Proof for $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$:

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

for all $n \in \mathbb{Z}_+$

Base case: when $n = 1$

$\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1+1}$

$\frac{1}{1(1+1)} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

So the statement is true for $n = 1$.

Induction step: if $k \in \mathbb{Z}_+$ and suppose the statement is true for $n = k$, then for $n = k+1$:

$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)}$

$= \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{(k+1)(k+2)}$

$= \frac{(k+1)}{(k+2)}$

$= \frac{(k+1)}{(k+1)+1} = \frac{n}{n+1}$

Which shows that $n = k+1$ is possible and satisfies the induction step.

Thus, thanks to the principle of induction, the statement is true for all $n \in \mathbb{Z}_+$.

Rosen 5.1, Exercise 18:

Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

a) What is the statement $P(2)$?

$P(2) = 2! < 2^2$

b) Show that $P(2)$ is true, completing the basis step of the proof.

$P(2) = 2! < 2^2$
$P(2) = 2 < 4$
So it is true

c) What is the inductive hypothesis?

$P(n) = n! = n^n$

d) What do you need to prove in the inductive step?

Show that $P(n)$ implies $P(n+1)$ are true

e) Complete the inductive step.

show that $P(n+1)$ is true:
$(n+1)! = (n+1) + n!$

$(n+1) + n! < (n+1) + n^n$
$(n+1) + n! < (n+1) + (n+1)^n$
$(n+1) + n! < (n+1)^{n+1}$

$(n+1)! < (n+1)^{n+1}$
So the induction step is true.

f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

Through the principle of mathematical induction, the fact that the base case of $P(2)$ is trua and the fact that $P(n)$ implies $P(n+!)$, it is possible to say that the inequality is true whenever n is an integer greater than 1.

Rosen 5.1, Exercise 40:

Prove that if $A_1, A_2, ..., A_n$ and B are sets, then
$(A_1 \cap A_2 \cap \cdots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B)$

Let $P(n) = (A_1 \cap A_2 \cap \cdots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B)$

Base case: when $n = 1$

$(A_1) \cup B = (A_1 \cup B)$

So the statement is true for $n = 1$.

Induction step: suppose the statement is true for $n = k$, then for $n = k + 1$:

$(A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}) \cup B$
$= ((A_1 \cap A_2 \cap \cdots \cap A_n) \cap A_{n+1}) \cup B$
$= ((A_1 \cap A_2 \cap \cdots \cap A_n) \cup B) \cap (A_{n+1} \cup B)$
$= (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B) \cap (A_{n+1} \cup B)$

Which shows that $n = k + 1$ is possible and satisfies the induction step.

Thus, thanks to the principle of induction, the statement is true for all $n \in \mathbb{Z}_+$.

Rosen 5.1, Exercise 62:

Show that n lines separate the plane into $\frac{(n^2+n+2)}{2}$ regions if no two of these lines are parallel and no three pass through a common point.

Let $P(n) =$ "n lines separate the plane into $\frac{(n^2+n+2)}{2}$ regions"

Base case: when $n = 1$

One line seperates a plane into two regions.

$\frac{(1^2+1+2)}{2} = 2$

So the statement is true for $n = 1$.

Induction step: suppose the statement is true for $n = k$, then for $n = k + 1$:

The $k + 1$ line will intersect the remaining k lines at some point as no two are parallel and it intesects each line where there is no other intersect. It alone, divides the region in two and the presence of k intersections will create an additional k regions so from $P(k)$ to $P(k + 1)$, the difference is $k + 1$ regions

$\frac{(k^2+k+2)}{2} + (k + 1)$
$= \frac{k^2+k+2+2(k+1)}{2}$
$= \frac{k^2+k+2+2k+2}{2}$
$= \frac{(k^2+2k+1)+(k+1)+2}{2}$
$= \frac{(k+1)^2+(k+1)+2}{2}$

Which shows that $n = k + 1$ is possible and satisfies the induction step.

Thus, thanks to the principle of induction, the statement is true for all $n \in \mathbb{Z}_+$.