Modified Euler

Saturday, May 14, 2022 6:40 PM

average out the slopes

$$x_{n+1} = x_n + \frac{\Delta t}{2}\left(f(t, x_n) + f(t_n + \Delta t, x_{n+1})\right)$$

what we're trying to solve

Predictor - corrector method

predictor

$$\hat{y}_{n+1} = y_n + \Delta t\, f(t_n, x_n)$$

$$y_{n+1} = y_n + \frac{\Delta t}{2}\left(f(t_n, x_n) + f(t_n + \Delta t, \hat{y}_{n+1})\right)$$

corrector

show this first

Runge Kutta Method

$\boxed{RK2}$ → stages

$$K_1 = \Delta t\, f(t_n, x_n)$$

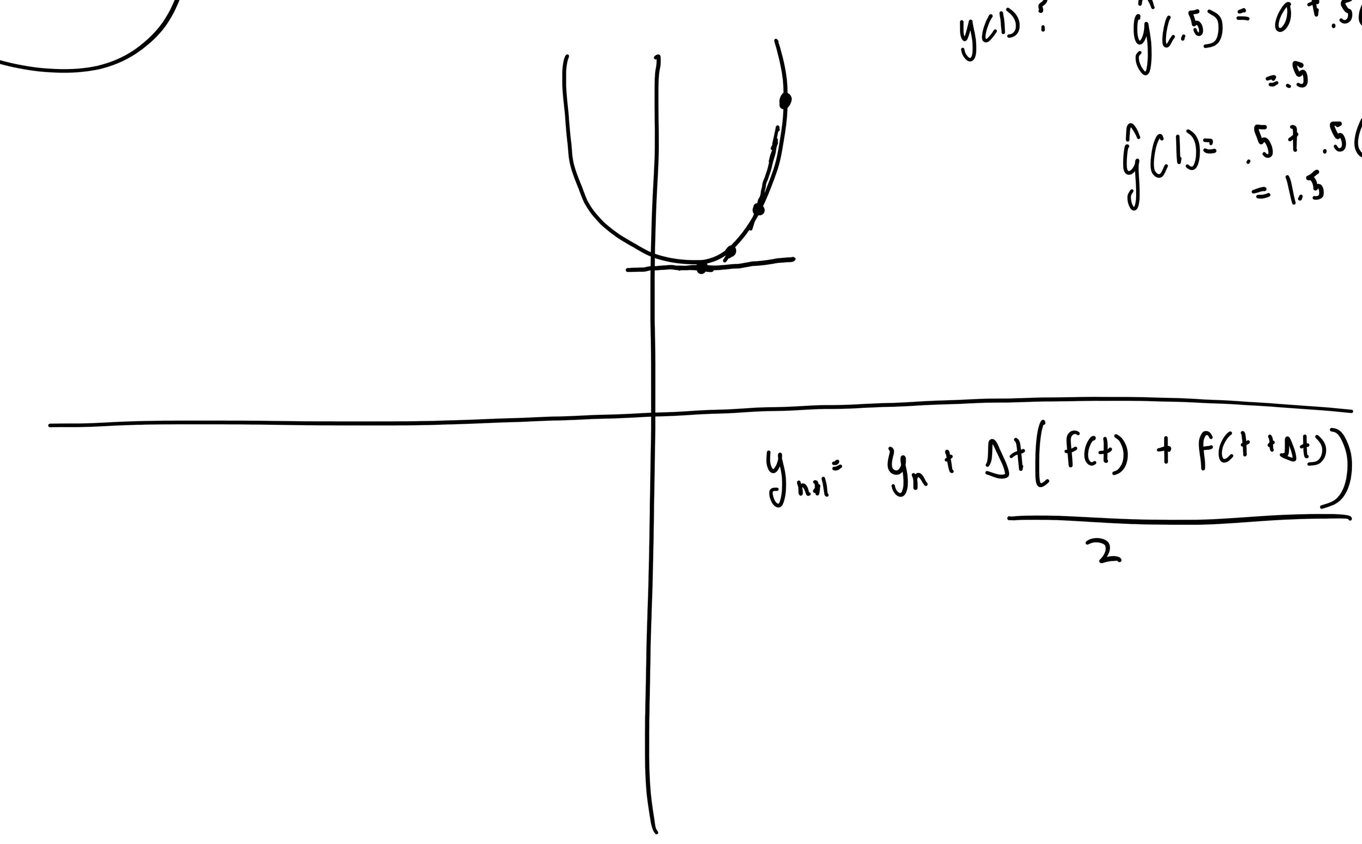
$$K_2 = \Delta t\, f(t_n + \Delta t, x_n + k_1)$$

$$\boxed{x_{n+1} = x_n + \frac{1}{2}(k_1 + k_2)}$$

$$\boxed{x_n + \frac{\Delta t}{2}(k_1 + k_2)}$$

- Basic Linearization
- Basic Taylor
- Basic Euler - Modify Euler

$$\frac{dy}{dt} = f(t)$$



$\frac{dy}{dt} = 2t$ $y(0) = 0$

$y(1)$? $\hat{y}(.5) = 0 + .5(1) = .5$

$\hat{y}(1) = .5 + .5(2) = 1.5$

$$y_{n+1} = y_n + \frac{\Delta t\left(f(t) + f(t + \Delta t)\right)}{2}$$

"centered at t"

what if we use derivative information at both t at $t + \Delta t$

$$y_{n+1} = y_n + \Delta t \cdot f(t)$$

$y(0) = 1$

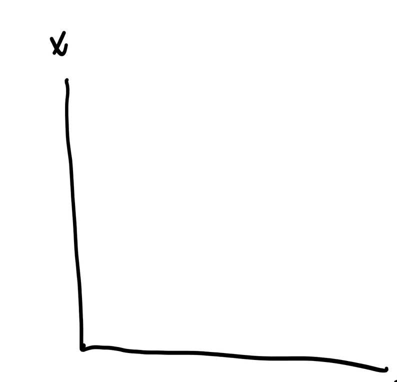
$\Delta t = 0.5$

$\hat{y}_1 = 1 + .5(0) = 1$

$\hat{y}_2 = 1 + .5(1) = \boxed{1.5}$

Modified Euler

$\frac{dy}{dt} = 2t$



$$f'(x) = xy$$

$$y' = -2xy^2$$

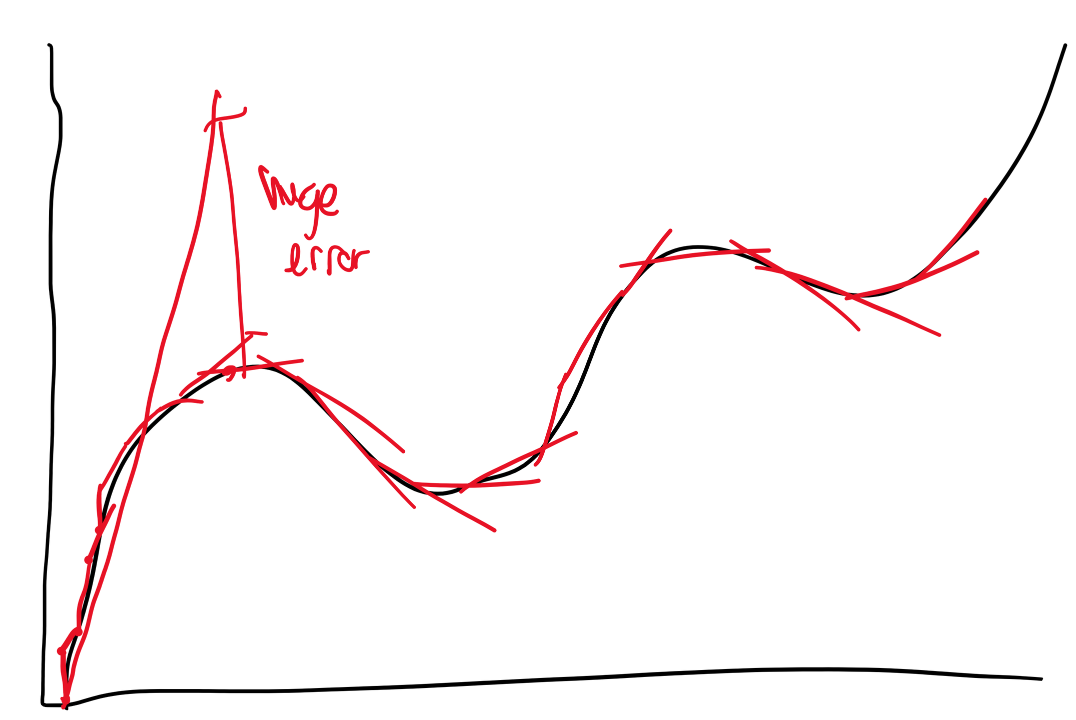
$y(0) = 1$

$$y' = \frac{dy}{dx} = f(t, y)$$

$\hat{y}_1 = 1 + .72$

$$y_{n+1} = y_n + \Delta t \cdot \frac{dy}{dx}$$

$$= y_n + \Delta t \cdot f(t, y)$$



error proportional to Δt

Talking about error

Taylor series

$$f(x) = f(a) + \frac{f'(a)}{1}(x-a) + \frac{f''(a)(x-a)^2}{2}$$

$$x - a = \Delta x$$

$$f(x) = f(a) + f'(a)\Delta x$$