MACHINE LEARNING

GRAPHICAL MODELS II

D-SEPARATION, INFERENCE ON A CHAIN, GENERAL INFERENCE, MARKOV BLANKET, IMAGE DENOISING

Example: Car with a faulty gas gauge G measuring fuel level F influenced by a battery B

$$p(B = 1) \ = \ 0.9$$
$$p(F = 1) \ = \ 0.9.$$

Given the state of the fuel tank and the battery, the fuel gauge reads full with probabilities given by

$$p(G = 1 | B = 1, F = 1) \ = \ 0.8$$
$$p(G = 1 | B = 1, F = 0) \ = \ 0.2$$
$$p(G = 1 | B = 0, F = 1) \ = \ 0.2$$
$$p(G = 1 | B = 0, F = 0) \ = \ 0.1$$

so this is a rather unreliable fuel gauge! All remaining probabilities are determined by the requirement that probabilities sum to one, and so we have a complete specification of the probabilistic model.

Before we observe any data, the prior probability of the fuel tank being empty is $p(F = 0) = 0.1$. Now suppose that we observe the fuel gauge and discover that it reads empty, i.e., $G = 0$, corresponding to the middle graph in Figure 8.21. We can use Bayes' theorem to evaluate the posterior probability of the fuel tank being empty. First we evaluate the denominator for Bayes' theorem given by

$$p(G = 0) = \sum_{B \in \{0,1\}} \sum_{F \in \{0,1\}} p(G = 0 | B, F) p(B) p(F) = 0.315 \qquad (8.30)$$

and similarly we evaluate

$$p(G = 0 | F = 0) = \sum_{B \in \{0,1\}} p(G = 0 | B, F = 0) p(B) = 0.81 \qquad (8.31)$$

and using these results we have

$$p(F = 0 | G = 0) = \frac{p(G = 0 | F = 0) p(F = 0)}{p(G = 0)} \simeq 0.257 \qquad (8.32)$$

Next suppose that we also check the state of the battery and find that it is flat, i.e., $B = 0$. We have now observed the states of both the fuel gauge and the battery, as shown by the right-hand graph in Figure 8.21. The posterior probability that the fuel tank is empty given the observations of both the fuel gauge and the battery state is then given by

$$p(F = 0|G = 0, B = 0) = \frac{p(G = 0|B = 0, F = 0)p(F = 0)}{\sum_{F \in \{0,1\}} p(G = 0|B = 0, F)p(F)} \simeq 0.111 \quad (8.33)$$

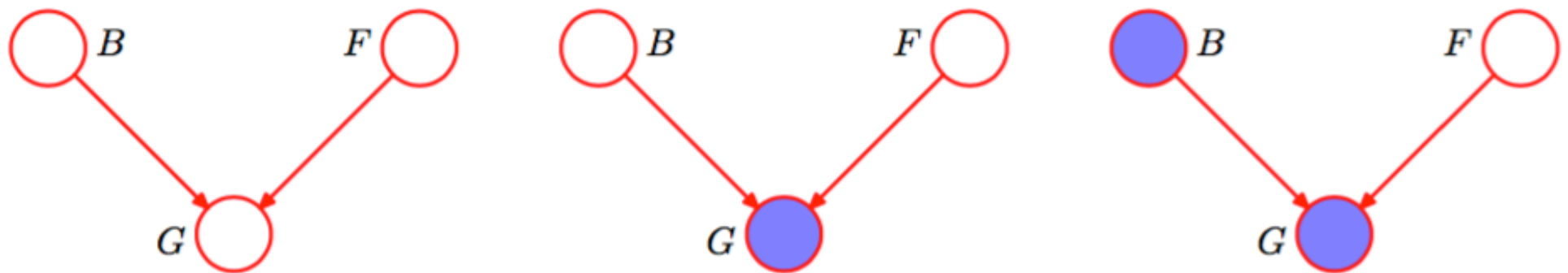**Figure 8.21** An example of a 3-node graph used to illustrate the phenomenon of 'explaining away'. The three nodes represent the state of the battery (B), the state of the fuel tank (F) and the reading on the electric fuel gauge (G). See the text for details.

4

All paths blocked = Cond. Indep.

Consider a general directed graph in which A, B, and C are arbitrary nonintersecting sets of nodes (whose union may be smaller than the complete set of nodes in the graph). We wish to ascertain whether a particular conditional independence statement $A \perp\!\!\!\perp B \mid C$ is implied by a given directed acyclic graph. To do so, we consider all possible paths from any node in A to any node in B. Any such path is said to be *blocked* if it includes a node such that either

(a) the arrows on the path meet either head-to-tail or tail-to-tail at the node, and the node is in the set C, or

(b) the arrows meet head-to-head at the node, and neither the node, nor any of its descendants, is in the set C.

If all paths are blocked, then A is said to be d-separated from B by C, and the joint distribution over all of the variables in the graph will satisfy $A \perp\!\!\!\perp B \mid C$.
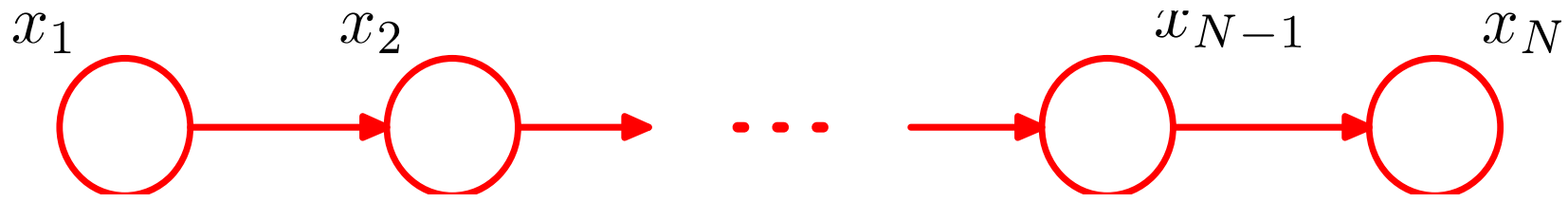
Two Exs ... Are any blocked?
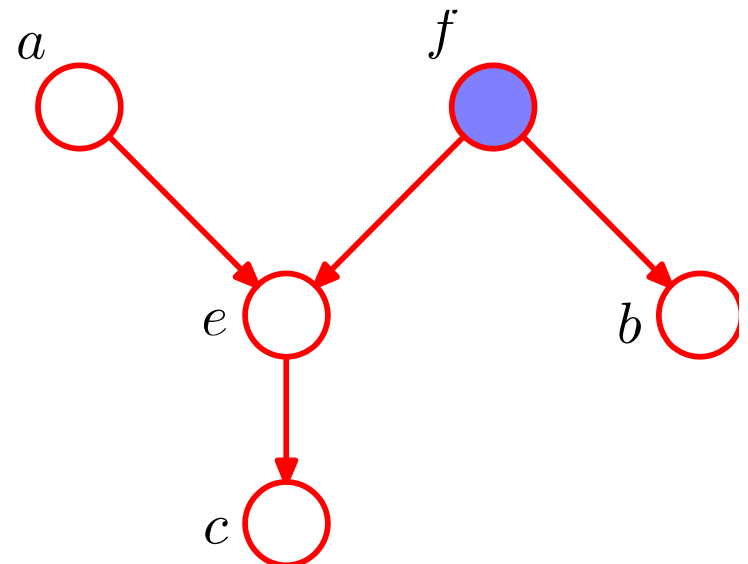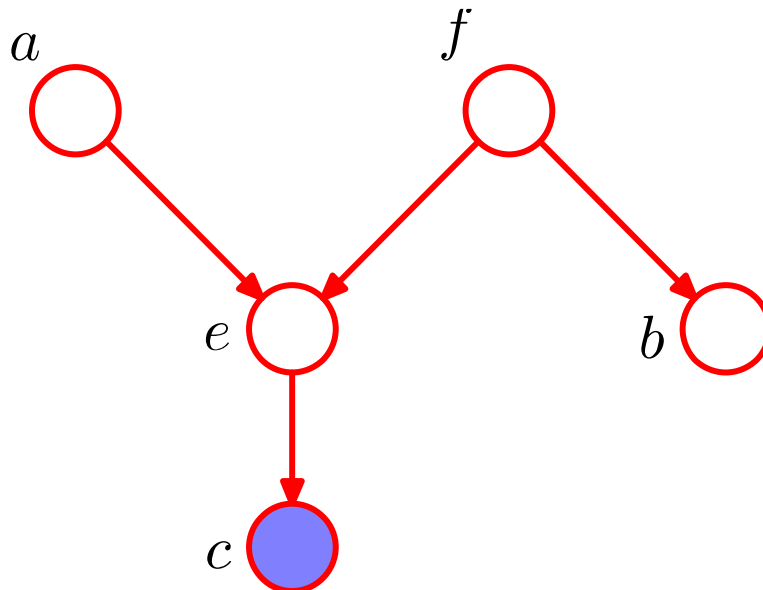
(a) the arrows on the path meet either head-to-tail or tail-to-tail at the node, and the node is in the set C, or

(b) the arrows meet head-to-head at the node, and neither the node, nor any of its descendants, is in the set C.

Inference on a chain

Directed converted to undirected

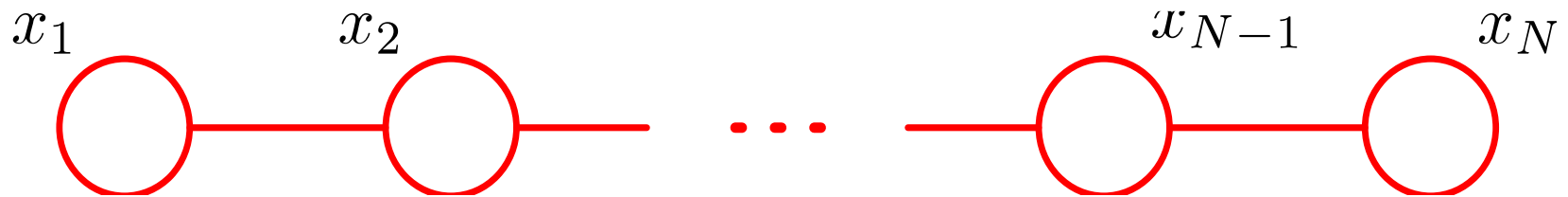$$p(\mathbf{x}) = p(x_1)p(x_2|x_1)p(x_3|x_2) \cdots p(x_N|x_{N-1}).$$

$$
\begin{aligned}
\psi_{1,2}(x_1, x_2) &= p(x_1)p(x_2|x_1) \\
\psi_{2,3}(x_2, x_3) &= p(x_3|x_2) \\
&\vdots \\
\psi_{N-1,N}(x_{N-1}, x_N) &= p(x_N|x_{N-1})
\end{aligned}
$$

Undirected graph

$$p(\mathbf{x}) = \frac{1}{Z}\psi_{1,2}(x_1, x_2)\psi_{2,3}(x_2, x_3)\cdots\psi_{N-1,N}(x_{N-1}, x_N)$$

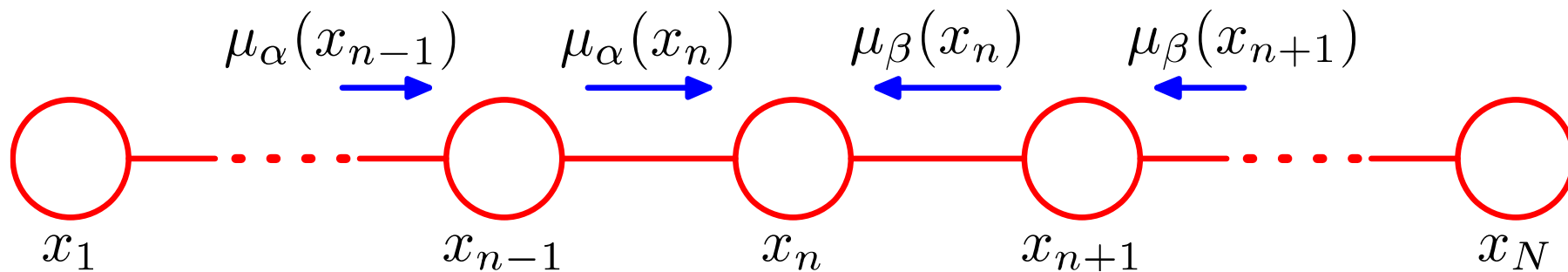# Computing P(x$_n$) could be expensive

$$p(x_n) = \sum_{x_1} \cdots \sum_{x_{n-1}} \sum_{x_{n+1}} \cdots \sum_{x_N} p(\mathbf{x}).$$

Grouping terms …

$$p(x_n) = \frac{1}{Z} \underbrace{\left[\sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \cdots \left[\sum_{x_2} \psi_{2,3}(x_2, x_3) \left[\sum_{x_1} \psi_{1,2}(x_1, x_2) \right] \right] \cdots \right]}_{\mu_\alpha(x_n)}$$

$$X \underbrace{\left[\sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \cdots \left[\sum_{x_2} \psi_{2,3}(x_2, x_3) \left[\sum_{x_1} \psi_{1,2}(x_1, x_2) \right] \right] \cdots \right]}_{\mu_\alpha(x_n)}$$

View the calculation of μs as message passing

$$\mu_\alpha(x_n) = \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \left[\sum_{x_{n-2}} \cdots \right]$$

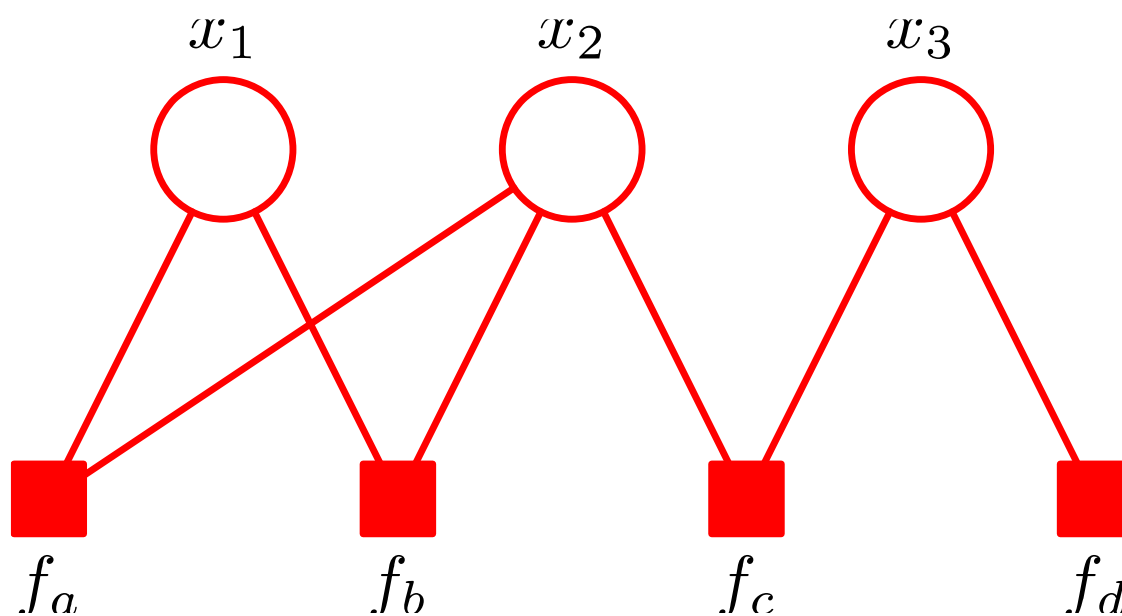$$= \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \mu_\alpha(x_{n-1}).$$

$$p(x_n) = \frac{1}{Z}$$

$$\underbrace{\left[\sum_{x_{n-2}} \psi_{n-2,n-1}(x_{n-2}, x_{n-1}) \cdots \left[\sum_{x_1} \psi_{1,2}(x_1, x_2)\right] \cdots \right]}_{\mu_\alpha(x_{n-1})} \psi_{n-1,n}(x_{n-1}, x_n)$$

$$\underbrace{\left[\sum_{x_{n+1}} \psi_{n,n+1}(x_n, x_{n+1}) \cdots \left[\sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N)\right] \cdots \right]}_{\mu_\beta(x_n)},$$

Factor graphs
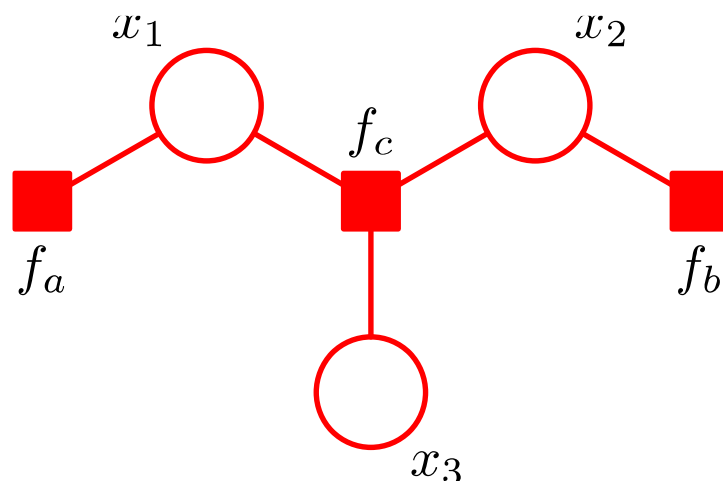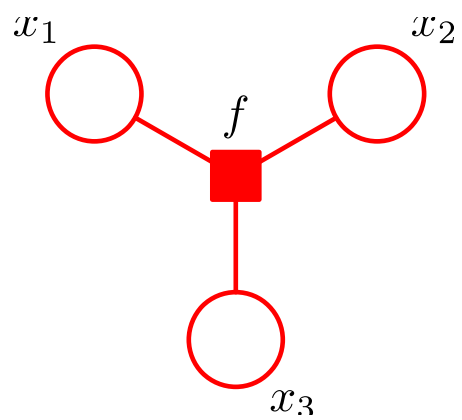
Need a notation that can handle message passing in graphs

$$p(\mathbf{x}) = \prod_s f_s(\mathbf{x}_s)$$

$$p(\mathbf{x}) = f_a(x_1, x_2) f_b(x_1, x_2) f_c(x_2, x_3) f_d(x_3)$$

$$\mu_{x \to f}(x) = 1$$

$$\mu_{f \to x}(x) = f(x)$$
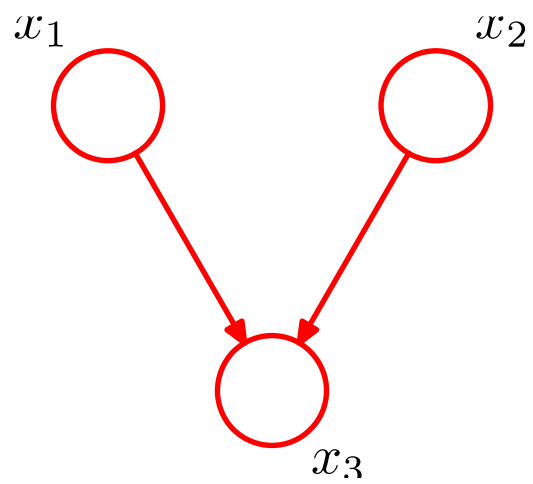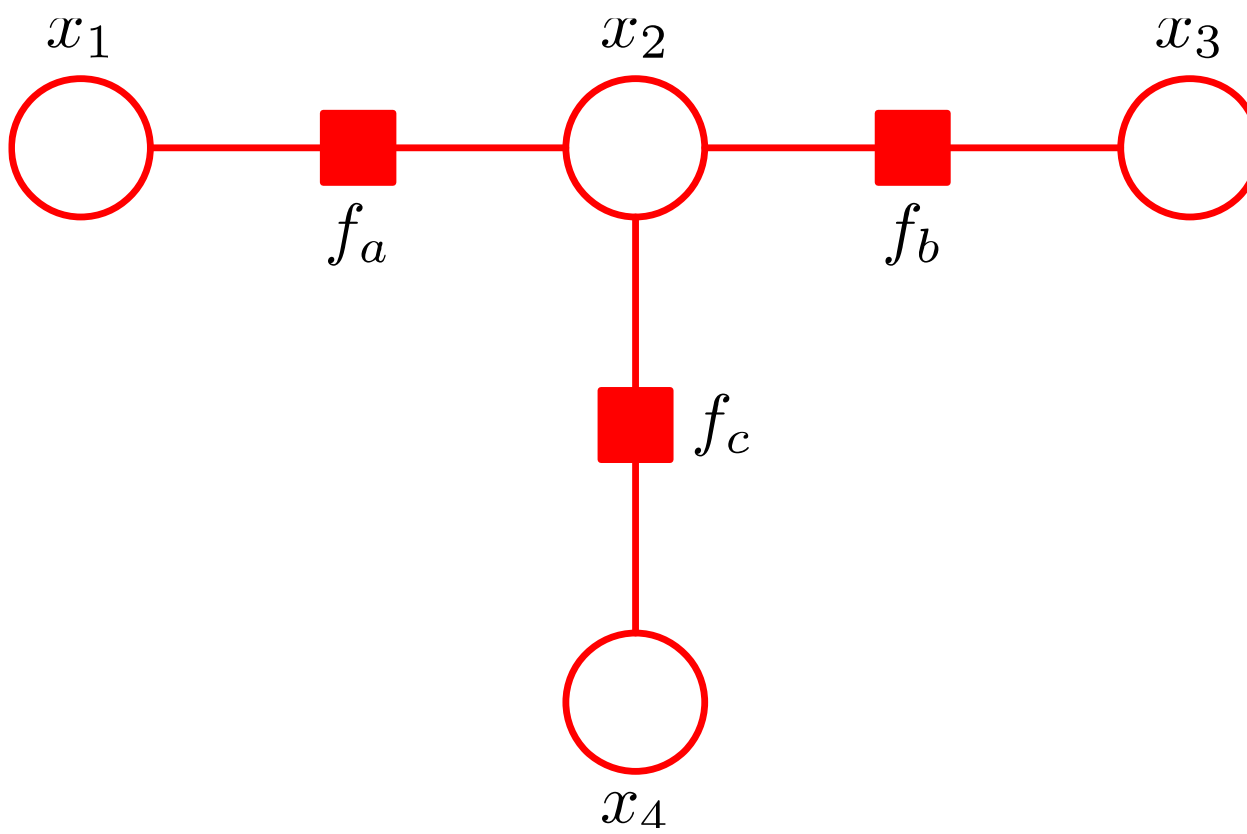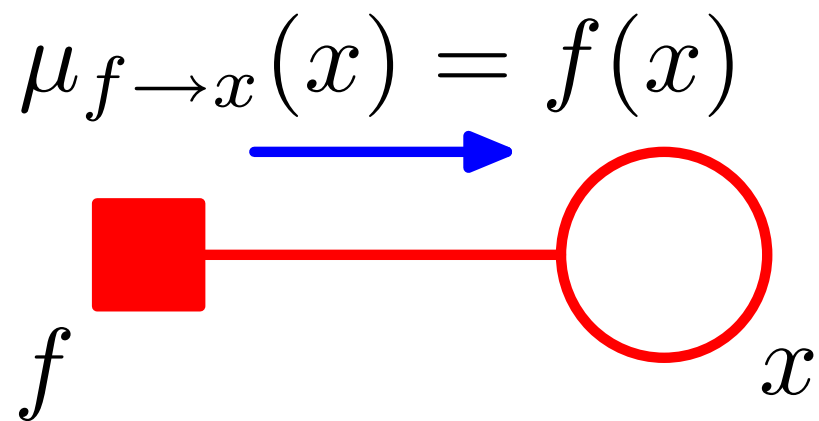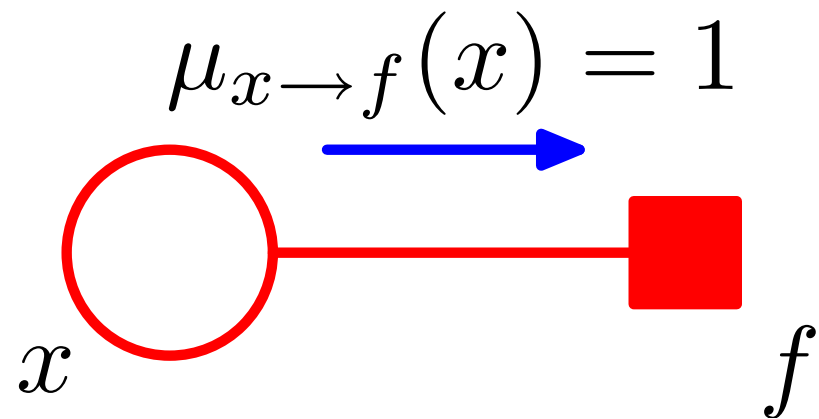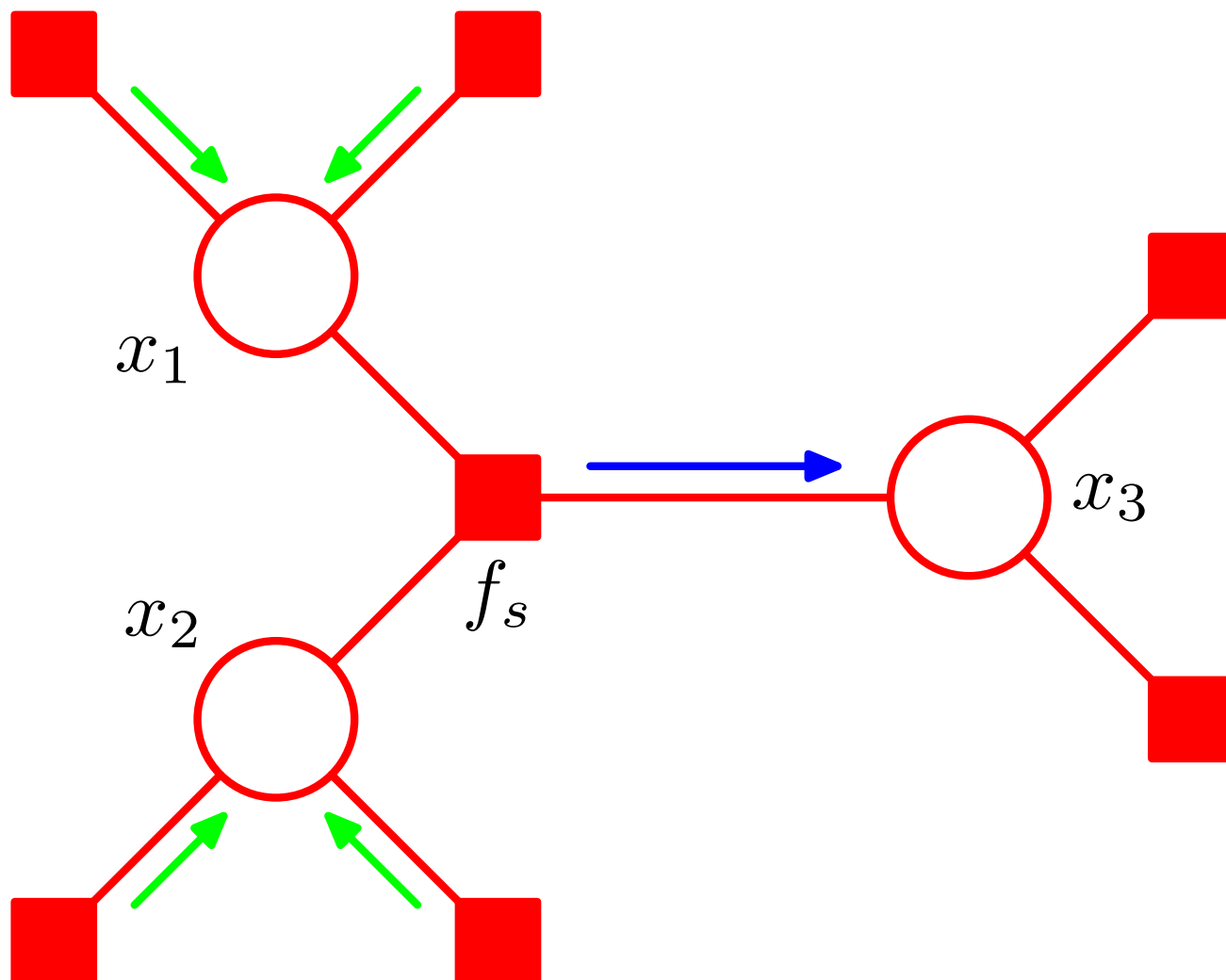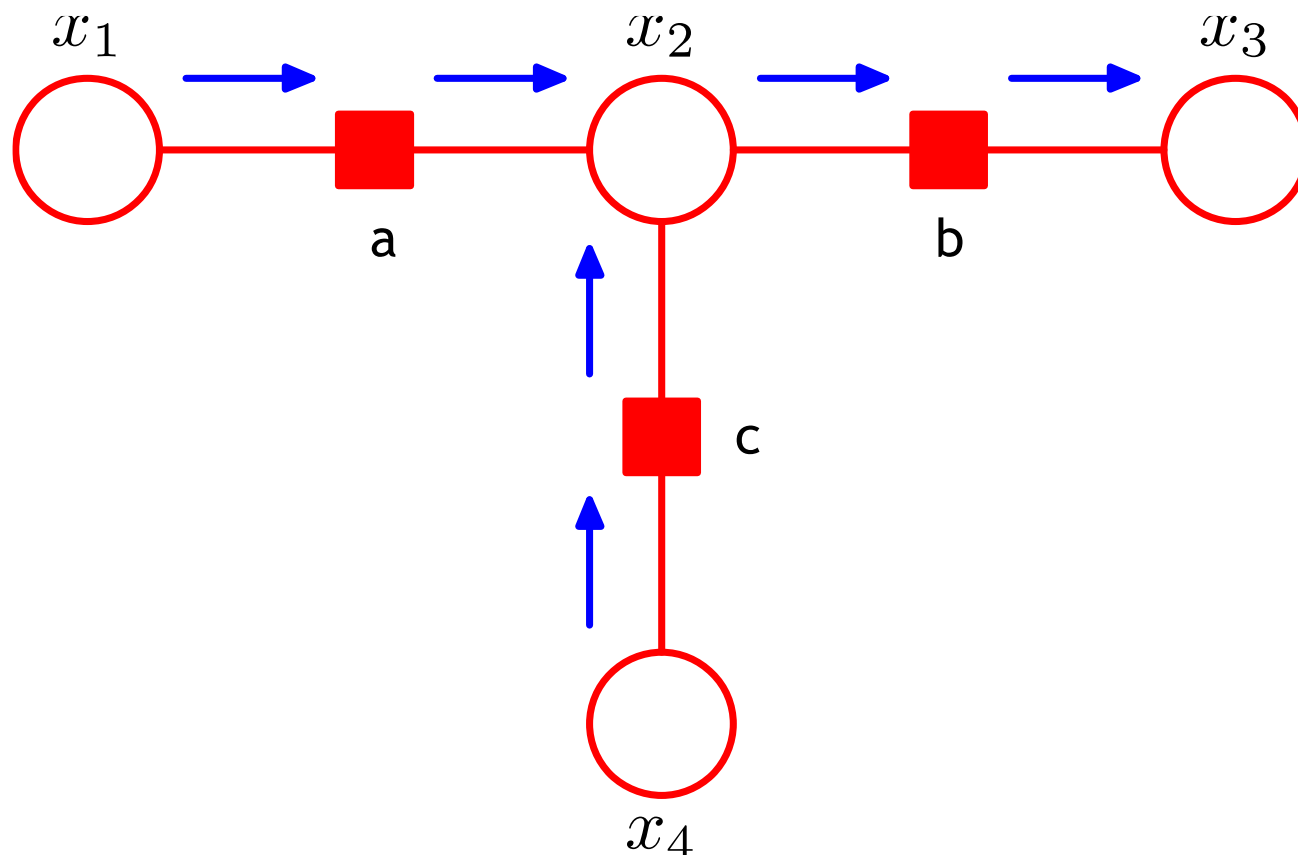
$$\mu_{x_1 \to f_a}(x_1) = 1$$

$$\mu_{f_a \to x_2}(x_2) = \sum_{x_1} f_a(x_1, x_2)$$

$$\mu_{x_4 \to f_c}(x_4) = 1$$

$$\mu_{f_c \to x_2}(x_2) = \sum_{x_4} f_c(x_2, x_4)$$

$$\mu_{x_2 \to f_b}(x_2) = \mu_{f_a \to x_2}(x_2) \mu_{f_c \to x_2}(x_2)$$

$$\mu_{f_b \to x_3}(x_3) = \sum_{x_2} f_b(x_2, x_3) \mu_{x_2 \to f_b}.$$
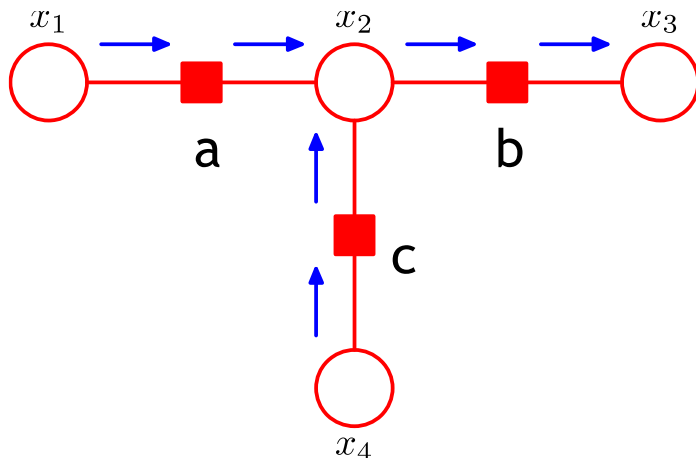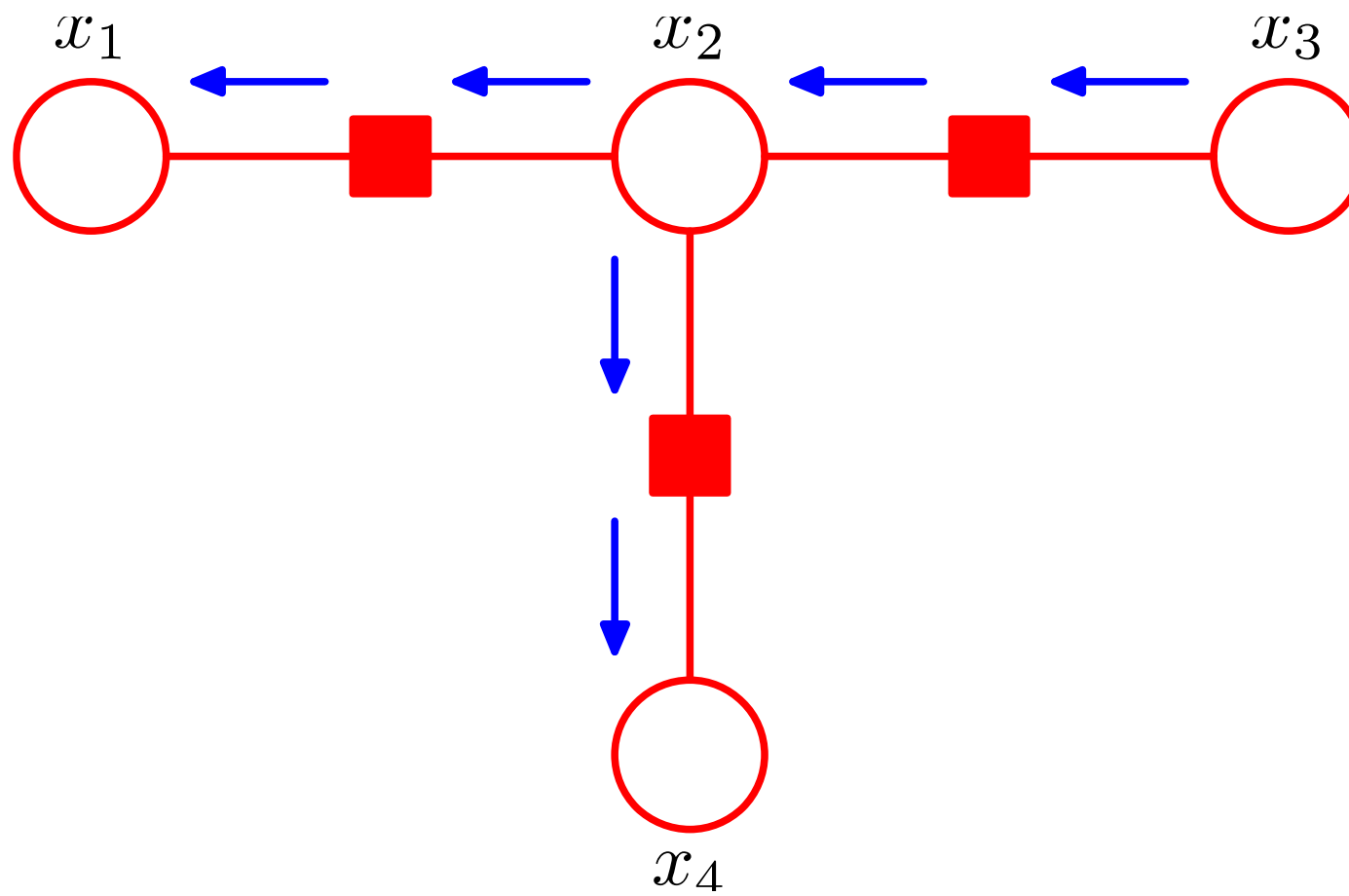
$$\mu_{x_3 \to f_b}(x_3) = 1$$

$$\mu_{f_b \to x_2}(x_2) = \sum_{x_3} f_b(x_2, x_3)$$

$$\mu_{x_2 \to f_a}(x_2) = \mu_{f_b \to x_2}(x_2)\mu_{f_c \to x_2}(x_2)$$

$$\mu_{f_a \to x_1}(x_1) = \sum_{x_2} f_a(x_1, x_2)\mu_{x_2 \to f_a}(x_2)$$

$$\mu_{x_2 \to f_c}(x_2) = \mu_{f_a \to x_2}(x_2)\mu_{f_b \to x_2}(x_2)$$

$$\mu_{f_c \to x_4}(x_4) = \sum_{x_2} f_c(x_2, x_4)\mu_{x_2 \to f_c}(x_2).$$
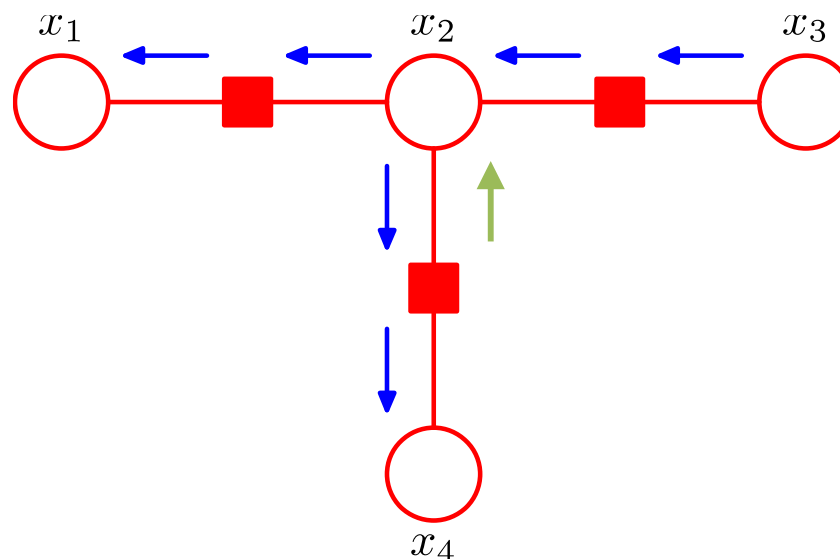
$$\widetilde{p}(x_2) = \mu_{f_a \to x_2}(x_2) \mu_{f_b \to x_2}(x_2) \mu_{f_c \to x_2}(x_2)$$

$$= \left[\sum_{x_1} f_a(x_1, x_2) \right] \left[\sum_{x_3} f_b(x_2, x_3) \right] \left[\sum_{x_4} f_c(x_2, x_4) \right]$$

$$= \sum_{x_1} \sum_{x_2} \sum_{x_4} f_a(x_1, x_2) f_b(x_2, x_3) f_c(x_2, x_4)$$

$$= \sum_{x_1} \sum_{x_3} \sum_{x_4} \widetilde{p}(\mathbf{x})$$

8.22 (⋆) Consider a tree-structured factor graph, in which a given subset of the variable nodes form a connected subgraph (i.e., any variable node of the subset is connected to at least one of the other variable nodes via a single factor node). Show how the sum-product algorithm can be used to compute the marginal distribution over that subset.

8.22 (⋆) Consider a tree-structured factor graph, in which a given subset of the variable nodes form a connected subgraph (i.e., any variable node of the subset is connected to at least one of the other variable nodes via a single factor node). Show how the sum-product algorithm can be used to compute the marginal distribution over that subset.

$$p(X_a) = \prod_{s_a} f_{s_a}(X_{s_a}) \prod_{s \in neX_a} \sum_{X_s} F_s(x_s, X_s)$$

$$= \prod_{s_a} f_{s_a}(X_{s_a}) \prod_{s \in neX_a} \mu_{f_s \to x_s}(x_s).$$

D:Ū95 H:33Ē V:163 FC:62443

Operation	Function	Parameter(s)
SearchObject	Find an object in both views	Object characteristics
Saccade	Conjugate eye movement	Yaw, pitch and roll
Vergence	Disconjugate eye movement	Angle
ImageMatching	Test two images' similarity	Labels of remembered images
Memorize	Remember a position or an image	Label to be assigned
MoveHand	Hand movement	Current fixation point
TurnHand	Turn hand (and object in hand)	Yaw, pitch and roll
Pickup/Dropoff	Object pickup/dropoff by hand	None

```
locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
```
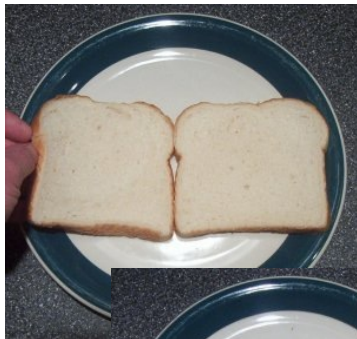
Subject A
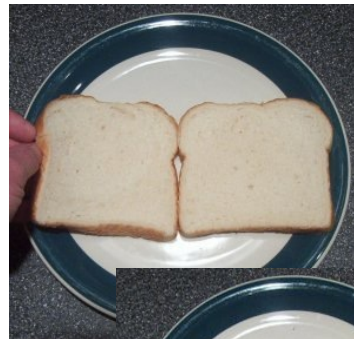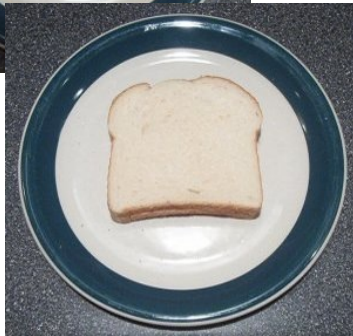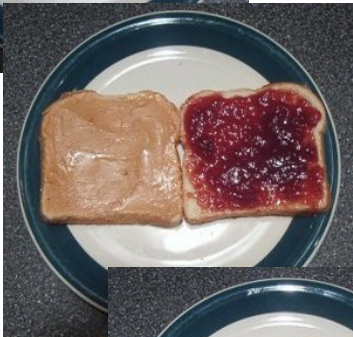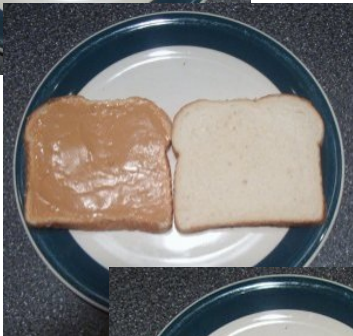
Subject B

time

locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
fixate loc_loaf
puthand right
pickup right
fixate 0.1 -0.2 0.78
remember loc_bread2
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
locate jar-lid
remember loc_jarlid
puthand right
pickup right
fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate jar
remember loc_jar
puthand right
fixate 0.1 -0.2 0.78
puthand right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate pbbottle
remember loc_bottle
puthand right
fixate -0.1 -0.2 0.78
puthand right
puthand right loc_bottle
fixate -0.1 -0.2 0.78
puthand right
fixate loc_bread1
puthand right
pickup right
turnhand right 0 0 -90
fixate loc_bread2
puthand right
dropoff right
fixate loc_knife
puthand right
dropoff right
locate jar
fixate loc_jarlid_on_table
puthand right
pickup right
fixate loc_jarlid
puthand right
dropoff right
fixate loc_bottlelid_on_table
puthand right
pickup right
fixate loc_bottlelid
puthand right
dropoff right

Left column:
```
locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
fixate loc_loaf
puthand right
pickup right
fixate 0.1 -0.2 0.78
remember loc_bread2
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
locate jar-lid
remember loc_jarlid
puthand right
pickup right
fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table
puthand right
dropoff right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate pbbottle
remember loc_bottle
puthand right
fixate -0.1 -0.2 0.78
puthand right
puthand right loc_bottle
fixate -0.1 -0.2 0.78
puthand right
locate jar
remember loc_jar
puthand right
fixate 0.1 -0.2 0.78
puthand right
fixate loc_knife
puthand right
dropoff right
locate jar
fixate loc_bread1
puthand right
pickup right
turnhand right 0 0 -90
fixate loc_bread2
puthand right
dropoff right
fixate loc_bottlelid_on_table
puthand right
pickup right
fixate loc_bottlelid
puthand right
dropoff right
fixate loc_jarlid_on_table
puthand right
pickup right
fixate loc_jarlid
puthand right
dropoff right
```
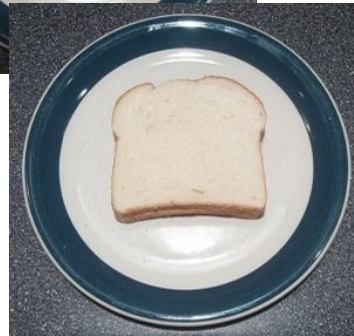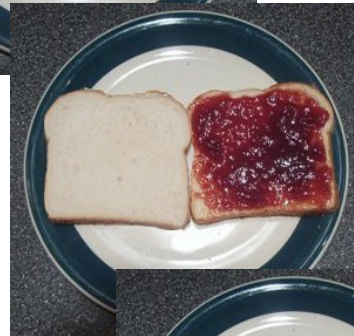
Right column:
```
locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
fixate loc_loaf
puthand right
pickup right
fixate 0.1 -0.2 0.78
remember loc_bread2
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
locate jar-lid
remember loc_jarlid
puthand right
pickup right
fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate jar
remember loc_jar
puthand right
fixate 0.1 -0.2 0.78
puthand right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate pbbottle
remember loc_bottle
puthand right
fixate -0.1 -0.2 0.78
puthand right
puthand right loc_bottle
fixate -0.1 -0.2 0.78
puthand right
fixate loc_bread1
puthand right
pickup right
turnhand right 0 0 -90
fixate loc_bread2
puthand right
dropoff right
fixate loc_knife
puthand right
dropoff right
locate jar
fixate loc_jarlid_on_table
puthand right
pickup right
fixate loc_jarlid
puthand right
dropoff right
fixate loc_bottlelid_on_table
puthand right
pickup right
fixate loc_bottlelid
puthand right
dropoff right
```

Magnified view:

Left:
```
puthand right
dropoff right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate pbbottle
```

Right:
```
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate jar
remember loc_jar
puthand right
fixate 0.1 -0.2 0.78
puthand right
locate pblid
remember loc_bottlelid
puthand right
pickup right
```

locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
fixate loc_loaf
puthand right
pickup right
fixate 0.1 -0.2 0.78
remember loc_bread2
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
locate jar-lid
remember loc_jarlid
puthand right
pickup right
fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table
puthand right
dropoff right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate pbbottle
remember loc_bottle
puthand right
fixate -0.1 -0.2 0.78
puthand right
puthand right loc_bottle
fixate -0.1 -0.2 0.78
puthand right
locate jar
remember loc_jar
puthand right
fixate 0.1 -0.2 0.78
puthand right
fixate loc_knife
puthand right
dropoff right
locate jar
fixate loc_bread1
puthand right
pickup right
turnhand right 0 0 -90
fixate loc_bread2
puthand right
dropoff right
fixate loc_bottlelid_on_table
puthand right
pickup right
fixate loc_bottlelid
puthand right
dropoff right
fixate loc_jarlid_on_table
puthand right
pickup right
fixate loc_jarlid
puthand right
dropoff right

locate bread
remember loc_loaf
puthand right
pickup right
fixate -0.1 -0.2 0.78
remember loc_bread1
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
fixate loc_loaf
puthand right
pickup right
fixate 0.1 -0.2 0.78
remember loc_bread2
puthand right
turnhand right 0 0 90
dropoff right
turnhand right 0 0 0
locate jar-lid
remember loc_jarlid
puthand right
pickup right
fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table
puthand right
dropoff right
locate khandle
remember loc_knife
puthand right
pickup right
locate jar
remember loc_jar
puthand right
fixate -0.1 -0.2 0.78
puthand right
puthand right loc_bottle
fixate -0.1 -0.2 0.78
puthand right
locate pblid
remember loc_bottlelid
puthand right
pickup right
fixate -0.3 -0.2 0.65
remember loc_bottlelid_on_table
puthand right
dropoff right
locate pbbottle
remember loc_bottle
puthand right
fixate -0.1 -0.2 0.78
puthand right
fixate loc_bread1
puthand right
pickup right
turnhand right 0 0 -90
fixate loc_bread2
puthand right
dropoff right
fixate loc_knife
puthand right
dropoff right
locate jar
fixate loc_jarlid_on_table
puthand right
pickup right
fixate loc_jarlid
puthand right
dropoff right
fixate loc_bottlelid_on_table
puthand right
pickup right
fixate loc_bottlelid
puthand right
dropoff right

Magnified detail:

puthand right   puthand right
dropoff right   dropoff right
locate pblid   locate khandle
remember loc_bottlelid   remember loc_knife
puthand right   puthand right
pickup right   pickup right
fixate -0.3 -0.2 0.65   locate jar
remember loc_bottlelid_on_table   remember loc_jar
puthand right   puthand right
dropoff right   fixate 0.1 -0.2 0.78
locate khandle   puthand right
remember loc_knife   locate pblid
puthand right   remember loc_bottlelid
pickup right   puthand right
locate pbbottle   pickup right

(left)	(right)
locate bread	locate bread
remember loc_loaf	remember loc_loaf
puthand right	puthand right
pickup right	pickup right
fixate -0.1 -0.2 0.78	fixate -0.1 -0.2 0.78
remember loc_bread1	remember loc_bread1
puthand right	puthand right
turnhand right 0 0 90	turnhand right 0 0 90
dropoff right	dropoff right
turnhand right 0 0 0	turnhand right 0 0 0
fixate loc_loaf	fixate loc_loaf
puthand right	puthand right
pickup right	pickup right
fixate 0.1 -0.2 0.78	fixate 0.1 -0.2 0.78
remember loc_bread2	remember loc_bread2
puthand right	puthand right
turnhand right 0 0 90	turnhand right 0 0 90
dropoff right	dropoff right
turnhand right 0 0 0	turnhand right 0 0 0
locate jar-lid	locate jar-lid
remember loc_jarlid	remember loc_jarlid
puthand right	puthand right
pickup right	pickup right
fixate -0.40 -0.2 0.65	fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table	remember loc_jarlid_on_table
puthand right	puthand right
dropoff right	dropoff right
locate pblid	locate khandle
remember loc_bottlelid	remember loc_knife
puthand right	puthand right
pickup right	pickup right
fixate -0.3 -0.2 0.65	locate jar
remember loc_bottlelid_on_table	remember loc_jar
puthand right	puthand right
dropoff right	fixate 0.1 -0.2 0.78
locate khandle	puthand right
remember loc_knife	locate pblid
puthand right	remember loc_bottlelid
pickup right	puthand right
locate pbbottle	pickup right
remember loc_bottle	fixate -0.3 -0.2 0.65
puthand right	remember loc_bottlelid_on_table
fixate -0.1 -0.2 0.78	puthand right
puthand right	dropoff right
puthand right loc_bottle	locate pbbottle
fixate -0.1 -0.2 0.78	remember loc_bottle
puthand right	puthand right
locate jar	fixate -0.1 -0.2 0.78
remember loc_jar	puthand right
puthand right	puthand right loc_bottle
fixate 0.1 -0.2 0.78	fixate -0.1 -0.2 0.78
puthand right	puthand right
fixate loc_knife	fixate loc_bread1
puthand right	puthand right
dropoff right	pickup right
locate jar	turnhand right 0 0 -90
fixate loc_bread1	fixate loc_bread2
puthand right	puthand right
pickup right	dropoff right
turnhand right 0 0 -90	fixate loc_knife
fixate loc_bread2	puthand right
puthand right	dropoff right
dropoff right	locate jar
fixate loc_bottlelid_on_table	fixate loc_jarlid_on_table
puthand right	puthand right
pickup right	pickup right
fixate loc_bottlelid	fixate loc_jarlid
puthand right	puthand right
dropoff right	dropoff right
fixate loc_jarlid_on_table	fixate loc_bottlelid_on_table
puthand right	puthand right
pickup right	pickup right
fixate loc_jarlid	fixate loc_bottlelid
puthand right	puthand right
dropoff right	dropoff right

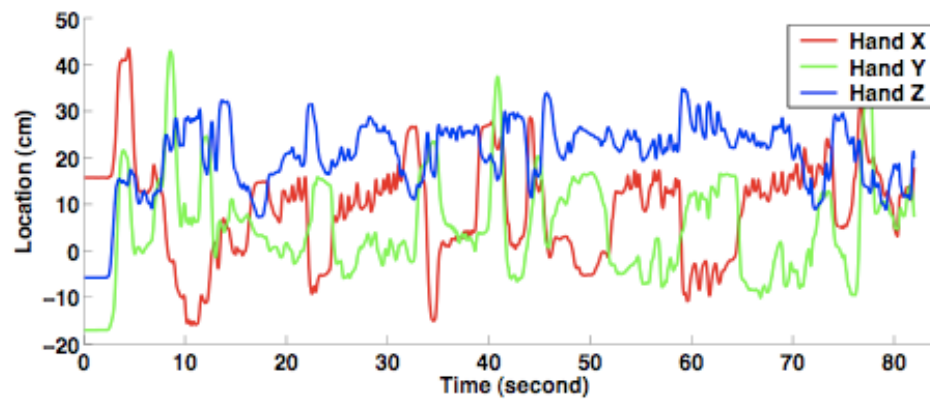| (left)	(right)
locate bread	locate bread
remember loc_loaf	remember loc_loaf
puthand right	puthand right
pickup right	pickup right
fixate -0.1 -0.2 0.78	fixate -0.1 -0.2 0.78
remember loc_bread1	remember loc_bread1
puthand right	puthand right
turnhand right 0 0 90	turnhand right 0 0 90
dropoff right	dropoff right
turnhand right 0 0 0	turnhand right 0 0 0
fixate loc_loaf	fixate loc_loaf
puthand right	puthand right
pickup right	pickup right
fixate 0.1 -0.2 0.78	fixate 0.1 -0.2 0.78
remember loc_bread2	remember loc_bread2
puthand right	puthand right
turnhand right 0 0 90	turnhand right 0 0 90
dropoff right	dropoff right
turnhand right 0 0 0	turnhand right 0 0 0
locate jar-lid	locate jar-lid
remember loc_jarlid	remember loc_jarlid
puthand right	puthand right
pickup right	pickup right
fixate -0.40 -0.2 0.65	fixate -0.40 -0.2 0.65
remember loc_jarlid_on_table	remember loc_jarlid_on_table
puthand right	puthand right
dropoff right	dropoff right
locate pblid	locate khandle
remember loc_bottlelid	remember loc_knife
puthand right	puthand right
pickup right	pickup right
fixate -0.3 -0.2 0.65	locate jar
remember loc_bottlelid_on_table	remember loc_jar
puthand right	puthand right
dropoff right	fixate 0.1 -0.2 0.78
locate khandle	puthand right
remember loc_knife	locate pblid
puthand right	remember loc_bottlelid
pickup right	puthand right
locate pbbottle	pickup right
remember loc_bottle	fixate -0.3 -0.2 0.65
puthand right	remember loc_bottlelid_on_table
fixate -0.1 -0.2 0.78	puthand right
puthand right	dropoff right
puthand right loc_bottle	locate pbbottle
fixate -0.1 -0.2 0.78	remember loc_bottle
puthand right	puthand right
locate jar	fixate -0.1 -0.2 0.78
remember loc_jar	puthand right
puthand right	puthand right loc_bottle
fixate 0.1 -0.2 0.78	fixate -0.1 -0.2 0.78
puthand right	puthand right
fixate loc_knife	fixate loc_bread1
puthand right	puthand right
dropoff right	pickup right
locate jar	turnhand right 0 0 -90
fixate loc_bread1	fixate loc_bread2
puthand right	puthand right
pickup right	dropoff right
turnhand right 0 0 -90	fixate loc_knife
fixate loc_bread2	puthand right
puthand right	dropoff right
dropoff right	locate jar
fixate loc_bottlelid_on_table	fixate loc_jarlid_on_table
puthand right	puthand right
pickup right	pickup right
fixate loc_bottlelid	fixate loc_jarlid
puthand right	puthand right
dropoff right	dropoff right
fixate loc_jarlid_on_table	fixate loc_bottlelid_on_table
puthand right	puthand right
pickup right	pickup right
fixate loc_jarlid	fixate loc_bottlelid
puthand right	puthand right
dropoff right	dropoff right

BreadOnTable	putting bread on table
PeanutButterLidOff	taking peanut butter lid off
JellyLidOff	taking jelly lid off
KnifeInHand	grabbing knife in hand
PeanutbutterOnBread	spreading peanut butter on bread
JellyOnBread	spreading jelly on bread
PeanutButterLidOn	putting peanut butter lid back on
JellyLidOn	putting jelly lid back on
KnifeOnTable	putting knife on table
FlipBread	flipping bread to make an sandwich

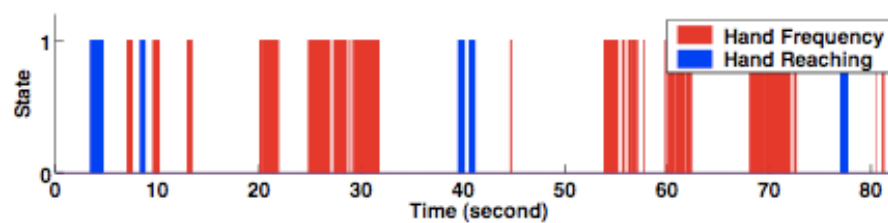|	1	2	3	4	5	6	7	8	9	10
BT	abc									
PLF		a	c		b					
JLF		bc				a				
KH			ab	c						
POB				a	c	b				
JOB				b		c	a			
PLO					a				b	c
JLO									c	ab
KT							c	ab		
FB							b	c	a	

Eye
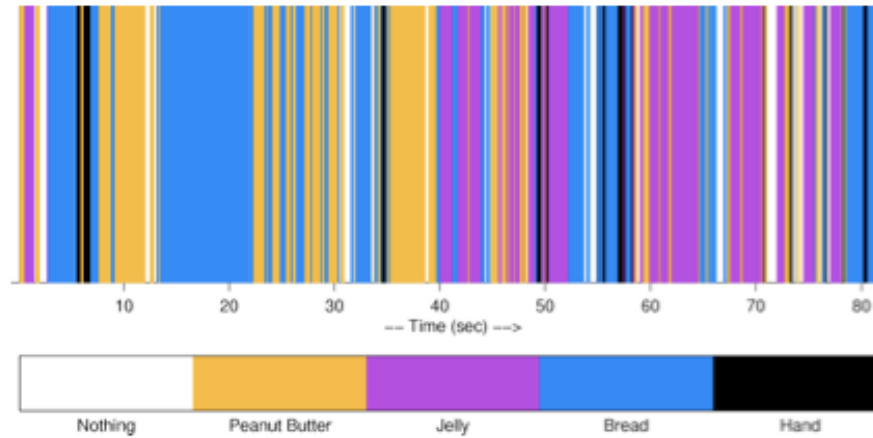
(a) Raw hand location data

Hand

(b) Observed hand movement

Gaze Object   ○ → ●   Recognized Object

Hand Object   ○   ●   Hand Reaching

Subtask   ○ → ●   Hand Frequency
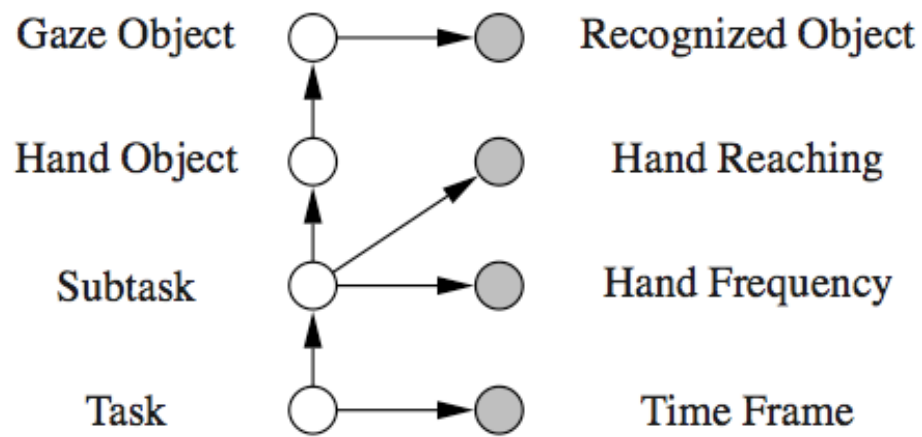
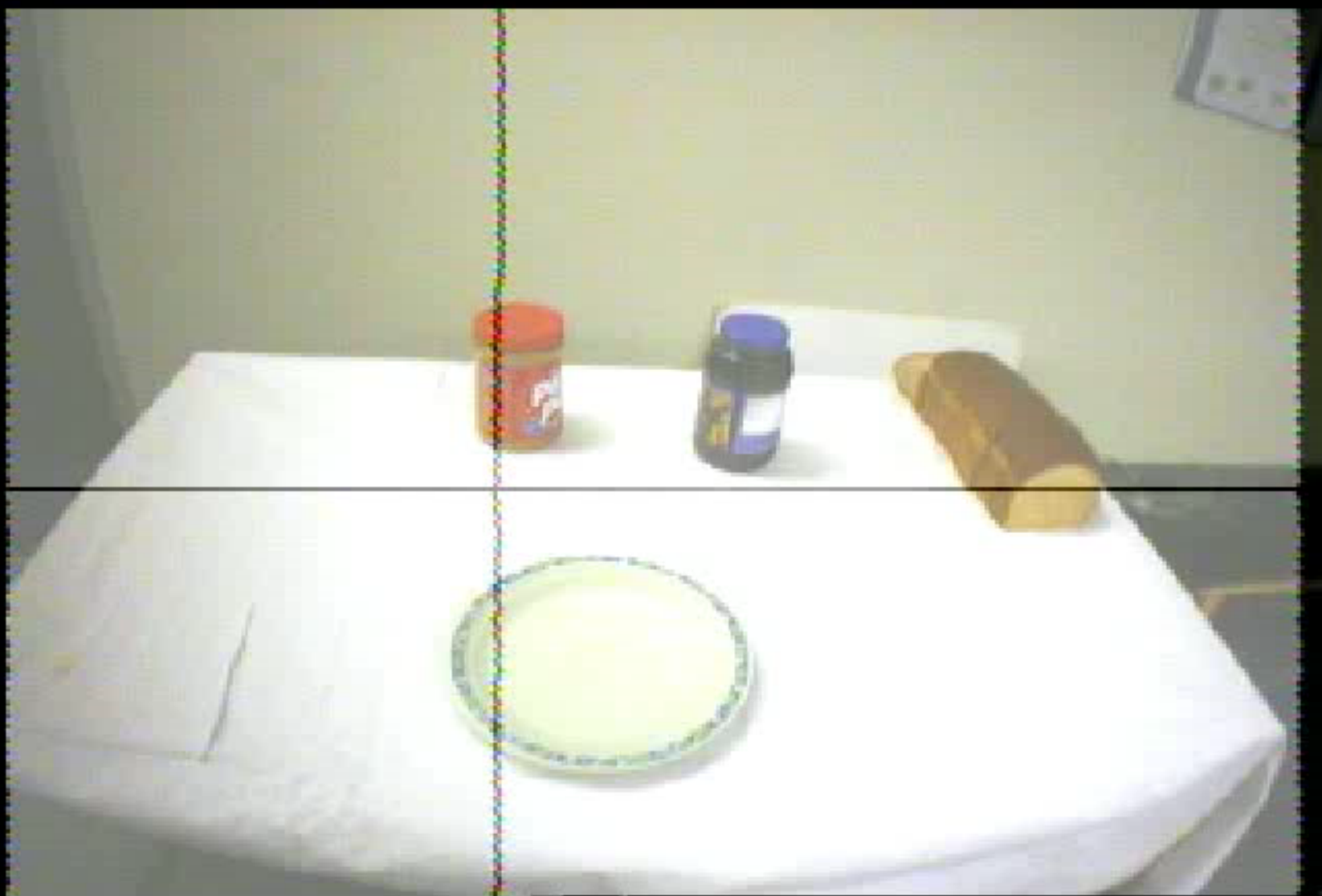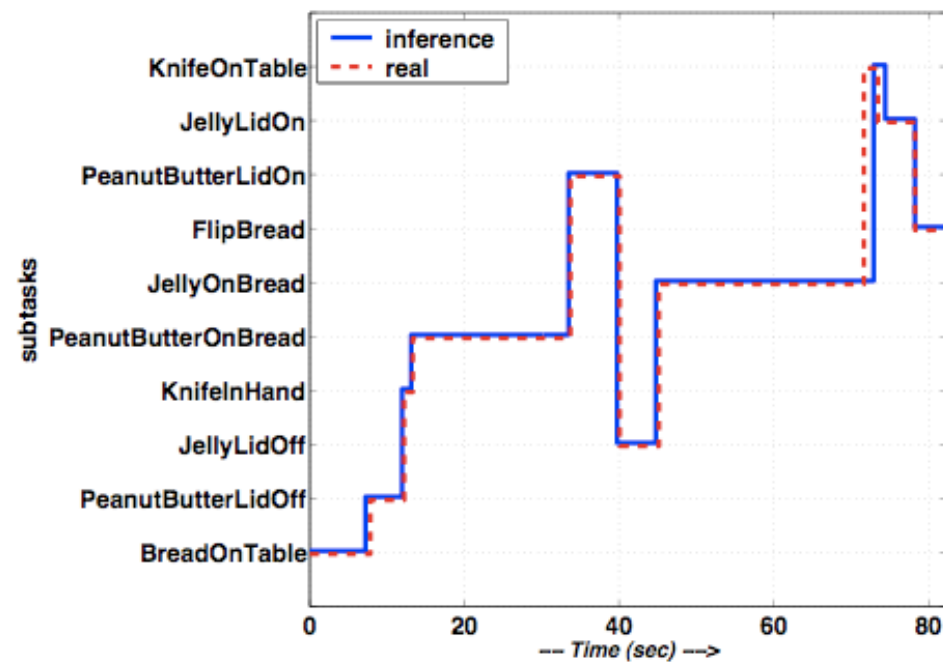Task   ○ → ●   Time Frame

Task:

$$(\emptyset, \{t_1\}) \xrightarrow{p} (\{t_1\}, \{t_1, t_2\}) \xrightarrow{q} (\{t_1, t_2\}, \{t_1, t_2, t_4\})$$

Time Slice i Time Slice $i+1$
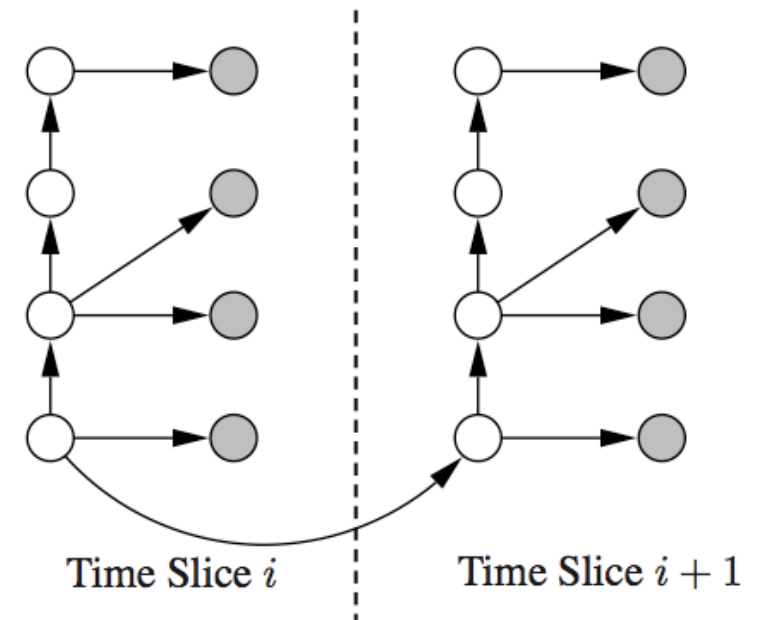
Node Name	# of States	Node Name	# of States
task	80	time frame	20
subtask	10	hand frequency	2
hand object	4	hand reaching	2
gaze object	5	recognized object	5

(a) Offline behavior recognition