

**Rosen 1.3, Exercise 12(b):**

*Proof.*  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$   
 Logical equivalences with conditionals:  
 $\equiv [(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (\neg p \vee r)$   
 $\equiv \neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$   
 De Morgan's law use twice:  
 $\equiv [\neg(\neg p \vee q) \vee \neg(\neg q \vee r)] \vee (\neg p \vee r)$   
 Double negation law used twice:  
 $\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r)$   
 Distributive law:  
 $\equiv [(p \vee (q \wedge \neg r)) \wedge (\neg q \vee (q \wedge \neg r))] \vee (\neg p \vee r)$   
 $\equiv [(p \vee q) \wedge (p \vee \neg r)) \wedge ((\neg q \vee q) \wedge (\neg q \vee \neg r))] \vee (\neg p \vee r)$   
 Negation law:  
 $\equiv [(p \vee q) \wedge (p \vee \neg r)) \wedge (T \wedge (\neg q \vee \neg r))] \vee (\neg p \vee r)$   
 Identity law:  
 $\equiv [(p \vee q) \wedge (p \vee \neg r)) \wedge (\neg q \vee \neg r)] \vee (\neg p \vee r)$   
 Associative law:  
 $\equiv [(p \vee q) \wedge (\neg q \vee \neg r) \wedge (p \vee \neg r)] \vee (\neg p \vee r)$   
 Distributive law:  
 $\equiv [(p \vee q) \wedge (\neg q \vee \neg r)) \vee (\neg p \vee r)] \wedge [(p \vee \neg r) \vee (\neg p \vee r)]$   
 Associative law:  
 $\equiv [(p \vee q) \wedge (\neg q \vee \neg r)) \vee (\neg p \vee r)] \wedge [(p \vee \neg p) \vee (r \vee \neg r)]$   
 Negation law:  
 $\equiv [(p \vee q) \wedge (\neg q \vee \neg r)) \vee (\neg p \vee r)] \wedge [T \vee (r \vee \neg r)]$   
 Domination law:  
 $\equiv [(p \vee q) \wedge (\neg q \vee \neg r)) \vee (\neg p \vee r)] \wedge T$   
 Identity law:  
 $\equiv ((p \vee q) \vee (\neg p \vee r)) \wedge ((\neg q \vee \neg r) \vee (\neg p \vee r))$   
 Distributive law:  
 $\equiv ((p \vee \neg p) \vee (q \vee r)) \wedge ((r \vee \neg r) \vee (\neg p \vee \neg q))$   
 Associative law:  
 $\equiv (T \vee (q \vee r)) \wedge (T \vee (\neg p \vee \neg q))$   
 Negation law:  
 $\equiv (T \vee (q \vee r)) \wedge (T \vee (\neg p \vee \neg q))$   
 Domination law:  
 $\equiv T \wedge T$   
 Identity law:  
 $\equiv T$   
 q.e.d.

□

**Rosen 1.3, Exercise 28:**

	p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg q$	
	T	T	T	T	
<i>Proof.</i>	T	F	F	F	q.e.d.
	F	T	F	F	
	F	F	T	T	

□

**Rosen 1.3, Exercise 62(c):**

This compound proposition is satisfiable when p, q and s are true and r is false

Put the steps and arguments you used to arrive at your answer here.

$$(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$$

In order to be satisfiable, there must be the possibility of a true output, That would require all the following statements to be true:

$$\begin{aligned} &(p \vee q \vee r) \\ &(p \vee \neg q \vee \neg s) \\ &(q \vee \neg r \vee s) \\ &(\neg p \vee r \vee s) \\ &(\neg p \vee q \vee \neg s) \\ &(p \vee \neg q \vee \neg r) \\ &(\neg p \vee \neg q \vee s) \\ &(\neg p \vee \neg r \vee \neg s) \end{aligned}$$

In the case that p, q and s are true and r is false, the statement will come out to be true in the end which will prove that it is satisfiable.

**Add-on :** Put the compound proposition from Rosen1.3, Exercise 12(b) in disjunctive normal form.

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

Your work goes here

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

p	q	r	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

The disjunctive normal form has all instances where the statement is true and this is a tautology so all instances of the statement will be true.

Therefore the disjunctive normal form will be:

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

**Rosen 1.4, Exercise 8(c):**

There exists at least one animal that, if it is a rabbit, then it hops.

Your work goes here

$$\exists x(R(x) \rightarrow H(x))$$

$R(x)$ : x is a rabbit

$H(x)$ : x hops

Domain: animals

There exists at least one animal that  $R(x) \rightarrow H(x)$

There exists at least one animal that, it is a rabbit  $\rightarrow$  it hops

There exists at least one animal that, if it is a rabbit, then it hops.

**Rosen 1.4, Exercise 10(c):**

$$\exists x(C(x) \wedge F(x) \wedge \neg D(x))$$

Your work goes here

$C(x)$ : x has a cat

$D(x)$ : x has a dog

$F(x)$ : x has a ferret

Domain: all students in class

Some student in your class has a cat and a ferret, but not a dog.

Some student in your class:  $\exists x$

has a cat:  $C(x)$

and a ferret:  $\wedge F(x)$

but not a dog:  $\wedge \neg D(x)$

$$\exists x(C(x) \wedge F(x) \wedge \neg D(x))$$

**Rosen 1.4, Exercise 18(e):**

$$\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$$

Your work goes here

$P(x)$ : Propositional function

Integers in function: 2, 1, 0, 1, and 2

$$\neg \exists x P(x)$$

Listing out all possible  $P(x)$  values

$$\neg \exists x (P(-2), P(-1), P(0), P(1), P(2))$$

$\exists x$  is converted to or

$$\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$$

**Rosen 1.4, Exercise 28(d):**

$$\neg \exists x (P(x) \wedge Q(x))$$

Your work goes here

Nothing is in the correct place and is in excellent condition.

$P(x)$ :  $x$  is in the correct place

$Q(x)$ :  $x$  is in excellent condition

Nothing is in the correct place and is in excellent condition.

$\neg \exists x (\text{in the correct place and is in excellent condition})$

$\neg \exists x (P(x) \wedge Q(x))$



**Rosen 1.4, Exercise 36(c):**

$$x = 0$$

Your work goes here

$$\forall x (|x| > 0)$$

Prove there exists a real number  $x$  such that  $|x| > 0$  is false.

for  $|x| > 0$  to be false, there must be  $|x| \leq 0$

for  $x = 0$ ,  $|x| = |0| = 0$  which satisfies  $|x| \leq 0$  so the counterexample is  $x = 0$

**Rosen 1.5, Exercise 4(c):**

Every student in the class has taken at least one computer science course.

Your work goes here

$P(x, y)$ : Student  $x$  has taken class  $y$

Domain for  $x$ : all students in the class

Domain for  $y$ : all computer science courses in school

$\forall x \exists y P(x, y)$

$P$ (every student in the class, there is a computer science course)

Every student in the class has taken at least one computer science course.

**Rosen 1.5, Exercise 10(e):**

$$\exists x \forall y F(x, y)$$

Your work goes here

$F(x, y)$ : x can fool y

Domain of x: all people in the world

Domain of y: all people in the world

Everyone can be fooled by somebody.

Somebody can fool everyone.

$F(\text{somebody}, \text{everyone})$

$\exists x \forall y F(x, y)$

**Rosen 1.5, Exercise 14(e):**

$$\exists x \forall y \exists z (P(x, y) \wedge Q(y, z))$$

Your work goes here

There is a student in this class who has taken every course offered by one of the departments in this school.

$P(x, y)$ : student  $x$  has taken course  $y$

$Q(y, z)$ : course  $y$  offered by department  $z$

Domain of  $x$ : all the students in the class

Domain of  $y$ : courses offered

Domain of  $z$ : departments in the school

$$\exists x \forall y \exists z (P(x, y) \wedge Q(y, z))$$

**Rosen 1.5, Exercise 24(d):**

For all pairs of numbers, both numbers are equal to 0 if and only if the product of both numbers is equal to 0.

Your work goes here

$$\forall x \forall y ((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$$

$\forall x \forall y$  (x and y are not equal to 0 if and only if the product of both numbers is equal to 0)

For all pairs of numbers, both numbers are equal to 0 if and only if the product of both numbers is equal to 0.

**Rosen 1.5, Exercise 38(d):**

All of the students in this class have not been in any of the rooms of one building on campus.

Your work goes here

There is a student in this class who has been in atleast one room of every building on campus.

$S(x, y, z)$ : student  $x$  has been in room  $y$  of building  $z$  on campus

Domain of  $x$ : all students in the class

Domain of  $y$ : all rooms

Domain of  $z$ : all building on campus

$$\exists x \exists y \forall z S(x, y, z)$$

for the negation:

$$\neg \exists x \exists y \forall z S(x, y, z)$$

using De Morgan's Law:

$$\forall x \forall y \exists z \neg S(x, y, z)$$

Back in English:

All of the students in this class have not been in any of the rooms of one building on campus.