Problem set 5

(First submission due on 4th December, midnight IST)

In this problem set, you will learn to fit computational models to behavioral data and how to carry out parameter recovery. The goal of model fitting is to answer the following question for a given model M and given some data D: "Assuming that the data D was produced by model M, what are the values of the free parameters of model M which maximize the probability of data D occurring ?" Please read the sections "Fit the parameters" and "Check that you can recover the parameters" (including Box 3 and 4) in Wilson & Collins (2013) before you solve this problem set.

Problem 1: Write a likelihood function for Model 2 (name it model2_loglike.m) and Model 3 (name is model3_loglike.m). A likelihood function is a **function** that takes as input

- choices made on each trial
- rewards obtained on each trial
- the experiment parameters (T mainly)
- model's free parameters (epsilon for model 2, alpha and beta for model 3),

and returns as output the ***negative log likelihood*** of the data given the model and its parameter values.

The ***log likelihood*** is given by the following equation:

$$\underbrace{\log p\,(d_{1:T} \mid \theta_m, m)}_{\text{log likelihood}} = \sum_{t=1}^{T} \log p\,(c_t \mid d_{1:t-1}, s_t, \theta_m, m)$$

Where $p(c_t \mid d_{1:t-1}, st, \theta_m, m)$ is the choice probability of the actual choice made (in the data) on trial t assuming that the choice was made by model m with parameters θ_m.

(Negative log likelihood is just the log likelihood multiplied by minus 1.)

Problem 2: In the folder real_data you are given a single file named pset5_real_data.mat. This file contains the data recorded from a single subject in a real 2-armed bandit experiment with 54 trials (i.e. T = 54 trials). It contains two variables:

- choices which is a 1 x 54 vector containing the choices made on each trial. Choices are coded as either 1 or 2.
- rewards which is a 1 x 54 vector containing the rewards obtained on each trial. Rewards are coded as either 1 or -1.

Fit both model 2 and model 3 to this data and find the maximum likelihood estimates of each model's parameters through exhaustive search. To do this, please compute the likelihood of the data under model 2 and model 3 for the following parameter values.

Model 2: epsilon = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]

Model 3: alpha = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
beta = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Note that this will mean 10 fits for model 2, and 100 fits for model 3. Report the maximum likelihood estimates for the parameters.

Bonus problem: Plot the likelihood surface for this dataset.

Problem 3:

For the data that was provided in pset5_real_data.mat, answer the following questions:

(a) for each value of the parameter epsilon of model 2, what will the value of the **trial** likelihood

$$p\left(c_t \mid d_{1:t-1}, s_t, \boldsymbol{\theta}_m, m\right)$$

be for the first trial (i.e. where $t = 1$)?

(b) Is there any circumstance in which the trial likelihood becomes zero and thus the trial log likelihood becomes infinity? What does that mean for the model's fit to the data?

(Optional) Problem 4: In the folder sim_data you are provided with 100 files named ps5_sim_data_XYZ.mat. Each file contains the results of a simulation of model 3. It contains three variables:

- choices which is a 1000 x 1 vector containing the choices made on each trial.
- rewards which is a 1000 x 1 vector containing the rewards obtained on each trial.
- alpha which is the learning rate parameter that was used to generate the data

Write a script to fit model 3 to each of these 100 files using fmincon and the likelihood function you wrote for Problem 1. A sample model fitting script with an example of how to use fmincon is provided (model_fitting_wrapper.m). Each model fit should give you a set of maximum likelihood estimates of alpha and beta for that dataset (one for each iteration). these maximum likelihood estimates.

Make a scatterplot which plots the true alpha (i.e. the alpha that was stored in the original data file) on the horizontal axis and the best fitting alpha (i.e. the maximum likelihood estimates of alpha) on the vertical axis.

Also report the mean and the standard deviation of the maximum likelihood estimates of the beta parameter. The original simulations fixed the beta parameter to the value of e (2.7183).

Interpret the scatterplot and the mean and standard deviations of beta in light of Box 4 Fig 1 in Wilson & Collins (2019).

References:
Wilson, R. C., & Collins, A. G. (2019). Ten simple rules for the computational modeling of behavioral data. Elife, 8, e49547.