Digital Electronic Circuits
Section 1 (EE, IE)

Lecture 6

Two-Variable Karnaugh Map

$A \backslash B$	$\bar{B}$ 0	B 1
$\bar{A}$ 0	m_0 $\bar{A}\bar{B}$	m_1 $\bar{A}B$
A 1	m_2 $A\bar{B}$	m_3 AB

$Y = F(A,B) = \sum m(2,3)$

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

	$\bar{B}$	B
$\bar{A}$	0	0
A	1	1

$Y = A$

$Y = F(A,B) = \sum m(1,2,3)$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

	$\bar{B}$	B
$\bar{A}$	0	1
A	1	1

$Y = A + B$

SOP: Simplification

Three Variable Karnaugh Map

- Largest logically adjacent group of size 2^i
- Minimum no. of groups to cover all 1s
- Each group gives one product term
- Variables remaining constant form product term (1:unprimed, 0:primed)
- All product terms are summed.

	$AB \backslash C$	$\overline{C}$ 0	C 1
$\overline{A}\,\overline{B}$	00	0	1
$\overline{A}B$	01	2	3
AB	11	6	7
$A\overline{B}$	10	4	5

F(*A*,*B*,*C*) minterm numbers

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$Y = F(A,B,C)$
$= \sum m(2,4,5,6,7)$

	$\overline{C}$	C
$\overline{A}\,\overline{B}$	0	0
$\overline{A}B$	1	0
AB	1	1
$A\overline{B}$	1	1

B.C'

A

$Y = A + B.C'$

Four-Variable Karnaugh Map

		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	1	3	2
$\bar{A}B$	01	4	5	7	6
AB	11	12	13	15	14
$A\bar{B}$	10	8	9	11	10

F(A,B,C,D)
minterm numbers

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

***Y* = *F*(*A*,*B*,*C*,*D*)**
= ∑ m(0,1,2,6,8,10,13,14)
= *B'D' + CD' + A'B'C' + ABC'D*

A'B'C'

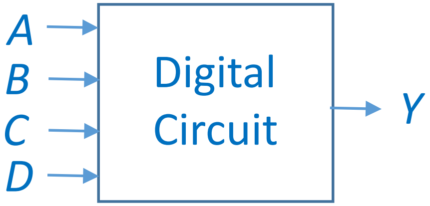
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	0	0	0	1
AB	0	1	0	1
$A\bar{B}$	1	0	0	1

CD'

B'D' **ABC'D**

Don’t Care in Karnaugh Map

Design: *Y* is H when BCD (Binary Coded Decimal) input is odd.



***Y* = *F*(*A*,*B*,*C*,*D*)**
= ∑ m(1,3,5,7,9) + d(10,11,12,13,14,15)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Not considering X

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	1	1	0
$\overline{A}B$	0	1	1	0
AB	X	X	X	X
$A\overline{B}$	0	1	X	X

Y* = *A’D* + *B’C’D

Considering X

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	1	1	0
$\overline{A}B$	0	1	1	0
AB	X	X	X	X
$A\overline{B}$	0	1	X	X

Y* = *D

- **If not considered, X = 0.**
- **If considered, X = 1.**
- **Consideration wherever helps.**

Karnaugh Map: POS

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$Y = F(A,B,C)$
$= \sum m(2,4,5,6,7)$
$= \prod M(0,1,3)$

(A + B)

AB \ C	0	1
00	0	0
01	1	0
11	1	1
10	1	1

(A + C′)

$Y = (A + B).(A + C')$

Distributive

$Y = A + B.C'$

SOP Simplification

$Y = F(A,B,C,D)$
$= \prod M(0,2,4,6,8).D(10, 11,12,13,14,15)$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	X	X	X	X
10	0	1	X	X

D

$Y = D$

- To cover all 0s
- Variables remaining constant form sum term (1:primed, 0:unprimed)
- Product of all sum terms generate output
- X is considered 0 wherever helps

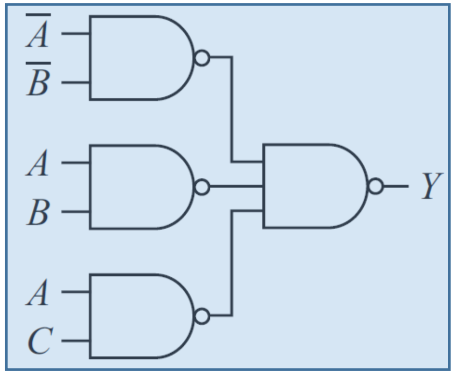
Dual Circuit

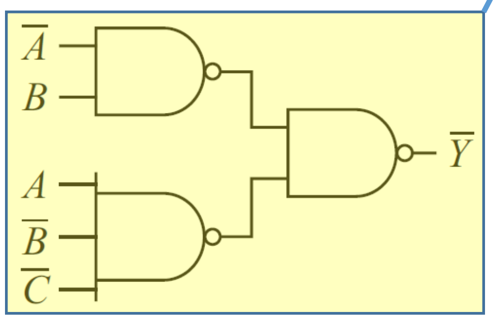
	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	1	1	1	1
$\overline{A}B$	0	0	0	0
AB	1	1	1	1
$A\overline{B}$	0	0	1	1

Y

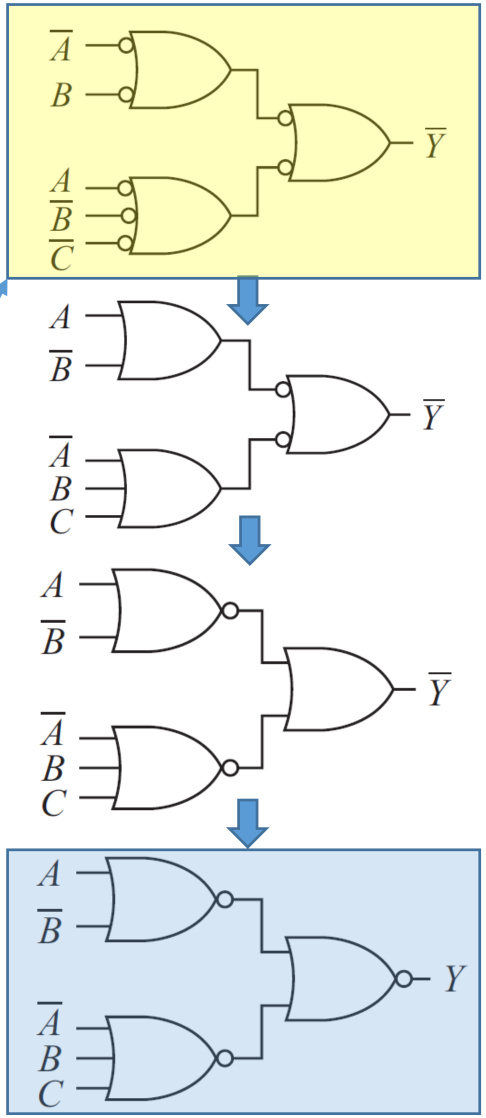
	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	0	0	0	0
$\overline{A}B$	1	1	1	1
AB	0	0	0	0
$A\overline{B}$	1	1	0	0

$\overline{Y}$





$$\overline{A.B} = \overline{A} + \overline{B}$$



- NAND → NOR, NOR → NAND, Complement all input and output
- AND → OR, OR → AND, Complement all input and output

Self-Dual Function

Consider, $F(A,B) = A.B' + A'.B$

Its dual, $F_D(A,B) = (A+B').(A'+B)$

(on simplification) $= A'.B' + A.B$

$F(A,B) \neq F_D(A,B)$

A	B	Y	
0	0	0	
0	1	1	$A'.B$
1	0	1	$A.B'$
1	1	0	

Consider, $F(A,B,C) = A.B + B.C + C.A$

Then, $F_D(A,B,C) = (A+B).(B+C).(C+A)$

(on simplification) $= A.B + B.C + C.A$

$F(A,B) = F_D(A,B)$: Self-dual

A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$A'.B.C$
1	0	0	0	
1	0	1	1	$A.B'.C$
1	1	0	1	$A.B.C'$
1	1	1	1	$A.B.C$

Conditions:

- Neutral: No. of minterms is same as no. of Maxterms
- Function not to contain any mutually exclusive minterm

$A.B.C \xleftrightarrow[\text{exclusive}]{\text{mutually}} A'.B'.C'$

References:

- Donald P. Leach, Albert P. Malvino, and Goutam Saha, Digital Principles & Applications 8e, McGraw Hill
- M. Morris Mano, and Michael D. Ciletti, Digital Design 5e, Pearson