Assignment 3

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Problem 1

Here there are two scenarios that will be inspected using Integer Linear Programming (ILP) to model total cost. Scenario 1 - plant in Baltimore, Scenario 2 - plant in Seattle.

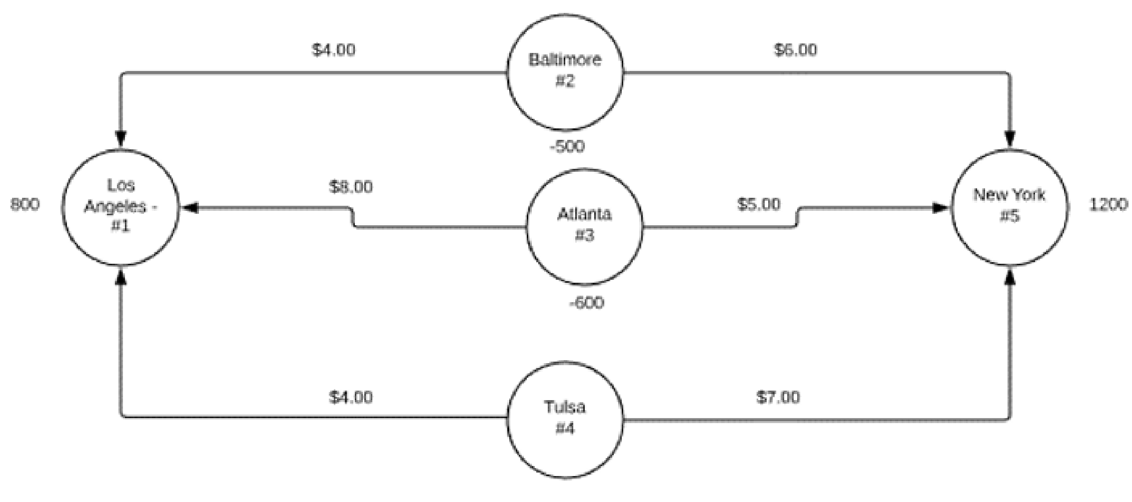
Executive Summary:

Both Scenarios yield a total cost of $9,900, therefore any location between Baltimore and Seattle can be chosen.

Scenario 1

Formulation:

The model is formulated as a network flow model as shown in the figure below, where nodes 1 through 5 are Los Angeles, Baltimore, Atlanta, Tulsa and New York respectively



Decision variables:

Let X_{ij} be the flow from node i to j where $i \in \{2, 3, 4\}$ and $j \in \{1, 5\}$

X_{ij} are the decision variables.

Other variables

Let C_{ij} be the cost variable for distribution of toys between X_ij Let D_i be the supply at i and D_j be the demand at j. D_i is denoted with a negative number and the D_j as positive number for modeling as a network flow problem.

Objective function

$Min\ TotalCost = X_{ij}C_{ij}$

Constraints:

Since the supply equals the demand ($\sum_i D_i + \sum_j D_j = 0$), the model will be constrained as $Inflow - Outflow = Supply\ or\ Demand$.

The constraints in explicit form are:

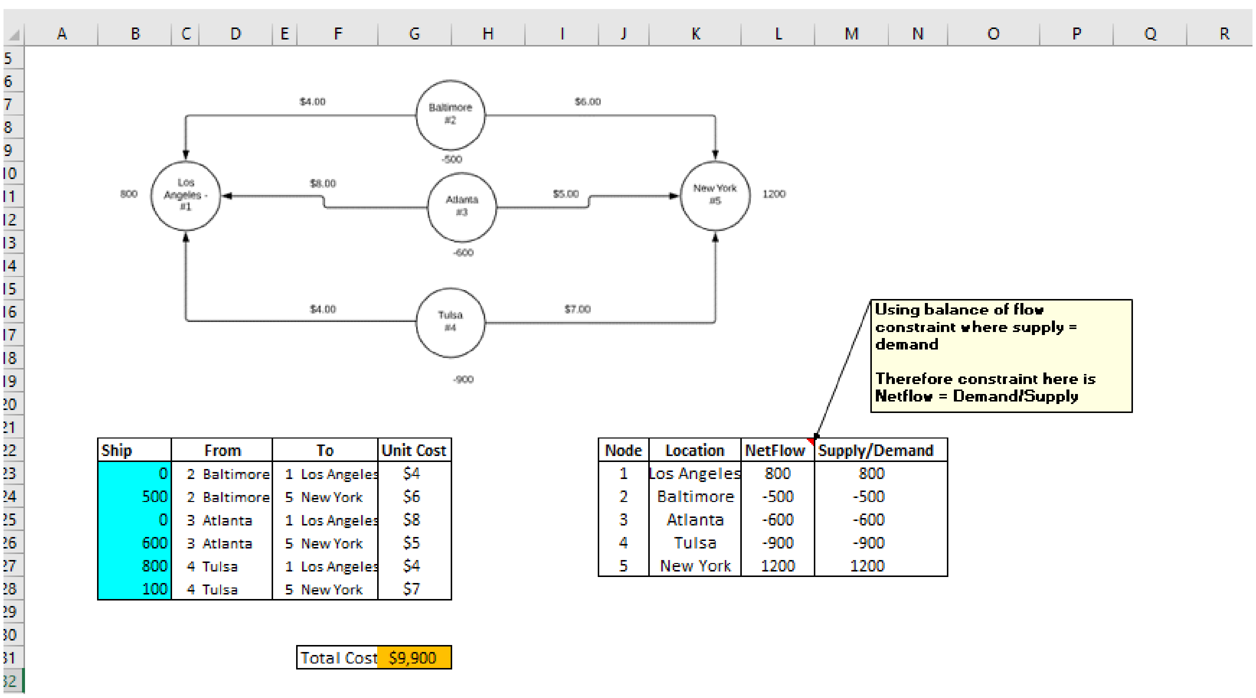
$X_{21} + X_{31} + X_{41} - 0 = D_1$ where $D_1 = 800$ $X_{25} + X_{35} + X_{45} - 0 = D_5$ where $D_5 = 1200$

$0 - X_{21} - X_{25} = D_2$ Where $D_2 = -500$ $0 - X_{31} - X_{35} = D_3$ Where $D_3 = -600$ $0 - X_{41} - X_{45} = D_4$ Where $D_2 = -900$

$X_{ij} >= 0$ and X_{ij} are integers

ASME Modeling

Figures 1 and 2 show the model set up in ASPE



Ship	From	To	Unit Cost
0	2 Baltimore	1 Los Angeles	$4
500	2 Baltimore	5 New York	$6
0	3 Atlanta	1 Los Angeles	$8
600	3 Atlanta	5 New York	$5
800	4 Tulsa	1 Los Angeles	$4
100	4 Tulsa	5 New York	$7

Node	Location	NetFlow	Supply/Demand
1	Los Angeles	800	800
2	Baltimore	-500	-500
3	Atlanta	-600	-600
4	Tulsa	-900	-900
5	New York	1200	1200

Figure 1: ASPE formulation - Baltimore scenario

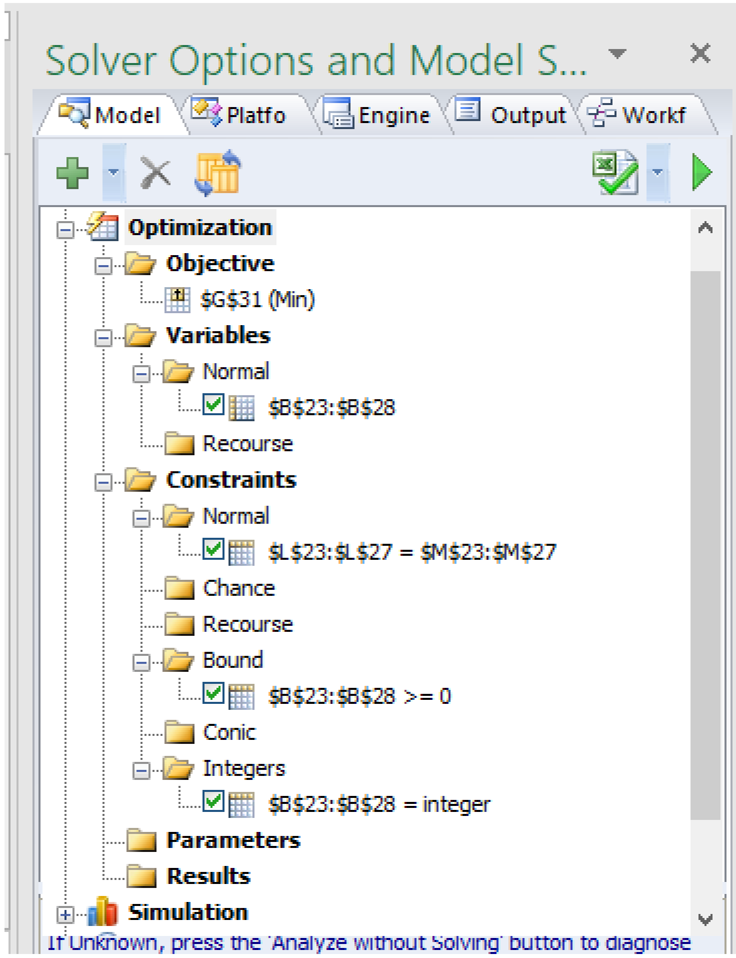


Figure 2: ASPE Model Setup - Baltimore scenario

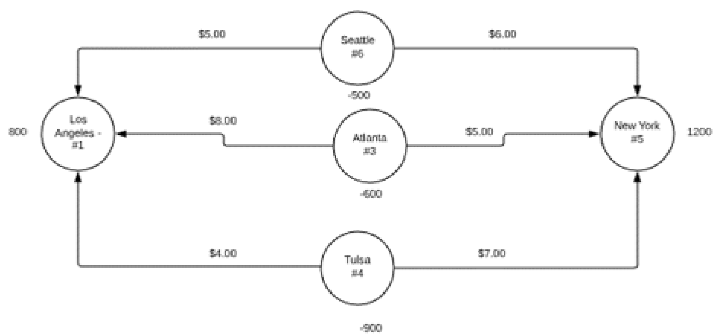
Scenario 1 Results

The Total cost for a plant in Baltimore was $9900

Scenario 2

This is scenario is identical to the previous scenario but the node 2 is replaced with node 6 (seattle) and its respective costs for distribution to Node 1 and 5. The formulation is not repeated here fro brevity. The model set up is shown in figures 3 and 4

The Total cost for a plant in Seattle was $9900. Therefore the either of the scenarios would work for the Toy company.



Using balance of flow constraint where supply = demand

Therefore constraint here is Netflow = Demand/Supply

Ship	From	To	Unit Cost
0	6 Seattle	1 Los Angeles	$5
500	6 Seattle	5 New York	$6
0	3 Atlanta	1 Los Angeles	$8
600	3 Atlanta	5 New York	$5
800	4 Tulsa	1 Los Angeles	$4
100	4 Tulsa	5 New York	$7

Node	Location	NetFlow	Supply/Demand
1	Los Angeles	800	800
6	Seattle	-500	-500
3	Atlanta	-600	-600
4	Tulsa	-900	-900
5	New York	1200	1200

Total Cost $9,900

Figure 3: ASPE formulation - Seattle scenario

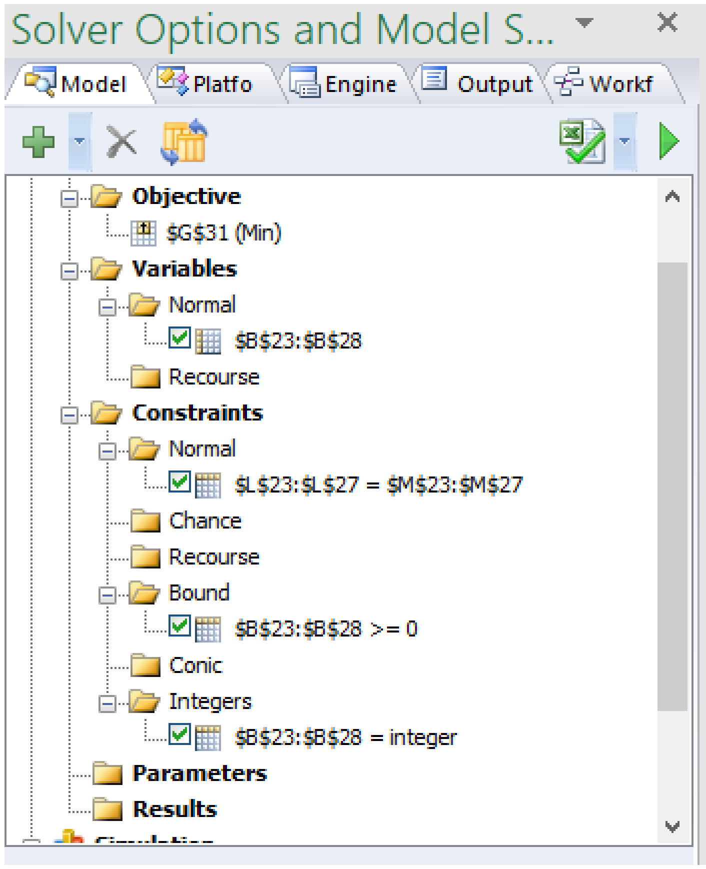


Figure 4: ASPE Model Setup - Seattle scenario

Problem 2

In this problem, the economical combination of four meats for hot dog is formuated as a linear program. The units of weight measures are standardized to grams and all figures are reported in grams except for calories and cost.

Executive Summary:

The economical combination would be 14.175 grams of beef and pork and 28.35 grams of turkey. The cost of 2 ounce hot dog would be $0.086

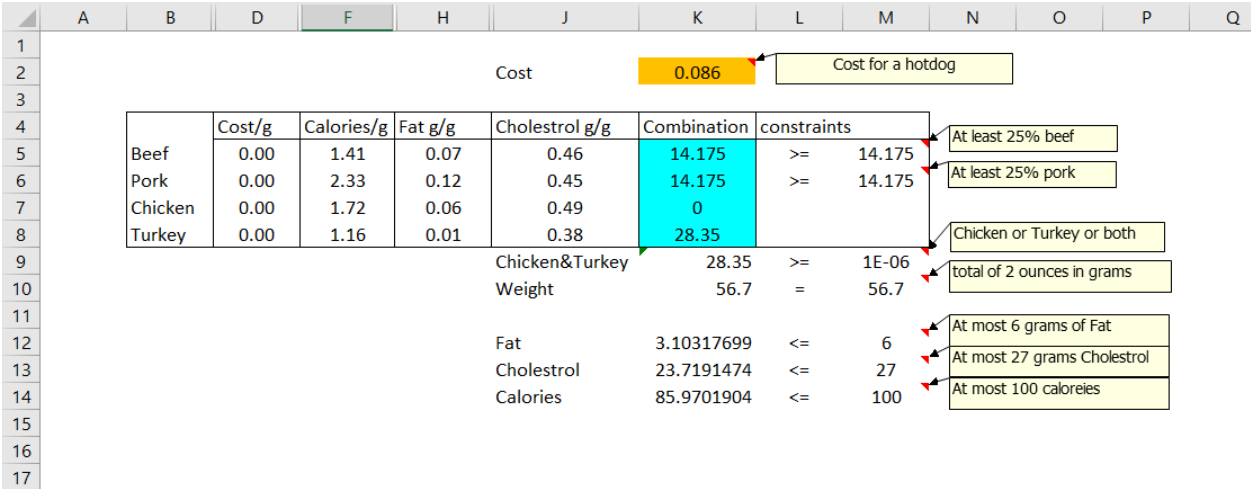
Formulation

Decision variables

Let X_i be the amount of meat used for hot dog where $i \in \{Beef, Pork, Chicken, Turkey\}$.

Other variables

Let D_i, K_i, F_i, L_i be the Cost, Calories, Fat(grams) and Cholestrol(grams) per gram of hot dog respectively where $i \in \{Beef, Pork, Chicken, Turkey\}$.



	Cost/g	Calories/g	Fat g/g	Cholestrol g/g	Combination	constraints	
Beef	0.00	1.41	0.07	0.46	14.175	>=	14.175
Pork	0.00	2.33	0.12	0.45	14.175	>=	14.175
Chicken	0.00	1.72	0.06	0.49	0		
Turkey	0.00	1.16	0.01	0.38	28.35		
				Chicken&Turkey	28.35	>=	1E-06
				Weight	56.7	=	56.7
				Fat	3.10317699	<=	6
				Cholestrol	23.7191474	<=	27
				Calories	85.9701904	<=	100

Figure 5: Problem 2 ASPE formulation

Objective function

$Min \sum_i X_i D_i$

Constraints

$\sum_i X_i = 56.7$ (2 ounces = 56.7 grams) $X_{Chicken} + X_{Turkey} > 0$ (Use of either Chicken or Turkey or both)

Note that in the ASPE, there is no greater than constraint and so greater than on equal to a very small number (0.0000001) was used to model.

$\sum_i X_i F_i <= 6$ Total Fat constraint

$\sum_i X_i L_i <= 27$ Total Cholestrol constraint

$\sum_i X_i K_i <= 100$ Total Calories constraint

$X_{ij} >= 0$

ASPE Modeling

Figures 5 and 6 show the model set up

Result

The economical combination would be 14.175 grams of beef and pork and 28.35 grams of turkey. The cost of 2 ounce hot dog would be \$0.086

Problem 3

Executive Summary

For 5 Surgery problem, the schedule is as shown in figure 6 with a total setup time of 58. The cost is shown next to the arcs

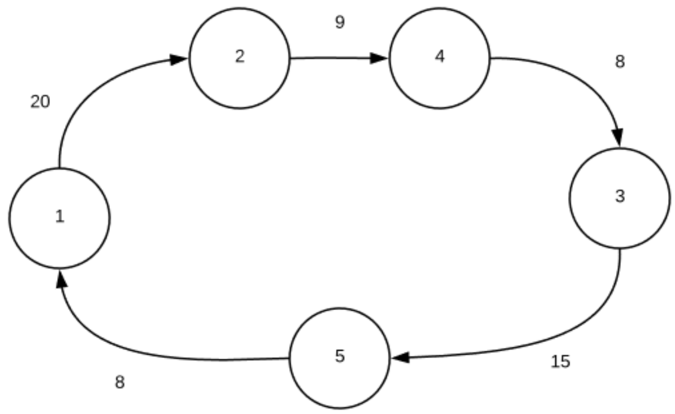


Figure 6: 5 surgery schedule

For the 10 surgery problem, the schedule is shown in figure 7 with a total setup time of 92.

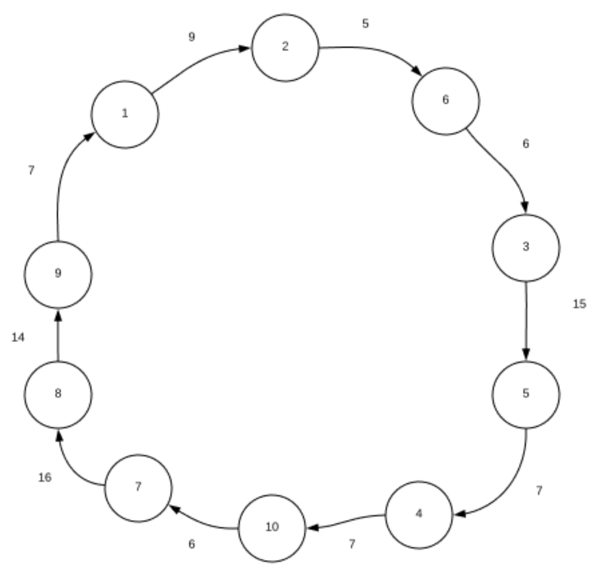


Figure 7: 10 surgery schedule

Formulation

The formulation is not reproduced here from the paper. However an attempt was made to make the schedule continuous in ASPE by using a dummy cost parameter.

$dummyCost_{i=j} = 1000 - 1000 * \sum_i X_{ij} \; \forall_j \in N$

That is, if a schedule ends in node 2, $\sum_i X_{i2} = 1$, then $dummyCost_{i=2} = 0$, which encourages the model to schedule a surgery starting from 2. Else the cost would be 1000.

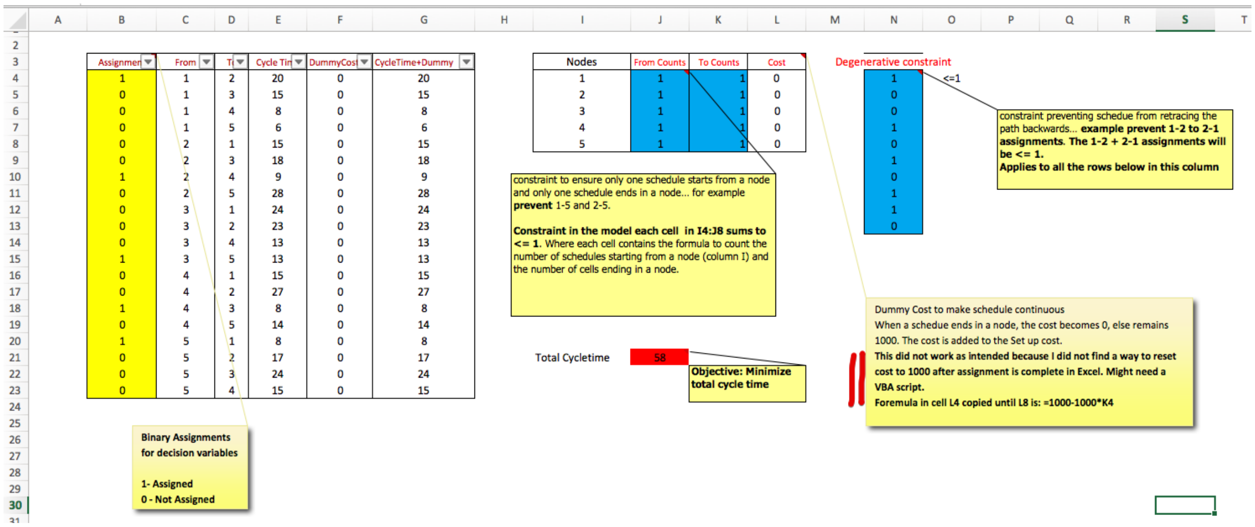
Also, We need to prevent schedules that re-traces a path example 1->5 -> 1

$X_{12} + X_{21} <= 1$ $X_{13} + X_{31} <= 1$ $X_{14} + X_{41} <= 1$ $X_{15} + X_{51} <= 1$ $X_{23} + X_{32} <= 1$

So on . . .

ASPE Model set up for 5 surgery problem

Figures 8 and 9 show the ASPE model set up



Assignmen	From	T	Cycle Tin	DummyCost	CycleTime+Dummy
1	1	2	20	0	20
0	1	3	15	0	15
0	1	4	8	0	8
0	1	5	6	0	6
0	2	1	15	0	15
0	2	3	18	0	18
1	2	4	9	0	9
0	2	5	28	0	28
0	3	1	24	0	24
0	3	2	23	0	23
0	3	4	13	0	13
1	3	5	13	0	13
0	4	1	15	0	15
0	4	2	27	0	27
1	4	3	8	0	8
0	4	5	14	0	14
1	5	1	8	0	8
0	5	2	17	0	17
0	5	3	24	0	24
0	5	4	15	0	15

Nodes	From Counts	To Counts	Cost
1	1	1	0
2	1	1	0
3	1	1	0
4	1	1	0
5	1	1	0

Degenerative constraint
1
0
0
1
0
1
0
1
1
0

Figure 8: 5 surgery ASPE formulation

5 Surgery problem result

The schedule is 1->2->4->3->5 With a total setup time of 58

ASPE Model set up for 10 surgery problem

The model set up identical to the 5 surgery problem (as shown in figure 11 and 12). For brevity the model is shown by filtering on $X_{ij} = 1$. The dummy cost did not help with keeping the schedule continuous. There was a degenerative schedule as a result of this type of model. the schedule is shown in figure 10.

When $X_{51} = 0$ constraint was added to the model, then a plausible schedule resulted.

10 Surgery problem result

As shown in figure 10, the schedule is

1->2 -> 6 -> 3 -> 5 -> 4 -> 10 -> 7 -> 8 -> 9 -> 1, with a total setup time of 92.

Problem 4

Data

	Tank	Truck	Turtle	Available
Plastic	1.5	2.0	1	16000
Rubber	0.5	0.5	1	5000
Metal	0.3	0.6	0	9000
Labor	2.0	2.0	1	40
Cost	7.0	5.0	4	164000

Goals

- Minimize over-utilization of Plastic, Rubber and Metal with twice the emphasis on Plastic.
- Minimize the under and over utilizations of the budget
- Maximize labor utilization

Decision Variables

Let X_1, X_2 and X_3 be the number of Tanks, Trucks and Turtles made.

Goal Constraint:

$1.5X_1 + 2X_2 + X_3 - d_p^+ + d_p^- = 16000$ (plastic usage)

$0.5X_1 + 0.5X_2 + X_3 - d_r^+ + d_r^- = 5000$ (rubber usage)

$0.3X_1 + 0.6X_2 - d_m^+ + d_m^- = 9000$ (metal usage)

$2X_1 + 2X_2 + X_3 - d_l^+ + d_l^- = 40$ (labor usage)

$7X_1 + 5X_2 + 4X_3 - d_c^+ + d_c^- = 164000$ (budget usage)

Where $d_p^+, d_r^+, d_m^+, d_l^+, d_c^+$ are over achieving deviational varibles and $d_p^-, d_r^-, d_m^-, d_l^-, d_c^-$ are under achieving deviational varibles

$d_p^+, d_r^+, d_m^+, d_l^+, d_c^+, d_p^-, d_r^-, d_m^-, d_l^-, d_c^- >= 0$ and integer $X_i >= 0$ and integer

Objective function

$Min\ w_r \frac{(d_p^+)}{5000} + w_m \frac{d_m^+}{9000} + w_p \frac{d_p^+}{16000} + w_l \frac{d_l^-}{40} + w_c \frac{d_c^+ + d_c^-}{164000}$

where the $w_r, w_m, w_l, w_c = 1$ and $w_p = 2$

Problem 5

Goals

- Achieve total exposures of atleast 750,000 persons
- Avoid expenditures of more than \$100,000

- Avoid expenditures of > \$70,000 for TV advertisements
- Achieve at least 1 Million total expenditures
- Reach at least 250,000 persons in each of the two age groups 18-21 and 25-30.
- In addition purchasing power of 25-30 is twice as much of 18-21 group.

Decision Variables

X_1 be the dollar amount spent on TV campaign **in 1000s of dollars**

X_2 be the dollar amount spent on Radio campaign **in 1000s of dollars**

Goal constraints

$5500X_1 + 4500X_2 + d_p^- - d_p^+ = 750000$ (goal 1)

$X_1 + X_2 + d_c^- - d_c^+ = 100$ (goal 2)

$X_1 + d_t^- - d_t^+ = 70$ (goal 3)

$10000X_1 + 7500X_2 + d_e^- - d_e^+ = 1000000$ (goal 4)

$2500X_1 + 3000X_2 + d_{18-21}^- + d_{18-21}^+ = 250000$ (goal 5)

$3000X_1 + 1500X_2 + d_{25-30}^- + d_{25-30}^+ = 250000$ (goal 5)

Objective function

$Min\ w_1 \frac{d_p^-}{750000} + w_2 \frac{d_c^+}{100} + w_3 \frac{d_t^+}{70} + w_4 \frac{d_e^-}{1000000} + w_5 \frac{0.5*d_{18-21}^- + d_{25-30}^-}{250000}$

$w_1 = 5, w_2 = 4, w_3 = 3, w_4 = 2, w_1 = 1$

Further there is a weight of 0.5 was given to d_{18-21}^- to account for purchasing power of 18-21 group being half of 25-30.

Extra Credit

a) The different patterns that may be used are

- 2 5ft pieces cut
- 3 3ft pieces cut
- 2 4ft pieces cut
- 2 3ft pieces and 1 4ft piece cut
- 1 4ft piece and 1 5ft piece cut
- 1 3ft piece and 1 5ft piece cut

b) Formulation

Decision Variables:

Let X_i be the number of boards used to cut in the pattern i, where i belongs to the patterns described in the same order as above.

Objective

$Min \sum_i X_i$

Constraints

$3X_2 + 2X_4 + x_6 >= 90$ (number of 3 feet board requirement)

$2X_3 + X_4 + X_5 >= 60$ (number of 4 feet board requirement)

$2X_1 + X_5 + X_6 >= 60$ (number of 5 feet board requirement)

$X_i > 0$ and $integers$

ASME Formulation

Result

- *30 boards* for 2 5ft pieces cut
- *None* for 3 3ft pieces cut
- *8 boards* for 2 4ft pieces cut
- *45 boards* for 2 3ft pieces and 1 4ft piece cut
- *None* for 1 4ft piece and 1 5ft piece cut
- *None* for 1 3ft piece and 1 5ft piece cut

The total boards required is 83

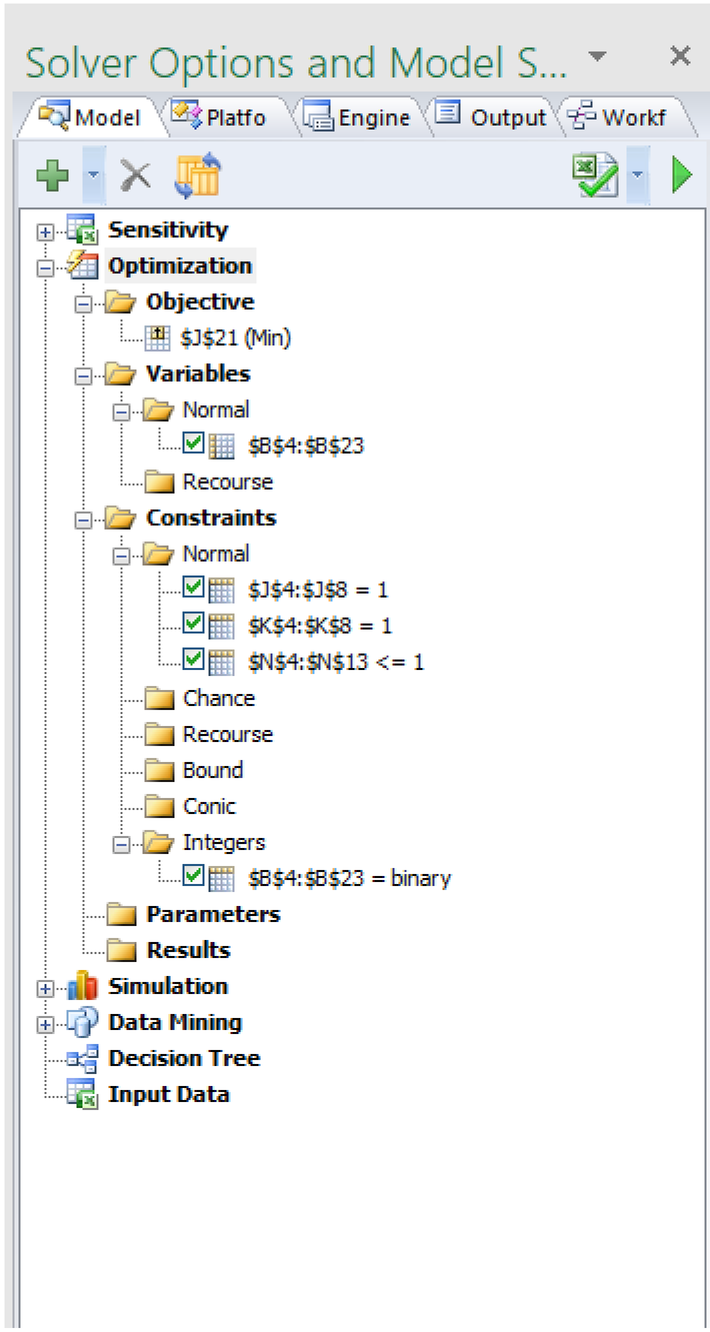


Figure 9: 5 surgery ASPE model set up

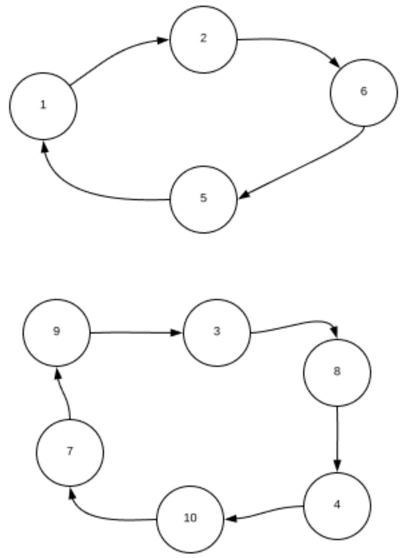


Figure 10: 10 surgery degenerative schedule

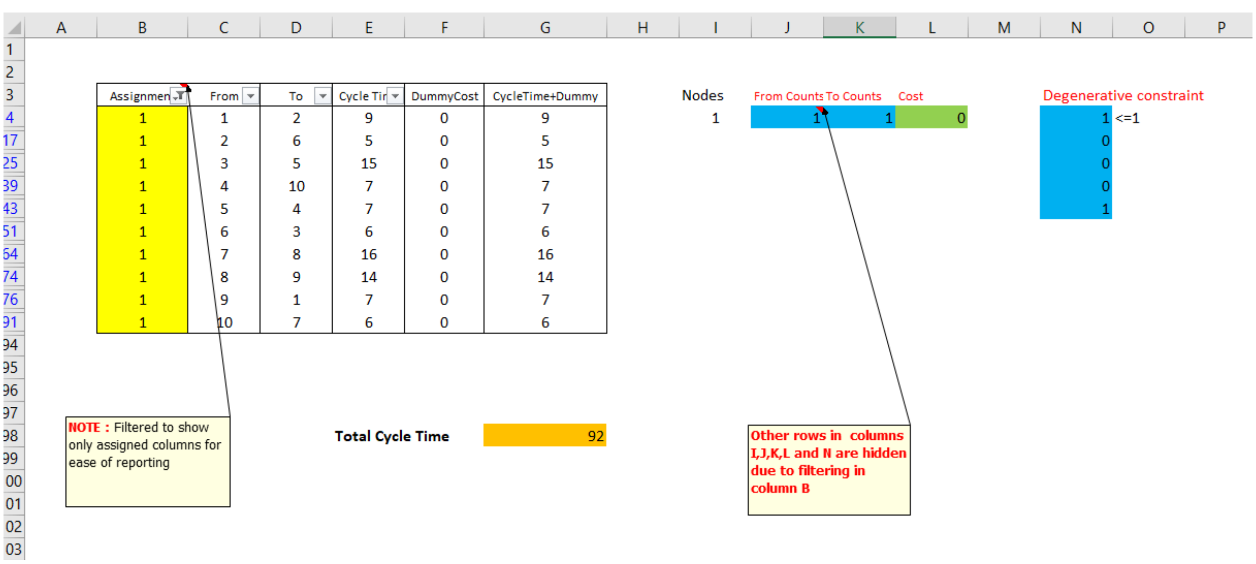


Figure 11: 10 surgery ASPE formulation

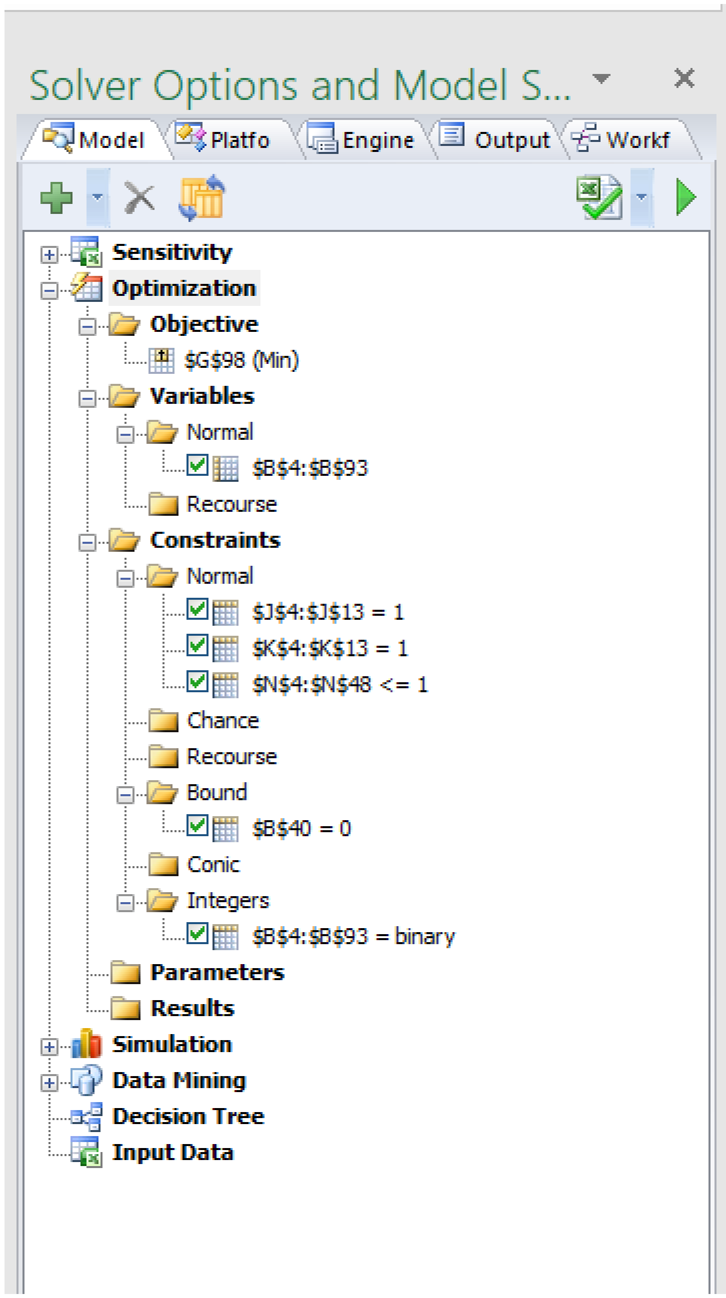


Figure 12: 10 surgery ASPE model set up

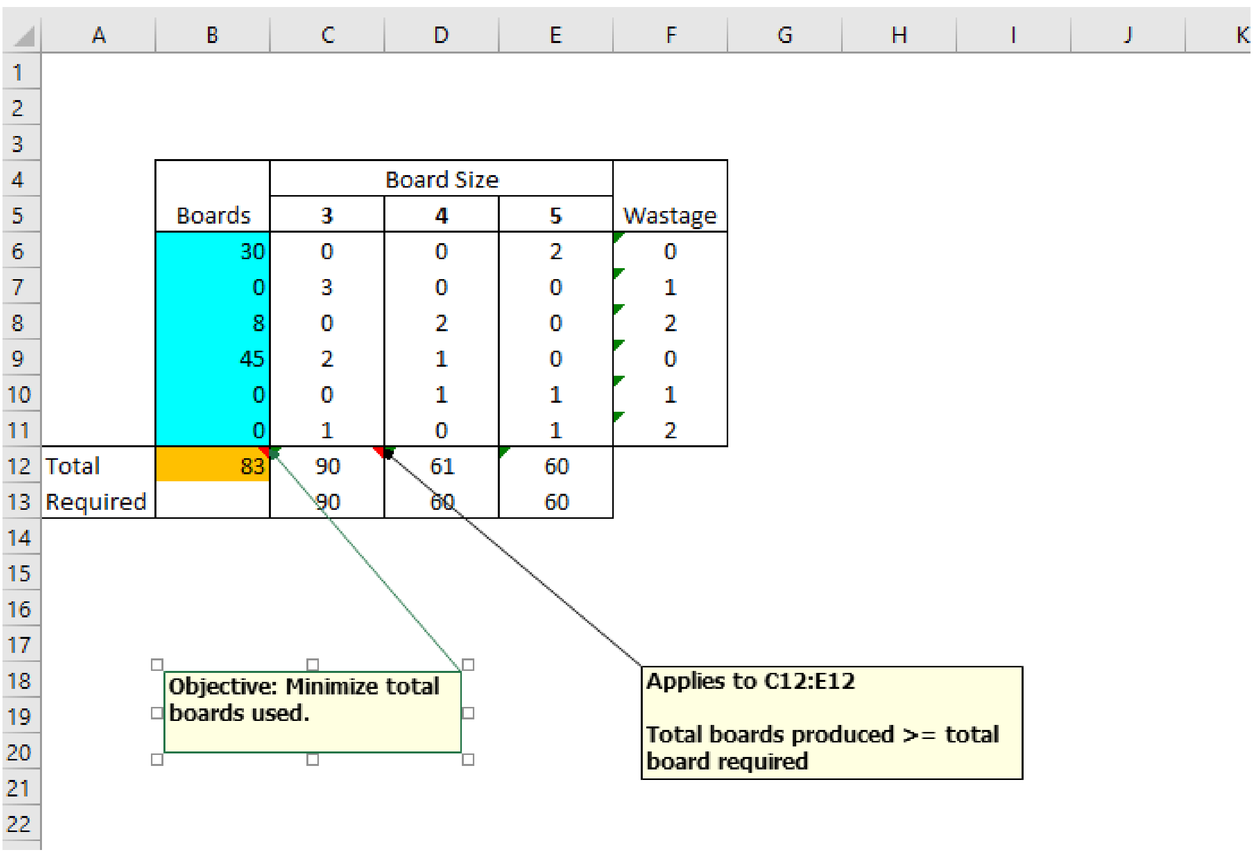


Figure 13: Lumberyard ASPE Model setup

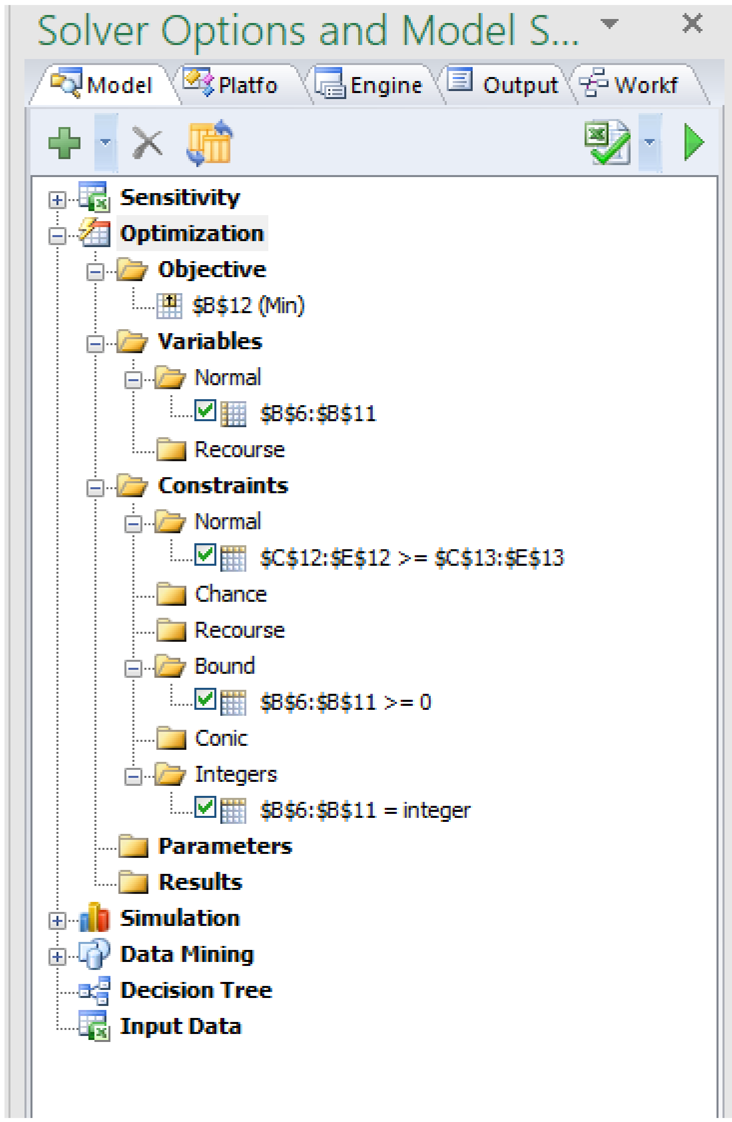


Figure 14: Lumberyard ASPE Model constraints