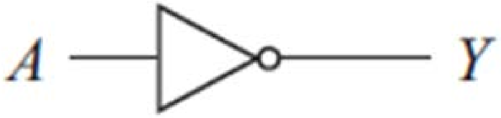
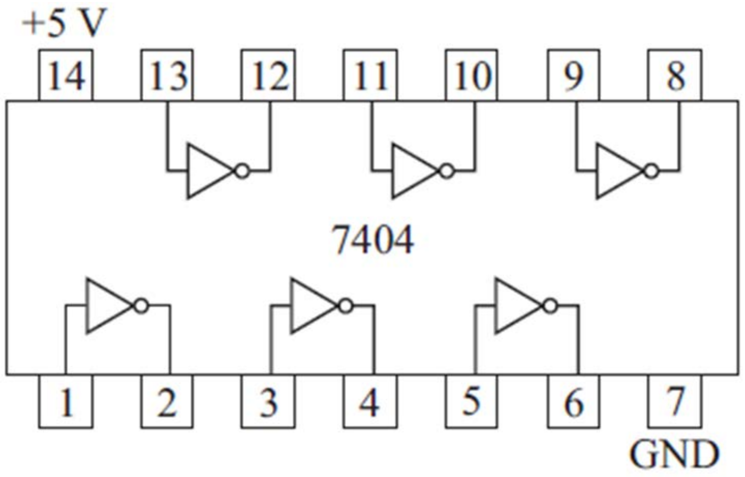
# Digital Electronic Circuits
# Section 1 (EE, IE)

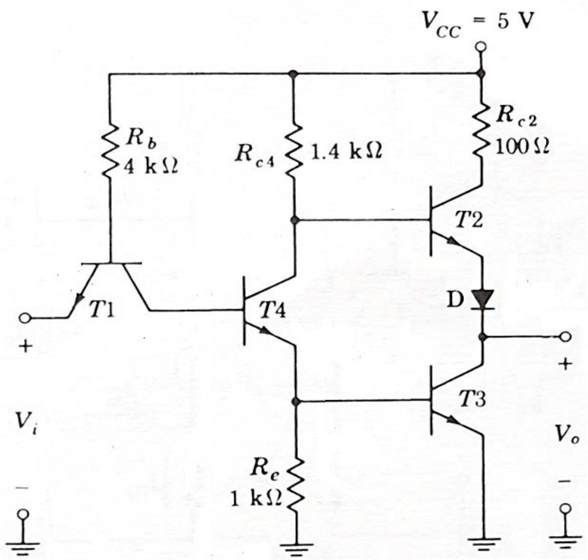
## Lecture 4

# Inverter (NOT Gate)

**$Y$ = not $A$**



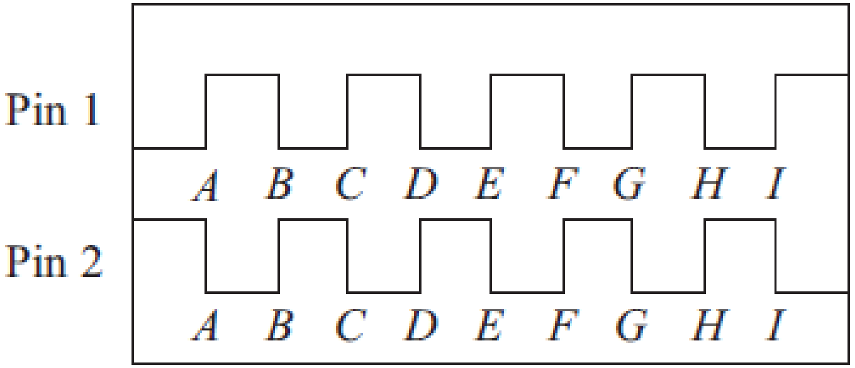




$$Y = \overline{A} \qquad Y = A'$$

| $A$ | $Y$ |
|---|---|
| $L$ | $H$ |
| $H$ | $L$ |

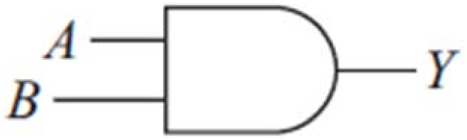
| $A$ | $Y$ |
|---|---|
| 0 | 1 |
| 1 | 0 |




**Delay in actual circuit i.e. not instantaneous.**

# AND Gate

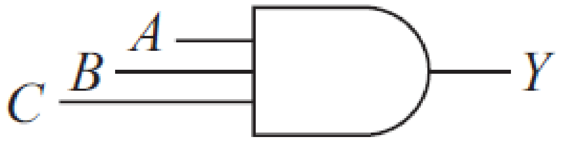
## $Y$ = $A$ AND $B$




$Y = A.B = A.B$

$= B.A$

(Commutative)



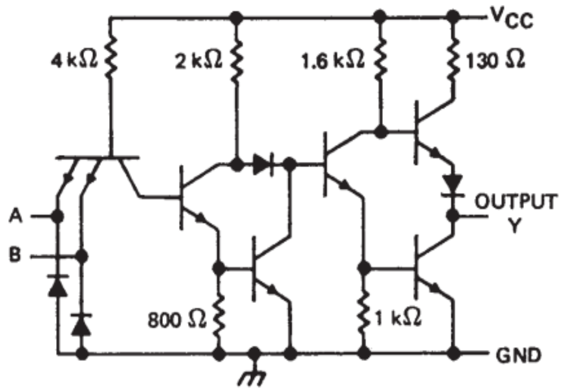

$Y = A.B.C$

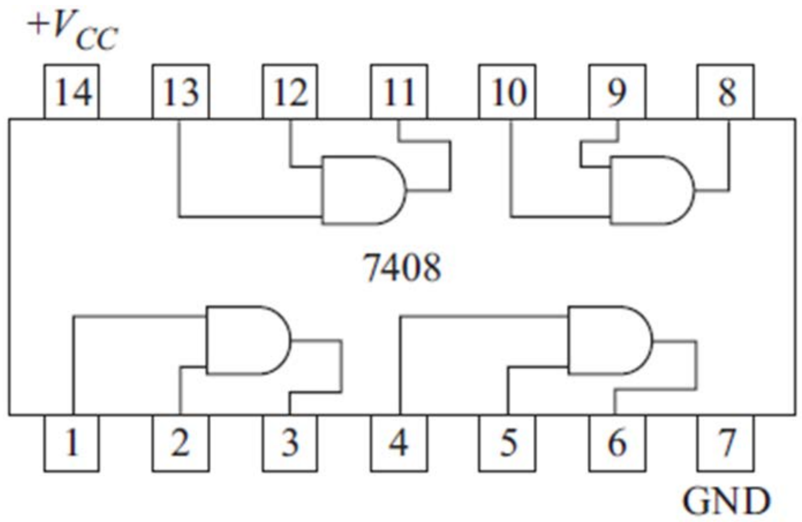
$= A.(B.C)$

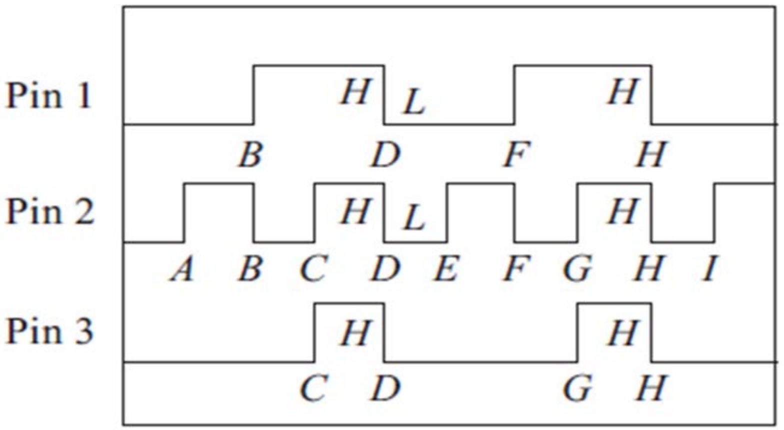
$= (A.B).C$

(Associative)

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

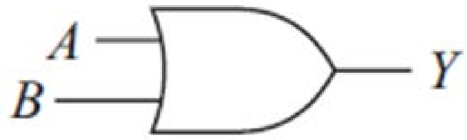


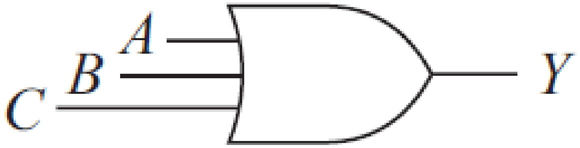
# OR Gate

## $Y$ = $A$ OR $B$




$$Y = A + B$$
$$= B + A$$

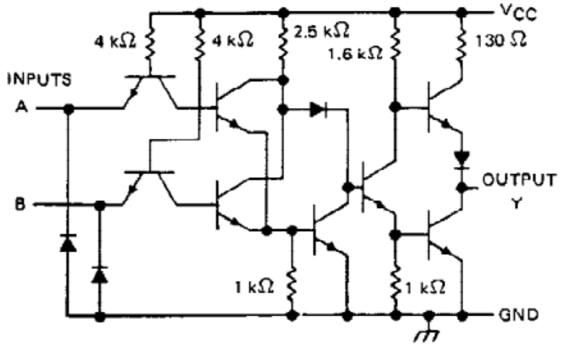
(Commutative)

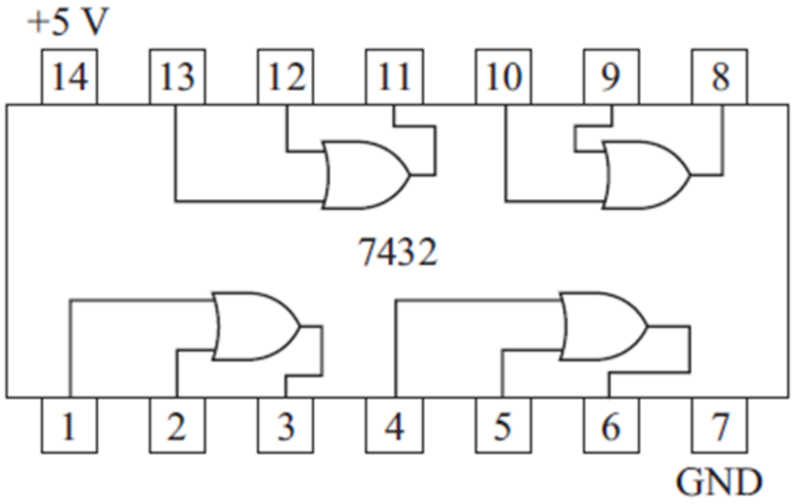


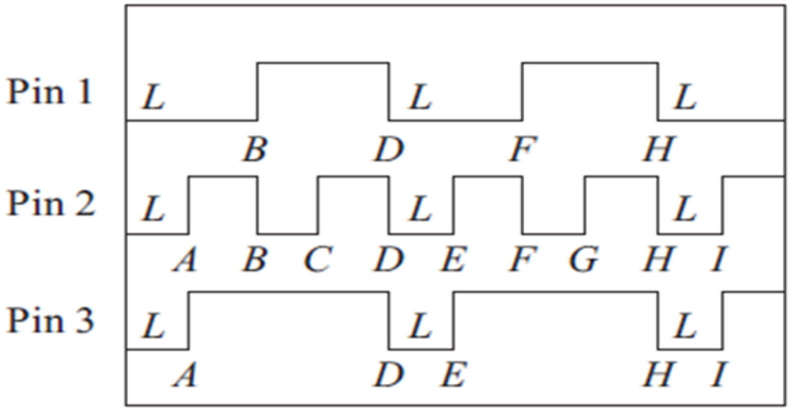

$$Y = A + B + C$$
$$= A + (B + C)$$
$$= (A + B) + C$$

(Associative)

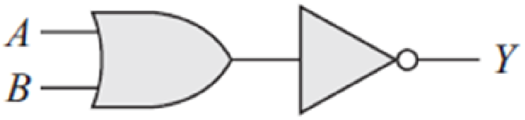
| $A$ | $B$ | $C$ | $Y$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

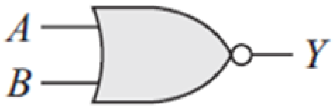


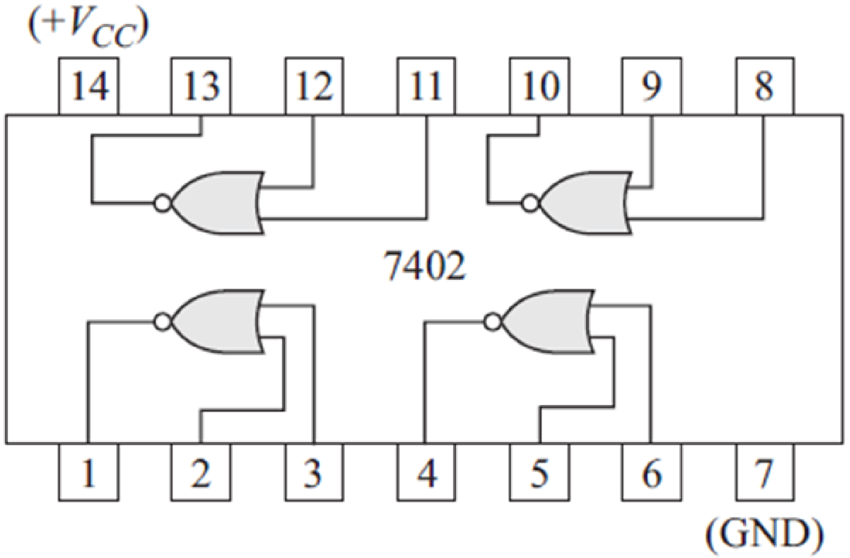



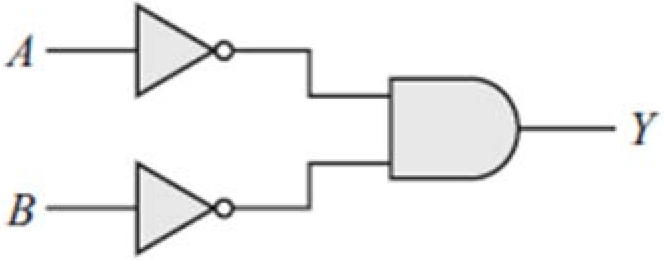
# NOR Gate




| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |



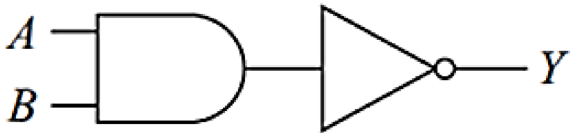

$$Y = \overline{A + B} \quad = \overline{A}\,\overline{B}$$

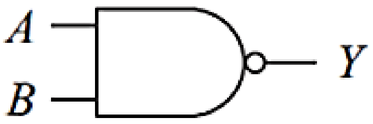
**(Commutative)**

## De Morgan's 1st Theorem:

$$\boldsymbol{Y = \overline{(A + B + C + ..)}}$$
$$\boldsymbol{= \overline{A}.\overline{B}.\overline{C}\ ...}$$

# NAND Gate

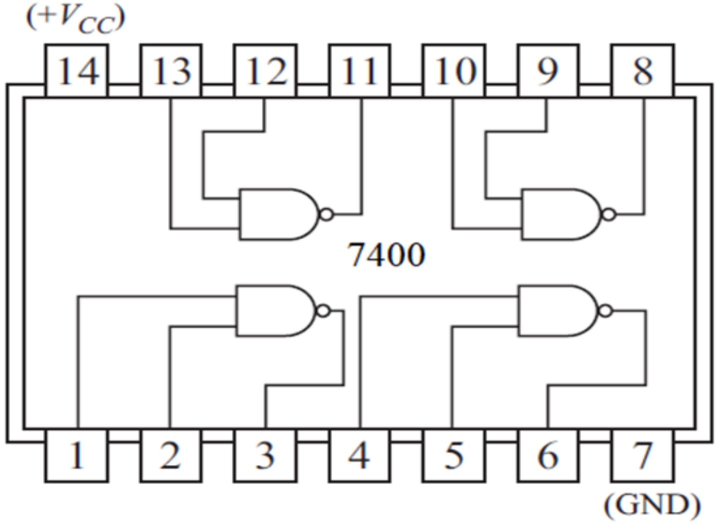


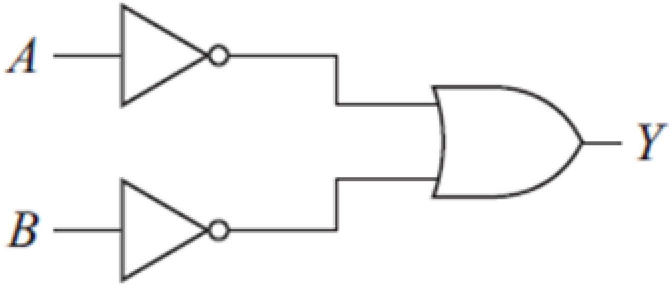




| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**De Morgan's 2nd Theorem:**

$$Y = \overline{ABC..}$$
$$= \bar{A} + \bar{B} + \bar{C} + \cdots$$



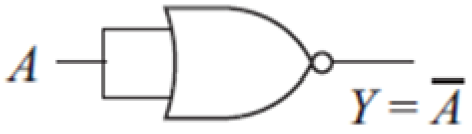



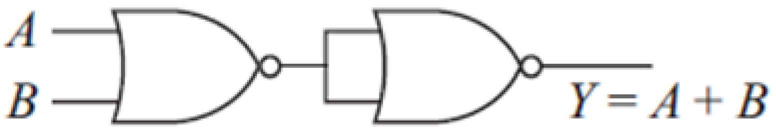

$$Y = \overline{AB} = \bar{A} + \bar{B}$$

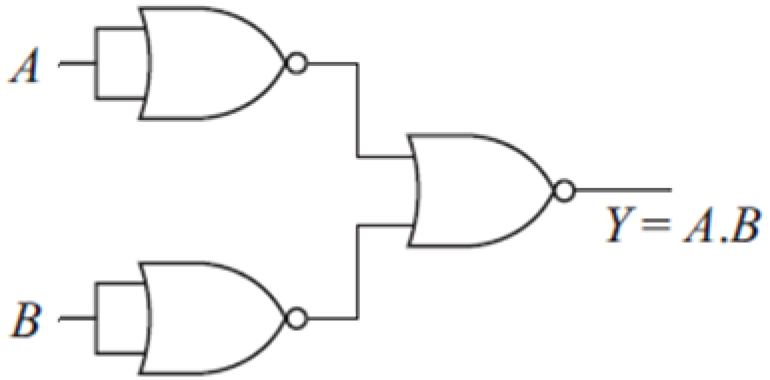
**(Commutative)**

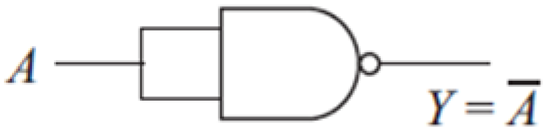
# Universality of NOR, NAND gate

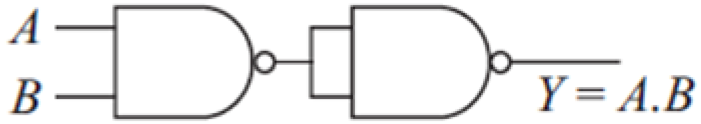
## Realization of AND, OR, NOT

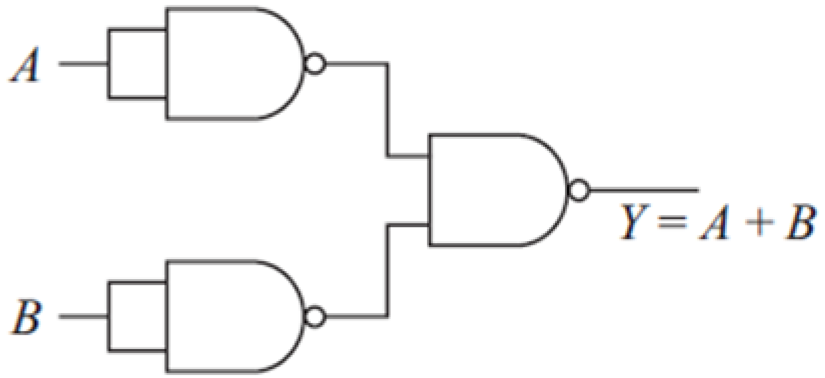


# Possible Logic Operations

| $x$ | $y$ | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | | | NOR | | NOT $x$ | | NOT $y$ | | NAND | AND | | | | | | OR | |

- $2^{2^n}$ possible functions with $n$ variables
- Formalization of representation and manipulation: Boolean Algebra

# Boolean Algebra: Huntington Postulates

| No. | Postulate | Description |
|---|---|---|
| 1 | **Closed with operators + and .** | Result of each is 1, 0 ϵ $B$ |
| 2 | **Identity element: 0 with +, 1 with .** | $x + 0 = x$; $x.1 = x$ |
| 3 | **Commutative w.r.t. +, .** | $x + y = y + x$; $x.y = y.x$ |
| 4 | **. is distributive over +,**<br>**+ is distributive over .** | $x.(y + z) = x.y + x.z$;<br>$x + (y.z) = (x + y).(x + z)$ |
| 5 | **For $x \in B$, there is $x' \in B$ s.t.**<br>**$x + x' = 1$ and $x.x' = 0$** | $0 + 0' = 0 + 1 = 1$, $1 + 1' = 1$;<br>$0.0' = 0.1 = 0$, $1.1' = 1.0 = 0$ |
| 6 | **At least 2 elements $x, y \in B$ s.t. $x \neq y$** | **B = {0,1}, 0 ≠ 1** |

**Associative Law:** $(x + y) + z = x + (y + z)$; $(x.y).z = x.(y.z)$ ⇐ From postulates

**1854:** Boolean Algebra, George Boole

**1904:** Postulates, E. V. Huntington

**1938:** Switching Algebra (2-valued), Claude. E. Shanon

# Postulates and Basic Theorems

| Name | (a) | (b) |
|---|---|---|
| Identity | $x + 0 = x$ | $x.1 = x$ |
| Null | $x + 1 = 1$ | $x.0 = 0$ |
| Complementarity | $x + x' = 1$ | $x.x' = 0$ |
| Idempotency | $x + x = x$ | $x.x = x$ |
| Involution | $(x')' = x$ | |
| Commutative | $x + y = y + x$ | $x.y = y.x$ |
| Associative | $(x + y) + z = x + (y + z)$ | $(x.y).z = x.(y.z)$ |
| Distributive | $x.(y + z) = x.y + x.z$ | $x + (y.z) = (x + y).(x + z)$ |

**Proof:** Null (a)

$$
\begin{aligned}
x + 1 &= (x + 1).1 \\
&= (x + 1).(x + x') \\
&= x + x'.1 \\
&= x + x' \\
&= 1
\end{aligned}
$$

# Duality and Boolean Expressions

| Name | (a) | (b) |
|---|---|---|
| **Absorption** | $x + x.y = x$ | $x.(x + y) = x$ |
| **Adsorption** | $x + x'.y = x + y$ | $x.(x' + y) = x.y$ |
| **Uniting** | $x.y + x.y' = x$ | $(x + y).(x + y') = x$ |
| **Consensus** | $x.y + x'.z + y.z = x.y + x'.z$ | $(x + y).(x' + z).(y + z) = (x + y).(x' + z)$ |
| **De Morgan's** | $(x_1 + x_2 + x_3 + \ldots x_N)' = x_1'.x_2'.x_3'. \ldots x_N'$ | $(x_1.x_2.x_3. \ldots x_N)' = x_1' + x_2' + x_3' + \ldots + x_N'$ |

**Duality:** Boolean Algebraic expression remains valid if the operators and identity elements are interchanged.

**Proof:** Null (b)
$x + 1 = 1$ (proven)
By Duality, $x.0 = 0$

**Operator precedence:** Parentheses, NOT, AND, OR

# More Proof

**Proof:** Idempotency (a)

$x + x = (x + x).1$ **:Identity**

$= (x + x).(x + x')$ **:Complem.**

$= x + x.x'$ **:Distribut.**

$= x + 0$ **:Complem.**

$= x$ **:Identity**

**Proof:** Idempotency (b)

$$
\begin{aligned}
x.x &= x.x + 0 \\
&= x.x + x.x' \\
&= x.(x + x') \\
&= x.1 \\
&= x
\end{aligned}
$$

[Also, by duality of (a),
$x + x = x$ ]

**Proof:** Involution

$$
\begin{aligned}
(x')' &= (x')' + 0 \\
&= (x')' + x.x' \\
&= [(x')' + x].[(x')' + x'] \\
&= [x + (x')'].[x' + (x')'] \\
&= [x + (x')'].1 \\
&= [x + (x')'].[x + x'] \\
&= x + (x')'.x' \\
&= x + x'.(x')' \\
&= x + 0 \\
&= x
\end{aligned}
$$

# More Proof

**Proof:** Consensus (a)

$$
\begin{aligned}
x.y + x'.z + y.z &= x.y + x'.z + y.z.1 \\
&= x.y + x'.z + y.z.(x + x') \\
&= x.y + x'.z + x.y.z + x'.y.z \\
&= (x.y + x.y.z) + (x'.z + x'.y.z) \\
&= (x.y + x.y.z) + (x'.z + x'.y.z) \\
&= x.y.(1 + z) + x'.z.(1 + y) \\
&= x.y.1 + x'.z.1 \\
&= x.y + x'.z
\end{aligned}
$$

**Proof:** De Morgan's Th. (a) 2 var.: $(x + y)' = x'.y'$

We know, $(x + y) + (x + y)' = 1$

To show, $(x + y) + x'.y' = 1$

$$
\begin{aligned}
(x + y) + x'.y' &= [(x + y) + x'].[(x + y) + y'] \\
&= [(y + x) + x'].[(x + y) + y'] \\
&= [y + (x + x')].[x + (y + y')] \\
&= [y + 1].[x + 1] \\
&= 1.1 \\
&= 1
\end{aligned}
$$

# More Characteristics

- Distributive Law, $x + (y.z) = (x + y).(x + z)$, is not valid for ordinary algebra.
- Complement is not available in ordinary algebra.
- Boolean algebra does not have subtraction, division.
- Boolean algebra (2-valued) has finite set of elements (0 and 1).

Difference with ordinary algebra

**Use of NAND (↑), NOR (↓) instead of AND (.), OR (+):**

- $(x\uparrow y)\uparrow z \neq x\uparrow(y\uparrow z)$; $(x\uparrow y)'\uparrow z = x\uparrow(y\uparrow z)'$
- $(x\downarrow y)\downarrow z \neq x\downarrow(y\downarrow z)$; $(x\downarrow y)'\uparrow z = x\downarrow(y\downarrow z)'$

Pseudoassociative

- $x\uparrow(y\downarrow z) \neq (x\downarrow y)\uparrow(x\downarrow z)$; $x\uparrow(y\downarrow z) = (x'\downarrow y)\uparrow(x'\downarrow z)$
- $x\downarrow(y\uparrow z) \neq (x\uparrow y)\downarrow(x\uparrow z)$; $x\downarrow(y\uparrow z) = (x'\uparrow y)\downarrow(x'\uparrow z)$

Pseudodistributive

# Boolean Function to Truth Table

$F(x,y) = x + x'.y$

$F(0,0) = 0 + 0'.0 = 0 + 1.0 = 0 + 0 = 0$
$F(0,1) = 0 + 0'.1 = 0 + 1.1 = 0 + 1 = 1$
$F(1,0) = 1 + 1'.0 = 1 + 0.0 = 1 + 0 = 1$
$F(1,1) = 1 + 1'.1 = 1 + 0.1 = 1 + 0 = 1$

| *x* | *y* | *F(x,y)* |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

| *x* | *y* | *z* | *F(x,y,z)* |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

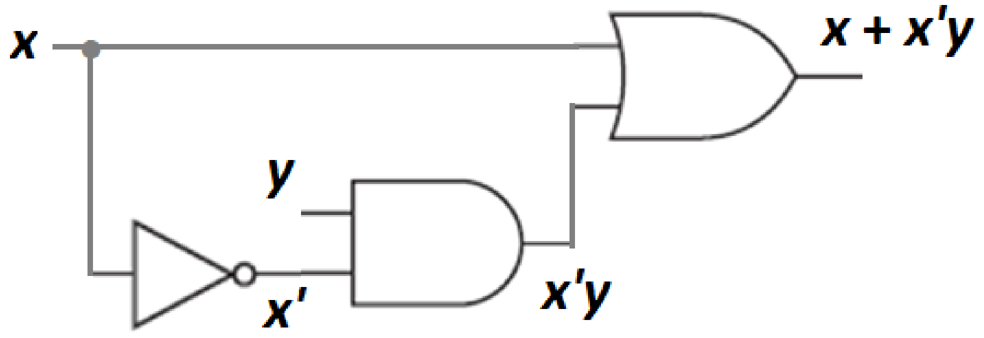
$F(x,y,z) = (x + y).(x + z)$

$F(0,0,0) = (0 + 0).(0 + 0)$
$= 0.0 = 0$
$F(0,0,1) = (0 + 0).(0 + 1)$
$= 0.1 = 0$

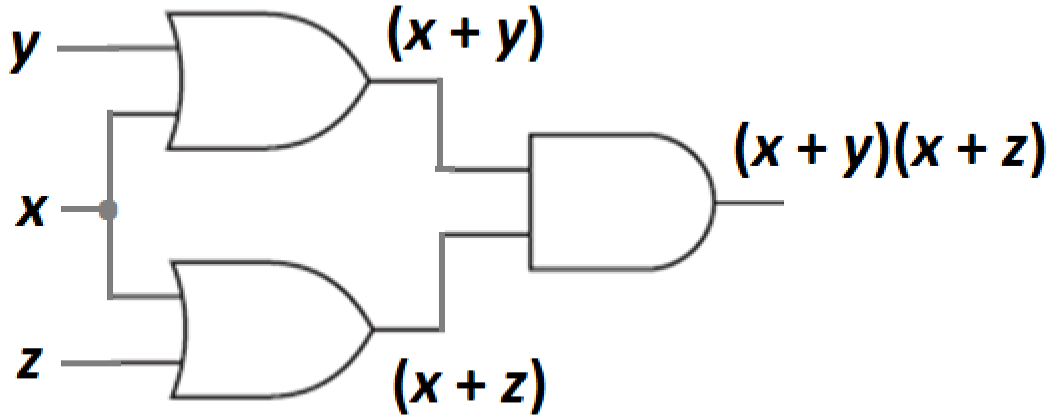
…

…

# Implementation of Boolean Function




$F(x,y) = x + x'.y$




$F(x,y,z) = (x + y).(x + z)$

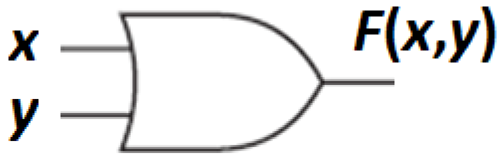
# Efficient Implementation

## Use of Boolean Algebra

Equivalence can be verified from Truth Table.

**From Adsorption Theorem:**

$F(x,y) = x + x'.y = x + y$

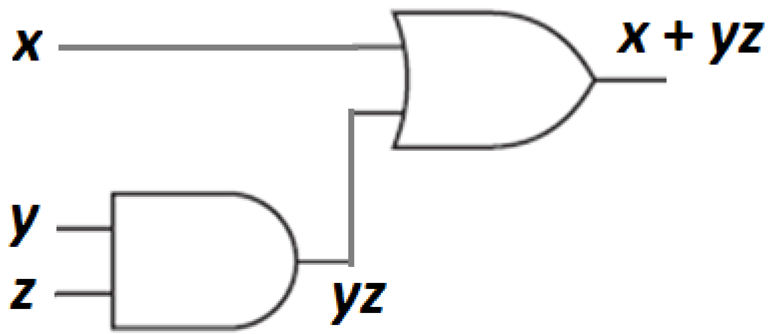



**Earlier:** One NOT, One 2 i/p AND, One 2 i/p OR

**Now:** One 2 i/p OR

**From Postulate (Distributive Law):**

$(x + y).(x + z) = x + y.z$




**Earlier:** One 2 i/p AND, Two 2 i/p OR

**Now:** One 2 i/p AND, One 2 i/p OR

# Algebraic Simplification

**Simplify,**

***Y* = *F*(*A*,*B*,*C*) = *A*(*A'* + *C*)(*A'B* + *C*)(*A'BC* + *C'*)**

$$
\begin{aligned}
Y &= (AA' + AC)\,(A'B + C)\,(A'BC + C') &&: \text{distribut.}\\
&= AC(A'B + C)\,(A'BC + C') &&: XX' = 0\\
&= (AC \cdot A'B + AC \cdot C)\,(A'BC + C') &&: \text{distribut.}\\
&= AC(A'BC + C') &&: XX' = 0\\
&= AC \cdot A'BC + AC \cdot C' &&: \text{distribut.}\\
&= 0 + 0 = 0 &&: XX' = 0
\end{aligned}
$$

**Note that, output *Y* is always L here and can be connected to LOW voltage directly.**

**Simplify, *Y* = (*A* + *B*)(*A'*(*B'* + *C'*))' + *A'*(*B* + *C*)**

$$
\begin{aligned}
Y &= (A + B)\,((A + (B' + C')') + A'(B + C) &&: \text{De Morgan's}\\
&= (A + B)\,(A + BC) + A'(B + C) &&: \text{De Morgan's}\\
&= (AA + ABC + AB + BBC) + A'(B + C)\\
&= (A + AB + ABC + BC) + A'(B + C)\\
&= A(1 + B + BC) + BC + A'(B + C)\\
&= A + BC + A'(B + C)\\
&= (A + A'(B + C)) + BC\\
&= A + B + C + BC\\
&= A + B + C(1 + B)\\
&= A + B + C
\end{aligned}
$$

# Shanon's Expansion Theorem

$$F(x_1, x_2, x_3, \ldots, x_N) = x_1'.F(0, x_2, x_3, \ldots, x_N) + x_1.F(1, x_2, x_3, \ldots, x_N)$$

**Proof:** $x_1$ can take only 2 values, 0 and 1

For $x_1 = 0$, LHS $= F(0, x_2, x_3, \ldots, x_N) = 0'.F(0, x_2, x_3, \ldots, x_N) + 0.F(1, x_2, x_3, \ldots, x_N)$
$= 1.F(0, x_2, x_3, \ldots, x_N) + 0$
$= F(0, x_2, x_3, \ldots, x_N) =$ RHS

For $x_1 = 1$, LHS $= F(1, x_2, x_3, \ldots, x_N) = 1'.F(0, x_2, x_3, \ldots, x_N) + 1.F(1, x_2, x_3, \ldots, x_N)$
$= 0.F(0, x_2, x_3, \ldots, x_N) + F(1, x_2, x_3, \ldots, x_N)$
$= 0 + F(1, x_2, x_3, \ldots, x_N)$
$= F(1, x_2, x_3, \ldots, x_N) =$ RHS

**Dual Form:**

$$F(x_1, x_2, x_3, \ldots, x_N) = [x_1' + F(1, x_2, x_3, \ldots, x_N)].[x_1 + F(0, x_2, x_3, \ldots, x_N)]$$

**Example:**

$F(x,y) = x + x'.y$
$F(0,y) = 0 + 0'.y = 1.y = y$
$F(1,y) = 1 + 1'.y = 1$
$F(x,y) = x'.F(0,y)+ x.F(1,y)$
$= x'.y + x.1$
$= x'.y + x$
$= x + x'.y$

# Example on Simplification

**Simplify,**

$$F(A, B, C, D, E) = A + \overline{A}.B + A.D.(B + E).(B.C + D.E)$$

**Choice of *A* as expansion variable as it is associated with more number of terms**

$$F(0, B, C, D, E) = 0 + \overline{0}.B + 0.D.(B + E).(B.C + D.E) = B$$

$$F(1, B, C, D, E) = 1 + \overline{1}.B + 1.D.(B + E).(B.C + D.E) = 1$$

$$\begin{aligned} F(A, B, C, D, E) &= \overline{A}.F(0, B, C, D, E) + A.F(1, B, C, D, E) \\ &= \overline{A}.B + A.1 = A + \overline{A}.B = A + B \end{aligned}$$

# More on Shanon's Expansion Theorem

$F(x_1, x_2, x_3, \ldots, x_N) = x_1'.F(0, x_2, x_3, \ldots, x_N) + x_1.F(1, x_2, x_3, \ldots, x_N)$

$F(x_1, x_2, x_3, \ldots, x_N) = x_1'.[x_2'.F(0, 0, x_3, \ldots, x_N) + x_2.F(0, 1, x_3, \ldots, x_N)]$
$+ x_1.[x_2'.F(1, 0, x_3, \ldots, x_N) + x_2.F(1, 1, x_3, \ldots, x_N)]$ ⇦ Nesting

$\mathbf{F(x,y)} = x'.F(0,y)+ x.F(1,y) = x'.[y'.F(0,0)+ y.F(0,1)]+ x.[y'.F(1,0)+ y.F(1,1)]$ ⇦ For 2 variables
$= x'.y'.F(0,0)+ x'.y.F(0,1)+ x.y'.F(1,0)+ x.y.F(1,1)$

**Example:**

$F(x,y) = x + x'.y$ ⇨ $F(0,0) = 0 + 0'.0 = 0;\ F(0,1) = 0 + 0'.1 = 1;$
$F(1,0) = 1 + 1'.0 = 1;\ F(1,1) = 1 + 1'.1 = 1;$

| *x* | *y* | *F(x,y)* |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$\mathbf{F(x,y)} = x'.y'.0 + x'.y.1+ x.y'.1+ x.y.1 = \mathbf{x'.y + x.y' + x.y}$ ⇦ In Truth Table, o/p is 1 in 3 rows
$= x'.y + x.(y + y') = x'.y + x.1 = \mathbf{x + x'.y}$

# References: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