\documentclass[10pt]{article}

% math fonts

\usepackage{amsmath,amsfonts,amsthm,amssymb}

% to insert graphics

\usepackage{graphicx}

% to change margins of the pages

\usepackage[margin=0.9in]{geometry}

% Makes equations flush left

\usepackage{fleqn}

% This generates a page header with your name in it.

\usepackage{fancyhdr}

\pagestyle{fancy}

\fancyhf{}

\lhead{FOCS Fall 2018}

\rhead{HW04 solution by Sriyuth Sagi}

\rfoot{Page \thepage}

% This package makes it easy to have boxes around large text.

\usepackage{framed}

\begin{document}

{\bf Rosen 1.8, Exercise 6:} \\

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The rule of even numbers is there is an even number x if there exists an integer n where x = 2n.\\

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The rule of odd numbers is there is an odd number y if there exists an integer m where y = 2m + 1.\\

\\

For 5x + 5y:\\

5x + 5y

= 5(2n) + 5(2m + 1)

= 10n + 10m + 5

= 5( 2n + 2m + 1 )

= 5( 2(n + m) + 1 )\\

\\

If n and m are both integers, there will also be an integer n + m.\\

Following the rule for odd numbers, 2(n + m) + 1, furthermore using the below proof, two odd number multiplied by one another will always return an odd number, so 5( 2(n + m) + 1 ) will be odd, proving that 5x + 5y is odd.\\

\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

Proof for product of odd numbers being odd\\

odd number x = 2n + 1\\

odd number y = 2m + 1\\

\\

xy = (2n + 1)(2m + 1)

= 4nm + 2n + 2m + 1

= 2(2nm + n + m) + 1\\

\\

Which satisfies the rule of odd numbers with 2nm + n + m being an integer.

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\noindent

{\bf Rosen 1.8, Exercise 10:} \\

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$2 \cdot 10^{500} + 15$ and $2 \cdot 10^{500} + 15$ are both consecutive integers which means that the only possibility is that they are consecutive squares. However,\\

\\

If we assume $2 \cdot 10^{500} + 15$ = x and that x is a perfect square, being equal to $n^2$ for some integer n. The consecutive greater perfect square would be:\\

\\

$(n + 1)^2$ = $n^2 + 2n + 1$

= $x + 2n + 1$\\

\\

$x + 2n + 1 > x + 1$\\

$2n + 1 > 1$\\

\\

This means that the difference between consecutive perfect squares will always be greater than 1 so consecutive numbers like $2 \cdot 10^{500} + 15$ and $2 \cdot 10^{500} + 15$ cannot both be perfect squares.

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\noindent

{\bf Rosen 1.8, Exercise 30:} \\

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$2x^2 + 5y^2 = 14$ where x and y are both integers.\\

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$x^2 \geq 0$ and $y^2 \geq 0$ \\

\\

$5y^2 \leq 14$\\

when y = 2: $5(2)^2 = 20$ and $20 > 14$\\

\\

$2x^2 \leq 14$\\

when x = 3: $2(3)^2 = 18$ and $18 > 14$\\

\\

So the only possible values for y are 0 and 1, while the only possible values for x are 0, 1 and 2.\\

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For f(x, y) = $2x^2 + 5y^2$\\

f(0, 0) = 0\\

f(1, 0) = 2\\

f(2, 0) = 8\\

f(0, 1) = 5\\

f(1, 1) = 7\\

f(2, 1) = 13\\

\\

\\

None of the possible combinations satisfy the statement therefore there are no integer solutions to the equation.

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{\bf Rosen 4.1, Exercise 14(f):} \\

Put your answer inside the box. Show your work outside the box. \\

If your answer is correct, but you don't show your work, you only get 80\% of the points. \\

\framebox(150,30){$c = 14$} \\

\\

$a \equiv 11 (mod 19)$\\

$b \equiv 3 (mod 19)$\\

$0 \leq c \leq 18$\\

\\

$c \equiv a^3 + 4b^3 (mod19)$

$= (11)^3 + 4(3)^3 (mod19)$

$= 1331 + 108 (mod19)$

$= 1439 (mod19)$

$= 14 (mod19)$

\vspace\*{3cm}

\noindent

{\bf Rosen 4.1, Exercise 28(a):} \\

Put your answer inside the box. Show your work outside the box. \\

If your answer is correct, but you don't show your work, you only get 80\% of the points. \\

\framebox(150,30){$37 \not\equiv 3 (mod 7)$} \\

\\

$37 - 3 = 34$\\

$34 \div 7 = 7 \cdot 4 + 6$

34 is not divisible by 7 so 37 is not congruent to $3 (mod 7)$.\\

\vspace\*{3cm}

\noindent

{\bf Rosen 4.1, Exercise 28(c):} \\

Put your answer inside the box. Show your work outside the box. \\

If your answer is correct, but you don't show your work, you only get 80\% of the points. \\

\framebox(150,30){$-17 \not\equiv 3 (mod 7)$} \\

\\

$-17 - 3 = -20$\\

$-20 \div 7 = 7 \cdot 2 - 6$

-20 is not divisible by 7 so -17 is not congruent to $3 (mod 7)$.\\

\vspace\*{2cm}

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\noindent

{\bf Rosen 4.1, Exercise 42:} Only prove the associativity property - {\bf not} all the properties requested. \\

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Associativity is defined by:\\

$(a + \_m b) + \_m c = a + \_m (b + \_m c)$\\

\\

Will use the properties:\\

a + $\_m$ b = (a + b) mod(m)\\

a + $\_m$ b = (a mod(m)) + $\_m$ b\\

a + $\_m$ b = a + $\_m$ (b mod(m))\\

(a + b) + c = a + (b + c)

\\

\\

$(a + \_m b) + \_m c$

$=((a + b) mod(m)) + \_m c$

$=(a + b) + \_m c$

$=((a + b) + c) mod(m)$

$=(a + (b + c)) mod(m)$

$=a + \_m (b + c)$

$=a + \_m ((b + c) mod(m))$

$=a + \_m (b + \_m c)$

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\noindent

{\bf Rosen 4.3, Exercise 34:} \\

Put your answer inside the box. Show your work outside the box. \\

If your answer is correct, but you don't show your work, you only get 80\% of the points. \\

\framebox(150,30){7} \\ \\\\\\

gcd(21, 34)\\

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$34 = (21 \cdot 1) + 13$\\

$21 = (13 \cdot 1) + 8$\\

$13 = (8 \cdot 1) + 5$\\

$8 = (5 \cdot 1) + 3$\\

$5 = (3 \cdot 1) + 2$\\

$3 = (2 \cdot 1) + 1$\\

$2 = 1 \cdot 2$\\

\\

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There are a total of 7 divisions for this particular gcd.

\end{document}