SACHIN SRIVASTAVA

1.a) $$\frac{1}{1+\frac{t}{\lfloor 2}+\frac{t^2}{\lfloor 3}+\dots} = \sum_{j=0}^{\infty} B_j \frac{t^i}{\lfloor i} = B_0 + \frac{B_1 t}{\lfloor 1} + \frac{B_2 t^2}{\lfloor 2} + \dots$$

$$\Rightarrow 1 = B_0 + B_1 t + \frac{B_2 t^2}{2} + \dots + B_0 \frac{t}{2} + \frac{B_1 t^2}{2} + \dots$$

$$= B_0 + \left(B_1 + \frac{B_0}{2}\right) + \left(\frac{B_2}{2} + \frac{B_1}{2} + \frac{B_0}{6}\right)t^2 + \dots$$

$$\Rightarrow B_0 = 1,\ B_1 + \frac{B_0}{2} = 0,\ \frac{B_2}{2} + \frac{B_1}{2} + \frac{B_0}{6} = 0$$

$$\Rightarrow B_1 = -\frac{1}{2},\ B_2 = \frac{1}{6},\ \dots$$

b) $$\int_0^n x^2 dx = \sum_{i=0}^{n} i^2 - \frac{1}{2}(n^2+0) - \frac{1}{2\times 6}(2n-0)$$

$$\Rightarrow \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} = \sum_{i=0}^{n} i^2$$

c) $$\int_0^n x^4 dx = \sum_{i=0}^{n} i^4 - \frac{1}{2}(n^4) - \frac{1}{12}(3n^3) + \frac{1}{30} \times \frac{1}{\lfloor 4}(12 \times 2n)$$

$$= \frac{n^5}{5}$$

$$\sum_{i=0}^{n} i^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

2. $\int_0^1 f(x)\,dx - \frac{1}{2}[f(0)+f(1)] = \int_0^1 [f(x) - P_1(x)]$

$$= \int_0^1 \frac{f''(\theta)\,x(x-1)}{2}\,dx = \frac{1}{2}\int_0^1 f''(\theta)\,x(x-1)\,dx$$

$$= \frac{1}{2} f''(\sigma) \int_0^1 x(x-1)\,dx = \frac{1}{2} f''(\sigma)\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2} f''(\sigma)\left[\frac{1}{3} - \frac{1}{2}\right] = -\frac{1}{12} f''(\sigma)$$

3 i) $I = T^{(1)}(h) + A_2^* h^4 + \dots$ (1)

and $I = T^{(1)}\left(\frac{h}{2}\right) + A_2^*\left(\frac{h}{2}\right)^4 + \dots$ (2)

$4^2 I = 4^2\, T^{(1)}\left(\frac{h}{2}\right) + A_2^* h^4 \dots$ (3)

Subtract (1) from (3)

$(4^2 - 1) I = 4^2\, T^{(1)}\left(\frac{h}{2}\right) - T^{(1)}(h) + A_{\cancel{2}}^{**} h^6 \dots$

Dividing by (4^2-1); Equivalently

$I = T^{(2)}(h) + A_{\cancel{2}}^* h^6 + \dots$

where $T^{(2)}(h) = \dfrac{4^2\, T^{(1)}\left(\frac{h}{2}\right) - T^{(1)}(h)}{4^2 - 1}$

3 ii) $I = T^{(2)}(h) + A_2^* h^6 + \dots$

and $I = T^{(2)}\left(\frac{h}{2}\right) + A_2^* h^6 + \dots$

$\Rightarrow 4^3 I = 4^3 T^{(2)}\left(\frac{h}{2}\right) + A_2^* h^6$

subtracting

$(4^3 - 1) I = 4^3 T^{(2)}\left(\frac{h}{2}\right) - T^{(2)}(h) + A_2^{**} h^8 + \dots$

$I = T^{(3)}(h) + A_2^* h^8 + \dots$

where, $T^{(3)}(h) = \dfrac{4^3 T^{(2)}\left(\frac{h}{2}\right) - T^{(2)}(h)}{4^3 - 1}$

3. iii)

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function integral=trapezoidal(a,b,h,index_f)

n=(b-a)/h;
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Midtem-2 practice-with solutions

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sumend = (f(a,index_f) +f(b,index_f))/2;

sum = 0;

for i=1:1:n-1

sum = sum + f(a+i*h,index_f);

end

integral = h*(sumend + sum);

function f_value = f(x,index)

switch index

case 1

f_value = x;

case 2

f_value = x^7;

end
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3. iv)

O(h^2)	O(h^4)	O(h^6)	O(h^8)
0.500000000000000			
0.253906250000000	0.171875000000000		
0.160339355468750	0.129150390625000	0.126302083333333	
0.134043693542480	0.125278472900390	0.125020345052083	0.125000000000000=1/8

4a) $\int_0^1 f(x)\,dx \approx A f(0) + B f\left(\frac{1}{2}\right) + C f(1)$

Substituting $f(x) = 1$

$A + B + C = 1$

Substituting $f(x) = x$

$\frac{B}{2} + C = \frac{1}{2}$

With x^2

$\frac{B}{4} + C = \frac{1}{3}$

Solving, $A = \frac{1}{6}$, $B = \frac{4}{6}$, $C = \frac{1}{6}$

We can find the error using $f(x) = x^3$

$$\int_0^1 x^3\,dx - \frac{1}{6}\left[0 + 4\left(\frac{1}{2}\right)^3 + 1\right] = 0$$

Taking $f(x) = x^4$

$$\int_0^1 x^4\,dx - \frac{1}{6}\left[0 + 4\left(\frac{1}{2}\right)^3 + 1\right] = \frac{1}{5} - \frac{1}{6}\left(\frac{5}{4}\right)$$

$= -0.0083$

4b) $\int_0^1 \sqrt{x}\, f(x)\, dx = A f(c) + B f(1)$

Chosing $f(x) = 1$, we get

$A + B = \frac{2}{3}$ — (1)

Chosing $f(x) = x$

$Ac + B = \frac{2}{5}$ — (2)

Similarly chosing $f(x) = x^2$

$Ac^2 + B = \frac{2}{7}$ — (3)

Solving (1), (2) & (3)

$c = \frac{15}{35}$, $A = \frac{35}{75}$, $B = \frac{45}{225}$

Error $\int_0^1 \sqrt{x}\, x^3\, dx - \frac{35}{75} \times \left(\frac{15}{35}\right)^3 - \frac{45}{225} = 0.38$

4c) $\int_0^1 x^3\, dx = \frac{1}{6}\left(0 + 4\left(\frac{1}{2}\right)^3 + 1\right) = \frac{1}{4}$