Applications of Derivatives

Maxima and Minima

Important Concepts

(Section 4)

by

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Lecture 9

First Derivative Test

First Derivative Test

(i) c is a point of local maximum if $f'(c) = 0$ and $f'(x)$ changes sign from positive to negative as x increases through c.

(ii) c is a point of local minimum if $f'(c) = 0$ and $f'(x)$ changes sign from negative to positive as x increases through c.

Working rule for finding points of local maxima or points of local minima :

Step 1: Find $f'(x)$

Step 2: Solve $f'(x) = 0$. Let these values of x be a,b,c, etc.
These are the candidates for maxima or minima.

Step 3: Test the function for each one of these values.

Consider $x = c$.

Step 4: Determine the sign of $f'(x)$ for values of x slightly less than c and for values of x slightly greater than c.

Conclusion:

(i) If $f'(x)$ changes sign from **positive to negative** the function is **maximum** for $x=c$.

(ii) If $f'(x)$ changes sign from **negative to positive** the function is **minimum** for $x=c$.

(iii) If $f'(x)$ does not change its sign,then $x=c$ gives a point of Inflection (Inflexion).

Repeat the steps with the other roots of $f'(x) = 0$ and examine them for maxima and minima.

SUMMARY

x	slightly <c	slightly >c	Nature of the turning point
f '(x)	+ ve	- ve	Maxima
f '(x)	- ve	+ ve	Minima
f '(x)	+ ve	+ ve	Neither maxima nor minima
f '(x)	- ve	- ve	Neither maxima nor minima

Lecture 10

Second Derivative Test

Second Derivative Test

Theorem:

Suppose f be twice differentiable on the open interval I at $c \in I$. Then

(i) $x=c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$

$f(c)$ is the local maximum value of f.

(ii) x=c is a point of local minima if
$f'(c) = 0$ and $f''(c) > 0$

$f(c)$ is the local minimum value of f.

(iii) The test fails if

$f'(c) = 0$ and $f''(c) = 0$

Working rule for finding points of local maxima or points of local minima :

Step 1: Find $f'(x)$

Step 2: Solve $f'(x)= 0$. Let these values of x be a,b,c, etc.
These are the candidates for maxima or minima.

Step 3: Test the function for each one of these values.

Consider $x = c$.

Step 4: Find $f''(x)$ and put $x=c$ to get $f''(c)$

Now,

if **$f''(c)<0$**, then $x=c$ is a **local maximum**

if **$f''(c)>0$**, then $x=c$ is a **local minimum**

if $f''(c)=0$, then use the first derivative test.

Repeat the steps with the other roots of $f'(x) = 0$ and examine them for maxima and minima.

SUMMARY

Second Derivative Test		
$f''(c)$	<0	**local maximum**
$f''(c)$	>0	**local minimum**
$f''(c)$	$=0$	use first derivative test.

Note: The Second derivative test fails if $f'(c)=0$ and $f''(c)=0$