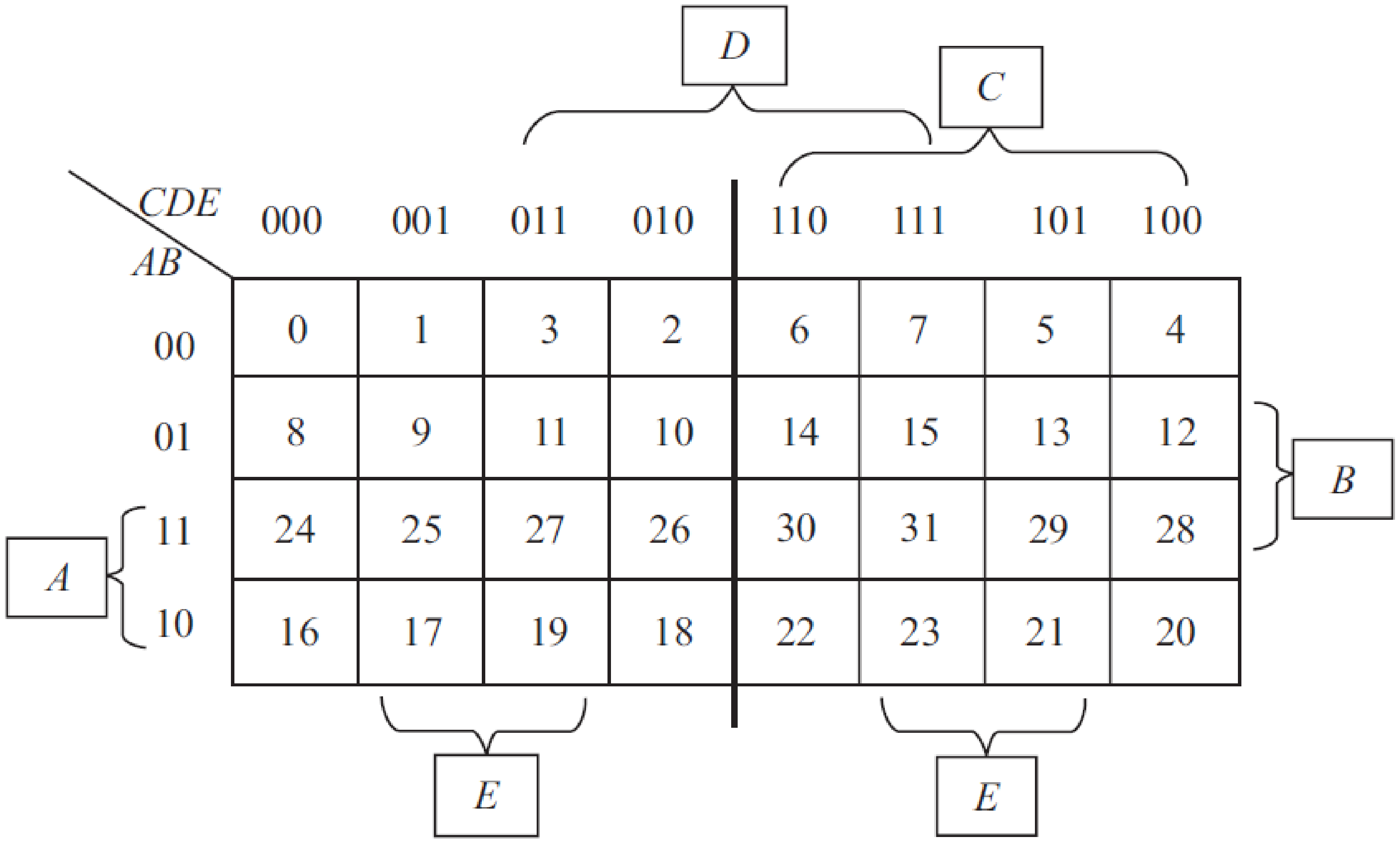
Digital Electronic Circuits
Section 1 (EE, IE)

Lecture 7

Five-Variable Karnaugh Map

Reflection Map



A	*B*	*C*	*D*	*E*	minterm	Notation
0	0	0	0	0	*A′B′C′D′E′*	m_0
0	0	0	0	1	*A′B′C′D′E*	m_1
…	…	…	…	…		
1	1	1	1	0	*ABCDE′*	m_{30}
1	1	1	1	1	*ABCDE*	m_{31}

Five-Variable Karnaugh Map

Example

$F(A,B,C,D,E) = \sum m(2,6,9,11,13,15,18,19,22,23,25,27,29,31)$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

AB \ CDE	000	001	011	010	110	111	101	100
00	0	0	0	1	1	0	0	0
01	0	1	1	0	0	1	1	0
11	0	1	1	0	0	1	1	0
10	0	0	1	1	1	1	0	0

B'.D.E'

B.E

A.D.E

A.B'.D

$F = B.E + A.B'.D + B'.D.E'$

Or

$F = B.E + A.D.E + B'.D.E'$

Entered Variable (EV)

	A	*B*	*C*	*Y*
Y=0	0	0	0	0
	0	0	1	0
Y=*C'*	0	1	0	1
	0	1	1	0
Y=0	1	0	0	0
	1	0	1	0
Y=1	1	1	0	1
	1	1	1	1

***F*(*A*,*B*,*C*) = ∑ m(2,6,7)**

A	*B*	*Y*
0	0	0
0	1	*C'*
1	0	0
1	1	1

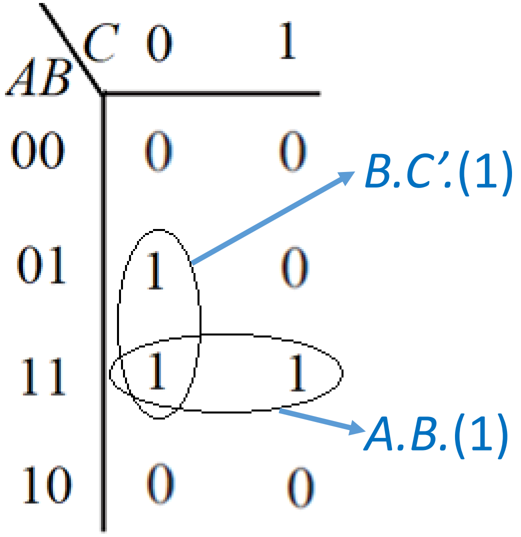
Y=0

Y=*A*

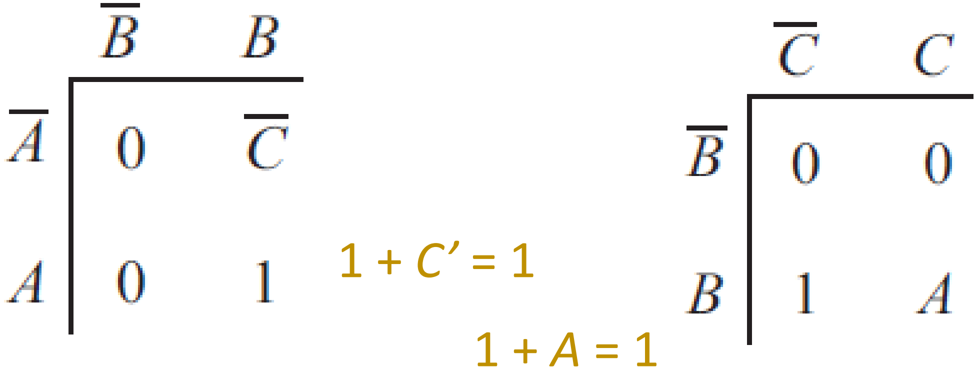
A	*B*	*C*	*Y*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

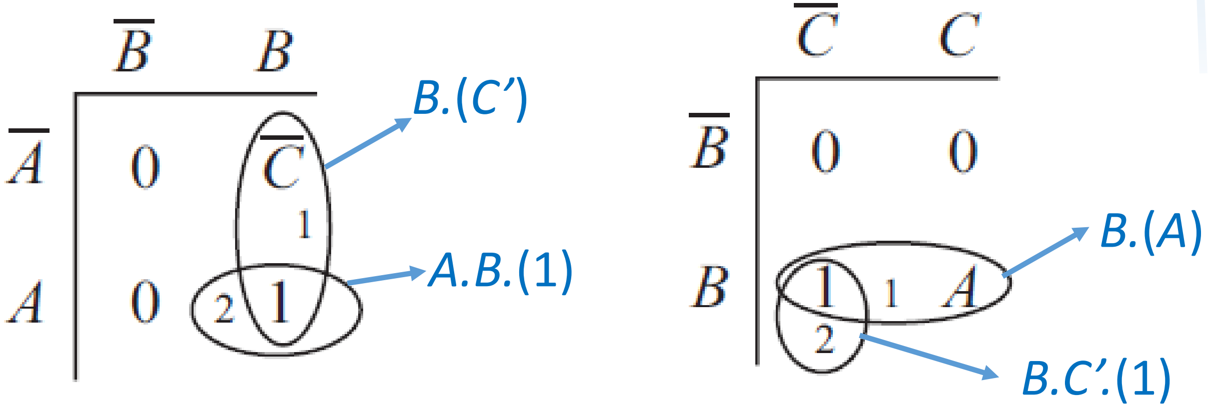
B	*C*	*Y*
0	0	0
0	1	0
1	0	1
1	1	*A*

EV Map: SOP Simplification

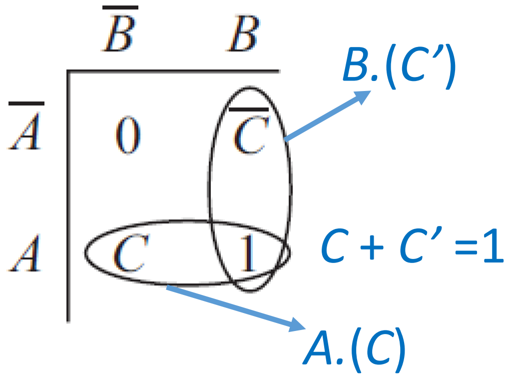


$F(A,B,C) = \sum m(2,6,7)$



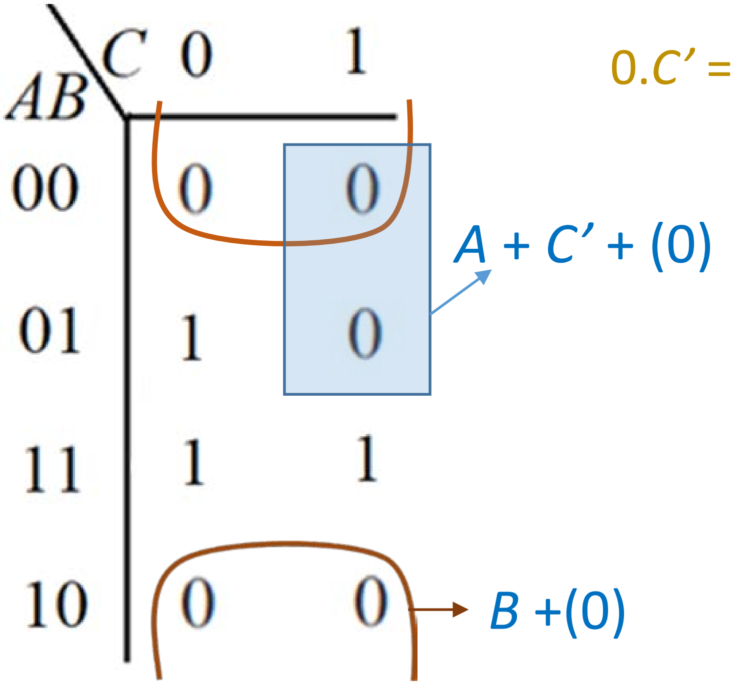


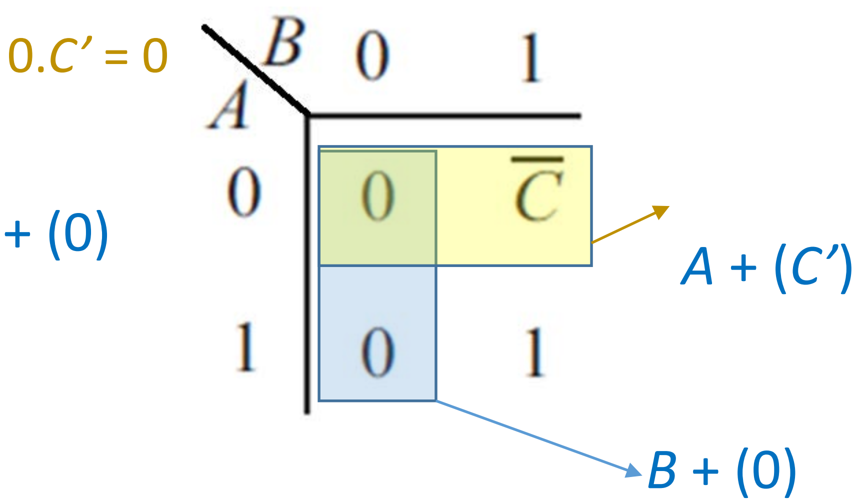
$F(A,B,C) = B.C' + A.B$

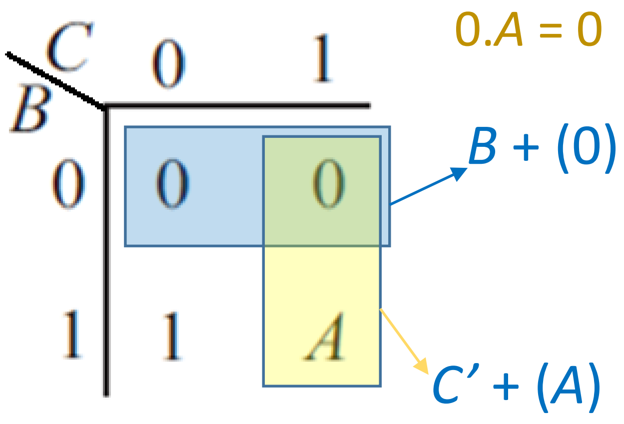


$F(A,B,C) = B.C' + A.C$

EV Map: POS Simplification







***F*(*A*,*B*,*C*) = *B*.(*A* + *C′*)**

Also, ***C*.*C′* = 0** can be considered by which ***C*** could be in one sum term and ***C′*** in another sum term to cover a **0**.

Simplification: Five-Variable

	A	B	C	D	E	Y
0	0	0	0	0	0	0
	0	0	0	0	1	0
E’	0	0	0	1	0	1
	0	0	0	1	1	0
0	0	0	1	0	0	0
	0	0	1	0	1	0
E’	0	0	1	1	0	1
	0	0	1	1	1	0

	A	B	C	D	E	Y
E	0	1	0	0	0	0
	0	1	0	0	1	1
E	0	1	0	1	0	0
	0	1	0	1	1	1
E	0	1	1	0	0	0
	0	1	1	0	1	1
E	0	1	1	1	0	0
	0	1	1	1	1	1

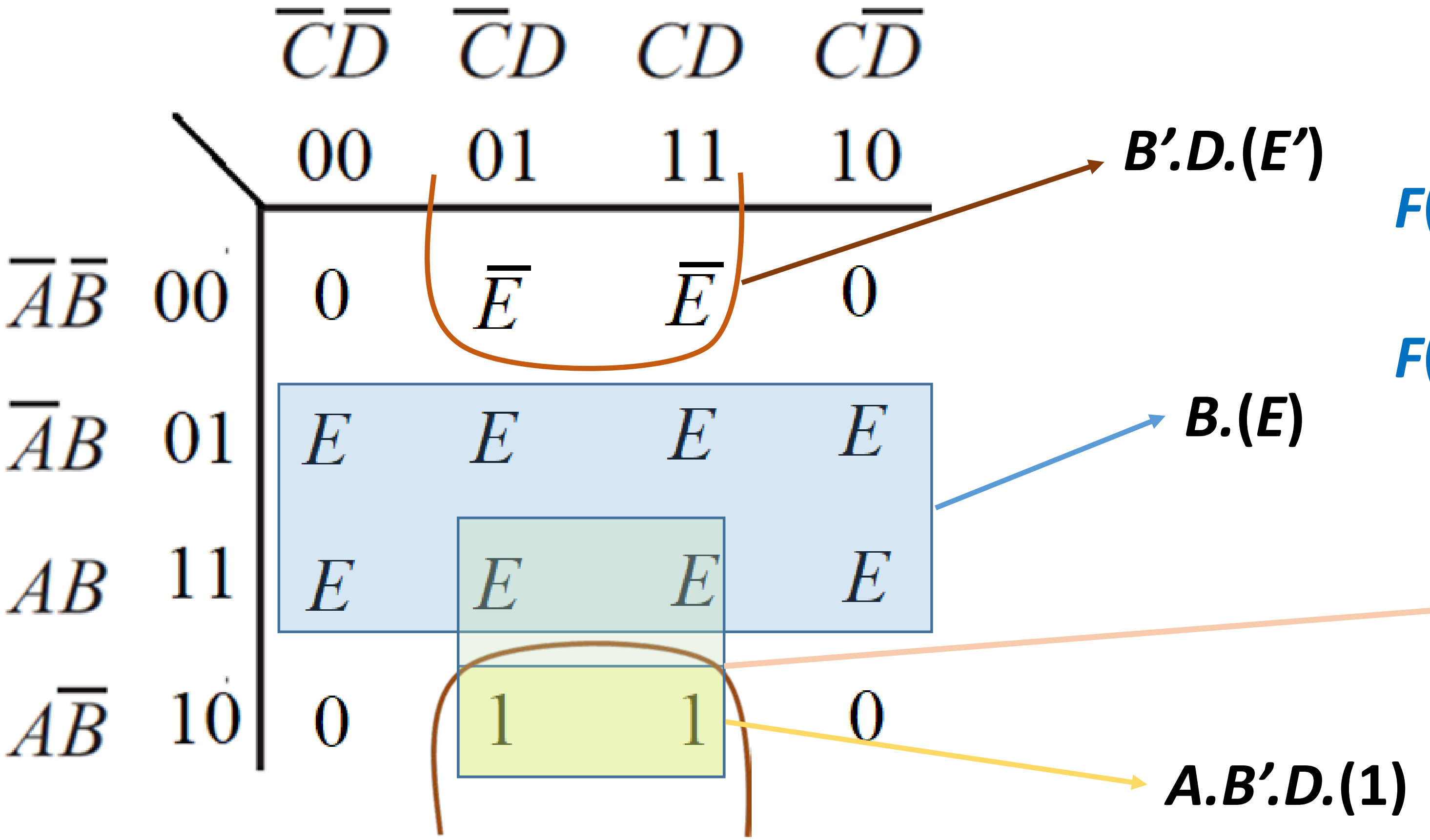
	A	B	C	D	E	Y
0	1	0	0	0	0	0
	1	0	0	0	1	0
1	1	0	0	1	0	1
	1	0	0	1	1	1
0	1	0	1	0	0	0
	1	0	1	0	1	0
1	1	0	1	1	0	1
	1	0	1	1	1	1

	A	B	C	D	E	Y
E	1	1	0	0	0	0
	1	1	0	0	1	1
E	1	1	0	1	0	0
	1	1	0	1	1	1
E	1	1	1	0	0	0
	1	1	1	0	1	1
E	1	1	1	1	0	0
	1	1	1	1	1	1

$F(A,B,C,D,E) = \sum m(2,6,9,11,13,15,18,19,22,23,25,27,29,31)$

Simplification: Five-Variable

A.D.E and *B'.D.E'* together cover 1s covered by *A.B'.D* where is considered as 1 = *E* + *E'*



F(A,B,C,D,E) = *B.E* + *A.B'.D* + *B'.D.E'*

Or

F(A,B,C,D,E) = *B.E* + *A.D.E* + *B'.D.E'*

QM Algorithm

- Finding Prime Implicants (P.I.) through an iterative process.
- P.I is a product term that cannot be combined with any other product term (least no. of literals)
- Finding Essential P.I. to cover all minterms.
- Selecting minimal set.

$Y = \sum m(0,1,2,3, 10,11,12, 13,14,15)$

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Stage 1	
ABCD	
0 0 0 0	(0)
0 0 0 1	(1)
0 0 1 0	(2)
0 0 1 1	(3)
1 0 1 0	(10)
1 1 0 0	(12)
1 0 1 1	(11)
1 1 0 1	(13)
1 1 1 0	(14)
1 1 1 1	(15)

Grouping according to no. of 1s input variable combi. have in T.T. for *Y* = 1

Numbering of minterms to ensure all are included as its order changes

QM Algorithm

Stage 1 ABCD	
0 0 0 0	(0)√
0 0 0 1	(1)√
0 0 1 0	(2)√
0 0 1 1	(3)√
1 0 1 0	(10)√
1 1 0 0	(12)√
1 0 1 1	(11)√
1 1 0 1	(13)√
1 1 1 0	(14)√
1 1 1 1	(15)√

Stage 2 ABCD	
0 0 0 -	(0,1)
0 0 - 0	(0,2)
0 0 - 1	(1,3)
0 0 1 -	(2,3)
- 0 1 0	(2,10)
- 0 1 1	(3,11)
1 0 1 -	(10,11)
1 - 1 0	(10,14)
1 1 0 -	(12,13)
1 1 - 0	(12,14)
1 - 1 1	(11,15)
1 1 - 1	(13,15)
1 1 1 -	(14,15)

- **Grouping of terms from adjacent blocks of Stage 1 which differ in only one position of input variable combinations.**
- **Writing '–' (eqv. to don't care) in those positions.**
- **Ticking terms of Stage 1 which could make it to Stage 2.**

QM Algorithm

Stage 1 ABCD	
0 0 0 0	(0)√
0 0 0 1	(1)√
0 0 1 0	(2)√
0 0 1 1	(3)√
1 0 1 0	(10)√
1 1 0 0	(12)√
1 0 1 1	(11)√
1 1 0 1	(13)√
1 1 1 0	(14)√
1 1 1 1	(15)√

Stage 2 ABCD	
0 0 0 -	(0,1)√
0 0 - 0	(0,2)√
0 0 - 1	(1,3) √
0 0 1 -	(2,3) √
- 0 1 0	(2,10)√
- 0 1 1	(3,11) √
1 0 1 -	(10,11)√
1 - 1 0	(10,14)√
1 1 0 -	(12,13)√
1 1 - 0	(12,14)√
1 - 1 1	(11,15)√
1 1 - 1	(13,15)√
1 1 1 -	(14,15)√

Stage 3 ABCD	
0 0 - -	(0,1,2,3)
0 0 - -	(0,2,1,3)
- 0 1 -	(2,10,3,11)
1 - 1 -	(10,11,14,15)
1 - 1 -	(10,14,11,15)
1 1 - -	(12,13,14,15)
1 1 - -	(12,14,13,15)

- **Similar grouping of Stage 2 terms in Stage 3.**
- **Ticking terms of Stage 2 which could make it to Stage 3.**
- **This continues as long as grouping possible for next stages.**
- **In each stage no. of blocks reduce.**

QM Algorithm

ABCD	Stage 3
0 0 - -	(0,1,2,3)
0 0 - -	(0,2,1,3)
- 0 1 -	(2,10,3,11)
1 - 1 -	(10,11,14,15)
1 - 1 -	(10,14,11,15)
1 1 - -	(12,13,14,15)
1 1 - -	(12,14,13,15)

- **For this example, no further grouping and thus, no Stage 4.**
- **Every term in Stage 3 remains unticked. There were no unticked term in Stage 2 and Stage 1.**
- **Each unticked term of any stage is to contribute to generation of P.I.**
- **For this example, 4 P.I.s which are mutually exclusive for *ABCD* combinations 00--, -01-, 1-1-, 11--.**

QM Algorithm

Stage 1

ABCD	
0 0 0 0	(0)√
0 0 0 1	(1)√
0 0 1 0	(2)√
0 0 1 1	(3)√
1 0 1 0	(10)√
1 1 0 0	(12)√
1 0 1 1	(11)√
1 1 0 1	(13)√
1 1 1 0	(14)√
1 1 1 1	(15)√

Stage 2

ABCD	
0 0 0 -	(0,1)√
0 0 - 0	(0,2)√
0 0 - 1	(1,3) √
0 0 1 -	(2,3) √
- 0 1 0	(2,10)√
- 0 1 1	(3,11) √
1 0 1 -	(10,11)√
1 - 1 0	(10,14)√
1 1 0 -	(12,13)√
1 1 - 0	(12,14)√
1 - 1 1	(11,15)√
1 1 - 1	(13,15)√
1 1 1 -	(14,15)√

Stage 3

ABCD	
0 0 - -	(0,1,2,3)
0 0 - -	(0,2,1,3)
- 0 1 -	(2,10,3,11)
1 - 1 -	(10,11,14,15)
1 - 1 -	(10,14,11,15)
1 1 - -	(12,13,14,15)
1 1 - -	(12,14,13,15)

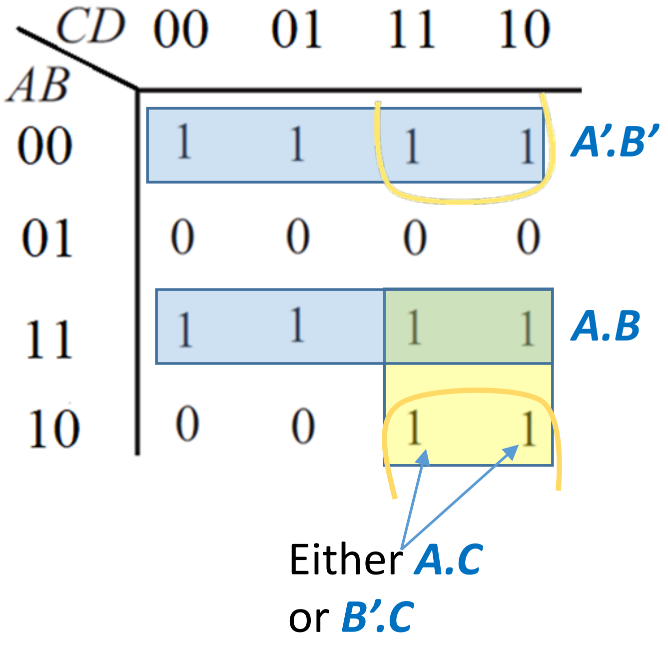
ABCD	P.I.	minterms
0 0 - -	A'.B'	0,1,2,3
- 0 1 -	B'.C	2,3,10,11
1 - 1 -	A.C	10,11,14, 15
1 1 - -	A.B	12,13,14,15

P.I. Table

(All stages together)

QM Algorithm

P.I.	minterms
A'.B'	**0,1**,2,3
B'.C	2,3,**10,11**
A.C	**10,11**,14, 15
A.B	**12,13**,14,15



Essential P.I. Table

P.I.	0	1	2	3	10	11	12	13	14	15
A'.B'	**√**	**√**	√	√						
B'.C			√	√	**√**	**√**				
A.C					**√**	**√**			√	√
A.B							**√**	**√**	√	√

Essential P.I. : ***A'.B'*, *A.B* and any one of *B'.C* or *A.C***

Minimal set: $Y = A'.B' + A.B + B'.C$ **or** $Y = A'.B' + A.B + A.C$

QM Algorithm with Don’t Care

- **Include don’t care in finding P.I.**
- **Exclude don’t care in finding Essential P.I.**

P.I.	minterms
A’.B’	(0),(1),2,3
B’.C	**2,3,10,11**
A.C	10,11,14, 15
A.B	**12,13,14,15**

Essential P.I. Table

P.I.	2	3	10	11	12	13	14	15
A’.B’	√	√						
B’.C	√	√	**√**	**√**				
A.C			**√**	**√**			√	√
A.B					**√**	**√**	√	√

Example:

$$Y = \sum m(2,3,10,11,12,13,14,15) + d(0,1)$$

P.I. as before

Essential P.I. : ***B’.C*** **and** ***A.B***

$$Y = A.B + B'.C$$

References:

- Donald P. Leach, Albert P. Malvino, and Goutam Saha, Digital Principles & Applications 8e, McGraw Hill