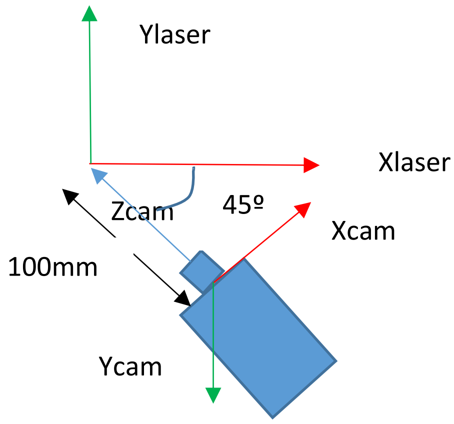
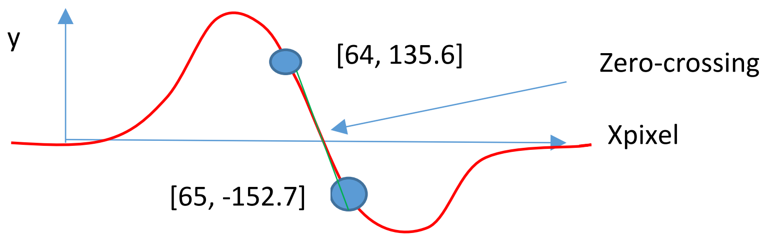
Exam Perception Test3

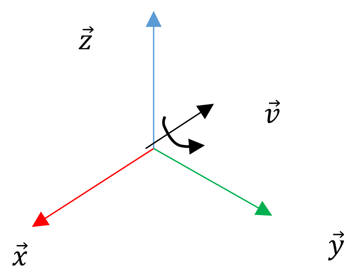
1. Given a camera of 1000 x 1000 pixels, f = 10 mm, and 1 micron of pixel size. Suppose that the optical center is at Cx = Cy = 1000 / 2. The camera is observing the laser plane as it is shown in the Figure. What are the Homography H between the laser and image plane, and what is the observation in pixels of the laser point (10, 20)?.



2. We like to find the maximum of the laser line with sub-pixel precision. For that, we use a zero crossing of the second derivative of the laser intensity. In the Figure we show it with 2 points at different sides of the zero-crossing. Calculate the maximum with subpixel precision, by performing the intersection of the line between these 2 points with the y = 0.



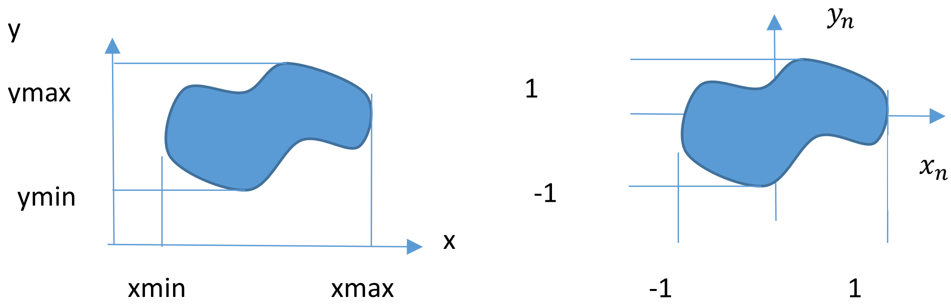
3. The unit quaternion corresponding to the axis-angle representation $(\vec{v}, \theta)$ of rotations is given by the formula: $q = [\cos\left(\frac{\theta}{2}\right), \vec{v} * \sin\left(\frac{\theta}{2}\right)]$, where $\cos\left(\frac{\theta}{2}\right)$ is the real part of the quaternion, and $\vec{v} * \sin\left(\frac{\theta}{2}\right)$ is the vectorial part of the quaternion, i.e. the one corresponding to $[i \quad j \quad k]$. What is the quaternion, if the rotation rotates $\vec{x}$ to $\vec{y}$, $\vec{y}$ to $\vec{z}$, and $\vec{z}$ to $\vec{x}$.



4. Prove or show why the equation of the epipolar line $\tilde{e}_{p1}$ that goes from the epipole $\tilde{e}_1$ to $\tilde{m}_1$, can be written as $\tilde{e}_{p1} = \tilde{e}_1 \times \tilde{m}_1$.
   Hint: The equation of the epipolar line is $\tilde{e}_{p1}^T * \tilde{p} = 0$, where $\tilde{p}$ is any point in the line.

5. In order to find the fundamental or essential matrices using the 8 points algorithm, it is better to do a normalization of the pixels by means of a transformation function T, such that:
$$\tilde{p}_n = [x_n \quad y_n \quad 1]^T = T * \tilde{p} = T * [x \quad y \quad 1]^T$$
   Where, $\tilde{p}_n$ are the homogeneous coordinates of the normalized point p.

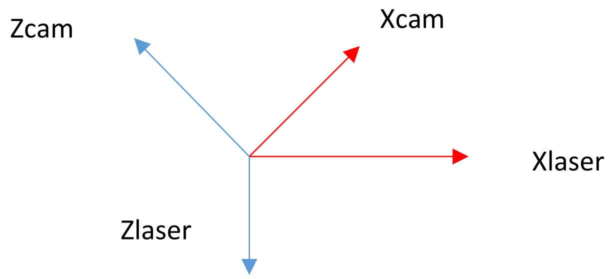
   Give the expression of T, if we like that the maximum value in x of all points is transformed to the value 1, the minimum value of x transformed to -1, the maximum value of y transformed to +1, and the minimum value of y to -1.



1. $K = \begin{bmatrix} 10000 & 0 & 500 \\ 0 & 10000 & 500 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & -1 & 0 \\ -0.7071 & 0 & -0.7071 \end{bmatrix}$

$$T = [0 \quad 0 \quad 100]^T; \; H = K * [R1, R2, T]$$
$$\tilde{p}_i = H * \tilde{p}_l = [2481.2, 500, 1]^T$$

The matrix R can be calculated from the projections:



This matrix $R = \left[{}^{cam}X_{laser} \quad {}^{cam}Y_{laser} \quad {}^{cam}Z_{laser}\right]$.

It can also be found by first rotating the cam frame by 180 degrees in the Xcam, so that the Ycam is in the opposite direction of the Ylaser. Then, we rotate it by 45 degrees in the Y direction.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} * \begin{bmatrix} c45 & 0 & s45 \\ 0 & 1 & 0 \\ -s45 & 0 & c45 \end{bmatrix} = \begin{bmatrix} c45 & 0 & -s45 \\ 0 & -1 & 0 \\ -s45 & 0 & -c45 \end{bmatrix}$$

2. Pmax = 64 + 135.6 / (152.7 + 135.6) = 64.4703

3. $\vec{v} = [1,1,1]/norm[1,1,1]$, $\theta = \frac{360}{3} = 120º$,

$$q = \cos(60) + \frac{[1,1,1]}{norm[1,1,1]} * \sin(60) = 0.5 + i * 0.5 + j * 0.5 + k * 0.5$$

4. $\tilde{e}_{p1}^T * \tilde{e}_1 = 0$ implies that $\tilde{e}_1$ is perpendicular to $\tilde{e}_{p1}^T$.

   $\tilde{e}_{p1}^T * \tilde{m}_1 = 0$ implies that $\tilde{m}_1$ is perpendicular to $\tilde{e}_{p1}^T$

   Thus, $\tilde{e}_{p1}^T$ is a vector in the direction of $\tilde{e}_1 \times \tilde{m}_1$. As $\tilde{e}_{p1}^T$ and $\lambda * \tilde{e}_{p1}^T$ represents the same line due to the fact that $\tilde{e}_{p1}^T * \tilde{p} = 0 = \lambda * \tilde{e}_{p1}^T * \tilde{p}$, then $\tilde{e}_{p1} = \tilde{e}_1 \times \tilde{m}_1$.

5. We try to formulate first the the x coordinate, as the y coordinate has the same treatment.

$$x_n = a * x + b$$

Where, x_n corresponds to the normalized x values. Then:

$$1 = a * xmax + b$$

$$-1 = a * xmin + b$$

Resulting in $a = \frac{2}{(xmax-xmin)}$, $b = \frac{-(xmax+xmin)}{(xmax-xmin)}$

The transformation T can be written:

$$T = \begin{bmatrix} \frac{2}{(xmax - xmin)} & 0 & \frac{-(xmax + xmin)}{(xmax - xmin)} \\ 0 & \frac{2}{(ymax - ymin)} & \frac{-(ymax + ymin)}{(ymax - ymin)} \\ 0 & 0 & 1 \end{bmatrix}$$