LaTeX Mini Project 3

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1. Problem 1

$$lnP(E) = ln\sum_H P(H,E)$$

$$= ln\sum_H Q(H|E).\frac{P(H,E)}{Q(H|E)}$$

$$>= \sum_H Q(H|E)ln\frac{P(H,E)}{Q(H|E)}$$

The difference between the inequality is given by KL divergence $D(Q||P)$. which is $= -\sum Q(H|E)ln\frac{P(H|E)}{Q(H|E)}$ as we have

$$lnP(E) = \sum_H Q(H|E)ln\frac{P(E,H)}{Q(H|E)} - \sum_H Q(H|E)ln\frac{P(H|E)}{Q(H|E)}$$

from the Bayes theorem.

We cannot consider $D(P||Q)$ as KL divergence is not symmetric, i.e. $D(P||Q)! = D(Q||P)$

2. Problem 2

3. Problem 3

For a fixed x_i, y_i are i.i.d random variables with y_i $N(w_1x_i + w_0, \sigma^2)$. So the probability distribution of y_1, y_2, ... is defined by:

$$f(y_1, ..., y_n|w_1, w_0) = \pi_{i=1}^n f(y_i|w_1, w_0)$$

$$= \pi_{i=1}^n \frac{1}{(2\sigma^2)^{\frac{n}{2}}} \exp^{\frac{-(y_i-w_1x_i-w_0)^2}{2\sigma^2}}$$

$$= \frac{1}{(2\sigma^2)^{\frac{n}{2}}} \exp^{\frac{1}{(2\sigma^2)^{\frac{n}{2}}}\sum_{i=1}^n (y_i-w_1x_i-w_0)^2}$$

To get the MLE estimates of w_1 and w_0 we will set $\frac{\partial f}{\partial w_1} = 0$ and $\frac{\partial f}{\partial w_0} = 0$ which leads to the equations:

$$\sum_{i=1}^n x_i(y_i - w_1x_i - w_0) = 0$$

$$\sum_{i=1}^{n}(y_i - w_1x_i - w_0) = 0$$

Solving the second equation for w_0 yields $w_0 = y - w_1x$ and replacing w_0 in the first equation yields:

$$\sum_{i=1}^{n}(x_i - \bar{x} + \bar{x})(y_i - w_1x_i - \bar{y} + w_1\bar{x}) = 0$$

which in turn leads to

$$w_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})}$$