Problem 8.

```
function [inverse] = GEpivinv(A)

[n,n] = size(A);
I = eye(n,n);
piv = 1:n;
tic
for k=1:n-1
    [maxv,r] = max(abs(A(k:n,k)));
    q = r+k-1;
    piv([k q]) = piv([q k]);
    A([k q],:) = A([q k],:);
    I([k q],:) = I([q k],:);
    if A(k,k) ~= 0
        A(k+1:n,k) = A(k+1:n,k)/A(k,k);
        A(k+1:n,k+1:n) = A(k+1:n,k+1:n) -A(k+1:n,k)*A(k,k+1:n);
        I(k+1:n,1:n) = I(k+1:n,1:n) -A(k+1:n,k)*I(k,1:n);
    end
end
```

```
for k=2:n
    if A(k,k) ~= 0
        A(1:k-1,k) = A(1:k-1,k)/A(k,k);
        A(1:k-1,k+1:n) = A(1:k-1,k+1:n) -A(1:k-1,k)*A(k,k+1:n);
        I(1:k-1,1:n) = I(1:k-1,1:n) -A(1:k-1,k)*I(k,1:n);
    end
end

for k = 1:n
    I(k,1:n)=I(k,1:n)/A(k,k);
end

toc
L = eye(n,n) + tril(A,-1);
inverse = I;
U = triu(A);
```

Problem 11.

```
function [X]=HW6_Problem5(A,b,x0)

D = tril(A)-tril(A,-1);
x1=inv(D)*b - inv(D)*((A-D)*x0);
N=1;

M1 = norm(inv(D)*(A-D),1);
M2 = norm(inv(D)*(A-D),1);
Minf = norm(inv(D)*(A-D),1);
Mnorm = min(M1,min(M2,Minf))

if Mnorm < 1
    while N>10^-5
        x0=x1;
        x1=inv(D)*b - inv(D)*((A-D)*x0);
        n1=norm(x1-x0,1);
        n2=norm(x1-x0,2);
        ninf=norm(x1-x0,inf);
        N=min(n1,min(n2,ninf));
    end
end

X=x1;
```

1\. Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ a_{31} & a_{32} & & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

$$||A||_\infty = \max \begin{bmatrix} a_{11} + a_{12} + \cdots + a_{1n}, \\ a_{21} + a_{22} + \cdots + a_{2n}, \\ \cdots a_{n1} + a_{n2} + \cdots + a_{nn} \end{bmatrix}$$

$$||A||_1 = \max \begin{bmatrix} a_{11} + a_{21} + \cdots + a_{n1}, \\ a_{12} + a_{22} + \cdots + a_{n2}, \\ \cdots a_{1n} + a_{2n} + \cdots + a_{nn} \end{bmatrix}$$

Since, we have a symmetric matrix so,

$a_{12} = a_{21}$, $a_{13} = a_{31}$, ... and so on

$\therefore$

$$|A||_\infty = \begin{bmatrix} a_{11} + a_{21} + \cdots & a_{n1}, \\ a_{12} + a_{22} + \cdots & a_{n2} \\ \cdots a_{nn} + a_{2n} + \cdots + a_{nn} \end{bmatrix}$$

2 a)

Therefore, we get $\|A\|_1 = \|A\|_\infty$

2 a) $PA = AP$, ~~TRUE~~ False
$A \Rightarrow n \times n$, $P \Rightarrow$ permutation matrix

For each $j \in \{1, \dots, n\}$, let $\sigma(j)$ be the unique element of $\{1, \dots, n\}$ such that $P_{\sigma(j), j} = 1$ (i.e., the unique 1 in the j^{th} column of X occurs in the $\sigma(j)$th row).
Since P is a permutation matrix, the numbers $\sigma(1), \dots, \sigma(n)$ are a permutation of the numbers $1, \dots, n$.

Fix a $j \in \{1, \dots, n\}$. For each i,

$$(AP)_{ij} = \sum_{k=1}^{n} A_{i,k} P_{k,j}$$

$$= A_{i,\sigma(j)} P_{\sigma(j), j} = A_{i, \sigma(j)}$$

Now, if we calculate $(PA)_{ij}$

$$= \sum_{k=1}^{n} P_{i,k} \; A_{k,j}$$

$$= P_{i,\sigma(j)} \; A_{\sigma(j),j} \;=\; A_{\sigma(j),j}$$

These two quantities are not equal

Take for examples

$$PA = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,1} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{bmatrix}$$

$$= \begin{bmatrix} A_{3,*} \\ A_{1,*} \\ A_{4,*} \\ A_{2,*} \end{bmatrix}$$

If we take,

$$AP = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

2 b)

$$= \begin{bmatrix} \cancel{A_{*,3}} \\ \cancel{A_{*,1}} \\ \cancel{A_{*,4}} \\ \cancel{A_{*,2}} \end{bmatrix} \begin{bmatrix} A_{*,2} & A_{*,4} & A_{*,1} & A_{*,3} \end{bmatrix}$$

2b) $PA = AP^T$, False

Now to find AP^T we have

$$AP^T = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{1,3} & a_{23} & a_{33} & a_{43} \\ a_{11} & a_{21} & a_{31} & a_{41} \\ a_{14} & a_{24} & a_{34} & a_{44} \\ a_{12} & a_{22} & a_{32} & a_{42} \end{bmatrix}$$

$$(P^t P)_{ij} = c_i^t c_j = \delta_{ij}$$

(The last equality follows from the observation that for $1 \le i, j \le n$, $e_i^t . c_j = \delta_{ij}$), So $P^t P = I$ ~~and hence~~ ~~P is~~,

2d) True, As proved above.

3a) Not every nonsingular A can be factored as $A = LU$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- first row of U, first column of L

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & l_{32} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$(P^t P)_{ij} = c_i^t c_j = \delta_{ij}$$

(The last equality follows from the observation that for $1 \le i, j \le n$, $e_i^t \cdot c_j = \delta_{ij}$), so $P^t P = I$ ~~and hence P is~~,

2d) True, As proved above.

3a) Not every nonsingular A can be factored as $A = LU$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

• first row of U, first column of L

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & l_{32} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Second row of U, second column of L

$$\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{22} & u_{23} \\ 0 & u_{33} \end{bmatrix}$$

$u_{22} = 0, \quad u_{23} = 2, \quad l_{32} \cdot 0 = 1 \,?$

4. $A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$

$\Rightarrow \underline{1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right]$

$\Rightarrow \underline{2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$

5.

6.

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

5. $$\begin{bmatrix} L1 & 0 \\ B & L2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$$

$\Rightarrow L1\, x = b$

$B\, x + L2\, y = c$

First solve $L1 x = b$, by forward substitution

6. $$\begin{bmatrix} 1 & a \\ c & b \end{bmatrix}$$

$m_{21} = c$

$$\Rightarrow \begin{bmatrix} 1 & a \\ 0 & b - ac \end{bmatrix}$$

$\therefore$ LU decomposition

$$\begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & b-ac \end{bmatrix}$$

To find inverse

$$\underline{c} \begin{bmatrix} 1 & a \\ c & b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & a \\ 0 & b-ac \end{bmatrix} \begin{matrix} 1 & 0 \\ -c & 1 \end{matrix}\Bigg]$$

$$\Rightarrow \quad \frac{a}{b-ac} \begin{bmatrix} 1 & a \\ 0 & b-ac \end{bmatrix} \begin{matrix} \frac{\cancel{1+ac}\; b}{b-ac} & \frac{-a}{b-ac} \\ -c & 1 \end{matrix}\Bigg]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\cancel{1+ac}\; b}{b-ac} & \frac{-a}{b-ac} \\ \frac{-c}{b-ac} & \frac{1}{b-ac} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \dfrac{b}{b-ac} & \dfrac{-a}{b-ac} \\[2ex] \dfrac{-c}{b-ac} & \dfrac{1}{b-ac} \end{bmatrix}$$

$$||A||_\infty = \max\left\{ 1+|a| \, , \, |b|+|c| \right\}$$

$$||A^{-1}||_\infty = \max\left\{ \frac{|b|-|a|}{|b|-|ac|} \; ; \; \frac{|1|-|c|}{|b|-|ac|} \right\}$$

The matrix is singular $|A| = 0$,
$b - ac = 0 \;\; ; \;\; b = ac$

$$||A||_1 = \text{Max}\left\{ 1+|c| \, , \, |a|+|b| \right\}$$

$$||A^{-1}|| = \text{Max}\left\{ \frac{|b|+|c|}{|b-ac|} \, , \, \frac{|a|+1}{|b-ac|} \right\}$$

$\text{Cond}\,(A, 1) = ||A||_1 \; ||A^{-1}||_1$
$\text{Max}\left[1+|c| \, , \, |a|+|b| \right] \times \text{Max}\left[\frac{|b|+|c|}{|b-ac|} \, , \, \frac{|a|+1}{|b-ac|} \right]$

$$\text{Cond}(A,\infty) = \|A\|_\infty \, \|A^{-1}\|_\infty$$
$$= \text{Max}\left[1+|a|,\ |d|+|c|\right] \times$$
$$\text{Max}\left[\frac{|a|+|b|}{|b-ac|},\ \frac{1+|c|}{|b-ac|}\right]$$

7. The complexity of LU factorization is $\frac{n^3}{3}$

	Elapsed time code given in HW7	Elapsed time code given in HW
$n=200$	0.025	0.239
$n=400$	0.020	2.208
$n=800$	0.070	23.672

There is a difference in elapsed time because code submitted by me uses temporary matrix 'I' to store multiplier. Also, the permutation matrix in our matrix is $n \times n$ matrix, where as the code given uses a $1 \times n$ vector to keep track of permutation

8. We know $A = LUP^T$. Multiplying with A^{-1} on both side we get $I = A^{-1}LUP^T$ (Since $AA^{-1} = I$) A^{-1} and P are thus related.

9. $x = [0 \quad 0.5 \quad 1 \quad 6 \quad 7 \quad 9]$

$y = [0 \quad 1.6 \quad 2 \quad 2 \quad 1.5 \quad 0]$

Functions $[1, x, x^2, x^3]$

Matrix 'A' will have a columns and 6 rows.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0.5 & 0.25 & 0.125 \\ 1 & 1 & 1 & 1 \\ 1 & 6 & 36 & 216 \\ 1 & 7 & 49 & 343 \\ 1 & 9 & 81 & 729 \end{bmatrix}_{6\times 4}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.5 & 1 & 6 & 7 & 9 \\ 0 & 0.25 & 1 & 36 & 49 & 81 \\ 0 & 0.125 & 1 & 216 & 343 & 729 \end{bmatrix}_{4\times 6}$$

$$(A^T A)_{4\times 4} = \begin{bmatrix} 6 & 23.5 & 167.25 & 1289.1 \\ 23.5 & 167.25 & 1289.1 & 10259 \\ 167.25 & 1289.1 & 10259 & 83633 \\ 1289.1 & 10259 & 83633 & 695747 \end{bmatrix}$$

$$(A^T y)_{4\times 1} = \begin{bmatrix} 7.1 \\ 25.3 \\ 147.9 \\ 948.7 \end{bmatrix}$$

$(A^T A) b = A^T y$ is given. We need to solve for b. This is similar to solving a system of linear equations. This can be done by LU factorization. For matrix, $A^T A$, L and U are given by.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.12974 & 1 & 0 & 0 \\ 0.00465 & 0.59905 & 1 & 0 \\ 0.01823 & 0.47202 & 0.36281 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1289.1 & 10259 & 83633 & 695747 \\ 0 & -41.87 & -591.4 & -6632.6 \\ 0 & 0 & 120.45 & 1891.5 \\ 0 & 0 & 0 & 20.44 \end{bmatrix}$$

and $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Let $A^T A = C$; $A^T Y = Q$

We are solving $C.b = Q$ (solving for b)

We know, $C = P^T LU \Rightarrow P^T L \underbrace{U\,b}_{z} = Q$

$\Rightarrow P^T L z = Q$

Multiply with P on both side $\Rightarrow$

$L z = PQ$ $[\because P^T P = I]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.12974 & 1 & 0 & 0 \\ 0.00465 & 0.579 & 1 & 0 \\ 0.0182 & 0.472 & 0.3628 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 948.7 \\ 147.9 \\ 7.1 \\ 25.3 \end{bmatrix}$$

Forward substitution $\Rightarrow z_1 = 948.7$

$$0.12974 z_1 + z_2 = 147.9 \Rightarrow z_2 = 24.8156$$

$$0.00465 z_1 + 0.57905 z_2 + z_3 = 7.1$$

$$\Rightarrow z_3 = -11.68$$

$$z_4 = 0.531$$

But $ub = z$

$$\Rightarrow \begin{bmatrix} 1289.1 & 10259 & 83633 & 695747 \\ 0 & -41.87 & -591.4 & -6632.6 \\ 0 & 0 & 120.45 & 1891.5 \\ 0 & 0 & 0 & 2044 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

Backward substitution

$b_4 = z_4 = 0.026$

$b_3 = -0.5033$

$b_2 = 2.425$

$b_1 = 0.1821$

$$\therefore\ b = \begin{bmatrix} 0.1821 \\ 2.425 \\ -0.5053 \\ 0.026 \end{bmatrix}$$

Problem 9

Below is output of the use of functions polyfit and polyval as well a computation of RMSE and absolute errors.

ErrorCalc

$$p = \begin{matrix} 0.026003853661861 \\ -0.505350102398000 \\ 2.425733973577638 \\ 0.182139732490656 \end{matrix}$$

RMSE = 0.466007

Absolute Error =
1.01727197700560 1

MATLAB code: Errorcalc.m

```
x = [0   0.5   1 6 7 9];
y = [0   1.6   2 2 1.5   0];
P = polyfit (x, y, 3)

Y = polyval (P, x)

Sum RMSE = 0
sum Error = 0

for i = 1:6
    Sum RMSE = Sum RMSE + (Y(i) - y(i))^2
    sum Error = Sum Error + abs (Y(i) - y(i)),
end.

RMSE = sqrt (Sum RMSE)
Absolute Error = Sum Error.
```

Problem

a) Let

b)

Problem 10

a) Let X denote the solution of $Ax = b$
i.e, $Ax = b \Rightarrow X = A^{-1}b$

Let there be another solution denoted by Y. i.e, $Y = A^{-1}b$. Introduce $R = X - Y$, the differences between the solutions
$= A^{-1}b - A^{-1}b = 0$

i.e., for any A, there exist a unique solution for $x \ni Ax = b$

b) For $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$+ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Convergence for Gauss Jacobi is given by $M = D^{-1}(L+U)$

$$D^{-1} = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.33 \end{bmatrix}$$

$$D^{-1}(L+U) = \begin{bmatrix} 0 & 0.25 & 0.25 \\ 0.5 & 0 & 0 \\ 0 & 0.667 & 0 \end{bmatrix} = M$$

$\|M\|_1 = 0.9167$ Norm of $M < 1$

$\|M\|_\infty = 0.667$ $\Rightarrow$ Converges

$\|M\|_2 = 0.718$

Convergence of Gauss-Siedel is given by

$M = (L+D)^{-1} U$

$$(L+D)^{-1} = \begin{bmatrix} 0.25 & 0 & 0 \\ -0.125 & 0.5 & 0 \\ 0.08 & -0.33 & 0.33 \end{bmatrix}$$

$$M = (L+D)^{-1}u = \begin{bmatrix} 0 & 0.25 & 0.25 \\ 0 & -0.125 & -0.125 \\ 0 & 0.083 & 0.083 \end{bmatrix}$$

$\|M\|_1 = 0.458$ Norm of $M < 1$

$\|M\|_\infty = 0.5$ $\Rightarrow$ converges

$\|M\|_2 = 0.412$

Let $f = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ be the polynomial fit for the data

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \\ 1 & x_5 & x_5^2 & x_5^3 \\ 1 & x_6 & x_6^2 & x_6^3 \end{bmatrix}$$

Need to solve $A^T A a = A^T y$

We have already solved this hence,

$a_0 = 0.1821$, $a_1 = 2.425$, $a_2 = -0.5053$,

$a_4 = 0.026$

MATLAB code polyfit(x,y,3) gives the same coefficients

c) $Ax = b$; $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$

Actual Solution $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$(D+L)x = b - Ux$

$(D+L)x^{k+1} = b - Ux^{k}$

Step 1: $x^{0} = [0; 0; 0]$

$\Rightarrow \begin{bmatrix} 4 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix} x^{k+1} = \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 1.9167 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 4.083 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow x^{k+1} = \begin{bmatrix} 1.020825 \\ 0.98958 \\ 1.006941 \end{bmatrix}$

Step 3

$$x^{k+1} = \begin{bmatrix} 1.00086 \\ 0.99956 \\ 1.000289 \end{bmatrix}$$

10(c) Gauss Seidel Iteration with
$x_0 = [0.5, 0.5, 0.5]$

1 $x_0 = [0.5, 0.5, 0.5]$ $x1 = [1.25, 0.875, 1.083]$
2 $x_0 = [1.25, 0.875, 1.083]$ $x1 = [1.0104, 0.994, 1.0031]$
3 $x_0 = [1.01041, 0.994, 1.00347]$
$x1 = [1.0004, 0.999, 1.0001]$
4 $x_0 = [1.0104, 0.99476, 1.0034]$
$x1 = [1.000, 0.999, 1.000]$

11 i) $||M|| = ||D^{-1}(A-D)|| = 1.5 > 1$.
The program output does not show a convergence to solution.

11 ii) $||M|| = ||D^{-1}(A-D)|| = 0.667 < 1$.
Hence the method converges to solution