CS 161 Homework 5

1.

(P => Q) = (-P v Q)
(-Q => -P) = (Q v -P)

	P	Q	P => Q	-Q =>-P	(P => Q) = (-Q => -P)
w_1	0	0	1	1	**1**
w_2	0	1	1	1	**1**
w_3	1	0	0	0	**1**
w_4	1	1	1	1	**1**

W(P => Q) = W(-Q => P) = {w_1, w_2, w_4}

	P	Q	P ⇔ Q	((P ^ -Q) v (-P ^ Q))	(P ⇔ Q) = ((P ^ -Q) v (-P ^ Q))
w_1	0	0	0	0	**1**
w_2	0	1	1	1	**1**
w_3	1	0	1	1	**1**
w_4	1	1	0	0	**1**

W(P ⇔ Q) = W((P ^ -Q) v (-P ^ Q)) = {w_2, w_3}

2.

a) Smoke = S, Fire = F, Heat = H
(S => F) => (-S => -F)
= (-S v F) => (S v -F)
= (S ^ -F) v (S v -F)

	S	F	(S => F) => (-S => -F)
w_1	0	0	1

w_2	0	1	0
w_3	1	0	1
w_4	1	1	1

$W((S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)) = \{w_1, w_3, w_4\}$

This sentence is not valid, since it is not satisfied by all worlds.

b) $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

$= (\neg S \vee F) \Rightarrow ((\neg S \wedge \neg H) \vee F)$

$= (S \wedge \neg F) \vee ((\neg S \wedge \neg H) \vee F)$

	S	F	H	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
w_1	0	0	0	1
w_2	0	0	1	0
w_3	0	1	0	1
w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

This sentence is not valid, since it is not satisfied by all 8 worlds (satisfied every world except for w_2)

c) $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow (\neg S \vee \neg H \vee F)$

	S	F	H	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
w_1	0	0	0	1
w_2	0	0	1	1
w_3	0	1	0	1

w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

This sentence is valid since it is satisfiable by all worlds.

3.

a) Mythical => Immortal
-Mythical => (-Immortal ^ Mammal)
(Immortal v Mammal) => Horned
Horned => Magical

b)

Symbol	Represented By
Mythical	Y
Immortal	I
Mammal	M
Horned	H
Magical	G

(Y => I) = (-Y v I)
(-Y => (-I ^ M)) = (Y v (-I ^ M)
((I v M) => H) = ((-I ^ -M) v H)
(H => G) = (-H v G)

CNF = (-Y v I) ^ (-I v Y) ^ (M v Y) ^ (-I v H) ^ (-M v H) ^ (-H v G)

c) -

i) Assume the Unicorn is not mythical.

#	Statement	Source
1	(-Y v I)	Knowledge Base
2	(-I v Y)	Knowledge Base
3	(M v Y)	Knowledge Base
4	(-I v H)	Knowledge Base

5	(-M v H)	Knowledge Base
6	(-H v G)	Knowledge Base
7	-Y	Knowledge Base
8	-I	(2 and 7)
9	M v I	(1 and 3)
10	Y v H	(3 and 5)
11	-I v G	(4 and 6)
12	M	(8 and 9)
13	H	(7 and 10)
14	G	(6 and 13)

Satisfied by -Y ^ -I ^ M ^ H ^ G.

Since not mythical is satisfiable, we cannot prove that the unicorn is mythical with the given knowledge base.

ii) Assume the Unicorn is not magical.

#	Statement	Source
1	(-Y v I)	Knowledge Base
2	(-I v Y)	Knowledge Base
3	(M v Y)	Knowledge Base
4	(-I v H)	Knowledge Base
5	(-M v H)	Knowledge Base
6	(-H v G)	Knowledge Base
7	-G	Knowledge Base
8	-H	(6 and 7)
9	Y v H	(3 and 5)
10	I v M	(1 and 3)
11	-M v G	(5 and 6)
12	Y	(8 and 9)

13	I v G	(10 and 11)
14	I	(7 and 13)
15	H	(4 and 14)
16	null	(8 and 15)

Line 16 presents a contradiction - therefore -G is unsatisfiable. Since -G is unsatisfiable, G must be valid, meaning the unicorn is magical.

iii) Assume the Unicorn is not horned.

#	Statement	Source
1	(-Y v I)	Knowledge Base
2	(-I v Y)	Knowledge Base
3	(M v Y)	Knowledge Base
4	(-I v H)	Knowledge Base
5	(-M v H)	Knowledge Base
6	(-H v G)	Knowledge Base
7	-H	Knowledge Base
8	-I	(4 and 7)
9	-M	(5 and 7)
10	-Y	(1 and 8)
11	M	(3 and 10)
12	null	(9 and 11)

Line 12 presents a contradiction - therefore -H is unsatisfiable. Since -H is unsatisfiable, H must be valid, meaning the unicorn is horned.

4.

a) Figure 1 is
   i) Decompasable, since for each AND gate, the subcircuits do not have overlapping variables.
   ii) Not deterministic, since the assignment (A, -B, C -D) results in two true inputs to the highest OR gate.
   iii) Not smooth, since the central OR gates on level 3 do not have the same variables on each side.

b) Figure 2 is

i) Decompasable, since for each AND gate, the subcircuits do not have overlapping variables.

ii) Not deterministic, since the assignment (-A, B) results in the two OR gates on level 3 to have two true inputs.

iii) Smooth, as for each OR gate, the subcircuits have the same variables.

5.

a)

	A	B	(-A ^ B) v (-B ^ A)
w_1	0	0	0
w_2	0	1	1
w_3	1	0	1
w_4	1	1	0

ω(-A)ω(B) + ω(A)ω(-B) = 0.9*0.3 + 0.1*0.7 = 0.34

b) (-A * B) + (-B & A)
(0.27) + (0.07) = 0.34
The count on the root and the WMC for the formula are equivalent.

c) ((-A * B) + (-B * A))*((C*D) + (-C * -D)) + ((-A * -B) + (A *B))*((C*-D) + (-C * D))
=(0.9*0.3 + 0.7*0.1)*(0.5*0.7 + 0.5*0.3) + (0.8*0.7 + 0.3*0.1)*(0.5*0.3 + 0.5*0.3)
= 0.5