Digital Electronic Circuits
Section 1 (EE, IE)

Lecture 5

Recap.: Shanon's Expansion Theorem

$$F(x_1, x_2, x_3, \ldots, x_N) = x_1'.F(0, x_2, x_3, \ldots, x_N) + x_1.F(1, x_2, x_3, \ldots, x_N)$$

$$F(x_1, x_2, x_3, \ldots, x_N) = x_1'.[x_2'.F(0, 0, x_3, \ldots, x_N) + x_2.F(0, 1, x_3, \ldots, x_N)] + x_1.[x_2'.F(1, 0, x_3, \ldots, x_N) + x_2.F(1, 1, x_3, \ldots, x_N)]$$

Nesting

$$F(x,y) = x'.F(0,y) + x.F(1,y) = x'.[y'.F(0,0) + y.F(0,1)] + x.[y'.F(1,0) + y.F(1,1)]$$

$$F(x,y) = x'.y'.F(0,0) + x'.y.F(0,1) + x.y'.F(1,0) + x.y.F(1,1)$$

Truth Table to Boolean Function: Minterm

$\boldsymbol{F(x,y) = x + x'.y}$

x	y	$F(x,y)$	
0	0	0	$x'.y'$
0	1	1	⇨ $\boldsymbol{x'.y}$
1	0	1	⇨ $\boldsymbol{x.y'}$
1	1	1	⇨ $\boldsymbol{x.y}$

$$\boldsymbol{F(x,y)} = x'.y'.F(0,0) + x'.y.F(0,1) + x.y'.F(1,0) + x.y.F(1,1)$$
$$= \boldsymbol{x'.y + x.y' + x.y}$$

- **Product term containing all variables of a function** (primed, unprimed): *fundamental product* or *standard product* or **minterm**.
- Minterms of $\boldsymbol{F(x,y)}$: $\boldsymbol{x'.y'}$, $\boldsymbol{x'.y}$, $\boldsymbol{x.y'}$, $\boldsymbol{x.y}$ (also designated as $\boldsymbol{m_0}$, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{m_3}$, respectively)
- The minterm designation $\boldsymbol{m_i}$ corresponds to **decimal eqv. of x,y combination** of a row.

  [Decimal Eqv. = $B_3.2^3 + B_2.2^2 + B_1.2^1 + B_0.2^0$]

- Boolean function: **Taking OR of minterms** associated with **1** in function output.

Minterm: More Example

x	y	z	minterm	Notation
0	0	0	$x'y'z'$	m_0
0	0	1	$x'y'z$	m_1
0	1	0	$x'yz'$	m_2
0	1	1	$x'yz$	m_3
1	0	0	$xy'z'$	m_4
1	0	1	$xy'z$	m_5
1	1	0	xyz'	m_6
1	1	1	xyz	m_7

$F(x,y,z) = (x + y).(x + z)$

x	y	z	$F(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Sum of Product (SOP): Canonical Form

$F(x,y,z) = x'yz + xy'z' + xy'z + xyz' + xyz$

$F(x,y,z) = m_3 + m_4 + m_5 + m_6 + m_7$

$F(x,y,z) = \sum m(3,4,5,6,7)$

Truth Table to Boolean Function: Maxterm

x	y	$F(x,y)$	
0	0	0	$\boldsymbol{x + y}$
0	1	1	$\boldsymbol{x + y'}$
1	0	1	$\boldsymbol{x' + y}$
1	1	1	$\boldsymbol{x' + y'}$

$$\boldsymbol{F(x,y)} = [x' + y' + F(1,1)].[x' + y + F(1,0)]$$
$$.[x + y' + F(0,1)].[x + y + F(0,0)]$$
$$= 1.1.1.(x + y) = \boldsymbol{x + y}$$

- **Sum term containing all variables of a function** (primed, unprimed): *fundamental sum* or *standard sum* or **Maxterm**.
- Maxterms of $\boldsymbol{F(x,y)}$: $\boldsymbol{x+y, x+y', x'+y, x'+y'}$ (also designated as $\boldsymbol{M_0, M_1, M_2, M_3}$, respectively)
- The Maxterm designation $\boldsymbol{M_i}$ corresponds to **decimal eqv. of *x,y* combination** of a row.
- Boolean function: **Taking AND of Maxterms** associated with **0** in function output.

$$\boldsymbol{F(x_1, x_2, x_3, \ldots, x_N) = [x_1' + F(1, x_2, x_3, \ldots, x_N)].[x_1 + F(0, x_2, x_3, \ldots, x_N)]}$$

Maxterm: More Example

x	y	z	Maxterm	Notation
0	0	0	$x+y+z$	M_0
0	0	1	$x+y+z'$	M_1
0	1	0	$x+y'+z$	M_2
0	1	1	$x+y'+z'$	M_3
1	0	0	$x'+y+z$	M_4
1	0	1	$x'+y+z'$	M_5
1	1	0	$x'+y'+z$	M_6
1	1	1	$x'+y'+z'$	M_7

$F(x,y,z) = (x + y).(x + z)$

x	y	z	$F(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Product of Sum(POS): Canonical Form

$F(x,y,z) = (x+y+z).(x+y+z').(x+y'+z)$

$F(x,y,z) = M_0.M_1.M_2$

$F(x,y,z) = \prod M(0,1,2)$

More on Canonical Forms

Conversion between Canonical Forms

We have seen, **(i)** $\boldsymbol{F(x,y) = x + x'.y} = \sum m(1,2,3) = \prod M(0)$
and **(ii)** $\boldsymbol{F(x,y,z) = (x + y).(x + z)} = \sum m(3,4,5,6,7) = \prod M(0,1,2)$

From **example(ii) Truth Table** (T.T.):
$\boldsymbol{F'(x,y,z)} = \sum m(0,1,2) = m_0 + m_1 + m_2$
$\boldsymbol{F(x,y,z) = (F'(x,y,z))'} = (m_0 + m_1 + m_2)' = m_0'.m_1'.m_2'$ (De Morgan's Th.)
We already noted, $F(x,y,z) = \prod M(0,1,2) = M_0.M_1.M_2$

Extending to any T.T., $\boldsymbol{m_i' = M_i}$

Ex. (ii) Truth Table

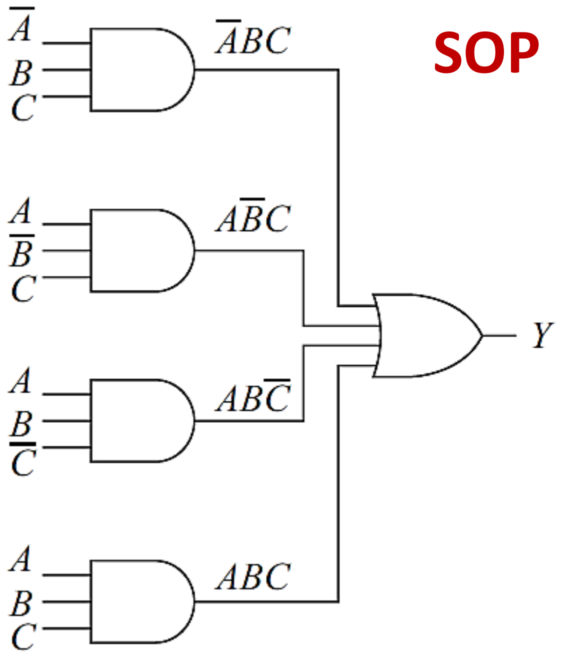
x	*y*	*z*	*F(x,y,z)*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$\boldsymbol{F(x,y) = x + x'.y} = x.(y + y') + x'.y = \boldsymbol{x'.y + x.y' + x.y}$
$\boldsymbol{F(x,y,z) = (x + y).(x + z)} = (x + y + z.z').(x + z + y.y')$
$= (x + y + z).(x + y + z').(x + z + y).(x + z + y')$
$\boldsymbol{= (x + y + z).(x + y + z').(x + y' + z)}$

Getting minterms and Maxterms algebraically

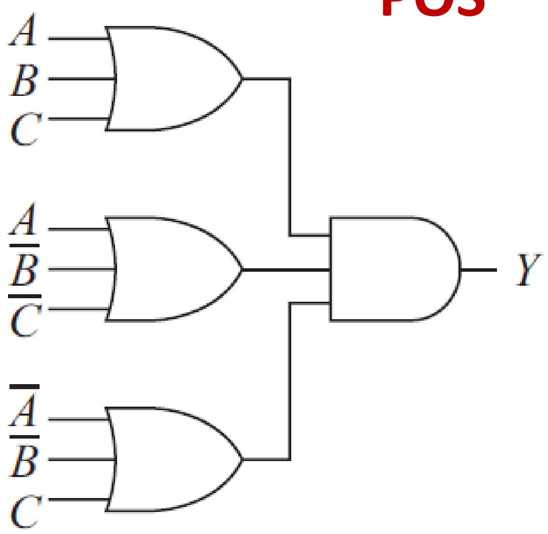
Two Level Implementation

SOP



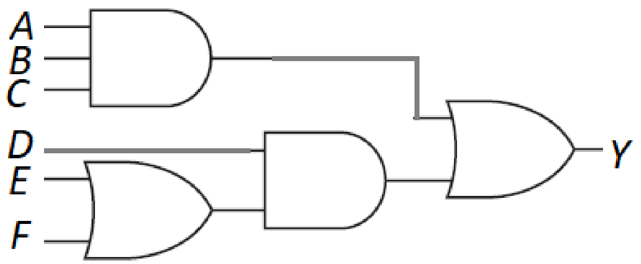
$Y = F(A,B,C) = \sum m(3,5,6,7)$

POS



$Y = F(A,B,C) = \prod M(0,3,6)$

SOP: 3-level implementation

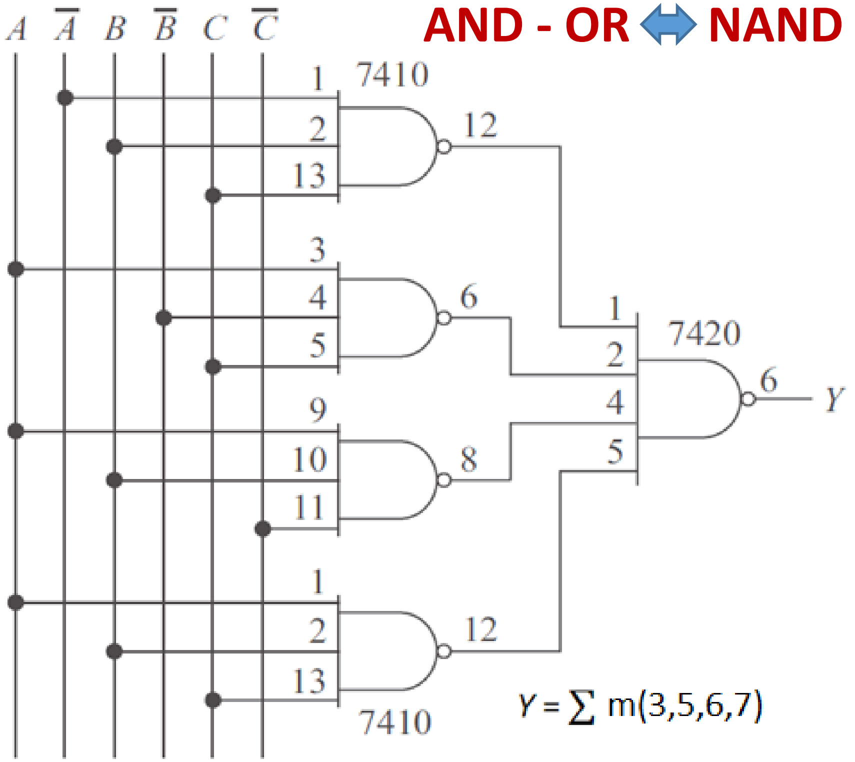


$Y = ABC + D(E + F)$

NAND - NAND, NOR - NOR Implementation

$$\mathbf{A.B + C.D + E.F} = ((A.B + C.D + E.F)')'$$
$$= \mathbf{((A.B)'.(C.D)'.(E.F)')'}$$

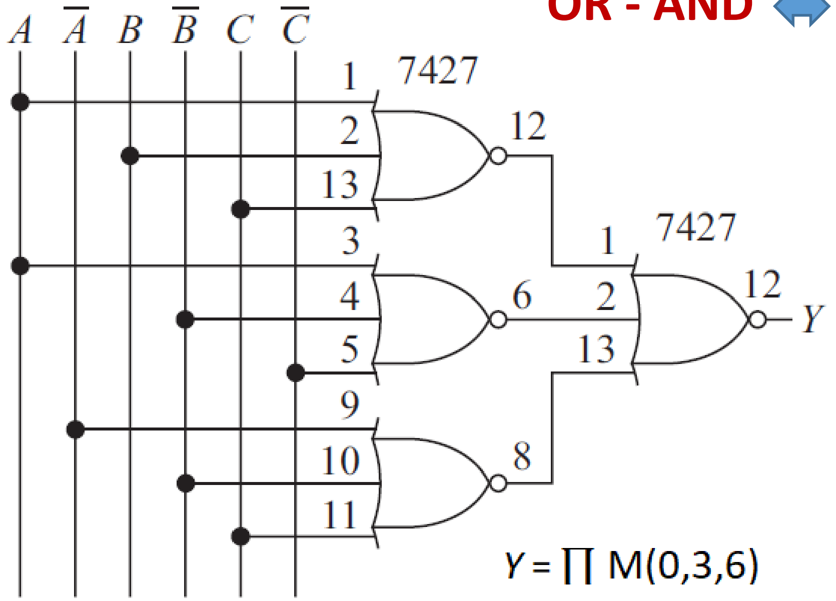
AND - OR ⇔ NAND - NAND



$Y = \sum m(3,5,6,7)$

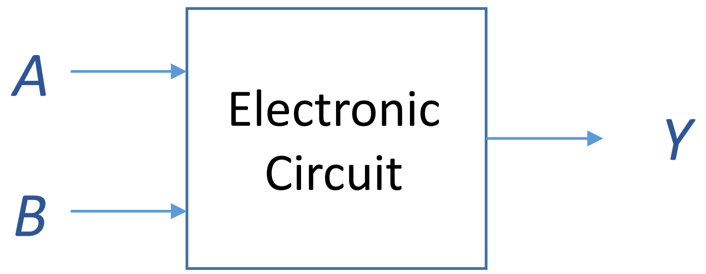
$$\mathbf{(A + B).(C + D).(E + F)} = (((A + B).(C + D).(E + F))')'$$
$$= \mathbf{((A + B)'+(C + D)'+(E + F)')'}$$

OR - AND ⇔ NOR - NOR



$Y = \prod M(0,3,6)$

Positive and Negative Logic



A	*B*	*Y*
Low	Low	Low
Low	High	High
High	Low	High
High	High	High

In terms of voltage levels

Positive Logic
H = 1
L = 0

A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

OR

Negative Logic
H = 0
L = 1

A	*B*	*Y*
1	1	1
1	0	0
0	1	0
0	0	0

AND

Positive Logic	Negative Logic
OR	AND
AND	OR
NAND	NOR
NOR	NAND

References:

- Donald P. Leach, Albert P. Malvino, and Goutam Saha, Digital Principles & Applications 8e, McGraw Hill
- M. Morris Mano, and Michael D. Ciletti, Digital Design 5e, Pearson