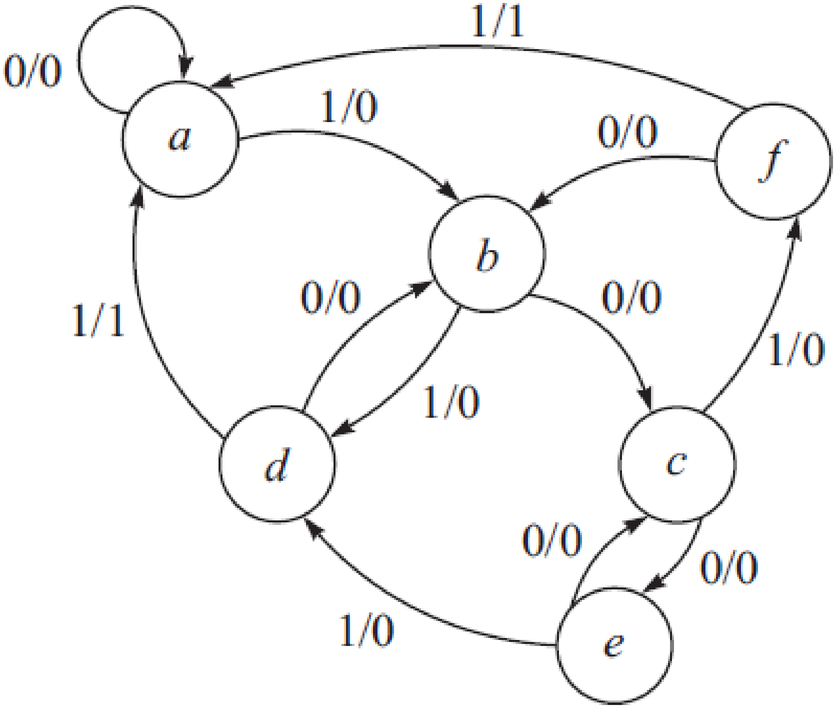
Digital Electronic Circuits
Section 1 (EE, IE)

Lecture 28

State Minimization



$a_0{}^0a_1{}^0b_0{}^0c_1{}^0f_0{}^0b_1{}^0d_1{}^1a_0{}^0a\ldots$

Present state	Next state $X=0$	Next state $X=1$	Present output $X=0$	Present output $X=1$
a	a	b	0	0
b	c	d	0	0
c	e	f	0	0
d	b	a	0	1
e	c	d	0	0
f	b	a	0	1

Two states are considered equivalent if they move to same or equivalent state for every input combination and also generate same output.

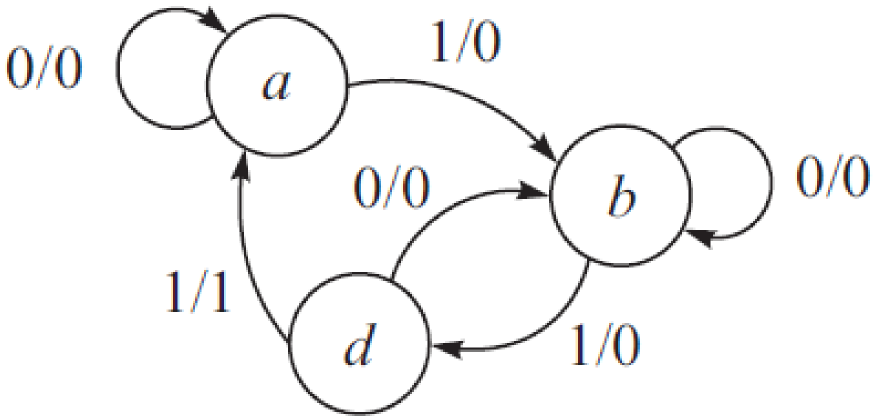
Row Elimination Method

Present state	Next state $X=0$	Next state $X=1$	Present output $X=0$	Present output $X=1$
a	a	b	0	0
b	c	d	0	0
c	e	f	0	0
d	b	a	0	1
e	c	d	0	0
f	b	a	0	1

Present state	Next state $X=0$	Next state $X=1$	Present output $X=0$	Present output $X=1$
a	a	b	0	0
b	c	d	0	0
c	b	f	0	0
d	b	a	0	1
f	b	a	0	1

One row eliminated

Row Elimination Method



Present state	Next state		Present output	
	$X=0$	$X=1$	$X=0$	$X=1$
a	a	b	0	0
b	c	d	0	0
c	b	d	0	0
d	b	a	0	1

Two rows eliminated $a_0{}^0a_1{}^0b_0{}^0b_1{}^0d_0{}^0b_1{}^0d_1{}^1a_0{}^0a..$

Tautology: *b* and *c* are equivalent if *c* and *b* are equivalent

Present state	Next state		Present output	
	$X=0$	$X=1$	$X=0$	$X=1$
a	a	b	0	0
b	b	d	0	0
d	b	a	0	1

Three rows eliminated

Partitioning Method

Partition P_1 : For each input, the output is identical for blocks formed due to partition.

Partition P_2 : If for each input, next states lie in single block of P_1.

Partition P_3 : If for each input, next states lie in single block of P_2.

…

until, $\mathbf{P_k = P_{k-1}}$ i.e. for each input, next states lie in single block of previous partition.

Partition (Output/Next State)	Partition Blocks
P_0	*a b c d e f*
Output for $X = 0$	0 0 0 0 0 0
Output for $X = 1$	0 0 0 1 0 1
P_1	*a b c e* | *d f*
Next state for $X = 0$	*a c e c* | *b b*
Next state for $X = 1$	*b d f d* | *a a*
P_2	*a* | *b c e* | *d f*
Next state for $X = 0$	*a* | *c e c* | *b b*
Next state for $X = 1$	*b* | *d f d* | *a a*
$P_3 = P_2$	*a* | *b c e* | *d f*

Present state	Next state $X=0$	Next state $X=1$	Present output $X=0$	Present output $X=1$
a	a	b	0	0
b	c	d	0	0
c	e	f	0	0
d	b	a	0	1
e	c	d	0	0
f	b	a	0	1

P: $(a)(bce)(df)$

Example with Moore Model

Present State	Next State $X=0$	Next State $X=1$	Present Output
a	b	e	1
b	a	f	1
c	f	c	0
d	b	c	1
e	f	e	0
f	c	a	0

Partition (Output/Next State)	Partition Blocks
P_0 Output	*a b c d e f* 1 1 0 1 0 0
P_1 Next state for $X = 0$ Next state for $X = 1$	*a b d* | *c e f* *b a b* | *f f c* *e f c* | *c e a*
P_2 Next state for $X = 0$ Next state for $X = 1$	*a b d* | *c e* | *f* *b a b* | *f f* | *c* *e f c* | *c e* | *a*
P_3 Next state for $X = 0$ Next state for $X = 1$	*a d* | *b* | *c e* | *f* *b b* | *a* | *f f* | *c* *e c* | *f* | *c e* | *a*
$P_4 = P_3$	*a d* | *b* | *c e* | *f*

P: (*ad*)(*b*)(*ce*)(*f*)

References

References:

- Donald P. Leach, Albert P. Malvino, and Goutam Saha, Digital Principles & Applications 8e, McGraw Hill