

Roll No.:.....

National Institute of Technology, Delhi

Name of the Examination: B.Tech.

Branch : Computer Science Semester : 3rd
Engineering

Title of the Course : Discrete Structures Course Code : CSL201

Time: 3 Hours

Maximum Marks: 50

Note : Make necessary assumptions wherever needed.

Guidelines:

1. The question paper is divided into three sections A, B and C and each section has following type of questions
 - a. Section A: Contains 10 questions of 01 mark each and all parts are compulsory.
 - b. Section B: Contains Five (05) questions of 5 marks each and any four (04) are to be attempted.
 - c. Section C: Contains Three (03) questions of ten (10) marks each and any two (02) are to be attempted.

Section – A

All questions are compulsory in this section.

1. Derive recurrence relation that arises in relation with searching of an element in given array of length n , using binary search method ?
2. Find total number of possible binary tree with 4 vertices
3. Is $(P \vee \neg P) \rightarrow Q$ tautology or not ?
4. Let A and B be sets with cardinalities m and n respectively, The number of injective mapping from A to B , when $m < n$ is
5. Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to.....
6. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes.....

7. set $\{1,2,4,7,8,11,13,14\}$ is a group under multiplication modulo 15. the inverses of 4 and 7 are respectively:

- (a) 3 and 13 (b) 2 and 11 (c) 4 and 13 (d) 8 and 14

8. A compound proposition that is always TRUE is

9. Is $(P \vee \neg P) \rightarrow Q$ tautology or not ?

10. Find total number of possible binary tree with 4 vertices

Section – B

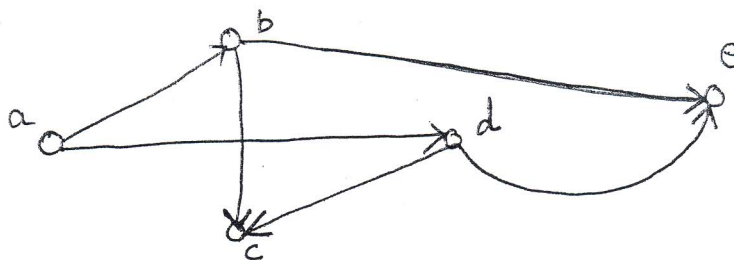
Do any 4 of the following questions :

(4x5 = 20 marks)

Question 1. (A) Define regular graph .How many vertices does a regular graph of degree four with 10 edges have ? (3 marks)

(B) Let a cycle on n vertices is isomorphic to its complement ,then find value of n .(2 marks)

Question 2. Write the adjacency matrix representation of given graph :-



(B) Show that if G is bipartite simple graph with vertices V and edges E then $E \leq V^2/4$ and find maximum number of edges in an acyclic undirected graph with n vertices. (3 marks)

Question 3. (A) Use generating function to determine the number of different ways 15 identical stuffed animals can be given to six children so that each child received at least one but no more than three stuffed animals . (2.5 marks)

(B) Find the generating function for the finite sequence 1,4 ,16,64,256..... (2.5 marks)

Question 4(A) How many ways can the digits 0,1,2,3,4,5,6,7,8,9 be arranged so that no even digits is in its original position (using generating function) (3 marks)

(B) What is the solution of recurrence relation:-

$$a_n = a_{n-1} + 2a_{n-2} \text{ with initial conditions } a_0 = 2, a_1 = 7 .. \quad (2 \text{ mark})$$

Question 5. (A) Let F be the set of one –to-one functions from the set $\{1,2,3,\dots,a\}$ to the set $\{1,2,3,\dots,b\}$ where $a \geq b \geq 1$.

(a) How many functions are members of F ?

- (b) How many functions f in F satisfy the property $f(i) = 1$ for some i , $1 \leq i \leq b$?
- (c) How many functions f in F satisfy the property $f(i) < f(j)$ for some $1 \leq i < j \leq a$? (5 marks)

Section – C

Do any 2 of the following questions :

Question 1. coding system encodes message using string of base 4 digits (digit from set $\{0,1,2,3\}$). A codeword is valid if and only if it contains an even number of 0s and even number of 1s .let a_n equal the number of valid codeword of length n . Furthermore ,let b_n , c_n , and d_n equal the number of strings of base 4 digits of length n with an even number of 0s and an odd number of 1s , with an odd number of 0s and an even number of 1s and with an odd number of 0s and odd number of 1s respectively .

(a) Show that $d_n = 4^n - a_n - b_n - c_n$. Use this to show that $a_{n+1} = 2a_n + b_n + c_n$,
 $b_{n+1} = b_n - c_n + 4^n$ and $c_{n+1} = c_n - b_n + 4^n$.

(b) Find values of a_3 , b_3 , c_3 and d_3 .

(c) Use recurrence relations and together with initials conditions ,generate generating functions

$A(x)$, $B(x)$ and $C(x)$.

(10 marks)

Question 2. (A) Show that every monoid $(M, *, e)$ is isomorphic to sub-monoid of (M^M, \oplus, ∇) where ∇ is the identity mapping of M and M^M is the set of all function from M to M .

(B) Express the statement “Everyone has exactly one best friend “ as a logical expression involving predicate , quantifiers with a domain consisting of all people , and logical connectives .
 (6+4) marks

Question 3. (A) Let G be a connected weighted graph in which every edge belongs to some circuit ,if e_1 is the edge with weight greater than that of any other edge in G ,show that no shortest spanning tree in G will contain e_1 .

(B) Show that a graph with n vertices and with vertex connectivity k must have atleast $kn/2$ edges.

(C) Construct a graph G with the following properties : Edge connectivity of $G=4$,vertex connectivity of $G=3$,and degree of every vertex of $G \geq 5$.
 (5+3+3)