Lecture Notes

Saturday, May 14, 2022 9:03 AM

Brown

"Differentiation" vs. "differencing"

discrete

alternative representation

Taylor Expansion

- $y(t+\Delta t) = y(t) + y'(t)\,\Delta t + y''(t)\,\frac{\Delta t^2}{2!} + \dots$

error term is $\boxed{\frac{1}{n!}\, y^{n}(\tau)\,\Delta t^{n}}$

Lagrange error bound!

Euler's Method

Start with Taylor but only use the first 2 terms

$y(t+\Delta t) \approx y(t) + y'(t)\,\Delta t$

$\frac{y(t+\Delta t) - y(t)}{\Delta t} \approx y'(t)$

Interesting, definition of derivative

A refinement - tack on an error term $E(\Delta t)$

$y'(t) = \frac{y(t+\Delta t) - y(t)}{\Delta t} + E(\Delta t)$

discretize this process, and run it iteratively

Loop
1 → start at a point
2 → Compute the derivative at that point
3 → take a tiny step according to the slope at that point

Ideally, $\lim_{\Delta t \to 0} E(\Delta t) = 0$

Depends on the nature of the function, and error can accumulate over iterations

Local Truncation Error

$R_1(x_1)$