QUIZ-2 practice LastName________________FirstName_________________

1. Understand Figure 3.6 Page 103 from the book.
2. A sufficient condition to prove convergence of an iterative method $x_n = g(x_{n-1}), n = 1,2,\ldots$ is to show that the error at the n^{th} step is less than the error at $n-1$ step, i.e. $|x_n - p| = |x_{n-1} - p|M_n, \quad M_n < 1,$ in some interval that contains the root p. However this condition is not necessary since convergence might also occur outside that interval as long as the sequence x_n monotonically winds up in the interval(see HW-2). Answer the following questions: The quadratic equation has two roots $2x^2 - x - 1 = 0$
   i. Find the roots using the quadratic equation
   ii. The equation can be rewritten as $x = 2x^2 - 1 = g(x)$. Will this iteration converge to any of the two roots?[Since you know the roots you can use the QUICK test to answer this question $|g'(p)| < 1$]
   iii. Another way of rewriting the equation is $x = \sqrt{\frac{x+1}{2}} = g(x)$. Will this iteration converge to a root? Which root? What is the interval of convergence?
   iv. Derive Newton's method for this equation. Will newton's method converge for all initial starting points?
3. The function $f(x) = x^2 + x - 2$ has two roots in the intervals $[0,3]$ and $[-3,0]$.
   i. What are the roots?
   ii. Perform 3 steps of the bisection method for the root in $[0, 3]$. How many steps will you need so that the error in the n^{th} iteration of the bisection method is less than 0.0001?
   iii. Perform 3 steps of the Regula Falsi iteration.
   iv. Perform 2 steps of Newton's method for both roots staring with x_0=2 and x_0=-2. How many steps will you need to get to 0.0001 error in Newton's method.
   v. A fixed point iteration can be derived by re-writing $x^2 + x - 2 = 0$ into its equivalent form $x = 2 - x^2 = g(x)$. Will this iteration converge to any of the two roots? Explain.
   vi. Another fixed point iteration can be derived by re-writing the equation as $x = \sqrt{2 - x}$. Will this iteration converge to any of the roots?
4. Given the iteration $x_{n+1} = \frac{x_n(x_n^2+3)}{3x_n^2+1} = g(x_n)$.
   i. What is the original equation that we are trying to solve?
   ii. Can you two propose alternatives to the above method that converge to the root?
   iii. What is Newton's method for this equation.
   iv. We know that for Newton's method the convergence is quadratic, i.e. $g'(p) = 0$. What is the order of convergence for the iteration above.