LQR Coupling and Steady-State Behavior – Reference Notes

1. System Overview

The system has four states: position, velocity, angle, and angular velocity. A Linear Quadratic Regulator (LQR) is used to stabilize the system using state feedback.

2. What is Coupling?

Coupling exists when one state appears in the differential equation of another state. In state-space form, this means off-diagonal elements in the A matrix.

3. Coupling in This System

The angular state (angle) appears in the translational dynamics. This means angle directly influences linear acceleration and hence position. However, position does not influence angle dynamics. This is one-way coupling.

4. Physical Interpretation

Physically, this can be imagined as a cart with a rotating arm. Tilting the arm causes the cart to move, but moving the cart does not automatically tilt the arm.

5. LQR Behavior in Coupled Systems

LQR minimizes a cost function balancing state errors and control effort. Because angle affects position, the controller can regulate position indirectly through angle. As a result, LQR may allow a small steady-state angle offset to keep position at zero with minimal effort.

6. Why Position Goes to Zero but Angle Does Not

In the presence of constant disturbances or step inputs, LQR (without integral action) does not guarantee zero steady-state error. Due to coupling, position is regulated strongly, while angle may settle at a non-zero equilibrium.

7. Why Increasing Q Does Not Fully Fix the Issue

Increasing the weight on angle in the Q matrix can reduce the offset but cannot eliminate it completely. This is because LQR has no memory of past errors and cannot remove steady-state bias.

8. Role of Integral Action (LQI)

Adding integral states for position and angle error forces the controller to eliminate any steady-state offset. This augmentation leads to an LQI controller, which guarantees zero steady-state error for constant references or disturbances.

9. Key Takeaway (Exam-Ready)

In coupled systems, LQR can regulate some states indirectly through others, allowing non-zero steady-state offsets. Integral augmentation is required to enforce zero steady-state error in all controlled outputs.