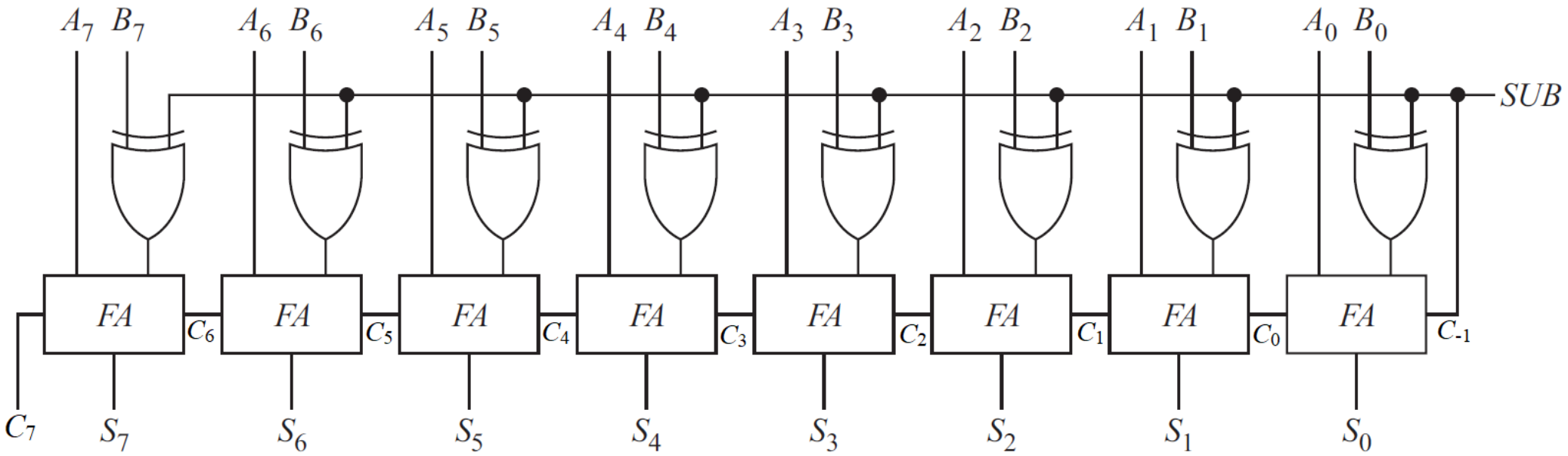
Digital Electronic Circuits
Section 1 (EE, IE)

Lecture 16

Adder – Subtract0r Circuit

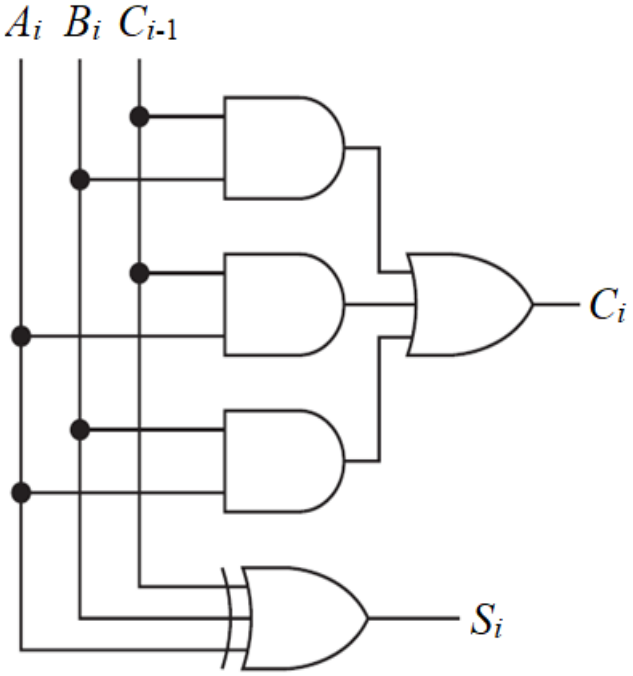


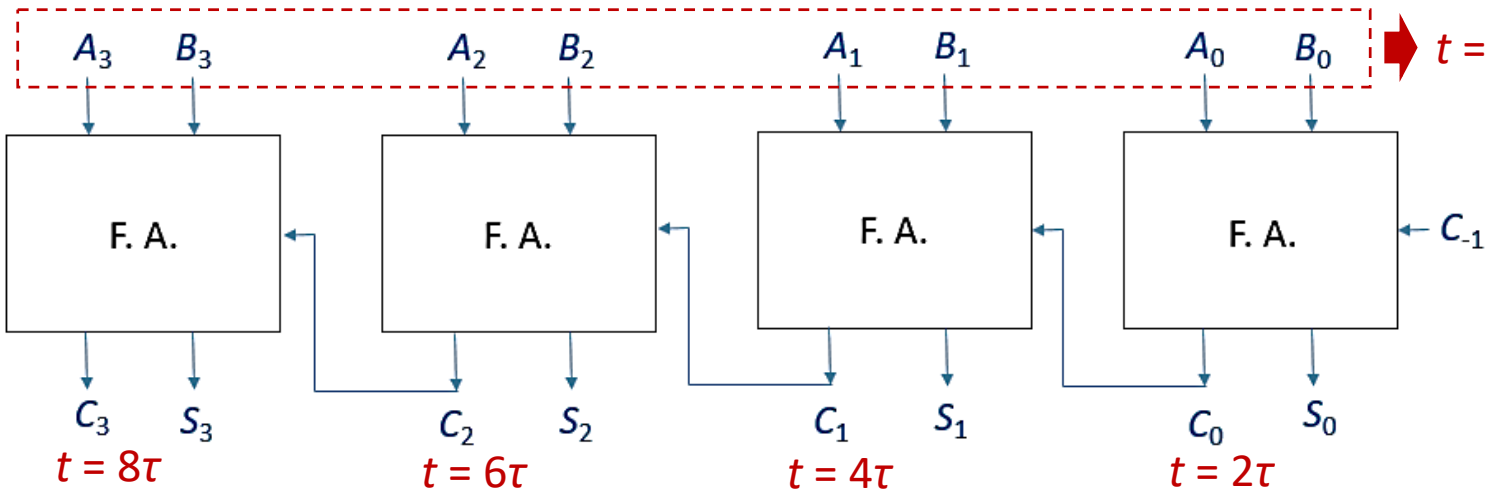
If SUB = 0, then $C_{-1} = 0$
Ex-OR gate output = $B_i \oplus 0 = B_i$
F.A. output = $A + B$

If SUB = 1, then $C_{-1} = 1$
Ex-OR gate output = $B_i \oplus 1 = B_i'$
F.A. output = $A + (-B) = A - B$

Delay in Ripple Carry Adder

Propagation delay is cumulative.
For *n*-bit addition, $2n\tau$ delay.





$C_i = A_i.B_i + A_i.C_{i-1} + B_i.C_{i-1}$

$S_i = A_i \oplus B_i \oplus C_{i-1}$

Each basic gate delay = τ
(AND, OR)

Each Ex-OR gate delay = 2τ

C_3 available after 8τ delay.

For S_3, $A_3 \oplus B_3$ result after 2τ.
This is Ex-ORed with C_2.
C_2 available after 6τ.
S_3 available after 8τ.

Carry: Serial to Parallel

$C_i = A_iB_i + A_iC_{i-1} + B_iC_{i-1}$

$C_i = A_iB_i + C_{i-1}(A_i + B_i)$

$C_i = G_i + P_iC_{i-1}$

Where, $G_i = A_iB_i$: **Generation term**

and $P_i = A_i + B_i$: **Propagation term**

$G_i = 1$: Carry is generated

$P_i = 1$: Input carry is propagated

$C_0 = G_0 + P_0C_{-1}$ … (1)

$C_1 = G_1 + P_1C_0$ … (2)

Substituting C_0 from Eq.(1) in Eq.(2),

$C_1 = G_1 + P_1(G_0 + P_0C_{-1})$

$C_1 = G_1 + P_1G_0 + P_1P_0C_{-1}$ … (3)

2 parallel AND delay: τ
1 OR delay : τ
For G_i and P_i, delay : τ

Total delay : 3τ

Carry Look Ahead Adder

$C_2 = G_2 + P_2C_1$ … (4)

Substituting C_1 from Eq.(3) in Eq.(4),

$C_2 = G_2 + P_2(G_1 + P_1G_0 + P_1P_0C_{-1})$

$C_2 = G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_{-1}$ … (5)

C_2 is generated after 3τ delay, too.

…

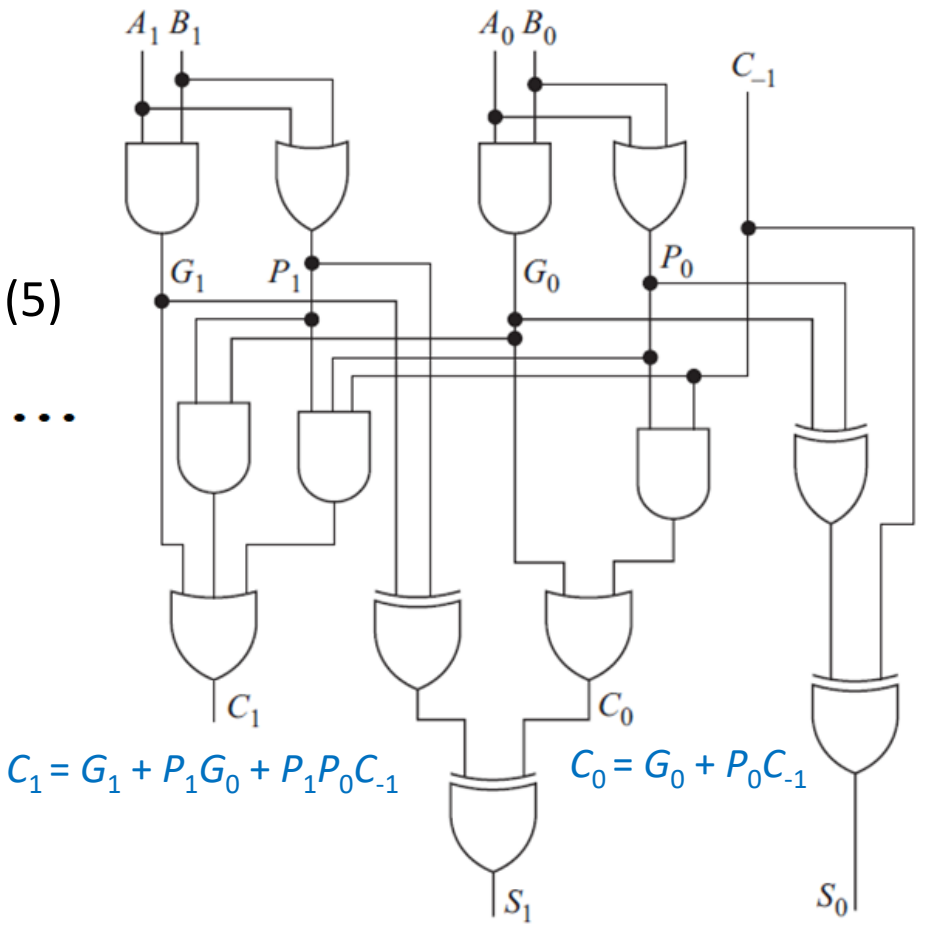
$C_3 = G_3 + P_3C_2$ … (6)

From Eq.(5) and Eq.(6)

$C_3 = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0$

$+ P_3P_2P_1P_0C_{-1}$ … (7)

C_3 is generated after 3τ delay, too.



$S_i = A_i \oplus B_i \oplus C_{i-1}$

$S_i = G_i \oplus P_i \oplus C_{i-1}$

(avoids loading of inputs)

All $G_i \oplus P_i$ in 3τ

All C_{i-1} in 3τ

All S_i in 3τ + 2τ = 5τ

Delay is not cumulative.

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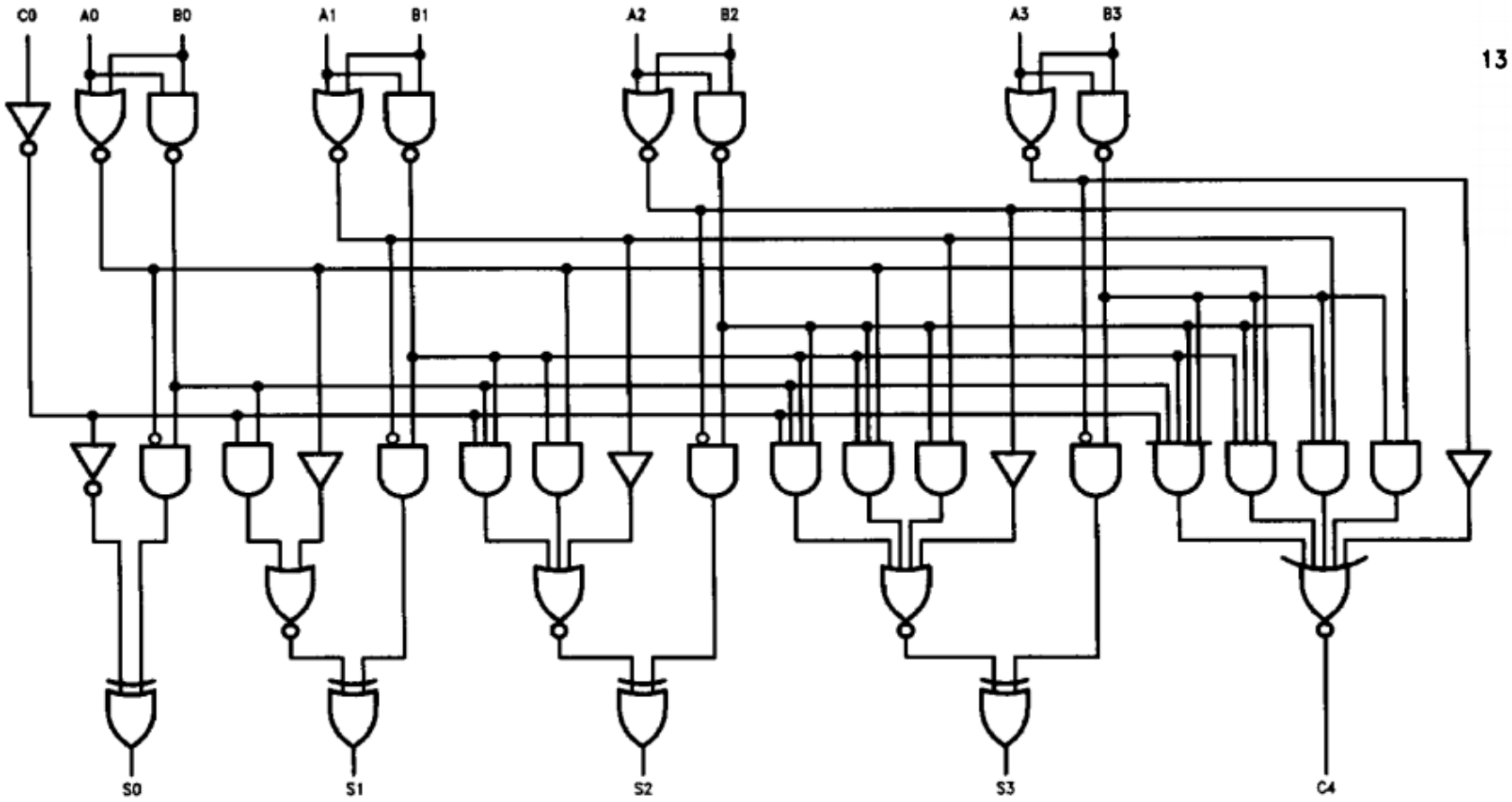
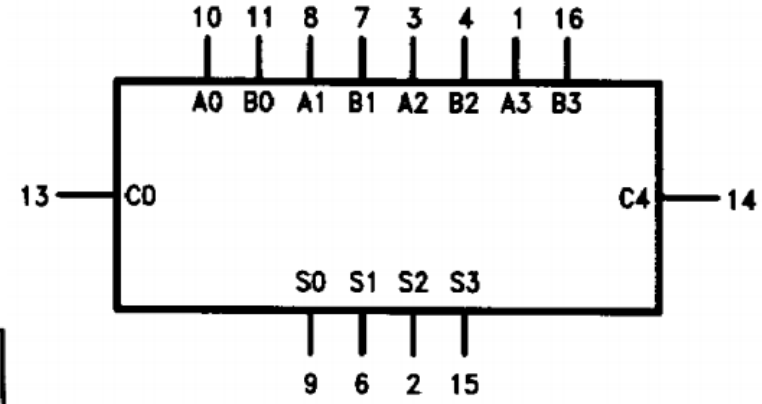
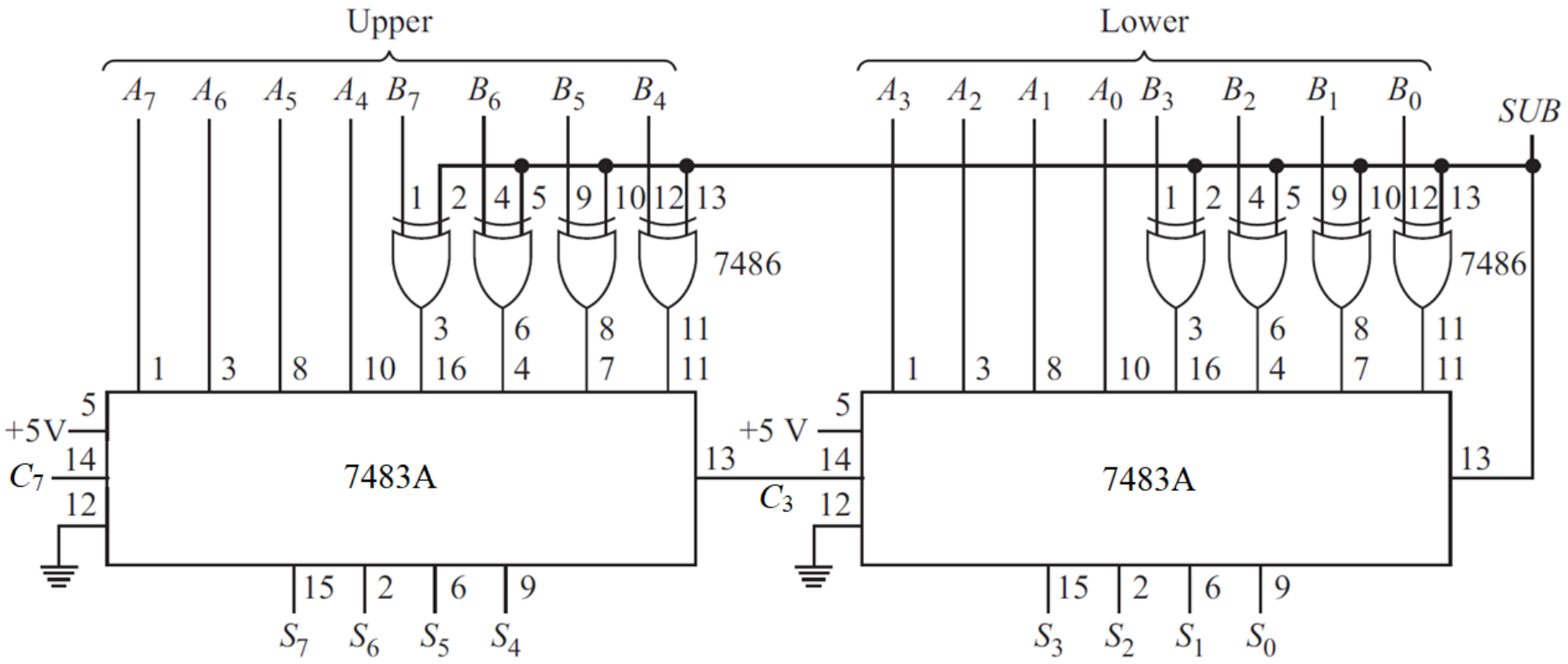




Different manufacturers use different notations.

Addition / Subtraction in Cascade



- Carry ripples from one IC to other.
- Carry look ahead within IC.

Group Carry Generation and Propagation

$C_3 = G_3 + P_2G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_{-1}$

$G_{3\text{-}0} = G_3 + P_2G_2 + P_3P_2G_1 + P_3P_2P_1G_0$: Group Carry Generation term

$P_{3\text{-}0} = P_3P_2P_1P_0$: Group Carry Propagation term

$C_3 = G_{3\text{-}0} + P_{3\text{-}0}C_{-1}$

$C_7 = G_{7\text{-}4} + P_{7\text{-}4}C_3$

$C_7 = G_{7\text{-}4} + P_{7\text{-}4}(G_{3\text{-}0} + P_{3\text{-}0}C_{-1}) = G_{7\text{-}4} + P_{7\text{-}4}G_{3\text{-}0} + P_{7\text{-}4}P_{3\text{-}0}C_{-1}$

Similarly,

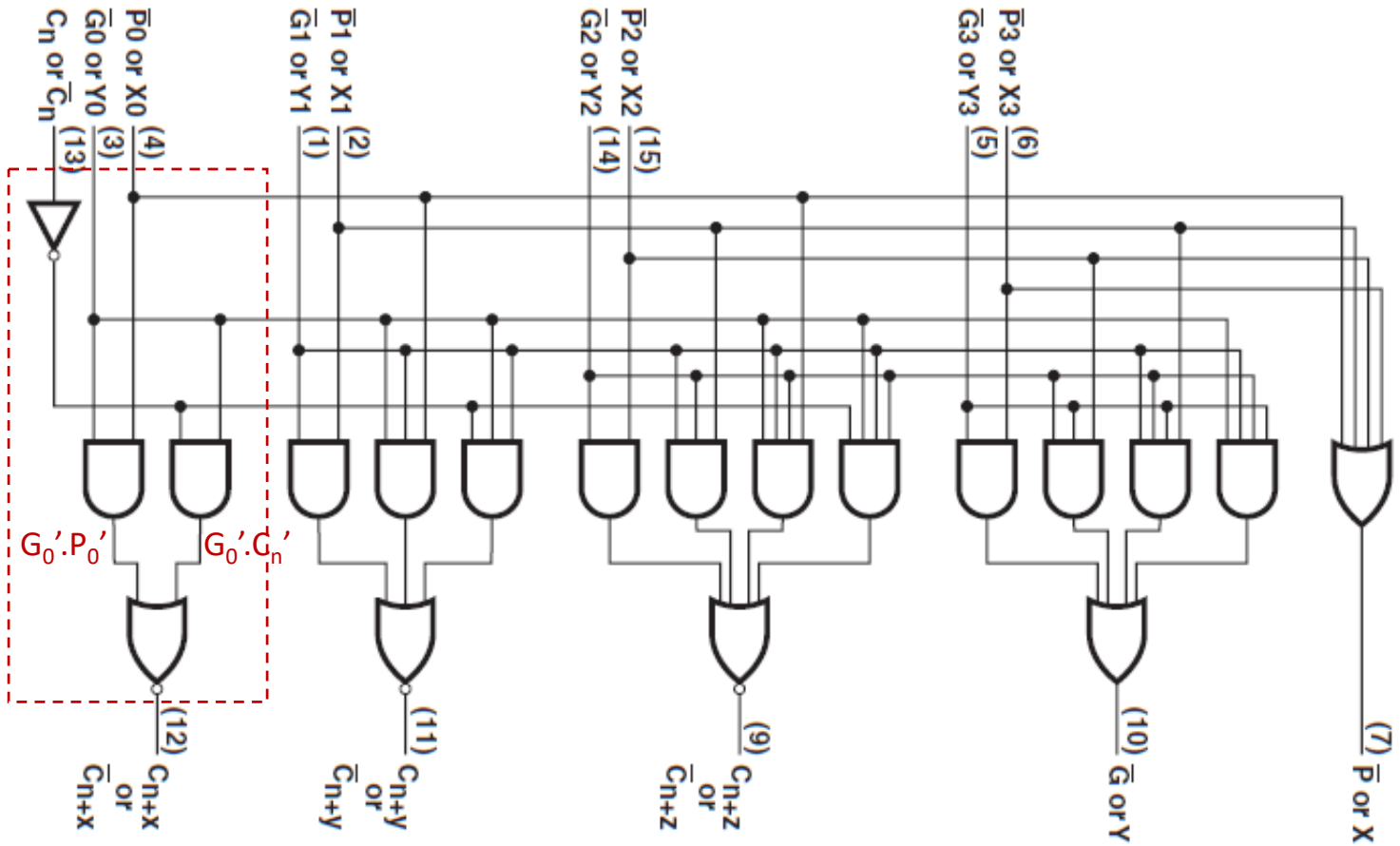
$C_{11} = G_{11\text{-}8} + P_{11\text{-}8}G_{7\text{-}4} + P_{11\text{-}8}P_{7\text{-}4}G_{3\text{-}0} + P_{11\text{-}8}P_{7\text{-}4}P_{3\text{-}0}C_{-1}$

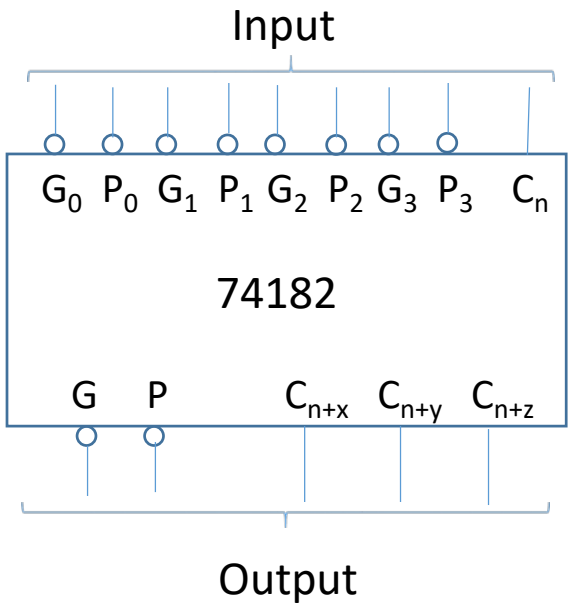
$C_{15} = G_{15\text{-}12} + P_{15\text{-}12}G_{11\text{-}8} + P_{15\text{-}12}P_{11\text{-}8}G_{7\text{-}4} + P_{15\text{-}12}P_{11\text{-}8}P_{7\text{-}4}G_{3\text{-}0}$
$+ P_{15\text{-}12}P_{11\text{-}8}P_{7\text{-}4}P_{3\text{-}0}C_{-1}$

$G_{(i+3)\text{-}i}$: in 3τ and $P_{(i+3)\text{-}i}$: in 2τ

Carry look ahead: in 5τ

Look Ahead Carry Generator





$C_{n+x} = (G_0'.P_0' + G_0'.C_n')'$
$= (G_0'.P_0')'.(G_0'.C_n')'$
$= (G_0+P_0).(G_0+C_n)$
$= G_0+G_0C_n+G_0P_0+P_0C_n$
$= G_0(1+C_n+P_0)+P_0C_n$
$= G_0+P_0C_n$

Overflow Detection for Adder

$A_3A_2A_1A_0$
$B_3B_2B_1B_0$

$[C_3]$ $S_3S_2S_1S_0$

0000 :	0	1000 :	-8
0001 :	1	1001 :	-7
0010 :	2	1010 :	-6
0011 :	3	1011 :	-5
0100 :	4	1100 :	-4
0101 :	5	1101 :	-3
0110 :	6	1110 :	-2
0111 :	7	1111 :	-1

Range: -8 to +7

+ve with +ve :

0010	(2)	0010	(2)	0111	(7)
0011	(3)	0111	(7)	1110	(-2)
------		------		------	
0101	(5)	1001	(-7)	⊥ 0101	(5)
		Overflow			

-ve with -ve :

1110	(-2)	1110	(-2)	0010	(2)
1101	(-3)	1001	(-7)	1001	(-7)
------		------		------	
⊥ 1011	(-5)	⊥ 0111	(7)	1011	(-5)
		Overflow			

+ve with -ve :

Overflow Flag: O

$O = A_3'B_3'S_3 + A_3B_3S_3'$

$= C_3 \oplus C_2$

References:

- Donald P. Leach, Albert P. Malvino, and Goutam Saha, Digital Principles & Applications 8e, McGraw Hill