Unit 1 Assignment

Sri Seshadri 9/24/2017

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Chapter 1. Question 1

a. Figure 1 shows the sctterplot of GPA vs SAT-Quant. There appears to be a linear relationship between the two variables. However the observations 2 and 5 in the plot appear to be influential points. If the two points are removed from the data, the straight line fit would have a steeper slope. Further analysis of the model is in below sections.

```
# read data in

GPA <- readr::read_csv(file = 'gpa.csv')

GPA1 <- head(GPA,8)

# GPA1 <- GPA1[c(-2,-5),] # To show that observations 2 and 5 are influential points
library(ggplot2)

ggplot(data = GPA1, mapping = aes(x = SAT_QUAN, y = GPA)) + geom_point() + geom_smooth(method = 'lm',set)</pre>
```

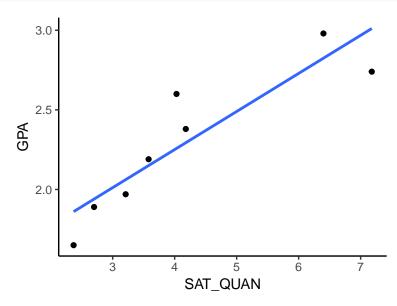


Figure 1: Scatterplot of GPA vs SAT-Quant

```
# Model for question b
model1.lm <- lm(data = GPA1, formula = GPA ~ SAT_QUAN)
coefs <- round(coefficients(model1.lm),2)
signs <- ifelse(sign(coefs)==1,"+", "-")
Betas <- paste(abs(coefs[2:length(coefs)]),"*",'SAT_QUANT')
model1eqn <- paste("GPA = ",paste(coefs[1],paste(paste(signs[2:2], Betas),collapse = " ")))
model1data <- broom::augment(model1.lm)</pre>
```

b. The GPA is modeled as:

```
GPA = 1.29 + 0.24 * SAT_QUANT
```

With Intercept as 1.29 and slope as 0.24

c. The Predicted values and the residuals are shown in the table below.

```
df <- model1data[,c('GPA','SAT_QUAN','.fitted','.resid')]
knitr::kable(df,caption = "Predicted values and residuals")</pre>
```

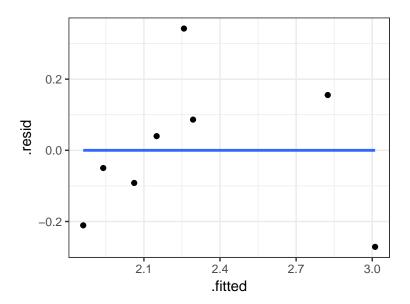


Figure 2: Residuals vs predicted GPA

Table 1: Predicted values and residuals

.resid	.fitted	SAT_QUAN	GPA
-0.0917210	2.061721	3.21	1.97
-0.2712494	3.011249	7.18	2.74
0.0397839	2.150216	3.58	2.19
0.3421548	2.257845	4.03	2.60
0.1553078	2.824692	6.40	2.98
-0.2108132	1.860813	2.37	1.65
-0.0497413	1.939741	2.70	1.89
0.0862784	2.293722	4.18	2.38

```
z <- anova(model1.lm)
SSE <- z$`Sum Sq`[2]
```

The R Squared value is found to be 0.8093426 and SSE = 0.2791225

d. Figure 2 shows the residuals plot against the predicted values of GPA. The residuals appear random.

```
ggplot(data = model1data, mapping = aes(x = .fitted, y = .resid)) + geom_point() + geom_smooth(method =
```

Chapter 1. Question 2

Figure 3 shows the relationship between quantitative SAT score against GPA for the full data. Observation 2 now doesn't seem as bad as an influential point in the context of the full data.

```
ggplot(data = GPA, mapping = aes(x = SAT_QUAN, y = GPA)) + geom_point() + geom_smooth(method = 'lm',se =
model2.lm <- lm(data = GPA, formula = GPA ~ SAT_QUAN)
coefs_2 <- round(coefficients(model2.lm),2)
signs_2 <- ifelse(sign(coefs_2)==1,"+", "-")
Betas_2 <- paste(abs(coefs_2[2:length(coefs_2)]),"*",'SAT_QUANT')</pre>
```

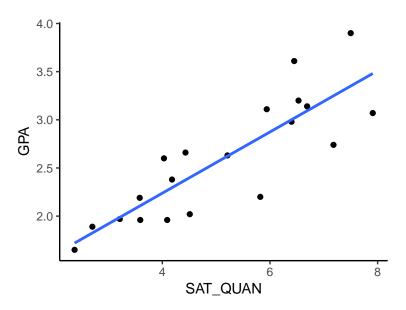


Figure 3: Scatterplot of GPA vs SAT-Quant - Full data

```
model2eqn <- paste("GPA = ",paste(coefs_2[1],paste(paste(signs_2[2:2], Betas_2),collapse = " ")))</pre>
model2data <- broom::augment(model2.lm)</pre>
Below is the summary for the model:
GPA = 0.97 + 0.32 * SAT_QUANT
summary(model2.lm)
##
## Call:
## lm(formula = GPA ~ SAT_QUAN, data = GPA)
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
                                            Max
## -0.61675 -0.18772 0.02693 0.18197
                                        0.59302
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                           0.24963
## (Intercept) 0.96699
                                     3.874 0.00111 **
## SAT_QUAN
                0.31783
                           0.04652
                                     6.832 2.15e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.337 on 18 degrees of freedom
## Multiple R-squared: 0.7217, Adjusted R-squared: 0.7062
## F-statistic: 46.68 on 1 and 18 DF, p-value: 2.146e-06
anova(model2.lm)
## Analysis of Variance Table
##
## Response: GPA
##
             Df Sum Sq Mean Sq F value
                                          Pr(>F)
## SAT_QUAN
             1 5.3015 5.3015 46.681 2.146e-06 ***
```

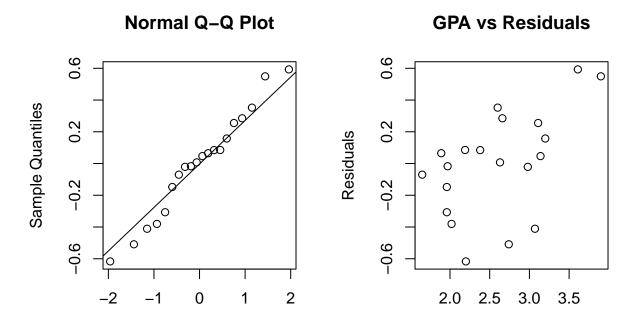


Figure 4: Residuals of model with full data

Theoretical Quantiles

```
## Residuals 18 2.0443  0.1136
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

z2 <- anova(model2.lm)
SSE2 <- z2$`Sum Sq`[2]
MSE2 <- SSE2/nrow(model2data)
modelcomparison <- data.frame(Question = c(1,2),model = c(model1eqn, model2eqn), RSquared = c(mosaic::r</pre>
```

GPA

Figure 4 shows the residual analysis of the model above. The residuals are fairly normally distributed and there is a hint of unequal variance in the residuals. May be we are missing another predictor. The next section will explore additional predictors.

```
par(mfrow = c(1,2))
qqnorm(y=model2.lm$residuals)
qqline(y=model2.lm$residuals)
plot(y = model2.lm$residuals,x = model2.lm$model$GPA, xlab = 'GPA', ylab = 'Residuals',main = 'GPA vs R
```

Model comparison

The models in question 1 and 2 are compared in the table below. We see that model for question 1 has a better R Squared value and MSE. However, the second model fits the data in question 1 better with much lower MSE. As mentioned above, the residuals for model 2 appear to have non-constant variance. May be another predictor is required.

pander::pandoc.table(modelcomparison,caption = "Model comparison")

## ##					
##	Question	model	RSquared	MSE	First_8obs_MSE
##	1	GPA = 1.29 + 0.24 * SAT_QUANT	0.8093	0.03489	0.03489
##	2	GPA = 0.97 + 0.32 * SAT_QUANT	0.7217	0.1022	0.0008923
##					

Table: Model comparison

Chapter 1 Question 3

a. Figure 5 shows the relationship between GPA and Highschool English scores. There isn't a strong linear relationship.

```
model3a <- lm(data = GPA, formula = GPA ~ HS_ENGL)
coefs_3a <- round(coefficients(model3a),2)
signs_3a <- ifelse(sign(coefs_3a)==1,"+", "-")
Betas_3a <- paste(abs(coefs_3a[2:length(coefs_3a)]),"*",'HS_ENGL')
model3aeqn <- paste("GPA = ",paste(coefs_3a[1],paste(paste(signs_3a[2:2], Betas_3a),collapse = " ")))
model3adata <- broom::augment(model3a)

ggplot(data = GPA, mapping = aes(x = HS_ENGL, y = GPA)) + geom_point() + theme_bw() + geom_smooth(method)</pre>
```

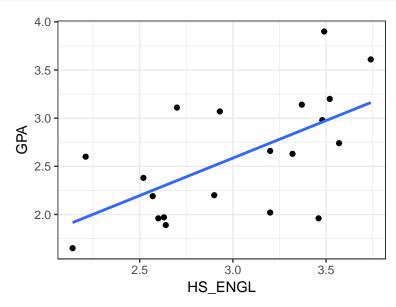


Figure 5: GPA vs HighSchool English

Below is the summary of the model:

```
GPA = 0.25 + 0.78 * HS\_ENGL
```

summary(model3a)

```
##
## Call:
## lm(formula = GPA ~ HS_ENGL, data = GPA)
## Residuals:
##
                  1Q
                      Median
## -0.98392 -0.30928 -0.07102 0.31163 0.93271
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                0.2487
                           0.7332
                                    0.339 0.73838
## HS_ENGL
                 0.7790
                           0.2407
                                    3.236 0.00458 **
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

b. Using Highschool english and SAT verbal scores as predictors

It'll be useful to understand the correlations that exist amongst the predictors. To better interpret the model in the context of variability of the regression coefficients (when the predictors are correlated). Figure 6 shows the correlations plot of GPA data. It's seen that the highschool english and verbal SAT scores are not highly correlated.

```
correlations <- cor(GPA)
corrplot::corrplot(correlations,tl.cex = 1)</pre>
```

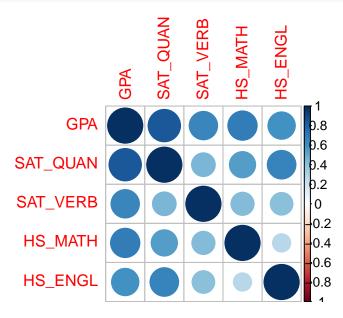


Figure 6: Correlation Plot

```
p <- ggplot(data = GPA, mapping = aes(x = SAT_VERB, y = GPA)) + geom_point() + geom_smooth(method = 'lm
p2 <- ggplot(data = GPA, mapping = aes(x = SAT_VERB, y = HS_ENGL)) + geom_point() + geom_smooth(method = library(gridExtra))
grid.arrange(p,p2, ncol = 2)

model3b <- lm(GPA~HS_ENGL + SAT_VERB, data = GPA)
Predictors <- c('HS_ENGL', 'SAT_VERB')
model3bdata <- broom::augment(model3b)
coefs3b <- round(coefficients(model3b),2)</pre>
```

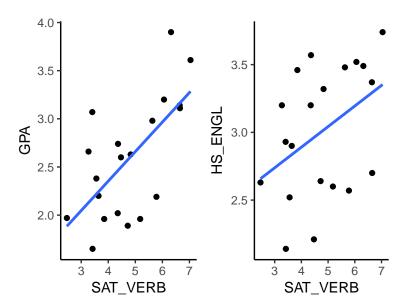


Figure 7: Relationship between predictors

```
signs3b <- ifelse(sign(coefs3b)==1,"+", "-")
Betas3b <- paste(abs(coefs3b[2:length(coefs3b)]),"*",Predictors)
model3beqn <- paste("GPA = ",paste(coefs3b[1],paste(paste(signs3b[2:length(signs3b)], Betas3b),collapse</pre>
```

Below is the summary for the model:

```
GPA = -0.06 + 0.52 * HS\_ENGL + 0.23 * SAT\_VERB
```

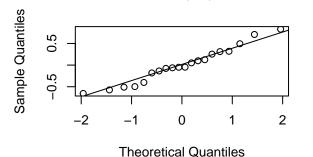
The model has an adjusted R squared of 51%, which is about half of the variation. Not a good explanatory model. The residuals vs actuals have a non-random relationship. The model might be missing a predictor.

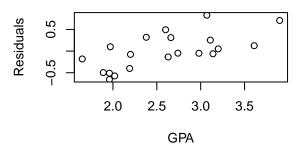
```
summary(model3b)
```

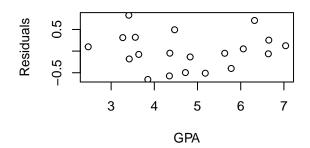
```
##
  lm(formula = GPA ~ HS_ENGL + SAT_VERB, data = GPA)
##
##
  Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
   -0.65511 -0.23661 -0.04908
                               0.26797
                                         0.83021
##
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05721
                            0.63789
                                     -0.090
                                              0.9296
## HS_ENGL
                0.51947
                            0.22682
                                      2.290
                                              0.0351 *
## SAT VERB
                                      2.745
                0.22726
                            0.08280
                                              0.0138 *
##
                           0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 0.4351 on 17 degrees of freedom
## Multiple R-squared: 0.5619, Adjusted R-squared: 0.5104
## F-statistic: 10.9 on 2 and 17 DF, p-value: 0.0008976
anova (model3b)
```

```
layout(matrix(c(1,2,3,4),byrow = 2, nrow = 2, ncol = 2))
qqnorm(model3bdata$.resid)
qqline(model3bdata$.resid)
plot(x = model3bdata$GPA, y = model3bdata$.resid, xlab = 'GPA', ylab = 'Residuals')
plot(x = model3bdata$SAT_VERB, y = model3bdata$.resid, xlab = 'GPA', ylab = 'Residuals')
plot(x = model3bdata$HS_ENGL, y = model3bdata$.resid, xlab = 'GPA', ylab = 'Residuals')
```

Normal Q-Q Plot







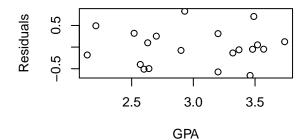


Figure 8: Residual analysis

```
# lattice::xyplot(.resid-GPA, data = model3bdata, type =c("p", "r"))
# plot.new()
# qqnorm(model3bdata$.resid)
# lattice::xyplot(.resid~HS_ENGL, data = model3bdata, type =c("p", "r"))
# lattice::xyplot(.resid~SAT_VERB, data = model3bdata, type =c("p", "r"))
# layout(matrix(c(1,2,3,4), 2, 2, byrow = TRUE))

model3c <- lm(GPA~HS_ENGL + SAT_VERB + SAT_QUAN, data = GPA)
Predictors <- c('HS_ENGL', 'SAT_VERB', 'SAT_QUAN')
model3cdata <- broom::augment(model3c)
coefs3c <- round(coefficients(model3c),2)
signs3c <- ifelse(sign(coefs3c)==1,"+", "-")
Betas3c <- paste(abs(coefs3c[2:length(coefs3c)]),"*",Predictors)
model3ceqn <- paste("GPA = ",paste(coefs3c[1],paste(paste(signs3c[2:length(signs3c)], Betas3c),collapse</pre>
```

c. Below is the summary for the model, with addition of SAT_QUAN as predictor to the above model in 3b.

```
GPA = 0.49 + 0.01 * HS\_ENGL + 0.16 * SAT\_VERB + 0.26 * SAT\_QUAN
```

It is seen that the residuals are fairly random and normally distributed. The intercept and HS_ENGL scores from the t -test are statistically insignificant. The SAT scores are key contributors to the model.

```
##
## Call:
## lm(formula = GPA ~ HS_ENGL + SAT_VERB + SAT_QUAN, data = GPA)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
## -0.42543 -0.15587 -0.01946
                              0.22889
                                        0.47949
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
               0.48633
                           0.44777
                                     1.086
## (Intercept)
                                             0.2935
## HS ENGL
                0.01115
                           0.18929
                                     0.059
                                             0.9538
## SAT_VERB
                0.15683
                           0.05811
                                     2.699
                                             0.0158 *
                                             0.0003 ***
## SAT_QUAN
                0.25862
                           0.05631
                                     4.593
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2945 on 16 degrees of freedom
## Multiple R-squared: 0.811, Adjusted R-squared: 0.7756
## F-statistic: 22.89 on 3 and 16 DF, p-value: 4.95e-06
anova(model3c)
## Analysis of Variance Table
##
## Response: GPA
             Df Sum Sq Mean Sq F value
                                          Pr(>F)
## HS ENGL
              1 2.7019 2.70193 31.144
                                       4.14e-05 ***
## SAT VERB
              1 1.4259 1.42594
                               16.436 0.0009210 ***
                                21.092 0.0003003 ***
## SAT QUAN
              1 1.8299 1.82987
## Residuals 16 1.3881 0.08676
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
layout(matrix(c(1,1,2,3,4,5),byrow = T, nrow = 3, ncol = 2))
plot(x = model3cdata$.fitted, y = model3cdata$.resid, xlab = 'fitted GPA', ylab = 'Residuals')
qqnorm(y = model3c$residuals)
qqline(model3c$residuals)
plot(x = model3cdata$HS_ENGL, y = model3cdata$.resid, xlab = 'HS_ENGL', ylab = 'Residuals')
plot(x = model3cdata$SAT_VERB, y = model3cdata$.resid, xlab = 'SAT_VERB', ylab = 'Residuals')
plot(x = model3cdata$SAT_QUAN, y = model3cdata$.resid, xlab = 'SAT_QUAN', ylab = 'Residuals')
```

Chapter 1 Question 4

summary(model3c)

- a. 78% of the Highschool english score contributes to the overall GPA at the end of 1st year of college.
- b. 26% of the SAT quantitative score contributes to the overall GPA at the end of 1st year of college.

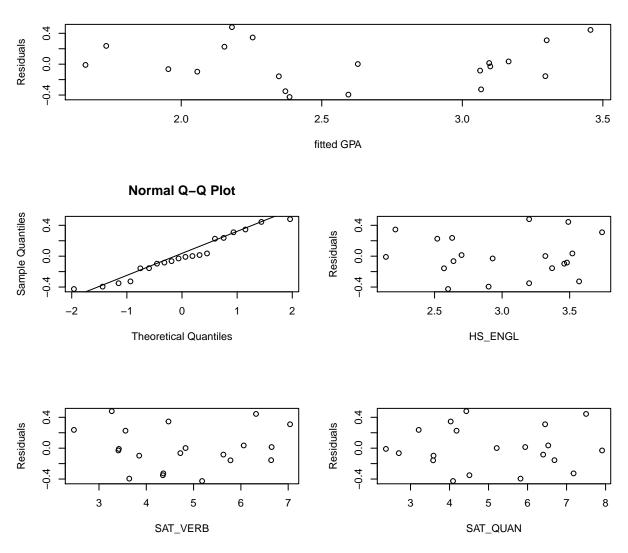


Figure 9: Model 3c residual analysis

Chapter 1 Question 5

The contribution of highschool english score to 1st year college GPA turned out to be negligible after the inclusion of the SAT scores. From test of individual regression coefficients' significance (t-test) in question 3c, it can be seen that the regression coefficient for HS_ENGL is not statistically different from zero. The t-test results is reproduced below:

```
summary(model3c)
```

```
##
## lm(formula = GPA ~ HS_ENGL + SAT_VERB + SAT_QUAN, data = GPA)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
  -0.42543 -0.15587 -0.01946 0.22889
##
                                        0.47949
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                0.48633
                           0.44777
                                     1.086
                                              0.2935
## HS ENGL
                           0.18929
                                     0.059
                                              0.9538
                0.01115
## SAT VERB
                0.15683
                           0.05811
                                     2.699
                                              0.0158 *
## SAT_QUAN
                                              0.0003 ***
                0.25862
                           0.05631
                                     4.593
## ---
                  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Residual standard error: 0.2945 on 16 degrees of freedom
## Multiple R-squared: 0.811, Adjusted R-squared: 0.7756
## F-statistic: 22.89 on 3 and 16 DF, p-value: 4.95e-06
```

Chapter 1 Question 6

- a. Figure 9 shows the qqplot of residuals, though the residuals are a bit left leaning/skewed; for practical purposes they seem normal.
- b. Figure 9 shows the raw residuals against fitted GPA. The residuals look to have a cyclical pattern. The same plot is reproduced below (Figure 10) with spline. However this may be purely random occurance, more sampling may help to confirm the pattern. The raw residuals can be misleading if there is an influential point that is influencing the regression coefficients to pull the fit closer to itself. Looking at studentized residuals can be helpful; shown in the right panel of figure 10.

The two plots in figure 10 are not different. This indicates that there aren't influential points. It'll be interesting to look at the Cook's distance.

```
res1 <- ggplot(data = model3cdata, mapping = aes(x = .fitted, y = .resid)) + geom_point() + geom_smooth
model3cdata$student.res <- rstudent(model3c)
res2 <- ggplot(data = model3cdata, mapping = aes(x = .fitted, y = student.res)) + geom_point() + geom_se
grid.arrange(res1,res2,ncol=2)</pre>
```

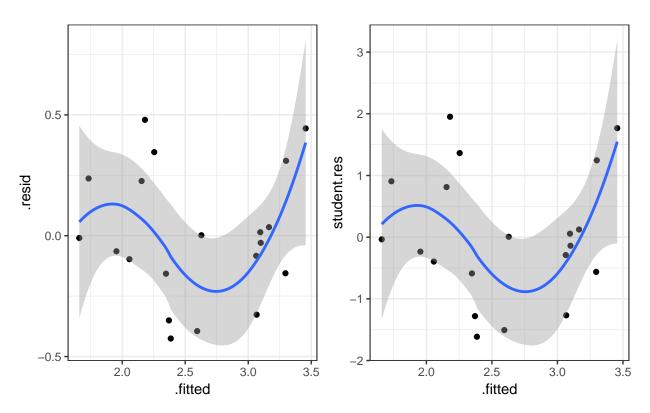


Figure 10: fitted vs residuals

c. The distribution of the studentized residuals, leverage and cook's distance is shown below.

The studentized residuals do not show outliers. The residuals are within \pm 2 standard deviations. Observation 17 has leverage of 0.5 which is greater than 2p/n = 0.4. This shows that this point is outlier in the x space. However the Cook's distance show that there is no effect of beta coefficients. Observation 17 is more a leverage point and not an influencial point.

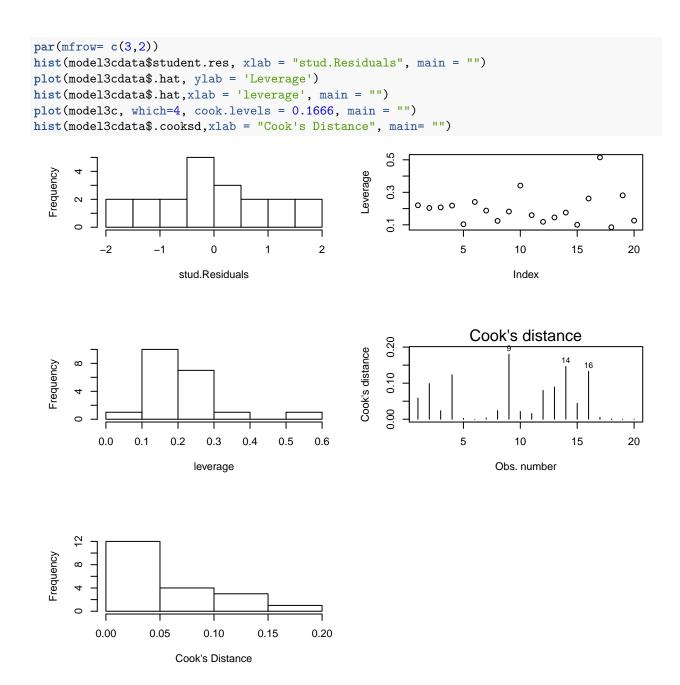


Figure 11: Leverage and Influence analysis

Chapter 2 Question 1

- a. Poisson distribution
- b. Binomial distribution
- c. Normal distribution?
- d. Multinomial
- e. Binomial
- f. Multinomial
- g. negative binomial

Chapter 2 Question 2

- a. Poisson distribution Natural logrithm link
- b. Binomial distribution Logit
- c. Normal distribution Identity
- d. Multinomial generalized logit
- e. Binomial logit
- f. Multinomial generalized logit
- g. negative binomial Logit

Chapter 2 Question 3

Question	What	Expected	Variance
a	Alcohol use	975.00	341.250
b.1	employed full time	975.00	49.500
b.2	employed part time	30.00	25.500
b.3	unemployed	20.00	18.000
b.4	non-participants	40.00	32.000
c.1	Low-Self Esteem	382.50	210.375
c.2	High-Self Esteem	467.50	210.375
d	Accidents per day	0.92	1.050

Chapter 2 Question 4

```
p <- 0.4

n <- 30

r <- 12

NBmean <- r/p

NBVar <- r*(1-p)/p^2

p30 <- choose(n-1,r-1)*p^r*(1-p)^(n-r)
```

The computation formula for mean, variance and probability can be see in the R Code above.

```
df <- data.frame(Mean = NBmean, Variance = NBVar, `Prob of Success by reaching 30` = round(p30,2))
knitr::kable(df)</pre>
```

Mean	Variance	Prob.of.Success.by.reaching.30
30	45	0.06

Chapter 2 Question 5

```
p <- (1:4)/10
probs <- function(p,n = 6, i = 2) {
   dbinom(i,n,p)
}
probabilities <- sapply(p,probs)
df5 <- data.frame(p = p, `probability when i equals 2` = round(probabilities,2))
knitr::kable(df5)</pre>
```

p	probability.when.i.equals.2
0.1	0.10
0.2	0.25
0.3	0.32
0.4	0.31

p = 0.3 is the most likely value of p

Chapter 2 Question 6

Though I understand the theory, the R part of the computation still confuses me. I am not fully confident about my answer to this question. Tammy had answered this question... I am yet to internalize it...

Still confused about the deviance and AI

```
usdata <- read.csv('usdata.csv')
null.lm <- glm(formula = VIOLRATE ~ 1, data = usdata)
DevianceMc <- deviance(null.lm)
ln_LMc <- DevianceMc/-2
summary(null.lm)</pre>
```

```
##
## Call:
```

```
## glm(formula = VIOLRATE ~ 1, data = usdata)
##
## Deviance Residuals:
##
           1Q Median
                                  3Q
      Min
                                          Max
## -452.68 -184.62
                    -26.84 167.65
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 539.42
                            38.08 14.17 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for gaussian family taken to be 72487.54)
##
##
      Null deviance: 3551890 on 49 degrees of freedom
## Residual deviance: 3551890 on 49 degrees of freedom
## AIC: 704.44
##
## Number of Fisher Scoring iterations: 2
anova(null.lm)
## Analysis of Deviance Table
##
## Model: gaussian, link: identity
## Response: VIOLRATE
##
## Terms added sequentially (first to last)
##
##
##
       Df Deviance Resid. Df Resid. Dev
## NULL
                          49
                                3551890
model2.6.lm <- glm(formula = VIOLRATE ~ UNEMPRAT + DENSITY + GSPROD, data = usdata)</pre>
summary(model2.6.lm)
##
## Call:
## glm(formula = VIOLRATE ~ UNEMPRAT + DENSITY + GSPROD, data = usdata)
## Deviance Residuals:
##
      Min
                    Median
                                  3Q
                1Q
## -419.24 -148.27
                   -25.04
                            140.42
                                       461.35
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.204e+01 1.380e+02 0.450 0.65506
## UNEMPRAT
              7.206e+01 2.626e+01
                                   2.744 0.00862 **
              4.212e-02 1.404e-01 0.300 0.76547
## DENSITY
## GSPROD
              6.587e-04 2.004e-04 3.286 0.00195 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for gaussian family taken to be 50893.5)
```

```
##
## Null deviance: 3551890 on 49 degrees of freedom
## Residual deviance: 2341101 on 46 degrees of freedom
## AIC: 689.6
##
## Number of Fisher Scoring iterations: 2

DevianceMu <- deviance(model2.6.lm)
ln_LMu <- DevianceMu/-2
Rsq_McF <- 1 - (ln_LMu - 4)/ln_LMc

Pseudo_R_Squared <- (DevianceMc - DevianceMu)/DevianceMc

AIC = DevianceMu + 8 # Not matching

BIC = DevianceMu - (nrow(usdata) - 4) * log(nrow(usdata))</pre>
```