Analysis of Probability of successful Packet delivery

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In this analysis, we find an expression for the probability of successful packet delivery considering RSU scheduling, packet collision and re transmission. The expression for probability can be split into 2 parts - namely probability of successful channel access, and then followed by the probability that RSU is on and can receive a packet.

Successful Channel access probability :

First, we define $P(l,n,w,k)$ ($1 \leq l \leq w$ and $1 \leq k \leq n$) is the probability that n vehicles select back-offs from a contention window of w slots, $(l-1)$ empty slots pass before the first transmission attempt, and k vehicles transmit in the l^{th} slot.

$$P(l,n,w,k) = \Big(1 - \frac{l-1}{w}\Big)^n . \binom{n}{k} . \Big(\frac{1}{w-l+1}\Big)^k . \Big(1 - \frac{1}{w-l+1}\Big)^{n-k}$$

Next , let's define $X(t,w,n)$ as the mean number of successful transmissions during an interval, given that there are t slots left in this interval, at most w contention slots left at the vehicles back-off counter and n vehicles have not attempted to transmit yet. In the below formula, s and c are the lengths (in units of time slots) of a successful and a collided transmission respectively.

$$X(t,w,n) = \sum_{l=1}^{min(w,t)} \Bigg\{ P(l,n,w,1)[1 + X(t-l+1-s, w-l, n-1)] + \sum_{k=2}^{n} P(l,n,w,k) X(t-l+1-c, w-l, n-k) \Bigg\}$$

The above analysis shows how we get $X(t,w,n)$ for one interval consisting for T time slots. So, if we just want to model the success probability of a packet without retransmissions, we can directly compute

$$P_{succ} = \frac{X(T,W,N)}{N}$$

Now, we take into account re transmissions due to collision. Here, we compute the probability of successful packet delivery by considering K transmission intervals. Assume at the start of the first analysis, there are N vehicles attempting to transmit N packets (and contention window of W). We now track the N packets that were attempted for transmission in the first slot, across K interval, each with T slots.

Define x_k as the number of packets of the initially considered N packets that get delivered successfully in the k_{th} time interval, let's denote $X(T,W,N)$ as X for simplicity of notation.

$$x_1 = X$$

To find x_2, we assume that N vehicles again transmit in the $2nd$ interval with contention window W, which is a reasonable assumption. Now, x_1 packets got delivered successfully in the $1st$ time interval, so $N - x_1$ packets will be retransmitted in this time interval. So, the number of packets of the initially considered N packets that get delivered successfully in the 2_{nd} time interval is given by :

$$x_2 = \Big(\frac{N - x_1}{N}\Big)X$$

Generalizing this, we get

$$x_k = \frac{\Big(N - \sum\limits_{i=1}^{k-1} x_i\Big)}{N}X$$

$$x_k = \frac{\sum\limits_{i=0}^{k-1} (-1)^{k-i+1} N^i X^{k-i}}{N^{k-1}}$$

Now, assuming the maximum number of attempts for any packet is K, we can find out the number of packets (of the initially considered N packets) delivered in successfully X' as :

$$X' = \sum_{k=1}^{K} x_k = \sum_{k=1}^{K} \frac{\Big(N - \sum\limits_{i=1}^{k-1} x_i\Big)}{N}X$$

.

Note that the above expression is obtained by further simplifying the expanded version of the summation.

The probability of successful packet delivery can be given as :

$$P_{succ} = \frac{X'}{N} = \sum_{k=1}^{K} \frac{x_k}{N} = \sum_{k=1}^{K} \frac{\Big(N - \sum\limits_{i=1}^{k-1} x_i\Big)}{N^2}X$$

To take into account RSU scheduling, we assume that the vehicles arrive as an exponential distribution. So, it's reasonable to assume that the probability of an RSU being OFF is also follows the same distribution. So, we define:

$$P(\text{RSU} = \text{OFF}) = \lambda e^{-\lambda n}$$

where there are n vehicles in the system.
So,

$$P(\text{RSU} = \text{ON}) = 1 - \lambda e^{-\lambda n}$$

So, we now incorporate this term directly :

$$P_{succ-new} = P_{succ} \times P(\text{RSU} = \text{ON})$$