MAE 271A Project
Calibration of an Accelerometer Using GPS Measurements

Due Date December 9, 2020

Consider that a vehicle accelerates in one dimension in an inertial frame. Assume that the acceleration is a harmonic of the form

$$a(t) = 10sin(2\pi\omega t) \quad \text{meters/sec}^2$$

where $\omega = .1\text{rad/sec}$. Suppose that the acceleration is measured by an accelerometer with a sample rate of 200 Hz at sample times t_j. The accelerometer is modelled with additive white Gaussian noise w with zero mean and variance $V = .0004(\text{meters/sec}^2)^2$. The accelerometer has a bias b_a with *a priori* statistics $b_a \sim N(0, .01(\text{meters/sec}^2)^2)$. The accelerometer a_c is modelled as

$$a_c(t_j) = a(t_j) + b_a + w(t_j)$$

A GPS receiver is used to measure position and velocity in an inertial space. The measurements which are available at a 5 Hz rate (synchronized with the accelerometer) are

$$\begin{aligned} z_{1i} &= x_i + \eta_{1i} \\ z_{2i} &= v_i + \eta_{2i} \end{aligned}$$

where x_i is the position and v_i is the velocity. Their *a priori* statistics are $x_0 \sim N(0\,\text{meters}, (10\,\text{meters})^2)$ and $v_0 \sim N(100\,\text{m/s}, (1\,\text{m/s})^2)$. The additive measurement noises are assumed to be white noise sequences and independent of each other with statistics

$$\begin{bmatrix} \eta_{1i} \\ \eta_{2i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1\text{meter}^2 & 0 \\ 0 & (4\text{cm/sec})^2 \end{bmatrix} \right)$$

- Using the difference between the integration of the accelerometer output and the GPS as a new measurement, determine an associated stochastic discrete time system that is approximately independent of the acceleration profile.
- For that stochastic system implement a minimum variance estimator.
- To test your filter implementation, from the above data simulate the exact stochastic system and do the test over a 30 sec. interval.
  - Show error estimates of position, velocity, and accelerometer bias for one realization. Include the one sigma bound obtained from the error variance computed in the Kalman filter.
  - Show that the error variance obtained from the simulation and averaged over an ensemble of realizations is close to the error variance used in the Kalman filter algorithm.
  - Show that the theoretical orthogonality properties, such as $E[(x_k - \hat{x}_k)\hat{x}_k^T]$ and $E[(x_k - H\bar{x}_k)(x_j - H\bar{x}_j)^T]$ $k > j$, are satisfied for this filter.

In plotting the states, errors, and variances, try to show values over the time interval, i.e. do not plot what looks like zero. Turn in a report having an abstract, and sections such as introduction, theory and algorithm, results and performance, and conclusions.