Applications of Derivatives

Maxima and Minima

Important Concepts

(Section 2)

by

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Lecture 2

Understanding

Maximum and Minimum
values of a function in its domain.

Maximum value of a function.

Definition:

Let f be a real valued function defined on an interval **I** **(** or on the domain **D**) . Then

f(x) is said to have the ***maximum value in I,*** if there exists a point ***a*** in **I** if
$f(x) \leq f(a)$ for all x in the interval **I**.

a is called the point of ***maximum.***

f(a) is called the ***maximum value*** of the function in the interval **I**.

Minimum value of a function.

Definition:

Let f be a real valued function defined on an interval **I** (or on the domain **D**) . Then

f(x) is said to have the ***minimum value in I,*** if there exists a point ***a*** in **I** if
$f(x) \geq f(a)$ for all x in the interval **I**.

a is called the point of ***minimum***

f(a) is called the ***minimum value*** of the function in the interval **I**.

Points to Remember:

A function f defined on an interval I

1. may attain the maximum value at a point in I but not the minimum value at any point in I.

2. may attain the minimum value at a point in I but not the maximum value at any point in I.

3. may attain both the maximum and minimum values at some points in I .

4. may not attain both the maximum and minimum values at some points in I .

Lecture 3

Understanding

Local Maximum and Local Minimum

Local maximum value of a function.

Definition:

f(x) is said to have the ***local maximum value*** at x = ***a*** if whenever x is near a, $f(x) \leq f(a)$

f(a) is a ***local maximum value*** for f if there is some $\varepsilon > 0$, so that whenever x is in $(a - \varepsilon, a + \varepsilon)$

$$f(x) \leq f(a)$$

a is called the point of local ***maximum***

f(a) is called the ***local maximum value*** of f(x) at x=a

Local minimum value of a function.

Definition:

f(x) is said to have the ***local minimum value*** at x = ***a***
if whenever x is near a, f(x) $\geq$ f(a)

f(a) is a ***local minimum value*** for f if there is
some $\varepsilon > 0$, so that whenever x is in $(a - \varepsilon, a + \varepsilon)$

$$f(x) \geq f(a)$$

a is called the point of local ***minimum***

f(a) is called the ***local minimum value*** of f(x) at x=a

Points to Remember:

1. Local maximum or Relative maximum

   Local minimum or Relative minimum

2. A function may have more than one local maxima and more than one local minima.

3. Local maximum and local minimum occur alternately.

4. A local maximum value at some point may be less than a local minimum value of the function at another point.

Lecture 4

Understanding

Global Maximum and Global Minimum

Global maximum value of a function.

Definition:

Let f be a real valued function defined on the domain D (subset of $\mathbb{R}$), then

f(x) is said to have the ***greatest value*** or ***global maximum value*** at a point ***a*** in its domain if
$f(x) \leq f(a)$ for all x in the domain of f(x).

a is called the point of ***maximum***

f(a) is called the ***global maximum value*** of the function on D

Global minimum value of a function.

Definition:

Let f be a real valued function defined on the domain D (subset of $\mathbb{R}$), then

f(x) is said to have the ***least value*** or ***global minimum value*** at a point ***a*** in its domain if
f(x) $\geq$ f(a) for all x in the domain of f(x).

a is called the point of ***minimum***

f(a) is called the ***global minimum value*** of the function on D

Points to Remember:

1

Global maximum

or

Absolute maximum

Global minimum

or

Absolute minimum

or

Extremum values

2

i. A local maximum value need not be the global maximum value.

ii. A global maxima can be a local maxima.

iii.A local minimum value need not be the global minimum value.

iv.A global minima can be a local minima.

Lecture 5

Absolute Maximum and Absolute Minimum values in a Closed Interval.

Theorem 1:

Let f be a continuous function on an interval $I=[a,b]$. Then

(i) f has the absolute maximum value and f attains it at least once in I.

(ii) f has the absolute minimum value and f attains it at least once in I.

Theorem 2:

Let f be a differentiable function on a closed interval I=[a,b] and let c be any interior point of I. Then

(i) $f'(c) = 0$, if f attains its absolute maximum value at c.

(ii) $f'(c) = 0$, if f attains its absolute minimum value at c.

Working Rule

Step 1. Find $f'(x)$.

Step 2. Put $f'(x) = 0$ and find values of x.
Let $c_1, c_2, c_3, \ldots\ldots, c_n$ be the values of x.

Step 3. Evaluate $f(x)$ at these points $c_1, c_2, c_3, \ldots\ldots, c_n$ where $f'(x) = 0$

i.e. Find $f(c_1), f(c_2), f(c_3), \ldots\ldots\ldots f(c_n)$.

Step 4. Evaluate $f(x)$ at the points where derivative fails to exist.

Step 5. Find f(a) and f(b)
(Evaluate f at the end points).

Step 6. Identify the maximum and minimum values of f out of the values calculated in steps 3,4,5.

This maximum value will be the absolute maximum (greatest) value of f in I=[a,b]

and

the minimum value will be the absolute minimum (least)value of f in I=[a,b]

Points to Remember:

Global maximum
or
Absolute maximum

Global minimum
or
Absolute minimum

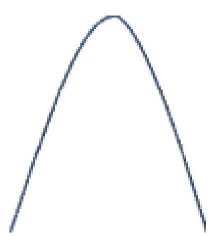
or
Extremum values

in the given interval

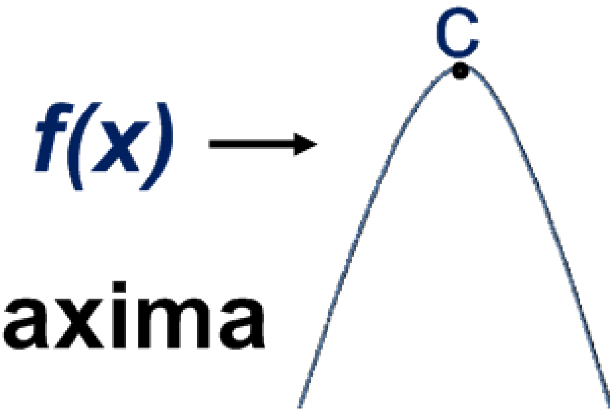
Lecture 6

Behaviour of **f '(x)** at

***L*ocal Maxima** and ***L*ocal Minima**



Case 1:



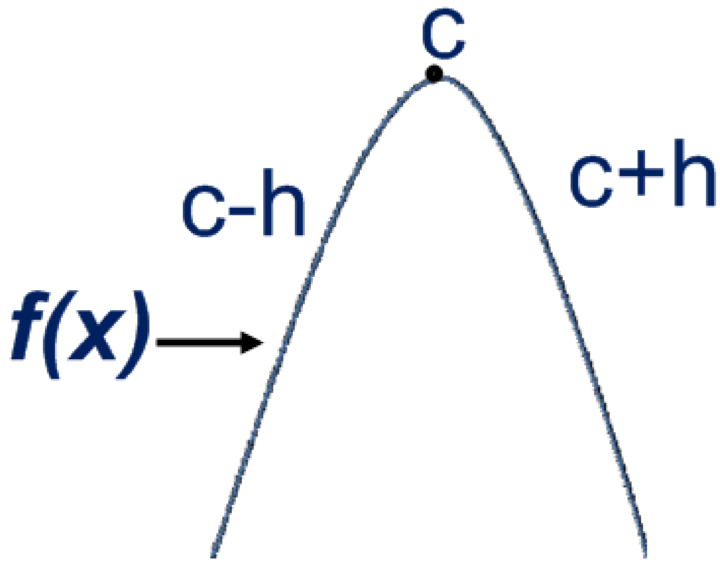
When c is a point of **Local maxima**

f(x) is increasing for values of x slightly less than c (in the left neighbourhood of c).

ceases to increase at x = c

f(x) is decreasing for values of x slightly greater than c (in the right neighbourhood of c).

$f'(x)>0$, for $x \in (c-h,c)$

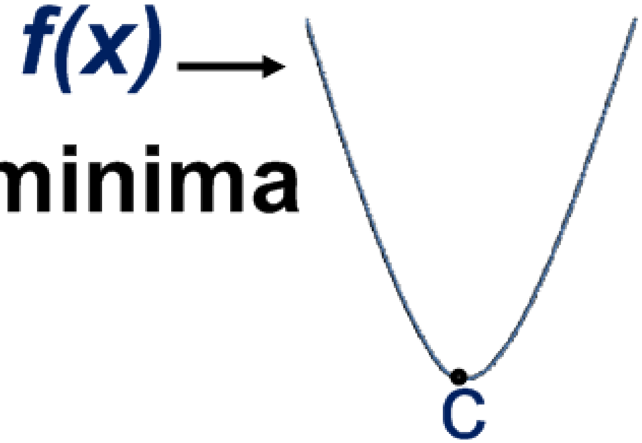


$f'(x)<0$, for $x \in (c,c+h)$

Thus $f'(x)$ changes sign from positive to negative as x increases through c

Case 2:

When c is a point of **Local minima**

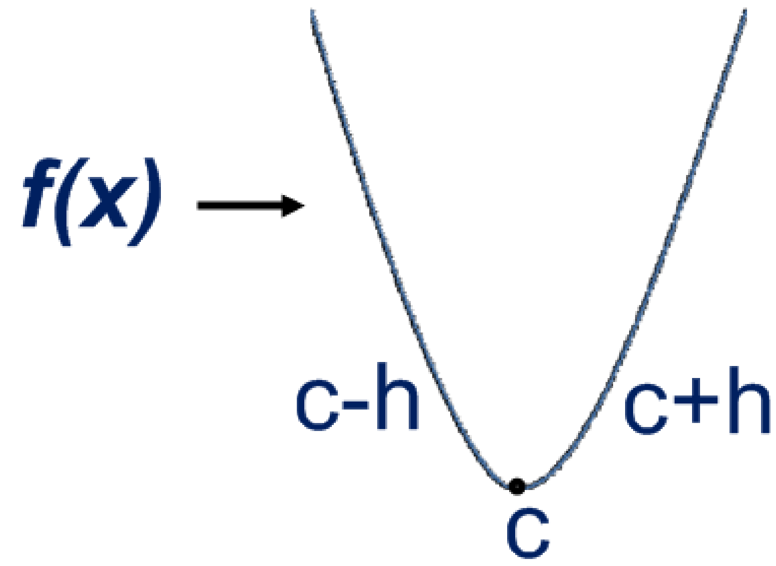


f(x) is decreasing for values of x slightly less than c (in the left neighbourhood of c).

ceases to decrease at x = c

f(x) is increasing for values of x slightly greater than c (in the right neighbourhood of c).

$f'(x)<0$, for $x \in (c-h,c)$



$f'(x)>0$, for $x \in (c,c+h)$

Thus $f'(x)$ changes sign from negative to positive as x increases through c

Theorem:

Let f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or a local minima at $x=c$, then either $f'(c) = 0$ or f is not differentiable at c.

(proof beyond the scope of this course)

Remark 1:

The **converse** of this theorem **is not true**.

i.e. a point at which the derivative vanishes need not be a point of local maxima or local minima.

Remark 2:

A function may attain a maximum or minimum value at a point without being derivable there at.

Remark 3:

Geometrical meaning:

The tangent to the curve $y = f(x)$ at a point where the ordinate is maximum or minimun is parallel to the x- axis.