MATHEMATICS FOR ELECTRONICS ENGINEERS (MEE) UE21EC241A
PROJECT TITLE – ANALYSIS OF WAITING LINES AT PES UNIVERSITY ADMISSION FOR OPTIMISING RESOURCES

Abstract –. The possibility of a long waiting line is always there in any scenario of verification of information, which requires time. The speed of the clerk, the processing time, the arrival of the customers or clients, all play a role in determining the density of a waiting line at different times of the day, month, and year. The use of queuing theory and simulation gives a generalized visualization of an otherwise non-countable model.

Case Study:

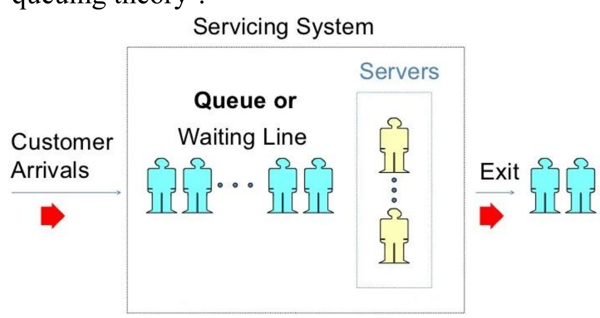
Consider the admission process for the batch of 2021-25 at PES University in September, 2021. There were 4 blocks at different locations that covered the entire process. A minimum of 4 counters were installed at each block location to reduce the long waiting lines common in such situations. The lack of information on the density of the line at different times of the day made some batches wait as long as 2-3 hrs, while others took relatively less to complete. We shall refer and construct a model that displays the randomness in the time between arrivals of students and the processing time of the counters.

Aim

To analyse the waiting time at the admission process lines in an attempt to reduce/minimize the waiting periods and idle periods at each counter, hence optimizing the resources available.

Approach

We shall use the 'waiting line' model to study the behavior of waiting lines.
A 'waiting line model' is a mathematical model analysts use to understand the characterisics of waiting lines. The other name for this domain is the 'queuing theory'.



We shall refer to a system where there is only one

admission counter or 'server' to process the entire admission. With this assumption, the mathematical queuing model now falls into a sub-category of name – 'M/M/1 modelling'.
The M/M/1 queuing model is the simplest waiting line comprising a single waiting line and a single server.
Different waiting line models have different KPIs or 'Key Performance Indicators' that measure the efficiency of queue performance and waiting time.
In the case of 'M/M/1 model' all KPIs can be identified as long as we know the probability distribution of the arrival process and the service process.

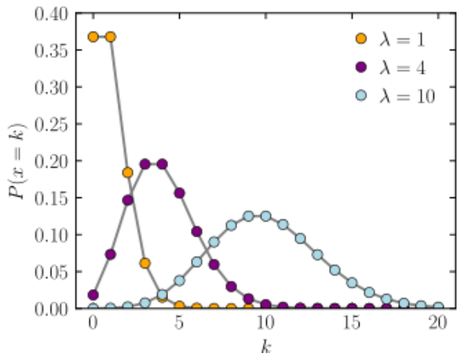
The Mathematical Random models

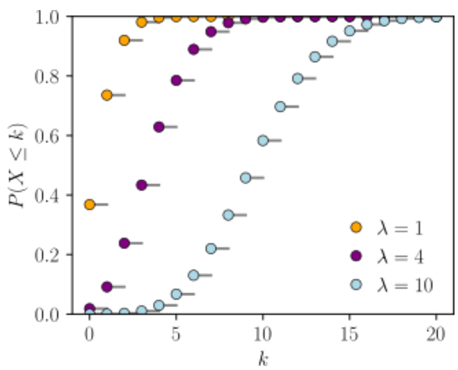
This project shall make use of some of the Random variable functions

1) Poisson distribution for inter-arrival time
2) Exponential distribution for service time
3) Rayleigh distribution for arrival time

The Poisson distribution is characterized by

$$\text{PMF(X=x)} = \frac{e^{-\lambda}.\lambda^{x}}{x!}$$

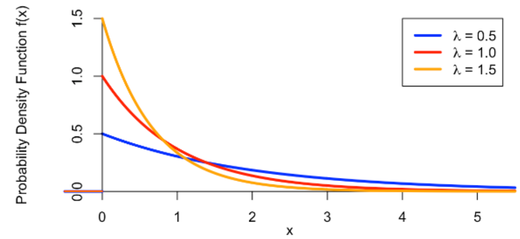


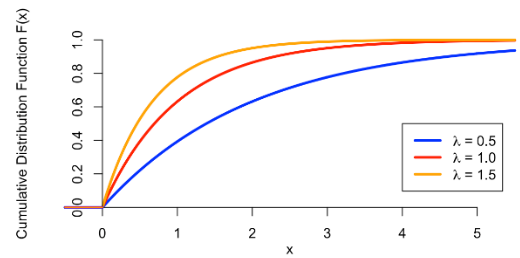


The famous Poisson distribution gives us the probability of a given number of events that will happen in each time, provided λ or event occurrence rate is given.

Some examples include:- Number of mistakes on this slide, Number of accidents that occurs at a junction, Number of phone calls that you get on a single day etc.

The Exponential distribution, on the other hand, could be called a ‘cousin’ of Poisson distribution. The Poisson distribution is mainly used for discrete random variables, while Exponential distribution is studied upon continuous random variables.

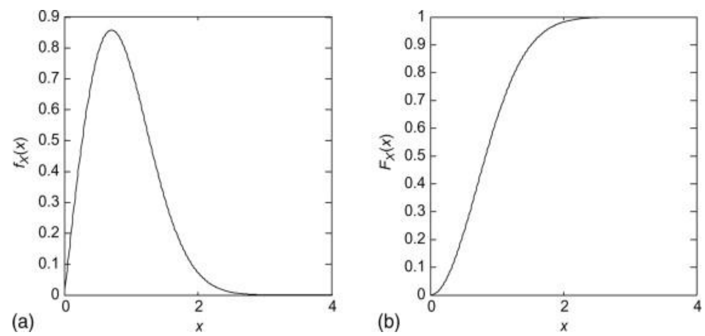




PDF $f_x = \lambda e^{-\lambda t}$

CDF $F_X = 1 - e^{-\lambda t}$

The Rayleigh distribution function is also used in this model as described later.



Methodology:

In our case study, a prior analysis and observation shows us that our arrivals follow a Poisson distribution (an assumption). We then plug the average inter-arrival rate into the Poisson distribution to find the probability of a certain number of arrivals in a fixed time frame. We calculate arrival times as cumulative sum of inter-arrivals and fit them into Rayleigh.

The reason we are taking this assumption is , in simplicity, the fact the variation of arrivals on waiting lines very often follow this probability.

The other criteria for an M/M/1 queue is that the duration of service has an Exponential distribution. This means that the duration of service has an average, and a variation around that average given by Exponential distribution.

Analysis and Computation:

1) Analysis of M/M/1 queue

- Average arrival rate, λ
- Average service time, 1/μ

2) Stability of M/M/1 queue

For the M/M/1 model to be stable, λ must remain smaller than μ. This implies that service is faster than arrival, concluding the fact that the waiting line would not grow too large.

3) Utilisation of M/M/1 queue

Utilisation – the average time that the server will be occupied with a customer or a client.

$\rho=\lambda/\mu$

4) The number of customers in our M/M/1 queuing model

The average number of customers is calculated:

$$\frac{\rho}{1-\rho}$$

The variation around the average number of customers is defined as:

$$\frac{\rho}{(1-\rho)^2}$$

5) The probability of x customers being in our M/M/1 queue

Improving on our previous analysis, we now compute the 'probability' of a 'x' number of customers in the waiting line.

$$\text{P(x customers)} = (1-\rho)\rho^x$$

6) Average response time in out M/M/1 queue

The time taken for a client from arrival to leaving after the processing of his request. The average response time is given as:

$$\text{Average response time} = \frac{1}{\mu-\lambda}$$

7) Average waiting time

$$\text{Average waiting time} = \frac{\rho}{\mu-\lambda}$$

Simulation:

The simulation of the 'M/M/1' queuing model was carried out in MATLAB to visualize the graphs obtained for the 7 KPIs discussed earlier in the project. The code for the same is shared below:

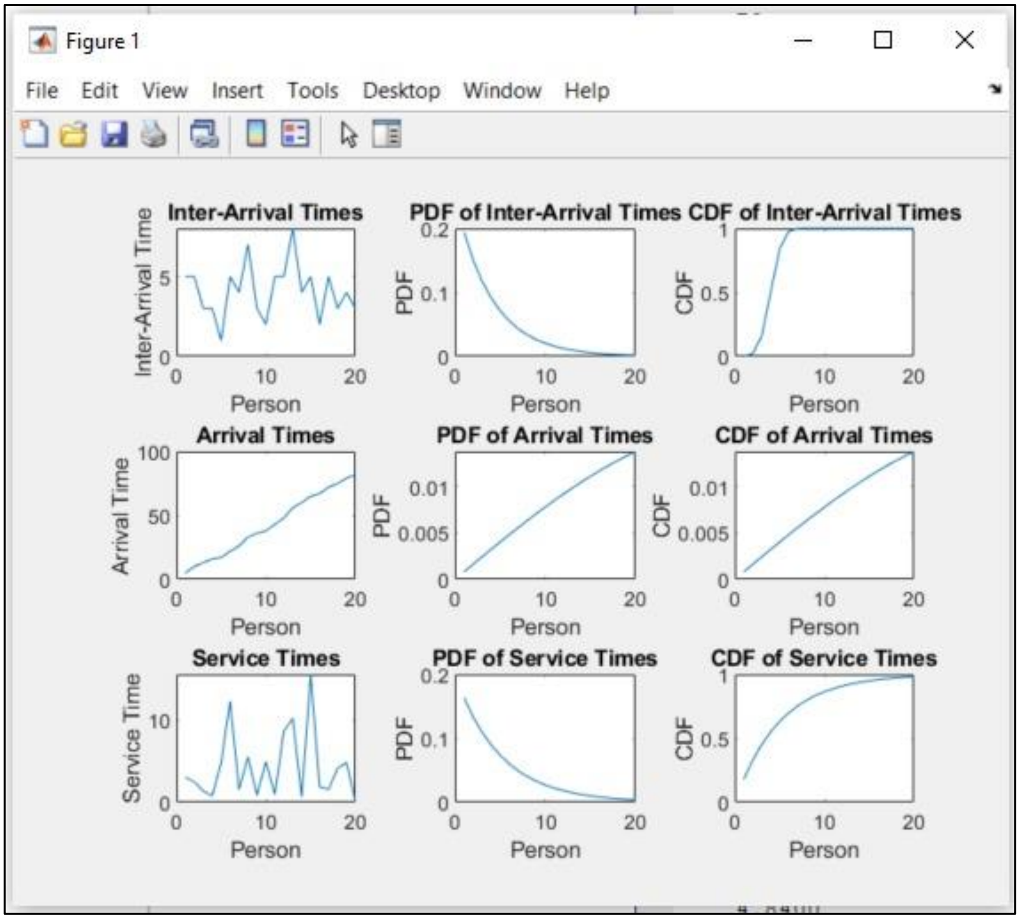
```
lambda = input('Enter the average arrival rate: ');
s = input('Enter the average service time: ');
myu = 1/s; % Service Rate
n = input('Enter the number of people: ');

x = 1:1:n;

% Modelling Inter-Arrival Times as Poisson Distribution
poissonpd = makedist('Poisson','lambda', lambda);
InterArrivalTimes = random(poissonpd,n,1);
fprintf('Inter-Arrival Times:\n')
disp(InterArrivalTimes)
subplot(3,3,1);
plot(x,InterArrivalTimes)
title('Inter-Arrival Times')
ylabel('Inter-Arrival Time')
xlabel('Person')

subplot(3,3,2);
plot(x, exppdf(x, lambda));
title('PDF of Inter-Arrival Times')
ylabel('PDF')
xlabel('Person')

subplot(3,3,3);
plot(x, normcdf(x, lambda));
title('CDF of Inter-Arrival Times')
ylabel('CDF')
xlabel('Person')

% Calculating Arrival times based on Inter-Arrival times
ArrivalTimes = cumsum(InterArrivalTimes);
fprintf('Arrival Times:\n')
disp(ArrivalTimes)
subplot(3,3,4);
plot(x,ArrivalTimes)
title('Arrival Times')
ylabel('Arrival Time')
xlabel('Person')
```

```
% Modelling Arrival Times as Rayleigh
B=raylfit(ArrivalTimes);
raypd=makedist('Rayleigh','B',B);
disp(B)

subplot(3,3,5);
plot(x, raylpdf(x, B));
title('PDF of Arrival Times')
ylabel('PDF')
xlabel('Person')

subplot(3,3,6);
plot(x, raylpdf(x, B));
title('CDF of Arrival Times')
ylabel('CDF')
xlabel('Person')

% Modelling Inter-Arrival Times as Poisson Distribution
ServiceTimes = exprnd(myu,n,1);
fprintf('Service Times:\n')
disp(ServiceTimes)

subplot(3,3,7);
plot(x,ServiceTimes)
title('Service Times')
ylabel('Service Time')
xlabel('Person')

subplot(3,3,8);
plot(x, exppdf(x, myu));
title('PDF of Service Times')
ylabel('PDF')
xlabel('Person')

subplot(3,3,9);
plot(x, expcdf(x, myu));
title('CDF of Service Times')
ylabel('CDF')
xlabel('Person')

% Analysis of M/M/1
if lambda < myu
    fprintf('Stable M/M/1 Queue!')
else
    fprintf('Unstable M/M/1 Queue!')
end

rho = lambda/myu;
fprintf('\nUtilisation of M/M/1 Queue: ');
disp(rho)
```

```
Avg = rho/(1-rho);
Var = rho/power((1-rho),2);
fprintf('\nAverage number of customers in system: ');
disp(Avg)
fprintf('\nVariation around average number of customers in system:');
disp(Var)

x = input('Enter Number of Customers in waiting line to find its Probability: ');
Pr = (1-rho)*(power(rho,x));
fprintf('Probability of a given number %d of customers in the waiting line: %.4f \n',x,
Pr);

Avg_Resp_Time = 1/(myu - lambda);
fprintf('Average response time: %f\n', Avg_Resp_Time);

Avg_Wait_Time = rho/(myu - lambda);
fprintf('Average waiting time: %f\n', Avg_Wait_Time);
```

The given code section works on receiving data from the user for average arrival rate, λ, average service time, and the number of customers, x. Using these data values, computation of the necessary KPIs are done and plotted to get a visual of our theory explained so far.

Results/ Output:




```
Stable M/M/1 Queue!
Utilisation of M/M/1 Queue:     0.8000


Average number of customers in system:     4.0000


Variation around average number of customers in system:   20.0000

Enter Number of Customers in waiting line to find its Probability: 5
Probability of a given number 5 of customers in the waiting line: 0.0655
Average response time: 1.000000
Average waiting time: 0.800000
fx >>
```

```
Command Window
>> project
Enter the average arrival rate: 4
Enter the average service time: 0.2
Enter the number of people: 20
Inter-Arrival Times:
     5
     6
     6
     6
     5
     8
     3
     4
     7
     4
     2
     4
     4
     6
     5
     4
     3
     7
     4
     9
```

```
Command Window
Arrival Times:
     5
    11
    17
    23
    28
    36
    39
    43
    50
    54
    56
    60
    64
    70
    75
    79
    82
    89
    93
   102

Service Times:
    2.9788
    1.0821
    1.7773
    0.6835
   14.7641
    7.5779
    3.8865
    0.2118
    1.1783
    3.9718
    5.4916
   14.1430
    1.4994
    3.3982
    8.0491
    4.2526
    8.8985
    1.4271
    4.9936
fx  0.2997
```

Acknowledgement:

The team would like to thank the Chairperson, Department of Electronics and Communication Engineering, for introducing the initiative of students taking on small projects outside the curriculum and apply their ideas and concepts learned in the course through the semester.
The team also thanks the Department of Mathematics for introducing interesting and well-defined project titles. This has given the team a specific directional point in choosing and working on this project.
Lastly, the team is grateful to Dr. Rajini. M, Head of department, Mathematics, for giving guidance in choosing relevant topics, and in areas where the team sought help.
This project is a culmination of the effort put in by the team members, who are happy to present this as an integration of all concepts and ideas learnt in the course of 'Mathematics for Electronics Engineers'.

References:

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