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MSc. ROBOTICS AND CONTROL SYSTEMS

(MRE002A) PERCEPTION

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LASER TRIANGULATION

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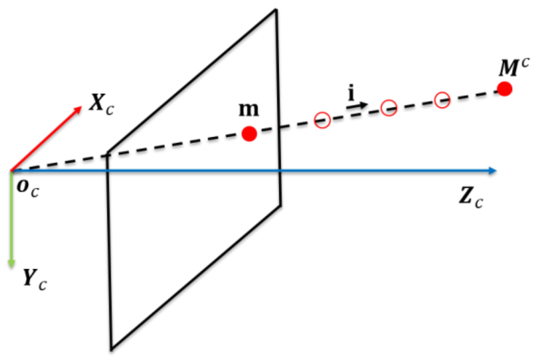
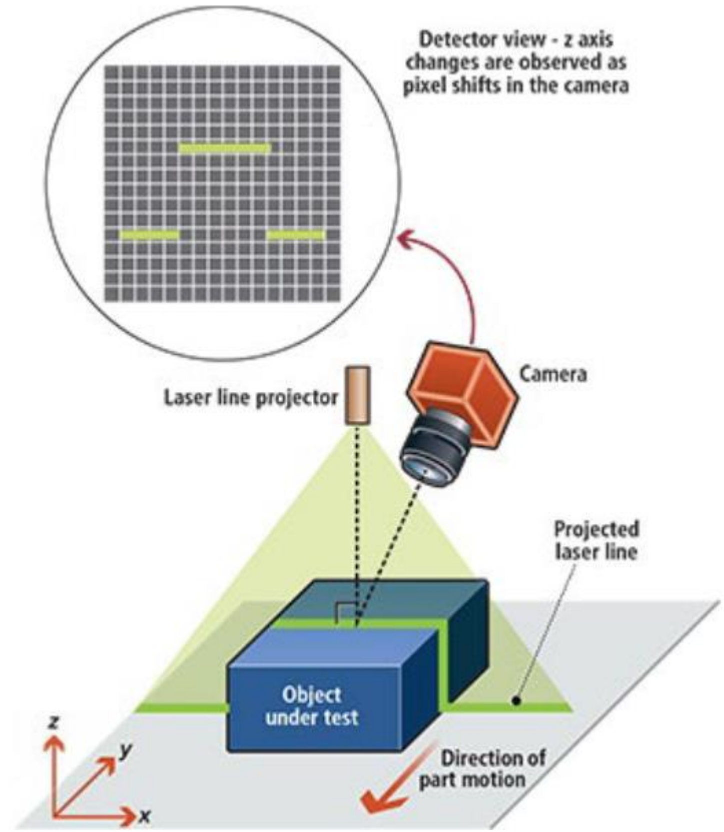
Sensor calibration

Recovering 3D Shape

- The distorted laser line encodes the shape of the surface
  - How can we 'decode' the shape from the image?
- Camera measures light and angles
  - But if the points lie on a known plane…
  - Ray-plane intersections give us 3D locations!





- System calibration
  - Camera Model
  - Laser Plane
  - Motion

Camera - Laser Plane Calibration

- If we assume an ideal pinhole model, the projection of a point in World coordinates to the image plane is given by

$$s\tilde{\boldsymbol{m}} = \boldsymbol{K} \cdot {}^{c}T_{\boldsymbol{w}} \cdot \tilde{\boldsymbol{M}}^{W}$$

$$s\begin{bmatrix} m_u \\ m_v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{instrinsics}} \cdot \underbrace{\begin{bmatrix} r_{11} & r_{21} & r_{31} & t_x \\ r_{12} & r_{22} & r_{32} & t_y \\ r_{13} & r_{23} & r_{33} & t_z \end{bmatrix}}_{\text{extrinsics}} \cdot \begin{bmatrix} M_x \\ M_y \\ M_z \\ 1 \end{bmatrix}$$

- Estimate the geometric relation between laser plane and camera coordinate system
  - Use a target with known geometry to stablish point correspondences

Calibration Targets

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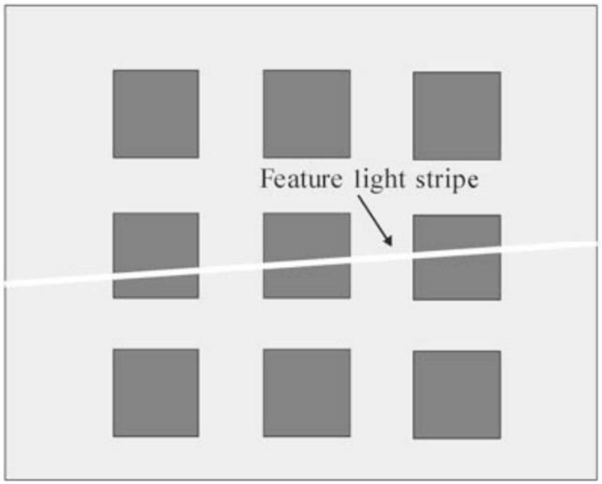
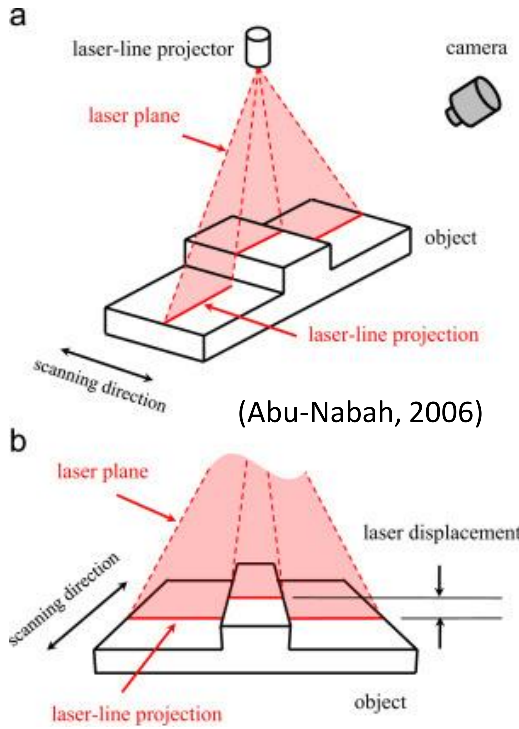
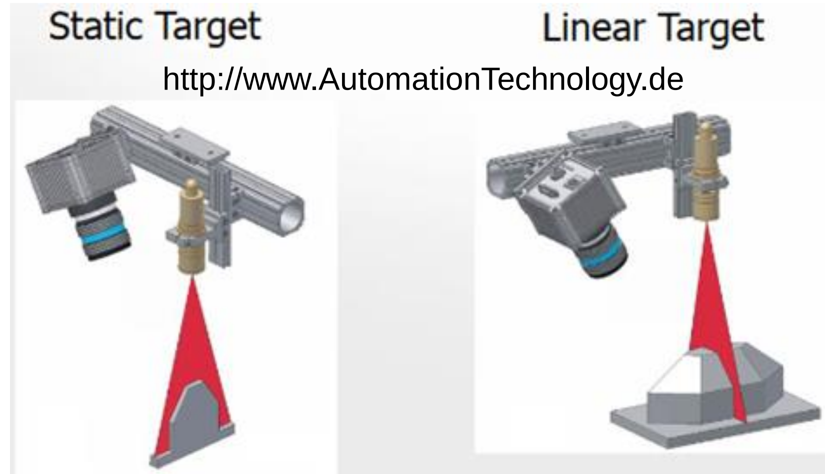
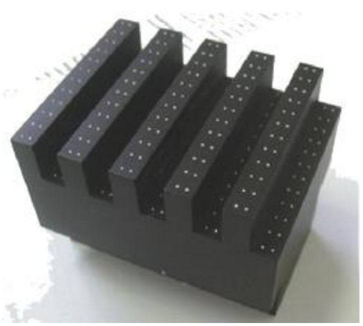
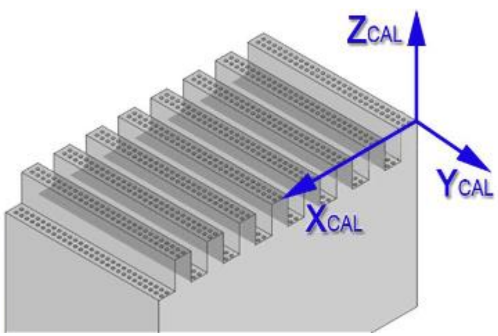




(Abu-Nabah, 2006)







Camera - Laser Plane Calibration

Two main approaches:

1. Calibrate the camera parameters and laser plane separately
   - Based on physical model, all parameters have a physical meaning.

2. Sensor as a black box
   - Homography
   - Laser plane and image plane as input and output to the mapping function

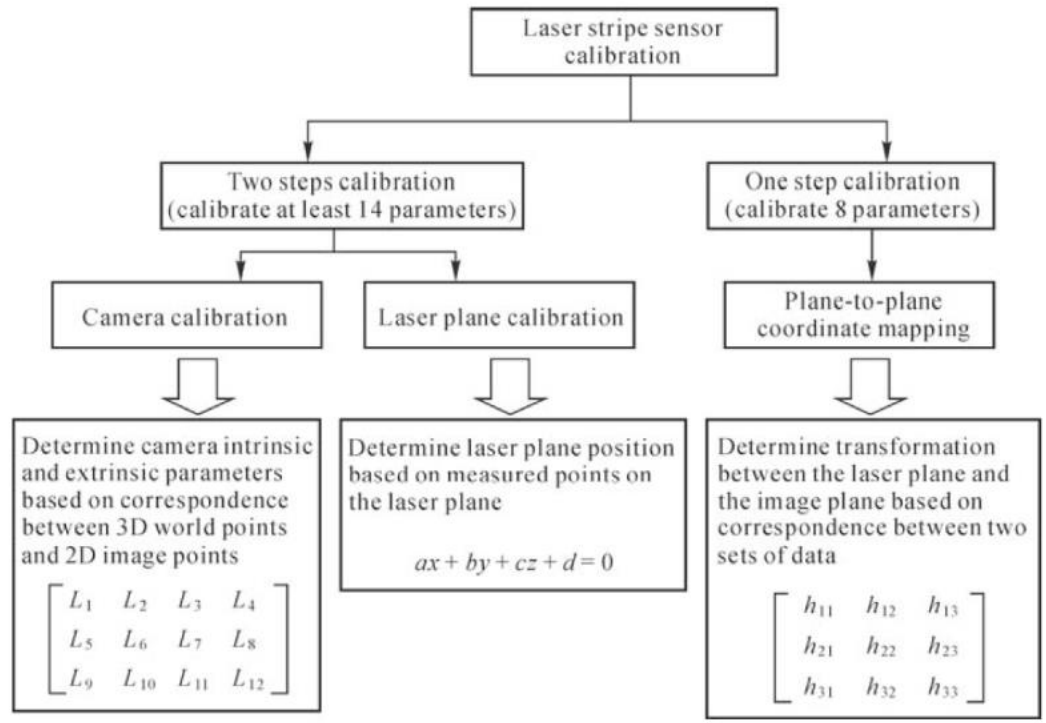


Fig. 3.1. Two methods of calibrating laser stripe sensor

Homography based Calibration

- The relation between the laser plane (a 2D plane in 3D space) and its projection in the image define a homography mapping
  - Laser plane WCS-> Y=0

$$s \cdot \begin{bmatrix} m_u \\ m_v \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & \cancel{r_{21}} & r_{31} & t_x \\ r_{12} & \cancel{r_{22}} & r_{32} & t_y \\ r_{13} & \cancel{r_{23}} & r_{33} & t_z \end{bmatrix} \cdot \begin{bmatrix} M_x \\ 0 \\ M_z \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{bmatrix} \cdot \begin{bmatrix} M_x \\ M_z \\ 1 \end{bmatrix}$$

- From the perspective projection we obtain 3 equations:

$$s \cdot \begin{bmatrix} m_u \\ m_v \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{bmatrix} \cdot \begin{bmatrix} M_x \\ M_z \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} h_{11} \cdot M_x + h_{21} \cdot M_z + h_{31} &= s \cdot m_u && (1) \\ h_{12} \cdot M_x + h_{22} \cdot M_z + h_{32} &= s \cdot m_v && (2) \\ h_{13} \cdot M_x + h_{23} \cdot M_z + h_{33} &= s && (3) \end{aligned}$$

Homography Estimation - DLT

- Dividing (1) and (2) by (3) we get

$$(h_{11} \cdot M_x + h_{21} \cdot M_z + h_{31})/(h_{13} \cdot M_x + h_{23} \cdot M_z + h_{33}) = m_u \qquad (4)$$
$$(h_{12} \cdot M_x + h_{22} \cdot M_z + h_{31})/(h_{13} \cdot M_x + h_{23} \cdot M_z + h_{33}) = m_v \qquad (5)$$

- Rearranging we get

$$-(h_{11} \cdot M_x + h_{21} \cdot M_z + h_{31}) + (h_{13} \cdot M_x + h_{23} \cdot M_z + h_{33}) \cdot m_u = 0 \qquad (6)$$
$$-(h_{12} \cdot M_x + h_{22} \cdot M_z + h_{31}) + (h_{13} \cdot M_x + h_{23} \cdot M_z + h_{33}) \cdot m_v = 0 \qquad (7)$$

- (6) and (7) can be represented in vector form as

$$\boldsymbol{A}_i \cdot \boldsymbol{h} = 0 \qquad (8)$$

where

$$\boldsymbol{A}_i = \begin{bmatrix} -M_x & -M_z & \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & M_x \cdot m_u & M_z \cdot m_u & m_u \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -M_x & -M_z & \mathbf{-1} & M_x \cdot m_v & M_z \cdot m_v & m_v \end{bmatrix}$$

$$\boldsymbol{h} = [h_{11} \quad h_{21} \quad h_{31} \quad h_{12} \quad h_{22} \quad h_{32} \quad h_{13} \quad h_{23} \quad h_{33}]^{\mathbf{T}}$$

Homography Estimation - DLT

- Since each point correspondence provides 2 equations, 4 correspondences are sufficient to solve for the 8 degrees of freedom of **H**.
  - The restriction is that no 3 points can be collinear!
- More than 4 correspondences can be used to ensure a more robust solution
  - From n points 2×9 $\boldsymbol{A}_i$ matrices (one per point correspondence) can be stacked on top of one another to get a single 2n×9 matrix $\boldsymbol{A}$, such that

$$\boldsymbol{A} \cdot \boldsymbol{h} = 0 \text{ , where } \boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \cdots \\ \boldsymbol{A}_n \end{bmatrix} \quad (9)$$

  - The solution vector **h** is given by the eigenvector corresponding to the smallest eigenvalue of $A^T \cdot A$
    - We minimize the norm $||\boldsymbol{A} \cdot \boldsymbol{h}||$ with the constraint that either $||\boldsymbol{h}|| = 1$ or $h_{33}=1$

Practical Exercise – Laser Stripe Simulation

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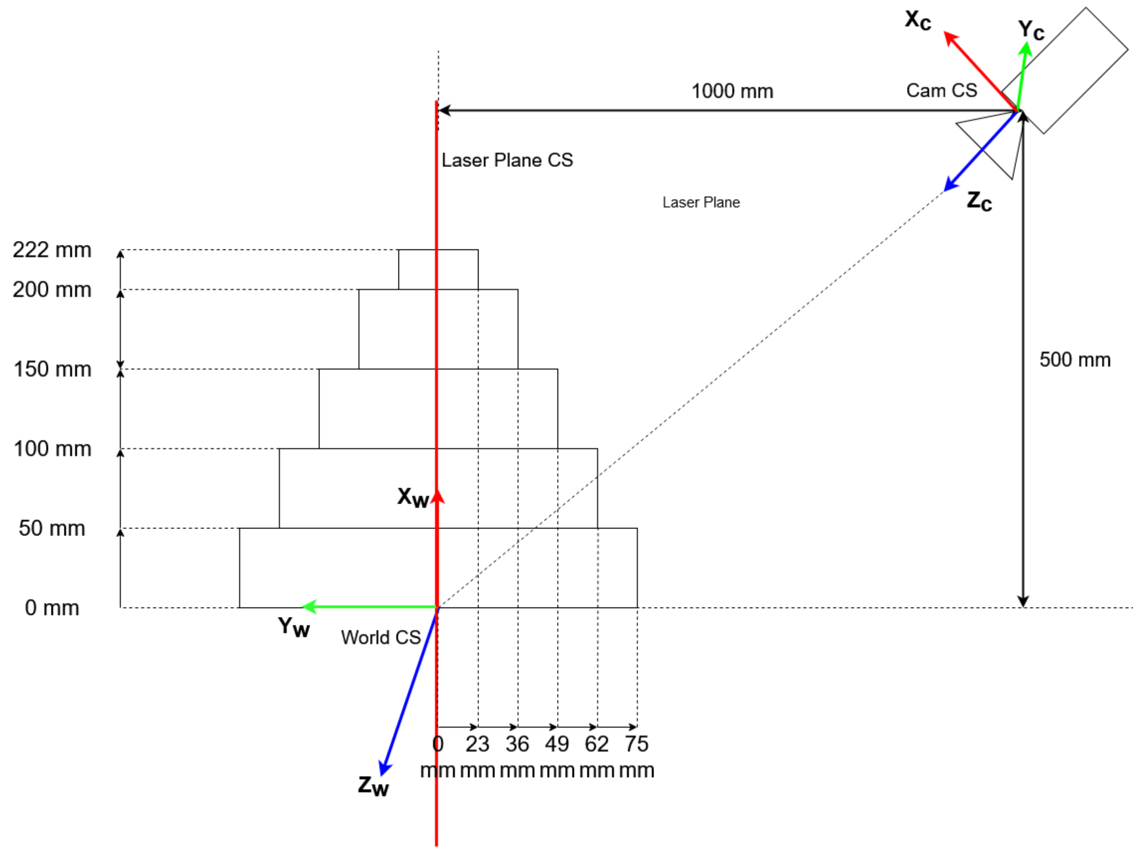
- Pinhole camera model
  - Distortion free
  - *f*=12mm
  - Skew = 0
  - Principal point in image center
  - Resolution 1280 x 1024 pix
  - Pixel size 5 μm

- Calibration object
  - Multiple disk pyramid



Exercise 1 – Laser Stripe Calibration

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a) Define the theoretical camera matrix $\boldsymbol{K}$, the extrinsic parameters $\boldsymbol{R}$ and $\boldsymbol{t}$, and compute the corresponding theoretical homography $\overline{\mathbf{H}}$.

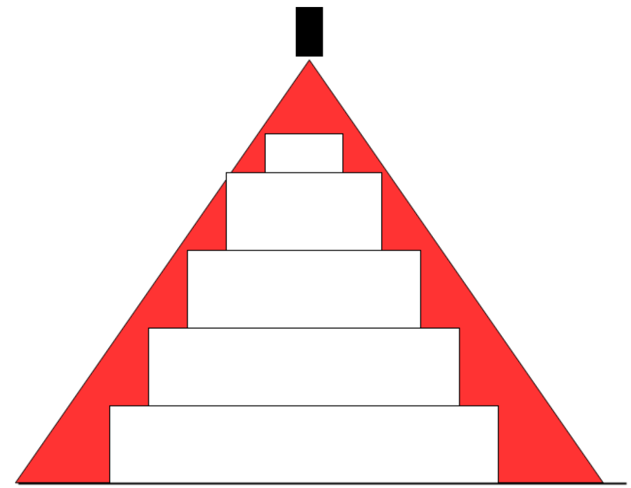
b) Using $\overline{\mathbf{H}}$, project 20 control points $\widetilde{\boldsymbol{M}}_i^W$ to the image and obtain the corresponding pixel coordinates $\boldsymbol{m}_i$.

c) Simulate the calibration procedure, by estimating the homography $\widetilde{\boldsymbol{m}}_i = \boldsymbol{H}_0 \cdot \widetilde{\boldsymbol{M}}_i^W$ from the observed image points.

d) Add Gaussian noise to observed image points $\boldsymbol{m}_i$, obtaining the noisy points $\boldsymbol{m}_i'$ and $\boldsymbol{m}_i''$ (σ=0.01 pix, σ=0.1 pix).

e) Estimate new homographies $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ from noisy points.

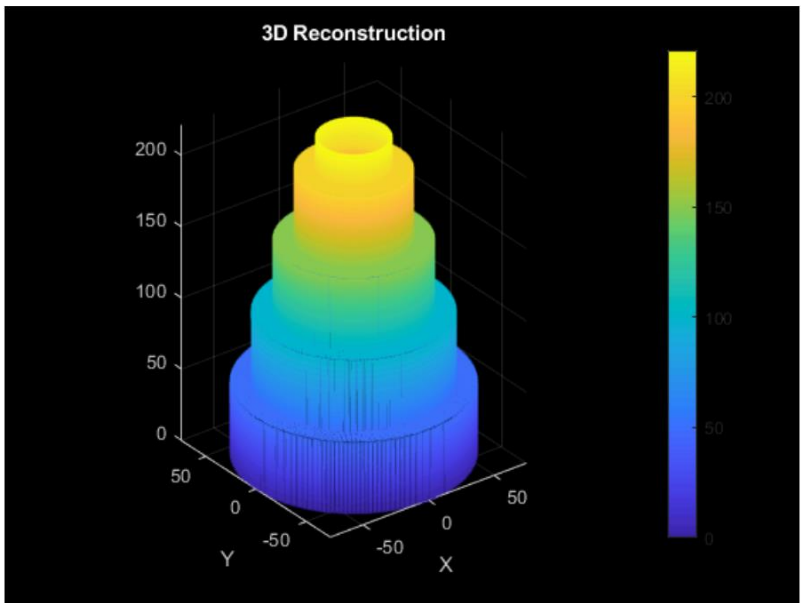
f) Compare errors in $\boldsymbol{H}_1$, $\boldsymbol{H}_2$, and $\boldsymbol{H}_3$.

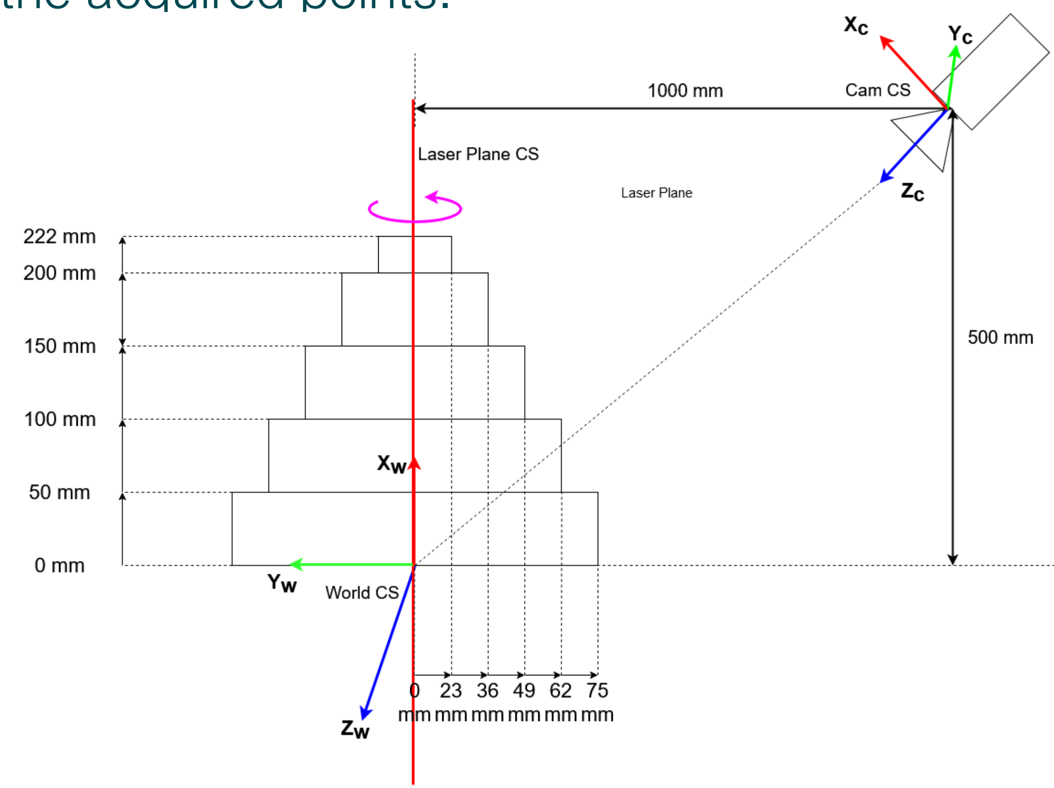


Exercise 2 - Reconstruction

The calibration object is rotated around its center, around the WCS X axis in 1deg steps .

- Simulate the acquisition of a dense line.
- Define the rotation matrix R_x.
- Reconstruct the surface by rotating the acquired points.
- Plot the resulting surface.





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THANK YOU

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