Request –

Using the data in US Term Structure, test the series m1, m2, m3, m6, Y1, Y2, Y3, Y4, Y5 for unit root. Using the Johansen test, check the cointegration rank of the whole vector (refer to Hall et all paper for the comment).
Describe how all the cointegration relations look like. Consider the pair of rates m1, Y1. Test, using the Engle/Granger methodology or the Johansen methodology, if the spread is the cointegrating vector. Discuss whether your findings may be consistent with implications that can be derived from the Expectations Hypothesis.
[Note about the dataset: the DF unit root test for m1 may have different outcomes (rejection or not) depending on the specified model. Choose the same order as the DF unit root test for y1]

Solution –

Note: Output Appendix is attached below

To test the series for unit root, Augmented Dickey-Fuller unit root test statistics were computed in EViews.

Unit Root Test – ADF

Series to be tested for unit root are:- m1, m2, m3, m6, Y1, Y2, Y3, Y4, Y5

Fig1. Represents the result of testing series m1 at the level.

Referring to the absolute value of t statistic 2.529 which is less than critical value at 5% level 2.866 it is not possible to reject the hypothesis and hence **m1 has a unit root at level form**.

Fig2. Checking the series m1 at 1^{st} difference form. The result shows that the series **m1 does not have unit root at 1^{st} difference form.**

The same procedure is followed for other series m2, m3, m6, Y1, Y2, Y3, Y4, Y5. The results are same as of series m1.

Based on ADF test - Series m1, m2, m3, m6, Y1, Y2, Y3, Y4, Y5 has a unit root at the level form.

Before performing the cointegration test VAR model estimation is required to select appropriate lags order for VAR by Information Criterion. From output **Fig3** based on SC (Bayes) criteria, the order lag chosen is 2 lags

Johansen Cointegration Test for the whole vector (check the cointegration rank)–

The **Fig4.** is for λ_{trace} and **Fig5**. is for λ_{max}

The λ_{trace} show that at r = 0 of 796.50 exceeds its critical value of 197.37 at 5% level and hence the null hypothesis is rejected of no cointegration equation. At r =1,2,3,4,5,6,7 the null hypothesis is rejected. But at r= 8 of λ_{trace} value of 2.799 is less than its critical value of 3.84 at 5% level, which means that the hypothesis cannot be rejected.

Hence, the Johansen Test based on λ_{trace} suggests that there are 8 cointegration relationships exist between m1, m2, m3, m6, Y1, Y2, Y3, Y4, Y5.

The lower panel based on λ_{max} shows a similar result. The λ_{max} value for r=0 is 248.29 which exceeds its critical value of 58.43 at 5% level and hence the null hypothesis is rejected of no cointegration equation. In the same order at r =1,2,3,4,5,6,7, the null hypothesis is rejected. But at r= 8 of λ_{max} value of 2.79 is less than its critical value of 3.84 at 5% level, which means that the hypothesis cannot be rejected.

With reference to Hall et paper Set of n yields are cointegrated with n −1 cointegrating vectors

Therefore, it can be concluded that there are 8 cointegrated vectors for the set of nine yields.

Fig4 and Fig 5 rank test clearly states that hypothesis of rank = 0, 1, 2, 3, 4, 5, 6, 7 are rejected however hypothesis that rank = 8 is not rejected. Hence, we can conclude that rank of whole vector is 8.

From the Johansen Test, conclusion is series are cointegrated. That is, they exhibit a long-run relationship which implies –

1. The series are related and can be combined linearly.
2. Even, if there are shocks in the short run, which may affect movement in the individual series, they would converge with time (in the long run)
3. Hence, estimate both short and long-run models

Lag Length Criteria for m1, y1

Fig 6 depicts the VAR lag length criteria as per Schwarz is 2.

Johansen Test for series m1, y1

From the Johansen test output **Fig7. & Fig8**, it can be concluded that both λ_{trace} and λ_{max} suggests that there is one cointegration relation which exists in between m1 and y1. Hence, we can say that spread is cointegrating vector because 1 cointegration relation is present.

Fig9. Represents the Vector Error Correction Model

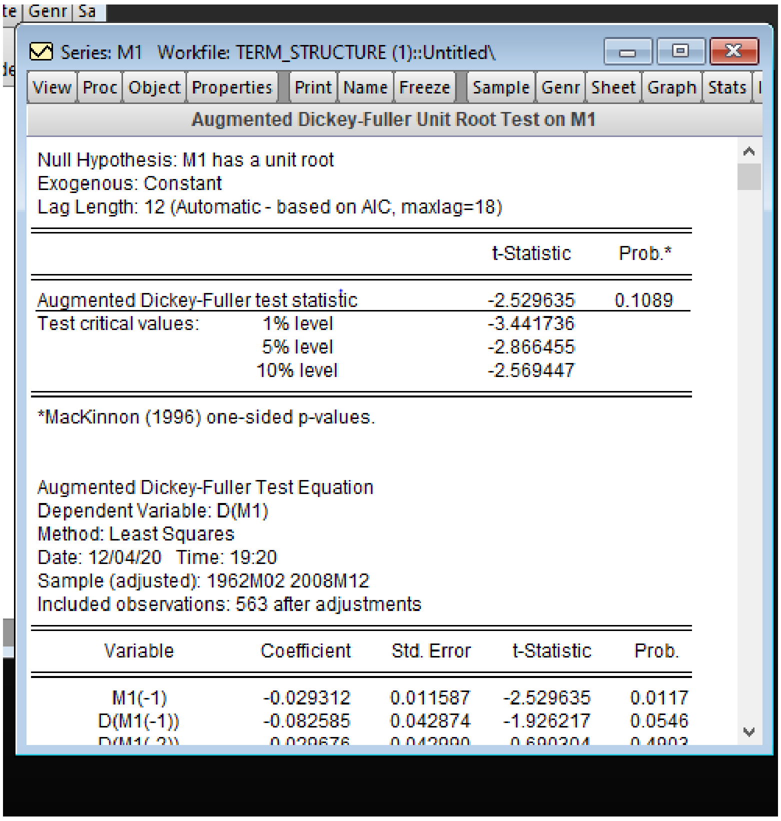
The estimated Cointegrating Equation is

$\boldsymbol{ECM_t = m_t - 0.9228_{y_t} + 0.3077B_c}$

The findings which are consistence with Expectation hypothesis are-

- The interest rates are I(1).
- The interest rates are cointegrated

Output Appendix



Series: M1 Workfile: TERM_STRUCTURE (1)::Untitled\

Augmented Dickey-Fuller Unit Root Test on M1

Null Hypothesis: M1 has a unit root
Exogenous: Constant
Lag Length: 12 (Automatic - based on AIC, maxlag=18)

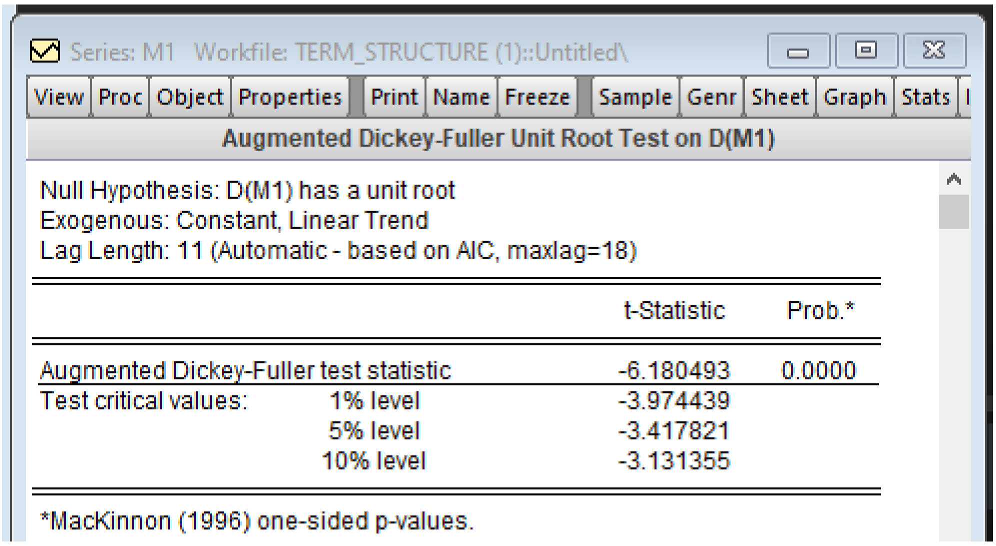
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.529635	0.1089
Test critical values:	1% level	-3.441736	
	5% level	-2.866455	
	10% level	-2.569447	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M1)
Method: Least Squares
Date: 12/04/20 Time: 19:20
Sample (adjusted): 1962M02 2008M12
Included observations: 563 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1(-1)	-0.029312	0.011587	-2.529635	0.0117
D(M1(-1))	-0.082585	0.042874	-1.926217	0.0546

Fig1. ADF Unit Root Test (M1)



Series: M1 Workfile: TERM_STRUCTURE (1)::Untitled\

Augmented Dickey-Fuller Unit Root Test on D(M1)

Null Hypothesis: D(M1) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 11 (Automatic - based on AIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.180493	0.0000
Test critical values:	1% level	-3.974439	
	5% level	-3.417821	
	10% level	-3.131355	

*MacKinnon (1996) one-sided p-values.

Fig 2. ADF Unit root Test for D(m1)

Var: VAR01 Workfile: TERM_STRUCTURE (1)::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Impulse | Resids

VAR Lag Order Selection Criteria
Endogenous variables: M1 M2 M3 M6 Y1 Y2 Y3 Y4 Y5
Exogenous variables: C
Date: 12/04/20 Time: 20:03
Sample: 1961M01 2008M12
Included observations: 568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-741.6143	NA	1.14e-10	2.643008	2.711809	2.669856
1	1633.916	4667.415	3.52e-14	-5.436323	-4.748311*	-5.167840
2	1797.610	316.4373	2.63e-14	-5.727500	-4.420277	-5.217382
3	1978.182	343.3418	1.86e-14	-6.078107	-4.151673	-5.326354*
4	2067.837	167.6282	1.80e-14	-6.108580	-3.562935	-5.115192
5	2186.005	217.1973	1.58e-14	-6.239455	-3.074599	-5.004433
6	2261.868	137.0342	1.62e-14	-6.221367	-2.437300	-4.744710
7	2358.660	171.7704	1.53e-14*	-6.276970*	-1.873692	-4.558678
8	2438.056	138.3845*	1.55e-14	-6.271324	-1.248834	-4.311397

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Fig3. VAR Model estimation for Lag Selection

Johansen Cointegration Test

Date: 12/08/20 Time: 02:57
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Trend assumption: Linear deterministic trend
Series: M1 M2 M3 M6 Y1 Y2 Y3 Y4 Y5
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.351650	796.5076	197.3709	0.0000
At most 1 *	0.233595	548.2127	159.5297	0.0000
At most 2 *	0.172223	395.7692	125.6154	0.0000
At most 3 *	0.134898	287.4659	95.75366	0.0000
At most 4 *	0.130489	204.4337	69.81889	0.0000
At most 5 *	0.096135	124.3140	47.85613	0.0000
At most 6 *	0.069071	66.39803	29.79707	0.0000
At most 7 *	0.038652	25.38689	15.49471	0.0012
At most 8	0.004874	2.799737	3.841465	0.0943

Trace test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Fig4. Johansen Cointegration Test – λtrace

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.351650	248.2949	58.43354	0.0000
At most 1 *	0.233595	152.4435	52.36261	0.0000
At most 2 *	0.172223	108.3032	46.23142	0.0000
At most 3 *	0.134898	83.03229	40.07757	0.0000
At most 4 *	0.130489	80.11962	33.87687	0.0000
At most 5 *	0.096135	57.91601	27.58434	0.0000
At most 6 *	0.069071	41.01114	21.13162	0.0000
At most 7 *	0.038652	22.58715	14.26460	0.0020
At most 8	0.004874	2.799737	3.841465	0.0943

Max-eigenvalue test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Fig5. Johansen Cointegration Test – λ_{max}

Var: UNTITLED   Workfile: TERM_STRUCTURE::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Impulse | Resids

VAR Lag Order Selection Criteria
Endogenous variables: M1 Y1
Exogenous variables: C
Date: 12/08/20   Time: 12:19
Sample: 1961M01 2008M12
Included observations: 568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1945.556	NA	3.260358	6.857591	6.872880	6.863558
1	-841.1474	2197.151	0.067688	2.982913	3.028781	3.000812
2	-819.4586	42.99574	0.063601	2.920629	2.997075*	2.950460*
3	-816.1221	6.590739	0.063750	2.922965	3.029989	2.964729
4	-814.9733	2.261155	0.064393	2.933005	3.070607	2.986701
5	-811.2910	7.222090	0.064466	2.934123	3.102304	2.999752
6	-803.0171	16.16904	0.063503	2.919074	3.117833	2.996636
7	-798.4259	8.939940	0.063372	2.916993	3.146330	3.006487
8	-790.9446	14.51467*	0.062600*	2.904735*	3.164650	3.006162

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Fig 6. Lag Length Criteria for M1 Y1

G Group: UNTITLED Workfile: TERM_STRUCTURE (1)::Untitled\

View | Proc | Object | Print | Name | Freeze | Sample | Sheet | Stats | Spec

Johansen Cointegration Test

Date: 12/05/20 Time: 00:00
Sample (adjusted): 1961M06 2008M12
Included observations: 571 after adjustments
Trend assumption: Linear deterministic trend
Series: M1 Y1
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.084467	53.23521	15.49471	0.0000
At most 1	0.004970	2.844833	3.841465	0.0917

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Fig7. Johansen Cointegration Test for series m1 & y1 - λtrace

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.084467	50.39037	14.26460	0.0000
At most 1	0.004970	2.844833	3.841465	0.0917

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Fig8. Johansen Cointegration Test for series m1&y1 – λmax

Var: UNTITLED Workfile: TERM_STRUCTURE::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Impulse | Resids

Vector Error Correction Estimates

Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
M1(-1)	1.000000	
Y1(-1)	-0.922844 (0.02342) [-39.4091]	
C	0.307691	
Error Correction:	**D(M1)**	**D(Y1)**
CointEq1	-0.300500 (0.04826) [-6.22720]	0.021596 (0.03898) [0.55399]
D(M1(-1))	-0.151750 (0.05848) [-2.59504]	-0.003912 (0.04724) [-0.08282]
D(M1(-2))	0.044204 (0.05249) [0.84214]	0.029159 (0.04240) [0.68767]
D(Y1(-1))	0.403301 (0.07043) [5.72647]	0.145071 (0.05689) [2.54990]
D(Y1(-2))	-0.173033 (0.06978)	-0.111019 (0.05637)

Fig9. Vector error correction model