Indian Institute of Technology Mandi, H. P. India
Engineering Mathematics - IC 110
Assignment-2
Odd Semester(2019-20)
Convergence of a sequence, Cauchy sequences and Series.

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1. (a) If the general term of a sequence $\{x_k\}$ is given as $x_k = \frac{3k-5}{9k+4}$ and it is also known that $\lim\limits_{k\to\infty} x_k = \dfrac{1}{3}$ then find the number of points x_k lying outside the open interval $I = (\frac{1}{3} - \frac{1}{2000}, \frac{1}{3} + \frac{1}{2000})$.

   (b) With the help of the definition of limit of a sequence prove that $\lim\limits_{k\to\infty} x_k = \dfrac{6}{7}$ if $x_k = \frac{6k^2+1}{7k^2-1}$. Beginning with which k is the inequality $|x_k - \frac{6}{7}| < 0.001$ fulfilled?

2. (a) Suppose that $0 < \alpha < 1$ and that $\{x_k\}$ is a sequence satisfying the condition: $|x_{k+1} - x_k| \leq \alpha^k, \forall k \in N$. Show that $\{x_k\}$ satises the Cauchy criterion.

   (b) With the help of statement of problem 1(a), show that the sequence $\{x_k\}$ converges if $x_1 = \frac{1}{1!}, x_2 = \frac{1}{1!} - \frac{1}{2!}, \ldots, x_k = \frac{1}{1!} - \frac{1}{2!} + \cdots + \frac{(-1)^{(k+1)}}{k!}$.

3. Let $\{x_k\}$ be a sequence of positive real numbers. Prove or disprove the following statements

   (a) $\{x_k\}$ is convergent if sub-sequences $\{x_{2k}\}$and $\{x_{2k-1}\}$ are convergent.

   (b) $\{x_k\}$ is convergent if sub-sequences $\{x_{2k}\}$ and $\{x_{3k}\}$ are convergent.

   (c) $\{x_k\}$ is convergent if sub-sequences $\{x_{kn}\}, k > 1$ are convergent.

   (d) $\{x_k\}$ is convergent if sub-sequences $\{x_{2k}\}$, $\{x_{3k}\}$ and $\{x_{2k+1}\}$ are convergent.

4. Test the convergence of Series

   (a) $\sum_{k=1}^{\infty} \frac{1}{k^{(1+\frac{1}{k})}}$

   (b) $\sum_{k=1}^{\infty} \frac{1}{3^k+x}$ for all positive x.