CS 323 : HW Solutions 3

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Contents

Problem 1

-	x	p(x)	p(,)	p(, ,)	p(, , ,)	p(, ...,)
x_0	-2	-5	-	-	-	-
x_1	-1	1	6	-	-	-
x_2	0	1	0	-3	-	-
x_3	1	1	0	0	1	-

$$\begin{aligned} P(x) &= -5+6(x+2)-3(x+2)(x+1)+1(x+2)(x+1)x \\ P(1) &= x^3-x+1 \end{aligned} \tag{1}$$

Problem 2

```
function interp(a,b,n,cheb,findex)

% Plots interpolation polynomials of some function
%
%INPUT [a,b] give the interpolation/plotting interval
%INPUT n is the number of sample points. If n is a vector,
%the function plots a polynomial for every value in n
%INPUT cheb indicates if equidistant/Chebyshev intepolation
% nodes should be used (0/1 respectively)
%INPUT findex specifies which equation should be used for
% interpolation (equations defined in fct() below)
    close all
    syms xp real;
    yp = fct(findex);
    x = a:0.02:b; %mesh points for plotting
    y = subs(yp,'xp',x);
    figure('Position',[100,100,1000,400]);
    subplot(1,2,1) %first subplot in 1x2 plot figure
    hold on
    plot(x,y,'Color',[0.6 0.75 0.15]);
    title('Graph of f(x) and P_n(x)');
    legend('f(x)');
    subplot(1,2,2) %second subplot in 1x2 plot figure
    hold on
    title('Error [f(x)-P_n(x)]');
    c(1,:) = linspace(1,0.7,length(n)); %R color for plotting
    c(2,:) = linspace(0.9,0.4,length(n)); %G color for plotting
    c(3,:) = linspace(0.7,0,length(n)); %B color for plotting
    for i=1:length(n) %n is a number
        if (cheb == 0)
           xi = linspace(a,b,n(i)); %equidistant nodes
          yi = subs(yp,'xp',xi);
            C = polyfit(xi,yi,n(i)-1); %get coeffs of P_{n-1}
            Interp = polyval(C,x); %evaluate P_{n-1}(x)
        else
            xi = cheby(a,b,n(i)); %Chebyshev intepolation nodes
            yi = subs(yp,'xp',xi);
            C = polyfit(xi,yi,n(i)); %get coeffs of P_{n-1}
```

```
            Interp = polyval(C,x); %evaluate P_{n-1}(x)
        end
        subplot(1,2,1) %first subplot in 1x2 plot figure
        plot(xi,yi,'o','Color',c(:,i)); %plot node points on f
        plot(x,Interp,'--','Color',c(:,i));
        subplot(1,2,2) %second subplot in 1x2 plot figure
        plot(x,y-Interp,'Color',c(:,i)) %plot error
    end
end

function fct = fct(index)
% function to define equation for interpolation
    syms xp real;
    switch index
    case 1
        fct = cos(xp);
    case 2
        fct = 1./(1+10*xp.^2);
    end
end

function xi = cheby(a,b,n)
% returns Chebyshev intepolation nodes
    i = 0:n;
    theta = (2*i+1)*pi/(2*n+2);
    xi = (b-a)*cos(theta)/2 + (a+b)/2;
end
```

Problem 3

i.
Newton Interpolation:

-	x	p(x)	p(,)	p(, ,)
x_0	0	1	-	-
x_1	1	2	1	-
x_2	2	3	1	0

So, from the table we get:

$$p(x) = 1 + 1x \tag{2}$$

$$p(x) = x + 1 \tag{3}$$

Lagrange Interpolation:

$$L_j(x) \quad = \prod_{i \neq j} \tfrac{x - x_i}{x_j - x_i} \tag{4}$$

$$p(x) \quad = \quad \sum_j L_j(x) f_j = L_0(x) f_0 + L_1(x) f_1 + L_2(x) f_2$$

$$
\begin{aligned}
&= \dots \\
&= 1 + x \qquad (5)
\end{aligned}
$$

ii
Repeat the same procedure

Problem 4

$$
\begin{aligned}
q(x) &= ax^2 + bx + c \\
q(0) &= -1 \\
q(1) &= -1
\end{aligned} \qquad (6)
$$

$$
\begin{aligned}
q'(x) &= 2ax + b \\
q'(1) &= 4 \qquad (7)
\end{aligned}
$$

So

$$
q(x) = -1 - 4x + 4x^2 \qquad (8)
$$

Problem 5

Let $x = a + sh$ where $s \in (0, n)$. Then,

$$\Omega_n(x) = (x - x_0)(x - x_1)\dots(x - x_n) \tag{1}$$

$$|\Omega_n(x)| = \prod_{i=0}^{n}(x - x_j) \tag{2}$$

$$= h^{n+1}\prod_{j=0}^{n}|s - j| \tag{3}$$

$$= h^{n+1}\prod_{j=0}^{n}|s - j| \quad (\text{ Assume}, i \leq s \leq i+1) \tag{4}$$

$$= h^{n+1}\underbrace{\prod_{j=0}^{i-1}|s - j|}_{\leq i!}\underbrace{|s - i||s - i - 1|}_{\leq 1/4}\underbrace{\prod_{j=i+1}^{n}|s - j|}_{\leq (n-i)!} \tag{5}$$

$$\leq h^{n+1}\frac{i!(n-i)!}{4} < h^{n+1}n! \tag{6}$$

$$\leq h^{n+1}\frac{n!}{4} < h^{n+1}n! \tag{7}$$

$$\max_{a\leq x\leq b}|f(x) - P_n(x)| \leq \max_{a\leq x\leq b}\frac{|\Omega_n(x)|}{(n+1)!}\max_{a\leq x\leq b}|f^{(n+1)}(x)| \tag{8}$$

$$\leq \frac{h^{n+1}}{n+1}\max_{a\leq x\leq b}|f^{(n+1)}(x)| \tag{9}$$