Rosen 1.6, Exercise 14(b):

Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.

P(x): x is one of the roommates
Q(x): x has taken a course in discrete mathematics
R(x): has taken a course in algorithms

Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics.
$\forall x(P(x) \rightarrow Q(x))$
Every student who has taken a course in discrete mathematics can take a course in algorithms.
$\forall x(Q(x) \rightarrow R(x))$

1.	$\forall x(P(x) \rightarrow Q(x))$	premise
2.	$\forall x(Q(x) \rightarrow R(x))$	premise
3.	$P(y) \rightarrow Q(y)$	universal instantiation from (1)
4.	$Q(y) \rightarrow R(y)$	universal instantiation from (2)
5.	$P(y) \rightarrow R(y)$	hypothetical syllogism from (3) and (4)
6.	$\forall x(P(x) \rightarrow R(x))$	universal generalization from (5)

Therefore, all five roommates can take a course in algorithms next year.
$\forall x(P(x) \rightarrow R(x))$

Rosen 1.6, Exercise 16(a,b):

16(a):

Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

P(x): x has lived in the dormitory
Q(x): x is enrolled in the university
m: Mia

Everyone enrolled in the university has lived in a dormitory.
$\forall x(P(x) \to Q(x))$
Mia has never lived in a dormitory.
$\neg Q(m)$

1.	$\forall x(P(x) \to Q(x))$	premise
2.	$\neg Q(m)$	premise
3.	$P(m) \to Q(m)$	universal instantiation from (1)
4.	$\neg P(m)$	Modus tollens from (2) and (3)

(4) shows that Mia is not in the university which agrees with the given conclusion so it is possible to conclude that the argument is true.

16(b):

A convertible car is fun to drive. Isaacs car is not a convertible. Therefore, Isaacs car is not fun to drive.

P(x): x is a convertible
Q(x): x is fun to drive
i: Isaac's car

A convertible car is fun to drive.
$\forall x(P(x) \to Q(x))$
Isaacs car is not a convertible.
$\neg P(i)$

1.	$\forall x(P(x) \to Q(x))$	premise
2.	$\neg P(i)$	premise
3.	$P(i) \to Q(i)$	universal instantiation from (1)

There is no rule of inference that can provide $\neg Q(i)$ so the conclusion can never be reached and the argument is therefore incorrect.

Rosen 1.6, Exercise 26:

1.	$\forall x(P(x) \to Q(x))$	premise
2.	$\forall x(Q(x) \to R(x))$	premise
3.	$P(y) \to Q(y)$	universal instantiation from (1)
4.	$Q(y) \to R(y)$	universal instantiation from (2)
5.	$P(y) \to R(y)$	hypothetical syllogism from (3) and (4)
6.	$\forall x(P(x) \to R(x))$	universal generalization from (5)

This shows that under the conditions of $\forall x(P(x) \to Q(x))$ and $\forall x(Q(x) \to R(x))$ being true, $\forall x(P(x) \to R(x))$ will also be true using universal instantiation, hypothetical syllogism and universal generalization.

Rosen 1.7, Exercise 4:

Assume there is even number x and any integer n. Using the property of even numbers where x = 2n for some integer n:
x = 2n

The additive inverse of x is -x:
-x = -2n

There is an integer m = -n:
-x = 2m

This will satisfy the definition for an even number where (-x) = 2m for some value m. This shows that the additive inverse, or negative, of an even number is an even number.

Rosen 1.7, Exercise 10:

The property of rational numbers where there exists a rational number x if there exist two integers, y and z that $x = \frac{y}{z}$.

If there are two variables x and p as well as four integers y, z, q and r such that:

$x = \frac{y}{z}$

$p = \frac{q}{r}$

The product of the two:

$x \cdot p = \frac{y}{z} \cdot \frac{q}{r} = \frac{y \cdot q}{z \cdot r}$

$y \cdot q$ and $q \cdot r$ will both return integers so, using the property of rational numbers, this shows that the product of two rational numbers will also be rational.

Rosen 1.7, Exercise 16:

The property of an odd number x is that there exists an integer n such that x = 2n + 1
m = 2a + 1
n = 2b + 1

mn = (2a + 1)(2b + 1)
= 4ab + 2a + 2b + 1
= 2(2ab + a + b) + 1
if there exists an integer d such that d = 2ab + a + b
=2d + 1 which shows that mn would be odd

Therefore, if m and n are both odd, mn would also be evaluated as odd, so for mn to be even, there must be at least on even number.

Rosen 1.7, Exercise 32:

(i) x is rational
(ii) x/2 is rational
(iii) $3x - 1$ is rational

To prove equivalence, prove that $(i) \to (ii) \to (iii) \to (i)$ is true with all of them evaluating to true.

$(i) \to (ii)$:
For a rational number x and two integers p and q such that $x = \frac{p}{q}$
$\frac{x}{2} = \frac{p}{2q}$
p and 2q are both integers so $\frac{x}{2}$ is rational and $(i) \to (ii)$ is true

$(ii) \to (iii)$:
For a rational number $\frac{x}{2}$ and two integers p and q such that $\frac{x}{2} = \frac{p}{q}$ or $x = \frac{2p}{q}$
$3x - 1 = \frac{6p}{q} - 1 = \frac{6p - q}{q}$
$6p - q$ and q are both integers so $3x - 1$ is rational and $(ii) \to (iii)$ is true

$(iii) \to (i)$:
For a rational number $3x - 1$ and two integers p and q such that $3x - 1 = \frac{p}{q}$
$3x = \frac{p}{q} + 1$
$x = \frac{p + q}{3q}$
$p + q$ and 3q are both integers so x is rational and $(iii) \to (i)$ is true