

Assignment 4

Sri Seshadri

8/19/2018

Problem 1

Linear Model

In this problem a linear model $y = ax + b$ is fit, where y is Average Revenue and x is Hours of operation. The strategy was to set a and b to be *decision variables* and the *minimize the mean absolute deviation* between the fitted model and the actual average revenue.

The *LSRG* solver in the educational license of ASPE was not available and hence a default non-linear optimization engine was chosen with multiple start points. The model set up is shown in figure 1.

The final model is $y = 4890 + 39.33x$, where y is Average Revenue and x is Hours of operation.

For 120 hours of operation the average revenue based on the linear model is \$9610

Non -linear model

Here the model $y = ax^b$ is fit to the same data, the strategy here was similar to the above, where we minimize the mean absolute deviation between fitted and actuals.

The value of $a = 1113.03$ and $b = 0.46$; thus

$y = 1113.03x^{0.46}$ where y is Average Revenue and x is Hours of operation.

For 120 hours of operation the average revenue based on the non linear model is \$9897.29

Comparing the Mean absolute deviation of either models; 678.5 for linear and 571.65 for non linear model, the non linear model prediction seems ideal. However one needs to be mindful of overfitting.

	A	B	C	D	E	F	G	H	I	J	K
1		Hours of Operation	Average Revenue	Fitted	Abs Deviation		Intercept	4890.00	Decision Variables: Intercept and Slope		
2		40	5958	6463.333333	505.3333333		Slope	39.33			
3		44	6662	6620.666667	41.33333333						
4		48	6004	6778	774						
5		48	6011	6778	767						
6		60	7250	7250	0						
7		70	8632	7643.333333	988.6666667						
8		72	6964	7722	758						
9		90	11097	8430	2667						
10		100	9107	8823.333333	283.6666667						
11		168	11498	11498	0						
12				Mean Abs Deviation	678.5	Objective Funtion : Minimize Mean absolute deviation					
13											
14		Revenue prediction for 120 hrs of operation				9610					
15											
16											
17											

Figure 1: Linear Model

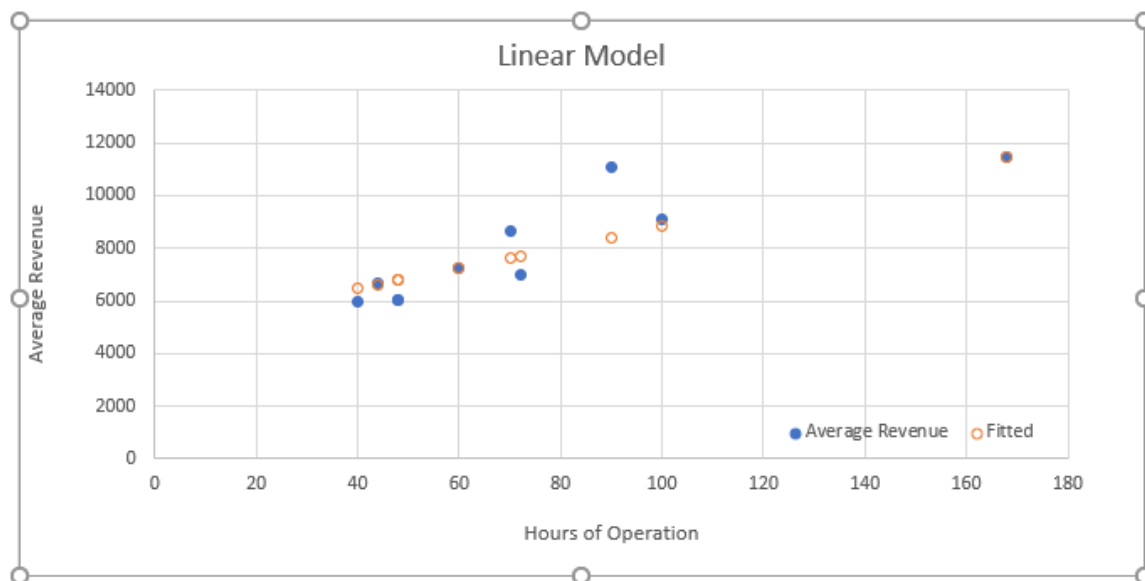


Figure 2: Linear Model plot

	A	B	C	D	E	F	G	H	I	J	K
1		Jours of Operation	Average Revenue	Fitted	Abs Deviation		a	1113.03	Decision variables for y = aX^b		
2		40	5958	5994.34	36.339		b	0.46			
3		44	6662	6260.87	401.135						
4		48	6004	6514.52	510.519						
5		48	6011	6514.52	503.519						
6		60	7250	7212.99	37.007						
7		70	8632	7738.78	893.222						
8		72	6964	7838.93	874.927						
9		90	11097	8679.40	2417.600						
10		100	9107	9106.99	0.006						
11		168	11498	11540.22	42.220						
12			Minimize MAE		571.65	Objective function - Minimize mean absolute deviation					
13											
14											
15											
16		Revenue prediction for 120 hrs of operation			9897.29						
17											
18											

Figure 3: Non Linear Model

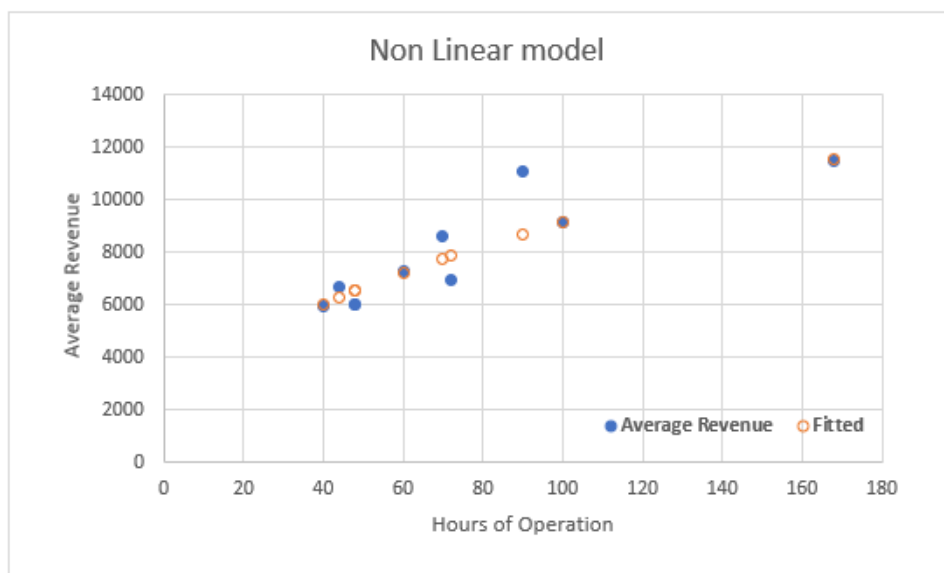


Figure 4: Non-Linear Model plot

	A	B	C	D	E	F	G	H	I	J
1		Effort	Sales	Fitted	Residuals	a	28.15937	Decision Variables, parameters of the model		
2		0	50	28.15937	21.84063	b	1073644			
3		25	53	59.48726	-6.48726	c	0.598183			
4		50	55	75.5831	-20.5831	d	235033.1			
5		75	75	88.59948	-13.5995					
6		100	100	99.94821	0.051794					
7		125	120	110.1988	9.801187					
8		150	127	119.6515	7.348495					
9		175	132	128.4884	3.511596					
10		200	135	136.8302	-1.83022					
11				SSE	1293.455	Objective function - Minimize SSE				
12		Predicted								
13		115		106.2076						
14										

Figure 5: S curve model

Problem 2

In this problem, an S curve model is fit for $S = a + \frac{(b-a)E^c}{(d+E^c)}$; where (a, b, c, d) are constants; E is effort in % and S being the Actual Sales (% of Current)

The optimization model is set such that a, b, c and d are *decision variables* and the objective function is sum of squared errors between the model fit and the actuals (residuals). The objective function is *minimized*.

a) Hence the model is :

$$S = 28.16 + \frac{(1073644 - 28.16)E^{0.598}}{(235033.1 + E^{0.598})}$$

b) The predicted value for 115% Effort based on this model is 106.2



Figure 6: S curve model plot

Problem 3

This problem is a modification of the example problem in Chapter 12. The *decision variables* are the Reorder point and Order Quantity. The goal is to find the optimal quantities for the decision variables that maximizes the average monthly profit.

Assumption made:

The Holding cost and ordering cost are NOT sunk in the profit margin (\$45 per unit)

Limitation of Education version of ASPE

Due to limitation of educational license of ASPE, the holding cost, ordering cost and opportunity cost are not shown in separate columns. They are collapsed into a single column where

The formula in K2 is $=E2*\$0\$7-(B2*\$0\$4+H2*\$0\$5+(MAX(D2-E2,0)*\$0\$6))$, which copied all the way down for column K. As shown in Figure 7.

Constraint

The Re-order point and Order quantity were constrained to take values ≥ 1 and constrained to be integers.

Uncertain inputs

The Demand and Lead time for delivery for orders were stochastic and is shown in Figures 8 and 9.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	Day	BeginInv	Received	Demand	Demand Satisfied	EndInv	Inventory	Order	LeadTime	Day of Arr	Profit			ReOrder Point	41				
2	1	50	0	6	6	44	44	0	0	0	255			Order Quantity	30				
3	2	44	0	3	3	41	41	0	0	0	121.8								
4	3	41	0	7	7	34	34	1	5	9	282.7			Holding cost per	0.3				
5	4	34	0	5	5	29	29	0	0	0	214.8			Ordering cost	20				
6	5	29	0	4	4	25	25	0	0	0	171.3			Stock out	65				
7	6	25	0	6	6	19	19	0	0	0	262.5			Profit per unit	45				
8	7	19	0	9	9	10	10	1	3	11	379.3								
9	8	10	0	8	8	2	2	0	0	0	357			Monthly Profit	5873.3				
10	9	2	30	8	8	24	24	0	0	0	359.4								
11	10	24	0	6	6	18	18	0	0	0	262.8			Average Monthly	6404.88				
12	11	18	30	9	9	39	39	1	4	16	379.6								
13	12	39	0	1	1	38	38	0	0	0	33.3								
14	13	38	0	2	2	36	36	0	0	0	78.6								
15	14	36	0	7	7	29	29	0	0	0	304.2								
16	15	29	0	6	6	23	23	0	0	0	261.3								
17	16	23	30	7	7	46	46	0	0	0	308.1								
18	17	46	0	5	5	41	41	0	0	0	211.2								
19	18	41	0	7	7	34	34	1	3	22	282.7								
20	19	34	0	3	3	31	31	0	0	0	124.8								
21	20	31	0	5	5	26	26	0	0	0	215.7								
22	21	26	0	5	5	21	21	0	0	0	217.2								
23	22	21	30	4	4	47	47	0	0	0	173.7								
24	23	47	0	5	5	42	42	0	0	0	210.9								

Figure 7: Inventory model

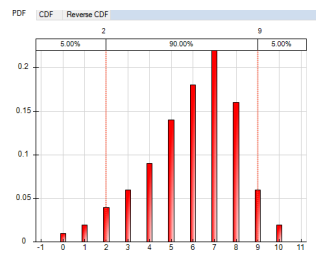


Figure 8: Inventory model - Demand

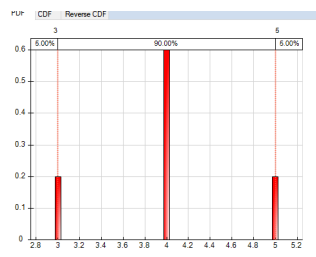


Figure 9: Inventory model - Lead Time

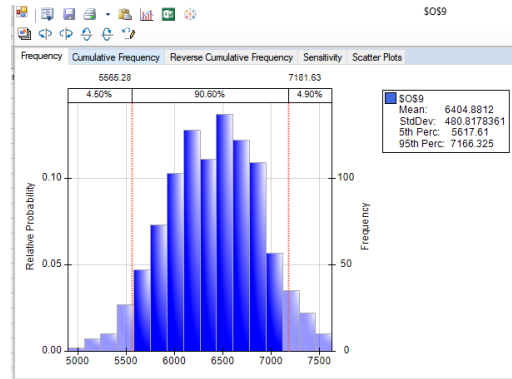


Figure 10: Inventory model - Lead Time

Objective function variability

The objective function variability is shown in Figure 10.

Optimal Values

The Reorder point of 41 and Order Quantity of 30 proved to optimal. (The values vary subtly based on seed of the random number generator, the seed used here is 123)

A	B	C	D	E	F	G	H
	Rooms Available	100					
	Price per Room	150					
	Var Cost per Occupancy	30					
	Prob. No Show	0.05					
	Compensation cost	200					
	Reservation Accepted	101					
	Rooms demanded	102					
	Rooms sold	101					
	Guests to check in	94					
	Revenue	14100					
	Var Cost	2820					
	Turning Away Cost	0					
	Marginal Profit	11280					

Figure 11: Hotel reservation model

Problem 4

In this problem, the goal is to find maximum number of reservations that can be accepted for hotel rooms. The hotel has a 100 rooms; there is a 5% chance of no show. When more guests show up than available rooms, there is a compensation of 200 dollars paid to the customer. It is assumed that the guest does not pay for the room that they are checked into. All guests pay (\$150) at check in if they have a room. There is variable cost of \$30 per room occupied.

Simulation set up

The simulation set up is shown in figure 11. There are 10 simulations run for different scenarios of accepted reservations ranging from 101 through 110. The Marginal profit (Revenue - variable cost - cost of turning away customers) is monitored for each of the simulation runs.

Assumptions

The demand for rooms is assumed to have a binomial distribution with a shift of 75. See figure 12.

Simulation out put

The results of the simulation is shown in figure 13. It is seen that the 5th simulation where maximum reservation accepted is 105 gives us the maximum marginal profit of \$11351.20.

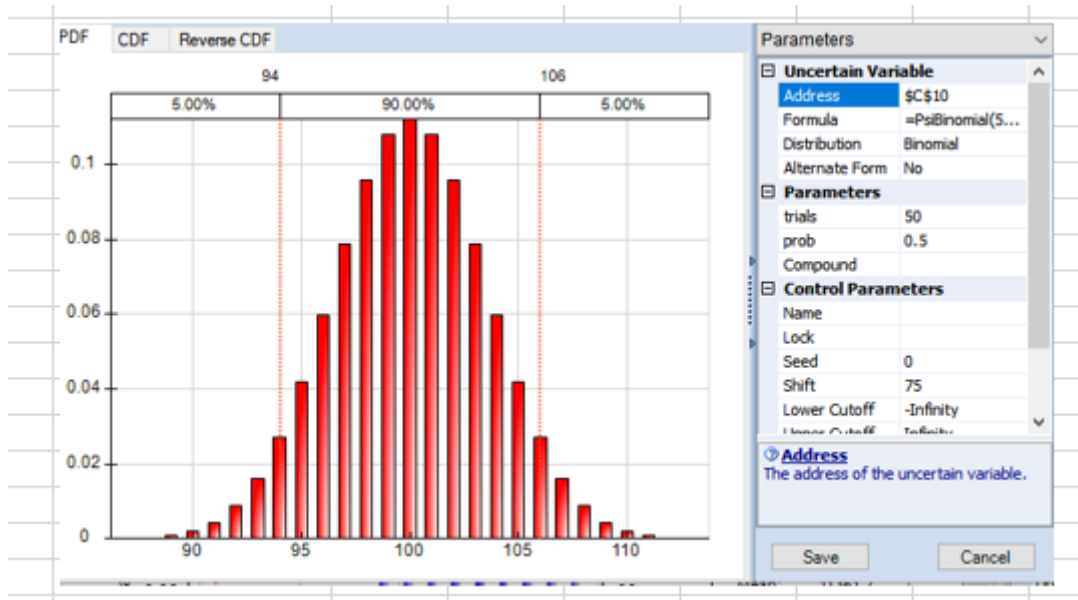


Figure 12: Hotel demand model

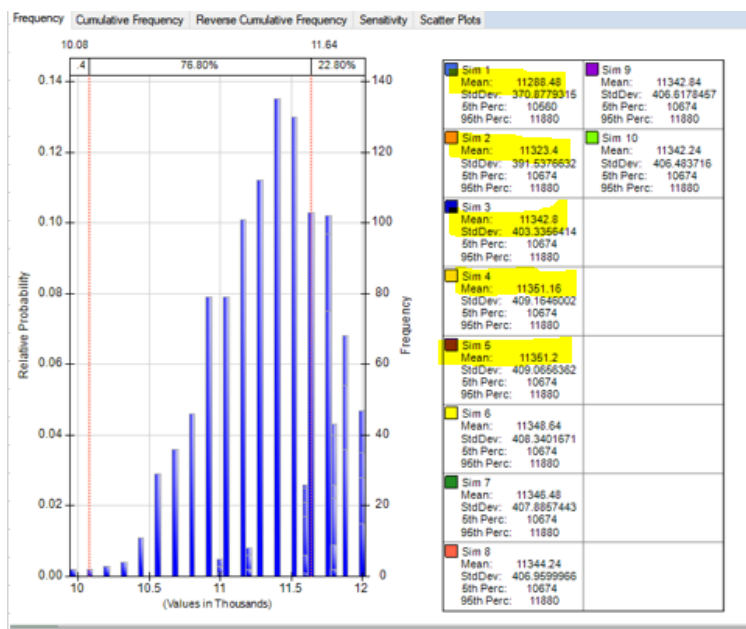


Figure 13: Simulation results

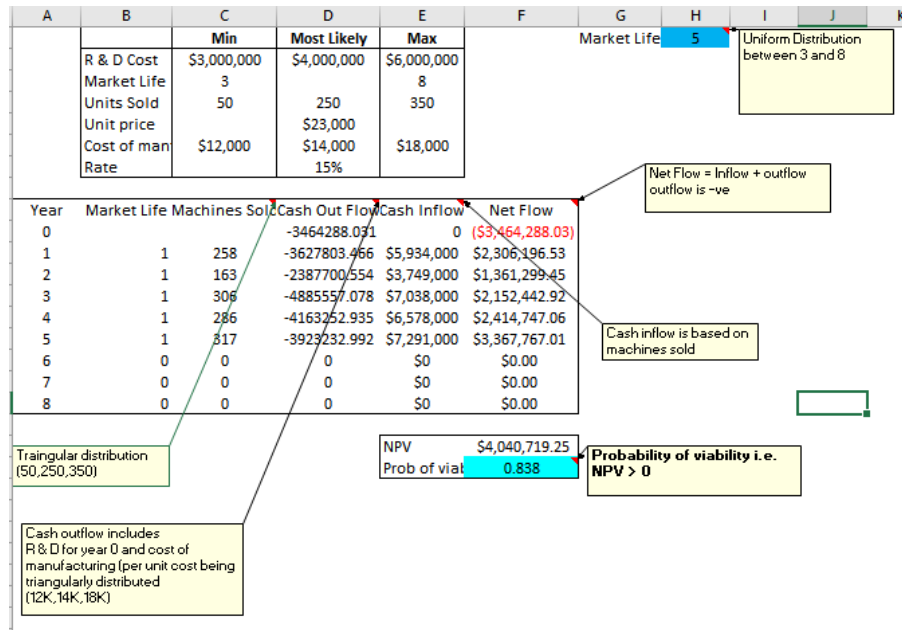


Figure 14: Product viability simulation model

Problem 5

In this problem the goal is see what is the probability of viability of introducing a prodcut to the market that has uncertain market life, manufacturing cost,unit to be sold per year. However the R & D investment, sale price and cost of capital are assumed to be certain. The net present value of the invenstment over the span of market life is simulated.

Assumption made.

The problem does not state clearly if the cost of manufacturing per machine randomly varies for each year of market life. The solution to this problem assumes it is so. Each year the manufacturing cost is assumed to have a triangular distribution of min = 12K, most likely = 14K and max = 18K.

The model set is shown in figure 14.

Simulation output

The Net Present Value (NPV) of the 1000 iterations of the simulation models is studied and the probability of NPV > 0 is evaluated.

It is seen that the probability of a viable product is 83.3%.

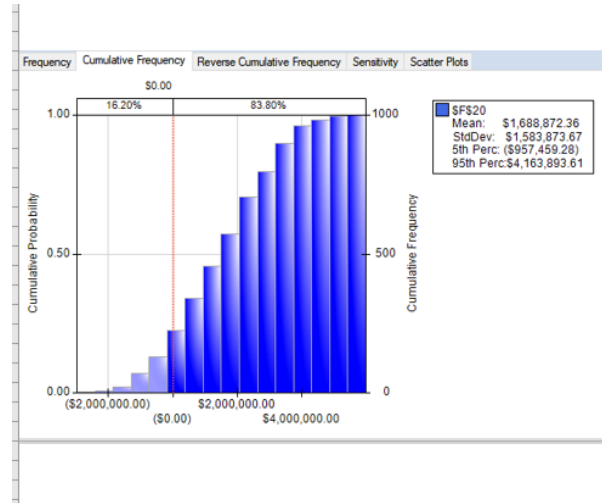


Figure 15: Product viability simulation result

Extra Credit

The below function **estimatepi**, takes in an argument for N and outputs a data frame that contain the estimate of π , standard error of the estimate and the 95% confidence interval around the estimate. The function generates N pairs of uniformly distributed random numbers between 0 and 1 for x and y coordinates. A subfunction **insidecircle** checks if the pair of points lie within a unit circle; by checking if $x^2 + y^2 \leq 1$. **insidecircle** returns a 1 if the pair of point is within the radius of the unit circle, else returns a 0.

Details of the code can be seen in the code chunk below.

```

#' This function estimates the value of pi using a unit circle.
#'
#' @param N, integer value denoting the number of random points to generate
#'
#' @return data frame with estimate of pi, standard error of estimate and the 95% confidence interval
#' @export
#'
#' @examples estimatepi(1000)
estimatepi <- function(N) {
  library(magrittr)
  library(dplyr)

  # Function to flag points that are inside unit circle.
  insidecircle <- function(x, y) {
    ifelse(x^2 + y^2 <= 1, 1, 0)
  }

  df <- data.frame(x = abs(runif(N)), y = abs(runif(N)))
  df %<>% dplyr::mutate(selected = insidecircle(x, y))

  # Get estimate of points that fall within a unit quarter-circle
  pi_estimate <- df %>% summarise(estimate = sum(selected)/n() *
    4) %>% as.numeric(.)
  # Store the proportion for ease of computation below
  prop <- sum(df$selected)/nrow(df)
  # CI <- prop.test(sum(df$selected),nrow(df))$conf.int

  StdErrorProp <- sqrt(prop * (1 - prop)/nrow(df))
  E <- qnorm(0.975) * StdErrorProp

  CI <- c(prop - E, prop + E)

  pi_se <- StdErrorProp * 4
  pi_CI <- CI * 4

  # Return estimate of pi, the standard error and the 95%
  # confidence interval

  data.frame(pi_estimate = pi_estimate, pi_StdError = pi_se,
    LowerConfLimit = pi_CI[1], UpperConfLimit = pi_CI[2])
}

```

The function `estimatepi` is run to estimate the value of π for N varying from 1000 to 10000 in steps of 500. The code chunk below shows the execution.

```

library(purrr)
library(ggplot2)
N <- seq(1000, 10000, 500)
set.seed(111)
results <- map_df(.x = N, .f = estimatepi) %>% mutate(N = N)

```

Figure 16 shows the confidence interval around the estimate of π against each value of N . The solid red line shows the true value of π and the dashed red lines show ± 0.05 (total band width of 0.1) from π .

It can be seen that from $N \geq 9000$, the estimate safely falls within ± 0.05 of true value of π .

Plot results

```
results %>% ggplot(mapping = aes(x = N, y = pi_estimate, ymin = LowerConfLimit,
  ymax = UpperConfLimit)) + geom_pointrange(alpha = 0.5) +
  geom_hline(yintercept = pi, col = "red") + geom_hline(yintercept = pi -
  0.05, lty = 2, col = "red") + geom_hline(yintercept = pi +
  0.05, lty = 2, col = "red") + geom_vline(xintercept = 8750,
  col = "blue") + theme_bw()
```

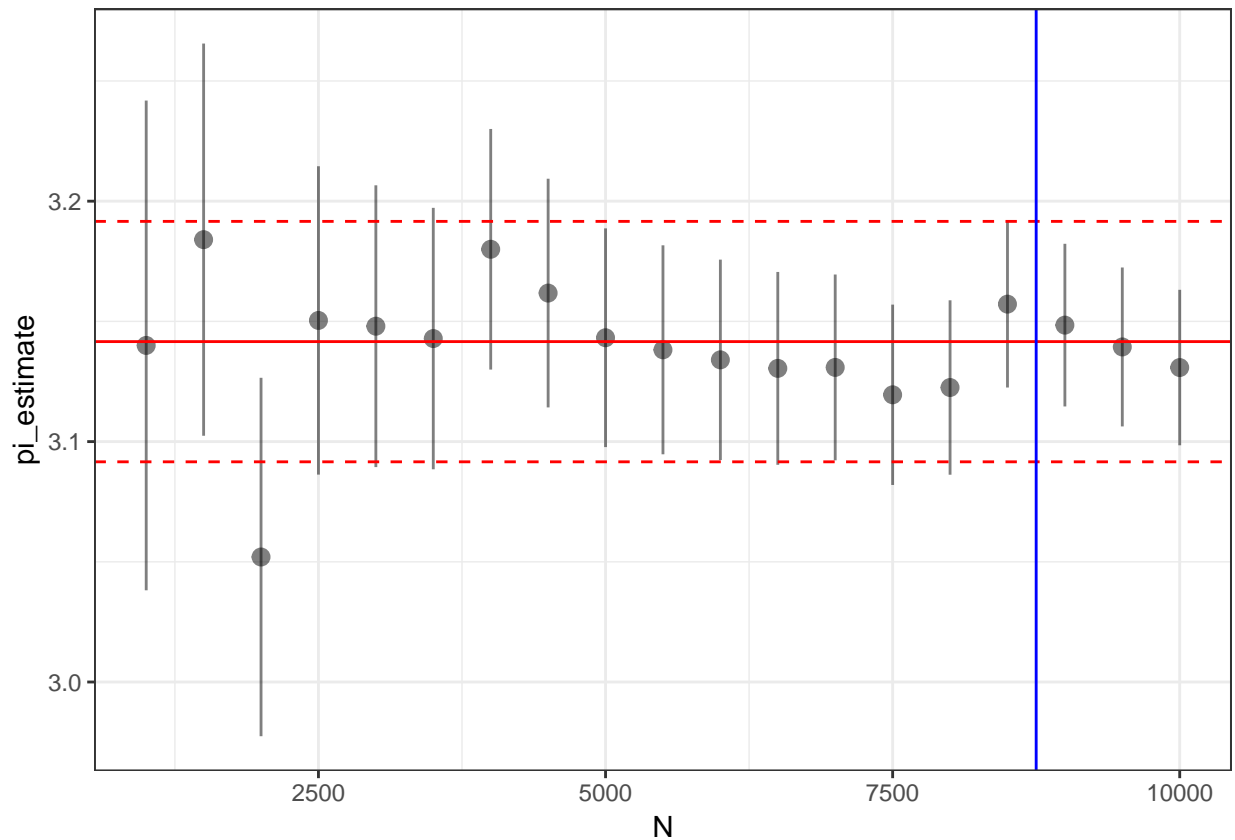


Figure 16: Estimate of π by N

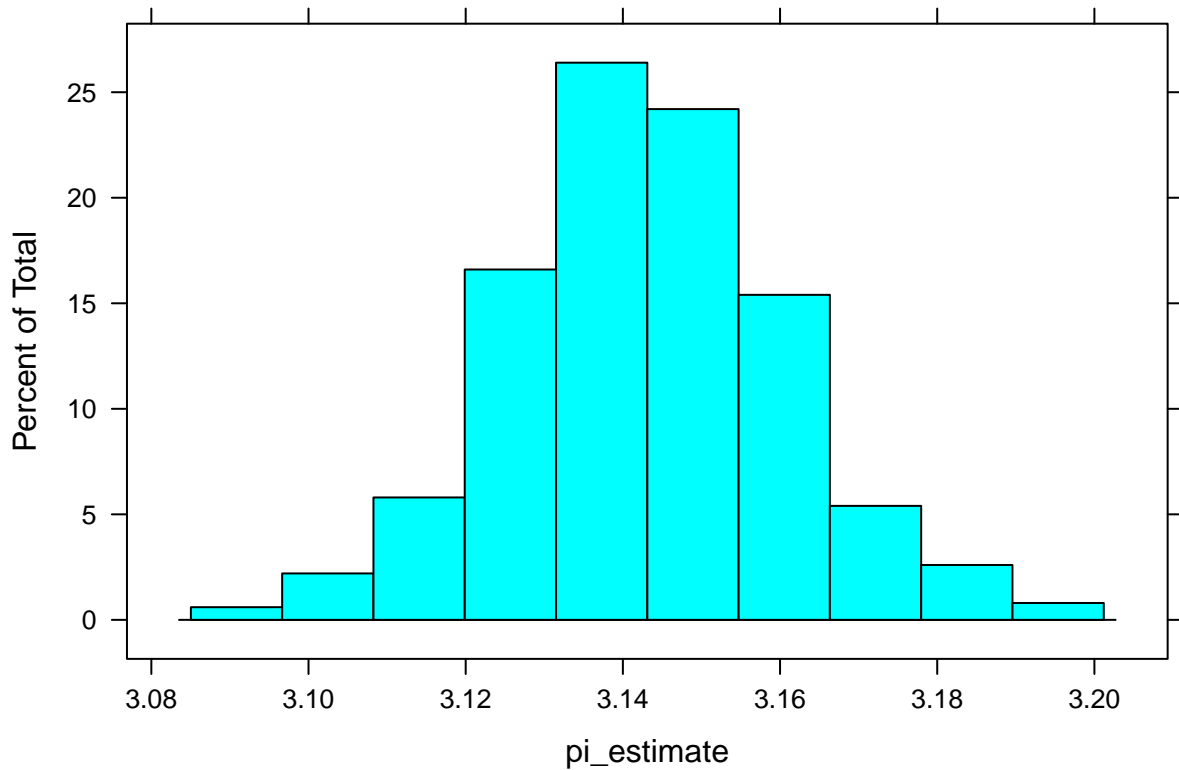
Repeated simulation for $N = 9000$

The estimate of π is simulated 500 times using a value of $N = 9000$. The histogram of the estimate is shown in figure 17. The distribution is normal looking as expected - due to central limit theorem.

```
N_optimal = rep(9000, 500)
```

500 simulation iterations

```
results_rep <- map_df(.x = N_optimal, .f = estimatepi)
lattice::histogram(x = ~pi_estimate, data = results_rep)
```



Validation

Table 1 shows the Average estimate of pi and the standard deviation of the estimate of pi. The standard error computed with $N = 9000$ previously is very close to that of the standard deviation of the estimate for the 500 simulations. of the estimate falls within the confidence interval estimated previously.

```
knitr::kable(data.frame(Average_Pi_estimate = mean(results_rep$pi_estimate),
  Pi_std.Dev = sd(results_rep$pi_estimate), pi_stdError = results$pi_StdError[results$N ==
    9000], PercentWithinCI = sum(between(results_rep$pi_estimate,
    results$LowerConfLimit[results$N == 9000], results$UpperConfLimit[results$N ==
    9000]))/500 * 100))
```

Average_Pi_estimate	Pi_std.Dev	pi_stdError	PercentWithinCI
3.14287	0.0177704	0.0172597	92.8