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1. Taylor's polynomial approximating a function is defined as follows

$$P_n(x) = f(a) + \sum_{j=1}^{n} \frac{f^{(j)}(a)(x-a)^j}{j!}$$

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(j)}(\mu), \qquad \alpha \le x \le \beta\,, and\ a \le \mu \le x.$$

$$f(x) = P_n(x) + R_n(x).$$

i. Find the Taylor's polynomial and error for $f(x) = \sin(x)\,,\ \ a = 0,\ \ \ 0 \le x \le \frac{\pi}{2}$.

ii. Find an upper bound for the error. Show that the error converges to zero as n goes to ∞.

2. What is the smallest positive integer number represented in 32 bit arithmetic? What is the largest integer number? Provide an explanation on how this is being computed. Repeat the same question for floating point real numbers.

3. a. Draw a graph for $f(x) = x^2 - 1$.

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b. Prove that there exist a root in the interval [0, 3].

c. Graphically show one step for each of the following methods for the positive root.

   i. Regular Falsi

   ii. Newton's method

4. For the following data [5 10 15 20 30 40 60] and key=10 show all steps of Binary(Bisection) method as it brackets the key.