Module : 2

Basic Statistics.

1) Binomial Distribution

defn :

$P(X=r) = {}^nC_r \, p^r q^{n-r}$,

$r = 0, 1, 2, \ldots n$, where $p+q=1$.

Mean & Variance of the Binomial Distribution

$E(X) = np$ & $E(X^2) = n(n-1)p^2 + np$

$Var(X) = E(X^2) - [E(X)]^2$
$= n(n-1)p^2 + np - n^2p^2$
$= np(1-p)$
$= npq$

Recurrence formula for the central moments of the Binomial Distribution

by definition, the k^{th} order central moment μ_k is given by

$\mu_k = E\{X - E(X)\}^k$

$$\mu_k = \sum_{r=0}^{n} (r-np)^k \cdot {}^nC_r \, p^r q^{n-r}$$

$$\mu_{k+1} = pq\left[\frac{d\mu_k}{dp} + nk\mu_{k-1}\right]$$

$\mu_0 = 1$ & $\mu_1 = 0$

$$\mu_2 = pq\left[\frac{d\mu_1}{dp} + n\mu_0\right]$$
$= npq$

$$\mu_3 = pq\left[\frac{d}{dp}\mu_2 + 2n\mu_1\right]$$

$$= npq(q-p)$$

$$\mu_4 = pq\left[\frac{d}{dp}\mu_3 + 3n\mu_2\right]$$

$$\therefore \mu_2 = npq\left[1+3pq(n-2)\right]$$

Note: μ_2 is the variance, μ_3 is a measure of skewness & μ_4 is a measure of kurtosis. sometimes the coefficients β_1 & β_2 are used to measure skewness & kurtosis respectively.

where $\mu_1 = \frac{\mu_3^2}{\mu_2^3}$ & $\mu_2 = \frac{\mu_4}{\mu_2^2}$

2) Poisson Distribution:

Def^n: If X is a discrete RV. that can assume the values 0, 1, 2, ..., such that it's probability mass function is given by.

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}, \quad r = 0, 1, 2, \ldots, \lambda > 0.$$

then X is said to follow a poisson distribution with parameter λ or symbolically X is said to follow $P(\lambda)$

Note: Poisson Distribution is a legitimate distribution :

$$\because \sum_{r=0}^{\infty} P(X=r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!}$$

$$= e^{-\lambda} \cdot e^{\lambda} = 1$$

Poisson Distribution as Limiting form of Binomial Distribution :-

Poisson distribution is a limiting case of binomial distribution under the following conditions.

(i) n, the number of trials is indefinitely large i.e., $n \to \infty$

(ii) p, the constant probability of success in each trial is very smal. i.e. $p \to 0$.

(iii) $np (= \lambda)$ is finite or $p = \frac{\lambda}{n}$ & $q = 1 - \frac{\lambda}{n}$, where λ is positive real number.

Mean & Variance of Poisson Distribution

$$E(X) = \lim_{\substack{n \to \infty \\ np = \lambda}} (np) = \lambda$$

$$\& \ \text{var}(X) = \lim_{\substack{p \to 0 \\ np = \lambda}} (npq) = \lim_{p \to 0} [\lambda(1-p)]$$

$$= \lambda.$$

$$E(X) = \lambda$$

$$E(X^2) = \lambda^2 + \lambda$$

$$var(X) = E(X^2) - [E(X)]^2$$
$$= \lambda^2 + \lambda - \lambda^2 = \lambda$$

Recurrence Formula for the central moments of the poisson Distribution:-

by def^n, the k^{th} order central moment μ_k is given by $\mu_k = E[X - E(X)]^k$

$$\mu_k = \sum_{r=0}^{\infty} (r-\lambda)^k e^{-\lambda} \frac{\lambda^r}{r!}$$

$$\mu_{k+1} = \lambda\left(\frac{d\mu_k}{d\lambda} + k\mu_{k-1}\right)$$

$$\mu_0 = 1 \ \& \ \mu_1 = 0$$

$$\mu_2 = 1$$

$$\mu_3 = \lambda, \ \mu_4 = \lambda(3\lambda + 1)$$

3) Geometric Distribution:

def^n: Let the R.V. X denote the number of trials of a random experiment required to obtain the first sucess (occurrence of an event A). obviously, X can assume the values 1, 2, 3, ...

Now $X = r$, if and only if the first $(r-1)$ trials result in failure (occurence of $\bar{A}$) & the r^{th} trial resuts in success (occurence A).

$\therefore P(X=r) = q^{r-1} \cdot p \quad , r = 1, 2, 3, \ldots \infty$

where $P(A) = p$ & $P(\bar{A}) = q$.

If X is a discrete R.V. that can assume the values $1, 2, 3, \ldots \infty$ such that it's probability mass function is given by.

$$P(X=r) = q^{r-1} \cdot p \quad , r = 1, 2, \ldots \infty$$

where $p + q = 1$.

then X is said to follow a geometric distribution.

Note: Geometric distribution is a legitimate probability distribution.

$$\sum_{r=1}^{\infty} P(X=r) = \sum_{r=1} q^{r-1} p = \frac{p}{1-q} = 1$$

Mean & variance of the Geometric distribution

$$E(X) = \frac{1}{p} \quad , E(X^2) = \frac{1}{p^2}(1+q).$$

$$\text{Var}(X) = E(X^2) - \{E(X)\}^2$$
$$= \frac{1}{p^2}(1+q) - \frac{1}{p^2} = \frac{q}{p^2}$$

4) Normal (or Gaussian) Distribution

defⁿ: A continuous R.V. X is said to follow a Normal distribution or Gaussian distribution with parameters μ & σ, if its probability density function is given by.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\cdot\sigma^2}$$

$$; -\infty < x < \infty$$

$$-\infty < \mu < \infty, \sigma > 0$$

symbolically, X follows $N(\mu, \sigma)$.
mean S.D
Sometimes it is also given as $N(\mu, \sigma^2)$,

$f(x)$ is a legitimate density function.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

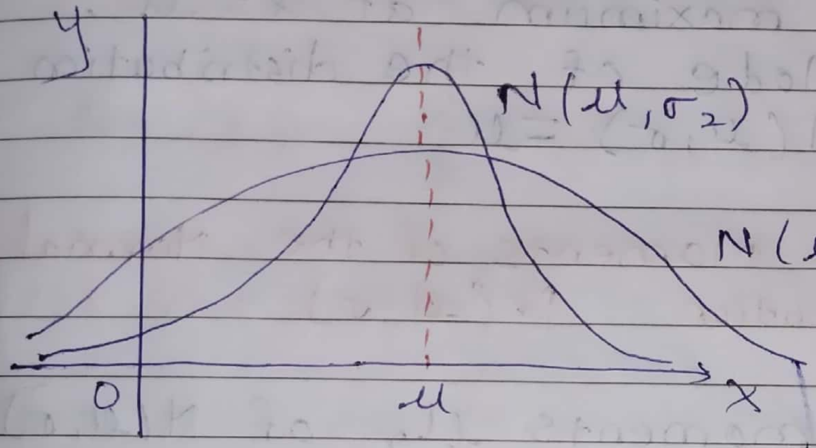
(a) Standard Normal Distribution.

The Normal distribution $N(0,1)$ is called the standardised Normal distribution whose density function is.

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} \quad , -\infty < z < \infty$$

This is obtained by putting $\mu = 0$ & $\sigma = 1$ & by changing x & f respectively into z & ϕ. if X has distribution $N(\mu, \sigma)$ & if $z = \frac{X - \mu}{\sigma}$, then we can prove that z has distribution $N(0,1)$

(b) Normal distribution curve :



(i) Properties of the Normal Distribution $N(\mu, \sigma)$

If X follows $N(\mu, \sigma)$ then $E(X) = \mu$ & $var(X) = \sigma^2$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \mu$$

$$E(X^2) = \mu^2 + \sigma^2$$

$$\therefore Var(X) = E(X^2) - \{E(X)\}^2 = \sigma^2$$

(ii) Median & mode of the Normal Distribution $N(\mu, \sigma)$

def^n: If X is a continuous R.V. with density function $f(x)$, then M is called the median value of X

$$\int_{M}^{\mu} f(x)\,dx = 0$$

$$\therefore M = \mu.$$

def^n: Mode of a continuous R.V. X is defined as the value of x for which the density function f(x) is maximum at $x = \mu$,

i.e. Mode of the distribution $N(\mu, \sigma) = \mu$

(iii) Central Moments of the Normal Distribution $N(\mu, \sigma)$

central moments μ_r of $N(\mu, \sigma)$ are given by $\mu_r = E(X-\mu)^r$.

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^r e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma t)^r \cdot e^{-r^2} dt$$

case (A) : r is an odd integer, that is, $r = 2n+1$

$\mu_{2n+1} = 0$ ($\because$ the integrand is an odd function of t)

case (B) : r is an even integer, i.e. $r=2n$

$$\mu_{2n} = (2n-1)\sigma^2 \cdot \mu_{2n-2}$$

Scanned with CamScanner

(iv). Mean Deviation about the mean of the Normal Distribution $N(\mu, \sigma)$.

Mean Deviation (M.D.) $= \frac{4}{5}\sigma$

(v) Quartile Deviation of the Normal Distribution $N(\mu, \sigma)$.

$Q.D = \frac{2}{3}\sigma$ (approximately)

(vi) Moment Generating Functing of $N(0,1)$ & $N(\mu, \sigma)$

The moment generating function of $N(0,1)$ is given by.

$$M_Z(t) = e^{t^2/2}$$

The moment generating function of $N(\mu, \sigma)$ is given by.

$$M_X(t) = e^{t\left(\mu + \frac{\sigma^2 t}{2}\right)}$$

$$M_X(t) = 1 + \frac{t}{1!}\left(\mu + \frac{\sigma^2 t}{2}\right) + \frac{t^2}{2!}\left(\mu + \frac{\sigma^2 t}{2}\right)^2 + \frac{t^3}{3!}\left(\mu + \frac{\sigma^2 t}{2}\right)^3 + \frac{t^4}{4!}\left(\mu + \frac{\sigma^2 t}{2}\right)^4 + \cdots + \infty$$

$E(X) =$ coefficient of $\frac{t}{1!} = \mu$.

$E(X^2) =$ coefficient of $\frac{t^2}{2!} = \sigma^2 + \mu^2$

$E(x^3) = \text{coefficient of } \frac{t^3}{3!} = 3\mu\sigma^2 + \mu^3$ &

$E(x^4) = \text{coefficient of } \frac{t^4}{4!}$

$= 3\sigma^4 + 6\mu^2\sigma^2 + \mu^4$

Examples:

1) The heights of men in a city are normally distributed with a mean of 171 cm & S.D. of 7cm, while the corresponding values for women in the same city are this city. find the probability that the woman is taller than the man.

→ Let $\overline{x_1}$ & $\overline{x_2}$ denote the mean heights of men & women respectively.

$\overline{x_1}$ follows a $N(171, 7)$ &

$\overline{x_2}$ follows a $N(165, 6)$

σ_1, σ_2.

$\therefore \overline{x_1} - \overline{x_2}$ also a follows a normal distribution.

$E(\overline{x_1} - \overline{x_2}) = E(\overline{x_1}) - E(\overline{x_2})$
$= 171 - 165 = 6.$

$V(\overline{x_1} - \overline{x_2}) = V(\overline{x_1}) + V(\overline{x_2}) = 49 + 36 = 85$
$= \sigma_1^2 + \sigma_2^2$

S.D. of $(\bar{x}_1 - \bar{x}_2) = \sqrt{85} = 9.22$

$\therefore \bar{x}_1 - \bar{x}_2$ follows a $N(6, 9.22)$

~~$\therefore \bar{x}_1 - \bar{x}_2$ follows a $N(6, 9.22)$~~

$P(\bar{x}_2 > \bar{x}_1) = P(\bar{x}_1 - \bar{x}_2 < 0)$

$= P\left\{\frac{(\bar{x}_1 - \bar{x}_2) - 6}{9.22} < \frac{-6}{9.22}\right\}$

$= P\{z < -0.65\}$, where z is the standard normal variate

$= P\{z > 0.65\}$, by symmetry

$= 0.5 - P(0 < z < 0.65)$

$= 0.5 - 0.2422 = 0.2578$

2) Two populations have the S.D. of one is twice that of the other. show that in samples, each of size 500, drawn under simple random conditions. the difference of the means will, in all probability, not exceed 0.3σ, where σ is the smaller S.D.

→ Let $\bar{x}_1$ & $\bar{x}_2$ be the means of the samples of size 500 each. Let their S.D's. be σ & 2σ respectively.

$\overline{x_1}$ follows a $N\left(\mu, \frac{\sigma}{\sqrt{500}}\right)$ f.

$\overline{x_2}$ follows a $N\left(\mu, \frac{2\sigma}{\sqrt{500}}\right)$.

$\therefore$ $\overline{x_1} - \overline{x_2}$ also follows a normal distribution.

$$E(\overline{x_1} - \overline{x_2}) = E(\overline{x_1}) - E(\overline{x_2})$$

$$= \mu - \mu = 0.$$

$$V(\overline{x_1} - \overline{x_2}) = V(\overline{x_1}) + V(\overline{x_2})$$

$$= \frac{\sigma^2}{500} + \frac{4\sigma^2}{500} = \frac{\sigma^2}{100}$$

$\therefore$ S.D. of $(\overline{x_1} - \overline{x_2}) = \frac{\sigma}{10}$

$(\overline{x_1} - \overline{x_2})$ follows a $N\left(0, \frac{\sigma}{10}\right)$

$\therefore$ $P\{|\overline{x_1} - \overline{x_2}| \leq 0.3\sigma\} =$

$$= P\left\{\frac{|(\overline{x_1} - \overline{x_2}) - 0|}{\sigma/10} \leq \frac{0.3\sigma}{\sigma/10}\right\}$$

$= P\{|z| \leq 3\}$, where z is the standard normal variate.

$$= 0.9974 \simeq 1$$

$\therefore$ $|\overline{x_1} - \overline{x_2}|$ will not exceed 0.3σ almost certainly

Examples on Binomial Distribution.

1) Find the mean of the probability distribution of the number of heads obtained in three flips of a balanced coin.

→ $p = \frac{1}{2}$, $n = 3$.

$\therefore$ mean $= E(X) = np = 3 \times \frac{1}{2} = 1.5$.

2) If X is Binomially distributed with $E(X) = 2$ & $var(X) = \frac{4}{3}$, find the probability distribution of X.

→ $E(X) = np = 2$ & $var(X) = npq = \frac{4}{3}$.

$\therefore \frac{npq}{np} = \frac{4/3}{2}$ $\quad \therefore q = \frac{2}{3}$

$\therefore p = 1 - q = \frac{1}{3}$

but $np = 2$, $n \cdot \frac{1}{3} = 2$. $\therefore n = 6$.

$\therefore$ The distribution is.

$P(X=x) = {}^nC_x p^x q^{n-x}$

$= {}^6C_x \left(\frac{1}{3}\right)^x \cdot \left(\frac{2}{3}\right)^{6-x}$

putting $x = 0, 1, 2, \ldots 6$

$\therefore$ Probability distribution of X.

X	0	1	2	3	4	5	6
$P(X=x)$	$\frac{64}{729}$	$\frac{192}{729}$	$\frac{240}{729}$	$\frac{160}{729}$	$\frac{60}{729}$	$\frac{12}{729}$	$\frac{1}{129}$

3) Prove that for all Binomial distributions with the same parameter n. the variance is maximum when $p=\frac{1}{2}$.

→ For Binomial distribution

$$P(x) = {}^{n}C_{x}\, p^{x} . q^{n-x}$$

And the variance is given by.

$$V = npq \quad , \text{ where } q = 1-p$$

$$\therefore V = np(1-p) = np - np^2$$

For maxima, $\frac{dV}{dp} = 0$ & $\frac{d^2V}{dp^2} = -ve.$

$\frac{dV}{dp} = n - 2np$ & $\frac{d^2V}{dp^2} = -2n$, always -ve.

$\therefore \frac{dV}{dp} = 0 \qquad \therefore n - 2np = 0$

$$\therefore p = \frac{1}{2},$$

$\therefore$ the variance is maximum when $p=\frac{1}{2}$.

4) The Ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is $\frac{1}{4}$. what is the probability of 4 successes in 6 independent trials.

→ For a Binomial distribution.

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

when $n=5, x=3$, $p(X=\cancel{x}) = {}^5c_{\cancel{x}} p^{\cancel{x}} q^{5-\cancel{x}}$

~~when~~. $P(x=3) = {}^5C_3 P^3 q^2$

when $n=5, x=2$, $P(x=2) = {}^5C_2 p^2 q^3$

The Ratio of these Probability is $\frac{1}{4}$.

$$\frac{p(x=3)}{p(X=2)} = \frac{{}^5c_3 p^3 q^2}{{}^5c_2 p^2 q^3} = \frac{1}{4}.$$

$\because {}^5c_3 = {}^5c_2$

$\frac{p}{q} = \frac{1}{4}$ $\therefore \frac{p}{1-p} = \frac{1}{4}$

$\therefore 4p = 1-p$

$\therefore 5p = 1$ $\therefore p = \frac{1}{5}$ $\therefore q = 1-p = \frac{4}{5}$

$\therefore P(X=x) = {}^nC_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{n-x}$.

when $n=6$, & $x=4$,

$p(x=4) = {}^6C_4 \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^2$.

5) The incidence of an occupational diseases in an industry is such that the workers have 20% chance of suffering from it. What is the Probability that out of 6 workers chosen

at random 4 or more will be suffering from the disease

$\rightarrow$

$p = 20\% = \frac{20}{100} = 0.2,$

$q = 1 - p = 0.8, \; n = 6$

$\therefore P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{6}C_{x} \left(\frac{1}{5}\right)^{x} \left(\frac{4}{5}\right)^{6-x}.$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= {}^{6}C_{4}\left(\frac{1}{5}\right)^{4}\left(\frac{4}{5}\right)^{2} + {}^{6}C_{5}\left(\frac{1}{5}\right)^{5}\left(\frac{4}{5}\right)^{1}$

$+ {}^{6}C_{6}\left(\frac{1}{5}\right)^{6}\left(\frac{4}{5}\right)^{0}.$

$\therefore P(X \geq 4) = \frac{1}{5^6}\left[15.4^{2} + 6.4 + 1\right]$

$= \frac{205}{5^6} = \frac{41}{3125}$

6) Let X, Y be two independent binomial variates with parameters $(n_1 = 6, p = \frac{1}{2})$ & $(n_2 = 4, p = \frac{1}{2})$ respectively. Evaluate $P(X+Y) = 3$ & Find $P(X+Y) \geq 3$

→ By the additive property of Binomial variates $z = x + y$ is a Binomial variate with parameters,

$n = n_1 + n_2 = 6 + 4 = 10$ & $p = \frac{1}{2}$.

$\therefore P(z) = {}^{n}C_{z}\, p^{z} \cdot q^{n-z}$.

$P(z=3) = {}^{10}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{7}$

$P(z=3) = \frac{15}{128} = 0.1172$

Now, To find $P(X+Y) \geq 3$

$P(z \geq 3) = 1 - [P(z=0) + P(z=1) + P(z=2)]$

$= 1 - \left[{}^{10}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{10} + {}^{10}C_{1} \left(\frac{1}{2}\right)^{1} \left(\frac{1}{2}\right)^{9} + {}^{10}C_{2} \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{8}\right]$

$= 1 - \left[\left({}^{10}C_{0} + {}^{10}C_{1} + {}^{10}C_{2}\right)\left(\frac{1}{2}\right)^{10}\right]$

$\therefore P(z \geq 3) = 0.945$

Examples on Poissons Distributions

1) If the variance of a Poisson Distribution is 2, find the probability of $r = 1, 2, 3, 4$ from the recurrence relation of poisson distribution

$\rightarrow$

$$P(X) = e^{-m} \frac{m^x}{x!}$$

$\because$ variance $= m = 2$

$$P(X) = e^{-2} \frac{2^x}{x!} \quad \text{when } x = 0$$

$$P(0) = e^{-2}$$

the recurrence relation is.

$$P(X=1) = \frac{m}{x+1} P(X)$$

Putting $x = 0$, $P(1) = \frac{2}{1} \cdot P(0) = 2e^{-2}$

putting $x = 1$, $P(2) = \frac{2}{2} \cdot P(1) = \frac{2}{2} \cdot 2e^{-2} = e^{-2}$

putting $x = 2$, $P(3) = \frac{2}{3} P(3) = \frac{2}{3} e^{-2}$

putting $x = 3$, $P(4) = \frac{2}{3} P(3) = \frac{1}{3} e^{-2}$

2) Find out the fallacy if any in the following statement.

"If X is a Poisson variate such that $P(X=2) = 9P(X=4) + 90 \cdot P(X=6)$ then mean of $X = 1$".

→ Let m be the mean of X.

$$\therefore P(X=x) = e^{-m} \cdot \frac{m^x}{x!}$$

$$e^{-m} \cdot \frac{m^2}{2!} = 9 \cdot e^{-m} \cdot \frac{m^4}{4!} + 90 \cdot e^{-m} \cdot \frac{m^6}{6!}$$

$$\therefore \frac{1}{2} = \frac{3m^2}{8} + \frac{m^4}{8}$$

$$\therefore m^2 = -4 \quad \text{or} \quad m^2 = 1.$$

$\therefore$ The mean is 1, since $m > 0$

$\therefore$ The statement is correct.

3) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as poisson variate with mean 1.5. calculate the proportion of days on which
(i) neither car is used
(ii) some demand is refused.

→

$$P(X) = e^{-m} \frac{m^x}{x!} = \frac{e^{-1.5} \cdot (1.5)^x}{x!}, \quad x = 0, 1, 2, \ldots$$

(i) Probability that there is no demand is

$$P(X=0) = e^{-1.5} \cdot \frac{(1.5)^0}{0!} = 0.2231.$$

Scanned with CamScanner

(ii) Probability that some demand is refused means there was demand for more than two cars.

$\therefore P(X>2) = P(X=3) + P(X=4) + \dots$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[e^{-1.5}\frac{(1.5)^0}{0!} + e^{-1.5}\frac{(1.5)^1}{1!} + e^{-1.5}\frac{(1.5)^2}{2!} \right]$

$= 1 - [0.2231 + 0.3347 + 0.2510] = 0.1912$

$\therefore$ Proportion of days on which
(i) neither car is used is 0.2231
(ii) some demand is refused is 0.1912.

4) If X_1, X_2, X_3 are three independent poisson variates with parameters $m_1 = 1$, $m_2 = 2$, $m_3 = 3$ respectively. Find
(i) $P[(X_1 + X_2 + X_3) \geq 3]$ &
(ii) $P[X_1 = 1 / (X_1 + X_2 + X_3) = 3]$

→ $Z = X_1 + X_2 + X_3$ is also a poisson distribution with parameter $m = m_1 + m_2 + m_3 = 6$.

$\therefore P(Z \geq 3) = 1 - P(Z \leq 2)$

$= 1 - \sum_{z=0}^{2} \frac{e^{-6} 6^z}{z!} = 1 - \left(e^{-6} + 6 \cdot e^{-6} + \frac{6^2 e^{-6}}{2!}\right)$

$= 1 - 25e^{-6} = 1 - 25(0.002478)$
$= 0.938$

By def^n of conditional Probability,

$P[X_1 = 1 / (X_1 + X_2 + X_3) = 3]$

$$= \frac{P(X_1 = 1 \ \& \ X_2 + X_3 = 2)}{P[(X_1 + X_2 + X_3) = 3]}$$

5) Fit a poisson distribution to the following data.

no. of deaths.	0	1	2	3	4
frequencies	123	59	14	3	1

→ Fitting Poisson distribution means Finding expected frequencies of $X = 0, 1, 2, 3, 4$.

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = m$$

$\therefore$ mean $= 123(0) + 59(1) + 14(2) + 3(3) + 1(4)$

$$= \frac{100}{200} = 0.5$$

Poisson distribution of X is.

$$P(X=x) = \frac{e^{-m} \times m^x}{x!} = \frac{e^{-0.5} \times (0.5)^x}{x!} \quad \text{--- (1)}$$

$$\text{Expected frequency} = N \times P(x) = 200 \times \frac{e^{-0.5} \times (0.5)^x}{x!}$$

Putting $X = 0, 1, 2, 3, 4$

The Expected frequency as 121, 61, 15, 2, 1.

or putting $X = 0$ in (1).

$$P(X=0) = e^{-0.5} \frac{(0.5)^0}{0!} = 0.6065$$

$\therefore$ Expected frequency $f(0) = Np$

$= 200 \times 0.6065$

$= 121.$

but $f(x+1) = \frac{m}{x+1} \cdot f(x) = \frac{0.5}{x+1} \cdot f(x)$

putting $x = 0$, $f(1) = \frac{0.5}{1} \cdot 121 = 61.$

putting $X = 2$, $f(3) = \frac{0.5}{3} \cdot 15 = 3.$

putting $x = 1$, $f(2) = \frac{0.5}{2} \cdot 60 = 15$

putting $x = 3$, $f(4) = \frac{0.5}{4} \cdot 3 = 1$

Examples on Normal Distribution.

1) For a Normal Distribution the mean is 50 & the S.D. is 15 find (i) Q_1 & Q_3 (ii) Mean Deviation. (Also the interquartile range).

→ (i) For a Normal Distribution:

$$Q_1 = m - \frac{2}{3}\sigma = 50 - \frac{2}{3}(15) = 40.$$

$$Q_3 = m + \frac{2}{3}\sigma = 50 + \frac{2}{3}(15) = 60.$$

(ii) The mean deviation of the Normal distribution is.

$$M.D. = \frac{4}{5}\sigma = \frac{4}{5} \times 15 = 12.$$

$$\text{Interquartile range} = Q_3 - Q_1$$
$$= 60 - 40 = 20.$$

2) The First & Third quartiles of a Normal Distribution are 36 & 44. find mean, S.D. & M.D.

→ $Q_1 = m - \frac{2}{3}\sigma$ & $Q_3 = m + \frac{2}{3}\sigma$.

$\therefore 36 = m - \frac{2}{3}\sigma$ & $44 = m + \frac{2}{3}\sigma$.

Adding $80 = 2m$ $\therefore \boxed{m = 40}$

$36 = 40 - \frac{2}{3}\sigma$ $\therefore \frac{2}{3}\sigma = 4.$

$\therefore \boxed{\sigma = 6}$

& Mean Deviation $= \frac{4}{5}\sigma = \frac{4}{5} \times 6 = \frac{24}{5}$

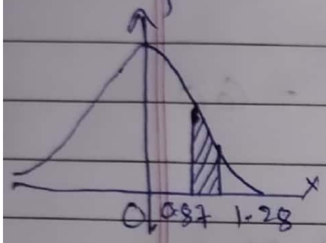
3) Find the Probability that a random variable having the standard normal distribution will take on a value between 0.87 & 1.28

→ $P(0.87 < z < 1.28) =$

$= \text{area bet}^n\ z = 0.87\ \&\ z = 1.28$

$= (\text{area from } z=0 \text{ to } z=1.28)$

$- (\text{area from } z=0 \text{ to } z=0.87)$



$= 0.3997 - 0.3078 = 0.0919$

4) Find the Probability that a random variable having standard normal Distribution will take a value between (i) 0.87 & 1.28 (ii) −0.34 & 0.62

→ (i) as in the above example

(ii) Area from $z = -0.34$ to 0 is the same as $z=0$ to $z = 0.34$ & 0.1331

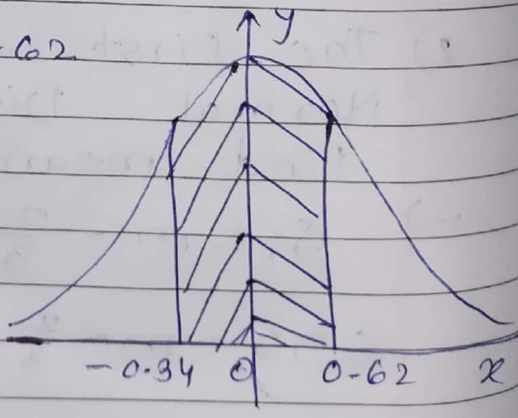
Area from $z=0$ to $z = 0.62$ is 0.2324

required area is the sum of the two.

$\therefore P(-0.34 < z < 0.62) =$

$= 0.1331 + 0.2324$

$= 0.3655$



Type-II

5) In a factory turning out blades in mass production. it was found that in a packet of 100 blades on an average 16 blades are defective. find the S.D. of the defective blades. can the distribution of defective blades be approximated to a normal distribution ? If so write it's equation.

→ The distribution of defective blades is a binomial distribution.

$$n = 100 \quad , \quad p = \frac{16}{100} = 0.16$$

Mean , $\bar{x} = np = 100 \times 0.16 = 16.$

$\therefore q = 1 - p = 0.84$

$\therefore nq = 100 \times 0.84 = 84$

$\because$ both np & nq are greater than 15 ~~as started in the~~

Binomial distribution can be approximated to Normal distribution.

$\bar{x} = 16$ & $\sigma = \sqrt{npq} = \sqrt{100 \times 0.16 \times 0.84}$

$= 3.67.$

The equation of the normal distribution is

$$y = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi} \cdot (3.67)} e^{-\frac{1}{2}\left(\frac{x-16}{3.67}\right)^2}$$