\documentclass[10pt]{article}

% math fonts

\usepackage{amsmath,amsfonts,amsthm,amssymb}

% to insert graphics

\usepackage{graphicx}

% to change margins of the pages

\usepackage[margin=0.9in]{geometry}

% Makes equations flush left

\usepackage{fleqn}

% This generates a page header with your name in it.

\usepackage{fancyhdr}

\pagestyle{fancy}

\fancyhf{}

\lhead{FOCS Fall 2018}

\rhead{HW05 solution by Sriyuth Sagi}

\rfoot{Page \thepage}

% This package makes it easy to have boxes around large text.

\usepackage{framed}

\begin{document}

{\bf Rosen 2.1, Exercise 24:} \\

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a) $\emptyset$\\

\framebox(400,30){Not a power set because a power set can not be empty}\\\\\\

b) $\{\emptyset, \{a\}\}$\\

\framebox(400,30){It is a power set of $\mathcal{P}\{\{a\}\}$}\\\\\\

c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$\\

\framebox(400,30){Not a power set because a power set always has a cardinality of a power of 2.}\\\\\\

d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$\\

\framebox(400,30){It is a power set of $\mathcal{P}\{\{a, b\}\}$}\\\\\\

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{\bf Rosen 2.1, Exercise 32(b):} \\

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C = \{0,1\}\\

B = \{x,y\}\\

A = \{a,b,c\}\\

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$C \times B \times A$ = \{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y),

\hspace{1.8cm}(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c,x), (1, c, y)\}

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{\bf Rosen 2.2, Exercise 16(a):} \\

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$(A \cap B) \subseteq A$\\\\

\begin{tabular}{c | c | c | c}

A & B & $(A \cap B)$ & $(A \cap B) \cap A$ \\ [0.5ex]

\hline

1 & 1 & 1 & 1 \\

\hline

1 & 0 & 0 & 0 \\

\hline

0 & 1 & 0 & 0 \\

\hline

0 & 0 & 0 & 0 \\

\end{tabular}\\

\\\\

$(A \cap B) \cap A$ proves that all elements of $(A \cap B)$ lie within set A. Hence, $(A \cap B) \subseteq A$ is true.

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{\bf Rosen 2.2, Exercise 30(c):} \\

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$A \cup C$ = $B \cup C$ and $A \cap C$ = $B \cap C$\\

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Case 1: x is an element of A\\\\

Let $x$ be an element in set A\\

$x \in A$

$x \in A \cup C$\hspace{1cm} (definition of union)

$x \in B \cup C$\hspace{1cm} ($A \cup C = B \cup C$)\\\\

So $x$ must be in B or C, assume it is in C\\

$x \in C$

$x \in A \cap C$\hspace{1cm} (definition of intersection)

$x \in B \cap C$\hspace{1cm} ($A \cap C = B \cap C$)\\\\

By definition of intersection, $x$ must be in B. Therefore all elements in set B will also be in set A by definition of subset.\\\\

$A \subseteq B$\\\\

Case 2: x is an element of B\\\\

Let $x$ be an element in set B\\

$x \in B$

$x \in B \cup C$\hspace{1cm} (definition of union)

$x \in A \cup C$\hspace{1cm} ($A \cup C = B \cup C$)\\\\

So $x$ must be in B or C, assume it is in C\\

$x \in C$

$x \in B \cap C$\hspace{1cm} (definition of intersection)

$x \in A \cap C$\hspace{1cm} ($A \cap C = B \cap C$)\\\\

By definition of intersection, $x$ must be in A. Therefore all elements in set A will also be in set B by definition of subset.\\\\

$B \subseteq A$\\\\\\

Since $A \subseteq B$ and $B \subseteq A$, we can conclude that A = B

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{\bf Rosen 2.3, Exercise 20(b):} \\

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\framebox(400,40){$f(n) = n/2$}\\

\\\\

It is onto because:\\

$f(2n) = n$ whenever n is a natural number\\

\\

It is not one-to-one because:\\

$f(2) = f(3) = 1$

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{\bf Rosen 2.3, Exercise 20(c):} \\

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\framebox(400,40){$

f(n) =

\begin{cases}

n + 1 $\hspace{0.5cm} if n is even$ \\

n - 1 $\hspace{0.5cm} if n is odd$

\end{cases}\\

$}

\\\\

It is onto because:\\

if n is even and n + 1 is odd: $f(n + 1) = n$\\

if n is odd and n - 1 is even: $f(n - 1) = n$\\

\\

It is one-to-one because:\\

$f(n) = f(n') \rightarrow n = n'$\\

\\

it is not an identity because $f(1) = 0$

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{\bf Rosen 2.3, Exercise 36:} \\

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$f(x) = x^2 + 1$\\

$g(x) = x + 2$\\\\\\

$f \circ g$:\\\\

$f \circ g = f(g(x))$

\hspace{0.3cm}$= (x + 2)^2 + 1$

\hspace{0.3cm}$= x^2 + 4x + 5$\\\\\\

$g \circ f$:\\\\

$g \circ f = g(f(x))$

\hspace{0.3cm}$= (x^2 + 1) + 2$

\hspace{0.3cm}$= x^2 + 3$

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{\bf Rosen 2.3, Exercise 74(c):} \\

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$\lceil \lceil x / 2 \rceil / 2 \rceil = \lceil x / 4 \rceil$ for all real numbers x.\\\\

Prove using proof by parts:\\\\

Let $x = 4n + c$ where n is an integer $0 \leq c < 4$\\\\

If c = 0, then x will already be a multiple of four and both sides equal n.\\\\

If $0 < c \leq 2$:\\

$\lceil x/2 \rceil = 2n + 1$\\

The LHS and RHS can both be evaluated to x + 1\\\\

If $2 < c < 4$:\\

$\lceil x/2 \rceil = 2n + 2$\\

The LHS and RHS can both be evaluated to x + 1\\\\\\

So this function has been proved true for all statements.

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{\bf Rosen 2.4, Exercise 16(d):} \\

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$a\_n = 2a\_{n - 1} - 3$\\

$a\_0 = -1$\\\\\\

$a\_n = 2^1a\_{n - 1} - 3$

$= 2^2a\_{n - 2} - (3 \cdot 2^0 + 3 \cdot 2^1)$

$= 2^3a\_{n - 3} - (3 \cdot 2^0 + 3 \cdot 2^1 + 3 \cdot 2^2)$\\

\\

$a\_n = 2^na\_{n - n} - \sum\_{i = 0}^{n - 1} 3 \cdot 2^i$\\

$= 2^na\_{0} - 3 \cdot \sum\_{i = 0}^{n - 1} 2^i$\\

$= 2^n \cdot( -1) - 3 \cdot \dfrac{2^n - 1}{2 - 1}$\\

$= -2^n - 3 \cdot (2^n - 1)$\\

$= -2^n - 3 \cdot 2^n + 3$\\

$= -4 \cdot 2^n + 3$\\

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{\bf Rosen 2.4, Exercise 18(b):} \\

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$a\_n = 1.09 \cdot a\_{n - 1}$\\

$a\_0 = 1000$\\\\\\

$a\_n = 1.09 \cdot a\_{n - 1}$

$= 1.09(1.09 \cdot a\_{n - 2})$

$= 1.09^2(1.09 \cdot a\_{n - 3})$\\

\\

$a\_n = 1.09^n \cdot a\_{n - n}$\\

$a\_n = 1.09^n \cdot a\_{0}$\\

$a\_n = 1000 \cdot 1.09^n$\\

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{\bf Rosen 2.4, Exercise 32(c):} \\

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$\sum\_{j = 0}^{8} (2 \cdot 3^j + 3 \cdot 2^j)$\\

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Use property:\\

$\sum\_{j = 0}^{n} q^j = \dfrac{1 - q^{n+1}}{1 - q}$, $q \neq 0, 1$\\\\\\

$\sum\_{j = 0}^{8} (2 \cdot 3^j + 3 \cdot 2^j) = 2 \cdot \dfrac{1 - 3^9}{1 - 3} + 3 \cdot \dfrac{1 - 2^9}{1 - 2} = 21215$

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\end{document}