QUIZ 3 Last Name-------------------------First Name--

1. We want to solve Ax=b. Solve the system by using Gaussian elimination with partial pivoting for the following linear systems:

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

10 i.   What is the one norm $\|A\|_1 =$

10 ii. What is the Maximum or infinity norm $\|A\|_\infty =$

30 iii. Will the iterative method $x^{k+1} = D^{-1}b - Mx^k, \quad M = D^{-1}(A - D)$ converge to the

solution of $Ax = b$. Where $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Explain.

30 iv. Find PLU and solve $PLUx = b$

10 v.   What is the det(A)

i) maximum ~~colu~~ column sum : 5

ii) — 11 — row sum : 5

v) $(3 \cdot 3 \cdot 2 + 0 + 0) - (0 + 2 + 3) = 18 - 5 = 3$

iii) $D^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ $D^{-1}b = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$M = D^{-1}(A-D) = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 & 0 \\ 1/3 & 0 & 1/3 \\ 0 & 1/2 & 0 \end{bmatrix}$

$x^{(k+1)} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 & 1/3 & 0 \\ 1/3 & 0 & 1/3 \\ 0 & 1/2 & 0 \end{bmatrix} x^k$

$\|M\|_\infty = 2/3 < 1$ will converge.

iv)
$$
\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}
$$

$$
\left[\begin{array}{ccc|c} 3 & 1 & 0 & 3 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 2 \end{array}\right]
\xrightarrow{\;K=1\;}
\left[\begin{array}{ccc|c} 3 & 1 & 0 & 3 \\ 1/3 & 8/3 & 1 & -1 \\ 0 & 1 & 2 & -2 \end{array}\right]
\xrightarrow{\;K=2\;}
$$

$$
\left[\begin{array}{ccc|c} 3 & 1 & 0 & 3 \\ 1/3 & 8/3 & 1 & -1 \\ 0 & 3/8 & 13/8 & -13/8 \end{array}\right]
$$

$$
L = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 0 & 3/8 & 1 \end{bmatrix}
\qquad
U = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 8/3 & 1 \\ 0 & 0 & 13/8 \end{bmatrix}
$$

$$
P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I
$$

Solution

$$
\begin{bmatrix} 3 & 1 & 0 \\ 0 & 8/3 & 1 \\ 0 & 0 & 13/8 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} 3 \\ -1 \\ -13/8 \end{bmatrix}
$$

$$x_3 = -1$$
$$x_2 = 0$$
$$x_1 = 1$$