RK2

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$k_1 = \Delta t \cdot f(t_i, y_i)$

$k_2 = \Delta t \cdot f(t_i + \alpha \Delta t, y_i + \beta k_1)$

4 parameters

$x_{n+1} = x_n + a k_1 + b k_2$

$x_{n+1} = x(t_n + \Delta t) = x(t_n)$

$\dot{x} = f(t, x)$

$k_1 = \Delta t f(t_n, x_n) \rightarrow$ Euler change term

$k_2 = \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$

steps put together

scalar multiple parameters

$x_{n+1} = x_n + a k_1 + b k_2$

Lines vs 4 parameters

- α, β
- $a\ b$

— constrain these to make Runge Kutta 2nd order in time

Taylor series

Compare taylor

$t_{n+1} = t_n + \Delta t$

$x(t_{n+1}) = x(t_n) + \Delta t \frac{dx}{dt}\Big|_{t_n} + \frac{\Delta t^2}{2} \frac{d^2 y}{dt^2}\Big|_{t_n} + O(\Delta t^3)$ → LTE

$\frac{d^2 x}{dt^2} = \frac{d f(t,x)}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial t}\frac{dx}{dx} = \frac{\partial f}{\partial t} + f \frac{\partial f}{\partial x}$

$x_{n+1} = x_n + \Delta t f(t_n, x_n) + \frac{\Delta t^2}{2}\left(\frac{\partial f}{\partial t} + f \frac{\partial f}{\partial x}\right)(t_n, x_n) + O(\Delta t^3)$

partial derivatives

Skip many steps time, etc

Taylor series to the Δt^2 term

(1) $x_{n+1} = x_n + \Delta t f(t_n, x_n) + \frac{1}{2}(\Delta t)^2 \left[f_t(t_n, x_n) + f(t_n, x_n) f_x(t_n, x_n) \right]$

$x_{n+1} = x_n + a \Delta t f(t_n, x_n) + b \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$

⋮

(2) $x_{n+1} = x_n + (a+b) \Delta t f(t_n, x_n) + (\Delta t)^2 \left[\alpha b f_t(t_n, x_n) + \beta b f(t_n, x_n) f_x(t_n, x_n) \right]$

$$\boxed{a + b = 1 \qquad \alpha b = 1/2 \qquad \beta b = 1/2}$$

Uses

Similar in accuracy

- accurate to 2nd order taylor

LTE proportional to Δt^3

Modified Euler

- $a = b = 1/2, \quad \alpha = \beta = 1$

gets modified Euler

Midpoint method

middle of the interval

$\alpha = 1/2$

then

$b = 1 \qquad a = 0$

$\beta = 1/2$