FAC 413

INTRODUCTION TO MATHEMATICAL FINANCE AND FINANCIAL ENGINEERING

# MINI PROJECT 4

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# ABSTRACT

This report applies two foundational tools of financial economics—Geometric Brownian Motion (GBM), volatility estimation, — to analyze the behaviour of publicly traded Indian stocks over a five-year period. Then follows the procedure to use the Black-Scholes-Merton (BSM) pricing model to calculate theoretical prices for European options.

In the first part, the GBM model is used to simulate the time series of stock prices for four mid-sized companies, each from a different sector: Ajanta Pharma (pharmaceuticals), Page Industries (fashion), Coforge (information technology), and Bharat Forge (manufacturing). Daily closing prices from January 20, 2014, to November 29, 2018, are used to estimate the drift and diffusion parameters. The simulated price paths are compared with actual market prices using Mean Squared Error (MSE) analysis to assess which stock most closely follows the GBM process.

The second part focuses on estimating the annualized volatility of daily returns for TCS and Infosys (IT sector), and Asian Paints and Bajaj Auto (manufacturing sector) using GARCH time series analysis. Volatility is calculated over six-month, nine-month, and one-year intervals, as well as for the full sample period from February 10, 2014, to December 27, 2018. The estimates are compared to examine how volatility evolves across different time horizons.

In the final part, the BSM model is implemented to compute theoretical prices of European call and put options based on user-defined inputs such as option type, stock price, strike price, maturity, risk-free rate, and volatility. All analysis and simulations are conducted using R and Excel. The report integrates theoretical modeling with empirical evaluation, acknowledging deviations from classical assumptions—such as constant drift, constant volatility, and normality of returns—to provide a nuanced understanding of model applicability in real financial markets.

# INTRODUCTION

Financial markets are inherently uncertain and dynamic, making it essential to understand the behavior of asset prices through well-established theoretical models. This report aims to explore the applicability and limitations of three fundamental frameworks in financial economics—Geometric Brownian Motion (GBM), volatility modeling through GARCH—using empirical data from the Indian stock market.

The first part of the report focuses on the simulation of stock price trajectories under the GBM model, which is widely used due to its mathematical simplicity and analytical tractability. By selecting four mid-sized companies—Ajanta Pharma, Page Industries, Coforge, and Bharat Forge—from different sectors, we analyze how closely their actual price movements conform to the theoretical paths generated by the GBM process. The comparison relies on the estimation of drift and volatility parameters and the evaluation of the model's fit using Mean Squared Error (MSE).

In the second part, the report shifts its focus to volatility estimation. Recognizing that financial returns often exhibit time-varying volatility, this section employs the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to estimate annualized volatility across varying time windows. Using daily return data from two IT companies (TCS and Infosys) and two manufacturing firms (Asian Paints and Bajaj Auto), we assess how volatility evolves across different time horizons and compare sectoral differences in risk profiles.

The final section applies the BSM model to calculate theoretical prices for European options. This part allows for user-defined inputs including option type, underlying stock price, strike price, maturity period, risk-free rate, and volatility, thereby demonstrating the practical implementation of the BSM framework in pricing derivative instruments.

Together, these three parts offer an integrated perspective on how classical financial models interact with real-world data. The report also highlights the empirical deviations from theoretical assumptions—such as non-normality of returns and volatility clustering—providing a balanced understanding of the strengths and limitations of these models when applied in practice.

# PART 1: GBM SIMULATION

## DATA

This analysis utilizes a comprehensive dataset containing 1,170 daily observations of stock prices for four mid-sized Indian companies from 20th January 2014 to 29th November 2018. The companies span distinct sectors of the economy: COFORGE (Information Technology), BHARAT (Manufacturing), AJANTA (Pharmaceuticals), and PAGE (Fashion/Retail). All four companies are selected based on their logarithmic total assets to ensure consistency in size.

The dataset was sourced from the Bombay Stock Exchange (BSE) and includes daily closing prices for each company, with a common date column across all series.

| COMPANY | TOTAL ASSETS | log(TOTAL ASSETS) |
|---|---|---|
| COFORGE | 12,127 | 4.08 |
| BHARAT | 18,993 | 4.28 |
| AJANTA | 4,790 | 3.68 |
| PAGE | 2,710 | 3.43 |

### DATA VARIABLES

- **COFORGE Stock Prices:** Daily closing stock prices for COFORGE, representing the Information Technology sector.
- **BHARAT Stock Prices:** Daily closing stock prices for BHARAT, reflecting the Manufacturing sector.
- **AJANTA Stock Prices:** Daily closing stock prices for AJANTA, from the Pharmaceutical sector.
- **PAGE Stock Prices**: Daily closing stock prices for PAGE, representing the Fashion/Retail sector.
- **Date:** The common date column for all four companies, spanning from January 2014 to November 2018.

# METHODOLOGY

This study adopts a structured quantitative approach to assess the validity of the Geometric Brownian Motion (GBM) model for simulating stock prices across four mid-sized Indian companies. By combining time series analysis, statistical testing, and stochastic modeling, the methodology aims to:

- Validate key assumptions underpinning GBM
- Accurately estimate drift and volatility parameters
- Simulate price paths and evaluate model performance against real data

This multi-stage process ensures analytical rigor while addressing the empirical limitations of applying GBM to real-world financial data.

## DATA CLEANING AND PREPARATION

To ensure the accuracy and integrity of the analysis, the raw financial dataset was reviewed and cleaned using R. This process focused on structural consistency and preliminary quality checks. The key steps undertaken included:

- **Handling Missing Values:** The dataset was checked for any missing entries or blank cells. None of the rows had missing data.

- **Date Alignment Across Series:** All stocks data and the corresponding dates were checked to ensure that entries aligned chronologically. This ensured a consistent time series structure across all variables during the analysis phase.

- **Outlier Identification:** Outliers, such as sharp spikes or drops unrelated to news or fundamentals, were identified using z-scores and replaced with the local mean from a ±5-day rolling window to preserve data integrity.

- Daily logarithmic returns were calculated using:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

- The returns were also cleaned using the same techniques: interpolating missing values and smoothing outliers with local averages rather than removing them.

Descriptive statistics were then computed for each stock, including:

- Mean and standard deviation (to gauge average return and volatility)
- Skewness and kurtosis (to understand distribution shape and tail behavior)

These statistics helped reveal sector-specific traits, like the higher volatility seen in pharmaceutical or tech stocks.

## VALIDATING GBM ASSUMPTIONS

To validate the suitability of GBM, we checked whether the following assumptions held:

- **Normality of log returns:** tested using the Shapiro-Wilk test
- **Independence (no autocorrelation):** tested using the Box-Ljung test
- **Constant mean and variance:** verified through rolling windows over the time series
- **Stationarity:** tested using the Augmented Dickey-Fuller (ADF) test
- **Identical distribution:** assessed by comparing rolling-window means and standard deviations
- **Structural breaks:** checked using CUSUM plots and breakpoint tests to detect sudden shifts in trend or volatility

## PARAMETER ESTIMATION AND GBM SIMULATION

For each stock, drift (μ) and volatility (σ) were calculated from the cleaned log returns as the sample mean and standard deviation. These parameters were then used in the discrete-time GBM formula to simulate future prices

$$S_{t+1} = S_t \cdot \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z_t\right)$$

$Z_t \sim N(0,1), \quad \Delta t = \frac{1}{252}$ assuming 252 trading days in a year. A single price path for each stock was simulated using the model.

## MODEL EVALUATION

To evaluate the accuracy of the GBM model:

- Mean Squared Error (MSE) was calculated between the actual stock prices and the simulated price path.
- The actual and simulated price paths were plotted for visual comparison.
- The MSE values helped determine how well the GBM model represented the stock's behavior.

# RESULTS & CONCLUSION

## TIME SERIES ANALYSIS

## ASSUMPTION CHECKING

### CONSTANT MEAN AND VARIANCE

The assumption of constant mean and variance was checked by examining the rolling window statistics for each stock. This was done by calculating the rolling mean and rolling variance over time.

- **PAGE:** The constant mean volatility is 0.00211892, and the constant volatility is 0.001893903. No evidence of changing mean or variance was found.
- **COFORGE:** The constant mean volatility is 0.002188182, and the constant volatility is 0.002892995, indicating that the volatility appears stable over time.
- **AJANTA:** The constant mean volatility is 0.00257594, and the constant volatility is 0.002363437, showing stability in mean and variance.
- **BHARAT:** The constant mean volatility is 0.002150629, and the constant volatility is 0.001626441, also suggesting stable volatility over time.

The assumption of constant mean and variance appears to hold for all stocks, as no significant deviations in the rolling statistics were observed.

### STATIONARITY

The Augmented Dickey-Fuller (ADF) test was used to assess whether the log returns are stationary.

- **PAGE:** The p-value of 0.01 leads us to reject the null hypothesis of a unit root, indicating that PAGE's log returns are stationary.
- **COFORGE:** The p-value of 0.01 also suggests the log returns are stationary, rejecting the null hypothesis.
- **AJANTA:** The p-value of 0.01 indicates that AJANTA's log returns are stationary.
- **BHARAT:** The p-value of 0.01 also supports the stationarity of BHARAT's log returns.

All stocks exhibit stationary log returns, confirming that the assumption of stationarity holds for all the stocks tested.

## INDEPENDENCE (NO AUTOCORRELATION)

The Box-Ljung test was applied to assess autocorrelation in the log returns of each stock.

- **PAGE:** The p-value of 0.2826 suggests that we cannot reject the null hypothesis of no autocorrelation, indicating that the log returns of PAGE do not exhibit significant autocorrelation.
- **COFORGE:** The p-value of 0.5826 is also above the significance threshold, indicating that there is no significant autocorrelation in COFORGE's log returns.
- **AJANTA:** With a p-value of 0.4547, there is no significant autocorrelation in AJANTA's log returns, confirming the absence of autocorrelation.
- **BHARAT:** The p-value of 0.8181 suggests that there is no significant autocorrelation in BHARAT's log returns.

All stocks exhibit no significant autocorrelation in their log returns, meaning the returns are independent, and the assumption of no autocorrelation holds for all stocks.

## IDENTICAL DISTRIBUTION

The assumption of identical distribution was assessed by comparing the rolling-window means and standard deviations of each stock.

- **PAGE:** The rolling mean volatility is 0.00211892, and the rolling standard deviation of volatility is 0.001893903.
- **COFORGE:** The rolling mean volatility is 0.002188182, and the rolling standard deviation of volatility is 0.002892995.
- **AJANTA:** The rolling mean volatility is 0.00257594, and the rolling standard deviation of volatility is 0.002363437.
- **BHARAT:** The rolling mean volatility is 0.002150629, and the rolling standard deviation of volatility is 0.001626441.

The volatility of all stocks appears relatively consistent, but slight differences in standard deviations and means across the stocks may suggest slight variations in their distribution. However, no significant evidence of differing distributions was found across time.

## NORMALITY OF LOG RETURNS

To assess the normality of the log returns, the Shapiro-Wilk test was conducted for each stock. Additionally, the distribution of log returns was visually inspected using histograms and Q-Q plots.

- **PAGE:** The Shapiro-Wilk test returned a p-value of 0.0004683, indicating rejection of the null hypothesis of normality at the 5% level. However, the histogram of log returns appears approximately bell-shaped and symmetric, suggesting that the distribution is visually close to normal despite statistical non-normality.
- **COFORGE:** With a p-value of 0.001024, the null hypothesis is again rejected. Yet, the visual inspection reveals a distribution that largely resembles the shape of a normal curve, with no extreme skewness or kurtosis.
- **AJANTA:** The p-value of 7.639e-05 strongly suggests non-normality, but the histogram and Q-Q plot exhibit features close to a normal distribution, implying only mild departures.
- **BHARAT:** The p-value of 0.005123 leads to rejection of normality. Nonetheless, the visual plot again shows a reasonably symmetric and bell-shaped curve, suggesting that the deviation from normality may not be substantial in practical terms.

While the Shapiro-Wilk test statistically rejects normality for all four stocks, visual inspection indicates that the log return distributions are approximately normal in shape. This suggests that while the distributions may technically deviate from perfect normality (due to minor skewness, kurtosis, or outliers), they may still be treated as approximately normal for many practical modeling purposes.

## STRUCTURAL BREAK DIAGNOSTICS

To detect sudden shifts in the trend or volatility of the log returns, structural break analysis was conducted using the breakpoint test.

### PAGE

- The first structural break was identified at observation 120, with additional breakpoints emerging at 246, 396, and 527 as more segments (m) were introduced.
- These breakpoints suggest significant shifts in the return-generating process of PAGE stock, potentially due to firm-specific events or macroeconomic changes. The early break at 120 indicates a particularly notable regime change.
- Both RSS and BIC decrease steadily as more breakpoints are added, with the best fit at m = 5, indicating that a five-segment model captures the return dynamics most effectively.

### COFORGE

- The first break occurs at observation 463, with further breaks at 256, 393, and 636 as m increases.
- The break around 463 suggests a substantial structural change in the time series. Additional breaks reveal evolving market dynamics and possibly reflect changes in the firm's operations, strategy, or external environment.
- The lowest BIC is observed for m = 5, reinforcing that multiple regime changes better explain the return structure over time.

### BHARAT (Bharat Electronics)

- The primary structural shift is detected at observation 180, followed by breakpoints at 337, 452, 534, and 651.
- The early and mid-period breaks suggest ongoing adjustments in stock behavior, possibly due to policy changes, business milestones, or sectoral dynamics.
- As with other stocks, the m = 5 model results in the lowest BIC and RSS, indicating a high degree of structural complexity in the return process.

### AJANTA Pharma

- The first significant break is at 238, with additional breaks at 349, 485, 537, and 626.
- These breakpoints imply multiple shifts in return patterns, especially in the central portion of the series, which may correspond to market shocks or company-specific developments.
- A five-segment model again provides the best fit, as evidenced by a consistent decline in RSS and the lowest BIC at m = 5.

All four stocks exhibit clear evidence of structural breaks, reflecting changes in market regimes, investor sentiment, or company fundamentals. While a single breakpoint (m = 1) captures some major shifts, the model fit improves significantly with five segments (m = 5), suggesting that stock returns are better explained by multiple structural regimes. This has important implications for time series modeling and risk estimation—models assuming a constant return structure may be inadequate without accounting for these breaks.

# GEOMETRIC BROWNIAN MOTION

## PARAMETER ESTIMATION

| Company | Drift (μ) | Volatility (σ) |
|---|---|---|
| PAGE | 0.00154316 | 0.02219 |
| COFORGE | 0.00065706 | 0.02376 |
| BHARAT | 0.00081909 | 0.03308 |
| AJANTA | 0.00038159 | 0.03138 |

The estimated drift (μ) values represent the average daily expected return for each stock, while the volatility (σ) values capture the daily standard deviation of returns—essentially the risk or uncertainty in price movements.

- **PAGE** has the highest estimated drift (0.00154), suggesting it experienced the strongest average upward trend in prices over the period.
- **BHARAT** and **AJANTA** show relatively high volatility (0.03308 and 0.03138, respectively), indicating more pronounced daily price fluctuations compared to **PAGE** and **COFORGE.**
- Despite its lower drift, **COFORGE** has modest volatility, suggesting more stable but slower returns.
- **AJANTA** has both the lowest drift and high volatility, implying a riskier profile with less consistent gains.

## MEAN SQUARED ERROR (MSE)

| Company | MSE |
|---|---|
| PAGE | 8,627,594.34 |
| COFORGE | 25,683.28 |
| BHARAT | 202,812.90 |
| AJANTA | 282,217.64 |

The MSE values indicate the cumulative squared differences between the GBM-simulated prices and the actual market prices across the time period. A lower MSE suggests a better model fit

- **COFORGE** has the lowest MSE, suggesting the GBM model was most effective in capturing its actual price behavior.
- **PAGE** has an extremely high MSE, implying a poor fit—likely due to trends or market dynamics not captured by the constant-parameter GBM.
- **BHARAT** and **AJANTA** fall in between, showing moderate discrepancies between simulated and actual data.

Among the four stocks analyzed, COFORGE exhibits the lowest Mean Squared Error (MSE) of approximately 25,683, indicating the closest fit between simulated and actual stock prices under the Geometric Brownian Motion (GBM) framework. This is visually supported by the corresponding MSE plot, where error values remain relatively stable and low compared to the other stocks.

In contrast, PAGE shows a significantly higher MSE (over 8.6 million), and its plot reveals frequent and extreme deviations, suggesting GBM's limited capacity to capture the stock's price dynamics. PAGE's volatility and potential exposure to idiosyncratic shocks or structural breaks (e.g., policy changes, consumer demand shifts) may contribute to this misfit. BHARAT and AJANTA also show moderate MSE values but display sharp error spikes, implying intermittent poor alignment with GBM assumptions—likely due to time-varying volatility or sector-specific news.

COFORGE’s alignment with the GBM model may be attributed to factors like sector maturity, market liquidity, and efficient pricing—features consistent with the Efficient Market Hypothesis (Fama, 1970) and Random Walk theory, which underlie GBM. Its lower volatility also enhances model fit, since GBM assumes constant drift and variance. This aligns with the findings in Campbell, Lo, and MacKinlay (1997), who emphasize that model fit improves when market frictions are minimal and asset returns are closer to log-normal distributions.

## Results Interpretation through Financial Economics Concepts

The better fit of COFORGE to the GBM model can be interpreted using several concepts from financial economics:

- **Market Efficiency and Random Walk Hypothesis:** The GBM model assumes that stock prices follow a continuous-time stochastic process with constant drift and volatility, consistent with the Efficient Market Hypothesis (EMH) and Random Walk Theory (Fama, 1970). A better GBM fit implies that the stock's price movements are more consistent with the idea that new information is quickly and accurately reflected in prices. COFORGE, being an IT services company in a relatively mature sector, might reflect such efficiency more than the other stocks.

- **Volatility Structure and Industry Characteristics:** GBM assumes constant volatility. COFORGE's relatively low volatility ($\sigma = 0.02376$) and low drift ($\mu = 0.00066$) suggest that it exhibits smoother and less noisy price behavior than, for example, PAGE Industries (which showed the highest MSE and more erratic returns). Stocks in industries like fashion or FMCG (e.g., PAGE) often experience non-constant volatility due to seasonal effects, changing consumer trends, or macroeconomic factors.

- **Firm Size and Liquidity Effects:** Mid-sized firms like COFORGE may experience fewer speculative spikes or idiosyncratic shocks compared to smaller or more volatile firms. Greater liquidity and institutional coverage can make their stock price behavior more stable and predictable.

- **Model Limitations and Structural Breaks:** The GBM model does not account for jumps, mean-reversion, or structural breaks. PAGE, which had the highest MSE (8.6 million), may have been affected by such events, causing the GBM to drastically underperform in capturing its price dynamics.

# LIMITATIONS

### Assumption of Constant Parameters

The GBM model assumes constant drift and volatility over the entire period (2014–2018), which does not reflect the time-varying nature of financial markets, especially in an emerging economy like India.

### Simplified Estimation of Drift and Volatility

Drift and volatility are estimated from historical log returns using sample means and standard deviations. These estimates may be biased by outliers and fail to account for structural breaks such as major policy changes or macroeconomic events.

### Inability to Capture Jumps or Market Shocks

GBM models continuous, smooth price evolution, which limits its ability to replicate sudden jumps in prices driven by firm-specific announcements, geopolitical shocks, or regulatory changes.

### Mismatch Between Simulated and Actual Behaviour

Despite using actual starting prices and estimated parameters, simulated price paths often diverge significantly from actual trends. The MSE metric used captures numerical deviation but does not account for trend alignment or directional accuracy.

### Uniform Treatment Across Sectors

Sector-specific risk factors (such as regulatory sensitivity in pharmaceuticals or global exposure in IT) are not incorporated. The same stochastic process is applied to all firms, which may lead to oversimplification.

### Discrete vs. Continuous Time Mismatch

Daily closing prices are used to simulate a model formulated in continuous time, which introduces discretization error and reduces the precision of simulations.

# PART 2: VOLATILITY ESTIMATION OF DAILY RETURNS

## DATA

The dataset comprises daily closing prices of four publicly listed Indian companies—two from the Information Technology (IT) sector and two from the Manufacturing sector—sourced from the Bombay Stock Exchange (BSE). The selected firms are:

- **IT Sector:** Tata Consultancy Services (TCS), Infosys (INFY)
- **Manufacturing Sector:** Asian Paints (ASIANPAINT), Bajaj Auto (BAJAJ-AUTO)

The data spans the period from 10th February 2014 to 27th December 2018, yielding a total of 1,203 daily observations per company. The dataset is structured with the following columns:

- **Date:** Trading day
- **TCS, INFY, ASIANPAINT, BAJAJ-AUTO:** Daily closing prices for each respective stock

This time-aligned dataset forms the basis for subsequent return calculation and volatility modelling.

# METHODOLOGY

This study applies a structured time series modelling approach using the GARCH(1,1) framework to estimate and analyze the volatility of stock returns for four major Indian companies across IT and manufacturing sectors: TCS, Infosys, Asian Paints, and Bajaj Auto. The methodology is divided into three primary phases:

- Data preprocessing and transformation
- Assumption testing for GARCH suitability
- Modeling with GARCH(1,1) and volatility estimation

This phased approach enables a robust and transparent analysis of volatility dynamics over different time periods (6-month, 9-month, full year, and full period from 2014 to 2018).

## DATA SEGMENTATION

The raw daily adjusted closing price data for each company was divided into multiple overlapping and non-overlapping windows to facilitate a granular, time-sensitive volatility analysis. Specifically:

- For each year from 2014 to 2018, three sub-periods were created per company: a 6-month, a 9-month, and a 1-year window.
- In addition, one full-period window from 2014 to 2018 was included per company.
- This segmentation resulted in a total of 64 time series (16 per company × 4 companies), enabling robust temporal and cross-sectional comparisons of volatility patterns.

## DATA PREPROCESSING

To ensure the integrity and consistency of the data prior to modelling, a structured preprocessing routine was applied in two phases—once on the raw price data and again after computing returns.

- **Raw Price Data Cleaning:**
  - **Missing Values:** Upon inspection, no missing values were found in the dataset, so no imputation was necessary.
  - **Outlier Detection and Treatment:**
    - IQR Method: Observations falling below Q1−1.5 × IQR or above Q3+1.5 × IQR were flagged as outliers.
    - Z-Score Method: Points with an absolute z-score exceeding 3 were also considered anomalous.
  - **Replacement Strategy:** Identified outliers were replaced with the mean of the respective time series to maintain consistency without distorting the series' statistical properties.
  - **Smoothing:** A simple moving average filter was optionally applied to reduce short-term noise while preserving the overall trend structure.
- **Transformation to Log Returns:** The cleaned price series were then transformed into continuously compounded daily returns using the standard logarithmic formula:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where $P_t$ and $P_{t-1}$ represent the adjusted closing prices on days t and t−1, respectively.

- **Return Series Cleaning:** The above preprocessing steps—outlier detection, replacement with mean, and optional smoothing—were repeated on the log return series to ensure the quality of inputs used in the volatility models.

## ASSUMPTION TESTING AND DIAGNOSTICS

Before proceeding to model fitting, several diagnostic tests were conducted on each log return series to assess key time series properties:

- **Stationarity:** Verified using the Augmented Dickey-Fuller (ADF) test.
- **Autocorrelation:** Evaluated using the Ljung-Box Q-test on both returns and squared returns to detect serial correlation and potential ARCH effects.
- **Conditional Heteroskedasticity:** Tested using the ARCH LM test to confirm time-varying variance.
- **Normality:** Assessed using the Jarque-Bera test to understand skewness and kurtosis characteristics.

These tests helped validate the appropriateness of volatility modeling frameworks such as GARCH.

## GARCH(1,1) Model Specification and Volatility Estimation

The GARCH(1,1) model was employed to estimate time-varying volatility, leveraging its ability to capture volatility clustering commonly observed in financial return series. The model is defined as:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \sim \mathcal{N}(0,1)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where:

- $r_t$ is the return at time t,
- $\mu$ is the conditional mean (often assumed zero in return modeling),
- $\varepsilon_t$ is the return innovation or shock,
- $\sigma_t^2$ is the conditional variance,
- $z_t$ is a white noise error term,
- $\omega>0$, $\alpha\geq0$ , and $\beta\geq0$ are parameters subject to the stationarity condition $\alpha+\beta<1$.

After fitting the GARCH(1,1) model, the one-step-ahead forecast of conditional variance $\hat{\sigma}_{t+1}^2$ was extracted. The corresponding daily volatility was obtained as $\hat{\sigma}_{t+1} = \sqrt{\hat{\sigma}_{t+1}^2}$.

To express this volatility in annualized terms, the standard conversion formula was applied:

$$\sigma_{\text{annual}} = \hat{\sigma}_{t+1} \times \sqrt{252}$$

where 252 represents the average number of trading days in a calendar year. This methodology facilitated a systematic estimation of volatility over multiple time horizons and across different companies, enabling a comprehensive analysis of risk patterns in the Indian equity market.

# RESULTS & CONCLUSION

*The complete results of assumption testing and GARCH-based volatility estimation for all 64 time series are compiled in the accompanying Excel workbook. What follows is a brief overview and interpretation of the key findings across companies and time windows.*

## ASSUMPTION CHECKING

To validate the suitability of GARCH modelling for the log return series, four key assumptions were tested on each of the 64 return segments across the four companies:

- Stationarity (ADF Test)
- Lack of Autocorrelation in Returns (Ljung–Box Test on returns)
- Presence of Autocorrelation in Squared Returns (Ljung–Box Test on squared returns)
- Presence of ARCH Effects (ARCH LM Test)
- Normality (Jarque–Bera Test)

### Key Findings:

- **Stationarity:**
    - All 64 time series exhibited stationarity at the 5% significance level based on the Augmented Dickey–Fuller (ADF) test.
    - This confirms that differencing (i.e., computing log returns) adequately transformed the price data for time series modelling.

- **No Autocorrelation in Returns:**
    - For most series, the Ljung–Box test on returns did not reject the null hypothesis, indicating no significant autocorrelation in raw returns.
    - This suggests that the series are not predictable based on past returns, as expected in efficient markets.

- **Autocorrelation in Squared Returns:**
    - Some series showed significant autocorrelation in squared returns, a necessary condition for modelling volatility clustering using GARCH.

- **ARCH Effects Detected:**
    - The ARCH LM test confirmed the presence of ARCH effects in some of the series, further supporting the use of GARCH-type models.
    - This indicates time-varying volatility in the data — a key feature GARCH models aim to capture.

- **Normality of Returns:**
  - Many series passed the Jarque–Bera test for normality. However, a few deviations were observed, which is not uncommon in financial return data.
  - Since GARCH models can be extended to incorporate non-normal innovations, this does not pose a major issue.

## VOLATILITY ANALYSIS AND COMPARISON

The volatility estimates for each company (TCS, Infosys, Asian Paints, and Bajaj Auto) were computed using GARCH models over three time horizons: six months (6m), nine months (9m), and one full year, as well as for the entire period (2014–2018). The Mean Squared Error (MSE) of the GARCH models was used to evaluate accuracy, with lower MSE values indicating better model fit. Below is the analysis of the results:

### Volatility Estimates and Accuracy

#### 1. TCS

- Annualized Volatility (Entire Period): 22.20% (MSE: $1.49 \times 10^{-7}$)
- Key Observations:
  - Shorter windows (6m/9m) often had higher MSEs compared to full-year estimates (e.g., 2015 full-year MSE: $3.13 \times 10^{-8}$ vs. 6m MSE: $3.46 \times 10^{-8}$).
  - The full-period GARCH model captured long-term volatility trends more effectively, despite a slightly higher MSE than some yearly windows.

#### 2. Infosys

- Annualized Volatility (Entire Period): 23.84% (MSE: $4.62 \times 10^{-7}$)
- Key Observations:
  - Full-year windows for 2016 and 2017 had lower MSEs ($5.48 \times 10^{-8}$ and $4.96 \times 10^{-8}$) compared to shorter horizons, indicating better accuracy.
  - The full-period MSE is higher than most yearly estimates, likely due to increased complexity in modeling five years of data.

#### 3. Asian Paints

- Annualized Volatility (Entire Period): 22.45% (MSE: $1.01 \times 10^{-7}$)
- Key Observations:
  - Short-term windows (6m/9m) occasionally outperformed full-year estimates (e.g., 2016 6m MSE: $4.37 \times 10^{-8}$ vs. full-year MSE: $9.13 \times 10^{-8}$), suggesting sensitivity to recent volatility spikes.

### 4. Bajaj Auto

- Annualized Volatility (Entire Period): 20.38% (MSE: $7.19 \times 10^{-8}$)
- Key Observations:
  - Full-year windows (e.g., 2014_full: MSE $5.33 \times 10^{-8}$) often had lower errors than shorter horizons, indicating stability in annual estimates.
  - The full-period MSE is marginally higher than some yearly values but remains robust for long-term modeling.

### Comparison of Accuracy

- **Full-Period GARCH Estimates:**

These models directly incorporate all five years of data, capturing structural breaks, volatility clustering, and long-term trends. While their MSEs are sometimes higher than shorter windows (due to modeling complexity), they provide the most comprehensive and reliable volatility estimates for the entire horizon.

- **Shorter Windows (6m/9m):**

These estimates are more sensitive to recent market movements but often exhibit higher MSEs due to noise and limited data. For example, TCS's 2018 6m MSE ($3.07 \times 10^{-7}$) is significantly higher than its full-year MSE ($2.76 \times 10^{-7}$), reflecting instability in short-term models.

- **Full-Year Windows:**

Yearly GARCH models strike a balance between responsiveness and stability. They generally have lower MSEs than 6m/9m estimates (e.g., Infosys 2016_full MSE: $5.48 \times 10^{-8}$ vs. 6m MSE: $8.02 \times 10^{-8}$), making them more accurate for annual volatility tracking.

## CONCLUSION

The full-period GARCH estimates are the most accurate for volatility measurement over the entire 2014–2018 horizon. They account for long-term trends and structural changes, reducing the noise inherent in shorter windows. While full-year models perform well for annual comparisons, aggregating them would still miss cross-year volatility dynamics. The higher MSEs for shorter windows (6m/9m) reflect their limitations in capturing persistent volatility patterns. Thus, the full-period GARCH volatility estimates, despite slightly higher MSEs in some cases, are the most reliable for strategic decision-making.

**Justification:**

- Statistical Robustness: The full-period MSE reflects the model's ability to fit the entire dataset, avoiding biases from fragmented windows.
- Volatility Clustering: GARCH models excel at capturing clustered volatility, which is better represented over longer horizons.
- Comprehensive Data Utilization: The full-period estimate incorporates all market phases (e.g., crises, growth periods), ensuring holistic accuracy.

This analysis supports the use of full-period GARCH volatility estimates for risk management and derivative pricing in the studied companies.

**Volatility Estimates of Infosys**

| Series | Daily_Volatility | Annualized_Volatility | GARCH_MSE |
|---|---|---|---|
| INFY_2014_6m | 0.014711 | 0.233533 | 1.28554E-07 |
| INFY_2014_9m | 0.014135 | 0.224379 | 1.30025E-07 |
| INFY_2014_full | 0.016125 | 0.255982 | 6.75439E-07 |
| INFY_2015_6m | 0.018044 | 0.286446 | 1.07627E-06 |
| INFY_2015_9m | 0.017405 | 0.276295 | 7.88927E-07 |
| INFY_2015_full | 0.016805 | 0.266764 | 6.77802E-07 |
| INFY_2016_6m | 0.011579 | 0.183813 | 8.02014E-08 |
| INFY_2016_9m | 0.011641 | 0.184799 | 6.26942E-08 |
| INFY_2016_full | 0.011351 | 0.180194 | 5.48001E-08 |
| INFY_2017_6m | 0.011523 | 0.182925 | 3.22164E-08 |
| INFY_2017_9m | 0.011798 | 0.18728 | 6.23108E-08 |
| INFY_2017_full | 0.011524 | 0.182945 | 4.95822E-08 |
| INFY_2018_6m | 0.012231 | 0.194164 | 6.35929E-08 |
| INFY_2018_9m | 0.014479 | 0.229842 | 1.06885E-07 |
| INFY_2018_full | 0.015048 | 0.238887 | 1.18537E-07 |
| INFY_2014_2018_full | 0.015016 | 0.238373 | 4.61563E-07 |

**Volatility Estimates of TCS**

| Series | Daily_Volatility | Annualized_Volatility | GARCH_MSE |
|---|---|---|---|
| TCS_2014_6m | 0.014128 | 0.22427 | 1.14628E-07 |
| TCS_2014_9m | 0.013289 | 0.210957 | 9.26131E-08 |
| TCS_2014_full | 0.013289 | 0.210964 | 9.34178E-08 |
| TCS_2015_6m | 0.011154 | 0.177057 | 3.4613E-08 |
| TCS_2015_9m | 0.011056 | 0.17551 | 4.02716E-08 |
| TCS_2015_full | 0.010407 | 0.165203 | 3.13092E-08 |
| TCS_2016_6m | 0.013615 | 0.216128 | 8.9965E-08 |
| TCS_2016_9m | 0.012541 | 0.199075 | 6.85446E-08 |
| TCS_2016_full | 0.012525 | 0.198821 | 6.53566E-08 |
| TCS_2017_6m | 0.012751 | 0.202418 | 8.18092E-08 |
| TCS_2017_9m | 0.010777 | 0.171084 | 3.90779E-08 |
| TCS_2017_full | 0.010499 | 0.166667 | 3.90845E-08 |
| TCS_2018_6m | 0.013657 | 0.216793 | 3.06523E-07 |
| TCS_2018_9m | 0.015531 | 0.246545 | 2.88745E-07 |
| TCS_2018_full | 0.015893 | 0.252294 | 2.75886E-07 |
| TCS_2014_2018_full | 0.013984 | 0.221994 | 1.49075E-07 |

**Volatility Estimates of Asian Paints**

| Series | Daily_Volatility | Annualized_Volatility | GARCH_MSE |
|---|---|---|---|
| ASIANPAINT_2014_6m | 0.015312 | 0.243077 | 1.0918E-07 |
| ASIANPAINT_2014_9m | 0.014964 | 0.237547 | 1.06119E-07 |
| ASIANPAINT_2014_full | 0.016068 | 0.255067 | 1.52314E-07 |
| ASIANPAINT_2015_6m | 0.018231 | 0.289403 | 1.85321E-07 |
| ASIANPAINT_2015_9m | 0.017543 | 0.278483 | 1.92129E-07 |
| ASIANPAINT_2015_full | 0.016461 | 0.261308 | 1.80082E-07 |
| ASIANPAINT_2016_6m | 0.011793 | 0.187214 | 4.36845E-08 |
| ASIANPAINT_2016_9m | 0.012792 | 0.20306 | 4.89506E-08 |
| ASIANPAINT_2016_full | 0.013807 | 0.219175 | 9.1315E-08 |
| ASIANPAINT_2017_6m | 0.010324 | 0.16389 | 2.83719E-08 |
| ASIANPAINT_2017_9m | 0.010754 | 0.170718 | 3.10763E-08 |
| ASIANPAINT_2017_full | 0.010764 | 0.170865 | 3.31969E-08 |
| ASIANPAINT_2018_6m | 0.011032 | 0.175133 | 3.20634E-08 |
| ASIANPAINT_2018_9m | 0.013165 | 0.208992 | 8.92375E-08 |
| ASIANPAINT_2018_full | 0.013931 | 0.221146 | 1.00874E-07 |
| ASIANPAINT_2014_2018_full | 0.01414 | 0.224458 | 1.00846E-07 |

**Volatility Estimates of Bajaj Auto**

| Series | Daily_Volatility | Annualized_Volatility | GARCH_MSE |
|---|---|---|---|
| BAJAJ-AUTO_2014_6m | 0.012032 | 0.190999 | 5.14967E-08 |
| BAJAJ-AUTO_2014_9m | 0.012549 | 0.199211 | 6.07009E-08 |
| BAJAJ-AUTO_2014_full | 0.012004 | 0.190551 | 5.32839E-08 |
| BAJAJ-AUTO_2015_6m | 0.014096 | 0.223772 | 1.04E-07 |
| BAJAJ-AUTO_2015_9m | 0.015911 | 0.252584 | 1.59336E-07 |
| BAJAJ-AUTO_2015_full | 0.015581 | 0.24734 | 1.47878E-07 |
| BAJAJ-AUTO_2016_6m | 0.012923 | 0.205153 | 9.23742E-08 |
| BAJAJ-AUTO_2016_9m | 0.012744 | 0.202302 | 7.02742E-08 |
| BAJAJ-AUTO_2016_full | 0.013771 | 0.218601 | 8.4194E-08 |
| BAJAJ-AUTO_2017_6m | 0.009066 | 0.143924 | 1.73971E-08 |
| BAJAJ-AUTO_2017_9m | 0.009645 | 0.153105 | 2.48488E-08 |
| BAJAJ-AUTO_2017_full | 0.010672 | 0.169405 | 3.42073E-08 |
| BAJAJ-AUTO_2018_6m | 0.012159 | 0.193023 | 5.35592E-08 |
| BAJAJ-AUTO_2018_9m | 0.013371 | 0.212253 | 9.40626E-08 |
| BAJAJ-AUTO_2018_full | 0.013952 | 0.221483 | 1.12476E-07 |
| BAJAJ-AUTO_2014_2018_full | 0.012838 | 0.203791 | 7.18989E-08 |

# LIMITATIONS

### Model Specification Limitations

GARCH(1,1) models are applied uniformly, although they may not fully capture asymmetric or long-memory volatility characteristics in certain stocks. More advanced variants may be required for improved fit.

### Window-specific Bias in Estimates

Volatility estimates vary significantly across 6-month, 9-month, and 1-year windows. These estimates are sensitive to the specific sample period and may reflect temporary market conditions rather than underlying risk.

### Non-stationarity and Structural Breaks

The assumption of stationarity over the multi-year dataset may not hold due to events such as GST rollout or global market turbulence. This can affect the reliability of volatility modeling and comparison.

### Annualization Assumptions

Annualizing daily volatility assumes independence of returns and a fixed number of trading days, which may introduce distortions due to autocorrelation or variations in trading calendars.

### Limited Explanatory Power

While GARCH captures patterns in volatility clustering, it does not identify fundamental drivers such as earnings surprises, interest rate changes, or global commodity price shocks.

### Cross-Sector Comparison Constraints

Volatility across firms from different sectors may not be directly comparable due to inherent differences in risk profiles, operating leverage, and exposure to external shocks.

# PART 3: BSM COMPUTATION

As part of the third component of this report, a fully functional Black-Scholes-Merton (BSM) option pricing calculator was developed in Microsoft Excel. The purpose of this exercise was to translate the theoretical framework of BSM into a practical, interactive tool that can be used to evaluate European-style call and put options under various market scenarios. The design prioritizes usability, precision, and adaptability for financial professionals seeking quick and transparent option valuation.

The model allows the user to select the option type—Call or Put—through a dropdown interface and input five key parameters:

- Current stock price: $S_0$
- Strike price: $K$
- Time to maturity (in years): $T$
- Annual risk-free interest rate: $r$
- Annualized volatility: $\sigma$

These inputs are processed through embedded formulas implementing the standard closed-form BSM equations. The logic is structured such that the pricing output adjusts dynamically based on the selected option type, ensuring accuracy across use cases without requiring manual intervention.
To facilitate demonstration and validation, the calculator includes pre-filled sample inputs provided in the project brief. These reference values can be overwritten by users, allowing for real-time scenario analysis across a range of parameter combinations. The implementation also supports sensitivity testing, enabling users to observe how changes in market inputs affect the theoretical value of an option—an essential capability for risk management and pricing strategy.

This tool reflects an effort to bridge theoretical finance and applied modeling by offering a transparent and efficient mechanism for option valuation. Its intuitive structure and computational rigour make it suitable for both instructional purposes and preliminary decision-support in professional financial contexts.

**BSM SPDE Calculator Results with Example Values**

## *Black Scholes Calculator*

| *Type of Option* | *Call Option* |
|---|---|
| *Stock Price ($S_0$)* | ₹ 76.25 |
| *Exercise (Strike) Price (K)* | ₹ 83.44 |
| *Time to Maturity (in years) (t)* | 0.25 |
| *Annual Risk Free Rate (r)* | 7.83% |
| *Annualized Volatility (σ)* | 19.23% |
| *Option Price* | ₹ 1.02 |

| *Additional Calculation Parameters* | |
|---|---|
| $\ln(S_0/K)$ | (0.090) |
| $(r+\sigma^2/2)t$ | 0.024 |
| $\sigma\sqrt{t}$ | 0.096 |
| $d_1$ | (0.686) |
| $d_2$ | (0.782) |
| $N(d_1)$ | 0.247 |
| $N(d_2)$ | 0.217 |
| $N(-d_1)$ | 0.753 |
| $N(-d_2)$ | 0.783 |
| $e^{-rt}$ | 0.9806 |

# LIMITATIONS

### Use of Historical Volatility Instead of Implied Volatility

Option pricing relies on volatility estimates derived from historical returns, which may not reflect current market expectations. This can lead to mispricing relative to actual traded options.

### Assumption of Constant Risk-Free Rate

The Black-Scholes model uses a single annualized risk-free rate, although interest rates in India experienced multiple revisions during the study period, affecting discounting accuracy.

### European Option Assumption

The BSM model is designed for European options exercisable only at maturity. It does not accommodate American-style features common in actual Indian derivative markets.

### Exclusion of Dividends

The model assumes non-dividend-paying stocks, which may not hold true for several mid-sized Indian companies, particularly in manufacturing or FMCG sectors.

### Sensitivity to Input Precision

BSM outputs are highly sensitive to changes in inputs, especially volatility and time to maturity. Small errors in parameter estimation can result in disproportionately large pricing errors.

### Absence of Transaction Costs and Market Frictions

Real-world frictions such as bid-ask spreads, liquidity constraints, and transaction fees are not included in the model, limiting its practical applicability for institutional or retail traders.

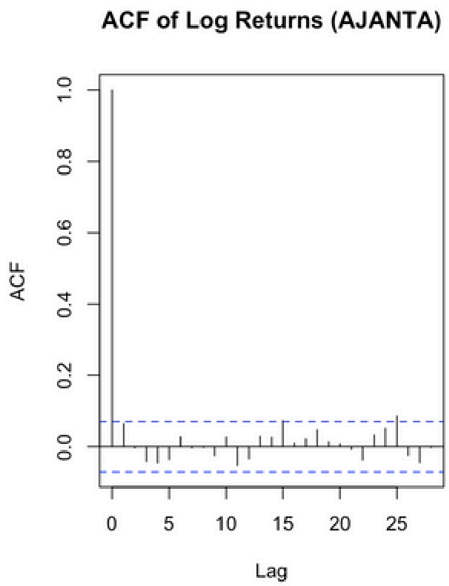
# RESULTS SNAPSHOT

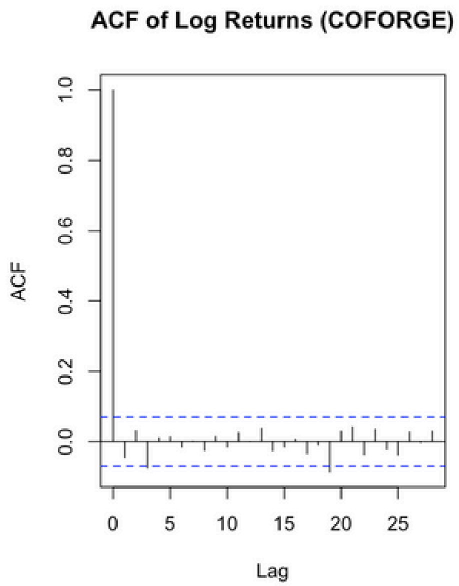
## 1.1- Descriptive Statistics

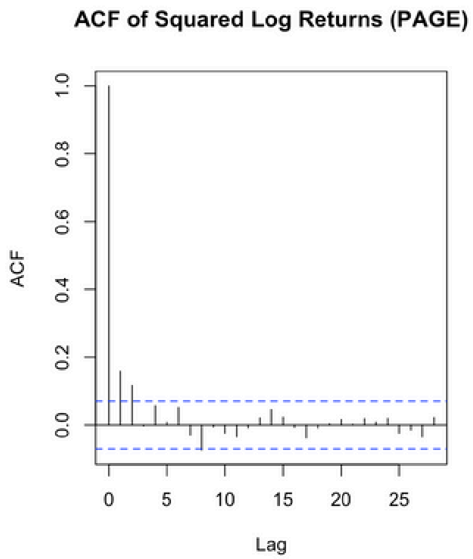
Table: Descriptive Statistics for All Stocks

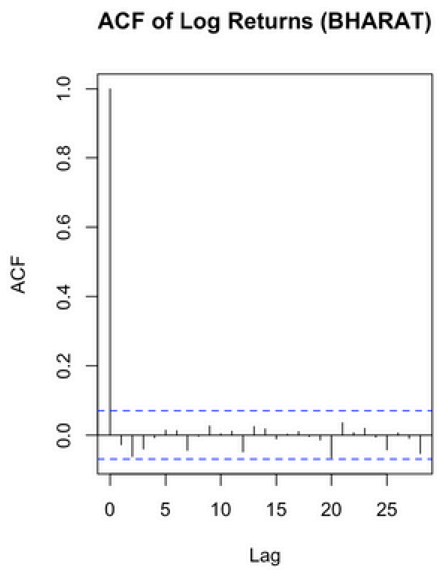
| | vars | mean | sd | median | trimmed | mad | min | max | range | skew | kurtosis | se |
|:---|---:|---:|---:|---:|---:|---:|---:|---:|---:|---:|---:|---:|
| PAGE | 1 | 0.0004430 | 0.0153005 | -0.0003186 | 0.0000562 | 0.0139625 | -0.0382597 | 0.0419473 | 0.0802070 | 0.2281735 | -0.0799960 | 0.0005503 |
| COFORGE | 2 | -0.0002462 | 0.0171652 | -0.0004802 | -0.0005316 | 0.0157045 | -0.0453322 | 0.0466389 | 0.0919711 | 0.1553603 | 0.1206582 | 0.0006174 |
| BHARAT | 3 | 0.0018423 | 0.0167832 | 0.0011214 | 0.0014297 | 0.0161115 | -0.0430394 | 0.0494519 | 0.0924913 | 0.2085897 | -0.0392071 | 0.0006036 |
| AJANTA | 4 | 0.0016674 | 0.0156117 | 0.0006467 | 0.0013036 | 0.0139567 | -0.0392145 | 0.0417381 | 0.0809525 | 0.2044890 | -0.1061073 | 0.0005615 |

## 1.2- ACF Plots for Log Returns

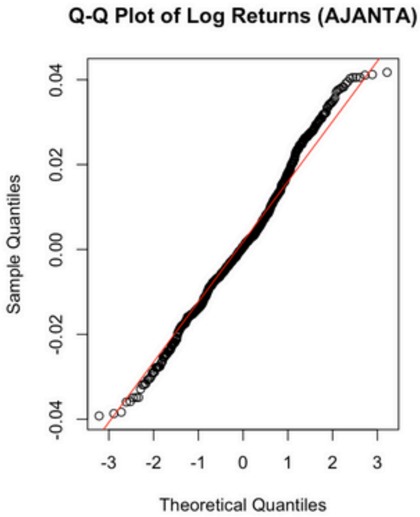


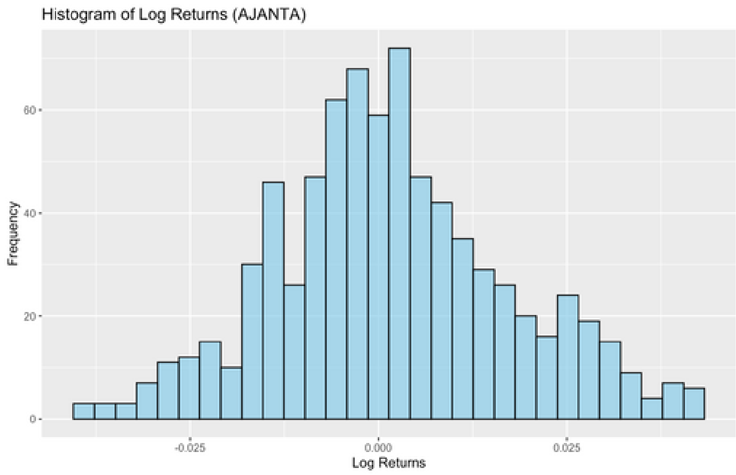



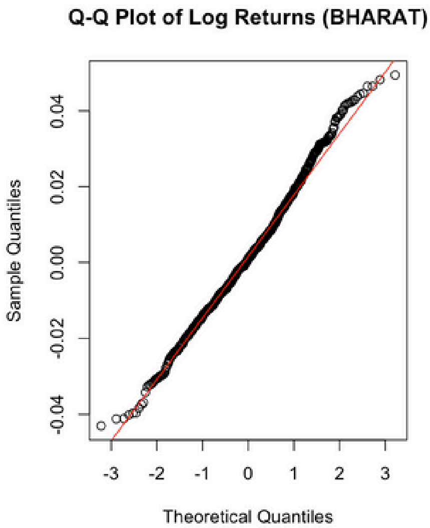



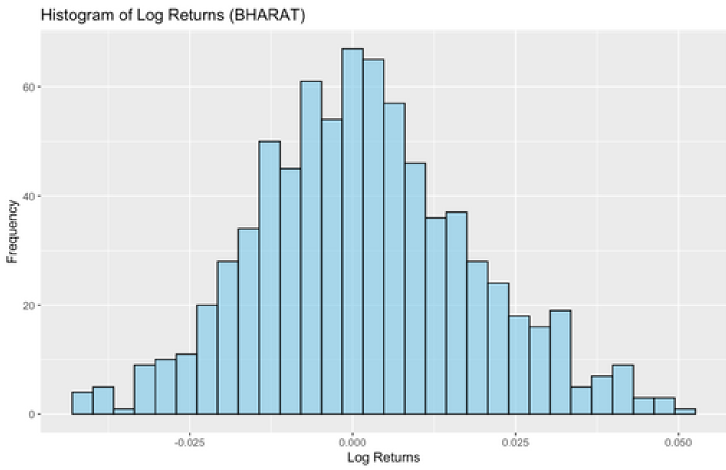



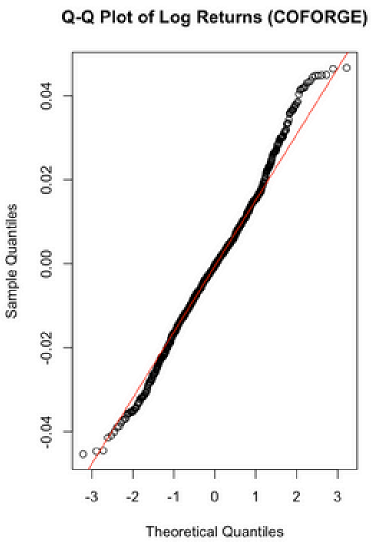
## 1.3- Histogram & Q-Q Plot of Log Returns

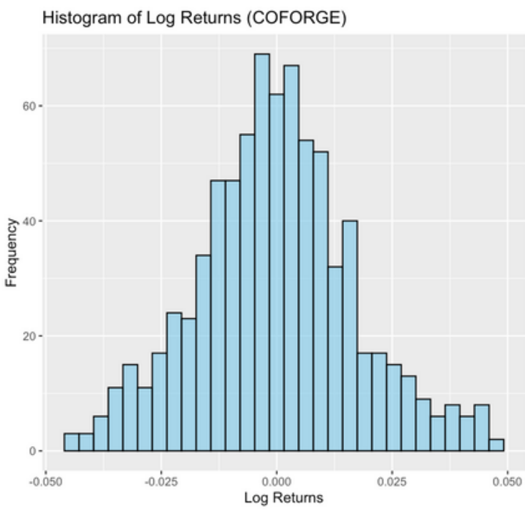


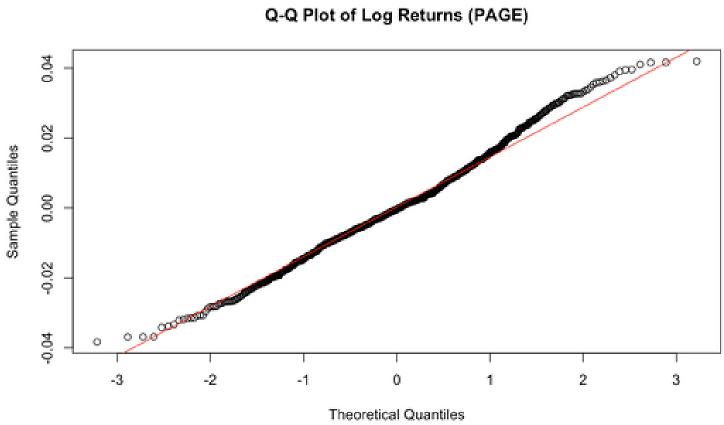



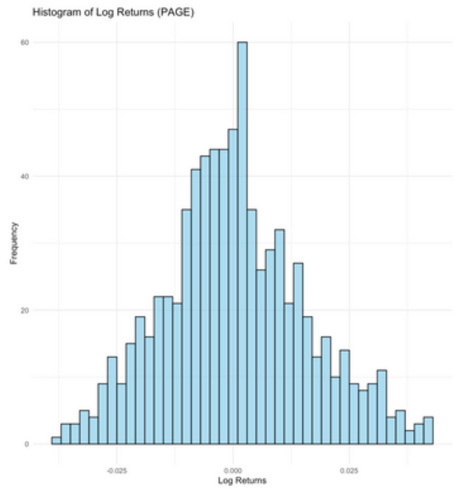



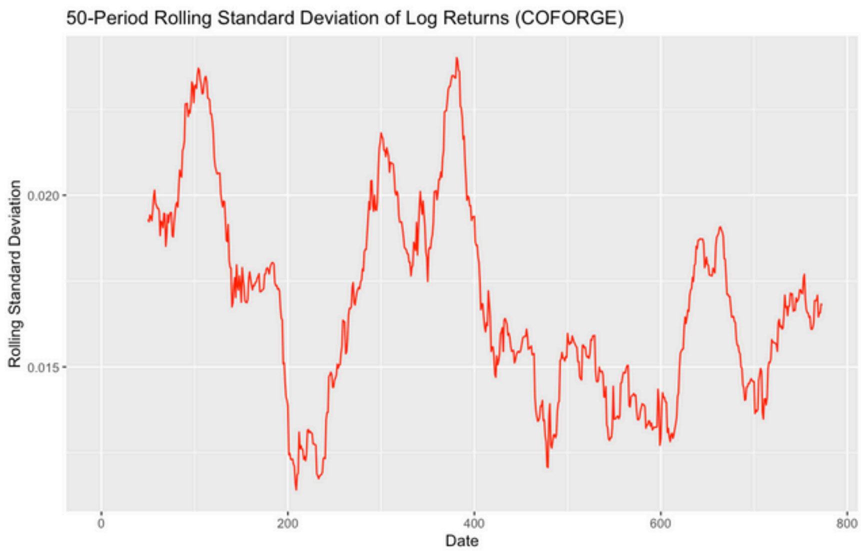



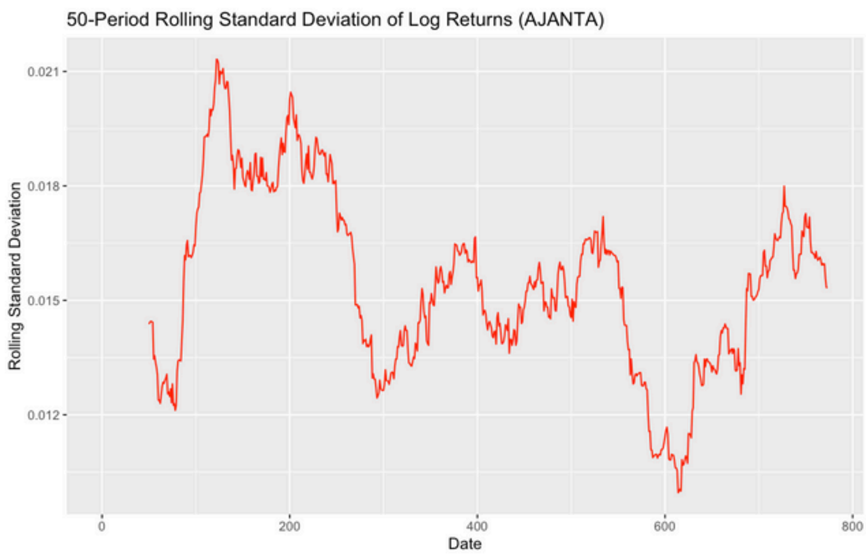



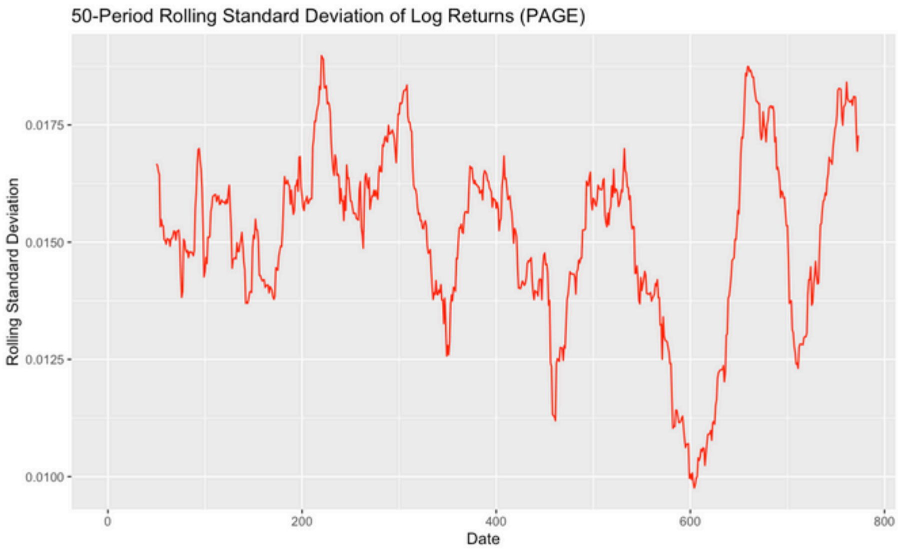


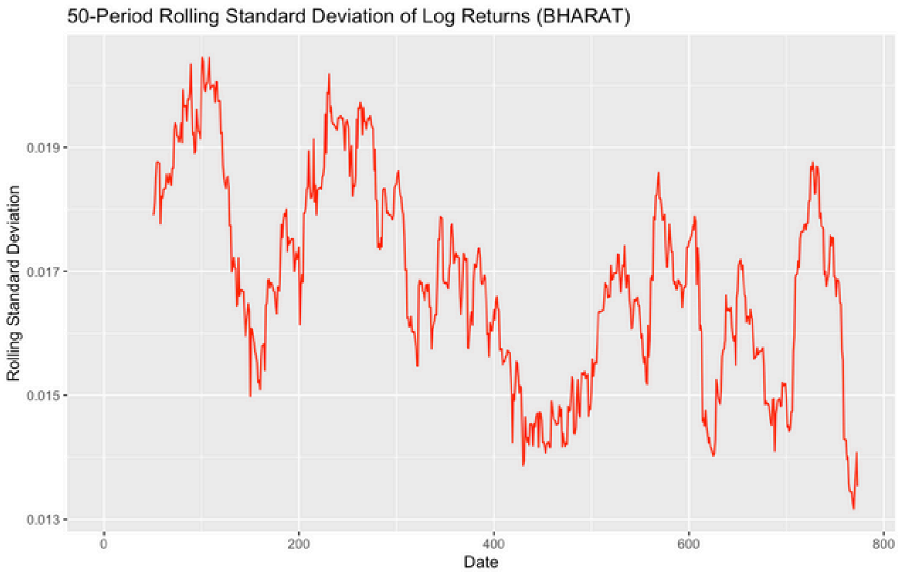




## 1.4- 50-Period Rolling Standard Deviation of Log Returns

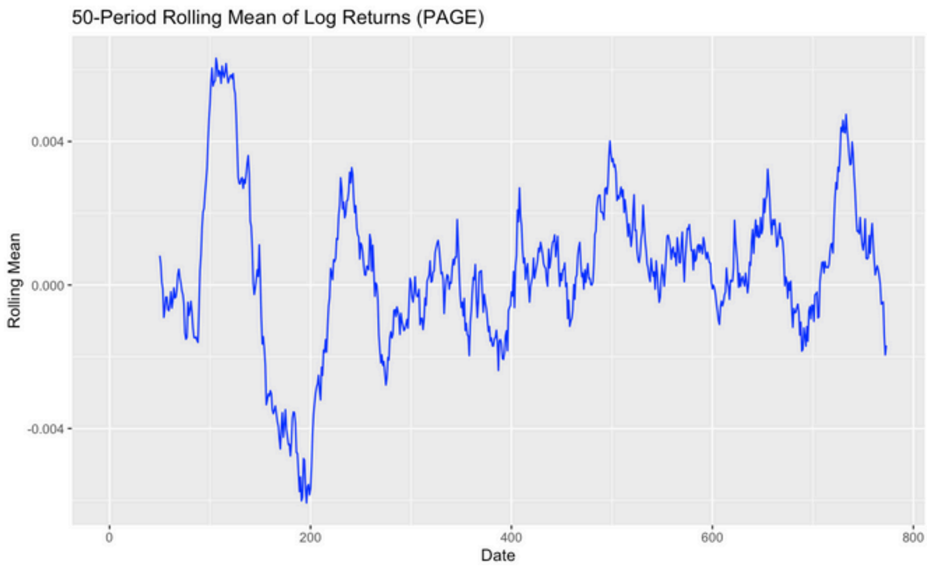


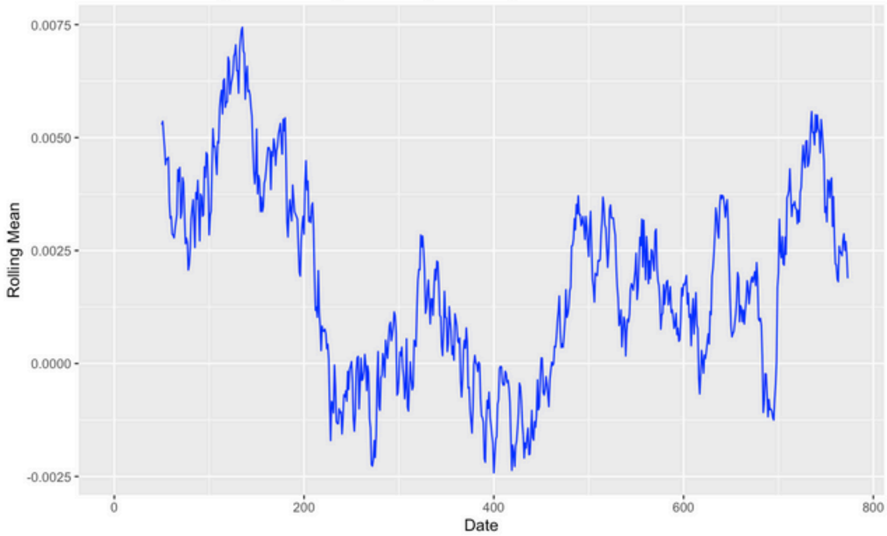


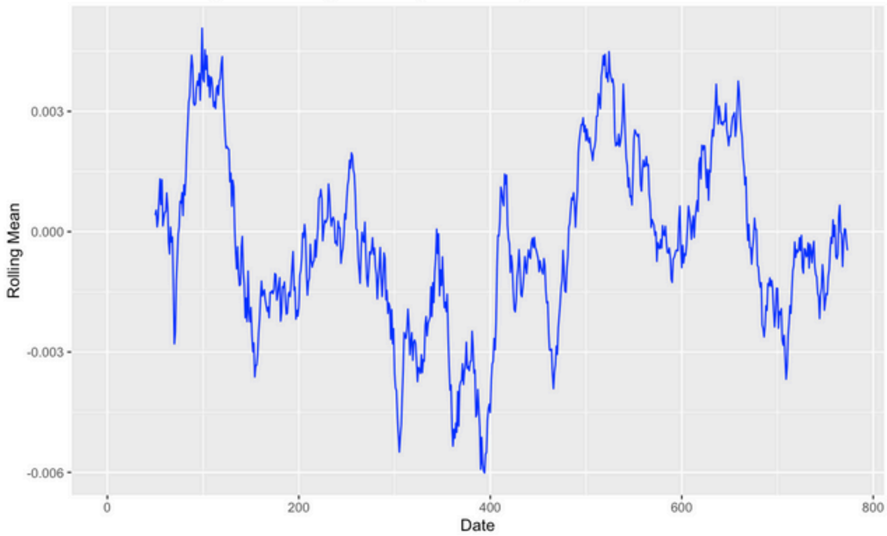

## 1.5- 50-Period Rolling Standard Mean of Log Returns



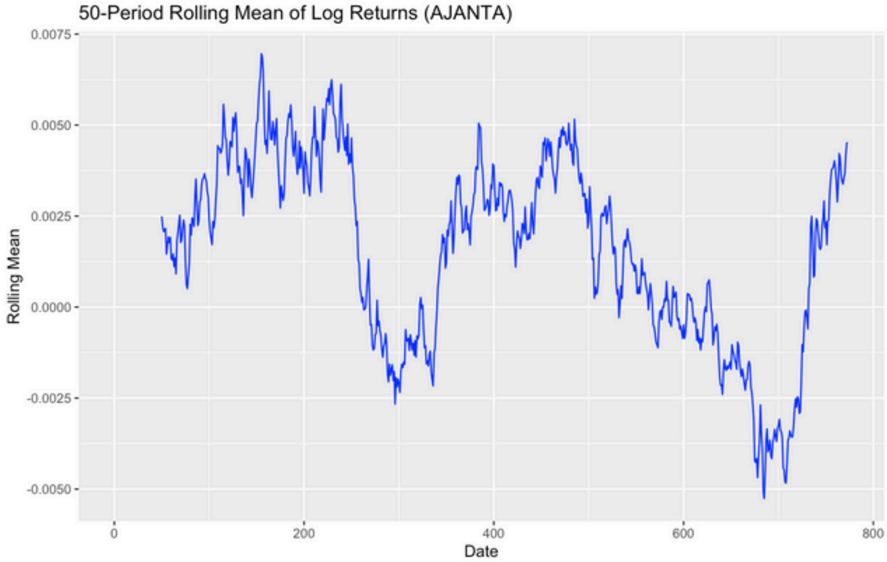
50-Period Rolling Mean of Log Returns (BHARAT)




50-Period Rolling Mean of Log Returns (COFORGE)




50-Period Rolling Mean of Log Returns (AJANTA)



## 1.6- Assumptions Tests Results

```
Checking assumptions for stock: AJANTA


        Shapiro-Wilk normality test

data:  log_returns[[stock]]
W = 0.9906, p-value = 7.639e-05


        Box-Ljung test

data:  log_returns[[stock]]
X-squared = 20.051, df = 20, p-value = 0.4547

Constant Mean Volatility: 0.00257594
Constant Volatility: 0.002363437

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  log_returns[[stock]]
Chi-squared = 18.029, df = 12, p-value = 0.1148


        Augmented Dickey-Fuller Test

data:  log_returns[[stock]]
Dickey-Fuller = -9.1874, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary

Identically Distributed Check - Mean Volatility: 0.00257594
Identically Distributed Check - SD Volatility: 0.002363437
```

```
Checking assumptions for stock: BHARAT


        Shapiro-Wilk normality test

data:  log_returns[[stock]]
W = 0.99428, p-value = 0.005123


        Box-Ljung test

data:  log_returns[[stock]]
X-squared = 14.24, df = 20, p-value = 0.8181

Constant Mean Volatility: 0.002150629
Constant Volatility: 0.001626441

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  log_returns[[stock]]
Chi-squared = 12.658, df = 12, p-value = 0.3944


        Augmented Dickey-Fuller Test

data:  log_returns[[stock]]
Dickey-Fuller = -9.0158, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary

Identically Distributed Check - Mean Volatility: 0.002150629
Identically Distributed Check - SD Volatility: 0.001626441
```

```
Checking assumptions for stock: PAGE


        Shapiro-Wilk normality test

data:  log_returns[[stock]]
W = 0.99227, p-value = 0.0004683


        Box-Ljung test

data:  log_returns[[stock]]
X-squared = 23.127, df = 20, p-value = 0.2826

Constant Mean Volatility: 0.00211892
Constant Volatility: 0.001893903

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  log_returns[[stock]]
Chi-squared = 39.548, df = 12, p-value = 8.545e-05


        Augmented Dickey-Fuller Test

data:  log_returns[[stock]]
Dickey-Fuller = -8.3395, Lag order = 9, p-value =
0.01
alternative hypothesis: stationary

Identically Distributed Check - Mean Volatility: 0.00211892
Identically Distributed Check - SD Volatility: 0.001893903
```

```
Checking assumptions for stock: COFORGE


        Shapiro-Wilk normality test

data:  log_returns[[stock]]
W = 0.99295, p-value = 0.001024


        Box-Ljung test

data:  log_returns[[stock]]
X-squared = 18.074, df = 20, p-value = 0.5826

Constant Mean Volatility: 0.002188182
Constant Volatility: 0.002892995

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  log_returns[[stock]]
Chi-squared = 40.355, df = 12, p-value = 6.276e-05


        Augmented Dickey-Fuller Test

data:  log_returns[[stock]]
Dickey-Fuller = -9.0449, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary

Identically Distributed Check - Mean Volatility: 0.002188182
Identically Distributed Check - SD Volatility: 0.002892995
```

## 1.7- Structural Break Tests Results

```
Checking structural breaks for stock: AJANTA
Breakpoints:  NA

         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = stock_data ~ 1)

Breakpoints at observation number:

m = 1       238
m = 2       238     537
m = 3       232 349 485
m = 4   120 238 368 485
m = 5   120 238 368 485 626

Corresponding to breakdates:

m = 1                     0.307891332470893
m = 2                     0.307891332470893                   0.694695989650712
m = 3                     0.30012936610608  0.451487710219922 0.627425614489004
m = 4   0.155239327296248 0.307891332470893 0.476067270375162 0.627425614489004
m = 5   0.155239327296248 0.307891332470893 0.476067270375162 0.627425614489004 0.809831824062096

Fit:

m   0         1         2         3         4         5
RSS     0.1882     0.1862     0.1857     0.1844     0.1841     0.1840
BIC -4224.9740 -4219.6234 -4208.7013 -4200.7146 -4188.6917 -4175.5452
```

```
Checking structural breaks for stock: BHARAT
Breakpoints:  NA

         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = stock_data ~ 1)

Breakpoints at observation number:

m = 1   180
m = 2   180     452
m = 3   180 337 452
m = 4   180 337 452 567
m = 5   180 298 419 534 651

Corresponding to breakdates:

m = 1   0.232858990944373
m = 2   0.232858990944373                   0.584734799482536
m = 3   0.232858990944373 0.435963777490298 0.584734799482536
m = 4   0.232858990944373 0.435963777490298 0.584734799482536 0.733505821474774
m = 5   0.232858990944373 0.385510996119017 0.542043984476067 0.690815006468305 0.842173350582147

Fit:

m   0         1         2         3         4         5
RSS     0.2175     0.2154     0.2143     0.2141     0.2140     0.2143
BIC -4113.1102 -4107.1343 -4097.8667 -4085.1999 -4072.4169 -4057.9540
```

```
Checking structural breaks for stock: PAGE
Breakpoints:  NA

         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = stock_data ~ 1)

Breakpoints at observation number:

m = 1   120
m = 2   120 296
m = 3   120     358 497
m = 4   120 246 448     655
m = 5   120 246 396 527 655

Corresponding to breakdates:

m = 1   0.155239327296248
m = 2   0.155239327296248 0.382923673997413
m = 3   0.155239327296248                   0.463130659767141 0.642949547218629
m = 4   0.155239327296248 0.318240620957309 0.579560155239327                   0.847347994825356
m = 5   0.155239327296248 0.318240620957309 0.51228978007762  0.681759379042691 0.847347994825356

Fit:

m   0         1         2         3         4         5
RSS     0.1807     0.1797     0.1788     0.1785     0.1784     0.1784
BIC -4256.0994 -4247.1854 -4238.0076 -4225.5789 -4213.0378 -4199.7270
```

```
Checking structural breaks for stock: COFORGE
Breakpoints:  NA

         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = stock_data ~ 1)

Breakpoints at observation number:

m = 1               463
m = 2       256 393
m = 3       256     463 636
m = 4   116 256     463 636
m = 5   116 241 361 477 636

Corresponding to breakdates:

m = 1                                                        0.598965071151358
m = 2                    0.33117723156533  0.508408796895213
m = 3                    0.33117723156533                    0.598965071151358 0.822768434670116
m = 4   0.15006468305304 0.33117723156533                    0.598965071151358 0.822768434670116
m = 5   0.15006468305304 0.311772315653299 0.467011642949547 0.617076326002587 0.822768434670116

Fit:

m   0         1         2         3         4         5
RSS     0.2275     0.2268     0.2255     0.2246     0.2244     0.2245
BIC -4078.3119 -4067.2291 -4058.4839 -4048.1703 -4035.6497 -4022.0550
```

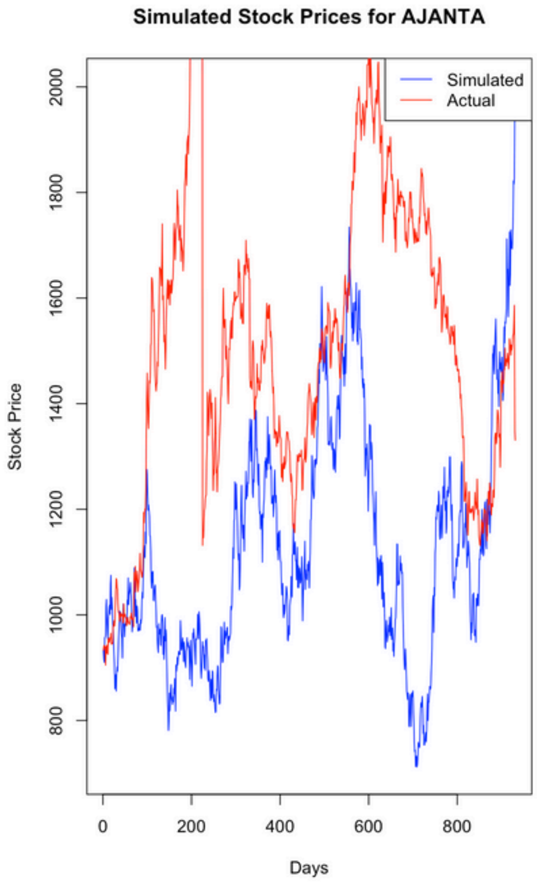
### 1.8- Parameter Estimation

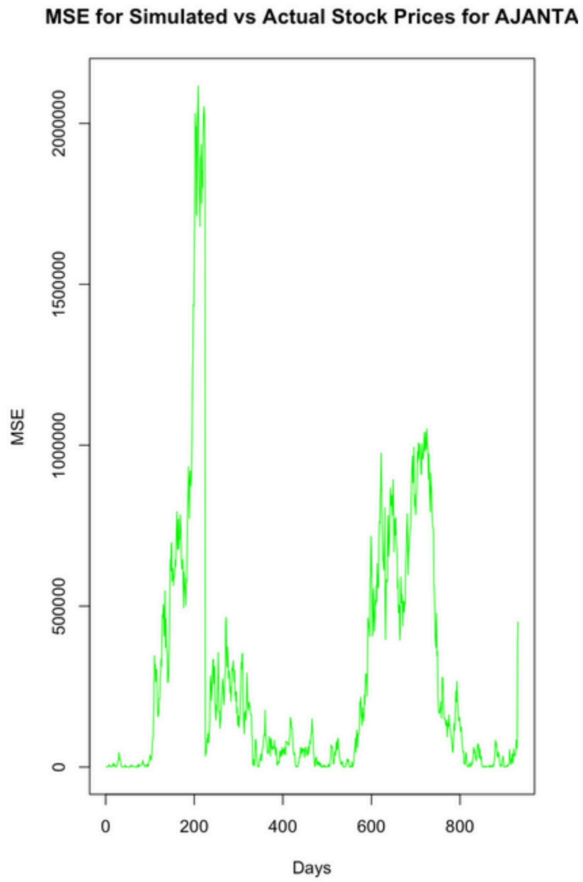
| | Stock | Drift_mu | Volatility_sigma |
|---|---|---|---|
| 1 | PAGE | 0.0015431646 | 0.02218753 |
| 2 | COFORGE | 0.0006570631 | 0.02376397 |
| 3 | BHARAT | 0.0008190898 | 0.03307974 |
| 4 | AJANTA | 0.0003815897 | 0.03137761 |

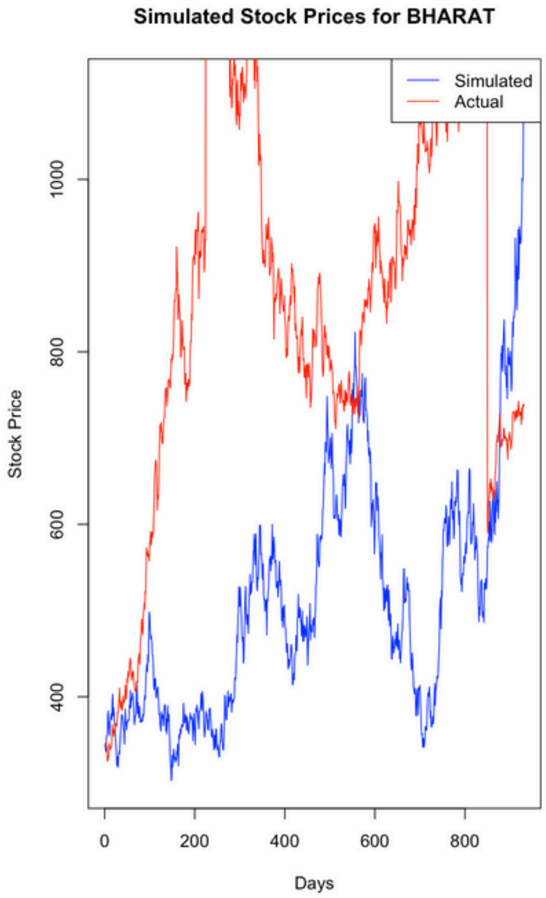
### 1.9- Mean Squared Error

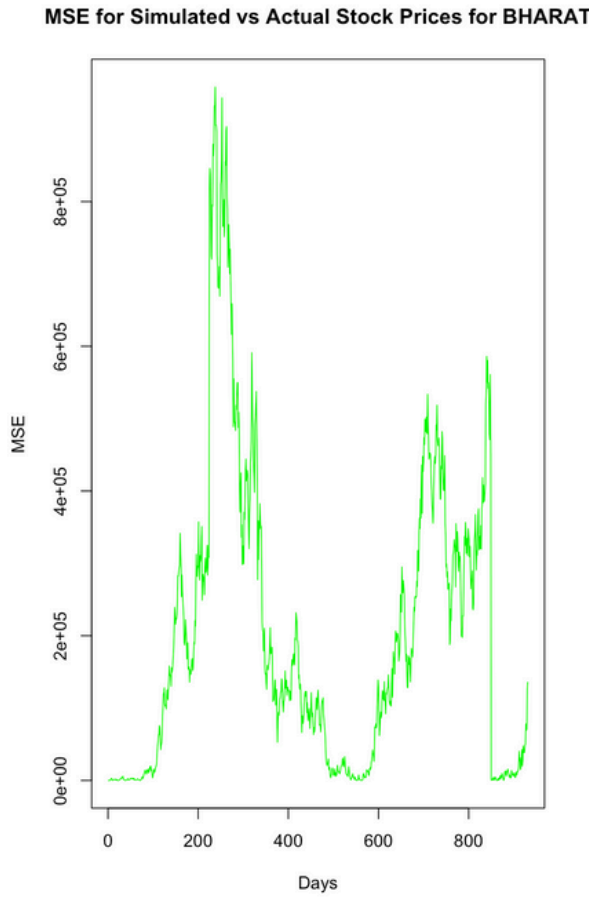
| | Stock | MSE |
|---|---|---|
| 1 | PAGE | 8627594.34 |
| 2 | COFORGE | 25683.28 |
| 3 | BHARAT | 202812.90 |
| 4 | AJANTA | 282217.64 |

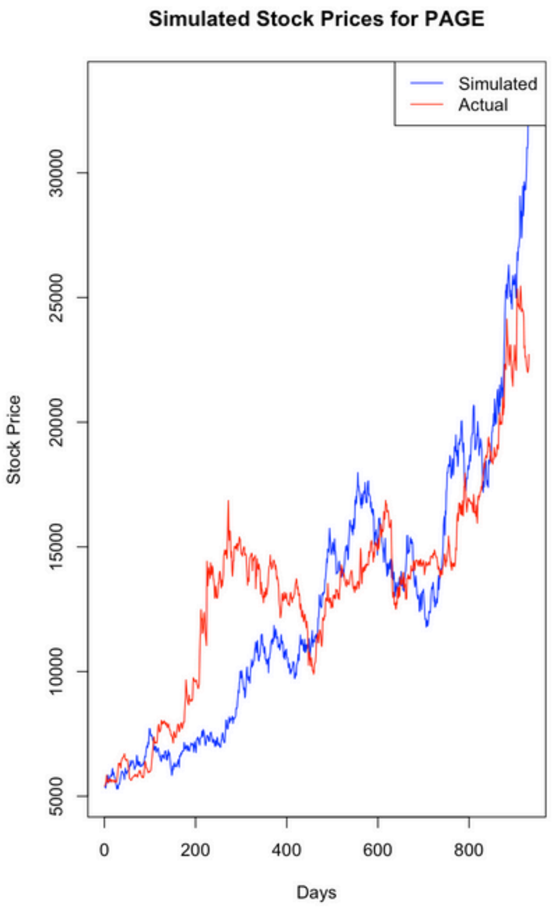
### 1.10- GBM Simulations VS Actual Prices

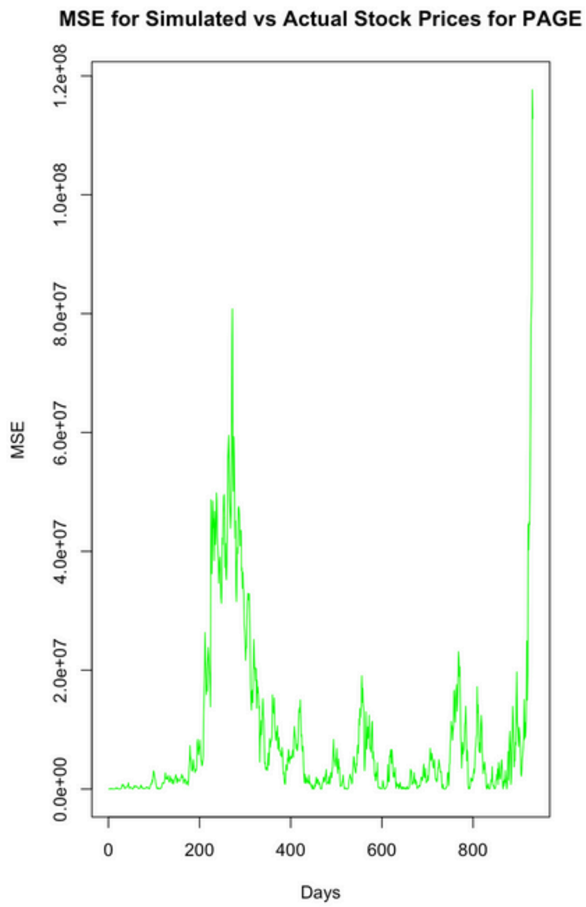


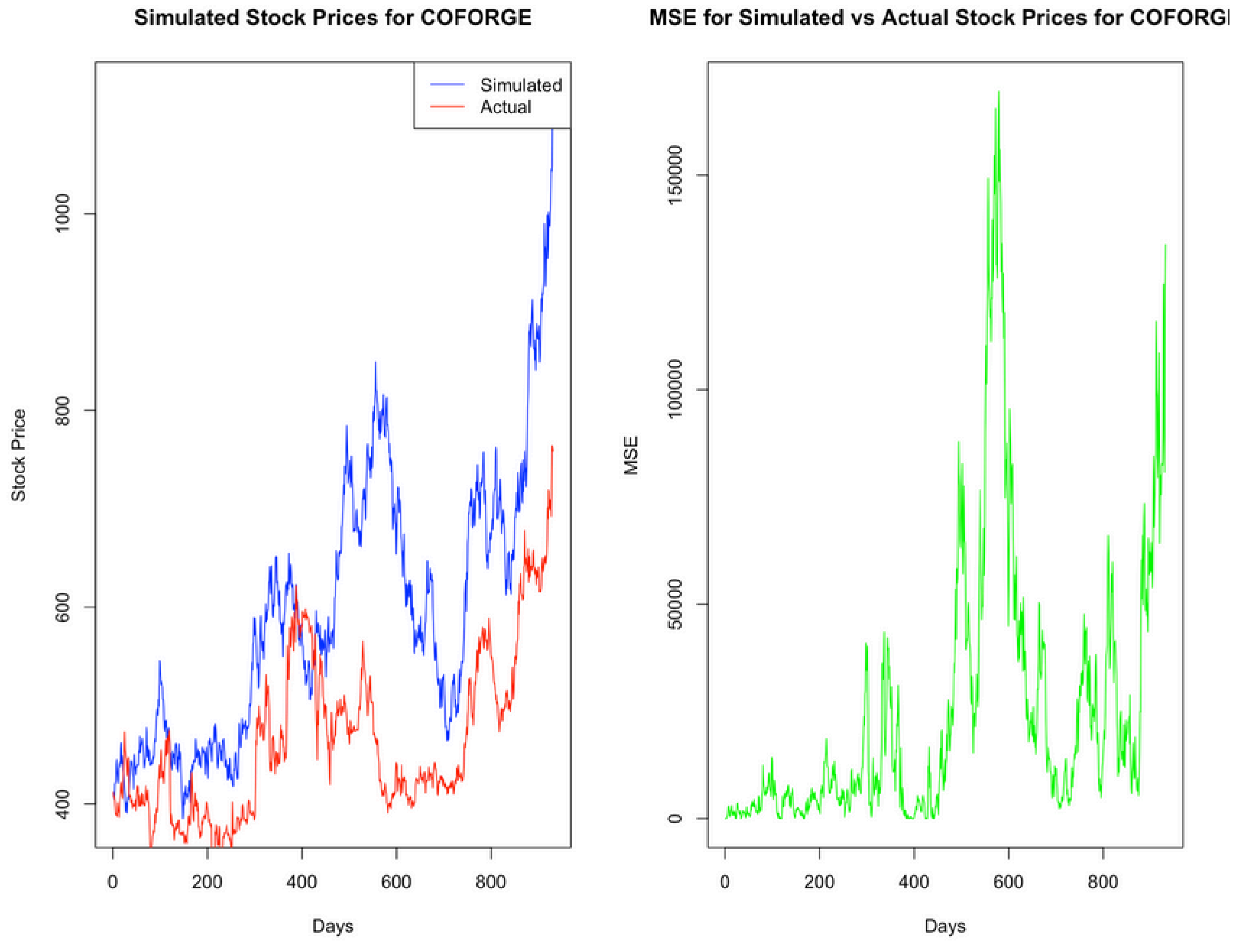




Simulated Stock Prices for BHARAT
Simulated
Actual
Stock Price
400
600
800
1000
0
200
400
600
800
Days

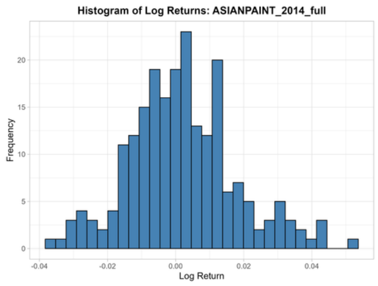

MSE for Simulated vs Actual Stock Prices for BHARAT
MSE
0e+00
2e+05
4e+05
6e+05
8e+05
0
200
400
600
800
Days

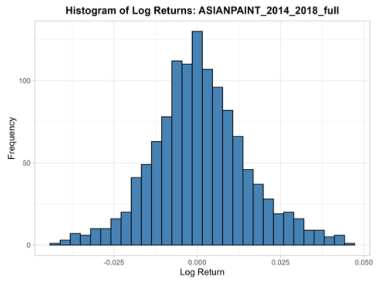

Simulated Stock Prices for PAGE
Simulated
Actual
Stock Price
5000
10000
15000
20000
25000
30000
0
200
400
600
800
Days

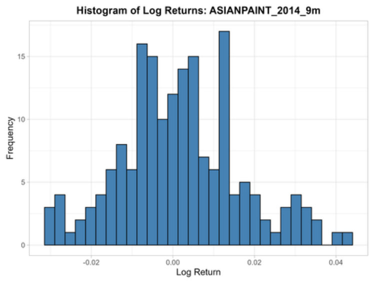

MSE for Simulated vs Actual Stock Prices for PAGE
MSE
0.0e+00
2.0e+07
4.0e+07
6.0e+07
8.0e+07
1.0e+08
1.2e+08
0
200
400
600
800
Days

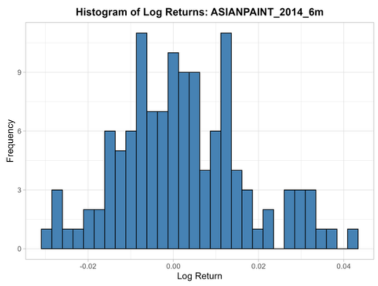

Simulated Stock Prices for COFORGE
Simulated
Actual
Stock Price
Days
MSE for Simulated vs Actual Stock Prices for COFORGI
MSE
Days

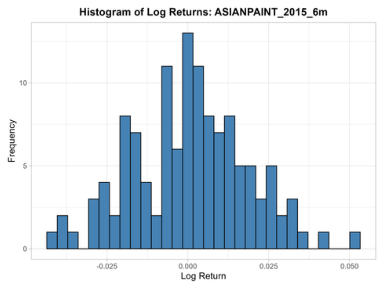
## 2.1- Historgrams of TS

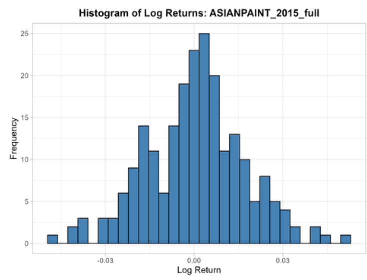


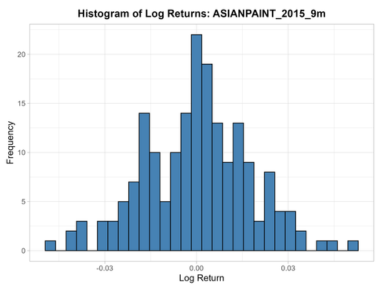



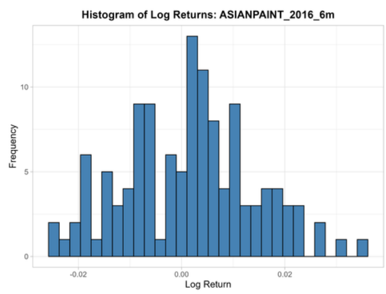



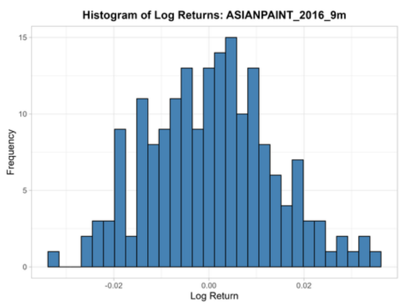



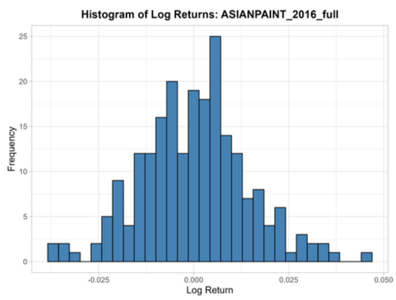



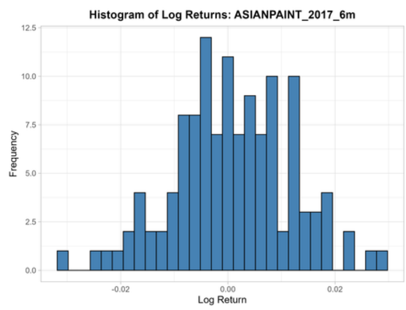



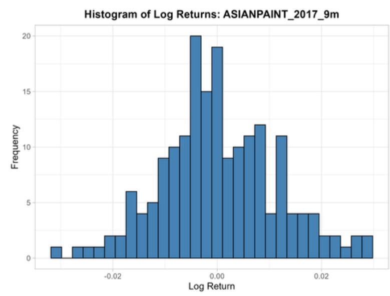



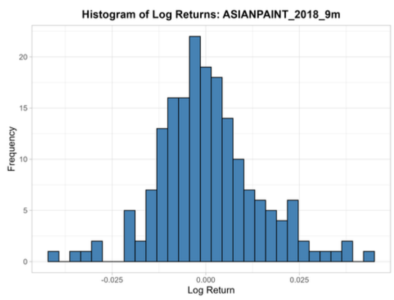



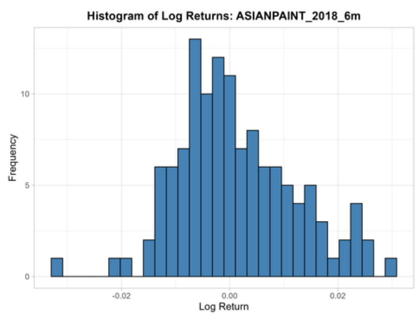



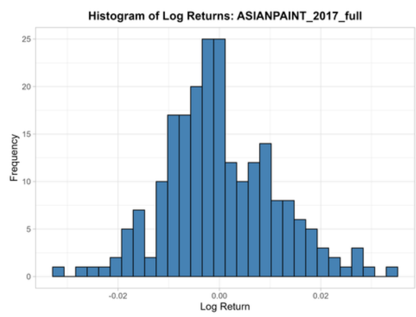



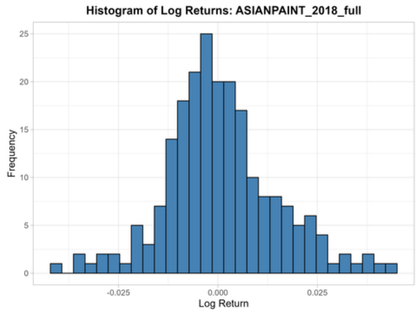



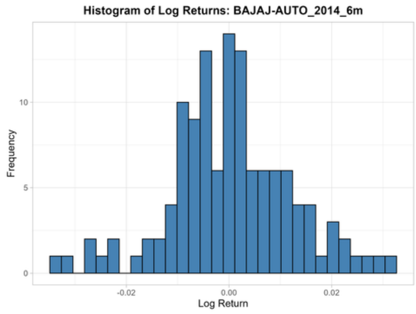



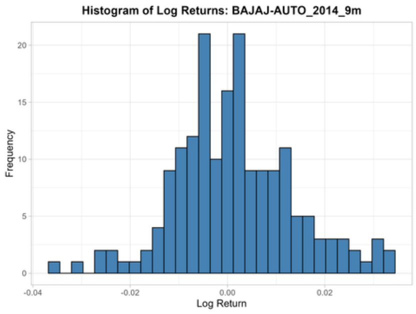



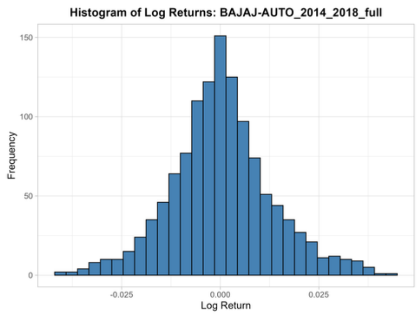




Histogram of Log Returns: ASIANPAINT_2018_full
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2014_6m
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2014_9m
Frequency
Log Return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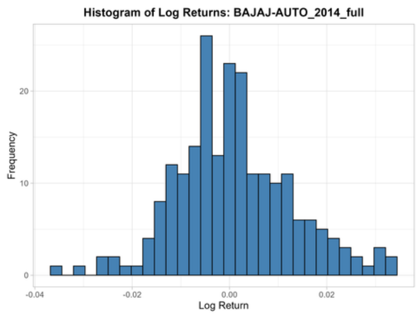
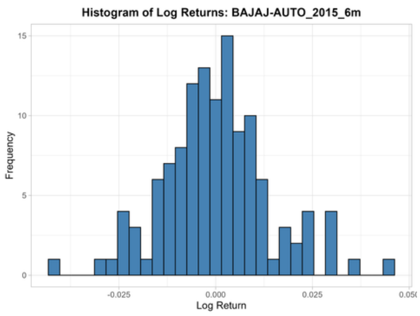
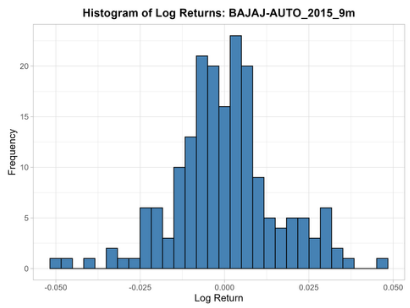
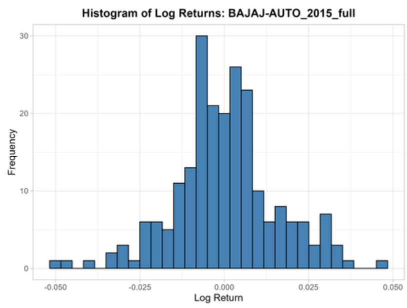
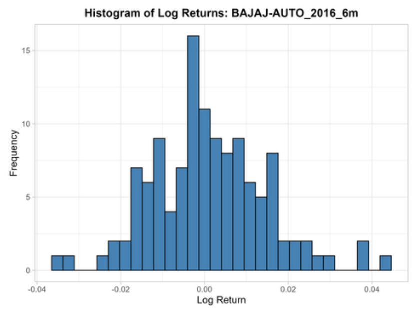
Frequency
Log Return

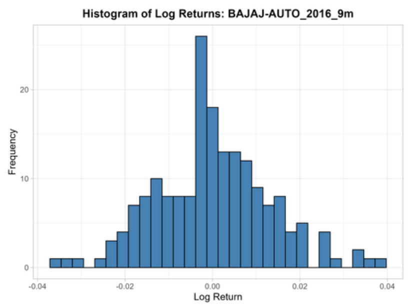

Histogram of Log Returns: BAJAJ-AUTO_2014_full
Frequency
Log Return

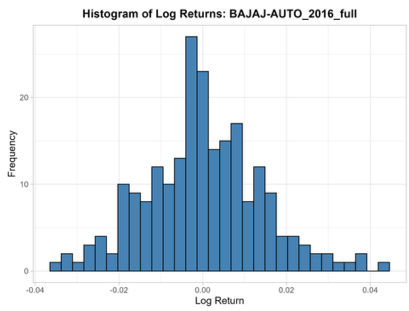

Histogram of Log Returns: BAJAJ-AUTO_2015_6m
Frequency
Log Return

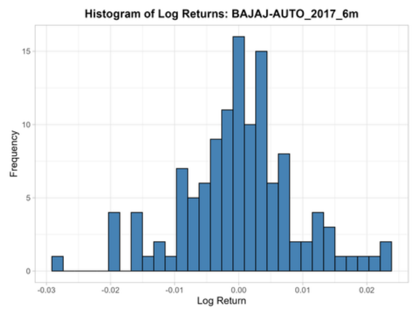

Histogram of Log Returns: BAJAJ-AUTO_2015_9m
Frequency
Log Return

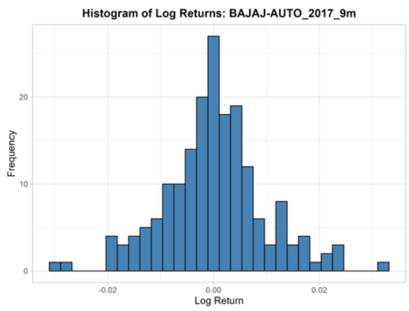

Histogram of Log Returns: BAJAJ-AUTO_2015_full
Frequency
Log Return

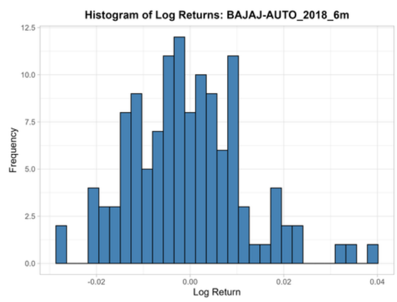

Histogram of Log Returns: BAJAJ-AUTO_2016_6m
Frequency
Log Return

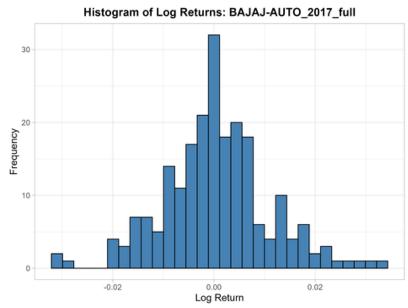

Histogram of Log Returns: BAJAJ-AUTO_2016_9m
Frequency
Log Return

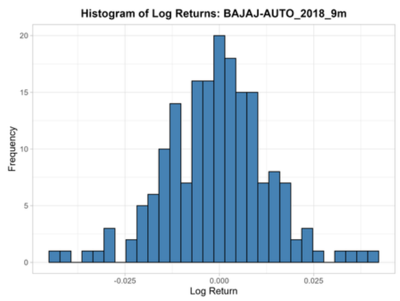

Histogram of Log Returns: BAJAJ-AUTO_2016_full
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2017_6m
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2017_9m
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2018_6m
Frequency
Log Return


Histogram of Log Returns: BAJAJ-AUTO_2017_full
Frequency
Log Return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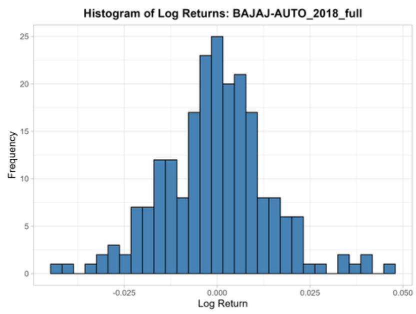
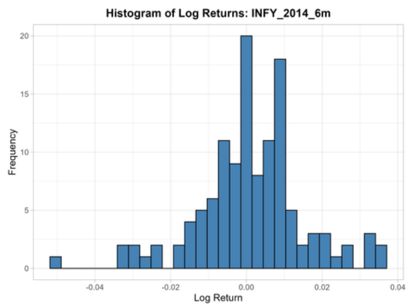
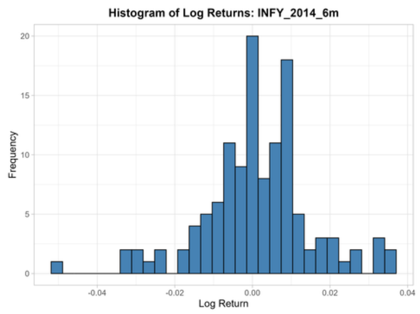
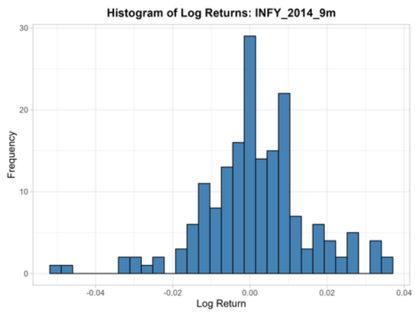
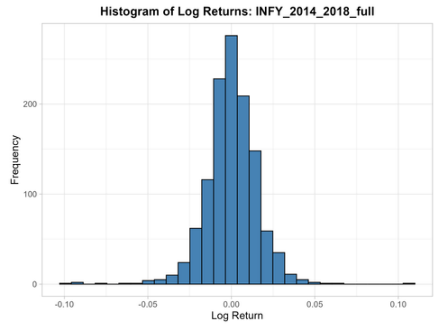
Frequency
Log Return

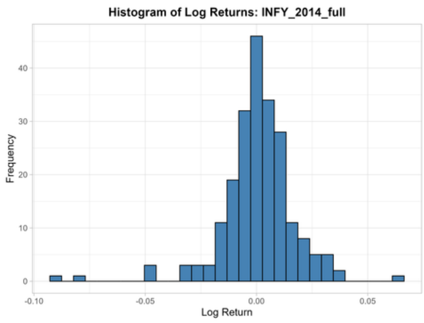

Histogram of Log Returns: BAJAJ-AUTO_2018_full
Frequency
Log Return

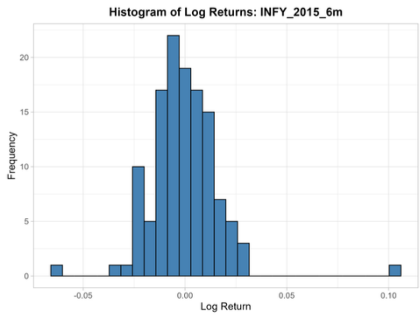

Histogram of Log Returns: INFY_2014_6m
Frequency
Log Return

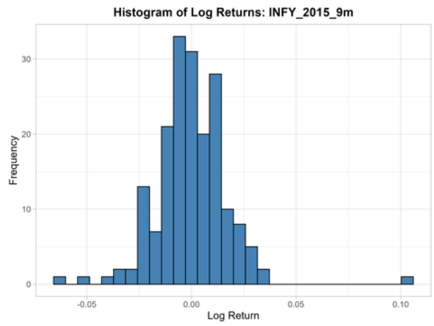

Histogram of Log Returns: INFY_2014_6m
Frequency
Log Return

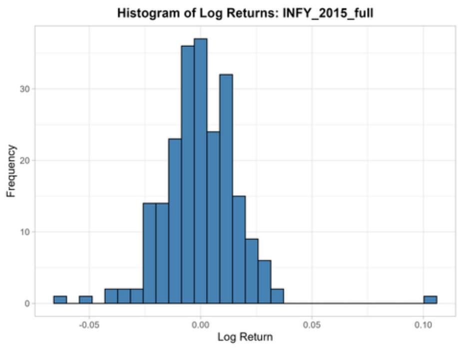

Histogram of Log Returns: INFY_2014_9m
Frequency
Log Return

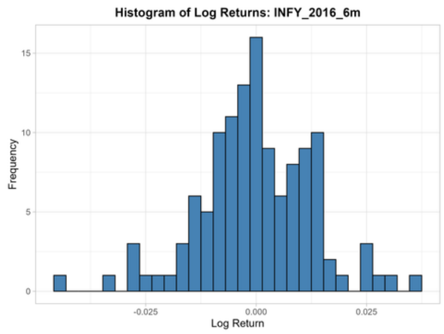

Histogram of Log Returns: INFY_2014_2018_full
Frequency
Log Return

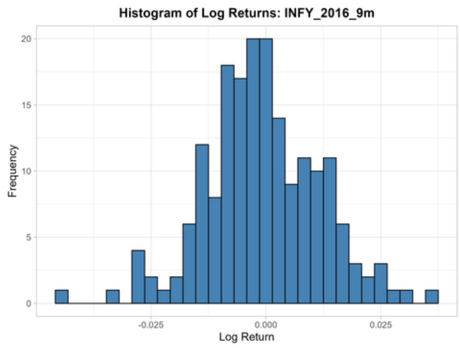

Histogram of Log Returns: INFY_2014_full
Frequency
Log Return

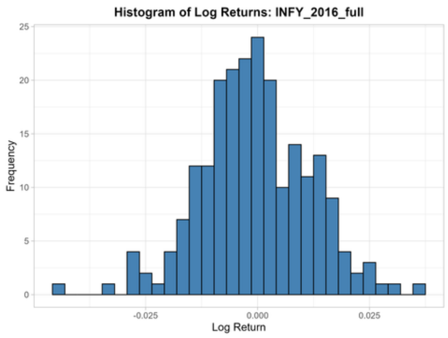

Histogram of Log Returns: INFY_2015_6m
Frequency
Log Return

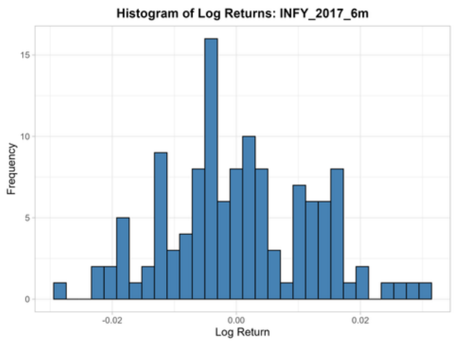

Histogram of Log Returns: INFY_2015_9m
Frequency
Log Return

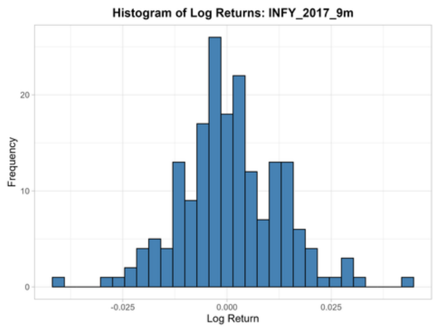

Histogram of Log Returns: INFY_2015_full
Frequency
Log Return


Histogram of Log Returns: INFY_2016_6m
Frequency
Log Return


Histogram of Log Returns: INFY_2016_9m
Frequency
Log Return


Histogram of Log Returns: INFY_2016_full
Frequency
Log Return


Histogram of Log Returns: INFY_2017_6m
Frequency
Log Return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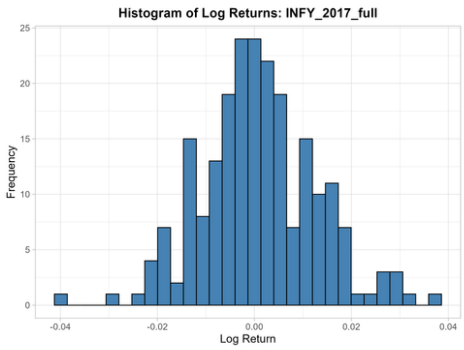
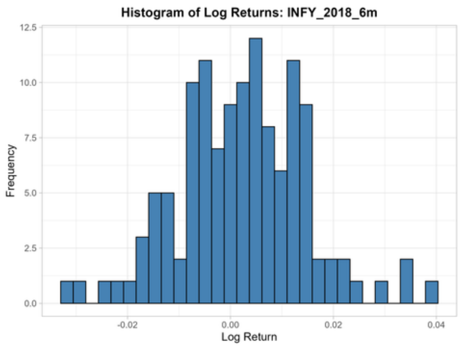
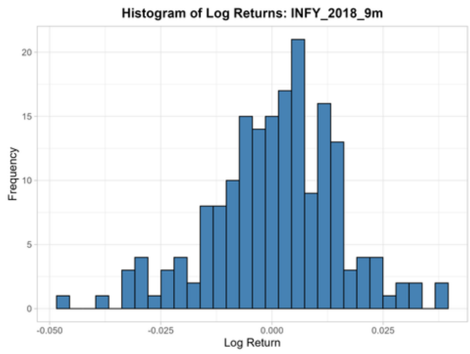
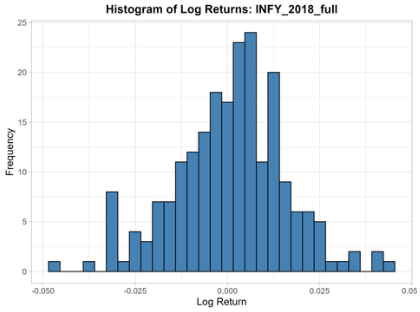
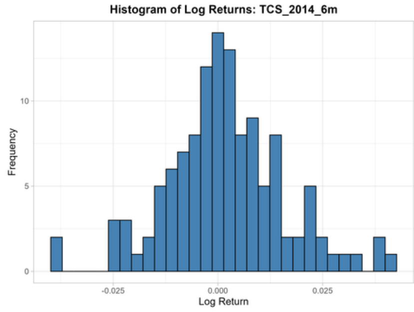
Frequency
Log Return

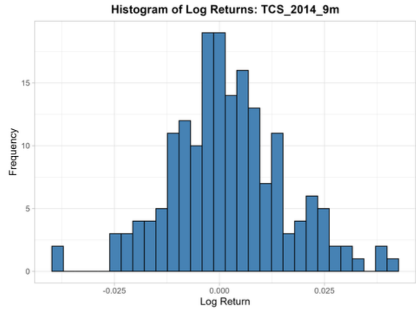

Histogram of Log Returns: INFY_2017_full
Frequency
Log Return

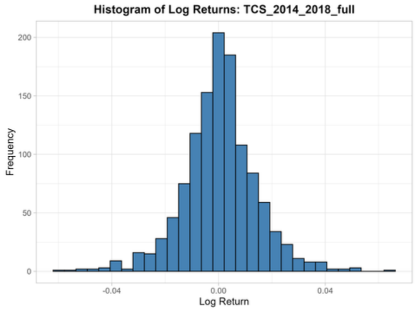

Histogram of Log Returns: INFY_2018_6m
Frequency
Log Return

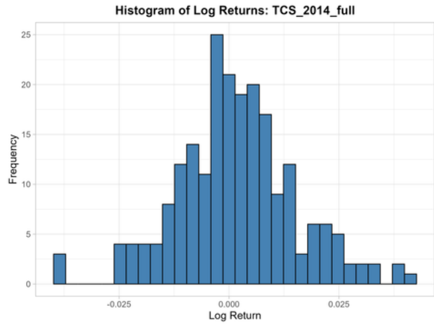

Histogram of Log Returns: INFY_2018_9m
Frequency
Log Return

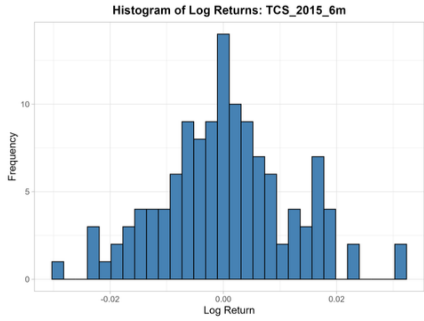

Histogram of Log Returns: INFY_2018_full
Frequency
Log Return

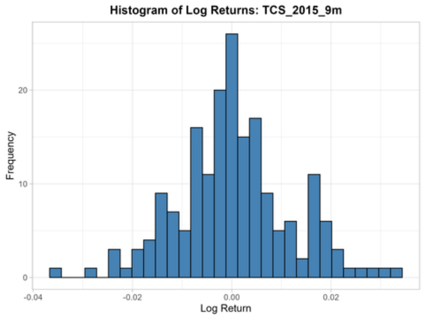

Histogram of Log Returns: TCS_2014_6m
Frequency
Log Return

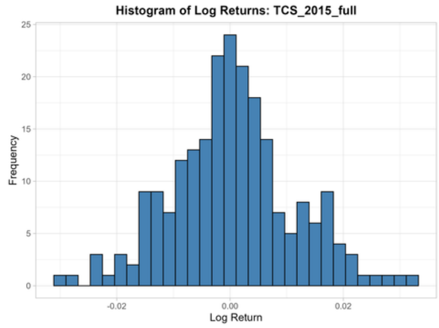

Histogram of Log Returns: TCS_2014_9m
Frequency
Log Return

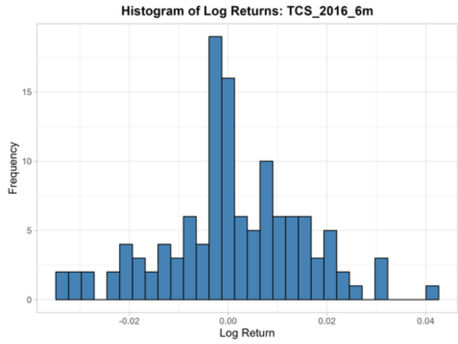

Histogram of Log Returns: TCS_2014_2018_full
Frequency
Log Return

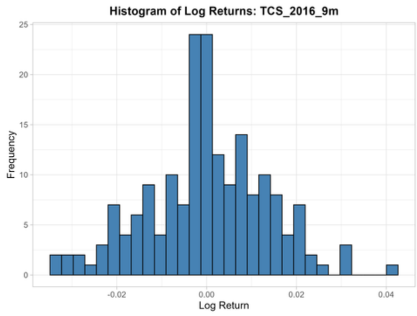

Histogram of Log Returns: TCS_2014_full
Frequency
Log Return

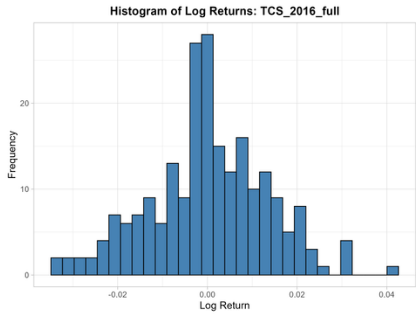

Histogram of Log Returns: TCS_2015_6m
Frequency
Log Return

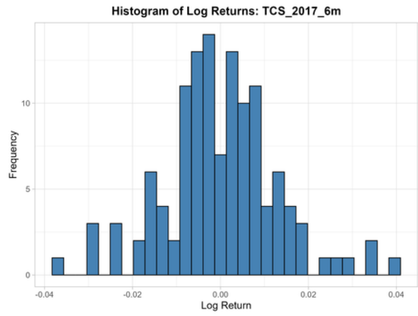

Histogram of Log Returns: TCS_2015_9m
Frequency
Log Return


Histogram of Log Returns: TCS_2015_full
Frequency
Log Return


Histogram of Log Returns: TCS_2016_6m
Frequency
Log Return


Histogram of Log Returns: TCS_2016_9m
Frequency
Log Return


Histogram of Log Returns: TCS_2016_full
Frequency
Log Return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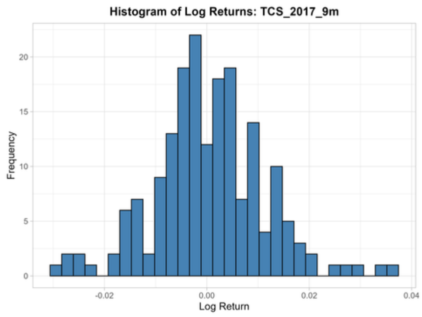
Frequency
Log Return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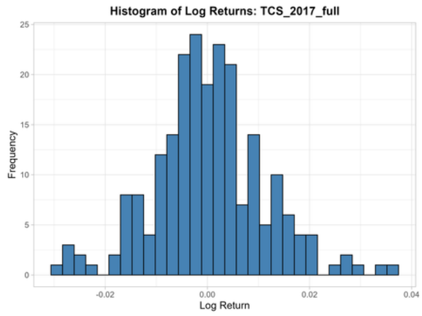
Frequency
Log Return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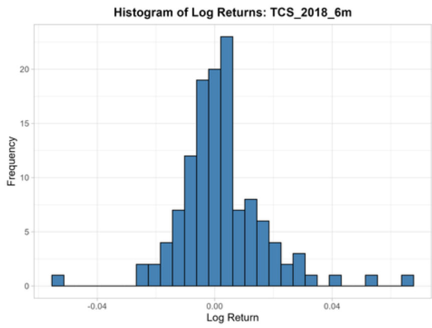
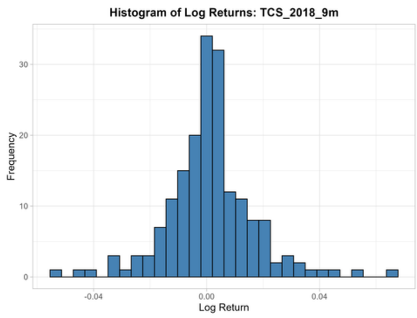
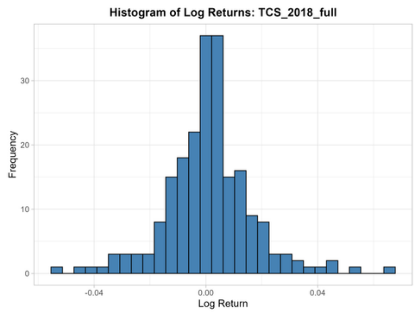
Frequency
Log Return


Histogram of Log Returns: TCS_2018_6m
Frequency
Log Return


Histogram of Log Returns: TCS_2018_9m
Frequency
Log Return


Histogram of Log Returns: TCS_2018_full
Frequency
Log Return

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