FAC 413

INTRODUCTION TO MATHEMATICAL FINANCE AND FINANCIAL ENGINEERING

MINI PROJECT 3

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ABSTRACT

This report presents a detailed financial analysis aimed at constructing an optimal investment strategy for Mr. SRK, based on sectoral performance within the Indian equity market. Using daily return data from the Fashion, Manufacturing, and Food sectors, the study evaluates both individual and portfolio-level risk-return dynamics. The Capital Asset Pricing Model (CAPM) was applied to estimate expected returns and sector-specific beta coefficients, providing insight into systematic risk. Rigorous testing of classical linear regression assumptions was conducted, with Newey-West and robust standard errors employed to address autocorrelation and heteroskedasticity, ensuring the reliability of estimates. Time series diagnostics confirmed the weak dependence assumption and guided appropriate methodological choices. To develop a portfolio aligned with Mr. SRK's investment objectives, the analysis incorporated Sharpe Ratio Maximization, the Minimum Variance Portfolio, and the Single Index Model. These approaches provided a nuanced understanding of risk-adjusted returns, diversification benefits, and sector-specific exposures, culminating in a robust and data-driven investment recommendation.

INTRODUCTION

In an increasingly complex and volatile financial environment, making informed investment decisions requires a comprehensive understanding of both risk and return dynamics. This report is tailored to address the investment needs of Mr. SRK, an investor seeking a well-reasoned strategy for allocating capital across key sectors of the Indian equity market. The analysis focuses on three prominent sectors—Fashion, Manufacturing, and Food—chosen for their diverse risk profiles and economic significance.

To evaluate the investment potential of these sectors, the study leverages daily return data and applies a blend of financial modeling techniques. The Capital Asset Pricing Model (CAPM) serves as the foundational framework to estimate expected returns based on market movements, while also quantifying systematic risk through the beta coefficient. To ensure statistical robustness, each regression model underwent classical assumption testing, and appropriate corrections—such as Newey-West and robust standard errors—were implemented wherever necessary.

Beyond individual sector analysis, the report integrates advanced portfolio optimization methods, including Sharpe Ratio Maximization, the Minimum Variance Portfolio (MVP), and the Single Index Model. These tools provide insights into risk-adjusted performance, optimal diversification, and efficient capital allocation strategies tailored to varying investor preferences.
By synthesizing asset pricing theory with real-world data and rigorous statistical validation, the report aims to deliver actionable insights and a reliable investment roadmap that aligns with Mr. SRK's financial goals and risk tolerance.

DATA

This analysis utilizes a comprehensive dataset containing 516 daily observations of excess returns for three key industry sectors: Fashion, Manufacturing, and Food, along with corresponding market portfolio returns and risk-free rates. The dataset enables robust quantitative analysis to support investment decision-making for our client, Mr. SRK.

DATA VARIABLES

Fashion Sector Excess Returns

Daily excess returns (sector returns minus risk-free rate) for the fashion industry, providing insights into its risk-adjusted performance characteristics.

Manufacturing Sector Excess Returns

Daily excess returns for the manufacturing sector, reflecting its cyclical performance and economic sensitivity.

Food Sector Excess Returns

Daily excess returns for the food industry, offering perspective on its defensive qualities during market fluctuations.

Market Portfolio Excess Returns

Daily benchmark returns (net of risk-free rate) representing broad market performance for systematic risk assessment.

Risk-Free Rate

Daily risk-free benchmark (T-bill equivalent) used for calculating excess returns across all series.

This dataset will support portfolio optimization, CAPM-based risk assessment, and time series analysis to provide Mr. SRK with tailored investment recommendations.

METHODOLOGY

This analysis employs a multi-method quantitative approach to evaluate investment opportunities across three key sectors (Fashion, Manufacturing, and Food) using daily excess return data. By integrating portfolio optimization techniques, CAPM-based risk assessment, and time series analysis, we aim to:

- Identify the **most pragmatic sector** allocation based on risk-adjusted returns
- Determine the **safest investment** for risk-averse preferences
- Recommend a **balanced option** combining moderate risk and return

The methodology rigorously tests both cross-sectional relationships (via CAPM) and temporal patterns (through trend/cyclicality analysis) to provide empirically grounded recommendations while addressing potential limitations of each approach.

DATA CLEANING AND PREPARATION

To ensure the accuracy and integrity of the analysis, the raw financial dataset was reviewed and cleaned using Microsoft Excel before modelling in R. This process focused on structural consistency and preliminary quality checks. The key steps undertaken included:

- **Handling Missing Values:** The dataset was checked for any missing entries or blank cells. None of the rows had missing data.

- **Date Alignment Across Series:** All sectoral data and the corresponding market index data were checked to ensure that entries aligned chronologically. This ensured a consistent time series structure across all variables during the analysis phase.

- **Initial Structural Validation:** Manual checks were conducted in Excel to detect formatting inconsistencies such as duplicate rows or misaligned data.

- **Outlier Identification:** Basic visual inspection through scatter plots and conditional formatting was used to highlight unusually large or small return values. These were flagged for further review but were largely retained for realism, given the inherent volatility in financial data.

DESCRIPTIVE STATISTICS AND EXPLORATORY DATA

To summarize the dataset and gain a foundational understanding of sectoral and market-level returns, several statistical and visual techniques were employed using R:

- **Computation of Summary Statistics:** The describe() function from the psych package was used to compute measures such as mean, median, standard deviation, and range for all key variables, including Fashion, Manufacturing, Food, Market Index, and Interest rates. This helped in identifying the basic distribution characteristics and variability within each series.

- **Variance and Standard Deviation Calculation:** Individual variance and standard deviation values were extracted to assess the volatility associated with each sector and the broader market environment.

- **Density and Histogram Plots:** Kernel density plots and histograms were generated to visually inspect the shape and spread of return distributions. These plots aided in detecting skewness, multimodality, and potential deviations from normality.

- **Scatter Plots and Trend Analysis:** Bivariate scatter plots were created to evaluate the relationship between sectoral returns and the Market Index. Additional scatter plots against time were used to observe trends, volatility clustering, or structural patterns across the time series.

- **Covariance and Correlation Matrix:** Covariance and correlation matrices were computed to understand the interrelationships among variables, especially relevant for diversification and portfolio optimization analysis.

- **Skewness and Kurtosis Computation:** Skewness and kurtosis statistics were used to assess asymmetry and tail behaviour in the data, providing early insight into potential issues with normality or outliers.

These descriptive and graphical tools ensured a deeper understanding of the data's structure and informed subsequent modelling decisions in portfolio optimization, CAPM estimation, and time series analysis.

TIME SERIES ANALYSIS

To examine the temporal behaviour of sectoral returns, a time series analysis was conducted on the Fashion, Manufacturing, and Food sectors. This involved analyzing patterns over time and testing for key assumptions like stationarity and autocorrelation that are essential for robust financial modelling.

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- **Trend Visualization:** Time series plots of sectoral returns were created to visually inspect the movement of returns over time. A linear trendline was added using ordinary least squares (OLS) to determine the presence of any deterministic trend component in the series.

- **Time Series Decomposition:** Each return series was decomposed into three primary components: trend, seasonal, and remainder using classical additive decomposition. This approach helped isolate long-term trends, periodic patterns, and irregular fluctuations to better understand the underlying structure of the data.

- **Autocorrelation Diagnostics:** To examine the presence of serial correlation in model residuals, two tests were conducted:

1. The **Ljung-Box test** was applied up to lag 10 to assess the overall presence of autocorrelation in the residuals.
2. The **Durbin-Watson test** was used to test for first-order autocorrelation specifically. If significant autocorrelation was detected, corrections such as Newey-West standard errors were planned to be applied.

- **Stationarity Testing:** The Augmented Dickey-Fuller (ADF) test was conducted on the residuals of each CAPM model to evaluate stationarity. This test helps determine whether the time series is weakly dependent and mean-reverting—an essential assumption for time series regressions.

SHARPE AND SORTINO RATIO CALCULATION

To evaluate sectoral performance in terms of risk-adjusted returns, two key measures were computed: the Sharpe ratio and the Sortino ratio. These metrics provide insight into how efficiently each sector compensates for risk taken.

- **Sharpe Ratio Calculation:** Sharpe ratios were computed to assess overall risk-adjusted returns using total return volatility.

- **Sortino Ratio Calculation:** Sortino ratios focus specifically on downside risk. The Sortino ratio provides a more refined view of performance under negative market movements by isolating returns that fall below the risk-free rate.

CAPM MODELLING AND ASSUMPTION TESTING

To apply the CAPM model across the Fashion, Manufacturing, and Food sectors, actual returns were calculated by adding the risk-free interest rate to the respective excess returns. All relevant columns were formatted as numeric to ensure proper regression analysis.

Before running the regressions, we verified the key assumptions of the multiple linear regression (MLR) framework:

- It was assumed that the given data set was a **random sample**.

- The **linearity in parameters** was considered fulfilled as the CAPM model is inherently linear

- **No perfect multicollinearity** was confirmed by checking that the Market Index had non-zero variance.

- **Zero conditional mean** was supported by residuals with mean values close to zero and no significant correlation with the Market Index.

- **Homoskedasticity** was tested using the Breusch-Pagan test. While the Fashion and Manufacturing sectors satisfied this assumption, the Food sector violated it. Therefore, robust standard errors were applied where necessary.

- **Normality** of residuals was checked using the Shapiro-Wilk and Kolmogorov-Smirnov tests, alongside histogram and Q-Q plots. Residuals were found to be approximately normal across sectors.

Based on these results, the CAPM model was estimated with appropriate adjustments:

- The Fashion sector used Newey-West standard errors to correct for autocorrelation detected during time series analysis.
- The Manufacturing and Food sectors used robust standard errors (HC3) to account for heteroskedasticity.

These corrections ensured that the CAPM estimations were statistically valid and robust for all three sectors.

PORTFOLIO OPTIMIZATION

To identify optimal sectoral allocations under different investor risk preferences, three complementary methods were employed: Sharpe Ratio Maximization, the Minimum Variance Portfolio (MVP), and the Single Index Model.

Sharpe Ratio Maximization:

- Calculated the Sharpe Ratio for each sector based on their monthly excess returns relative to the risk-free rate.
- The Sharpe Ratio, defined as the excess return per unit of risk (standard deviation), was used to identify the most efficient risk-adjusted performance sector.
- This measure guided the selection of a portfolio that maximizes return for a given level of risk, aligning with the preferences of moderately risk-tolerant investors.

Minimum Variance Portfolio (MVP):

- Constructed a portfolio that minimizes total variance while maintaining full capital allocation across sectors.
- Used historical excess return data to estimate the variance-covariance matrix and determine optimal weights.
- This approach was particularly useful for identifying the safest possible portfolio configuration, suitable for highly risk-averse investors.

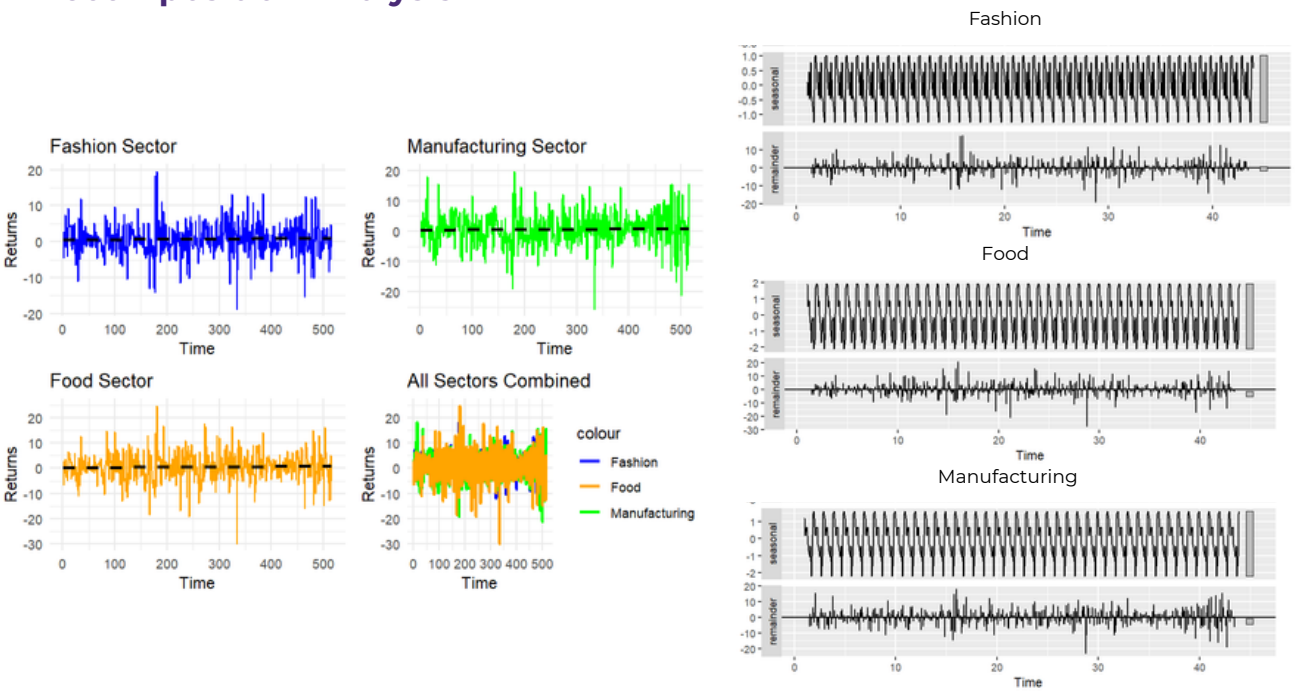
Single Index Regression:

- Conducted linear regressions of each sector's excess returns against the market index to estimate beta (systematic risk) and alpha (sector-specific outperformance).
- Extracted the idiosyncratic volatility (residual risk) from the regression model to quantify firm-specific risk that cannot be diversified away.
- These metrics formed the basis for evaluating each sector's contribution to overall portfolio risk and return and aligning recommendations with the investor's risk appetite.

RESULTS

TIME SERIES ANALYSIS

Decomposition Analysis



The decomposition of daily sectoral return data into trend, seasonality, and residual components revealed important characteristics for each sector:

- All three sectors — Fashion, Manufacturing, and Food — **exhibited notable volatility and seasonality in their return series with no long term trend**.
- The Fashion sector displayed the most frequent seasonal oscillations, suggesting shorter cyclical intervals and more rapid shifts in return patterns.
- The Manufacturing sector also showed consistent seasonal patterns, though with slightly longer cycles and moderate irregular fluctuations.
- The Food sector demonstrated well-defined seasonal waves with stable periodicity, albeit accompanied by noticeable random disturbances.

While seasonality is evident across all sectors, especially around regular intervals, none of the series shows a clear upward or downward trend. This suggests that no single sector appears to structurally outperform the others over time. The presence of seasonal cycles may offer limited short-term insights, but from a long-term investment perspective, predictability remains low, reinforcing the importance of diversification and risk-adjusted evaluation rather than timing sectoral entry.

Autocorrelation Diagnostics:

To assess the presence of autocorrelation in the daily excess returns of each sector, both the Ljung-Box Q-test and the Durbin-Watson test were applied.

Ljung-Box Test Results:

- Fashion Sector: With a p-value of 0.0008, the test rejects the null hypothesis of no autocorrelation at the 5% significance level. This provides strong evidence of autocorrelation in the Fashion sector's excess returns, indicating a time-dependent structure in the data.

- Manufacturing Sector: The p-value of 0.068 exceeds the 5% threshold, so the null hypothesis cannot be rejected. This suggests that there is no significant autocorrelation in the Manufacturing sector's return series.

- Food Sector: With a high p-value of 0.7829, the null hypothesis is comfortably retained, confirming the absence of autocorrelation in the Food sector's excess returns.

Durbin-Watson Test Results:

- Fashion Sector: A p-value of 0.0016 leads to rejecting the null hypothesis of no first-order autocorrelation. This corroborates the Ljung-Box result, reaffirming the presence of significant autocorrelation in the Fashion sector.

- Manufacturing Sector: The p-value of 0.3292 does not indicate statistical significance. Therefore, we do not reject the null hypothesis, implying the absence of first-order autocorrelation.

- Food Sector: With a p-value of 0.1975, there is again insufficient evidence to reject the null hypothesis, confirming no significant first-order autocorrelation.

These findings confirm the **existence of autocorrelation, specifically in the Fashion sector**'s return series. This necessitates using Newey-West standard errors in the CAPM estimation for the Fashion sector to obtain robust and reliable coefficient estimates. For the **Manufacturing and Food sectors, the lack of autocorrelation** supports the validity of standard error estimation methods that do not account for serial dependence.

Stationarity Analysis

To ensure the validity of time series modelling and regression-based analysis, the Augmented Dickey-Fuller (ADF) test was applied to the excess return series of each sector to check for stationarity.

- The ADF test yielded a p-value of 0.01 for all three sectors—Fashion, Manufacturing, and Food.

- Since the p-value is less than the 5% significance level, the null hypothesis of a unit root (non-stationarity) is rejected in each case.

This indicates that the **excess return series for the Fashion, Manufacturing, and Food sectors are stationary.** As a result, the data's statistical properties, such as mean and variance, remain constant over time. This reinforces the suitability of these series for time series modeling and regression-based analyses like CAPM, ensuring that inferences drawn from historical data are both valid and robust.

CAPM RESULTS ANALYSIS

The Capital Asset Pricing Model (CAPM) quantifies each sector's sensitivity to market movements through the beta (β) coefficient — a crucial indicator of systematic risk and return potential. The estimated betas were as follows:

- Fashion sector: $\beta = 0.7795$
- Manufacturing sector: $\beta = 1.1072$
- Food sector: $\beta = 1.1531$

These results offer clear insights into the sectors' relative risk profiles. With the highest beta, the **Food sector exhibits strong responsiveness to market trends**, making it an attractive option for investors seeking higher returns during market upswings. However, its heightened exposure also implies greater risk during downturns.

Similarly, the **Manufacturing sector displays a beta slightly above 1, indicating a solid correlation with the overall market.** It balances growth potential with a moderate risk level, suitable for investors seeking exposure to cyclical gains without excessive volatility.

In contrast, the **Fashion sector, with a beta under 1, appears less reactive to broad market fluctuations**. This lower sensitivity may appeal to more conservative investors looking to preserve capital during market corrections, suggesting a more stable return profile even in turbulent conditions.

These CAPM estimates guide sectoral allocation decisions by aligning risk exposure with investment preferences. A higher-beta portfolio may appeal to aggressive investors aiming for amplified returns, whereas incorporating lower-beta sectors like Fashion could provide diversification and downside protection within a balanced investment strategy.

PORTFOLIO OPTIMIZATION ANALYSIS

Sharpe Ratio Maximization

The analysis revealed that the **Fashion sector offered the highest expected excess return** at 0.665%, outperforming the Manufacturing (0.525%) and Food (0.428%) sectors. When assessed risk-adjusted, the **Fashion sector also exhibited the most favourable Sharpe Ratio of 0.15**, driven by a portfolio standard deviation of 4.54%.

As a result, the **optimal allocation for maximizing the Sharpe Ratio consisted of a 100% investment in the Fashion sector**, with zero weightage assigned to the Manufacturing and Food sectors. This implies that, among the three, the Fashion sector provided the most efficient return per unit of risk and would be the most suitable standalone investment for a moderately risk-tolerant investor aiming to maximize performance.

Despite higher irregularity observed during time series decomposition, the superior Sharpe Ratio reinforces the Fashion sector's relative advantage regarding risk-adjusted returns in the current market context.

Single Index Model

	Alpha	Beta	Idiosyncratic Volatility
Fashion	0.340	0.783	2.880
Manufacturing	0.064	1.111	2.956
Food	-0.052	1.157	2.568

The Single Index Model was employed to decompose sectoral returns into systematic and unsystematic components, providing critical insights into return attribution and risk exposure. The analysis considered **alpha** (sector-specific return unexplained by the market), **beta** (market sensitivity), and **idiosyncratic volatility** (firm-specific risk not captured by market movements).

- The **Fashion** sector delivered the most favorable performance, with a notably high alpha of 0.34, indicating the potential to **outperform market expectations** through stock-specific factors. Its beta of 0.78 suggests **lower sensitivity to market fluctuations**, positioning it as a stabilizing force in a diversified portfolio. The idiosyncratic volatility of 2.88 reflects **moderate sector-specific risk**, offering a compelling balance between return potential and risk containment.

- The **Manufacturing** sector registered a positive alpha of 0.06, indicating **limited outperformance** beyond market trends. However, its beta of 1.11 highlights **greater-than-market responsiveness**, making it more reactive to macroeconomic shifts. With the highest idiosyncratic volatility at 2.96, this sector exhibits **elevated firm-level uncertainty**, suggesting that portfolio exposure should be approached with careful risk management.

- The **Food** sector presented a negative alpha of -0.05, pointing to a tendency to **underperform relative to what its market exposure would predict.** Despite this, its beta of 1.16 confirms **strong market sensitivity**. Importantly, the lowest idiosyncratic volatility (2.57) among the three sectors implies **more consistent performance at the firm level**, which may support diversification objectives even in the presence of lower alpha.

Collectively, the model underscores that:

- Fashion is well-suited for investors seeking alpha generation and downside protection.
- Manufacturing offers market-aligned returns but demands greater tolerance for idiosyncratic risk.
- Food can enhance diversification, though its lack of alpha limits its attractiveness as a primary investment vehicle.

Minimum Variance Portfolio

The Minimum Variance Portfolio (MVP) was constructed to identify the optimal allocation of capital across the Fashion, Manufacturing, and Food sectors that minimizes overall portfolio risk. The solution is grounded in mean-variance optimization, leveraging the variance–covariance structure of sectoral returns.

Based on the optimization results:

- The **Fashion sector commands the dominant allocation, with a weight of 83.04%**, reflecting its comparatively lower individual and co-variances with other sectors. This allocation emphasizes its role as the anchor of portfolio stability.

- The **Manufacturing sector is allocated 15.81%, contributing additional diversification benefits**, despite its relatively higher variance. Its inclusion indicates that while riskier in isolation, it enhances risk-adjusted performance when paired with Fashion.

- The **Food sector receives a minimal allocation of 1.15%**, suggesting that its variance profile and co-movements with the other sectors do not significantly enhance diversification, and thus it plays a marginal role in minimizing portfolio volatility.

The computed portfolio variance stands at 0.00205, confirming the efficacy of this allocation in risk minimization. This configuration is particularly valuable for risk-averse investors seeking stability over aggressive return pursuit, highlighting Fashion's utility as a low-risk core holding.

RECOMMENDATIONS AND CONCLUSION

THE MOST PRAGMATIC CHOICE FOR THE INVESTMENT

Based on an integrated analysis of sector performance using CAPM, portfolio optimization techniques, and time series diagnostics, the **Fashion sector emerges as the most pragmatic and well-rounded investment choice** for Mr. SRK. This conclusion is supported by the following considerations:

Superior Risk-Adjusted Returns

- Under the Sharpe Ratio Maximization approach, the Fashion sector received a 100% allocation, driven by its highest expected excess return (0.665%) among the three sectors and a Sharpe Ratio of 0.15. This clearly demonstrates its dominance in terms of return per unit of risk.

Central Role in Risk Minimization

- Even within the Minimum Variance Portfolio, designed to minimize overall portfolio volatility, the Fashion sector was assigned a substantial weight of 83.04%. This reinforces its utility not only for maximizing returns but also for stabilizing the portfolio through relatively lower variance and favourable covariance properties with other sectors.

Favourable Market Sensitivity

- The CAPM analysis revealed that Fashion has a beta of 0.7795, the lowest among the sectors. This indicates lower sensitivity to overall market fluctuations, making it a more defensive option suitable for investors seeking a balance between market participation and downside protection.

Strong Individual Characteristics

- Results from the Single Index Model showed Fashion had the highest positive alpha (0.3396) and moderate idiosyncratic volatility (2.8797). While alpha was not the core focus of our CAPM analysis, this outperformance signals the potential for returns exceeding those implied by market risk alone.

Predictable and Stable Return Pattern

- Time series decomposition further highlighted Fashion's clear seasonal structure, likely driven by consistent consumer trends and festive cycles. Despite some randomness, the lack of a dominant trend and some autocorrelation (as confirmed by Ljung-Box and Durbin-Watson tests) suggest a reliable and stable return behaviour that is valuable for strategic allocation.

Considering its strong performance across multiple models — including highest return-to-risk ratio, risk minimization dominance, market defensiveness, and structural return stability — the Fashion sector offers the most compelling investment opportunity. **For Mr. SRK, who seeks a data-driven and balanced approach to investing, allocating a significant portion of the portfolio to Fashion represents a prudent, evidence-backed decision.**

THE SAFEST INVESTMENT FOR MR. SRK IF HE'S HIGHLY RISK-AVERSE

If Mr. SRK identifies as a highly risk-averse investor, prioritizing capital preservation and stability over aggressive growth, **the Minimum Variance Portfolio (MVP) is the most suitable investment strategy.** This recommendation is substantiated by the following factors:

Objective Alignment with Risk Aversion

- The Minimum Variance Portfolio is explicitly designed to minimize total portfolio risk by identifying an optimal allocation that reduces overall volatility, regardless of return-maximizing potential.
- With a portfolio variance of just 0.00205, this strategy offers the lowest risk exposure among all the portfolio configurations considered.

Diversified Allocation to Offset Sector-Specific Risk

- Unlike the Sharpe-maximizing portfolio, which concentrated entirely on the Fashion sector, the MVP distributes investments across:
  - Fashion: 83.04%
  - Manufacturing: 15.81%
  - Food: 1.15%
- This diversification helps cushion against shocks in any single sector, especially critical for risk-averse investors looking to mitigate idiosyncratic or sector-specific risks.

Balanced Risk-Return Trade-Off

- Although the expected return of the MVP would be slightly lower than the return-maximizing portfolio, the reduction in volatility more than compensates for it from a risk-adjusted standpoint.
- The allocation still leverages Fashion's favourable characteristics but tempers exposure with stabilizing weights in Manufacturing and Food.

Support from Time Series Stability

- Time series decomposition and diagnostic tests showed limited autocorrelation and no stationarity concerns across sectors, reinforcing the reliability of underlying return behaviours.
- The MVP's allocation to sectors with consistent seasonal behaviour contributes to its time-stable return profile, offering a structured cushion against unpredictable market fluctuations.

For a risk-averse investor like Mr. SRK, the Minimum Variance Portfolio offers the best balance between return reliability and downside protection. With its strategic diversification and minimized volatility, this portfolio structure provides a conservative yet data-backed path to long-term capital preservation, making it the safest investment choice under the given risk preferences.

AN INVESTMENT CONTAINING SOME RISK AND RETURN

For an investor like Mr. SRK, who is willing to tolerate a moderate level of risk in pursuit of enhanced returns, **the Sharpe Ratio Maximization portfolio—fully allocated to the Fashion sector—stands out as an optimal choice.** This recommendation is supported by the following key findings:

Optimal Risk-Return Trade-Off

- Within the Sharpe-based optimization framework, the entire allocation was directed toward the Fashion sector. This outcome reflects its ability to offer favourable returns relative to its volatility, making it especially suitable for investors looking to balance performance and prudence.

Controlled Market Sensitivity

- Fashion's CAPM beta of 0.7795 indicates moderate sensitivity to market fluctuations. This level of exposure allows the portfolio to benefit from market upswings while still maintaining a degree of protection against broader downturns—ideal for an investor seeking a balanced risk-return profile.

Consistent Outperformance Indicators

- The positive alpha (0.3396) from the Single Index Model highlights the Fashion sector's historical outperformance relative to market expectations. Coupled with moderate idiosyncratic volatility (2.8797), it reflects the sector's potential to generate superior returns without excessive randomness.

Seasonally Driven Return Structure

- Time series decomposition revealed a well-defined seasonal pattern in Fashion's returns, likely driven by cyclical consumer demand and festive retail peaks. This predictable return behaviour adds a layer of structural reliability, making aligning with mid-risk investment strategies easier.

These results point to the Fashion sector as a strong candidate for Mr. SRK's portfolio when seeking an investment that combines moderate risk with meaningful return potential. Its balanced market exposure, historical outperformance, and structured return patterns make it a confident, data-driven choice for an investor with a moderate risk appetite.

LIMITATIONS

Assumption of Rationality and Efficiency

The study assumes that investors act rationally and that markets are efficient (as per the Efficient Market Hypothesis). However, behavioural biases, information asymmetries, and market frictions often challenge these assumptions, potentially distorting actual investor behaviour and return outcomes.

Single-Factor Model Limitation

The CAPM framework relies solely on the market index to explain returns, omitting other critical factors such as firm size, value, momentum, or sector-specific shocks. This limits the model's ability to capture the multifaceted nature of asset returns.

Stationarity and Time-Series Dependence

Although standard diagnostic tests such as the Augmented Dickey-Fuller (ADF), Ljung-Box, and Durbin-Watson were conducted, key assumptions like strict stationarity and weak dependence may still be at risk of violation. Financial time series are inherently prone to structural breaks and evolving dynamics, particularly during periods of macroeconomic volatility or regulatory change.

Simplified Sectoral Aggregation

The analysis relies on sector-level indices, which may mask firm-level differences, skewed intra-sector weightings, and company-specific risk profiles. This aggregation reduces the granularity of insights and may dilute actionable takeaways for more nuanced portfolio construction.

Estimation Risk and Sampling Period Constraints

The study is based on 516 daily returns. While this high-frequency data allows for detailed modelling, it may introduce short-term volatility and noise. Furthermore, the limited time horizon may not fully capture long-term cyclical trends, crises, or structural changes.

Assumption of Constant Beta and Market Conditions

The CAPM assumes a stable relationship between asset returns and market movements. In reality, beta values and market sensitivities are likely to evolve due to changes in firm strategy, macroeconomic regimes, or investor sentiment, potentially leading to inaccurate risk estimates.

Measurement Error in Market Proxy

The chosen market index may not fully represent the theoretical "market portfolio," especially if it excludes smaller or international firms. Any mismatch between the proxy and the true investable universe could skew beta estimates and model outputs.

Lack of Macroeconomic and External Variables

The analysis does not incorporate external macroeconomic indicators such as inflation, exchange rates, or policy changes, which can materially influence sectoral returns and investor behaviour.

No Consideration of Transaction Costs or Market Frictions

All portfolio models assume frictionless trading and zero transaction costs. In real markets, execution costs, bid-ask spreads, and slippage can meaningfully erode returns, particularly for strategies involving frequent rebalancing or high turnover.

Newey-West and Robust SE Corrections May Not Fully Resolve Issues

While heteroskedasticity and autocorrelation were addressed through the use of robust standard errors and Newey-West adjustments, these methods primarily enhance inference accuracy. They cannot fully resolve deeper issues such as model misspecification, omitted variable bias, or underlying nonlinear relationships between variables.

RESULTS SNAPSHOT

1- Descriptive Statistics

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
Fashion	1	516	0.66	4.54	0.68	0.72	3.73	-18.79	19.56	38.35	-0.14	2.07	0.20
Manufacturing	2	516	0.53	5.80	0.50	0.60	5.01	-25.74	19.74	45.48	-0.22	1.37	0.26
Food	3	516	0.43	5.79	0.47	0.45	4.75	-29.81	24.67	54.48	-0.18	2.34	0.25
Market Index	4	516	0.42	4.48	0.73	0.57	4.26	-23.09	16.05	39.14	-0.47	1.79	0.20
Interest	5	516	0.47	0.22	0.43	0.45	0.18	0.11	1.35	1.24	1.21	1.98	0.01

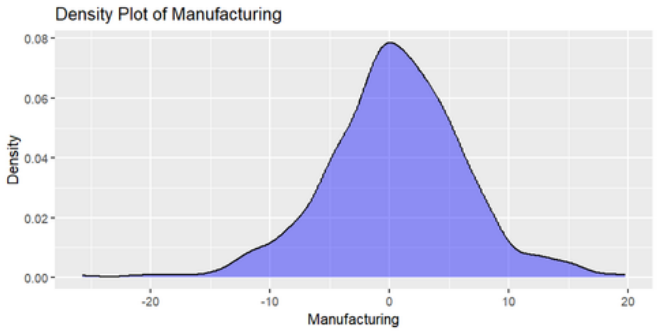
2- Variance of Original Variables

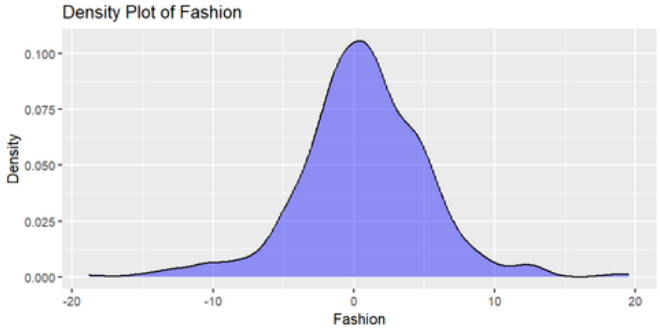
Fashion	Manufacturing	Food	Market Index	Interest
20.64948282	33.58549986	33.52691998	20.11492019	0.04861831

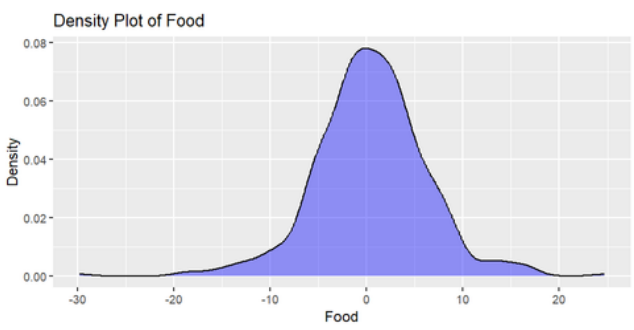
3- Standard Deviation of Original Variables

Fashion	Manufacturing	Food	Market Index	Interest
4.5441702	5.7952998	5.7902435	4.4849660	0.2204956

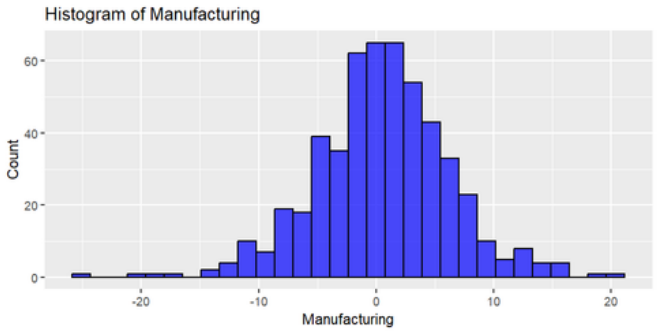
4- Density Plots

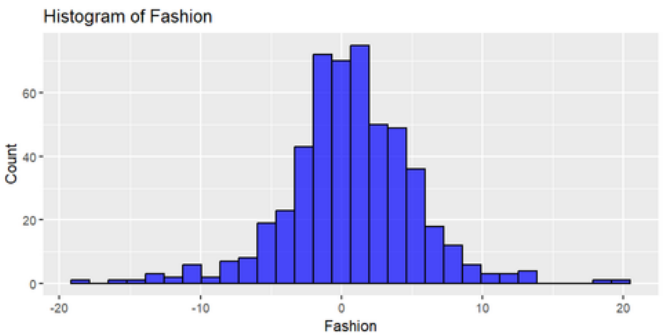


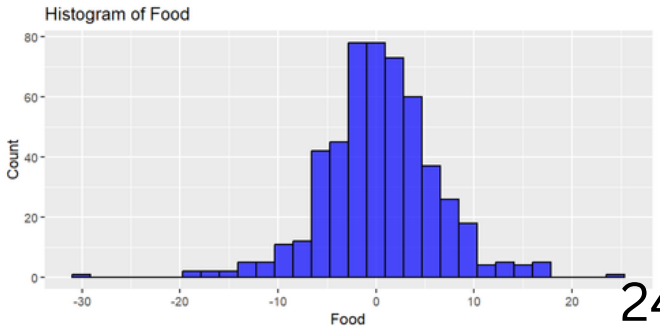




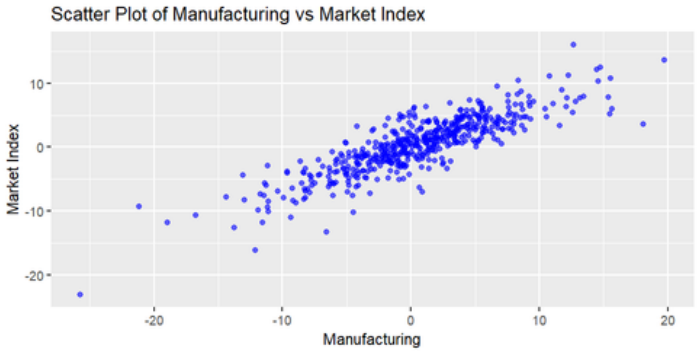
5- Histogram



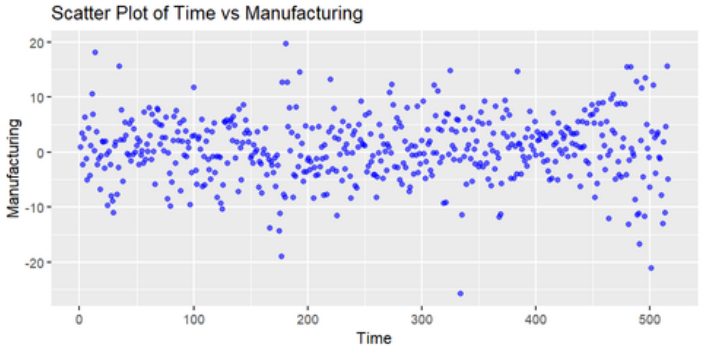


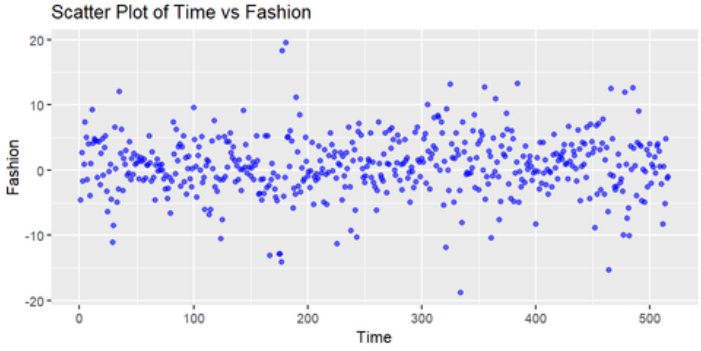


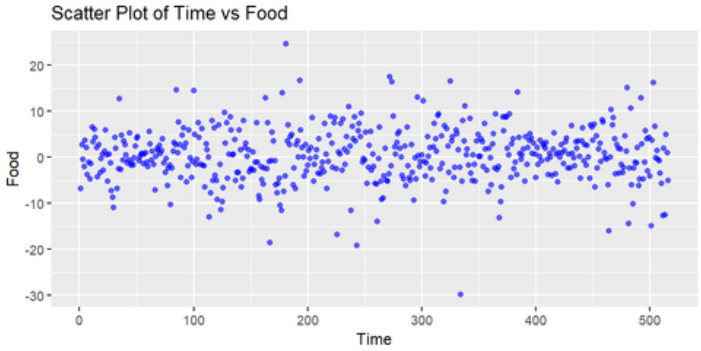
6- Scatter Plots Against Market Index



7- Scatter Plots Against Time







7- Covariance Matrix

	Fashion	Manufacturing	Food	Market Index
Fashion	20.64948282	17.6141169	18.96906998	15.75544399
Manufacturing	17.61411691	33.5854999	26.34206199	22.34793320
Food	18.96906998	26.3420620	33.52691998	23.27084432
Market Index	15.75544399	22.3479332	23.27084432	20.11492019
Interest	-0.02696835	-0.1273404	-0.09205402	-0.07579356

	Interest
Fashion	-0.02696835
Manufacturing	-0.12734039
Food	-0.09205402
Market Index	-0.07579356
Interest	0.04861831

8- Correlation Matrix

	Fashion	Manufacturing	Food	Market Index
Fashion	1.00000000	0.66885253	0.72093253	0.77306658
Manufacturing	0.66885253	1.00000000	0.78501333	0.85980962
Food	0.72093253	0.78501333	1.00000000	0.89609933
Market Index	0.77306658	0.85980962	0.89609933	1.00000000
Interest	-0.02691534	-0.09965299	-0.07210179	-0.07664313

	Interest
Fashion	-0.02691534
Manufacturing	-0.09965299
Food	-0.07210179
Market Index	-0.07664313
Interest	1.00000000

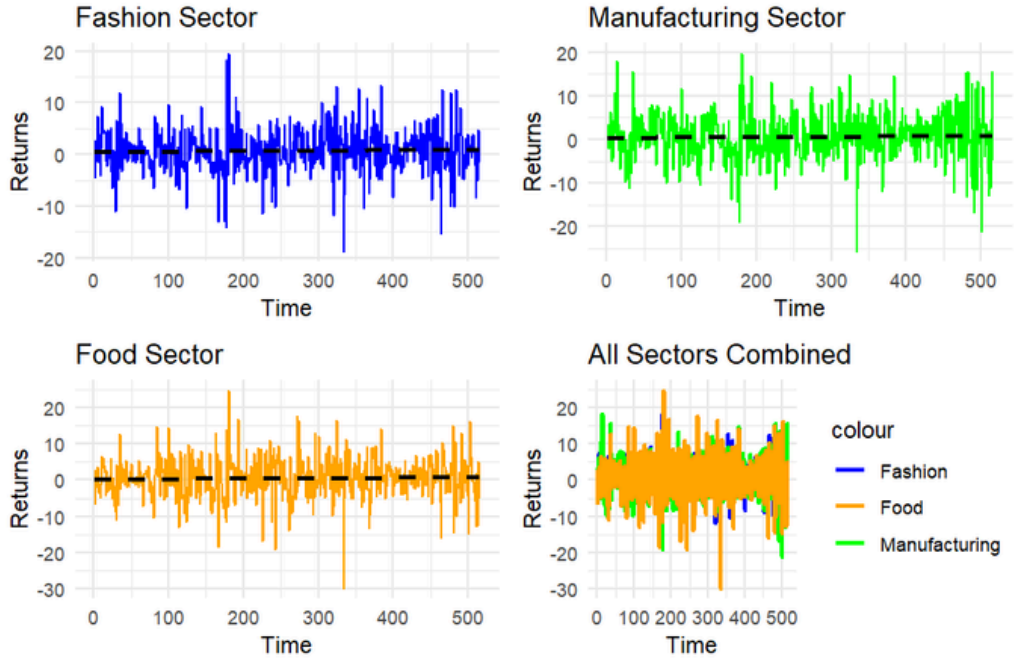
9- Skewness

Fashion	Manufacturing	Food	Market Index	Interest
-0.1376669	-0.2247961	-0.1765498	-0.4725187	1.2095524

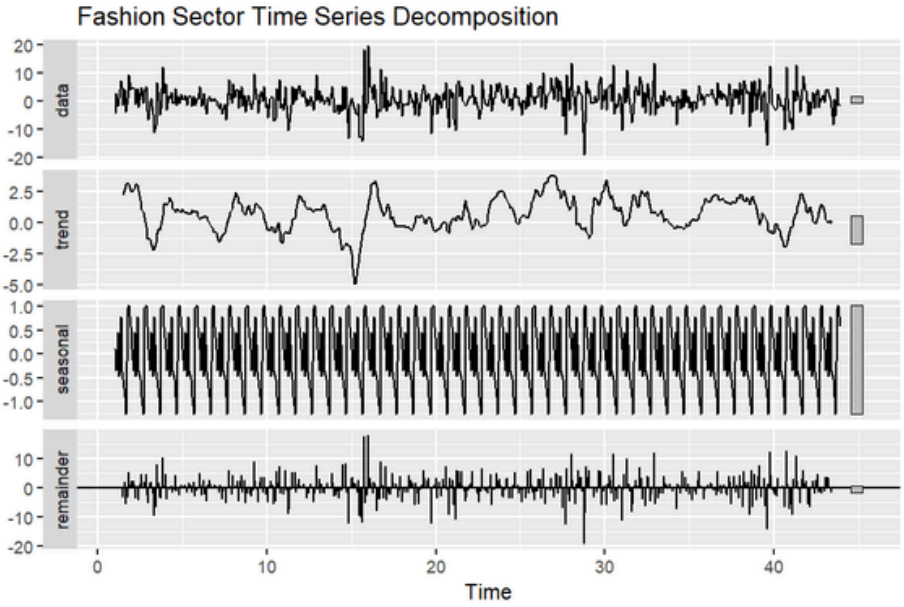
10- Kurtosis

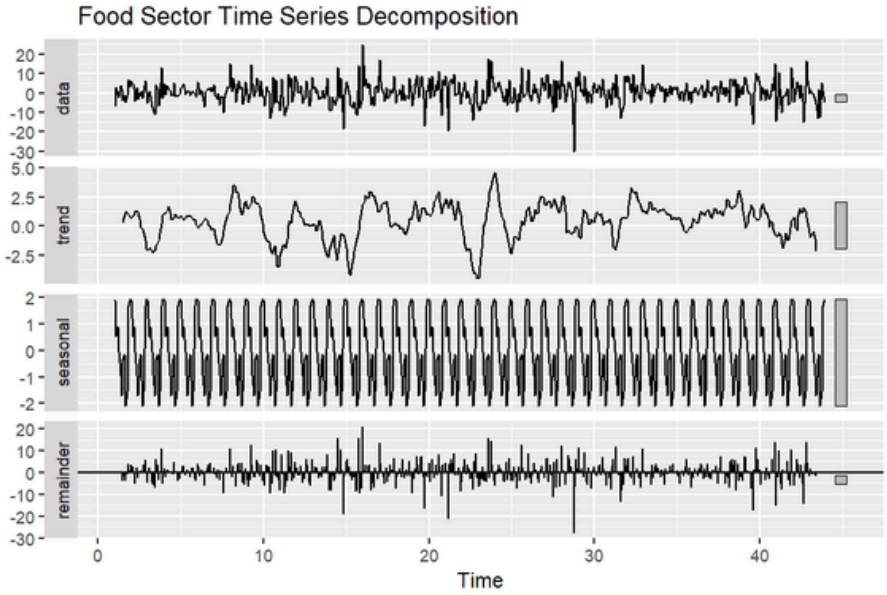
Fashion	Manufacturing	Food	Market Index	Interest
5.090290	4.385594	5.356746	4.812626	4.999371

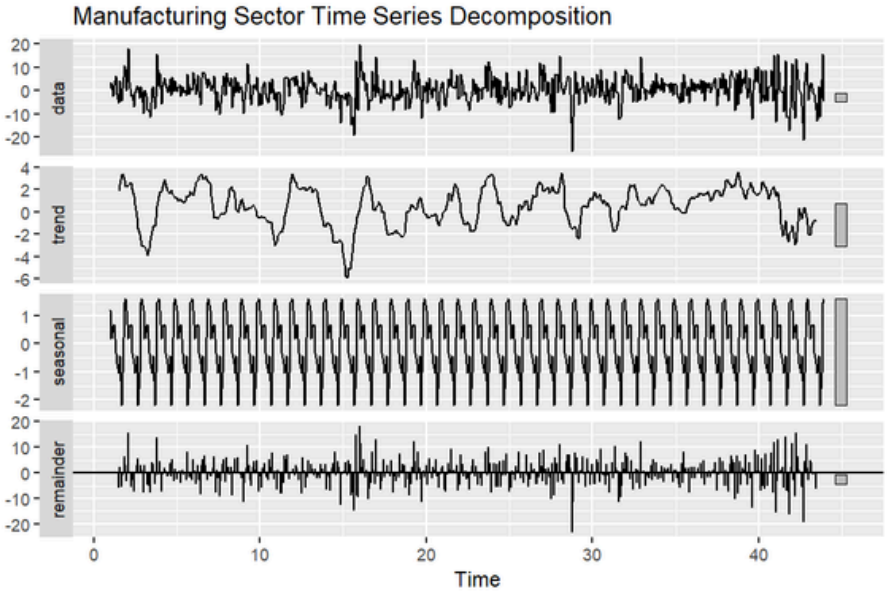
11- Fitting the Trend



12 - Decomposition of Time Series







13 - Ljung-Box Test

```
data:  residuals(fashion_model)
X-squared = 30.173, df = 10, p-value = 0.0008025

data:  residuals(manufacturing_model)
X-squared = 17.299, df = 10, p-value = 0.068

data:  residuals(food_model)
X-squared = 6.3739, df = 10, p-value = 0.7829
```

14- Durbin-Watson test

```
data:  fashion_model
DW = 1.7411, p-value = 0.001594

data:  manufacturing_model
DW = 1.9614, p-value = 0.3292

data:  food_model
DW = 1.9255, p-value = 0.1975
```

15- Augmented Dickey-Fuller Test

```
data:  residuals(fashion_model)
Dickey-Fuller = -5.6772, Lag order = 8, p-value = 0.01
alternative hypothesis: stationary

data:  residuals(manufacturing_model)
Dickey-Fuller = -6.9827, Lag order = 8, p-value = 0.01
alternative hypothesis: stationary

data:  residuals(food_model)
Dickey-Fuller = -6.7394, Lag order = 8, p-value = 0.01
alternative hypothesis: stationary
```

16- Sharpe Ratio

Fashion	Manufacturing	Food	Market Index
0.14523127	0.08983727	0.07305697	0.09148009

17- Sortino Ratio

Fashion	Manufacturing	Food	Market Index
0.2088201	0.1319082	0.1074010	0.1301784

18 - Checking for Perfect Collinearity Assumption

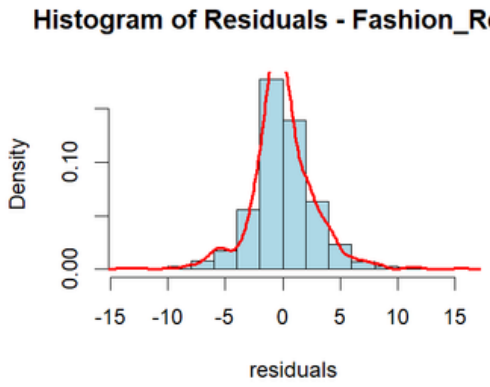
```
Variance of 'Market Index': 20.11492
No issues with perfect collinearity detected.
```

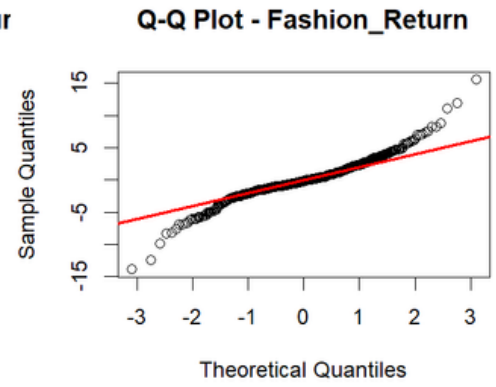
19- Breusch Pagan Test

```
(Fashion): BP = 0.1181574 p-value: 0.7310416
(Manufacturing): BP = 0.1946107 p-value: 0.6591065
(Food): BP = 2.374597 p-value: 0.1233234
```

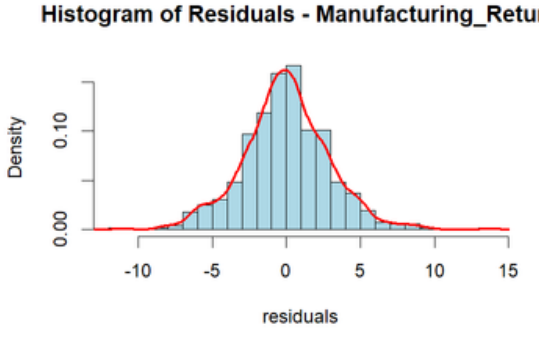
20- Tests for Normality Assumption

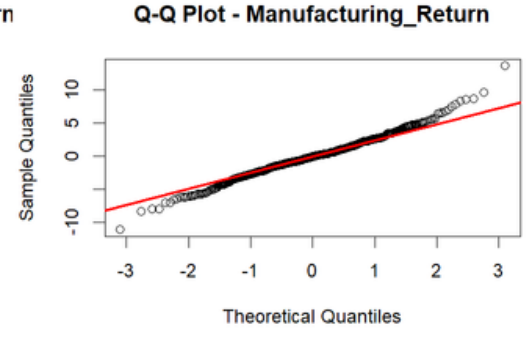
```
------------------------------
Checking Normality for: Fashion_Return
------------------------------
Shapiro-Wilk Test: W = 0.9422345 , p-value = 2.788854e-13
Kolmogorov-Smirnov Test: D = 0.0887937 , p-value = 0.0005852493
Skewness: 0.1929637
Kurtosis: 4.092942
```



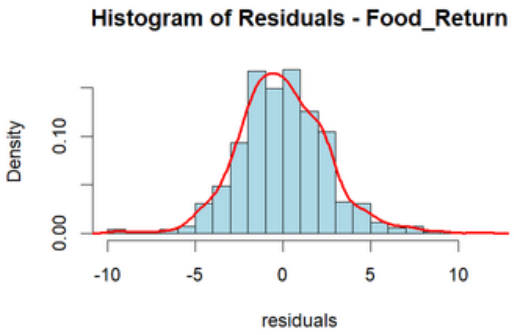


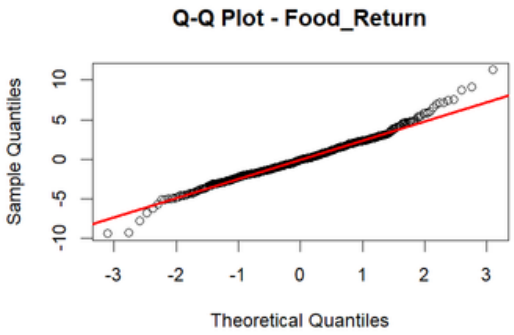

```
------------------------------
Checking Normality for: Manufacturing_Return
------------------------------
Shapiro-Wilk Test: W = 0.9872537 , p-value = 0.0001766465
Kolmogorov-Smirnov Test: D = 0.04527484 , p-value = 0.2407461
Skewness: 0.1816562
Kurtosis: 1.353631
```



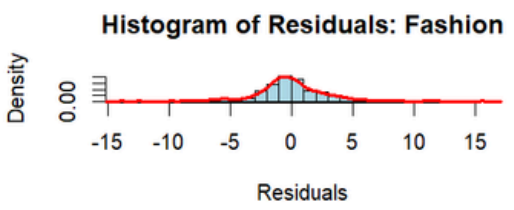


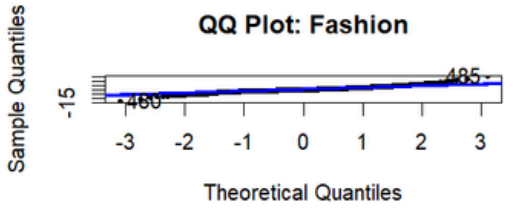
```
------------------------------
Checking Normality for: Food_Return
------------------------------
Shapiro-Wilk Test: W = 0.9827833 , p-value = 8.678589e-06
Kolmogorov-Smirnov Test: D = 0.03730019 , p-value = 0.4694348
Skewness: 0.3165938
Kurtosis: 1.515061
```

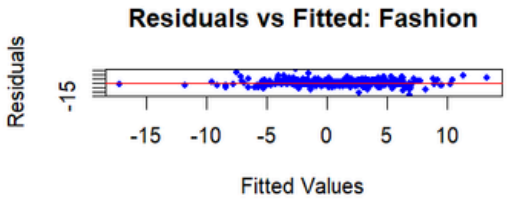


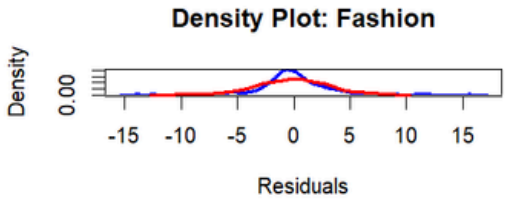


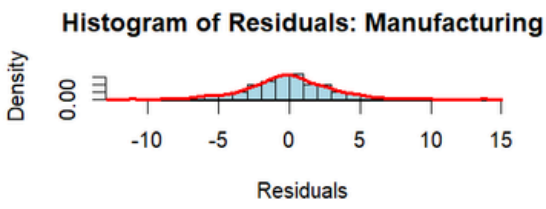
21- Residual Plots

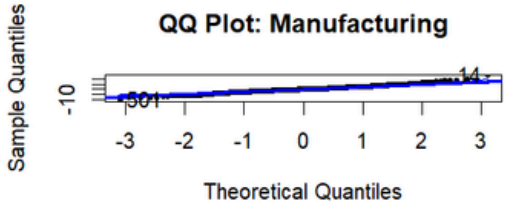


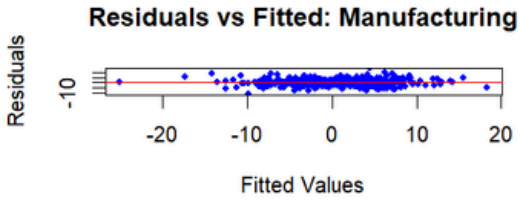


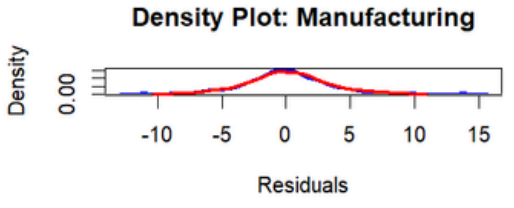


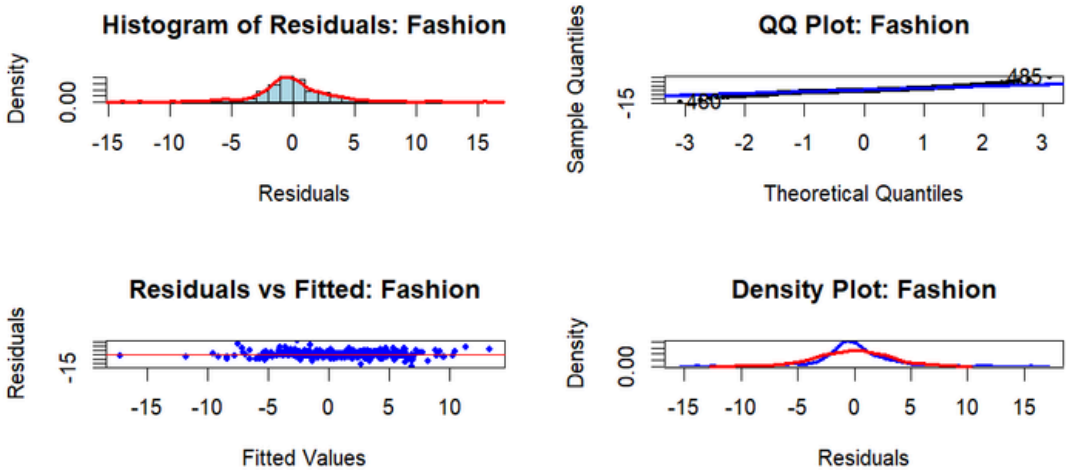












22- CAPM Model

Fashion

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.814611   0.159966  5.0924 4.971e-07 ***
`Market Index` 0.779503   0.067724 11.5100 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Manufacturing

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.539270   0.131810  4.0913  4.98e-05 ***
`Market Index` 1.107245   0.035774 30.9512 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Food

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.422612   0.113442  3.7254 0.0002166 ***
`Market Index` 1.153127   0.032921 35.0276 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

23- Sharpe Ratio Maximization

	Fashion	Manufacturing	Food
Expected Excess Returns	0.665%	0.525%	0.428%

Var and CoVar matrix

		Fashion	Manufacturing	Food
	Weights	*100%*	*0%*	*0%*
Fashion	100%	0.002060946	0.001757998	0.00189323
Manufacturing	0%	0.001757998	0.003352041	0.0026291
Food	0%	0.001893231	0.002629101	0.00334619
Contribution to Var		0.2061%	0.0000%	0.0000%

Sum of Weights	100%
Return	0.665%
Std Dev of Portfolio	4.54%
Sharpe Ratio	0.15

24- Single Index Model

Fashion

Alpha	0.33961706
Beta	0.78327151
Idiosyncratic Volatility	2.87968579

Manufacturing

Alpha	0.06427639
Beta	1.11101277
Idiosyncratic Volatility	2.95629674

Food

Alpha	-0.05238178
Beta	1.15689469
Idiosyncratic Volatilit	2.56752866

25- Minimum Variance Portfolio

XTX			
	Fashion	**Manufacturing**	**Food**
Fashion	1.09	0.93	0.99
Manufacturing	0.93	1.74	1.37
Food	0.99	1.37	1.74

Variance - Covariance			
	Fashion	**Manufacturing**	**Food**
Fashion	0.002105	0.001793	0.001922
Manufacturing	0.001793	0.003380	0.002652
Food	0.001922	0.002652	0.003364

Weights	
Fashion	0.8304
Manufacturing	0.1581
Food	0.0115
Total	1.0

Weights Transpose Sigm	0.00205366	0.002053657	0.00205366
Portfolio Variance	0.00205366		

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