HW-7

1. For a symmetric matrix A, is it always the case that $||A||_1 = ||A||_\infty$? Prove it or disprove it.
2. True or False with a Proof: If A is any n x n matrix and P is any permutation matrix of the same size then (a) PA=AP (b) $PA=AP^T$ (c) $PP^T = P$ (d) $P^TP=I$
3. (a) Can every nonsingular n x n matrix be written as a product A= LU, where L is a lower triangular and U is an upper triangular matrix? (b) If so what is an algorithm fo accomplishing this? If not, give a counterexample to illustrate.
4. What is the inverse of the following matrix: (hint solve with attached Unit matrix using Gaussian elimination performed in ALL rows of the expanded matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

5. How would you solve the partitioned linear system of the form

$$\begin{bmatrix} L1 & 0 \\ B & L2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$$

Where L1 and L2 are lower triangular systems (nonsingular).

6. What is the LU factorization of the following matrix and under what conditions the matrix is singular. What is the inverse? And what is the condition number $cond(A) = ||A^{-1}|| * ||A||$?

$$\begin{bmatrix} 1 & a \\ c & b \end{bmatrix}$$

7. Compare the following matlab program with your program in terms of time of execution using tic/toc to measure elapsed time and a randomly generated matrix $A = rand(n)$ using $n = 200{,}400{,}800$. Does the complexity behave like $O(n^3)$? If there are differences in time provide an explanation.

```
function [L,U,piv] = GEpiv(A)
% [L,U,piv] = GE(A)
%
% The LU factorization with partial pivoting. If A is n-by-n, then
% piv is a permutation of the vector 1:n and L is unit lower triangular
% and U is upper triangular so A(piv,:) = LU. |L(i,j)|<=1 for all i and j.

[n,n] = size(A);
piv = 1:n;
for k=1:n-1
   [maxv,r] = max(abs(A(k:n,k)));
   q = r+k-1;
   piv([k q]) = piv([q k]);
   A([k q],:) = A([q k],:);
   if A(k,k) ~= 0
      A(k+1:n,k) = A(k+1:n,k)/A(k,k);
            A(k+1:n,k+1:n) = A(k+1:n,k+1:n) -A(k+1:n,k)*A(k,k+1:n);
   end
end
L = eye(n,n) + tril(A,-1);
U = triu(A);
```

HW-7

8. Modify the previous program to find the inverse of a matrix A using the Gauss Jordan method. How is output of the program relates to A^{-1} and P the permutation matrix .
9. Derive the normal equations $A'Ab = A'y$ for the data x=[0 0.5 1.0 6.0 7.0 9.0] and y=[0 1.6 2.0 2.0 1.5 0.0] and the basis functions $\{1, x, x^2, x^3\}$. Solve the system above. Also derive the maximum error as well the RMSE . You can use matlab for help but do this problem by hand. Then use P = POLYFIT(X,Y,N) as well polyval to compute the same polynomial using the matlab functions. Plot these data and functions.
10. Consider the iterative solution of $Ax = b$ with

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

    a. Show that for any right-hand side vector b the system has a unique solution.
    b. When the Gauss-Jacobi and Gauss-Seidel methods are used do you expect convergence for both? Explain!
    c. If the answer is YES for b. then can you solve the system for $b = [6\ 3\ 5]^T$. How many iterations does it take to converge to 4 significant digits of accuracy? You can use matlab to compute the iterations. Show all iteration steps in a table.

11. The Gauss-Jacobi iterative method is defined as follows: $Ax = b ==> [D + (A - D)]x = b ==> Dx = b - (A - D)x ==> x = D^{-1}b - D^{-1}(A - D)x$ so we can derive an iterative method: $x^{k+1} = D^{-1}b - D^{-1}(A - D)x^k$ Write a matlab program that implements this method using Matlab matrix definitions e.g. $D = tril(A) - tril(A, -1);\ x(k+1) = inv(D) * (b - (A - D) * x(k))$, you can use as stopping criterion $norm(x(k+1) - x(k), \gamma), \gamma = 2, or\ \gamma = 1, or\ \gamma = inf$ for the 3 norms that we learned in class, the 2 norm or the 1 norm or the infinity or maximum norm.

Here is an example on how the iterations in your program should work

```
A =[

   1   1  -1
   1   2  -2
  -2   1   1 ]
>> D=tril(A)-tril(A,-1)

D =

   1   0   0
   0   2   0
   0   0   1
b =[

   0
```

HW-7

```
   -1
   -1]
>> x0=zeros(3,1)

x0 =

     0
     0
     0
x1=inv(D)*(b-(A-D)*x0)

x1 =

         0
   -0.5000
   -1.0000
```

Now if you continue iterating like that or use your program you will see that this iteration does not converge for this matrix. The reason being is that the norm of the iteration matrix $D^{-1}(A-D)$ is greater than one.

```
>> norm(inv(D)*(A-D),2)

ans =

    2.3284
>> norm(inv(D)*(A-D),1)

ans =

    2.5000
>> norm(inv(D)*(A-D),inf)

ans =

     3
```

i. Will this method converge for $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ check your program and explain if YES or NO.

ii. Repeat the same question with $A = \begin{bmatrix} -6 & 1 & 1 \\ 1 & 4 & -2 \\ 1 & 1 & 3 \end{bmatrix}$ $b = \begin{bmatrix} -4 \\ 3 \\ 5 \end{bmatrix}$

HW-7