Assignment # 3

1 (Weight: 30%) Describe an O(n) algorithm that, given a set S of n distinct numbers and a positive integer k<=n, determines the k numbers in S that are closest to the median of S.

2 (Weight: 30%) Find an optimal parenthesization of a matrix chain multiplication whose sequence of dimensions is (7, 10, 9, 5, 12, 6).

3 (Weight: 40%) Suppose n activities apply for using a common resource. Activity a_i ($1 \le i \le n$) has a starting time $S[i]$ and a finish time $F[i]$ such that $0 < S[i] < F[i]$. Two activities a_i and a_j ($1 \le i, j \le n$) are compatible if intervals $[S[i], F[i])$ and $[S[j], F[j])$ do not overlap. We assume the activities have been sorted such that $S[1] \le S[2] \le \ldots \le S[n]$.

(a) Design an $O(n^2)$ dynamic programming algorithm to find a set of compatible activities such that the total amount of time the resource is used by these compatible activities is maximized. You need to define the sub-problems, establish inductive formula, and show the initial conditions. Pseudo code is not required.

(b) Apply your algorithm to the following set of activities

i	1	2	3	4	5	6	7	8	9	10	11
$S[i]$	2	3	5	6	7	9	10	12	13	14	16
$F[i]$	6	5	7	10	8	13	16	14	14	18	20