CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

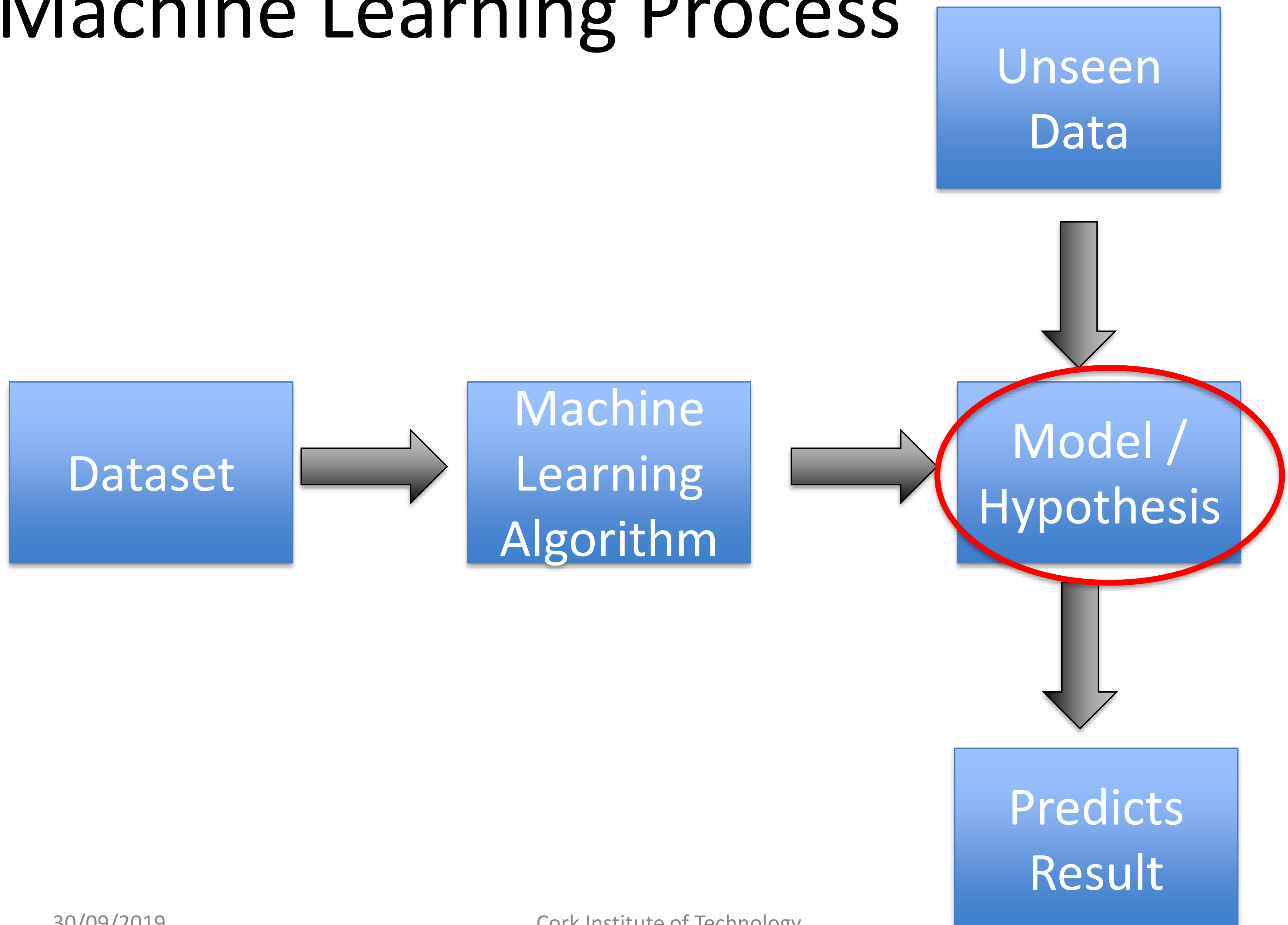
# Machine Learning

**Machine Learning**

Lecture: Bayesian Classification

Ted Scully

# Machine Learning Process



# Naïve Bayes Classifier Example (Weather Dataset)

In order to see the probability estimates in action we will look at a simple dataset called the weather dataset. We will look at the process by which it creates a model and then classifies unseen instances of data such as the following.

Outlook =sunny, Temp = cold, Humidity = high, Windy = true: **Play = ?**

**Anyone for Tennis?**

| ID | Outlook | Temp | Humidity | Windy | Play? |
|---|---|---|---|---|---|
| A | sunny | hot | high | false | no |
| B | sunny | hot | high | true | no |
| C | overcast | hot | high | false | yes |
| D | rainy | mild | high | false | yes |
| E | rainy | cool | normal | false | yes |
| F | rainy | cool | normal | true | no |
| G | overcast | cool | normal | true | yes |
| H | sunny | mild | high | false | no |
| I | sunny | cool | normal | false | yes |
| J | rainy | mild | normal | false | yes |
| K | sunny | mild | normal | true | yes |
| L | overcast | mild | high | true | yes |
| M | overcast | hot | normal | false | yes |
| N | rainy | mild | high | true | no |

Outlook =<u>sunny</u>, Temp = <u>cold</u>, Humidity = <u>high</u>, Windy = <u>true</u>: **Play = ?**

$$c_{NB} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{x \in X} P(x \mid c)$$

Outlook =<u>sunny</u>, Temp = <u>cold</u>, Humidity = <u>high</u>, Windy = <u>true</u>: **Play = ?**

**P(Play =** y **| Outlook** = sunny, **Temp** = cold, **Humidity** = high, **Windy** = true**)** =

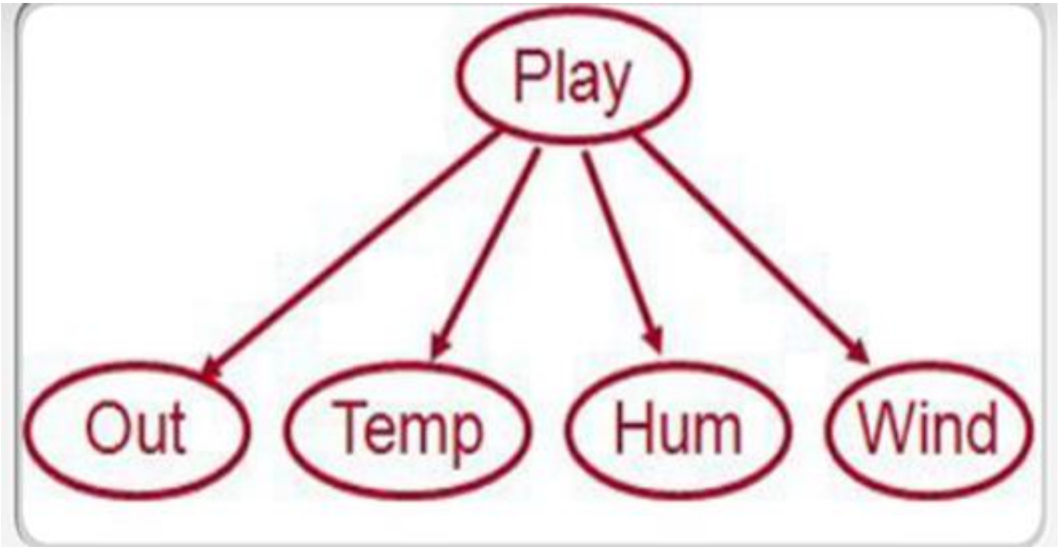
P(Play = y) * P(Outlook=s | Play=y)*P(Temp=c | Play=y)*P(Humidity = h | Play=y)*P(Windy=t | Play=y)

**P(Play** = n **| Outlook** = sunny, **Temp** = cold, **Humidity** = high, **Windy** = true**)** =

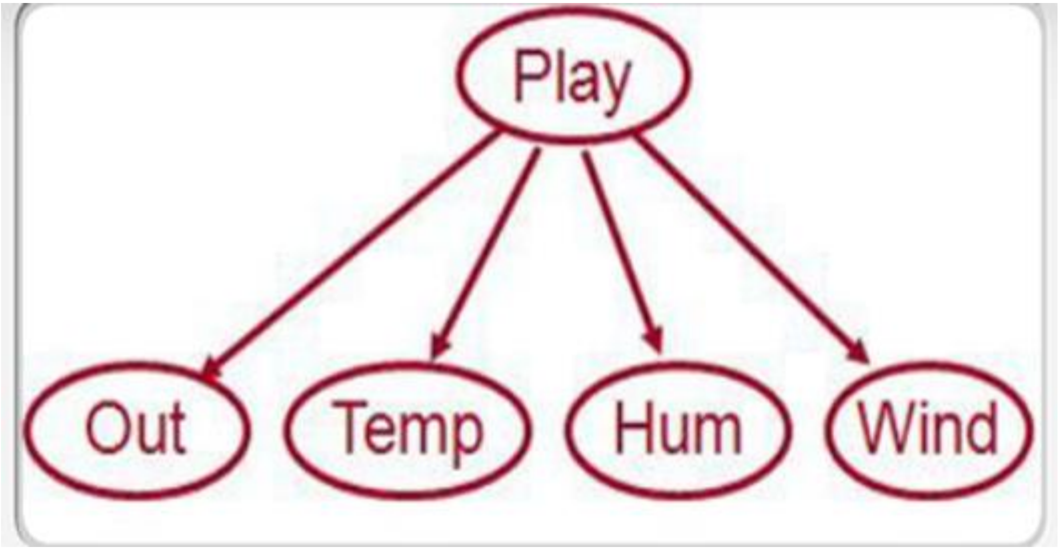
P(Play = n) * P(Outlook=s | Play=n)*P(Temp=c | Play=n)*P(Humidity = h | Play=n)*P(Windy=t | Play=n)

$$c_{NB} = \operatorname*{argmax}_{c \in C} P(c) \prod_{x \in X} P(x \mid c)$$

# Conditional Probabilities for Play = Y



# Conditional Probabilities for Play = Y




P(Out = Sunny | Play = Y)

P(Out = Overcast | Play = Y)

P(Out = Rainy | Play = Y)

P(Temp = Hot | Play = Y)

P(Temp = Mild | Play = Y)

P(Temp = Cool | Play = Y)

P(Hum= High | Play = Y)

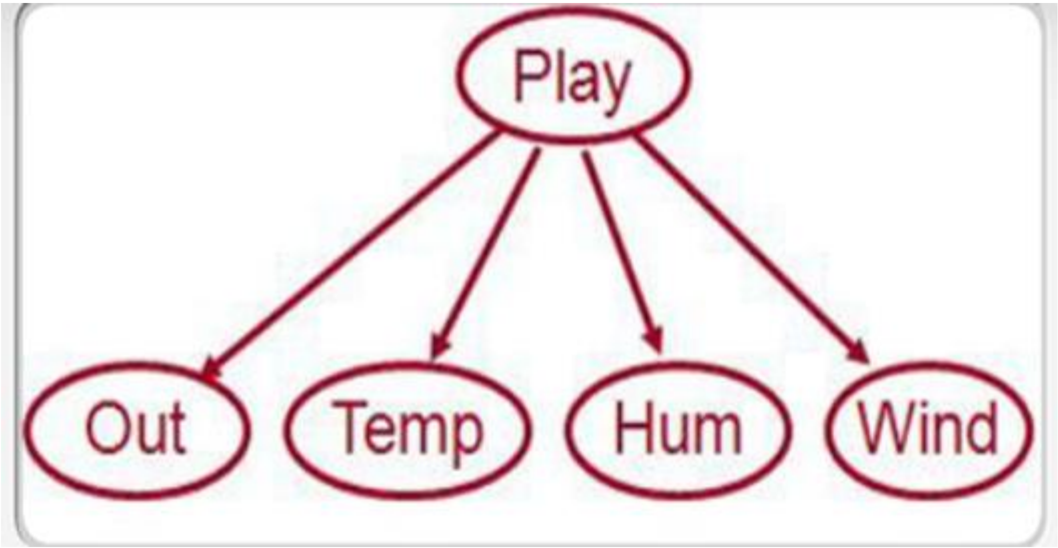
P(Hum= Normal | Play = Y)

P(Win= False | Play = Y)

P(Win= True | Play = Y)

# Conditional Probabilities for Play = N

Conditional Probabilities also need to be worked out for Play = N




P(Out = Sunny | Play = N)

P(Out = Overcast | Play = N)

P(Out = Rainy | Play = N)

P(Temp = Hot | Play = N)

P(Temp = Mild | Play = N)

P(Temp = Cool | Play = N)

P(Hum= High | Play = N)

P(Hum= Normal | Play = N)

P(Win= False | Play = N)

P(Win= True | Play = N)

| ID | Outlook | Temp | Humidity | Windy | Play? |
|---|---|---|---|---|---|
| A | sunny | hot | high | false | no |
| B | sunny | hot | high | true | no |
| C | overcast | hot | high | false | yes |
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| E | rainy | cool | normal | false | yes |
| F | rainy | cool | normal | true | no |
| G | overcast | cool | normal | true | yes |
| H | sunny | mild | high | false | no |
| I | sunny | cool | normal | false | yes |
| J | rainy | mild | normal | false | yes |
| K | sunny | mild | normal | true | yes |
| L | overcast | mild | high | true | yes |
| M | overcast | hot | normal | false | yes |
| N | rainy | mild | high | true | no |

- We will first look at calculating the probabilities needed for the class 'play' and the feature '**windy**'. Lets work out the **P(Wind = t | Play = y)**

- $P(X = x_1 | C = c_1) = \frac{N_{x1c1}}{N_{c1}}$
  - $N_{x1c1}$ = counts of cases where $X=x_1$ and $C=c_1$
  - $N_{c1}$ = count of cases where $C=c_1$

| ID | Outlook | Temp | Humidity | Windy | Play? |
|---|---|---|---|---|---|
| A | sunny | hot | high | false | no |
| B | sunny | hot | high | true | no |
| C | overcast | hot | high | false | yes |
| D | rainy | mild | high | false | yes |
| E | rainy | cool | normal | false | yes |
| F | rainy | cool | normal | true | no |
| G | overcast | cool | normal | true | yes |
| H | sunny | mild | high | false | no |
| I | sunny | cool | normal | false | yes |
| J | rainy | mild | normal | false | yes |
| K | sunny | mild | normal | true | yes |
| L | overcast | mild | high | true | yes |
| M | overcast | hot | normal | false | yes |
| N | rainy | mild | high | true | no |

- We will first look at calculating the probabilities needed for the class ‘play’ and the attribute ‘**windy**’. Lets work out the P(Wind = t | Play = y)

As we can see from the image, there are 9 cases where **`Play=y`**. In 3 of these, **`Wind=t`**, and in the other 6, **`Wind=f`**. Therefore, the probability of **`Wind=t`** given that **`Play=y`** is 3/9, according to these observations.

```
P(Wind=t | Play=y) = 3/9
P(Wind=f | Play=y) = 6/9
```

| ID | Outlook | Temp | Humidity | Windy | Play? |
|---|---|---|---|---|---|
| A | sunny | hot | high | false | no |
| B | sunny | hot | high | true | no |
| C | overcast | hot | high | false | yes |
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| K | sunny | mild | normal | true | yes |
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| M | overcast | hot | normal | false | yes |
| N | rainy | mild | high | true | no |

Next work out **P(Wind = t | Play = n)** and **P(Wind = f | Play = n)**

As we can see from the image, there are 5 cases where `Play=n`. In 3 of these, `Wind=t`, and in the other 2, `Wind=f`. Therefore:

```
P(Wind=t | Play=n) = 3/5
P(Wind=f | Play=n) = 2/5
```

We now have all four probabilities we need for the arc between play and windy. Next we need to apply the same method to calculate the probabilities for each of the other arcs.

Our Naïve Bayes algorithm takes as input the data set and produces the following **model.**

```
P(Outlook=s | Play=y) = 2/9  P(Outlook=s | Play=n) = 3/5
P(Outlook=o | Play=y) = 4/9  P(Outlook=o | Play=n) = 0/5
P(Outlook=r  | Play=y) = 3/9 P(Outlook=r  | Play=n) = 2/5
```

```
P(Wind=t | Play=y) = 3/9   P(Wind=t | Play=n) = 3/5
P(Wind=f | Play=y) = 6/9   P(Wind=f | Play=n) = 2/5
```

```
P(Temp=h | Play=y) = 2/9  P(Temp=h  | Play=n) = 2/5
P(Temp=m | Play=y) = 4/9  P(Temp=m | Play=n) = 2/5
P(Temp=c  | Play=y) = 3/9 P(Temp=c  | Play=n) = 1/5
```

**P(Humidity =high| Play = yes) = 3/9**
**P(Humidity =normal| Play = yes) = 6/9**

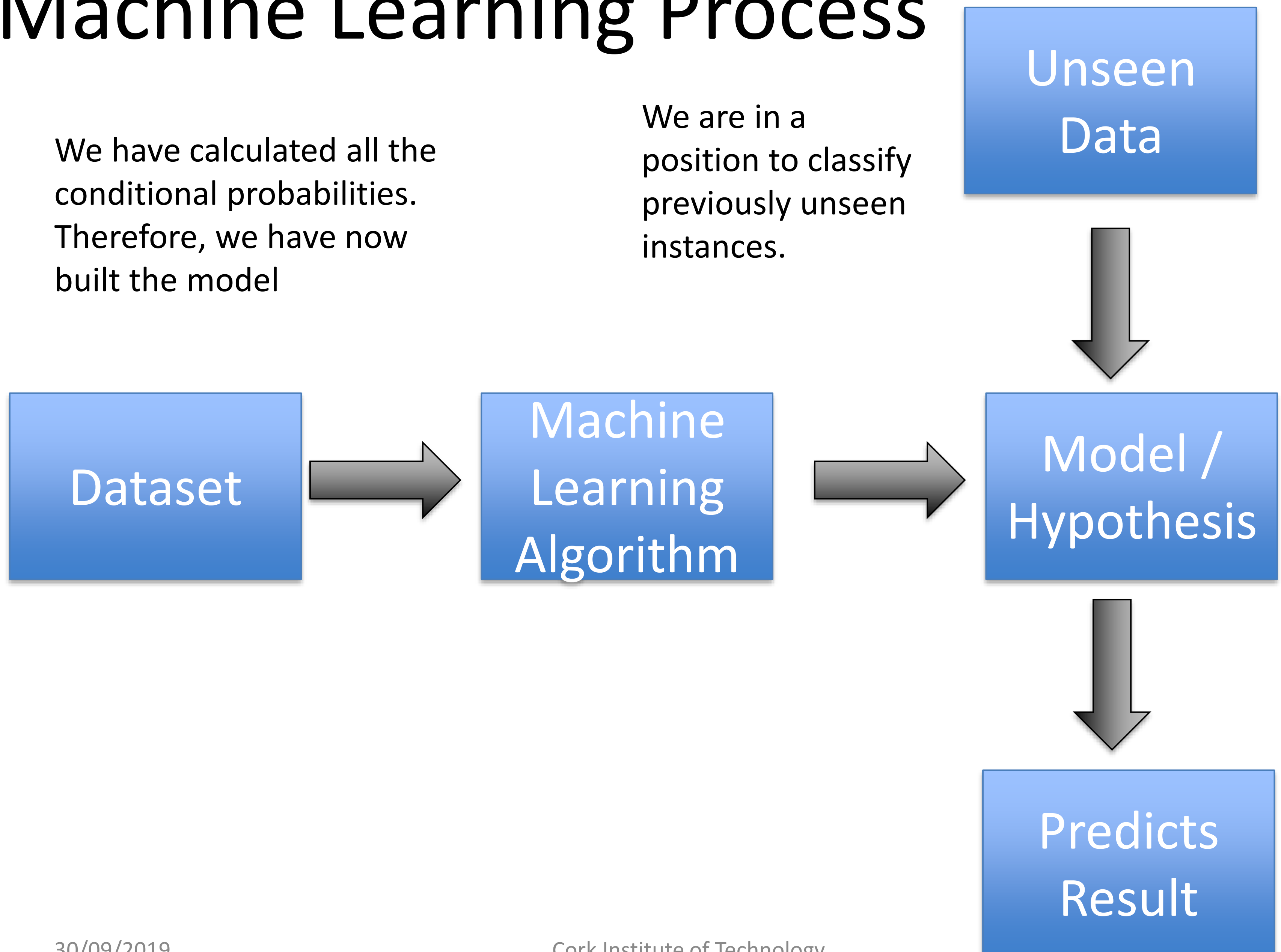
**P(Humidity =high| Play = no) = 4/5**
**P(Humidity =normal| Play = no) = 1/5**

```
Play=y) = 9/14
Play=n) = 5/14
```

# Machine Learning Process

We have calculated all the conditional probabilities. Therefore, we have now built the model

We are in a position to classify previously unseen instances.



$$P(c_j)\prod_{x\in X} P(x \mid c)$$

# Classify a New Instance

- *Will I play tennis under the following conditions:*
  - ***Outlooks = sunny, Temp = cool, Humidity = high, Windy = true, Play = ?***

Play is y or n. Evaluate probability of each given data.

**P(Play = y | Outlook = s, Temp =c, Humidity = h, Wind = t)** =
P(Play = y) * P(Outlook = s | Play = y) * P(Temp=c | Play = y) * P(Humidity= h | Play = y) * P(Wind = t | Play = y)

**P(Play = n | Outlook = s, Temp =c, Humidity = h, Wind = t)** =
P(Play = n) * P(Outlook = s | Play = n) * P(Temp=c | Play = n) * P(Humidity= h | Play = n) * P(Wind = t | Play = n)

$$P(c_j)\prod_{x \in X} P(x \mid c)$$

# Classify a New Instance

- *Will I play tennis under the following conditions:*
  - ***Outlooks = sunny, Temp = cool, Humidity = high, Windy = true, Play = ?***

Play is y or n. Evaluate probability of each given data.

**P(Play = y | Outlook = s, Temp =c, Humidity = h, Wind = t)** =
P(Play = y) * P(Outlook = s | Play = y) * P(Temp=c | Play = y) * P(Humidity= h | Play = y) * P(Wind = t | Play = y)
= 9/14 * 2/9 * 3/9 * 3/9 * 3/9 = **0.005291**

**P(Play = n | Outlook = s, Temp =c, Humidity = h, Wind = t)** =
P(Play = n) * P(Outlook = s | Play = n) * P(Temp=c | Play = n) * P(Humidity= h | Play = n) * P(Wind = t | Play = n)
= 5/14 * 3/5 * 1/5 * 4/5 *3/5 = **0.020571**

$$P(c_j)\prod_{x\in X} P(x \mid c)$$

# Normalise the Results

**P(Play = y | data) = 0.005291**
**P(Play = n | data) = 0.020571**

**(Why do above probabilities not add to 1?)**

**P(Play = y | data ) = (0.005291*100) / (0.005291+ 0.020571) = 20.5%**
**P(Play = n | data ) = (0.020571 *100) / (0.005291+ 0.020571) = 79.5%**

Conclusion: more likely NOT to play tennis today.

From the calculations, it is seen that the probability of `Play=yes` is 20.5%, whereas the probability of `Play=no` is 79.5%. Selecting the outcome with the higher probability, the classification is that Play=no.

$$P(c_j)\prod_{x\in X} P(x \mid c)$$

Consider the following data instance:

Outlook =**overcast**, Temp = mild, Humidity = normal, Windy = false: Play = ?

```
P(Outlook=s | Play=y) = 2/9  P(Outlook=s | Play=n) = 3/5
P(Outlook=o | Play=y) = 4/9  P(Outlook=o | Play=n) = 0/5
P(Outlook=r  | Play=y) = 3/9 P(Outlook=r  | Play=n) = 2/5
```

```
P(Wind=t | Play=y) = 3/9   P(Wind=t | Play=n) = 3/5
P(Wind=f | Play=y) = 6/9   P(Wind=f | Play=n) = 2/5
```

```
P(Temp=h | Play=y) = 2/9 P(Temp=h  | Play=n) = 2/5
P(Temp=m | Play=y) = 4/9 P(Temp=m | Play=n) = 2/5
P(Temp=c  | Play=y) = 3/9 P(Temp=c  | Play=n) = 1/5
```

**P(Humidity =high| Play = yes) = 3/9**
**P(Humidity =normal| Play = yes) = 6/9**

**P(Humidity =high| Play = no) = 4/5**
**P(Humidity =normal| Play = no) = 1/5**

```
Play=y) = 9/14
Play=n) = 5/14
```

$$P(c_j)\prod_{x\in X} P(x \mid c)$$

Consider the following data instance:

Outlook =overcast, Temp = mild, Humidity = normal, Windy = false: Play = ?

```
P(Outlook=s | Play=y) = 2/9  P(Outlook=s | Play=n) = 3/5
P(Outlook=o | Play=y) = 4/9  P(Outlook=o | Play=n) = 0/5
P(Outlook=r  | Play=y) = 3/9 P(Outlook=r  | Play=n) = 2/5
```

# Problem with Using Frequencies for Probability Calculations

- So far we estimated probabilities using the following:
- $P(X = x_1 | C = c_1) = \frac{N_{x1c1}}{N_{c1}}$
  - $N_{x1c1}$ = counts of cases where $X=x_1$ and $C=c_1$
  - $N_{c1}$ = count of cases where $C=c_1$

- To avoid the problem of zero probabilities we can applying basic smoothing techniques to the above formula.

# Avoiding Zeros

- To avoid the problem outlined on the previous slide we typically use +1 or laplace smoothing.

- Often some basic softening of the equation is performed. For example (**+1 smoothing**), $(N_{x1c1} + \mathbf{1}) / (N_{c1} + \mathbf{2})$

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- ***Laplace* Smoothing** (m-estimate) : $(N_{x1c1} + \mathbf{1}) / (N_{c1} + \mathbf{|X|})$

  - $N_{x1c1}$ = counts of cases where X=x1 and C=c1
  - $N_{c1}$ = count of cases where C=c1
  - |X| = count of cases of X (number of features(attributes))

# Avoiding Zeros

- Remember we worked out P(Outlook = o | Play = n) = 0/5

- +1 smoothing $(N_{x1c1} + 1) / (N_{c1} + 2)$
- If we use +1 smoothing **P(Outlook = o | Play = n) would be (0+1)/(5+2) = 1/7**

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- *Laplace* Smoothing (m-estimate) : $(N_{x1c1} + 1) / (N_{c1} + |X|)$
- **P(Outlook = o | Play = n) would be (0+1)/(5+4) = 1/9**

    - Remember |X| is the number of features

# Problems with Probabilities for Naïve Bayes

$$P(c_j)\prod_{x\in X} P(x \mid c)$$

Can you see any computational problem that may occur from this formula? Hint: What might happen if you have a large amount of features?

The computation issue is that of underflow: doing too many multiplications of small numbers.

When we go to calculate the product p(w0|ci)p(w1|ci)p(w2|ci)...p(wN|ci) and many of these numbers are very small, we'll get underflow (multiply many small numbers in a programming language and eventually it rounds off to 0.)

# Using Log

- The most common solution to the problem on the previous slide is to calculate the logarithm of this product.

- Doing this allows us to avoid the underflow or round-off error problem. Why? Because we end up adding the individual probabilities rather than multiplying them

- In other word we now get the log of the Bayes equation

$$\mathbf{log}(P(c)\prod_{x\in X} P(x|c))$$

    - We now use

$$\mathbf{log}\,P(c) + \sum_{x\in X} \mathbf{log}\,P(x \mid c)$$

- Word of caution about Naïve Bayes probability estimates.