Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	**: CSE**	**Semester**	**: III**
Title of the Course	**: Probability and Statistics**	**Course Code**	**: MAL241**

Time: 3 Hours **Maximum Marks: 50**

Note1: This question paper is divided into three sections A, B and C, and each section must be solved with rules given as follows:.

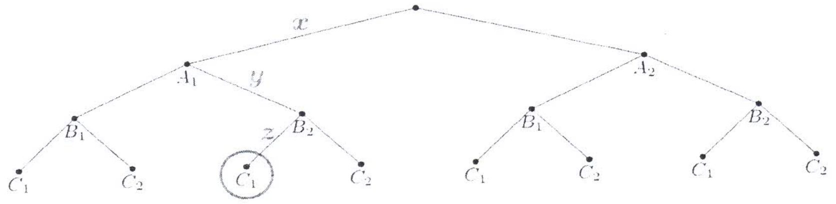
Note2: Answer the questions in sequence as Section A, Section B and Section C, respectively otherwise problem will be not checked.

Section A

Q. 1.

(*a*) In 3 tosses of a coin which of following equals the event "exactly two heads" and give proper reason.
E_1 = {THH, HTH, HHT, HHH}, E_2 = {THH, HTH, HHT}; E_3 = {HTH, THH}.

(*b*) These questions all refer to the following tree as mentioned below. For each one circle the best answer.



(*i*) The probability x represents **A.** $P(A1)$ **B.** $P(A1|B2)$ **C.** $P(B2|A1)$ **D.** $P(C1|B2 \cap A1)$.

(*ii*) The probability y represents **A.** $P(B2)$ **B.** $P(A1|B2)$ **C.** $P(B2|A1)$ **D.** $P(C1|B2 \cap A1)$.

(c) As given above tree, circled the best answer with proper reason.

(*iii*) The probability z represents A. $P(C1)$ B. $P(B2|C1)$ C. $P(C1|B2)$ D. $P(C1|B2 \cap A1)$.

(d) Define the Chi-Square distribution and its density function.

(e) Consider the following joint pdf's for the random variables X and Y. Write down the ones where X and Y are independent and why.

A. $f(x, y) = 4x^2y^3$ **B.** $f(x, y) = 12(xy^3 + xy^3)$ **C.** $f(x, y) = 6e^{-3x-6y}$

(*f*) How many distinct permutations can be made from the letter of word INFINITY?

(*g*) State the Gamma distribution with density function.

(*h*) State the central limit theorem.

(*j*) Define basic difference between Type-I and Type-II error.

Section B

Q. 2. I have a bag with 3 coins in it. One of them is a fair coin, but the others are biased trick coins. When flipped, the three coins come up heads with probability 0.5, 0.6, 0.1 respectively. Suppose that I pick one of these three coins uniformly at random and flip it three times.

(a) What is P (*HTT*)? (That is, it comes up heads on the first flip and tails on the second and third flips.)
(b) Assuming that the three flips, in order, are ***HTT***, what is the probability that the coin that I picked was the fair coin? (Hint: Use Bay's rule) [2+3]

Q. 3. Let X be binomial random variable with probability distribution $b(x;n,p)$ when $n \to \infty$ and $p \to 0$, and $np \to \mu$, remains constant, show that $b(x;n,p) \to p(x;\mu)$ as $n \to \infty$.

Q. 4. Q. 3. Suppose that two dimensional continuous random variable (X,Y) has joint p.d.f. is given by:

$$f(x, y) = 6x^2 y \quad 0 < x < 1, 0 < y < 1$$

Find

I. P(X+Y<1)
II. P(X>Y)
III. P(X<1/Y<2) [2+1+2]

Q. 5. Suppose X and Y have joint pdf $f(x, y) = c(x^2 + xy)$ on $[0,1] \times [0,1]$.
(a) Find c and the joint cdf $F(x, y)$
(b) Find the marginal pdf f_X and f_Y
(c) Find the covariance and correlation of X and Y.

[2+1+2]

Q. 6. Derive the mean, variance and moment generating function of binomial distribution.

Section C

Q.7. You independently draw 100 data points from a normal distribution.
(a) Suppose you know the distribution is $N(\mu, 4)$ and you want to test the null hypothesis $H_0 : \mu = 3$ against the alternative hypothesis $H_1 : \mu \neq 3$.
If you want a significance level of $\alpha = 0.05$. What is your rejection region?
You must clearly state what test statistic you are using.
(b) Suppose the 100 data points have sample mean 5. What is the p-value for this data? Should you reject H_0?
(c) Determine the power of the test using the alternative H_1: $\mu = 4$.

[2+3+5]

Q. 8 . (a) A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximation probability that a box will fail to meet the guaranteed quality?

(b) An urn contains 3 red balls and 2 blue balls. A ball is drawn. If the ball is red, it is kept out of the urn and a second ball is drawn from the urn. If the ball is blue, then it is put back in the urn and a red ball is added to the urn. Then a second ball is drawn from the urn.

(a) What is the probability that both balls drawn are red?

(b) If the second drawn ball is red, what is the probability that the first drawn ball was blue?

Qu.9. Suppose X and Y are random variables with $P(X = 1) = P(X = -1) = 1/2$; $P(Y = 1) = P(Y = -1) = 1/2$. Let $C = P(X = 1 \text{ and } Y = 1)$.
(a) Determine the joint distribution of X and Y, Cov(X, Y), and Cor(X, Y).
(b) For what values of C are X and Y independent? For what values of C are X and Y 100% correlated?