\documentclass[10pt]{article}

% math fonts

\usepackage{amsmath,amsfonts,amsthm,amssymb}

% to insert graphics

\usepackage{graphicx}

% to change margins of the pages

\usepackage[margin=0.9in]{geometry}

% Makes equations flush left

\usepackage{fleqn}

% This generates a page header with your name in it.

\usepackage{fancyhdr}

\pagestyle{fancy}

\fancyhf{}

\lhead{FOCS Fall 2018}

\rhead{HW03 solution by Sriyuth Sagi}

\rfoot{Page \thepage}

% This package makes it easy to have boxes around large text.

\usepackage{framed}

\begin{document}

{\bf Rosen 1.6, Exercise 14(b):} \\

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\\

“Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.”\\

\\

\\

P(x): x is one of the roommates\\

Q(x): x has taken a course in discrete mathematics\\

R(x): has taken a course in algorithms\\

\\

Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics.\\

$ \forall x (P(x) \rightarrow Q(x))$\\

Every student who has taken a course in discrete mathematics can take a course in algorithms.\\

$ \forall x (Q(x) \rightarrow R(x))$

\\

\begin{tabular}{c | c | c}

1. & $ \forall x (P(x) \rightarrow Q(x))$ & premise \\

\hline

2. & $ \forall x (Q(x) \rightarrow R(x))$ & premise \\

\hline

3. & $ P(y) \rightarrow Q(y)$ & universal instantiation from (1) \\

\hline

4. & $ Q(y) \rightarrow R(y)$ & universal instantiation from (2) \\

\hline

5. & $ P(y) \rightarrow R(y)$ & hypothetical syllogism from (3) and (4) \\

\hline

6. & $ \forall x (P(x) \rightarrow R(x))$ & universal generalization from (5) \\

\end{tabular}

\\

\\

Therefore, all five roommates can take a course in algorithms next year.\\

$ \forall x (P(x) \rightarrow R(x))$

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{\bf Rosen 1.6, Exercise 16(a,b):} \\

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16(a):\\

\\

Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.\\

\\

P(x): x has lived in the dormitory\\

Q(x): x is enrolled in the university\\

m: Mia\\

\\

Everyone enrolled in the university has lived in a dormitory.\\

$ \forall x (P(x) \rightarrow Q(x))$\\

Mia has never lived in a dormitory.\\

$ \lnot Q(m) $\\

\\

\begin{tabular}{c | c | c}

1. & $ \forall x (P(x) \rightarrow Q(x))$ & premise \\

\hline

2. & $ \lnot Q(m)$ & premise \\

\hline

3. & $ P(m) \rightarrow Q(m) $ & universal instantiation from (1) \\

\hline

4. & $ \lnot P(m)$ & Modus tollens from (2) and (3) \\

\end{tabular}

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(4) shows that Mia is not in the university which agrees with the given conclusion so it is possible to conclude that the argument is true.\\

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16(b):\\

\\

A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.\\

\\

P(x): x is a convertible\\

Q(x): x is fun to drive\\

i: Isaac's car\\

\\

A convertible car is fun to drive.\\

$ \forall x (P(x) \rightarrow Q(x))$\\

Isaac’s car is not a convertible.\\

$ \lnot P(i) $\\

\\

\begin{tabular}{c | c | c}

1. & $ \forall x (P(x) \rightarrow Q(x))$ & premise \\

\hline

2. & $ \lnot P(i)$ & premise \\

\hline

3. & $ P(i) \rightarrow Q(i) $ & universal instantiation from (1) \\

\end{tabular}

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\\

There is no rule of inference that can provide $ \lnot Q(i)$ so the conclusion can never be reached and the argument is therefore incorrect.

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{\bf Rosen 1.6, Exercise 26:} \\

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\begin{tabular}{c | c | c}

1. & $ \forall x (P(x) \rightarrow Q(x))$ & premise \\

\hline

2. & $ \forall x (Q(x) \rightarrow R(x))$ & premise \\

\hline

3. & $ P(y) \rightarrow Q(y)$ & universal instantiation from (1) \\

\hline

4. & $ Q(y) \rightarrow R(y)$ & universal instantiation from (2) \\

\hline

5. & $ P(y) \rightarrow R(y)$ & hypothetical syllogism from (3) and (4) \\

\hline

6. & $ \forall x (P(x) \rightarrow R(x))$ & universal generalization from (5) \\

\end{tabular}

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This shows that under the conditions of $ \forall x (P(x) \rightarrow Q(x))$ and $ \forall x (Q(x) \rightarrow R(x))$ being true, $ \forall x (P(x) \rightarrow R(x))$ will also be true using universal instantiation, hypothetical syllogism and universal generalization.

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{\bf Rosen 1.7, Exercise 4:} \\

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\\

Assume there is even number x and any integer n. Using the property of even numbers where x = 2n for some integer n:\\

x = 2n\\

\\

The additive inverse of x is -x:\\

-x = -2n\\

\\

There is an integer m = -n:\\

-x = 2m\\

\\

This will satisfy the definition for an even number where (-x) = 2m for some value m. This shows that the additive inverse, or negative, of an even number is an even number.

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{\bf Rosen 1.7, Exercise 10:} \\

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The property of rational numbers where there exists a rational number x if there exist two integers, y and z that $x = \dfrac{y}{z}$.\\

\\

If there are two variables x and p as well as four integers y, z, q and r such that:\\\\

$x = \dfrac{y}{z}$\\\\

$p = \dfrac{q}{r}$\\\\

The product of the two:\\\\

$x \cdot p = \dfrac{y}{z} \cdot \dfrac{q}{r} = \dfrac{y \cdot q}{z \cdot r}$\\\\

$y \cdot q$ and $q \cdot r$ will both return integers so, using the property of rational numbers, this shows that the product of two rational numbers will also be rational.

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{\bf Rosen 1.7, Exercise 16:} \\

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The property of an odd number x is that there exists an integer n such that x = 2n + 1\\

m = 2a + 1\\

n = 2b + 1\\

\\

mn = (2a + 1)(2b + 1)\\

= 4ab + 2a + 2b + 1\\

= 2(2ab + a + b) + 1\\

if there exists an integer d such that d = 2ab + a + b\\

=2d + 1 which shows that mn would be odd\\

\\

Therefore, if m and n are both odd, mn would also be evaluated as odd, so for mn to be even, there must be at least on even number.

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{\bf Rosen 1.7, Exercise 32:} \\

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(i) x is rational\\

(ii) x/2 is rational\\

(iii) $3x - 1$ is rational\\

\\\\

To prove equivalence, prove that $(i) \rightarrow (ii) \rightarrow (iii) \rightarrow (i)$ is true with all of them evaluating to true.\\

\\

$(i) \rightarrow (ii)$:\\

For a rational number x and two integers p and q such that $x = \dfrac{p}{q}$\\\\

$\dfrac{x}{2} = \dfrac{p}{2q}$\\\\

p and 2q are both integers so $\dfrac{x}{2}$ is rational and $(i) \rightarrow (ii)$ is true\\

\\\\

$(ii) \rightarrow (iii)$:\\

For a rational number $\dfrac{x}{2}$ and two integers p and q such that $\dfrac{x}{2} = \dfrac{p}{q}$ or $x = \dfrac{2p}{q}$\\\\

$3x - 1 = \dfrac{6p}{q} - 1 = \dfrac{6p - q}{q}$\\\\

$6p - q$ and q are both integers so $3x - 1$ is rational and $(ii) \rightarrow (iii)$ is true\\

\\\\

$(iii) \rightarrow (i)$:\\

For a rational number $3x - 1$ and two integers p and q such that $3x - 1 = \dfrac{p}{q}$\\\\

$3x = \dfrac{p}{q} + 1$\\\\

$x = \dfrac{p + q}{3q}$\\\\

$p + q$ and 3q are both integers so x is rational and $(iii) \rightarrow (i)$ is true\\

\end{document}