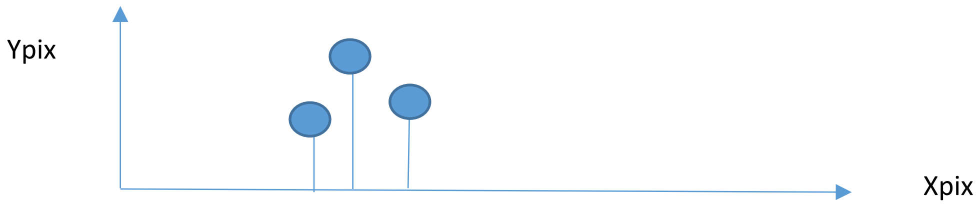
ExamPerceptionTest1

1. Suppose that we have a 1000x1000 pixel camera of 25 mm and and 5 microns of pixel size. The optical center is at Cx = Cy = 1000 / 2. Give the matrix K of the camera. A laser plane is placed parallel to the camera (Xlaser = Xcam, Ylaser = Ycam, Zlaser = Zcam) and the origin of the laser frame respect to the camera frame is placed at ${}^{cam}P_{laser} = [10 \quad 0 \quad 100]^T$. What is the homography between the laser and camera planes? Given a point in the laser plane ${}^{laser}P = [10 \quad 0]^T$, give the observed point I the image plane.

2. Calculate the subpixel maximum based on the center of gravity of these 3 pixels in the Figure. The 3 points are: $p1 = [260 \quad 130]^T, p2 = [261 \quad 150]^T, p1 = [262 \quad 140]^T$



3. The solution of the hand-eye is given by equations of the form:
$$A * X = X * B, A = \begin{bmatrix} R_A & T_A \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} R_B & T_B \\ 0 & 1 \end{bmatrix}, X = \begin{bmatrix} R_X & T_X \\ 0 & 1 \end{bmatrix}$$
There are two equations one for the rotation and the second for the translation:
$$R_A * R_X = R_X * R_B$$
$$R_A * T_X + T_A = R_X * T_B + T_X$$

   How do you solve the equation for the rotation with quaternions? Once you solve the rotation Rx, how do you solve Tx?

4. How do you multiply the quaternions $q1 = 1 + 2 * i + 3 * j + 4 * k$ and $q2 = -1 + 3 * i - 2 * j - k$? Are they unit-quaternions? Why?

5. What are dual quaternion and dual numbers? What is their use in the equation $A * X = X * B$?

6. The fundamental and essential matrices are given by the formulations:
$$\boldsymbol{\tilde{m}_2^T} * \boldsymbol{F} * \boldsymbol{\tilde{m}_1} = \boldsymbol{0}$$
$$\boldsymbol{\hat{m}_2^T} \ * \boldsymbol{E} * \boldsymbol{\hat{m}_1} = \boldsymbol{0}$$

   What relation exists between $\boldsymbol{\hat{m}_1}, \boldsymbol{\tilde{m}_1}$ and $\boldsymbol{\hat{m}_2}, \boldsymbol{\tilde{m}_2}$? Based on it, what relation exists between the fundamental and essential matrices?

1. $K = \begin{bmatrix} 5000 & 0 & 500 \\ 0 & 5000 & 500 \\ 0 & 0 & 1 \end{bmatrix}, H = K * [R1, R2, T] =$

$$\begin{bmatrix} 5000 & 0 & 500 \\ 0 & 5000 & 500 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix} = \begin{bmatrix} 5000 & 0 & 100000 \\ 0 & 5000 & 50000 \\ 0 & 0 & 100 \end{bmatrix}.$$

$$\tilde{p_i} = H * \tilde{p_l} = \begin{bmatrix} 5000 & 0 & 100000 \\ 0 & 5000 & 50000 \\ 0 & 0 & 100 \end{bmatrix} * \begin{bmatrix} 10 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 150000 \\ 50000 \\ 100 \end{bmatrix} = \begin{bmatrix} 1500 \\ 500 \\ 1 \end{bmatrix}$$

2. $\bar{x} = \frac{\sum_i (x_i * y_i)}{\sum_i y_i} = \frac{260*130+261*150+262*140}{130+150+150} = 261.0238$

3. $q_A * q_X = q_X * q_B$. This allows a equation $C * q_X = 0$. From q_X, we obtain R_X. The second equation can be written as:

$$(R_A - I) * T_X = (R_X * T_B - T_A)$$

This equation can be put in the way: $D * T_X = E$, the solution been: $T_X = D^+ * E$.

4. $(1 + 2*i + 3*j + 4*k)* (-1 + 3*i - 2*j - k) = (-1 + 3*i - 2*j - k) * (-2*i + 6*i*i - 4*i*j - 2*i*k) + (-3*j + 9*j*i - 6*j*j - 3*j*k) + (-4*k + 12*k*i - 8*k*j - 4*k*k) = (-1 - 6 + 6 + 4) + i*(3 - 2 - 3 + 8) + j*(-2 + 2 - 3 + 12) + k*(-1 - 4 - 9 - 4) = 3 + 6*i + 9j - 18k$.

With the constraints of $i^2 = j^2 = k^2 = -1; i*j = k; j*k = i; k*i = j; j*i = -k; k*j = -i; i*k = -j$.

They are not unitary quaternions because the sum of squares is not 1.

5. A dual number is define as a real number plus a dual quantity, defined as $a + \varepsilon * b$, where $\varepsilon^2 = 0$. A dual quaternion is a quaternion, where each element is a dual number. The equation $A * X = X * B$ in dual quaternions is equal to: $\hat{q}_A * \hat{q}_X = \hat{q}_X * \hat{q}_B$, where $\hat{q}_A$is the unit dual quaternion corresponding to A (contains the information about the rotation and translation part). This equation is linear in the unit dual quaternion $\widehat{q}_X$. The solution to this system is in the form $C * \hat{q}_X = 0$. However it is more complicated, as the unicity of the dual quaternion has to be respected.

6. $\widehat{\boldsymbol{m}}_1 = \boldsymbol{K}_1^{-1} * \widetilde{\boldsymbol{m}}_1 \qquad \widehat{\boldsymbol{m}}_2 = \boldsymbol{K}_2^{-1} * \widetilde{\boldsymbol{m}}_2$

$$\widehat{\boldsymbol{m}}_2^T * \boldsymbol{E} * \widehat{\boldsymbol{m}}_1 = \widetilde{\boldsymbol{m}}_2^T * \boldsymbol{K}_2^{-T} * \boldsymbol{E} * \boldsymbol{K}_1^{-1} * \widetilde{\boldsymbol{m}}_1 = \boldsymbol{0}$$

$$\boldsymbol{F} = \boldsymbol{K}_2^{-T} * \boldsymbol{E} * \boldsymbol{K}_1^{-1}$$