EXAM-2 PRACTICE

1. Given the matrix A = $\begin{bmatrix} -2 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & 2 & -1 \end{bmatrix}$

   a. What is $||A||_1$ and $||A||_\infty$
   b. Find the inverse A^{-1}
   c. Find the condition number $cond(A,'1')$ and $cond(A,'\infty')$?
   The $cond(A,'n') = ||A||_n \, ||A^{-1}||_n$. Where n is a norm, 1 or ∞.
   d. For the matrix A above find an $A = LU$
   e. Solve $Ax = b$, where , $b = \left[0,2,\frac{3}{2}\right]^T$ Using $x = A^{-1} * b$
   f. Solve $Ax = b$, where $b = \left[0,2,\frac{3}{2}\right]^T$, Using $Ly = b$, $Ux = y$.
   g. For a general nxn system which method is faster in terms of operations (e) or (f). Explain by deriving the number of operations for each approach.

2. Calculate the inverse for the matrix

$$A = \begin{pmatrix} 1 & c \\ c & 1 \end{pmatrix}$$

Where c is an unknown constant.

(a) Then calculate the condition number.

(b) If you use single precision accuracy, i.e the error is less or equal to ½ $*10^{-7}$ for what c the entire precision of 7 digits will be lost?

(c) What is the relation of the condition number and the determinant of this matrix?

3. a. Derive the normal equations $A^T Ab = A^T y$ for the data x=[1 2 3 4] and y=[1 17 49 97] and the basis functions $\{1, 2x - 1, 8x^2 - 8x + 1\}$.

   b. Solve the system above.

   c. What is the Least squares error for these data?

4. Consider the iterative solution of $Ax = b$ with

$$A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$$

   a. Show that the system has a unique solution .
   b. Derive the Gauss-Jacobi method and prove that it will converge for any initial value x_0.
   c. Derive the Gauss-Seidel method and prove that it will converge for any initial value x_0 .
   d. Perform 4 iteration of Gauss-Seidel and Gauss-Jacobi methods with $b = [5\ 5]^T$. How fast do they converge? (Ratio of the error between consecutive steps).

EXAM-2 PRACTICE

5. Given a tri-diagonal matrix.

A= [b_1 c_1 0 … 0

a_2 b_2 c_2 … 0

0 a_3 b_3 c_3 … 0

… … ..

a_{n-1} b_{n-1} c_{n-1}

0 … 0 a_n b_n]

a. Write a Matlab program that the finds the L and U so that $A = LU$ using only one loop.(as opposed to the kij or kji for general LU decomposition that uses triple loop).
b. What is the number of operations required to find L and U for this A.
c. If $b_i = 2$ and $c_i = a_i = 1$ for all $i = 1{:}n$ Show L and U for the any n.
d. Assume we want to solve $Ax = b$ and that both L and U and A^{-1} are already computed for the tri-diagonal matrix. You are asked to choose between the following two methods $LUx = b$ or $x = A^{-1} * b$ to find the solution. Which method will you choose and why…EXPLAIN.

6. Given the matrix

A= [-2 1 1

1 2 -2

1 2 -1]

h. What is the **$||A||_1$** and **$||A||_{inf}$**
i. Find the **inv(A)=A^{-1}**
j. Find the condition number **cond(A,1)** and **cond(A,'inf')** ?

The **cond(A)=$||A||$ $||A^{-1}||$.**

k. For the matrix **A** above find an **L** and **U** so that **A=LU.**
l. Solve **Ax=b,** where **, b=[0,2,3/2]'** Using **x=inv(A)*b**
m. Solve **Ax=b,** where **, b=[0,2,3/2]'** Using **Ly=b , Ux=y.**

EXAM-2 PRACTICE

n. For a general *nxn* system which method is faster in terms of operations (e) or (f). Explain.