Rosen 2.1, Exercise 24:

a) $\emptyset$

Not a power set because a power set can not be empty

b) $\{\emptyset, \{a\}\}$

It is a power set of $\mathcal{P}\{\{a\}\}$

c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$

Not a power set because a power set always has a cardinality of a power of 2.

d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

It is a power set of $\mathcal{P}\{\{a, b\}\}$

Rosen 2.1, Exercise 32(b):

C = {0,1}
B = {x,y}
A = {a,b,c}

$C \times B \times A$ = {(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y),
(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c,x), (1, c, y)}

Rosen 2.2, Exercise 16(a):

$(A \cap B) \subseteq A$

A	B	$(A \cap B)$	$(A \cap B) \cap A$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

$(A \cap B) \cap A$ proves that all elements of $(A \cap B)$ lie within set A. Hence, $(A \cap B) \subseteq A$ is true.

Rosen 2.2, Exercise 30(c):

$A \cup C = B \cup C$ and $A \cap C = B \cap C$

Case 1: x is an element of A

Let x be an element in set A

$x \in A$
$x \in A \cup C$ (definition of union)
$x \in B \cup C$ $(A \cup C = B \cup C)$

So x must be in B or C, assume it is in C

$x \in C$
$x \in A \cap C$ (definition of intersection)
$x \in B \cap C$ $(A \cap C = B \cap C)$

By definition of intersection, x must be in B. Therefore all elements in set B will also be in set A by definition of subset.

$A \subseteq B$

Case 2: x is an element of B

Let x be an element in set B

$x \in B$
$x \in B \cup C$ (definition of union)
$x \in A \cup C$ $(A \cup C = B \cup C)$

So x must be in B or C, assume it is in C

$x \in C$
$x \in B \cap C$ (definition of intersection)
$x \in A \cap C$ $(A \cap C = B \cap C)$

By definition of intersection, x must be in A. Therefore all elements in set A will also be in set B by definition of subset.

$B \subseteq A$

Since $A \subseteq B$ and $B \subseteq A$, we can conclude that A = B

Rosen 2.3, Exercise 20(b):

$$\boxed{f(n) = n/2}$$

It is onto because:
$f(2n) = n$ whenever n is a natural number

It is not one-to-one because:
$f(2) = f(3) = 1$

Rosen 2.3, Exercise 20(c):

$$\boxed{f(n) = \begin{cases} n+1 & \text{if n is even} \\ n-1 & \text{if n is odd} \end{cases}}$$

It is onto because:
if n is even and n + 1 is odd: $f(n+1) = n$
if n is odd and n - 1 is even: $f(n-1) = n$

It is one-to-one because:
$f(n) = f(n') \rightarrow n = n'$

it is not an identity because $f(1) = 0$

Rosen 2.3, Exercise 36:

$f(x) = x^2 + 1$
$g(x) = x + 2$

$f \circ g$:

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= (x+2)^2 + 1 \\ &= x^2 + 4x + 5 \end{aligned}$$

$g \circ f$:

$$\begin{aligned} g \circ f &= g(f(x)) \\ &= (x^2 + 1) + 2 \\ &= x^2 + 3 \end{aligned}$$

Rosen 2.3, Exercise 74(c):

$\lceil \lceil x/2 \rceil /2 \rceil = \lceil x/4 \rceil$ for all real numbers x.

Prove using proof by parts:

Let $x = 4n + c$ where n is an integer $0 \leq c < 4$

If c = 0, then x will already be a multiple of four and both sides equal n.

If $0 < c \leq 2$:
$\lceil x/2 \rceil = 2n + 1$
The LHS and RHS can both be evaluated to x + 1

If $2 < c < 4$:
$\lceil x/2 \rceil = 2n + 2$
The LHS and RHS can both be evaluated to x + 1

So this function has been proved true for all statements.

Rosen 2.4, Exercise 16(d):

$a_n = 2a_{n-1} - 3$
$a_0 = -1$

$$
\begin{aligned}
a_n &= 2^1 a_{n-1} - 3 \\
&= 2^2 a_{n-2} - (3 \cdot 2^0 + 3 \cdot 2^1) \\
&= 2^3 a_{n-3} - (3 \cdot 2^0 + 3 \cdot 2^1 + 3 \cdot 2^2)
\end{aligned}
$$

$$
\begin{aligned}
a_n &= 2^n a_{n-n} - \textstyle\sum_{i=0}^{n-1} 3 \cdot 2^i \\
&= 2^n a_0 - 3 \cdot \textstyle\sum_{i=0}^{n-1} 2^i \\
&= 2^n \cdot (-1) - 3 \cdot \frac{2^n - 1}{2 - 1} \\
&= -2^n - 3 \cdot (2^n - 1) \\
&= -2^n - 3 \cdot 2^n + 3 \\
&= -4 \cdot 2^n + 3
\end{aligned}
$$

Rosen 2.4, Exercise 18(b):

$a_n = 1.09 \cdot a_{n-1}$
$a_0 = 1000$

$$\begin{aligned} a_n &= 1.09 \cdot a_{n-1} \\ &= 1.09(1.09 \cdot a_{n-2}) \\ &= 1.09^2(1.09 \cdot a_{n-3}) \end{aligned}$$

$a_n = 1.09^n \cdot a_{n-n}$

$a_n = 1.09^n \cdot a_0$

$a_n = 1000 \cdot 1.09^n$

Rosen 2.4, Exercise 32(c):

$\sum_{j=0}^{8}(2 \cdot 3^j + 3 \cdot 2^j)$

Use property:
$\sum_{j=0}^{n} q^j = \dfrac{1-q^{n+1}}{1-q}$, $q \neq 0, 1$

$\sum_{j=0}^{8}(2 \cdot 3^j + 3 \cdot 2^j) = 2 \cdot \dfrac{1-3^9}{1-3} + 3 \cdot \dfrac{1-2^9}{1-2} = 21215$