Analysis of the Probability of Successful Packet Delivery

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May 18, 2015

1 System model

We have a 1-D road situation where vehicles have bi-directional movement in the Vehicular ad hoc networks (VANET) scenario. The vehicles have On Board Units (OBUs) which are used to communicate with other vehicles and with stationary radio units placed along the road called Road Side Units (RSUs). The RSUs are placed at specific locations on the road. A packet which is generated by a vehicle can reach any of the RSUs present, typically it reaches the closest one. For packet transmission, we consider an uplink scenario. Optimal placement and scheduling of RSUs is done using the Rainbow Product Ranking algorithm [5]. Specifically, we consider an ON-OFF scheduling of the RSUs based on the existing traffic conditions, so as to minimize energy usage. The model followed in the system is as per [3].

We follow the basic 802.11p MAC protocol [1], where the system uses the DSRC channels allocated at 5.9 GHz. A two-ray path loss signal propagation model is assumed . We use Packet Delivery Ratio (PDR) as the main metric of evaluation, in addition to the energy consumptions when the RSU scheduling is used. We show that the PDR obtained using the simulation matches closely with the probability of successful packet delivery obtained from analysis that we have below. In this analysis, we find an expression for the probability of successful packet delivery considering RSU sleep scheduling, packet collision retransmission and hidden terminals. The expression for probability can be split into 2 parts - namely probability of successful channel access, and then followed by the probability that the RSU is on and can receive a packet, i.e the SINR (Signal to Noise Ratio) of the RSU is greater than a specific threshold.

2 Original Model

First, we define $P(l,n,w,k)$ ($1 \leq l \leq w$ and $1 \leq k \leq n$) as the probability that n vehicles select backoffs from a contention window of w slots, $(l-1)$ empty slots pass before the first transmission attempt, and k vehicles transmit in the l^{th} slot.

$$P(l,n,w,k) = \left(1 - \frac{l-1}{w}\right)^n . \binom{n}{k} . \left(\frac{1}{w-l+1}\right)^k . \left(1 - \frac{1}{w-l+1}\right)^{n-k} \tag{1}$$

Here, the first term in the probability of no transmission in the first $l-1$ slots, the second term is the number of ways we can choose k vehicles out of n vehicles, the third term in the probability of k vehicles choosing uniformly at random the l^{th} slot to transmit out of the remaining $w-l+1$ slots, while the final term is the probability that the remaining $n-k$ vehicles choose some other slot for transmission. Next, we define $X(t,w,n)$ as the mean number of successful transmissions during an interval, given that there are t slots left in this interval, at most w contention slots left at the vehicles' back-off counter and n vehicles have not attempted to transmit yet. If we take the transmission time (given in terms of number of time slots) for successful transmission and collision as s and c respectively,

$$X(t, w, n) = \sum_{l=1}^{min(w,t)} \Big\{ P(l, n, w, 1)[1 + X(t - l + 1 - s, w - l, n - 1)] + \sum_{k=2}^{n} P(l, n, w, k) X(t - l + 1 - c, w - l, n - k) \Big\} \quad (2)$$

The above equation computes the expectation of the number of successful transmissions, with $P(l, n, w, 1)$ being the probability that only one vehicle transmits in the l^{th} slot, a success, and $P(l, n, w, k)$ being the probability that more than k ($k > 1$) vehicles transmit in the l^{th} time slot, a failure due to collision. In the former case, the number of successful transmissions is the mean number of successful transmissions of the remaining $n - 1$ vehicles in the leftover $t - l + 1 - s$ time slots (since l slots have elapsed and s slots are used up during the successful transmission, preventing any other vehicle from transmitting) with at most $w - l$ contention slots left at the vehicles' back-off counter. The latter case deals with a failure due to collision, in which case the number of successful transmissions is equal to the number of successes in the remaining $t - l + 1 - c$ time slots (c is used instead of s because of the collision) by the remaining $n - 1$ vehicles. Thus, $X(T, W, N)$ gives us the number of successful transmissions of N vehicles in T time slots, with a maximum contention window size of W.

Note : We compute s and c as :

$$s = \frac{T_s}{\sigma}$$
$$c = \frac{T_c}{\sigma}$$

Here, σ denotes the slot time, T_s and T_c are the times taken for a successful and a collided transmission respectively. They are computed as :

$$T_s = T_h + L/R + AIFS$$
$$T_c = T_h + L/R + EIFS$$

Here, L is the size of the packet, R is the data rate or the bandwidth of the channel and T_h the duration of the physical layer header. As per the 802.11 protocol, a node is allowed to transmit only if it detects the channel idle for a time duration equal to AIFS (Arbitrary InterFrame Space) seconds, and we use EIFS if whenever the physical layer indicates an unsuccessful transmission event.

3 Incorporating Hidden Terminals

The earlier analysis was done under the assumption that no hidden terminals exist. If we were to incorporate hidden terminals as well into our model, $X(t,w,n)$ is rewritten as

$$\begin{aligned} X(t,w,n) = \sum_{l=1}^{\min(w,t)} \Bigg\{ & P(l,n,w,1)[1 + X(t-l+1-s, w-l, n-1)] \prod_r (1 - P_s(l,w)) \\ & + P(l,n,w,1) X(t-l+1-c, w-l, n-1) \sum_r P_s(l,w) \\ & + \sum_{k=2}^{n} P(l,n,w,k) X(t-l+1-c, w-l, n-k) \Bigg\} \end{aligned} \quad (3)$$

where $P_s(l,w)$ is the probability of there being an ongoing transmission at one of the r neighbouring RSUs during time slot l, out of a contention window of size w (Assumption: Max contention window size for all RSUs is W, time slots are all in sync so time slot l will happen at the same time everywhere, leading to similar slot updates everywhere). The original equation has been rewritten to allow a successful transmission (ie., $P(l,n,w,1)$) only if there is no node currently transmitting at a neighbouring RSU (Hence the product of probabilities of no transmission at all neighbouring nodes). If there is a concurrent transmission happening, then the transmission for the node under consideration fails, and is accounted as a collision, giving rise to the similar $X(t-l+1-c, w-l, n-1)$ as in the $P(l,n,w,k)$ case (This happens if there is a simultaneous transmission at even one neighbouring RSU, and hence the summation over r). As $P_s(l,w)$ is the probability of a transmission happening (at a neighbouring RSU) **during** the l^{th} slot from a contention window of w slots,

$$P_s(l,w) = \sum_{i=0}^{s} \sum_{j=0}^{N} P_t(l-i, j, w) \quad (4)$$

(where $P_t(l,n,w)$ is the probability of a successful transmission starting **in** the l^{th} slot when there are n vehicles, selecting backoffs from a contention window of w slots), since a successful transmission lasts for the following s time slots. Now, $P_t(l,n,w)$ is given by:

$$\begin{aligned} P_t(l,n,w) &= \binom{n}{1} \frac{1}{w-l+1} \left(1 - \frac{1}{w-l+1}\right)^{n-1} P\,(\text{no successful transmission in the previous } s \text{ slots}) \\ &= \frac{n}{w-l+1} \left(1 - \frac{1}{w-l+1}\right)^{n-1} (1 - P\,(\text{successful transmission in the previous } s \text{ slots})) \\ &= \frac{n}{w-l+1} \left(1 - \frac{1}{w-l+1}\right)^{n-1} \left(1 - \sum_{i=1}^{s} P\,(\text{successful transmission in slot } l-i)\right) \\ &= \frac{n}{w-l+1} \left(1 - \frac{1}{w-l+1}\right)^{n-1} \left(1 - \sum_{i=1}^{s} P_t(l-i, n+1, w+i)\right) \end{aligned} \quad (5)$$

The above equation arises as a result of choosing one of n vehicles, and choosing the l^{th} slot uniformly at random from the remaining slots. Thus,

$$\begin{aligned} P_s(l,w) &= \sum_{i=0}^{s} \sum_{j=0}^{N} P_t(l-i, j, w) \\ &= \sum_{i=0}^{s} \sum_{j=0}^{N} \frac{j}{w-l+i+1} \left(1 - \frac{1}{w-l+i+1}\right)^{j-1} \left(1 - \sum_{k=1}^{s} P_t(l-i-k, j+1, w+i)\right) \end{aligned} \quad (6)$$

(Another alternative, simpler non-recursive formulation I came up with is (I don't know if the two are equivalent, or if either is correct) assuming n is the no. of vehicles that have already transmitted, instead of it being the no. of vehicles left to transmit, as above. Then,

$$P_t(l,n,w) = \begin{cases} \left(\frac{l-1-s}{w}\right)^n \left(1 - \frac{s}{w-l+s}\right)^{N-n} \binom{N-n}{1} \frac{1}{w-l+1} \left(1 - \frac{1}{w-l+1}\right)^{N-n-1} & l > s \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Here, we say that the n vehicles should have transmitted in the first $l-1-s$ slots, the following s slots shouldn't have any transmission by the remaining $N-n$ vehicles, then we choose 1 out of $N-n$ vehicles and make it transmit in the l^{th} slot, and finally require the remaining $N-n-1$ vehicles to not transmit in the l^{th} slot. If $l \leq s$, then whatever be the value of n, it is impossible for the transmission to have ended by the l^{th} slot, in which case a successful transmission cannot start in the l^{th} slot. Hence, the value is zero. Now,

$$P_s(l,w) = \sum_{i=0}^{s} \sum_{j=0}^{N} P_t(l-i,j,w)$$

$$\implies P_s(l,w) = \begin{cases} \sum\limits_{i=0}^{s} \sum\limits_{j=0}^{N} \left(\frac{l-i-1-s}{w}\right)^j \left(1 - \frac{s}{w-l+i+s}\right)^{N-j} \binom{N-j}{1} \frac{1}{w-l+i+1} \left(1 - \frac{1}{w-l+i+1}\right)^{N-j-1} & l > s \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Here we don't end up with a recursive formulation, and it is also $O(sN)$ as opposed to $O(s^2N)$ for the previous formulation.)

Now, if we just want to model the success probability of a packet without retransmissions, we can directly compute

$$P_{succ} = \frac{X(T,W,N)}{N}$$

4 Including Retransmissions

Now, we take into account re transmissions due to collision. Here, we compute the probability of successful packet delivery by considering K transmission intervals. Assume at the start of the first analysis, there are N vehicles attempting to transmit N packets (and contention window of W). We now track the N packets that were attempted for transmission in the first slot, across K interval, each with T slots.

Define x_k as the number of packets of the initially considered N packets that get delivered successfully in the k_{th} time interval, let's denote $X(T,W,N)$ as X for simplicity of notation.

$$x_1 = X$$

To find x_2, we assume that N vehicles again transmit in the $2nd$ interval with contention window W, which is a reasonable assumption. Now, x_1 packets got delivered successfully in the $1st$ time interval, so $N - x_1$ packets will be retransmitted in this time interval. So, the number of packets of the initially considered N packets that get delivered successfully in the 2_{nd} time interval is given by:

$$x_2 = \left(\frac{N - x_1}{N}\right) X$$

Generalizing this, we get

$$x_k = \frac{\left(N - \sum\limits_{i=1}^{k-1} x_i\right)}{N} X$$

We see that

$$x_{i+1} = (N - x_1 - x_2 - \cdots - x_i)\, X/N$$

Replacing x_i with $(N - x_1 - x_2 - \cdots - x_{i-1})\, X/N$, we get

$$x_{i+1} = \left(\frac{N^2 - Nx_1 - \cdots - Nx_{i-1} - (N - x_1 - x_2 - \cdots - x_{i-1})\, X}{N} \right) \frac{X}{N}$$

$$= \frac{(N - X)\, X}{N^2} (N - x_1 - x_2 - \cdots - x_{i-1})$$

$$= \frac{(N - X)\, X}{N^2} \left(x_i \frac{N}{X} \right) = \left(\frac{N - X}{N} \right) x_i$$

Since $x_1 = X$, we obtain

$$x_k = \left(\frac{N - X}{N} \right)^{k-1} X \tag{9}$$

Now, assuming the maximum number of attempts for any packet is K, we can find out the number of packets (of the initially considered N packets) delivered in successfully X' as:

$$X' = \sum_{k=1}^{K} x_k = \sum_{k=1}^{K} \frac{\left(N - \sum_{i=1}^{k-1} x_i \right)}{N} X = \sum_{k=1}^{K} \left(\frac{N - X}{N} \right)^{k-1} X$$

We see that

$$X' = X \left[1 + \left(\frac{N - X}{N} \right) + \left(\frac{N - X}{N} \right)^2 + \cdots + \left(\frac{N - X}{N} \right)^{K-1} \right]$$

which is the sum of geometric progression, given by $S_n = \frac{a(1-r^n)}{1-r}$. Hence,

$$X' = \frac{X \left(1 - \left(\frac{N-X}{N} \right)^K \right)}{1 - \left(\frac{N-X}{N} \right)} = \frac{N^K - (N - X)^K}{N^{K-1}}$$

The probability of successful packet delivery can be given as:

$$P_{succ} = \frac{X'}{N} = \frac{N^K - (N - X)^K}{N^K} = 1 - \left(\frac{N - X}{N} \right)^K \tag{10}$$

It is interesting to note that we could have arrived at the same result if we were to track a single packet instead, over K retransmissions. In this case, in each retransmission, only X of the N packets are successfully transmitted, giving us a failure probability of $(N - X)\,/N$ for each retransmission attempt. Thus, for a single packet to fail after K retransmissions, it must fail on each retransmission, giving a total failure probability of

$$P_{fail} = \left(\frac{N - X}{N} \right)^K$$

$$\implies P_{succ} = 1 - P_{fail} = 1 - \left(\frac{N - X}{N} \right)^K$$

We could also have computed x_k as the expected number of packets to succeed on the k^{th} try, after failing for the first $k - 1$ tries. Thus,

$$x_k = \left(\frac{N - X}{N} \right)^{k-1} \left(\frac{X}{N} \right) \cdot N = \left(\frac{N - X}{N} \right)^{k-1} X$$

5 Final Model

To take into account RSU scheduling, we assume that the vehicles arrive as an exponential distribution. So, it's reasonable to assume that the probability of an RSU being OFF is also follows the same distribution.
So, we define:

$$P(\text{RSU} = \text{OFF}) = \lambda e^{-\lambda N}$$

where there are N vehicles in the system. Since $P(\text{RSU} = \text{OFF}|N = 0) = \lambda$, and we know that the RSU is shut off when there are no vehicles, we get $\lambda = 1$. Thus, $P(\text{RSU} = \text{ON}) = 1 - e^{-N}$.
The SINR at each RSU depends on the vehicle distribution at that RSU as well as whether the RSU is switched on. Since the vehicle arrival distribution as well as the RSU turning on/off is exponential, the probability of the SINR being a certain value is also exponential. Since the probability of the SINR being greater than the threshold δ is 1 minus the exponential CDF, we have:

$$P(\text{SINR} > \delta) = e^{-\lambda N}$$

Since the two events, successful packet transmission and RSU being on/off are independent, we can simply multiply the two probabilities to obtain the final probability of successful packet delivery. Thus,

$$P_{succ-new} = P_{succ} \times P(\text{RSU} = \text{ON}) \times P(\text{SINR} > \delta)$$

$$\implies P_{succ-new}(T, W, N, K) = \left(1 - \left(\frac{N - X(T, W, N)}{N}\right)^K\right)\left(1 - e^{-N}\right)\left(e^{-\lambda N}\right)$$

6 Experimental setup (for analysis)

The parameters that were used for getting the analytical results are as shown below :

Slot time	16 μs
SIFS time	32 μs
AIFSN	2
EIFS	188 μs
Header duration (T_h)	40 μs
Packet size (L)	500 bytes
Data rate (R)	3 Mbps
CCH interval (T_{CCH})	50 ms
RSU's within range (r)	4
Radio power during reception	10.0W
Radio power during idle state	9.0W
Min. contention window	15ms
Max. contention window	1023ms
Hop limit	3
Switching energy	50J
Grid energy cost	0.06$kW^{-1}h^{-1}$
Solar energy cost	0.01$kW^{-1}h^{-1}$
Battery value	240Whr
Min. battery value	12Whr
PV Panel output	40W
Noise constant	-127.0dBm
Transmission power (SINR computatioon)	33dBm
SINR Threshold	0.001

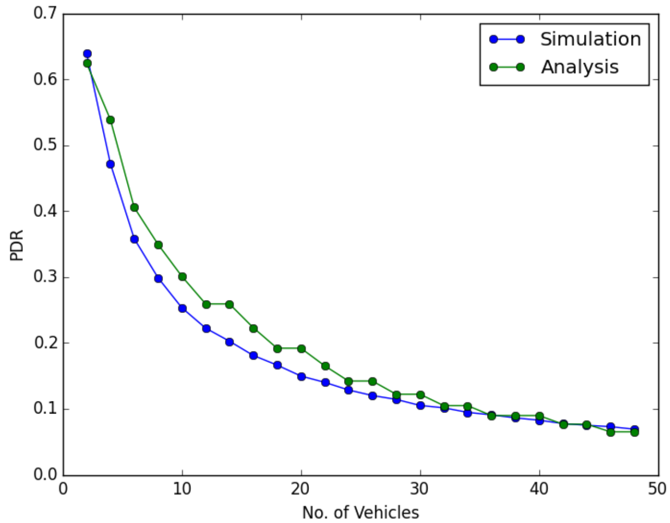

Figure 1: Variation in PDR with number of vehicles

Vehicle, velocity, packet distributions were sampled from exponential distributions. The value of AIFS is computed as :

$$\text{AIFS} = 2\sigma + \text{SIFS}$$

The number of vehicles n were varied from 1 to 60 (beyond which the values were computationally infeasible), and the contention window W was varied in powers of 2 from 4 to 64.

7 Results

The analysis is verified with the help of the simulation, using the parameters given in the previous section. The RMSE of the values obtained from the analysis, compared to the simulation values is **0.03057**. The contention window was set to 8, and the range of vehicles considered was 2 to 50. Larger values are computationally infeasible for the analysis. The RPR algorithm is run to obtain the RSU scheduling, and the energy and OPEX graphs are given below.

References

[1] D. Jiang and L. Delgrossi, IEEE 802.11p: Towards an international standard for wireless access in vehicular environments, in Proceedings of the IEEE Vehicular Technology Conference - Spring, 2008, pp. 2036 2040.

[2] Karagiannis, Georgios, et al. "Vehicular networking: A survey and tutorial on requirements, architectures, challenges, standards and solutions." Communications Surveys & Tutorials, IEEE 13.4 (2011): 584-616.

[3] Vageesh, D. C., Moumita Patra, and C. Siva Ram Murthy. "Joint placement and sleep scheduling of grid-connected solar powered road side units in vehicular networks." Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2014 12th International Symposium on. IEEE, 2014.

[4] Campolo, Claudia, et al. "Modeling broadcasting in IEEE 802.11 p/WAVE vehicular networks." Communications Letters, IEEE 15.2 (2011): 199-201.

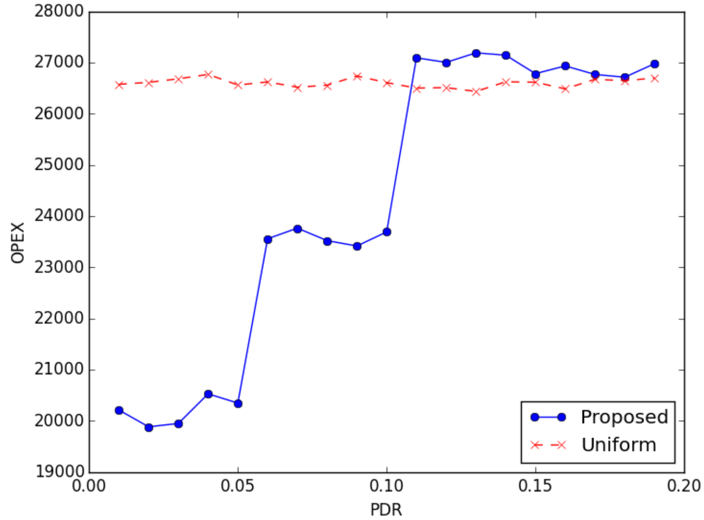

Figure 2: Variation in OPEX with required PDR values

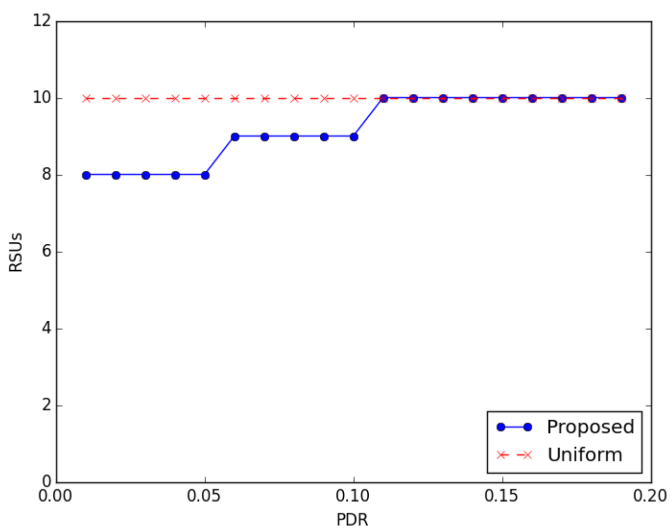

Figure 3: Variation in PDR with number of RSUs

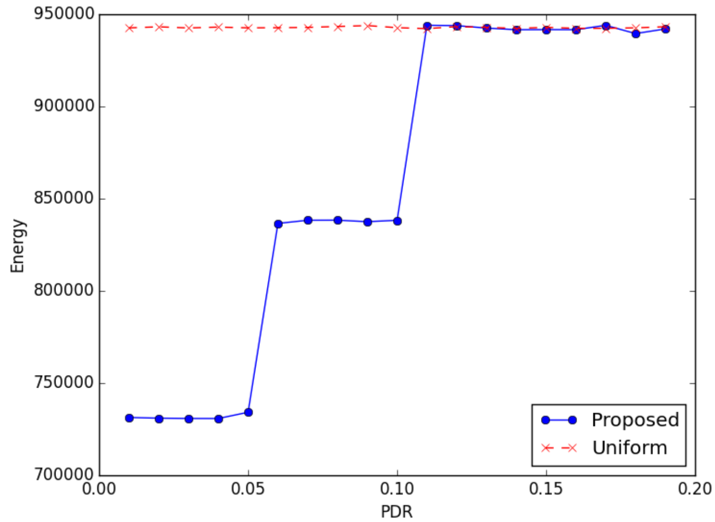

Figure 4: Energy variation with required PDR values

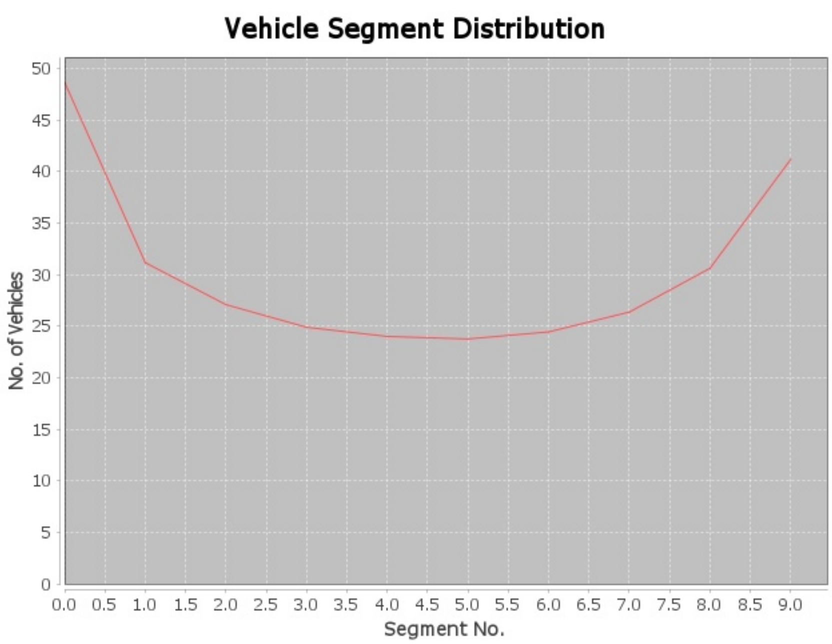

Figure 5: Average density variation with segment number

[5] Q. Feng, K. Hwang, and Y. Dai, Rainbow product ranking for upgrading e-commerce, IEEE Internet Computing, vol. 13, no. 5, pp. 7280, 2009