Tutorial 3

EC31003: Digital Electronic Circuits

IIT Kharagpur, Autumn Semester 2020-21

26th September 2020

Time limit (25 min.)

1. Identify the type of hazard and show the corresponding hazard free implementation for the following Boolean expression $F(A,B,C,D) = (A+C)(A'+D')(B'+C'+D)$.

2. a. Represent the following expression using two-input NAND gate: $Y = AB+B'C+CD'E+DEF'$

   b. Find the total cost of the following expression?
   $Y = ((P+Q)'+(((R+S)')'+T)')'$

   c. Find the value of Gate input cost G and GN from the following POS function:
   $F(A,B,C,D) = \prod M(0, 1, 4, 7).D(6, 11, 14, 15)$

3. a. Draw the KMap for the function $F(A,B,C,D)= \Pi\ M(2, 3, 6, 8, 9, 12, 13, 14)$

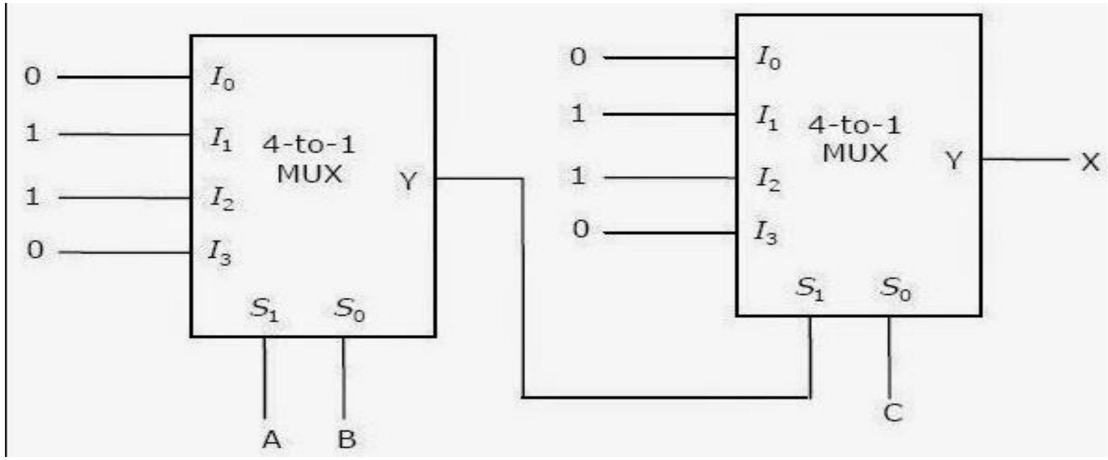
   b. Determine the minimized function in product of sums form.

   c. Are there any static-0 hazards in the minimized function? If yes, find them and eliminate them.

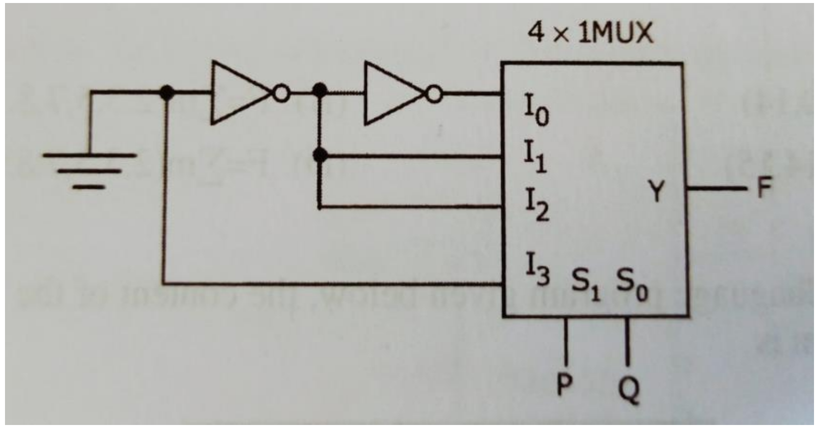
Time limit (20 min.)

4. Write the truth table for the following Boolean expression , and realize it using only an 8:1 multiplexer.
   $F(A,B,C,D)=\overline{AB}+BD+AB\overline{C}$.

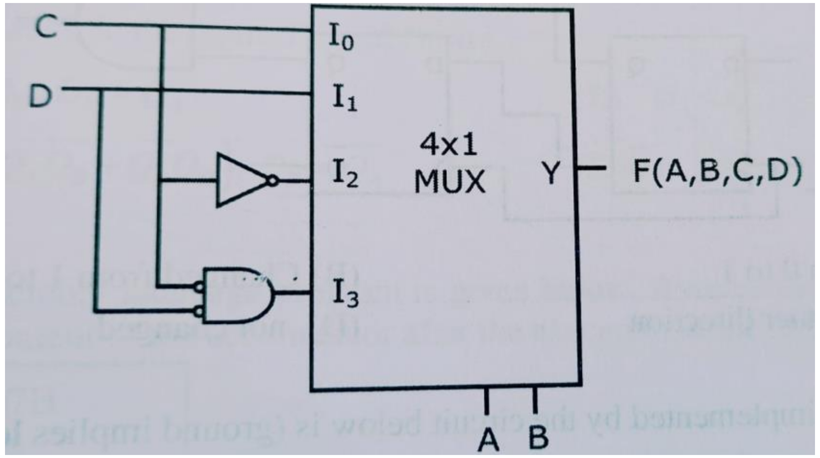
5. a. In the following circuit X is given by -



b. Find the logic function F implemented by the below multiplexer.



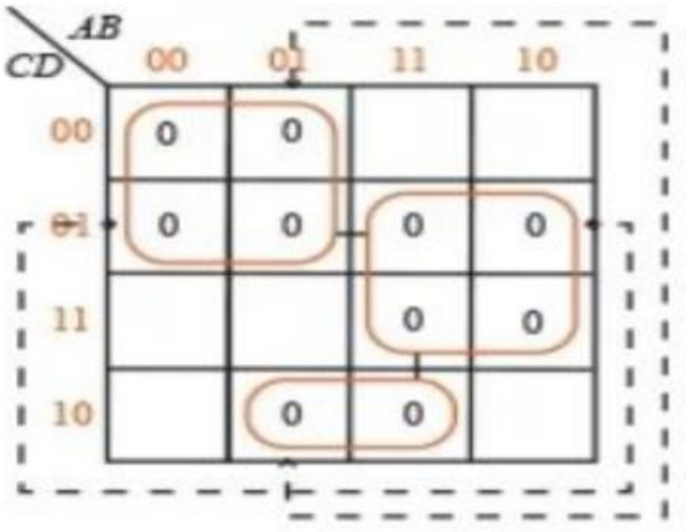
c. Find the canonical POS form solution of the below diagram.



Solution 1

Static 0 hazard.

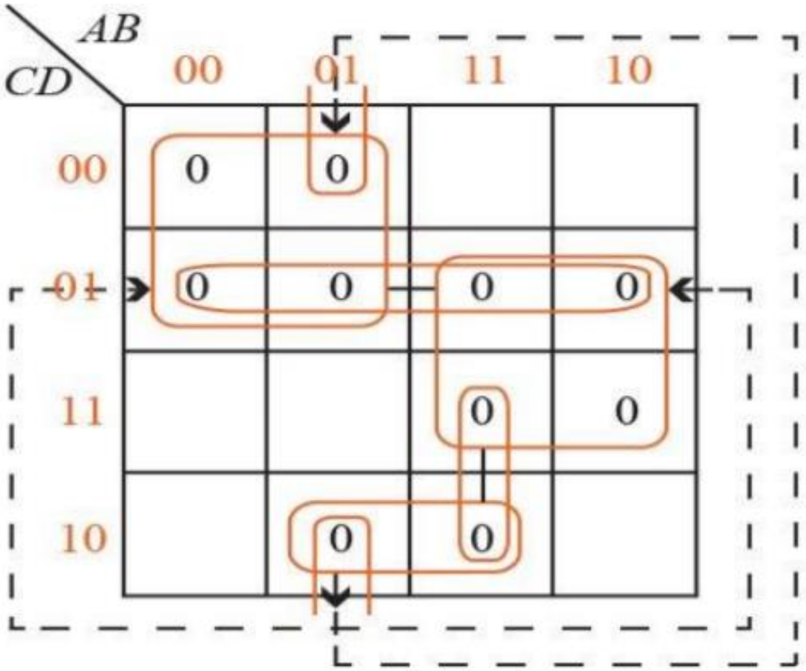
The K-map implementation for the expression is shown below. It has four pairs of adjacent 0's that are not covered by a common loop. The arrows indicate each 0 which is not being looped which causes a 0 hazard. If A=0, B=1, D=0 and C changes from 0 to 1, there is a chance that spike can appear at the output for any combination of gate delays.



Solution 1 (cont.)

Elimination of this hazard can be done using three additional loops in the K Map which includes the redundant terms and this will eliminate the 0-hazards,

$F = (A + C)(A' + D')(B' + C' + D)(C + D')(A + B' + D)(A' + B' + C')$



The final K Map

Solution 2 (a)

a. $Y = ((AB)'.\ (B'C)'.\ (((CD')')'E)'.\ (((DE)')'F')')'$

Solution 2 (b)

b. Total Cost(TC) = Number of gates + Number of gate Inputs

= 5 + 10 = 15

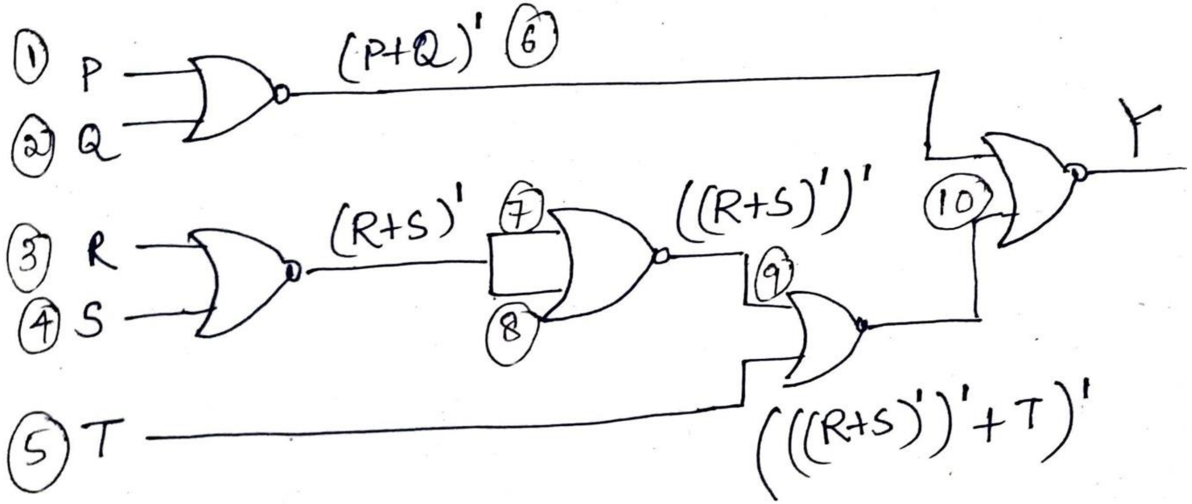
Note: Here, we have considered 2-input NOR gate implementation. If we consider two-level with no restriction in fan-in of logic gates then the expression will be (P+Q)(R+S+T) and number of gate inputs will be 5+2=7. As, here we used two-input NOR gate implementation so the no. of gate inputs =5 x 2 =10

2. b. $Y = ((P+Q)' + (((R+S)')' + T)')'$ (2-input NOR gates)

Total cost = Number of gates + Number of gate inputs

No. of gates = 5

No. of gate inputs = $(2 \times 5) = 10$



∴ No. of gate inputs = 10

Total cost = 5 + 10 = 15

Solution 2 (c)

2c. G (Gate input cost without counting NOT gates)

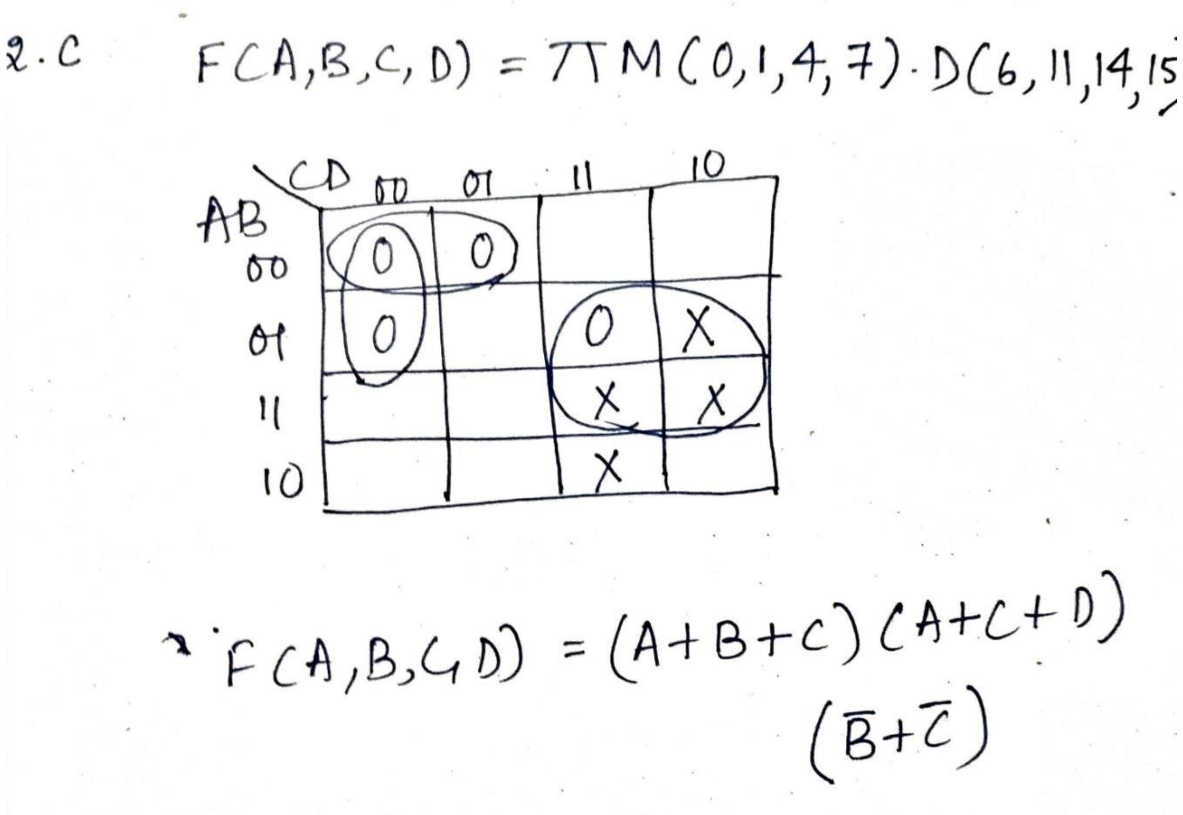
GN (Gate input cost counting NOT gates)

$F = \prod M(0, 1, 4, 7).D(6, 11, 14, 15)$

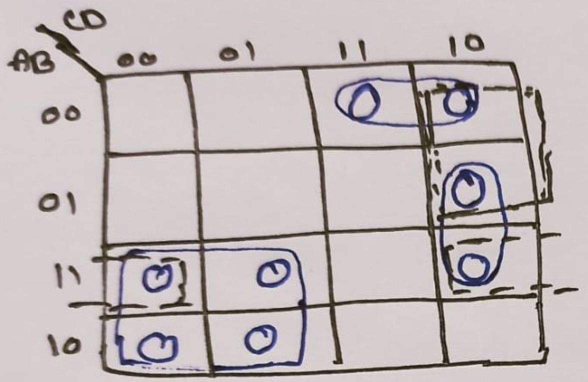
$= (A+B+C)(A+C+D)(B'+C')$

$L = 8, T = 3, N = 2,$

$G = L+T = 11$ and $GN = L+T+N = 13$



Solution 3



→ $(A+B+C')(B'+C'+D)(A'+C)$ → Minimized POS

→ $(A+B+C')(B'+C'+D)(A'+C)\underbrace{(A+C'+D)(A'+B'+D)}_{\text{Hazard free}}$

Solution 4

Given function is

$$f = \overline{AB} + BD + AB\overline{C}$$

$$= \overline{A} + (\overline{B} + B)(\overline{B} + D) + AB\overline{C}$$

$$= \overline{A} + \overline{B} + D + AB\overline{C}$$

Truth table for the given function f is

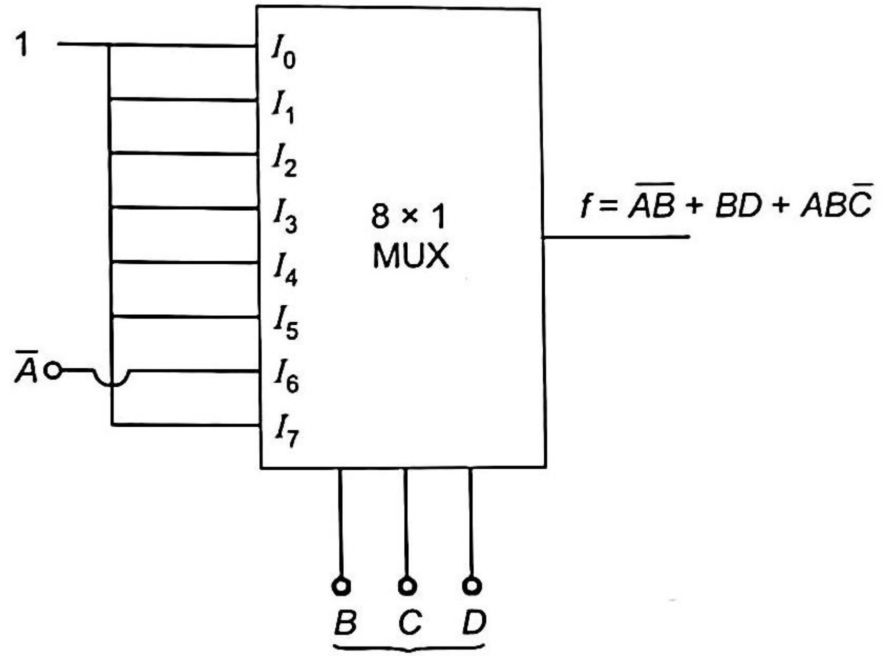
A	B	C	D	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

k-map

$AB \backslash CD$	00	01	11	10
00	1_0	1_1	1_3	1_2
01	1_4	1_5	1_7	1_6
11	1_{12}	1_{13}	1_{15}	0_{14}
10	1_8	1_9	1_{11}	1_{10}

realization of given function using 8 × 1 MUX

A \ BCD	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
0	0	1	2	3	4	5	6	7
1	8	9	10	11	12	13	14	16
	1	1	1	1	1	1	$0 \to \overline{A}$	1



Solution 5 (a)

<u>MUX 1</u>

Y=A'B'.0+A'B.1+AB'.1+AB.0

Y=A'B+AB'

<u>MUX 2</u>

X=Y'C'.0+Y'C.1+YC'.1+YC.0

X=Y'C+YC'

X=(A'B'+AB)C+(A'B+AB')C'

X=A'B'C+ABC+A'BC'+AB'C'

Solution 5 (b) & (c)

Sol. 5 (b) - Output is 1 only when the select lines are PQ' or P'Q.
Hence,
F=P'Q+PQ'-→XOR(P,Q)

Sol. 5 (c) -

→

A	B	Y
0	0	C
0	1	D
1	0	$\bar{C}$
1	1	$\bar{C}\bar{D}$

→

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	1	0	0	0
10	1	1	0	0

$$Y = \sum m(2,3,5,7,8,9,12)$$

$$\Rightarrow \prod M(0,1,4,6,10,11,13,14,15)$$