Lab 3 - IMU Noise Analysis

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1 Introduction

This report describes the noise analysis of a VectorNav VN-100 IMU. A short dataset of 14.2 minutes was used to plot the data of the accelerometers, gyroscopes and magnetometer for each axes and analyze the data distribution. A much longer dataset of almost 5 hours was taken to do an Allan Variance test on the accelerometer and gyroscope. Both the datasets were collected in a closet with minimum external interference. The long dataset can be found here

2 Short Dataset

2.1 Visualization

The time plots for Yaw, Pitch, Roll, Magnetic field - X,Y and Z axes, Linear Acceleration - X,Y and Z axes and Angular Velocities - X,Y and Z axes are given below. The mean, standard deviation and variances for each plot is also mentioned.

Figure 1: Yaw, Pitch, Roll time plots

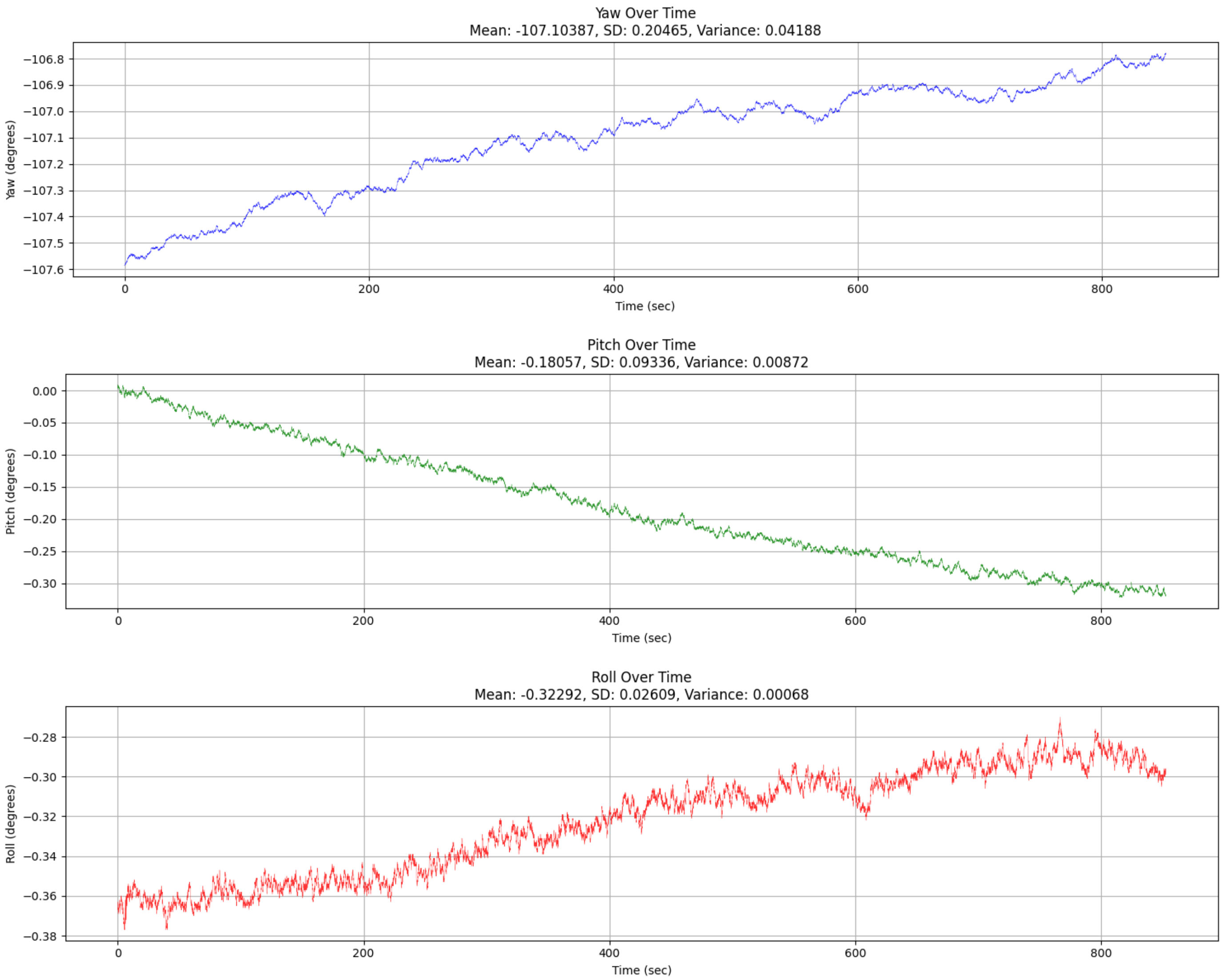


Figure 2: Magnetic field - X,Y and Z time plots

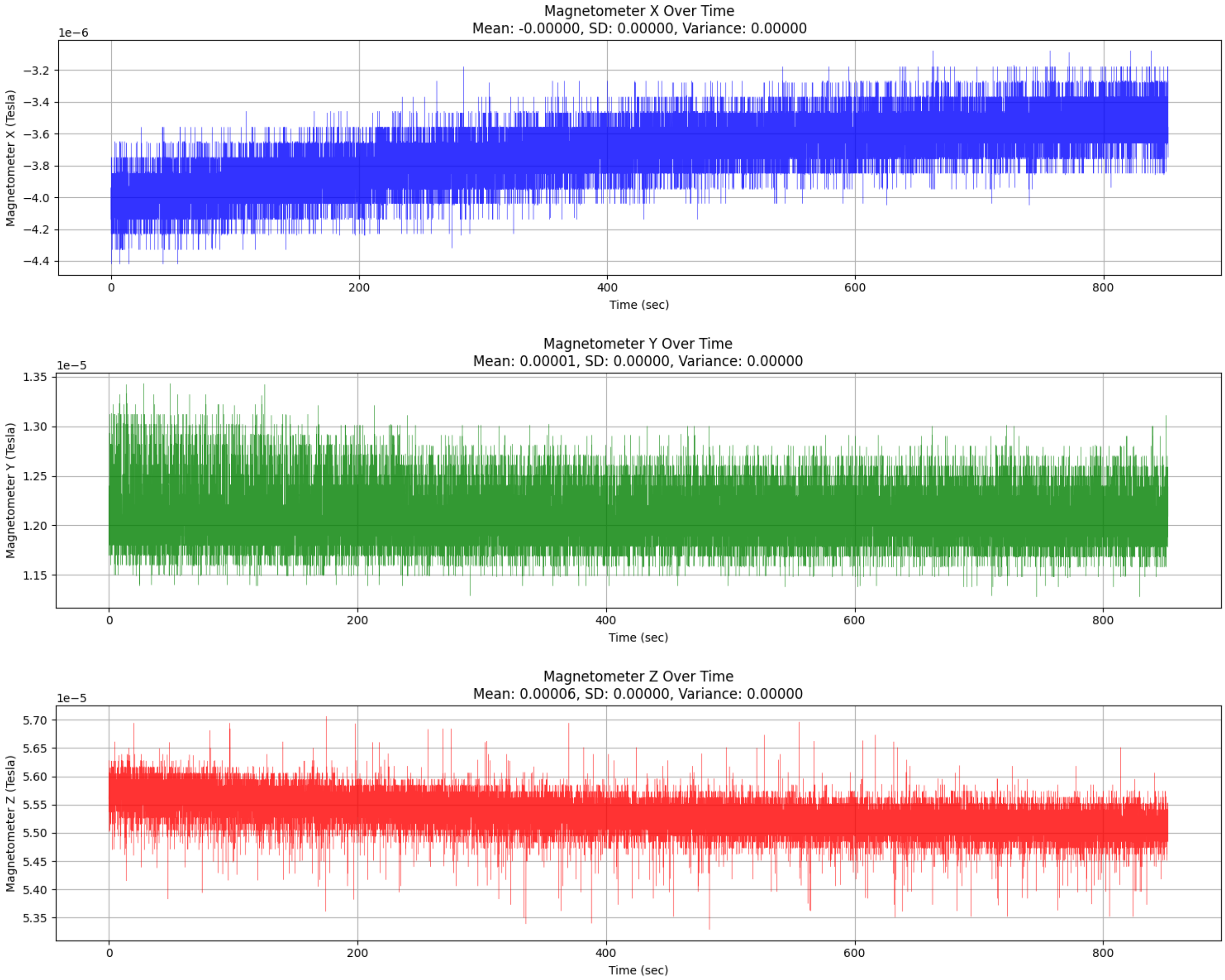


Figure 3: Linear Acceleration - X,Y and Z time plots

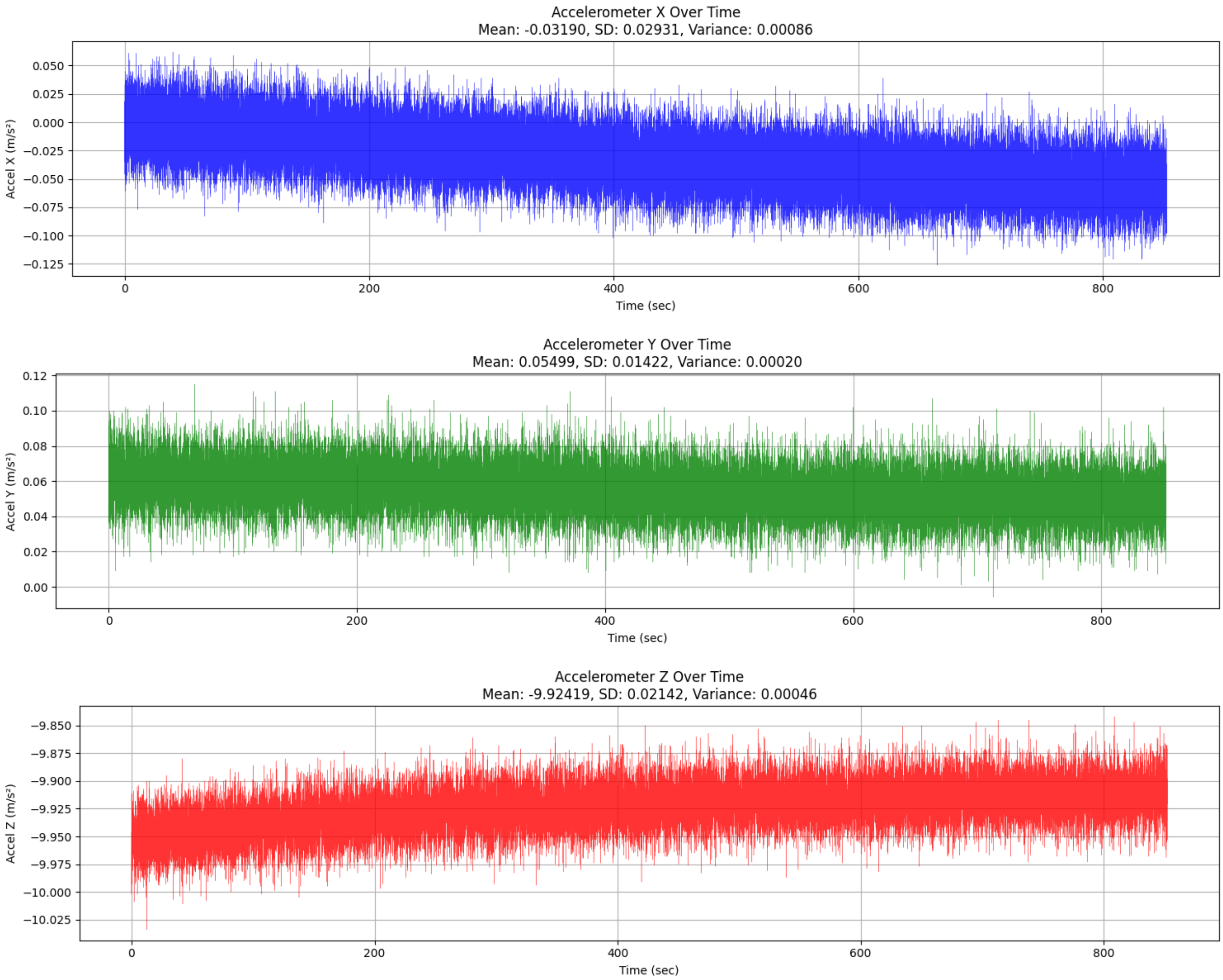
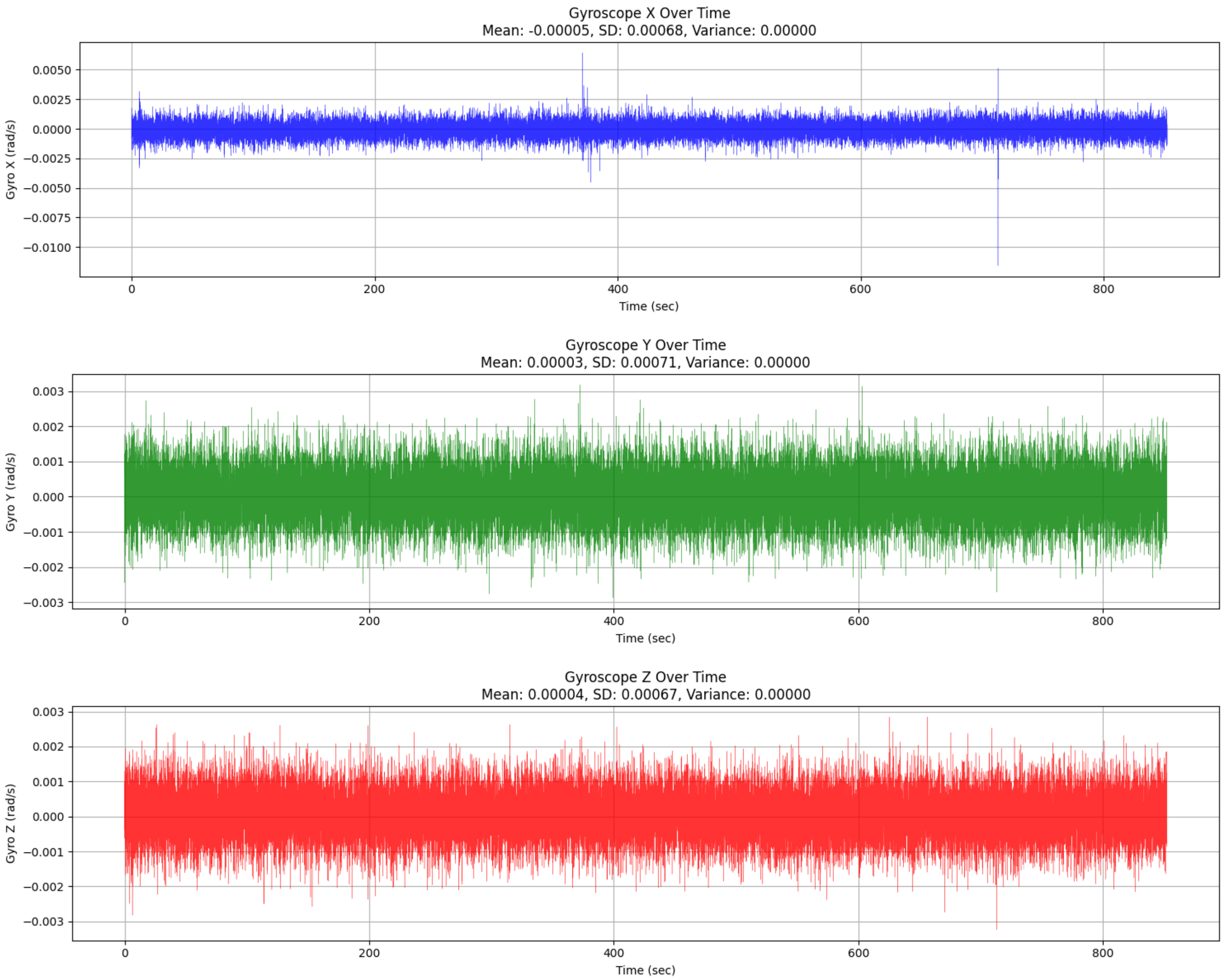


Figure 4: Angular Velocity - X,Y and Z time plots



2.2 Noise Distribution Analysis

The inferences that can be made from time plots is very limited. but just looking at the plots, a few observations can be made. All the plots except roll, pitch and yaw are very stable and minimal drift can be observed. This is because the yaw, pitch and roll are calculated using the sensor data which is why the error is more apparent. A few spikes can be observed in the X-axis Gyroscope plot which is due to external vibrations.

To properly analyze the distributions, histograms need to be plotted. The error of each sample is calculated as the difference of the value from the mean and the densities were plotted into 15, 30 and 100 bins for Magnetic field, Linear acceleration and Angular velocity respectively (The amount of bins was chosen according to how discrete the data was). scipy.stats library in python was used to find the normal distribution that is the closest fit to each histogram to see how close to a normal distribution each sensor data is. The histograms with their closest fits are given below

Figure 5: Yaw, Pitch, and Roll histograms with fitted normal distribution

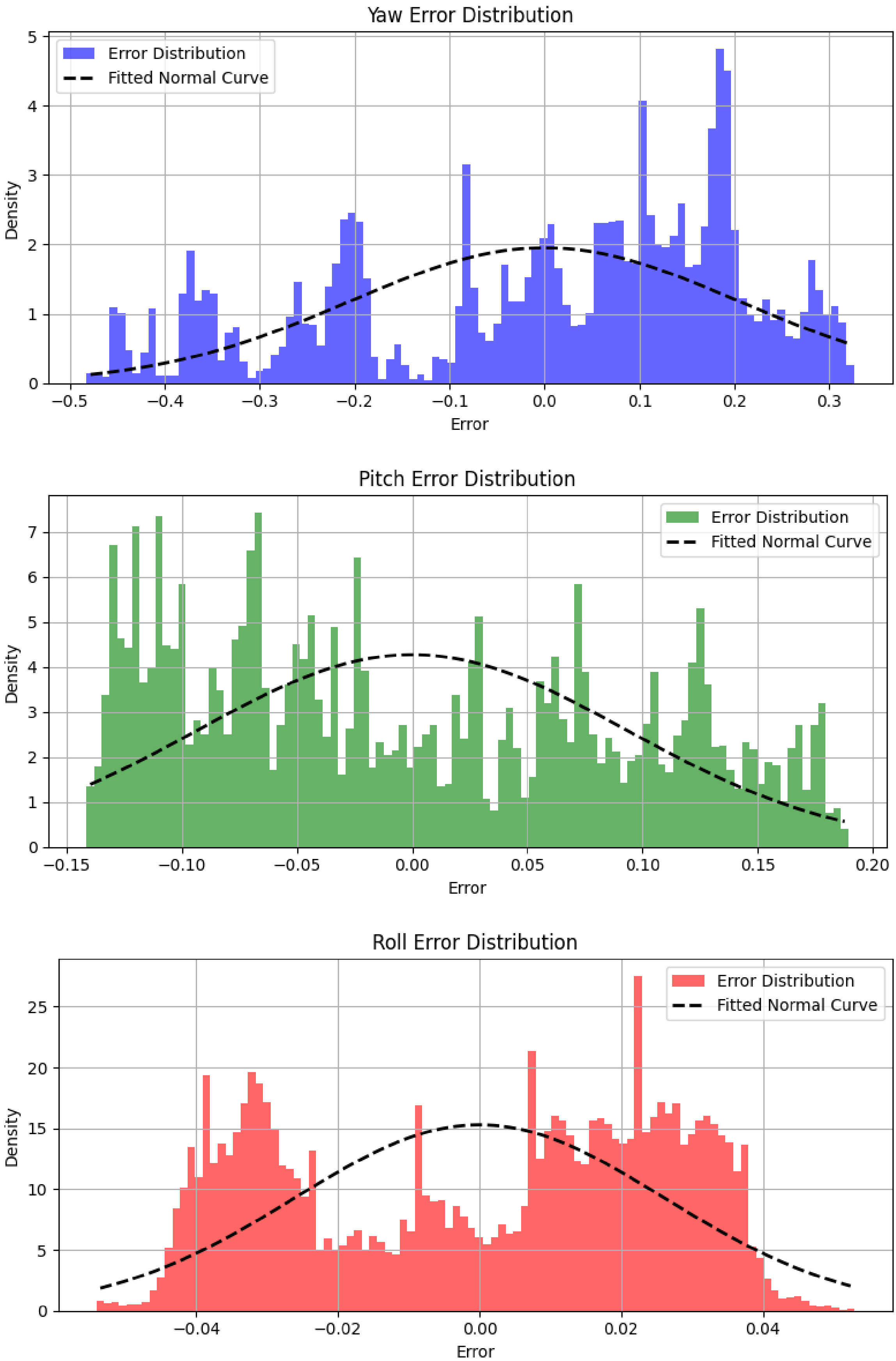
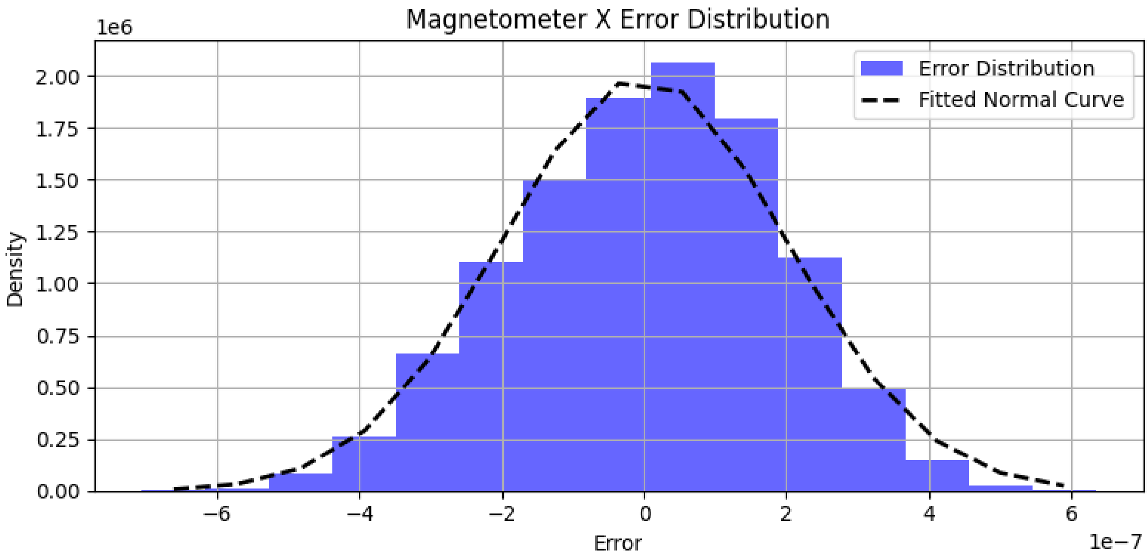
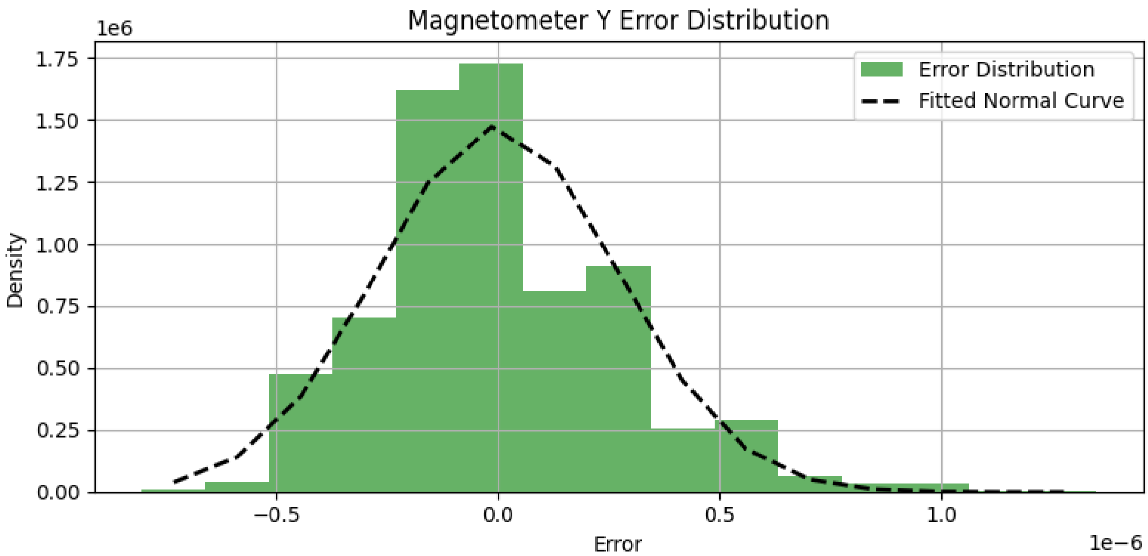


Figure 6: Magnetic Field - X, Y, and Z histograms with fitted normal distribution





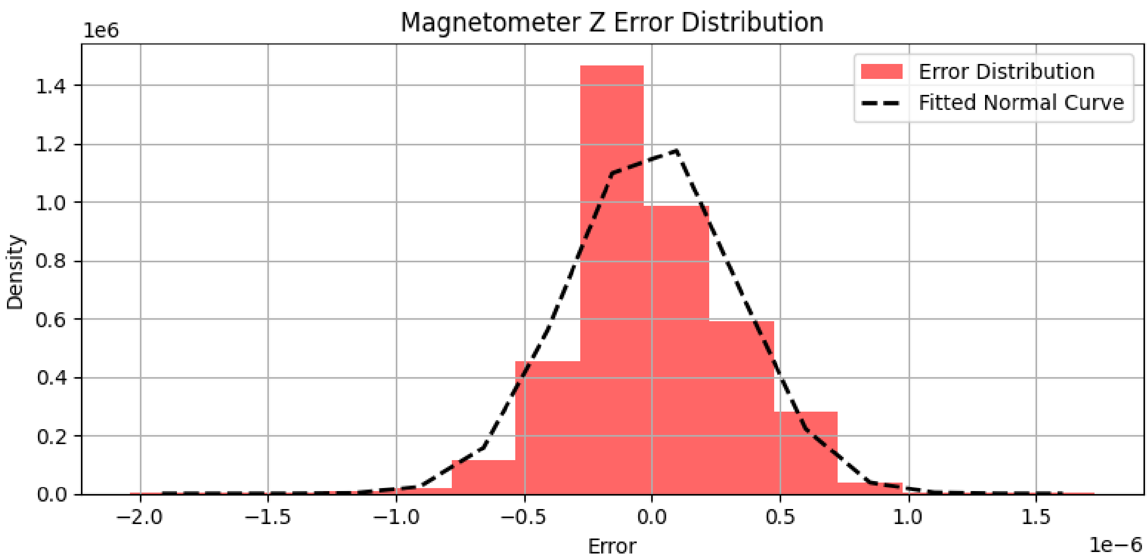


Figure 7: Linear Acceleration - X, Y, and Z histograms with fitted normal distribution

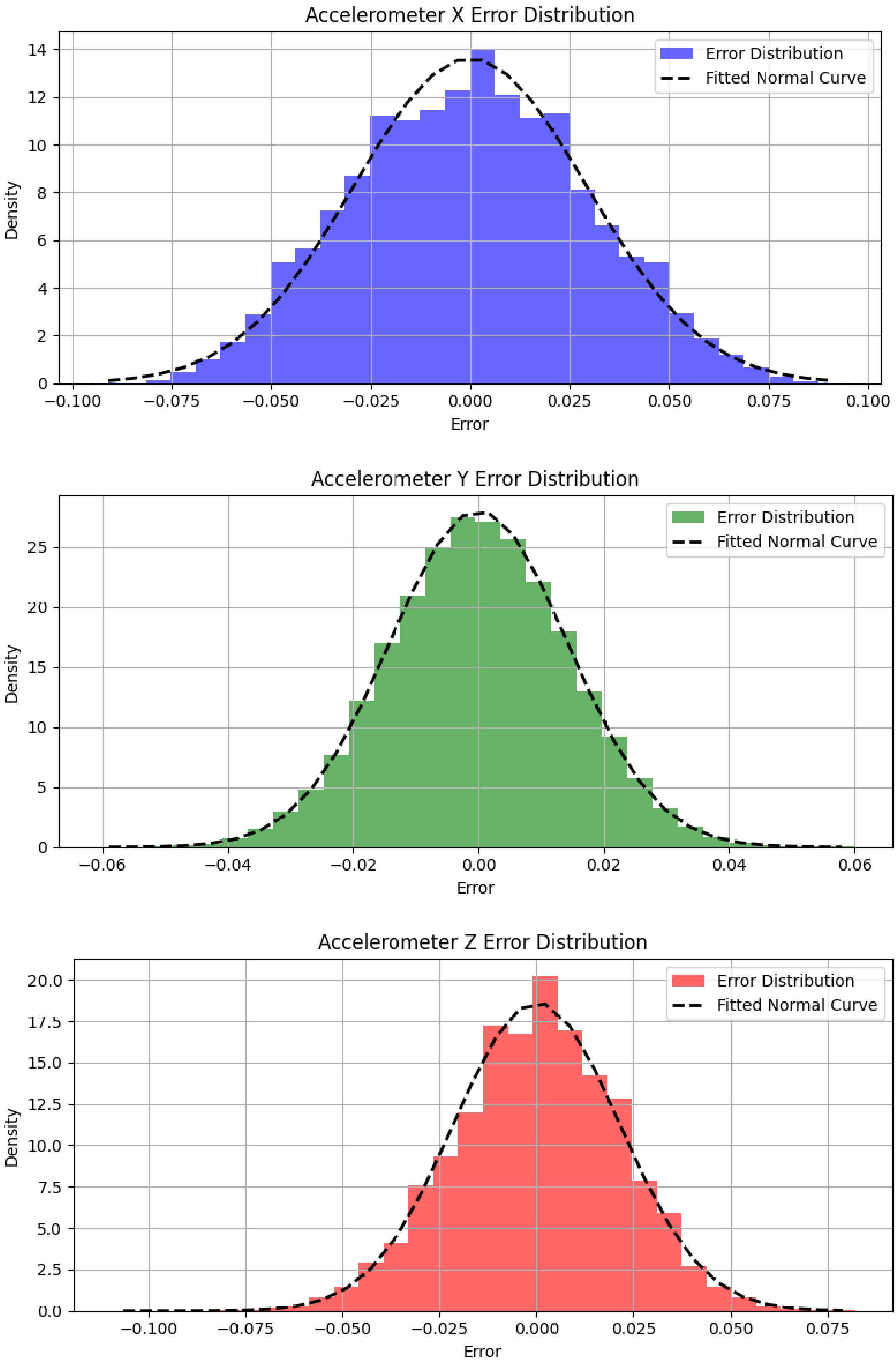
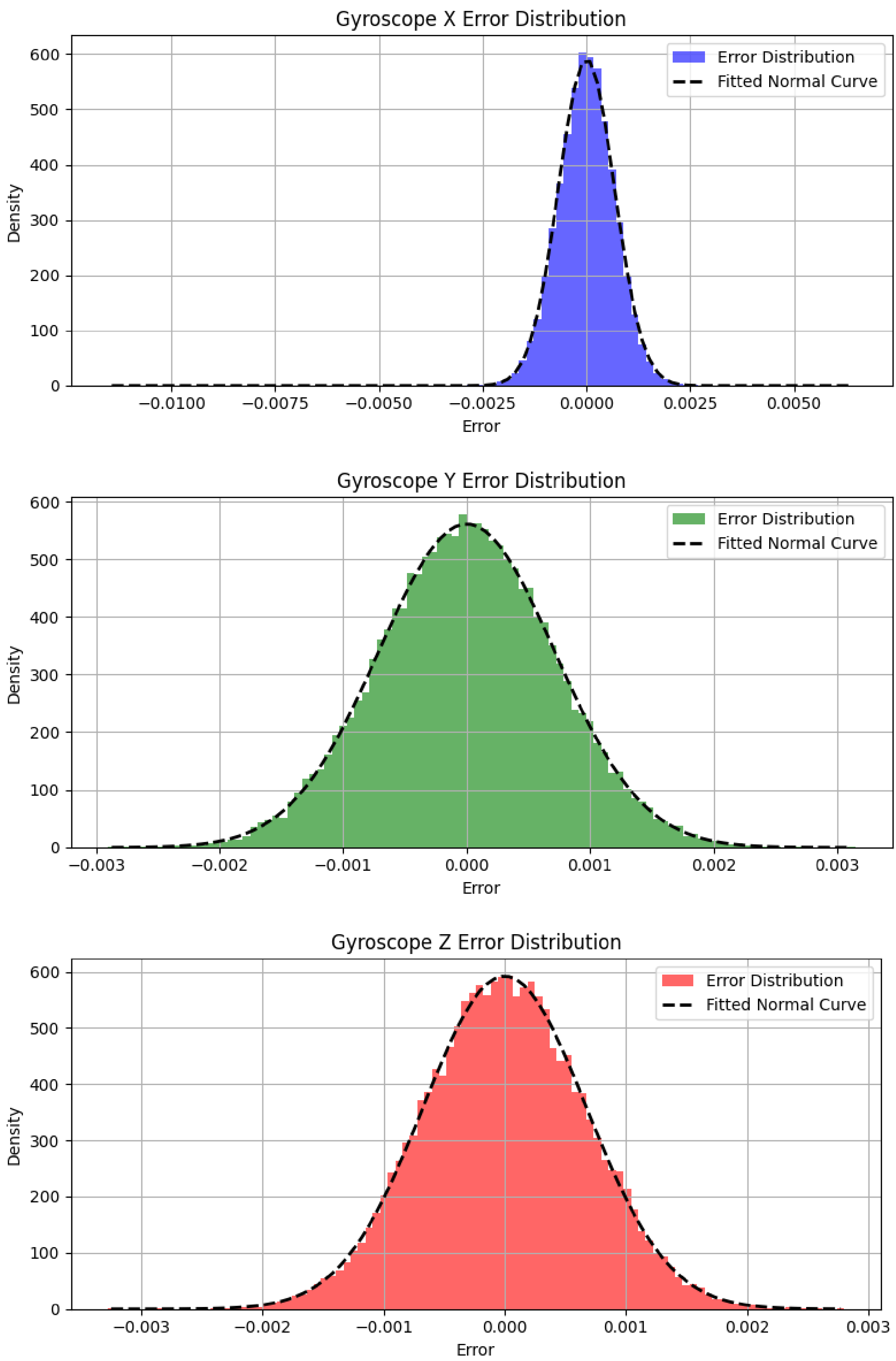


Figure 8: Angular Velocity - X, Y, and Z histograms with fitted normal distribution



From the figures, it is apparent that the sensor data follows normal/gaussian distribution. The angular velocity histogram aligns almost perfectly with the fitted normal curve where as the fit is not as perfect for the magnetic field. This is because of the lower bin size taken for the magnetometer data due to how discrete the readings were. When the bin size was higher, it was observed that the density was always concentrated in around 8 bins and the bins in between had low densities. This interferes with the curve fitting so the number of bins were adjusted for each plot. Magnetic field is also the most susceptible out of the three due to external interference from electronic devices and absence of calibration. Regardless of this, it is clear that it follows a normal distribution too. Yaw, pitch and roll do not fit the normal distribution at all because it is obtained by doing non-linear operations on the raw sensor data along with filtering. Therefore, it does not directly follow any distribution.

3 Allan Deviation Test - Long Dataset

The in-run bias stability, rate random walk and noise density for the gyroscope and accelerometer data collected is given in the tables below. Note that the noise density and in-run bias stability have been converted into the same units used in the datasheet and the random walk is left as is from the matlab code used.

Table 1: Gyroscope Noise Characterization

Axis	X	Y	Z
Bias Instability (deg/hr)	0.6093	0.7221	0.2772
Random Walk (deg/s$\sqrt{\text{Hz}}$)	4.2415×10^{-6}	4.4419×10^{-6}	1.7604×10^{-6}
Noise Density (deg/s/$\sqrt{\text{Hz}}$)	0.0056	0.0063	0.0061

Table 2: Acceleration Noise Characterization

Axis	X	Y	Z
Bias Instability (mg)	0.0453	0.0520	0.0419
Random Walk (m/s$^2\sqrt{\text{Hz}}$)	3.0125×10^{-5}	5.6574×10^{-5}	2.3717×10^{-5}
Noise Density (mg/$\sqrt{\text{Hz}}$)	0.2145	0.1923	0.2942

3.1 What kind of errors/sources of noise are present?

- **White noise**: Random fluctuations in the gyroscope output with a constant power spectral density
- **Red/Brownian noise** : Fluctuations that accumulate over time with a power spectral density that decreases with increasing frequency following a 1/f^2 relationship. This leads to cumulative drift
- **Pink/Flicker noise** : Fluctuations that accumulate over time with a power spectral density that decreases with increasing frequency following a 1/f relationship. This leads to long-term bias instability.
- **Quantization noise** : Occurs due to the sensor sending a digital signal of discrete data. Depends on resolution of the sensor
- **Temperature drift** : Changes in temperature affects performance and bias
- **Mechanical Vibrations** : External vibrations can introduce noise intermittently

3.2 How do we model them? Where do we measure them?

- **White noise**: Modelled as Noise density. The PSD is represented by

  $$S_\Omega(f) = N^2$$

  where N is the noise density or angle random walk coefficient. The Allan variance relationship for this noise can be expressed as:

  $$\sigma^2(\tau) = \frac{N^2}{\tau}$$

  From this relationship we can see that as the averaging time(τ) increases, the effect of white noise decreases. Plotting the Allan deviation against the averaging time on a log-log scale, the section with a slope of -0.5 represents white noise and the intersection of this line with $\tau = 1$ gives us the noise density N

- **Red/Brownian noise** : Modelled as Rate angular walk. The PSD is represented as

$$S_\Omega(f) = \left(\frac{K}{2\pi}\right)^2 \frac{1}{f^2}$$

where K is the rate random walk coefficient. The Allan variance relationship for this noise can be expressed as:

$$\sigma^2(\tau) = \frac{K^2\tau}{3}$$

From this relationship we can see that the variance increases inversely with the averaging time, which shows how the error accumulates over time. Plotting the Allan deviation against the averaging time on a log-log scale, the section with a slope of 0.5 represents white noise and the intersection of this line with $\tau = 3$ gives us the rate random walk K.

- **Pink/Flicker noise** : Modelled as in-run bias stability. The PSD is represented as

$$S_\Omega(f) = \begin{cases} \left(\frac{B^2}{2\pi}\right)\frac{1}{f} & : f \leq f_0 \\ 0 & : f > f_0 \end{cases}$$

where B is the bias instability coefficient and f_0 is the cut-off frequency.

For longer averaging times, the Allan variance can be approximated as:

$$\sigma^2(\tau) = \frac{2B^2}{\pi}\ln 2$$

$$B = 0.664\sigma^2(\tau)$$

This relationship indicates that the variance approaches a constant value. Plotting the Allan deviation against the averaging time on a log-log scale, the in-run bias stability B can be calculated by looking at the point where the slope is 0

So, from the allan deviation test, we can model the sensor as

$$X(t) = X_{\text{Ideal}}(t) + \text{Bias}_B(t) + \text{Bias}_K(t) + \text{Bias}_N(t)$$

where in this case X could be Angular velocity for the Gyro and linear acceleration for the accelerometer.

Since the temperature was not controlled during data collection, the quantization must have been part of the white noise (since it is consistent and time invariant) and temperature drift must have affected the bias and drift. Ideally, the effects of these two must be negated while sampling the data and the bias caused due to the temperature must be measured as a function of temperature and added to the model from the allen deviation test.

3.3 Can you relate your measurements to the datasheet for the VN100?

Looking at the datasheet, the specifications are as in the table below

Table 3: VN-100 Specifications

Sensor	Specification	Value
Gyroscope	In-Run Bias Stability	<10°/hr
	Noise Density	0.0035°/s /$\sqrt{\text{Hz}}$
Accelerometer	In-Run Bias Stability	<0.04 mg
	Noise Density	0.14 mg/$\sqrt{\text{Hz}}$

For the Gyroscope, the bias instability can be observed to be much lower than the specification value. This is because the dataset collection time was not enough to collect significant long term drift. The noise distribution is slightly higher which means there was more white noise during dataset collection.

For the accelerometer, both the bias stability and noise density are slightly more than the datasheet with the z axis noise density being significantly more. This is because of vibrations since it would explain why the z axis noise density is more pronounced (vibrations travel through the mounting surface and primarily affect the Z axis)

Other than the above reasons, sensor aging and temperature drift should have also caused bias to change.

Figure 9: Allen Deviation Plots for Gyroscope

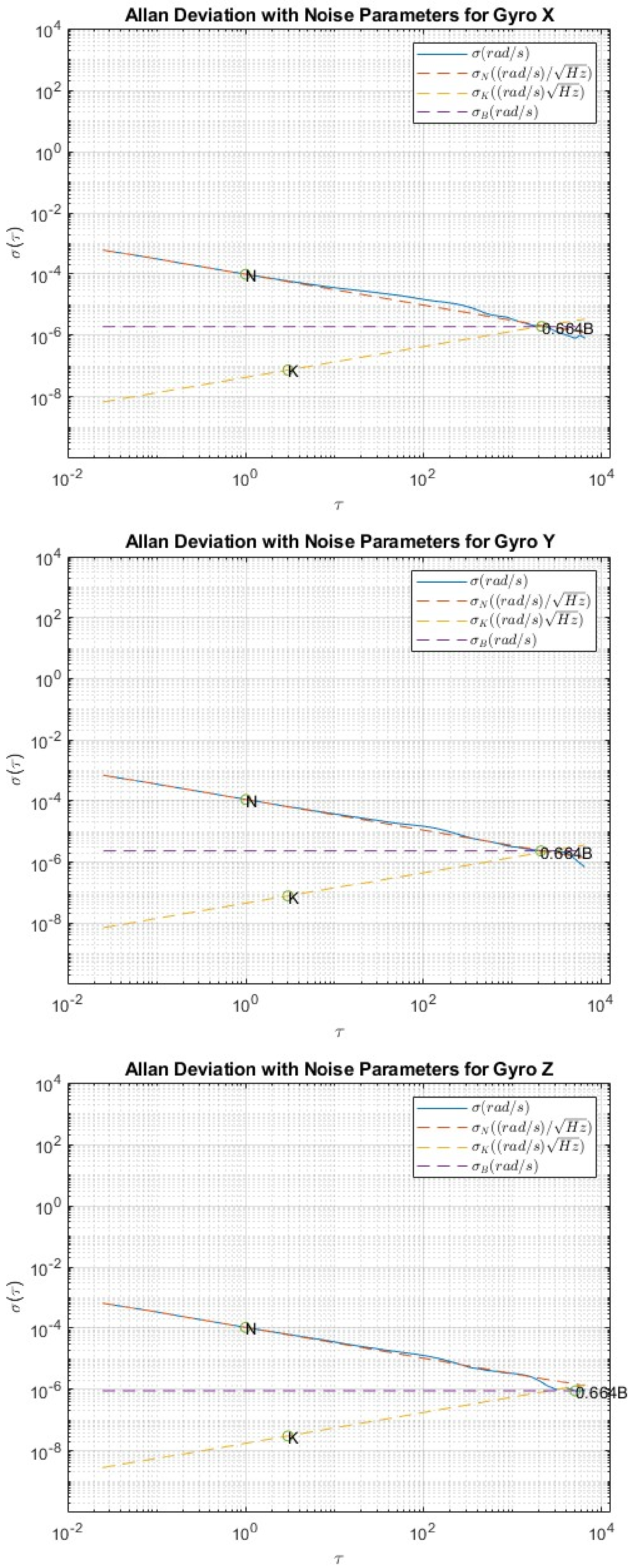


Figure 10: Allen Deviation Plots for Accelerometer