National Institute of Technology, Delhi

Name of the Examination: B.Tech.

Branch   :CSE/ECE   Semester   :III

Title of the Course   :Probability and Statistics or   Course Code   :MAL241/ECL205

Probability Th. and Sto. Pro.

Time: 2 Hours   Maximum Marks: 25

Note:Answers all the questions.

Qu.1. I have a bag with 3 coins in it. One of them is a fair coin, but the others are biased trick coins. When flipped, the three coins come up heads with probability 0.5, 0.6, 0.1 respectively. Suppose that I pick one of these three coins uniformly at random and flip it three times.

(a) What is $P(HTT)$? (That is, it comes up heads on the first flip and tails on the second and third flips.
(b) Assuming that the three flips, in order, are HTT, what is the probability that the coin that I picked was the fair coin?(Hint: Use Bay's rule).

[2+2]

Qu. 2. let X be a continuous random variable with probability density function (pdf) given by

$$f(x) = \begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx + 3k, & 2 \le x < 3 \\ 0, & \text{otherwise} \end{cases}$$

a. Determine the constant k.
b. Determine F(x), the cumulative density function (CDF).   [2+1]

Qu.3. The joint density function of X & Y is given by

$$f(x,y) = e^{-(x+y)}, \qquad x \ge 0, \ y \ge 0$$

a. Find P(X<Y|X<2y)
b. Find P(1<X+Y<2)   [2+2]

Qu.4. Two discrete random variable X and Y have the joint density function

$$f(x,y) = \frac{A^x e^{-A} p^y (1-p)^{x-y}}{y!\,(x-y)!}, \quad y = 0,1,2,3 \dots x \ \& \ x = 0,1,2,3 \dots$$

Where A, p are constant with $A>0$ and $0<p<1$, Find the marginal distribution of X & Y.

[2+2]

Qu. 5. Prove the following statement $cov(X,Y) = E[XY] - \mu_X \mu_Y$ [2]

Qu. 6. Consider an experiment that consists of 2 rolls of balanced die. If X is the number of 4s and Y is the number 5s obtained in 2 rolls of the die. Find the
(a) Joint probability distribution of X and Y.
(b) P[(X,Y)∈ A], where A is the region as A={(x, y)|2x+y<3}   [2+2]

Qu. 6. Two pen are selected at random form a box containing 3 blue, 3 red and 2 green pens. If X is the number of blue pen and Y is the number of red pen selected. Find the joint p.d.f. and P[(X,Y)∈ A], where A is the region as A={(x, y)|x+y<2}.

[2+2]