Lab 4 - Dead Reckoning with IMU and GPS

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The driver code and rosbags are in my Gitlab account. Username - srirangam.r

1 How did you calibrate the magnetometer from the data you collected? What were the sources of distortion present, and how do you know?

Magnetometer X and Y data from a car driving in circles was used to calibrate the magnetometer. Only circular motion data was retained, with non-circular segments removed. Figure 1 shows the raw X and Y magnetometer data, where retained circular data is in blue and excluded points are in red.

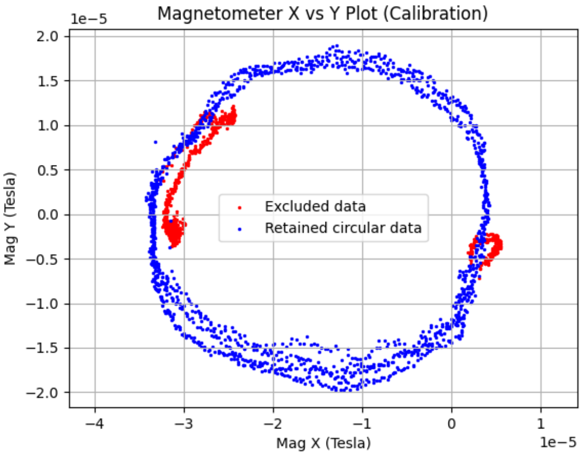


Figure 1: Raw Magnetometer data (Calibration dataset)

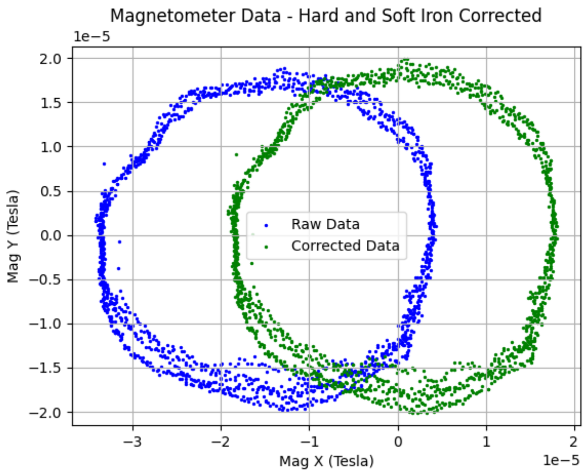


Figure 2: Original vs Calibrated Magnetometer data

Ideally, since the data contains the magnetometer going in a circle, the data should be a circle centered on the origin. But due to distortions, this is not usually the case. There are mainly two kinds of distortions for magnetometer data - Hard iron and Soft iron. Hard iron distortions are caused by objects with magnetic fields and they cause a permanent bias in the data. These can be corrected by translating the data to be centered around the origin. Soft iron distortions are caused by metals that stretch the magnetic field. These can be corrected by fitting the data into an ellipse and scaling the data along the major and minor axes of the fitted ellipse to turn it into a circle. Using this, the calibration matrix can be written as

$$\mathbf{M}(CalibrationMatrix) = SoftIronCalibrationMatrix \times HardIronTranslationMatrix$$

$$SoftIronCalibrationMatrix = \mathbf{R}(\theta) \times \mathbf{S} \times \mathbf{R}(-\theta)$$

$$= \begin{bmatrix} \sqrt{\frac{b}{a}}\cos^2(\theta) + \sqrt{\frac{a}{b}}\sin^2(\theta) & \left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)\sin(\theta)\cos(\theta) \\ \left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)\sin(\theta)\cos(\theta) & \sqrt{\frac{b}{a}}\sin^2(\theta) + \sqrt{\frac{a}{b}}\cos^2(\theta) \end{bmatrix}$$

Where, a and b are the semi major and semi minor axis lengths; θ is the angle between the major axis and X axis; $\mathbf{R}(\theta)$ is a rotation about θ; S is the Scaling matrix

The scaling factors were chosen to be $\sqrt{\frac{b}{a}}$ and $\sqrt{\frac{a}{b}}$ for along the major and minor axes respectively to preserve the area. The specific scaling factors used does not matter (as long as they are in the same ratio) since the magnetometer data is solely used to compute yaw and during which the Y and X components will be divided, canceling out the scaling factor. The factors chosen make sure the correction can properly be compared when plotted on a single plot.

To calibrate the magnetometer data, the raw data was first fitted into an ellipse using direct linear least square fitting method [2]. The equation of the fitted ellipse for the data is

$$0.6548x^2 - 0.0535xy + 0.7539y^2 + 1.8875 \times 10^{-5}x - 1.8993 \times 10^{-7}y - 9.9772 \times 10^{-11} = 0$$

From the equation of the fitted ellipse, the coefficients can be used to calculate the center, a, b and θ [3]. The ellipse parameters are

$$Centre = \left(-1.4428 \times 10^{-5}, -3.8560 \times 10^{-7}\right)$$

$$\theta = 14.175°$$

$$a = 1.9079 \times 10^{-5}, \quad b = 1.7611 \times 10^{-5}$$

As mentioned above, the hard iron distortion can be corrected by translating the data from the center to the origin and the soft iron distortion can be corrected using the matrix derived above. The calibrated data along with the original data for comparison is plotted in Figure 2. To check how good the correction is, an ellipse was fitted into the corrected data and it had an eccentricity of 8.5601×10^{-8} which means it is almost a perfect circle.

Looking at the data and the calibration, it can be seen that hard iron distortions are much more significant than soft iron distortions. The hard iron distortions are caused due to a combination of the magnetic field generated due to the electronics(laptops, phones and car's electronics and motors etc.) in the car and the other sensors/circuitry inside the IMU. The soft iron effects are a combination of the magnetic materials in the car like the body, batteries etc [5]. The calibration only corrects the time invariant distortions i.e if the sources are in the car. There could be external magnetic fields or magnetic materials outside the car that distort the magnetic field only at certain positions like the distortion that can be seen at the top-left of Figure 1.

2 How did you use a complementary filter to develop a combined estimate of yaw? What components of the filter were present, and what cutoff frequency(ies) did you use?

The Calibration Matrix obtained above was used to correct the magnetometer data for the driving dataset as well. The X-Y plot of this dataset before and after correction is shown in Figure 3

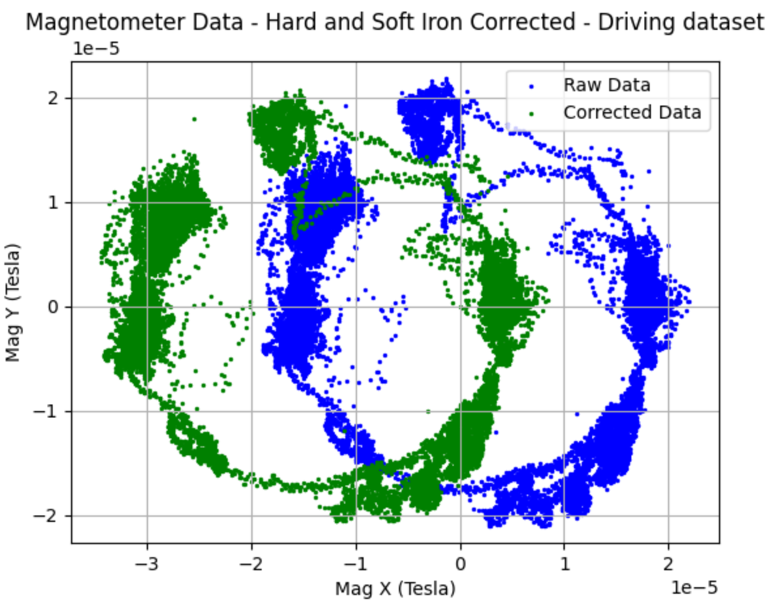


Figure 3: Original vs Calibrated Magnetometer X-Y data - Driving dataset

The magnetometer data can be used to compute the yaw of the IMU with respect to the earth's magnetic axis by applying atan2(y, x). The yaw computed before and after correction is given in Figure 4. The angular velocity about Z axis obtained from the gyroscope was integrated using cumulative trapezoidal integration to get an estimate of the yaw. Since this yaw is relative to the starting orientation of the IMU, The initial yaw of the gyroscope estimate was made equal to the initial yaw from the corrected magnetometer data. The Magnetometer and Gyroscope yaw estimates are plotted together in Figure 5. Note that the angles are in degrees and have been unwrapped for visualization and comparison.

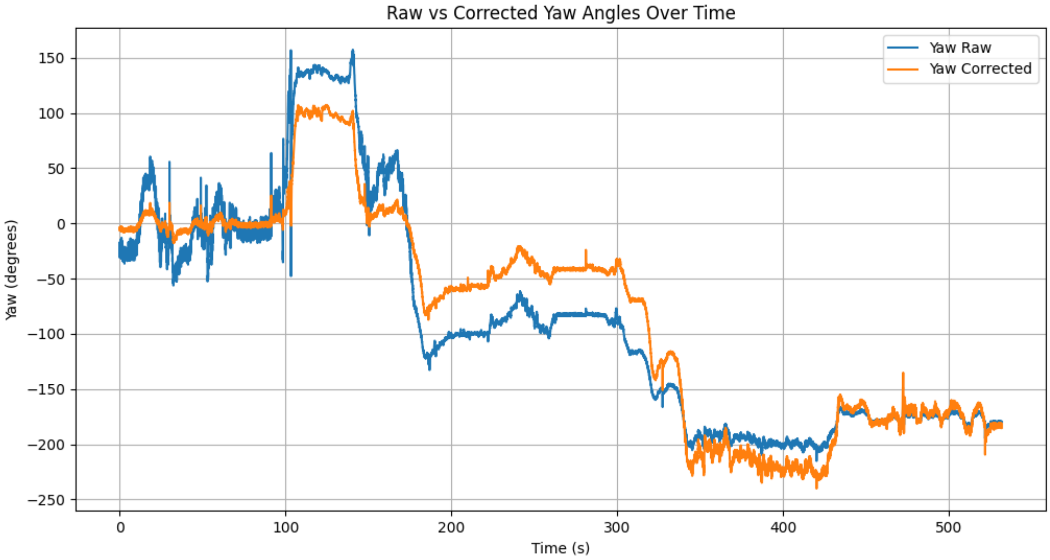


Figure 4: Original vs Calibrated Magnetometer Yaw

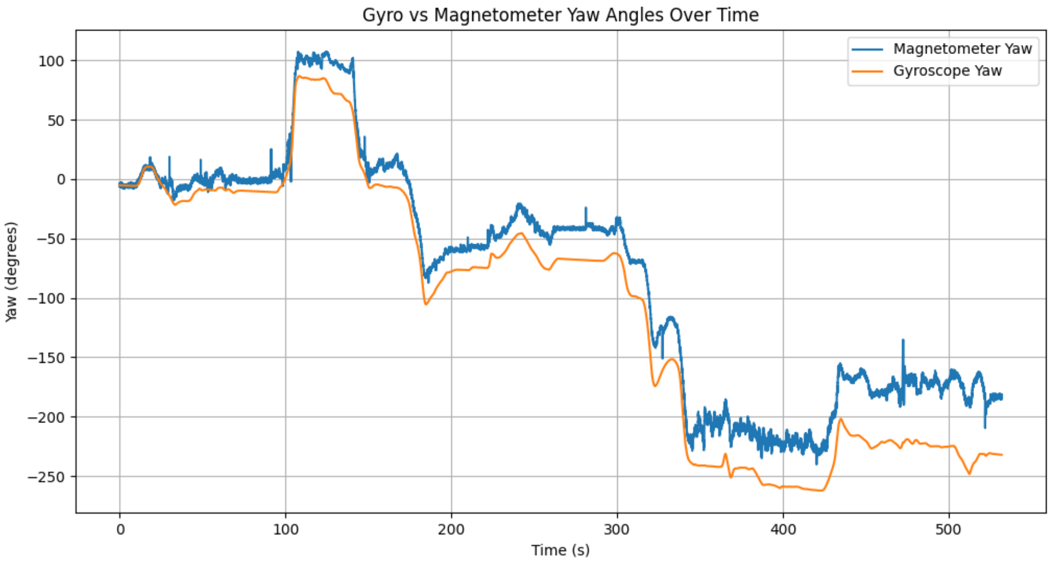


Figure 5: Gyroscope vs Magnetometer Yaw estimates

The magnetometer data is noisy but it does not accumulate measurement bias. Gyroscope on the other hand, is not very noisy but accumulates bias over time due to integration[1]. In other words, the noise characteristics observed in the magnetometer yaw data are predominantly high-frequency, while the noise in the gyroscope yaw data is primarily low-frequency. So in order to get the best yaw estimate, the high frequencies were filtered out from the magnetometer yaw using a low-pass filter with a cutoff frequency of 0.075 (chosen from plotting the FFT). Since all high-frequency data, whether signal or noise, has been filtered out, the remaining high-frequency components can be estimated using the gyroscope data, as it exhibits minimal high-frequency noise. This was done by filtering the gyroscope data using a high-pass filter with the same cutoff frequency and adding the result to the filtered magnetometer yaw. This approach is different from the typical complementery filtering approach of assigning weights to the filtered yaw estimates and adding them. Typically, the weighted method is effective when combining gyroscope data free of bias and magnetometer data devoid of noise. However, since we are combining low frequency and high frequency components without any overlap they can directly be added. The high frequency data obtained from a high pass filter on the gyroscope yaw and the low frequency data obtained from a low pass filter on the magnetometer yaw along with the combined complementary filter combining the two as explained above are shown in Figure

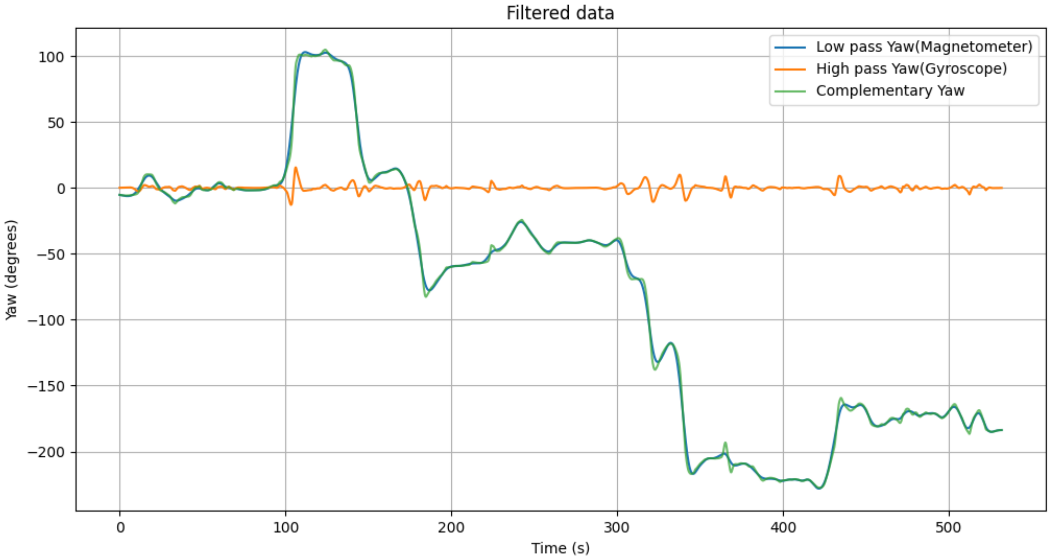


Figure 6: LPF, HPF, and CF yaw plots

3 Which estimate or estimates for yaw would you trust for navigation? Why?

The complementary filter estimate is the most reliable estimate for the yaw since it combines the strengths of the gyroscope and the magnetometer. A useful comparison for the complementary filter yaw estimate is the yaw directly output by the IMU. Both estimates are plotted together for comparison in Figure 7. Though the starting points are not the same for both the estimates due different reference frames, a simple way of judging them is by looking at the initial and final yaw estimates. Regardless of the reference frame, since the car eventually returned to the same road it started from, the difference between the initial and final yaws should be close to 180 degrees. The difference for the IMU estimate is 130°whereas for the complementary filter yaw, the difference comes out to be 178.5°. From this, it can be concluded that the complementary filter yaw is reliable. The unreliability of the IMU yaw is due to the lack of magnetometer calibration.

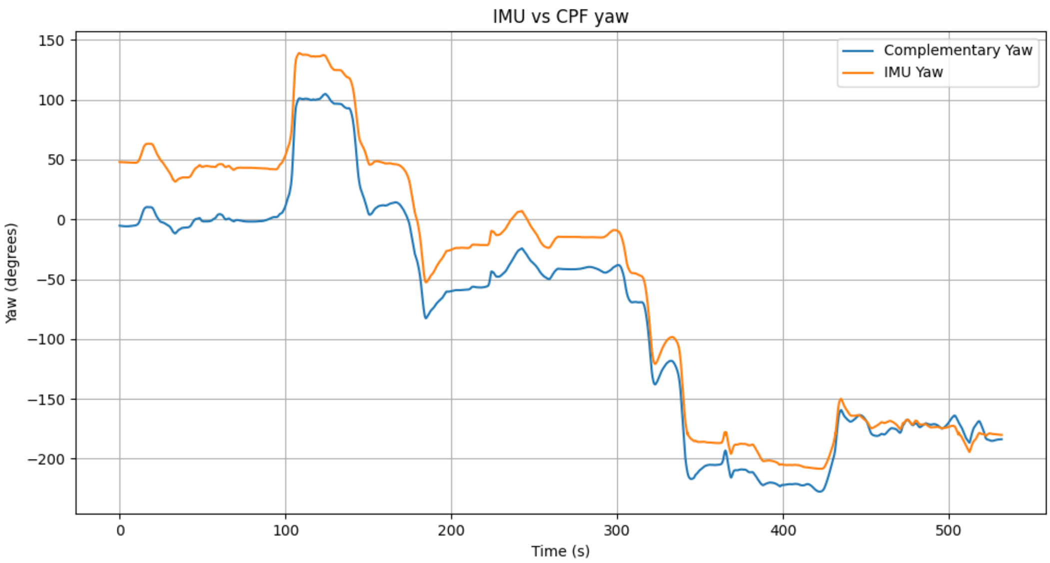


Figure 7: Complementary Filter Yaw vs IMU Yaw

4 What adjustments did you make to the forward velocity estimate, and why?

The forward velocity can be estimated by integrating the X acceleration from the accelerometer. But the acceleration needs to be corrected before integrating because the errors/biases in the acceleration will result in an extremely biased velocity estimate.

First, a low pass filter was applied on the acceleration to reduce noise. Then, measurements where the car was stationary were found using the rolling standard deviation with windows of 1 second to find indices with low variability (<0.005). This is because when the acceleration is almost constant for a period of time, the car is almost certainly stationary. These stationary periods are then used to find the bias for the stationary periods. These biases are then linearly interpolated across all the data to estimate the bias for the non stationary data. Subtracting the filtered data with the interpolated biases gives us corrected data. Additionally, since the car was only moving forward, the cumulative velocity during integration was constrained to never fall below zero, and it was set to zero at the stationary indices to ensure accuracy.

The problem with this method arose due to the fact that for the specific dataset collected, there were no stationary points for the last 40% of the data. Since the bias was being estimated using the stationary points, this gave a pretty inaccurate bias for this part of the data. A few methods were attempted to fix this issue such as using a spline interpolation, making the bias zero after 30 sec when there is no stationary point and using jerk to estimate how long to apply a constant bias. But none of these were able to find an accurate bias. As a last resort, two arbitrary stationary points were manually added. This approach is specific to this dataset, as no other method was identified due to the lack of stationary points in that segment of the data. Therefore, please note that the subsequent data has been slightly adjusted manually. Attempts were also made to generalize addition of stationary points but since there was no method found other than trial and error to find the right bias at the arbitrary stationary points, generalization was not possible. The filtered and corrected accelerations along with the estimated bias used for correction are shown in Figure 8. The integrated forward velocity is shown in the next section.

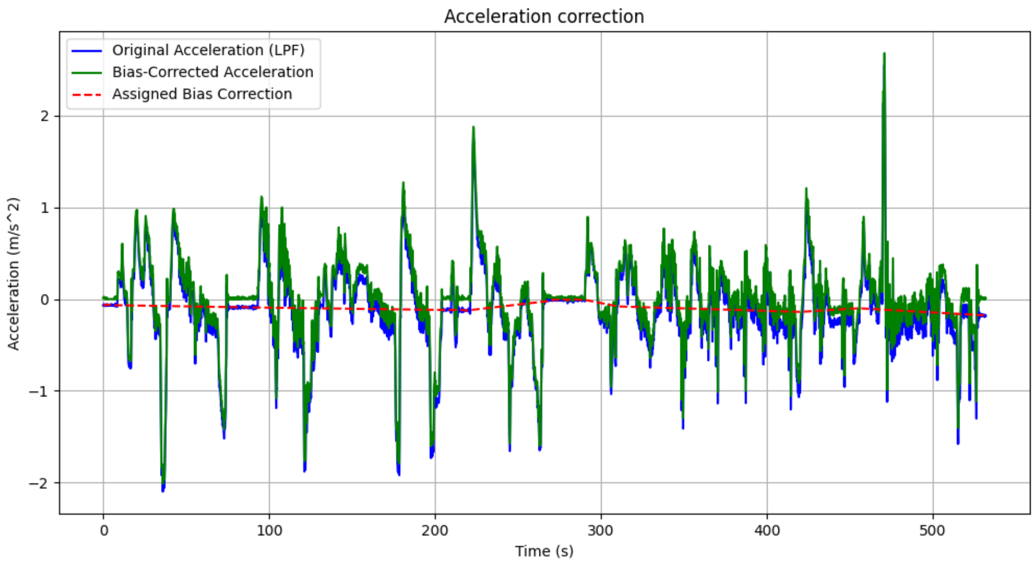
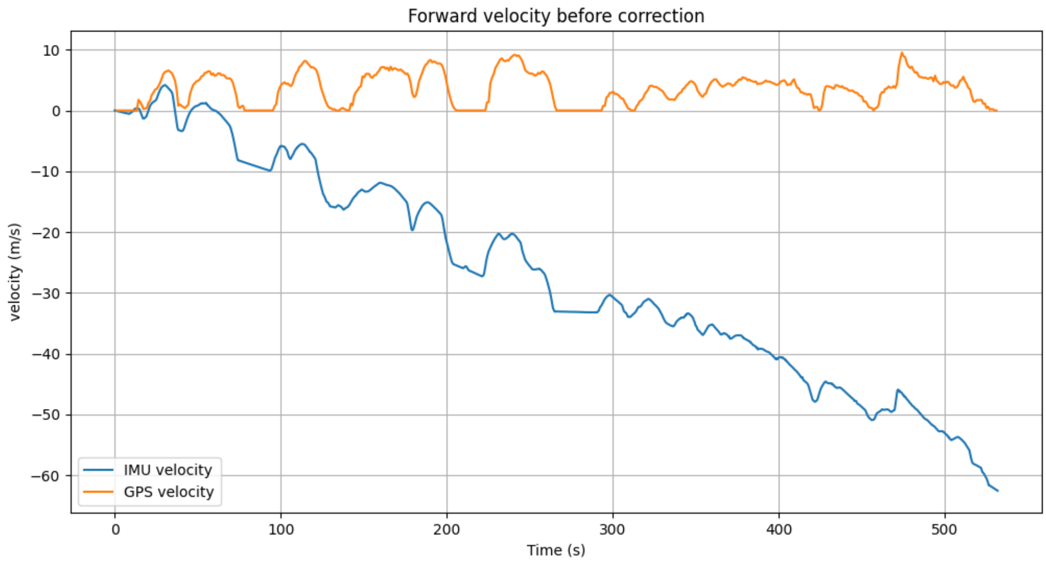


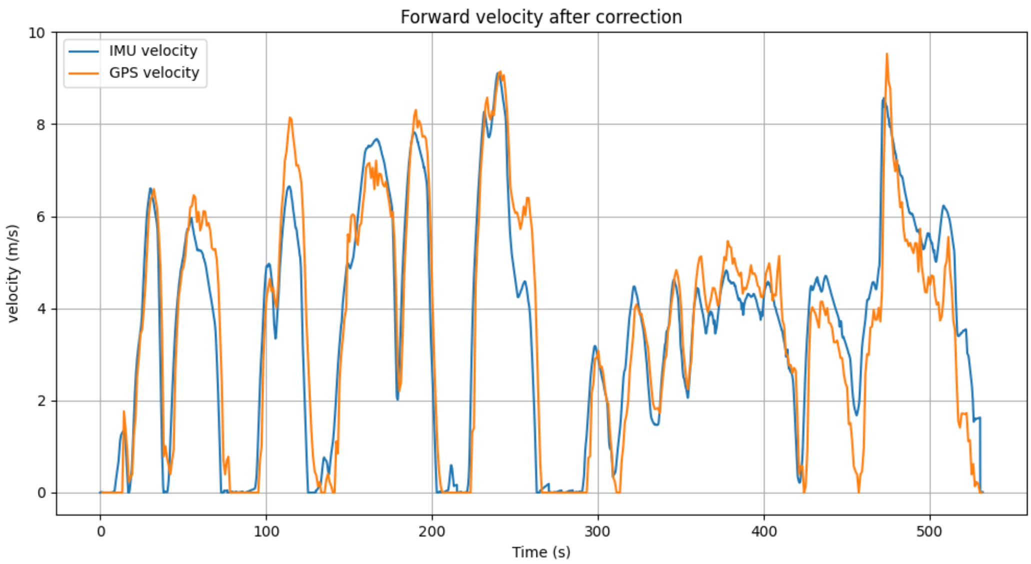
Figure 8: Bias correction of acceleration

5 What discrepancies are present in the velocity estimate between accelerometer and GPS. Why?

The integrated forward velocity before and after correction along with the GPS velocity are shown in Figure 9. The importance of the acceleration correction can be seen by comparing Figures 9a and 9b. Though the GPS and IMU estimates follow similar trends, the specific velocity values are rarely matching up. Even with the correction and manual adjustment, there are discrepancies because there is no way to precisely estimate the bias as discussed in the previous section. Due to the assumption that the forward acceleration is equal to the acceleration in the IMU's X direction, any error in the placement of the IMU also causes errors. It is also assumed that the car remains on a flat surface, without going up or down a slope. While this assumption is mostly accurate, there are a few slight elevations and descents that may introduce errors. A moving car is also prone to a lot of vibrations which can cause acceleration errors. Since errors compound over time due to integration, an error in the acceleration as little as 0.01 could cause large discrepancies. Additionally, GPS also has errors due to multipath though these are not nearly as significant as the errors caused due to the IMU.



(a) Before Correction



(b) After Correction

Figure 9: GPS vs IMU velocity estimates

6 Compute $\omega\dot{x}$ and compare it to $\ddot{y}_{\text{obs}}$. How well do they agree? If there is a difference, what is it due to?

Since the car is only moving forwards, all the the acceleration in the Y direction should be due to the centripetal force. Since centripetal acceleration $= \omega^2 r$ and $\omega = \dot{x}/r$, the centripetal acceleration becomes $\omega\dot{x}$. $\omega\dot{x}$ using the estimated forward velocity from the IMU and observed Y acceleration (Low-pass filtered for visibility) are compared in Figure 10. From the figure, it can be seen that the general trend seems to be similar and the two values match closely at times but are significantly off at points. The most significant reason for this are the errors in all three values used - angular velocity (Z), estimated IMU forward velocity and the observed Y acceleration. The errors are most clearly noticeable with the observed Y acceleration. For example, from Figure 9 it can be seen that the car was stationary for a short time after 200 seconds and a similar amount of time before 300 seconds. Though the acceleration needs to be zero here and the estimated centripetal acceleration agrees with that, there is a significant bias that can be seen in those periods for the observed Y acceleration. All of the causes of errors discussed in Section 5 can be applied here to explain the difference but out of them the most significant cause of error is the IMU placement. If the forward velocity is not exactly along X, its component along Y will be added to the observed Y acceleration. The difference could also be caused due to the effect of x_c which is discussed in Section 8.

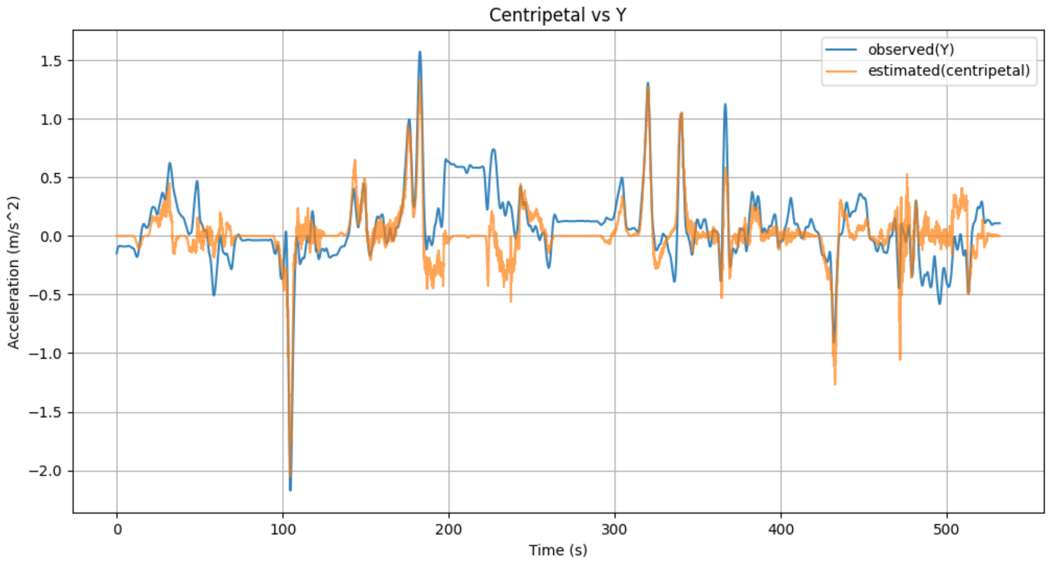


Figure 10: Observed Y acceleration vs estimated centripetal acceleration

7 Estimate the trajectory of the vehicle (xe,xn) from inertial data and compare with GPS by plotting them together

The IMU trajectory can be generated using the estimated forward velocity and heading from the complementary filter. Figure 11 shows the GPS trajectory and the IMU trajectory. Both plots start at the origin and have the same heading. The yaw from the complementary filter were shifted to be relative to easting instead of the magnetic axis. Though the heading is almost accurate and the trajectories line up for almost half the data (this is due to the manual adjustment), the final points are 134.6m apart. This shows how unreliable IMU data by itself without position fix is for navigation. It can be used to determine heading but another sensor should be used along with an IMU for positioning

8 Estimate x_c and explain your calculations

The given equations for the measured acceleration with the actual Y velocity and acceleration assumed to be zero (no skidding) are

$$\ddot{x}_{\text{obs}} = \ddot{x} - \omega^2 x_c$$

$$\ddot{y}_{\text{obs}} = \omega \dot{x} + \dot{\omega} x_c$$

The first equation gives $\ddot{x}$, which can be integrated to get $\dot{x}$. This can be substituted in the second equation and solved for x_c to get

$$x_c = \frac{\ddot{y}_{\text{obs}} - (\omega \dot{x}_{\text{obs}})}{\dot{\omega} + \omega \int \omega^2 \, \mathrm{d}t}$$

Using this equation, the value of x_c can be computed for each sample. The estimated forward velocity and low-pass filtered Y acceleration were taken for the calculations. Looking at all of the computed values, it was observed that almost all the values were very close to each other except for a few extreme outliers (due to the denominator being near zero sometimes). Therefore, the value of x_c can be estimated to be the median of these values which is -2.18 cm. Since this is small and the effect due to this is negligible, it was not used to correct the Y acceleration.

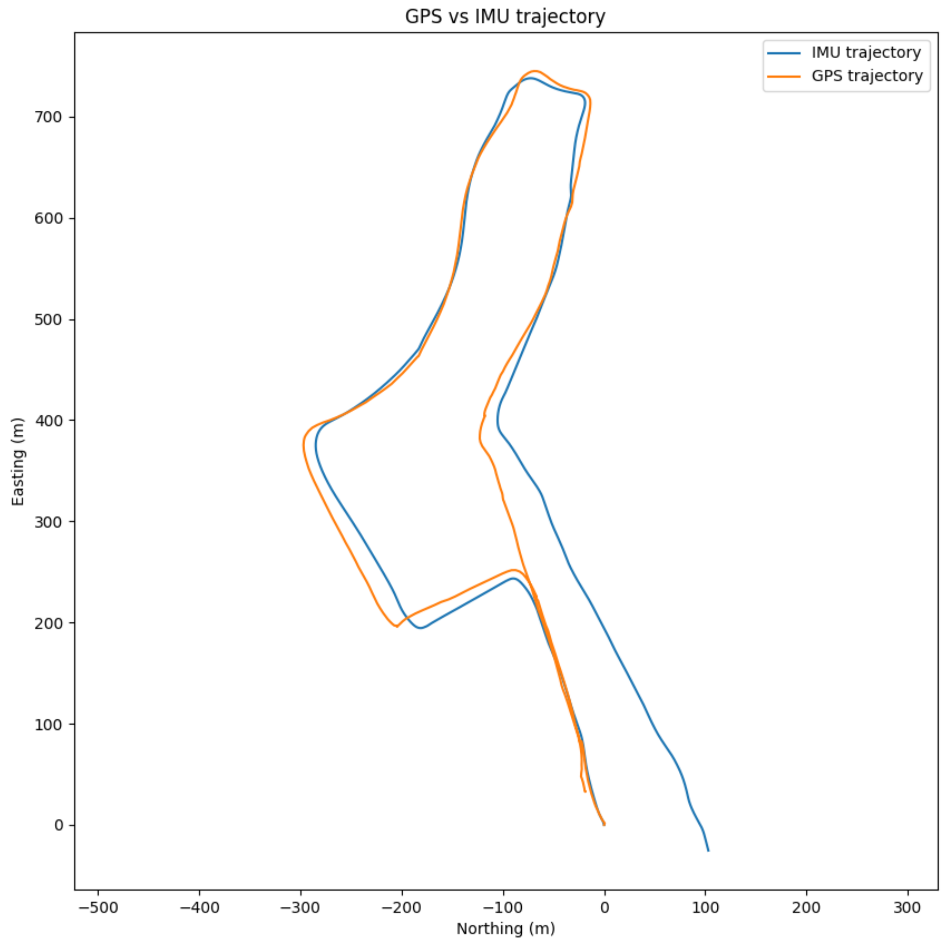


Figure 11: GPS trajectory vs Estimated IMU trajectory

9 Given the specifications of the VectorNav, how long would you expect that it is able to navigate without a position fix? For what period of time did your GPS and IMU estimates of position match closely? (within 2m) Did the stated performance for dead reckoning match actual measurements? Why or why not?

A navigation grade IMU with an accelerometer in-run bias stability of 0.01mg accumulates an error of 100m over 10 min in a purely static condition[4]. The VectorNav has an in-run bias stability of 0.04mg along with a magnetometer accuracy of 2°[6]. Since the IMU was run in a dynamic system and was running for almost 9 minutes, the error would have been a lot more if it was not corrected. Considering the rate of error accumulation [4] and that it was a dynamic system, an accurate position should not be expected for more than 60 seconds. In the collected data, the error between GPS and IMU positions exceeded 2 meters in just over 60 seconds, which is expected. At times, manual adjustments to the velocity corrections reduced this error, allowing the positions to align temporarily.

References

[1] *Complementary Filters*. URL: https://vanhunteradams.com/Pico/ReactionWheel/Complementary_Filters.html.

[2] *Direct linear least squares fitting of an ellipse*. URL: https://scipython.com/blog/direct-linear-least-squares-fitting-of-an-ellipse/.

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