Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	**:CSE**	**Semester**	**:III**
Title of the Course	**:Discrete Structure**	**Course Code**	**:CSL 201**

Time: 3 Hours **Maximum Marks: 50**

Section A: Carry only one (01) question of 10 parts of 01 mark each and all parts are compulsory.

Q1.

a. What Is Cardinality Of A Set?
b. What Are The Types Of Normal Forms?
c. How the relations are represented using Graphs?
d. Let f(x)=x+2 and g(x)=2x+1, find (fog)(x) and (gof)(x).
e. Prove $(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$ is a contradiction.
f. Draw the Hasse diagram for poset of subsets of $\{1,2,3\}=\{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$.
g. Define mathematical induction.
h. Explain the properties of homomorphism.
i. What are the different applications of Graph Colouring?
j. Minimized the Boolean expression : $F(A,B,C)=A'B+BC'+BC+AB'C'$

Section B: Contains Five (05) questions of 5 marks each and any four (04) are to be attempted.

1. Prove that direct product of two lattices is itself a lattice.

2. There are 16 points in a plane no three of which are collinear.
   i. Find the number of straight lines
   ii. Number of Triangles formed by joining them.

3. Let (G,*) be a finite group(i.e. |G| is finite), and a be any element of G. Show that the inverse of a, denoted by a^{-1}, belonges to $\{a^1,a^2,a^3,\ldots,a^k\ldots\}$, where a^1 is defined as a and a^k for k>1 is defiend as $a*a^{k-1}$.

4. The adjacency matrix of a graph is given below
   i. Draw the graph defined by this adjacency matrix. Label the vertices of your graph 1, 2, . . . , 6 so that vertex i corresponds to row and column i of the matrix.
   ii. What is the diameter of this graph? Explain why.
   iii. Find a cycle in this graph of maximum length and explain why it has maximum length.

5. Let B_n denote the butterfly network with $N = 2^n$ inputs and N outputs. We will show that the congestion of B_n is exactly $\sqrt{N}$ when n is even.
For the butterfly network, there is a unique path from each input to each output, so the congestion is the maximum number of messages passing through a vertex for any matching of inputs to outputs. If v is a vertex at level i of the butterfly network, there is a path from exactly 2^i input vertices to v and a path from v to exactly 2^{n-i} output vertices. At which level of the butterfly network must the congestion be worst? What is the congestion at the node whose binary representation is all 0^s at that level of the network?
   i. Show that the congestion of B^n is at most $\sqrt{N}$ when n is even.
   ii. That the congestion achieves $\sqrt{N}$ somewhere in the network and conclude that the congestion of B^n is exactly $\sqrt{N}$ when n is even.

Section C: Contains Three (03) questions of ten (10) marks each and any two (02) are to be attempted.

1. Show that the set G of all real valued functions defined on the closed intervel [0,1] is a commutative ring with unity with respect to pointwise addition and pointwise multiplication of functions, define as

$$(f+g)(x) = f(x) + g(x)$$
$$(fg)(x) = f(x)g(x)$$

for all f, g blongs to G

2. This relate to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies (called normals by Smullyan [Sm78]) who can either lie or tell the truth. You encounter three people, *A*, *B*, and *C*. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

   i. A says "C is the knave," B says, "A is the knight," and C says "I am the spy."
   ii. A says "I am the knight," B says "I am the knave," and C says "B is the knight."
   iii. A says "I am the knave," B says "I am the knave," and C says "I am the knave."
   iv. A says "I am the knight," B says "A is telling the truth," and C says "I am the spy."
   v. A says "I am the knight," B says, "A is not the knave," and C says "B is not the knave."
   vi. A says "I am the knight," B says "I am the knight," and C says "I am the knight."
   vii. A says "I am not the spy," B says "I am not the spy," and C says "A is the spy."
   viii. A says "I am not the spy," B says "I am not the spy," and C says "I am not the spy."

3. Consider the graph (V, E), where
   V = {a, b, c, d, e, f},
   E = {ab, ac, ad, ae, bc, bd, bf, ce, cf, de, df, ef}.
   Answer the following questions concerning this graph. **[Justify all your answers.]**
   i. Is the graph complete?
   ii. Is the graph regular?
   iii. Is the graph connected?
   iv. Does the graph have an Eulerian circuit?
   v. Does the graph have a Hamiltonian circuit?
   vi. Calculate the number of spanning tree in given graph
   vii. Given an example of an isomorphism between the graph described above and the graph (V 0 , E0), where V 0 = {p, q, r, s, t, u}, E 0 = {pq, ps, pt, pu, qr, qt, qu, rs, rt, ru, st, su}.

Q-4 Section-B

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ & 0 & 1 & 0 & 0 & 1 \\ & & 0 & & & \\ & & & 0 & 1 & 1 \\ & & & & 0 & 1 \\ & & & & & 0 \end{bmatrix}$$