Runge Kutta Notes

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$$\boxed{\frac{dy}{dt} = e^{-t^2}}$$

$$y = \int e^{-t^2}\,dx$$

what does it mean to solve?

m → "solution" cannot be expressed in terms of commonly understood functions

- The problem is that sometimes, we can't solve differential equations.

can't solve analytically

Physical Examples of derivatives and integrals →

$a = \frac{dv}{dt}$ $v = \frac{dx}{dt}$

$\int v\,dx = x$

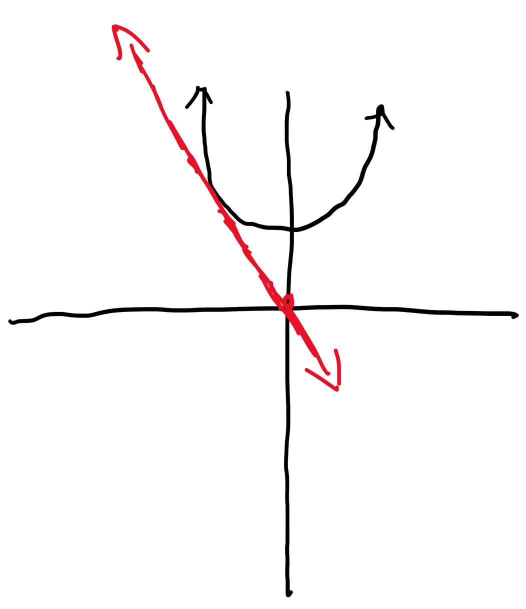
Consider a particle moving in 1-D with velocity $v(t) = e^{-t^2}$. Given that at $t=0$, $x=0$, what is the position of the particle at time $t=5$?

This can happen, this is real, there's just a lack of expressability. Sometimes, that's not our problem!

- In many fields, finding these exact solutions is not important!
  - Sometimes, its more important to broadly understand how the function behaves.
- So let's look at some ways of approximating functions. We have already seen a lot!
- Runge-Kutta Methods are ways of "numerically integrating" differential equations.

Prior experience

- Linearization?
  - Use tangent lines

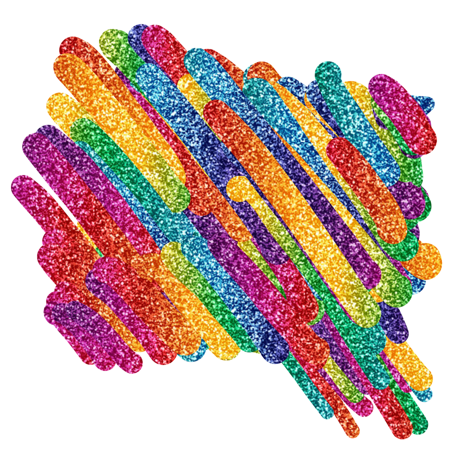


- Problems with linearization
  - only concerned with information at a single point

but its the right idea!

Euler's Method

- an iterative linearization process



Taylor Series

- Taylor methods →

advantages

- error can be bounded

Lagrange Error bound

Disadvantages

- Doesn't always converge

Higher order approximation methods

RK4

RK2

$k_1 =$