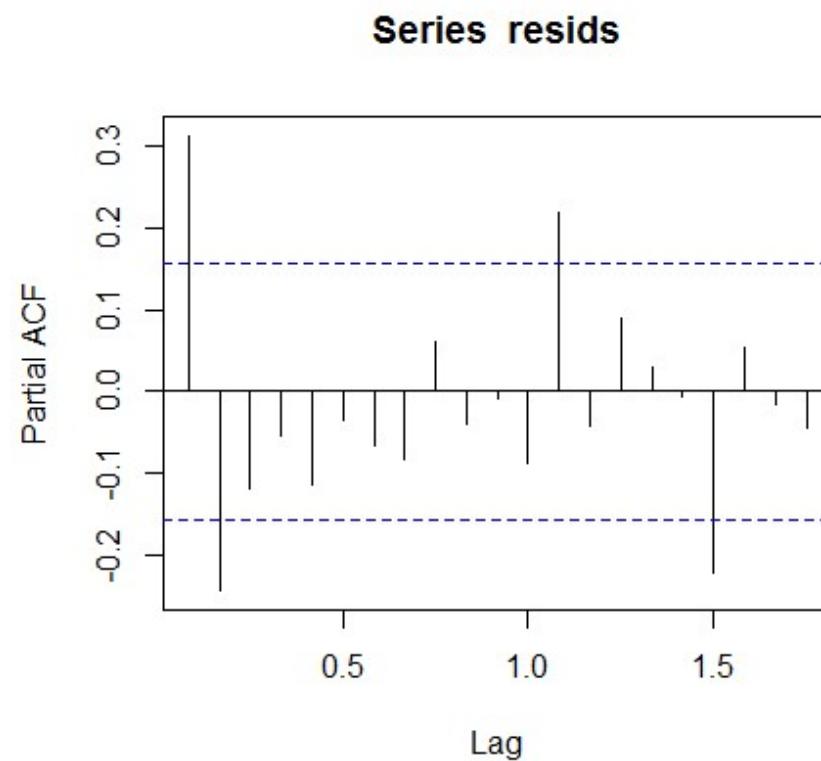
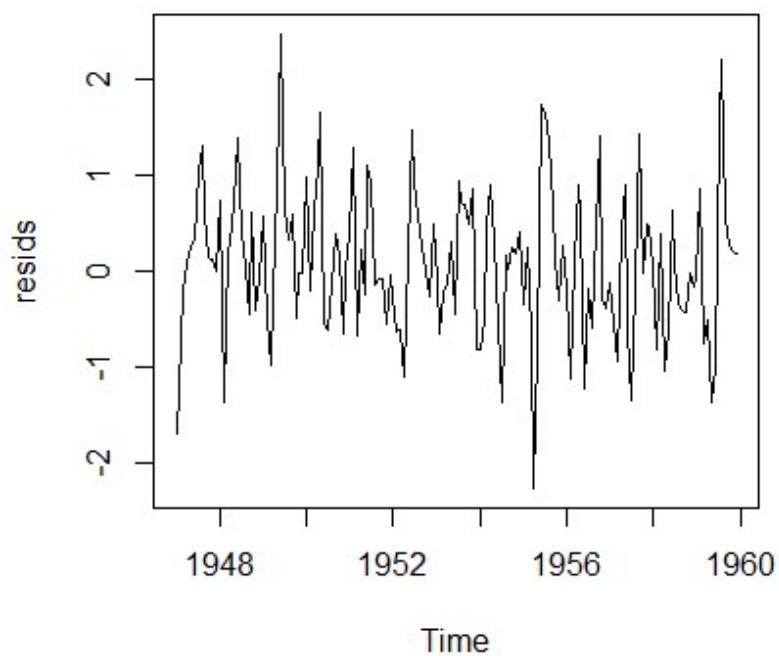


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Residual Plots are useful

```
> resids <- birthstimeseries - birthsforecast$fitted[,1]  
> plot(resids)  
> pacf(resids)  
> |
```



If the residuals are random and have no auto-correlation, then it means all useful information has been extracted. If the residuals show auto-correlation, it means more extractable information exists in the residual.

AR, MA AND ARIMA MODELS

AR(p) models

- Auto-regressive model of order p

$$\hat{y}_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{t-p}$$

- We find the best value of parameters (β_1, β_2, \dots) that minimize the errors in forecast of \hat{y}_t .
- The order of the model p is determined based on the number beyond which PACF terms are zero.



Moving Average or MA(q) models

- Model attempts to predict future values using past error in predictions $\varepsilon_1 = \hat{y}_1 - y_1$
- So MA(2) model is

$$\hat{y}_t = \mu + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2}$$

Where, μ is the average value of the time series

- Again, the parameters (ϕ_1, ϕ_2) are determined so that prediction error is minimized.
- The number of terms, q , is determined from the ACF plot. Its the maximum lag beyond which the ACF is 0



ARMA(p,q) model

- Combines both AR(p) and MA(q) models

- So a ARMA(2,1) model is

$$\hat{y}_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \phi_1 \varepsilon_{t-1}$$

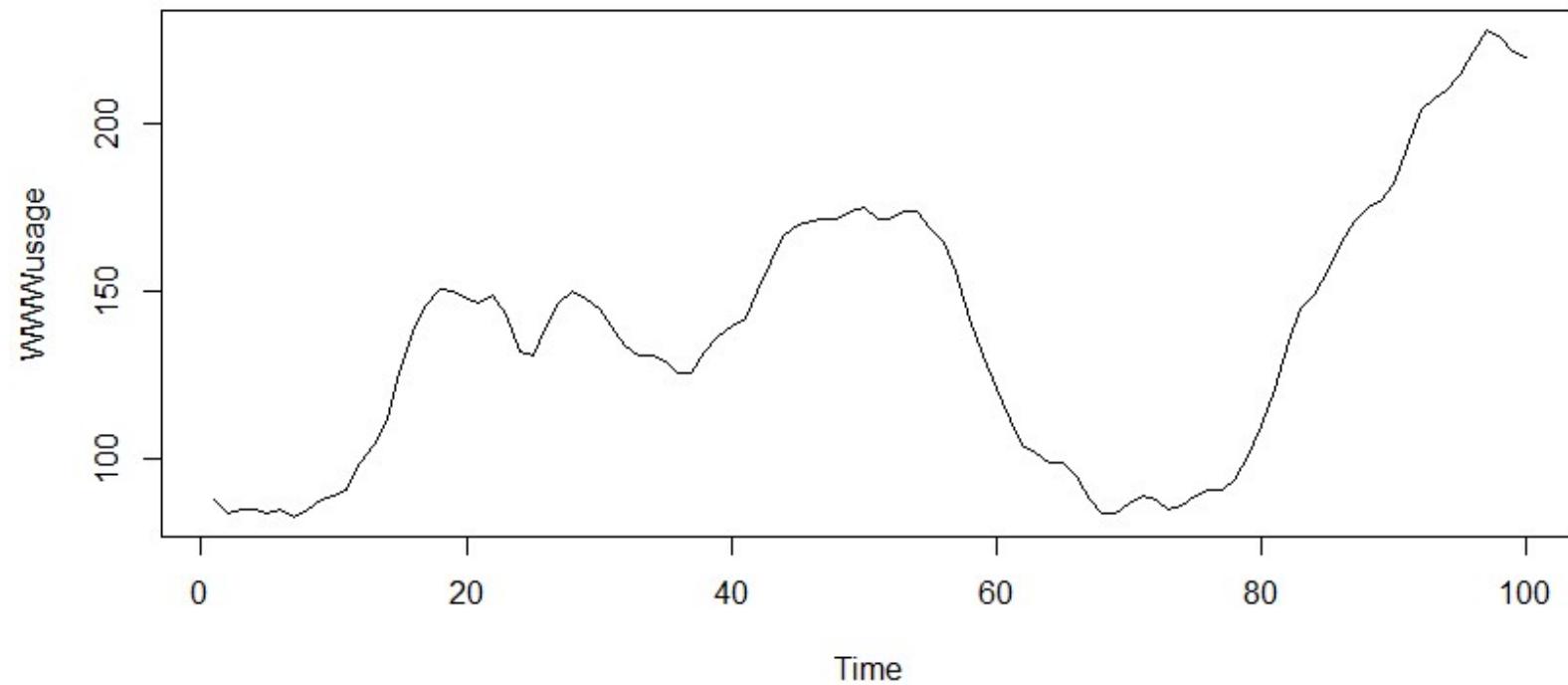


ARIMA(p,d,q) Model

- p is the number of autoregressive terms (a linear regression of the current value of the series against one or more prior values of the series)
 - Maximum lag beyond which PACF is 0
- d is the number of non-seasonal differences (order of the differencing) used to make the time series stationary
- q is the number of past prediction error terms used for the future forecasts.



Using ARIMA to forecast

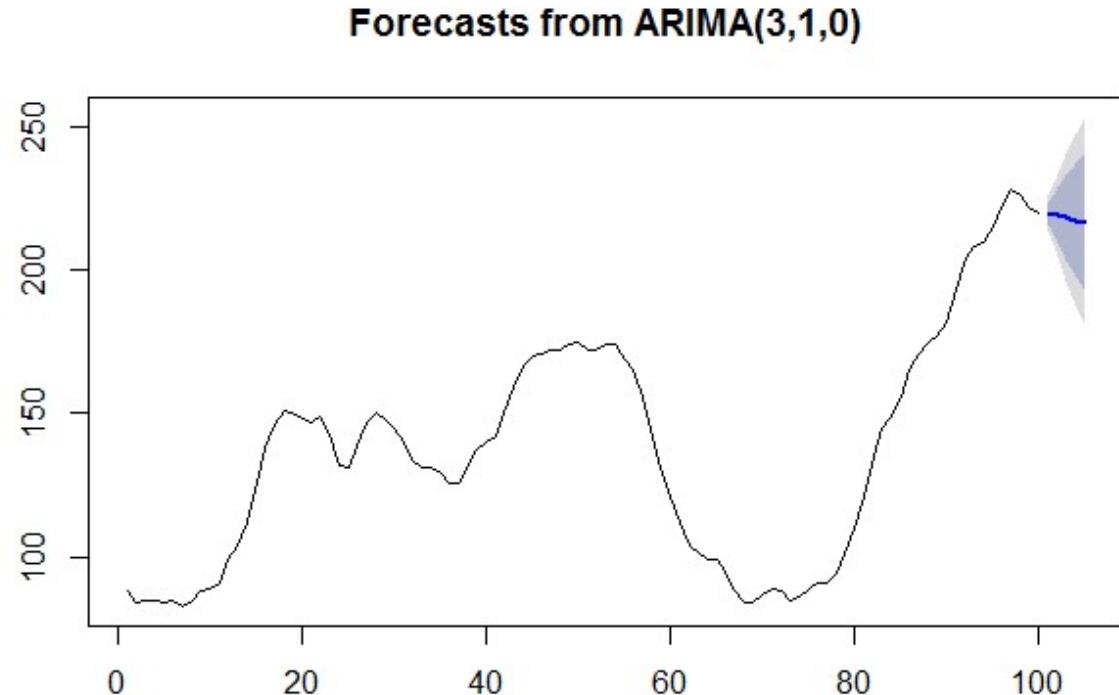


A time series of the numbers of users connected to the Internet through a server every minute.

Using ARIMA to forecast

Let us see what the forecast looks like if we use Arima(3,1,0) model

```
>  
> fit <- Arima(WWWusage,c(3,1,0))  
> plot(forecast(fit,h=5))  
>
```



The forecast is plotted in dark blue. The dark grey and light grey regions represent the 80% and 95% confidence intervals.



Model Identification

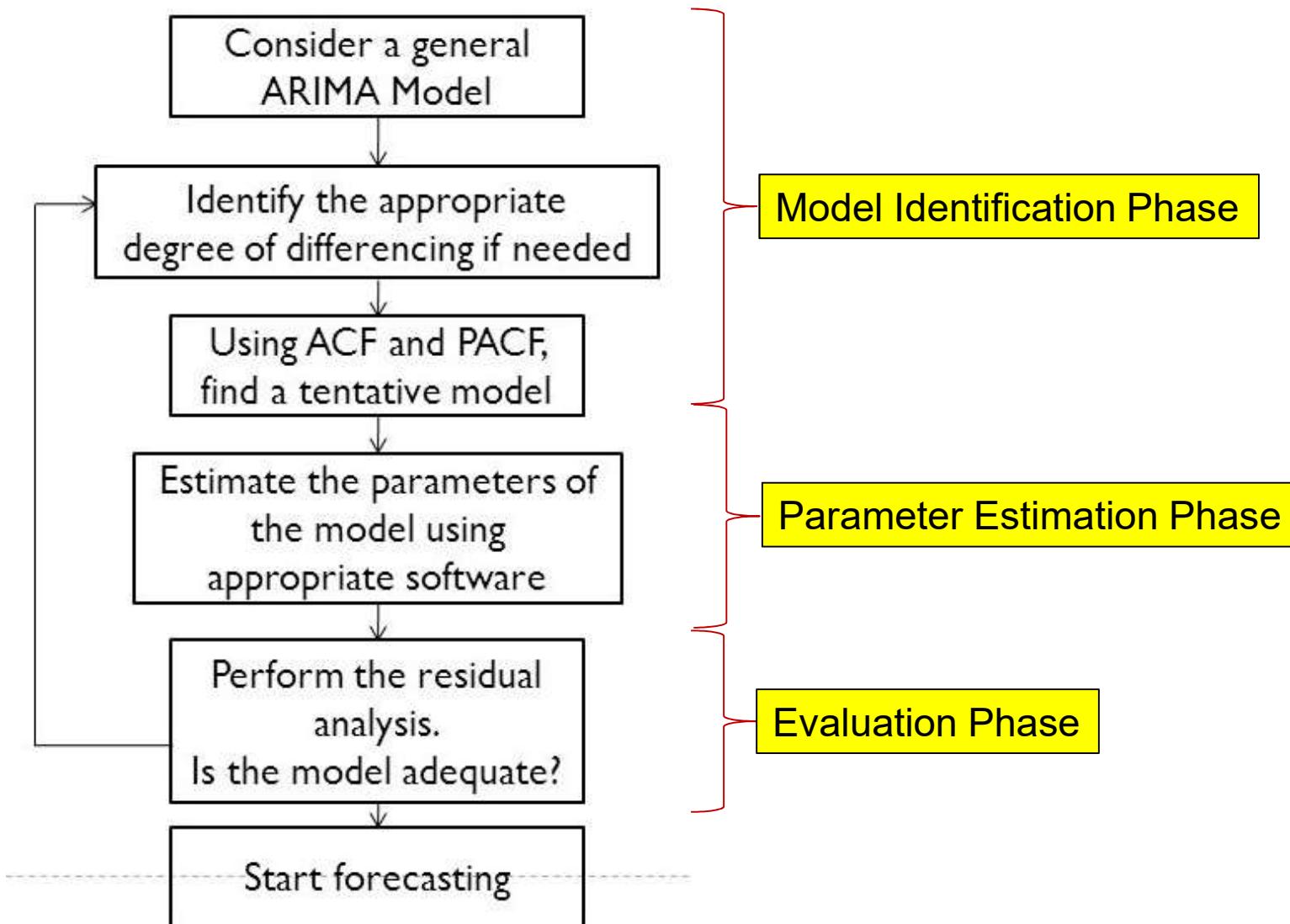
- Before Automated functions were available, one used to use ACF plots to determine the best value of (p,d,q) for a given dataset
- Box–Jenkins Methodology: Model identification and model selection
 - Make sure variables are stationary. Difference as necessary to get a constant mean and transformations to get constant variance.
 - Check for seasonality: Decays and spikes at regular intervals in ACF plots.
- Parameter estimation
 - Compute coefficients that best fit the selected model.
- Model checking
 - Check if residuals are independent of each other and constant in mean and variance over time (white noise).

http://www.ncss.com/wp-content/themes/ncss/pdf/Procedures/NCSS/The_Box-Jenkins_Method.pdf

- Non-seasonal ARIMA models are denoted $\text{ARIMA}(p,d,q)$
- Seasonal ARIMA (SARIMA) models are denoted $\text{ARIMA}(p,d,q)(P,D,Q)_m$, where m refers to the number of periods in each season and (P,D,Q) refer to the autoregressive, differencing and moving average terms of the seasonal part of the ARIMA model.



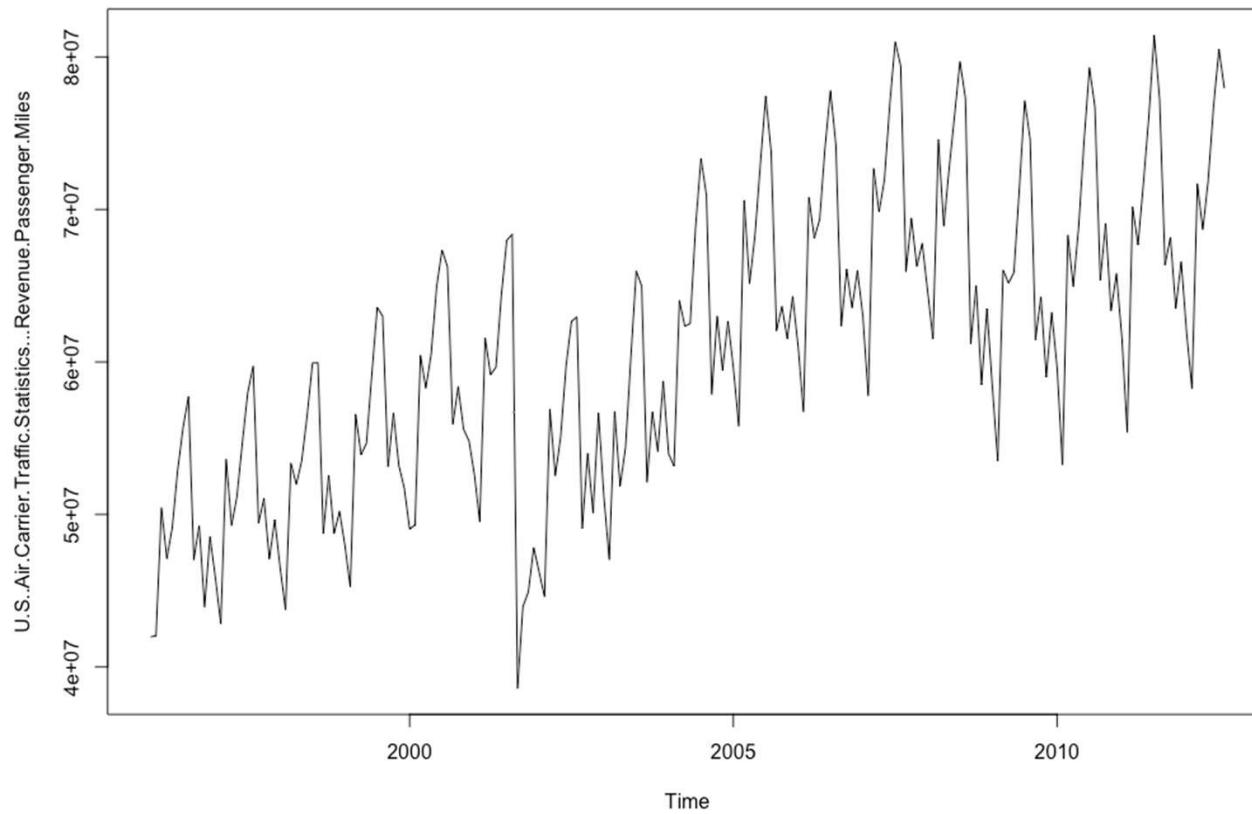
Time Series Model Building Using ARIMA



Time Series Model Building Using ARIMA

Identification Phase

Step 1: Plot the data (transform data to stabilize variance, if required)



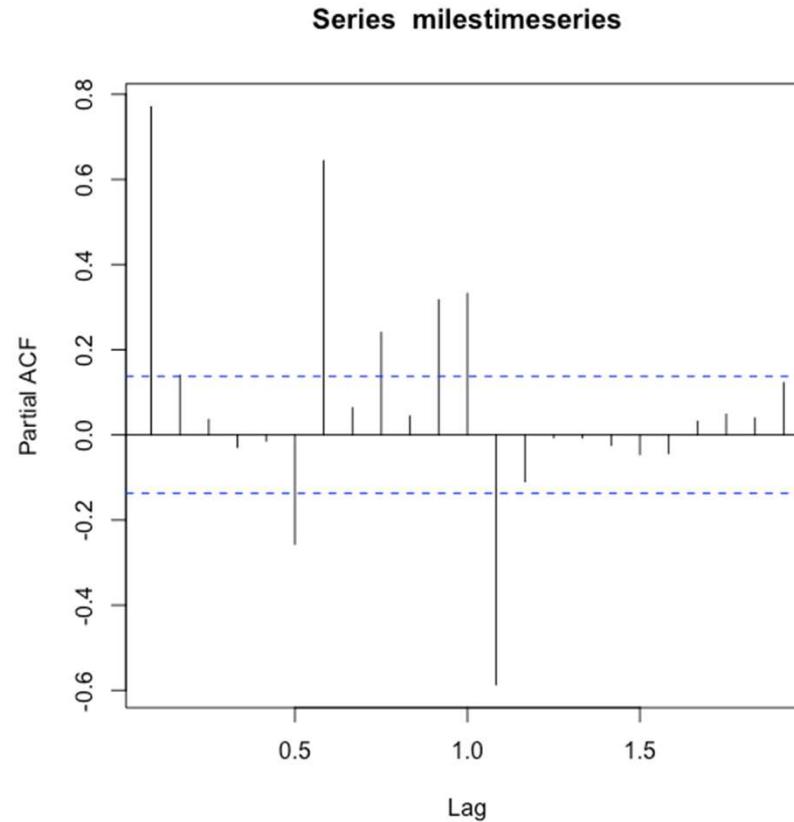
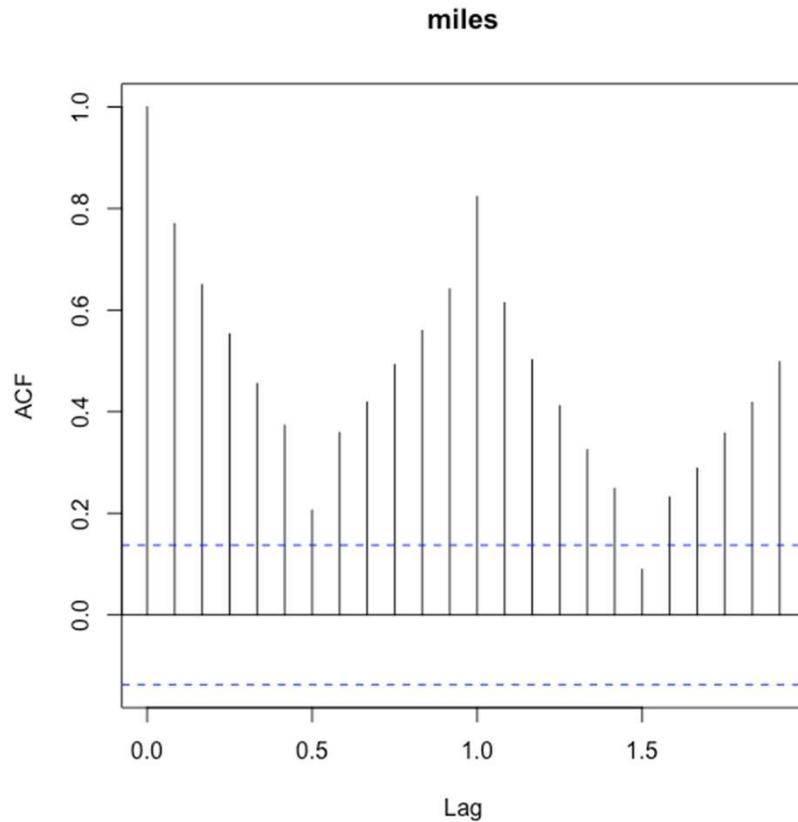
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Time Series Model Building Using ARIMA

Identification Phase

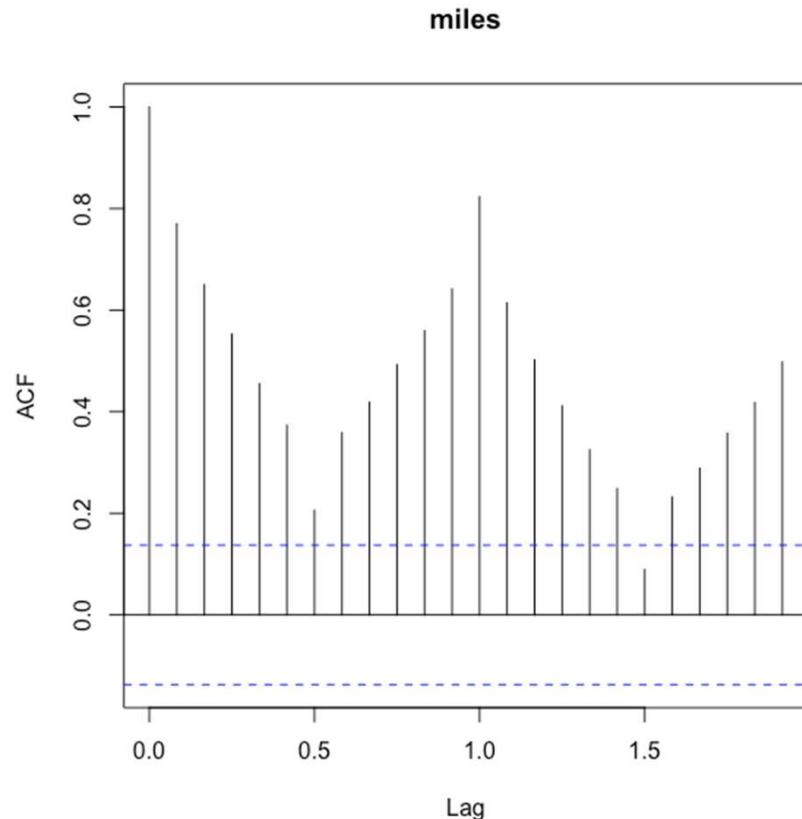
Step 2: Plot ACF and PACF to get preliminary understanding of the processes involved.



Time Series Model Building Using ARIMA

Identification Phase

Step 2: Plot ACF and PACF to get preliminary understanding of the processes involved.

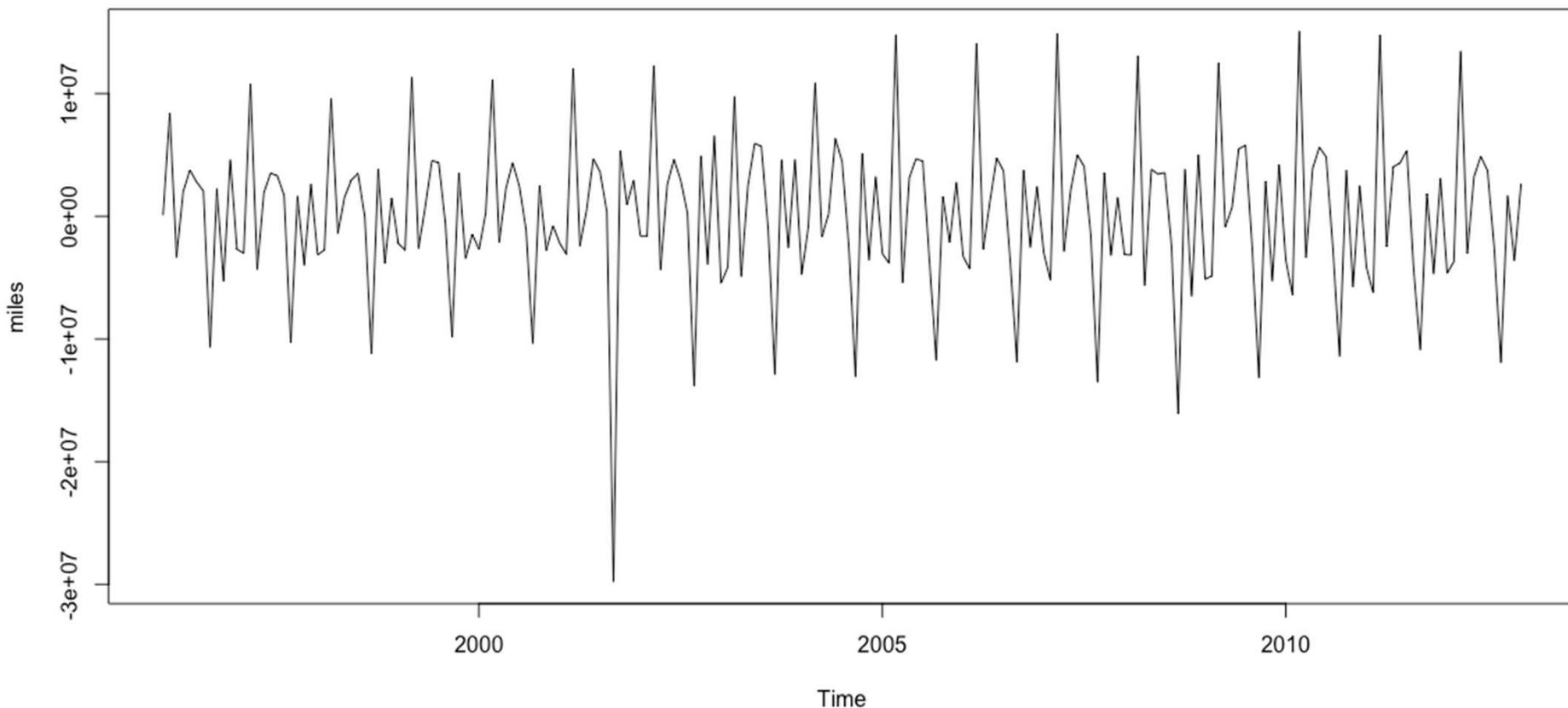


The suspension bridge pattern in ACF (also, positive and negative spikes in PACF) suggests non-stationarity and strong seasonality.

Time Series Model Building Using ARIMA

Identification Phase

Step 3: Perform a non-seasonal difference. It is the same as an ARIMA(0,1,0) model.



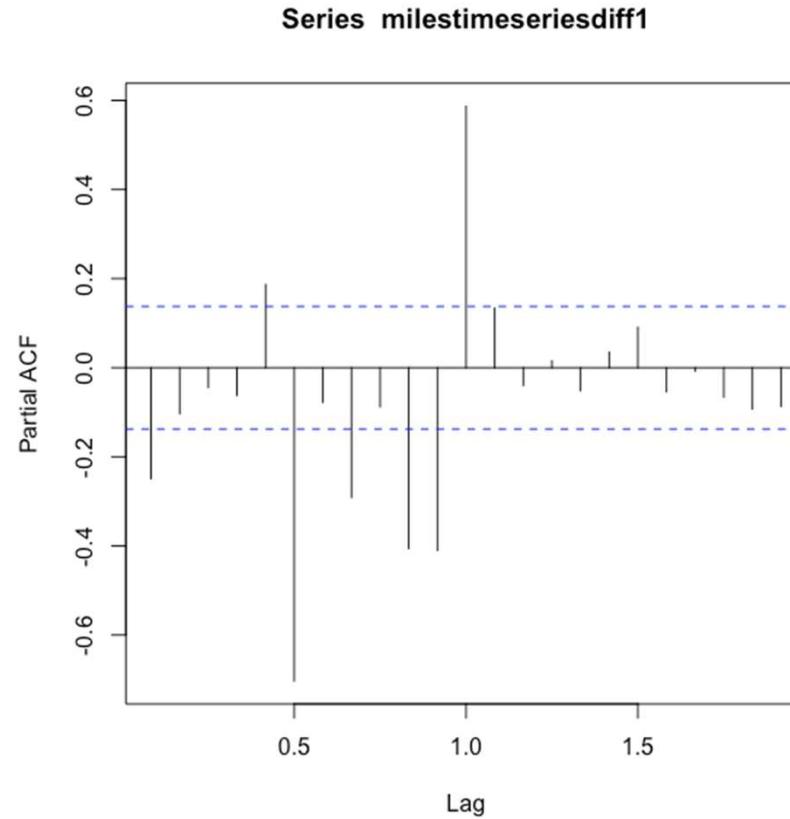
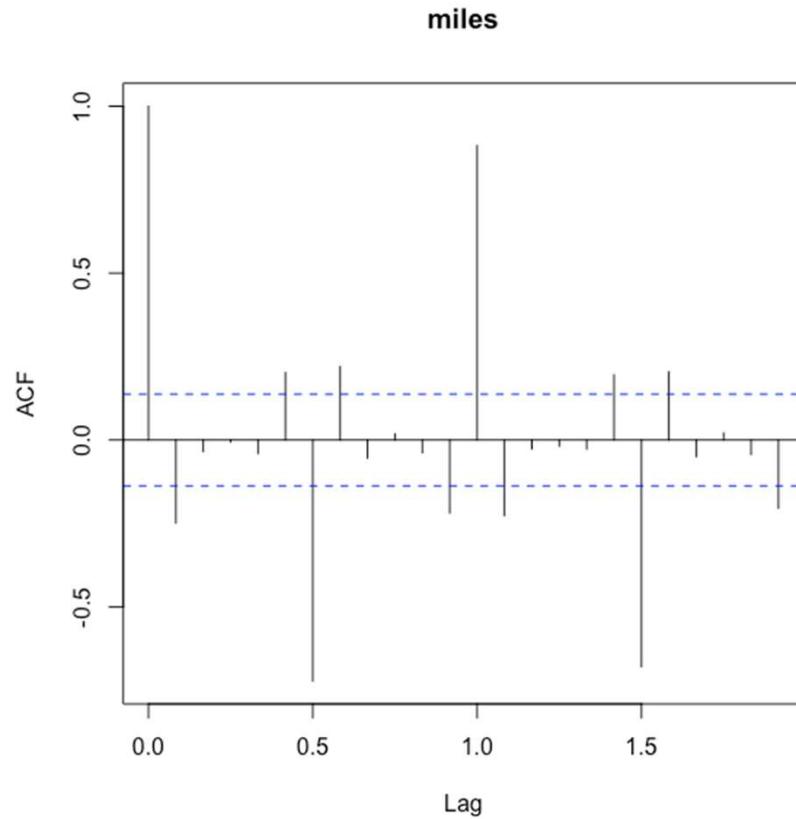
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Time Series Model Building Using ARIMA

Identification Phase

Step 4: Check ACF and PACF of differenced data to explore remaining dependencies.

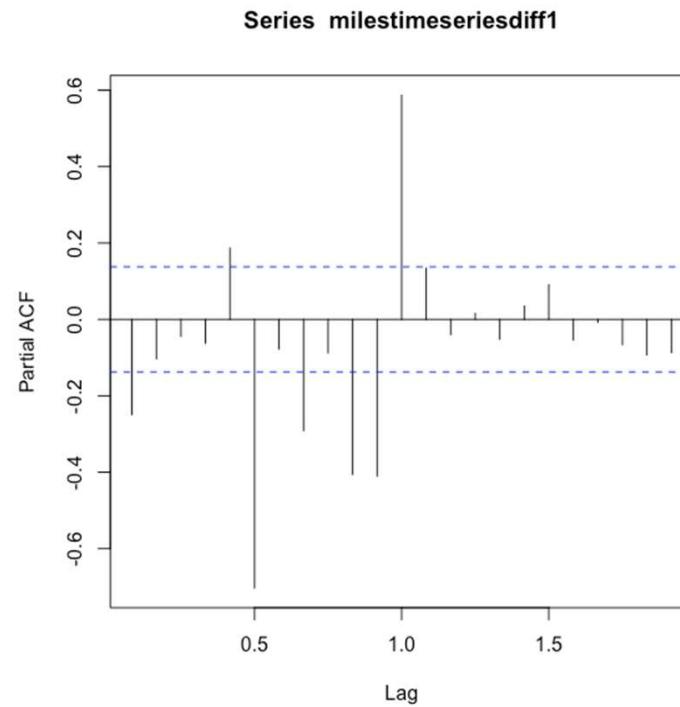
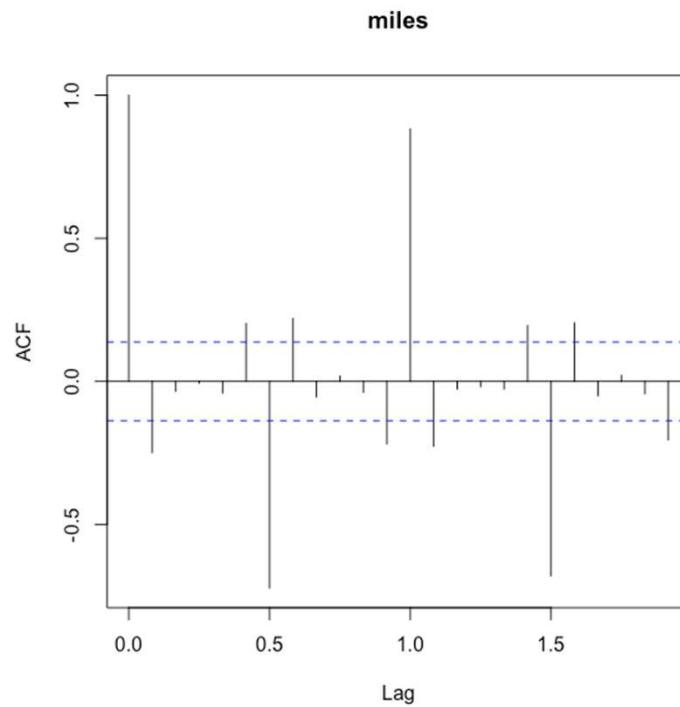


Time Series Model Building Using ARIMA

Identification Phase

Step 4: Check ACF and PACF of differenced data to explore remaining dependencies.

The differenced series looks somewhat stationary but has strong seasonal lags.



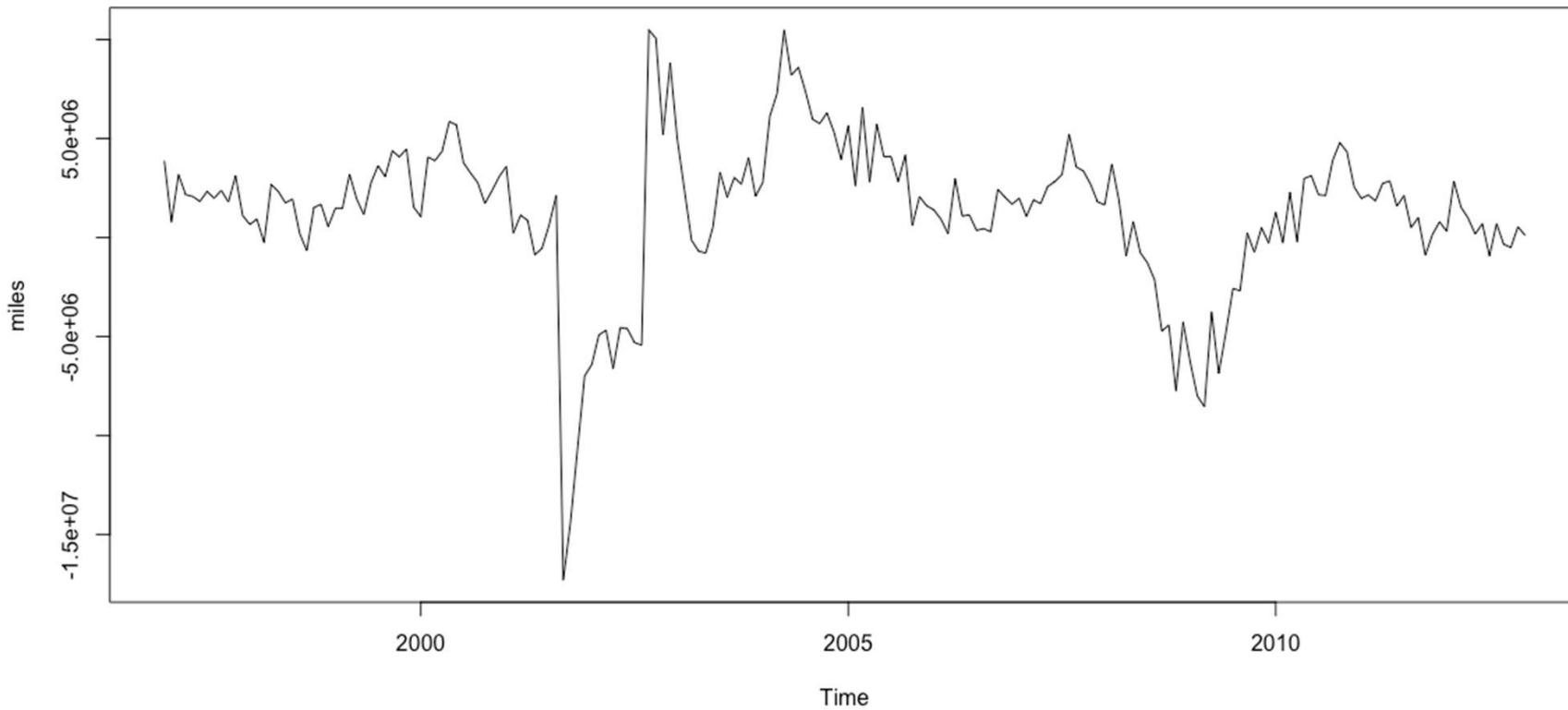
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Time Series Model Building Using ARIMA

Identification Phase

Step 5: Perform seasonal differencing (t_0-t_{12}, t_1-t_{13} , etc.) on the **original** time series to get seasonal stationarity. This is the same as an ARIMA(0,0,0)(0,1,0)₁₂ model.



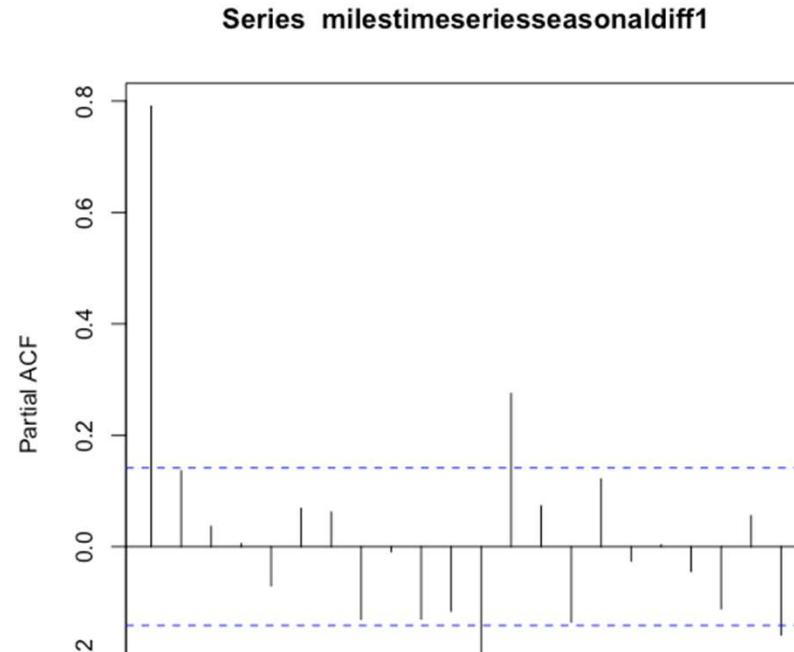
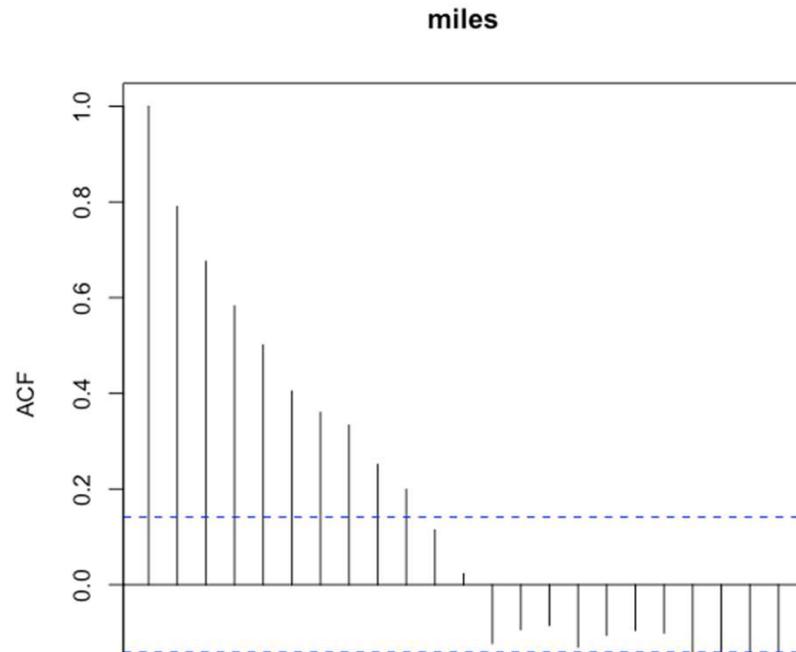
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Time Series Model Building Using ARIMA

Identification Phase

Step 6: Check ACF and PACF of seasonally differenced data to explore remaining dependencies and identify model(s).



Strong positive autocorrelation indicates need for either an AR term or a non-seasonal differencing.

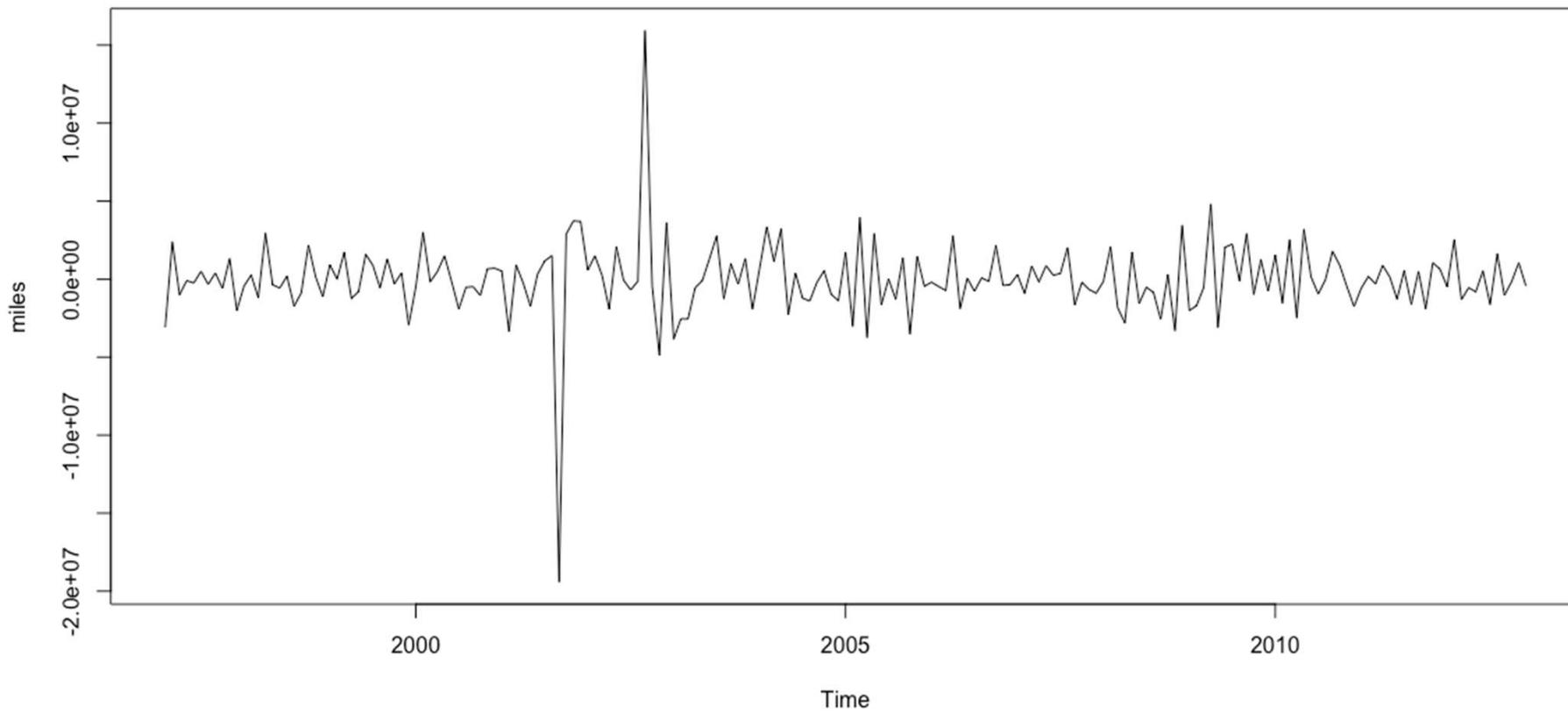
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Time Series Model Building Using ARIMA

Identification Phase

Step 7: Perform a non-seasonal differencing on seasonally differenced data. This is like an ARIMA(0,1,0)(0,1,0)₁₂ model.



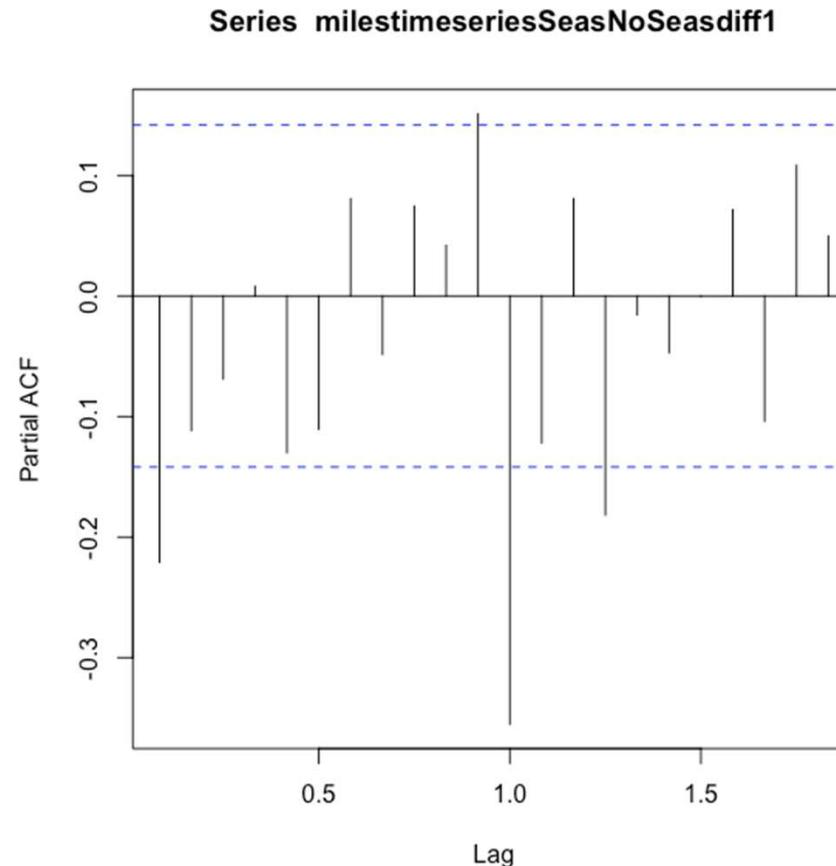
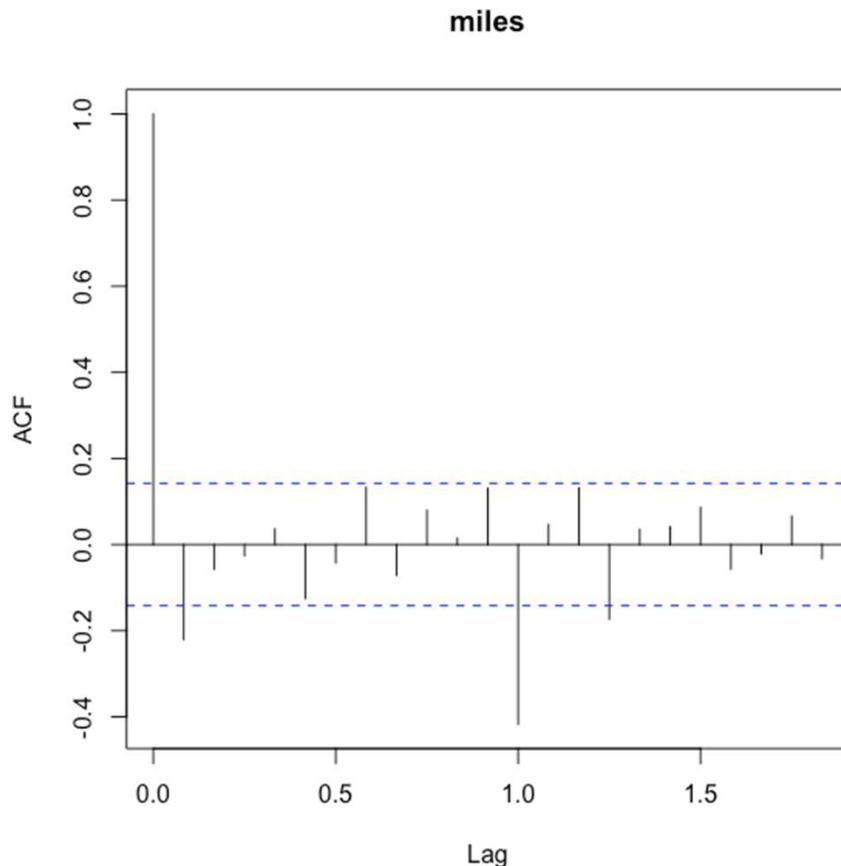
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Time Series Model Building Using ARIMA

Identification Phase

Step 8: Check ACF and PACF to explore remaining dependencies.



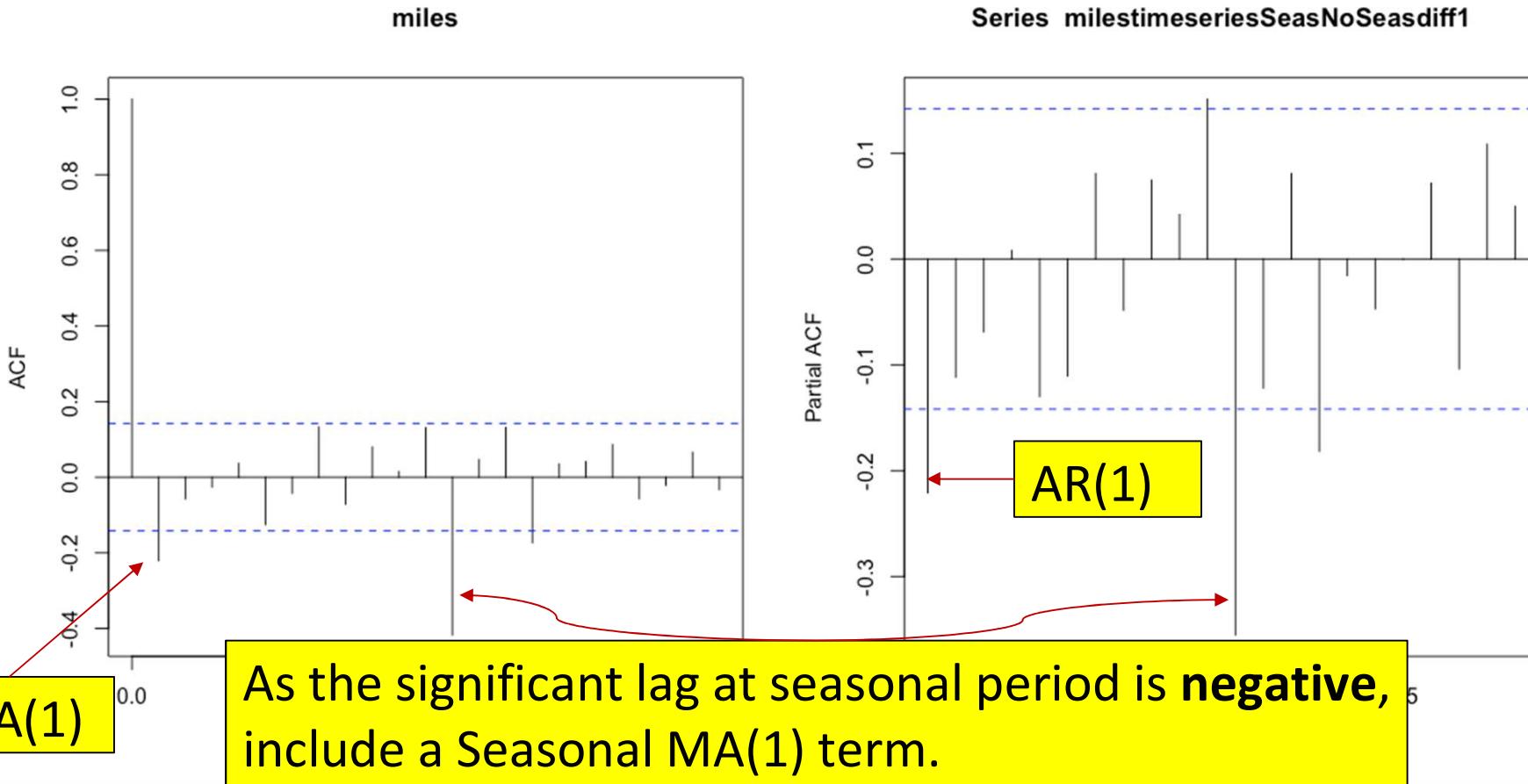
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Time Series Model Building Using ARIMA

Identification Phase

Step 8: This indicates an ARIMA(1,1,1)(0,1,1)₁₂ model.



Time Series Model Building Using ARIMA

Parameter Estimation Phase

Step 9: Calculate parameters using the identified model(s). Use AIC to pick the best model.

```
Series: milestoneseries
ARIMA(1,1,1)(0,1,1)[12]
```

Coefficients:

	ar1	ma1	sma1
	0.4501	-0.7035	-0.7393
s.e.	0.1755	0.1407	0.0641

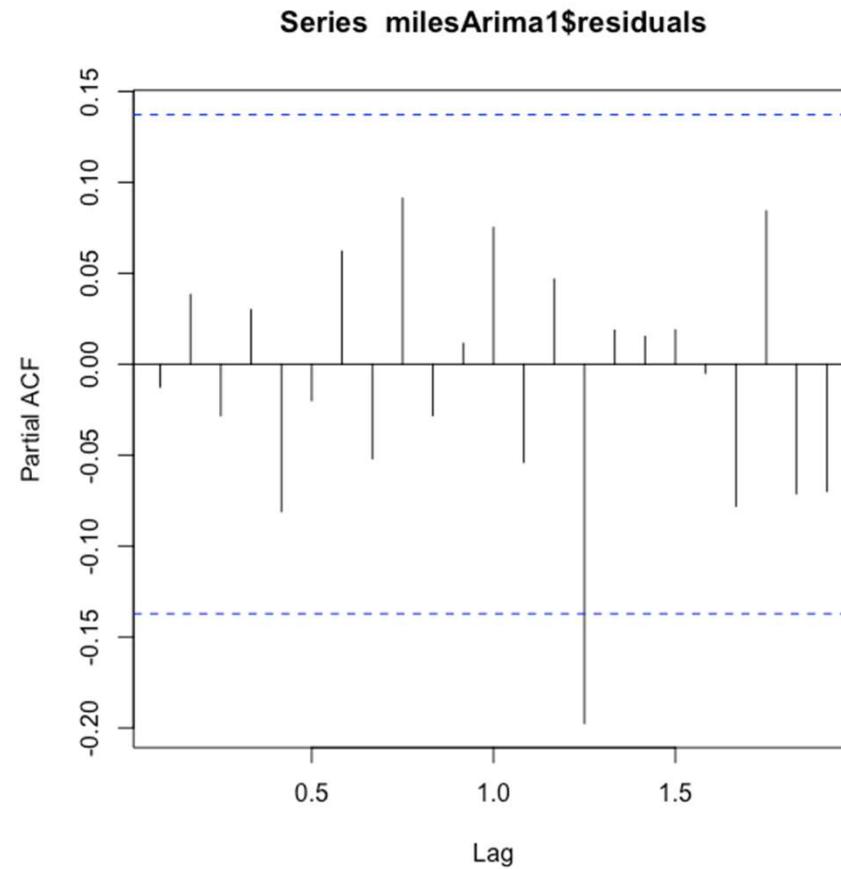
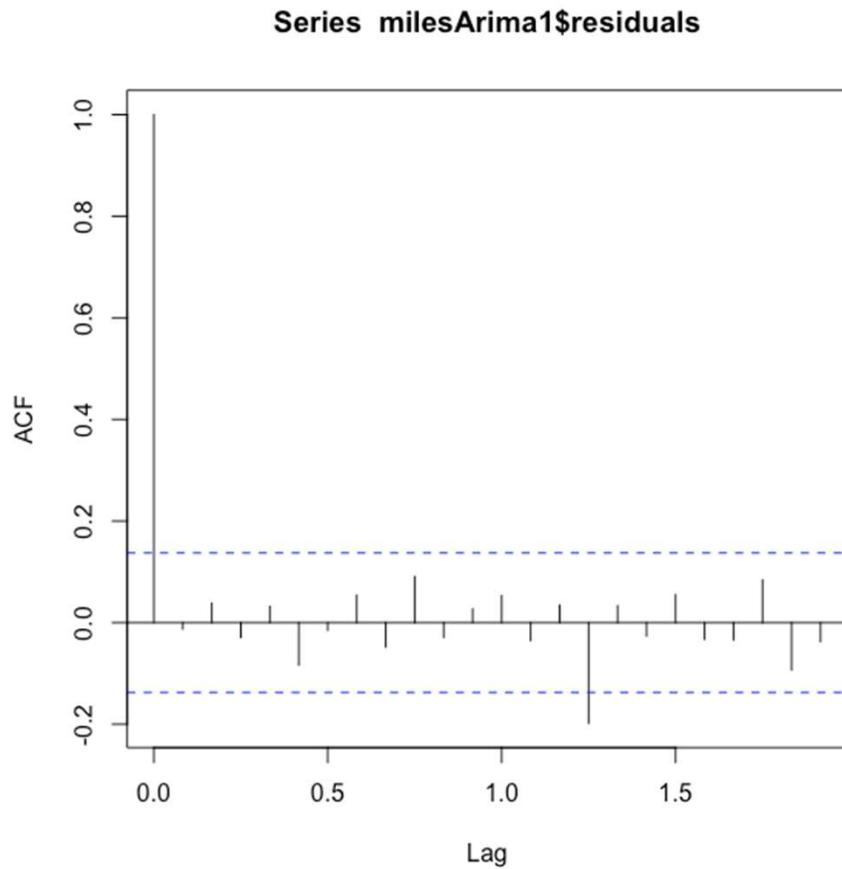
```
sigma^2 estimated as 3.917e+12: log likelihood=-3043.49
AIC=6094.99    AICc=6095.2    BIC=6107.99
```



Time Series Model Building Using ARIMA

Evaluation Phase

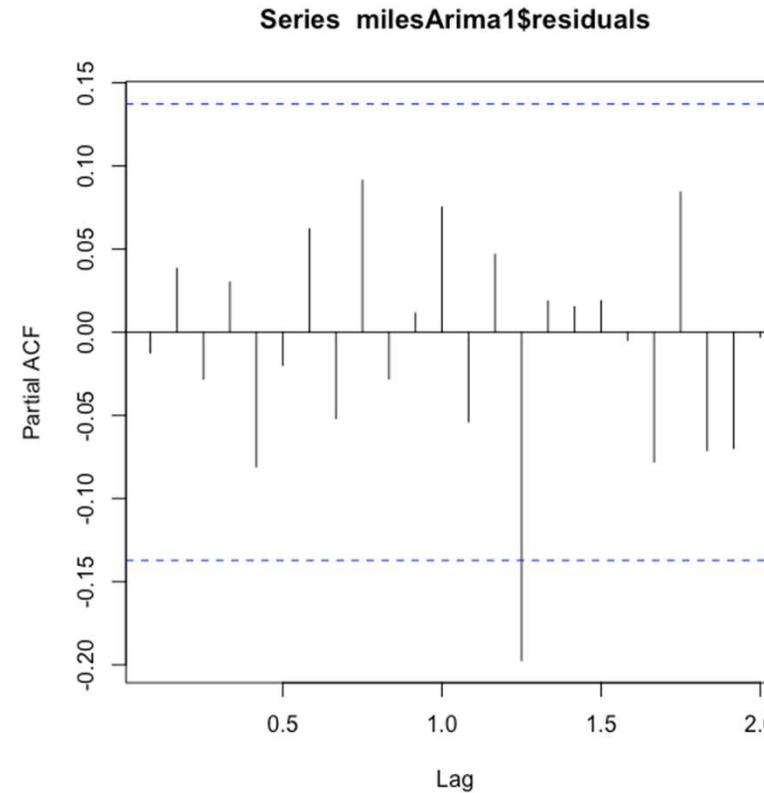
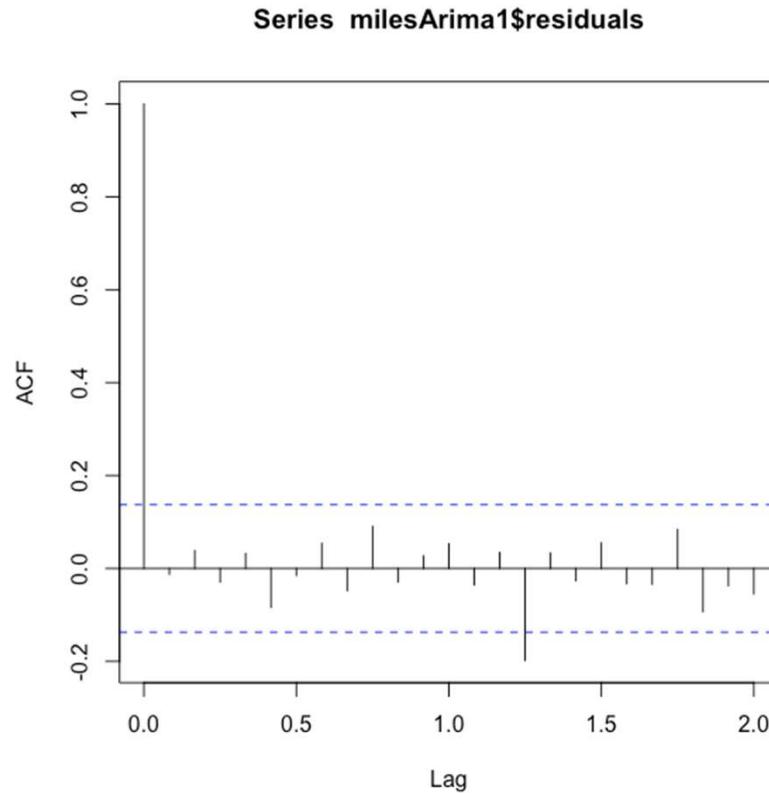
Step 10: Check ACF and PACF of the residuals to evaluate model.



Time Series Model Building Using ARIMA

Evaluation Phase

Step 10: The residuals indicate white noise. Indicates a good model that can be used for forecasting.



Time Series Model Building Using ARIMA

Evaluation Phase

Step 10: The residuals indicate white noise. Can be checked using Ljung-Box test.

$$Q^* = n(n + 2) \sum_{k=1}^h \frac{r_k^2}{n - k}$$

h is the maximum lag being considered
 n is the # of observations (length of the time series)
 r_k is the autocorrelation

* For non-seasonal time series, use $h = \min(10, n/5)$
For seasonal time series, use $h = \min(2m, n/5)$, where m is the seasonal period

Box-Ljung test

```
data: milesArima1$residuals
X-squared = 21.65, df = 24, p-value = 0.6002
```

If residuals are white noise (purely random), then Q has a χ^2 distribution with $h-p$ degrees of freedom, where p is the number of parameters estimated in the model

* <http://robjhyndman.com/hyndtsight/ljung-box-test/>

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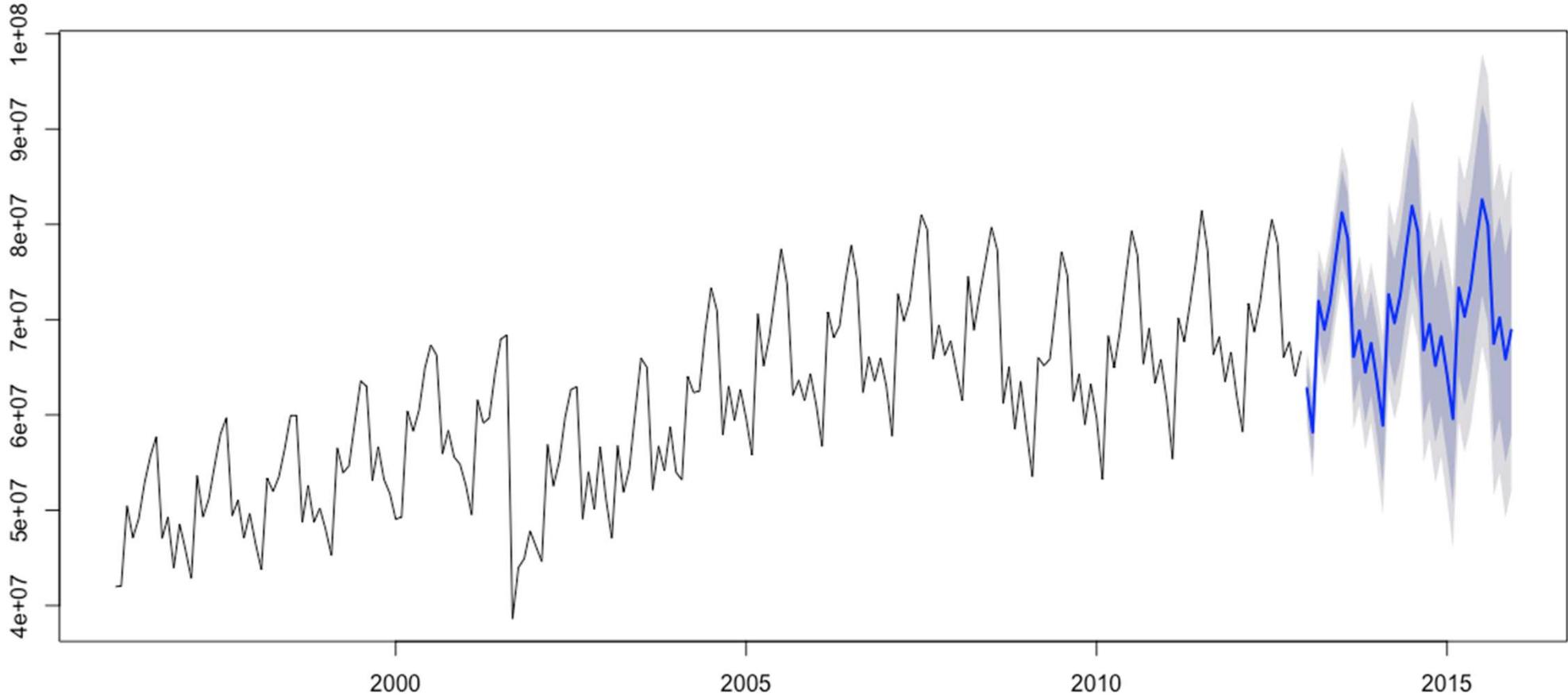


Time Series Model Building Using ARIMA

Forecasting Phase

Step 11: Start forecasting.

Forecasts from ARIMA(1,1,1)(0,1,1)[12]



Time Series Model Building Using ARIMA

A nice summary of rules for identifying ARIMA models

<http://people.duke.edu/~rnau/arimrule.htm>

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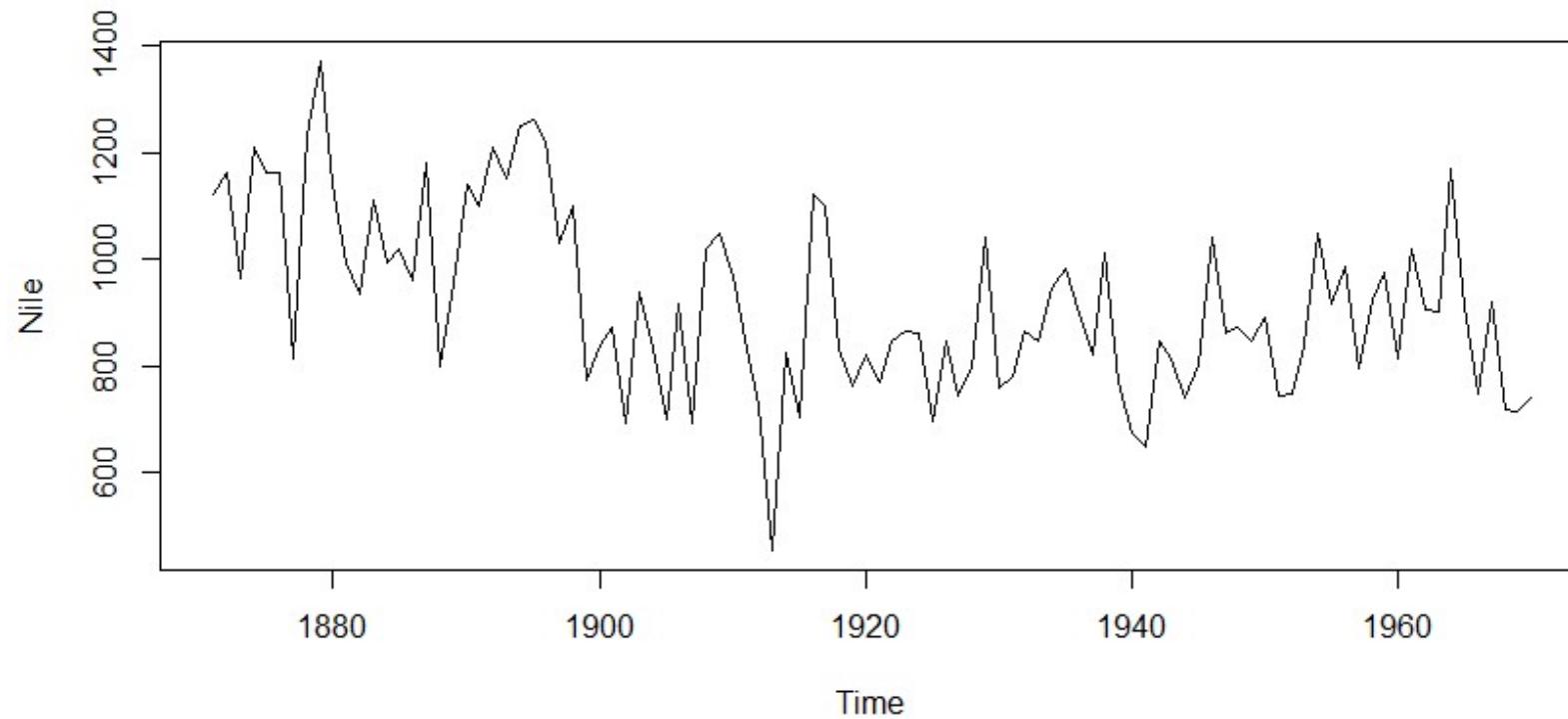


Model Selection

- The number of parameters (p,d,q) needed to fit, depends on the dataset
- There are techniques that automate model selection
- *auto.Arima* command in R picks the best p,d & q parameters for ARIMA(p,d,q)



Auto.Arima: Annual Flow in River Nile



```
>  
> plot(Nile)  
> |
```

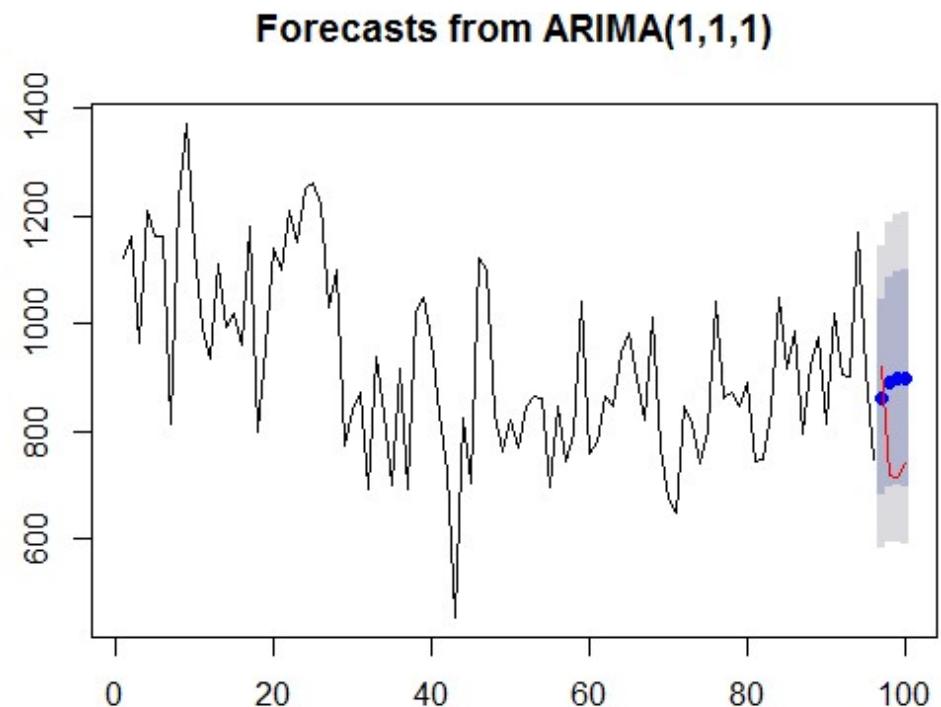


Auto.Arima: Annual Flow in River Nile

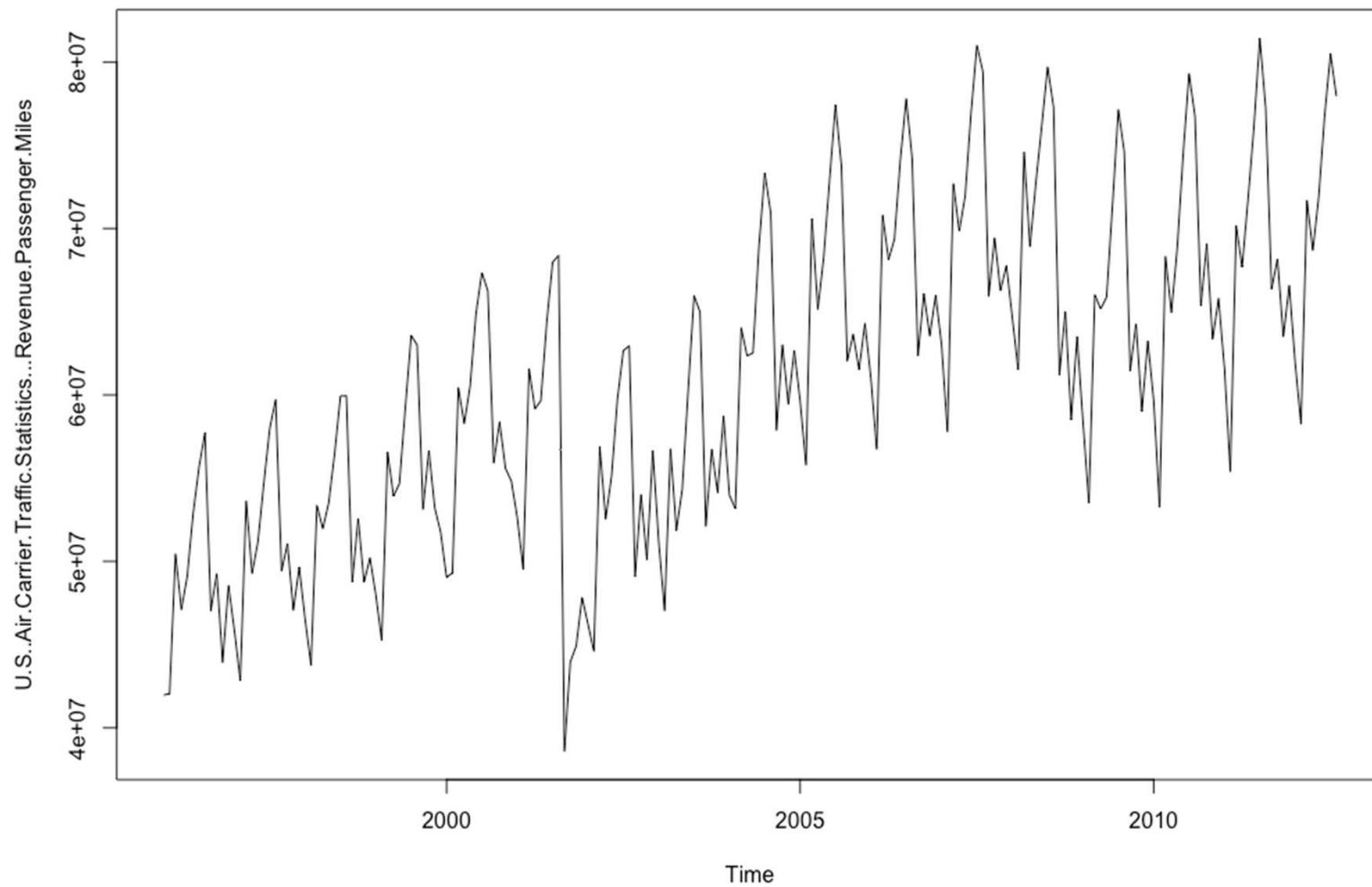
```
> # Fit auto.arima to the first 96 points
> fitNile <- auto.arima(Nile[1:96])
> fitNile
Series: Nile[1:96]
ARIMA(1,1,1)

Coefficients:
      ar1      ma1
    0.2389 -0.8711
  s.e.  0.1214  0.0585

sigma^2 estimated as 20372:  log likelihood=-605.59
AIC=1217.17  AICc=1217.44  BIC=1224.84
>
> #Now we predict last 4 points using the fit
> plot(forecast(fitNile,h=4))
> lines(97:100,Nile[97:100],col="red")
```



Time Series Model Building Using ARIMA - RPM



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Time Series Model Building Using ARIMA - RPM

Auto ARIMA

```
Series: milestoneseries
ARIMA(1,0,1)(0,1,1)[12] with drift

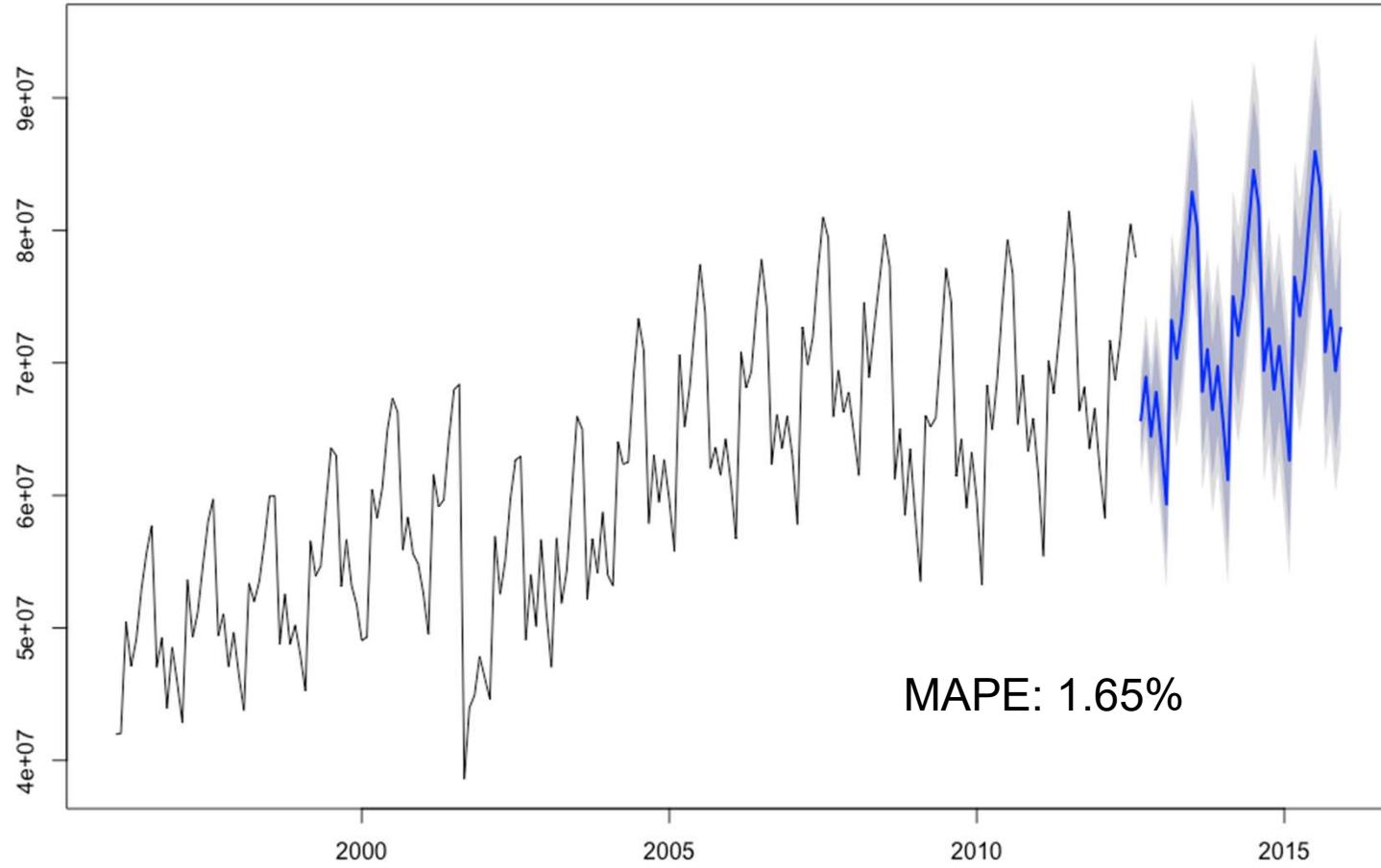
Coefficients:
            ar1      ma1     sma1      drift
            0.9078  -0.2093  -0.7266  110280.44
s.e.    0.0364   0.0885   0.0682   31856.26

sigma^2 estimated as 3.901e+12: log likelihood=-2994.93
AIC=5999.86  AICc=6000.19  BIC=6016.04
```



Time Series Model Building Using ARIMA - RPM Forecast

Forecasts from ARIMA(1,0,1)(0,1,1)[12] with drift



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Manufacturing Case Study

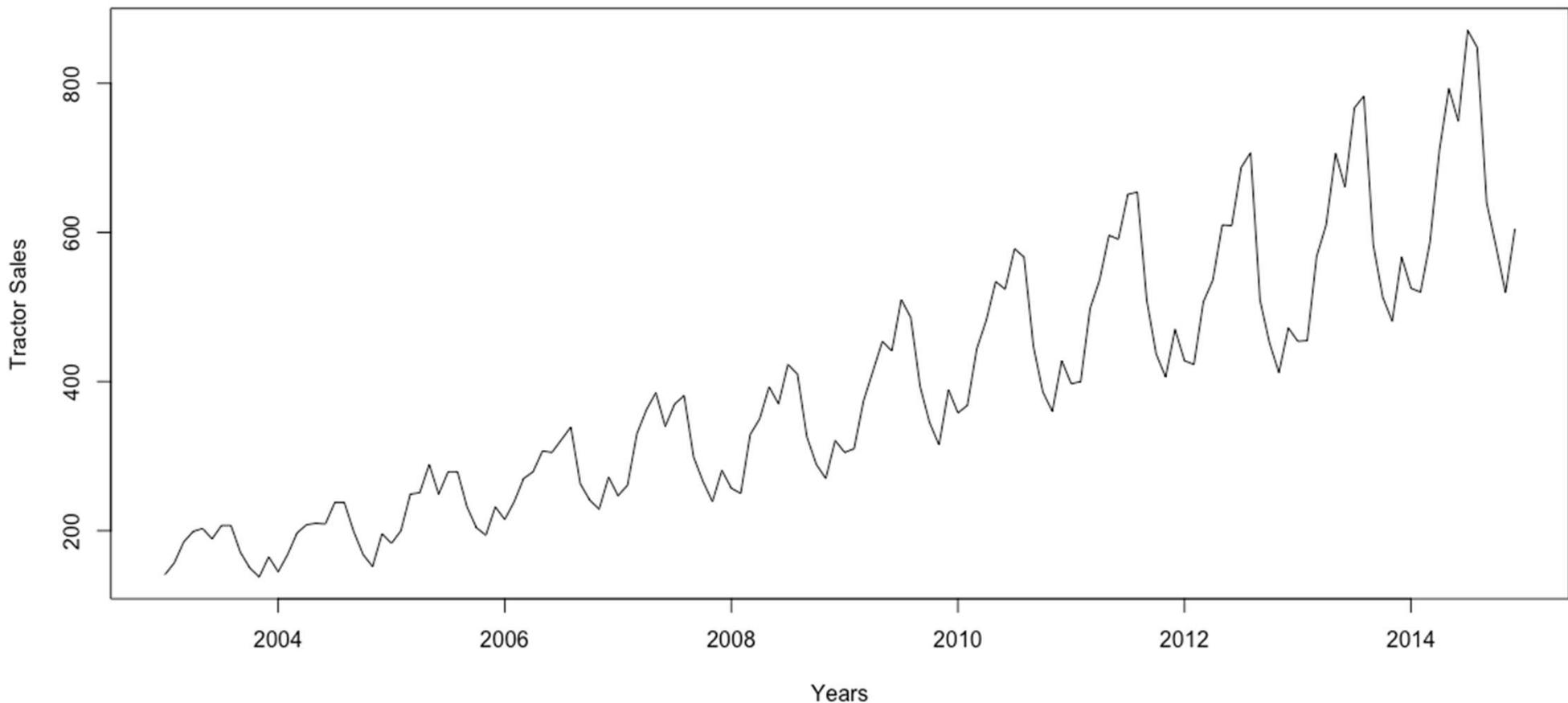
FORECASTING TRACTOR SALES

CSE 7202C



Tractor Sales Case Study

Step 1: Convert data into time series, and plot and decompose into components to understand it.

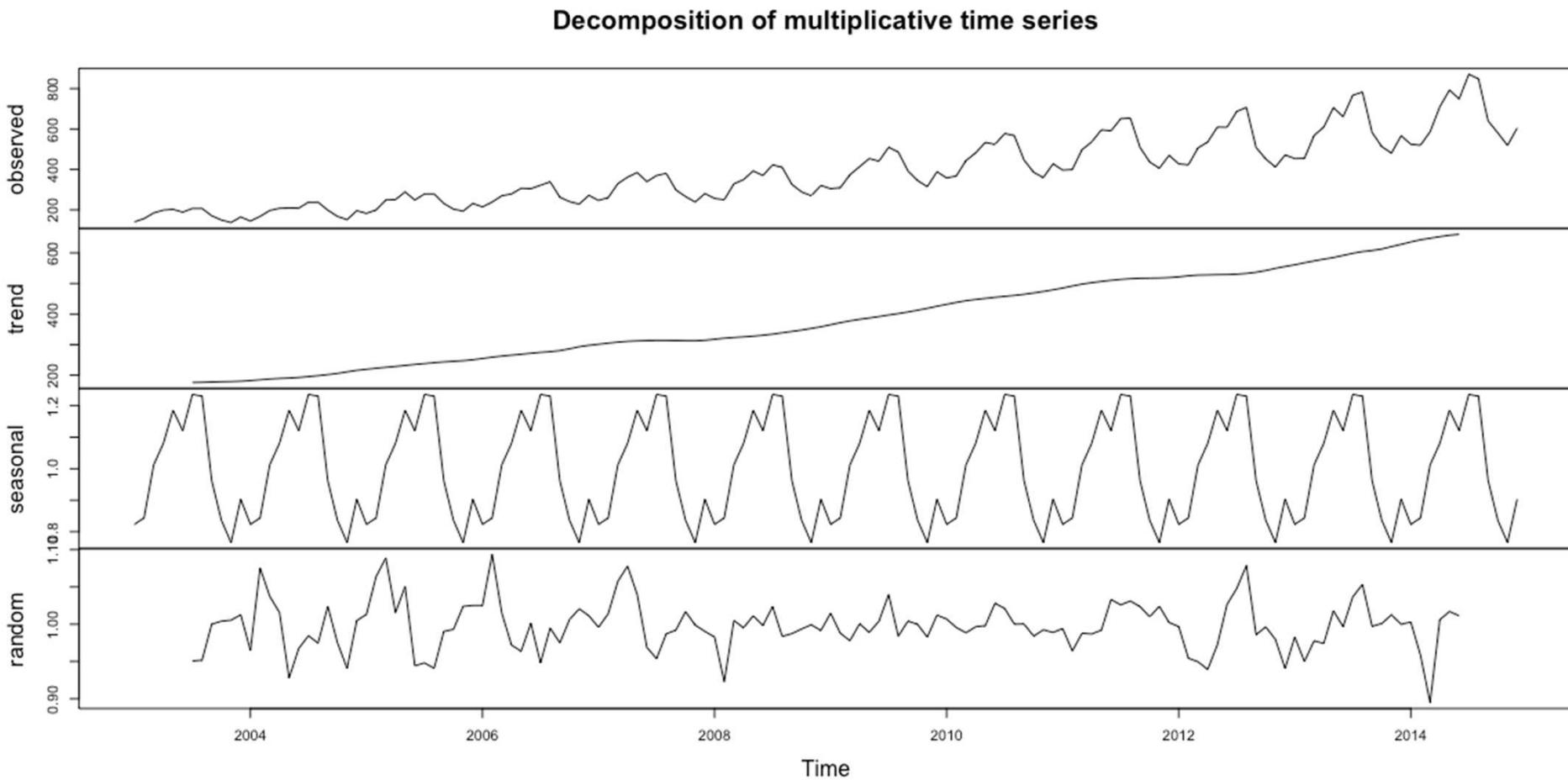


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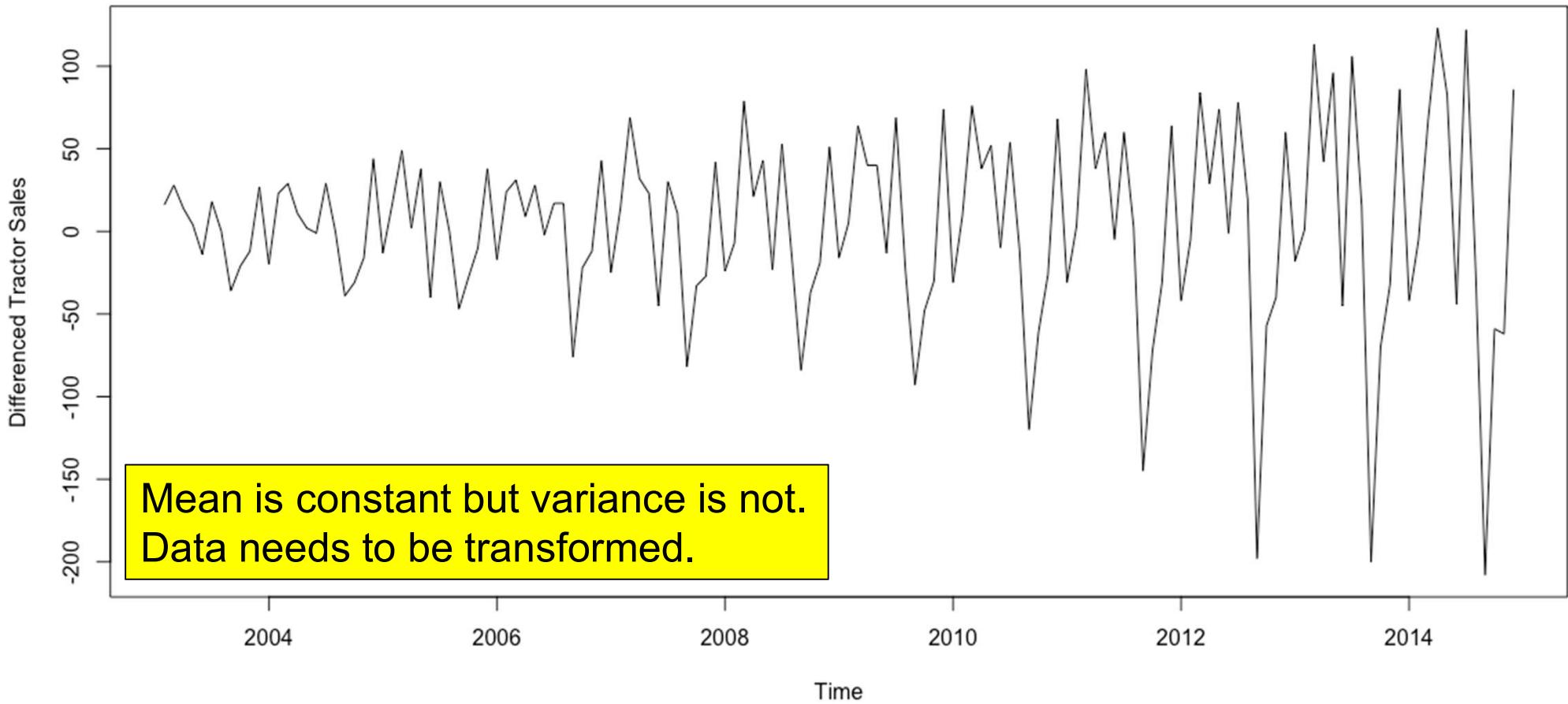
Tractor Sales Case Study

Step 1: Convert data into time series, and plot and decompose into components to understand it.



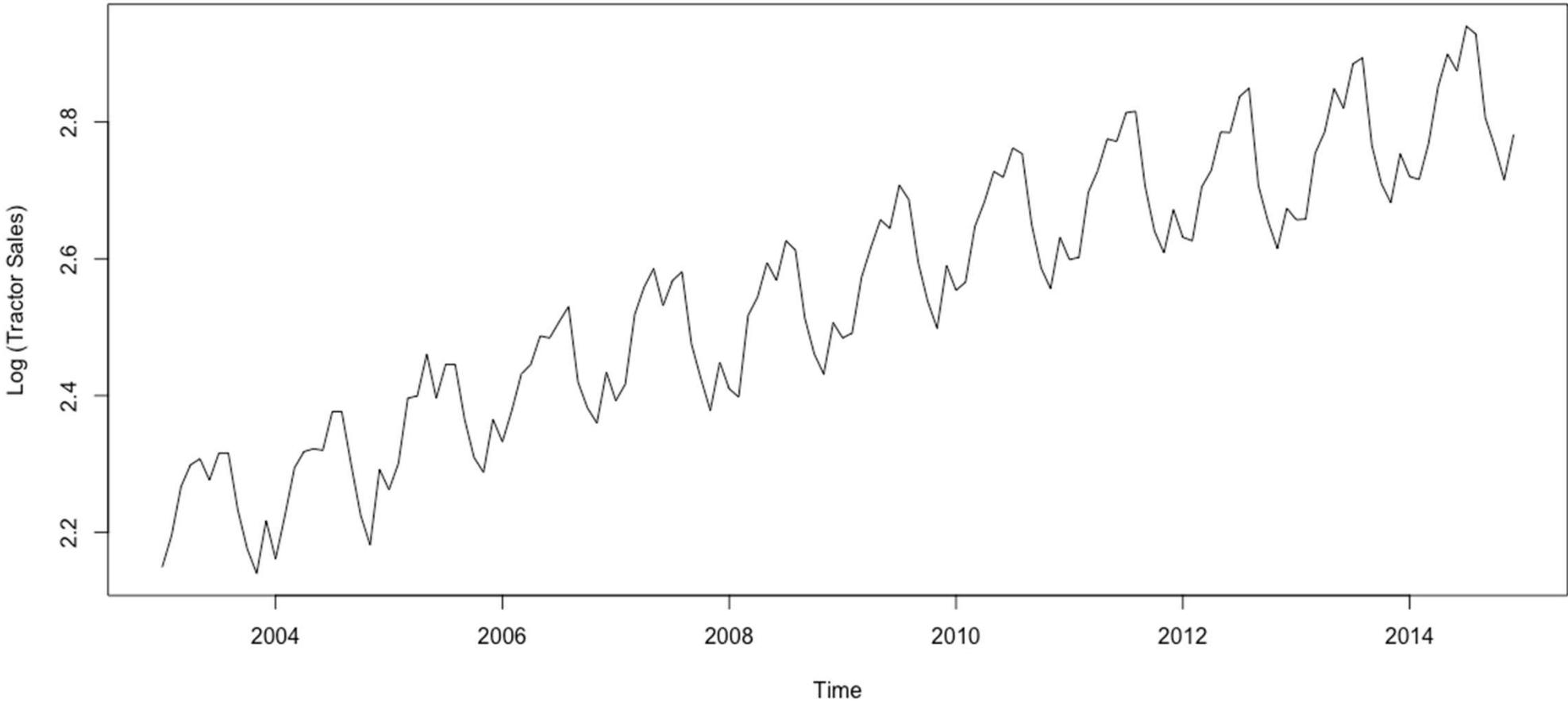
Tractor Sales Case Study

Step 2: Difference the data to make it stationary.



Tractor Sales Case Study

Step 3: Log transform the data.

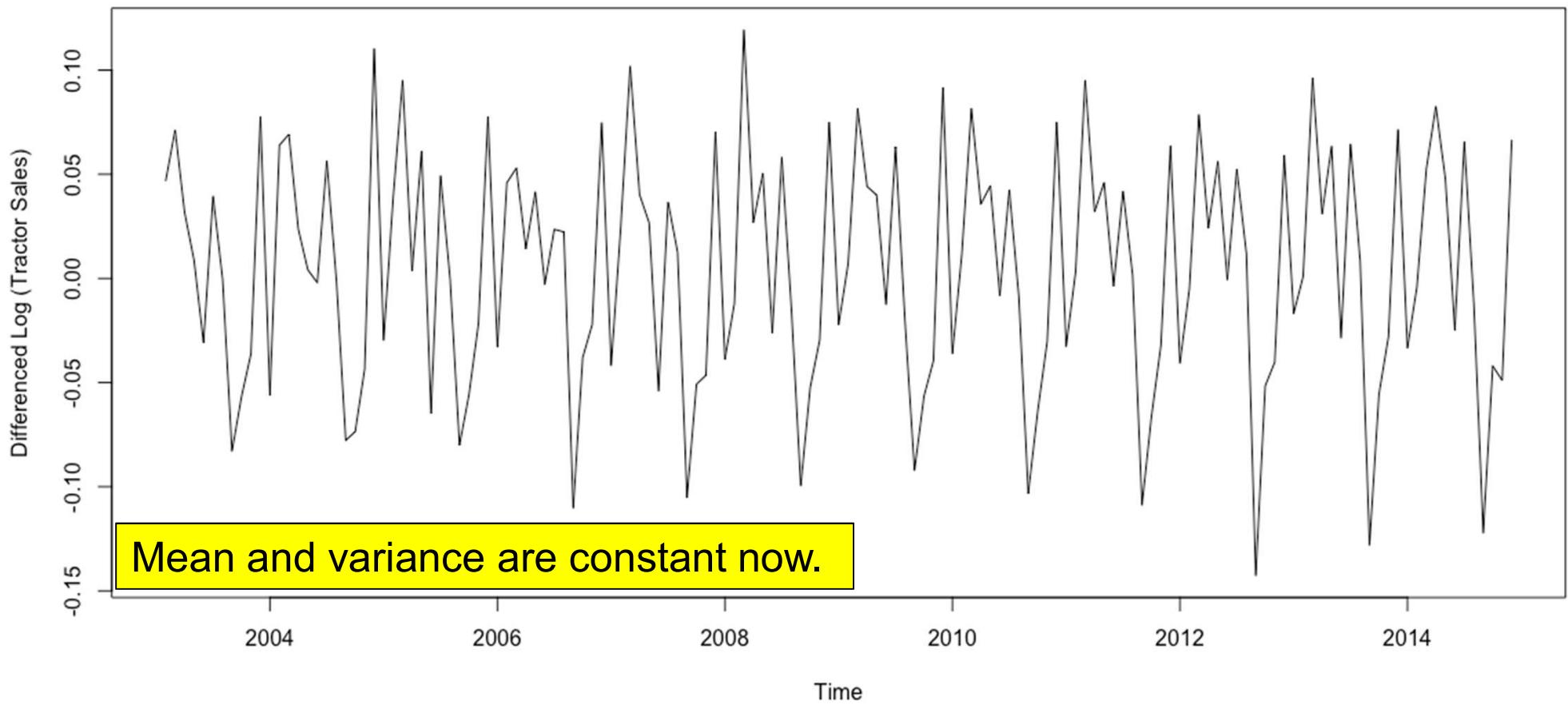


INSOFE



Tractor Sales Case Study

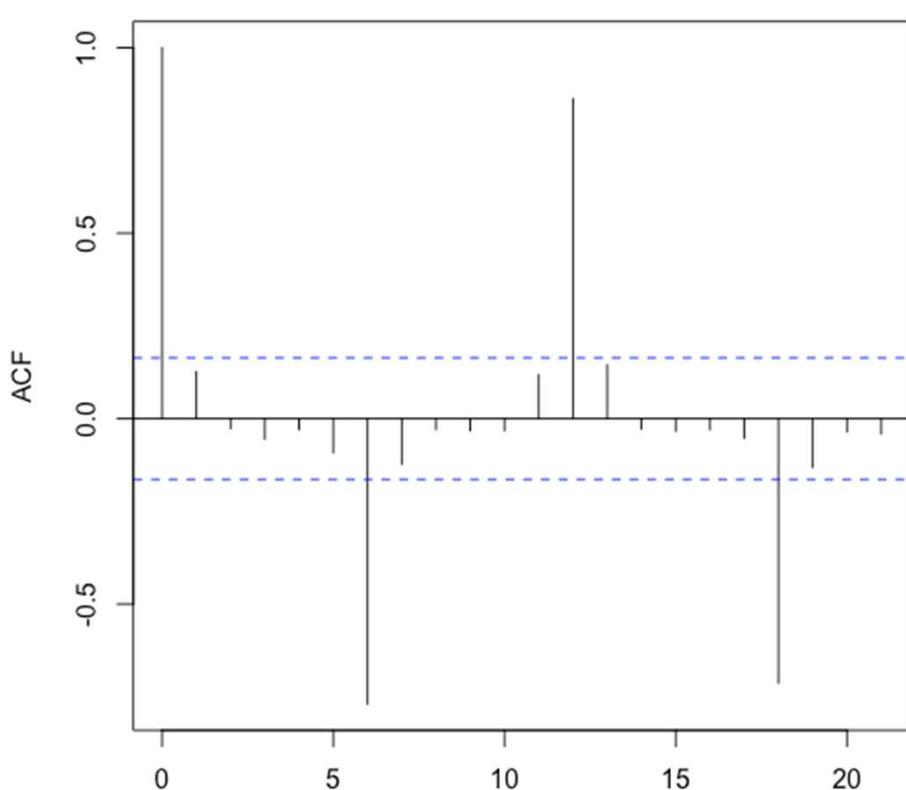
Step 4: Difference the log transformed data to check for stationarity.



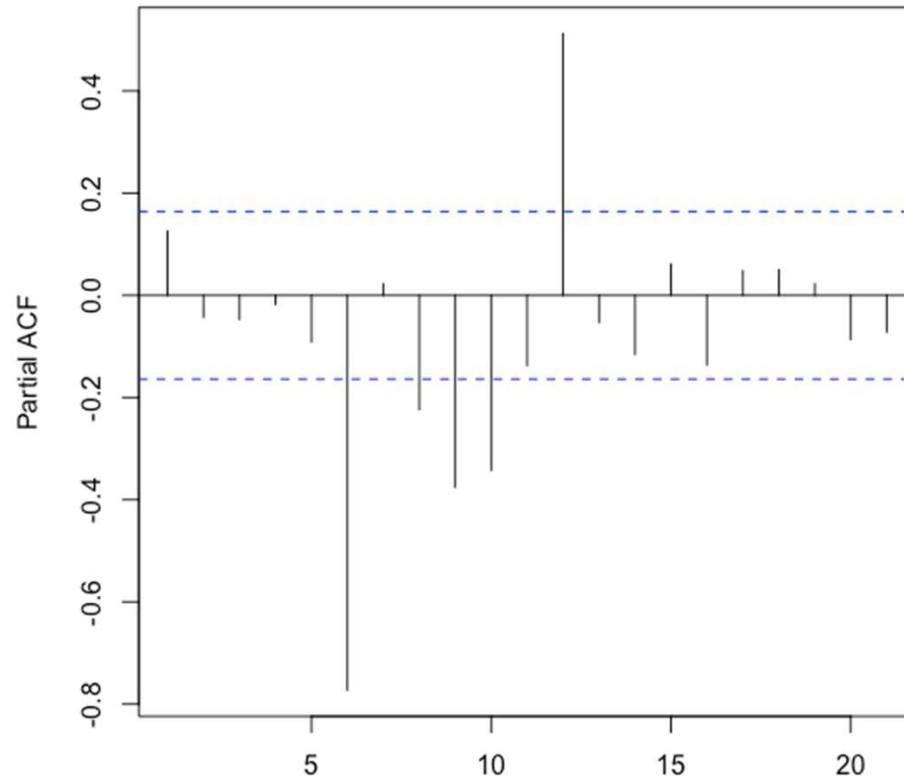
Tractor Sales Case Study

Step 5: Check ACF and PACF to explore remaining dependencies.

ACF Tractor Sales



PACF Tractor Sales



ACF has cutoff but PACF shows some decay. This indicates an MA process.
Also, there is strong seasonality.

Tractor Sales Case Study

Step 6: Run Auto ARIMA.

```
Series: log10(TractorSalesTS)
ARIMA(0,1,1)(0,1,1)[12]
```

Coefficients:

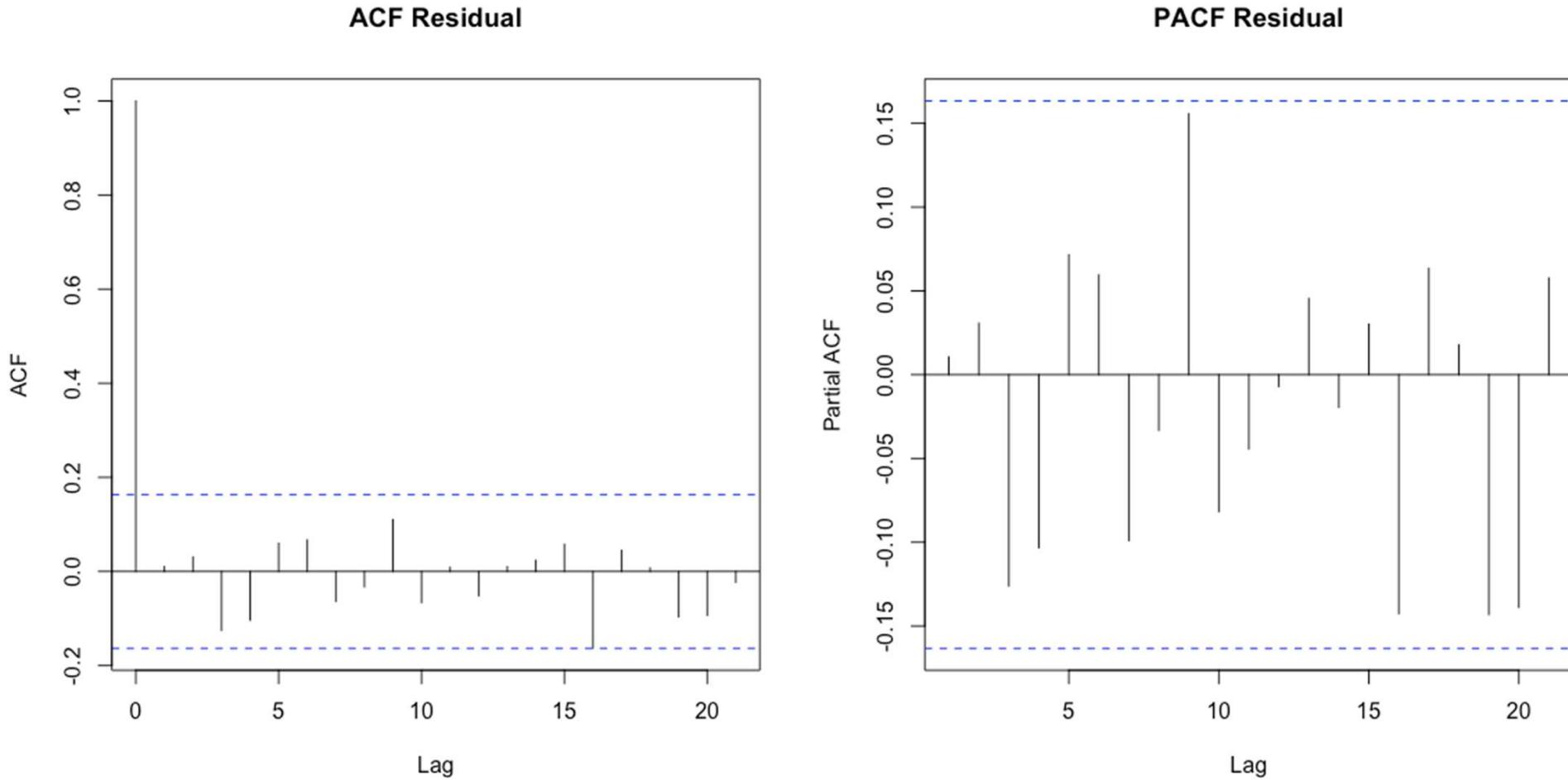
	ma1	sma1
-	-0.4047	-0.5529
s.e.	0.0885	0.0734

```
sigma^2 estimated as 0.0002571: log likelihood=354.4
AIC=-702.79    AICc=-702.6    BIC=-694.17
```



Tractor Sales Case Study

Step 7: Check ACF and PACF of residuals to ensure they are white noise.



Tractor Sales Case Study

Step 7: Use Box-Ljung test to verify residuals are white noise. *It is good enough to do the verification visually on ACF and PACF.*

Box-Ljung test

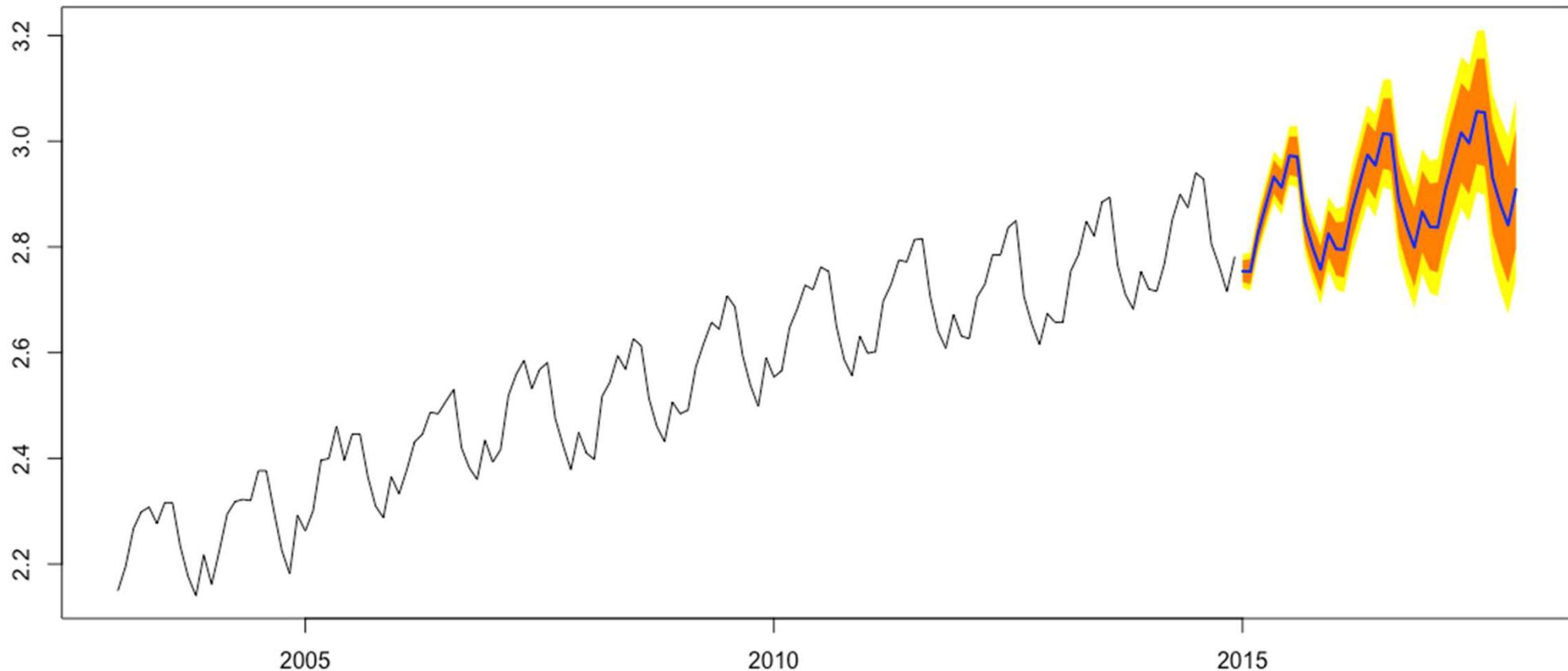
```
data: LogTractorSalesARIMA$residuals  
X-squared = 26.219, df = 24, p-value = 0.3422
```



Tractor Sales Case Study

Step 8: Forecast.

Forecasts from ARIMA(0,1,1)(0,1,1)[12]

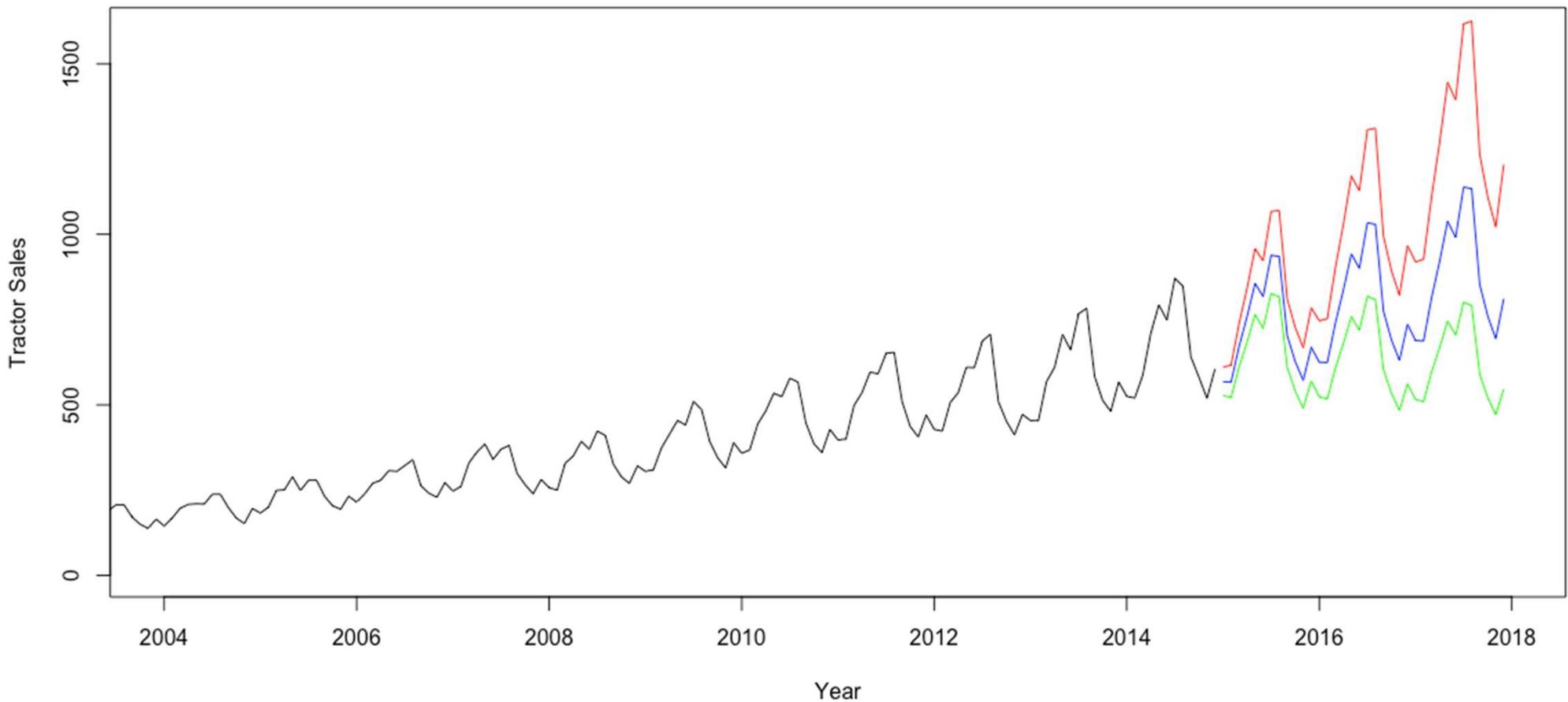


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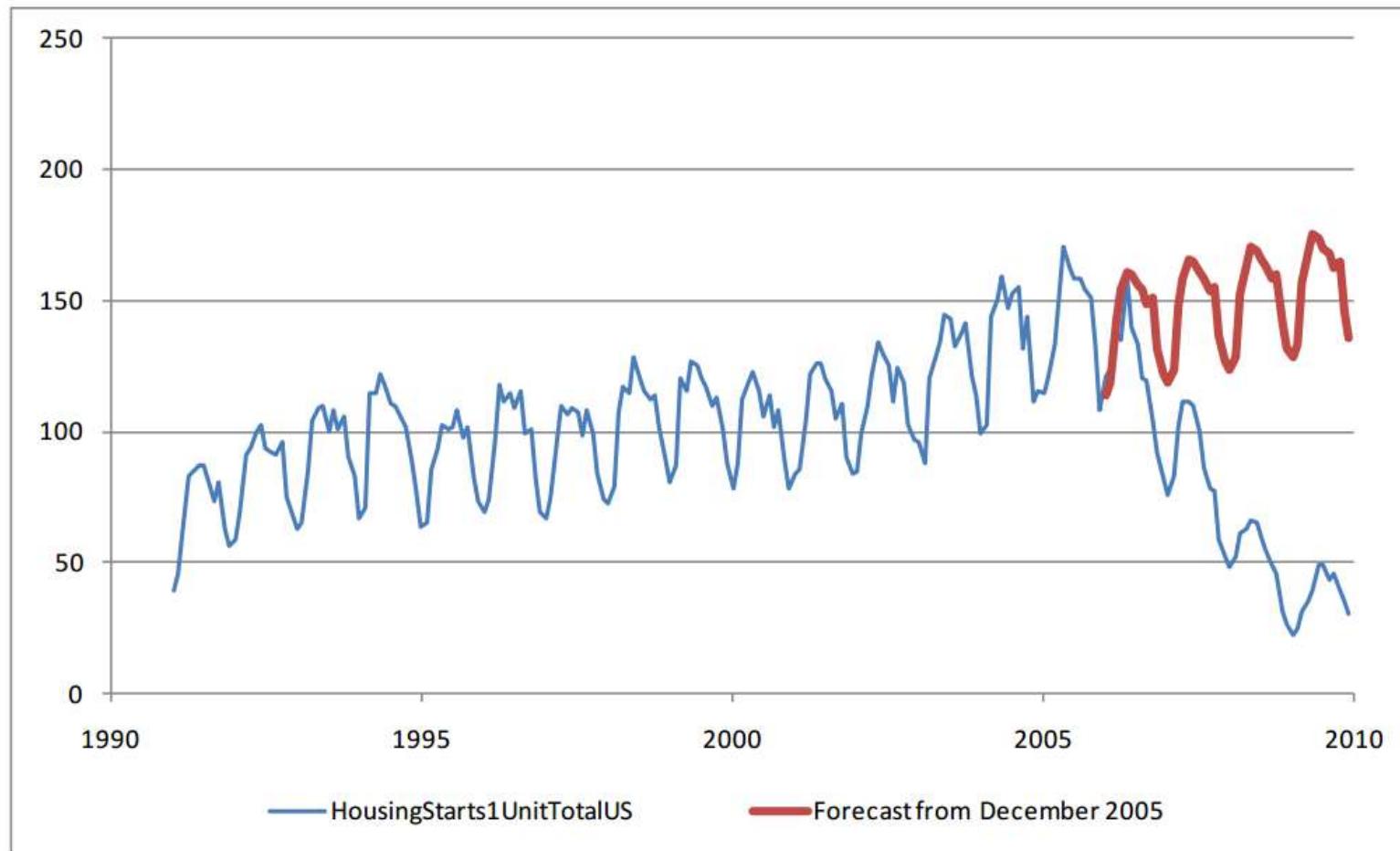


Tractor Sales Case Study

Step 8: Forecast on the original scale and not the logged data.



Caution: Forecasting is Risky!



"Prediction is very difficult, especially if it's about the future."

--Niels Bohr, Nobel laureate in Physics

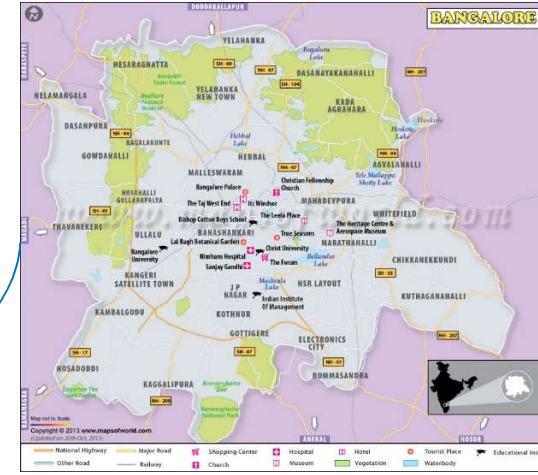
Resources

- <https://www.otexts.org/fpp>

An good open online book on Forecasting methods and practices

- <http://a-little-book-of-r-for-time-series.readthedocs.io/en/latest/src/timeseries.html> A short condensed summary on time-series
- <https://www.analyticsvidhya.com/blog/2015/12/complete-tutorial-time-series-modeling/> A short tutorial on using ARIMA models





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