Applications of Derivatives

Maxima and Minima

Important Concepts

(Section 3)

by

SURYA KUMARI

Lecture 7

To understand

Stationary or Turning points

Stationary or Turning values

Critical points

Points of Inflection (Inflexion)

Stationary or Turning points / values

The values of x for which $f'(x)=0$ are called **Stationary or Turning points** and the corresponding values of $f(x)$ are called **Stationary or Turning values.**

A point (or points) in the graph of a function that tells that the function itself is neither decreasing nor increasing is called a stationary point.

At the stationary points...

- The derivative is zero
- The tangent line is horizontal
- The function is neither

  increasing (positive slope)

  nor

  decreasing (negative slope).

Note:

1. Not all functions have stationary points.

2. A line has no stationary points.

3. Circles do not have stationary points.

Critical point

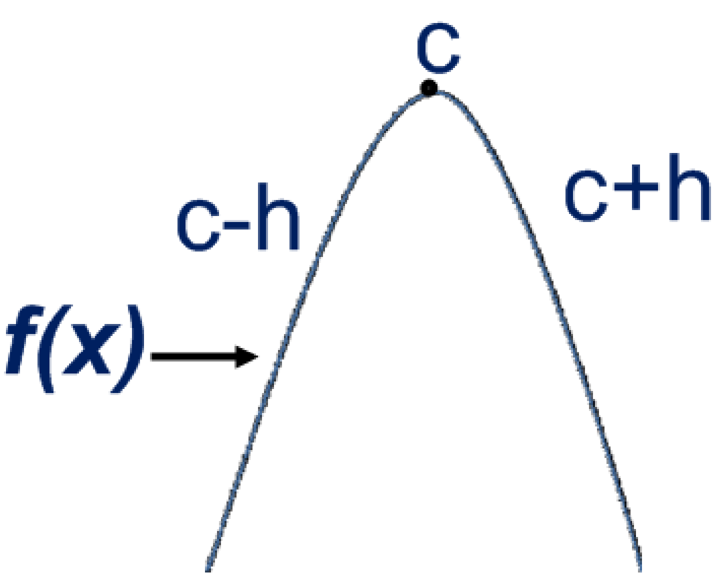
The values of x for which $f'(x)=0$ or, $f\,'(x)$ does not exist are known as **Critical points**.

Note:

A critical point can occur when $f'(x)$ does not exist, but stationary points only occur when $f'(x) = 0$.

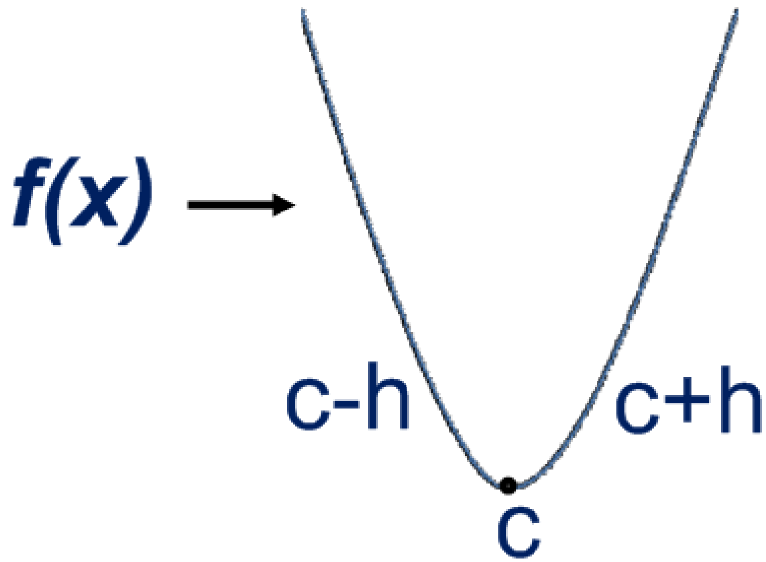
Point(s) of Inflexion (Inflection)

When c is a point of **Local maxima**



f '(x) changes sign from positive to negative as x increases through c

When c is a point of **Local minima**



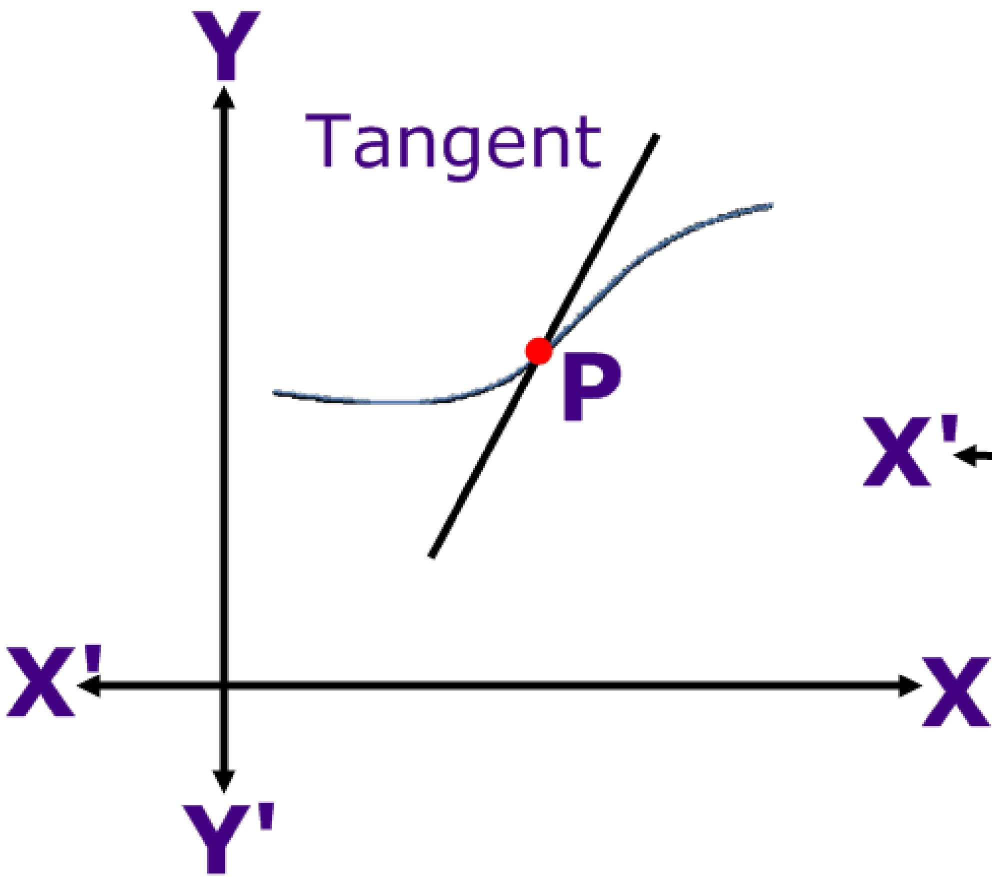
f '(x) changes sign from
negative to positive
as x increases through c

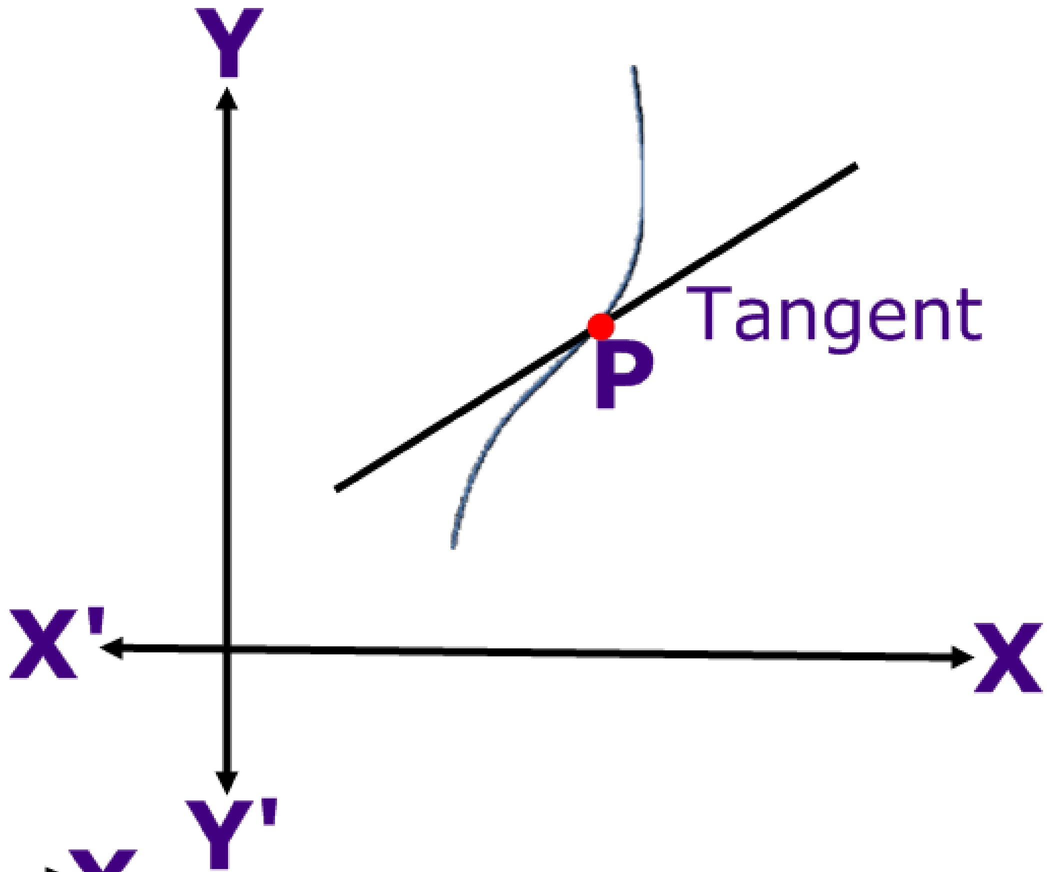
If $f'(x)$ does not change sign as x increases through c

c is neither a point of local maxima nor a point of local minima.

It is a point of Inflexion (Inflection)

P:Point of Inflexion





The point where the curve crosses the tangent.

One side of the curve lies below the tangent at P and on the other side, it lies above the tangent at P.

Lecture 8

To understand

Concavity

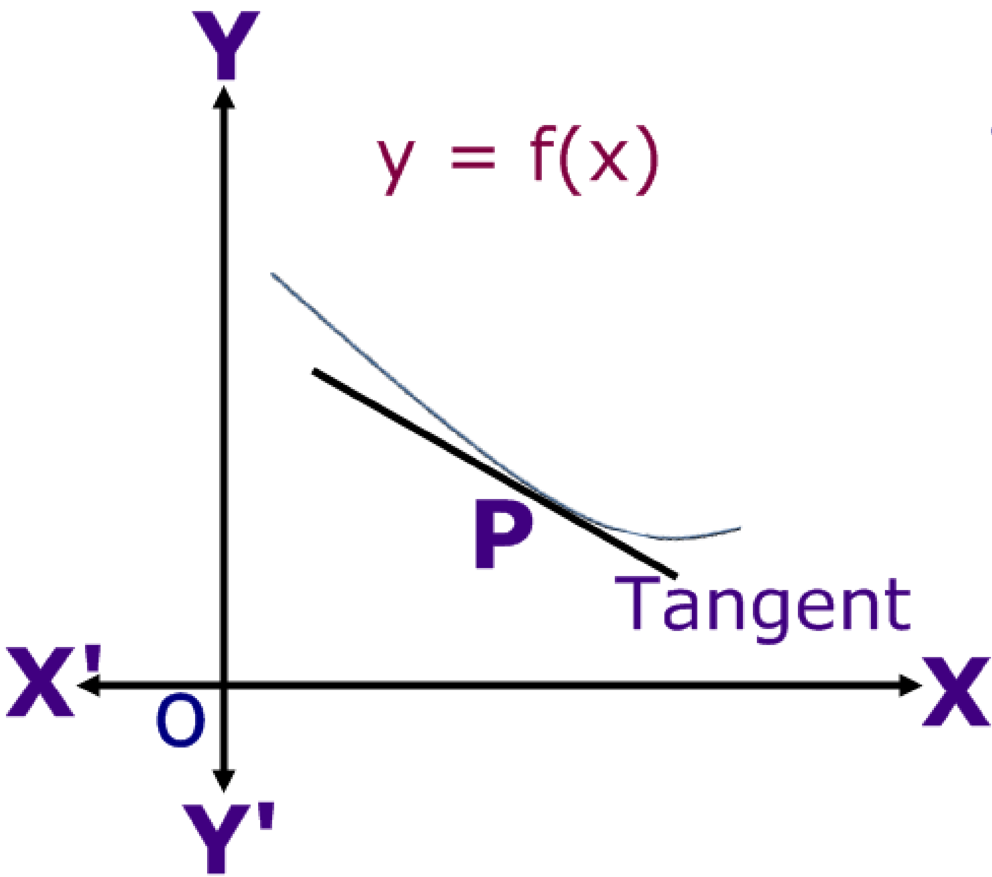
and

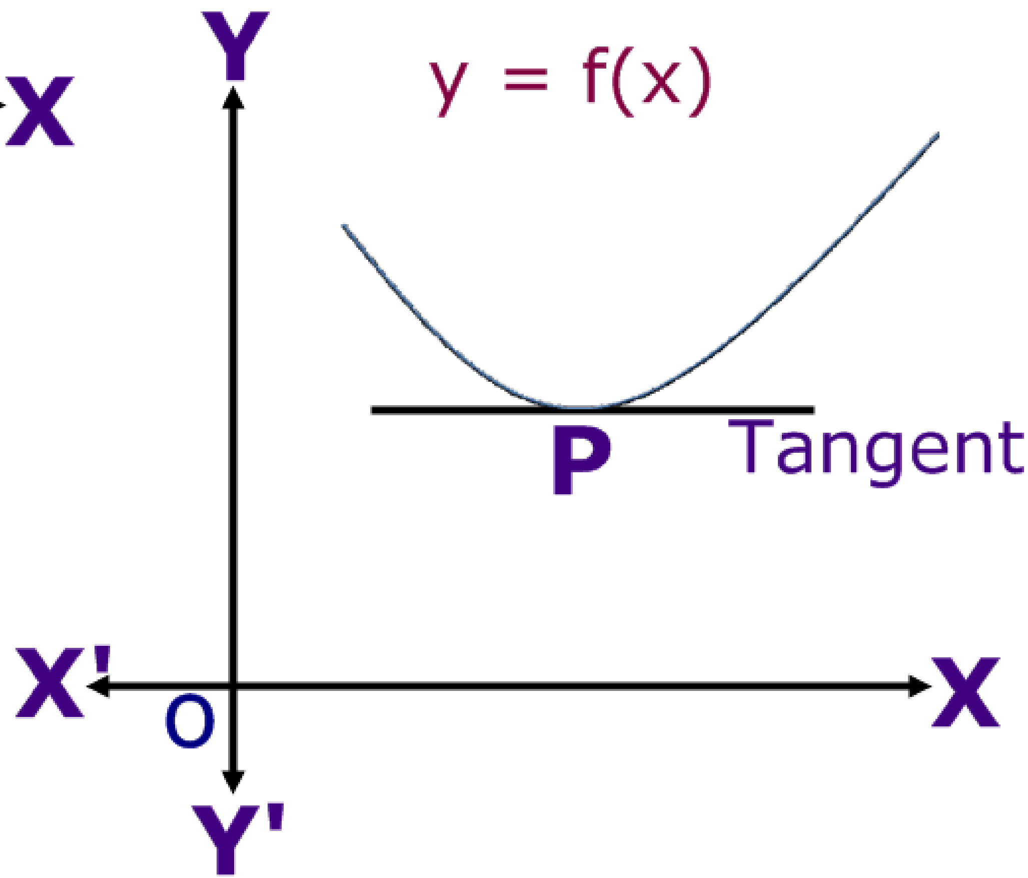
more on Points of Inflection

Concave upward:

An arc of a curve $y=f(x)$ is called concave upward if, at each of its points, the arc lies above the tangent at the point.

Concave upward:





If $y=f(x)$ is a concave upward curve, then as x increases

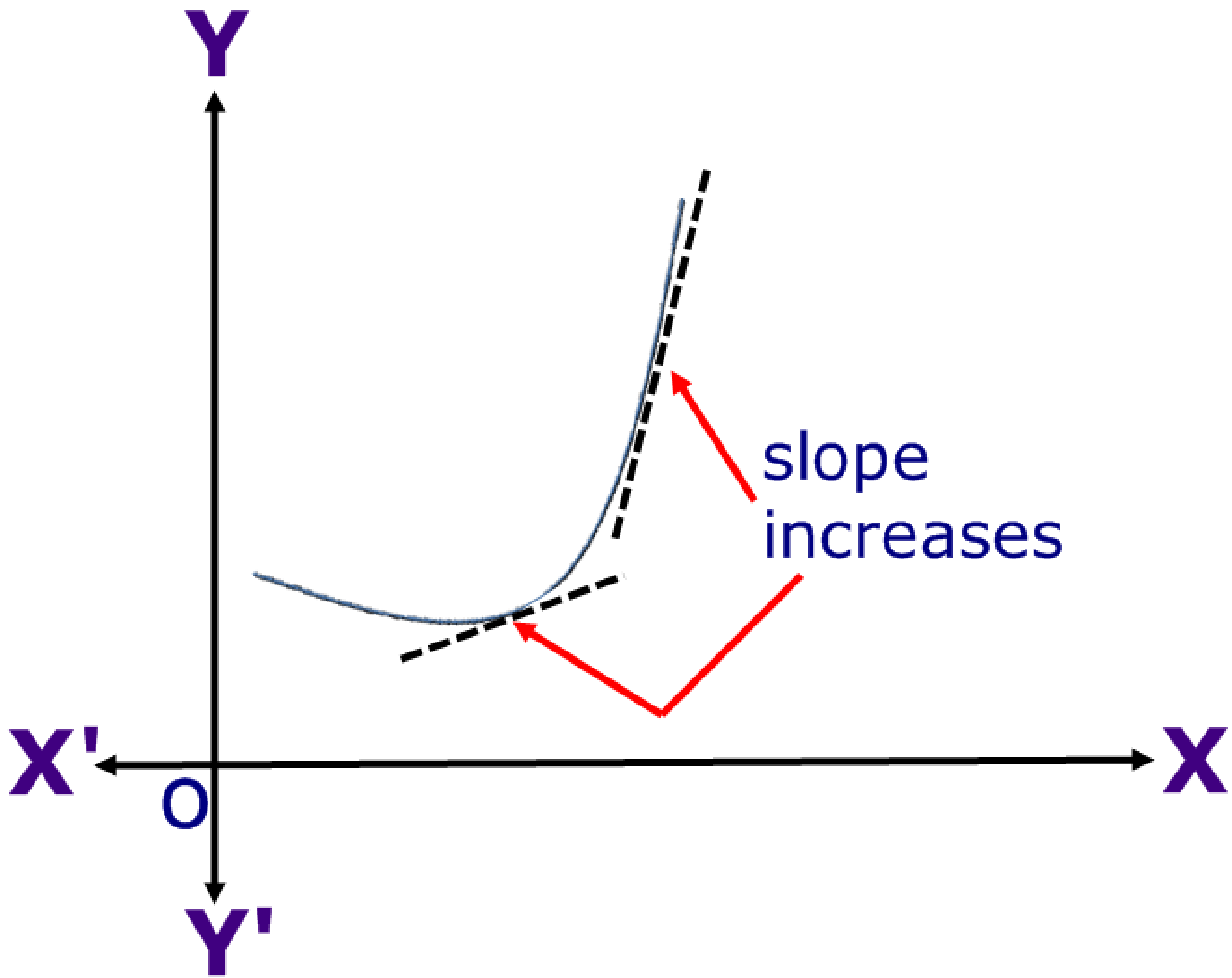
1. $f'(x)$ either is of the same sign and increasing or
2. changes sign from negative to positive.

In both the cases $f'(x)$ is increasing and so $f''(x)>0$.

Hence for a concave upward curve
$f''(x)>0$

Concave upward is when the slope increases:

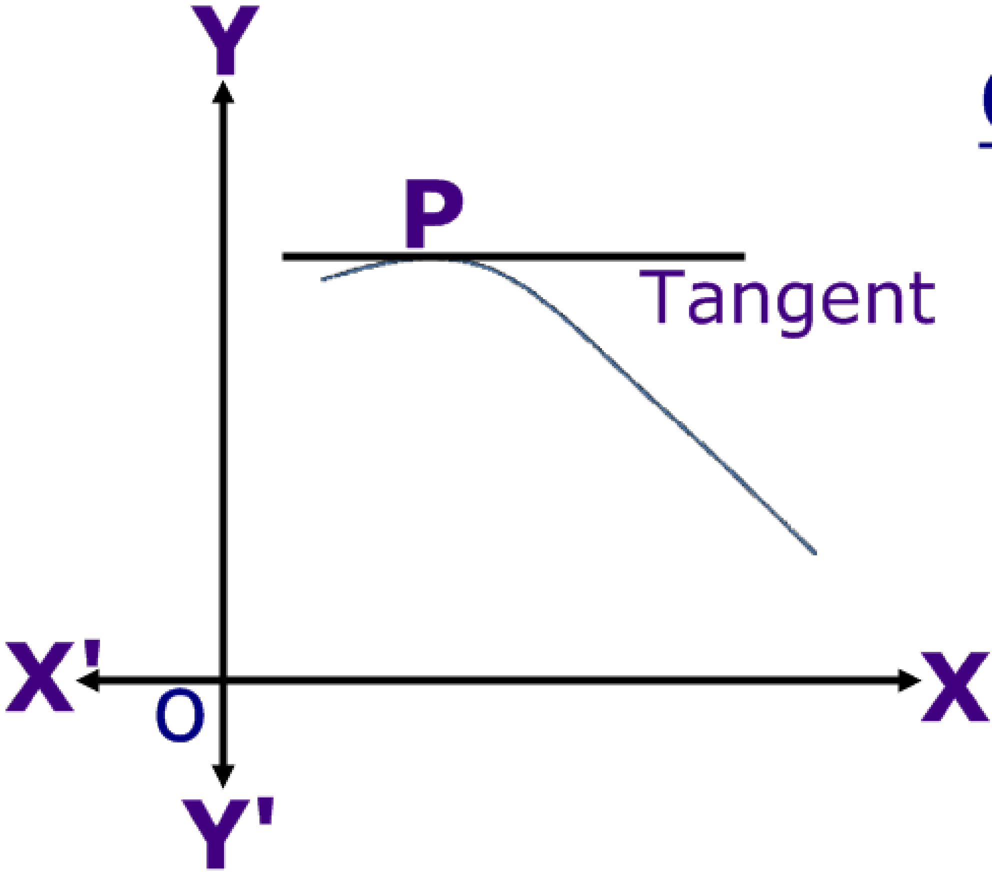
$y = f(x)$

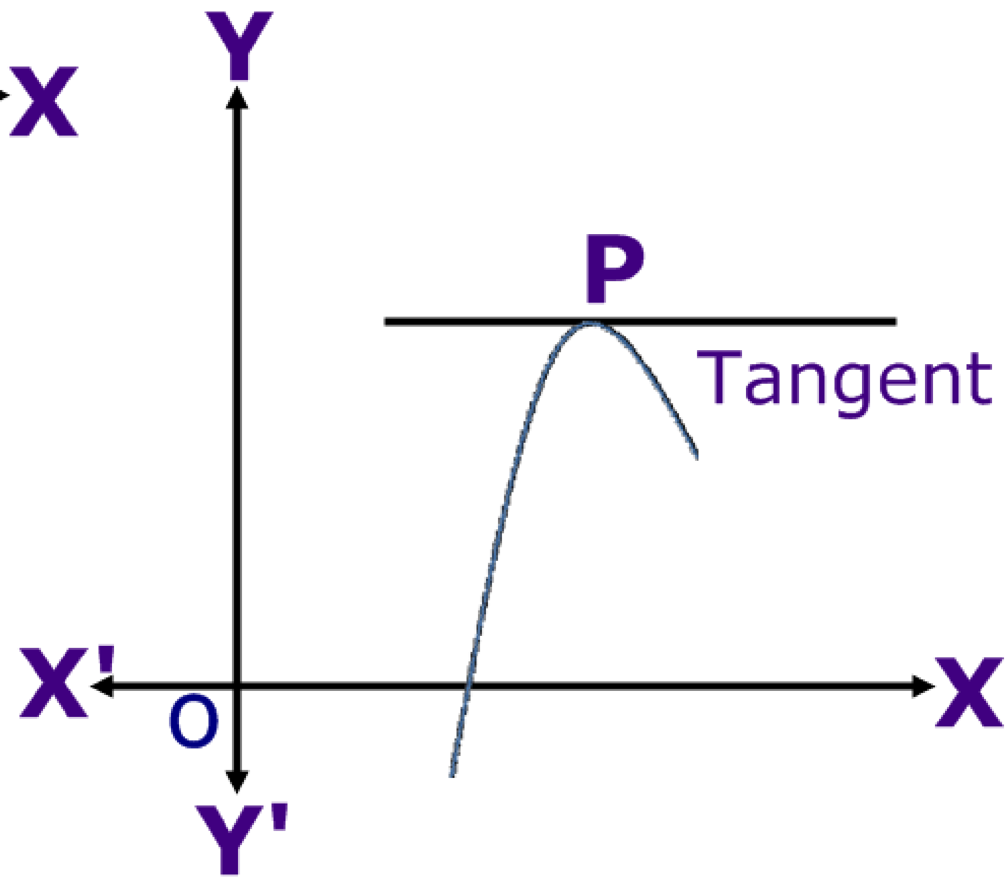


Concave downward:

An arc of a curve $y=f(x)$ is called concave downward if, at each of its points, the arc lies below the tangent at the point.

<u>Concave downward:</u>

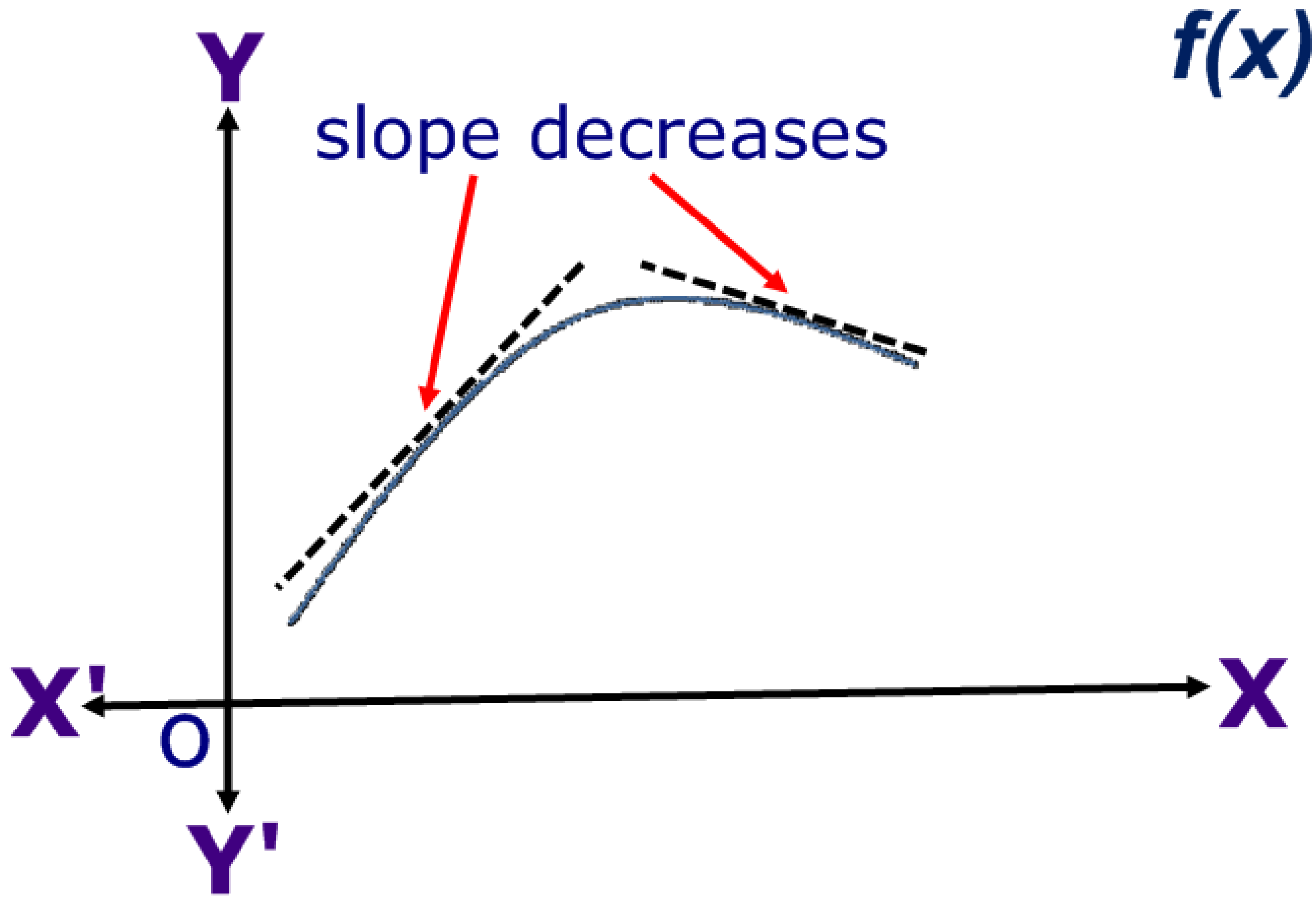




If $y=f(x)$ is a concave downward curve, then as x increases, $f'(x)$ either is of the same sign and decreasing or changes sign from positive to negative.
In both the cases $f'(x)$ is decreasing and so $f''(x)<0$.

Hence for a concave downward curve $\boldsymbol{f''(x)<0}$

Concave downward is when the slope decreases:

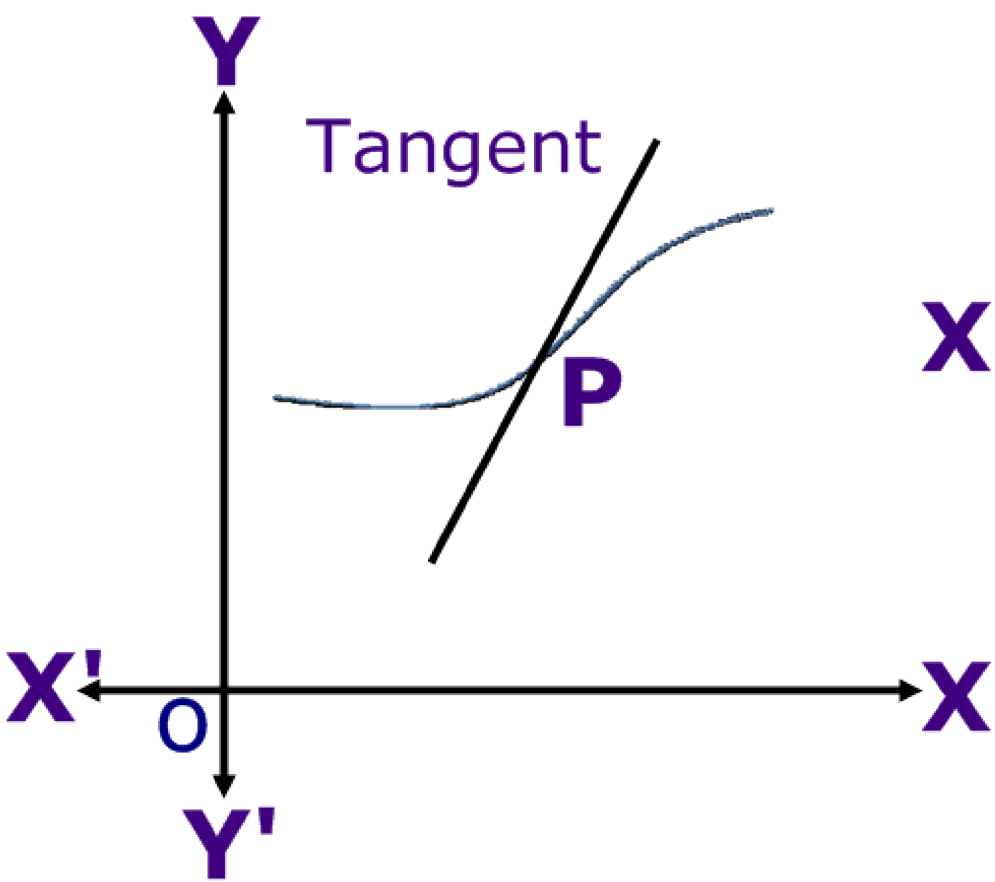


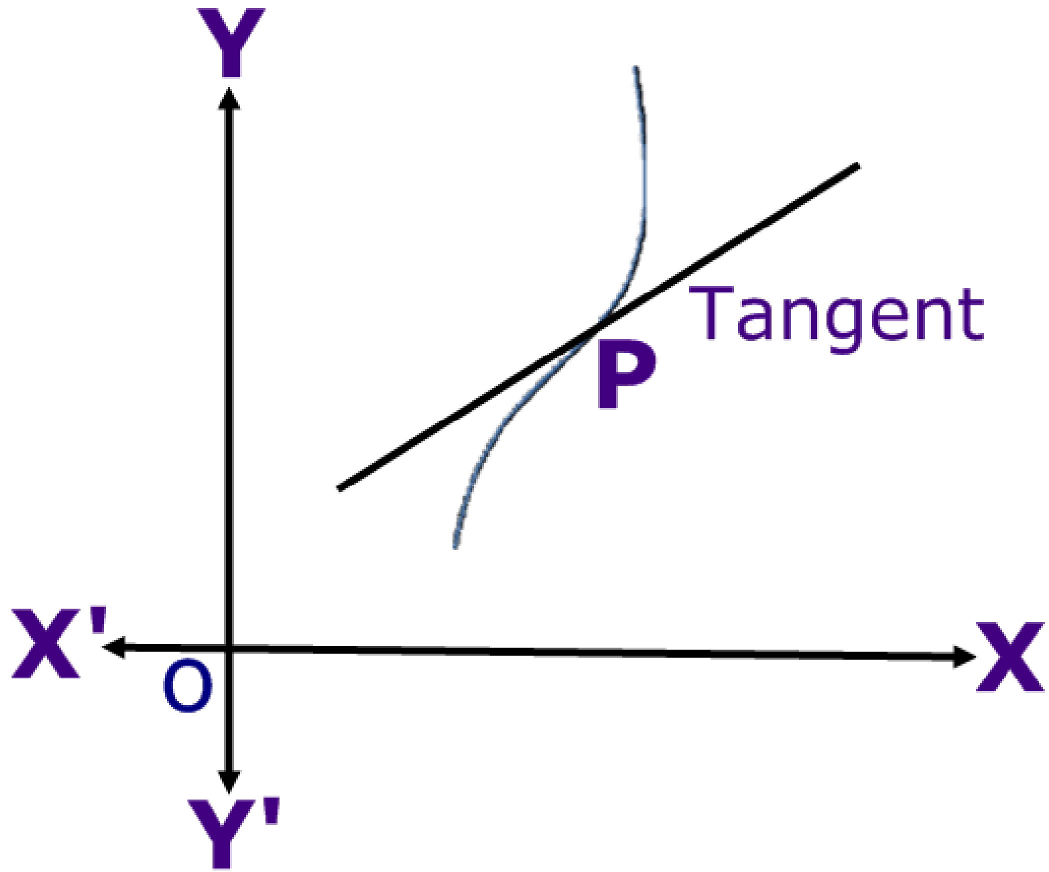
Point of Inflection / Inflexion

If $f'(c)=0$ and $f'(x)$ has the same sign in the complete neighbourhood of c, then c is neither a point of local maximum value nor a point of local minimun value. Such a point is called a **point of inflection.**

One side of the curve lies below the tangent at P and on the other side, it lies above the tangent at P.

P:point of Inflexion





The point where the curve crosses the tangent.

A point of inflection is a point at which a curve is changing concave upward to concave downward

or

A point at which a curve is changing concave downward to concave upward.

A **point of inflection** of the graph of a function $f(x)$ is a point where the *second* derivative $f''(x)$ is 0.

Theorem:

A function f (or the curve $y=f(x)$) has a point of inflexion at $x=c$ iff $f'(c)=0$, $f''(c)=0$ and $f'''(c) \neq 0$

Example : $f(x) = x^3$

$f'(x) = 3x^2$ $\quad$ $f''(x) = 6x$ $\quad$ $f'''(x) = 6$

$f'(0) = 0$ $\quad$ $f''(0) = 0$ $\quad$ $f'''(0) = 6 \neq 0$

0 is neither a point of local maxima nor a point of local minima.