**Rosen 1.8, Exercise 6:**

The rule of even numbers is there is an even number x if there exists an integer n where x = 2n.

The rule of odd numbers is there is an odd number y if there exists an integer m where y = 2m + 1.

For 5x + 5y:
5x + 5y
= 5(2n) + 5(2m + 1)
= 10n + 10m + 5
= 5( 2n + 2m + 1 )
= 5( 2(n + m) + 1 )

If n and m are both integers, there will also be an integer n + m.
Following the rule for odd numbers, 2(n + m) + 1, furthermore using the below proof, two odd number multiplied by one another will always return an odd number, so 5( 2(n + m) + 1 ) will be odd, proving that 5x + 5y is odd.

Proof for product of odd numbers being odd
odd number x = 2n + 1
odd number y = 2m + 1

xy = (2n + 1)(2m + 1)
= 4nm + 2n + 2m + 1
= 2(2nm + n + m) + 1

Which satisfies the rule of odd numbers with 2nm + n + m being an integer.

**Rosen 1.8, Exercise 10:**

$2 \cdot 10^{500} + 15$ and $2 \cdot 10^{500} + 15$ are both consecutive integers which means that the only possibility is that they are consecutive squares. However,

If we assume $2 \cdot 10^{500} + 15 =$ x and that x is a perfect square, being equal to $n^2$ for some integer n. The consecutive greater perfect square would be:

$$\begin{aligned}(n+1)^2 &= n^2 + 2n + 1 \\ &= x + 2n + 1\end{aligned}$$

$$x + 2n + 1 > x + 1$$
$$2n + 1 > 1$$

This means that the difference between consecutive perfect squares will always be greater than 1 so consecutive numbers like $2 \cdot 10^{500} + 15$ and $2 \cdot 10^{500} + 15$ cannot both be perfect squares.

**Rosen 1.8, Exercise 30:**

$2x^2 + 5y^2 = 14$ where x and y are both integers.

$x^2 \geq 0$ and $y^2 \geq 0$

$5y^2 \leq 14$
when y = 2: $5(2)^2 = 20$ and $20 > 14$

$2x^2 \leq 14$
when x = 3: $2(3)^2 = 18$ and $18 > 14$

So the only possible values for y are 0 and 1, while the only possible values for x are 0, 1 and 2.

For f(x, y) = $2x^2 + 5y^2$
f(0, 0) = 0
f(1, 0) = 2
f(2, 0) = 8
f(0, 1) = 5
f(1, 1) = 7
f(2, 1) = 13

None of the possible combinations satisfy the statement therefore there are no integer solutions to the equation.

**Rosen 4.1, Exercise 14(f):**
Put your answer inside the box. Show your work outside the box.
If your answer is correct, but you don't show your work, you only get 80% of the points.

$\boxed{c = 14}$

$a \equiv 11(mod 19)$
$b \equiv 3(mod 19)$
$0 \leq c \leq 18$

$c \equiv a^3 + 4b^3(mod 19)$
$= (11)^3 + 4(3)^3(mod 19)$
$= 1331 + 108(mod 19)$
$= 1439(mod 19)$
$= 14(mod 19)$

**Rosen 4.1, Exercise 28(a):**
Put your answer inside the box. Show your work outside the box.
If your answer is correct, but you don't show your work, you only get 80% of the points.

$\boxed{37 \not\equiv 3(mod 7)}$

$37 - 3 = 34$
$34 \div 7 = 7 \cdot 4 + 6$ 34 is not divisible by 7 so 37 is not congruent to $3(mod 7)$.

**Rosen 4.1, Exercise 28(c):**
Put your answer inside the box. Show your work outside the box.
If your answer is correct, but you don't show your work, you only get 80% of the points.

$\boxed{-17 \not\equiv 3(mod 7)}$

$-17 - 3 = -20$
$-20 \div 7 = 7 \cdot 2 - 6$ -20 is not divisible by 7 so -17 is not congruent to $3(mod 7)$.

**Rosen 4.1, Exercise 42:** Only prove the associativity property - **not** all the properties requested.

Associativity is defined by:
$(a +_m b) +_m c = a +_m (b +_m c)$

Will use the properties:
a + $_m$ b = (a + b) mod(m)
a + $_m$ b = (a mod(m)) + $_m$ b
a + $_m$ b = a + $_m$ (b mod(m))
(a + b) + c = a + (b + c)

$(a +_m b) +_m c$
$= ((a + b)mod(m)) +_m c$
$= (a + b) +_m c$
$= ((a + b) + c)mod(m)$
$= (a + (b + c))mod(m)$
$= a +_m (b + c)$
$= a +_m ((b + c)mod(m))$
$= a +_m (b +_m c)$

**Rosen 4.3, Exercise 34:**
Put your answer inside the box. Show your work outside the box.
If your answer is correct, but you don't show your work, you only get 80% of the points.

| |
|---|
| 7 |

gcd(21, 34)

$34 = (21 \cdot 1) + 13$
$21 = (13 \cdot 1) + 8$
$13 = (8 \cdot 1) + 5$
$8 = (5 \cdot 1) + 3$
$5 = (3 \cdot 1) + 2$
$3 = (2 \cdot 1) + 1$
$2 = 1 \cdot 2$

There are a total of 7 divisions for this particular gcd.