Third-Order Bessel Low Pass Filter

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1. Objectives

Design and demonstration of Third-Order Bessel Low Pass Filter
Nodal and Mesh Analysis
Transient Analysis
Frequency Response
Thevenin's Theorem

2. Introduction

A low-pass filter is a filter that passes signals with a frequency lower than a selected cutoff frequency fH and attenuates signals with frequencies higher than the cutoff frequency. The exact frequency response of the filter depends on the filter design. The filter is sometimes called a high-cut filter, or treble-cut filter in audio applications. A low-pass filter is the complement of a high-pass filter.

Low-pass Bessel filters have a monotonically decreasing magnitude response. Compared to the Butterworth, Chebyshev, and Elliptic filters, the Bessel filter has the slowest rolloff and requires the highest order to meet an attenuation specification.

This project is a case study of Bessel Low Pass Filter of Order 3.

3. Circuit Diagram

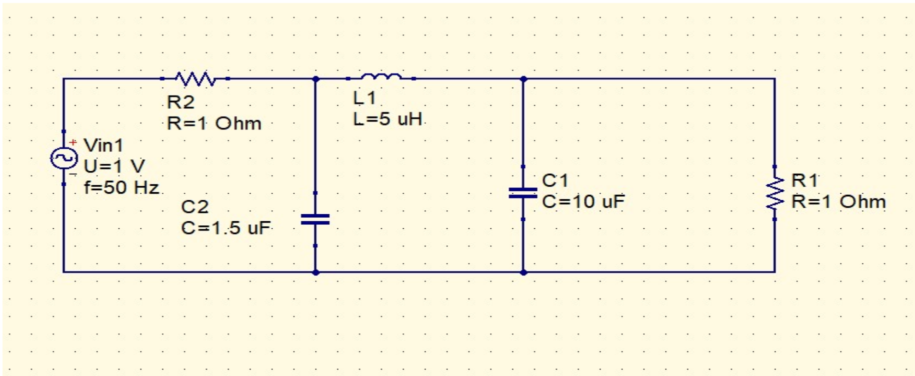
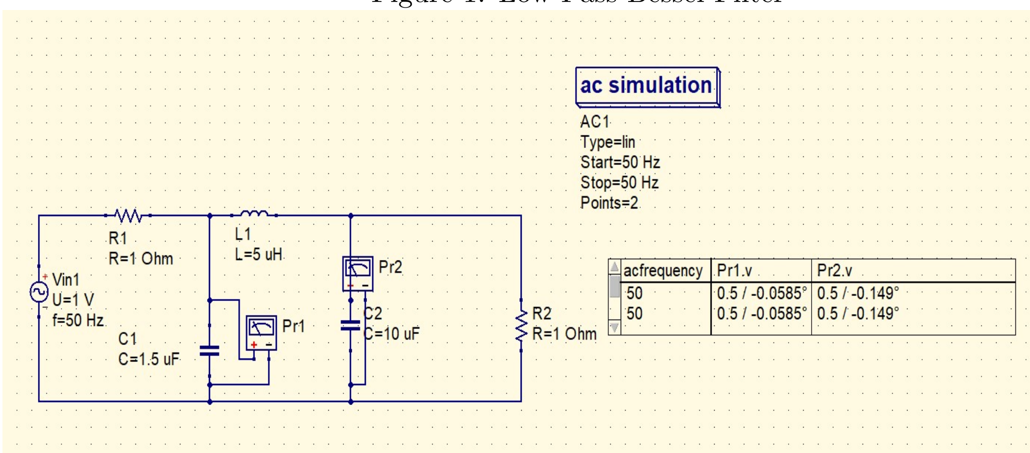
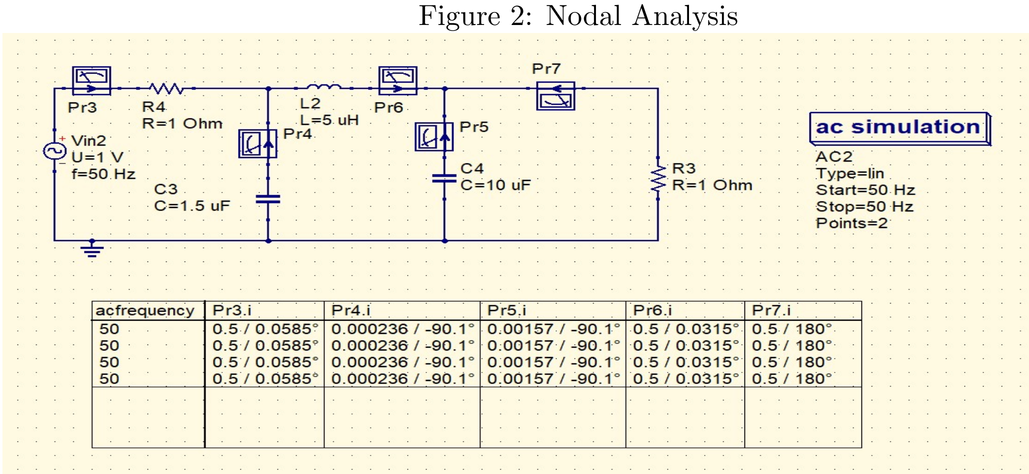


Figure 1: Low Pass Bessel Filter



acfrequency	Pr1.v	Pr2.v
50	0.5 / -0.0585°	0.5 / -0.149°
50	0.5 / -0.0585°	0.5 / -0.149°

Figure 2: Nodal Analysis



acfrequency	Pr3.i	Pr4.i	Pr5.i	Pr6.i	Pr7.i
50	0.5 / 0.0585°	0.000236 / -90.1°	0.00157 / -90.1°	0.5 / 0.0315°	0.5 / 180°
50	0.5 / 0.0585°	0.000236 / -90.1°	0.00157 / -90.1°	0.5 / 0.0315°	0.5 / 180°
50	0.5 / 0.0585°	0.000236 / -90.1°	0.00157 / -90.1°	0.5 / 0.0315°	0.5 / 180°
50	0.5 / 0.0585°	0.000236 / -90.1°	0.00157 / -90.1°	0.5 / 0.0315°	0.5 / 180°

Figure 3: Mesh Analysis

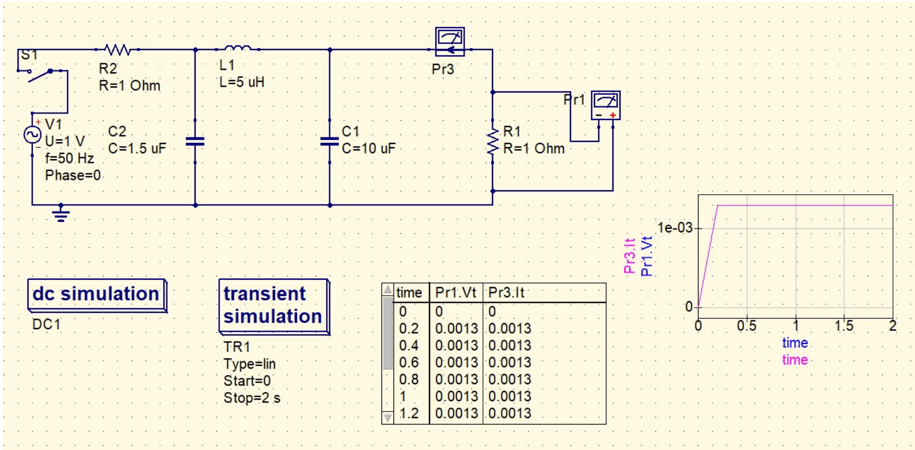


Figure 4: Transient Analysis

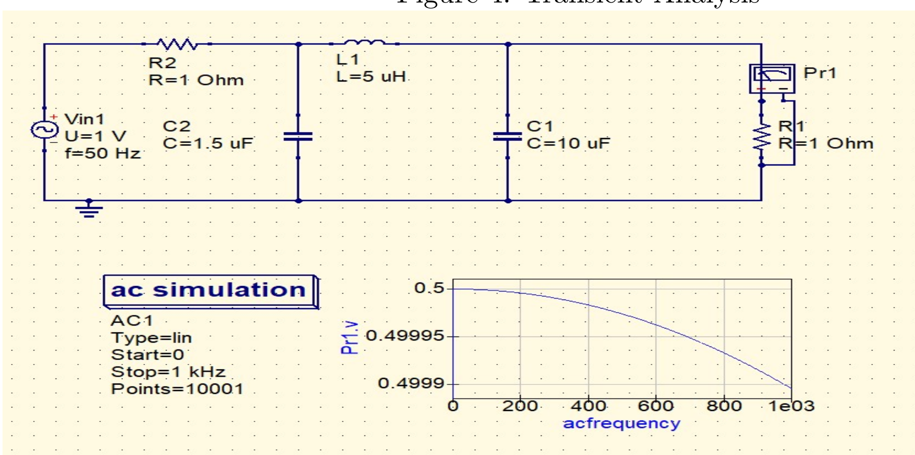


Figure 5: Frequency Response

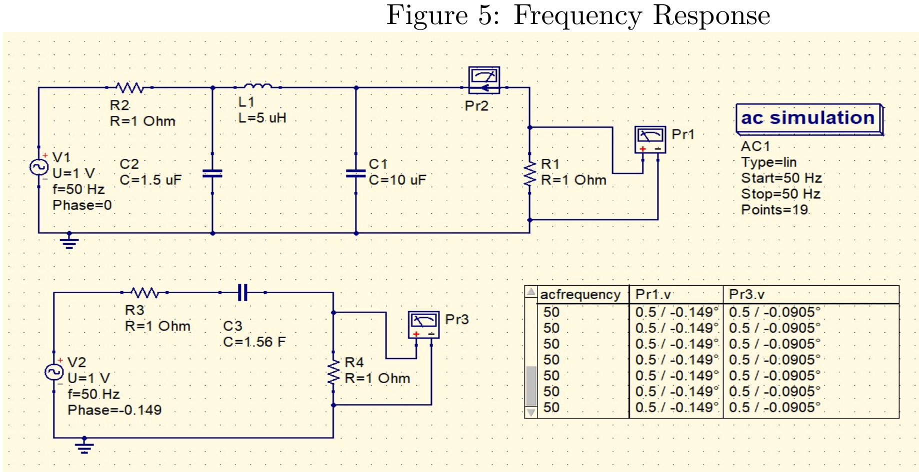


Figure 6: Thevenins Theorem

4. Calculation

Nodal Analysis:

$X_{C2} = -2123.14j \quad X_L = 0.00157j \quad X_{C1} = -318.47j$

At Node 1,

$$\frac{V_1 - 1}{1} + \frac{V_1}{-2123.14j} + \frac{V_1 - V_2}{0.00157j} = 0$$

$$V_1(1 - 636.939j) + 636.94j = 1$$

At Node 2,

$$\frac{V_2 - V_1}{(0.00157j)} + \frac{V_2}{-316.47j} + \frac{V_2}{1} = 0$$

$$636.94j\, V_1 + (1 - 636.936j)\, V_2 = 0$$

$$V_1 = \frac{\begin{vmatrix} 1 & 636.94j \\ 0 & 1 - 636.936j \end{vmatrix}}{\begin{vmatrix} 1 - 636.936j & 636.94j \\ 636.94j & 1 - 636.93j \end{vmatrix}}$$

$$= \frac{1 - 636.936j}{8.0063 - 1273.86j} = 0.4999\angle -0.585^\circ$$

$$\simeq 0.5\angle -0.585^\circ\ V$$

$$V_2 = \frac{\begin{vmatrix} 1 & 1 - 636.939j \\ 0 & 636.94j \end{vmatrix}}{8.0063 - 1273.86j}$$

$$= 0.4999\angle -0.1487$$

$$\simeq 0.5\angle -0.149\ V$$

Figure 7: Nodal Analysis

Mesh Analysis

$$-V + (Z_1 + Z_2) I_1 - Z_2 I_2 = 0$$

$$-1 + (1 - 2123.14j) I_1 + 2123.14j\, I_2 = 0$$

$$(1 - 2123.14j) I_1 + 2123.14j\, I_2 = 1 \quad \text{— (1)}$$

$$(-2123.14j - 318.47j + 0.00157j) I_2 + 2123.14j\, I_1 + 318.47j\, I_3 = 0$$

$$2123.14j\, I_1 + 2441.608j\, I_2 + 318.47j\, I_3 = 0 \quad \text{— (2)}$$

$$(-318.47j + 1) I_3 + 318.47j\, I_2 = 0 \quad \text{— (3)}$$

$$\begin{bmatrix} 1 - 2123.14j & 2123.14j & 0 \\ 2123.14j & 2441.608j & 318.47j \\ 0 & 318.47j & 1 - 318.47j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = 0.5 \angle 0.0585 \text{ A}$$

$$I_2 = 0.5 \angle 0.0315 \text{ A}$$

$$I_3 = -0.5 \angle 180 \text{ A}$$

Figure 8: Mesh Analysis

Thevenin' Equivalent.

$Z_1 = 1\,\Omega$, $Z_2 = \dfrac{-j}{2\pi \times 50 \times 1.5 \times 10^{-6}} = -2122j\,\Omega$,

$Z_3 = j2\pi \times 50 \times 5 \times 10^{-6} = 1.57 \times 10^{-3}j\,\Omega$

$Z_4 = \dfrac{-j}{2\pi \times 50 \times 10 \times 10^{-6}} = -318.3j\,\Omega$

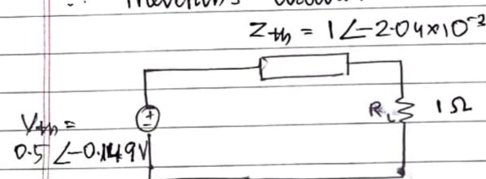
$Z_{th} = ((1 \,//\, -2122j) + 1.57 \times 10^{-3}j) \,//\, -318.3j\,\Omega$

$= 1\angle -2.04 \times 10^{-3}\,\Omega$

From nodal analysis,

$V_{th} = V_2 = 0.5\angle -0.149\ V$

$\therefore$ Thevenin's circuit:



$R_{th} + X_{c\,th}$

$= 1\,\Omega + -2.04 \times 10^{-3}j\,\Omega$.

$C = \dfrac{1}{2\pi f\, j X_c} = \dfrac{1}{2\pi \times 50 \times j X_c} = \dfrac{1}{0.64} = 1.56\ F$

Figure 9: Thevenin's Theorem

5. Results

1. On performing Nodal analysis on Low Pass Bessel Filter, we get V1=$0.5\angle{-0.0505} V and V2 = 0.5\angle - 0.149V.$

2. On performing Mesh analysis on Low Pass Bessel Filter, we get I1=$0.5\angle 0.0585A, I2 = 0.5\angle 0.0315A and I3 = -0.5\angle 180A.$

3. On performing Transient analysis on Low Pass Bessel Filter The graph is plotted for the response of the circuit, current IL=0.0013 A and voltage VL=0.0013 V across load over a period of time as switch is closed.

4. On performing Frequency Response on Low Pass Bessel Filter,
The graph for frequency response characteristics is found between gain (dB) = Vout/Vin where Vin=1 V and frequency (Hz).
At the low frequencies, the gain of the LPF is higher than the passband gain of the filter, At high frequencies, the gain of the LPF i s less than its passband gain and it falls to -20dB. As the frequency increases, the output voltage falls 70.71 % below the input voltage.

5. On performing Thevenin's Theorem on Low Pass Bessel Filter, we get Vth = 0.5 $\angle - 0.149V, Zth = 1\angle - 2.04x10^{-3} ohm.$

$By this experiment we can demonstrate the design and working of a Low Pass Bessel Filter.$

6. Applications

- Used as hiss filters in audio speakers to reduce the high frequency hiss produced in the system and these are used as inputs for sub woofers.
- Used in equalizers and audio amplifiers. In analog to digital conversion these are used as anti-aliasing filters to control signals. In digital filters these are used in blurring of images, smoothing sets of data signals. In radio transmitters to block harmonic emissions.
- Used to filter the high frequency signals in acoustics from the transmitting sound which will cause echo at higher sound frequencies.

References

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