Roll No.

National Institute of Technology, Delhi

Name of the Examination: B.Tech

Mid-Semester Examination September, 2019

Branch: CSE | **Semester: III**

Title of the Course: Discrete Structures | **Course Code: CSL 201**

Time: 2 hours | **Maximum Marks: 25**

Note: All questions are compulsory

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1. Express each of these sentences using quantifiers (write the predicates clearly):

(a) No traveler loves the city they live in.

(b) Every student in this class has taken exactly two mathematics classes at this school.

(1.5+1.5=3 marks)

2. **(a)** Use rules of inference to show that the hypotheses "Bob failed the course, but attended every lecture", "Everyone who did the homework every week passed the course", and "If a student passed the course, then he/she did some of the homework" imply the conclusion "Not every student did/submitted every homework assignment". Write the predicates and the names of the inference rules clearly.

(b) Use rules of inference to show that the hypotheses "$p \rightarrow q$", "$\neg p \rightarrow r$", and "$r \rightarrow s$", imply the conclusion "$\neg q \rightarrow s$". Write the names of inference rules clearly. (2+2=4 marks)

3. **(a)** Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on.

(b) Prove or disprove the following statement: "Every integer can be expressed as the sum of two squares". (3+2=5 marks)

4. Show that if C_1 and C_2 are conditions the elements of the n-ary relation R may satisfy, then $s_{C_1 \wedge C_2}(R) = s_{C_1}(s_{C_2}(R))$. (Hint: Prove that $s_{C_1 \wedge C_2}(R)$ is a subset of $s_{C_1}(s_{C_2}(R))$ and $s_{C_1}(s_{C_2}(R))$ is a subset of $s_{C_1 \wedge C_2}(R)$) (4 marks)

5. **(a)** Prove that $3^n < n!$ if n is an integer greater than 6.

(b) Let a and b be real numbers. Using proof by contradiction, prove that if $a < b + \epsilon$ for all $\epsilon > 0$, then $a \leq b$. (2+3 = 5 marks)

6. Given the directed graphs representing two relations, how can the directed graph of the symmetric difference and composition of these relations be found? Illustrate your answer using the digraphs of $R1$ and $R2$ given on page 2.
(4 marks)

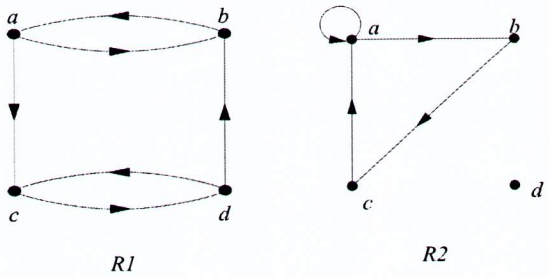

Figure 1: Digraphs of $R1$ and $R2$.