CS 510: Homework 3

Due on September, 30, 2013

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Contents

Problem 1

Use the Matlab program in Sakai that implements the fixed point iteration method, $x_{k+1} = g(x_k)$, $k = 1, 2, \ldots$ with a stopping criterion, the $|x_k - x_{k-1}| < \epsilon$, and a max number of iterations k_{max}. Then, solve the equation $f(x) = x^2 - 3x + 2$ by using the following iteration functions below. (Select the initial points by using ezplot to first plot f(x) and identify the regions for the root. Do these methods converge, if YES or NOT provide an explanation. If they converge identify the interval of convergence, i.e. any initial starting point in that region will converge to a root. What are the rates of convergence for each method? If they converge?) (**50 points = 5 $\times$ 10 points each**)

Solution:

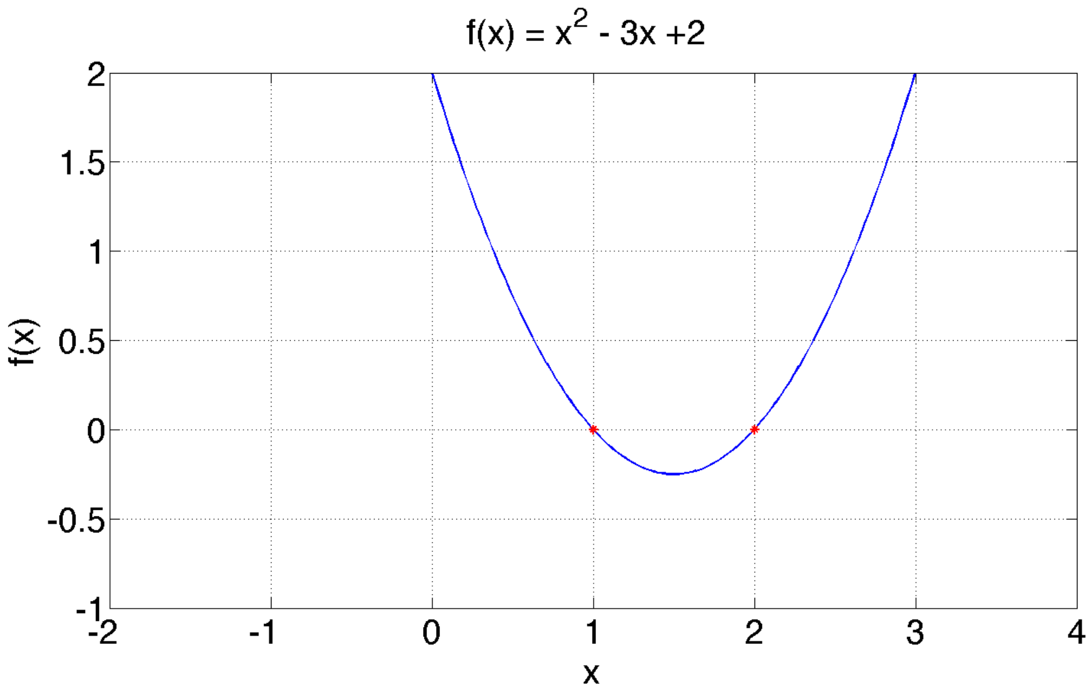

Figure 1: *ezplot* figure for $f(x) = x^2 - 3x + 2$. It is clear that roots are $\alpha \in \{1, 2\}$

Problem 1 (a)

Newton's method for $g(x) = x - \frac{f(x)}{f(x)}$

Solution: YES, convergence in $\mathbb{R} - \{1.5\}$, quadratic because $g''(\alpha) \neq 0$

Problem 1 (b)

$g(x) = \frac{x^2+2}{3}$

Solution: YES, convergence in $(-1.5, 1.5)$, linear because $g'(\alpha) \neq 0$

Problem 1 (c)

$g(x) = \sqrt{3x-2}$

Solution: YES, convergence in $(\frac{17}{12}, \infty)$, linear because $g'(\alpha) \neq 0$

Problem 1 continued on next page...

Problem 1 (d)

$g(x) = 3 - \frac{2}{x}$

Solution: YES, convergence in $\mathbb{R} - [-\sqrt{2}, \sqrt{2}]$, linear because $g'(\alpha) \neq 0$

Problem 1 (e)

$g(x) = \frac{(x^2-2)}{(2x-3)}$

Solution: YES, convergence in $\mathbb{R} - \{1.5\}$, quadratically because $g''(\alpha) \neq 0$

Problem 2

We would like to solve $e^{-x} = x$, determine the rate of convergence for Newton's method for points that are farther to the root and points near the root. Will newton's method converge for any starting point? Can you propose a method that is faster than Newton's method? (**25 points = 10 + 10 + 5 points each**)

Solution: We have, $f(x) = x - e^x$, $g(x) = x - \frac{x-e^x}{1+e^x}$ and $g'(x) = \frac{1-xe^x}{(e^x+1)^2}$. So since $|g(x)| < 1$ for all x, it converges for all values of x **(10 points)**and the convergence is quadratic because **(10 points)** $g''(\alpha) \neq 0$. A faster method for this could be a third order method, for example *Halleys method* **(5 points)**.

Problem 3

Show that $x_{n+1} = \frac{x_n(x_n^2+3a)}{3x_n^2+a}$ is a third order method for computing $\sqrt{a}$. Implement the above method by modifying Newton's program in Sakai. Compare newton's method bisection method and the above third order method for the computation of $\sqrt{a}$ within $\epsilon = 10^{-12}$. Write conclusions of your comparison. (**25 points = 10 + 10 + 5 points each**)

Solution: We have $g(x) = \frac{x(x^2-3a)}{3x^2+a}$ and $g'(x) = \frac{3(a-x^2)^2}{(a+3x^2)^2}$ and $g''(x) = \frac{48ax(x^2-a)^2}{(a+3x^2)^3}$ and $g'''(x) = \frac{48a(a^2-18ax^2-9x^4)^2}{(a+3x^2)^4}$. So we see $g(\sqrt{a}) = g'(\sqrt{a}) = g''(\sqrt{a}) = 0$ and $g'''(\sqrt{a}) \neq 0$ **(10 points)**. So it is a third order method. In comparison with Newton's method, which is second order method, the above method converges faster**(5 points)**. This should be shown empirically **(10 points)**.