\documentclass[10pt]{article}

% math fonts

\usepackage{amsmath,amsfonts,amsthm,amssymb}

% to insert graphics

\usepackage{graphicx}

% to change margins of the pages

\usepackage[margin=0.9in]{geometry}

% Makes equations flush left

\usepackage{fleqn}

% This generates a page header with your name in it.

\usepackage{fancyhdr}

\pagestyle{fancy}

\fancyhf{}

\lhead{FOCS Fall 2018}

\rhead{HW02 solution by Sriyuth Sagi}

\rfoot{Page \thepage}

% This package makes it easy to have boxes around large text.

\usepackage{framed}

\begin{document}

{\bf Rosen 1.3, Exercise 12(b):} \\

\begin{proof}

\begin{framed}

$[(p \rightarrow q) \wedge (q \rightarrow r) ] \rightarrow (p \rightarrow r)$\\

Logical equivalences with conditionals:\\

$\equiv [(\lnot p \vee q) \wedge (\lnot q \vee r) ] \rightarrow (\lnot p \vee r)$\\

$\equiv \lnot [(\lnot p \vee q) \wedge (\lnot q \vee r) ] \vee (\lnot p \vee r)$\\

De Morgan's law use twice:\\

$\equiv [\lnot (\lnot p \vee q) \vee \lnot (\lnot q \vee r) ] \vee (\lnot p \vee r)$\\

Double negation law used twice:\\

$\equiv [(p \wedge \lnot q) \vee (q \wedge \lnot r) ] \vee (\lnot p \vee r)$\\

Distributive law:\\

$\equiv [(p \vee (q \wedge \lnot r)) \wedge (\lnot q \vee (q \wedge \lnot r)) ] \vee (\lnot p \vee r)$\\

$\equiv [((p \vee q) \wedge (p \vee \lnot r)) \wedge ((\lnot q \vee q) \wedge (\lnot q \vee \lnot r)) ] \vee (\lnot p \vee r)$\\

Negation law:\\

$\equiv [((p \vee q) \wedge (p \vee \lnot r)) \wedge (T \wedge (\lnot q \vee \lnot r)) ] \vee (\lnot p \vee r)$\\

Identity law:\\

$\equiv [((p \vee q) \wedge (p \vee \lnot r)) \wedge (\lnot q \vee \lnot r) ] \vee (\lnot p \vee r)$\\

Associative law:\\

$\equiv [(p \vee q) \wedge (\lnot q \vee \lnot r) \wedge (p \vee \lnot r) ] \vee (\lnot p \vee r)$\\

Distributive law:\\

$\equiv [((p \vee q) \wedge (\lnot q \vee \lnot r)) \vee (\lnot p \vee r) ] \wedge [(p \vee \lnot r) \vee (\lnot p \vee r)]$\\

Associative law:\\

$\equiv [((p \vee q) \wedge (\lnot q \vee \lnot r)) \vee (\lnot p \vee r) ] \wedge [(p \vee \lnot p) \vee (r \vee \lnot r)]$\\

Negation law:\\

$\equiv [((p \vee q) \wedge (\lnot q \vee \lnot r)) \vee (\lnot p \vee r) ] \wedge [T \vee (r \vee \lnot r)]$\\

Domination law:\\

$\equiv [((p \vee q) \wedge (\lnot q \vee \lnot r)) \vee (\lnot p \vee r) ] \wedge T$\\

Identity law:\\

$\equiv ((p \vee q) \vee (\lnot p \vee r)) \wedge ((\lnot q \vee \lnot r) \vee (\lnot p \vee r))$\\

Distributive law:\\

$\equiv ((p \vee \lnot p) \vee (q \vee r)) \wedge ((r \vee \lnot r) \vee (\lnot p \vee \lnot q))$\\

Associative law:\\

$\equiv (T \vee (q \vee r)) \wedge (T \vee (\lnot p \vee \lnot q))$\\

Negation law:\\

$\equiv (T \vee (q \vee r)) \wedge (T \vee (\lnot p \vee \lnot q))$\\

Domination law:\\

$\equiv T \wedge T$\\

Identity law:\\

$\equiv T$\\

\\

q.e.d.

\end{framed}

\end{proof}

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\noindent

{\bf Rosen 1.3, Exercise 28:} \\

\begin{proof}

\begin{framed}

\begin{tabular}{c | c | c | c}

p & q & $p \leftrightarrow q$ & $\lnot p \leftrightarrow \lnot q$ \\

\hline\hline

T & T & T & T\\

\hline

T & F & F & F\\

\hline

F & T & F & F\\

\hline

F & F & T & T\\

\end{tabular}

q.e.d.

\end{framed}

\end{proof}

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{\bf Rosen 1.3, Exercise 62(c):} \\

\begin{framed}

This compound proposition is satisfiable when p, q and s are true and r is false

\end{framed}

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\noindent

Put the steps and arguments you used to arrive at your answer here.\\

\\

$(p \vee q \vee r) \wedge (p \vee \lnot q \vee \lnot s) \wedge (q \vee \lnot r \vee s) \wedge (\lnot p \vee r \vee s) \wedge (\lnot p \vee q \vee \lnot s) \wedge (p \vee \lnot q \vee \lnot r) \wedge (\lnot p \vee \lnot q \vee s) \wedge (\lnot p \vee \lnot r \vee \lnot s)$ \\

\\

In order to be satisfiable, there must be the possibility of a true output, That would require all the following statements to be true:\\

$(p \vee q \vee r)$ \\

$(p \vee \lnot q \vee \lnot s)$ \\

$(q \vee \lnot r \vee s)$ \\

$(\lnot p \vee r \vee s)$ \\

$(\lnot p \vee q \vee \lnot s)$ \\

$(p \vee \lnot q \vee \lnot r)$ \\

$(\lnot p \vee \lnot q \vee s)$ \\

$(\lnot p \vee \lnot r \vee \lnot s)$ \\

In the case that p, q and s are true and r is false, the statement will come out to be true in the end which will prove that it is satisfiable.

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{\bf Add-on :} Put the compound proposition from Rosen1.3, Exercise 12(b) in disjunctive normal form.\\

\begin{framed}

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \lnot r) \vee (p \wedge \lnot q \wedge r) \vee (p \wedge \lnot q \wedge \lnot r) \vee (\lnot p \wedge q \wedge r) \vee (\lnot p \wedge q \wedge \lnot r) \vee (\lnot p \wedge \lnot q \wedge r) \vee (\lnot p \wedge \lnot q \wedge \lnot r)$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

$[(p \rightarrow q) \wedge (q \rightarrow r) ] \rightarrow (p \rightarrow r)$\\

\\

\begin{tabular}{c | c | c | c}

p & q & r & $[(p \rightarrow q) \wedge (q \rightarrow r) ] \rightarrow (p \rightarrow r)$ \\

\hline\hline

T & T & T & T\\

\hline

T & T & F & T\\

\hline

T & F & T & T\\

\hline

T & F & F & T\\

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F & T & T & T\\

\hline

F & T & F & T\\

\hline

F & F & T & T\\

\hline

F & F & F & T\\

\end{tabular}\\

\\

The disjunctive normal form has all instances where the statement is true and this is is a tautology so all instances of the statement will be true.\\

Therefore the disjunctive normal form will be:\\

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \lnot r) \vee (p \wedge \lnot q \wedge r) \vee (p \wedge \lnot q \wedge \lnot r) \vee (\lnot p \wedge q \wedge r) \vee (\lnot p \wedge q \wedge \lnot r) \vee (\lnot p \wedge \lnot q \wedge r) \vee (\lnot p \wedge \lnot q \wedge \lnot r)$

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{\bf Rosen 1.4, Exercise 8(c):} \\

\begin{framed}

There exists at least one animal that, if it is a rabbit, then it hops.

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

$\exists x (R(x) \rightarrow H(x))$\\

\\

R(x): x is a rabbit \\

H(x): x hops \\

Domain: animals \\

\\

There exists at least one animal that $R(x) \rightarrow H(x)$ \\

There exists at least one animal that, it is a rabbit $\rightarrow$ it hops \\

There exists at least one animal that, if it is a rabbit, then it hops.

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{\bf Rosen 1.4, Exercise 10(c):} \\

\begin{framed}

$\exists x (C(x) \wedge F(x) \wedge \lnot D(x))$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

C(x): x has a cat \\

D(x): x has a dog \\

F(x): x has a ferret \\

Domain: all students in class \\

\\

Some student in your class has a cat and a ferret, but not a dog.\\

\\

Some student in your class: $\exists x$ \\

has a cat: C(x) \\

and a ferret: $\wedge$ F(x) \\

but not a dog: $\wedge$ $\lnot$ D(x) \\

\\

\\

$\exists x (C(x) \wedge F(x) \wedge \lnot D(x))$

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{\bf Rosen 1.4, Exercise 18(e):} \\

\begin{framed}

$\lnot (P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

P(x): Propositional function \\

Integers in function: −2, −1, 0, 1, and 2 \\

\\

$\lnot \exists xP(x)$ \\

Listing out all possible P(x) values \\

$\lnot \exists x$ (P(-2), P(-1), P(0), P(1), P(2)) \\

$\exists x$ is converted to or \\

$\lnot (P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$

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{\bf Rosen 1.4, Exercise 28(d):} \\

\begin{framed}

$\lnot \exists x(P(x) \wedge Q(x))$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

Nothing is in the correct place and is in excellent condition.\\

P(x): x is is the correct place \\

Q(x): x is in excellent condition\\

\\

Nothing is in the correct place and is in excellent condition.\\

$\lnot \exists x$(in the correct place and is in excellent condition)\\

$\lnot \exists x(P(x) \wedge Q(x))$

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{\bf Rosen 1.4, Exercise 36(c):} \\

\begin{framed}

x = 0

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

$\forall x(|x| > 0)$\\

Prove there exists a real number x such that $|x| > 0$ is false. \\

for $|x| > 0$ to be false, there must be $|x| \leq 0$ \\

for x = 0, $|x| = |0| = 0$ which satisfies $|x| \leq 0$ so the counterexample is x = 0\\

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{\bf Rosen 1.5, Exercise 4(c):} \\

\begin{framed}

Every student in the class has taken at least one computer science course.

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

P (x, y): Student x has taken class y\\

Domain for x: all students in the class\\

Domain for y: all computer science courses in school\\

\\

$\forall x \exists y P (x, y)$\\

P(every student in the class, there is a computer science course) \\

Every student in the class has taken at least one computer science course. \\

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{\bf Rosen 1.5, Exercise 10(e):} \\

\begin{framed}

$\exists x \forall y F (x, y)$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here\\

\\

F(x, y): x can fool y \\

Domain of x: all people in the world \\

Domain of y: all people in the world \\

\\

Everyone can be fooled by somebody. \\

Somebody can fool everyone.\\

F(somebody, everyone) \\

$\exists x \forall y F (x, y)$ \\

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{\bf Rosen 1.5, Exercise 14(e):} \\

\begin{framed}

$\exists x \forall y \exists z (P(x, y) \wedge Q(y, z))$

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here \\

\\

There is a student in this class who has taken every course offered by one of the departments in this school. \\

\\

P(x, y): student x has taken course y \\

Q(y, z): course y offered by department z \\

Domain of x: all the students in the class \\

Domain of y: courses offered \\

Domain of z: departments in the school \\

\\

$\exists x \forall y \exists z (P(x, y) \wedge Q(y, z))$

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{\bf Rosen 1.5, Exercise 24(d):} \\

\begin{framed}

For all pairs of numbers, both numbers are equal to 0 if and only if the product of both numbers is equal to 0.

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here \\

\\

$\forall x \forall y ((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$ \\

$\forall x \forall y ($x and y are not equal to 0 if and only if the product of both numbers is equal to 0) \\

For all pairs of numbers, both numbers are equal to 0 if and only if the product of both numbers is equal to 0. \\

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{\bf Rosen 1.5, Exercise 38(d):} \\

\begin{framed}

All of the students in this class have not been in any of the rooms of one building on campus.

\end{framed}

\vspace\*{1cm}

\noindent

Your work goes here \\

\\

There is a student in this class who has been in atleast one room of every building on campus. \\

S(x, y, z): student x has been in room y of building z on campus \\

Domain of x: all students in the class \\

Domain of y: all rooms \\

Domain of z: all building on campus \\

\\

$\exists x \exists y \forall z S(x, y, z)$ \\

\\

for the negation: \\

$\lnot \exists x \exists y \forall z S(x, y, z)$ \\

using De Morgan's Law: \\

$\forall x \forall y \exists z \lnot S(x, y, z)$ \\

\\

Back in English: \\

All of the students in this class have not been in any of the rooms of one building on campus. \\

\end{document}