CS 510: Homework 7

Due on November, 24, 2013

A. Gerasoulis 3:00 pm

Chetan Tonde

Contents

Problem 1 (i)

Solution: (**20 points**)

Listing 1: Sample program for column-wise LU factorization

```
function [L U] = kji(a,n)
    l=eye(n);
    tic
    for k=1:n-1
        l(i,k) = a(i,k)/a(k,k);
         for j = k:n
             for i = k+1:n
                a(i,j)=a(i,j)-l(i,k)*a(k,j);
             end
         end
    end
    toc
    U=a; L=l;
end
```

Problem 1 (ii)

Solution: (**20 points**)

Listing 2: Sample program to count time

```
clear all;
N = [500, 1000, 2000];
for i = 1:length(N)
     n = N(i);
    a=ones(n,n)+n*eye(n,n);
    tic;
    [L U] = kij(a, n);
     toc;
end


for i = 1:length(N)
     n = N(i);
    a=ones(n,n)+n*eye(n,n);
    tic;
     [L U] = kji(a, n);
    toc;
end
```

Problem 1 (iii)

Solution: (**10 points**) Matlab performs column-wise operations faster than row wise. See `http://www.mathworks.com/company/newsletters/articles/programming-patterns-maximizing-code-performanc html`. Hence, column-wise implementation is slightly faster.

Problem 2 (i)

Solution: (**25 points**)

Listing 3: Sample column-wise LU program with pivoting

```
function [L, U, P] = lup(A)
    a = A;
    n = size(A,1);
    l = zeros(n);
    perm = eye(n);
    idx = [];

    for k = 1:n-1
         loc = a(1:n, k);
         if ~isempty(idx)
            loc(idx) = 0;
         end
         [~, I] = max(abs(loc));
         idx = [idx; I];
         for j = k:n
             for i = 1:n
                 if j == k && isempty(find(idx == i, 1))
                    l(i,k) = a(i,k)/a(I,k);
                 end
                 if isempty(find(idx == i, 1))
                     a(i,j) = a(i,j)-l(i,k)*a(I,j);
                 end
             end
         end
    end

    for itr2 = 1:n
         if isempty(find(idx == itr2, 1))
         idx = [idx; itr2];
     end
    end

    L = l(idx, :)+eye(n);
    U = a(idx, :);
    P = perm(idx, :);
end
```

Problem 2 (ii)

Solution: (**25 points**)
Procedure:

- Factorize, $A = P'LU$
- Let $Ux = z$, solve, $Lz = Pb$ by forward substitution
- Then solve, $Ux = z$ by backward substitution

For example code `https://ece.uwaterloo.ca/~ece204/howtos/backward/` and `https://ece.uwaterloo.ca/~ece204/howtos/forward/`.

For large matrices the condition number, $cond(A) = ||A^{-1}||||A||$ becomes large. Hence, the solutions becomes progressively inaccurate.