# Assignment 1

*Sri Seshadri*

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## Problem 1

### Decision Variable:

$S_{ij}$ = number of servings of category $i$ and item $j$. Where $i \in \{1 = Vegetables, 2 = Meat, 3 = Desert\}$ and $j$ for $S_{1j}$ is such that $j \in \{1 = peas, 2 = Greenbeans, 3 = Okra, 4 = Corn, 5 = Macaroni, 6 = Rice\}$; $j$ for $S_{2j}$ is such that $j \in \{1 = Chicken, 2 = Beef, 3 = Fish\}$ and $j$ for $S_{3j}$ is such that $j \in \{1 = Orange, 2 = Apple, 3 = Pudding, 4 = Jello\}$

### Objective function

If the cost for each item is denoted by $c_{ij}$, then the objective function can be $Minimize\ \Sigma_{i=1}^{3}\Sigma_{j=1}^{J}S_{ij}C_{ij}$

#### in explicit form

$Minimize\ z = 0.1S_{11} + 0.12S_{12} + 0.13S_{13} + 0.09S_{14} + 0.1S_{15} + 0.07S_{16} + 0.7S_{21} + 1.2S_{22} + 0.63S_{23} + 0.28S_{31} + 0.42S_{32} + 0.15S_{33} + 0.12S_{34}$

### Constraints

Let Carbs for item $I_{ij}$ be $Cr_{ij}$; likewise vitamins be $V_{ij}$; Protein be $P_{ij}$; Fat be $F_{ij}$

Minimal requirement for Carbs in diet $\Sigma_{i=1}^{3}\Sigma_{j=1}^{J}S_{ij}Cr_{ij} >= 5$ Minimal requirement for Vitamins $\Sigma_{i=1}^{3}\Sigma_{j=1}^{J}S_{ij}V_{ij} >= 10$ Minimal requirement for protein $\Sigma_{i=1}^{3}\Sigma_{j=1}^{J}S_{ij}P_{ij} >= 10$ Minimal requirement for fat $\Sigma_{i=1}^{3}\Sigma_{j=1}^{J}S_{ij}F_{ij} >= 2$

Atleast one equivalent serving per category

$\Sigma_{j=1}^{J}S_{1j} >= 1$, $\Sigma_{j=1}^{J}S_{2j} >= 1$, $\Sigma_{j=1}^{J}S_{3j} >= 1$

#### constraints in explicit form

$S_{11} + S_{12} + S_{13} + 2S_{14} + 4S_{15} + 5S_{16} + 2S_{21} + 3S_{22} + 3S_{23} + S_{31} + S_{32} + S_{33} + S_{34} >= 5$

$3S_{11} + 5S_{12} + 5S_{13} + 6S_{14} + 2S_{15} + 1S_{16} + 1S_{21} + 8S_{22} + 6S_{23} + 3S_{31} + 2S_{32} + 0S_{33} + 0S_{34} >= 10$

$S_{11} + 2S_{12} + S_{13} + S_{14} + S_{15} + S_{16} + 3S_{21} + 5S_{22} + 6S_{23} + 1S_{31} + 0S_{32} + 0S_{33} + 0S_{34} >= 10$

$0S_{11} + 0S_{12} + 0S_{13} + 2S_{14} + 1S_{15} + 1S_{16} + 1S_{21} + 2S_{22} + 1S_{23} + 0S_{31} + 0S_{32} + 0S_{33} + 0S_{34} >= 2$

$S_{11} + S_{12} + S_{13} + S_{14} + S_{15} + S_{16} >= 1$

$S_{21} + S_{22} + S_{23} >= 1$

$S_{31} + S_{32} + S_{33} + S_{34} >= 1$

## Problem 2

### Decision Variable:

$X_{ij}$ = Tons of fuel of type $i \in \{1, 2, .., m\}$ used in plant $j \in \{1, 2, ..., n\}$

### Other variables:

$b_j$ = Total energy used in plant j BTU/day

$e_{ij}$ = effluent emission per ton of fuel type $i$ at plant $j$

$c_i$ = cost per ton of fuel of type $i$

$a_{ij}$ = BTU generated at plant $j$ using one ton of fuel of type $i$

### Objective function:

$Minimize\ z = \sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} C_i$

### Constraints:

Total energy needed in a plant :

$\sum_{i=1}^{m} X_{ij} a_{ij} = b_j \forall j \in \{1, 2, ..., n\}$

Air pollution per region constraint:

$\sum_{j=1}^{n} \gamma_j \sum_{i=1}^{m} e_{ij} X_{ij} <= b$

Non-negative constraints:

$X_{ij} >= 0$

$b_j >= 0$

$e_{ij} >= 0$

$a_{ij} >= 0$

$c_i >= 0$

## Problem 3

### Decision variables:

$X_a$,$X_b$,$X_c$ and $X_d$ be the numbers of products A,B,C and D respectively to be produced.

### Objective function:

$Maximize\ z = 18X_a + 15X_b + 13X_c + 14X_d$

### Constraints:

Contractual constraints and non -negative constraints on the number to be produced:

$X_a >= 200$ ; $X_d >= 300$ ; $X_b >= 0$ ; $X_c >= 0$

Production time constraints:

$3X_a + X_b + 2X_c + X_d <= 40 * 60$

$8X_a + 12X_b + 6X_c + 7X_d <= 80 * 60$

$10X_a + 6X_b + 9X_c + 7X_d <= 80 * 60$

$X_a + X_b + X_c + X_d <= 20 * 60$

$3X_a + 5X_b + 3X_c + 2X_d <= 40 * 60$

## Problem 4(a)

$Maximize\ z = \frac{4x_1+x_2-3x_4+1}{2x_1+x_3+4x_4+3}$

subject to $x_1 - 2x_2 + x_3 + 2x_4 <= 10$

$x_2 - x_3 + 5x_4 <= 12$

$x_i >= 0, i = 1, ..4$

let r $= \frac{1}{2x_1+x_3+4x_4+3}$

let $y_1 = x_1 r, y_2 = x_2 r, y_4 = x_4 r$

### Therefore objective function is :

$Maximize\ 4y_1 + y_2 - 3y_4 + r$

Now $x_3$ in terms are r can be written as :

$r(2x_1 + x_3 + 4x_4 + 3) = 1$

$x_3 r = 1 - 2y_1 - 4y_4 - 3r$

### Rewritting the first constraint in terms of y

$y_1 + 2y_2 + 2y_4 >= 1 - 13r$

### likewise the second constraint is

$y_2 + 9y_4 + 2y_1 - 1 <= 9r$

#### Also the non-zero constraints in terms of y are:

$y_1 >= 0, y_2 >= 0, y_4 >= 0$

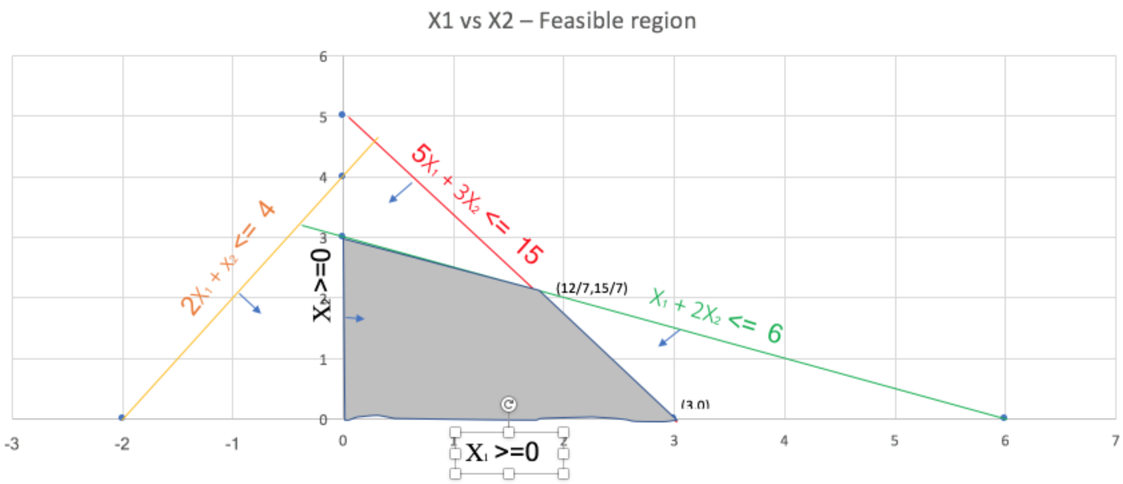


Figure 1: Graphical method for LP solution

# Problem 4(b)

### Objective :

$Minimize\ 5X_1 + 4X_2$

### subject to

$X_1 + 2X_2 <= 6$

$-2X_1 + x_2 <= 4$

$5X_1 + 3X_2 <= 15$

$X_1, X_2 >= 0$

**The objective function is maximized at (12/7,15/7) yielding a value of 17.14.**

# Problem 5

### Decision variables :

$x_1, x_2, x_3$ and $x_4$ as number of stocks to be invested in stock 1 through 4 respectively.

### Objective :

$Minimize\ 0.82x_1 + 3.04x_2 + 1.08x_3 + 8x_4$ -> minimum risk investment investment

## Subject to

### Total investment constraint:

$30x_1 + 45x_2 + 27x_3 + 53x_4 <= 100000$

let $r = \frac{1}{30x_1+45x_2+27x_3+53x_4}$

### at least 10% return constraint

$\frac{2.9X_1+5.42x_2+2.6x_3+20x_4}{30x_1+45x_2+27x_3+53x_4} >= 0.1$

if we let $y_1 = x_1 r$; $y_2 = x_2 r$; $y_3 = x_3 r$ and $y_4 = x_4 r$

then $2.9y_1 + 5.42y_2 + 2.6y_3 + 20y_4 >= 0.1$

### at least 10% of investment in Stock 4

$\frac{53x_4}{30x_1+45x_2+27x_3+53x_4} >= 0.1$ or

$53x_4 r >= 0.1$

$y_4 >= \frac{1}{530}$

## Non negative constraints

$y_1, y_2, y_3$ and $y_4 >= 0$

## re-write objective

$Minimize\ 0.82\frac{y_1}{r} + 3.04\frac{y_2}{r} + 1.08\frac{y_3}{r} + 8\frac{y_4}{r}$

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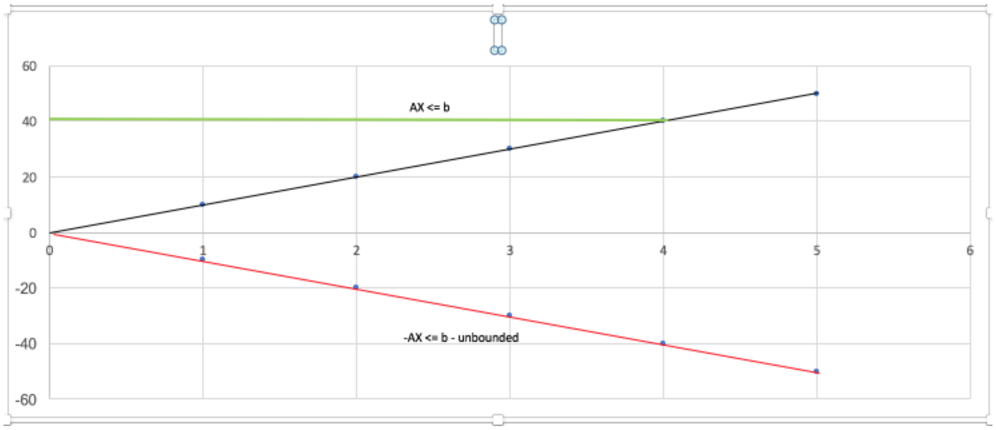


Figure 2:

## Extra credit

The objective is to maximize $cx$, where $c \neq 0$

Subject to $Ax <= b$, $x >= 0$

if $A < 0\$$ then we have an unbounded constraint as shown below as a red line. Hence any point other than infinite would not be optimal.

if $A > 0$ then x would be maximum when $Ax = b$ there by increasing the objective function $cx$. Any point $x_0$ such that $Ax_0 < b$ would be less than $x$ when $Ax = b$. Hence $x_0$ cannot be the optimal point. This scenario is illustrated by the black line in the plot below with b as a positive upper boundary condition.